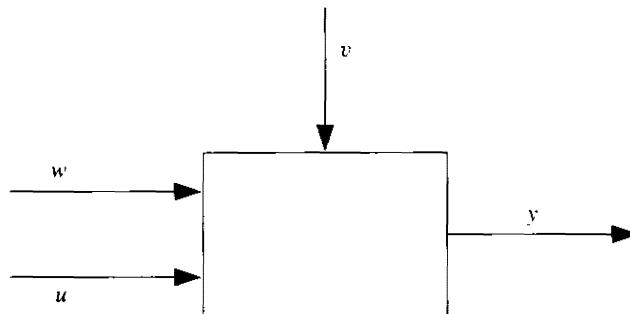


# INTRODUCTION

Inferring models from observations and studying their properties is really what science is about. The models (“hypotheses,” “laws of nature,” “paradigms,” etc.) may be of more or less formal character, but they have the basic feature that they attempt to link observations together into some pattern. System identification deals with the problem of building mathematical models of dynamical systems based on observed data from the system. The subject is thus part of basic scientific methodology, and since dynamical systems are abundant in our environment, the techniques of system identification have a wide application area. This book aims at giving an understanding of available system identification methods, their rationale, properties, and use.

## 1.1 DYNAMIC SYSTEMS

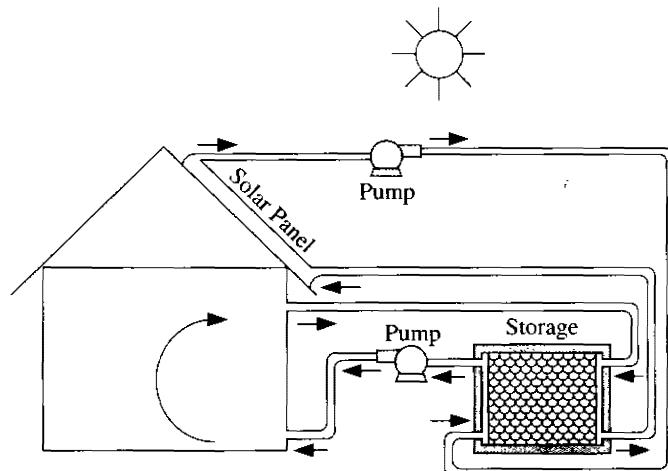
In loose terms a *system* is an object in which variables of different kinds interact and produce observable signals. The observable signals that are of interest to us are usually called *outputs*. The system is also affected by external stimuli. External signals that can be manipulated by the observer are called *inputs*. Others are called *disturbances* and can be divided into those that are directly measured and those that are only observed through their influence on the output. The distinction between inputs and measured disturbances is often less important for the modeling process. See Figure 1.1. Clearly the notion of a system is a broad concept, and it is not surprising that it plays an important role in modern science. Many problems in various fields are solved in a system-oriented framework. Instead of attempting a formal definition of the system concept, we shall illustrate it by a few examples.



**Figure 1.1** A system with output  $y$ , input  $u$ , measured disturbance  $w$ , and unmeasured disturbance  $v$ .

### Example 1.1 A Solar-Heated House

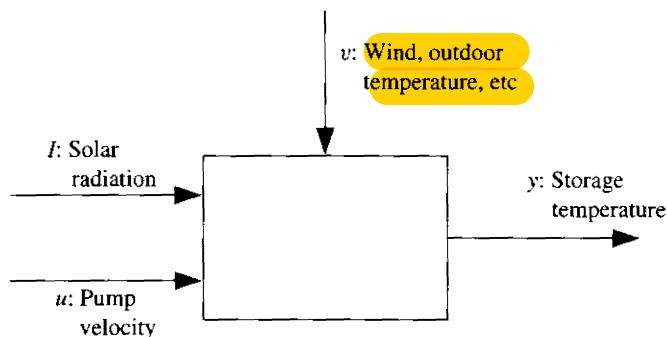
Consider the solar-heated house depicted in Figure 1.2. The system operates in such a way that the sun heats the air in the solar panel. The air is then pumped into a heat storage, which is a box filled with pebbles. The stored energy can later be transferred to the house. We are interested in how solar radiation and pump velocity affect the temperature in the heat storage. This system is symbolically depicted in Figure 1.3. Figure 1.4 shows a record of observed data over a 50-hour period. The variables were sampled every 10 minutes.  $\square$



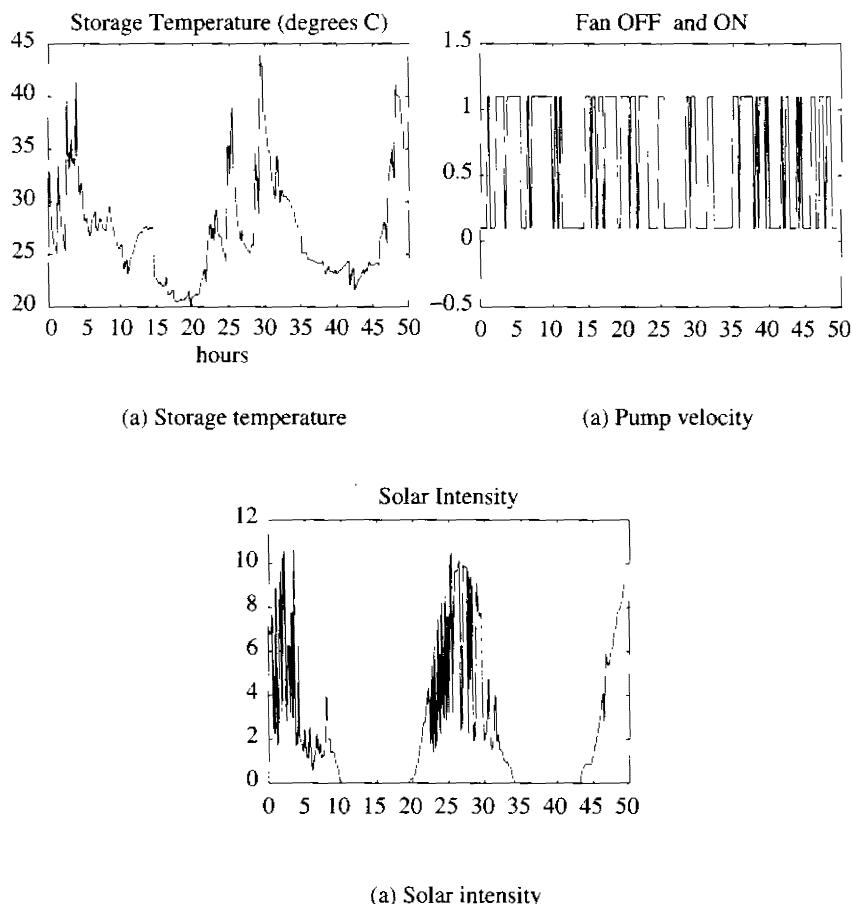
**Figure 1.2** A solar-heated house.

### Example 1.2 A Military Aircraft

For the development of an aircraft, a substantial amount of work is allocated to construct a mathematical model of its dynamic behavior. This is required both for the simulators, for the synthesis of autopilots, and for the analysis of its properties. Substantial physical insight is utilized, as well as wind tunnel experiments, in the course of this work, and a most important source of information comes from the test flights.



**Figure 1.3** The solar-heated house system:  $u$ : input;  $I$ : measured disturbance;  $y$ : output;  $v$ : unmeasured disturbances.

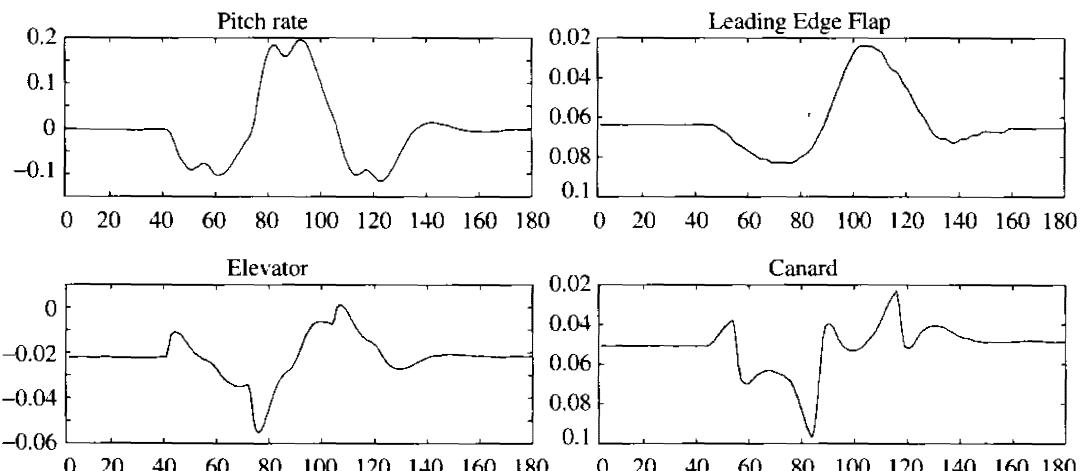


**Figure 1.4** Storage temperature  $y$ , pump velocity  $u$ , and solar intensity  $I$  over a 50-hour period. Sampling interval: 10 minutes.



**Figure 1.5** The Swedish fighter aircraft JAS-Gripen.

Figure 1.5 shows the Swedish aircraft JAS-Gripen, developed by SAAB AB, Sweden. and Figure 1.6 shows some results from test flights. Such data can be used to build a model of the pitch channel, i.e., how the pitch rate is affected by the three control signals: elevator, canard, and leading edge flap. The elevator in this case corresponds to aileron combinations at the back of the wings, while separate action is achieved from the ailerons at the leading edge (the front of the wings). The canards are a separate set of rudders at the front of the wings.

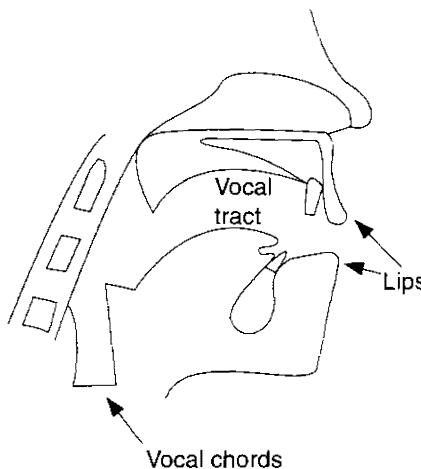


**Figure 1.6** Results from test flights of the Swedish aircraft JAS-Gripen, developed by SAAB AB, Sweden. The pitch rate and the elevator, leading edge flap, and canard angles are shown.

The aircraft is unstable in the pitch channel at this flight condition, so clearly the experiment was carried out under closed loop control. In Section 17.3 we will return to this example and identify models based on the measured data.  $\square$

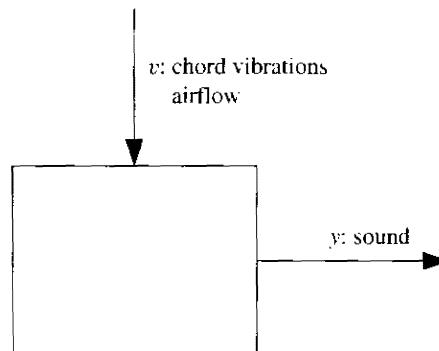
### Example 1.3 Speech

The sound of the human voice is generated by the vibration of the vocal chords or, in the case of unvoiced sounds, the air stream from the throat, and formed by the shape of the vocal tract. See Figure 1.7. The output of this system is sound vibration (i.e., the air pressure), but the external stimuli are not measurable. See Figure 1.8. Data from this system are shown in Figure 1.9.  $\square$

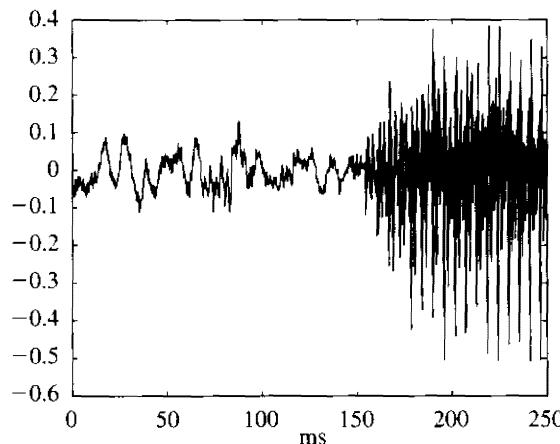


**Figure 1.7** Speech generation.

The systems in these examples are all *dynamic*, which means that the current output value depends not only on the current external stimuli but also on their earlier values. Outputs of dynamical systems whose external stimuli are not observed (such as in Example 1.3) are often called *time series*. This term is especially common in economic applications. Clearly, the list of examples of dynamical systems can be very long, encompassing many fields of science.



**Figure 1.8** The speech system:  $y$ : output;  $v$ : unmeasured disturbance.



**Figure 1.9** The speech signal (air pressure). Data sampled every 0.125 ms. (8 kHz sampling rate).

## 1.2 MODELS

### Model Types and Their Use

When we interact with a system, we need some concept of how its variables relate to each other. With a broad definition, we shall call such an assumed relationship among observed signals a *model* of the system. Clearly, models may come in various shapes and be phrased with varying degrees of mathematical formalism. The intended use will determine the degree of sophistication that is required to make the model purposeful.

No doubt, in daily life many systems are dealt with using *mental models*, which do not involve any mathematical formalization at all. To drive a car, for example, requires the knowledge that turning the steering wheel to the left induces a left turn, together with subtle information built up in the muscle memory. The importance and degree of sophistication of the latter should of course not be underestimated.

For certain systems it is appropriate to describe their properties using numerical tables and/or plots. We shall call such descriptions *graphical models*. Linear systems, for example, can be uniquely described by their impulse or step responses or by their frequency functions. Graphical representation of these are widely used for various design purposes. The nonlinear characteristics of, say, a valve are also well suited to be described by a graphical model.

For more advanced applications, it may be necessary to use models that describe the relationships among the system variables in terms of mathematical expressions like difference or differential equations. We shall call such models *mathematical (or analytical) models*. Mathematical models may be further characterized by a number of adjectives (time continuous or time discrete, lumped or distributed, deterministic or stochastic, linear or nonlinear, etc.) signifying the type of difference or differential equation used. The use of mathematical models is inherent in all fields of engineering and physics. In fact, a major part of the engineering field deals with how to make good

designs based on mathematical models. They are also instrumental for simulation and forecasting (**prediction**), which is extensively used in all fields, including nontechnical areas like economy, ecology and biology.

The model used in a computer simulation of a system is a program. For complex systems, this program may be built up by many interconnected subroutines and lookup tables, and it may not be feasible to summarize it analytically as a mathematical model. We use the term *software model* for such computerized descriptions. They have come to play an increasingly important role in decision making for complicated systems.

### Building Models

Basically, a model has to be constructed from observed data. The mental model of car-steering dynamics, for example, is developed through driving experience. Graphical models are made up from certain measurements. Mathematical models may be developed along two routes (or a combination of them). One route is to split up the system, figuratively speaking, into subsystems, whose properties are well understood from previous experience. This basically means that we rely on earlier empirical work. These subsystems are then joined mathematically and a model of the whole system is obtained. This route is known as *modeling* and does not necessarily involve any experimentation on the actual system. The procedure of modeling is quite application dependent and often has its roots in tradition and specific techniques in the application area in question. Basic techniques typically involve structuring of the process into block diagrams with blocks consisting of simple elements. The reconstruction of the system from these simple blocks is now increasingly being done by computer, resulting in a software model rather than a mathematical model.

The other route to mathematical as well as graphical models is directly based on experimentation. Input and output signals from the system, such as those in Figures 1.4, 1.6, and 1.9, are recorded and subjected to data analysis in order to infer a model. This route is *system identification*.

### The Fiction of a True System

The real-life actual system is an object of a different kind than our mathematical models. In a sense, there is an impenetrable but transparent screen between our world of mathematical descriptions and the real world. We can look through this window and compare certain aspects of the physical system with its mathematical description, but we can never establish any exact connection between them. The question of nature's susceptibility to mathematical description has some deep philosophical aspects, and in practical terms we have to take a more pragmatic view of models. Our acceptance of models should thus be guided by "usefulness" rather than "truth." Nevertheless, we shall occasionally use a concept of "the true system," defined in terms of a mathematical description. Such a fiction is helpful for devising identification methods and understanding their properties. In such contexts we assume that the observed data have been generated according to some well-defined mathematical rules, which of course is an idealization.

### 1.3 AN ARCHETYPICAL PROBLEM—ARX MODELS AND THE LINEAR LEAST SQUARES METHOD

In this section we shall consider a specific estimation problem that contains most of the central issues that this book deals with. The section will thus be a preview of the book. In the following section we shall comment on the general nature of the issues raised here and how they relate to the organization of the book.

#### The Model

We shall generally denote the system's input and output at time  $t$  by  $u(t)$  and  $y(t)$ , respectively. Perhaps the most basic relationship between the input and output is the *linear difference equation*:

$$y(t) + a_1y(t-1) + \dots + a_ny(t-n) = b_1u(t-1) + \dots + b_mu(t-m) \quad (1.1)$$

We have chosen to represent the system in *discrete time*, primarily since observed data are always collected by sampling. It is thus more straightforward to relate observed data to discrete time models. In (1.1) we assume the *sampling interval* to be one time unit. This is not essential, but makes notation easier.

A pragmatic and useful way to see (1.1) is to view it as a way of *determining the next output value given previous observations*:

$$y(t) = -a_1y(t-1) - \dots - a_ny(t-n) + b_1u(t-1) + \dots + b_mu(t-m) \quad (1.2)$$

For more compact notation we introduce the vectors:

$$\theta = [a_1 \ \dots \ a_n \ \ b_1 \ \dots \ b_m]^T \quad (1.3)$$

$$\varphi(t) = [-y(t-1) \ \dots \ -y(t-n) \ \ u(t-1) \ \ \dots \ \ u(t-m)]^T \quad (1.4)$$

With these, (1.2) can be rewritten as

$$y(t) = \varphi^T(t)\theta$$

To emphasize that the calculation of  $y(t)$  from past data (1.2) indeed depends on the parameters in  $\theta$ , we shall rather call this calculated value  $\hat{y}(t|\theta)$  and write

$$\hat{y}(t|\theta) = \varphi^T(t)\theta \quad (1.5)$$

#### The Least Squares Method

Now suppose for a given system that we do not know the values of the parameters in  $\theta$ , but that we have recorded inputs and outputs over a time interval  $1 \leq t \leq N$ :

$$Z^N = \{u(1), y(1), \dots, u(N), y(N)\} \quad (1.6)$$

An obvious approach is then to select  $\theta$  in (1.1) through (1.5) so as to fit the calculated values  $\hat{y}(t|\theta)$  as well as possible to the measured outputs by the least squares method:

$$\min_{\theta} V_N(\theta, Z^N) \quad (1.7)$$

where

$$V_N(\theta, Z^N) = \frac{1}{N} \sum_{t=1}^N (y(t) - \hat{y}(t|\theta))^2 = \frac{1}{N} \sum_{t=1}^N (y(t) - \varphi^T(t)\theta)^2 \quad (1.8)$$

We shall denote the value of  $\theta$  that minimizes (1.7) by  $\hat{\theta}_N$ :

$$\hat{\theta}_N = \arg \min_{\theta} V_N(\theta, Z^N) \quad (1.9)$$

("arg min" means the minimizing argument, i.e., that value of  $\theta$  which minimizes  $V_N$ .)

Since  $V_N$  is quadratic in  $\theta$ , we can find the minimum value easily by setting the derivative to zero:

$$0 = \frac{d}{d\theta} V_N(\theta, Z^N) = \frac{2}{N} \sum_{t=1}^N \varphi(t)(y(t) - \varphi^T(t)\theta)$$

which gives

$$\sum_{t=1}^N \varphi(t)y(t) = \sum_{t=1}^N \varphi(t)\varphi^T(t)\theta \quad (1.10)$$

or

$$\hat{\theta}_N = \left[ \sum_{t=1}^N \varphi(t)\varphi^T(t) \right]^{-1} \sum_{t=1}^N \varphi(t)y(t) \quad (1.11)$$

Once the vectors  $\varphi(t)$  are defined, the solution can easily be found by modern numerical software, such as MATLAB.

#### Example 1.4 First Order Difference Equation

Consider the simple model:

$$y(t) + ay(t-1) = bu(t-1).$$

This gives us the estimate according to (1.4), (1.3) and (1.11):

$$\begin{bmatrix} \hat{a}_N \\ \hat{b}_N \end{bmatrix} = \begin{bmatrix} \sum y^2(t-1) & -\sum y(t-1)u(t-1) \\ -\sum y(t-1)u(t-1) & \sum u^2(t-1) \end{bmatrix}^{-1} \times \begin{bmatrix} -\sum y(t)y(t-1) \\ \sum y(t)u(t-1) \end{bmatrix}$$

All sums are from  $t = 1$  to  $t = N$ . A typical convention is to take values outside the measured range to be zero. In this case we would thus take  $y(0) = 0$ .  $\square$

The simple model (1.1) and the well known least squares method (1.11) form the archetype of System Identification. Not only that—they also give the most commonly used parametric identification method and are much more versatile than perhaps perceived at first sight. In particular one should realize that (1.1) can directly be extended to several different inputs (this just calls for a redefinition of  $\varphi(t)$  in (1.4)) and that the inputs and outputs do not have to be the raw measurements. On the contrary—it is often most important to think over the physics of the application and come up with suitable inputs and outputs for (1.1), formed from the actual measurements.

### Example 1.5 An Immersion Heater

Consider a process consisting of an immersion heater immersed in a cooling liquid. We measure:

- $v(t)$ : The voltage applied to the heater
- $r(t)$ : The temperature of the liquid
- $y(t)$ : The temperature of the heater coil surface

Suppose we need a model for how  $y(t)$  depends on  $r(t)$  and  $v(t)$ . Some simple considerations based on common sense and high school physics (“Semi-physical modeling”) reveal the following:

- The change in temperature of the heater coil over one sample is proportional to the electrical power in it (the inflow power) minus the heat loss to the liquid
- The electrical power is proportional to  $v^2(t)$
- The heat loss is proportional to  $y(t) - r(t)$

This suggests the model:

$$y(t) = y(t-1) + \alpha v^2(t-1) - \beta(y(t-1) - r(t-1))$$

which fits into the form

$$y(t) + \theta_1 y(t-1) = \theta_2 v^2(t-1) + \theta_3 r(t-1)$$

This is a two input ( $v^2$  and  $r$ ) and one output model, and corresponds to choosing

$$\varphi(t) = [-y(t-1) \quad v^2(t-1) \quad r(t-1)]^T$$

in (1.5). We could also enforce the suggestion from the physics that  $\theta_1 + \theta_3 = -1$  by another choice of variables.  $\square$

### Linear Regressions

Model structures such as (1.5) that are linear in  $\theta$  are known in statistics as *linear regressions*. The vector  $\varphi(t)$  is called the *regression vector*, and its components are the *regressors*. “Regress” here alludes to the fact that we try to calculate (or describe)  $y(t)$  by “going back” to  $\varphi(t)$ . Models such as (1.1) where the regression vector— $\varphi(t)$ —contains old values of the variable to be explained— $y(t)$ —are then

partly *auto-regressions*. For that reason the model structure (1.1) has the standard name ARX-model: *Auto-Regression with eXtra inputs (or eXogeneous variables)*.

There is a rich statistical literature on the properties of the estimate  $\hat{\theta}_N$  under varying assumptions. We shall deal with such questions in a much more general setting in Chapters 7 to 9, and the following section can be seen as a preview of the issues dealt with there. See also Appendix II.

### Model Quality and Experiment Design

Let us consider the simplest special case, that of a *Finite Impulse Response (FIR)* model. That is obtained from (1.1) by taking  $n = 0$ :

$$y(t) = b_1 u(t-1) + \dots + b_m u(t-m) \quad (1.12)$$

Suppose that the observed data really have been generated by a similar mechanism

$$y(t) = b_1^0 u(t-1) + \dots + b_m^0 u(t-m) + e(t) \quad (1.13)$$

where  $e(t)$  is a white noise sequence with variance  $\lambda$ , but otherwise unknown. (That is,  $e(t)$  can be described as a sequence of independent random variables with zero mean values and variances  $\lambda$ .) Analogous to (1.5), we can write this as

$$y(t) = \varphi^T(t)\theta_0 + e(t) \quad (1.14)$$

The input sequence  $u(t)$ ,  $t = 1, 2, \dots$  is taken as a given, deterministic, sequence of numbers. We can now replace  $y(t)$  in (1.12) by the above expression, and obtain

$$\begin{aligned} \hat{\theta}_N &= R(N)^{-1} \left[ \sum_{t=1}^N \varphi(t)\varphi^T(t)\theta_0 + \sum_{t=1}^N \varphi(t)e(t) \right] \\ R(N) &= \sum_{t=1}^N \varphi(t)\varphi^T(t) \end{aligned}$$

or

$$\tilde{\theta}_N = \hat{\theta}_N - \theta_0 = R(N)^{-1} \sum_{t=1}^N \varphi(t)e(t) \quad (1.15)$$

Since  $u$  and hence  $\varphi$  are given, deterministic variables,  $R(N)$  is a deterministic matrix. If  $E$  denotes *mathematical expectation*, we therefore have

$$E\tilde{\theta}_N = E \left[ R(N)^{-1} \sum_{t=1}^N \varphi(t)e(t) \right] = R(N)^{-1} \sum_{t=1}^N \varphi(t)Ee(t) = 0$$

since  $e(t)$  has zero mean. The estimate is consequently *unbiased*.

We can also form the expectation of  $\tilde{\theta}_N \tilde{\theta}_N^T$ , i.e., the covariance matrix of the parameter error

$$\begin{aligned}
 P_N &= E\tilde{\theta}_N \tilde{\theta}_N^T = ER(N)^{-1} \sum_{t,s=1}^N \varphi(t)e(t)e(s)\varphi^T(s)R(N)^{-1} \\
 &= R(N)^{-1} \sum_{t,s=1}^N \varphi(t)\varphi^T(s)R(N)^{-1} Ee(t)e(s) \\
 &= R(N)^{-1} \sum_{t,s=1}^N \varphi(t)\varphi^T(s)R(N)^{-1}\lambda\delta(t-s) \\
 &= R(N)^{-1}\lambda \sum_{t=1}^N \varphi(t)\varphi^T(t)R(N)^{-1} = \lambda R(N)^{-1}
 \end{aligned} \tag{1.16}$$

where we used the fact that  $e$  is a sequence of independent variables so that  $Ee(t)e(s) = \lambda\delta(t-s)$ , with  $\delta(0) = 1$  and  $\delta(\tau) = 0$  if  $\tau \neq 0$ .

We have thus computed the covariance matrix of the estimate  $\hat{\theta}_N$ . It is determined entirely by the input properties  $R(N)$  and the noise level  $\lambda$ . Moreover, define

$$\bar{R} = \lim_{N \rightarrow \infty} \frac{1}{N} R(N) \tag{1.17}$$

This will correspond to the covariance matrix of the input, i.e., the  $i - j$ -element of  $\bar{R}$  is

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N u(t-i)u(t-j)$$

If the matrix  $\bar{R}$  is non-singular, we find that the covariance matrix of the parameter estimate is approximately given by

$$P_N = \frac{\lambda}{N} \bar{R}^{-1} \tag{1.18}$$

and the approximation improves as  $N \rightarrow \infty$ . A number of things follow from this.

- The covariance decays like  $1/N$ , so the parameters approach the limiting value at the rate  $1/\sqrt{N}$ .
- The covariance is proportional to the Noise-To-Signal ratio. That is, it is proportional to the noise variance  $\lambda$  and inversely proportional to the input power.
- The covariance does not depend on the input's or noise's signal shapes, only on their variance/covariance properties.
- Experiment design, i.e., the selection of the input  $u$ , aims at making the matrix  $\bar{R}^{-1}$  “as small as possible.” Note that the same  $\bar{R}$  can be obtained for many different signals  $u$ .

## 1.4 THE SYSTEM IDENTIFICATION PROCEDURE

### Three Basic Entities

As we saw in the previous section, the construction of a model from data involves three basic entities:

1. A data set, like  $Z^N$  in (1.6).
2. A set of candidate models: a *Model Structure*, like the set (1.1) or (1.5).
3. A rule by which candidate models can be assessed using the data, like the Least Squares selection rule (1.9).

Let us comment on each of these:

1. *The data record.* The input-output data are sometimes recorded during a specifically designed identification experiment, where the user may determine which signals to measure and when to measure them and may also choose the input signals. The objective with *experiment design* is thus to make these choices so that the data become maximally informative, subject to constraints that may be at hand. Making the matrix  $\bar{R}^{-1}$  in (1.18) small is a typical example of this, and we shall in Chapter 13 treat this question in more detail. In other cases the user may not have the possibility to affect the experiment, but must use data from the normal operation of the system.
2. *The set of models or the model structure.* A set of candidate models is obtained by specifying within which collection of models we are going to look for a suitable one. This is no doubt the most important and, at the same time, the most difficult choice of the system identification procedure. It is here that *a priori* knowledge and engineering intuition and insight have to be combined with formal properties of models. Sometimes the model set is obtained after careful *modeling*. Then a model with some unknown physical parameters is constructed from basic physical laws and other well-established relationships. In other cases standard linear models may be employed, without reference to the physical background. Such a model set, whose parameters are basically viewed as vehicles for adjusting the fit to the data and do not reflect physical considerations in the system, is called a *black box*. Model sets with adjustable parameters with physical interpretation may, accordingly, be called *gray boxes*. Generally speaking, a model structure is a parameterized mapping from past inputs and outputs  $Z^{t-1}$  (cf (1.6)) to the space of the model outputs:

$$\hat{y}(t|\theta) = g(\theta, Z^{t-1}) \quad (1.19)$$

Here  $\theta$  is the finite dimensional vector used to parameterize the mapping. Chapters 4 and 5 describe common model structures.

3. *Determining the “best” model in the set, guided by the data.* This is the *identification method*. The assessment of model quality is typically based on how the models perform when they attempt to reproduce the measured data. The basic approaches to this will be dealt with independently of the model structure used. Chapter 7 treats this problem.

### Model Validation

After having settled on the preceding three choices, we have, at least implicitly, arrived at a particular model: the one in the set that best describes the data according to the chosen criterion. It then remains to test whether this model is “good enough,” that is, whether it is valid for its purpose. Such tests are known as *model validation*. They involve various procedures to assess how the model relates to observed data, to prior knowledge, and to its intended use. Deficient model behavior in these respects make us reject the model, while good performance will develop a certain confidence in it. A model can never be accepted as a final and true description of the system. Rather, it can at best be regarded as a good enough description of certain aspects that are of particular interest to us. Chapter 16 contains a discussion of model validation.

### The System Identification Loop

The system identification procedure has a natural logical flow: first collect data, then choose a model set, then pick the “best” model in this set. It is quite likely, though, that the model first obtained will not pass the model validation tests. We must then go back and revise the various steps of the procedure.

The model may be deficient for a variety of reasons:

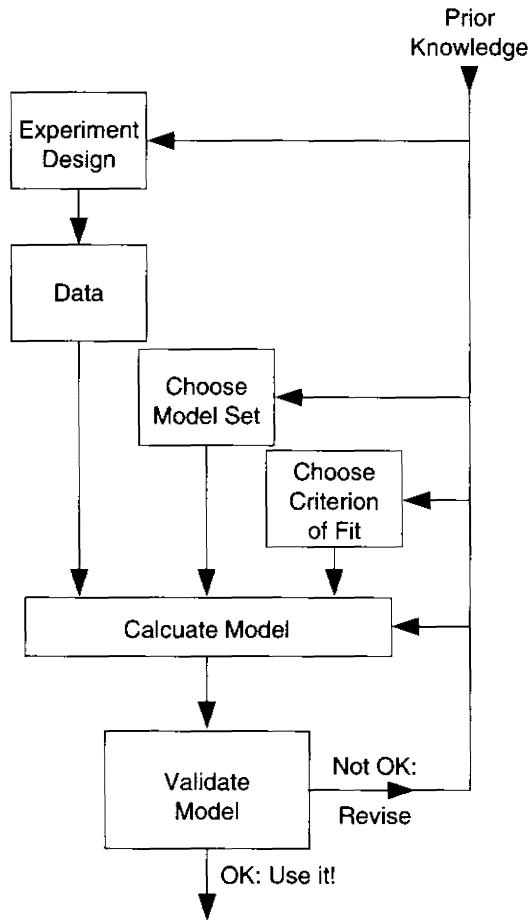
- The numerical procedure failed to find the best model according to our criterion.
- The criterion was not well chosen.
- The model set was not appropriate, in that it did not contain any “good enough” description of the system.
- The data set was not informative enough to provide guidance in selecting good models.

The major part of an identification application in fact consists of addressing these problems, in particular the third one, in an iterative manner, guided by prior information and the outcomes of previous attempts. See Figure 1.10. Interactive software obviously is an important tool for handling the iterative character of this problem.

## 1.5 ORGANIZATION OF THE BOOK

To master the loop of Figure 1.10, the user has to be familiar with a number of things.

1. Available techniques of identification and their rationale, as well as typical choices of model sets.
2. The properties of the identified model and their dependence on the basic items: data, model set, and identification criterion.
3. Numerical schemes for computing the estimate.
4. How to make intelligent choices of experiment design, model set, and identification criterion, guided by prior information as well as by observed data.



**Figure 1.10** The system identification loop.

In fact, a user of system identification may find that he or she is primarily a user of an interactive software package. Items 1 and 3 are then part of the package and the important thing is to have a good understanding of item 2 so that task 4 can be successfully completed. This is what we mean by “Theory for the User” and this is where the present book has its focus.

The idea behind the book’s organization is to present the list of common and useful model sets in Chapters 4 and 5. Available techniques are presented in Chapters 6 and 7, and the analysis follows in Chapters 8 and 9. Numerical techniques for off-line and on-line applications are described in Chapters 10 and 11. Task 4, the user’s choices, is discussed primarily in Chapters 13 through 16, after some preliminaries in Chapter 12. In addition, Chapters 2 and 3 give the formal setup of the book, and Chapter 17 describes and assesses system identification as a tool for practical problems.

Figure 1.11 illustrates the book’s structure in relation to the loop of system identification.

Available Choices	User's Choices	Analysis
	Chapter 13	Experiment design
	Chapter 14	Data
Chapters 4 and 5	Chapter 16	Choose model set
Chapters 6 and 7	Chapter 15	Choose criterion of fit
Chapters 10 and 11		Calculate model
	Chapter 16	Validate model

Figure 1.11 Organization of the book.

### About the Framework

The system identification framework we set up here is fairly general. It does not confine us to linear models or quadratic criteria or to assuming that the system itself can be described within the model set. Indeed, this is one of the points that should be stressed about our framework. Nevertheless, we often give proofs and explicit expressions only for certain special cases, like single-input, single-output systems and quadratic criteria. The purpose is of course to enhance the underlying basic ideas and not conceal them behind technical details. References are usually provided for more general treatments.

Parameter estimation and identification are usually described within a probabilistic framework. Here we basically employ such a framework. However, we also try to retain a pragmatic viewpoint that is independent of probabilistic interpretations. That is, the methods we describe and the recommendations we put forward should make sense even without the probabilistic framework that may motivate them as "optimal solutions." The probabilistic and statistical environments of the book are described in Appendices I and II, respectively. These appendices may be read prior to the other chapters or consulted occasionally when required. In any case, the book does not lean heavily on the background provided there.

### 1.6 BIBLIOGRAPHY

The literature on the system identification problem and its ramifications is extensive. Among general textbooks on the subject we may mention Box and Jenkins (1970), Eykhoff (1974), Spriet and Vansteenkiste (1982), Ljung and Glad (1994a), and Johansson (1993) for treatments covering several practical issues, while Goodwin and Payne (1977), Davis and Vinter (1985), Hannan and Deistler (1988), Caines

(1988), Chen and Guo (1991), and Söderström and Stoica (1989) give more theoretically oriented presentations. Kashyap and Rao (1976), Rissanen (1989) and Bohlin (1991) emphasize the role of model validation and model selection in their treatment of system identification, while Söderström and Stoica (1983) focuses on instrumental-variable methods. A treatment based on frequency domain data is given in Schoukens and Pintelon (1991), and the so-called subspace approach is thoroughly discussed in Van Overschee and DeMoor (1996). Texts that concentrate on recursive identification techniques include Ljung and Söderström (1983), Solo and Kong (1995), Haykin (1986), Widrow and Stearns (1985), and Young (1984). Spectral analysis is closely related, and treated in many books like Marple (1987), Kay (1988) and Stoica and Moses (1997). Statistical treatments of time-series modeling such as Anderson (1971), Hannan (1970), Brillinger (1981), and Wei (1990) are most relevant also for the system identification problem. The so-called behavioral approach to modeling is introduced in Willems (1987).

Among edited collections of articles, we may refer to Mehra and Lainiotis (1976), Eykhoff (1981), Hannan, Krishnaiah, and Rao (1985), and Leondes (1987), as well as to the special journal issues Kailath, Mehra, and Mayne (1974), Isermann (1981), Eykhoff and Parks (1990), Kosut, Goodwin, and Polis (1992) and Söderström and Åström (1995). The proceedings from the IFAC (International Federation of Automatic Control) Symposia on Identification and System Parameter Estimation contain many articles on all aspects of the system identification problem. These symposia are held every three years, starting in Prague 1967.

Philosophical aspects on mathematical models of real-life objects are discussed, for example, in Popper (1934). Modeling from basic physical laws, rather than from data, is discussed in many books: see, for example, Wellstead (1979), Ljung and Glad (1994a), Frederick and Close (1978) and Cellier (1990) for engineering applications. Such treatments are important complements to the model set selection (see Section 1.3 and Chapter 16).

Many books discuss modeling and identification in various application areas. See, for example, Granger and Newbold (1977) or Malinvaud (1980) (econometrics), Godfrey (1983) (biology), Robinson and Treitel (1980), Mendel (1983) (geoscience), Dudley (1983) (electromagnetic wave theory), Markel and Gray (1976) (speech signals) and Beck and Van Straten (1983) (environmental systems). Rajbman (1976, 1981) has surveyed the Soviet literature.