1. (10 points) Find the (shortest) distance between the two parallel planes
$$2x - y + 2z = 3$$
and
$$2x - y + 2z = 12.$$

2. (2 points) **True or False:** Let V and W be linear subspaces of $\mathbb{R}^{51}$ and consider the set $U = \{\mathbf{v}+\mathbf{w} : \mathbf{v} \in V, \mathbf{w} \in W\}$. Then, U is a linear subspace of $\mathbb{R}^{51}$.

3. (2 points) **True or False:** Let $\mathbf{u}$, $\mathbf{v}$ be fixed vectors in $\mathbf{R}^n$ such that $W = \mathrm{span}(\mathbf{u}, \mathbf{v})$ is a linear subspace of dimension 2. Suppose $\mathbf{x}$ is a vector in $\mathbf{R}^n$. Then it is always the case that
$$\|\,\mathbf{Proj}_W(\mathbf{x})\| \geq \|\,\mathbf{Proj}_{\mathbf{u}}(\mathbf{x})\|.$$

4. (3 points) Consider the three vectors
$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$$
and the three planes
$$P_1 = \mathrm{span}(\mathbf{v}_2, \mathbf{v}_3), \;\; P_2 = \mathrm{span}(\mathbf{v}_1, \mathbf{v}_3), \;\; P_3 = \mathrm{span}(\mathbf{v}_1, \mathbf{v}_2).$$
Note that
$$\mathbf{v}_1 \cdot \mathbf{v}_2 = 0, \;\; \mathbf{v}_1 \cdot \mathbf{v}_3 = 0.$$
What is $\mathrm{span}\big(\mathbf{Proj}_{P_1}(\mathbf{v}_1), \mathbf{Proj}_{P_2}(\mathbf{v}_2), \mathbf{Proj}_{P_3}(\mathbf{v}_3)\big)$?

   a) The plane P_1. b) The plane P_2. c) The plane P_3. d) All of $\mathbf{R}^3$.

5. (3 points) For each of the following sets V, if it is a linear subspace, determine its dimension $\dim(V)$; if it is not a linear subspace, write 0.

   (A) $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbf{R}^2 : x^2 = 4y^2 \right\}$

   (B) $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbf{R}^3 : x + y = z + 1 \right\}$

   (C) $V = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \in \mathbf{R}^4 : x + w = y + z \right\}$

   (D) $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \in \mathbf{R}^5 : x_1 + 2x_2 + 3x_3 = 0 \text{ and } x_1 + x_2 + x_3 = x_4 + x_5 \right\}$