

SUMMARY OF WINDOW FUNCTIONS IN FIR FILTER DESIGN.

Reference: "Digital Filters: Theory and Applications", N.K. Bose,
Elsevier Science Publishing Co., Inc., New York, 1985.

WINDOW TYPE	WINDOW FUNCTION $W(k) \quad -N \leq k \leq N$ $W(k) = 0 \quad \forall k > N.$
RECTANGULAR	1
HANN	$\frac{1}{2} [1 + \cos((2\pi k)/(2N))]]$
HAMMING	$0.54 + 0.46 \cos((2\pi k)/(2N))$
GENERALIZED HAMMING	$\alpha + (1-\alpha) \cos((2\pi k)/(2N)) \quad 0 < \alpha < 1$
FEJER - CESARO / BARTLETT / TRIANGULAR	$1 - ((2 k)/(2N))$
LANCZOS	$\left\{ \sin[(2k\pi)/(2N)] / [(2k\pi)/(2N)] \right\}^L$ $L > 0$
DOLPH - CHEBYSHEV (Fourier Transform)	$W(e^{j\omega}) = \frac{\cos[(2N) \cos^{-1}(\alpha \cos(\omega/2))]}{\cosh(2N \cosh^{-1} \alpha)}$ $\alpha > 0$
PAPOULIS	$\frac{1}{\pi} [\sin[(2\pi k)/(2N)]] + \frac{2 k }{2N} \cos \frac{2\pi k}{2N}$
KAISER	$\frac{I_0[\beta N \sqrt{1 - (k/N)^2}]}{I_0(\beta N)} \quad \beta > 0.$ $I_0(x) =$ modified Bessel function of first-kind and order 0 in x $I_0(x) \triangleq 1 + \sum_{l=1}^{\infty} \left[\frac{(x/2)^l}{l!} \right]^2$
TUKEY	$1 \quad \forall k \leq \alpha N \quad 0 < \alpha < 1$ $\frac{1}{2} [1 + \cos\{[(k - \alpha N)\pi]/[(1 - \alpha)N]\}]$ $\alpha N \leq k \leq N.$
PARZEN	$m \triangleq \frac{1}{2} k/N.$ $1 - 24m^2 + 48 m \quad m \leq \frac{1}{4}$ $\frac{1}{2} [1 - 2 m] \quad \frac{1}{4} \leq m \leq \frac{1}{2}$
BLACKMAN	$0.42 + 0.5 \cos\left(\frac{2\pi k}{2N}\right) + 0.08 \cos\left(\frac{4\pi k}{2N}\right)$

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- Window functions - Comparison of commonly used windows in FIR filter design

Window Type (Name)	Peak relative sidelobe amplitude (dB)	Approximate mainlobe width	Peak error in approximation $20 \log_{10} \delta$ (dB)	Equivalent α in Kaiser window *	Equivalent Kaiser window Transition width **
Rectangular	- 13	$4\pi/(2N+1)$	- 21	0	$1.81\pi/2N$
Bartlett	- 25	$4\pi/N$	- 25	1.33	$2.37\pi/2N$
Hann	- 31	$4\pi/N$	- 44	3.86	$5.01\pi/2N$
Hamming	- 41	$4\pi/N$	- 53	4.86	$6.27\pi/2N$
Blackman	- 57	$6\pi/N$	- 74	7.04	$9.19\pi/2N$

δ = peak ripple in passband and stopband

$\alpha = \beta.N$ in Kaiser window

* : to get the same δ as the corresponding window

** : accordingly $\Delta\omega_T$ from empirical equations below.

- Empirical design equations for Kaiser window

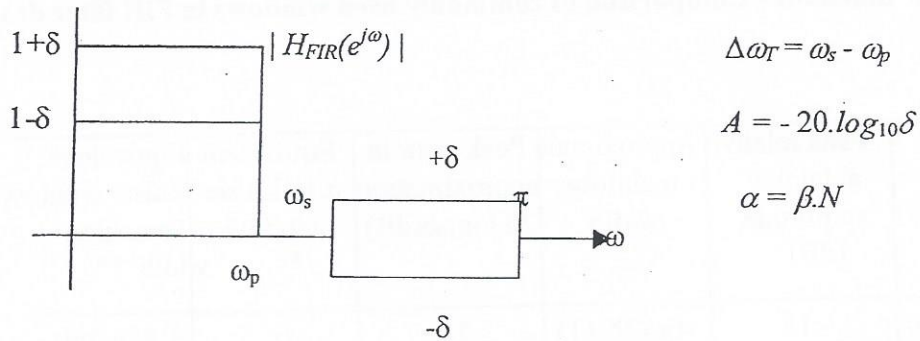


Fig. : Prototype Specifications for Low Pass FIR filter

Reference : "Discrete – Time Signal Processing", Oppenheim and Schaffer, pp.450-454

Design steps :

1. Choose N according to $(2N+1) \geq 1 + \frac{A-8}{2.285\Delta\omega_T}$
2. Now choose α and hence β according to

$$\alpha = \begin{cases} 0.1102 (A - 8.7) & \text{for all } A > 50 \\ 0.5842 (A - 21)^{0.4} + 0.07886 (A - 21) & \text{for } 21 \leq A \leq 50 \\ 0 & \text{for } A < 21 \end{cases}$$

Remember $\beta = \alpha/N$

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• Park Mc Clellan Algorithm for FIR filters (odd length, symmetric)

To determine optimum filter of the form, $H_{FIR}(e^{j\omega})$ as below,
Optimum filter is bound to satisfy - Eqn 1 below

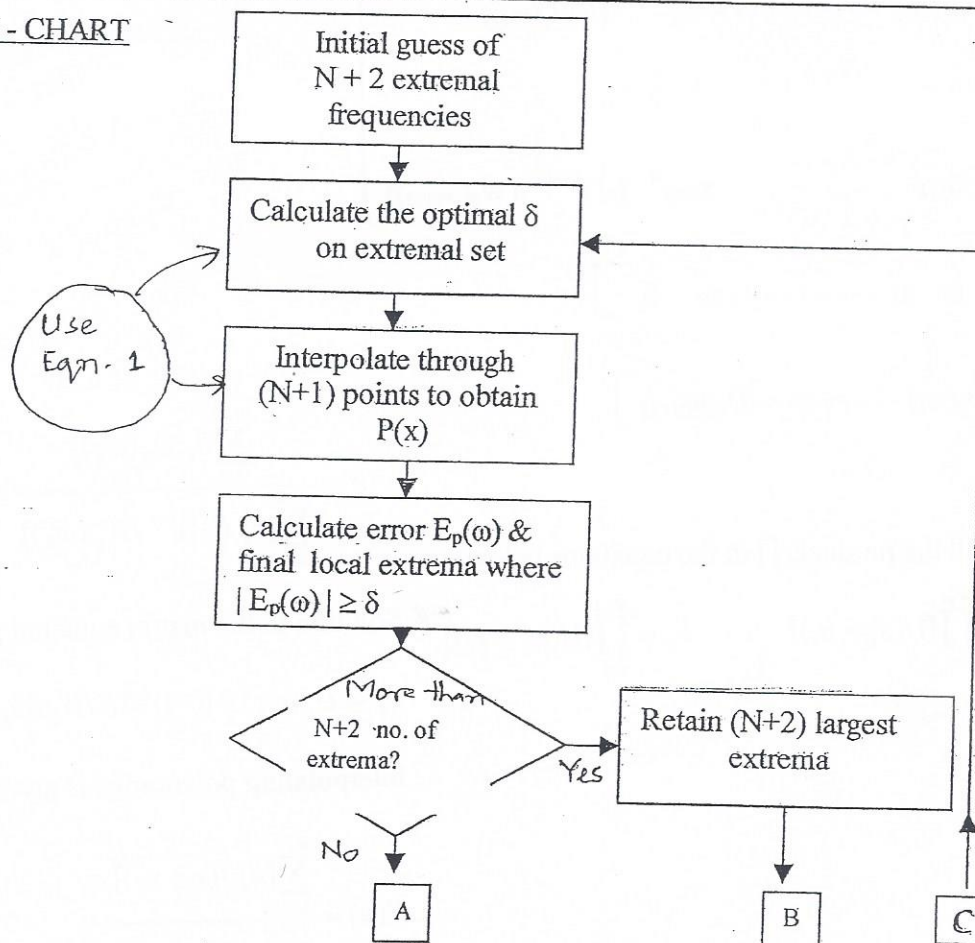
$$H_{FIR}(e^{j\omega}) = \sum_{n=-N}^N h_{FIR}[n] \cdot e^{-j\omega n}$$

$$\text{Eqn.1: } E_p(\omega_i) = W_p(\omega_i) \cdot [D_p(\omega_i) - P(\cos \omega_i)] = (-1)^{i+1} \delta \quad i = 1, \dots, N+2.$$

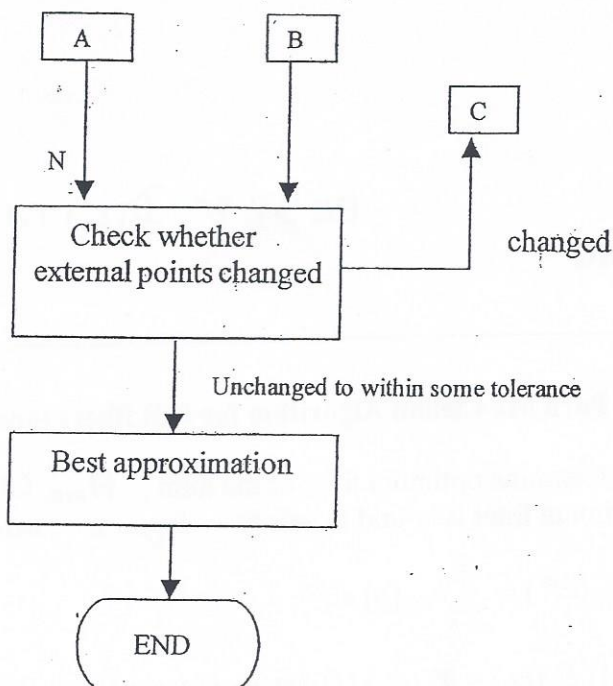
$$H_{FIR}(e^{j\omega}) = P(\cos \omega) = P(x) = \sum_{k=0}^N a_k \cdot x^k \quad x = \cos \omega$$

where ω_i , $i = 1, \dots, N+2$ are the external frequencies

FLOW - CHART



Flow - chart (contd...)



Eq n 1) gives the system

$$x.A = H$$

$$x = \begin{pmatrix} 1 & x_1 & x_1^2 & \dots & x_1^N & 1/(W_p(\omega_1)) \\ 1 & x_2 & x_2^2 & \dots & x_2^N & -1/(W_p(\omega_2)) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_{N+2} & & & x_{N+2}^N & (-1)^{1+N+2}/(W_p(\omega_{N+2})) \end{pmatrix}$$

$$A = \begin{bmatrix} a_0 & a_1 & \dots & a_N & \delta \end{bmatrix}^T$$

$$H = \begin{bmatrix} D_p(\omega_1) & \dots & D_p(\omega_{N+2}) \end{bmatrix}$$

$i \neq k$ for all the products \prod in the equations below

$$d_k = \prod_{i=1}^{N+1} [1/(x_k - x_i)]$$

$$b_k = \prod_{i=1}^{N+2} [1/(x_k - x_i)]$$

$$\delta = \frac{\sum_{k=1}^{N+2} b_k \cdot D_p(\omega_k)}{\sum_{k=1}^{N+2} [b_k \cdot (-1)^{k+1} / W_p(\omega_k)]}$$

Solution to the matrix equation given by $x.A = H$ is :

$$c_k = D_p(\omega_k) - [(-1)^{k+1} \delta / W_p(\omega_k)]$$

Interpolating polynomial is given by

$$P(x) = \frac{\sum_{k=1}^{N+1} [d_k / (x - x_k)] \cdot c_k}{\sum_{k=1}^{N+1} [d_k / (x - x_k)]}$$

References:
Discrete Time Signal Processing
"Oppenheim and Schaffer" pp. 468 - 480