Notes on Compressed Sensing

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# Introduction

In many practical problems we need to infer quantities of interest from measured information.

In signal and image processing we want to reconstruct signal from measured data. When the information acquisition process is linear, the problem reduces to solving a linear system of equations. Let us denote with the signal of interest, and with the observed data. The observed data is connected to the signal of interest via

(1)

The matrix models the linear measurement (information) process. We want to recover the vector . If , then (1) is underdetermined and without additional information it is impossible to recover from .

Shannon’s sampling theorem states that the sampling rate of a continuous-time signal must be twice its highest frequency to guarantee lossless reconstruction.

However, under certain assumption it is possible to achieve lossless reconstruction of the signal even when the number of available measurements is smaller than the signal length .

The assumption which makes this possible is sparsity. The research area associated with this phenomenon is known as *compressive sensing*, *compressed sensing*, *compressive sampling* or *sparse recovery*.

***Sparsity***

A signal is sparse if most of its components are zero. Many real-world signals are compressible in the sense that they are well approximated by sparse signals , often after appropriate change of basis.

Let us consider the acquisition of a signal and the resulting measured data. With the additional knowledge that the signal is sparse (that is, compressible) , the traditional approach of taking at least as many measurements as the signal length is wasteful; at first, we measure all possible entries of the signal and then most coefficients are discarded in the compressed version. Instead, one would want to acquire the compressed version of a signal directly via significantly fewer measured data than the signal strength. In other words, we want to compressively sense a compressible signal – this is the basic goal of compressive sensing.

The main difficulty here lies in the location of the non-zero entries of the vector x not being known beforehand. Not knowing the non-zero locations of the vector to be reconstructed introduces nonlinearity

# References

[1] [Compressed Sensing, David L. Donoho, Stanford U., 2004](https://github.com/dimitarpg13/spectral_analysis/blob/main/literature/articles/compressed_sensing/CompressedSensingDonoho2004.pdf)

[2] [Mathematical Introduction to Compressed Sensing, Simon Foucart, Holger Rauhut, Springer, 2010](https://github.com/dimitarpg13/spectral_analysis/blob/main/literature/articles/compressed_sensing/MathematicalIntroductionToCompressedSensingRauhut.pdf)

[3] Wavelet Tour of Signal Processing, Stephane Mallat, 2004