# Overview of Probabilistic Causality Concepts and Theories

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//TODO: the labels of the definitions in both appendices are mixed up and out of order – do not forget to fix them

### Review on Good’s, Reichenbach’s and Suppes’ Causality theories presented in Salmon’s essays

Twenty-six of W. C. Salmon’s essays are collected in the anthology “*Causality and Explanation*” see (Salmon, Causality and Explanation, 1998).

#### Good’s Causal Calculus overview in (Salmon, Probabilistic Causality, 1980)

The Good’s Causal Calculus idea was presented in two parts paper (Good, 1961).

Salmon feels there are two problems with Good’s approach to Causality – the first one is Good’s formalism to assign a degree of strength to a causal chain on the basis of the individual links in the chain.

The second objection of Salmon is against something which Good’s, Reichenbach’s and Suppes’ theories share – it concerns cases in which an effect is introduced in an improbable fashion. Both Good and Suppes are aware of this rather familiar difficulty, and they try to deal with it in ways that are different but complementary.

The causal framework is presented in the following context:

Let us suppose there are aggregates of events, denoted by , among which certain physical probability relations hold. The particular events are located in space-time. Like Suppes, but unlike Reichenbach, Good stipulates that cause temporally precede their effects – that is, temporal priority is used to define causal priority, not vice versa. Good’s aim is to examine certain types of networks of events that join an initial event F to a final event E, usually by way of various intermediate events , and to define a measure of “the degree to which caused ” or “the contribution to the causation of provided by ” (Good, 1961). The specification of this measure, and various related measures, involves 24 axioms and 18 theorems. One particularly important special case of a causal net is a *causal chain*. In a causal chain, all of the constituent events , are linearly ordered. It is assumed that the adjacent events and are spatiotemporally contiguous (or approximately so), that they do not overlap too much, and that does not depend on the occurrence of any event in the chain prior to (p. 45 of part II in (Good, 1961)). In order to arrive at the measure for a wider class of nets, Good defines a measure of the strength of the causal chain joining to . A particularly simple type of causal chain is one consisting only of the two events and . A measure of the strength of a chain of this sort can be used, according to Good, to define a measure of the strength of longer chains. It is this aspect of Good’s approach – the attempt to compound the strengths of the individual “links” of the chain in order to ascertain the strength of the entire chain – that is the locus of the first problem. Good introduces a measure which is to stand for “the tendency of to cause ” (p. 307 of part I in (Good, 1961)). Salmon points out that this informal rendering in words is misleading as in fact is the measure of statistical relevance. Events of type are statistically relevant to events of type , if the occurrence of an event of type makes a difference to the probability that an event of type B will occur. We say that the relation is one of positive relevance if the occurrence of a member of A increases the probability that a member of will occur. The relation is of negative relevance if the occurrence of decreases the probability of . As we all recognize, a mere correlation does not necessarily constitute a causal relation – not even a tendency to cause. The falling barometric reading has no tendency at all to cause a storm, though the barometric reading is highly relevant statistically to the onset of stormy weather. *Note*: In his essay “*Intuitions – Good and Not So Good*”, (Salmon, Causality and Explanation, 1998), W. Salmon changes his stance on this topic.

There are many different measures of statistical relevance. Although the first five axions A1-A5 (Good, 1961) do not fix the precise form of Q, they do show what sort of measure it is. According to A1, is a function is a function of , , , and alone. According to A5, has the same sign as and according to A3 and A4 increases continuously with if and are held constant, and it decreases continuously as increases if and are held constant. may be a real number, it may assume the value or , or under special circumstances it may be indeterminate.

### Review on Reichenbach’s and Suppes’ Causality frameworks presented in Otte’s thesis

In this section we will look into Reichenbach’s and Suppes’ probabilistic interpretations of Causality discussed in R. Otte’s PhD thesis (Otte, 1982).

#### Reichenbach’s Treatment of Causality

##### Definition of Causal Betweenness

The relation “causally between” should capture the structure of a causal net and is supposed to reveal when two events are causally connected in a certain way. When we say that is causally between and , we are saying that there are causal processes that connect and on one side, and then and on the other side. Reichenbach (Reichenbach, 1956) gives two examples: a causal chain and causal fork. Both arrangements exemplify the relation of causal betweenness, although in different ways.

Reichenbach’s (Reichenbach, 1956) definition of the relation “causally between” consists of three requirements:

**Definition A1**: An event is causally between the events and if the relations hold:



We will denote the between relation with .

Discussion on **Definition A1**:

*Positive Relevance:*

This definition formalizes the principle that closer one gets in a causal chain to an effect the better one is able to predict the occurrence of the effect. Cond. 1 and cond. 2 in Definition A1 claim that regardless of the direction from which we approach an event, the closer we get to the event, the higher its probability becomes. Let us consider a simple example which illustrates what Reichenbach was trying to communicate.

Suppose that we have three events which are causally connected in such a way that causes and causes . Now suppose that we know that has occurred; then we can predict with probability that event will occur. This will be greater than 0 and less than 1. As we get closer in the causal chain to event , then we are able to predict with higher probability that event will occur. Thus, since is closer to than , then . The probability of the prediction will increase towards 1 as we get closer to . So Reichenbach’s basic intuition, as expressed in the first two equations, is that the closer one gets to an event in the causal chain, the higher its probability becomes.

A simple example may help to illustrate this intuition. Suppose that we are launching a missile and hope to hit a target some distance away. Knowing that the missile was launched (event ) certainly raises the probability of it hitting the target (event ). However, knowing that the missile is still on target when it crosses a certain tracking station between the launch site and the target event (event ), enables us to infer with even higher probability that the missile will hit the target. If we were to determine that the missile was on target at a later stage in its trajectory, we would be even more confident that it would hit the target. It appears that our confidence in it hitting the target is raised because we believe that there is less that can go wrong and cause the missile to veer off course as it gets closer to the target.

*Screening Off:*

The other part of Reichenbach’s basic intuition is captured by cond. 3 of **Definition A1** and is a version of the Markov property. This tells us that if we have a causal chain, an earlier event cannot affect later events except through the intermediate links. One can view this as a partial limitation on action at a distance: if there is a causal chain connecting two events, the only way the earlier event can affect the later event is through the intermediate links.

Another way of looking at this is that events that happen before are irrelevant to making a prediction of whether will occur, once we know that has occurred. Imagine that it is possible to know the complete state of a system at time ; then states of the system before are irrelevant to any prediction about the future of the system. Similarly, if we knew the complete state of the universe today, we could make certain predictions about the state of the universe tomorrow; Reichenbach’s claim is that the state of the universe yesterday would not help us make the prediction of what the universe would be like tomorrow, once we know what the universe is like today. The idea, captured in cond. 3 of **Definition A1** is known is the *screening off relation*. If the probability of given and , , just equals the probability of given , , then the knowledge of is irrelevant to a prediction of whether will occur, once we know that has occurred. When an event is screened off from another event, it is no longer predictively informative about that event. This is the other basic intuition that Reichenbach was trying to capture in his definition of causal betweenness.

*Causal Betweenness and the Causal Net*

The relation of causal betweenness enables us to determine the structure of the causal net. Reichenbach says that if we determine that the following hold,

then we will know that the four events have the causal structure diagrammed in Figure A.1 and not on Figure A.2.

Figure A.1: Causal Betweenness in a Causal Fork. Figure A.2: Causal Betweenness in a Causal Chain

The betweenness relations of forks and chains differ considerably. In the chain diagrammed on Figure A.2 the relation is false, whereas it is true of the fork diagrammed on Figure A.1. Using probability relations to pick out causal relations finds its roots in this idea of Reichenbach’s.

*Critical Discussion of Causal Betweenness*

Since the definition of causal betweenness formalizes the idea that the closer one gets to another event in a causal chain, the higher its probability will be, a rejection of of the definition of causal betweenness will most likely involve a rejection of that principle. Salmon rejects the causal betweenness principle (Salmon, Causality and Explanation, 1998) and points out that not all events in a causal chain need to be positively relevant to later events in that chain; sometimes things happen “the hard way”.

#### Suppes’ Causality Framework

**Definition A3**: Suppes’ definition of *prima facie* cause

An event is a *prima facie* cause of event *iff*:

We should interpret this as being for all and where . That is, the probability of A occurring at any time after B is greater than the marginal probability of A occurring at any time. Thus, the conditions 1-3 do not refer to specific values of and but rather describe the relationship between and . In some cases, these causes may turn out to be false. Even if something meets the criterion of being a *prima facie* cause, this may be due only to common cause of it and the effect. Suppes introduces two ways in which something may be a false, or spurious cause. In each, the idea is that there is some earlier event than the *prima facie* cause that accounts equally well for the effect, so that his other information is known, the spurious cause does not have any influence on the effect.

**Definition A4**: Suppes’ first definition of *spurious cause*

An event , a prima facie cause of event , is a *spurious cause* in sense one iff and such that:

While is a possible cause of , there may be another, earlier, event that has more explanatory relevance to . However, condition 2 of the definition above is very strong and perhaps counterintuitive. It means that there exists an event that completely eliminates the effectiveness of the cause for predicting the effect. One way of relaxing this condition is to find not individual events but rather kinds of events. In Suppes’ second definition of spurious causes there will be a set of nonempty sets that cover the full sample space, and which are mutually exclusive (pairwise disjoint). Thus, only one of these sets can be true *and* together they cover all possibilities.

**Definition A5**: Suppes’ second definition of *spurious cause*

An event , a prima facie cause of event , is a *spurious cause* in sense two iff there is a partition where and for every in :

Distinction between these two kinds of spuriousness is made with an example given by (Otte, 1982) on pp63:

*For now on I will abbreviate “spurious in sense two” by and “spurious in sense one” by . This definition makes an event if the world can be partitioned in such a way that the above conditions are satisfied. Thus, if we can observe a certain kind of event given by the partition, the observation of the later event is uninformative, which makes it a cause. Suppes proves that if an event is a cause, then it is a cause. The converse of this theorem, however, is not necessarily true: it is possible for an event to be a cause and not be a cause.*

*As an example of a cause, let us take the case of decreasing air pressure causing not only rain but a falling barometer reading. The falling barometer reading is a prima facie cause of rain; given that the barometer reading is dropping, the probability that it will rain rises. Letting denote rain, denote a falling barometer reading, and denote decreasing air pressure, the probability of rain given that the barometer reading, and the air pressure are decreasing, , is equal to the probability of rain given that the air pressure is decreasing, ; thus the second condition of the second definition of spurious cause is satisfied. The third condition is likewise satisfied, since the probability of rain given decreasing air pressure and a falling barometer reading is a least as great as the probability of rain given a falling barometer reading, . Thus, by the second definition a falling barometer reading is a cause of rain. The falling barometer reading is a cause of rain. If we let be our partition (decreasing air pressure, non-decreasing air pressure), then*

*So the falling barometer reading is a cause of the rain.*

**Definition A6**: Suppes’ definition of *genuine cause*

All non-spurious prima facie causes i.e., prima facie causes which do not meet the **Definition A4** and **Definition A5**.

Looking again at **Definition A4** and **Definition A5** for spurious causes, the stipulation that means that some causes may not be deemed spurious, despite meeting all the conditions, if there is a small difference in the probabilities on either side of this equality. To address this issue, Suppes introduced the concept of an -spurious cause.

**Definition A7**: Suppes’ definition of *-spurious cause*

An event is an -spurious cause of event iff and a partition such that for every of :

This definition means that a genuine cause that has a small effect on the probability of the event being caused will be ruled spurious. The partition separates off the past prior to the possibly spurious cause . Note that there is no set value for other than it being small //Can we improve on that?

One issue that arises when using these definitions to determine the true cause of an effect is that we may find an earlier and earlier causes that make the later ones spurious, and the cause may be quite removed from the effect in time (not to mention space). Suppes introduces the idea of *direct cause* to account for this issue. This is a concept very similar to screening off and spurious causes, except here we must consider whether there is some event coming temporarily between cause and effect. Note that there is no link between spurious and indirect causes. A direct cause may still be remote in space (and perhaps in time), but this can rule out indirect remote causes.

One of the first problems which we encounter with these definitions is in handling of causal chains. As discussed by (Otte, 1982), pp64:

*Closely related to the notion of spurious cause is the idea of indirect cause. We will first define a direct cause:*

**Definition A8***:* Otte’s definition of *direct cause*

An event is a direct cause of iff is a prima facie cause of and there is no and no partition such that for every in

We will then define an indirect cause to be a prima facie cause that is not direct. One immediately notices the similarity between **Definition A5** and **Definition A8**. The main difference is that falls between and in **Definition A8**. Although Suppes does not do so, this similarity suggests that a definition of a direct cause could also be developed using the analysis of a cause.

**Definition A9**: Otte’s first definition of direct cause

An event is a direct cause in sense one of if and only if is a prima facie cause of and for every , there is no such that

The conditions of **Definition A9** are similar to those of **Definition A4** with the difference that . We will call the definition of direct cause given by **Definition A8** direct cause in sense two. We abbreviate “direct cause in sense one” with and direct cause in sense two by . **Definitions A8** and **A9** say that a cause is a direct cause if and only if there is no later event (or kind of event) that will account for as well as does. Whereas an event is if a certain kind of event does not exist, an event is if a certain event does not exist. This mirrors the difference between and causes. Recall, that Suppes proves that if a cause is a cause, then it is also a cause. A similar proof can be constructed to show that if a prima facie cause is cause, then it is also a cause, and if it is an cause then it is cause.

Additionally, Suppes defines supplementary causes:

**Definition A10**: *Suppes’ definition of supplementary cause*

Events and are supplementary causes of iff:

1. is prima facie cause of
2. is prima facie cause of

Two causes are supplementary causes if the probability of an event occurring given both is higher than it would have been either one alone. Thus, consuming drugs and consuming alcohol are supplementary causes of death, because the probability of dying given one has consumed drugs and alcohol is greater than either the probability of dying given one has consumed drugs or the probability of dying given one has consumed alcohol.

**Theorem A1** no spurious cause of A can be a supplementary cause of A

*Proof*: If according to condition 2. of **Definition A4** or **Definition A5** , then it is not the case that condition 4. of **Definition A10** can be satisfied, so and will not be supplementary causes.

Sufficient causes are viewed as those limiting cases in which the conditional probability of an event reaches one:

**Definition A11**: Suppes’ definition of *sufficient (or determining) cause*

An event is a sufficient (or determining) cause of iff is a prima facie cause of and

Suppes’ framework implies that the sufficient cause relation is transitive, that is if is a sufficient cause of , and if is a sufficient cause of , then is a sufficient cause of . This is captured by the following theorem

**Theorem A2**: *transitivity* of sufficient cause relation based on Suppes’ causality framework

If , , , then

*Proof of Theorem A2*:

If and then *(Suppes’ Theorem 1, 1970, p. 35)*

*Proof of Suppes’ Theorem 1:*

Obviously, . We have .

Thus, we get:

Assuming that then we can write:

Finally, is a subset of event with probability zero, namely so we conclude that

. Thus, we get . **QED** (*Suppes’ Theorem 1*).

Using

We find that .

Alternatively,

( *by total probability*)

Then from from where it follows that . **QED**.

Another important idea in causation is that of a necessary cause or condition.

**Definition A12**: Otte’s definition of *necessary causes*

An event is a necessary cause (or condition) of iff the probability of given the absence of is equal to zero. That is, is a prima facie cause of and .

**Theorem A3**: transitivity of *necessary causes*

If , , , and then

*Proof of Theorem A3:*

**Lemma A1**: if and then

*Proof of Lemma A1*:

Obviously, since we have

since is strictly positive.

Finally, is a subset of event with probability zero, namely so we conclude that

. **QED** *(Lemma A1)*

( *by total probability*)

Since and we conclude . **QED**.

In conjunction with the transitivity of sufficient causes, the transitivity of necessary causes that if a chain of necessary and sufficient causes is present, then any member of that chain at is a necessary and sufficient cause of any member of that chain at , for all .

##### Additional Discussion on and causes

We will assess the definitions of and causes (**Definition A4** and **A9**) with the use of several examples. Let us consider the adequacy of the definition of cause. It seems reasonable to believe that the world is composed of both deterministic and probabilistic causes; presumably if there are indeterminate events they will be intermingled with determinate events and thus there will be causal chains consisting of both deterministic and probabilistic causes.

*Example 1*: consider the causal chain , where is a probabilistic cause of ; is necessary and sufficient cause of ; is necessary and sufficient cause of , etc. The first thing to notice is that is a cause of since the following conditions are satisfied:

We know that condition 1 is satisfied because . We can also show that condition 2 is satisfied because .

*Proof of cond. 2., Example 1*:

We have:

Since we have . But because is the only direct probabilistic cause of .

Now it is easy to see that . This is true because .. Also since .

From it follows that from where it follows that . **QED**.

We will show that 3. Is satisfied because

*Proof of cond. 3., Example 1*:

//TODO: Finish Otte’s overview on Probabilistic Causality

### Overview on Causality foundations in Samantha Kleinberg’s work

In her PhD thesis (Kleinberg, An Algorithmic Enquiry Concerning Causality, 2010) and book (Kleinberg, Causality, Probability, and Time, 2012) Samantha Kleinberg has provided detailed overview spanning from discussion on the philosophical foundations of causality to looking into each of the contributors to the modern probabilistic causality theory.

#### Discussion on Hume

The first modern attempt to understand the question “*Why?*” comes from David Hume. Hume defines a causal relationship between and to mean that is a cause of if and only if every event of type is followed by an event of type . These relations are to be inferred from observations and are subjective due to belief and perception. That is, based on experience, we reason about what will happen, have expectations based on our perceptions, and may establish whether our beliefs are true or false through experimentation and observation. For example, when we hear a noise outside in the morning, we may believe that a garbage truck is outside. Since in the past we heard this noise and saw a garbage truck outside the window, we expect to go to the window and see the same thing this time. This belief may turn out to be false, as perhaps today there is instead a street sweeper causing the noise. The important point here is that without empirical evidence, we could not have made *any* predictions about the cause of the noise.

Hume examined causality in terms of (1) what is meant when we use the term and (2) what is needed to infer such a relation from empirical evidence. First, addressing the concept of causality, Hume defined three essential relations: contiguity, temporal priority, and necessary connection. The contiguity condition asserts that a cause and its effect must be nearby in time and space. While it may seem this condition does not always hold true, Hume states that any relationships between distant causes and effects can be found to be “linked by a chain of causes, which are contiguous among themselves.” The second quality, temporal priority, means that a cause must precede its effect. While Hume traces the chain of events that would occur if we allow cause and effect to be co-temporary, ending with the “utter annihilation of time”, it suffices to say that if we do allow cause and effect to be co-temporary, we could not distinguish the cause from the effect and would in fact only be able to determine a correlation between the pair. Finally, necessary connection is the defining feature that allows us to make the distinction between causal and non-causal relationships. Here it is stipulated that both the cause and effect must occur. That is, the cause always produces the effect, and the effect is not produced without the cause. Hume then empirically defines a cause as:

**Definition A15**: An object precedent and contiguous to another, and where all the objects resembling the former are placed in a like relation of priority and contiguity to those objects that resemble the latter.

Necessary connection is replaced here by *constant conjunction*, whereby we may observe two events as being conjoined, but this does not mean that they are necessarily so and nor do we have any basis for being able to make such a statement. One common counterexample to this theory of causality is that “day causes night” satisfies all three criteria, though we would not call day a cause of night. Cases may be made against each of the three criteria; however, they represent he first step toward a theory of causality that may be verified through empirical data. The main effect of Hume’s work was “when I assert “Every event of class causes an event of class ,” do I mean merely, “Every event of class A is followed by an event of class B,” or do I mean something more? Before Hume, the latter view was always taken; since Hume, most empiricists have taken the former.”

#### Regularity and Mackie’s work

Refining Hume’s work in 1974 John Leslie Mackie formalized the ideas of necessity and sufficiency of causes. Here, an event is a *necessary condition* of an event if whenever an event of type occurs, an event of type also occurs, and is a *sufficient condition* of if whenever an event of type occurs an event of type also occurs. Thus, Mackie states that a cause is an *INUS* condition: “an insufficient but non-redundant part of an unnecessary but sufficient condition”. That is, there are some sets of conditions that result in the effect, , and the cause, , is a necessary part of one of those sets.

**Definition A16**: is a *minimal sufficient condition* for if no conjunct is redundant (i.e. no part, such as , is itself sufficient for ), and is sufficient for .

**Definition A17**: is an *INUS* condition of iff for some and for some , is a necessary and sufficient condition of , but is not a sufficient condition of and is not a sufficient condition of . That is,

1. is sufficient for ,
2. is not necessary since could also cause ,
3. alone is insufficient for E,
4. is a non-redundant part of

For example, a lit match () may be the cause of a house fire. There are, however, many other situations in which a match is lit and does not cause a fire, as well as other situations () in which a fire occurs without a lit match (). In the case of the match causing the fire, there is some set of circumstances (), each one necessary, which together are sufficient for the fire to occur.

Mackie analyzes an event as a cause of an event on a particular occasion (what is also referred to as token, or singular causality) thusly:

1. is at least an INUS condition of ,
2. was present on the occasion in question,
3. The components of , if there are any, were present on the occasion in question
4. Every disjunct in not containing as a conjunct was absent on the occasion in question

**Definition A18**: is at least an INUS condition of iff either is an INUS condition for , or is minimum sufficient condition for , or is a necessary and sufficient condition for , or is part of some necessary and sufficient condition for .

Using the house fire example, a lit match was the cause of specific fire if it was present, and there was oxygen, flammable material and the other conditions needed for a lit match to create a fire, and there was no unattended cooking, faulty electrical wiring, or other factors that cause fires in the absence of lit matches. That is, the third and fourth conditions above ensure that the other factors necessary for to cause are present, while avoiding the problem of overdetermination. For example, if there was a lit match and the house was struck by lightning, we would violate the fourth condition and in fact neither would be deemed the cause of the fire.

#### Counterfactuals and Lewis’s work

Counterfactuals provide another approach to causality by saying that had the cause not taken place, the effect would not have happened either. This relates to Hume’s work in which a cause is “*an object, followed by another, and where all the objects similar to the first are followed by objects similar to the second*”. Or in other words “*where, if the first object had not been, the second never had existed*”. Though they are supposed restatements of the same theory, the first part, known as the “regularity definition” of causality is quite different from the second part, the “counterfactual definition”. If we were to use only the first part of the definition, we would again have the problem of day being a cause of the night, as one regularly removes the causal relationship, as had day not been, night would still exist (consider the case of Polar night at the arctic circle where the sun does not rise at all).

David Lewis developed the primary counterfactual theory of causality, discussing how we can use these conditional statements to distinguish genuine cause from effects and other factors (Lewis, Counterfactuals, 1973).

In this work, Lewis limits causes and effects to events, and looks only at the analysis of causes in terms of particular cases (what is termed token, or singular causality). He begins by introducing the notion of *possible worlds*, and *comparative similarity* between possible worlds, which may be thought of as maximally consistent sets of propositions true in those worlds. Then, one world is *closer to actuality* than another is if it resembles the actual world more than the other world does. Lewis introduces two constraints on this relation, namely, (1) it involves a weak ordering of the worlds, so any two worlds may be compared, but they may be equal in similarity to the actual world; (2) the actual world is closest to actuality, as it resembles itself more than any other world resembles it.

Then, we can take the *counterfactual* of two propositions, and . This assertion is represented by and it means that if were true, would be true. Then, the truth condition for this statement is: is true (in the actual world ) iff (1) there are no possible -worlds or (2) some -world where holds is closer (to ) than any -world where does not hold. That is, in the non-vacuous case (2), the counterfactual is true iff “it takes less of a departure from actuality to make the consequent true along with the antecedent than it does to make the antecedent true without the consequent”.

Let us look at a sequence events instead of logical propositions and define causal dependence between them.

The dependence defined here means that *whether* e occurs depends on *whether or not* c occurs.

//TODO: Finish Kleinberg’s overview on the modern foundations of Causality

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