Notes on Wavelets and related Transforms

compiled by D.Gueorguiev, 3/2/25

# Introduction

## Time-invariant and Transient Phenomena

Classical signal processing has devoted much of its efforts to the design of time-invariant and space-invariant operators that modify stationary signal properties. With most notable representative the Fourier transform the classical approach does not address many information-processing scenarios and the related use cases.

The search of the optimal basis in absolute terms is a hopeless quest; instead, we have a multitude of different transforms and bases each having their strengths and weaknesses , therefore each suitable in specific subset of contexts.

The Fourier transform is used in time-invariant signal processing because sinusoidal waves are eigenvectors of linear time-invariant operators. A linear time-invariant operator is entirely specified by the eigenvalues :

(1)

To compute , a signal is decomposed as sum of sinusoidal eigenvectors :

(2)

If has finite energy, one can show that the amplitude of each sinusoidal wave is the Fourier transform of :

(3)

Applying the operator to f in (2) and inserting the eigenvector expression (1) gives

(4)

The operator amplifies or attenuates each sinusoidal component of by . It represents a *frequency filtering* of .

The Fourier transform provides solution to a variety of problems in which linear time-invariant operator is finding the relevant for the problem at hand pattern. However, if we are interested in transient phenomena the Fourier transform cannot find the relevant patterns for those problems at hand. The Fourier coefficient is obtained in (3) by correlating with a sinusoidal wave . Since the support of covers the whole real line, depends on the values for all times . This globally encoded information makes it difficult to analyze any local property of from .

## Focusing on the Transients

# References

[1] Wavelet Tour of Signal Processing, Stephane Mallat, 2004

[2] Ten Lectures of Wavelet, Ingrid Daubechies, Bell Labs, Rutgers U., 1992