1. (3 points) For the following pairs of matrices, which of them satisfy $AB = BA$? Select all that apply.

   a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

   b) $A = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$

   c) $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

   d) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   e) none of these

2. (1 point) **True or False:** If

$$A = \begin{bmatrix} 2 & 3 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ -1 & -2 \\ 0 & 15 \end{bmatrix},$$

   then $AB = BA$.

3. (4 points) Consider the effect of a linear transformation on the following image of the bear flag without the black background.

   For each of the following matrices M_i, identify which picture shows the output when M_i is applied to the original image, i.e. the bear flag without the black background.

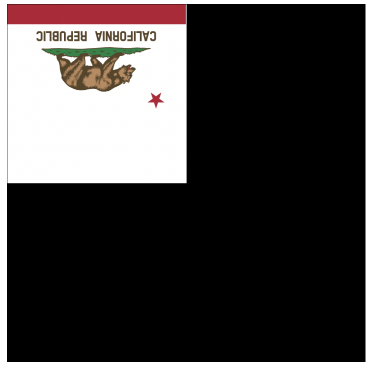
   Note that we are not specifying where the images are with respect to the origin, as this problem can be solved without this information.

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$

$$M_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad M_4 = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}.$$

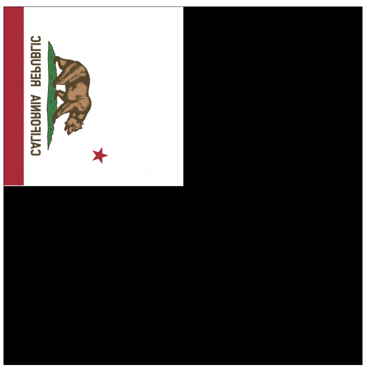
(A)

(B)

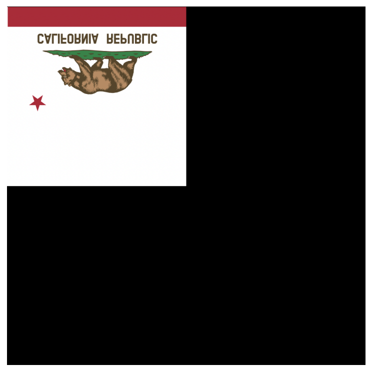


(C)

(D)



(E)



(F)

4. (3 points) $f : \mathbf{R}^2 \to \mathbf{R}^3$ is given by

$$f(x,y) = \begin{bmatrix} x^2y \\ x - y + xy \\ 3y - x^3 \end{bmatrix}$$

Which of the following represents the linear approximation to f at the point $(1,2)$?

(a)

$$f(x,y) \approx \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 4 & 3 & -3 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x-1 \\ y-2 \end{bmatrix}$$

(b)

$$f(x,y) \approx \begin{bmatrix} 4 \\ 3 \\ -5 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 3 & 0 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x-2 \\ y-1 \end{bmatrix}$$

(c)

$$f(x,y) \approx \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 3 & 0 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} x-1 \\ y-2 \end{bmatrix}$$

(d)

$$f(x,y) \approx \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 3 & 0 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

5. (4 points) Let P be the parallelogram with corners at $\mathbf{0}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, and $\mathbf{v} + \mathbf{w} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, as shown in Figure 1:

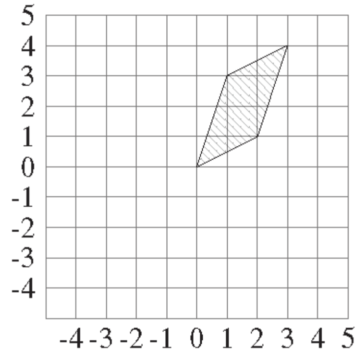
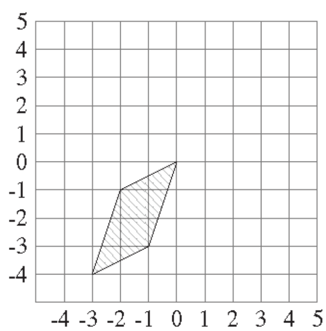


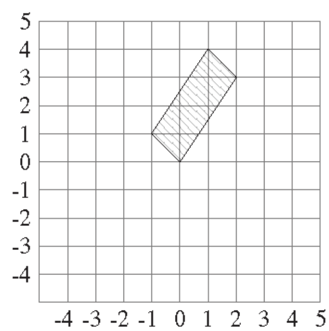
Figure 1: The parallelogram P

For each of the following four matrices M, determine which of the six pictures below shows the output when the corresponding matrix is applied to all points of P.

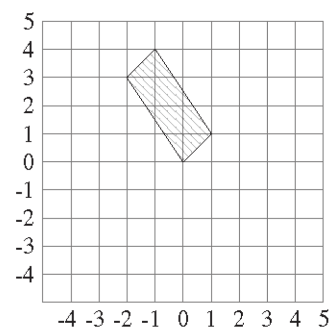
$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \_\_\_\_ \qquad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \_\_\_\_ \qquad C = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix} \_\_\_\_ \qquad D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \_\_\_\_$$



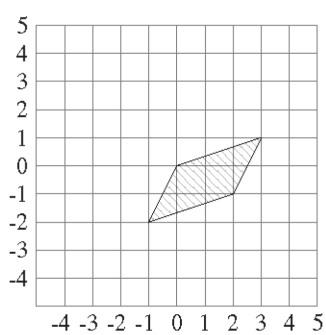
(1)



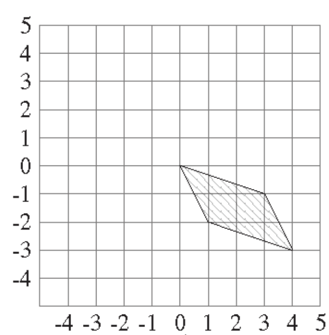
(2)



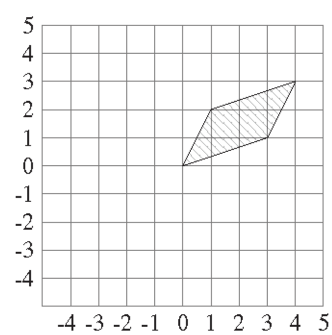
(3)



(4)



(5)



(6)

6. (4 points) Suppose f is a linear function where

$$f\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \qquad f\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \qquad f\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Then $f(\mathbf{x}) = A\mathbf{x}$ where the matrix A must be

a) $\begin{bmatrix} 3 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$ f) $\begin{bmatrix} 3 & 3 \\ 2 & 2 \\ 2 & 1 \end{bmatrix}$

7. (3 points) Suppose f is a linear function where

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} y - z \\ y - x \end{bmatrix}$$

Then $f(\mathbf{x}) = A\mathbf{x}$ where the matrix A must be ____________.

8. (3 points) Suppose the linear transformation $F\colon \mathbf{R}^2 \to \mathbf{R}^2$ first rotates a vector $\begin{bmatrix} x \\ y \end{bmatrix}$ counterclockwise by 45 degrees, *then* stretches it by a factor of 2 in the y-direction. Then $F\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = M\begin{bmatrix} x \\ y \end{bmatrix}$, where the matrix M must be

a) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \sqrt{2} \\ -\frac{\sqrt{2}}{2} & \sqrt{2} \end{bmatrix}$ b) $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\sqrt{2} \\ \frac{\sqrt{2}}{2} & \sqrt{2} \end{bmatrix}$ c) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}$ d) $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$