1. (10 points) Given three points $\mathbf{a} = (6, 3)$, $\mathbf{b} = (4, 5)$, $\mathbf{c} = (10, 9)$, and a linear combination $\mathbf{p} = \frac{1}{5}\mathbf{a} + \frac{1}{5}\mathbf{b} + \frac{3}{5}\mathbf{c}$.

   The line through $\mathbf{p}$ and $\mathbf{c}$ intersects the line segment between $\mathbf{a}$ and $\mathbf{b}$ in a single point, which we call $\mathbf{X} = (x, y)$.

   Compute the values of x and y. Your answers should be non-negative integers.

   [Hint: Write $\mathbf{p}$ as a linear combination of $\mathbf{c}$ with another point that is located on the line segment between $\mathbf{a}$ and $\mathbf{b}$.]

2. (2 points) **True or False:** For two non-zero vectors $\mathbf{v}$ and $\mathbf{w}$, $\|\mathbf{v} + 2\mathbf{w}\|$ is always greater than $\|\mathbf{v}\|$.

3. (2 points) **True or False:** the line through the two points $(1, 0, 1)$ and $(3, 1, 2)$ is parallel to the plane $x - y - z = 4$.

4. (3 points) The planes $2x + y - 2z = 2$ and $x - y + 2z = 1$ intersect in a line L with parametric form:

   (i) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + t_1 \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} + t_2 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

5. (3 points) Let $\mathbf{v}$ be a fixed nonzero vector in $\mathbf{R}^3$, and $d \in \mathbf{R}$ a scalar. Geometrically, the collection of vectors $\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbf{R}^3$ satisfying the condition

   $$\mathbf{v} \cdot \mathbf{w} = d$$

   is a

   a) line. b) plane. c) $\mathbf{R}^3$. d) might take different shapes depending on what $\mathbf{v}$ and $\mathbf{w}$ are.