

1. (10 points) The best line of fit through the 6 data points  $(x_1, y_1), \dots, (x_6, y_6)$

$$(4, -2), (2, -1), (2, 0), (1, 5), (-1, 7), (-2, 9)$$

is

$$y = mx + b.$$

Find  $m$  and  $b$ .

2. (2 points) Let

$$f(x, y) = (y^2 + 1) \cos(x) + x \cos(y) + x$$

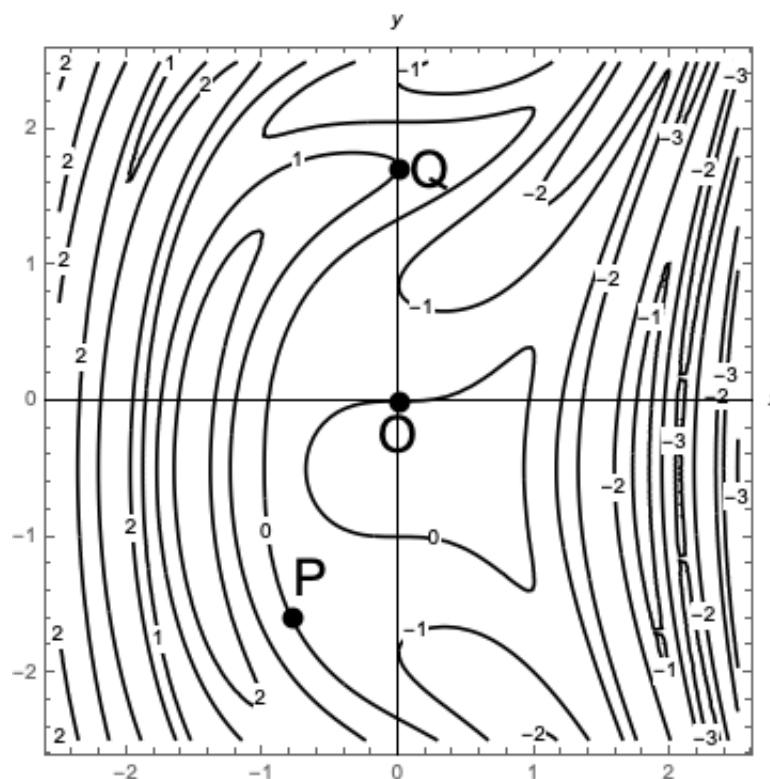
Compute  $\frac{\partial^2 f}{\partial x \partial y}$ . At which points  $(a, b)$  is the value

$$\frac{\partial^2 f}{\partial x \partial y}(a, b) > 0?$$

Select all that apply.

- $(a, b) = (0, 0)$
- $(a, b) = (0, -\pi/2)$
- $(a, b) = (-\pi/2, \pi)$
- $(a, b) = (\pi/2, \pi/2)$
- None of the these four

3. (2 points) Below is a contour plot of a function  $g(x, y)$  over the region of points  $(x, y)$  where  $-2.5 \leq x \leq 2.5$  and  $-2.5 \leq y \leq 2.5$ .



For the points labeled  $O, P, Q$ , determine whether the following is negative, positive or 0.

a)  $\frac{\partial g}{\partial x}(O) = g_x(O)$       b)  $\frac{\partial g}{\partial y}(O) = g_y(O)$       c)  $\frac{\partial g}{\partial x}(P) = g_x(P)$       d)  $\frac{\partial g}{\partial y}(Q) = g_y(Q)$

4. (3 points) Define the functions  $f(x, y)$  and  $g(x, y)$  on  $\mathbf{R}^2$  by

$$f(x, y) = e^x(x \cos(y) - y \sin(y)), \quad g(x, y) = e^x(y \cos(y) + x \sin(y)).$$

Compute  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$ ,  $\frac{\partial g}{\partial x}$ , and  $\frac{\partial g}{\partial y}$ . Answer each of the following two questions:

1. Which among a, b, c, d below equals  $\frac{\partial f}{\partial x}$ ?
2. Which among a, b, c, d below equals  $\frac{\partial f}{\partial y}$ ?

a)  $\frac{\partial g}{\partial x}$       b)  $\frac{\partial g}{\partial y}$       c)  $-\frac{\partial g}{\partial x}$       d)  $-\frac{\partial g}{\partial y}$

5. (3 points) Consider a function  $f(x, y)$  satisfying

$$\left| \frac{\partial f}{\partial y}(a, b) \right| = \left| \frac{\partial f}{\partial y}(a, -b) \right| = \left| \frac{\partial f}{\partial y}(-a, b) \right|$$

for all  $(a, b) \in \mathbf{R}^2$ . Which contour plot is most likely to correspond to  $f(x, y)$ ?

Note that the contour plots below all have uniform increments in  $f$ -values: the gaps between  $f$ -values for successive level curves are the same.

