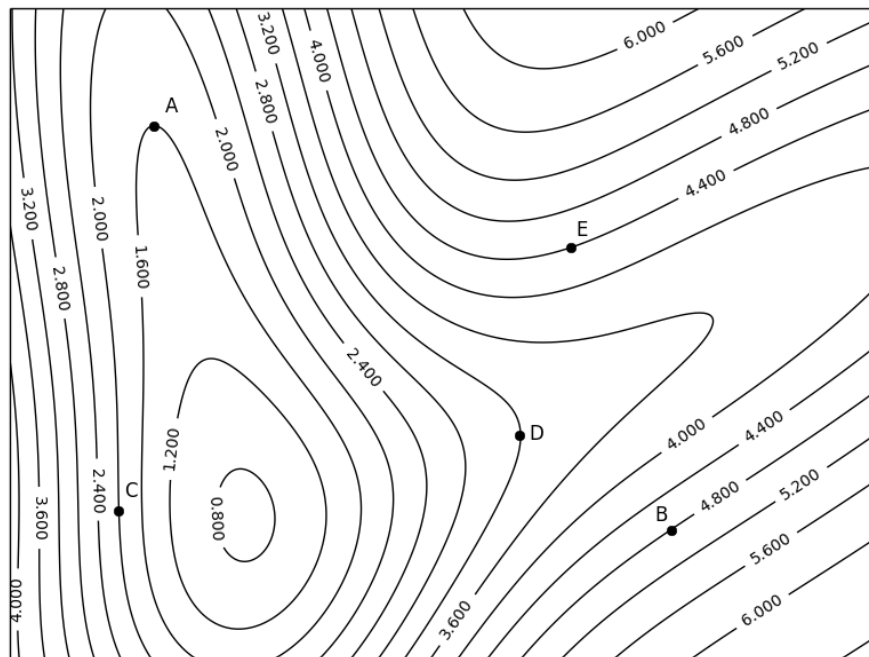


Problem 1: Visually interpreting derivatives

Below is a collection of level sets of a function $f: \mathbf{R}^2 \rightarrow \mathbf{R}$. (As usual, x is horizontal and y is vertical, and the length scales in the x - and y -directions are equal.)



- (a) (Choose one) $\frac{\partial f}{\partial y}$ at **A** is: NEGATIVE ZERO POSITIVE
- (b) (Choose one) $\frac{\partial f}{\partial y}$ at **B** is: NEGATIVE ZERO POSITIVE
- (c) (Choose one) $\frac{\partial f}{\partial y}$ at **C** is: NEGATIVE ZERO POSITIVE
- (d) (Choose one) $\frac{\partial f}{\partial x}$ at **D** is: NEGATIVE ZERO POSITIVE
- (e) Which partial derivative is larger, in *absolute value*? $|f_y(\mathbf{A})|$ $|f_y(\mathbf{B})|$
- (f) Which partial derivative is larger, in *absolute value*? $|f_x(\mathbf{E})|$ $|f_y(\mathbf{E})|$
- (g) At what point(s) (not necessarily labeled) in the region depicted does f reasonably seem to have a local minimum? a local maximum? What can you say about the value taken by f at each of these points?

A picture to help you visualize contour plots.
This is not part of the problem.



Problem 2: Partial derivative practice

Compute the first and second partial derivatives in general, verifying equality of mixed partials directly, and evaluate the first partials at the indicated point \mathbf{a} .

(a) $g(x_1, x_2) = \sin(x_1 x_2 - x_1 + x_2)$, $\mathbf{a} = (\sqrt{\pi}, \sqrt{\pi})$.

(b) $h(x, y) = e^x(x - y)^2$, $\mathbf{a} = (0, 1)$.

Problem 3: Finding candidates for local extrema

For each of the following functions $f : \mathbf{R}^2 \rightarrow \mathbf{R}$, find all critical points.

(a) $x_1^2 + 4x_1 x_2 + 5x_2^2 - 4x_1 + 2x_2$.

(b) $x^4 y^4 - 2x^2 - 2y^2$.

(c) $\cos(\pi(x^2 + y^2))$.

(d) $x_1^3 - 3x_1 x_2^2 + 3x_2^2$.

Problem 4: Computing extrema on a region I

Find the global extreme values of $f(x, y) = 2x^2 + y^2 + 5y$ on the disk of points (x, y) satisfying $x^2 + y^2 \leq 16$.

Problem 5: Computing extrema on a region II

Find the global extreme values of $f(x, y) = x^4 y^4 - 2x^2 - 2y^2$ on the region of points (x, y) that lies on or inside the triangle with vertices $(-2, -2)$, $(-2, 2)$, $(2, -2)$. (Sketch this triangle first, to get oriented.)