

1. (10 points) What is the minimum value attained by the function $f(x, y) = y^2 - x + 5$ on the curve $x^3 + 3x - y^2 = 0$?

You may assume that a global minimum exists.

2. (2 points) **True or False:** All scalar-valued functions $f(x, y, z)$ has at least one critical point.

3. (2 points) **True or False:** The circle $x^2 + y^2 = 4$ has two points at which the tangent line is vertical.

4. (3 points) Let

$$f(x, y) = x^2y - y^2 - 2x^2.$$

$f(x, y)$ has at least one critical point (a, b) that is not $(0, 0)$. Find the value of $f(a, b)$.

- a) -8 b) -4 c) 0 d) 4 e) 8

5. (3 points) (Short answer) Let S be the graph of the function $f(x, y) = \ln\left(\frac{y}{x}\right)$ where $x, y > 0$. Suppose the tangent plane to S at the point $P = (a, b, c)$ on S is parallel to the plane $2x - y + 2z = 0$. What are a, b and c ?