1. (10 points) Consider the following 3-vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ -4 \\ 7 \end{bmatrix}.$$

Find the triple of scalars (a, b, c) so that the vector $\mathbf{x} = \begin{bmatrix} 8 \\ -7 \\ 16 \end{bmatrix}$ satisfies

$$\mathbf{x} = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3.$$

Compute the product abc.

2. (2 points) **True or False:** Let V be a linear subspace of $\mathbf{R}^n$ and $\mathbf{x}$ a vector in $\mathbf{R}^n$. If $\mathbf{y} = \mathrm{Proj}_V(\mathbf{x})$, then it's always the case that

$$\|\mathbf{x} - \mathbf{y}\|^2 = \|\mathbf{x}\|^2 - \|\mathbf{y}\|^2.$$

3. (2 points) **True or False:** Let $V = \mathrm{span}(\mathbf{v}, \mathbf{w})$, where $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$. The projection of $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ onto V is given by the formula

$$\mathbf{Proj}_V(\mathbf{u}) = \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\mathbf{v} + \frac{\mathbf{u} \cdot \mathbf{w}}{\mathbf{w} \cdot \mathbf{w}}\mathbf{w}.$$

4. (3 points) For each of the following linear subspaces V, determine its dimension.

(A) $V = \mathrm{Span}\left(\begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$

(B) $V = \mathrm{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix} \right)$

(C) $V = \mathrm{Span}\left(\begin{bmatrix} 9 \\ -2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 8 \\ 7 \end{bmatrix} \right)$

5. (3 points) Let $\mathbf{u}$ be a fixed nonzero vector in $\mathbf{R}^3$, $V = \{\mathbf{x} \in \mathbf{R}^3 : \mathbf{x} \cdot \mathbf{u} = 0\}$, and $\mathbf{v}$ a fixed nonzero vector in V. Geometrically, the collection of vectors $\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbf{R}^3$ satisfying the condition

$$\mathbf{Proj}_V(\mathbf{w}) = \mathbf{v}$$

is

a) line.

b) plane.

c) point.

d) $\mathbf{R}^3$.

e) a linear subspace.

f) might take different shapes depending on what $\mathbf{u}$ and $\mathbf{v}$ are.