1. (10 points) Suppose $g(x, y)$ is a function where $f(1, 1) = 4$ and linear approximation of $f(x, y)$ near $(1, 1)$ yields

$$f(1.1, 1.1) \approx 4.2, \qquad f(0.9, 1.1) \approx 3.4$$

Estimate $A = f_x(1, 1)$ and $B = f_y(1, 1)$ from these data.

2. (4 points) Suppose $D = \{(x, y) \in \mathbf{R}^2 \mid x^2 + y^2 \leq 1\}$ is the unit disk, and let $g\colon D \to \mathbf{R}$ be the function given by $g(x, y) = \cos(x^2 + y^2)$. Complete the following two statements:

(a) (2 points) g has a critical point at _____ point(s) in the interior of D .

(b) (2 points) g has a constrained local extremum at _____ point(s) on the boundary of D.

Choices for fill-ins:

a) zero b) one c) two d) four e) infinitely many

3. (3 points) Suppose C is the curve in $\mathbf{R}^2$ given by the equation

$$xy^3 - yx^4 = -6$$

At which of the following points P is the tangent line to C at P parallel to the x-axis? Choose all that apply.

a) $(1, -2)$ b) $(2, -3)$ c) $(3, -3)$ d) none of these

4. (3 points) Let

$$g(x, y, z) = x^2 + y^4 + z^6, \qquad f(x, y, z) = 4 + g(x, y, z)$$

How many solution(s) does the Lagrange Multiplier system for maximizing $f(x, y, z)$ under the constraint

$$g(x, y, z) = 2021$$

have?

You may assume that the number of solution(s) is at least 1.

a) 1 b) 2 c) 4 d) 6

e) 2021 f) infinitely many g) not enough information to tell