Review

Problem 1

(a) Using that perpendicularity is governed by dot products being equal to 0, find a nonzero vector in $\mathbb{R}^3$ that is perpendicular to $\vec{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Then find another

$\left[\, 1 \,\right]$ that is not a scalar multiple.

(b) Find an equation in x, y, z that characterizes when $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is perpendicular to $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$. What does this collection of vectors look like?

(c) (Extra) What does the collection of nonzero vectors $\vec{w} = \begin{bmatrix} x \\ y \end{bmatrix}$ making an angle of at most $60°$ against $\vec{v} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ look like? Describe this region with a pair of conditions $ax^2 + bxy + cy^2 \geq 0$ and $y \leq (3/4)x$ (away from origin).

Problem 2

(a) For $\vec{a} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 6 \\ -4 \\ -1 \end{bmatrix}$ show that

$$\vec{a} \cdot (\vec{b} - \vec{c}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}.$$

(b) Give an example of 2-vectors $\vec{a}, \vec{b}, \vec{c}$ for which

$$(\vec{a} \cdot \vec{b})\vec{c} \neq (\vec{a} \cdot \vec{c})\vec{b}$$

(c) (Extra) Explain in terms of variables why

$$\vec{v}\cdot(\vec{w}_1+\vec{w}_2)=\vec{v}\cdot\vec{w}_1+\vec{v}\cdot\vec{w}_2 \text{ for } \vec{v}=\begin{bmatrix} x\\ y\\ z\end{bmatrix},\ \vec{w}_1=\begin{bmatrix} a_1\\ b_1\\ c_1\end{bmatrix},\ \vec{w}_2=\begin{bmatrix} a_2\\ b_2\\ c_2\end{bmatrix}.$$

Why does it follow that

$$(\vec{v}_1+\vec{v}_2)\cdot(\vec{w}_1+\vec{w}_2)=\vec{v}_1\cdot\vec{w}_1+\vec{v}_2\cdot\vec{w}_1+\vec{v}_1\cdot\vec{w}_2+\vec{v}_2\cdot w_2?$$

Does this work for n-vectors for any n?

(d) For n-vectors $\vec{w}_1$ and $\vec{w}_2$, verify that

$$\|\vec{w}_1+\vec{w}_2\|^2=\|\vec{w}_1\|^2+2(\vec{w}_1\cdot\vec{w}_2)+\|\vec{w}_2\|^2$$

by using the relation $\|\vec{w}\|^2=\vec{w}\cdot\vec{w}$ and general properties of dot products as stated in (c).

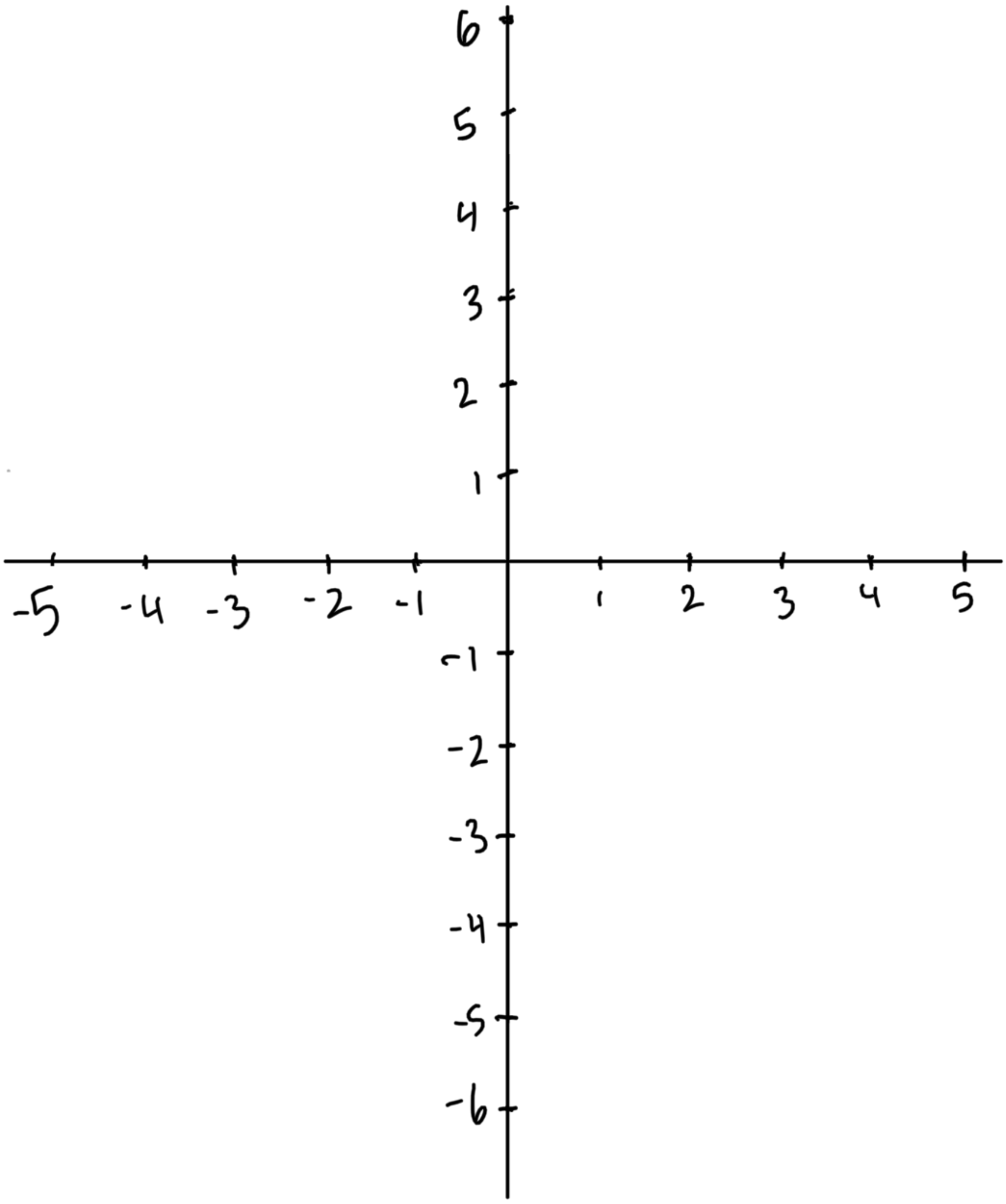
Problem 3 Correlation coefficients

Consider the collection of data points:

$$(-2,5), (-1,3), (0,0), (1,-2), (2,-6).$$

Plot the points and see if they look close to a line

(a) Plot the points and see if they look close to a line.



(b) Compute the correlation coefficient exactly. Using a calculator, approximate it to 3 decimal digits to see if its nearness to ±1 fits well with the visual quality of fit of the line to the data plot in (a).

$$(-2,5), (-1,3), (0,0), (1,-2), (2,-6).$$

$\boxed{\text{Problem 4}}$ (Extra)

(a) For the 2-vectors $\vec{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, describe the set of all possible vectors $r\vec{a} + s\vec{b} + t\vec{c}$, where $r+s+t=1$ with $0 \leq r,s,t \leq 1$. Which points correspond to $t=0$? $s=0$? $r=0$?

(b) Try the same thing using the 3-vectors $\vec{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(c) Can you explain why your description in (a) applies to any three 2-vectors $\vec{a}, \vec{b}, \vec{c}$ not on a common line?

(d) Is there a version for a triple of 3-vectors not all on a common line? Why does it work?