# 01_linear_dimension_reductionV2

April 17, 2025

Download the necessary libraries.

Link to data - https://raw.githubusercontent.com/DataSlingers/clustRviz/master/data/authors.rda

Install these if necessary.

[48]:
```
# %pip install scikit-learn --quiet
# %pip install adjustText --quiet
# %pip install umap-learn --quiet
# %pip install wordcloud
```

[49]:
```
## Basics
import pandas as pd
import numpy as np
import os
import matplotlib.pyplot as plt
import seaborn as sns

## ML Packages
import umap
from sklearn.decomposition import FastICA, NMF, KernelPCA, PCA, TruncatedSVD
from sklearn.preprocessing import StandardScaler, LabelEncoder
from scipy.spatial.distance import pdist, squareform
from sklearn.manifold import SpectralEmbedding, TSNE
from sklearn.model_selection import train_test_split
from sklearn.manifold import MDS

## Msc
from adjustText import adjust_text
from itertools import combinations
from wordcloud import WordCloud
```

Load dataset.

[50]:
```
df = pd.read_csv('00_authors.csv').rename(columns = {'Unnamed: 0': 'Author'}).
  ↪drop(columns = 'BookID')
X = df.copy().drop(['Author'], axis=1)
authors = df['Author'].values
df
```

[50]:
```
          Author   a  all  also  an  and  any  are  as  at  ...  was  were  \
0         Austen  46   12     0   3   66    9    4  16  13  ...   40    11
1         Austen  35   10     0   7   44    4    3  18  16  ...   27    13
2         Austen  46    2     0   3   40    1   13  11   9  ...   24     6
3         Austen  40    7     0   4   64    3    3  20  13  ...   26    10
4         Austen  29    5     0   6   52    5   14  17   6  ...   23     5
..           ...  ..  ...   ...  ..  ...  ...  ...  ..  ..  ...  ...   ...
836  Shakespeare  32    4     0   6   33    0    7   8   4  ...    0     1
837  Shakespeare  16    5     0   5   49    1    6  10   3  ...    1     1
838  Shakespeare  22   15     0   3   48    0    9  10   2  ...    4     0
839  Shakespeare  25    4     0   8   59    3    6   7   3  ...    3     4
840  Shakespeare  26    4     0   2   62    0    4   7   4  ...    5     0

     what  when  which  who  will  with  would  your
0       7     5      6    8     4     9      1     0
1       5     7      7    3     5    14      8     0
2      10     4      6    4     5    15      3     9
3       3     6     10    5     3    22      4     3
4       8     4     13    2     4    21     10     0
..    ...   ...    ...  ...   ...   ...    ...   ...
836    13     2      3    3    11    17      5    10
837     6     5      6    0    11    20      2     7
838    16     2      2    0    12    15      1    10
839    11     2      2    2    22    23      4     5
840    13     2      5    3    11    19      0     3

[841 rows x 70 columns]
```

[51]:
```python
book_id = df['Author']
book_id.value_counts() # 4 different books ; w/ 317 - Austen, ..., 55 - Milton.
```

[51]:
```
Author
Austen         317
London         296
Shakespeare    173
Milton          55
Name: count, dtype: int64
```

- Unsupervised learning: drop columns ['Authors'] and determine patterns with words across chapters using unsupervised learning methods.
- We will later come back to these labels we dropped to validate our results.
- Note, a row represents a book chapter with each column representing the word counts of key words in that chapter.

# 1 Linear Methods

## 1.1 PCA

**PCA Theory**

Given a centered data matrix $X \in \mathbb{R}^{n \times p}$ with: - $n$: observations (chapters), - $p$: features (words),

PCA uses the SVD: $X = U \Sigma V^\top$

- $U\Sigma \to$ principal component scores (embedding of chapters),
- $V$ (or `pca.components_`) $\to$ principal axes (directions for words).

**No need to transpose** the data to get word embeddings.
Using `fit_transform(X)` for chapters, and `components_.T` for words — where both come from the same PCA fit.

Also note that when fitting a PCA in this dataset: **WE SHOULD NOT SCALE DATA SINCE IT IS WORD COUNT!!!**

### 1.1.1 Observations

[52]:
```
pca = PCA()
X_pca = pca.fit_transform(X)
word_loadings = pca.components_.T

pca_df = pd.DataFrame(X_pca)
cols = [f'PC{j+1}' for j in range(pca_df.shape[1])]
pca_df = pca_df.rename(columns = {i:cols[i] for i in range(pca_df.shape[1])})
pca_df
```

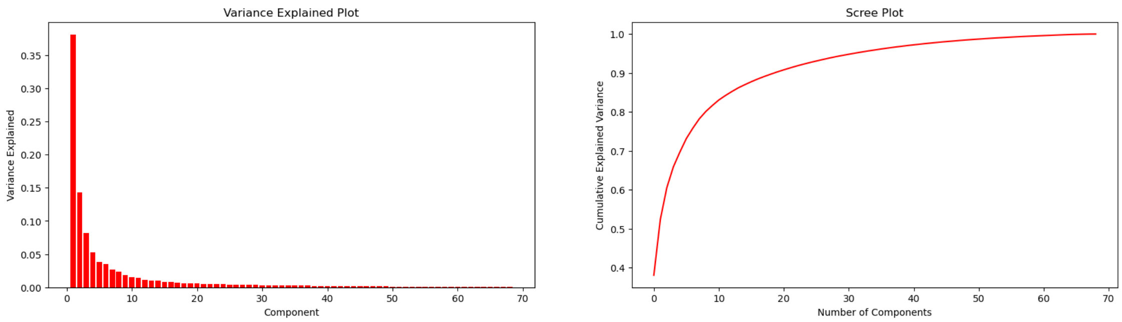
[52]:
```
           PC1        PC2        PC3        PC4        PC5        PC6  \
0    -2.265044  43.499301   5.196950  -2.333575  23.359407  22.309224
1    -2.604648  25.086417  -9.488717   7.748273  19.244916   1.052493
2   -33.199533   8.667765 -14.833418   3.971572  -6.913595   4.173856
3     8.098653  21.760546   6.962558   6.683136  15.262020   7.640936
4    10.031814   6.801164   0.035520  22.731646  10.465248  -5.248171
..         …          …          …          …          …          …
836 -64.400961 -28.132705 -21.267908   0.131106  -0.337083  12.068621
837 -58.313001 -23.417273   4.887866   2.462693  -1.275957  -2.603434
838 -47.898865 -31.938566  -7.364020  -9.133520  10.442357  14.851150
839 -39.844905 -29.936659   0.614003  -2.261258   5.007836  14.369278
840 -34.807687 -38.141056   6.289341  -2.114649   1.435612  13.845645

           PC7        PC8       PC9       PC10  …      PC60      PC61  \
0    13.585777  -0.891477 -2.178907   0.062718  … -2.043904 -3.175850
1     6.765152   3.650037 -0.746511  -1.753312  … -2.269775  0.991903
2     7.094662 -11.728066  1.002140  -4.270366  … -2.055041  0.864406
3    11.974431  -5.748289 -1.011280  -4.502115  …  0.590874  0.830834
4     7.489147  -6.171808 -9.885234 -14.616535  …  1.616164  2.706626
..         …          …         …          …  …         …         …
```

```
836  -5.848328   3.920184 -7.233151   0.239586  … -0.089364 -0.674634
837 -11.452006   4.745220  7.397146  -1.948260  … -0.264725 -1.237238
838 -10.872174  10.228351 -5.141056   3.876664  …  0.938558  1.771321
839  -3.165040   3.620264  1.479972   2.361646  … -0.011889  1.010582
840   3.620534   3.312984 -2.660270   0.521845  …  1.273284  4.486729

         PC62      PC63      PC64      PC65      PC66      PC67      PC68  \
0    1.578725 -2.575981  1.463553 -1.107755 -0.341962  2.639943  3.214014
1    0.013898  1.037391 -1.077602  0.044514 -1.312650  0.800142  0.183083
2   -2.179206  2.564626 -3.672920  0.798756 -0.664101 -1.319190 -0.493611
3   -1.969229  0.937902 -3.603491 -1.244561  1.283435 -0.912942 -0.271062
4   -0.708168 -1.272716 -0.278721 -1.525001  1.746847 -1.004067 -1.272581
..        …         …        …        …        …        …        …
836 -1.165301 -0.470178 -1.348415 -0.240363  0.167233  0.514576 -0.123753
837 -0.170168  1.335993  0.455795  0.942986 -0.266152 -2.256417 -0.182997
838  1.542836 -1.795202  1.242780  2.648138 -0.017259 -2.002290  0.787364
839  1.514307 -0.011713  1.135751  0.803847 -0.028491 -0.803976 -1.307802
840  3.058903  0.497595  0.257034 -0.099739 -0.201420 -0.168980 -0.437193

         PC69
0   -0.594109
1   -0.194564
2    0.165242
3   -0.743841
4   -0.229321
..        …
836 -0.036760
837 -0.021365
838  0.071339
839 -0.026705
840 -0.209892

[841 rows x 69 columns]
```

[53]:

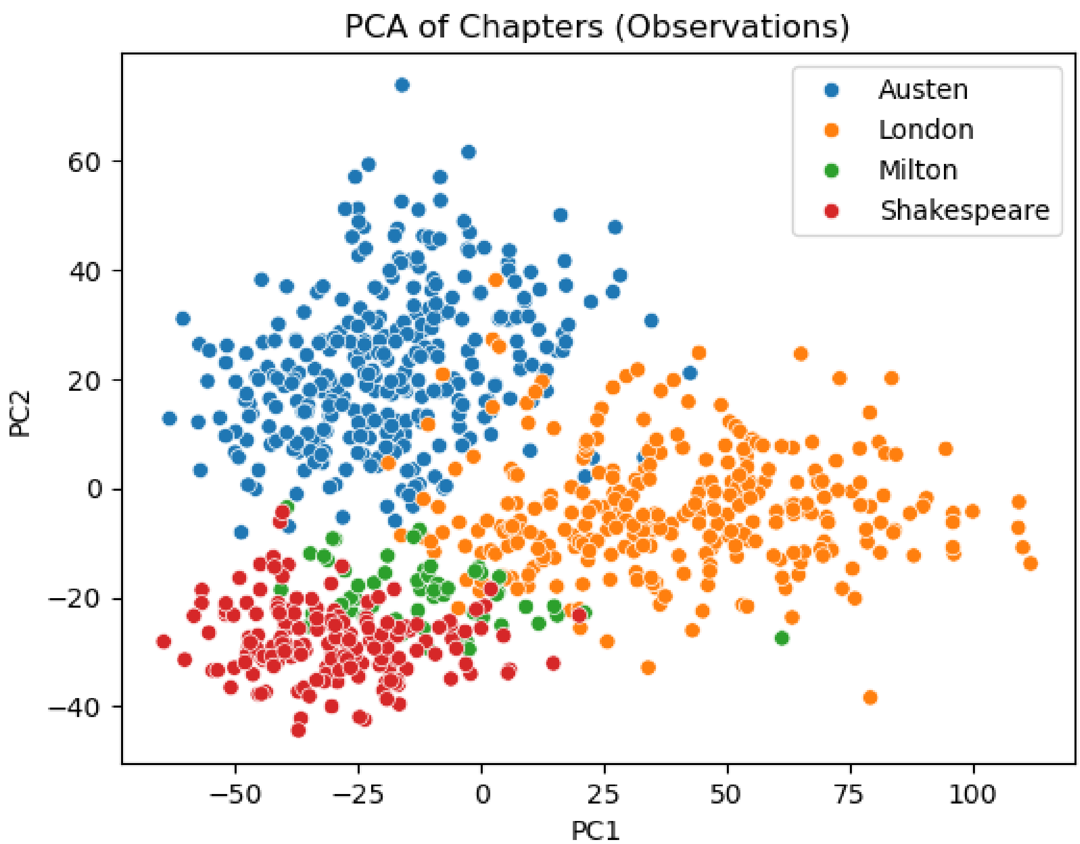
```python
fig, ax = plt.subplots(1,2, figsize=(20,5))

ax[0].bar(np.arange(1, pca_df.shape[1]),pca.explained_variance_ratio_[0:pca_df.
 ↪shape[1]-1], color = 'red')
ax[0].set_xlabel('Component')
ax[0].set_ylabel('Variance Explained')
ax[0].set_title('Variance Explained Plot')

ax[1].plot(np.cumsum(pca.explained_variance_ratio_), color = 'red')
ax[1].set_xlabel('Number of Components')
ax[1].set_ylabel('Cumulative Explained Variance')
ax[1].set_title('Scree Plot')
```

```
plt.savefig('Media/viz/01/01_pca_var_explained_and_screeplot')
plt.show()

print(f'PC1 and PC2 explain {np.sum(pca.explained_variance_ratio_[0:2])*100:.
 ↪3f}% of the variance.')
```




PC1 and PC2 explain 52.396% of the variance.

[54]:
```
pca_chapters_df = pca_df.loc[:,['PC1','PC2']] # Nested and ordered; extract␣
 ↪first 2 Principal Components
pca_chapters_df['Author'] = df['Author']

sns.scatterplot(data=pca_chapters_df, x='PC1', y='PC2', hue='Author',␣
 ↪palette='tab10')
plt.title('PCA of Chapters (Observations)')
plt.xlabel('PC1')
plt.ylabel('PC2')
plt.legend()
plt.savefig('Media/viz/01/01_pca_chapters')
plt.show()
```



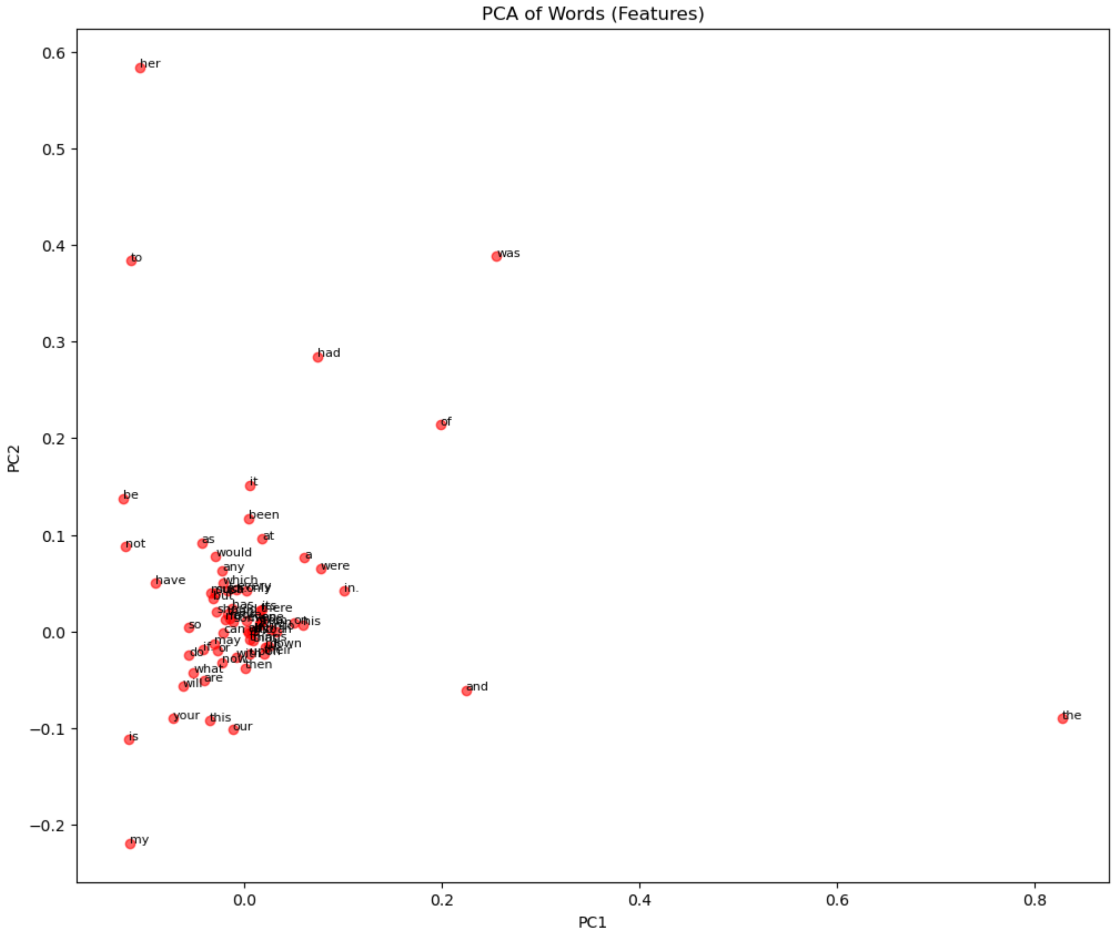

### 1.1.2 Features

[55]:
```python
pca_words_df = pd.DataFrame(word_loadings)
cols = [f'PC{j+1}' for j in range(pca_words_df.shape[1])]
pca_words_df = pca_words_df.rename(columns = {i:cols[i] for i in range(pca_words_df.shape[1])})
pca_words_df.index = X.columns
pca_words_df
```

[55]:
```
            PC1       PC2       PC3       PC4       PC5       PC6       PC7  \
a      0.060676  0.076522 -0.315737 -0.055225 -0.078926  0.263739  0.484181
all    0.003003  0.000487  0.053015 -0.032618  0.066769 -0.029175 -0.009633
also   0.007791 -0.000515  0.007859  0.001355 -0.008722  0.000714  0.000660
an     0.032094 -0.000843 -0.326638 -0.223061  0.188812 -0.163141  0.137205
and    0.223970 -0.060692  0.667811 -0.081825 -0.015979  0.543313 -0.035569
…           …         …         …         …         …         …         …
who    0.003630 -0.001030  0.052789  0.002280  0.002378 -0.013840 -0.007167
will  -0.062244 -0.056680 -0.001381  0.124291  0.004152  0.024512 -0.127001
with  -0.008282 -0.026201  0.128355 -0.102281  0.099969 -0.044243  0.033596
would -0.029127  0.078012 -0.025317  0.047541 -0.050614  0.034905 -0.062669
```

```
your  -0.072387 -0.090066 -0.053952  0.107079 -0.015571  0.009243 -0.028564

            PC8       PC9      PC10  …      PC60      PC61      PC62  \
a     -0.525683  0.264041  0.219213  … -0.005026  0.002127 -0.020590
all    0.042661  0.058567 -0.200501  …  0.007117 -0.043006  0.021078
also   0.007541  0.003267 -0.006968  … -0.002700  0.018310 -0.007202
an     0.060527  0.235075 -0.152501  … -0.012955 -0.026053 -0.005756
and   -0.145092  0.219987 -0.003553  … -0.002065  0.003563  0.004656
…           …         …         …  …       …         …         …
who   -0.006857  0.004959 -0.064097  … -0.030093 -0.166349  0.078814
will  -0.070483 -0.042921  0.024465  …  0.029968  0.013908  0.020761
with  -0.015710  0.041990  0.047793  … -0.006375 -0.009951  0.001977
would -0.037895 -0.032398 -0.054165  … -0.024179 -0.008643 -0.011250
your   0.019719  0.015591  0.159339  …  0.010928 -0.015083 -0.011472

           PC63      PC64      PC65      PC66      PC67      PC68      PC69
a     -0.020622 -0.005487 -0.001468  0.006522 -0.002964  0.020486  0.001386
all    0.017012 -0.001458  0.008575  0.012527  0.033792 -0.014598  0.001157
also  -0.008381 -0.001825  0.006400 -0.019838 -0.093516  0.056643  0.989992
an     0.020056  0.008385 -0.027075  0.000523  0.005918  0.007497  0.005771
and   -0.002794  0.001242 -0.004503  0.005302 -0.000100  0.000272 -0.005046
…           …         …         …         …         …         …         …
who    0.125654 -0.021114 -0.080313  0.096907  0.061265 -0.000161 -0.017311
will   0.013775  0.006232 -0.059988 -0.017531 -0.003613  0.022391 -0.003819
with  -0.007391  0.009002  0.006421 -0.005216  0.012884  0.008480 -0.000474
would  0.021813  0.022624 -0.031247  0.009858 -0.018687 -0.006684 -0.001789
your  -0.023488 -0.021720  0.023567  0.006809 -0.026835  0.019442 -0.000493

[69 rows x 69 columns]
```

[56]:
```python
plt.figure(figsize=(12, 10))
plt.scatter(pca_words_df['PC1'], pca_words_df['PC2'], alpha=0.6, color = 'red')

for _, row in pca_words_df.iterrows():
    plt.text(row['PC1'], row['PC2'], row.name, fontsize=8)  # ← fixed here
plt.title('PCA of Words (Features)')
plt.xlabel('PC1')
plt.ylabel('PC2');
plt.savefig('Media/viz/01/01_pca_words')
```




This method offers poor visualization and little interpretability. There are other methods that will provide more interpretability on the features such as NMF.

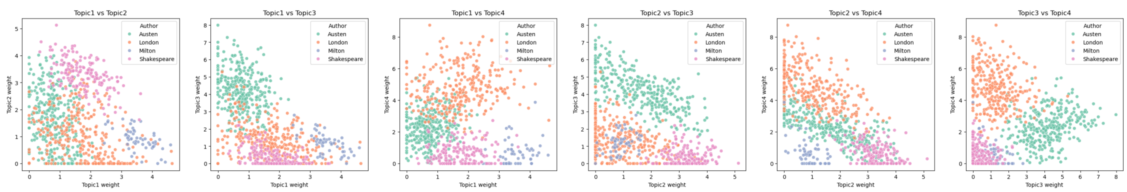
## 1.2 NMF

NMF may perform strongly here because our word frequencies are non-negative! Furthermore, this method will provide semantic intuition by creating topics.

[57]:
```
k = 4 # hyperparameter -> clusters
model = NMF(n_components=k, init='random', random_state=0, max_iter=500)

# dimensions -> (841 chapters, k topics)
W = model.fit_transform(X)

# dimensions -> (k topics, 69 words)
H = model.components_
```

### 1.2.1 Observations

[58]:
```
W_df = pd.DataFrame(W, index=X.index, columns=[f'Topic{i+1}' for i in␣
↪range(k)]) # Turn W into a DataFrame with word labels
topic_df = W_df.reset_index(drop=True)
topic_df["Author"] = df["Author"].reset_index(drop=True)
topics = W_df.columns.tolist() # List of topic names
topic_pairs = list(combinations(topics, 2)) # All pairs of topics

n_plots = len(topic_pairs)
fig, axes = plt.subplots(nrows=1, ncols=n_plots, figsize=(6 * n_plots, 5))

if n_plots == 1:
    axes = [axes]

for i, (x_topic, y_topic) in enumerate(topic_pairs):
    ax = axes[i]
    sns.scatterplot(data=topic_df, x=x_topic, y=y_topic, hue="Author",␣
↪palette="Set2", alpha=0.8, ax=ax)
    ax.set_title(f"{x_topic} vs {y_topic}")
    ax.set_xlabel(f"{x_topic} weight")
    ax.set_ylabel(f"{y_topic} weight")
    ax.legend().set_title("Author")

plt.savefig('Media/viz/01/01_nmf_observations')
```



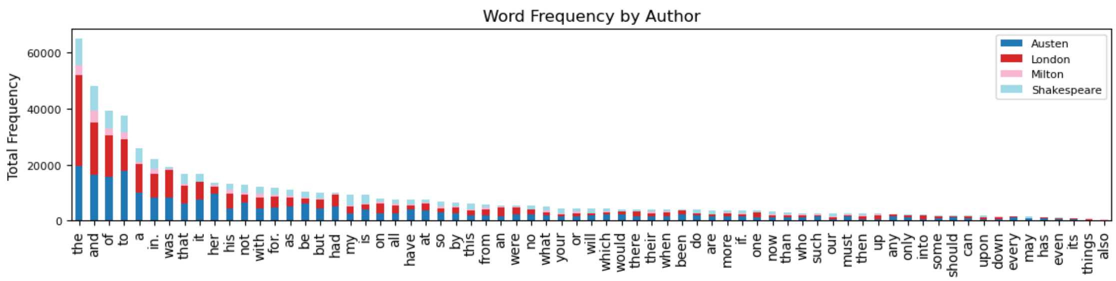

This provides poor interpretability so it is clearly not our first choice when visualizing chapters/observations.

### 1.2.2 Features

First of all the most simple and easy/inuitive vizualization of the features/words is just a simple total count!

[59]:
```
X_df = X.copy()
X_df['Author'] = authors
author_word_freq = X_df.groupby('Author').sum().T
author_word_freq = author_word_freq.loc[author_word_freq.sum(axis=1).
↪sort_values(ascending=False).index]
```

```python
fig, ax = plt.subplots(figsize=(len(author_word_freq)/6, 3))  # adjust based on # of words
author_word_freq.plot(
    kind='bar',
    stacked=True,
    ax=ax,
    colormap='tab20'
)

plt.title("Word Frequency by Author", fontsize=12)
plt.ylabel("Total Frequency", fontsize=10)
plt.xticks(rotation=90, fontsize=10)
plt.yticks(fontsize=8)
plt.legend(loc='upper right', fontsize=8)
plt.tight_layout()
plt.savefig("Media/viz/01/01_all_word_freq_by_author_stackedbar_condensed.png", dpi=300)
plt.show()
```




This is where NMF will truly shine.

[60]:
```python
n = 10 # Set here -> top words per topic!
```

[61]:
```python
word_topic_df = pd.DataFrame(H.T, index=X.columns, columns=[f"Topic{i+1}" for i in range(H.shape[0])]) # Transpose H to get words as rows, topics as columns
columns = list(word_topic_df.columns)
highest_n_weighted_words_per_topic = {}
for col in columns:
    words = list(word_topic_df[col].sort_values(ascending=False).head(n).index)
    highest_n_weighted_words_per_topic[col] = words
print(highest_n_weighted_words_per_topic)
```

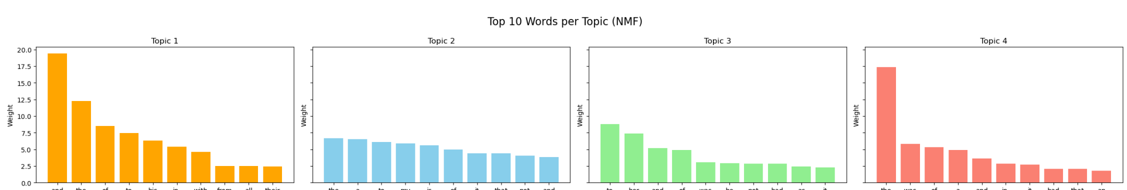
```
{'Topic1': ['and', 'the', 'of', 'to', 'his', 'in.', 'with', 'from', 'all',
'their'], 'Topic2': ['the', 'a', 'to', 'my', 'is', 'of', 'it', 'that', 'not',
'and'], 'Topic3': ['to', 'her', 'and', 'of', 'was', 'be', 'not', 'had', 'as',
'it'], 'Topic4': ['the', 'was', 'of', 'a', 'and', 'in.', 'it', 'had', 'that',
'an']}
```

Below are semantic interpretations of each topic based on the top contributing words:

- **Topic 1 – Structural & Possessive Language**
  Emphasizes grammatical connectors, possession, and sentence scaffolding — likely reflecting narrative structure or formal exposition.
- **Topic 2 – Determiners & Negation**
  Centers on articles, pronouns, and simple negations, suggesting basic sentence formation and assertive language.
- **Topic 3 – Past-Tense Narration**
  Highlights auxiliary verbs and past-tense forms, indicating descriptive or event-driven storytelling.
- **Topic 4 – Temporal & Descriptive Grammar**
  Focuses on narrative tense, articles, and function words often used in unfolding sequences or descriptive prose.

These topics reflect broad grammatical and stylistic features common in text, helping differentiate author styles or narrative structures.

[62]:
```python
fig, axes = plt.subplots(1, H.shape[0], figsize=(6 * H.shape[0], 4),␣
↪sharey=True)

colors = ['orange', 'skyblue', 'lightgreen', 'salmon']  # or however many␣
↪topics you have
for topic_idx, ax in enumerate(axes):
    topic_weights = H[topic_idx]
    top_word_indices = topic_weights.argsort()[::-1][:n]
    top_words = [X.columns[i] for i in top_word_indices]
    top_scores = topic_weights[top_word_indices]
    ax.bar(top_words, top_scores, color=colors[topic_idx % len(colors)])
    ax.set_title(f"Topic {topic_idx + 1}")
    ax.set_ylabel("Weight")

fig.suptitle(f"Top {n} Words per Topic (NMF)", fontsize=16)
plt.tight_layout()
plt.savefig('Media/viz/01/01_nmf_top_words_per_topic')
```




### Extra Feature Vizuals (not scientific)

```
[63]: word_freq_dict = X.sum(axis=0).to_dict()

wordcloud = WordCloud(width=1000, height=500, background_color='white').
  ↪generate_from_frequencies(word_freq_dict)

plt.figure(figsize=(15, 7))
plt.imshow(wordcloud, interpolation='bilinear')
plt.axis('off')
plt.title("Word Cloud of All Features", fontsize=16)
plt.savefig('Media/viz/01/01_word_cloud')
plt.show()
```

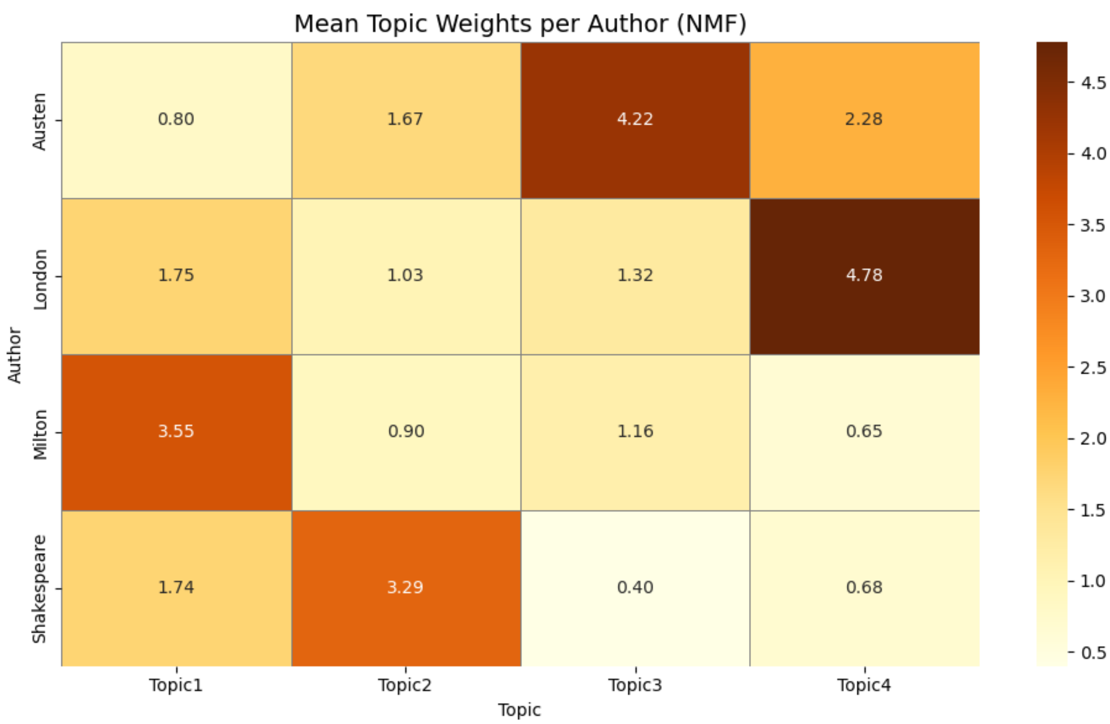

Now let us try and use this to determine which chapters correspond to which topics and predict the author based on the semantics of each topic.

Establish a mapping by grouping by using the labels, grouping by Author and then evaluating the mean topic weights per author! First we should split our data into a training and testing.

```
[64]: W_df['Authors'] = df['Author']

# Split the data
train_df, test_df = train_test_split(W_df, test_size=0.2, random_state=42,␣
  ↪stratify=W_df['Authors'])

print(f"Training samples: {len(train_df)}")
print(f"Testing samples: {len(test_df)}")
```

```
Training samples: 672
Testing samples: 169
```

[65]:
```python
author_topic_means = train_df.groupby('Authors').mean() # Compute mean topic weights grouped by author
plt.figure(figsize=(10, 6))
sns.heatmap(author_topic_means, annot=True, fmt=".2f", cmap="YlOrBr", linewidths=0.5, linecolor='gray')
plt.title("Mean Topic Weights per Author (NMF)", fontsize=14)
plt.xlabel("Topic")
plt.ylabel("Author")
plt.tight_layout()
plt.savefig("Media/viz/01/01_nmf_author_topic_heatmap", dpi=300)
```




### Mean Topic Weights per Author (NMF)

The heatmap above shows the average topic weights across all chapters written by each author. It is computed by grouping the NMF document-topic matrix (`W`) by author and taking the mean.

**Interpretation:** - Each cell reflects how strongly a given author tends to express a specific topic. - Higher values indicate that the author frequently uses patterns or structures associated with that topic. - While no topic is exclusive to a single author, we observe distinct preferences: - **Milton** leans heavily on Topic 1 - **Shakespeare** shows strong usage of Topic 2 - **Austen** favors Topic 3 - **London** stands out on Topic 4

This unsupervised representation helps reveal stylistic or grammatical tendencies across authors.

[66]:
```
mapping = {'Topic3': 'Austen', 'Topic4': 'London', 'Topic1': 'Milton', 'Topic2':
 ↪ 'Shakespeare'}
```

Now lets evaluate this mapping on the test set!

[67]:
```
test_df['Highest Weighted Topic'] = test_df.iloc[:, :4].idxmax(axis=1).values
test_df['Likely Author'] = test_df['Highest Weighted Topic'].apply(lambda x:␣
 ↪mapping[x])
correct_proportion = (test_df['Likely Author'] == test_df['Authors']).mean()
print(f'This method was correct {correct_proportion*100:.4f}% of the time')
```

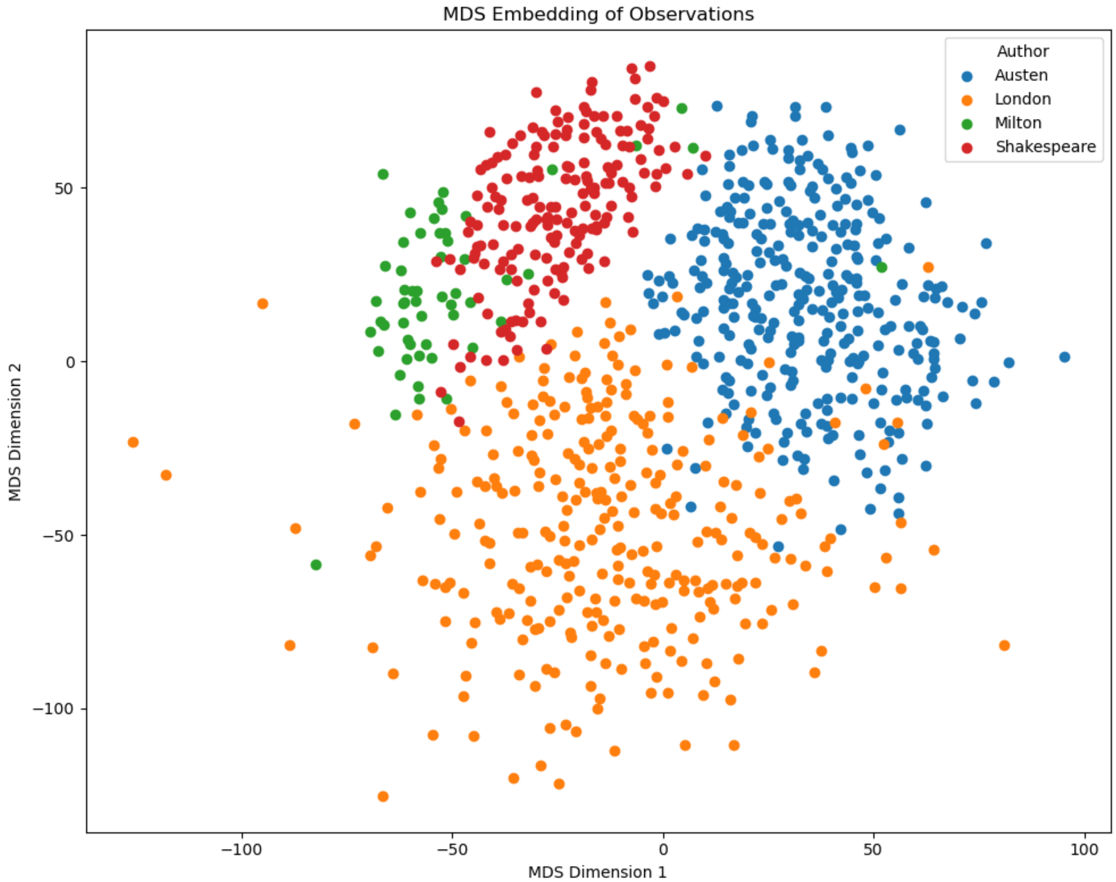
```
This method was correct 91.7160% of the time
```

This yields pretty good results!!

## 1.3 MDS

### 1.3.1 Observations

[68]:
```
model = MDS(n_components=2, random_state=42)
X_mds = model.fit_transform(X)

unique_authors = sorted(set(authors))

# Plot
plt.figure(figsize=(10, 8))
for i, author in enumerate(unique_authors):
    mask = authors == author
    plt.scatter(X_mds[mask, 0], X_mds[mask, 1], label=author)

plt.xlabel("MDS Dimension 1")
plt.ylabel("MDS Dimension 2")
plt.title("MDS Embedding of Observations")
plt.legend(title="Author", loc="best")
plt.tight_layout()
plt.savefig("Media/viz/01/01_mds_observation_viz")
plt.show()
```



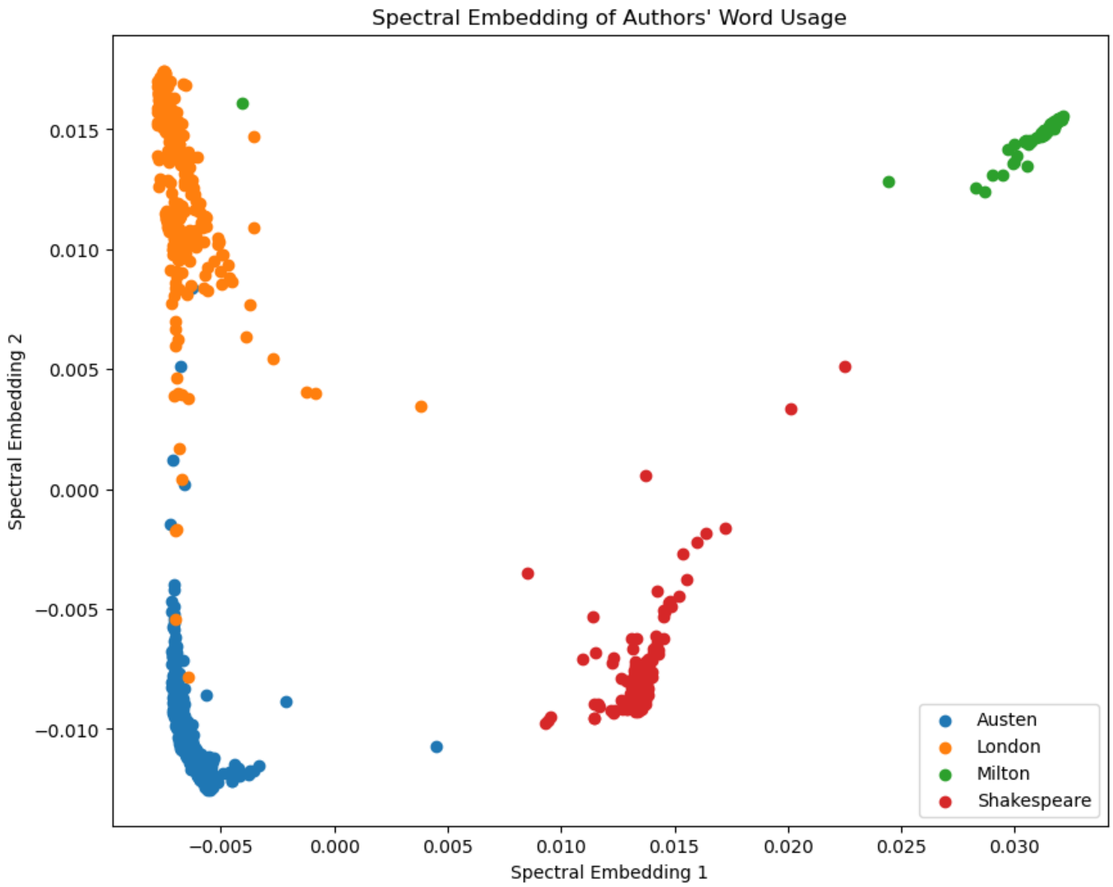

## 2 Non-Linear Methods

### 2.1 Spectral Embeddings

#### 2.1.1 Observations

[69]:
```
spectral = SpectralEmbedding(n_components=2, affinity='nearest_neighbors',␣
  ↪n_neighbors=10)
X_spec = spectral.fit_transform(X)  # shape: (n_samples, 2)
spectral_df = pd.DataFrame(X_spec, columns=['Dim1', 'Dim2'])

plt.figure(figsize=(10, 8))
for author in ['Austen', 'London', 'Milton', 'Shakespeare']:
    mask = (authors == author)
    plt.scatter(X_spec[mask, 0], X_spec[mask, 1], label=author)
plt.xlabel("Spectral Embedding 1")
plt.ylabel("Spectral Embedding 2")
plt.title("Spectral Embedding of Authors' Word Usage")
plt.legend(loc="lower right")
```

```
plt.savefig('Media/viz/01/01_spectal_obs_viz')
plt.show()
```




### 2.1.2 Features

Unlike PCA and NMF there are no components we can use to visualize the features in Spectral Embedding. However, we can apply a transpose to our data and then reapply Spectral Embedding.

This procedure will yield results where:

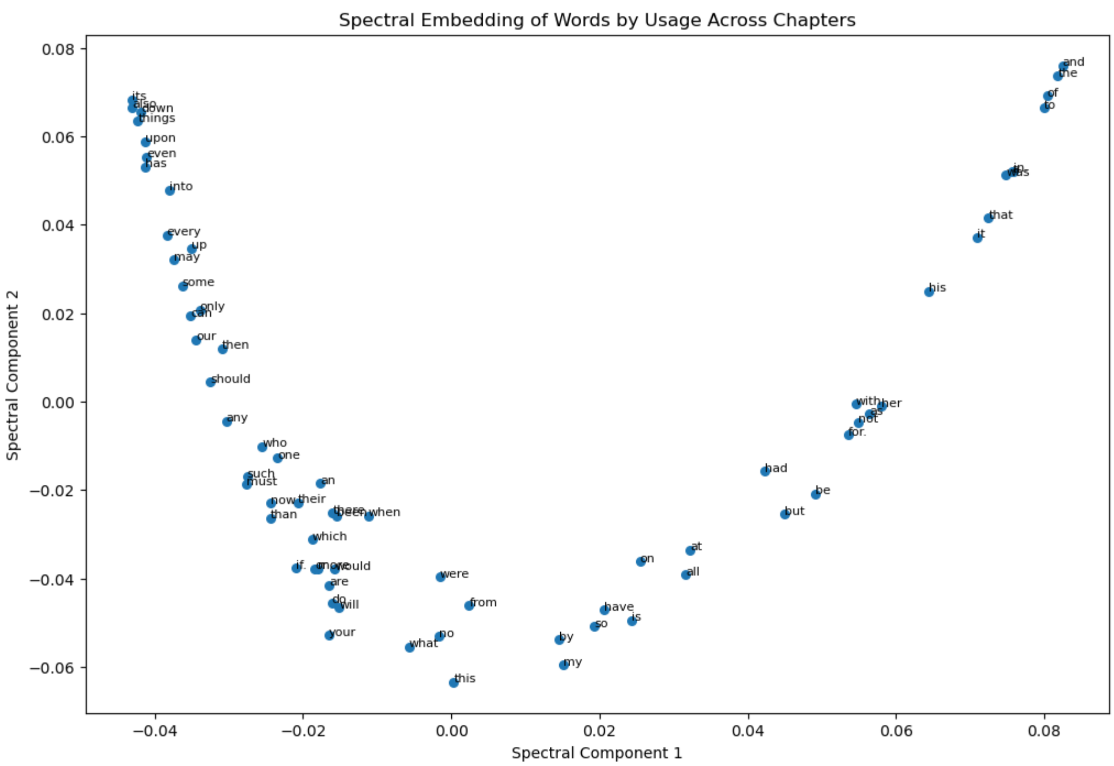
- Two words will be close in the embedding if they tend to appear in the same chapters.
- This gives you a co-occurrence-like structure, driven by chapter usage.
- This is conceptually similar to Latent Semantic Analysis, just via graph-based distance instead of SVD.

[70]:
```
X_transpose = X.T
X_transpose = X_transpose.rename(columns = {i:f'Chapter{i}' for i in range(df.
    ↪shape[0])})
X_transpose
```

[70]:
```
       Chapter0  Chapter1  Chapter2  Chapter3  Chapter4  Chapter5  Chapter6  \
a            46        35        46        40        29        27        34
all          12        10         2         7         5         8         8
also          0         0         0         0         0         0         0
an            3         7         3         4         6         3        15
and          66        44        40        64        52        42        44
…             …         …         …         …         …         …         …
who           8         3         4         5         2         6         4
will          4         5         5         3         4         3         9
with          9        14        15        22        21        18        11
would         1         8         3         4        10         4         6
your          0         0         9         3         0         5         4

       Chapter7  Chapter8  Chapter9  …  Chapter831  Chapter832  Chapter833  \
a            38        34        54  …          46          48          39
all           6        12         8  …           4           2           5
also          1         0         0  …           0           0           0
an            2         5         6  …           3           9          10
and          67        50        44  …          43          45          38
…             …         …         …  …           …           …           …
who           6         1         3  …           1           0           2
will          7         2         5  …           7          10           8
with         15        13        15  …          18          11          26
would         3        12         6  …           2           6           2
your          5         5         2  …           3          11          16

       Chapter834  Chapter835  Chapter836  Chapter837  Chapter838  Chapter839  \
a              22          28          32          16          22          25
all            13           7           4           5          15           4
also            0           0           0           0           0           0
an              5           7           6           5           3           8
and            47          45          33          49          48          59
…               …           …           …           …           …           …
who             4           2           3           0           0           2
will            9           7          11          11          12          22
with           12           8          17          20          15          23
would           6           3           5           2           1           4
your            7           7          10           7          10           5

       Chapter840
a              26
all             4
also            0
an              2
and            62
…               …
who             3
```

```
will          11
with          19
would          0
your           3

[69 rows x 841 columns]
```

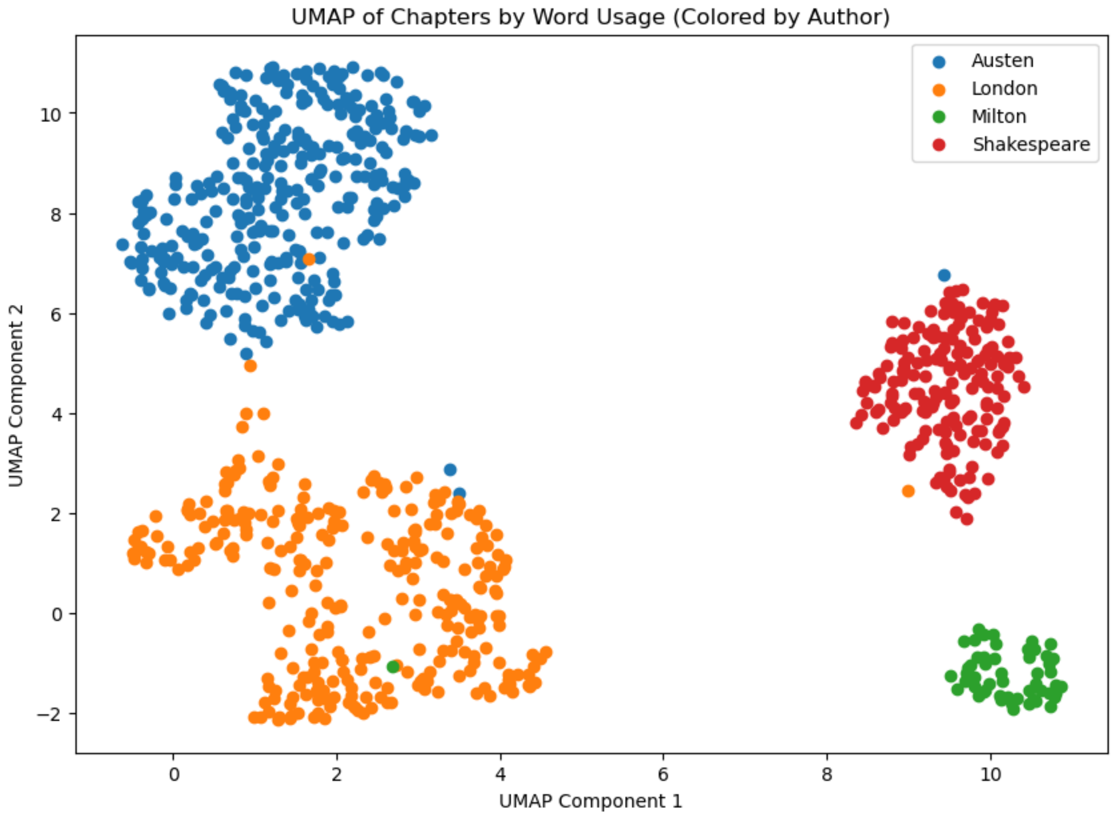
[71]:
```python
X_words = X_transpose.to_numpy()
spectral = SpectralEmbedding(n_components=2, affinity='nearest_neighbors', n_neighbors=10, random_state=0)
X_words_spec = spectral.fit_transform(X_words)

plt.figure(figsize=(12, 8))
plt.scatter(X_words_spec[:, 0], X_words_spec[:, 1], s=30)
for i, word in enumerate(X_transpose.index):
    plt.text(X_words_spec[i, 0], X_words_spec[i, 1], word, fontsize=8)
plt.title("Spectral Embedding of Words by Usage Across Chapters")
plt.xlabel("Spectral Component 1")
plt.ylabel("Spectral Component 2")
plt.savefig('Media/viz/01/01_spectral_feature_viz')
plt.show()
```



## 2.2 UMAP

### 2.2.1 Observations

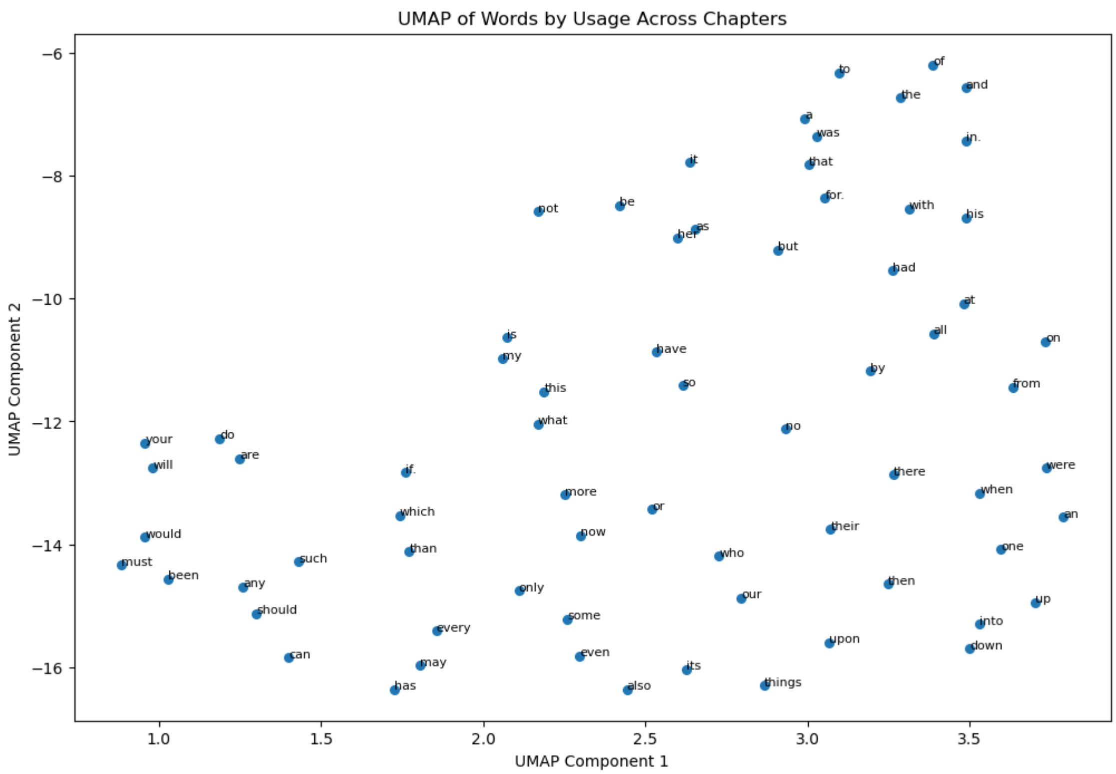
[84]:
```
umap_model = umap.UMAP(n_neighbors=10, min_dist=0.3, n_jobs=-1) # Fit UMAP
X_umap = umap_model.fit_transform(X.to_numpy())

plt.figure(figsize=(10, 7))
for author in ['Austen', 'London', 'Milton', 'Shakespeare']:
    mask = (authors == author)
    plt.scatter(X_umap[mask, 0], X_umap[mask, 1], label=author)
plt.xlabel("UMAP Component 1")
plt.ylabel("UMAP Component 2")
plt.title("UMAP of Chapters by Word Usage (Colored by Author)")
plt.legend(loc="upper right")
plt.savefig('Media/viz/01/01_umap_obs_viz')
plt.show()

plt.figure(figsize=(10, 7))
plt.scatter(X_umap[:,0],X_umap[:,1], color = 'purple')
plt.xlabel("UMAP Component 1")
plt.ylabel("UMAP Component 2")
plt.title("UMAP")
plt.savefig('Media/viz/01/01_umap_viz_unlabeled');
plt.close()
```




UMAP does a very good job at creating seperable clusters! This visualization appears to be the best so far when observing observations (chapters)!

#### 2.2.2 Features

[73]:
```
umap_model = umap.UMAP(n_neighbors=10, min_dist=0.3, n_jobs=-1) # Fit UMAP
X_words_umap = umap_model.fit_transform(X_transpose.to_numpy())

# Plot w/ word labels
plt.figure(figsize=(12, 8))
plt.scatter(X_words_umap[:, 0], X_words_umap[:, 1], s=30)
for i, word in enumerate(X_transpose.index):
    plt.text(X_words_umap[i, 0], X_words_umap[i, 1], word, fontsize=8)
plt.title("UMAP of Words by Usage Across Chapters")
plt.xlabel("UMAP Component 1")
plt.ylabel("UMAP Component 2")
plt.savefig('Media/viz/01/01_umap_feature_viz')
plt.show()
```




Not very interpretable/helpful.

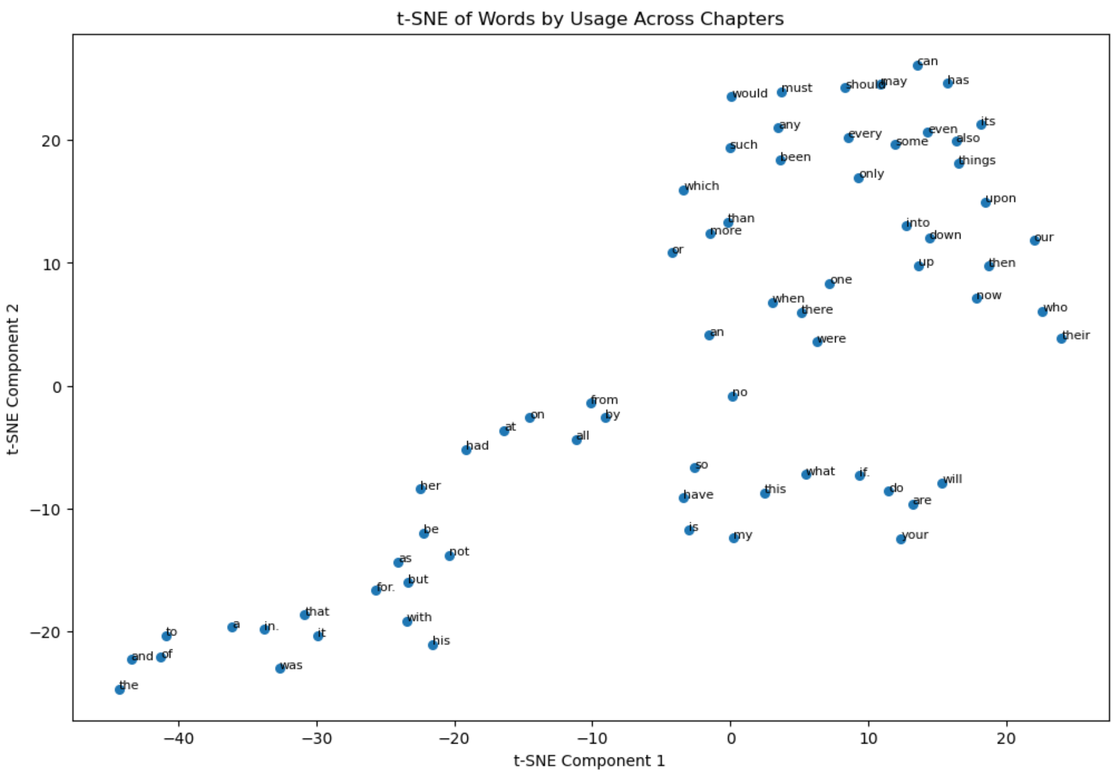
## 2.3 tSNE

### 2.3.1 Observations

[74]:
```
tsne = TSNE(n_components=2, perplexity=30, learning_rate='auto') # Fit t-SNE
X_tsne = tsne.fit_transform(X.to_numpy())

# Plot chapters with author labels
plt.figure(figsize=(10, 7))
for author in ['Austen', 'London', 'Milton', 'Shakespeare']:
    mask = (authors == author)
    plt.scatter(X_tsne[mask, 0], X_tsne[mask, 1], label=author)

plt.xlabel("t-SNE Component 1")
plt.ylabel("t-SNE Component 2")
plt.title("t-SNE of Chapters by Word Usage (Colored by Author)")
plt.legend(loc="upper right")
plt.savefig('Media/viz/01/01_tsne_obs_viz')
plt.show()
```


### 2.3.2 Features

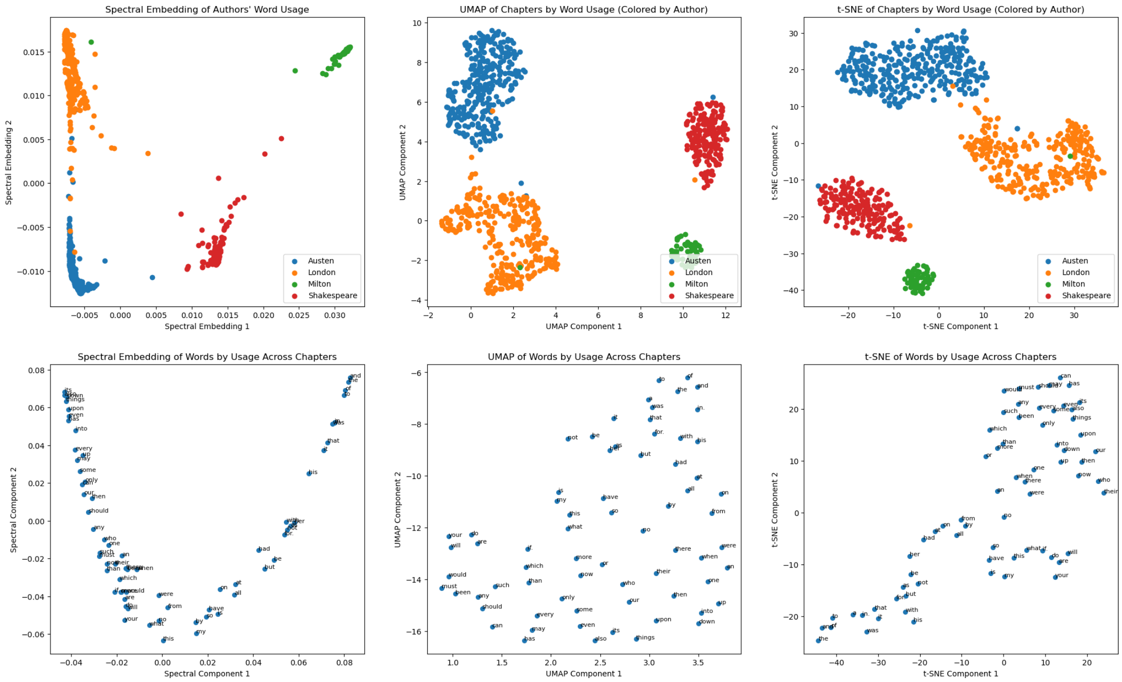
[75]:
```
tsne = TSNE(n_components=2, perplexity=5, learning_rate='auto') # Fit t-SNE
X_words_tsne = tsne.fit_transform(X_words)

# Plot w/ word labels
plt.figure(figsize=(12, 8))
plt.scatter(X_words_tsne[:, 0], X_words_tsne[:, 1], s=30)
for i, word in enumerate(X_transpose.index):
    plt.text(X_words_tsne[i, 0], X_words_tsne[i, 1], word, fontsize=8)

plt.title("t-SNE of Words by Usage Across Chapters")
plt.xlabel("t-SNE Component 1")
plt.ylabel("t-SNE Component 2")
plt.savefig('Media/viz/01/01_tsne_feature_viz')
plt.show()
```




## 3 Combined Visualizations

[76]:
```
fig, ax = plt.subplots(2, 3, figsize=(25, 15), sharey=False)  # 1 row, 3 columns

# Spectral Embedding
for author in ['Austen', 'London', 'Milton', 'Shakespeare']:
    mask = (authors == author)
    ax[0,0].scatter(X_spec[mask, 0], X_spec[mask, 1], label=author)
ax[0,0].set_xlabel("Spectral Embedding 1")
ax[0,0].set_ylabel("Spectral Embedding 2")
ax[0,0].set_title("Spectral Embedding of Authors' Word Usage")
ax[0,0].legend(loc="lower right")

ax[1,0].scatter(X_words_spec[:, 0], X_words_spec[:, 1], s=30)
for i, word in enumerate(X_transpose.index):
    ax[1,0].text(X_words_spec[i, 0], X_words_spec[i, 1], word, fontsize=8)
ax[1,0].set_title("Spectral Embedding of Words by Usage Across Chapters")
ax[1,0].set_xlabel("Spectral Component 1")
ax[1,0].set_ylabel("Spectral Component 2")

# UMAP
for author in ['Austen', 'London', 'Milton', 'Shakespeare']:
```

```python
    mask = (authors == author)
    ax[0,1].scatter(X_umap[mask, 0], X_umap[mask, 1], label=author)
ax[0,1].set_xlabel("UMAP Component 1")
ax[0,1].set_ylabel("UMAP Component 2")
ax[0,1].set_title("UMAP of Chapters by Word Usage (Colored by Author)")
ax[0,1].legend(loc="lower right")

ax[1,1].scatter(X_words_umap[:, 0], X_words_umap[:, 1], s=30)
for i, word in enumerate(X_transpose.index):
    ax[1,1].text(X_words_umap[i, 0], X_words_umap[i, 1], word, fontsize=8)
ax[1,1].set_title("UMAP of Words by Usage Across Chapters")
ax[1,1].set_xlabel("UMAP Component 1")
ax[1,1].set_ylabel("UMAP Component 2")

# tSNE
for author in ['Austen', 'London', 'Milton', 'Shakespeare']:
    mask = (authors == author)
    ax[0,2].scatter(X_tsne[mask, 0], X_tsne[mask, 1], label=author)
ax[0,2].set_xlabel("t-SNE Component 1")
ax[0,2].set_ylabel("t-SNE Component 2")
ax[0,2].set_title("t-SNE of Chapters by Word Usage (Colored by Author)")
ax[0,2].legend(loc="lower right")

ax[1,2].scatter(X_words_tsne[:, 0], X_words_tsne[:, 1], s=30)
for i, word in enumerate(X_transpose.index):
    ax[1,2].text(X_words_tsne[i, 0], X_words_tsne[i, 1], word, fontsize=8)
ax[1,2].set_title("t-SNE of Words by Usage Across Chapters")
ax[1,2].set_xlabel("t-SNE Component 1")
ax[1,2].set_ylabel("t-SNE Component 2")

plt.savefig('Media/viz/01/01_nonlinear_viz')
plt.show()
```




UMAP offers the best balance between global and local structure, preserves neighborhood quality, and gives interpretable groupings without as much distortion as t-SNE.

Clusters of words in the UMAP embedding reflect similar usage patterns across chapters. Since word usage is shaped by topic and syntax, these local neighborhoods can be interpreted as reflecting semantic or grammatical similarity. While UMAP does not preserve global distances, local groupings are meaningful.