Notes on Fourier series and Integrals (Dym and McKean, 1972)

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# The Lebesgue Integral

The Lebesgue integral is a generalization of the Riemann’s notion of integral.

Let us consider a positive-continuous function defined on an interval .

To form a Riemann sum we subdivide the interval with the following points of subdivision:

form the Riemann sum

in which is any point between and . It can be verified that this sum approaches the limit – the Riemann integral

as and the biggest of the lengths () tends to 0.

In the Lebesgue integral definition, we subdivide/quantize the range of the function instead of the domain.

Lebesgue’s technique subdivides the vertical axis by a series of points

to form the sum

in which is the sum of the lengths of the subintervals of on which the stated inequality takes place, and finally to verify that this sum approaches the same number as and the biggest of the lengths () tends to 0. The point is that by now extending the idea of *measure* from unions of disjoint subintervals to the wider class of “*measurable*” subsets of the interval , we can integrate a much wider class of functions compared to the Riemann’s definition.

More formal definition of the Lebesgue integral

Fix an interval , which may be bounded

# References

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