ENGR 2420: Lab 1 Prelab Solution

January 30, 2015

Prelab

1. **Electrical Measurement Concepts**.

   (a) An ideal voltage meter has **infinite** internal resistance and should be connected in **parallel** with the circuit or device being tested.

   (b) An ideal current meter has **zero** internal resistance and should be connected in **series** with the circuit or device being tested.

2. **Resistive Divider Accuracy**. We would like to determine how the tolerences of the two resistances that go into a resistive voltage or current divider affect the divider ratio. Let us call the nominal divider ratio γ, which we shall assume is given by

$$\gamma = \frac{R_1}{R_1 + R_2}.$$

For sufficiently small changes in the resistances about their nominal values, we can express the change in the divider ratio, $\delta\gamma$, in terms of the changes in the resistances, δR_1 and δR_2, as

$$\begin{aligned}\delta\gamma &= \frac{\partial\gamma}{\partial R_1} \cdot \delta R_1 + \frac{\partial\gamma}{\partial R_2} \cdot \delta R_2 \\ &= \frac{R_2}{(R_1 + R_2)^2} \cdot \delta R_1 - \frac{R_1}{(R_1 + R_2)^2} \cdot \delta R_2.\end{aligned}$$

We can express the change in γ in a normalized way by dividing both sides of this equation by γ, which yields

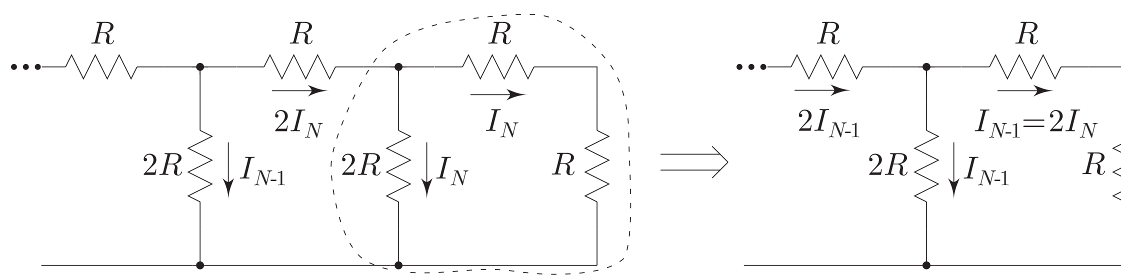
$$\begin{aligned}\frac{\delta\gamma}{\gamma} &= \frac{R_2}{R_1 + R_2} \cdot \frac{\delta R_1}{R_1} - \frac{1}{R_1 + R_2} \cdot \delta R_2 \cdot \frac{R_2}{R_2} \\ &= \frac{R_2}{R_1 + R_2} \cdot \left(\frac{\delta R_1}{R_1} - \frac{\delta R_2}{R_2}\right) \\ &= (1 - \gamma)\left(\frac{\delta R_1}{R_1} - \frac{\delta R_2}{R_2}\right),\end{aligned}$$

where the quantities $\delta R_1/R_1$ and $\delta R_2/R_2$ are directly related to the tolerances of the resistor values. Because the sign of $\delta R_1/R_1$ and $\delta R_2/R_2$ can be either positive or negative, we obtain the largest normalized variation in the divider ratio when these quantities have opposite signs. We might express this worst-case scenario as

$$\left|\frac{\delta\gamma}{\gamma}\right| = (1 - \gamma)\left(\left|\frac{\delta R_1}{R_1}\right| + \left|\frac{\delta R_2}{R_2}\right|\right).$$

For example, on this analysis, a nominal divider ratio of $\frac{1}{3}$ constructed from $\pm 5\,\%$ resistors would be subject to a $\pm 6.67\,\%$ tolerance.

3. **R-2R Ladder Network**. Consider the rightmost end of the R-$2R$ ladder network, as shown below. The two R resistors connected in series are equivalent to a single resistor whose value is $2R$. This equivalent resistance appears in parallel with another resistor whose value is also $2R$. Because the voltage drop across each of these $2R$ resistors is the same, they will carry equal currents, I_N, as indicated. By KCL, the current flowing horizontally through the next R resistor to the left will be given by the sum of these two currents, $2I_N$, as indicated on the schematic. Now, we can collapse the two $2R$ resistors connected in parallel into a single equivalent resistance whose value is $2R\| (R+R) = 2R\|2R = R$, as shown on the right. This situation, however, is self similar to the one with which we started. Therefore, a precisely parallel argument holds once again on the collapsed network from which we would conculde that $I_{N-1} = 2I_N$, as shown below on the right.



By repeating this process, we would in general find that $I_{n-1} = 2I_n$, and so on. From this recurrence relation, we can show that $I_n = I_1/2^{n-1}$. By a repetition of the collapse process just outlined, we would also have that the current flowing out of the V voltage source is given by $2I_1$, which by Ohm's law must also be given by $V/2R$, so $I_1 = V/4R$. By combining these results, we have that

$$I_n = \frac{1}{2^{n+1}} \cdot \frac{V}{R}.$$

4. **Accurate 2:1 Resistor Ratios**. We can connect two of our unit resistors in series and let them play the role of $2R$. The third unit resistor would then play the role of R. Alternatively, we can take two of our unit resistors and connect them in parallel and let them play the role of R. In this case the third unit resistor would play the role of $2R$. Each of these alternatives requires three unit resistors per stage. The only difference is in the termination structure (i.e., the two resistors that appear in series at the right end of the network). In the series method, the termination requires two unit resistors. In the series method, each R resistor is a implemented as a parallel combination of unit resistors, which seems to require two more unit resistors than does the series method. However, we can collapse this network of four resistors into a single unit resistor, which leaves the currents through the rungs of the ladder unchanged. Thus, the parallel method requires one fewer resistor than does the series method.