Last time

- applications of projections
- introduction to multivariable functions

Today

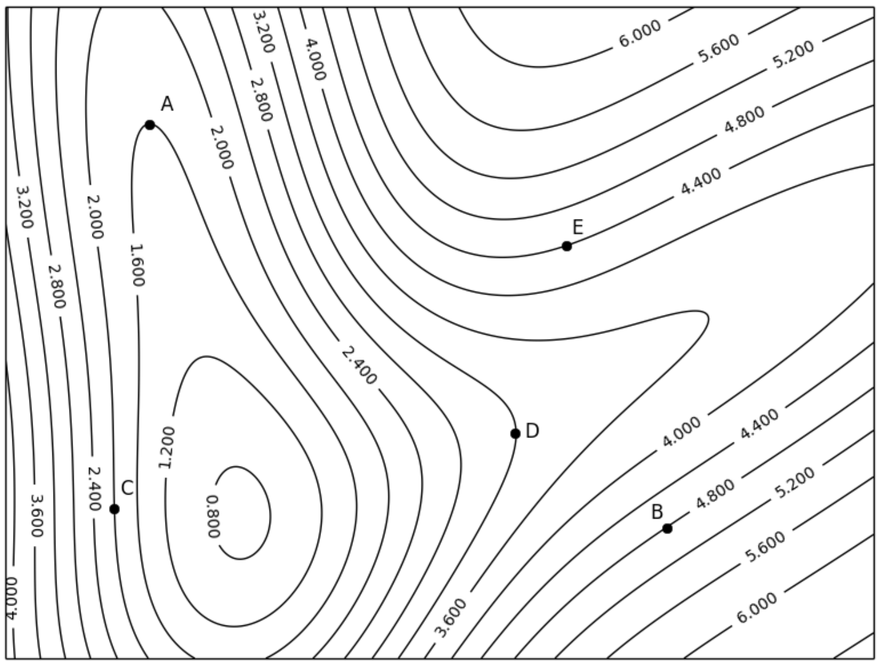
- reading partial derivatives off of a contour plot
- computing partial derivatives
- finding critical points and local extrema

(• short feedback survey on my teaching)

Notation: Partial derivatives

Problem 1: Visually interpreting derivatives

Below is a collection of level sets of a function $f\colon \mathbf{R}^2 \to \mathbf{R}$. (As usual, x is horizontal and y is vertical, and the length scales in the x- and y-directions are equal.)



(a) (Choose one) $\dfrac{\partial f}{\partial y}$ at **A** is: NEGATIVE ZERO POSITIVE

(b) (Choose one) $\dfrac{\partial f}{\partial y}$ at **B** is: NEGATIVE ZERO POSITIVE

(c) (Choose one) $\dfrac{\partial f}{\partial y}$ at **C** is: NEGATIVE ZERO POSITIVE

(d) (Choose one) $\dfrac{\partial f}{\partial x}$ at **D** is: NEGATIVE ZERO POSITIVE

(e) Which partial derivative is larger, in *absolute value*? $|f_y(\mathbf{A})|$ $|f_y(\mathbf{B})|$

(f) Which partial derivative is larger, in *absolute value*? $|f_x(\mathbf{E})|$ $|f_y(\mathbf{E})|$

(g) At what point(s) (not necessarily labeled) in the region depicted does f reasonably seem to have a local minimum? a local maximum? What can you say about the value taken by f at each of these points?

A picture to help you visualize contour plots.
This is not part of the problem.

Review:

Problem 2: Partial derivative practice

Compute the first and second partial derivatives in general, verifying equality of mixed partials directly, and evaluate the first partials at the indicated point $\mathbf{a}$.

(a) $g(x_1, x_2) = \sin(x_1 x_2 - x_1 + x_2)$, $\mathbf{a} = (\sqrt{\pi}, \sqrt{\pi})$.

(b) $h(x, y) = e^x(x - y)^2$, $\mathbf{a} = (0, 1)$.

Problem 3: Finding candidates for local extrema

For each of the following functions $f : \mathbf{R}^2 \to \mathbf{R}$, find all critical points.

(a) $x_1^2 + 4x_1x_2 + 5x_2^2 - 4x_1 + 2x_2$.

(b) $x^4y^4 - 2x^2 - 2y^2$.

(c) $\cos(\pi(x^2 + y^2))$.

(d) $x_1^3 - 3x_1x_2^2 + 3x_2^2$.

Problem 4: Computing extrema on a region I

Find the global extreme values of $f(x, y) = 2x^2 + y^2 + 5y$ on the disk of points (x, y) satisfying $x^2 + y^2 \leq 16$.

Problem 5: Computing extrema on a region II

Find the global extreme values of $f(x, y) = x^4y^4 - 2x^2 - 2y^2$ on the region of points (x, y) that lies on or inside the triangle with vertices $(-2, -2)$, $(-2, 2)$, $(2, -2)$. (Sketch this triangle first, to get oriented.)

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