Assignment 2

EE 735

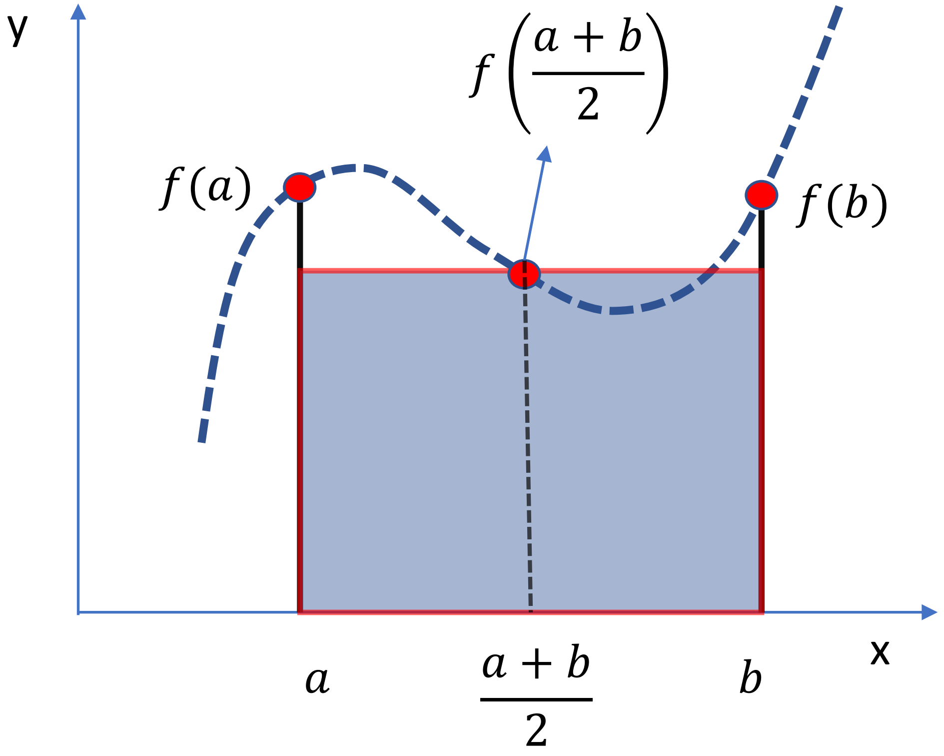
Outline

- Numerical integration

- System of linear equations and its numerical solution

- System of non linear equation its numerical solution

Numerical Integration

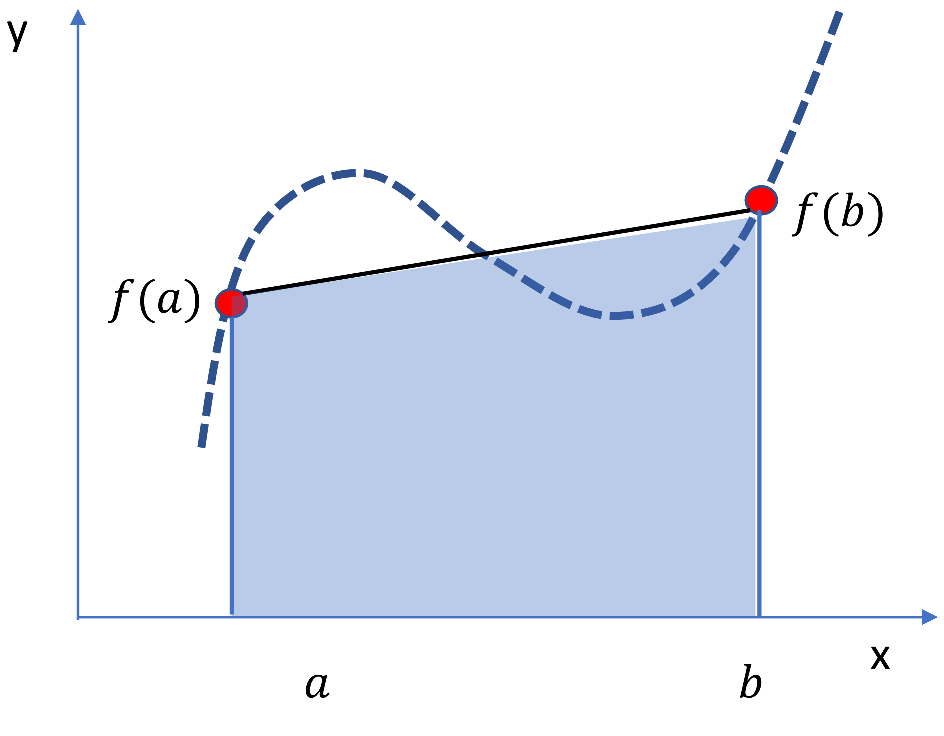


$$y = f(x)$$

Definite integral

$$\int_a^b f(x)\,dx \approx (b-a)f\left(\frac{a+b}{2}\right)$$

Trapezoidal Rule



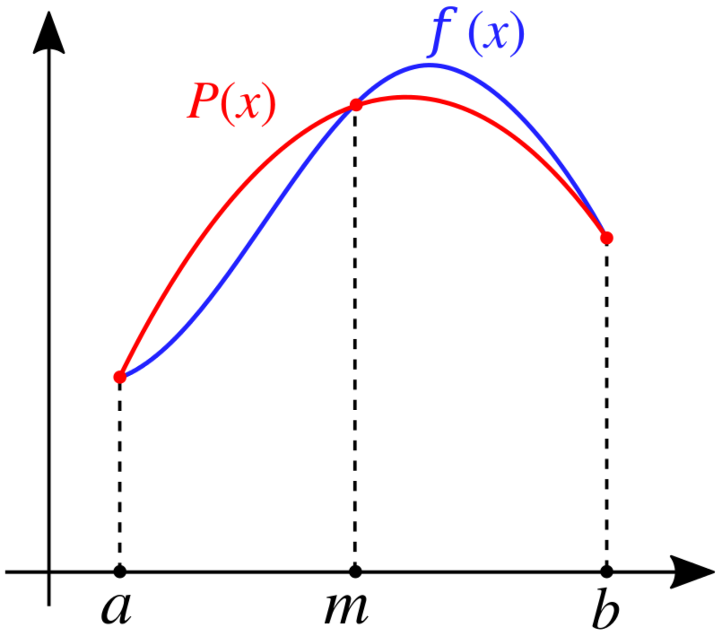
$$y = f(x)$$

$$\int_a^b f(x)\,dx \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

Numerical Integration

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{N} \Delta x_i \left[\frac{f(x_{i-1})+f(x)}{2}\right]$$

Simpson's Rule



Lagrange polynominal

$$P(x) = f(a)\frac{(x-m)(x-b)}{(a-m)(a-b)} + f(m)\frac{(x-a)(x-b)}{(m-a)(m-b)} + f(b)\frac{(x-a)(x-m)}{(b-a)(b-m)}$$

$$\int_a^b P(x)\,dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Numerical Integration

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 4f(x_{n-1}) + f(x_n)\right),$$

$$Simpon's\ merhod\ is\ the\ weighted\ average\ of\ Mid\ point\ and\ trapezoidal\ methods\ S = \frac{2M + T}{3}$$

Assignment

Question 1.

Solve the following for abrupt and linearly graded pn junction diode.

(a) Plot the Electric field profile by solving the Gauss Law using numerical integration techniques such as Trapezoidal or Simpson's methods. Compare the result with the inbuilt MATLAB functions for integration. Check the accuracy of numerically integrated results by varying the grid spacing.

$E(x) = \int \frac{\rho(x)}{\epsilon_s}\, dx$, for $-x_p \leq x \leq x_n$

System of linear equations

$$\nabla^2 V(x) = -\frac{q}{\epsilon_s}(N_D - N_A)$$

Using central difference approximation

$$V_{i-1} - 2V_i + V_{i+1} = -\frac{h^2 q}{\epsilon_s}(N_D - N_A)$$

In terms of coefficient matrix A

$$[A]_{n\times n}[V]_{n\times 1} = [b]_{n\times 1}$$

Numerical methods to solve Ax=b

- Direct methods:
  1. Gauss elimination
  2. LU decomposition

- Indirect methods: Iterative
  1. Gauss Seidel

A has to be Strictly diagonal dominant

$$|a_{kk}| > \sum_{j=1, j\neq k}^{N} |a_{kj}|$$

L U decomposition method for AV=b

The operation count of LU decomposition is much lesser than Gauss elimination.

- decompose $A\ into\ L\ and\ U$ matrix

$$\mathbf{A} = \mathbf{LU} \rightarrow \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

- $\boldsymbol{Ax} = \boldsymbol{b} \ \rightarrow \ LUV = b$

- $\boldsymbol{Ly} = \boldsymbol{b}\ and\ \boldsymbol{Ux} = \boldsymbol{y}$ $\rightarrow$ these can be solved for V vector, by forward and backward substitutions respectively.

L U decomposition method for AV=b

$$\textbf{\textit{Forward sustitution}}$$
$$\boldsymbol{Ly = b}$$

$$y_1 = b_1$$
$$m_{21}y_1 + y_2 = b_2$$
$$m_{31}y_1 + m_{32}y_2 + y_3 = b_3$$

$$\textbf{\textit{Backward sustitution}}$$
$$\boldsymbol{Ux = y}$$

$$u_{11}x_1 + u_{12}x_2 + x_3 = y_1$$
$$u_{22}x_2 + u_{23}x_3 = y_2$$
$$u_{33}x_3 = y_3$$

L U factorization

$$\mathbf{A} = \begin{pmatrix} 4 & 3 & -1 \\ -2 & -4 & 5 \\ 1 & 2 & 6 \end{pmatrix} \longrightarrow \mathbf{A} = \mathbf{IA} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \underline{\mathbf{4}} & 3 & -1 \\ -2 & -4 & 5 \\ 1 & 2 & 6 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/4 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 & -1 \\ 0 & \underline{\mathbf{-2.5}} & 4.5 \\ 0 & 1.25 & 6.25 \end{pmatrix}$$

$R2 \to R2 + 2R1$
$R3 \to R3 - 4R1$

$$\mathbf{A} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/4 & -1/2 & 1 \end{pmatrix}}_{\boldsymbol{L}} \underbrace{\begin{pmatrix} 4 & 3 & -1 \\ 0 & -2.5 & 4.5 \\ 0 & 0 & 8.5 \end{pmatrix}}_{\boldsymbol{U}}$$

$R3 \to R3 + 0.5R2$

System of non-linear equations

- Charge neutrality equation: $N_D^+ - N_A^- = n - p$

$$N_D^+ = \frac{N_D}{1 + 2\exp\left(\frac{E_F - E_D}{kT}\right)}$$

$$N_A^- = \frac{N_A}{1 + 4\exp\left(\frac{E_A - E_F}{kT}\right)}$$

$$\frac{N_D}{1 + 2\exp\left(\frac{\boldsymbol{E_F} - E_D}{kT}\right)} - \frac{N_A}{1 + 4\exp\left(\frac{E_A - \boldsymbol{E_F}}{kT}\right)} = -N_v \exp\left(\frac{E_V - \boldsymbol{E_F}}{kT}\right) + N_c \exp\left(\frac{\boldsymbol{E_F} - E_c}{kT}\right)$$

Newton Raphson Method

- Numerical analysis to find the roots of real valued functions

x_o is the initial guess for a root of f.

$$x_1 = x_o - \frac{f(x_o)}{f'(x_o)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$