(15) a) $x > 0$   b) $-x > 0 \rightarrow \underline{x < 0}$   c) $x - 3 > 0 \rightarrow x > 3 \rightarrow$ ...

d) $\cap \ 3x+1^{\#} > 0 \quad 3x > -1 \quad x > -\tfrac{1}{3}$
   $\cap \ 3x-1 > 0 \quad 3x > 1 \quad x > \tfrac{1}{3}$   $\Big\} \quad \underline{\underline{x > \tfrac{2}{3} \text{ or } \tfrac{2}{3} > x > \tfrac{1}{3}}}$
   $\cap \ 3x-1 \neq 1 \quad 3x \neq 2 \quad x \neq \tfrac{2}{3}$

(19) a) D: $4.5x > 0 \cap 2x-1 > 0 \cap 2x-1 \neq 1$

$x > 0 \cap x > \tfrac{1}{2} \cap x \neq 1 \qquad \underline{\underline{x > 1 \text{ OR } 1 > x > \tfrac{1}{2}}}$

$(2x-1)^2 = 4.5x$

$4x^2 - 4x + 1 = 4.5x$

$4x^2 - 8.5x + 1 = 0$

$8x^2 - 17x + 2 = 0$

$(8x - 1)(x - 2) = 0$

$(x - \tfrac{1}{8})(x - 2) = 0$

$x = \tfrac{1}{8} \text{ or } \boxed{2}$

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b) D: $\underline{x > \tfrac{2}{3} \text{ or } \tfrac{2}{3} > x > \tfrac{1}{3}}$   (see 15(d))

$\log_{(3x-1)}(3x+1) = 2 \qquad (3x-1)^2 = (3x+1) \qquad 9x^2 - 6x + 1 = 3x + 1$

$9x^2 - 9x = 0 \qquad x = \tfrac{0}{} \text{ or } \boxed{1}$

c) $\# \ D: \underline{x > 0} \qquad 2x^2 + x = 10 \longrightarrow \left( \left( 5^{\log_5 \sqrt{10}} \right)^2 \right)$

$2x^2 + x - 10 = 0 \qquad (2x+5)(x-2) = 0 \qquad x = \boxed{2} \text{ or } -\tfrac{5}{2}$

d) D: $\underline{\underline{x > 0}}$

$3x^2 + x = 30 \qquad (3x + 10)(3x - 3) = 0 \qquad x = \boxed{3} \text{ or } -\tfrac{8}{3}$

e) D $1+x^2 > 0 \rightarrow \boxed{x \in \mathbb{R}}$

$\log_4^2(1+x^2) = \tfrac{1}{4} \qquad \log_4(1+x^2) = \pm \tfrac{1}{2} \qquad 1 + x^2 = 4^{\pm \frac{1}{2}} = 2 \text{ or } \tfrac{1}{2}$

$x^2 = 1 \text{ or } -\tfrac{1}{2} \qquad \boxed{x = \pm 1}$

f) D: $x+2>0$    $x>-2$
      $x+3>0$    $x>-3$ $\Big\}$ $\underline{\underline{x>-2}}$

$\log_{1/2}(x+2)+\log_{1/2}(x+3)=\log_{1/2}3-1$

$\log_{1/2}\left[\dfrac{(x+2)(x+3)}{3}\right]=-1$

$\left[\dfrac{(x+2)(x+3)}{3}\right]=\frac{1}{2}^{-1}$

$(x+2)(x+3)=6$

$x^2+5x+6=6$

$x^2+5x=0$

$\boxed{x=5\text{ or }0}$

20   a) $\underline{\underline{x>5}}$   b) $-x^2>0$   $x^2<0$   $\underline{\underline{x\in\emptyset}}$   c) $x^2+3>0$   $\underline{\underline{x\in\mathbb{R}}}$

d) $3x-5>0$   $\underline{\underline{x>\frac{5}{3}}}$   e) $5-\sqrt{x}>0$   $5>\sqrt{x}$   $25>x$ $\Big\}$ $\underline{\underline{25>x>0}}$
                         $x>0$

f) $12x-x^2-20>0$   $-(x-2)(x-10)>0$   $\underline{\underline{10>x>2}}$ → $\boxed{10>x>2}$

21   $\log x=\log 8+\log 20-\log 40$

$x=\dfrac{8\cdot 20}{40}=\boxed{4}$

22   a) $\log_{12}3+\log_{12}4=\log_{12}12=\boxed{1}$   b) $\log_3 15-\log_3 5=\log_3 3=\boxed{1}$

c) $\log_2 3+\log_2 \frac{2}{3}=\log_2 2=\boxed{1}$   d) $\log_{1/4}\left(\frac{1}{16}\right)^{-2}=\frac{1}{2}\cdot(-2)\cdot(-4)\log_2 2=\boxed{-4}$

23   a) D: $x+2>0$   $x>-2$
          $x>0$ $\Big\}$ $\underline{\underline{x>0}}$

$\sqrt{x}\cdot\log_3(x+2)=0$   $\sqrt{x}=0$   or   $\log_3(x+2)=0$

$x=0$   or   $x+2=1$    $\cancel{x=0}$ or $\cancel{x}$

b) D: $x>0$
   $\cdot 1, x^2>0$ $\Big\}$ $\underline{\underline{x\geq0}}$    $\log(1x^2)=\log x=1$    ✱ $[(-1)+2\cos(x)]\log(x)=1$

Remember $\cos x=\cos_{-x}$   $[\log(-1)+2\cos(x)]\log(x)=1$    $2\cos^2(x)+-\cos(x)-1=0$

$$2\log^2(x) - \log(x) - 1 = 0$$

$$(2\log(x) + 1)(\log(x) - 1) = 0$$

$$\log(x) = 1 \text{ or } -\tfrac{1}{2}$$

$$\boxed{x = 10 \text{ or } \tfrac{1}{\sqrt[2]{10}}}$$

c) D: $5^{x+1} - 20 > 0$

$$5^{x+1} > 20$$

$$x+1 > \log_5 20$$

$$x+1 > \log_5 5 \cdot \log_5 4$$

$$\underline{x > \log_5 4}$$

$\log_5(5^{x+1} - 20) = x$

$$5^x = 5^{x+1} - 20$$

$$5^x = 5 \cdot 5^x - 20$$

$$0 = 4 \cdot 5^x - 20$$

$$5^x = 5 \quad \boxed{x = 1}$$

(It's ok b/c
$1 > \log_5 4$ b/c
$\log_5 5 > \log_5 4$)

d) D: $x^2 + 4x + 1 > 0 \quad (x - (-2+\sqrt{3}))(x + (-2-\sqrt{3})) > 0 \quad x > -2+\sqrt{3} \text{ or } -2-\sqrt{3} > x$

$6x + 2 > 0 \quad 6x > -2$

$x > -\tfrac{1}{3}$

$x_{1,2} = \dfrac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 1}}{2} =$

$= -2 \pm \sqrt{3}$

$$\boxed{x > \sqrt{3} - 2}$$

$\log_2(x^2 + 4x + 1) + 1 = \log_2(6x + 2)$

$$\log_2\left[\frac{x^2 + 4x + 1}{6x + 2}\right] = -1$$

$$\frac{x^2 + 4x + 1}{6x + 2} = \tfrac{1}{2}$$

$$2x^2 + 8x + 2 = 6x + 2$$

$$2x^2 + 2x = 0$$

$$\boxed{x = 2 \text{ or } 0}$$

e) D: $x + 1 > 0 \atop x - 1 > 0 \Big\} \boxed{x > 1}$

$$\log_2(x+1) - \log_2(x-1) = 1$$

$$\log_2 \frac{x+1}{x-1} = 1 \qquad \frac{x+1}{x-1} = 2 \qquad x+1 = 2x - 2$$

$$x - 3 = 0 \quad \boxed{x = 3}$$

(24) a) $D: \underline{x \in \mathbb{R}}$

$$4^{x+1\frac{1}{2}} + 2^{x+2} = 4$$

$$2^{2x+3} + 2^{x+2} = 4$$

$$8 \cdot 2^{2x} + 4 \cdot 2^{x} = 4$$

$$2 \cdot 2^{2x} + 2^{x} - 1 = 0$$

$$(2 \cdot 2^{2x} - 1)(2^{x} + 1) = 0$$

$$2^{x} = \tfrac{1}{2} \text{ or } \cancel{X}$$

$$\boxed{x = -1}$$

b) $D: \underline{x \in \mathbb{R}}$

$$3^{x^2 - 4x + \frac{1}{2}} = \sqrt{3}$$

$$x^2 - 4x + \tfrac{1}{2} = \tfrac{1}{2}$$

$$x^2 - 4x = 0$$

$$\boxed{x = 4 \text{ or } 0}$$

c) $D: \underline{x \in \mathbb{R}}$

$$3^{x}\left(\tfrac{1}{3}\right)^{x-3} = \left(\tfrac{1}{27}\right)^{x}$$

$$3^{x} \cdot 3^{(3-x)} = 3^{-3x}$$

$$\cancel{\text{..........}} \quad x + 3 - x = -3x$$

$$\cancel{\text{..........}} \quad 3 = -3x$$

$$\boxed{\cancel{X}} \qquad \boxed{x = -1}$$

1) $D: x - 2 > 0 \quad > \underline{\underline{x > 2}}$
$\quad\quad x > 0$

$$\log_3(x-2) + \log_3 x = \log_3 8$$

$$\log_3\left[\frac{(x-2) \cdot x}{8}\right] = 0$$

$$\frac{(x-2) \cdot x}{8} = 1 \qquad x^2 - 2x = 8 \qquad x^2 - 2x - 8 = 0 \qquad (x-4)(x+2) = 0$$

$$x = \cancel{-2} \text{ or } \boxed{4}$$

2) $D: x - 9 > 0 \quad x > 9$
$\quad\quad \sqrt{2x-1} > 0 \Big\} \; x > \tfrac{1}{2} \Big\} \; \underline{\underline{x > 9}}$
$\quad\quad 2x - 1 > 0$

$$\log(x-9) + 2\log(\sqrt{2x-1}) = 2 \qquad (x-9)(2x-1) = 100$$

$$\log[(x-9)(2x-1)] = 2 \qquad 2x^2 - 18x - x + 9 = 100$$

$$\frac{19 \pm \sqrt{19^2 + 8 \cdot 91}}{4}$$

$$\frac{19 \pm \sqrt{1009}}{4}$$

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$$\boxed{\frac{19 + \sqrt{1009}}{4}}$$

3) $D: \left.\begin{array}{c} 30-x > 0 \\ x > 0 \end{array}\right\}$  $\underline{\underline{30 > x > 0}}$

$\log_3 x - \log_3 11 = \log_3 19 - \log_3 30 - x$

$$\frac{x}{11} = \frac{19}{30-x}$$

$30x - x^2 = 209$

$x^2 - 30x + 209 = 0$

$(x-11)(x-19) = 0$

$\boxed{x = 11 \text{ or } 19}$

(25)  $\underline{3\log 7 + 5 \log 3 - 2\log 5}$

(28)  a) $D: \underline{\underline{x > 0}}$

$x = \dfrac{3^4 \cdot 6^2}{27^{2/3}} = \dfrac{81 \cdot 36}{9} = 9 \cdot 36 \boxed{324}$

b)  $\underline{\underline{x > 0}}$

$x = \dfrac{6^2 \cdot (\sqrt[3]{20})^3}{100^{1/2}} = \dfrac{36 \cdot 20}{10} = \boxed{72}$

(29)  a) $D: \underline{\underline{x > 0}}$

$\log_9 x + 2\log_3 x = 5$

$\log_3 \sqrt{x} + \log_3 x^2 = 5$

$\log_3 x^{5/2} = 5 \qquad x^{5/2} = 3^5$

$x = 3^2 = \boxed{9}$

b) $D: 9^{2x-3} > 0 \quad \underline{\underline{x \in \mathbb{R}}}$

$(2x-3) \log_2 9 = 3^{\log_3 2}$

$2(2x-3) = 2$

$2x - 3 = 1$

$\boxed{x = 2}$

c) $\left.\begin{array}{c} 3-x > 0 \\ 4-x > 0 \end{array}\right\}$  $\underline{\underline{3 \geq x}}$

$\dfrac{(3-x)(4-x)}{2^2} = 3$

$12 - 7x + x^2 = 12$

$x = \cancel{7} \text{ or } \boxed{0}$