1. (10 points) Consider the function

$$f(x,y) = \ln\left(\frac{y}{x}\right).$$

Compute

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$$

and

$$x^2\frac{\partial^2 f}{\partial x^2} - y^2\frac{\partial^2 f}{\partial y^2}.$$

2. (2 points) Below is a contour plot of a function $g(x,y)$ over the region of points $(x,y)$ with $-3 \le x \le 1$ and $-2 \le y \le 2$, with the dashed line $y = 0$ drawn over it.

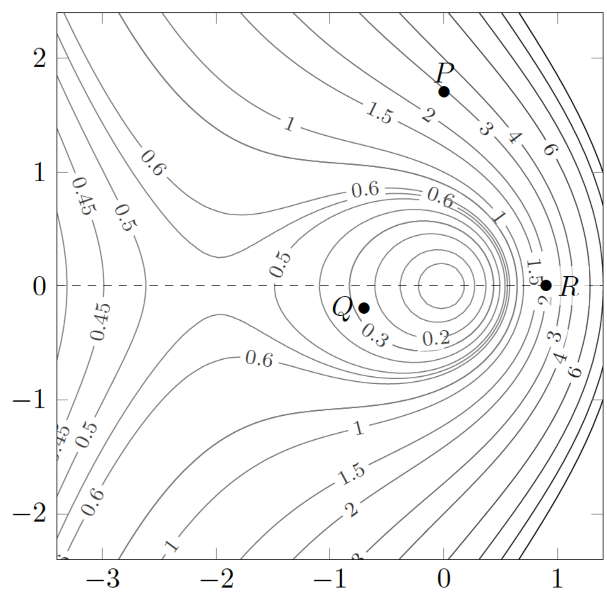



Figure 1: A contour plot for a function $g(x,y)$

For the points labeled $P, Q, R$, determine the if each of $g_x(P), g_y(Q), g_x(R)$, and $g_y(R)$ is positive, negative, or 0.

3. (2 points) Which compositions of the function

$$f(x,y) = (x^2+y^2, x^2-y^2) \text{ and } g(x,y,z) = (xy, xz)$$

are possible?

a) $f \circ g$ is defined, but $g \circ f$ is not.

b) $g \circ f$ is defined, but $f \circ g$ is not.

c) Both $f \circ g$ and $g \circ f$ are defined.

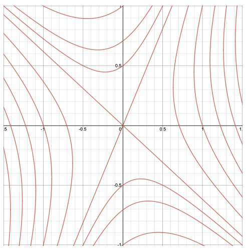
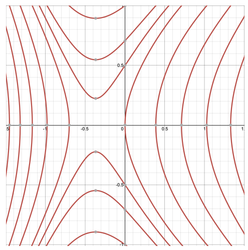
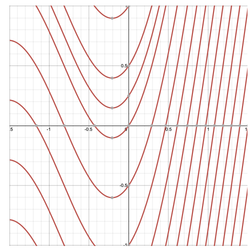
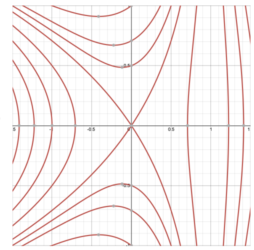
d) Neither $f \circ g$ nor $g \circ f$ is defined.

4. (3 points) Consider a function $f(x, y)$ satisfying

$$\left|\frac{\partial f}{\partial x}(a, b)\right| \neq \left|\frac{\partial f}{\partial x}(-a, -b)\right|$$

for all $(a, b) \in \mathbf{R}^2$. Which contour plot is most likely to correspond to $f(x, y)$?

Note that the contour plots below all have uniform increments in $f$-values: the gaps between $f$-values for successive level curves are the same.

a)  b)  c)  d) 

5. (3 points) The line of best fit for a collection of data points $(x_1, y_1), \ldots, (x_{100}, y_{100})$ is

$$y = -4x + 30.$$

Suppose the $x$-coordinates and the $y$-coordinates have the same mean, i.e. $\bar{x} = \bar{y}$. What is $\bar{x}$?

a) 0 b) 6 c) 7.5 d) 30 e) $-10$