Computers and math: A trade off

@dnikolos

1 The problem

Find a,b,c,d,e,f,g so that:

$$
\begin{array}{ccccc}
ab & \cdot & bc & = & dad \\
\times & & \div & & \times \\
ec & \div & f & = & f \\
\| & & \| & & \| \\
bcd & \times & g & = & bbea
\end{array}
\tag{1}
$$

From equation 1, we can derive six expressions.

$$
\begin{array}{rcl}
(10 \times a + b) \times (10 \times b + c) & = & (100 \times d + 10 \times a + d) \\
(10 \times e + c)/f & = & f \\
(100 \times b + 10 \times c + d) \times g & = & (1000 \times b + 100 \times b + 10 \times e + a) \\
(10 \times a + b) \times (10 \times e + c) & = & (100 \times b + 10 \times c + d) \\
(10 \times b + c)/f & = & g \\
(100 \times d + 10 \times a + d) \times f & = & (1000 \times b + 100 \times b + 10 \times e + a)
\end{array}
\tag{2}
$$

2 First solution

The first and slow solution is to check all possible combinations against the equations. Since f appears is the denominator of $bc \div f$ it cannot be zero. Here is the first solution written in Python:

```python
for a in range(0,10):
  for b in range(0,10):
    for c in range(0,10):
      for d in range(0,10):
        for e in range(0,10):
          for f in range(1,10):
            for g in range(0,10):
              if ((10*a+b)*(10*b+c)==(d*100+a*10+d)):
                if ((10*e+c)/f == f):
                  if ((100*b+10*c+d)*g==1000*b+100*b+10*e+a):
                    if ((10*a+b)*(10*e+c)==(100*b+10*c+d)):
                      if ((10*b+c)/f == g):
                        if ((100*d+10*a+d) * f == (1000*b+100*b+10*e+a)):
                          print(''.join([str(a),str(b),str(c),
                            str(d),str(e),str(f),str(g)]))
```

Now we have the answer which is $a = 2, b = 3, c = 6, d = 8, e = 1, f = 4, g = 9$ but this method is rather slow. In `http://www.codeskulptor.org/#user37_v9I1M7qi3q_0.py`, it exceeds time limit for my system.

3   Second solution

If we start solving it we will see that the equation $(10 \times e + c)/f = f$ is the easiest to begin with. We get that $10 \times e + c = f^2$ and we can compute e and c for $f = 1..9$. For example if $f = 4, f^2 = 16$, $e = 1$ and $c = 6$. If we incorporate this in the Python program we get:

```
posf = [1,2,3,4,5,6,7,8,9]
pose = [0,0,0,1,2,3,4,6,8]
posc = [1,4,9,6,5,6,9,4,1]
for (i,f) in enumerate(posf):
  e = pose[i]
  c = posc[i]
  for a in range(0,10):
    for b in range(0,10):
      for d in range(0,10):
        for g in range(0,10):
          if ((10*a+b)*(10*b+c)==(d*100+a*10+d)):
            if ((10*e+c)/f == f):
              if ((100*b+10*c+d)*g==1000*b+100*b+10*e+a):
                if ((10*a+b)*(10*e+c)==(100*b+10*c+d)):
                  if ((10*b+c)/f == g):
                    if ((100*d+10*a+d) * f == (1000*b+100*b+10*e+a)):
                      print(''.join([str(a),str(b),str(c),str(d),str(e),
                        str(f),str(g)]))
```

This program is faster and in `http://www.codeskulptor.org/#user37_v9I1M7qi3q_1.py` it runs and gives the solution.