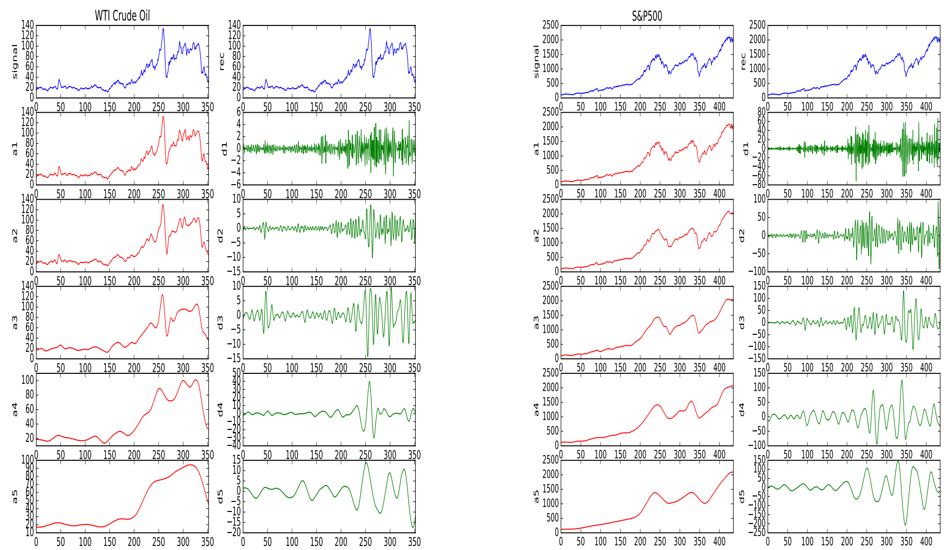
Results

WTI Crude Oil



(a) (b)

Figure 1: (a) Wavelet decomposition into 5 levels for (a) WTI crude oil prices and (b) S&P500 prices.

We first tried to apply the wavelet based method to the oil market, specifically we worked with monthly averaged WTI crude oil spot prices for the period from January 2, 1986 to March 1, 2016. However, despite the claims made in reference "Yousefi, S., Weinreich, I., and Reinarz" we were not successful in beating the futures market to predict future spot prices.

Figure 2 shows the predicted spot oil prices versus actual realized spot oil prices alongside with the corresponding R-values for 1 and 4 months ahead as well as the same plot but for futures prices versus actual realized spot oil prices. R-value can be used as a proxy for predictive power of our method, R-value of 1 corresponds to the perfect prediction, while zero R-value means no predictive power at all. Figure 2 clearly suggests that current futures prices predict future realized spot oil prices better then the wavelet forecasting method, which means that we cannot successfully apply the wavelet decomposition method to oil market.

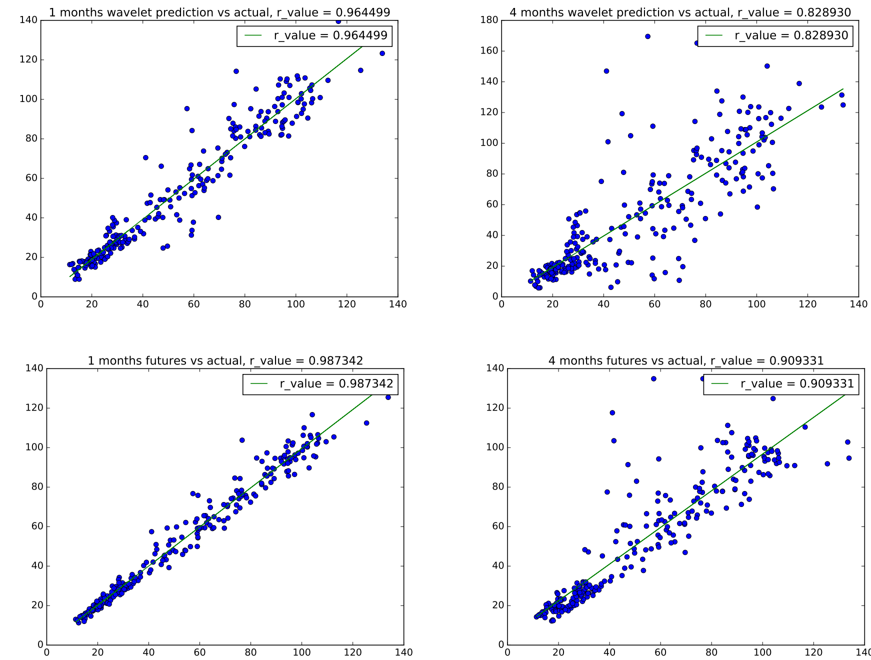


Figure 2: The wavelet predicted spot oil prices versus actual realized spot oil prices and futures prices versus actual realized spot oil prices alongside with the corresponding R-values for 1 and 4 months ahead.

S&P 500

Failed to apply wavelet based predictions to the oil market we were looking to the other asset classes that might be suitable for our procedure. We decided to apply wavelet forecasting to S&P 500 index instead and found the results to be quite promising.

Figure 3 shows the predicted value of S&P 500 index versus actually observed S&P 500 index alongside with the corresponding R-values for 1, 2, 3 and 4 months ahead. We can see that R-values are much higher than those for the oil market suggesting that the wavelet analysis should work for S&P 500 prices. We came up with the following trading strategy.

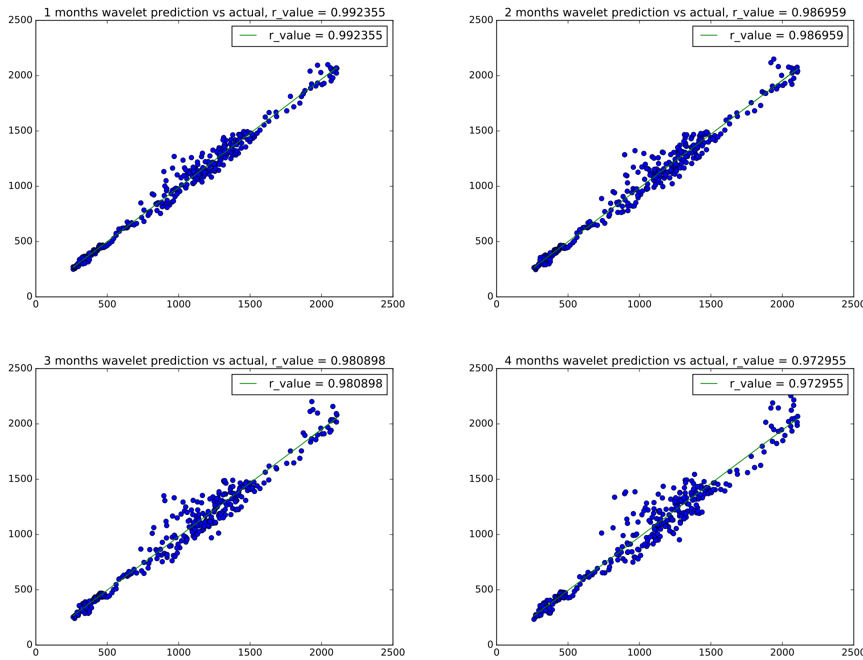


Figure 3: The wavelet predicted S&P 500 prices versus actual realized S&P 500 prices alongside with the corresponding R-values for 1, 2, 3 and 4 months ahead.

Trading strategy

1. *Historical prices.* We got S&P 500 historical monthly prices for the period from January 2, 1980 to April 1, 2016 and divided them into 333 samples of the length 100 each so that the start date of the backtesting trading is April, 1988.

2. *Decomposition.* For each input sample of historical prices we performed 5 level wavelet decomposition (see figure 1 (b))

$$f = a_5 + d_5 + d_4 + d_3 + d_2 + d_1.$$

3. *Extrapolation.* We extrapolated 4 steps into the future the approximation level a_5 and the highest detail level d_5 using spline fitting of order k, while for extrapolation of d_1, d_2, d_3, d_4 we used Fourier fitting with 10 harmonics.

4. *Reconstruction.* We obtained wavelet based prediction of S&P 500 index 4 steps into the future by reconstructing the extrapolated series for a_5, d_5, d_4, d_3, d_2, d_1. Based on this prediction we forecasted the expected return $r_{prediction}$.

Strategy	Sharpe	Annualized return, %	Max drawdown, %	Win rate, %	No trades, %
1 month	1.10	7.86	15.16	72.20	11.11
4 months	0.43	4.06	52.56	57.31	23.72
S&P500	0.60	7.76	51.72	62.46	-

Table 1: Summary for 1 month wavelet strategy, 4 months wavelet strategy and for S&P 500 index.

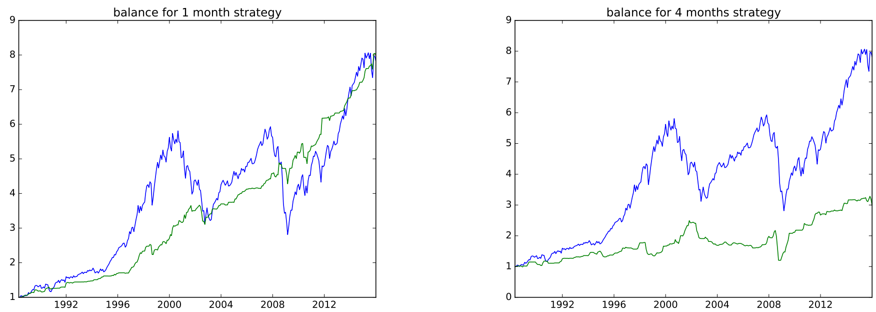
5. *Adjustments.* We found that the expected return is sensitive to the order of spline fitting k used to extrapolate a_5 and d_5, but it is quite robust to the number of harmonics used for Fourier fitting of d_4, d_3, d_2, d_1. To mitigate this issue we did steps 3-4 for several values of parameter k. If different forecasts agreed on the sign of the expected return, then we took their arithmetic average as the predicted return, if forecasts disagreed on the sign of future returns, then we deemed our forecast unreliable and did not trade at that month. To further protect us from misforecasting and potential huge losses we ceased trading whenever the absolute value of the expected return was larger then 7%.

6. *Sizing.*We made a bet based on our prediction of the expected return with the size adjusted according to the Kelly criterion. We made a reasonable assumption that future returns obey normal distribution with the average around $r_{prediction}$ and with standard deviation equal to the historical rolling volatility, we also assumed that possible gain and loss are equal. Then the probabilities of loss and gain can be easily calculated yielding

$$\text{Trade size} = \Pi_{size} \cdot \text{Max Drawdown} \left[1 - 2N \left(-\frac{r_{prediction}}{\sigma_{historical}} \right) \right],$$

where N is the standard normal cumulative distribution function.

7. *Transaction costs.* To model the effect of transaction costs, we introduced simple model with friction of 1% for each trade.



(a) (b)

Figure 4: (a) Value of \$1 invested in 1 month wavelet forecasting strategy (green line) and in S&P 500 index (blue line). (b) Value of \$1 invested in 4 months wavelet forecasting strategy (green line) and in S&P 500 index (blue line).

The results of this strategy are shown in figure 4 and table 1, they provide summary for 1 month wavelet trading strategy, 4 months wavelet trading strategy and for the passive investing in S&P 500 index. We can see that 1 month strategy outperformed the index producing similar cumulative return but with Sharpe ratio almost two times higher and with the maximum drawdown of only approximately 15% instead of 56% for the benchmark. The success rate for the bets made (i.e. excluding months when we did not trade) was approximately 72%, while in 11% we did not trade because we were not confident enough in our predictions (step 5). At the same time 4 months trading strategy failed to outperform the index, it had only 57% win rate and had no trades (unreliable prediction) in about 24% of cases, which is consistent with the intuition that it is harder to predict distant future and so the prediction power diminishes as we try to forecast further into the future.