ENGR 2420: Lab 2 Prelab Solution

February 6, 2015

Prelab

1. **Diode-Connected Transistors**.

   (a) If we assume that I_{s} is no more than $10\,\mathrm{fA}$, then, in order for the current through the diode-connected transistor to be greater than $1\,\mathrm{nA}$, the first term in the parentheses must be at least 10^5, which means that neglecting the second term in the parentheses introduces an error of less than $10\,\mathrm{ppm}$, or equivalently, $0.001\,\%$, which is an excellent approximation.

   (b) If we were to change the current by one e-fold, the voltage across the diode-connected transistor would change by U_{T}, which is approximately $25\,\mathrm{mV}$. If we were to change the current by one decade, the voltage across the diode-connected transistor would change by $U_{\mathrm{T}} \log 10 \approx 25\,\mathrm{mV} \times 2.303 \approx 60\,\mathrm{mV}$.

   (c) To find the incremental diode resistance, we can invert the current–voltage characteristic of the forward-biased diode-connected transistor, thereby obtaining

   $$V = U_{\mathrm{T}} \log \frac{I}{I_{\mathrm{s}}},$$

   and differentiate it with respect to I, to obtain an expression for r_{d} as

   $$r_{\mathrm{d}} = \frac{\partial V}{\partial I} = \frac{U_{\mathrm{T}}}{I/I_{\mathrm{s}}} \cdot \frac{1}{I_{\mathrm{s}}} = \frac{U_{\mathrm{T}}}{I}.$$

   (d) No, the situation would not change if we were to force the voltage and measure the current instead of forcing the current and measuring the voltage. The voltage and current pairs that develop should be consistent between these two alternative experiments.

   (e) We can transform the exponential current–voltage characteristic of the forward-biased diode-connected transistor into a linear one by taking the log of both sides of the equation, to obtain

   $$\begin{aligned} I &= I_{\mathrm{s}} e^{V/U_{\mathrm{T}}} \\ \log I &= \log\left(I_{\mathrm{s}} e^{V/U_{\mathrm{T}}}\right) \\ \log I &= \log I_{\mathrm{s}} + \log e^{V/U_{\mathrm{T}}} \\ \underbrace{\log I}_{y} &= \underbrace{\log I_{\mathrm{s}}}_{b} + \underbrace{\frac{1}{U_{\mathrm{T}}}}_{m} \cdot \underbrace{V.}_{x} \end{aligned}$$

   We can use linear regression on the log of the current and the voltage to find a best-fit line to the data. From the slope of this line, we can extract a value of U_{T} as $1/m$, and from the intercept, we can extract a value of I_{s} as e^b.

2. **Characteristics of a Resistor and a Diode in Series**.

   (a) With a current, I, flowing through the diode-connected transitor, the voltage drop across it will be given by $V = U_\mathrm{T} \log\left(I/I_\mathrm{s}\right)$. With a current, I, flowing through the resistor, the voltage drop across it will be given by $V_\mathrm{r} = IR$. The voltage drop across the input current source is given by the sum of these two voltage drops, $V_\mathrm{in} = IR + U_\mathrm{T} \log\left(I/I_\mathrm{s}\right)$.

   (b) If we were to increase the current I by a small amount, δI, the voltage across the diode-connected transistor would increase by $\delta V = r_\mathrm{d} \cdot \delta I$, the voltage across the resistor would increase by $\delta V_\mathrm{r} = R \cdot \delta I$, and the input voltage would increase by $\delta V_\mathrm{in} = \delta V_\mathrm{r} + \delta V = \left(r_\mathrm{d} + R\right) \delta I$.

   (c) By equating δV_r and δV, we find that

$$\begin{aligned}
\delta V_\mathrm{r} &= \delta V \\
R \cdot \delta I &= r_\mathrm{d} \cdot \delta I \\
R &= r_\mathrm{d} \\
R &= \frac{U_\mathrm{T}}{I_\mathrm{on}},
\end{aligned}$$

   which implies that $I_\mathrm{on} = U_\mathrm{T}/R$.

   (d) We can obtain an expression for V_on as a function of I_on, I_s, and U_T by inverting the diode-connected transistor's current–voltage characterisitc. Doing so, we find that $V_\mathrm{on} = U_\mathrm{T} \log\left(I_\mathrm{on}/I_\mathrm{s}\right)$.

   (e) We can calculate the fraction of δV_in that appears across the diode-connected transistor directly as

$$\frac{\delta V}{\delta V_\mathrm{in}} = \frac{r_\mathrm{d} \cdot \delta I}{\left(r_\mathrm{d} + R\right) \delta I} = \frac{r_\mathrm{d}}{r_\mathrm{d} + R} = \frac{\dfrac{U_\mathrm{T}}{I}}{\dfrac{U_\mathrm{T}}{I} + R} = \frac{\dfrac{U_\mathrm{T}}{R}}{\dfrac{U_\mathrm{T}}{R} + I} = \frac{I_\mathrm{on}}{I_\mathrm{on} + I}.$$

   The rest of δV_in drops across the resistor, so we can obtain $\delta V_\mathrm{r}/\delta V_\mathrm{in}$ as

$$\frac{\delta V_\mathrm{r}}{\delta V_\mathrm{in}} = 1 - \frac{\delta V}{\delta V_\mathrm{in}} = 1 - \frac{I_\mathrm{on}}{I_\mathrm{on} + I} = \frac{I}{I_\mathrm{on} + I}.$$

   (f) If $I \ll I_\mathrm{on}$, then $\delta V/\delta V_\mathrm{in} \approx 1$ and $\delta V_\mathrm{r}/\delta V_\mathrm{in} \approx 0$. Consequently, practically all of δV_in results from δV. Because V changes logarithmically with I, V_in should also increase logarithmically with I in this regime. Also, becaue I is so small in this regime, $V_\mathrm{r} \approx 0$ and $V_\mathrm{in} \approx V$. On the other hand, if $I \ll I_\mathrm{on}$, then $\delta V/\delta V_\mathrm{in} \approx 0$ and $\delta V_\mathrm{r}/\delta V_\mathrm{in} \approx 1$. Thus, in this regime, practically all of δV_in results from δV_r. Because V_r changes linearly with I, in this regime, V_in should also increase linearly with I. Moreover, because $\delta V \approx 0$ in this regime, we have that V essentially does not change (i.e., remains constant) with increases in I.

(g) As with the diode-connected transistor by itself, the situation that we have just described would not change if we were to force the input voltage and measure the current instead of forcing the current and measuring the input voltage. The input voltage and current pairs that develop should be consistent between these two alternative experiments. Because $V_{\text{in}} = V_{\text{r}} + V$ and $V_{\text{r}} > 0$, it follows directly that $V < V_{\text{in}}$, so that if $V_{\text{in}} < V_{\text{on}}$, then $V < V_{\text{on}}$. Given the way that we have defined I_{on} and V_{on}, we have that $I < I_{\text{on}}$ when $V_{\text{in}} < V_{\text{on}}$. Given the steepness of the current–voltage characteristic of the diode-connected transistor, if $V_{\text{in}} < V_{\text{on}}$ by more than a few U_{T}, then $I \ll I_{\text{on}}$, and the situation will be the first one described in part f. In this case, we should expect I to increase exponentially with V_{in}. If $V_{\text{in}} > V_{\text{on}}$ by more than a few U_{T}, we should expect to be in the second situation described in part f. Thus, we should expect that I will increase linearly with V_{in}.