

Chapter 1

D a) $2x^2 - 3x - 5 = 0$

$$(2x - 5)(x + 1) = 0$$

$$2(x - 2\frac{1}{2})(x + 1) = 0$$

$$\boxed{x = 2\frac{1}{2} \text{ or } -1}$$

b) $(x-1)(2-x)(x+4) = 0$

$$\boxed{x = 1, 2 \text{ or } -4}$$

c) $a(x-b) - b(x+a) = 0$

~~$$ax - ab - bx + ab = 0$$~~

~~$$ax - bx$$~~

$$ax - bx - ab = 0$$

$$ax - bx = 2ab$$

$$x(a-b) = 2ab$$

$$\boxed{x = \frac{2ab}{a-b}}$$

d) $4 - 9x^2 = 0$

$$(2-3x)(2+3x) = 0$$

$$9(\frac{2}{3} - x)(\frac{2}{3} + x) = 0$$

$$\boxed{x = \frac{2}{3} \text{ or } -\frac{2}{3}}$$

e) $4x^2(x-2) + 3x(x-2) = 0$

$$(4x^2 + 3x)(x-2) = 0$$

$$x(4x + 3)(x-2) = 0$$

$$4|x(x + \frac{3}{4})(x-2) = 0$$

$$\boxed{x = 0, -\frac{3}{4} \text{ or } 2}$$

f) $a(x-c) - b(3c-2x) = c(a+b)$

$$ax - ac - 3bc + 2bx = ac + bc$$

$$ax + 2bx = 2ac + 4bc$$

$$x = \frac{2ac + 4bc}{a + 2b} = \frac{2c(a + 2b)}{a + 2b}$$

$$\boxed{x = 2c} \quad \boxed{a \neq -2b}$$

$$g) x(2x+5)=12$$

$$2x^2+5x=12$$

$$2x^2+5x-12=0$$

$$(2x-3)(x+4)=0$$

$$2(x-1\frac{1}{2})(x+4)=0$$

$$\boxed{x = 1\frac{1}{2} \text{ or } -4}$$

$$h) x^3+3x^2-4x=12$$

$$x^3+3x^2-4x-12=0$$

$$x^2(x+3)-4(x+3)=0$$

$$(x^2-4)(x+3)=0$$

$$(x-2)(x+2)(x+3)=0$$

$$\boxed{x = 2, -2 \text{ or } -3}$$

$$i) (x-25)^2-a^2=0$$

$$((x-25)-a)((x-25)+a)=0$$

$$\boxed{x = 25+a \text{ or } 25-a}$$

$$\textcircled{2} a) 9x(x-4)=2x(x-4) \quad b) m(2m-x)=3(x+6)$$

$$7x(x-4)=0$$

$$\boxed{x = 0 \text{ or } 4}$$

$$2m^2-xm=3x+18$$

$$2m^2-xm-3x-18=0$$

$$(2m^2-18)=(xm+3x)$$

$$(2m^2-18)=x(m+3)$$

$$x = \frac{2m^2-18}{m+3} = \frac{2(m^2-9)}{m+3} = \frac{2(m-3)(m+3)}{m+3} = 2(m-3)$$

$$\boxed{x = 2m-6} \quad \boxed{m \neq -3}$$

$$c) \frac{12(x+1)}{x} = \frac{21(x+1)}{12} \rightarrow \boxed{x \neq 0}$$

$$144(x+1)=21x(x+1)$$

$$(144-21x)(x+1)=0$$

$$21\left(\frac{144}{21}-x\right)(x+1)=0$$

$$\boxed{x = -1 \text{ or } 6\frac{4}{7}}$$

$$e) 6x^2-x=1$$

$$6x^2-x-1=0$$

$$(3x+1)(2x-1)=0$$

$$6(x+\frac{1}{3})(x-\frac{1}{2})=0$$

$$\boxed{x = -\frac{1}{3} \text{ or } \frac{1}{2}}$$

$$d) x-1+x(x-1)=3(x-1)$$

$$1(x-1)+x(x-1)-3(x-1)=0$$

$$(1+x-3)(x-1)=0$$

$$(x-2)(x-1)=0$$

$$\boxed{x = 1, 2}$$

③ a) $x(x+1)(x+3) = 2(x+3)$
 $x(x+1)(x+3) - 2(x+3) = 0$
 $(x^2+x)(x+3) - 2(x+3) = 0$
 $(x^2+x-2)(x+3) = 0$
 $(x+2)(x-1)(x+3) = 0$
 $\boxed{x = -2, 1, -3}$

b) $9x^2 - 6x = 1$
 $9x^2 - 6x - 1 = 0$
 $x = \frac{6 \pm \sqrt{36 + 36}}{2 \cdot 9} =$
 $= \frac{6 \pm 6\sqrt{2}}{18} =$
 $= \frac{1 \pm \sqrt{2}}{3} = \boxed{\frac{1}{3} + \sqrt{\frac{2}{9}} \text{ or } \frac{1}{3} - \sqrt{\frac{2}{9}}}$

c) $\frac{x+2}{4} = \frac{(x+1)(x+2)}{2}$
 $x+2 = 2(x+1)(x+2)$
 $(2x+2)(x+2) - 1(x+2) = 0$
 $(2x+1)(x+2) = 0$
 $2(x+1/2)(x+2) = 0$
 $\boxed{x = -1/2, -2}$

e) $x^2 + 2x - 1 = 0$
 $x = \frac{-2 \pm \sqrt{4 + 4}}{2} =$
 $= \frac{-2 \pm 2\sqrt{2}}{2} =$
 $= -1 \pm \sqrt{2} =$
 $\boxed{\sqrt{2} - 1 \text{ or } -\sqrt{2} - 1}$

d) $x(x^2 - 9) = 4(9 - x^2)$
 $x(x^2 - 9) + 4(x^2 - 9) = 0$
 $(x+4)(x^2 - 9) = 0$
 $(x+4)(x+3)(x-3) = 0$
 $\boxed{x = -3, 3 \text{ or } -4}$

f) ~~$q(x+1)(x+2) = 2x+q$~~
 $\frac{2x+q}{2} = \frac{2x+q}{p}$
 $(2x+q)p = (2x+q)q \quad | q, p \neq 0$
 $2xp + p^2 = 2xq + q^2$
 $2xp - 2xq = q^2 - p^2$
 $2x(p-q) = (q+p)(q-p)$
 $2x = -q \cdot p$
 $\boxed{x = -\frac{q}{2} - \frac{p}{2}} \quad \boxed{q \neq p}$

$$g) x^2 - 6x + 1 = 161$$

$$x^2 - 6x - 160 = 0$$

$$(x+10)(x-16) = 0$$

$$\boxed{x = -10, 16}$$

$$h) 36x^2 - 4 = 0$$

$$(6x-2)(6x+2) = 0$$

$$36(x - 1/3)(x + 1/3) = 0$$

$$\boxed{x = 1/3, -1/3}$$

$$④ a) \frac{x+2}{x+2} + \frac{2}{2-x} = \frac{8}{x^2-4}$$

$$\boxed{x \neq \pm 2}$$

$$x(x-2) + 2(x+2) = 8$$

change
sign:

$$x^2 - 2x - 2x - 4 = 8$$

$$2x \rightarrow -(-2) \quad x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$$\boxed{x = -2, 6} \quad \text{NOT IN DOMAIN!}$$

$$\boxed{x = 6}$$

⊗

$$c) \frac{x+3}{x-1} + \frac{2x-1}{1-x} = \frac{x-1}{x+1}$$

$$\frac{x+3}{x-1} - \frac{2x-1}{x-1} = \frac{x-1}{x+1}$$

$$\frac{x+3-2x+1}{x-1} = \frac{x-1}{x+1}$$

$$\frac{-x+4}{x-1} = \frac{x-1}{x+1} \quad \boxed{x \neq \pm 1}$$

$$(4-x)(x+1) = (x-1)(x-1)$$

$$4x+4-x^2-x = x^2-2x+1$$

$$2x^2 - 5x - 3 = 0$$

$$(2x+1)(x-3) = 0$$

$$2(x+1/2)(x-3) = 0$$

$$\boxed{x = -1/2, 3}$$

$$b) \frac{x+1}{x-1} - \frac{3x+9}{6-x} = \frac{2x^2}{x^2-7x+6}$$

$$\frac{x+1}{x-1} - 3 \frac{x+3}{6-x} = \frac{2x^2}{(x-6)(x-1)}$$

$$\boxed{x \neq 1, 6}$$

$$(x+1)(x-6) - 3(x+3)(x-1) = 2x^2$$

$$x^2 - 5x - 6 + 3x^2 + 6x - 9 = 2x^2$$

$$2x^2 + x - 15 = 0$$

$$(2x-5)(x+3) = 0$$

$$2(x-2\frac{1}{2})(x+3) = 0$$

$$\boxed{x = 2\frac{1}{2}, -3}$$

$$d) \frac{28}{x^2-9} + \frac{x}{3-x} + \frac{7}{3+x} = 1$$

$$\boxed{x \neq \pm 3}$$

$$28 - x(x+3) + 7(x-3) = x^2-9$$

$$28 - x^2 - 3x + 7x - 21 = x^2 - 9$$

$$2x^2 - 4x - 16 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$\boxed{x = -2, 4}$$

5) a) $\sqrt{3-2x} + \sqrt{1-x} \geq 2$

$$3-2x \geq 0 \text{ AND } 1-x \geq 0$$

$$3 \geq 2x \text{ AND } 1 \geq x$$

$$1\frac{1}{2} \geq x \text{ AND } 1 \geq x$$

⇓

$$\boxed{x \leq 1}$$

d) $\sqrt{2x+2} + \sqrt{6-4x}$

$$2x+2 \geq 0 \quad 6-4x \geq 0$$

$$2x \geq -2 \quad 6 \geq 4x$$

$$x \geq -1 \quad 1\frac{1}{2} \geq x$$

$$\boxed{-1 \leq x \leq 1\frac{1}{2}}$$

b) $\sqrt{x} - \sqrt{3x-1}$

$$x \geq 0 \text{ AND } 3x-1 \geq 0$$

$$3x \geq 1$$

$$x \geq \frac{1}{3}$$

$$\boxed{x \geq \frac{1}{3}}$$

c) $\sqrt{6-x} - \sqrt{3x-9}$

$$6-x \geq 0 \quad 3x-9 \geq 0$$

$$6 \geq x \quad 3x \geq 9$$

$$x \geq 3$$

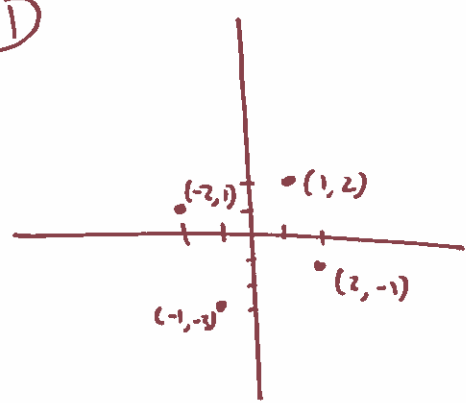
AND

⇓

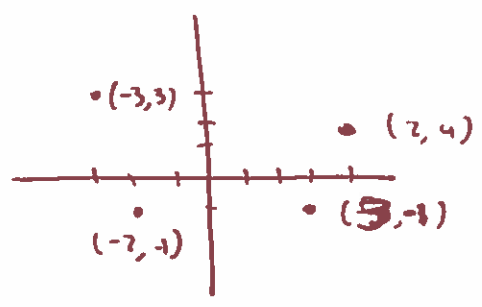
$$\boxed{3 \leq x \leq 6}$$

Chapter 21

①



②



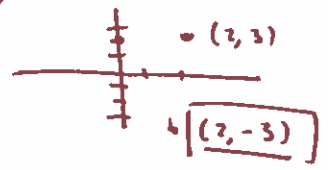
③ $y = 2$



④ $x = 2$



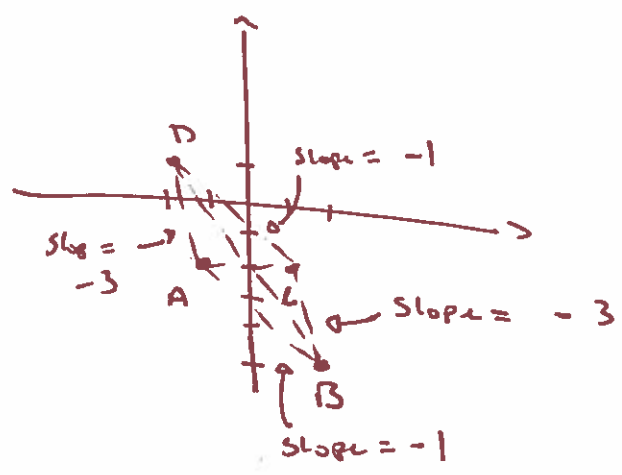
⑤



- ⑧ a) (3, 2) b) (-1, 3) c) (1, 1)

- ⑨ a) (-2, 3) b) (3, -5) c) (-4, 4)

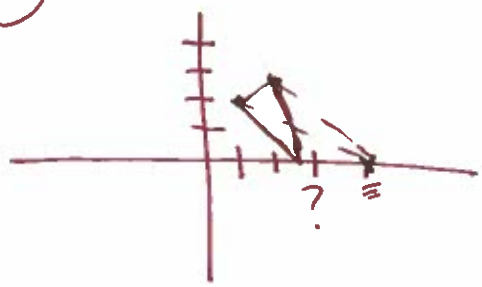
⑩



mid[A, C] = (0, -2)
mid[D, B] = (0, -2)

(0, -2)

⑪



$$\begin{aligned} (x, 0) \rightarrow (1, 2) &= \sqrt{(x-1)^2 + 2^2} \\ (x, 0) \rightarrow (2, 3) &= \sqrt{(x-2)^2 + 3^2} \end{aligned} \quad \Rightarrow \quad \boxed{(4, 0)}$$

$$(x-1)^2 + 2^2 = (x-2)^2 + 3^2$$

$$x^2 - 2x + 1 + 4 = x^2 - 4x + 4 + 9$$

$$2x = 8 \quad \Rightarrow \quad \underline{\underline{x = 4}}$$

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$$(x, y) \rightarrow (3, 6) = \sqrt{(x-3)^2 + (y-6)^2}$$

$$(x, y) \rightarrow y \text{ axis} = x$$

$$(x, y) \rightarrow x \text{ axis} = y$$

$$\sqrt{(x-3)^2 + (y-6)^2} = x = y$$

$$(x-3)^2 + (y-6)^2 = x^2 = y^2$$

$$x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 = y^2$$

$$\text{for } x = y$$

$$x^2 - 6x + 9 + x^2 - 12x + 36 = x^2$$

$$2x^2 - 18x + 45 = x^2$$

$$x^2 - 18x + 45 = 0$$

$$x = 15, 3 = y \rightarrow \boxed{(3, 3) \text{ OR } (15, 15)}$$

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$$\sqrt{(4-1)^2 + (0-k)^2} = \sqrt{(-4-1)^2 + (8-k)^2}$$

$$3^2 + k^2 = 5^2 + (8-k)^2$$

$$9 + k^2 = 25 + 64 - 16k + k^2$$

$$9 = 89 - 16k$$

$$\boxed{k = 5}$$

$$(14) \sqrt{(5+1)^2 + (k-2)^2} = \sqrt{(5-3)^2 + (k-0)^2}$$

$$6^2 + (k-2)^2 = 2^2 + k^2$$

$$36 + k^2 - 4k + 4 = 4 + k^2$$

$$36 - 4k = 0$$

$$\boxed{k = 9}$$

15

$$\sqrt{(-1-2)^2 + (1-k)^2} = 5$$

$$3^2 + (1-k)^2 = 25$$

$$9 + 1 - 2k + k^2 = 25$$

$$k^2 - 2k - 15 = 0$$

$$(k-5)(k+3) = 0$$

$$\boxed{k = -3, 5}$$

$$(16) \sqrt{(k+3)^2 + (5+1)^2} = 10$$

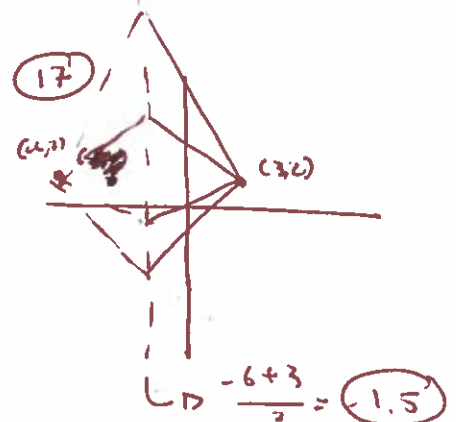
$$(k+3)^2 + 6^2 = 100$$

$$k^2 + 6k + 9 + 36 = 100$$

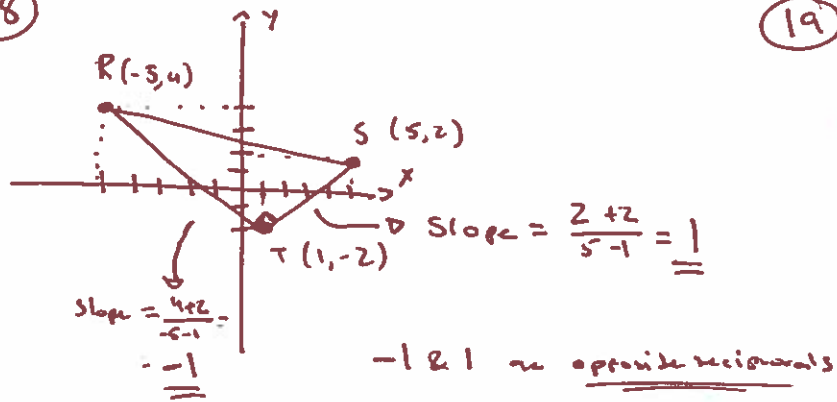
$$k^2 + 6k - 55 = 0$$

$$(k+11)(k-5) = 0$$

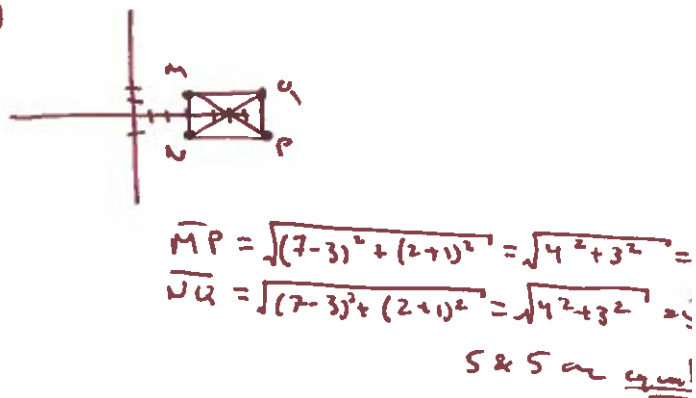
$$\boxed{k = 5, -11}$$



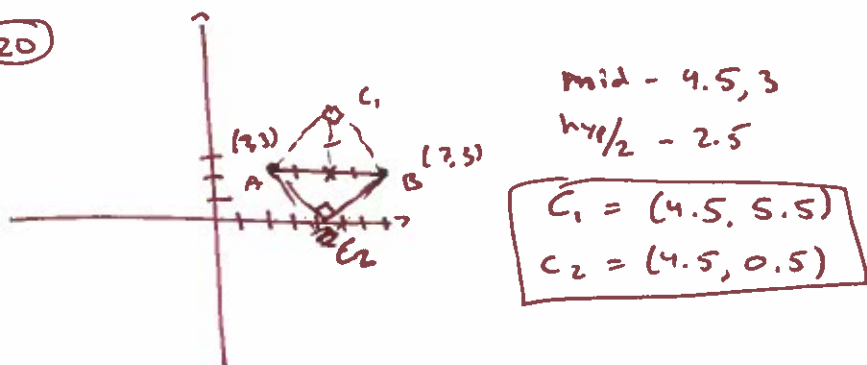
18)



19)



20)



21)