CHAPTER 13
ELECTRO-OPTIC MODULATORS

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13.1 GLOSSARY

$\bar{A}$	general symmetric matrix
$\bar{a}$	orthogonal transformation matrix
b	electrode separation of the electro-optic modulator
D	displacement vector
d	width of the electro-optic crystal
E	electric field
H	magnetic field
IL:	insertion loss
k	wave vector, direction of phase propagation
L	length of the electro-optic crystal
L/b	aspect ratio
N	number of resolvable spots
n_m	refractive index of modulation field
n_x, n_y, n_z	principal indices of refraction
R	global rotation axis of the index ellipsoid
r_{ijk}	third-rank linear electro-optic coefficient tensor
S	Poynting (ray) vector, direction of energy flow
s_{ijkl}	fourth-rank quadratic electro-optic coefficient tensor
T	transmission or transmissivity
V	applied voltage
V_π	half-wave voltage
v	phase velocity
v_m	modulation phase velocity

v_s	ray velocity
w	beamwidth
w_o	resonant frequency of an electro-optic modulator circuit
$\mathbf{X}$	position vector in cartesian coordinates
$\mathbf{X}'$	electrically perturbed position vector in cartesian coordinates
(x, y, z)	unperturbed principal dielectric coordinate system
(x', y', z')	new electro-optically perturbed principal dielectric coordinate system
(x'', y'', z'')	wave vector coordinate system
(x''', y''', z''')	eigenpolarization coordinate system
β_1	polarization angle between x''' and x''
β_2	polarization angle between y''' and x''
Γ	phase retardation
Γ_m	amplitude modulation index
Δn	electro-optically induced change in the index of refraction or birefringence
$\Delta(1/n^2)$	electro-optically induced change in an impermeability tensor element
$\Delta\phi$	angular displacement of beam
Δv	bandwidth of a lumped electro-optic modulator
δ	phase modulation index
$\bar{\boldsymbol{\epsilon}}$	permittivity tensor
ϵ_o	permittivity of free space
$\bar{\boldsymbol{\epsilon}}^{-1}$	inverse permittivity tensor
$\epsilon_x, \epsilon_y, \epsilon_z$	principal permittivities
$\bar{\varepsilon}$	dielectric constant tensor
$\bar{\varepsilon}^{-1}$	inverse dielectric constant tensor
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	principal dielectric constants
η_m	extinction ratio
θ	half-angle divergence
θ_k, ϕ_k	orientation angles of the wave vector in the (x, y, z) coordinate system
ϑ	optic axis angle in biaxial crystals
λ	wavelength of the light
$\bar{\lambda}$	diagonal matrix
v_{tw}	bandwidth of a traveling wave modulator
ξ	modulation efficiency
ρ	modulation index reduction factor

τ	transit time of modulation signal
Φ	global rotation angle of the index ellipsoid
ϕ	phase of the optical field
Ω	plane rotation angle
ϖ	beam parameter for bulk scanners
ω_d	frequency deviation
ω_e	stored electric energy density
ω_m	modulation radian frequency
$\overline{1/n^{2}\prime}$	electro-optically perturbed impermeability tensor
$\overline{1/n^{2}}$	inverse dielectric constant (impermeability) tensor

13.2 INTRODUCTION

The electro-optic effect is one of several means to impose information on, or modulate, a light wave carrier. Electro-optic devices have been developed for application in communications,[1–4] analog and digital signal processing,[5] information processing,[6] optical computing,[6,7] and sensing.[5,7] Example devices include phase and amplitude modulators, multiplexers, switch arrays, couplers, polarization controllers, deflectors,[1,2] Givens rotation devices,[8] correlators,[9] A/D converters,[10] multichannel processors,[11] matrix-matrix and matrix-vector multipliers,[11] and sensors for detecting temperature, humidity, and radio-frequency electrical signals.[5,7] The electro-optic effect allows for much higher modulation frequencies than other methods, such as mechanical shutters, moving mirrors, or acousto-optic devices, due to a faster response time.

The basic idea behind electro-optic devices is to alter the optical properties of a material with an applied voltage in a controlled way. The changes in the optical properties, particularly the permittivity tensor, translate into a modification of some parameter of a light wave carrier, such as phase, amplitude, frequency, polarization, or position, as it propagates through the device. Therefore, understanding how light propagates in these materials is necessary for the design and analysis of electro-optic devices. The following section gives an overview of light propagation in anisotropic materials that are homogeneous, nonmagnetic, lossless, optically inactive, and nonconducting. The third section, "The Electro-optic Effect," gives a geometrical and mathematical description of the linear and quadratic electro-optic effects. A geometrical approach using the index ellipsoid is presented to illustrate the details of how the optical properties change with applied voltage. A mathematical approach is offered to determine the perturbed principal dielectric axes and indices of refraction of any electro-optic material for any direction of the applied electric field as well as the phase velocity indices and eigenpolarization orientations for a given wave vector direction. Finally, basic bulk electro-optic modulators are described in the fourth section, "Modulator Devices," including some design considerations and performance criteria.

The discussion presented in this chapter applies to any electro-optic material, any direction of the applied voltage, and any direction of the wave vector. Therefore, no specific materials are described explicitly, although materials such as lithium niobate ($LiNbO_3$), potassium dihydrogen phosphate (KDP), and gallium arsenide (GaAs) are just a few of several materials commonly used. Emphasis is placed on the general fundamentals of the electro-optic effect and bulk modulator devices.
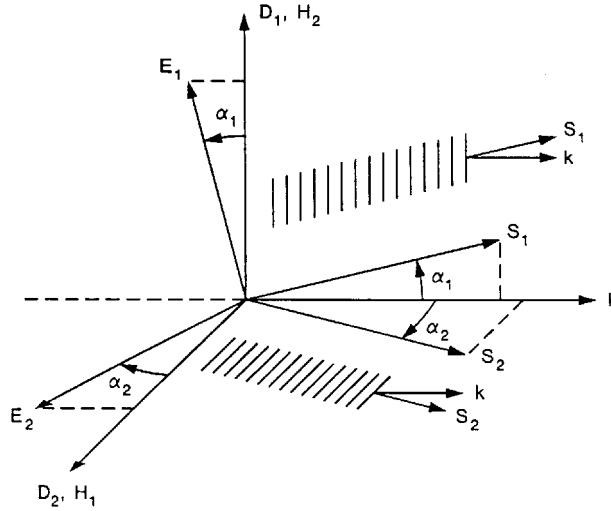
FIGURE 1 The geometric relationships of the electric quantities **D** and **E** and the magnetic quantities **B** and **H** to the wave-vector **k** and the ray vector **S** are shown for the two allowed extraordinary-like waves propagating in an anisotropic medium.[12]

13.3 CRYSTAL OPTICS AND THE INDEX ELLIPSOID

For any anisotropic (optically inactive) crystal class there are two allowed orthogonal lineraly polarized waves propagating with differing phase velocities for a given wave vector **k**. Biaxial crystals represent the general case of anisotropy. Generally, the allowed waves exhibit *extraordinary-like* behavior; the wave vector and ray (Poynting) vector directions differ. In addition, the phase velocity, polarization orientation, and ray vector of each wave change distinctly with wave vector direction. For each allowed wave, the electric field **E** is not parallel to the displacement vector **D** (which defines polarization orientation) and, therefore, the ray vector **S** is not parallel to the wave vector **k** as shown in Fig. 1. The angle α between **D** and **E** is the same as the angle between **k** and **S**, but for a given **k**, $\alpha_1 \neq \alpha_2$. Furthermore, for each wave **D** $\perp$ **k** $\perp$ **H** and **E** $\perp$ **S** $\perp$ **H**, forming orthogonal sets of vectors. The vectors **D**, **E**, **k**, and **S** are coplanar for each wave.[12]

The propagation characteristics of the two allowed orthogonal waves are directly related to the fact that the optical properties of an anisotropic material depend on direction. These properties are represented by the constitutive relation **D** = $\bar{\epsilon}$**E**, where $\bar{\epsilon}$ is the permittivity tensor of the medium and **E** is the corresponding optical electric field vector. For a homogeneous, nonmagnetic, lossless, optically inactive, and nonconducting medium, the permittivity tensor has only real components. Moreover, the permittivity tensor and its inverse $\bar{\epsilon}^{-1} = 1/\epsilon_0 (\overline{1/n^2})$, where n is the refractive index, are symmetric for all crystal classes and for any orientation of the dielectric axes.[13–15] Therefore, the matrix representation of the permittivity tensor can be diagonalized, and in principal coordinates the constitutive equation has the form

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{1}$$

where reduced subscript notation is used. The principal permittivities lie on the diagonal of $\bar{\epsilon}$.

The index ellipsoid is a construct with geometric characteristics representing the phase velocities and the vibration directions of **D** of the two allowed plane waves corresponding to a given optical wave-normal direction **k** in a crystal. The index ellipsoid is a quadric surface of the stored electric energy density ω_e of a dielectric:[13,16]

$$\omega_e = \tfrac{1}{2}\mathbf{E} \cdot \mathbf{D} = \tfrac{1}{2}\sum_i \sum_j E_i \epsilon_{ij} E_j = \tfrac{1}{2}\epsilon_0 \mathbf{E}^T \bar{\epsilon} \mathbf{E} \qquad i, j = x, y, z \qquad (2a)$$

or

$$\omega_e = \tfrac{1}{2}\epsilon_0 (E_x^2 \varepsilon_x + E_y^2 \varepsilon_y + E_z^2 \varepsilon_z) \qquad (2b)$$

in principal coordinates, where T indicates transpose. The stored energy density is positive for any value of electric field; therefore, the quadric surface is always given by an ellipsoid.[13,16–18]

With the constitutive equation, Eq. (2b) assumes the form $(D_x^2/\varepsilon_x) + (D_y^2/\varepsilon_y) + (D_z^2/\varepsilon_z) = 2\omega_e\epsilon_o$. By substituting $x = D_x/(2\omega_e\epsilon_o)^{1/2}$ and $n_x^2 = \varepsilon_x$, and similarly for y and z, the ellipsoid is expressed in cartesian principal coordinates as

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \qquad (3)$$

Equation (3) is the general index ellipsoid for an optically biaxial crystal. If $n_x = n_y$, the surface becomes an ellipsoid of revolution, representing a uniaxial crystal. In this crystal, one of the two allowed eigenpolarizations will always be an *ordinary* wave with its Poynting vector parallel to the wave vector and **E** parallel to **D** for any direction of propagation. An isotropic crystal ($n_x = n_y = n_z$) is represented by a sphere with the principal axes having equal length. Any wave propagating in this crystal will exhibit ordinary characteristics. The index ellipsoid for each of these three optical symmetries is shown in Fig. 2.

For a general direction of propagation, the section of the ellipsoid through the origin
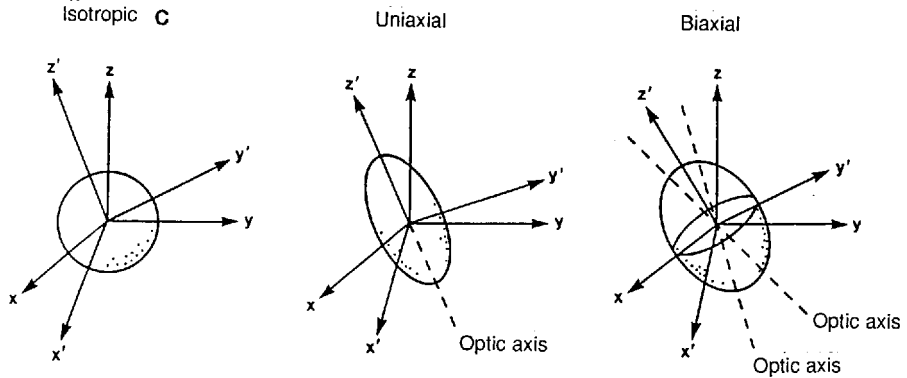

FIGURE 2 The index ellipsoids for the three crystal symmetries are shown in nonprincipal coordinates (x', y', z') relative to the principal coordinates (x, y, z). For isotropic crystals, the surface is a sphere. For uniaxial crystals, it is an ellipsoid of revolution. For biaxial crystals it is a general ellipsoid.[30]
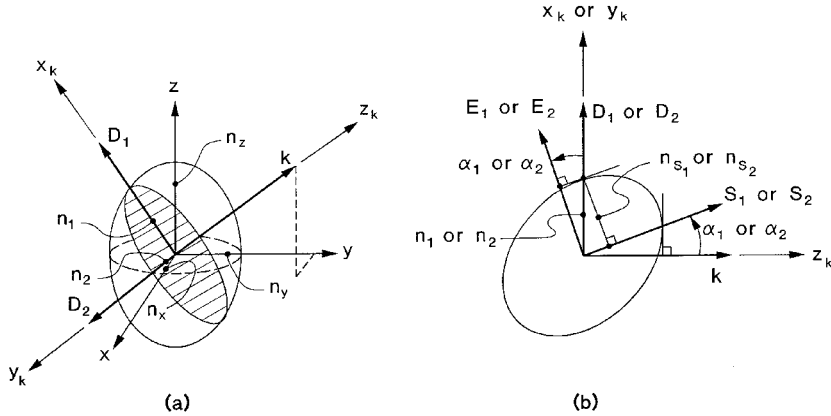
FIGURE 3 (*a*) The index ellipsoid cross section (crosshatched) that is normal to the wave vector **k** has the shape of an ellipse. The major and minor axes of this ellipse represent the directions of the allowed polarizations **D**$_1$ and **D**$_2$; (*b*) for each eigenpolarization (1 or 2) the vectors **D**, **E**, **S**, and **k** are coplanar.[30]

perpendicular to this direction is an ellipse, as shown in Fig. 3*a*. The major and minor axes of the ellipse represent the orthogonal vibration directions of **D** for that particular direction of propagation. The lengths of these axes correspond to the phase velocity refractive indices. They are, therefore, referred to as the "fast" and "slow" axes. Figure 3*b* illustrates the field relationships with respect to the index ellipsoid. The line in the (**k**, **D**$_i$) plane (*i* = 1 or 2) that is tangent to the ellipsoid at **D**$_i$ is parallel to the ray vector **S**$_i$; the electric field **E**$_i$ also lies in the (**k**, **D**$_i$) plane and is normal to **S**$_i$. The line length denoted by n_{s_i} gives the ray velocity as $v_{s_i} = c/n_{s_i}$ for **S**$_i$. The same relationships hold for either vibration, **D**$_1$ or **D**$_2$.

In the general ellipsoid for a biaxial crystal there are two cross sections passing through the center that are circles. The normals to these cross sections are called the *optic axes* (denoted in Fig. 2 in a nonprincipal coordinate system), and they are coplanar and symmetric about the *z* principal axis in the *x, z* plane. The angle ϑ of an optic axis with respect to the *z* axis in the *x, z* plane is

$$\tan \vartheta = \frac{n_z}{n_x} \sqrt{\frac{n_y^2 - n_x^2}{n_z^2 - n_y^2}} \tag{4}$$

The phase velocities for **D**$_1$ and **D**$_2$ are equal for these two directions: $v_1 = v_2 = c/n_y$. In an ellipsoid of revolution for a uniaxial crystal there is one circular cross section perpendicular to the *z* principal axis. Therefore, the *z* axis is the optic axis, and $\vartheta = 0°$ in this case.

13.4 *THE ELECTRO-OPTIC EFFECT*

At an atomic level an electric field applied to certain crystals causes a redistribution of bond charges and possibly a slight deformation of the crystal lattice.[14] In general, these alterations are not isotropic; that is, the changes vary with direction in the crystal. Therefore, the inverse dielectric constant (impermeability) tensor changes accordingly. Crystals lacking a center of symmetry are noncentrosymmetric and exhibit a linear (Pockels) electro-optic effect. The changes in the impermeability tensor elements are linear in the applied electric field. On the other hand, all crystals exhibit a quadratic (Kerr)

electro-optic effect. The changes in the impermeability tensor elements are quadratic in the applied field. When the linear effect is present, it generally dominates over the quadratic effect.

The linear electro-optic effect is represented by a third rank tensor r_{ijk}. The permutation symmetry of this tensor is $r_{ijk} = r_{jik}$, $i, j, k = 1, 2, 3$. Therefore, the tensor can be represented by a 6×3 matrix; i.e., $r_{ijk} \Rightarrow r_{ij}$, $i = 1, \ldots, 6$ and $j = 1, 2, 3$. Generally, the r_{ij} coefficients have very little dispersion in the optical transparent region of a crystal.[19] The electro-optic coefficient matrices for all crystal classes are given in Table 1. References 14, 19, 20, and 21, among others, contain extensive tables of numerical values for indices and electro-optic coefficients for different materials. The quadratic electro-optic effect is represented by a fourth rank tensor s_{ijkl}. The permutation symmetry of this tensor is $s_{ijkl} = s_{jikl}, = s_{ijlk}$, $i, j, k, l = 1, 2, 3$. The tensor can be represented by a 6×6 matrix; i.e., $s_{ijkl} \Rightarrow s_{kl}$, $k, l = 1, \ldots, 6$. The quadratic electro-optic coefficient matrices for all crystal classes are given in Table 2. Reference 14 contains a table of quadratic electro-optic coefficients for several materials.

The Linear Electro-optic Effect

An electric field applied in a general direction to a noncentrosymmetric crystal produces a linear change in the constants $(1/n^2)_i$ due to the linear electro-optic effect according to

$$\Delta(1/n^2)_i = \sum_j r_{ij} E_j \qquad \begin{matrix} i = 1, \ldots, 6 \\ j = x, y, z = 1, 2, 3 \end{matrix} \tag{5}$$

where r_{ij} is the ijth element of the linear electro-optic tensor in contracted notation. In matrix form Eq. (5) is

$$\begin{pmatrix} \Delta(1/n^2)_1 \\ \Delta(1/n^2)_2 \\ \Delta(1/n^2)_3 \\ \Delta(1/n^2)_4 \\ \Delta(1/n^2)_5 \\ \Delta(1/n^2)_6 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{6}$$

E_x, E_y, and E_z are the components of the applied electric field in principal coordinates. The magnitude of $\Delta(1/n^2)$ is typically on the order of less than 10^{-5}. Therefore, these changes are mathematically referred to as perturbations. The new impermeability tensor $\overline{1/n^{2\prime}}$ in the presence of an applied electric field is no longer diagonal in the reference principal dielectric axes system. It is given by

$$\overline{1/n^{2\prime}} = \begin{pmatrix} 1/n_x^2 + \Delta(1/n^2)_1 & \Delta(1/n^2)_6 & \Delta(1/n^2)_5 \\ \Delta(1/n^2)_6 & 1/n_y^2 + \Delta(1/n^2)_2 & \Delta(1/n^2)_4 \\ \Delta(1/n^2)_5 & \Delta(1/n^2)_4 & 1/n_z^2 + \Delta(1/n^2)_3 \end{pmatrix} \tag{7}$$

However, the field-induced perturbations are symmetric, so the symmetry of the tensor is not disturbed. The new index ellipsoid is now represented by

$$(1/n^2)_1' x^2 + (1/n^2)_2' y^2 + (1/n^2)_3' z^2 + 2(1/n^2)_4' yz + 2(1/n^2)_5' xz + 2(1/n^2)_6' xy = 1 \tag{8}$$

TABLE 1 The Linear Electro-optic Coefficient Matrices in Contracted Form for All Crystal Symmetry Classes[14]

Centrosymmetric ($\bar{1}$, 2/m, mmm, 4/m, 4/mmm, $\bar{3}$, $\bar{3}$m6/m, 6/mmm, m3, m3m):

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Triclinic:

1*

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix}$$

Cubic:

$\bar{4}$3m, 23

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{pmatrix}$$

432

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Monoclinic:

2 (2 ∥ x_2)

$$\begin{pmatrix} 0 & r_{12} & 0 \\ 0 & r_{22} & 0 \\ 0 & r_{32} & 0 \\ r_{41} & 0 & r_{43} \\ 0 & r_{52} & 0 \\ 6_{61} & 0 & r_{63} \end{pmatrix}$$

2 (2 ∥ x_3)

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ r_{41} & r_{42} & 0 \\ r_{51} & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

Tetragonal:

4

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\bar{4}$

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & -r_{13} \\ 0 & 0 & 0 \\ r_{41} & -r_{51} & 0 \\ r_{51} & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

422

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

m (m ⊥ x_2)

$$\begin{pmatrix} r_{11} & 0 & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & r_{53} \\ 0 & r_{62} & 0 \end{pmatrix}$$

m (m ⊥ x_3)

$$\begin{pmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ r_{31} & r_{32} & 0 \\ 0 & 0 & r_{43} \\ 0 & 0 & r_{53} \\ r_{61} & r_{62} & 0 \end{pmatrix}$$

4mm

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\bar{4}$2m (2 ∥ x_1)

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

Orthorhombic:

222

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

2mm

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Trigonal:

3

$$\begin{pmatrix} r_{11} & -r_{22} & r_{13} \\ -r_{11} & -r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ -r_{22} & -r_{11} & 0 \end{pmatrix}$$

32

$$\begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}$$

Hexagonal:

6

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

6mm

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

622

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3m (m ⊥ x_1)

$$\begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}$$

3m (m ⊥ x_2)

$$\begin{pmatrix} r_{11} & 0 & r_{13} \\ -r_{11} & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}$$

$\bar{6}$

$$\begin{pmatrix} r_{11} & -r_{22} & 0 \\ -r_{11} & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & r_{11} & 0 \end{pmatrix}$$

$\bar{6}$m2 (m ⊥ x_1)

$$\begin{pmatrix} 0 & -r_{22} & 0 \\ 0 & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}$$

$\bar{6}$m2 (m ⊥ x_2)

$$\begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}$$

* The symbol over each matrix is the conventional symmetry-group designation.

TABLE 2 The Quadratic Electro-optic Coefficient Matrices in Contracted Form for all Crystal Symmetry Classes[14]

Triclinic:

$$1,\bar{1}$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} \\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} & s_{46} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} \end{pmatrix}$$

Monoclinic:

$$2,m,2/m$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & s_{15} & 0 \\ s_{21} & s_{22} & s_{23} & 0 & s_{25} & 0 \\ s_{31} & s_{32} & s_{33} & 0 & s_{35} & 0 \\ 0 & 0 & 0 & s_{44} & 0 & s_{46} \\ s_{51} & s_{52} & s_{53} & 0 & s_{55} & 0 \\ 0 & 0 & 0 & s_{64} & 0 & s_{66} \end{pmatrix}$$

Orthorhombic:

$$2mm,222,mmm$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{21} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{31} & s_{32} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{pmatrix}$$

Tetragonal:

$$4,\bar{4},4/m$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & s_{16} \\ s_{12} & s_{11} & s_{13} & 0 & 0 & -s_{16} \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & s_{45} & 0 \\ 0 & 0 & 0 & -s_{45} & s_{44} & 0 \\ s_{61} & -s_{61} & 0 & 0 & 0 & s_{66} \end{pmatrix}$$

$$422,4mm,\bar{4}2m,4/mm$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{pmatrix}$$

Trigonal:

$$3,\bar{3}$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & -s_{61} \\ s_{12} & s_{11} & s_{13} & -s_{14} & -s_{15} & s_{61} \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ s_{41} & -s_{41} & 0 & s_{44} & s_{45} & -s_{51} \\ s_{51} & -s_{51} & 0 & -s_{45} & s_{44} & s_{41} \\ s_{61} & -s_{61} & 0 & -s_{15} & s_{14} & \frac{1}{2}(s_{11}-s_{12}) \end{pmatrix}$$

$$32,3m,\bar{3}m$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & 0 & 0 \\ s_{12} & s_{11} & s_{13} & -s_{14} & 0 & 0 \\ s_{13} & s_{13} & s_{33} & 0 & 0 & 0 \\ s_{41} & -s_{41} & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & s_{41} \\ 0 & 0 & 0 & 0 & s_{14} & \frac{1}{2}(s_{11}-s_{12}) \end{pmatrix}$$

Hexagonal:

$$6,\bar{6},6/m$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & -s_{61} \\ s_{12} & s_{11} & s_{13} & 0 & 0 & s_{61} \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & s_{45} & 0 \\ 0 & 0 & 0 & -s_{45} & s_{44} & 0 \\ s_{61} & -s_{61} & 0 & 0 & 0 & \frac{1}{2}(s_{11}-s_{12}) \end{pmatrix}$$

$$622,6mm,\bar{6}m2,6/mmm$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(s_{11}-s_{12}) \end{pmatrix}$$

Cubic:

$$23,m3$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{13} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{13} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{44} \end{pmatrix}$$

$$432,m3m,\bar{4}3m$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{12} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{44} \end{pmatrix}$$

Isotropic:

$$\begin{pmatrix} s_{11} & s_{12} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{12} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(s_{11}-s_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(s_{11}-s_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(s_{11}-s_{12}) \end{pmatrix}$$

or equivalently, $\mathbf{X}^T \ \overline{1/n^2}' \ \mathbf{X} = 1$, where $\mathbf{X} = [x \ y \ z]$.[17,22] The presence of cross terms indicates that the ellipsoid is rotated and the lengths of the principal dielectric <u>axes</u> are changed. Determining the new orientation and shape of the ellipsoid requires that $\overline{1/\mathbf{n}^2}'$ be diagonalized, thus determining its eigenvalues and eigenvectors. The perturbed ellipsoid will then be represented by a square sum:

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1 \tag{9}$$

The eigenvalues of $\overline{1/n^2}'$ are $1/n_{x'}^2$, $1/n_{y'}^2$, $1/n_{z'}^2$. The corresponding eigenvectors are $\mathbf{x}' = [x_{x'} \ y_{x'} \ z_{x'}]^T$, $\mathbf{y}' = [x_{y'} \ y_{y'} \ z_{y'}]^T$, and $\mathbf{z}' = [x_{z'} \ y_{z'} \ z_{z'}]^T$, respectively.

The Quadratic Electro-optic Effect

An electric field applied in a general direction to any crystal, centrosymmetric or noncentrosymmetric, produces a quadratic change in the constants $(1/n^2)_i$ due to the quadratic electro-optic effect according to

$$\begin{pmatrix} \Delta(1/n^2)_1 \\ \Delta(1/n^2)_2 \\ \Delta(1/n^2)_3 \\ \Delta(1/n^2)_4 \\ \Delta(1/n^2)_5 \\ \Delta(1/n^2)_6 \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} \\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} & s_{46} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} \end{pmatrix} \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ E_y E_z \\ E_x E_z \\ E_x E_y \end{pmatrix} \tag{10}$$

E_x, E_y, and E_z are the components of the applied electric field in principal coordinates. The perturbed impermeability tensor and the new index ellipsoid have the same form as Eqs. (7) and (8).

A Mathematical Approach

Although the eigenvalue problem is a familiar one, obtaining accurate solutions has been the subject of extensive study.[23–26] A number of formalisms are suggested in the literature to address the specific problem of finding the new set of principal dielectric axes relative to the zero-field principal dielectric axes. Most approaches, however, do not provide a consistent means of labeling the new axes. Also, some methods are highly susceptible to numerical instabilities when dealing with very small off-diagonal elements as in the case of the electro-optic effect. In contrast to other methods,[13,23,24,27–29] a similarity transformation is an attractive approach for diagonalizing a symmetric matrix for the purpose of determining its eigenvalues and eigenvectors.[23,24,26,30,31]

A symmetric matrix $\bar{A}$ can be reduced to diagonal form by the transformation $\bar{a}\bar{A}\bar{a}^T = \bar{\lambda}$, where $\bar{\lambda}$ is a 3×3 diagonal matrix and $\bar{a}$ is the orthogonal transformation matrix. Since the eigenvalues of $\bar{A}$ are preserved under similarity transformation, they lie on the diagonal of $I\bar{\lambda}$, as in Eq. (1).

The problem of determining the new principal axes and indices of refraction of the index ellipsoid in the presence of an external electric field is analogous to the problem of finding the transformation matrix $\bar{a}$ that will diagonalize the perturbed impermeability tensor. The lengths of the semiaxes are the reciprocals of the square roots of the eigenvalues of $\overline{1/\mathbf{n}^2}'$. Generally, this matrix reduction requires a *sequence* of similarity transformations. Since similarity is a transitive property, several transformation matrices can be multiplied to generate the desired cumulative matrix.[23,30]

The Jacobi method is a form of similarity transformation which has been shown to

provide both accurate eigenvalues and orthogonal eigenvectors. It produces reliable results for matrices with very small off-diagonal elements.[25,30] It is a systematic procedure for ordering the solutions to provide consistent labeling of the principal axes.

Principal Axes and Principal Refractive Indices

The Jacobi method utilizes the concepts of rigid-body rotation and the properties of ellipsoids to determine the principal axes and indices of a crystal.[30] From these concepts, a geometric interpretation of the electro-optic effect is developed. The Jacobi method is an iterative procedure that consists of a simple elementary plane rotation at each step to zero an off-diagonal element. The goal is to produce a diagonal matrix by minimizing the norm of the off-diagonal elements to within a desired level of accuracy. The transformation matrix is simply the product of the plane rotation matrices multiplied in the order in which they are applied. For step m the transformation is represented by

$$\overline{1/n_m^2} = \bar{a}_m \, \overline{1/n_{m-1}^2} \, \bar{a}_m^T \tag{11}$$

The perturbed impermeability matrix of Eq. (7) for a specific crystal is determined given the unperturbed principal refractive indices, the electro-optic coefficients, and the direction of the applied field defined in the principal coordinate system. The first step is to select the largest off-diagonal element $(1/n^2)_{ij}$ and execute a rotation in the (i, j) plane, $i < j$, so as to zero that element. The required rotation angle Ω is given by

$$\tan(2\Omega) = \frac{2(1/n^2)_{ij}}{(1/n^2)_{ii} - (1/n^2)_{jj}} \qquad i, j = 1, 2, 3 \tag{12}$$

For example, if the largest off-diagonal element is $(1/n^2)_{12}$, then the plane rotation is represented by

$$\bar{a} = \begin{pmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{13}$$

If $(1/n^2)_{ii} = (1/n^2)_{jj}$, which can occur in isotropic and uniaxial crystals, then $|\Omega|$ is taken to be 45°, and its sign is taken to be the same as the sign of $(1/n^2)_{ij}$. The impermeability matrix elements are updated with the following equations:

$$(1/n^2)'_{\Omega_{ii}} = (1/n^2)_{ii} \cos^2\Omega + (1/n^2)_{jj} \sin^2\Omega + 2(1/n^2)_{ij} \cos\Omega \sin\Omega$$

$$(1/n^2)'_{\Omega_{jj}} = (1/n^2)_{ii} \sin^2\Omega + (1/n^2)_{jj} \cos^2\Omega - 2(1/n^2)_{ij} \cos\Omega \sin\Omega$$

$$(1/n^2)'_{\Omega_{kk}} = (1/n^2)_{kk}$$

$$(1/n^2)'_{\Omega_{ij}} = [(1/n^2)_{jj} - (1/n^2)_{ii}] \cos\Omega \sin\Omega + (1/n^2)_{ij}(\cos^2\Omega - \sin^2\Omega) \tag{14}$$

$$= (1/n^2)'_{\Omega_{ji}} = 0$$

$$(1/n^2)'_{\Omega_{ik}} = (1/n^2)_{ik} \cos\Omega + (1/n^2)_{jk} \sin\Omega = (1/n^2)'_{\Omega_{ki}}$$

$$(1/n^2)'_{\Omega_{jk}} = -(1/n^2)_{ik} \sin\Omega + (1/n^2)_{jk} \cos\Omega = (1/n^2)'_{\Omega_{kj}}$$

Once the new elements are determined, the next iteration step is performed, selecting the

new largest off-diagonal element and repeating the procedure. The process is terminated when all of the off-diagonal elements are reduced below the desired level (typically 10^{-10}).

The next step is to determine the cumulative transformation matrix $\bar{a}$ by either of two ways. The first way is to multiply the plane rotation matrices in order, either as $\bar{a} = \bar{a}_n \cdots \bar{a}_2 \bar{a}_1$ or equivalently for the transpose of $\bar{a}$ as

$$\bar{a}^T = \bar{a}_1^T \bar{a}_2^T \cdots \bar{a}_n^T \tag{15}$$

A much simpler way is to find each eigenvector individually be a series of matrix-vector multiplications and then to construct the cumulative matrix by placing these vectors in the rows of $\bar{a}$.[23] For example, to find the first eigenvector, the first matrix-vector multiplication is $\bar{a}_n^T [1 \ 0 \ 0]^T = \mathbf{a_n}$ (a vector), followed by $\bar{a}_{n-1}^T \mathbf{a_n} = \mathbf{a_{n-1}}$, etc., until all matrix-vector multiplications are performed to obtain

$$\bar{a}_1^T \bar{a}_2^T \cdots \bar{a}_n^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix} \tag{16}$$

which is the first column of $\bar{a}^T$. Alternatively, a_{11}, a_{12}, and a_{13} are the elements of the first row of $\bar{a}$, and they represent the components of the first eigenvector $\mathbf{x}'$ of $\overline{1/n^2}'$. This eigenvector corresponds to the new x' axis. Likewise, the second ($\mathbf{y}'$) and third ($\mathbf{z}'$) eigenvectors may be found by multiplying the successive rotation matrixes by $[0 \ 1 \ 0]^T$ and $[0 \ 0 \ 1]^T$, respectively. All three eigenvectors are automatically in normalized form, since the rotation matrices are orthonormal.

The reorientation of the index ellipsoid can be described by a single rotation of Φ about a global rotation axis $\mathbf{R}$. Both Φ and $\mathbf{R}$ can be determined from $\bar{a}$ as shown in Fig. 4.[30,32] The cumulative transformation matrix always has an eigenvalue of $+1$, since it represents a rigid-body rotation. The global rotation axis is the eigenvector of $\bar{a}$ corresponding to this eigenvalue, and its direction cosines are unchanged by the transformation. Therefore, the rotation axis may be found by

$$(\bar{a} - \bar{I})\mathbf{R} = \mathbf{0} \tag{17}$$

where $\mathbf{R}$ is the vector of direction cosines for the rotation axis. The sense of $\mathbf{R}$ is not unique; $-\mathbf{R}$ is also a solution. To provide consistency in the orientation of $\mathbf{R}$, the z
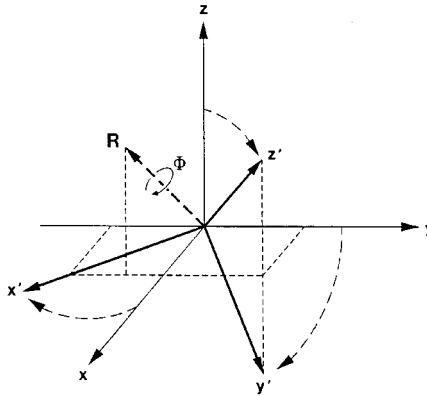

FIGURE 4   The transformation of the unperturbed (x, y, z) coordinate system to the (x', y', z') coordinate system can be described by one global rotation of angle Φ about a global rotation axis $\mathbf{R}$.[30]

component of **R** is set to $+1$ before normalizing. To determine the global rotation angle, a similarity transformation with orthogonal matrix $\overline{a'}$ is possible where the z axis is made to align with **R**, and the rotation is taken in the (x, y) plane perpendicular to **R**:

$$\overline{a'} = \begin{pmatrix} \cos \Phi & \sin \Phi & 0 \\ -\sin \Phi & \cos \Phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{18}$$

Since the trace of a matrix is invariant under similarity transformation, the magnitude of Φ may be found by

$$\sum_{i=1}^{3} a_{ii} = 1 + 2 \cos \Phi \tag{19}$$

where a_{ii} are the diagonal elements of $\overline{a}$. Using the a_{ii} elements as obtained by the general Jacobi method, Φ is automatically the *minimum* rotation angle required to reorient the zero-field principal axes to those of the perturbed index ellipsoid (x', y', z'). Other information that can be obtained from $\overline{a}$ is the set of Euler angles, which also defines the orientation of a rigid body.[32,33] These angles are given in the Appendix. Several examples for using the Jabobi method are given in Ref. 30.

Eigenpolarizations and Phase Velocity Indices of Refraction

For a general direction of phase propagation, there are two allowed linear orthogonal polarization directions. These waves are the only two that can propagate with unchanging orientation for the given wave vector direction. Figure 5a depicts these axes for a crystal in the absence of an applied field. Figure 5b depicts the x''' and y''' axes, which define the fast and slow axes, when a field is applied in a direction so as to reorient the index ellipsoid. A field, in general, rotates the allowed polarization directions in the plane perpendicular to the direction of phase propagation as shown in Fig. 5b.
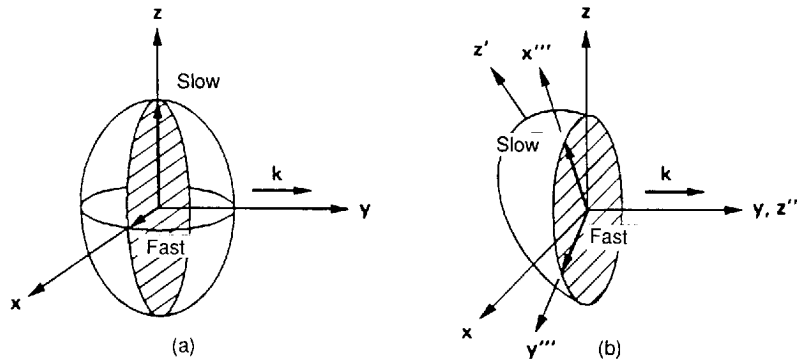

FIGURE 5 (*a*) The cross-section ellipse for a wave propagating along the *y* principal axis is shown with no field applied to the crystal; (*b*) with an applied electric field the index ellipsoid is reoriented, and the eigenpolarizations in the plane transverse to **k** are rotated, indicated by x''' and y'''.[30]

The perturbed index ellipsoid resulting from an external field is given by Eq. (8). For simplicity, the coefficients may be relabeled as

$$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gxz + 2Hxy = 1 \tag{20}$$

where x, y, and z represent the original dielectric axes with no applied field. The wave vector direction $\mathbf{k}$ may be specified by the spherical coordinates angles θ_k and ϕ_k in the (x, y, z) coordinate system as shown in Fig. 6. Given $\mathbf{k}$, the cross section ellipse through the center of the perturbed ellipsoid of Eq. (20) may be drawn. The directions of the semiaxes of this ellipse represent the fast and slow polarization directions of the two waves $\mathbf{D}_1$ and $\mathbf{D}_2$ that propagate independently. The lengths of the semiaxes are the phase velocity indices of refraction. The problem is to determine the new polarization directions x''' of $\mathbf{D}_1$ and y''' of $\mathbf{D}_2$ relative to the (x, y, z) axes and the corresponding new indices of refraction $n_{x'''}$ and $n_{y'''}$.

A new coordinate system (x'', y'', z'') may be defined with z'' parallel to $\mathbf{k}$ and x'' lying in the (z, z'') plane. The (x'', y'', z'') system is, of course, different from the (x', y', z') perturbed principal axes system. The (x'', y'', z'') system may be produced first by a counterclockwise rotation ϕ_k about the z axis followed by a counterclockwise rotation θ_k about y'' as shown in Fig. 6. This transformation is described by

$$x = x'' \cos \theta_k \cos \phi_k - y'' \sin \phi_k + z'' \sin \theta_k \cos \phi_k$$

$$y = x'' \cos \theta_k \sin \phi_k + y'' \cos \phi_k + z'' \sin \theta_k \sin \phi_k \tag{21}$$

$$z = -x'' \sin \theta_k + z'' \cos \theta_k$$

The equation for the cross section ellipse normal to $\mathbf{k}$ is determined by substituting Eqs. (21) into Eq. (20) and setting $z'' = 0$, giving
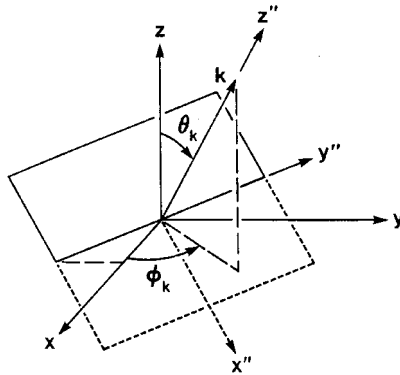
$$A'' x''^2 + B'' y''^2 + 2H'' x'' y'' = 1 \tag{22}$$

FIGURE 6   The coordinate system (x'', y'', z'') of the wave vector $\mathbf{k}$ is defined with its angular relationship (ϕ_k, θ_k) with respect to the unperturbed principal dielectric axes coordinate system (x, y, z).[30]
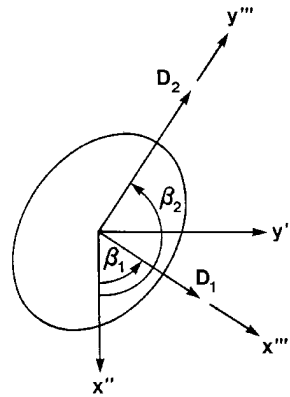
FIGURE 7   The polarization axes (x''', y''') are the *fast* and *slow* axes and are shown relative to the (x'', y'') axes of the wave vector coordinate system. The wave vector $\mathbf{k}$ and the axes z'' and z''' are normal to the plane of the figure.[30]

where $A'' = \cos^2 \theta_k [A \cos^2 \phi_k + B \sin^2 \phi_k + H \sin 2\phi_k] + C \sin^2 \theta_k$

$\qquad - \sin 2\theta_k [F \sin \phi_k + G \cos \phi_k]$

$\quad B'' = A \sin^2 \phi_k + B \cos^2 \phi_k - H \sin 2\phi_k$ (23)

$\quad 2H'' = \sin 2\phi_k \cos \theta_k [B - A] + 2 \sin \theta_k [G \sin \phi_k - F \cos \phi_k]$

$\qquad + 2H \cos \theta_k \cos 2\phi_k$

The polarization angle β_1 of x''' ($\mathbf{D}_1$) with respect to x'', as shown in Fig. 7, is given by

$$\beta_1 = \tfrac{1}{2} \tan^{-1} \left[\frac{2H''}{(A'' - B'')} \right]$$ (24)

The polarization angle β_2 of y''' ($\mathbf{D}_2$) with respect to x'' is $\beta_1 + \pi/2$. The axes are related by

$$x'' = x''' \cos \beta_1 - y''' \sin \beta_1$$
$$y'' = x''' \sin \beta_1 + y''' \cos \beta_1$$ (25)

The refractive index $n_{x'''}$ may be found by setting $y''' = 0$ and substituting Eqs. (25) into Eq. (22) and solving for $n_{x'''} = x'''$ giving

$$n_{x'''} = [A'' \cos^2 \beta_1 + B'' \sin^2 \beta_1 + 2H'' \cos \beta_1 \sin \beta_1]^{-1/2}$$ (26)

Similarly, the refractive index $n_{y'''}$ may be found by setting $x''' = 0$ and solving for $n_{y'''} = y'''$, giving

$$n_{y'''} = [A'' \sin^2 \beta_1 + B'' \cos^2 \beta_1 - 2H'' \cos \beta_1 \sin \beta_1]^{-1/2}$$ (27)

The larger index corresponds to the slow axis and the smaller index to the fast axis.

13.5 MODULATOR DEVICES

An electro-optic modulator is a device with operation based on an electrically-induced change in index of refraction or change in natural birefringence. Depending on the device configuration, the following properties of the light wave can be varied in a controlled way: phase, polarization, amplitude, frequency, or direction of propagation. The device is typically designed for optimum performance at a single wavelength, with some degradation in performance with wideband or multimode lasers.[14,34,35]

Electro-optic devices can be used in analog or digital modulation formats. The choice is dictated by the system requirements and the characteristics of available components (optical fibers, sources/detectors, etc.). Analog modulation requires large signal-to-noise ratios (SNR), thereby limiting its use to narrow-bandwidth, short-distance applications. Digital modulation, on the other hand, is more applicable to large-bandwidth, medium to long distance systems.[34,35]

Device Configurations

An electro-optic modulator can be classified as one of two types, *longitudinal* or *transverse,* depending on how the voltage is applied relative to the direction of light propagation in the device. Basically, a bulk modulator consists of an electro-optic crystal sandwiched between a pair of electrodes and, therefore, can be modeled as a capacitor. In general, the
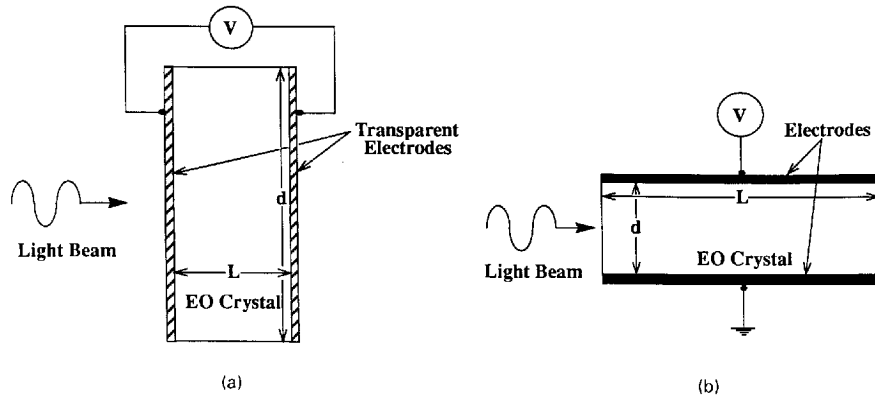
FIGURE 8 (*a*) A longitudinal electro-optic modulator has the voltage applied parallel to the direction of light propagation; (*b*) a transverse modulator has the voltage applied perpendicular to the direction of light propagation.[14]

input and output faces are parallel for the beam to undergo a uniform phase shift over the beam cross section.[14]

In the longitudinal configuration, the voltage is applied parallel to the wave vector direction as shown in Fig. 8*a*.[14,21,36–39] The electrodes must be transparent to the light either by the choice of material used for them (metal-oxide coatings of SnO, InO, or CdO) or by leaving a small aperture at their center at each end of the electro-optic crystal.[21,37–39] The ratio of the crystal length L to the electrode separation b is defined as the *aspect ratio*. For this configuration $b = L$, and, therefore, the aspect ratio is always unity. The magnitude of the applied electric field inside the crystal is $E = V/L$. The induced phase shift is proportional to V and the wavelength λ of the light but not the physical dimensions of the device. Therefore, for longitudinal modulators, the required magnitude of the applied electric field for a desired degree of modulation cannot be reduced by changing the aspect ratio, and it increases with wavelength. However, these modulators can have a large acceptance area and are useful if the light beam has a large cross-sectional area.

In the transverse configuration, the voltage is applied perpendicular to the direction of light propagation as shown in Fig. 8*b*.[14,36–39] The electrodes do not obstruct the light as it passes through the crystal. For this case, the aspect ratio can be very large. The magnitude of the applied electric field is $E = V/d$, $(b = d)$, and d can be reduced to increase E for a given applied voltage, thereby increasing the aspect ratio L/b. The induced phase shift is inversely proportional to the aspect ratio; therefore, the voltage necessary to achieve a desired degree of modulation can be greatly reduced. Furthermore, the interaction length can be long for a given field strength. However, the transverse dimension d is limited by the increase in capacitance, which affects the modulation bandwidth or speed of the device, and by diffraction for a given length L, since a beam with finite cross section diverges as it propagates.[14,37,40]

Modulation of Light Parameters

The modulation of phase, polarization, amplitude, frequency, and position of light can be implemented using an electro-optic bulk modulator with polarizers and passive birefringent elements. Three assumptions are made in this section. First, the modulating field is *uniform* throughout the length of the crystal; the change in index or birefringence is

uniform unless otherwise stated. Second, the modulation voltage is dc or very low frequency ($\omega_m \ll 2\pi/\tau$); the light experiences the same induced Δn during its transit time τ through the crystal of length L, and the capacitance is negligible. Finally, light propagation is taken to be along a principal axis, before and after the voltage is applied; therefore, the equations are presented in terms of the *optical electric field* **E**, rather than the *displacement vector* **D**, which is common practice in various optical references. For other general configurations the equations should be expressed in terms of the eigenpolarizations **D**$_1$ and **D**$_2$. However, the electric field will determine the direction of energy flow. References 14 and 37, among others, provide examples of modulator devices using potassium dihydrogen phosphate (KDP), lithium niobate (LiNbO$_3$), lithium tantalate (LiTaO$_3$), gallium arsenide (GaAs), and barium titanate (BaTiO$_3$).

Phase Modulation.   A light wave can be phase modulated, without change in polarization or intensity, using an electro-optic crystal and an input polarizer in the proper configuration. An example of a longitudinal device is shown in Fig. 9. In general, an applied voltage V will rotate the principal axes in the crystal cross section. For phase modulation, the input polarizer must be aligned parallel to one of the principal axes when the voltage is on or off. Figure 9 indicates a polarizer along x' with an input optical electric field $E_{i_{x'}}(t) = E_i \cos \omega t$.

The optical wave at the output of the crystal at $z = L$ is

$$E_o(t) = E_i \cos\left(\omega t - \phi\right) \tag{28}$$

where

$$\phi = \frac{2\pi}{\lambda}(n_{x'} + \Delta n_{x'})L = \phi_o + \Delta\phi_{x'} \tag{29}$$

is the total phase shift consisting of a natural phase term $\phi_o = (2\pi/\lambda)Ln_{x'}$, with $n_{x'}$ being the unperturbed index in the x' direction, and an electrically-induced phase term $\Delta\phi_{x'} = (2\pi/\lambda)L\Delta n_{x'}$ for a polarization along x'. The change in index is $\Delta n_{x'} \approx \frac{1}{2}n_{x'}^3 rE$, where r is the corresponding electro-optic coefficient.

For a longitudinal modulator the applied electric field is $E = V/L$, and the induced phase shift is $\Delta\phi_{x'} = \dfrac{\pi}{\lambda}n_{x'}^3 rV$, which is independent of L and is linearly related to V. For a
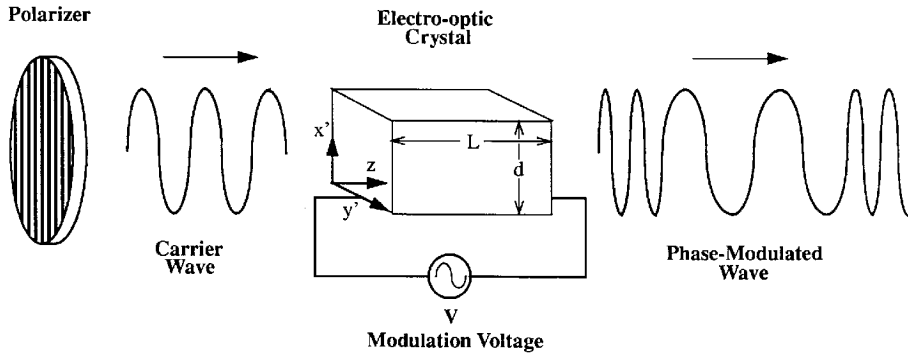

FIGURE 9 A longitudinal phase modulator is shown with the light polarized along the new x' principal axis when the modulation voltage V is applied.[14]

transverse modulator $E = V/d$, and the induced phase shift is $\Delta\phi_{x'} = \frac{\pi}{\lambda}n_{x'}^3 rV(L/d)$, which

is a function of the aspect ratio L/d and V. The voltage that would produce an induced phase shift of $\Delta\phi_{x'} = \pi$ is the *half-wave voltage*. It is $V_\pi = \lambda/n_{x'}^3 r$ for a longitudinal modulator and $V_\pi = (\lambda/n_{x'}^3 r)(d/L)$ for a transverse modulator.

If a dc voltage is used, one of two possibilities is required for a crystal and its orientation. The first possibility is a crystal having principal axes which will not rotate with applied voltage V; an example is $LiNbO_3$ with V applied in the z direction and an input polarization along the $x' = x$ axis propagating along $z' = z$. The second possibility is a crystal having a characteristic plane perpendicular to the direction of propagation. If a field is applied such that the axes rotate in this plane, the input wave must be polarized along one of the new principal axes. Therefore, it will always be polarized along a principal axis, whether the voltage is on or off. An example is KDP with V along the z axis and an input wave polarized along the new principal axis x' and propagating along $z' = z$. Phase modulation is then achieved by turning the voltage on and off.

If the applied modulation voltage is sinusoidal in time ($V = V_m \sin \omega_m t$), the corresponding electric field can be represented by

$$E = E_m \sin \omega_m t \tag{30}$$

The magnitude of the field varies only with time, not space; it is a stationary wave applied in the same direction for all time. In other words, this time-varying voltage signal is to be distinguished from a traveling wave voltage which will be discussed in the next section. In this case

$$\phi = \left(\frac{2\pi}{\lambda}\right)(n_{x'} - \tfrac{1}{2}n_{x'}^3 rE_m \sin \omega_m t)L$$

$$= \left(\frac{2\pi}{\lambda}\right)n_{x'}L - \delta \sin \omega_m t \tag{31}$$

The parameter $\delta = (\pi/\lambda)n_{x'}^3 rE_m L = \pi V_m/V_\pi$, where V_π is the half-wave voltage for a given configuration, is the *phase modulation index* or *depth-of-phase modulation*. By neglecting the constant phase term ϕ_o, applying the identity $\cos(\delta \sin \omega_m t) + j \sin(\delta \sin \omega_m t) = \exp[j\delta \sin \omega_m t] = \sum_{l=-\infty}^{\infty} J_l(\delta) \exp[jl\omega t]$, and equating the real and imaginary parts, the output light wave becomes

$$E_o(t) = E_i[J_0(\delta) \cos \omega t + J_1(\delta) \cos(\omega + \omega_m)t - J_1(\delta) \cos(\omega - \omega_m)t$$

$$+ J_2(\delta) \cos(\omega + 2\omega_m)t + J_2(\delta) \cos(\omega - 2\omega_m)t + \cdots] \tag{32}$$

The output consists of components at frequencies ω and $(\omega + n\omega_m)$, $n = \pm 1, \pm 2, \ldots$. For no modulation, $\delta = 0$ and $J_0(0) = 1$, $J_n(0) = 0$ for $n \neq 0$ and $E_o(t) = E_i \cos \omega t = E_{i_x}(t)$.[14] For $\delta \approx 2.4048$, $J_0(\delta) = 0$ and all the power is transferred to harmonic frequencies.[37]

Polarization Modulation (Dynamic Retardation).   Polarization modulation involves the coherent addition of two orthogonal waves, resulting in a change of the input polarization state at the output. As with a phase modulator, the basic components for an electro-optic polarization modulator (or dynamic retardation plate or polarization state converter) is an electro-optic crystal and an input polarizer. The crystal and applied voltage V (dc assumed) are configured to produce dynamically the fast and slow axes in the crystal cross section. In this case, however, the polarizer is positioned such that the input light wave is
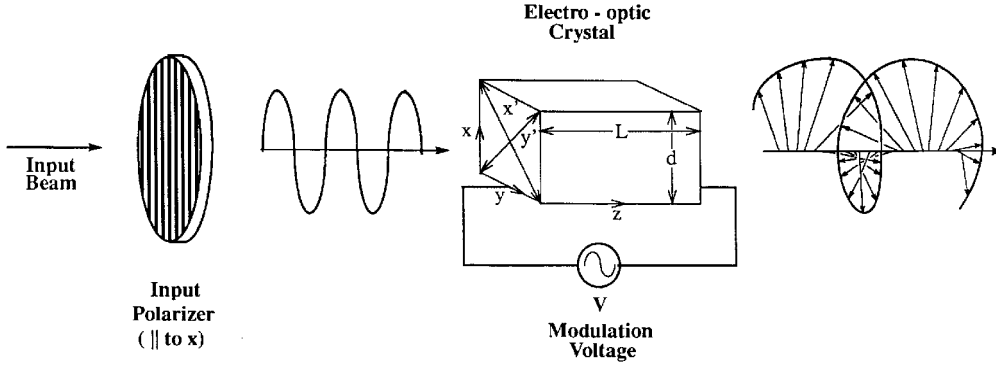
FIGURE 10 A longitudinal polarization modulator is shown with the input polarizer oriented along the x principal axis at 45° with respect to the perturbed x' and y' axes.

decomposed equally into the two orthogonal linear eigenpolarizations along these axes as shown in Fig. 10. If the light is polarized along the x axis and propagates along the z principal axis, for example, the propagating fields are

$$E_{x'} = E_0 \cos\left[\omega t - (2\pi/\lambda)n_{x'}z\right]$$

$$E_{y'} = E_0 \cos\left[\omega t - (2\pi/\lambda)n_{y'}z\right]$$

$$(33)$$

where the fast and slow axes are x' and y'. The corresponding refractive indices are

$$n_{x'} \approx n_x - \tfrac{1}{2}r_x n_x^3 E = n_x - \Delta n_x$$

$$n_{y'} \approx n_y - \tfrac{1}{2}r_y n_y^3 E = n_y - \Delta n_y$$

$$(34)$$

where n_x, n_y are the indices in the absence of an applied field and r_x, r_y are the appropriate electro-optic coefficients for the material being used and the orientation of the applied voltage. As the two polarizations propagate at different speeds through the crystal, a phase difference (relative phase) or *retardation* Γ evolves between them as a function of length:

$$\Gamma = \frac{2\pi}{\lambda}(n_{x'} - n_{y'})L$$

$$= \frac{2\pi}{\lambda}(n_x - n_y)L - \frac{\pi}{\lambda}(r_x n_x^3 - r_y n_y^3)EL = \Gamma_o + \Gamma_i \qquad (35)$$

where Γ_o is the natural phase retardation in the absence of an applied voltage and Γ_i is the induced retardation linearly related to V.

For a longitudinal modulator the applied electric field is $E = V/L$, and the induced retardation is $\Gamma_i = \left(\frac{\pi}{\lambda}\right)(r_y n_y^3 - r_x n_x^3)V$, which is independent of L and linearly related to V.

For a transverse modulator $E = V/d$, and the induced retardation is $\Gamma_i = \left(\frac{\pi}{\lambda}\right)(r_y n_y^3 - r_x n_x^3)V\left(\frac{L}{d}\right)$, which is dependent on the aspect ratio L/d and V.
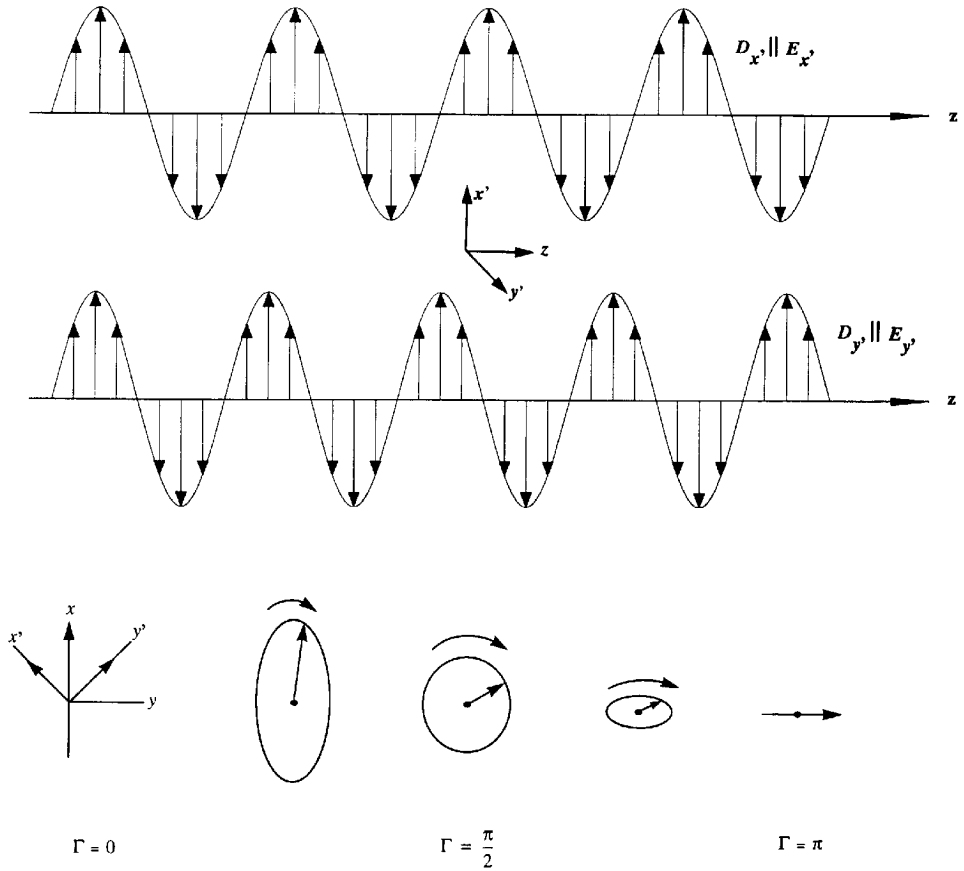
FIGURE 11   The polarization state of an input vertical linear polarization is shown as a function of crystal length L or applied voltage V. The retardation $\Gamma = \pi$ for a given length L_π in a passive $\lambda/2$ wave plate or applied voltage V_π in an electro-optic polarization modulator.[14]

The optical fields at the output can be expressed in terms of Γ:

$$
\begin{aligned}
E_{x'} &= \cos \omega t \\
E_{y'} &= \cos (\omega t - \Gamma)
\end{aligned}
\tag{36}
$$

Therefore, the desired output polarization is obtained by applying the appropriate voltage magnitude. Figure 11 illustrates the evolution of the polarization state as a function of propagation distance z. In terms of an active device, Fig. 11 also can be interpreted as a change in polarization state as a function of applied voltage for fixed length. The eigenpolarizations $E_{x'}$ and $E_{y'}$ are in phase at $z = 0$. They have the same frequency but different wavelengths. Light from one polarization gradually couples into the other. In the absence of natural birefringence, $n_x - n_y = 0$, the voltage that would produce a retardation of $\Gamma = \Gamma_i = \pi$, such that a vertical polarization input becomes a horizontal polarization output, is the *half-wave voltage* V_π. For a longitudinal modulator $V_\pi = \dfrac{\lambda}{r_x n_x^3 - r_y n_y^3}$, which is independent of L. For a transverse modulator $V_\pi = \dfrac{\lambda}{r_x n_x^3 - r_y n_y^3}\left(\dfrac{d}{L}\right)$, which is dependent

on the aspect ratio L/d. The total retardation in terms of V_π (calculated assuming no birefringence) is

$$\Gamma = \Gamma_o + \pi\left(\frac{V}{V_\pi}\right) \tag{37}$$

To cancel the effect of natural birefringence, the phase retardation Γ_o can be made a multiple of 2π by slightly polishing the crystal to adjust the length or by applying a bias voltage. If birefringence is present, an effective V_π can be calculated that would give a total retardation of $\Gamma = \pi$.

To achieve polarization modulation, a birefringence must exist in the crystal cross section. If the cross section is a characteristic plane, then the input polarization propagates through the crystal unchanged when $V = 0$. If an applied voltage causes the axes to rotate $45°$ in this cross section with respect to the input polarization, as in Fig. 10, then the input will decompose into two equal components and change polarization state at the output. If the cross section has natural birefringence, then the input polarization state will change with $V = 0$ as well as with an applied voltage.

Amplitude Modulation. The intensity (optical energy) of a light wave can be modulated in several ways. Some possibilities include using (1) a dynamic retarder configuration with a crossed polarizer at the output, (2) a dynamic retarder configuration with a parallel polarizer at the output, (3) a phase modulator configuration in a branch of a Mach-Zehnder interferometer, or (4) a dynamic retarder with push-pull electrodes. The intensity modulator parameter of interest is the *transmission* $T = I_o/I_i$, the ratio of output to input intensity.

An intensity modulator constructed using a dynamic retarder with crossed polarizers is shown in Fig. 12. The transmission for this modulator is

$$T(V) = \sin^2\left(\frac{\Gamma}{2}\right) = \sin^2\left(\frac{\Gamma_o}{2} + \frac{\pi V}{2V_\pi}\right) \tag{38}$$

using Eq. (37). For linear modulation, where the output is a replica of the modulating voltage signal, a fixed bias of $\Gamma_o = \pi/2$ must be introduced either by placing an additional phase retarder, a $\lambda/4$ wave plate (Fig. 12), at the output of the electro-optic crystal or by applying an additional dc voltage of $V_\pi/2$. This bias produces a transmission of $T = 0.5$ in the absence of a modulating voltage. If the crystal cross section has natural birefringence, then a variable compensator (Babinet-Soleil) or a voltage less than $V_\pi/2$ must be used to tune the birefringence to give a fixed retardation of $\pi/2$.

For a sinusoidal modulation voltage $V = V_m \sin \omega_m t$, the retardation at the output of the crystal, including the bias, is

$$\Gamma = \Gamma_o + \Gamma_i = \frac{\pi}{2} + \Gamma_m \sin \omega_m t \tag{39}$$

where $\Gamma_m = \pi V_m/V_\pi$ is the *amplitude modulation index* or *depth-of-amplitude modulation* and V_π is the half-wave voltage as presented in the previous section for polarization modulators. The transmission becomes

$$T(V) = \sin^2\left(\frac{\pi}{4} + \frac{\Gamma_m}{2}\sin \omega_m t\right)$$

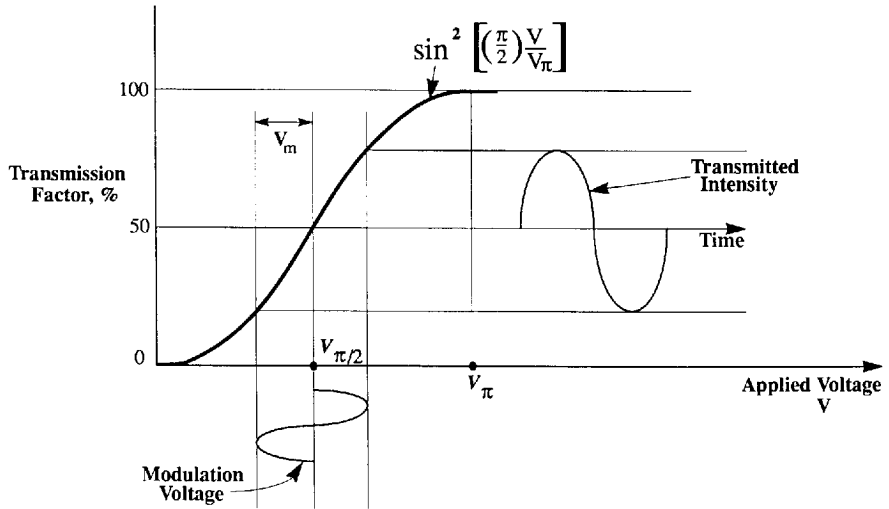$$= \frac{1}{2}\left[1 - \cos\left(\frac{\pi}{2} + \Gamma_m \sin \omega_m t\right)\right] \tag{40}$$
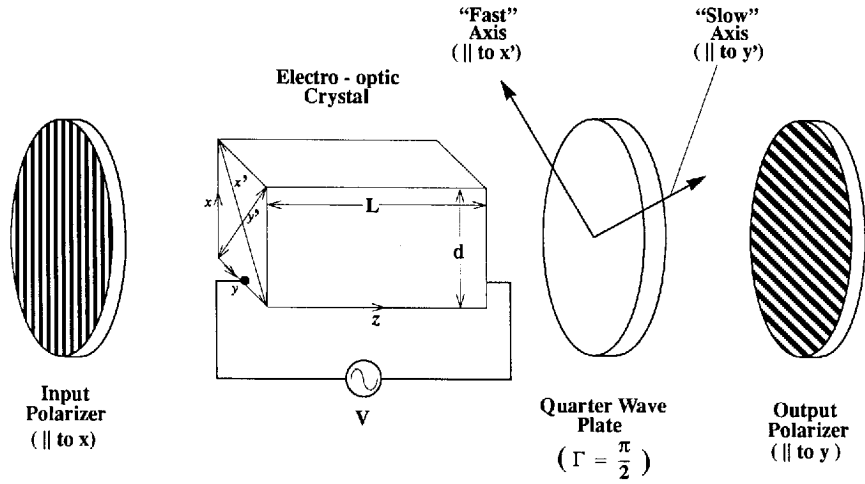
FIGURE 12 A longitudinal intensity modulator is shown using crossed polarizers with the input polarization along the x principal axis. A $\lambda/4$ wave plate is used as a bias to produce linear modulation.[14]

If the modulation voltage is small ($V_m \ll 1$), then the modulation depth is small ($\Gamma_m \ll 1$) and

$$T(V) = \tfrac{1}{2}[1 + \Gamma_m \sin \omega_m t] \qquad (41)$$

Therefore, the transmission or output intensity is linearly related to the modulating voltage. If the signal is large, then the output intensity becomes distorted, and higher-order odd harmonics appear.[14]
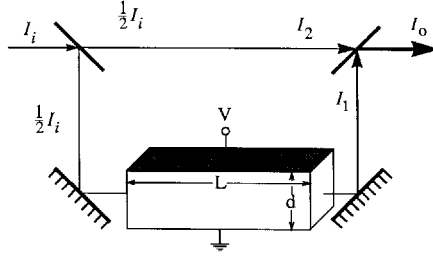
FIGURE 13 An intensity modulator is shown implementing a Mach-Zehnder interferometer configuration with a phase modulator in one branch.[38]

The dynamic retarder with parallel polarizers has a transmission of[14]

$$T = \cos^2\left(\frac{\Gamma}{2}\right) = \cos^2\left(\frac{\pi}{4} + \frac{\Gamma_m}{2}\sin\omega_m t\right)$$

$$= \frac{1}{2}\left[1 + \cos\left(\frac{\pi}{2} + \Gamma_m\sin\omega_m t\right)\right] \tag{42}$$

For small modulation, $T(V) = (1/2)[1 - \Gamma_m \sin \omega_m t]$, and again, the output is a replica of the modulating voltage.

Similarly, the output of a Mach-Zehnder interferometer is given by

$$I_o = I_1 + I_2 = \tfrac{1}{2}[I_i \cos \Gamma_o + I_i] = I_i \cos^2\left(\frac{\Gamma_o}{2}\right) \tag{43}$$

where Γ_o is the relative phase shift between the two branches. An intensity modulator is produced by placing a phase modulator in one branch[38] as shown in Fig. 13. The total retardation is $\Gamma = \Gamma_o + \Gamma_i$, as before. The transmission is

$$T = \frac{I_o}{I_i} = \cos^2\left(\frac{\Gamma}{2}\right) \tag{44}$$

The push-pull modulator is based on the Mach-Zehnder interferometer. In this case, a phase modulator is placed in each branch with opposite polarity voltages applied to the arms; the phase modulators are driven 180° out-of-phase. This configuration requires lower drive voltages and provides a shorter transit time for the light for a defined degree of modulation.[41]

Frequency Modulation. In frequency modulation a shift or deviation in the frequency by ω_d from the optical carrier instantaneous frequency ω is desired. One approach to achieve a shift in frequency is to use an intensity modulator configuration of an electro-optic crystal between left- and right-hand circular polarizers. The electrodes on the modulator must be designed to produce an applied circular electric field.[42]

A crystal and its orientation are selected such that there is no birefringence in the crystal cross section at zero voltage. When a circular electric field with frequency ω_m is applied, however, a birefringence is induced in the crystal cross section, and the induced principal axes rotate with angular velocity $\omega_m/2$ in the opposite sense with respect to the modulating field. The relative rotation between the axes and the modulating field creates a frequency shift in the optical electric field as it propagates through the crystal.
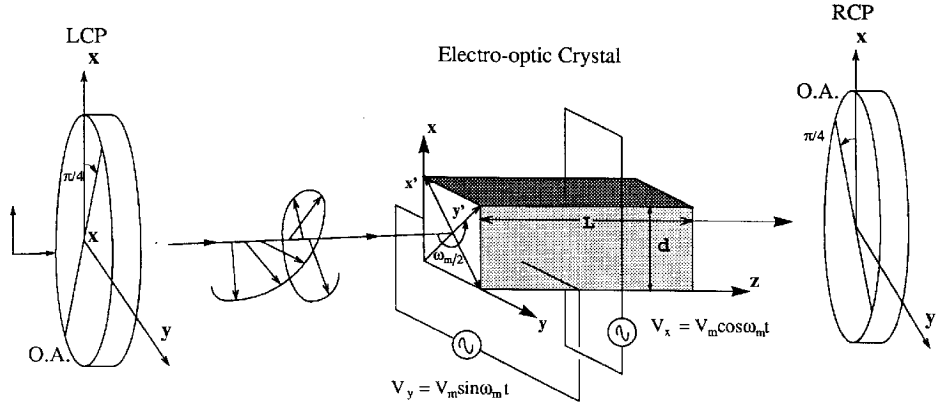
FIGURE 14 A frequency modulator using a phase modulator with two pairs of transverse electrodes set 90° out of phase to produce a circular applied electric field. The phase modulator is placed between left and right circular polarizers to create a frequency deviation in the output optical field.

An example of such a device[42] is shown in Fig. 14. There are two sets of electrodes in the transverse configuration. The applied voltages are 90° out of phase to produce a left circular modulating field:

$$E_x = E_m \cos \omega_m t$$
$$E_y = E_m \sin \omega_m t \tag{45}$$

The principal axes in the crystal cross section rotate through an angle

$$\beta_1(t) = -\tfrac{1}{2}(\omega_m t + \Phi) \tag{46}$$

where Φ is a fixed angle that depends on the electro-optic coefficients r_{ij} of the crystal and not on the electric field magnitude.

The input optical wave (after the left circular polarizer) has field components

$$E_{i_x} = E_i \cos \omega t$$
$$E_{i_y} = E_i \sin \omega t \tag{47}$$

The induced retardation $\Gamma_i = \Gamma = (2\pi/\lambda)\Delta n L$ is independent of time, since Δn is constant although the principal axes are rotating at constant angular velocity. The optical field components along the induced principal axes at the output of the crystal are

$$E_{o_{x'}} = E_i \cos\left(\omega t - \beta_1 + \frac{\Gamma}{2}\right)$$
$$E_{o_{y'}} = E_i \sin\left(\omega t - \beta_1 - \frac{\Gamma}{2}\right) \tag{48}$$

In terms of the original stationary x, y axes, the optical field components at the output are

$$E_{o_x} = E_i \cos (\Gamma/2) \cos \omega t - E_i \sin (\Gamma/2) \sin [(\omega + \omega_m)t + \Phi]$$
$$E_{o_y} = E_i \cos (\Gamma/2) \sin \omega t - E_i \sin (\Gamma/2) \cos[(\omega + \omega_m)t + \Phi] \tag{49}$$

The first terms in E_{o_x} and E_{o_y} represent left circular polarization at the original optical

frequency ω with constant amplitude $E_i \cos(\Gamma/2)$ and phase independent of the rotating principal axes (i.e., of β_1). The second terms represent right circular polarization at frequency $(\omega + \omega_m)$ with constant amplitude $E_i \sin(\Gamma/2)$ and phase proportional to $2\beta_1 = \omega_m t$. Therefore, the frequency shift ω_d is the modulation frequency ω_m. If the retardation $\Gamma = \pi$, the frequency of the light has complete deviation to $(\omega + \omega_m)$. If Γ is very small, the component optical fields at frequency $(\omega + \omega_m)$ are linearly related to Γ and therefore, to the applied voltage.

A shift in frequency to $(\omega - \omega_m)$ is obtained if the optical and the applied modulating electric fields rotate in the opposite sense.

Scanners. The position of an optical beam can be changed dynamically by using an electro-optic deflecting device or scanner. Analog scanners are based on refraction phenomena: (1) refraction at a dielectric interface (prism) and (2) refraction by an index gradient that exists perpendicular to the direction of light propagation. Digital scanners or switches are based on birefringence.

One of the most important parameters characterizing the performance of a scanner is its resolution, the number of independent resolvable spots it can scan, which is limited by the diffraction occurring at the aperture of the device. The Rayleigh criterion states that two spots are just resolved when the angular displacement of the beam $\Delta\varphi$ is equal to the half-angle divergence θ due to diffraction.[13] Therefore, the total number of resolvable spots is given by the ratio of the total deflection angle φ to the half-angle divergence θ

$$N = \frac{\varphi}{\theta} \qquad (50)$$

The half-angle divergence is $\theta = \varpi/w$, where w is the beamwidth, $\varpi = 1$ for a rectangular beam of uniform intensity, $\varpi = 1.22$ for a circular beam of uniform intensity, and $\varpi = 1.27$ for a beam of Gaussian intensity distribution.[43]

An analog scanner can be constructed by a prism of electro-optic material with electrodes on the crystal faces as shown in Fig. 15. The resolution is maximum in this isosceles-shaped prism when the beam is transmitted through at the minimum deviation angle and is[43,44]

$$N = \Delta n \left(\frac{l}{\varpi \lambda}\right)\left(\frac{w}{W}\right) \qquad (51)$$

where l is the base length of the prism and w/W is the ratio of the beamwidth to input
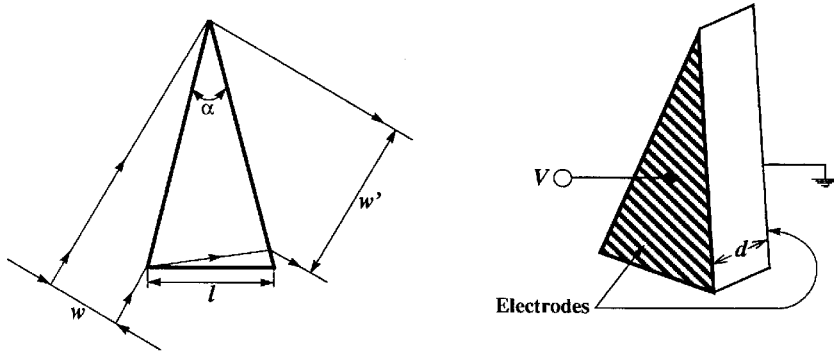

FIGURE 15 An analog scanner can be constructed using an isosceles-shaped prism with the beam transmitted at the minimum deviation angle.[44]
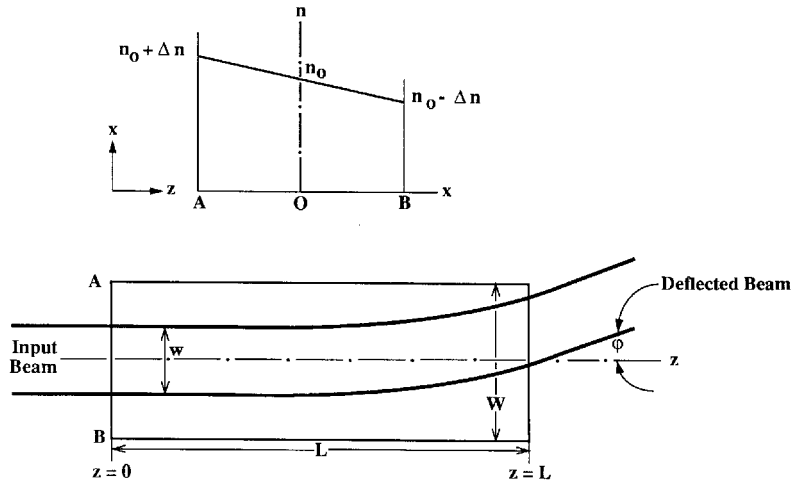
FIGURE 16   An analog scanner based on an index gradient is shown.[45]

aperture of the prism. This type of device typically requires a voltage much higher than the half-wave voltage V_π to resolve N spots.[38]

An analog scanner based on a gradient index of refraction[45] is shown in Fig. 16. The voltage is applied to a crystal such that the change in index is linear with distance perpendicular to the direction of light propagation; i.e., $n(x) = n_o + (2\Delta n/W)x$, where W is the width of the crystal. For linear gradient and small refraction angles, the wavefront remains planar. The (small) deflection angle of the ray after propagating a distance L in the crystal is approximated to be[43]

$$\varphi = L\frac{dn}{dx} \qquad (52)$$

and the resolution is

$$N = \frac{\varphi}{\theta} = 2\Delta n\left(\frac{L}{\varpi\lambda}\right)\left(\frac{w}{W}\right) \qquad (53)$$

A large crystal length L is needed to obtain appreciable deflection, since Δn is very small ($\sim 10^{-4}$). Laser beams, however, are very narrow to make such a device practical.

Digital light deflection can be implemented with a number of *binary units*,[38,46] each of which consists of a polarization modulator followed by a birefringent crystal (or discriminator) as shown in Fig. 17. The polarizer is oriented to give a horizontal polarization input to the modulator. With the voltage off, the light is polarized along a principal axis of the crystal, and it propagates through without change in polarization. It then enters the birefringent crystal, such as calcite, and passes through undeflected. With an applied voltage, the principal axes rotate 45° and the input is then decomposed into orthogonal components of equal amplitude. The voltage magnitude is set to produce a retardation of π, thereby rotating the polarization by 90°. The vertical polarization then enters the birefringent crystal which deflects it.[43,47]

The number of binary units n would produce a deflector of 2^n possible deflected positions at the output. An example of a three-stage deflector is shown in Fig. 18.[46–48] The on–off states of the three voltages determine at what position the output beam would be deflected. For example, if all three voltages are off, the input polarization remains horizontal through the system and is undeflected at the output. However, if all three voltages are on, the horizontal input is rotated 90° and becomes vertical after the first
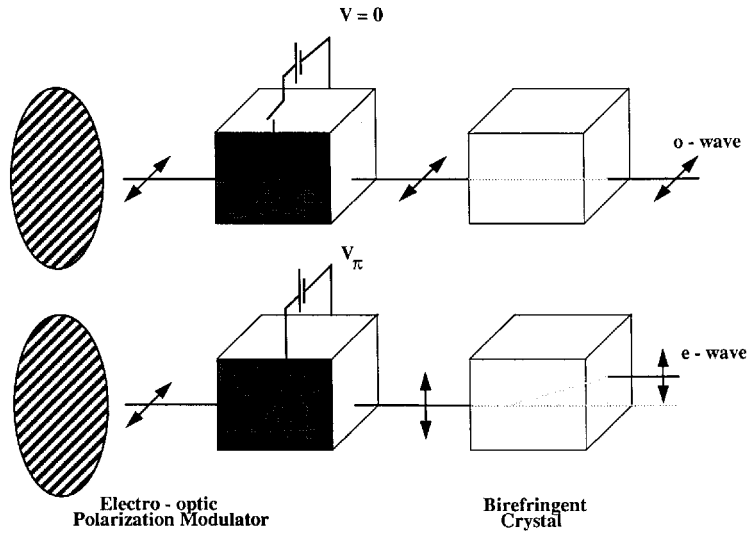
FIGURE 17 The component of a digital scanner is a binary unit. It consists of a polarization modulator followed by a birefringent crystal which serves as a discriminator.[38]
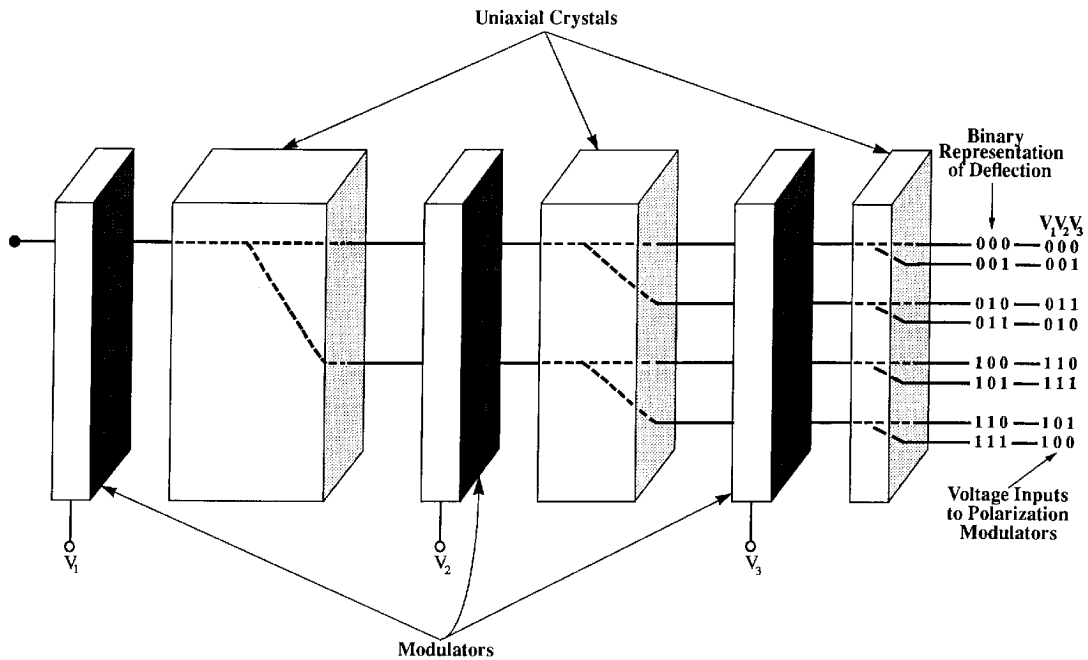

FIGURE 18 A three-stage digital scanner produces 2^3 possible deflected positions at the output.[46]

polarization modulator and is deflected. The polarization is rotated 90° after the second modulator, becoming horizontal, and therefore, propagates straight through the birefringent crystal. Finally, the polarization is rotated 90° after the third modulator, becoming vertical, and is deflected. The corresponding output represents a binary five; that is, 101.

Design Considerations

Two fundamental considerations by which to design an electro-optic modulator are signal voltage and frequency response. The goal is to achieve a desired depth of modulation with the smallest amount of power and at the same time obtain the largest possible bandwidth. In general, when designing bulk modulators, the field of the modulation signal and optical wave are assumed to have 100 percent overlap.

Choice of Material and Device Geometry.   An electro-optic material must be selected that would perform ideally at a given wavelength and in a given environment. The material should have high resistivity, a high electro-optic coefficient, good homogeneity, and temperature and mechanical stability. Some materials with natural birefringence, such as KDP and ferroelectric $LiTaO_3$ and $LiNbO_3$, are sensitive to temperature variations and would induce a temperature-dependent phase shift in the light. Some of these materials have acoustic resonances that cause undesirable peaks in the frequency response.[20,41,48,49] The half-wave voltage V_π required for a desired degree of modulation is an important parameter to consider when selecting a crystal, and V_π should be as small as possible. The material should have small dielectric dissipation, good optical quality in the proper size, and good ohmic contacts.[20,50]

When a modulator is operating in the transverse configuration, the half-wave voltage is inversely proportional to the aspect ratio L/d. Half-wave voltages on the order of tens of volts are typical for this configuration as compared to $\sim$10,000 V for a longitudinal device.[37] However, the transverse dimension d is limited by diffraction of the finite cross-section light beam and by capacitance of the device. For optimum performance, the aperture should be just large enough to pass the light through. The dimensions L and d cannot be selected independently.[20,37,50]

For phase modulation, a crystal orientation is required that would give the maximum change in index of refraction. For amplitude modulation a crystal orientation must produce the maximum birefringence.

Transit Time Limitations.   The transit time of the light is the time for it to pass through the crystal:

$$\tau = \frac{nL}{c} \tag{54}$$

where n is the index seen by the light. This parameter has no relevance for modulation frequencies $\omega_m \ll 2\pi/\tau$; the modulation field appears uniform in the crystal at very low frequencies. A rule of thumb for defining the limiting frequency such that τ can be neglected is that the length of the crystal be less than $\frac{1}{10}$ the wavelength of the modulating field[51] or $L \ll 2\pi c/\omega_m\sqrt{\varepsilon}$.[37] The electro-optic crystal is modeled as a lumped capacitor at low frequencies.

As the modulating frequency becomes larger, the transit time must be taken into account in evaluating modulator performance. The modulation electric field has the form $E = E_m \sin \omega_m t$, and the optical phase can no longer follow the time-varying index of

refraction adiabatically. The result is a reduction in the modulation index parameters, δ for phase modulation and Γ_m for amplitude modulation, by a factor[14,37,40]

$$\rho = \frac{\sin\left(\frac{1}{2}\omega_m\tau\right)}{\frac{1}{2}\omega_m\tau} \tag{55}$$

Therefore, the phase modulation index at high frequencies becomes

$$\delta_{RF} = \delta \cdot \rho = \delta \cdot \left[\frac{\sin\left(\frac{1}{2}\omega_m\tau\right)}{\frac{1}{2}\omega_m\tau}\right] \tag{56}$$

and the amplitude modulation index at high frequencies becomes

$$\Gamma_{m_{RF}} = \Gamma_m \cdot \rho = \Gamma_m \cdot \left[\frac{\sin\left(\frac{1}{2}\omega_m\tau\right)}{\frac{1}{2}\omega_m\tau}\right] \tag{57}$$

If $\tau = 2\pi/\omega_m$ such that the transit time of the light is equal to the time period of the modulation signal, then there is no retardation; the retardation produced in the first half of the crystal is exactly canceled by the retardation produced in the second half.[37] The maximum modulation frequency for a given crystal length L is determined by the allowable ρ parameter set by the designer.

Traveling Wave Modulators.   The limitation of the transit time on the bandwidth of the modulator can be overcome by applying the voltage as a traveling wave, propagating collinearly with the optical wave. Figure 19 illustrates a transverse traveling wave configuration. The electrode is designed to be an extension of the driving transmission line to eliminate electrode charging time effects on the bandwidth. Therefore, the transit time problem is addressed by adjusting the phase velocity of the modulation signal to be equal to the phase velocity of the optical signal.[14,37,40]

The applied modulation electric field has the form

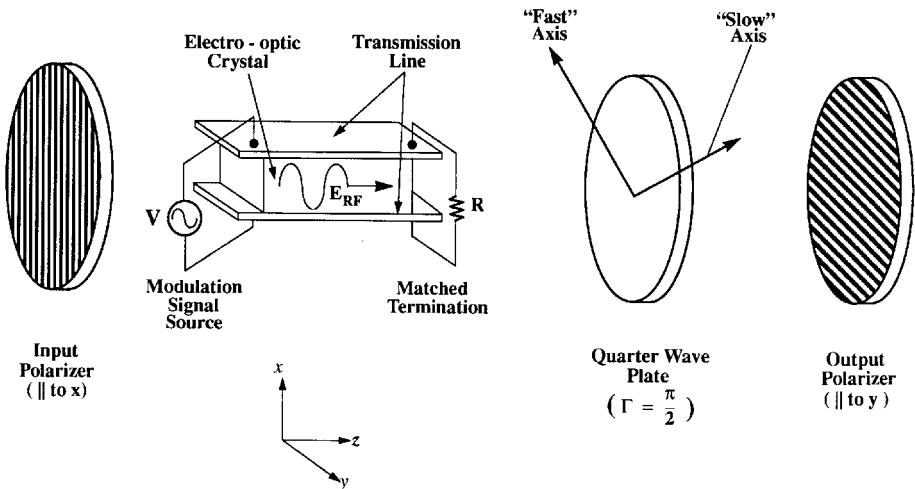$$E_{RF}(t, z) = E_m \sin\left(\omega_m t - k_m z\right) \tag{58}$$

FIGURE 19   A transverse traveling wave modulator has a modulating voltage polarized in the same orientation as the input light and propagating collinearly.[14]

for propagation along the z axis of a crystal. The direction of field vibration is the direction of the applied field, not the direction it is traveling, and is along x in Fig. 19. The parameter k_m is the wave-vector magnitude of the modulation field and is

$$k_m = \frac{\omega_m}{v_m} = \frac{\omega_m n_m}{c} \tag{59}$$

where v_m and $n_m = \sqrt{\varepsilon}$ are the phase velocity and index of refraction of the modulating signal. A mismatch in the phase velocities of the modulating signal and optical wave will produce a reduction in the modulation index δ or Γ_m by a factor

$$\rho_{tw} = \frac{\sin\left[\dfrac{\omega_m}{2c}(n - n_m)L\right]}{\dfrac{\omega_m}{2c}(n - n_m)L} = \frac{\sin(\Delta L)}{\Delta L} \tag{60}$$

In the case of amplitude modulation Eq. (60) holds only if there is no natural birefringence in the cross section of the crystal. The eigenpolarization magnitudes are functions of time and space. Therefore, they satisfy the coupled wave equations, which are more complicated when birefringence is present.[14]

With no phase velocity mismatch, the phase modulation index is $\delta_{RF} = \delta$ and is linearly proportional to the crystal length L. Likewise, $\Gamma_{RF} = \Gamma_m$ for amplitude modulation. With a mismatch, the maximum possible phase modulation index is $\delta_{RF_{max}} = \delta/(\Delta L)$, likewise for the amplitude modulation index $\Gamma_{RF_{max}}$. The modulation index becomes a sinusoidal function of L. This maximum index can be achieved for crystal lengths $L = \pi/2\Delta$, $3\pi/2\Delta$, etc. The ratio n/n_m is approximately $1/2$ for LiNbO$_3$, producing a walk-off between the optical and modulation waves.[40] Therefore, for a given length L the modulation frequency is greatly affected by the velocity mismatch.

Performance Criteria

The following parameters are indicators of modulator performance. Basically, the tradeoffs occur between the aspect ratio L/b, the drive voltage, and the electrode configuration (lumped or traveling wave) to achieve a desired depth of modulation with the smallest amount of power and at the same time obtain the largest possible bandwidth.

Modulation Bandwidth.   The bandwidth is the difference between the two closest frequencies at which the modulation index, δ or Γ, falls to 50 percent of the maximum value.[52,53] Therefore, the 3 dB optical bandwidth is determined by setting the modulation reduction factor ρ or ρ_{tw} to 0.5.[40,52,53] Modulation speed depends on the electrode type, lumped or traveling wave, and the modulator capacitance per length, which depends on the RF dielectric constant ε and the geometry of the electrodes. For a lumped modulator the bandwidth is limited by the optical (c/Ln) or electrical ($c/L\sqrt{\varepsilon}$) transit time, whichever is smaller, or the time constant of the lumped-circuit parameters ($1/RC$), where R is the resistance in the circuit and C is the capacitance of the modulator. For a traveling wave modulator the bandwidth v_{tw} is limited by the velocity mismatch between the optical and modulation waves, $v_{tw} = c/Ln(1 - \sqrt{\varepsilon}/n)$.[14]

Power per Unit Bandwidth (Specific Energy).   To apply the modulating signal efficiently to a *lumped* electro-optic modulator, a resonant *RLC* circuit can be created by adding an inductance and resistance in parallel to the modulator, which is modeled by a capacitor C.[14,37] The terminating resistance R ensures that more of the voltage drop occurs over the

modulator rather than the internal resistance of the source R_s. The impedance of the resonant circuit is high over a bandwidth (determined by the RC time constant) of $\Delta v = \Delta \omega_m / 2\pi \simeq 1/2\pi RC$ centered at the resonant frequency of $\omega_o = 1/\sqrt{LC}$, where the impedance of the parallel RLC circuit is exactly equal to R. A peak voltage V_m must be applied to achieve a desired peak retardation $\Gamma_m = \pi V_m / V_\pi$. Therefore, $V_m = \Gamma_m (V_\pi / \pi)$. The power required to achieve Γ_m is $P = V_m^2 / 2R$, where $1/2R = \pi C \Delta v$, giving

$$P/\Delta v = \frac{1}{\pi} \Gamma_m^2 V_\pi^2 C \qquad (61)$$

The power per unit bandwidth (joules or mW/MHz) depends on the modulator capacitance C and the peak modulation voltage V_m to give the desired depth of modulation Γ_m. The required drive power increases with modulation frequency.[37,53] Since the capacitance is a function of the modulator active area dimensions, the required power is also a function of the modulator dimensions.[53] Furthermore, since V_π is directly proportional to the wavelength λ, a higher input power (voltage) is required at longer wavelengths for a given peak retardation.

Extinction Ratio.   The extinction ratio is the maximum depth of intensity modulation for a time-varying voltage at the output when the optical bias is adjusted properly.[49,52] If for no voltage the output intensity is I_0 and for maximum applied voltage the output intensity is I_m, then the extinction ratio is defined as[52]

$$\eta_m = \frac{|I_m - I_0|}{I_0} \qquad I_m \leq I_0$$

$$\eta_m = \frac{|I_m - I_0|}{I_m} \qquad I_m \geq I_0 \qquad (62)$$

Material effects, such as crystal imperfections, temperature sensitivities, and birefringence, can degrade the extinction ratio,[14, 34, 41] thereby affecting the signal-to-noise ratio at the detector.[52] Another definition is in terms of the transmission T, $\eta_m = T_{\max}/T_{\min}$.[38,54] In general, $T_{\max} < 1$ due to absorption, reflection, and scattering losses, and $T_{\min} > 0$ due to beam-divergence angle, residual crystal birefringence, crystal inhomogeneity, electric field uniformity, background scattered light, and polarizer-analyzer alignment.[36] Extinction ratio also can be applied to phase modulation, since phase changes can be related to equivalent changes in intensity.[52]

Maximum Frequency Deviation.   A similar figure of merit as exists for intensity modulators likewise exists for frequency modulators. The maximum deviation of a frequency modulator is defined as[53]

$$D_{\max} = \frac{|\omega_d - \omega|}{\omega} \qquad (63)$$

where ω_d is the frequency shift when the maximum voltage is applied.

Percent Modulation.   Percent modulation is an intensity modulation parameter. Basically, it is the transmission $T = I_o / I_i$ times 100 percent at a specific wavelength. For a device with total retardation of $\Gamma = \dfrac{\pi}{2}$ radians at no voltage, the transmission $T = 0.5$. Then 70 percent modulation is achieved with an analog signal if a voltage is applied such that $\Gamma = 2$ radians; i.e., $\sin^2(1) = 0.70$.[52] 100 percent intensity modulation is the output intensity varying between the input intensity and zero intensity.[37] A peak voltage of $V = V_\pi / 2$ is

required to achieve this level of performance for a linear modulator. Another definition of percent modulation is the ratio of the peak output intensity at the modulation frequency to the maximum output intensity at dc voltage.[49] Reference 41 defines percent modulation as the peak modulation voltage output per dc voltage output, assuming no temperature variations.

Degree of Modulation. For a sinusoidal reference of an optical electric field, $E(t) = E_i \sin \omega t$, an amplitude-modulated signal is typically expressed as

$$E(t) = E_i(1 + m \sin \omega_m t) \sin \omega t \tag{64}$$

where m is the degree of modulation at a specific wavelength.[17] For $m \ll 1$ the intensity is

$$I = \tfrac{1}{2} E_i^2 (1 + 2m \sin \omega_m t) \tag{65}$$

The amplitude modulation index is then $\Gamma_m = 2m$, a function of the degree of modulation. The degree of modulation is often referred to as percent modulation for intensity modulation ($70 - 100$ percent nominal) and as modulation index for phase modulation (1 radian nominal).[34]

Modulation Efficiency. Modulation efficiency is defined as the percentage of total power which conveys information.[55,56] The total power of an amplitude-modulated optical carrier is a function of the modulation frequency and is proportional to $1 + m^2 \langle \sin^2 \omega_m t \rangle = 1 + \tfrac{1}{2} m^2$. Therefore, the modulation efficiency is[55]

$$\xi = \frac{\tfrac{1}{2} m^2}{1 + \tfrac{1}{2} m^2} \tag{66}$$

The maximum efficiency is achieved when m is maximum; i.e., $m = \tfrac{1}{2} \Gamma_m = \pi V / 2V_\pi$. In terms of the input power P the modulation efficiency is[56]

$$\xi = \frac{\Gamma^2}{P} \tag{67}$$

where $\Gamma = \pi V / V_\pi$ for maximum efficiency.

Optical Insertion Loss. For an external modulator, in particular, the optical insertion loss must be minimized when coupling light into and out of the device. For an input light intensity I_{in} the insertion loss IL is[52]

$$IL = 1 - \frac{I_m}{I_{in}} \qquad I_m \geq I_0$$

$$IL = 1 - \frac{I_0}{I_{in}} \qquad I_m \leq I_0 \tag{68}$$

where I_0 and I_m are the output intensities at no voltage and maximum voltage, respectively. If a beam has a large cross-sectional area, the modulator must have a large acceptance area to couple all or most of the light into the device. A longitudinal modulator typically has a large acceptance area. Minimizing insertion loss in a transverse modulator is more of a challenge due to its narrow width, particularly for wide beams. However, a transverse modulator requires less voltage for a desired degree of modulation.

13.6   *APPENDIX*: *EULER ANGLES*

Euler angles represent a set of three independent parameters which specify the orientation of a rigid body, in this case the index ellipsoid. An orthogonal transformation matrix consisting of direction cosines and having a determinant of $+1$ corresponds to defining the orientation of a rigid body.

The transformation matrix is developed by a specific sequence of three plane rotations (not in principal planes) in a defined order. There are 12 possible conventions for defining a set of Euler angles in a right-handed coordinate system.[32] One convention that has been proposed as a standard is the y convention.[57] The transformation evolves by an initial counterclockwise rotation ζ about the z axis, followed by a counterclockwise rotation η about the intermediate y' axis, and finally a counterclockwise rotation ω about z''. The resulting transformation matrix is

$$\bar{a} = \begin{pmatrix} \cos \omega & \sin \omega & 0 \\ -\sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \eta & 0 & -\sin \eta \\ 0 & 1 & 0 \\ \sin \eta & 0 & \cos \eta \end{pmatrix} \begin{pmatrix} \cos \zeta & \sin \zeta & 0 \\ -\sin \zeta & \cos \xi & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{(A.1)}$$

To find the Euler angles (ζ, η, ω), Eq. (A.1) is rearranged as[33]

$$\begin{pmatrix} \cos \omega & -\sin \omega & 0 \\ \sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} \cos \eta & 0 & -\sin \eta \\ 0 & 1 & 0 \\ \sin \eta & 0 & \cos \eta \end{pmatrix} \begin{pmatrix} \cos \zeta & \sin \zeta & 0 \\ -\sin \xi & \cos \zeta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{(A.2)}$$

where $\bar{a}$ is the cumulative transformation matrix of the normalized eigenvectors. Multiplying the matrices, the Euler angles are related to the elements of $\bar{a}$:

$$a_{11} \cos \omega - a_{21} \sin \omega = \cos \eta \cos \zeta \qquad \text{(A.3}a\text{)}$$

$$a_{12} \cos \omega - a_{22} \sin \omega = \cos \eta \sin \zeta \qquad \text{(A.3}b\text{)}$$

$$a_{13} \cos \omega - a_{23} \sin \omega = -\sin \eta \qquad \text{(A.3}c\text{)}$$

$$a_{11} \sin \omega + a_{21} \cos \omega = -\sin \zeta \qquad \text{(A.3}d\text{)}$$

$$a_{12} \sin \omega + a_{22} \cos \omega = \cos \zeta \qquad \text{(A.3}e\text{)}$$

$$a_{13} \sin \omega + a_{23} \cos \omega = 0 \qquad \text{(A.3}f\text{)}$$

$$a_{31} = \sin \eta \cos \zeta \qquad \text{(A.3}g\text{)}$$

$$a_{32} = \sin \eta \sin \zeta \qquad \text{(A.3}h\text{)}$$

$$a_{33} = \cos \eta \qquad \text{(A.3}i\text{)}$$

From Eq. (A.3f), the angle ω is $\omega = \tan^{-1}(-a_{23}/a_{13})$. From Eq. (A.3$c$), the angle η is $\eta = \sin^{-1}(a_{23} \sin \omega - a_{13} \cos \omega)$. From Eq. (A.3$d$), the angle ζ is $\zeta = \sin^{-1}(-a_{11} \sin \omega - a_{21} \cos \omega)$.[30]

13.7   *REFERENCES*

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