# Notes on the Theory of Super-Resolution

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## Introduction

### Uncertainty Principles and Signal Recovery

Uncertainty Principle for Continuous Functions:

if a function is zero everywhere outside of interval of length and its Fourier transform is zero outside of interval of length then

(1)

A more general principle utilizing measure set theory holds:

Measure Set Uncertainty Principle:

If a function is zero almost everywhere outside of a measurable set and is zero almost everywhere outside a measurable set , then

(2)

where and denote the measures of the sets and , and is a small number dependent on the measure of the almost-everywhere-zero set.

In words, and cannot be both highly concentrated no matter what sets and we pick in which the concentration occurs.

Uncertainty Principle for Discrete Functions

Let be a sequence of length and let be its discrete Fourier transform.

Let is not zero at points and is not zero at points. Then

(3)

The inequality (3) holds on all kinds of sets where and are nonzero: these may be intervals or any other sets.

## References

[Decoding by Linear Programming, Emmanuel Candes, Terence Tao, Caltech, 2004](https://github.com/dimitarpg13/spectral_analysis/blob/main/literature/articles/compressed_sensing/DecodingByLinearProgrammingCandes2005.pdf)

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