MBEYA UNIVERSITY OF SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF BUILT ENVIRONMENT ENGINEERING

TEST 1-SEMESTER 1

GSB 3103: DIFFERENTIAL EQUATIONS

Time 1:00 HR **DATE: 5TH December, 2017**

INSTRUCTIONS

1. This test consists of **three (3)** questions.
2. Attempt **ALL** questions.
3. Every question carries **Equal Marks**.
4. Programmable Calculators and Cellular Phones are **NOT** allowed in the examination room.
5. This test carries **15 Marks.**
6. **Guess work** or **copy work** will not be marked.

QUESTION 1

Find and classify singular points on the differential equations

(a) $x^2(x+1)^2y'' + (x^2-1)y' + 3y = 0$

(b) $(x^2-x)y'' + (3x-1)y' + y = 0$

(c) Solve the DE $x^2(x+1)^2y'' + (x^2-1)y' + 3y = 0$ at $x_0 = 0$

QUESTION 2

Solve the DE $x(x-1)y'' + (3x-1)y' + y = 0$ at $x_0 = 0$

QUESTION 3

Show that the solution of the differential equation $y'' + y = 0$ at $x_0 = 0$ is given by $y(x) = Asinx + Bcosx$ where A and B are arbitrary constants

TEST 01

① Solns

$$x^2(x+1)^2 y'' + (x^2-1)y' + 3y = 0$$

$\downarrow$

Normalized form

$$y'' + \frac{x^2-1}{x^2(x+1)^2} y' + \frac{3}{x^2(x+1)^2} y = 0$$

$$P(x) = \frac{x^2-1}{x^2(x+1)^2}$$

$$Q(x) = \frac{3}{x^2(x+1)^2}$$

$x^2(x+1)^2 = 0$.
$x = 0$
$x = -1$

$\therefore$ $x = 0$ and $x = -1$ are singular points.

classify

$q_1 = (x-x_0)P(x)$

$q_2 = (x-x_0)^2 Q(x)$

$x_0 = 0$

$$q_1 = x\left(\frac{x^2-1}{x^2(x+1)^2}\right) = \frac{x^2-1}{x(x+1)^2} = \frac{-1}{0} = \infty \text{ Not analytic}$$

$$q_2 = \cancel{x^2}\left(\frac{3}{\cancel{x^2}(x+1)^2}\right) = \frac{3}{(x+1)^2} = 3. = \text{Analytic}$$

$\therefore$ $x = 0$, Irregular singular point.

$X_0 = -1$

$$q_1 = (X+1)\frac{(X^2-1)}{X^2(X+1)^2} = \frac{(X^2+1)}{X^2(X+1)} = \frac{0}{0} = \text{Analytic}$$

$$q_2 = (X+\cancel{1})^2\left(\frac{3}{X^2(X+\cancel{1})^2}\right) = \frac{3}{X^2} = 3. \quad \text{Analytic}$$

$X = -1$, Are regular singular point.

Soln b

$$(X^2-X)y'' + (3X-1)y' + y = 0$$

Normalized form.

$$y'' + \frac{(3X-1)}{X^2-X}y' + \frac{y}{(X^2-X)} = 0$$

$$P(X) = \frac{3X-1}{X^2-X}$$

$$Q(X) = \frac{1}{X^2-X}$$

$X^2 - X = 0$

$X(X-1) = 0$

$X = 0, \; \& \; X = 1$

$\therefore X = 0$ and $X = 1$ Are singular point.

Classify

$X_0 = 0$

$q_1 = (X - X_0)P(x)$

$$q_1 = X\left(\frac{3X-1}{X^2-X}\right) = \frac{3X-1}{X-1} = 1 \quad \text{Analytic}$$

$Q_2 = (X - X_0)^2 Q(x)$

$Q_2 = X^2 \cdot \frac{1}{X^2 - X} = \frac{X}{X-1} = 0$ Analytic

$X = 0$, Regular singular point

$X_0 = 1$

$Q_1 = (\cancel{X-1})\left[\frac{3X-1}{X(\cancel{X-1})}\right] = \frac{3X-1}{X} = 2$ Analytic when $X=1$

$Q_2 = (X-1)^2\left[\frac{1}{X^2-X}\right] = \frac{X-1}{X} = 0$ Analytic when $X=1$

$X = 1$, Regular singular point.

Soln c

From (a) above, when $X_0 = 0$, Are Irregular singular point, hence D.E can't be solved.

(2) Soln

From qn 1 (b) above, when $X_0 = 0$, Are regular singular point hence D.E can be solved by Frobenius method

Assume the soln.

$$y = \sum_{n=0}^{\infty} a_n x^{n+r} = a_0 x^r + a_1 x^{r+1} + a_2 x^{r+2} + \cdots + a_n x^{n+r}$$

$$y' = \sum_{n=0}^{\infty} (n+r) a_n x^{n+r-1} = r a_0 x^{r-1} + (r+1) a_1 x^{r} + \cdots + (n+r) a_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1)a_n x^{n+r-2} = r(r-1)a_0 x^{r-2} + (1+r)(r)a_1 x^{r-1} + \dots + (n+r)(n+r-1)a_n x^{n+r-2}$$

$$\underbrace{x^2y'' - xy'' + 3xy' - y' + y} = 0$$

$$x^2y'' = r(r-1)a_0 x^r + (1+r)r a_1 x^{r+1} \dots + (n+r)(n+r-1)a_n x^{n+r}$$

$$-xy'' = -r(r-1)a_0 x^{r-1} + (1+r)(r)a_1 x^r + \dots + (n+r)(n+r-1)a_n x^{n+r-1}$$

$$3xy' = 3ra_0 x^r + 3(r+1)a_1 x^{r+1} + \dots + 3(n+r)a_n x^{n+r}$$

Sum of coefficient of lowest power of x^{r-1}

$$-r(r-1)a_0 + ra_0 = 0$$

$a_0 \neq 0$

$-r^2 + r + r = 0$ $\qquad -r^2 = 0$

$-r^2 + 2r = 0$ $\qquad r = 0,$

$2r - r^2 = 0$ $\qquad r_1 = 0$ $\qquad$ CASE III

$r(2-r) = 0$ $\qquad r_2 = 0$

$r = 0$, or $r = 2$ $\qquad$ ~~CASE II~~

Sum of the nth term.

$$(n+r)(n+r-1)a_n x^{n+r} - (n+r)(n+r-1)a_n x^{n+r-1} + 3(n+r)a_n x^{n+r} - (n+r)a_n x^{n+r-1} + a_n x^{n+r} = 0$$

Make power of x the same

$\boxed{n = n+1}$

$$(n+r)(n+r-1)a_n x^{n+r} - (n+r+1)(n+r)a_{n+1} x^{n+r} + 3(n+r)a_n x^{n+r} - (n+r+1)a_{n+1} x^{n+r} + a_n x^{n+r} = 0$$

$\cancel{x^{r+r}} = 0$

$$(n+r)(n+r-1)a_n - (n+r+1)(n+r)a_{n+1} + 3(n+r)a_n$$
$$- (n+r+1)a_{n+r} + a_n = 0$$

$$[(n+r)(n+r-1) + 3(n+r) + 1]a_n = [(n+r+1) + \cancel{(n+r+1)(n+r)}]a_{n+1}$$

$$[(n+r+1) + (n+r+1)(n+r)]$$

$$a_{n+1} = \frac{[(n+r)(n+r+1) + 3(n+r) + 1]a_n}{[(n+r+1) + [n+r+1](n+r)]}$$

$$a_{n+1} = \frac{[(n+r)[n+r-1+3] + 1]a_n}{(n+r+1)[1+n+r]}$$

$$a_{n+r} = \frac{[(n+r)(n+r+2) + 1]a_n}{(n+r+1)^2} \quad \text{where } \underline{r=0}$$

$$a_{n+1} = \frac{[n(n+2) + 1]a_n}{(n+1)^2} \quad n = 0, 1, 2, 3 \ldots$$

Recurrence Relation

$n=0$ $\qquad$ $n=2$

$a_1 = a_0$ $\qquad$ $a_3 = a_2 = a_0$

$\underline{n=1}$

$a_2 = a_1 = a_0$

$$y_1(x) = \sum_{n=0}^{\infty} a_n x^{n+r} = a_0 x^r + a_1 x^{r+1} + a_2 x^{r+2} + a_3 x^{r+3}$$

$$y_1(x) = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3$$

$$y_1(x) = a_0 + a_0 x + a_0 x^2 + a_0 x^3$$

$$y_1(x) = a_0[1 + x + x^2 + x^3 + \ldots]$$

$$y_2(x) = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{(y_1(x))^2}\,dx$$

$$y(x) = y_1(x) + y_2(x)$$

$$y_1(x) = a_0\left[1 + x + x^2 + x^3\right]$$

$1 + x + x^2 + x^3 =$ From Maclaurin series

$$\frac{1}{1-x}$$

$$y_2(x) = \frac{1}{1-x} \int \frac{e^{-\int P(x)\,dx}}{((1-x)^{-1})^2}\,dx$$

$$-\int P(x)\,dx = -\int \frac{3x-1}{x^2-x}\,dx$$

Partial

$$= \int \frac{3x-1}{x(x-1)}\,dx = \int \frac{A}{x} + \int \frac{B}{(x-1)}$$

$$\frac{3x-1}{x\cancel{(x-1)}} = \frac{A(x-1) + Bx}{x\cancel{(x-1)}}$$

$$3x-1 = A(x-1) + Bx.$$

$x=1$ $\qquad$ $x=0$

$B=2$ $\qquad$ $A=1$

$$\boxed{\begin{aligned} 3 &= A + B \\ -1 &= -A \\ A &= 1 \\ B &= 2 \end{aligned}}$$

$$-\int P(x)\,dx = -\ln x - 2\ln(x-1)$$

$$= -\ln x + \ln(x-1)^{-2}$$

$$= \ln(x-1)^{-2} - \ln x$$

$$= \ln\left(\frac{(x-1)^{-2}}{x}\right)$$

$$= \ln\left(\frac{1}{x(x-1)^2}\right)$$

$$y_2(x) = \frac{1}{1-x}\int \frac{e^{\ln\left(\frac{1}{x(x-1)^2}\right)}}{(1-x)^{-2}}\,dx$$

$$= \frac{1}{1-x}\int \frac{(1-x)^2}{x(x-1)^2}\,dx$$

$$= \frac{1}{1-x}\int \frac{(-1(-1+x))^2}{x(x-1)^2}\,dx$$

$$= \frac{1}{1-x}\int \frac{\cancel{(x-1)^2}}{x\cancel{(x-1)^2}}\,dx$$

$$= \frac{1}{1-x}\int \frac{1}{x}\,dx$$

$$y_2(x) = \frac{1}{1-x}\ln x$$

$$y(x) = y_1(x) + y_2(x)$$

$$y(x) = (1-x)^{-1} + (1-x)^{-1}\ln x$$

3 Soln

$y'' + y = 0 \qquad x_0 = 0$

$\downarrow$

$P(x) = 0 \qquad Q(x) = 1$

Power Series

Assum the solution

$$y = \sum_{n=0}^{\infty} a_n x^n =$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2} n(n-1) a_n x^{n-2}$$

substitute y'', $\cancel{y'}$ and y into eqn above

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n = 0$$

Let $k = n-2$, $\quad k = n$,
$n = k+2 \qquad k = n$

$$\sum_{k=0}^{\infty} (k+2)(k+1) a_{k+2} x^k + \sum_{k=0}^{\infty} a_k x^k = 0$$

$$\sum_{k=0}^{\infty} (k+2)(k+1) a_{k+2} + \sum_{k=0}^{\infty} a_k = 0$$

$$a_{k+2} = \frac{-a_k}{(k+2)(k+1)} \qquad \text{R.R} \qquad k = 0, 1, 2, 3, 4$$

$k = 0$

$a_2 = \frac{-a_0}{2}$

$k = 2$

$a_4 = \frac{-a_2}{12} = \frac{a_0}{24}$

$k = 1$

$a_3 = \frac{-a_1}{6} = \frac{-a_1}{6}$

$k = 3$

$a_5 = \frac{-a_3}{20} = \frac{a_1}{120}$

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

$$= a_0 + a_1 x - \frac{a_0}{2} x^2 - \frac{a_1}{6} x^3 + \frac{a_0}{24} x^4 + \frac{a_1}{120} x^5$$

$$= a_0 \left[1 - \frac{x^2}{2} + \frac{x^4}{24} + \ldots\right] + a_1 \left[x - \frac{x^3}{6} + \frac{x^5}{120} + \ldots\right]$$

Let $a_0 = B$, $a_1 = A$

$$= B\left[1 - \frac{x^2}{2} + \frac{x^4}{4!} + \ldots\right] + A\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right]$$

but $1 - \frac{x^2}{2!} + \frac{x^4}{4!} = \cos x$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} = \sin x$$

$$y(x) = B\cos x + A\sin x$$

$\therefore y(x) = A\sin x + B\cos x$ hence shown

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DEPARTMENT OF BEE
TEST II SEMESTER 1
GSB 3103: DIFFERENTIAL EQUATIONS

Time 1:00 HR **DATE: 9th January, 2018**

INSTRUCTIONS

1. This test consists of **three (3)** questions.
2. Attempt **ALL THREE (3)** questions.
3. Every question carries **Equal Marks**.
4. Programmable Calculators and Cellular Phones are **NOT** allowed in the examination room.
5. This test carries **15 Marks.**
6. **Guess work** or **copy work** will not be marked.

QUESTION 1

a) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\, cosx$

b) Use Rodriguez's formula to derive Legendre polynomials $P_4(x)$

QUESTION 2

(a) Using substitution $z = x^2$ reduce the DE $x^2y'' + xy' + (4x^4 - 1/4)y = 0$ to the Bessel's equation and then write the general solutions of new and original DE.

(b) Write the general solution of the Bessel's equation $x^2y'' + xy' + (x^2 - 9/16)y = 0$

QUESTION 3

(a) Write $f(x) = x^4 + 2x^2 + 1$ as a series of Legendre polynomials

b) If z is a complex number, Show that

i) $Z\bar{Z} = x^2 + y^2$

ii) $Z + \bar{Z} = 2ReZ$

iii) $Z - \bar{Z} = 2ImZ$

$\bar{z} = x - iy$
$z = x + iy$

(1)

TEST 02

1. Soln.

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x.$$

from

$$J_v(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(n+v+1)} \left(\frac{x}{2}\right)^{2n+v}$$

$$J_v(x) = \left(\frac{x}{2}\right)^v \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(n+v+1)} \left(\frac{x}{2}\right)^{2n}$$

$$J_{-\frac{1}{2}}(x) = \left(\frac{x}{2}\right)^{-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(n+\frac{1}{2})} \left(\frac{x}{2}\right)^{2n}$$

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{x}} \left[\frac{1}{\Gamma\frac{1}{2}} - \frac{x^2}{4\Gamma\frac{3}{2}} + \frac{x^4}{16 \cdot 2\Gamma\frac{5}{2}} + \text{———}\right]$$

$$= \sqrt{\frac{2}{x}} \left[\frac{1}{\sqrt{\pi}} - \frac{x^2}{2\sqrt{\pi}} + \frac{x^4}{24\sqrt{\pi}} - -\right]$$

$\frac{3}{2} \times \frac{1}{2}\sqrt{\pi}$

$\frac{3}{4}$

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - --\right]$$

But $1 - \frac{x^2}{2!} + \frac{x^4}{4!} = \cos x$

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$$ hence shown

Soln b

$$P_n(x) = \frac{1}{2^n . n!} \frac{d^n}{dx^n} [x^2-1]^n$$

$$P_4(x) = \frac{1}{2^4 . 4!} \frac{d^4}{dx^4} [x^2-1]^4$$

$$P_4(x) = \frac{1}{384} \frac{d^4}{dx^4} [x^2-1]^4$$

By triangular Method

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

$$[x^2-1]^4 = (x^2)^4 - 4(x^2)^3 + 6(x^2)^2 - 4(x^2)^1 + 1$$

$$[x^2-1]^4 = x^8 - 4x^6 + 6x^4 - 4x^2 + 1$$

$$\frac{d^4}{dx^4} = 8x^7 - 24x^5 + 24x^3 - 8x$$

$$\frac{d^3}{dx^3} = 56x^6 - 120x^4 + 72x^2 - 8$$

$$\frac{d^2}{dx^2} = 336x^5 - 480x^3 + 144x$$

$$\frac{d}{dx} = 1680x^4 - 1440x^2 + 144$$

$$P_4(x) = \frac{1}{384}[1680x^4 - 1440x^2 + 144]$$

$$P_4(x) = \frac{1}{8}[35x^4 - 30x^2 + 3]$$

2. soln a

$$x^2y'' + xy' + \left(4x^4 - \frac{1}{4}\right)y = 0$$

$$z = x^2$$

$$\frac{\partial z}{\partial x} = 2x$$

$$\frac{\partial y}{\partial x} = \frac{\partial z}{\partial x} \cdot \frac{\partial y}{\partial z}$$

$$\frac{\partial y}{\partial x} = 2x \frac{\partial y}{\partial z} \quad \text{---- i)}$$

$$\cancel{\frac{\partial^2 y}{\partial x^2} = 2x \cdot \frac{\partial^2 y}{\partial z^2} + 2\frac{\partial y}{\partial z}}$$

$$\frac{\partial^2 y}{\partial x^2} = 2\frac{\partial y}{\partial z} + 2x \frac{\partial}{\partial x}\left[\frac{\partial y}{\partial z}\right].$$

$$a = \frac{\partial y}{\partial z}$$

$$\frac{\partial a}{\partial x} = \frac{\partial a}{\partial z} \cdot \left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial a}{\partial x} = 2x \frac{\partial a}{\partial z}$$

$$\frac{\partial^2 y}{\partial x^2} = 2\frac{\partial y}{\partial z} + 2x\left[2x \frac{\partial a}{\partial z}\right]$$

$$\frac{\partial^2 y}{\partial x^2} = 2\frac{\partial y}{\partial z} + 4x^2\left[\frac{\partial}{\partial z}\left[\frac{\partial y}{\partial z}\right]\right].$$

$$\frac{\partial^2 y}{\partial x^2} = 2\frac{\partial y}{\partial z} + 4x^2 \frac{\partial^2 y}{\partial z^2}$$

$$x^2y'' + xy + \left(4x^4 - \frac{1}{4}\right)y = 0$$

$$x^2\left[2\frac{dy}{dz} + 4x^2\frac{d^2y}{dz^2}\right] + 2x^2\frac{dy}{dz} + \left(4x^4 - \frac{1}{4}\right)y = 0$$

$$2x^2\frac{dy}{dz} + 4x^4\frac{d^2y}{dz^2} + 2x^2\frac{dy}{dz} + \left(4x^4 - \frac{1}{4}\right)y = 0$$

$$4x^4\frac{d^2y}{dz^2} + 4x^2\frac{dy}{dz} + \left(4x^4 - \frac{1}{4}\right)y = 0$$

but $\left[\begin{matrix} z = x^2 \\ z^2 = x^4 \end{matrix}\right]$

$$4z^2\frac{d^2y}{dz^2} + 4z\frac{dy}{dz} + \left(4z^2 - \frac{1}{4}\right)y = 0$$

divide by 4 both side

$$z^2\frac{d^2y}{dz^2} + z\frac{dy}{dz} + \left(z^2 - \frac{1}{16}\right)y = 0$$

$$V = \frac{1}{4}.$$

$$z^2\frac{d^2y}{dx^2} + z\frac{dy}{dz} + \left(z^2 - \left(\frac{1}{4}\right)^2\right)y = 0$$

Bessel eqn

General soln.

$$y(x) = AJ_v(x) + BJ_{-v}(x)$$

$$y(z) = AJ_{1/4}(z) + BJ_{-1/4}(z)$$

$$\therefore y(z) = AJ_{\frac{1}{4}}(x^2) + BJ_{-1/4}(x^2)$$

2. Soln b

$$x^2y'' + xy' + \left(x^2 - \frac{9}{16}\right)y = 0$$

$$x^2y'' + xy' + \left(x^2 - \left(\frac{3}{4}\right)^2\right)y = 0$$

$$V = \frac{3}{4}$$

$$y(x) = AJ_{3/4}(x) + BJ_{-3/4}(x) \qquad \text{G.S}$$

3. Soln a

$$f(x) = x^4 + 2x^2 + 1.$$

from

$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x)$$

$$f(x) = a_0 P_0(x) + a_1 P_1(x) + a_2 P_2(x) + a_3 P_3(x)$$

$$x^4 + 2x^2 + 1 = a_0 + a_1 x + \frac{1}{2}(3x^2 - 1)a_2 + \frac{1}{2}(5x^3 - 3x)a_3 + \frac{1}{8}(35x^4 + 30x^2 + 3)a_4$$

$$x^4 + 2x^2 + 1 = a_0 + a_1 x + \frac{3a_2 x^2}{2} - \frac{a_2}{2} + \frac{5x^3 a_3}{2} - \frac{3x a_3}{2} + \frac{35x^4 a_4}{8} - \frac{30}{8}x^2 a_4 + \frac{3a_4}{8}$$

$$= \left(a_0 - \frac{a_2}{2} - \frac{3}{2}a_3 + \frac{3a_4}{8}\right) + \left(a_1 x + \frac{3}{2}x a_3\right) + \frac{5x^3 a_3}{2} + \left(\frac{3a_2 x^2}{2} - \frac{30}{8}x^2 a_4\right) + \frac{35}{8}x^4 a_4$$

Comparison

$$\cancel{0} = \frac{5}{2}a_3$$

$$\underline{a_3 = 0}$$

$$1 = a_0 - \frac{a_2}{2} - \frac{3}{2}a_3 \quad \text{--- i}$$

$$1 = a_0 - \frac{a_2}{2}$$

$$a_1 = 0$$

$$2 = \frac{3}{2}a_2 - \frac{30}{8}a_4 \quad \text{--- ii}$$

$$1 = \frac{35}{8}a_4$$

$$\underline{a_4 = 8/35}$$

$$2 = \frac{3}{2}a_2 - \frac{30}{\cancel{8}} \cdot \frac{\cancel{8}}{35}$$

$$\frac{2}{1} + \frac{30}{35} = \frac{3}{2}a_2$$

$$\frac{70+35}{35} = \frac{3}{2}a_2$$

$$\frac{105}{35} = \frac{3}{2}a_2$$

$$\underline{a_2 = 40/21}$$

$$1 = a_0 - \frac{40}{42}$$

$$a_0 = \frac{1}{1} + \frac{40}{42} = \frac{82}{42} \quad \frac{41}{21}$$

$$\underline{\underline{a_0 = 41/21}}$$

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Then

$$f(x) = a_0P_0(x) + a_1P_1(x) + a_2P_2(x) + a_3P_3(x) + a_4P_4(x)$$

$$x^4+2x^2+1 = \frac{41}{21}P_0(x) + \frac{40}{21}P_2(x) + \frac{8}{35}P_4(x)$$

3. Soln

show that

$Z\bar{Z} = x^2+y^2$

(i) $Z = x+iy.$

$\bar{Z} = x-iy.$

$$Z\cdot\bar{Z} = (x+iy)(x-iy)$$

$$= x^2 - i^2y^2 \qquad \text{but } i^2 = -1.$$

$\therefore Z\cdot\bar{Z} = x^2+y^2$ hence shown

(ii) $Z+\bar{Z} = 2Re(Z)$

$$(x+iy)+(x-iy) = x+iy+x-iy$$

$$= 2x$$

$\therefore Z+\bar{Z} = 2Re(Z)$ hence shown

(iii) $Z-\bar{Z} = 2Im(Z)$

$$(x+iy)-(x-iy) = 2iy$$

$Z-\bar{Z} = 2Im(Z)$ hence shown