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Институт компьютерных наук и технологий
Высшая школа программной инженерии

# Лабораторная работа №1
# по дисциплине «Вычислительная математика»

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Санкт-Петербург
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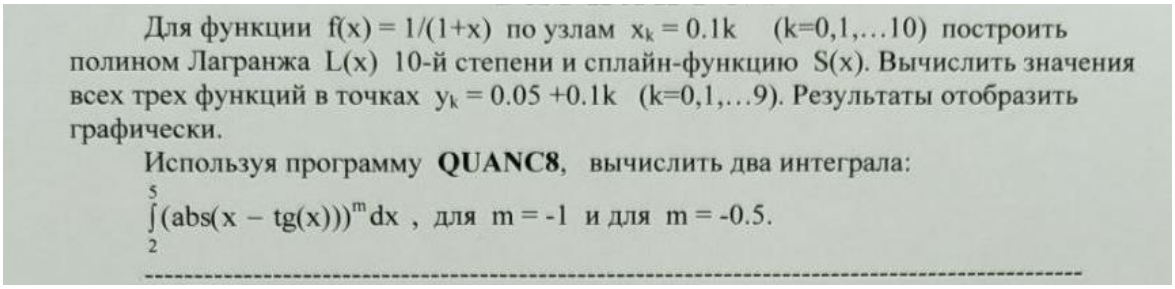
# Оглавление



# Задание



Для функции $f(x) = 1/(1+x)$ по узлам $x_k = 0.1k$ $(k=0,1,\ldots 10)$ построить полином Лагранжа $L(x)$ 10-й степени и сплайн-функцию $S(x)$. Вычислить значения всех трех функций в точках $y_k = 0.05 + 0.1k$ $(k=0,1,\ldots 9)$. Результаты отобразить графически.

Используя программу **QUANC8**, вычислить два интеграла:

$$\int_2^5 (\mathrm{abs}(x - \mathrm{tg}(x)))^m dx \text{ , для } m = -1 \text{ и для } m = -0.5.$$

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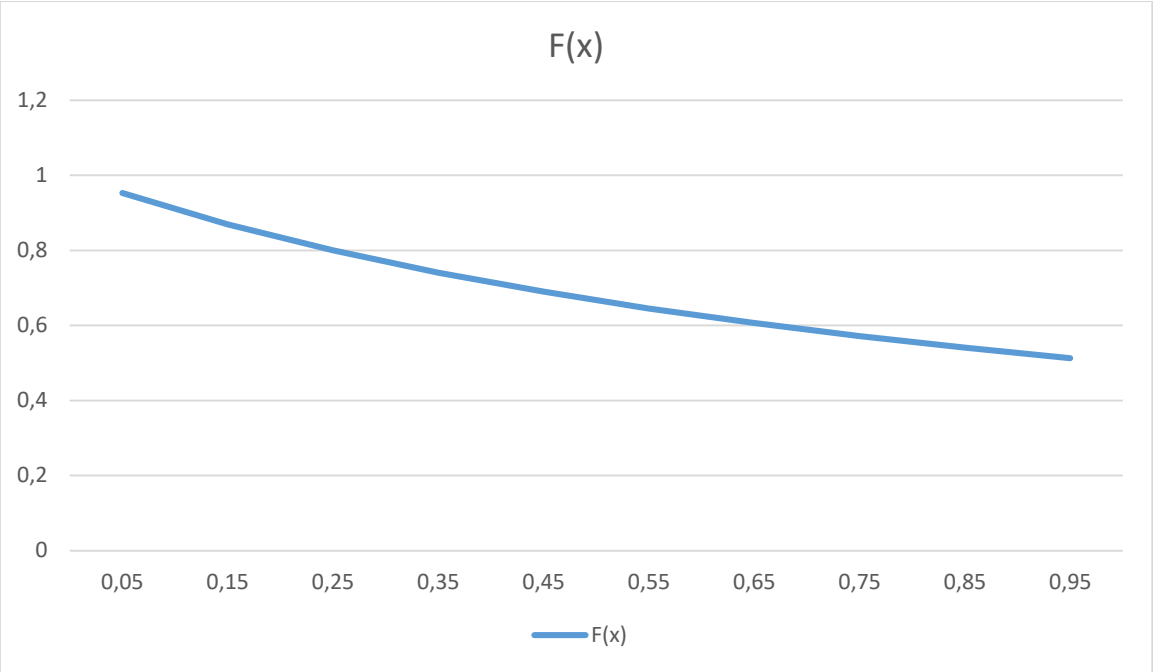
# Результаты

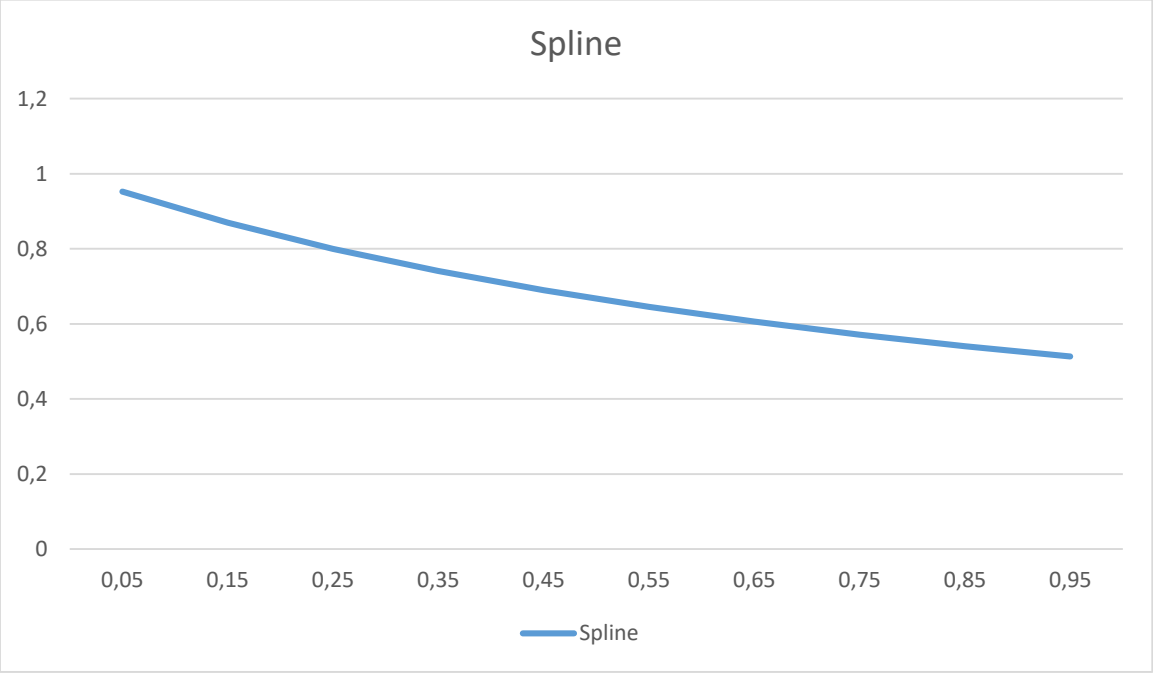
```
X    | Langrage | spline | F(x)
0.05 | 0.952381 | 0.952427 | 0.952381
0.15 | 0.869565 | 0.869549 | 0.869565
0.25 | 0.8 | 0.800002 | 0.8
0.35 | 0.740741 | 0.740738 | 0.740741
0.45 | 0.689655 | 0.689654 | 0.689655
0.55 | 0.645161 | 0.645161 | 0.645161
0.65 | 0.606061 | 0.60606 | 0.606061
0.75 | 0.571429 | 0.571428 | 0.571429
0.85 | 0.540541 | 0.540539 | 0.540541
0.95 | 0.51282 | 0.512824 | 0.512821



Result for m = -1: 1.37281
Error: 5.45525e-05
NoFun: 145
Flag: 0


Result for m = -0.5: 1.75731
Error: 6.48105e-06
NoFun: 145
Flag: 0
```

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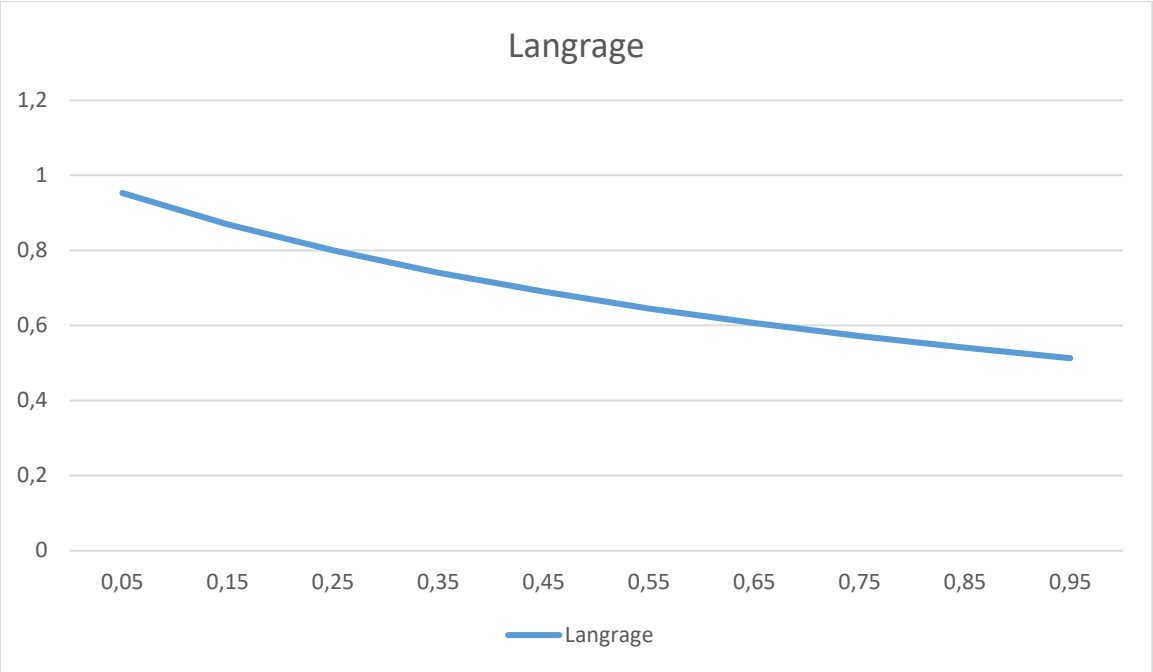


При eps = 0.0001

```
Result for m = -1: 1.37281
Error: 5.45525e-05
NoFun: 145
Flag: 0


Result for m = -0.5: 1.79034
Error: 6.61843e-05
NoFun: 81
Flag: 0
```

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При eps = 0.00001

```
Result for m = -1: 12.3138
Error: 0.00916075
NoFun: 497
Flag: 2


Result for m = -0.5: 1.75731
Error: 6.48105e-06
NoFun: 145
Flag: 0
```

При eps = 0.000001

```
Result for m = -1: 12.3138
Error: 0.00916075
NoFun: 497
Flag: 2


Result for m = -0.5: 1.76514
Error: 4.87317e-07
NoFun: 209
Flag: 0
```

# Вывод

В результате работы видно, что интерполяционный полином Лагранжа приближает лучше, чем интерполирование сплайном, потому что

- Функция гладкая и монотонно убывающая
- В ней нет сильных колебаний
- Точек интерполяции не слишком много и они распределены таким образом, что не вызывают сильных колебаний

# Код программы

<DIR>/computational_mathematics/first_lab/Langrage.cpp

```
#include "Langrage.h"

#include <stdexcept>
#include <cstddef>

dimkashelk::Langrage::Langrage(const std::vector<std::pair<double, double> >
&points):
    points_(points)
{}

double dimkashelk::Langrage::operator()(const double x) const {
    double res = 0.0;
    for (size_t i = 0; i < points_.size(); i++) {
        res += calculateFraction(i, x) * points_[i].second;
    }
    return res;
}

double dimkashelk::Langrage::calculateFraction(const std::size_t index, const
double x) const {
    double res = 1.0;
    double x_cur = points_[index].first;
    for (size_t i = 0; i < points_.size(); i++) {
        if (i != index) {
            res *= (x - points_[i].first) / (x_cur - points_[i].first);
        }
    }
    return res;
}
```

<DIR>/computational_mathematics/first_lab/Langrage.h

```
#ifndef COMPUTATIONAL_MATHEMATICS_LANGRAGE_H
#define COMPUTATIONAL_MATHEMATICS_LANGRAGE_H

#include <vector>

namespace dimkashelk {
    class Langrage {
    public:
        explicit Langrage(const std::vector<std::pair<double, double>>
&points);

        double operator()(double x) const;

    private:
        const std::vector<std::pair<double, double> > points_;
```

```
        double calculateFraction(std::size_t index, double x) const;
    };
}
#endif
```

<DIR>/computational_mathematics/first_lab/Quanc8.cpp

```
#include "Quanc8.h"

#include <algorithm>
#include <cmath>

dimkashelk::Quanc8::Quanc8(const std::function<double(double)> &fun, double
a, double b, double abs_err, double rel_err): fun_(fun),
    a_(a),
    b_(b),
    abs_err_(abs_err),
    rel_err_(rel_err),
    result_(0.0),
    error_(0.0),
    no_fun_(0),
    flag_(0.0)
{
    double QRIGHT[32], F[17], X[17], FSAVE[9][31], XSAVE[9][31];
    int LEVMIN, LEVMAX, LEVOUT, NOMAX, NOFIN, LEV, NIM, J, I;
    double W0, W1, W2, W3, W4, COR11, AREA, X0, F0, STONE, STEP;
    double QLEFT, QNOW, QDIFF, QPREV, TOLERR, ESTERR;

    LEVMIN = 1;
    LEVMAX = 30;
    LEVOUT = 6;
    NOMAX = 5000;
    NOFIN = NOMAX - (8 * (LEVMAX - LEVOUT + static_cast<int>(std::pow(2,
LEVOUT + 1))));

    W0 = 3956.0 / 14175.0;
    W1 = 23552.0 / 14175.0;
    W2 = -3712.0 / 14175.0;
    W3 = 41984.0 / 14175.0;
    W4 = -18160.0 / 14175.0;

    flag_ = 0.0;
    result_ = 0.0;
    COR11 = 0.0;
    error_ = 0.0;
    AREA = 0.0;
    no_fun_ = 0;
    if (a_ == b_) { return; }
    LEV = 0;
    NIM = 1;
```

```
    X0 = a_;
    X[16] = b_;
    QPREV = 0.0;
    F0 = fun_(X0);
    STONE = (b_ - a_) / 16.0;
    X[8] = (X0 + X[16]) / 2.0;
    X[4] = (X0 + X[8]) / 2.0;
    X[12] = (X[8] + X[16]) / 2.0;
    X[2] = (X0 + X[4]) / 2.0;
    X[6] = (X[4] + X[8]) / 2.0;
    X[10] = (X[8] + X[12]) / 2.0;
    X[14] = (X[12] + X[16]) / 2.0;
    for (J = 2; J <= 16; J = J + 2) {
        F[J] = fun_(X[J]);
    }
    no_fun_ = 9;

trenta:
    X[1] = (X0 + X[2]) / 2.0;
    F[1] = fun_(X[1]);
    for (J = 3; J <= 15; J = J + 2) {
        X[J] = (X[J - 1] + X[J + 1]) / 2.0;
        F[J] = fun_(X[J]);
    }
    no_fun_ = no_fun_ + 8;
    STEP = (X[16] - X0) / 16.0;
    QLEFT = (W0 * (F0 + F[8]) + W1 * (F[1] + F[7]) + W2 * (F[2] + F[6]) + W3
* (F[3] + F[5])
             + W4 * F[4]) * STEP;
    QRIGHT[LEV + 1] = (W0 * (F[8] + F[16]) + W1 * (F[9] + F[15]) + W2 *
(F[10] + F[14])
                       + W3 * (F[11] + F[13]) + W4 * F[12]) * STEP;
    QNOW = QLEFT + QRIGHT[LEV + 1];
    QDIFF = QNOW - QPREV;
    AREA = AREA + QDIFF;

    ESTERR = fabs(QDIFF) / 1023.0;
    if (abs_err > (rel_err * fabs(AREA)) * (STEP / STONE))
        TOLERR = abs_err;
    else
        TOLERR = (rel_err * fabs(AREA)) * (STEP / STONE);
    if (LEV < LEVMIN)
        goto cinquanta;
    if (LEV >= LEVMAX)
        goto sessantadue;
    if (no_fun_ > NOFIN)
        goto sessanta;
    if (ESTERR <= TOLERR)
        goto settanta;

cinquanta:
    NIM = 2 * NIM;
```

```
    LEV = LEV + 1;

    for (I = 1; I <= 8; I++) {
        FSAVE[I][LEV] = F[I + 8];
        XSAVE[I][LEV] = X[I + 8];
    }

    QPREV = QLEFT;
    for (I = 1; I <= 8; I++) {
        J = -I;
        F[2 * J + 18] = F[J + 9];
        X[2 * J + 18] = X[J + 9];
    }
    goto trenta;

sessanta:
    NOFIN = 2 * NOFIN;
    LEVMAX = LEVOUT;
    flag_ = flag_ + ((b_ - X0) / (b_ - a_));
    goto settanta;

sessantadue:
    flag_ = flag_ + 1.0;

settanta:
    result_ = result_ + QNOW;
    error_ = error_ + ESTERR;
    COR11 = COR11 + QDIFF / 1023.0;

    while (NIM % 2 != 0) {
        NIM = NIM / 2;
        LEV = LEV - 1;
    }
    NIM = NIM + 1;
    if (LEV <= 0)
        goto ottanta;

    QPREV = QRIGHT[LEV];
    X0 = X[16];
    F0 = F[16];
    for (I = 1; I <= 8; I++) {
        F[2 * I] = FSAVE[I][LEV];
        X[2 * I] = XSAVE[I][LEV];
    }
    goto trenta;

ottanta:
    result_ = result_ + COR11;
    if (error_ == 0.0)
        return;
    while (std::fabs(result_) + (error_) == std::fabs(result_))
        error_ = 2.0 * (error_);
```

```
}

double dimkashelk::Quanc8::getResult() const {
    return result_;
}

double dimkashelk::Quanc8::getError() const {
    return error_;
}

int dimkashelk::Quanc8::getNoFun() const {
    return no_fun_;
}

double dimkashelk::Quanc8::getFlag() const {
    return flag_;
}
```

<DIR>/computational_mathematics/first_lab/Quanc8.h

```
#ifndef QUANC8_H
#define QUANC8_H
#include <functional>

namespace dimkashelk {
    class Quanc8 {
    public:
        /**
         * \brief
         * \param fun user functions with one double argument
         * \param a lower bound of integration
         * \param b upper bound of integration
         * \param abs_err absolute error
         * \param rel_err intermediate error
         */
        Quanc8(const std::function<double (double)> &fun, double a, double b,
double abs_err, double rel_err);
        double getResult() const;
        double getError() const;
        int getNoFun() const;
        double getFlag() const;
    private:
        std::function<double (double)> fun_;
        const double a_;
        const double b_;
        const double abs_err_;
        const double rel_err_;
        double result_;
        double error_;
        int no_fun_;
        double flag_;
```

```
    };
}
#endif
```

<DIR>/computational_mathematics/first_lab/Spline.cpp

```
#include "Spline.h"

#include <stdexcept>

dimkashelk::Spline::Spline(const std::vector<std::pair<double, double> >
&points) {
    std::vector<double> B(points.size());
    std::vector<double> C(points.size());
    std::vector<double> D(points.size());

    const size_t count_minus_1 = points.size() - 1;
    if (points.size() < 2) { throw std::logic_error("Check points"); }
    if (points.size() > 2) {
        D[0] = points[1].first - points[0].first;
        C[1] = (points[1].second - points[0].second) / D[0];
        for (size_t i = 2; i <= count_minus_1; i++) {
            D[i - 1] = points[i].first - points[i - 1].first;
            B[i - 1] = 2.0 * (D[i - 2] + D[i - 1]);
            C[i] = (points[i].second - points[i - 1].second) / D[i - 1];
            C[i - 1] = C[i] - C[i - 1];
        }
        B[0] = -D[0];
        B[points.size() - 1] = -D[points.size() - 2];
        C[0] = 0.0;
        C[points.size() - 1] = 0.0;
        if (points.size() != 3) {
            C[0] = C[2] / (points[3].first - points[1].first) - C[1] /
(points[2].first - points[0].first);
            C[points.size() - 1] = C[points.size() - 2] /
                                   (points[points.size() - 1].first -
points[points.size() - 3].first) -
                                   C[points.size() - 3] /
                                   (points[points.size() - 2].first -
points[points.size() - 4].first);
            C[0] = C[0] * D[0] * D[0] / (points[3].first - points[0].first);
            C[points.size() - 1] = -C[points.size() - 1] * D[points.size() -
2] * D[points.size() - 2] /
                                   (points[points.size() - 1].first -
points[points.size() - 4].first);
        }
        for (size_t i = 2; i <= points.size(); i++) {
            const double current = D[i - 2] / B[i - 2];
            B[i - 1] = B[i - 1] - current * D[i - 2];
            C[i - 1] = C[i - 1] - current * C[i - 2];
```

```
        }
        C[points.size() - 1] = C[points.size() - 1] / B[points.size() - 1];
        for (size_t i = 1; i <= count_minus_1; i++) {
            const size_t d = points.size() - i;
            C[d - 1] = (C[d - 1] - D[d - 1] * C[d]) / B[d - 1];
        }
        B[points.size() - 1] = (points[points.size() - 1].second -
points[count_minus_1 - 1].second) /
                               D[count_minus_1 - 1] + D[count_minus_1 - 1] *
                               (C[count_minus_1 - 1] + 2. * C[points.size() -
1]);
        for (size_t i = 1; i <= count_minus_1; i++) {
            B[i - 1] = (points[i].second - points[i - 1].second) / D[i - 1] -
D[i - 1] * (C[i] + 2. * C[i - 1]);
            D[i - 1] = (C[i] - C[i - 1]) / D[i - 1];
            C[i - 1] = 3.0 * C[i - 1];
        }
        C[points.size() - 1] = 3. * C[points.size() - 1];
        D[points.size() - 1] = D[points.size() - 2];
    } else {
        B[0] = (points[1].second - points[0].second) / (points[1].first -
points[0].first);
        C[0] = 0.0;
        D[0] = 0.0;
        B[1] = B[0];
        C[1] = 0.0;
        D[1] = 0.0;
    }
    points_ = points;
    for (const auto &point: points) {
        a.push_back(point.second);
    }
    b = B;
    c = C;
    d = D;
}

double dimkashelk::Spline::operator()(const double number) const {
    size_t i = 1;
    size_t j = points_.size() + 1;
    do {
        const size_t k = (i + j) / 2;
        if (number < points_[k - 1].first) {
            j = k;
        }
        if (number >= points_[k - 1].first) {
            i = k;
        }
    } while (j > i + 1);
    const double DX = number - points_[i - 1].first;
    return a[i - 1] + DX * b[i - 1] + DX * DX * c[i - 1] + DX * DX * DX * d[i
- 1];
```

```
}
```

<DIR>/computational_mathematics/first_lab/Spline.h

```
#ifndef SPLINE_H
#define SPLINE_H

#include <vector>

namespace dimkashelk {
    class Spline {
    public:
        explicit Spline(const std::vector<std::pair<double, double> >
&points);

        double operator()(double number) const;

    private:
        std::vector<std::pair<double, double>> points_;
        std::vector<double> a_;
        std::vector<double> b_;
        std::vector<double> c_;
        std::vector<double> d_;
    };
}
#endif
```

<DIR>/computational_mathematics/first_lab/main.cpp

```
#include <cmath>
#include <iostream>
#include <vector>
#include <functional>
#include "Langrage.h"
#include "Quanc8.h"
#include "Spline.h"

double f(const double x) {
    return 1 / (1 + x);
}

double func(const double x, const double m) {
    return std::pow(std::abs(x - std::tan(x)), m);
}

int main() {
    std::vector<std::pair<double, double>> function;
    function.reserve(11);
    for (int i = 0; i < 11; i++) {
        function.emplace_back(0.1 * i, f(0.1 * i));
    }
    const dimkashelk::Langrage langrage(function);
```

```
    const dimkashelk::Spline spline(function);
    std::cout << "X   | Langrage | spline | F(x)\n";
    for (int i = 0; i < 10; i++) {
        const double x = 0.05 + 0.1 * i;
        std::cout << x << " | " << langrage(x) << " | " << spline(x) << " | "
<< f(x) << "\n";
    }

    std::cout << "\n\n\n";

    auto f1 = std::bind(func, std::placeholders::_1, -1);
    const dimkashelk::Quanc8 func1(f1, 2, 5, 0.0001, 0.00001);
    std::cout << "Result for m = -1: " << func1.getResult() << "\n"
              << "Error: " << func1.getError() << "\n"
              << "NoFun: " << func1.getNoFun() << "\n"
              << "Flag: " << func1.getFlag() << "\n\n\n";

    auto f2 = std::bind(func, std::placeholders::_1, -0.5);
    const dimkashelk::Quanc8 func2(f2, 2, 5, 0.00001, 0.00001);
    std::cout << "Result for m = -0.5: " << func2.getResult() << "\n"
              << "Error: " << func2.getError() << "\n"
              << "NoFun: " << func2.getNoFun() << "\n"
              << "Flag: " << func2.getFlag() << "\n";
    return 0;
}
```