Problem 1: Parametric form of a plane

Let P be the plane in $\mathbf{R}^3$ containing the points $(1, 1, 1)$, $(1, 2, 3)$, and $(3, 2, 1)$.

(a) Find a parametric representation of P. (Extra: can you write down many other parametrizations?)

(b) Use the dot product to find a normal vector to P. (Hint: Think about why it is the same as a vector perpendicular to two different "directions" within the plane, and then form some displacement vectors.)

(c) Find an equation for P of the form $ax + by + cz = d$ for some a, b, c, d in $\mathbf{R}$. (You can do this with or without (b).)

Problem 2: Equation of a plane

(a) Consider the distinct points $A = (0, 1, 1)$, $B = (3, 4, 4)$, and $C = (1, -1, -4)$. Compute the nonzero displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ to confirm these are not scalar multiples of each other, so these three points lie in a unique common plane P. Find an equation for P of the form $ax + by + cz = d$.

(b) Find a *unit* vector (i.e., a vector of length 1) that is normal to the plane whose equation is $6x - 2y - 3z = 4$. Your answer should have entries that are fractions (no ugly square roots).

(c) Are the planes in (a) and (b) parallel to each other? How do you know?

Problem 3: What sets can be linear subspaces, and what cannot?

For each of the following subsets of $\mathbf{R}^2$ or $\mathbf{R}^3$, write down a collection of finitely many vectors whose span is that set or explain why there is no such collection.

(a) The line $x + y = 1$ (b) The line $x + y = 0$ (c) The unit disk $x^2 + y^2 \leq 1$ (d) $\{\mathbf{0}\}$ (e) The plane $x + y + z = 0$

Problem 4: More recognizing and describing linear subspaces

Which of the following subsets S of $\mathbf{R}^3$ are linear subspaces? If a set S is a linear subspace, exhibit it as a span. If it is not a linear subspace, describe it geometrically and explain why it is not a linear subspace.

(a) The set S_1 of points (x, y, z) in $\mathbf{R}^3$ with both $z = x + 2y$ and $z = 5x$.

(b) The set S_2 of points (x, y, z) in $\mathbf{R}^3$ with either $z = x + 2y$ or $z = 5x$.

(c) The set S_3 of points (x, y, z) in $\mathbf{R}^3$ of the form $t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t' \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ for some scalars t and t' (which are allowed to be anything, depending on the point (x, y, z)).

Problem 5: Visualizing a span

For each collection of vectors in $\mathbf{R}^2$, sketch its span: is it a point, a line, or all of $\mathbf{R}^2$?

(a) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

For each collection of vectors in $\mathbf{R}^3$ sketch its span: is it a point, a line, a plane, or all of $\mathbf{R}^3$?

(f) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ (g) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ (h) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ (i) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

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