Review: Planes in $\mathbb{R}^3$

$\boxed{\text{Problem 1}}$

Let P be the plane in $\mathbb{R}^3$ containing $(1,1,1)$, $(1,2,3)$, and $(3,2,1)$

(a) Find a parametric representation of P.

(b) Use the dot product to find a normal vector to P.

(c) Find an equation for P of the form $ax+by+cz=d$ for a,b,c,d in $\mathbb{R}$.

Problem 2

(a) Consider the distinct points $A=(0,1,1)$, $B=(3,4,4)$, $C=(1,-1,-4)$. Compute the displacement vectors $\vec{AB}$ and $\vec{AC}$ to confirm these are not scalar multiples of one another, and find an equation of the form $ax+by+cz=d$ for the plane they lie in.

(b) Find a $\boxed{\text{unit}}$ vector that is normal to the plane whose equation is $6x-2y-3z=4$.

(c) Are the planes in (a) and (b) parallel? Why?

Review: Spans and subspaces

Problem 3

For each of the following subsets of $\mathbb{R}^2$ or $\mathbb{R}$

write down a collection of finitely many vectors whose span is that set, or explain why there is no such collection.

(a) The line $x+y=1$

(b) The line $x+y=0$

(c) The unit disc $x^2+y^2 \leq 1$

(d) $\{\vec{0}\}$

(e) The plane $x+y+z=0$.

Problem 4

Which of the following subsets S of $\mathbb{R}^3$ are linear subspaces? If S is a linear subspace, write it as a span. If not, describe it geometrically and explain why not.

(a) The set S_1 of points (x, y, z) in $\mathbb{R}^3$ with both $z = x + 2y$ and $z = 5x$.

(b) The set S_2 of points (x,y,z) in $\mathbb{R}^3$ with either $z = x+2y$ or $z = 5x$.

(c) The set S_3 of points (x, y, z) in $\mathbb{R}^3$ of the form

$$t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t' \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$ for some scalars t

and t'.

Problem 5

For each collection of vectors in $\mathbb{R}^2$, sketch its span. Is it a point, line, or all of $\mathbb{R}^2$?

(a) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

For each collection of vectors in $\mathbb{R}^3$, sketch its span. Is it a point, line, plane, or all of $\mathbb{R}^3$

(f) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(g) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(h) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(i) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$