Computers and math: A trade off

@dnikolos

1 The problem

Find a,b,c,d,e,f,g so that:

$$\begin{array}{ccccc} ab & \cdot & bc & = & dad \\ \times & & \div & & \times \\ ec & \div & f & = & f \\ \shortparallel & & \shortparallel & & \shortparallel \\ bcd & \times & g & = & bbea \end{array} \tag{1}$$

From equation 1, we can derive six expressions.

$$\begin{array}{rcl} (10\times a+b)\times(10\times b+c) & = & (100\times d+10\times a+d) \\ (10\times e+c)/f & = & f \\ (100\times b+10\times c+d)\times g & = & (1000\times b+100\times b+10\times e+a) \\ (10\times a+b)\times(10\times e+c) & = & (100\times b+10\times c+d) \\ (10\times b+c)/f & = & g \\ (100\times d+10\times a+d)\times f & = & (1000\times b+100\times b+10\times e+a) \end{array} \tag{2}$$

2 First solution

The first and slow solution is to check all possible combinations against the equations. Since f appears as the denominator of $bc\div f$ and it cannot be zero. Here is the first solution written in Python:

```
for a in range(0,10):
  for b in range(0,10):
    for c in range(0,10):
      for d in range(0,10):
        for e in range(0,10):
          for f in range(1,10):
            for g in range(0,10):
              if (((10*a+b)*(10*b+c)==d*100+a*10+d)
                and ((10*e+c)/f == f)
                and ((100*b+10*c+d)*g==1000*b+100*b+10*e+a)
                and ((10*a+b)*(10*e+c)==100*b+10*c+d)
                and ((10*b+c)/f == g)
                and ((100*d+10*a+d) * f == 1000*b+100*b+10*e+a)):
                  print(' '.join([str(a),str(b),str(c),str(d),str(e),
                    str(f),str(g)]))
```

Now we have the answer which is $a=2$,$b=3$,$c=6$,$d=8$,$e=1$,$f=4$,$g=9$ but this method is rather slow. In `http://www.codeskulptor.org/#user37_ceiMDMtEWQ_0.py`, it exceeds time limit for my system.

3 Second solution

If we start solving it we will see that the equation $(10 \times e + c)/f = f$ is the easiest to begin with. We get that $10 \times e + c = f^2$ and we can compute e and c for $f = 1..9$. For example if $f = 4$,$f^2 = 16$, $e = 1$ and $c = 6$. If we incorporate this in the Python program we get:

```
for a in range(0,10):
  for b in range(0,10):
    for d in range(0,10):
      for f in range(1,10):
        for g in range(0,10):
          c = (f*f) % 10
          e = (f*f) / 10
          if (((10*a+b)*(10*b+c)==d*100+a*10+d)
            and ((10*e+c)/f == f)
            and ((100*b+10*c+d)*g==1000*b+100*b+10*e+a)
            and ((10*a+b)*(10*e+c)==100*b+10*c+d)
            and ((10*b+c)/f == g)
            and ((100*d+10*a+d) * f == 1000*b+100*b+10*e+a)):
              print(''.join([str(a),str(b),str(c),str(d),str(e),
                str(f),str(g)]))
```

This program is faster and in `http://www.codeskulptor.org/#user37_SjLnh6PxQs_0.py` it runs and gives the solution.