Resistors and Diodes

2.1 Objectives

In this lab, you will examine the current–voltage characteristics of a diode-connected *npn* bipolar transistor, which closely follows an exponential current–voltage relationship over many decades of current. You will also examine the current–voltage characteristics of a series connection of a resistor and a diode-connected *npn* bipolar transistor. In the process, you should develop some intuition for how regimes of different qualitative behavior can arise in simple electronic circuits.

2.2 Prelab

The following prelab questions have been constructed to help you prepare to do the lab efficiently. Please complete these questions *before* you come to lab. While you may discuss the prelab questions with your lab partner or with other students in the class, each student in a lab group should complete the prelab assignment individually, so that you each understand the circuit(s) that you will be testing and what you will be doing in the lab.

1. **Diode-Connected Transistor Characteristics**. An *npn* bipolar transistor is an electronic device with three terminals, the *emitter*, the *base*, and the *collector*, as shown in Fig. 2.1. If the base and collector are connected together, as shown in Fig. 2.2, the transistor is said to be *diode-connected* and it behaves exactly as would a *pn* junction diode, because that is what it is under these circumstances. The current–voltage characteristics of such a device are well described by the *ideal diode equation*, which given by

$$I = I_{\mathrm{s}}\left(e^{V/U_{\mathrm{T}}} - 1\right), \tag{1}$$

where I_{s} is the *saturation current* of the diode, which is typically on the order of a few femptoamps at room temperature for the devices that you will be using in this lab, and U_{T} is the *thermal voltage*, kT/q, which has a value of about 25 mV at room temperature. (You will be using a diode-connected *npn* bipolar transistor in this lab instead of a signal diode because signal diodes tend to deviate from the ideal diode equation at low currents because they are doped with metalic impurities.)

   (a) Suppose that we force a current greater than a nanoamp into the diode-connected transistor, as shown in Fig. 2.2a, so that a positive voltage develops between the base/collector and the emitter. Under these conditions, how good of an approximation would it be to neglect the second term in the parentheses in Eq. 1?

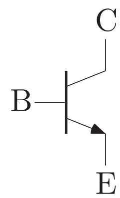

Figure 2.1: Emitter, base, and collector terminal locations of the *npn* bipolar transistor in (a) a circuit schematic symbol and (b) a 2N3904 signal transistor in a TO-92 package.

(b) In this regime, by how much would the applied voltage change if we were to increase the current one *e*-fold (i.e., a factor of e)? By how much would the voltage change if we were to increase the current by one decade?

(c) In this regime, derive an expression for the *incremental diode resistance*, which is given by

$$r_{\mathrm{d}} = \frac{\partial V}{\partial I}.$$

Express your answer in terms of the current flowing through the diode-connected transistor, I. This quantity is sometimes called the diode's *dynamic resistance* or *slope resistance*.

(d) Would you expect the situation to differ if you were instead to apply a positive voltage between the base/collector and the emitter and measure the current, as shown in Fig. 2.2b?

(e) Explain how using linear regression you could extract values of I_{s} and U_{T} from experimental measurements of a diode-connected transistor's current–voltage characteristics under the conditions described in part a.

2. **Characteristics of a Resistor and a Diode in Series**. Consider the situation shown in Fig. 2.3a, in which we force a current through a resistor, R, connected in series with a diode-connected *npn* transistor.

   (a) Find an expression for the voltage drop across the diode-connected transistor, V, the voltage drop across the resistor, and the total voltage drop that develops across the input current source, V_{in}, in terms of the applied input current, I.

   (b) Now, suppose that we increase the applied current, I, by a small amount, δI. As a result, the voltages across the resistor, the diode-connected transistor, and the

Figure 2.2: A diode connected *npn* bipolar transistor in two experimental situations: (a) source current/measure voltage and (b) source voltage/measure current.

input current source will all increase. Find an expression for the increase in each of these three voltage drops as a function of δI.

(c) At some applied input current, the increase in the voltage across the resistor will be just equal to the increase in the voltage across the diode-connected transistor. We shall call this particular value of input current the *turn-on current* of the diode, I_{on}. Find an expression for I_{on} as a function of R.

(d) Let us call the value of V that develops when I is equal to I_{on} the *turn-on voltage* of the diode, denoted by V_{on}. Find an expression for V_{on} in terms of I_{on}, I_{s}, and U_{T}.

(e) Find an expression for the fraction of δV_{in} that appears across the resistor and for the fraction of δV_{in} that appears across the diode-connected transistor. Express your answers in terms of the applied input current, I, and the turn-on current, I_{on}.

(f) Suppose that $I \ll I_{\text{on}}$, how much of δV_{in} is contributed by δV? Under these circumstances, with what functional form do you think V_{in} will change with I? Now, suppose that $I \gg I_{\text{on}}$, how much of δV_{in} is contributed by δV? Under these circumstances, by how much is V changing relative to V_{in}? How do you think V_{in} will change with I?

(g) Suppose that we were to apply the input voltage and measure the current, as shown in Fig. 2.3b. How do you think V would change with V_{in} when $V_{\text{in}} < V_{\text{on}}$ by more than a few U_{T}? How do you think I would depend on V_{in} under these circumstances? How do you think V would change with V_{in} when $V_{\text{in}} > V_{\text{on}}$ by more than a few U_{T}? How do you think I would depend on V_{in} in this situation?

2.3 Experiments

You will be doing two experiments in this lab. In the first experiment, you will be measuring the voltage–current and current–voltage characteristics of a diode-connected *npn* bipolar transistor, which follows an exponential current–voltage relationship over many decades of

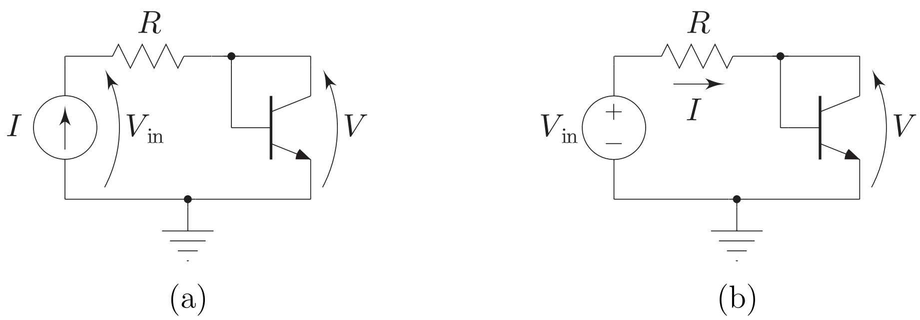

Figure 2.3: A resistor in series with a diode connected *npn* bipolar transistor in two experimental situations: (a) source current/measure voltage and (b) source voltage/measure current.

current. In the second experiment, you will be examining the behavior of a series connection of a resistor and the diode-connected transistor. In both experiments, you will be using a 2N3904 *npn* bipolar transistor, whose pinout is shown in Fig. 2.1.

In your lab report, you should include graphs of all theoretical and experimental curves. In general, you should plot the measurements in a point style so the individual points are distinguishable. Any theoretical fits to the data should be plotted on the same graph as the experimental data in a line style.

2.3.1 Experiment 1: Diode-Connected Transistor Characteristics

Obtain a 2N3904 bipolar transistor, diode connect it, and measure its voltage–current characteristic (i.e., force the into it and measure the resulting voltage, as shown in Fig. 2.2a) by logarithmically sweeping the current from about 1 nA to 10 mA. Next, measure the diode-connected transistor's current–voltage characteristic (i.e., force the voltage across the diode and measure the current that flows into it, as shown in Fig. 2.2b) over the same range of currents. Are there any substantial differences between the voltage–current characteristic and the current–voltage characteristic? If so, how might you account for these? Fit an exponential curve to the current–voltage characteristic and extract values of the saturation current, I_{s}, and the thermal voltage, U_{T}. How well does the exponential model fit the data? Make a single semilog plot for your report showing the voltage–current characterisitc, the current–voltage characteristic, and the theoretical fit.

Next, extract incremental resistance of the diode as a function of the current flowing through the diode. If your data is dense and relatively clean, you should be able to obtain reasonable results using a finite-difference approximation to the parital derivative with MATLAB's `diff` command and the `./` operator as follows:

$$r_{\text{d}} = \frac{\partial V}{\partial I} \approx \texttt{diff(V)./diff(I)}.$$

For your report, make a log-log plot showing the incremental resistance of your diode-connected transistor as a function of the current flowing through it, along with a theoretical fit to the data. Does the theoretical fit match the data?

2.3.2 Experiment 2: Characteristics of a Resistor and Diode in Series

Next, obtain at least three resistors, whose resistance values span at least two orders of magnitude and the least of which is at least $100\,\Omega$. Put each of these resistors, in turn, in series with your diode-connected 2N3904 and measure both the current flowing into the series combination and the voltage across the diode-connected transistor as you sweep the input voltage over a few volts from zero, as shown in Fig. 2.3b. You should observe two qualitatively different regions of operation. In doing your measurements, you should make sure that your data is dense in both regimes. Does the circuit behave qualitatively as you expected based on your prelab assignment? For your report, make a single plot showing the voltage across the diode-connected transistor as a function of the applied input voltage for all of the resistors that you used (i.e., one plot with at least three curves on it). Also, make a single semilog plot showing the measured current flowing into the circuit as a function of the applied input voltage. Finally, make a linear plot showing the input current as a function of the applied input voltage for each of the three resistors. For each plot, include theoretical fits that you believe make sense. For each resistor value, extract a value for the turn-on current, I_{on}, and the turn-on voltage, V_{on}. For your report, make plots showing I_{on} and V_{on} as a function of R. Do these parameters vary with R as you expected from your prelab analysis?

2.4 Postlab

In doing Experiment 2, you probably noticed two regimes in which the current–voltage characteristics of the resistor and diode-connected transistor exhibited qualitatively different behavior. In each regime, the current–voltage characteristic follows a relatively simple law. After a few moments of analysis and some algebraic effort, we quickly run up against a function that we cannot invert to explicitly solve for the intermediate node voltage, V, to find an equation relating the current flowing into the circuit, I, as a function of the input voltage, V_{in}. In this postlab assignment, you will develop an approximate explicit quantitative model for this circuit's current–voltage characteristic that captures the two regimes of operation and that transitions between them smoothly.

1. Consider the circuit shown in Fig. 2.3b, in which an input voltage, V_{in}, is applied across the resistor in series with the diode-connected transistor, as you did in Experiment 2, resulting in some voltage developing across the diode-connected transistor, V, and some current, I, flowing into the circuit.

   (a) Apply KCL to the intermediate node and derive a nonlinear funciton relating V and V_{in} that you would need to invert to eliminate V from our description of the current flowing into the circuit. Express your answer in terms of V_{on} and U_{T}.

   (b) What approximation can you make in order to invert this nonlinear relationship when $V < V_{\text{on}}$ by more than a few U_{T}? Obtain an approximate expression for I as a function of V_{in} in this regime. Does this model match the data that you obtained in Experiment 2?

(c) What approximation can you make in order to invert this nonlinear relationship when $V > V_{\text{on}}$ by more than a few U_{T}? Obtain an approximare expression for I as a function of V_{in} in this regime. Does this model match the data that you obtained in Experiment 2?

2. Next, consider the *soft minimum function*, given by

$$V = -U_{\text{T}} \log\left(e^{-V_1/U_{\text{T}}} + e^{-V_2/U_{\text{T}}}\right) = V_1 - U_{\text{T}} \log\left(1 + e^{(V_1 - V_2)/U_{\text{T}}}\right).$$

Later in the class, we will see simple circuits that embody this sort of function explicitly.

(a) Show that $V \approx V_1$ if $V_1 < V_2$ by more than a few U_{T} and that $V \approx V_2$ if $V_1 > V_2$ by more than a few U_{T}.

(b) Suggest how you might use this funciton to approximate the behavior of the voltage across the diode-connected transistor, V, as a function of the applied input voltage, V_{in}.

(c) From this approximate explicit relationship between V and V_{in}, and the current–voltage characteristic of the resistor, obtain an explicit expression for the input current, I, as a function of V_{in}. How does this expression behave when $V_{\text{in}} < V_{\text{on}}$ by more than a few U_{T}? How does this expression behave when $V_{\text{in}} > V_{\text{on}}$ by more than a few U_{T}? Do these asymptotic limits match the data that you obtained in Experiment 2?

(d) From this approximate explicit relationship between V and V_{in}, and the current–voltage characteristic of the diode-connected transistor, obtain an explicit expression for the input current, I, as a function of V_{in}. How does this expression behave when $V_{\text{in}} < V_{\text{on}}$ by more than a few U_{T}? How does this expression behave when $V_{\text{in}} > V_{\text{on}}$ by more than a few U_{T}? Do these asymptotic limits match the data that you obtained in Experiment 2?