Deep Reinforcement Learning for Portfolio Optimisation

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Introduction

Research Objectives

Objective:

- 🏁 Minimum Spanning Tree to reduce a universe of assets to an uncorrelated subset
- 🏁 Applicability of Deep Reinforcement Learning in portfolio optimisation

📝 Portfolio Optimisation

Strategic allocation of wealth into a fixed number of financial instruments in order to achieve an objective, such as maximising returns or minimising risk

Portfolio Optimisation

Financial Time Series I

A portfolio of m assets $w_t = [w_{1,t}, \dots w_{m,t}]$, at time t, should satisfy:

$$\sum_{i=1}^{m} w_{i,t} = 1.$$

Gross or simple returns of a portfolio p that contains m assets at time t:

$$R_{p,t} = \sum_{i=1}^{m} w_i R_{i,t} = w_1 R_{1,t} + w_2 R_{2,t} + \dots$$

For the log returns it holds:

$$r_{p,t} = \log\left[1 + \sum_{i=1}^{m} w_{i,t} R_{i,t}\right] = \log\left[1 + w_{1,t} R_{1,t} + w_{2,t} R_{2,t} + \dots\right]$$
$$= \log\left[\sum_{i=1}^{m} \frac{P_{i,t}}{P_{i,t-1}}\right] = \log\left[\sum_{i=1}^{m} P_{i,t}\right] - \log\left[\sum_{i=1}^{m} P_{i,t-1}\right].$$

Financial Time Series II

Examples of returns' distributions:

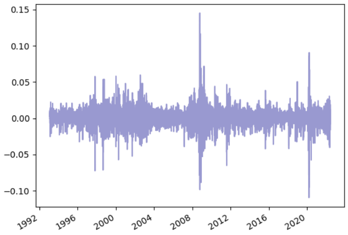


Figure: SPY ETF Returns

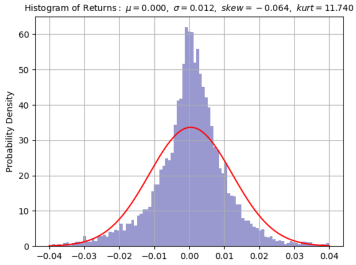


Figure: Distribution of SPY ETF Returns

Financial Time Series III

Covariance and correlation of assets' returns:

Figure: Covariance matrix of returns

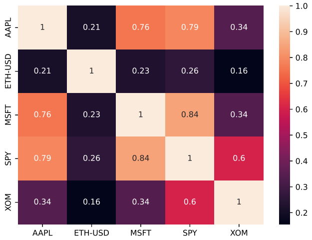


Figure: Correlation matrix of returns

Portfolio Objectives

	PnL	$\mu_{returns}$	$\sigma_{returns}$	$Skew$	$Kurt$	SR
[MSFT, SPY]	1.001	0.0001	0.014	−0.406	0.646	0.118
[MSFT, SPY, XOM]	1.199	0.0008	0.012	−0.603	1.503	1.073

Table: Statistics for two different portfolio constructions

Investment Advice

- **IA1:** Large standard deviation ⇒ returns fluctuate a lot [Rice, 2006]
- **IA2:** Risk-averse investors ⇒ positively skewed returns [Harvey et al., 2010]
- **IA3:** Excess kurtosis ⇒ fat-tail risk [Naqvi et al., 2017, Khan et al., 2020]
- **IA4:** Prefer uncorrelated securities to assets that belong in the same sector[1] [Rice, 2006]
- **IA5:** Choose the alternative with the higher Sharpe Ratio [Dowd, 2000]

[1] assuming this would imply a strong correlation

Modern Portfolio Theory & CAPM

[Markowitz, 1952]

$$\underset{w}{\text{Minimize}} \quad \frac{1}{2} w^T \sum_{i,j=1}^{m} w_i w_j \sigma_{i,j}$$

$$\text{subject to} \quad \sum_{i}^{m} w_i \bar{r}_i = \bar{r}_{target}$$

$$\text{and} \quad \sum_{i=1}^{m} w_i = 1$$

Capital Market Line

CML: $\bar{r} = r_f + \frac{\bar{r}_M - r_f}{\sigma_M} \sigma$

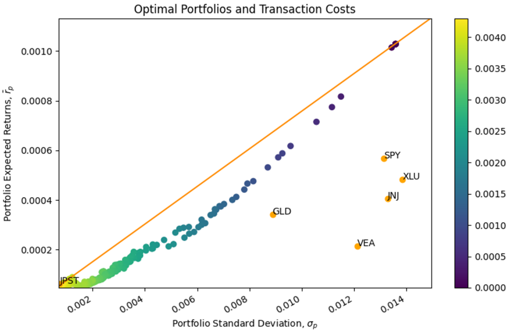


Figure: Efficient Frontier without short-selling

Asset Decorrelation

Minimum Spanning Tree I

Edge Distance

Clusters of assets with strong positive correlations

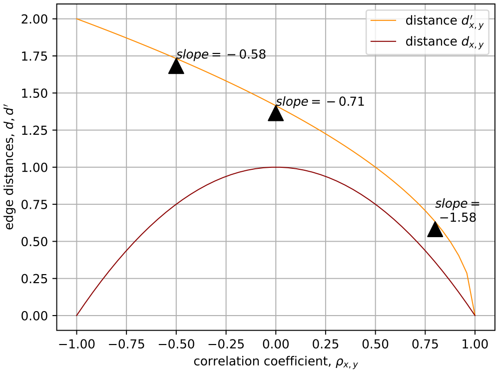


Figure: $d'(x, y) = \sqrt{2(1 - \rho_{x,y})}$ shown in orange; and $d(x, y) = 1 - \rho^2_{x,y}$ shown in dark red.

Minimum Spanning Tree II

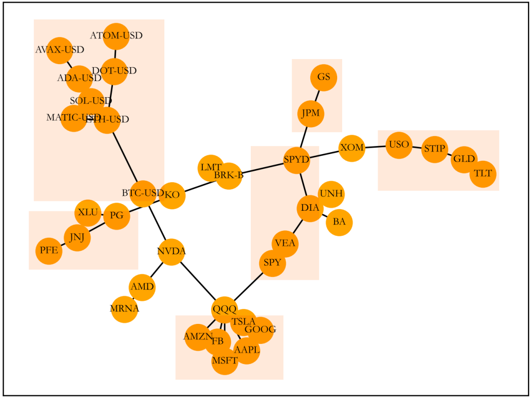


Figure: Universe of assets in a MST

Minimum Spanning Tree III

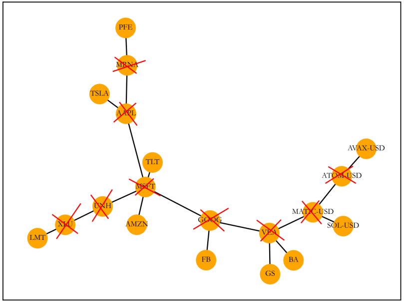


Figure: Node reduction

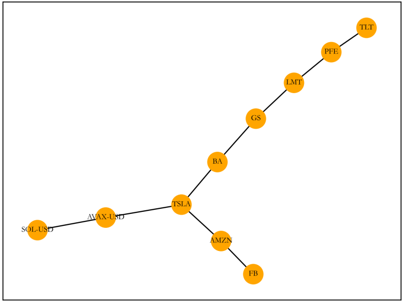


Figure: Final subset

Reinforcement Learning

Why Reinforcement Learning ?

- ✏️ Learns from scratch, by trial and error, without prior financial knowledge or a market model
- ✏️ Does not have any inherent assumptions about the market
- ✏️ Discounted rewards that give importance to future returns
- ✏️ Design of objectives/reward functions (e.g. profits or risk-adjusted)

Agent - Environment interaction in MDPs

$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots .$$

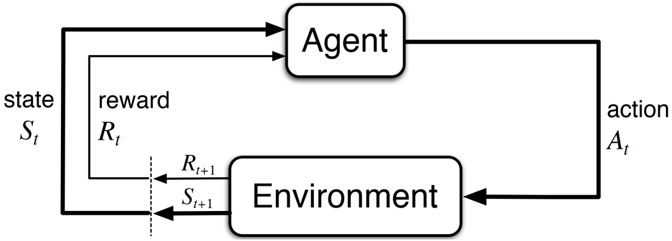


Figure: Agent - Environment setting in a MDP, source: [Sutton and Barto, 2018]

Key Elements

- **policy π:** Mapping from states to actions, stimulus-response rule, determines the agent's behaviour
- **state-value function V_π:** Value of a state (reward expected to be accumulated in the future), how good is to be in a given state

$$u_\pi(s) = \mathbb{E}_\pi[G_t \mid S_t = s] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right], \text{ for all } s \in \mathcal{S}$$

- **action-value function Q_π:** Value of an action (expected return of action α), how good is to perform a particular action in a given state

$$q_\pi(s) = \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right], \text{ for all } s \in \mathcal{S}$$

Actor - Critic

Architecture

Actor $\Rightarrow$ policy π_θ
Critic $\Rightarrow$ action-value function Q_π

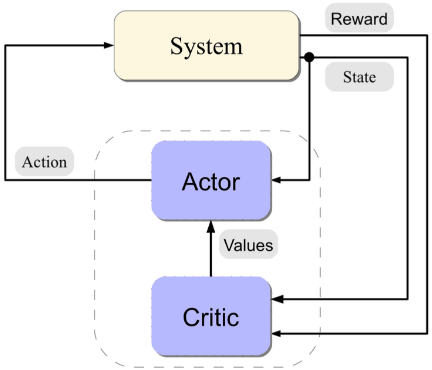


Figure: The Actor-Critic framework[2], source: [Szepesvari, 2010]

[2] System or Environment

Twin Delayed Deep Deterministic policy gradient algorithm (TD3)

1 Clipped Double Q-Learning

$$y_1 = r + \gamma Q_{\theta_2'}\left(s', \pi_{\phi_1}(s')\right)$$

$$y_2 = r + \gamma Q_{\theta_1'}\left(s', \pi_{\phi_2}(s')\right)$$

$$\Downarrow$$

$$y = r + \gamma \min_{i=1,2} Q_{\theta_i'}\left(s', \pi_{\phi_1}(s')\right)$$

📌 Taking the minimum between two target networks to limit overestimation bias

2 Delayed policy updates

A deterministic target policy is learned by:

$$\max_{\phi} \mathbb{E}_{s\sim\mathcal{B}}\left[Q_{\theta_1}\left(s, \mu_{\phi_{target}}(s)\right)\right]$$

📌 Policy updates are delayed for higher quality policies

3 Target policy smoothing

To avoid overfitting, the target is updated as:

$$y = r + \gamma Q_{\theta'}\left(s', \pi_{\phi'}(s') + \epsilon\right)$$

$$\epsilon \sim \text{ clip } \left(\mathcal{N}(0,\sigma), -c, c\right)$$

📌 Policy smoothing averts overfitting and leads to actions more robust to noise

Experiments

Trading Agents I

Algorithm 1: General setup of TD3 as a trading agent

input : Normalised prices of assets,
Objective function $\mathcal{J}(\phi) = \mathbb{E}_{\pi_\phi}\left[\sum_{t=0}^{T} \gamma^t r(s,a)\right]$

output : Parameters for the actor network ϕ and for the critic networks θ_1 and θ_2

for $t = 1, \ldots, T$ time steps in each episode **do**

Observe normalised prices of assets

Select portfolio weights / actions $\alpha \sim \pi_\phi(s)$, and observe reward r and new state s'

Store transition tuple (s, a, r, s') in replay buffer $\mathcal{B}$

Sample mini-batch of N transitions (s, a, r, s') from $\mathcal{B}$

Training:

Select action: $\tilde{\alpha} \leftarrow \pi_{\phi'}(s') + \epsilon, \quad \epsilon \sim \text{ clip } (\mathcal{N}(0, \tilde{\sigma}), -c, c)$

Target network: $y \leftarrow r + \gamma \min_{i=1,2} Q_{\theta'_i}(s', \tilde{\alpha})$

Update critics $\theta_i \leftarrow \arg\min_{\theta_i} N^{-1} \sum \left(y - Q_{\theta_i}(s,a)\right)^2$

if $t \mod d$ **then**

Update ϕ by the deterministic policy gradient:
`% [Silver et al., 2014]`

$\nabla_\phi J(\phi) = N^{-1} \sum \nabla_\alpha Q_{\theta_1}(s,a) \,|_{\alpha=\pi_\phi(s)} \nabla_\phi \pi_\phi(s)$

Update target networks: θ_i' and ϕ' `% Adaptive gradient optimiser ADAM [Kingma and Ba, 2014]`

end

end

Trading Agents II

Normalised price time series - Training input:

$$P_t = \frac{p_t - p_{min}}{p_{max} - p_{min}}, \quad P_t \in [0, 1].$$

The final portfolio wealth is derived as:

$$\mathbb{W} = \prod_{t=1}^{T} \Big[(1 - c_t) r_t \cdot w_{t-1} + 1 \Big].$$

Reward functions:

1 Simple returns

$$\rho_t = \sum_{i=1}^{m} (1 - c_t)\, r_{i,t} \cdot w_{i,t-1},$$

$$i = 1, \cdots m \text{ assets}$$

2 Log returns

$$R_t = \log\left(\sum_{i=1}^{m} w_i p_{i,t}\right) - \log\left(\sum_{i=1}^{m} w_i p_{i,t-1}\right)$$

3 Differential Sharpe Ratio

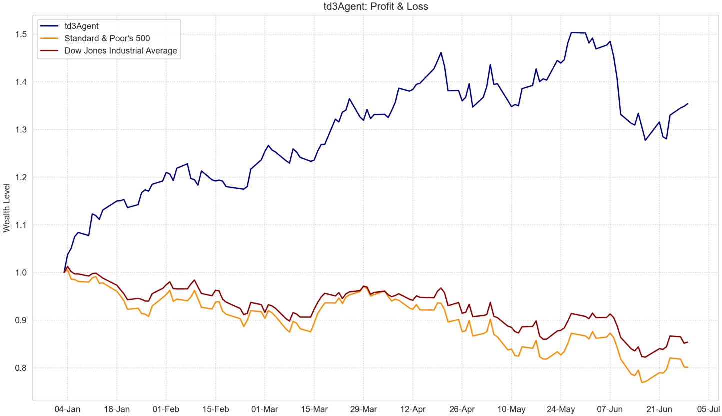
[Moody and Saffell, 1998]

$$\frac{\partial D_\eta(t)}{\partial R_t} = \frac{B_{t-1} - A_{t-1} R_t}{\left(B_{t-1} - A_{t-1}^2\right)^{\frac{3}{2}}}$$

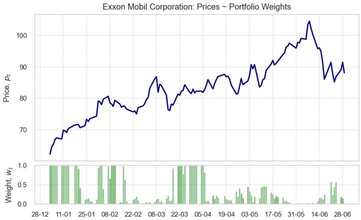
Results I

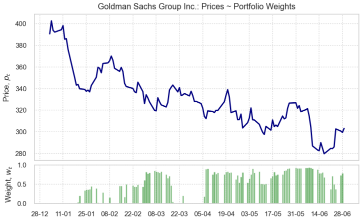
	Start date	End date
Training	02/01/15	30/12/20
Back-test	03/01/22	28/06/22

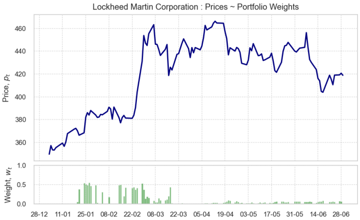
	TD3	S&P500	DJIA
PnL	1.35	0.80	0.85
Std. deviation	0.02	0.02	0.01
Skewness	-0.21	-0.21	-0.21
Kurtosis	0.43	-0.34	0.03
Sharpe ratio	1.69	-1.17	-1.07
MDD	0.33	0.24	0.19
CVaR −99%	-0.04	-0.04	-0.03



Results II


Exxon Mobil Corporation: Prices ~ Portfolio Weights
Price, p_t
100
90
80
70
Weight, w_t
1.0
0.5
0.0
28-12 11-01 25-01 08-02 22-02 08-03 22-03 05-04 19-04 03-05 17-05 31-05 14-06 28-06


Goldman Sachs Group Inc.: Prices ~ Portfolio Weights
Price, p_t
400
380
360
340
320
300
280
Weight, w_t
1.0
0.5
0.0
28-12 11-01 25-01 08-02 22-02 08-03 22-03 05-04 19-04 03-05 17-05 31-05 14-06 28-06

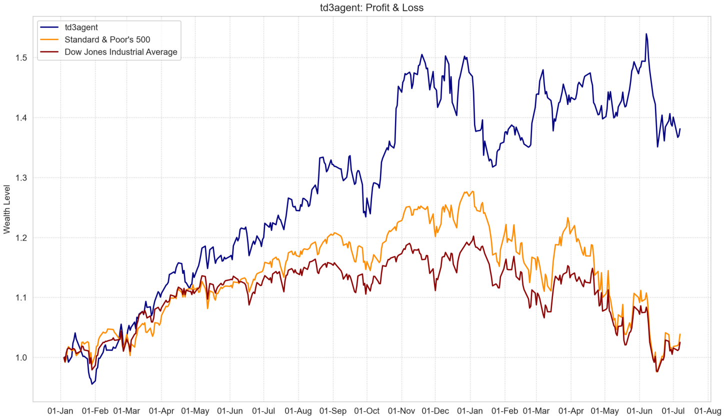

Lockheed Martin Corporation : Prices ~ Portfolio Weights
Price, p_t
460
440
420
400
380
360
Weight, w_t
1.0
0.5
0.0
28-12 11-01 25-01 08-02 22-02 08-03 22-03 05-04 19-04 03-05 17-05 31-05 14-06 28-06

Experiments

Results III

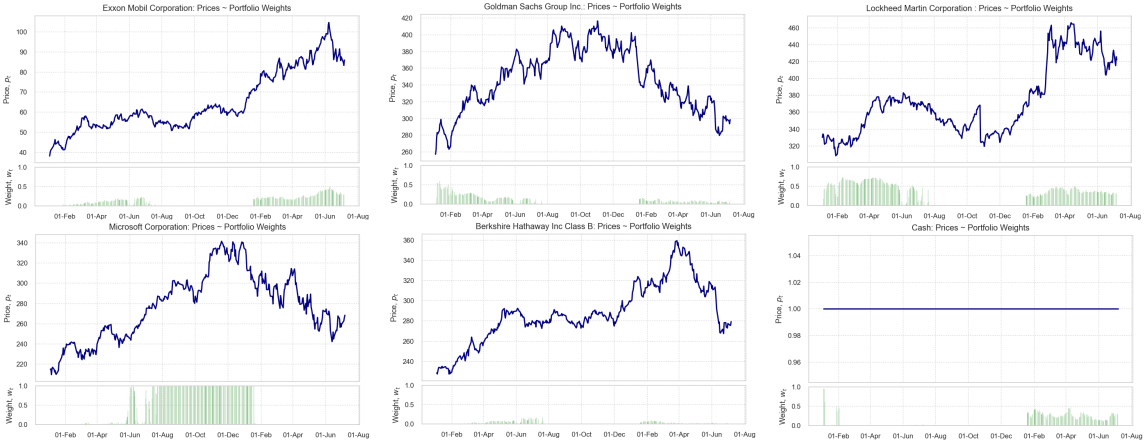
	Start date	End date
Training	02/01/15	30/12/20
Back-test	04/01/21	07/07/22

	TD3	S&P500	DJIA
PnL	1.38	1.04	1.03
Std. deviation	0.01	0.01	0.01
Skewness	-0.27	-0.49	-0.42
Kurtosis	1.23	1.16	1.02
Sharpe ratio	1.53	0.28	0.23
MDD	0.38	0.24	0.19
CVaR −99%	-0.04	-0.04	-0.03



Experiments

Results IV



Experiments

Reward functions comparison

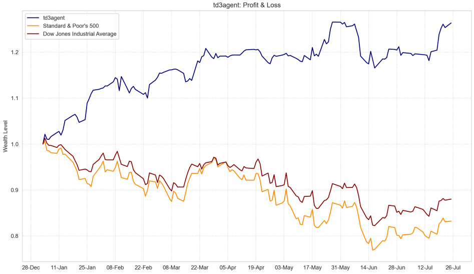


Figure: Differential Sharpe Ratio

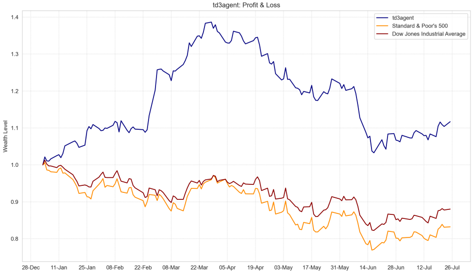


Figure: Log returns

Conclusions

Summary of Results

1. Minimum Spanning Tree used as an aid to topologically reveal clusters of assets with similar returns' patterns
2. Node reduction to obtain a smaller less correlated subset
3. Deep Reinforcement Learning finds patterns in the returns' series
4. Differential Sharpe Ratio, as a reward function, leads to more stable portfolio performances than log returns

Addressing Limitations

- MST selects for the least correlated asset's; albeit not taking their performance into account
- Interpretability; neural networks often described as "black-box" architectures
- Reproducibility issues

Future Work

- Optimise for simplicity in deep architectures to avoid overfitting and improve generalisation ability
- Reward signals that integrate financial knowledge; inclusion of technical indicators, sentiment analysis and fundamentals [Sato, 2019, Hambly et al., 2022, Filos, 2019]
- Learning through imitation; observing experts
- Offline reinforcement learning where data collection is expensive; the agent does not interact with the environment; safer for testing purposes [Fujimoto and Gu, 2021, Levine et al., 2020]

Questions ?

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