

Université d'Ottawa  
Faculté de génie  
SIGE



University of Ottawa  
Faculty of Engineering  
EECS

**ITI 1100-B****Digital Systems I****Mid-Term Examination**

First Name: Ali

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Student #: Sample Solution

Signature: \_\_\_\_\_

**Professor: Ali Karime****Exam Duration: 90 minutes****Instructions**

- Closed book examination. The maximum score is 100 points.
- Use the provided space to answer the questions. If more space is required, use the back of the page.
- Show all the steps to obtain marks.
- Calculators are **NOT allowed**.
- It is strongly recommended that you write down your solutions step by step. This is the only way it is possible for you to earn full marks.
- Cheating will be penalized in accordance with University of Ottawa regulations.

Question 1	(35)	
Question 2	(10)	
Question 3	(55)	
Total	/100	

(a) Convert the following two decimal numbers: A = 12.25 and B = 16.50 to signed binary numbers (use a signed magnitude presentation). Use the necessary bits to represent the integer part and use **2 bits** for the fractional part and an additional bit for the sign. **[15 points]**

Handwritten diagram illustrating the steps of the Euclidean algorithm for finding the GCD of 12 and 2:

$$\begin{array}{r}
 12 \overline{) 2} \\
 \underline{12} \phantom{0} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 12 \overline{) 6} \\
 \underline{12} \phantom{0} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 6 \overline{) 2} \\
 \underline{6} \phantom{0} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 6 \overline{) 3} \\
 \underline{6} \phantom{0} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 2 \overline{) 1} \\
 \underline{2} \phantom{0} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 1 \overline{) 2} \\
 \underline{1} \phantom{0} \\
 0
 \end{array}$$

An arrow points from the final result (1) to the first step.

$$0.5 \times 2 = 1 \quad 1$$

5 points

$$\begin{array}{r} 16 \overline{) 2} \\ 16 \overline{) 8} \quad 2 \\ \textcircled{0} \quad 8 \overline{) 4} \quad 2 \\ \textcircled{0} \quad 4 \overline{) 2} \quad 2 \\ \textcircled{0} \quad 2 \overline{) 1} \quad 2 \\ \textcircled{0} \quad 0 \overline{) 2} \\ \textcircled{1} \end{array}$$

$$B = (10000, 1)_2$$

5 points

$A = 001100.01$  and  $B = 010000.10$   
 2.5 points      2.5 points

$A = 001100.01$  and  $B = 010000.10$   
 Here, we use 1 sign bit, 5 bits for the integer part and 2 bits for fractional part (8 bits in total)

(b) Perform the following binary arithmetic operations using **1's complement**.

(i)  $C = A - B$  [7.5 points]

(ii)  $D = A + B$  [7.5 points]

i)  $1's \{B\} = 1's \{010000.10\} = 10111.01$

$A - B = A + 1's \{B\}$

$$\begin{array}{r} 001100.01 \\ + 101111.01 \\ \hline \end{array}$$

7.5

$$\begin{array}{r} 111011.10 \\ \hline \end{array}$$

Sign      1's complement magnitude form

So the answer in magnitude is  $-00100.01$

ii)  $001100.01$

$+ 010000.10$

$$\begin{array}{r} 001100.01 \\ + 010000.10 \\ \hline 011100.11 \end{array}$$

7.5

(b) Convert the binary number  $(10\ 0110.11)_2$  to hexadecimal. [5]

$$( \underbrace{0010}_2 \underbrace{0110}_6 \underbrace{.1100}_C )$$

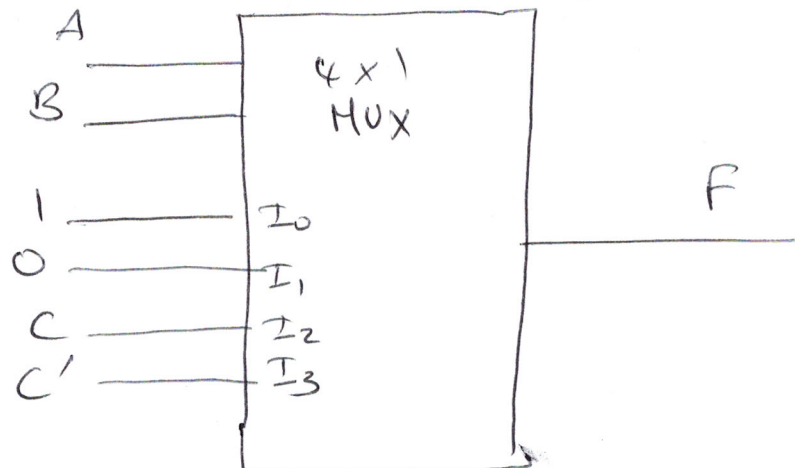
### Question 2 (10 points)

Consider the following truth table A, B and C are input variables and F is the output function

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

4 points for properly determining the volume of  $F$  at each pair of combinations

a) Implement the function F using a 4-to-1 multiplexer.



6. points  
for properly  
drawing the  
arclet.

**Question 3: (55 points)**

Given the logic function  $F(A,B,C,D) = \sum m(1, 3, 5, 12)$  together with the *don't care* conditions  $d(A,B,C,D) = \sum d(2, 7, 8, 11, 14, 15)$ ,

- (a) Write the truth table of the logic function. Use the Karnaugh-map method to find the simplest sum-of-products expression of function  $F$ . [20 points]

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	X
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	X
1	0	0	0	X
1	0	0	1	0
1	0	1	0	0
1	0	1	1	X
1	1	0	0	1
1	1	0	1	0
1	1	1	0	X
1	1	1	1	X

AB \ CD	CD			
	00	01	11	10
00	0	1	1	X
01	0	1	X	0
11	1	0	X	X
10	X	0	X	0

The simplified  $f$  is not unique

①  $F = A'D + AC'D'$

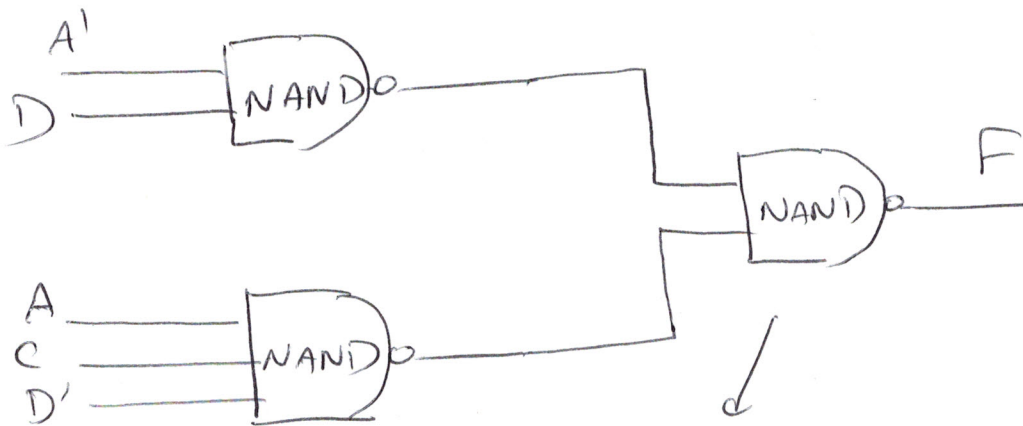
or


②  $F = A'D + ABD'$

Student may give any of the 2 answers.



- (b) Implement the minimized function with NAND gates only. Note: You can use NAND gates with any number of inputs you may need. Assume, as well, that the input variables are available in both true and complemented form. [10 points]



(Student may use the following symbol here )

10 pts

- (c) Express the same logic function in a product of sums form (give the non-simplified form of the function). [5 points]

Looking in the truth table for the maxterms, we find

$$F = (A + B + C + D)(A + B' + C + D)(A + B' + C' + D)(A' + B + C + D') \\ (A' + B + C' + D)(A' + B' + C + D')$$

5 pts

(d) Simplify your function in product of sums. [10 points]

We simplify by grouping 0's first and then finding  $F'$   
(N.B: simplification is not unique)

$$F' = A'D' + AD + AC$$

or  $F' = A'D' + AD + AB'$  or  $F' = A'D' + AD + CD'$

if  $F = A'D' + AD + AC$  is chosen, then

$$F = F'' = (A + D)(A' + D')(A' + C')$$

10pts

(e) Implement your function  $F$  employing a 4-to-16 **active low decoder** (see the following block diagram) and AND gate (with required number of inputs) only. [10 points]

$$F = \pi(0, 4, 6, 9, 10, 13)$$

10pts

