# Chapter 2

# Boolean Algebra and Logic Gates

# Topics

- Boolean Algebra (Analysis Tools)
  - Boolean Expressions: functions
  - Truth Tables
  - Boolean Identities
  - Standard Forms: Sum of products (SOP) and Product of sums (POS)
- Logic Gates (Hardware)
  - Basic: AND, OR, NOT gates and Binary signals
  - Other gates: NOR, NAND, XOR, XNOR
  - Implementation of Boolean expressions
- Examples
  - Half-Adder, Full-Adder, Deriving SOP/POS from Truth Tables, Simplifying SOP/POS with Boolean Algebra

# Binary Logic

- Binary logic deals with

1 - **Variables** that can take on **two** discrete **values**

→*Values can be called **True**, **False**, **yes**, **no**, etc.*

2 - **Operations** that assume **LOGICAL** Meaning

→ *Binary logic is equivalent to **Boolean algebra***

# Boolean Algebra

•Basic mathematics required for the description of digital circuits

• Used to describe the different interconnections of digital circuits

• the variable used in the Boolean algebra are called Boolean variables

→ We will study two-valued Boolean algebra and functions with simplifications using basic Boolean Identities

# Two-valued Boolean Algebra

- It consists of

**1- Boolean Variables**

- Designated by letters of the alphabet such as A, B, C, x, y, z etc.

- Each variable can have two and only two distinct values: 1 and 0 (True, False)

- Can be a Function of some other Boolean variables (F=ABC)

**2- Boolean Operations**

-There are three Basic logical operations:

AND, OR, and NOT

# Basic Boolean Operations- *AND operation*

- AND operator is a dot or by the absence of an operator

*Example:* $x \bullet y =$ *or* $xy = z$

*read:* *x AND y is equal to z*

*Interpretation:* $z = 1$ *if and only if* $x = 1$ *AND* $y = 1$

*Otherwise* $z = 0$

Truth table:

| x | y | xy |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

*Don't confuse this with binary multiplication operation*

*Truth table gives the value of xy (i.e. x•y) for all possible values of x and y*

# **Basic Boolean Operations-** *OR operation*

• *OR Operator is a plus sign (+)*

*Example:* $x + y = z$

*read:* *x OR y is equal to z*

*Interpretation:* $z = 1$ *if* $x= 1$ *or if* $y= 1$ *or if both* $x =1$ *and* $y =1$. $z = 0$ *if* $x = 0$ *and* $y=0$

Truth table:

| x | y | x+y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

*Don't confuse this with binary addition operation*

*Truth table gives the value of x+y for all possible values for x and y*

# **Basic Boolean Operations-** *NOT operation*

- *Represented by a prime or an overbar (also called complement)*

*Example:* $x' = z \quad (\text{or } \overline{x} = z)$

*read:* *Not x is equal to z*

*Interpretation:* $z =$ *"what x is not"*

$x = 1$ *then* $z=0$; $x = 0$ *then* $z=1$

Truth table:

| $x$ | $x'$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

*Truth table gives the value of $x'$ for all possible values for $x$*

# Binary Logic and Binary Signals

- For simplicity, we often still write digits instead:

  - 1 is true
  - 0 is false

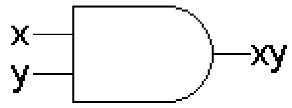
- We will use this interpretation along with special operations to *design functions* and *logic circuits* for doing arbitrary computations.

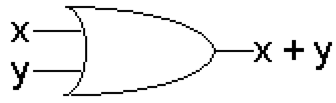
# Logic Gates

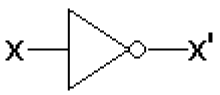
- **Logic gates are electronic circuits that operate on one or more input signal to produce an output signal**
- Basic operations can be implemented in hardware using a Basic logic gate.
  - Symbols for each of the logic gates are shown below.
  - These gates output the product, sum or complement of their inputs

| | | | |
|---|---|---|---|
| Logic Operation: | AND (product) of two inputs | OR (sum) of two inputs | NOT (complement) With one input |
| Representation: | x.y, or xy | x + y | x' |

Logic gate:

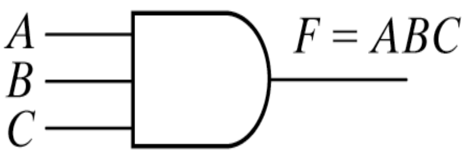


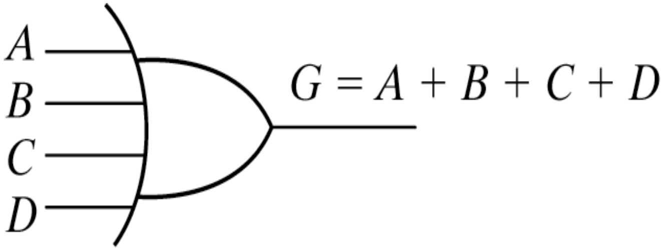



# Gates with Multiple Inputs

- AND and OR Gates may have more than 2 input signals




(a) Three-input AND gate

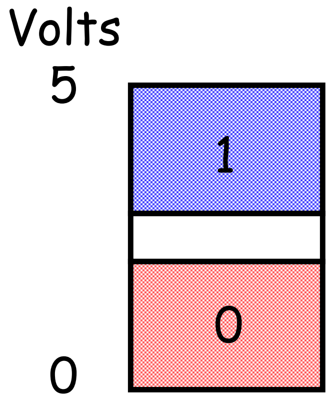



(b) Four-input OR gate

# Binary Signals

- Computers use voltages to represent information.
- Two voltage levels are used to represent a binary value "1" and "0"
- Some digital systems for example may define that:
  - Binary '0" is equal to 0 Volt
  - Binary "1" is equal to 5 Volt

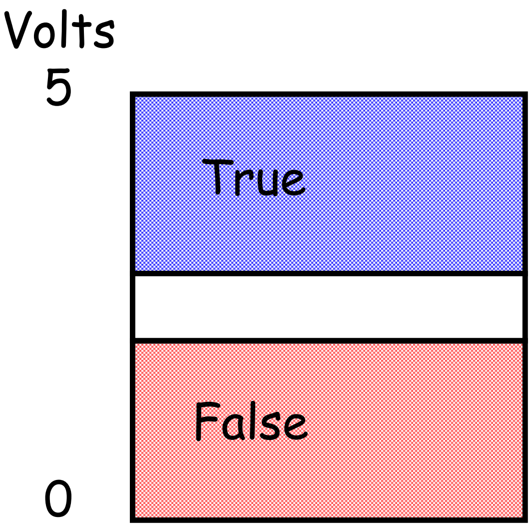
→ *It's convenient for us to translate these voltages into values 1 and 0.*



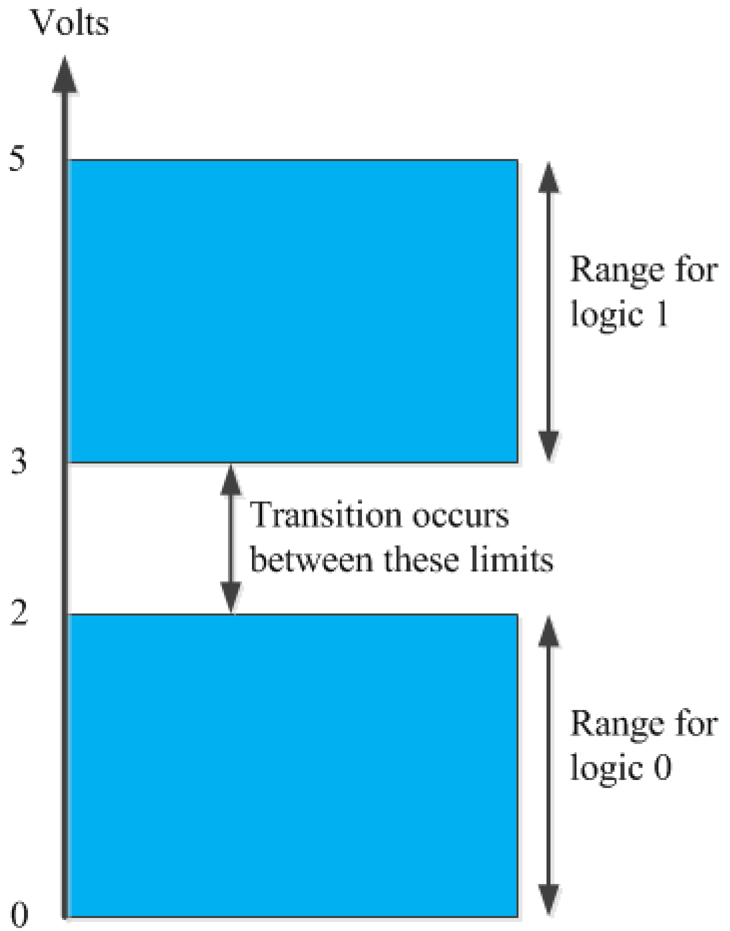
# Binary Logic and Binary Signals

- It's also possible to think of voltages as representing two *logical* values, ***true*** and ***false***.

→ These logical values are called Boolean values

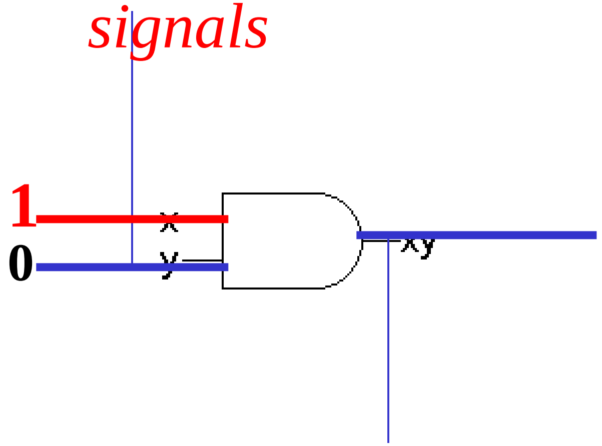


# Logic Gates - Signals

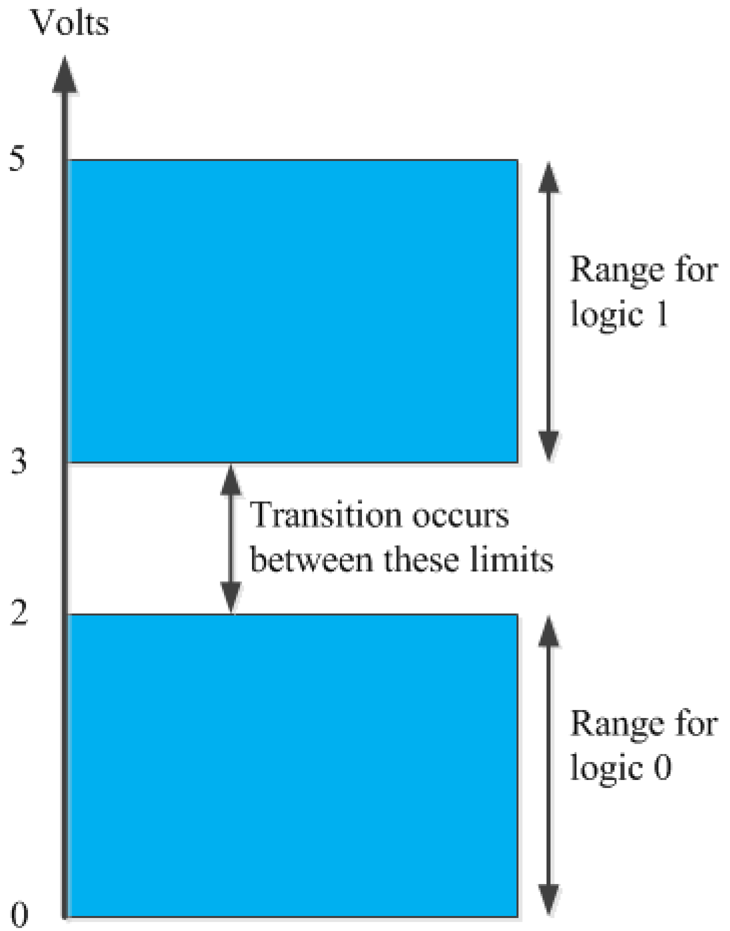



Binary Signal

*Example*

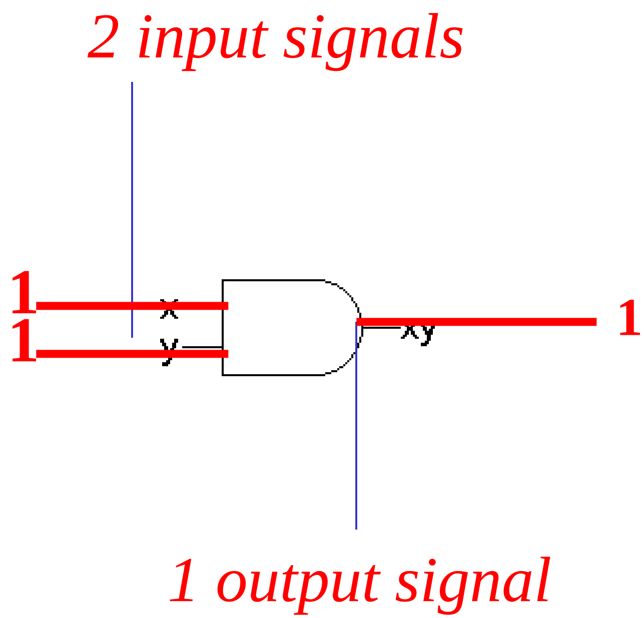


# Logic Gates - Signals

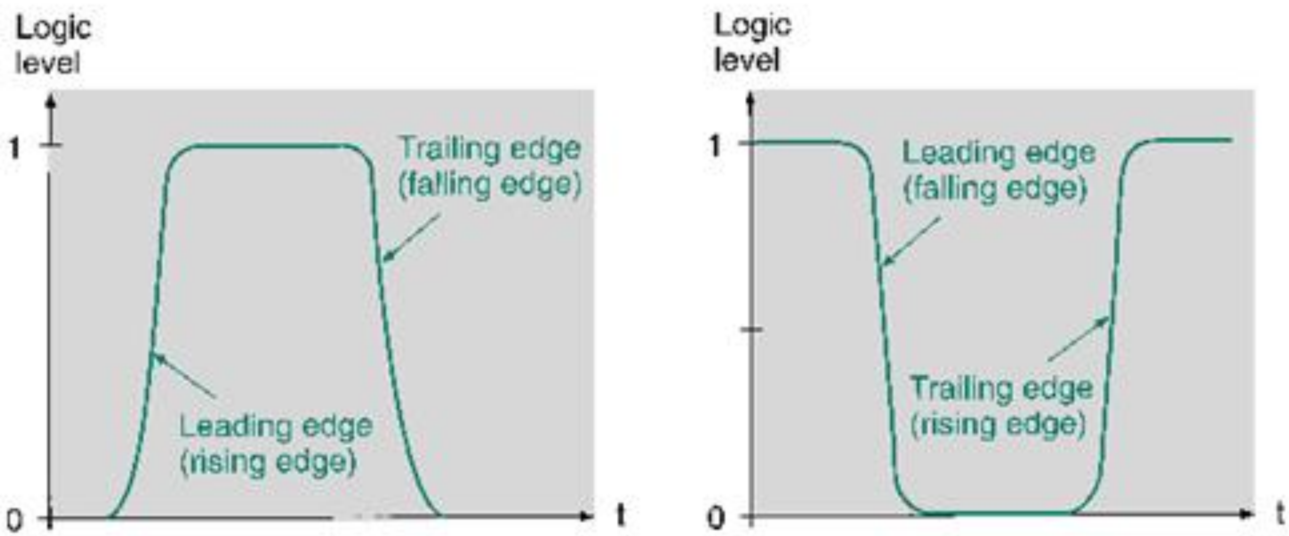



Binary Signal

*Example*



# Timing Diagram –Input and output signals




a. Positive-going pulse    b. Negative-going pulse

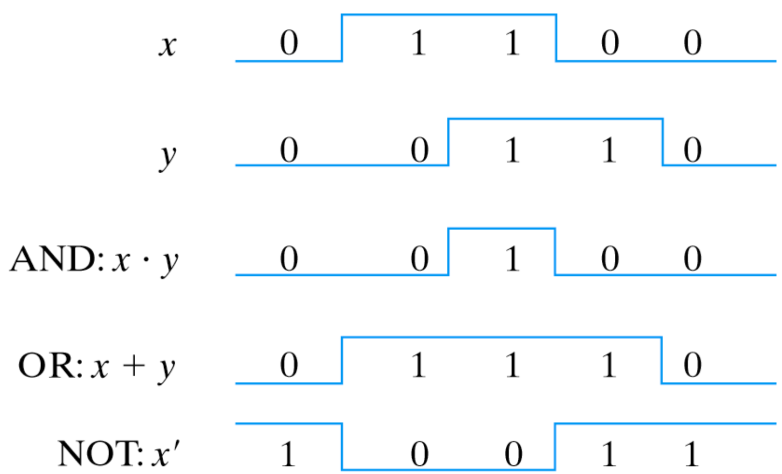



Fig. 1-5 Input-output signals for gates

# Boolean expressions (functions)

- **We can use the basic operations to form more complex expressions:**

$$f(x,y,z) = x\,y' + z\,x'$$

- **Some terminology and notation:**
  - **f** is the name of the function.
  - ***Term*** is an implementation with a gate (e.g. AND term, OR term): in this example f has two AND terms $x\,y'$ and $z\,x'$
  - ***( x,y,z )*** are the ***input variables***, each representing 1 or 0.
  - *A* ***literal*** is any occurrence of an input variable or its complement. The function above has four literals: $x$, $y'$, $z$, and $x'$.

# Precedence for Evaluation of Boolean Expression

- Precedence are important.
  - Parentheses first (if any) then

    NOT has the highest precedence, followed by AND, and then OR.

    $\rightarrow f(x,y,z) = (x + y')z + x'$
  - Fully parenthesized, the function above would be kind of messy:

$$f(x,y,z) = (((x + (y'))z) + x')$$

# Truth Table

- A truth table shows all possible inputs and outputs of a function. Each input variable represents either 1 or 0.
- A function with n variables has 2 power n possible combinations of inputs.
- Inputs are listed in binary order-example,  from 000 to 111.

$f(x,y,z) = (x + y')z + x'$

$f(0,0,0) = (0 + 1)0 + 1 = 1$
$f(0,0,1) = (0 + 1)1 + 1 = 1$
$f(0,1,0) = (0 + 0)0 + 1 = 1$
$f(0,1,1) = (0 + 0)1 + 1 = 1$
$f(1,0,0) = (1 + 1)0 + 0 = 0$
$f(1,0,1) = (1 + 1)1 + 0 = 1$
$f(1,1,0) = (1 + 0)0 + 0 = 0$
$f(1,1,1) = (1 + 0)1 + 0 = 1$

| x | y | z | f(x,y,z) |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

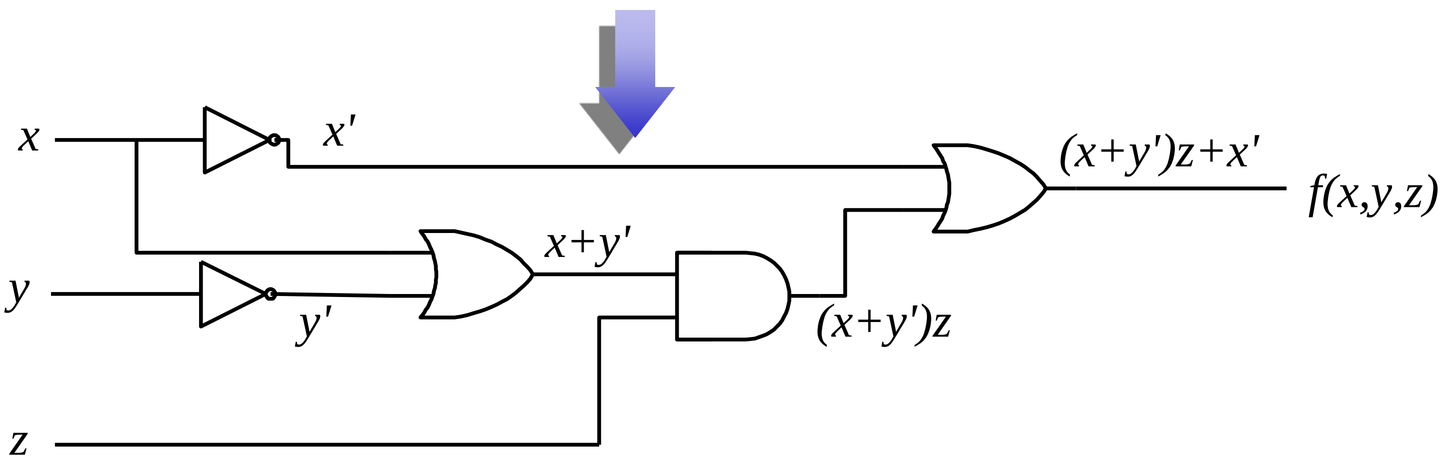
# Boolean Expression and Logic Circuits

- *A* Boolean expression (function) can be converted into a circuit by *combining* basic gates.
- Example:

  - The diagram below shows the inputs and outputs of each gate.

  - The precedences are explicit in a circuit.

**f(x,y,z) = (x + y')z + x'**



# Obtaining Boolean Expressions

- **Expressions may be obtained from:**
  - English language description
  - Truth table;
  - Logic circuit.

# Obtaining Boolean Expressions

- The Boolean expression (un-simplified) can be obtained from the truth table: Consider the following arbitrary Truth Table

| A | B | C | $F_1$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| 0 | 0 | 1 | 0 | |
| 0 | 1 | 0 | 1 | $A'BC'$ |
| 0 | 1 | 1 | 1 | $A'BC$ |
| 1 | 0 | 0 | 1 | $AB'C'$ |
| 1 | 0 | 1 | 1 | $AB'C$ |
| 1 | 1 | 0 | 1 | $ABC'$ |
| 1 | 1 | 1 | 1 | $ABC$ |

We can also write the function as:

$$F_1(A,B,C) = A'BC' + A'BC + AB'C' + AB'C + ABC' + ABC$$

# Obtaining Boolean Expressions

## *Using the false terms in the truth table*

•Sometimes it is easier to work with the terms that describe when the function is false.

•For example, if a function has four variables then there are sixteen possible states.

→If for instance thirteen out of sixteen were true, then only three out of sixteen are false. Fewer terms makes it easier

• In our example, two out of eight are false.

# Obtaining Boolean Expressions

- The Boolean expression (un-simplified) can be obtained from the truth table using false terms

So we can also write the function **NOT** $F_1$ as:

$$F_1'(A,B,C) = A'B'C' + A'B'C$$

| A | B | C | $F_1$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | *A'B'C'* |
| 0 | 0 | 1 | 0 | *A'B'C* |
| 0 | 1 | 0 | 1 | |
| 0 | 1 | 1 | 1 | |
| 1 | 0 | 0 | 1 | |
| 1 | 0 | 1 | 1 | |
| 1 | 1 | 0 | 1 | |
| 1 | 1 | 1 | 1 | |

# Boolean Identities

- Boolean algebra is used in digital design to reduce any logical function (expression) to its simplest form

→ the minimization of the number of literals and the number of terms

→ a circuit with less equipment

- It is a hard problem (no specific rules to follow)

# Boolean Identities

| | | |
|---|---|---|
| 1. | $x + 0 = x$ | |
| 2. | $x \cdot 1 = x$ | |
| 3. | $x + x' = 1$ | |
| 4. | $x \cdot x' = 0$ | |
| 5. | $x + x = x$ | Basic to Boolean algebra |
| 6. | $x \cdot x = x$ | |
| 7. | $x + 1 = 1$ | |
| 8. | $x \cdot 0 = 0$ | |
| 9. | $(x')' = x$ | |
| 10. | $x + y = y + x$ | Commutative |
| 11. | $xy = yx$ | Commutative |
| 12. | $x + (y + z) = (x + y) + z$ | Associative |
| 13. | $x(yz) = (xy)z$ | Associative |
| 14. | $x(y + z) = xy + xz$ | Distributive |
| 15. | $x + yz = (x+y)(x+z)$ | Associative |
| 16. | $(x + y)' = x'y'$ | DeMorgan |
| 17. | $(xy)' = x' + y'$ | DeMorgan |
| 18. | $x + xy = x$ | Absorption |
| 19. | $x(x + y) = x$ | Absorption |

# Verifying Boolean Identities-Examples

Theorem : $x+x = x$

$$
\begin{aligned}
x+x &= (x+x)\,1 \\
&= (x+x)\,(x+x') \\
&= x+xx' \\
&= x+0 \\
&= x
\end{aligned}
$$

$x \bullet 1 = x$

$x + x' = 1$

$x+yz = (x+y)(x+z)$

$x \bullet x' = 0$

$x + 0 = x$

Theorem : $x\,x = x$

$$
\begin{aligned}
xx &= x\,x + 0 \\
&= xx + xx' \\
&= x\,(x + x') \\
&= x\,1 \\
&= x
\end{aligned}
$$

# Verifying Boolean Identities-Examples

- DeMorgan's Theorems
  - → $(x+y)' = x'\,y'$
  - → $(x\,y)' = x' + y'$
- By means of truth table

| x | y | x+y | (x+y)′ | x′ | y′ | x′y′ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

# Verifying Boolean Identities-Examples

- Theorem $x + xy = x$

By means of truth table

| x | y | xy | x + xy |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

# Simplifying Boolean Expressions

## → Use the Rules of Boolean Algebra

We can simply the function as:

$$F_1(A,B,C) = A'BC' + A'BC + AB'C' + AB'C + ABC' + ABC$$

$$= A'B(C'+C) + AB'(C+C') + AB(C'+C)$$

$$= A'B + AB' + AB$$

$$= A'B + A(B'+B)$$

$$= A + A'B$$

Is it really the
simplest xpression?

# Simplifying Boolean Expressions

## Function with four variables

- Giving the following function:

$$
\begin{aligned}
F_{2a}(A,B,C,D) &= (\,AB'\,(C + BD) + A'B'\,)\,C \\
&= (\,AB'C + \mathbf{AB'BD} + A'B'\,)\,C \\
&= (\,AB'C + \mathbf{A0D} + A'B'\,)\,C \\
&= (AB'C + \mathbf{0} + A'B'\,)\,C \\
&= (AB'C + A'B'\,)\,C \\
&= AB'\mathbf{CC} + A'B'C \\
&= AB'C + A'B'C \\
&= (\mathbf{A+A'})B'C \\
&= B'C
\end{aligned}
$$

$$\boldsymbol{F_{2b}(A,B,C,D)} = B'C$$

→ the two expressions are equivalent!

→ **$F_{2a}$ requires more logic gates than $F_{2b}$**

# Basic and Other Logic gates

- **Basic Logic gate**
  - AND
  - OR
  - NOT

These are called "fundamental logic gates" as all other gates and digital Circuits can be created from these gates.

- **Other Logic gates**
  - NAND
  - NOR

These are called "Universal logic gates" as any digital circuit can be designed by just using these gates

  - XOR
  - XNOR

# The NAND & NOR Gates

• We can use a NAND and NOR gates to implement all three of the *basic operations* (AND,OR,NOT).

→ They are said to be **functionally complete**

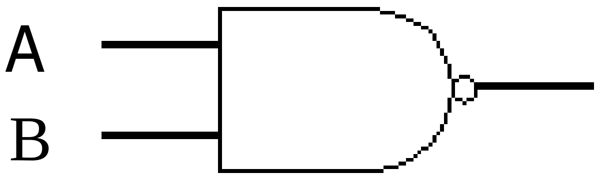
→ Both NAND and NOR gates are very valuable as any design can be realized using either one.

•It is easier to build digital circuits using all NAND or NOR gates than to combine AND,OR, and NOT gates.

•NAND/NOR gates are typically faster and cheaper to produce.

# The NAND Gate

- The NAND gate is a combination of an AND gate followed by an inverter (NOT gate).
- We can use a NAND gate to implement all three of the *basic operations* (AND,OR,NOT).
- Such a gate is said to be **functionally complete**.

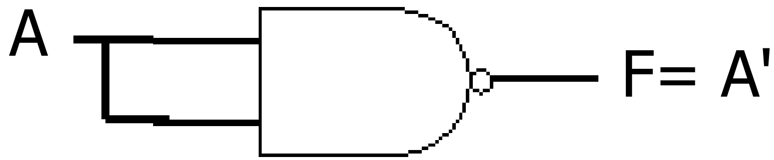



F= (A•B)' = A'+B'

| **A** | **B** | **F** |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

# The NAND Gate

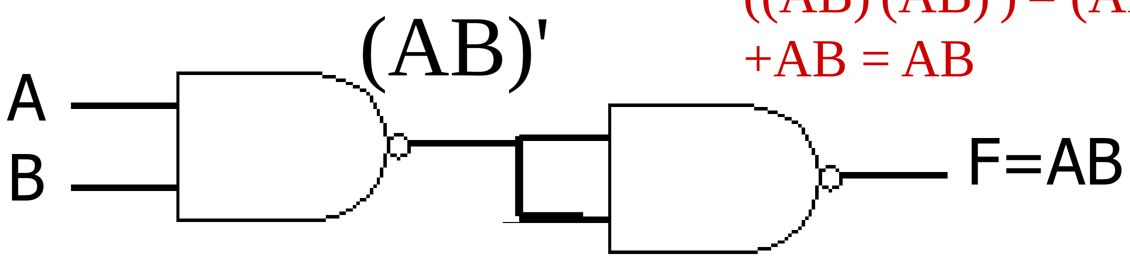
→ a NAND gate with both of its inputs driven by the same signal is equivalent to a NOT gate




NOT Gate

→a NAND gate whose output is complemented is equivalent to an AND gate

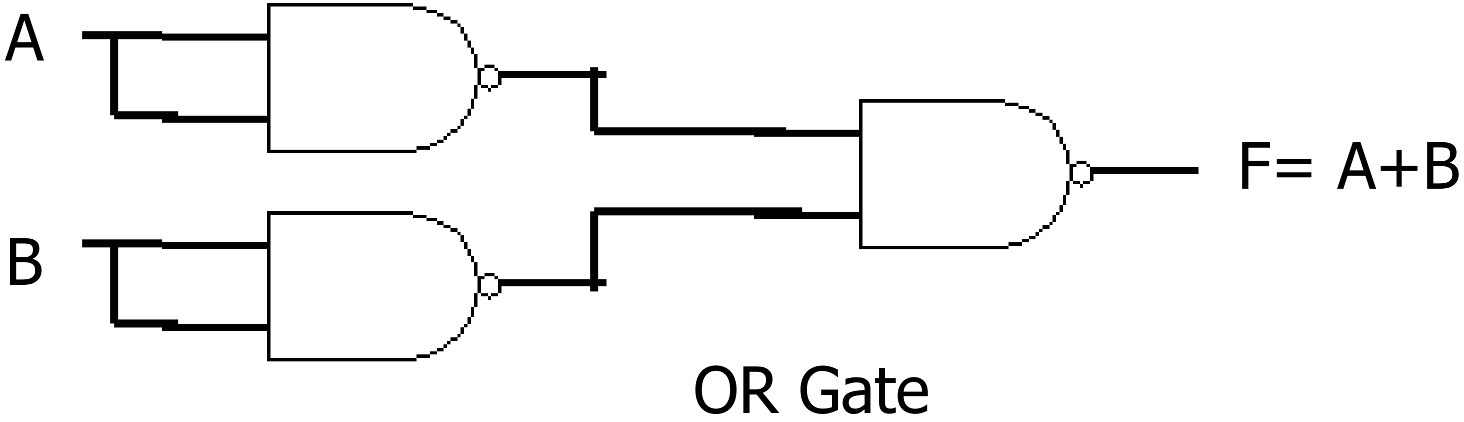
$((AB)'(AB)')' = (AB)'' + (AB)'' = AB + AB = AB$



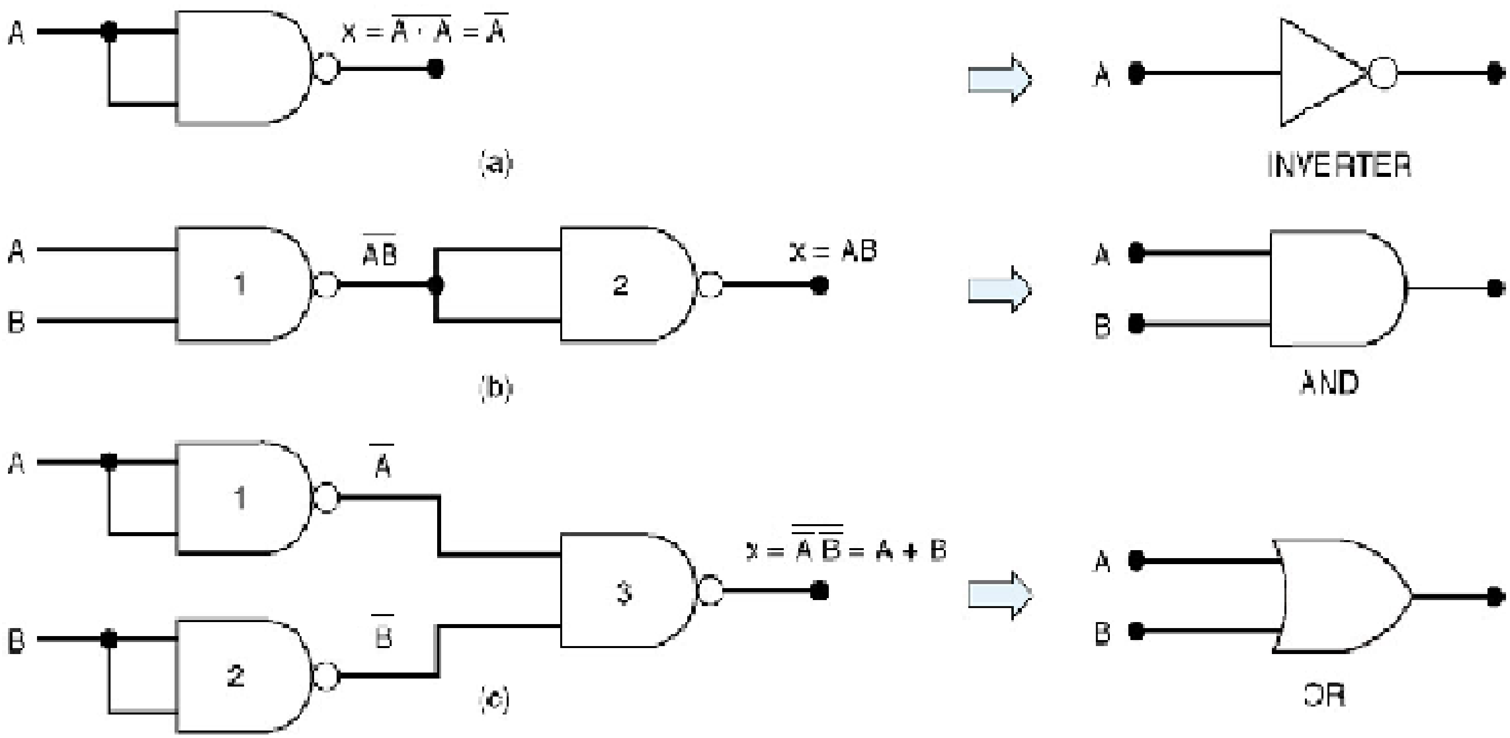

AND Gate

# The NAND Gate

- a NAND gate with complemented inputs acts as an OR gate.

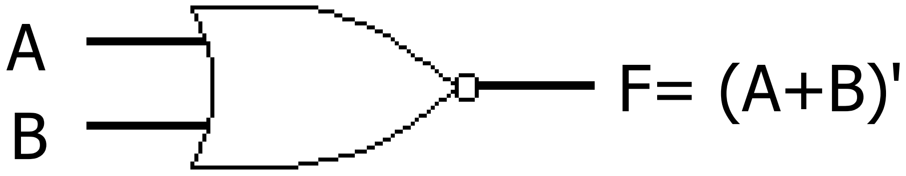


# Universality of NAND



# The NOR Gate

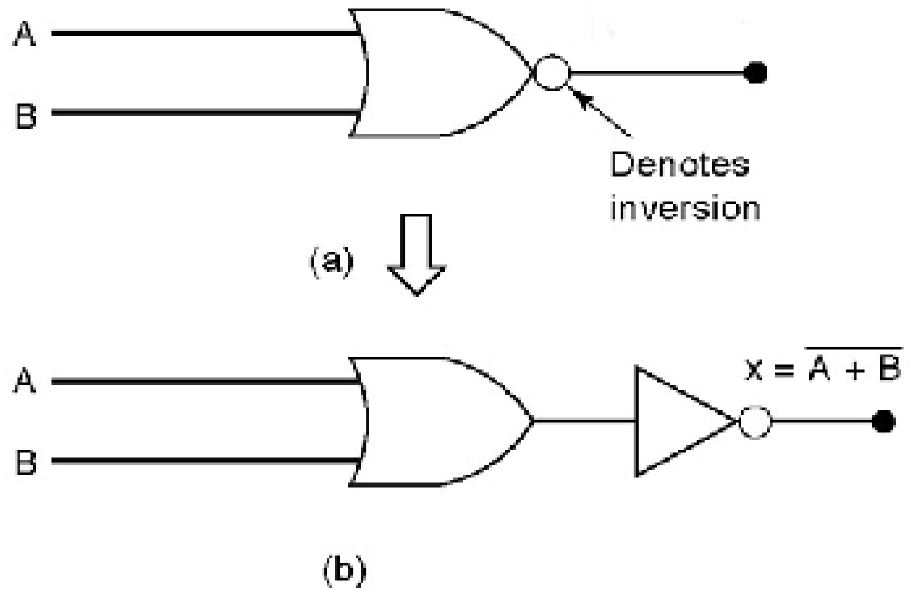
- This is a NOR gate. It is a combination of an OR gate followed by an inverter.
- like the NAND gate, the NOR gate is **functionally complete** →any logic function can be implemented using just NOR gates.




| A | B | F |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

# NOR Gate Equivalence

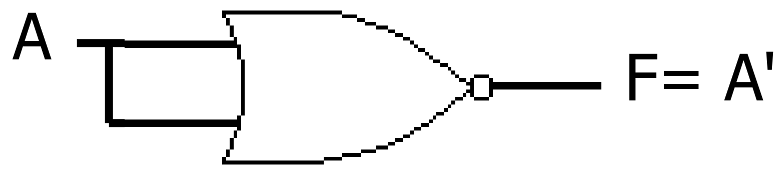
- NOR Symbol, Equivalent Circuit, Truth Table



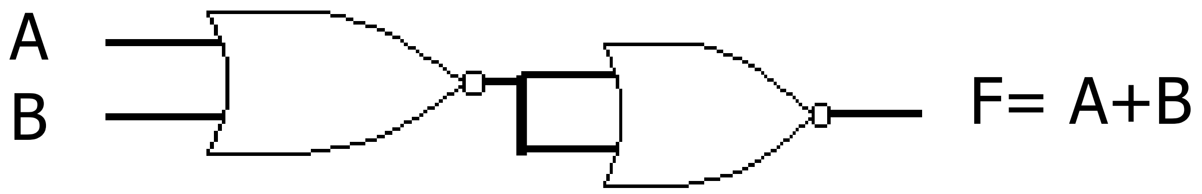

| | | OR | NOR |
|---|---|---|---|
| A | B | A + B | $\overline{A + B}$ |
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

(c)

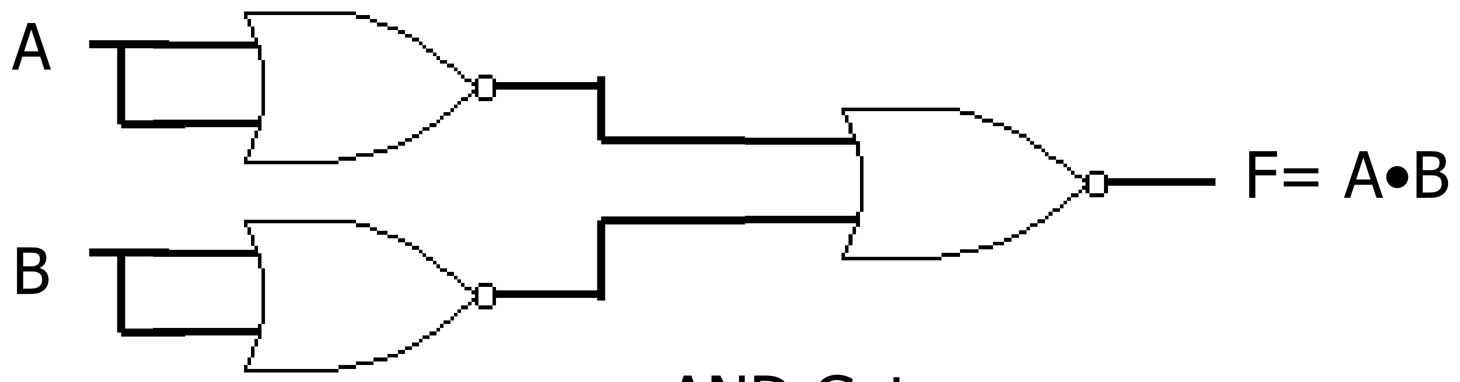
# NOR Gates-**functionally complete**




NOT Gate

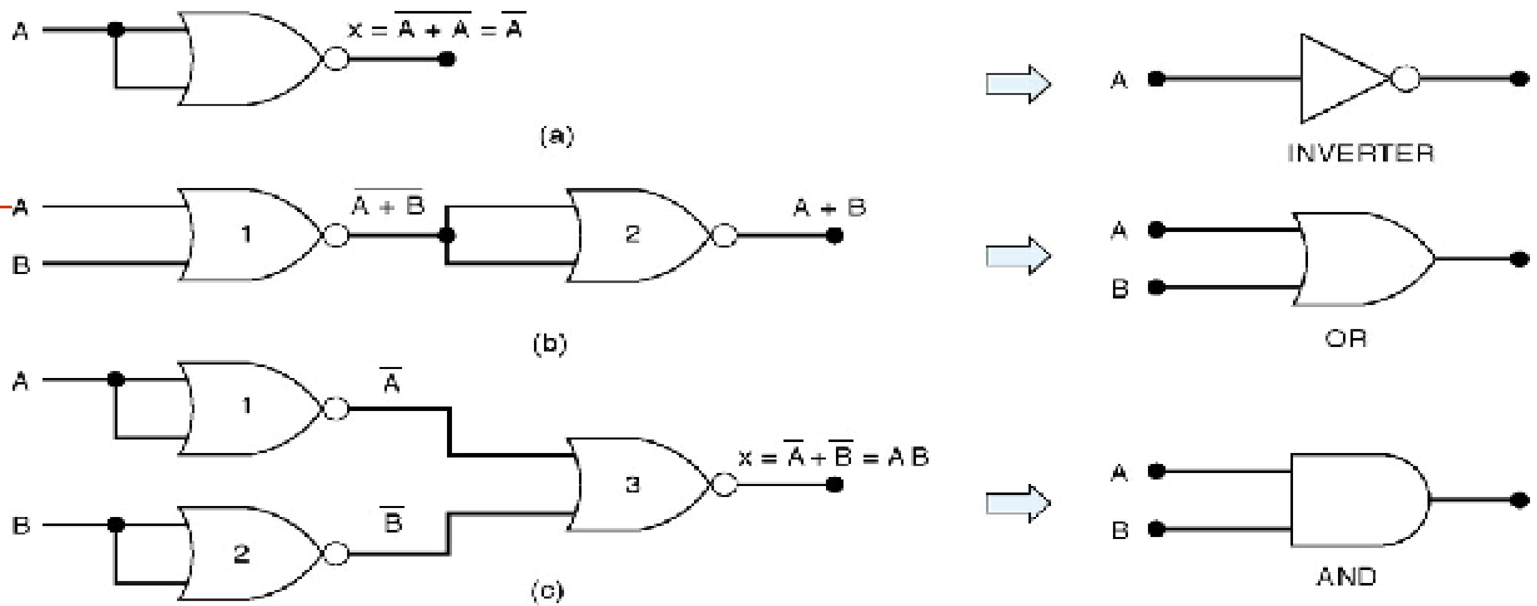



OR Gate




AND Gate

# Universality of NOR gate




- Equivalent representations of the AND, OR, and NOT gates

(A+A)' = A'A' = A'

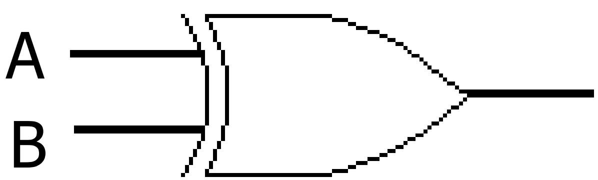
**((A+B)' + ( A + B)')' = (A+B)''( A+B)'' = (A'B')' (A'B')'**

**= (A''+B'') (A''+B'') = (A+B) (A+B) = (A+B)**

# The XOR Gate (Exclusive-OR)

- This is a XOR gate.
- XOR gates assert their output when exactly one of the inputs is asserted, hence the name.
- The operator symbol for this operation is $\oplus$

  $1 \oplus 1 = 0$ and $1 \oplus 0 = 1$.



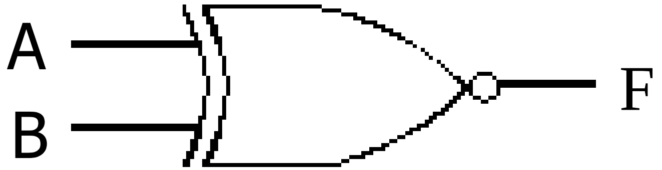

$F = A \oplus B = AB' + A'B$

| **A** | **B** | **F** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

# The XNOR Gate

- This functions as an exclusive-NOR gate, or simply the complement of the XOR gate.
- The symbol for this operation is $\odot$

$1 \odot 1 = 1$ and $1 \odot 0 = 0$.




$$F = \overline{A \oplus B} = (AB) + (\overline{A} \cdot \overline{B}) = AB + A'B'$$

| **A** | **B** | **F** |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# Standard Forms

- We have seen how to interpret truth tables, obtain Boolean expressions (functions) then build logic circuits.

- We have simplified Boolean expressions using Boolean algebra.

→**There is a "standard" way of writing Boolean expressions (Functions):**

- **The standard Sum of Products (SOP)**
- **The standard Product of Sums (POS)**

# The standard Sum of Product-Minterms

- A Minterm is one in which all variables appear (only) once.
- Each Minterm represents exactly one combination (row) in truth table.
- n variables give $2^n$ Minterms.

Truth Table

| Decimalvalue | A | B | C | F | Minterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | $m_0$ |
| 1 | 0 | 0 | 1 | 0 | $m_1$ |
| 2 | 0 | 1 | 0 | 1 | $m_2$ |
| 3 | 0 | 1 | 1 | 1 | $m_3$ |
| 4 | 1 | 0 | 0 | 1 | $m_4$ |
| 5 | 1 | 0 | 1 | 1 | $m_5$ |
| 6 | 1 | 1 | 0 | 1 | $m_6$ |
| 7 | 1 | 1 | 1 | 1 | $m_7$ |

# The standard Sum of Products-Function

- SOP are expressions of the form:

**F (A,B,C, …) = ( …) + ( … ) + ( … ) + …**

- Brackets can contain single or multiple variables

- Such expressions can be implemented using:

**F (A,B,C, …) = (AND’s) OR (AND’s) OR (AND’s) OR ...**

# The standard Sum of Product-**Function**

- SOP form not unique, and doesn’t necessarily contain all variables, for example:

**F (A,B,C) = A'B'C' + A'BC + C'A'B + C'AB' + BAC + BAC**

and F (A,B,C) = B + B'C'

**are both valid SOP expressions.**

# The standard Sum of Product-**Function**

- We Can obtain SOP from truth table (below)

F(A,B,C) = A'BC' + A'BC + AB'C' + AB'C + ABC' + ABC

A simpler notation is to write as

$$F(A,B,C) = m_2 + m_3 + m_4 + m_5 + m_6 + m_7$$

$$= \sum m_i (2, 3, 4, 5, 6, 7)$$

| Decimal value | A | B | C | F | Minterm | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | $m_0$ | A'B'C' |
| 1 | 0 | 0 | 1 | 0 | $m_1$ | A'B'C |
| 2 | 0 | 1 | 0 | 1 | $m_2$ | A'BC' |
| 3 | 0 | 1 | 1 | 1 | $m_3$ | A'BC |
| 4 | 1 | 0 | 0 | 1 | $m_4$ | AB'C' |
| 5 | 1 | 0 | 1 | 1 | $m_5$ | AB'C |
| 6 | 1 | 1 | 0 | 1 | $m_6$ | ABC' |
| 7 | 1 | 1 | 1 | 1 | $m_7$ | ABC |

# Product of Sums: **Function**

- **From truth table we have** **$F'(A,B,C) = (A'B'C' + A'B'C)$**
- **Therefore we obtain F from $\overline{F}$:**

$$F(A,B,C) = [F'(A,B,C)]' = (A'B'C' + A'B'C)'$$

$$= (A'' + B'' + C'') \cdot (A'' + B'' + C') = (A + B + C) \cdot (A + B + C')$$

**Form Compact** $F = M_0 \bullet M_1 = \prod Mi\ (0, 1)$

Truth Table

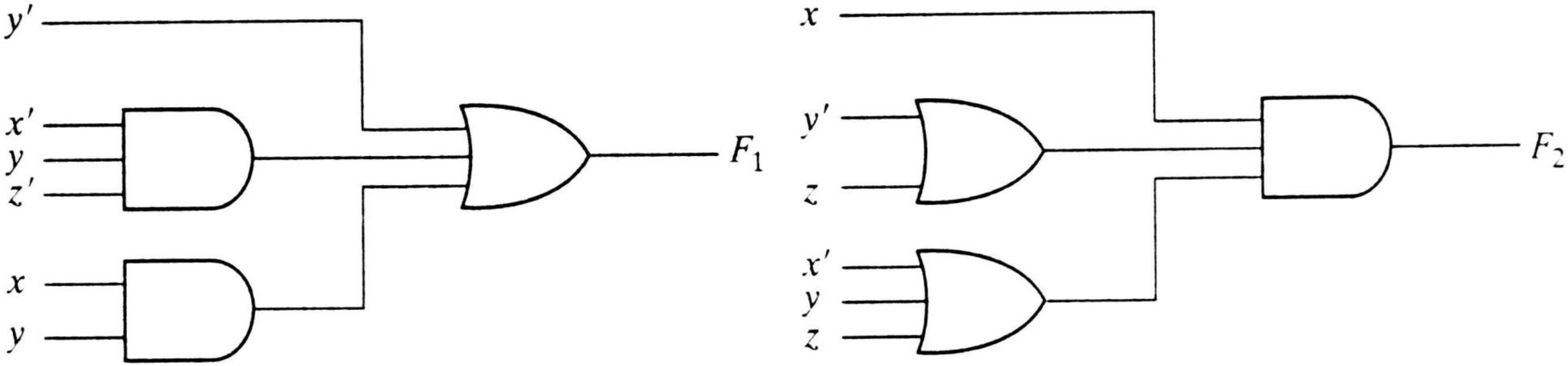
| Decimal value | A | B | C | F | Minterm | Maxterm | $M_i = \overline{m_i}$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | $m_0$ | $M_0$ | A+B+C |
| 1 | 0 | 0 | 1 | 0 | $m_1$ | $M_1$ | A+B+C' |
| 2 | 0 | 1 | 0 | 1 | $m_2$ | $M_2$ | A+B'+C |
| 3 | 0 | 1 | 1 | 1 | $m_3$ | $M_3$ | A+B'+C' |
| 4 | 1 | 0 | 0 | 1 | $m_4$ | $M_4$ | A'+B+C |
| 5 | 1 | 0 | 1 | 1 | $m_5$ | $M_5$ | A'+B+C' |
| 6 | 1 | 1 | 0 | 1 | $m_6$ | $M_6$ | A'+B'+C |
| 7 | 1 | 1 | 1 | 1 | $m_7$ | $M_7$ | A'+B'+C' |

# Obtain SOP and POS from a given expression

- Given an arbitrary Boolean expression
- Work out number of terms ($2^n$) for n inputs.
- Generate truth table and identify terms for which the function is true - the Minterms.
- Write function as: $$F = \sum_{i=0}^{n-1} m_i \quad \text{where } m_i \text{ is } 1$$
- Alternatively, identify terms for which the function is false and use a Maxterm description.
- Write function : $$F = \prod_{j=0}^{n-1} M_j \quad j \neq i \text{ (i.e. } M_j = 0)$$

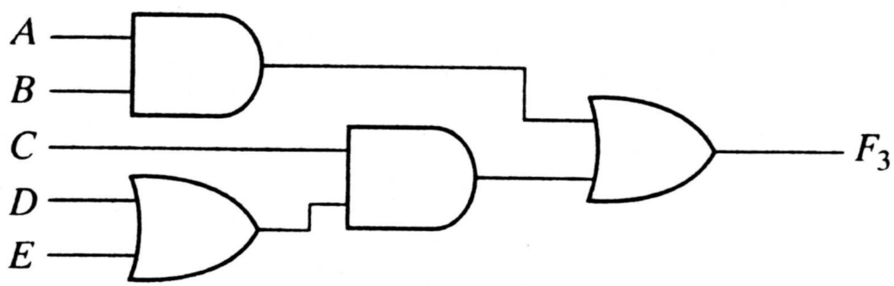
# SOP & POS Implementation using AND and OR

- Two-level implementation




(a) Sum of Products (b) Product of Sums

- Multi-level implementation




(a) $AB + C(D + E)$

## Truth Table

| A | B | C | $C_o$ | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$\begin{aligned} C_0 &= m_3 + m_5 + m_6 + m_7 \\ &= A'BC + AB'C + ABC' + ABC \\ &= M_0 M_1 M_2 M_4 \\ &= (A+B+C)(A+B+C')(A+B'+C)(A'+B+C) \end{aligned}$$

$$\begin{aligned} S &= m_1 + m_2 + m_4 + m_7 \\ &= (A'B'C) + (A'BC') + (AB'C') + (ABC) \\ &= M_0 M_3 M_5 M_6 \\ &= (A+B+C)(A+B'+C')(A'+B+C')(A'+B'+C) \end{aligned}$$

# Examples

1- Half Adder

2- Full Adder

3- Deriving SOP and POS from a truth table

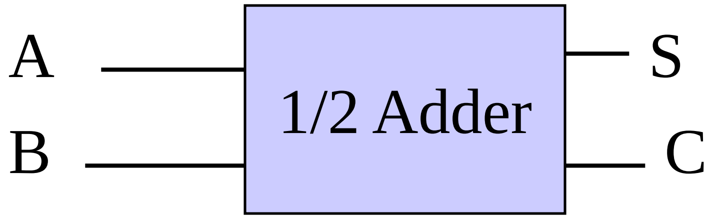
4- Simplifying SOP & POS using Boolean identities

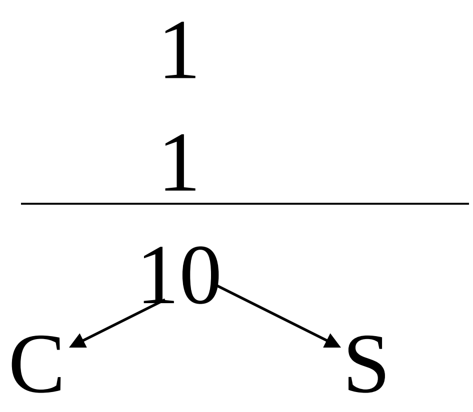
# Half Adder

→The half-adder accepts two binary digits on its inputs and produces two binary digits on its outputs: a sum bit and a carry bit.

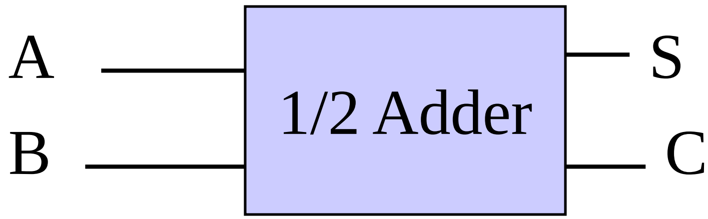
**Truth Table**

| A | B | C | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |



# Half-Adder




**Truth Table**

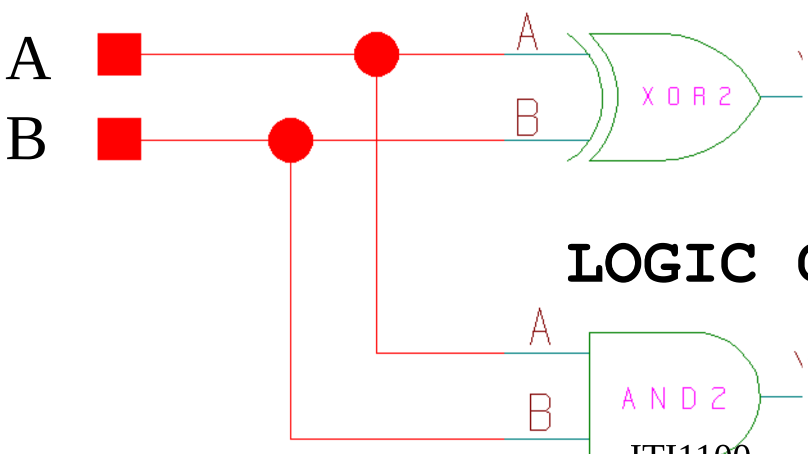
| A | B | C | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

**Logic Function**

$$\mathbf{S = A'B + AB'}$$

$$\mathbf{S = A \oplus B}$$

$$\mathbf{C = A \bullet B}$$




$\mathbf{S = A \oplus B}$

**LOGIC CIRCUIT**

$\mathbf{C = A \bullet B}$

# Full Adder

**Truth Table**

| A | B | C | $C_o$ | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

→ The Full-adder accepts two input bits and an input carry and generates a sum output and an output carry

→ Basic difference between a full and a half adder is that the full adder accepts an input carry

# Full-Adder

## Logic FUNCTION

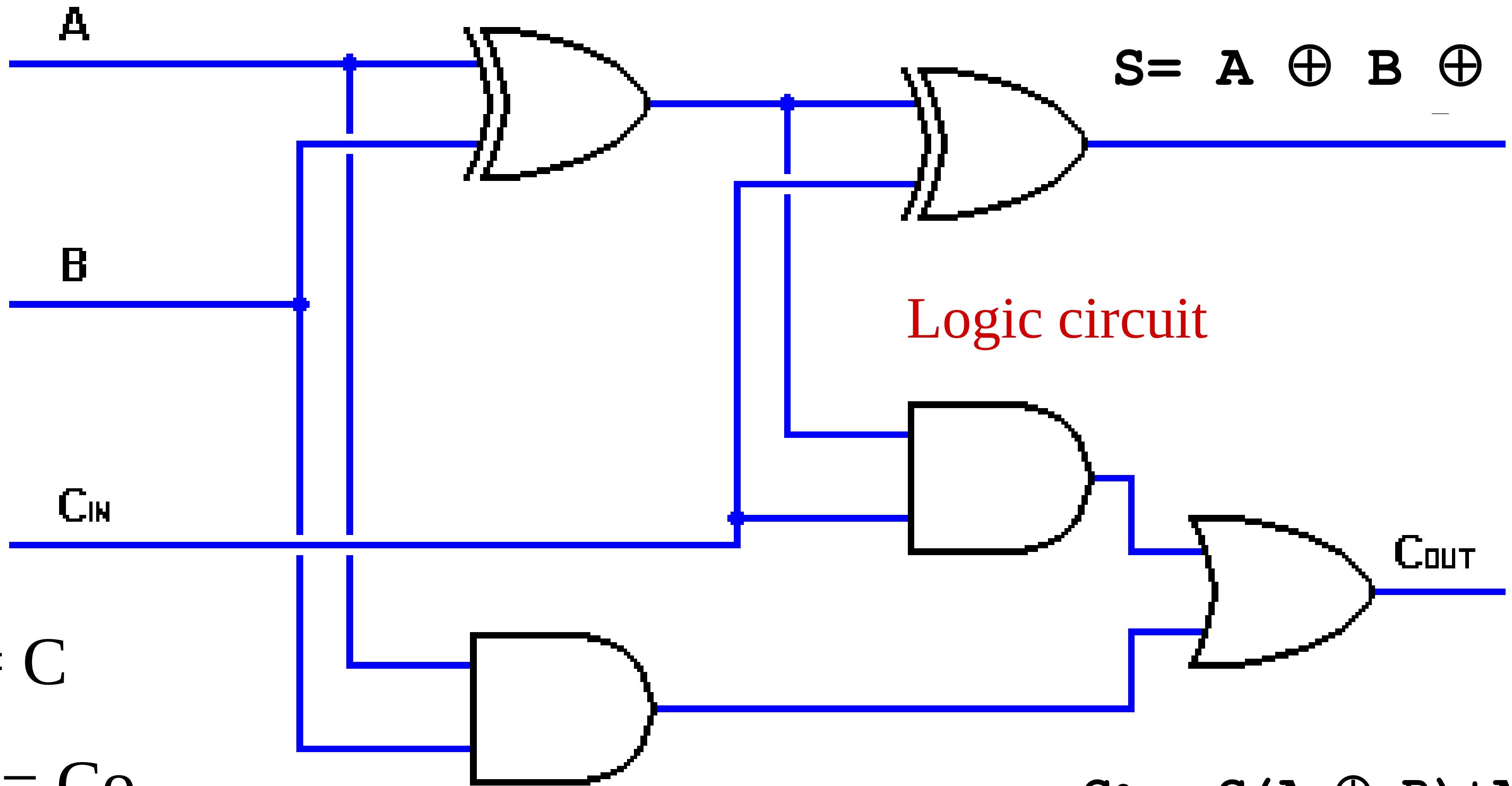
**Truth Table**

| A | B | C | $C_o$ | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$C_o = A'BC + AB'C + ABC' + ABC$$
$$= C[A'B + AB'] + AB[C' + C]$$
$$= C[A \oplus B] + AB \bullet 1$$
$$= C(A \oplus B) + AB$$

$$S = A'B'C + A'BC' + AB'C' + ABC$$
$$= A'[B'C + BC'] + A[B'C' + BC]$$
$$= A'\underbrace{[B \oplus C]}_{[X]} + A\underbrace{[B \oplus C]'}_{[X]'}$$
$$= A'X + AX'$$
$$= A \oplus X$$
$$= A \oplus B \oplus C$$

# Full Adder




$S = A \oplus B \oplus C$

Logic circuit

$Co = C(A \oplus B) + AB$

Cin = C

Cout = Co

# Two bit parallel adder

**1** ← Carry bit from right column

  1 1

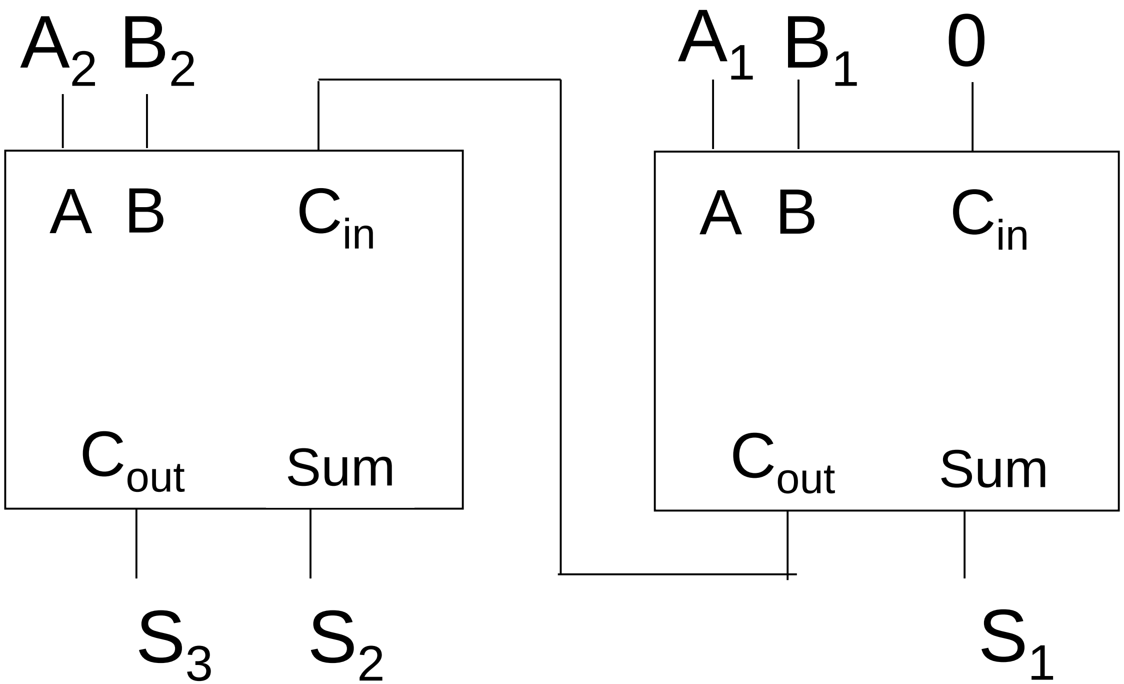
\+ 0 1

---

**1** 0 0

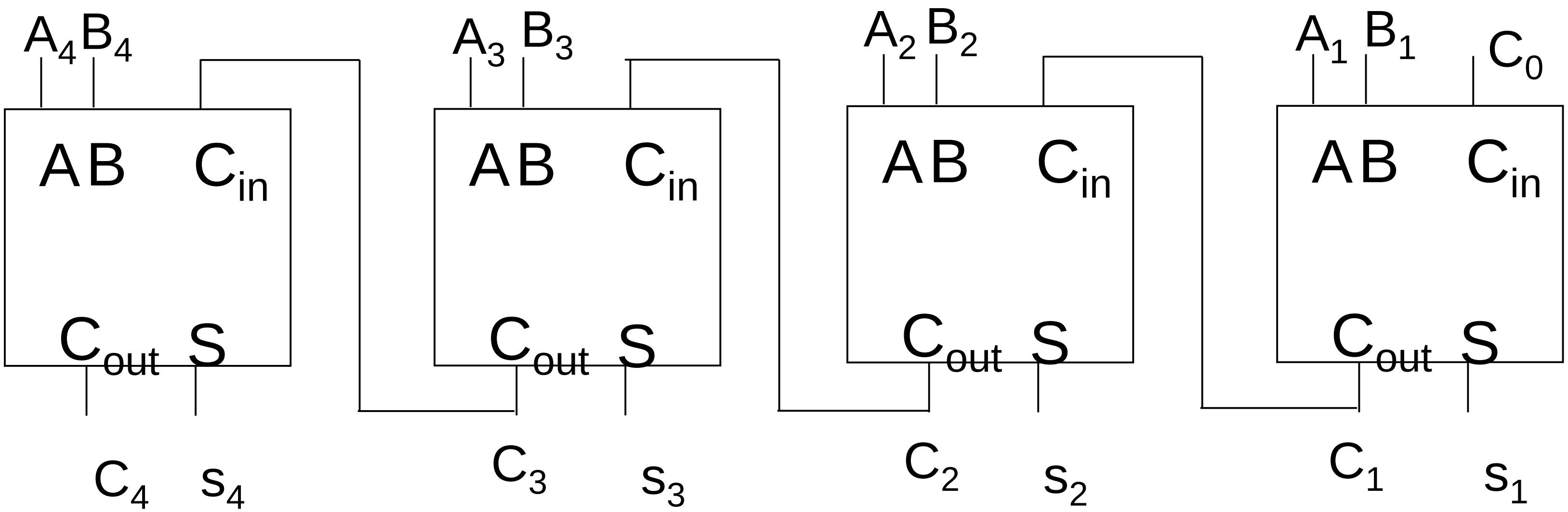
Carry bit from second column becomes a sum bit

# Two bit parallel adder




$$
\begin{array}{r}
A_2A_1 \\
+\ B_2B_1 \\
\hline
S_3\,S_2\,S_1
\end{array}
$$

# Four bit parallel adder



# Overflow Examples (review from chapter 1)

- In a 6-bit register

  + 17 = 010001

  + 16 = +<u>010000</u>

  =100001 → Overflow

- **100001 = 2's : - (11111) = -$(31)_{10}$** instead of **+ $(33)_{10}$**
- Same with a 7-bit register

  + 17 = 0 010001

  + 16 = +<u>0 010000</u>

  = 0 100001

  **0100001 = + 33 No Overflow**

# Four bit parallel adder

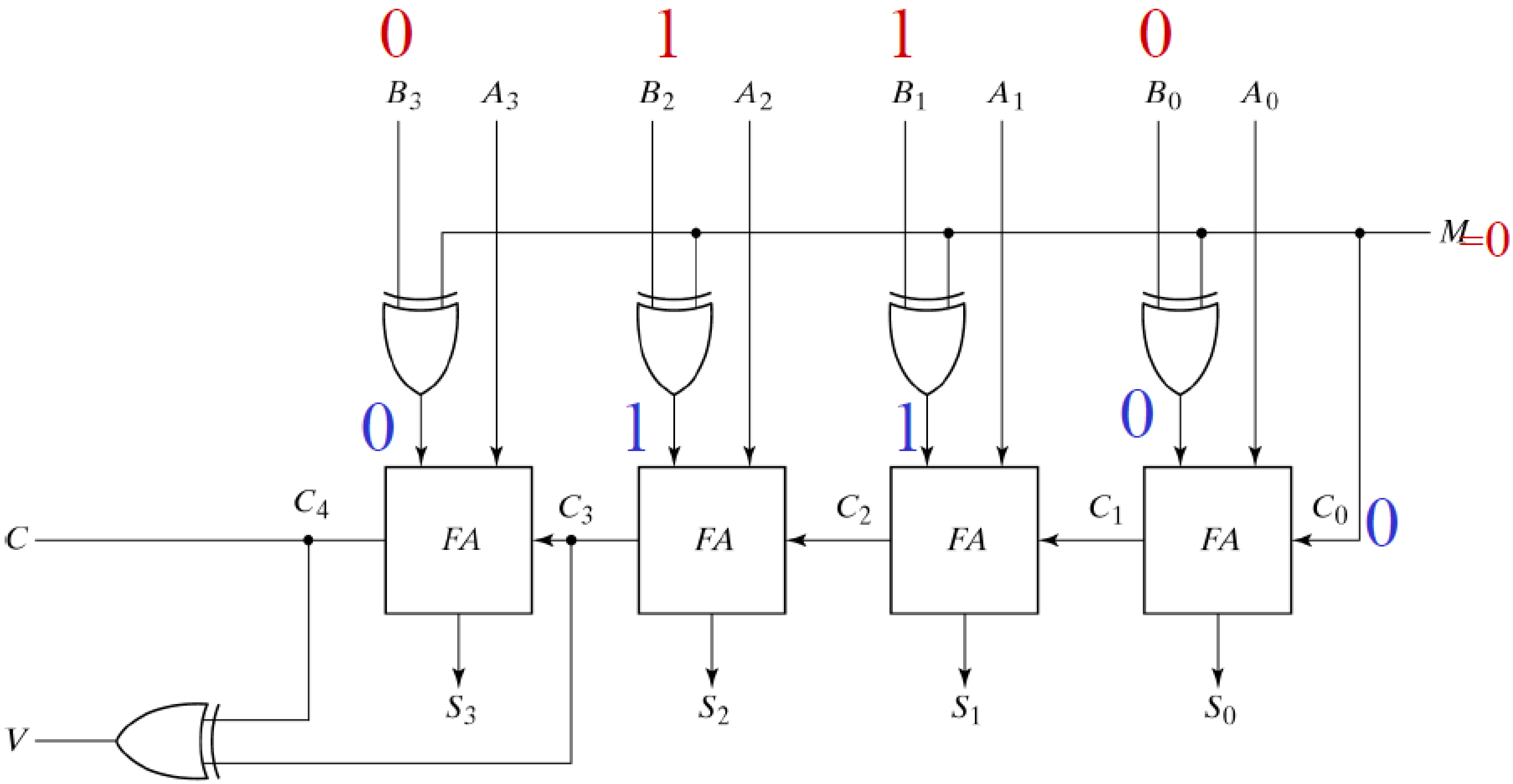



Fig. 4-13 4-Bit Adder Subtractor

# Four bit parallel adder

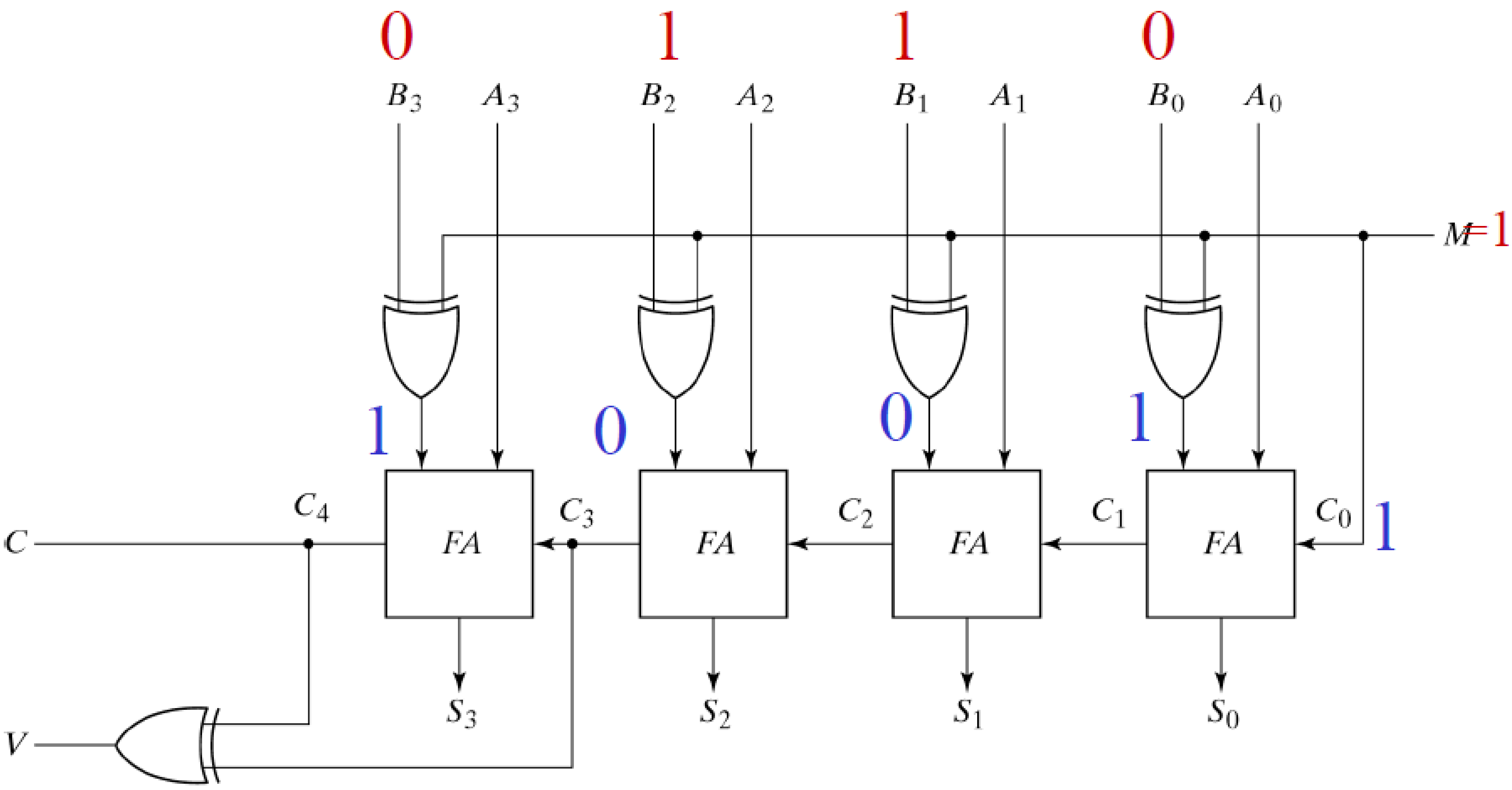



Fig. 4-13 4-Bit Adder Subtractor

# Four bit parallel adder: more examples

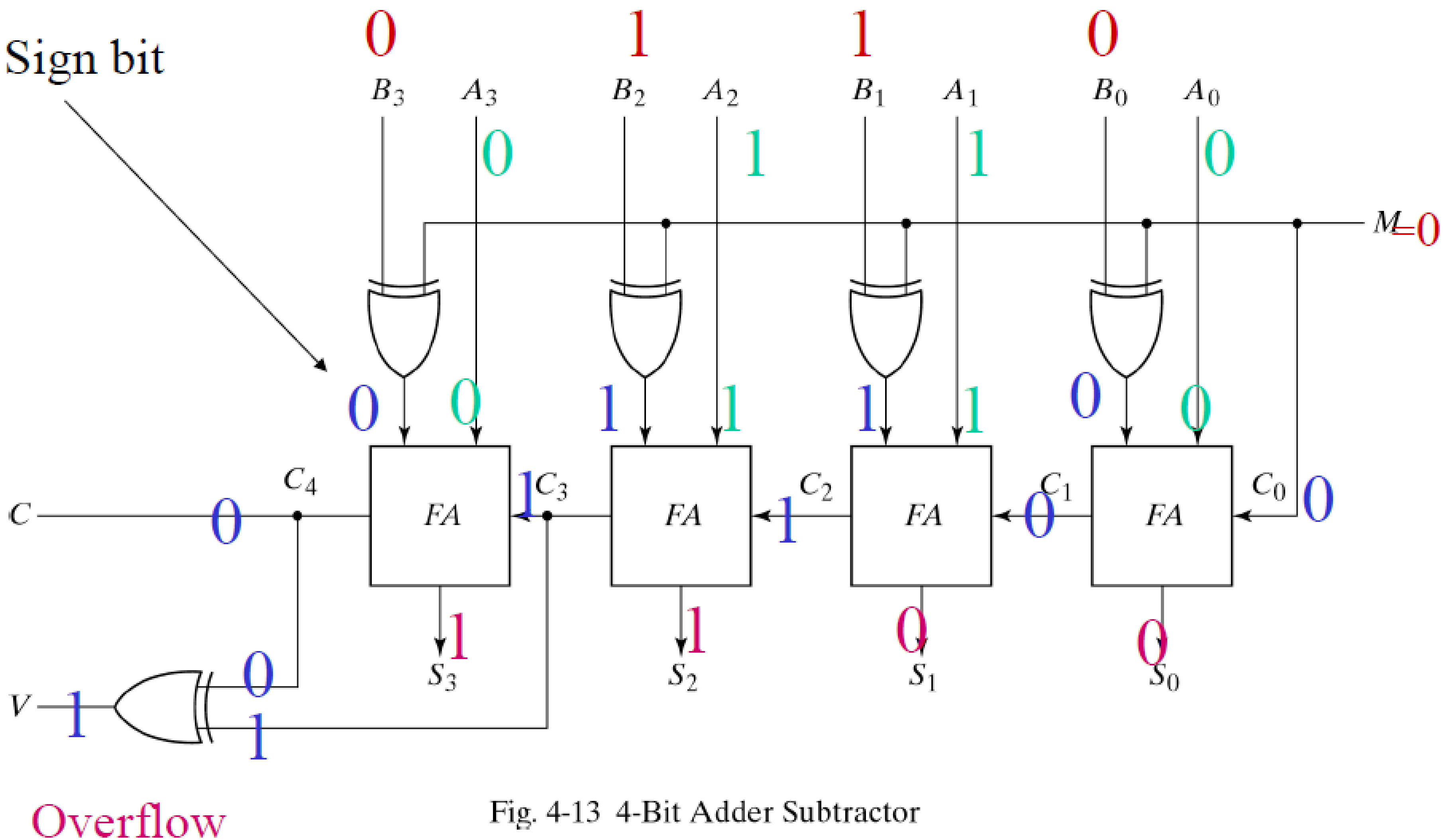



Fig. 4-13 4-Bit Adder Subtractor

# Four bit parallel adder: more examples

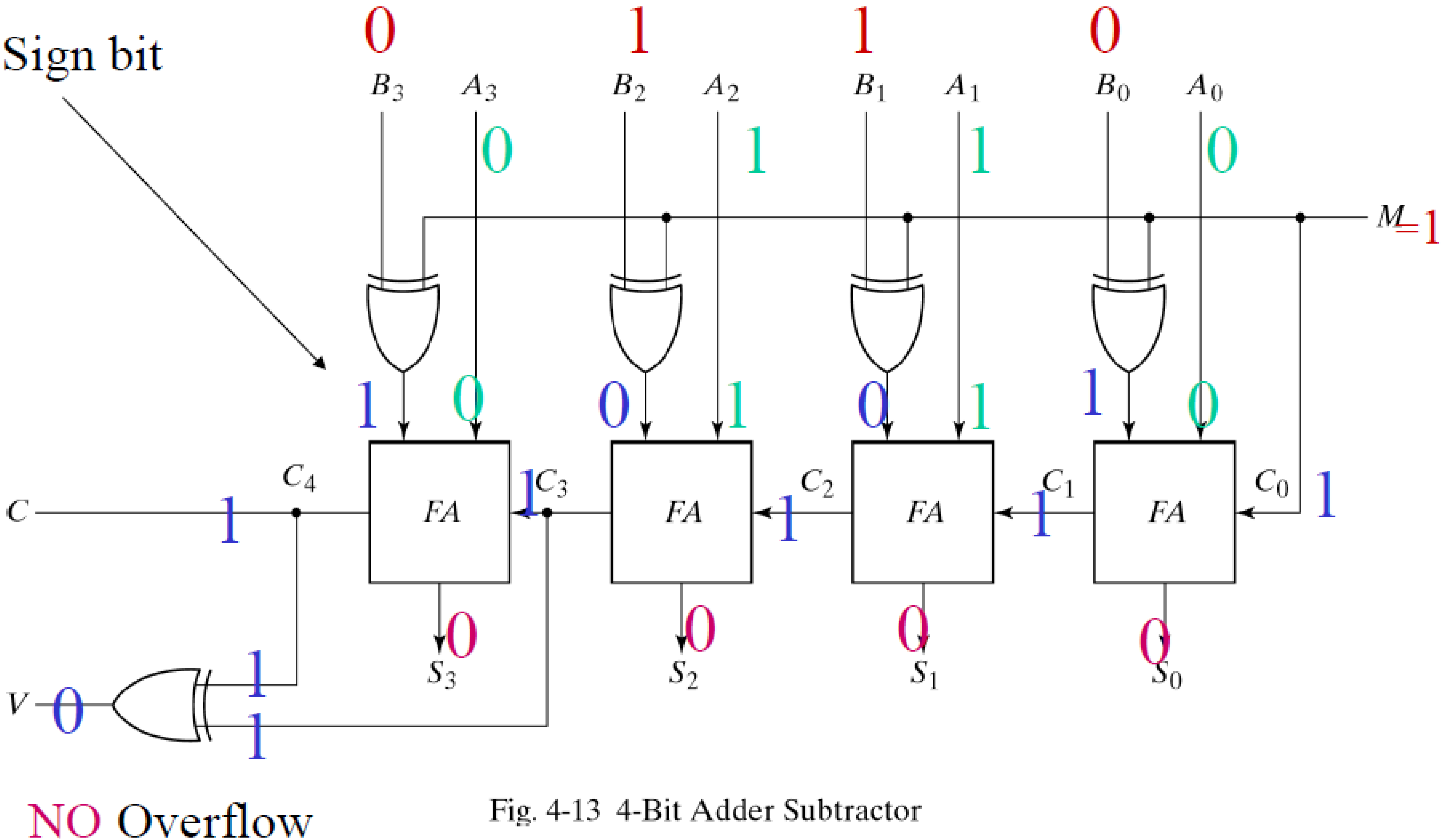



Fig. 4-13 4-Bit Adder Subtractor

# SOP & POS Standard Forms- Example

From an arbitrary Truth table (next slide)

- **Part one**

1- obtain SOP representation for F

2- obtain the two level implementation for F without simplification

3- simplify F using Boolean identities

4- obtain two level-implementation for F

5- compare the design obtained in question 4 with the one of question 2

- **Part two**

Repeat part one using POS

# Deriving SOP and POS from a truth table

Consider the following arbitrary Truth Table

i) SOP

ii) POS

| i | A | B | C | F | Minterms |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 1 | 0 | |
| 2 | 0 | 1 | 0 | 1 | **→ $m_2$ = A'BC'** |
| 3 | 0 | 1 | 1 | 1 | **→ $m_3$ = A'BC** |
| 4 | 1 | 0 | 0 | 0 | |
| 5 | 1 | 0 | 1 | 1 | **→ $m_5$ = AB'C** |
| 6 | 1 | 1 | 0 | 0 | |
| 7 | 1 | 1 | 1 | 1 | **→ $m_7$ = ABC** |

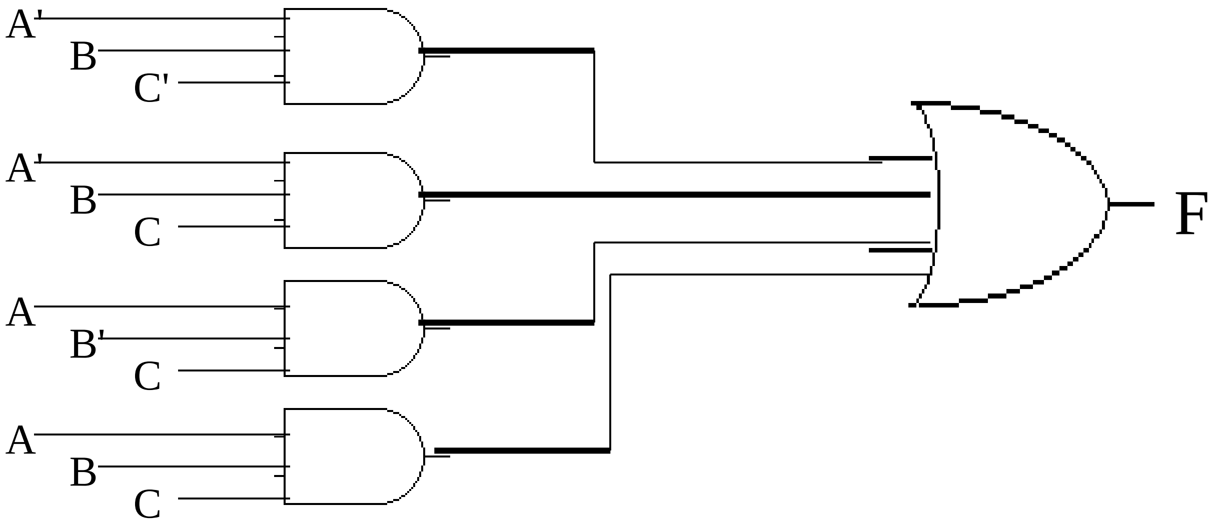
# Deriving SOP from a truth table

1- **Expression sum of products**

**a)** F = m2 + m3 + m5 + m7

= A'BC' + A'BC + AB'C + ABC

**b)** Implementation with logic gates (unsimplified )




Two level-Implementation

# Simplifying SOP using Boolean identities

## c) Simplification

Using the Boolean identity absorption $xy + xy' = x$

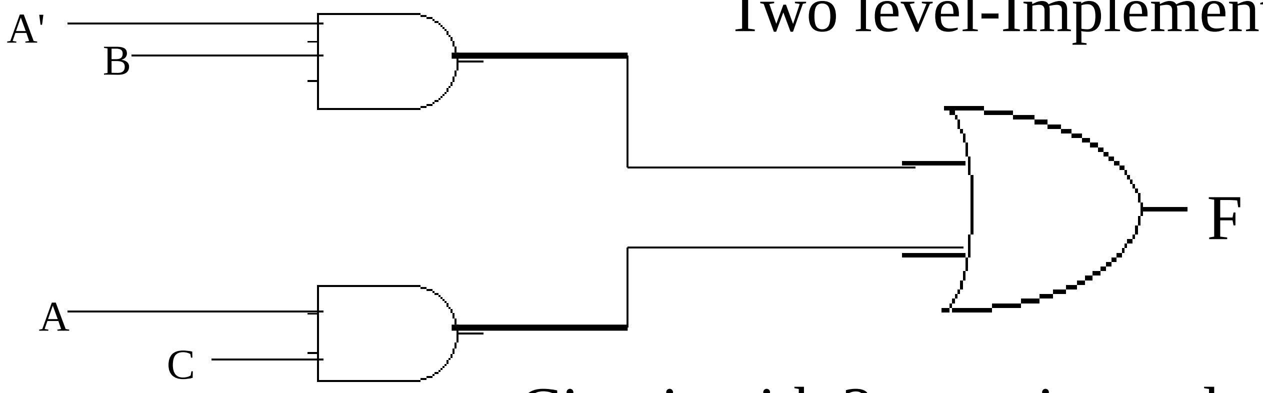
We can simplify $m_2$ with $m_3$ and $m_5$ with $m_7$

$m_2 + m_3 \quad = (A'B)\,C' + (A'B)\,C = A'B$

$m_5 + m_7 \quad = (AC)B' + (AC)B = AC$

**Therefore** **$F = A'B + AC$**

Two level-Implementation




Circuit with 3 gates instead of 5

# Deriving POS from a truth table

**2- Expression Product of sums**

| i | A | B | C | F | Maxterms |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | → $M_0 = A+B+C$ |
| 1 | 0 | 0 | 1 | 0 | → $M_1 = A+B+C'$ |
| 2 | 0 | 1 | 0 | 1 | |
| 3 | 0 | 1 | 1 | 1 | |
| 4 | 1 | 0 | 0 | 0 | → $M_4 = A'+B+C$ |
| 5 | 1 | 0 | 1 | 1 | |
| 6 | 1 | 1 | 0 | 0 | → $M_6 = A'+B'+C$ |
| 7 | 1 | 1 | 1 | 1 | |

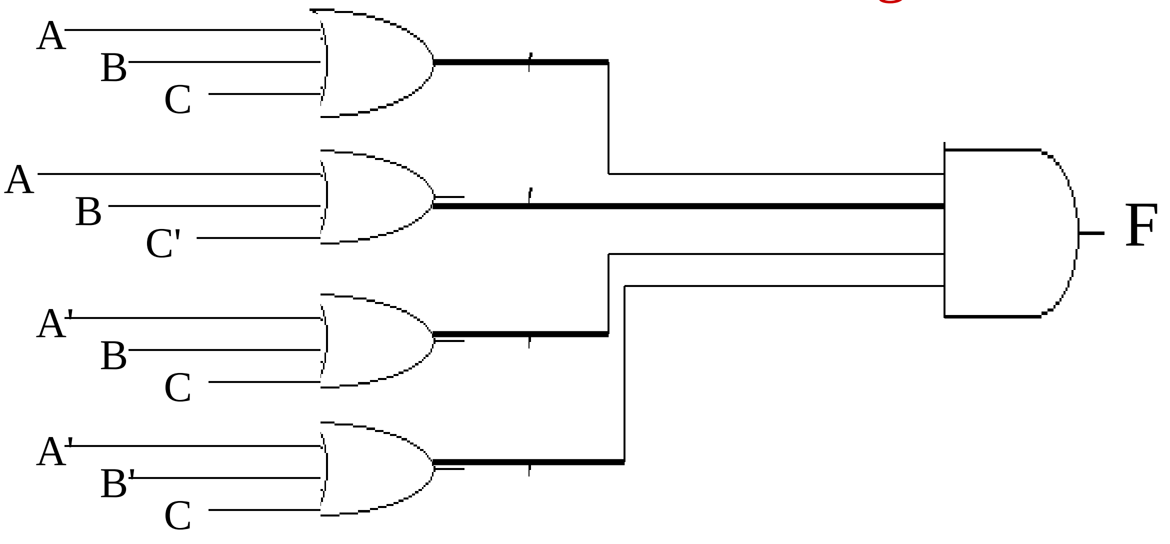
# Deriving SOP from a truth table

## a) Function

$F = M_0 \;.\; M_1 \;.\; M_4 \;.\; M_6$

$= (A+B+C)\,(A+B+C')(A'+B+C)(A'+B'+C)$

Logic circuit




Two level-Implementation

# Simplifying POS using Boolean identities

## b) Simplification

Using (X + Y) (X+Y') = X

Verification

| X | Y | Y' | X+ Y | X + Y' | **(X + Y) (X+Y')** |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |

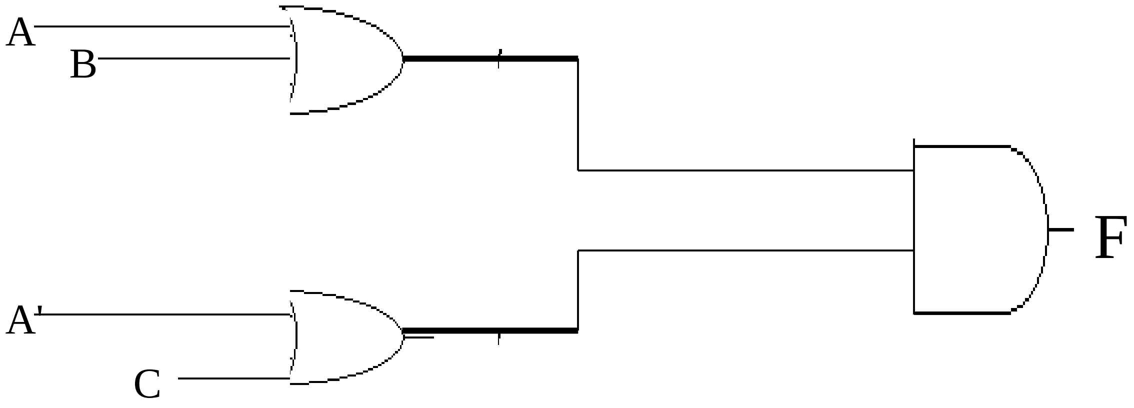
M0•M1 = [ (A+B) + C] [ (A+B) + C']= **A+B**

M4•M6 = [ A' + C) + B] [(A'+C) + B'] = **A'+C**

**F = (A+B)(A'+C)**

# Simplifying POS using Boolean identities

**C) Two level Implementation**



# Obtaining The Truth Table - Example

Design a digital circuit that will be used to control an Alarm bell. This Alarm bell is to be installed in a room to protect it from unauthorized entry.

Sensor devices provide the following logic signals

**C = 1** The control system is active

**D = 1** The room door is closed

**M = 1** There is a motion in the room

**Q = 1** The room is open to the public

i) Obtain the truth table

ii) derive the Boolean expression using **true** terms

# Truth table

| C | D | M | Q | Alarm |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

**Alarm= CD'M'Q' + CD'MQ' + CDMQ'**

Door should not be open

room is closed to the public → door open + motion

room is closed to the public + door closed → motion