Chapter 3

④ b) $\log_9 \log_4 64 = \log_9 \left(\log_2 2^6\right) = \log_9 3 = \boxed{1/2}$

⑤ b) $\log_8 \log_{14} 196 - \log_7 \sqrt{7} = \log_8 2 - 1/2 = 1/3 - 1/2 = \boxed{1/6}$

⑥ d) $\log_9 (x^2 - 2x) = \log_9 (2x+12)$

D: $x^2 - 2x > 0$ $2x+12 > 0$

$x(x-2) > 0$ $x+6 > 0$

$\boxed{-6 < x < 0 \text{ OR } 2 < x}$

$x^2 - 2x = 2x + 12$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$\boxed{x = -2, \; 6}$

c) $\log_4 (x^{-1} - 2) = 1$

D: $x^{-1} - 2 > 0$, $x \neq 0$

$x^{-1} > 2$

$x < 1/2$

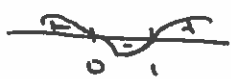$\boxed{x < 0 \text{ OR } 0 < x < 1/2}$

$4^1 = x^{-1} - 2$

$4 + 2 = 1/x$

$\boxed{x = 1/6}$

f) $\log_{1/2}(x^2 - x) = -1$ $\frac{1}{2}^{-1} = x^2 - x$

D: $x^2 - x > 0$

$x(x-1) > 0$

$\boxed{x < 0 \text{ OR } 1 < x}$

$2 = x^2 - x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2 \text{ or } -1}$

⑦ a) $4^x = 64$ $x = \log_4 64 = \boxed{3}$ b) $3^x = 1/9$ $\log_3 1/9 = \boxed{-2}$

c) $25^{-x} = 1/5$ $-x = \log_{25} 1/5 = -1/2$ $\boxed{x = 1/2}$ d) $.5^x = 1/64$ $x = \log_{1/2} 1/64 = \boxed{6}$

e) $\sqrt[3]{128} = 4^{2x}$ $2x = \log_4 \sqrt[3]{128} = \log_{2^2} 2^{7/3} = 7/6$ $\boxed{x = 7/12}$

f) $10^{x^2 + x - 2} = 1$ $\log_{10} 1 = x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $\boxed{x = 1, -2}$

g) $3^{x^2 - 4x - 1/2} = 81\sqrt{3}$ $x^2 - 4x - 1/2 = \log_3 81\sqrt{3} = 4.5$ $x^2 - 4x - 5 = 0$ $(x-5)(x+1) = 0$ $\boxed{x = -1, 5}$

1) $\log_2 x = 3$ $x = 2^3 = \boxed{8}$ 2) $\log_3 x = -1$ $x = 3^{-1} = \boxed{1/3}$ 3) $\log_5 (2x) = 1$ $2x = 5^1$ $\boxed{x = 2}$

4) $\log_7 x = 0$ $x = 7^0 = \boxed{1}$ 5) $\log_2 (-x) = -3$ $-x = 2^{-3}$ $x = \boxed{-1/8}$

⑦ 6) $\log_4 (x-1)^2 = 0$ $(x-1)^2 = 4^0$ $x^2 - 2x+1 = 1$ $x(x-2) = 0$ $\boxed{x = 0, 2}$

(both satisfy domain)

7) $\log_2 \log_3 x = 1$ $\log_3 x = 2^1$ $\mathbf{3}$ $x = 3^2 = \boxed{9}$

⑧ a) $\boxed{0}$ b) $\boxed{1}$ c) $\boxed{2}$ d) $\boxed{4}$ e) $\boxed{-1}$ f) $\boxed{-1\frac{1}{2}}$ g) $\boxed{3\frac{1}{2}}$ h) $\boxed{\frac{2}{3}}$

⑨ a) $3^{\frac{1}{2}\log_3 5} = \left(3^{\log_3 5}\right)^{\frac{1}{2}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$ b) $2^{-\log_2 3} = \left(2^{\log_2 3}\right)^{-1} = 3^{-1} = \boxed{\frac{1}{3}}$

⑩ a) $\left(\frac{1}{2}\right)^{1+2\log_2 3} = \left(2^{-1}\right)^{1+2\log_2 3} = 2^{(-1-2\log_2 3)} = 2^{-1} \cdot 2^{-2\log_2 3} =$

$= \frac{1}{2} \cdot \left(2^{\log_2 3}\right)^{-2} = \frac{1}{2} \cdot 3^{-2} = \frac{1}{2} \cdot \frac{1}{9} = \boxed{\frac{1}{18}}$

b) $3^{2\log_3 2 + \log_3 5} = \left(3^{\log_3 2}\right)^2 \cdot 3^{\log_3 5} = 2^2 \cdot 5 = \boxed{20}$

c) $\log_2 \log_2 4 = \log_2 2 = \boxed{1}$

⑪ a) $\log_x 9 = \frac{2}{3}$ D: $\boxed{x>0, x\neq 1}$ $x^{2/3} = 9$ $x^{1/3} = 3$ $\boxed{x = 27}$

b) $x^2 + 2^{\log_2 x} = 12$ D: $\boxed{x>0}$ $x^2 + x = 12$ $x^2 + x - 12 = 0$ $(x+4)(x-3) = 0$ Domain $\boxed{x = \cancel{-4}, 3}$

c) $x^2 + 3^{\log_3 x} = 20$ D: $\boxed{x>0}$ $x^2 + x = 20$ $x^2 + x - 20 = 0$ $(x+5)(x-4) = 0$ $\boxed{x = \cancel{-5}, 4}$

⑫ a) $\log_3 \log_3 \sqrt[3]{27} = \log_3 \log_3 3 = \log_3 1 = \boxed{0}$

b) $(\sqrt{3})^{-\log_3 2} = (3^{1/2})^{-\log_3 2} = \left(3^{\log_3 2}\right)^{-1/2} = 2^{-1/2} = \boxed{\frac{1}{\sqrt{2}}}$

c) $\log_4 \log_2 \log_3 81 = \log_4 \log_2 4 = \log_4 2 = \boxed{\frac{1}{2}}$

⑬ a) $\log_5 (x^2+1) = 1$ D: $x^2+1 > 0 \to \boxed{x \in \mathbb{R}}$

$x^2 + 1 = 5$ $x^2 - 4 = 0$ $\boxed{x = 2, -2}$

b) $\log_3 (\sqrt{x}+1) = 1$ D: $\sqrt{x}+1 > 0, \boxed{x>0}$

$\sqrt{x}+1 = 3$ $\sqrt{x} = 2$ $\boxed{x = 4}$

d) $\log(2x^2+3x) = \log(6x+2)$

D: $2x^2+3x>0$ $6x+2>0$

$2(x+1.5)x > 0$ $6(x+\frac{1}{3})>0$

$x<-1.5, 0<x$ $\cap$ $-\frac{1}{3}<x$

$\boxed{x>0}$

$2x^2+3x = 6x+2$

$2x^2-3x-2 = 0$

$(2x+1)(x-2)=0$

$\boxed{x = -\frac{1}{2}, 2}$

c) $\log(4x+5) - \log(5x+2) = 0$

D: $4x+5>0 \to 4x>-5 \to x>-\frac{5}{4}$
$5x+2>0 \to 5x>-2 \to x>-\frac{2}{5}$

$\to x>-\frac{2}{5}$

$4x+5=5x+2$

$\boxed{3=x}$

e) D: $x^2+2>0$ $x^2 > -2$ true for $\boxed{x \in \mathbb{R}}$

$x^2 + 2 = 6$ $x^2 = 4$ $\boxed{x = \frac{1}{2}}$

f) D: $x^2-3x>0$ $\boxed{x<0 \text{ or } 3<x}$

$\left(\frac{1}{10}\right)^{-1} = x^2 - 3x$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2)=0$

$\boxed{x = -2, 5}$