# Causality, Probability and Time Notes

Book by Samantha Kleinberg

## Notes on Types of Causes and their Representation

### Insignificant Causes – Intuitions

The conditions for *prima facie* causality are insufficient to distinguish between causes and non-causes, and the primary difference between probabilistic theories of causality is in how exactly they make this distinction. The two main types of methods ar those based on information and those based on manipulation. The information-based theories use the idea that a cause provides some information about an effect that cannot be gained in other ways and set about finding evidence for that. Manipulation theories hold that a cause is a way of bringing about an effect and can be understood in terms of how the probability or value of the effect changes when manipulating the cause to be true. One approach is not inherently superior to the other – there are counterexamples to both manipulation and information-based methods and neither subsumes the other. Manipulation is not possible in many cases, and therefore it is undesirable to require it. On other side, methods that aim to infer causal relationships from observational data – because it is readily available or because manipulations are costly, unfeasible, or unethical – generally use some variant of the information-based approach. The distinguishing feature between all variants of the information-based methods is how to quantify the information provided about the effect. The basic idea is that of holding fixed some set of information and then see how likely the effect is when the cause is present and absent relative to that set of information.

Desiderata for quantifying causal influence

Consider the following set of relationships (an abstraction of DTMC):

This is a subset of a full system that we may have a model of or observe. Here can cause in two time units through two paths, directly or through d: of the time causes d at , of the time it causes directly at t+2, and  of the time it does neither. Now assuming the marginal probability of is much lower than how can we determine how significant and are for ?

One approach is to find the earliest cause that best accounts for the effect. This is Suppes’ method which says that a cause is spurious if there is other information that predicts the effect at least as well and, once this information is known, the probability of the effect is unchanged by knowledge of the spurious cause. In this example this method would determine erroneously that is a spurious cause since occurs earlier than and .

Similarly, using Granger causality one would also fail to find as a cause of since the probability of is unchanged once the information about is included. This is incorrect since d is in fact a cause of e and accounts for e exactly as well as c does

## Appendix

### Granger causality

#### Notation:

denotes the value of the variable at time

denotes the set of measurements of up to time t i.e.,

denotes the set of all possible knowledge up to time t (including both and )

**Definition 2.4.1** Granger-causes if

(2.5)

As this is not exactly causality, it has come to be called Granger causality. Granger-s definition takes temporal priority as a given and does not make claims about how much of a difference makes to the probability of (or whether this difference is positive or negative). may not be the best or only predictor of , rather it is simply found to be informative after accounting for other information.