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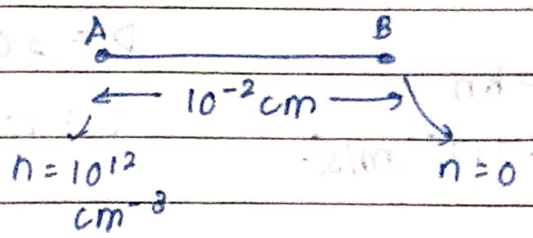
16D070010

Analytical Solutions.

Q1.a)



$D = 30 \, cm^2/s$
$\tau = 10^{-7}$ s

$$\frac{D\,d^2 n}{dx^2} = \frac{n}{\tau}$$

Let $n(x) = Ae^{\alpha x} + Be^{-\alpha x}$

$$D\alpha^2 (Ae^{\alpha x} + Be^{-\alpha x}) = \frac{1}{\tau}(Ae^{\alpha x} + Be^{-\alpha x})$$

$$\therefore \alpha^2 = \frac{1}{D\tau} \Rightarrow \alpha = \frac{1}{\sqrt{D\tau}} = \frac{1}{\sqrt{30 \times 10^{-7}}} = 577.35$$

Boundary conditions:

$n(0) = A + B = 10^{12}$

$n(10^{-2}) = Ae^{5.7735} + Be^{-5.7735} = 0.$

$\Rightarrow 321.669 A + (3.1088 \times 10^{-3}) B = 0.$

$\cancel{321.6569}$

$\cancel{321.6569 B} = 321.669 \times 10^{12}$

$\therefore B = 1.0000096 \times 10^{12}$

$\therefore A = -9.665 \times 10^{6}.$

$$n(x) = 10^6 \left[-9.665\, e^{577.35x} + 1.0000096 \times 10^6 \times e^{-577.35x} \right]$$

$$\therefore n(x) = 10^6 \left[-9.665\, e^{577.35x} + 1.0000096 \times 10^6 e^{-577.35x} \right]$$

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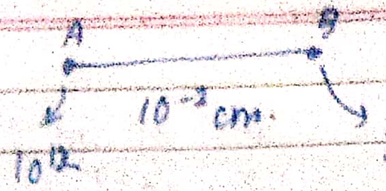
Q 1 (b)



$J = kn$

$k = 10^{3}$ cm/s.

$D = 30\ cm^2/s$

$\tau = 10^{-7}$.

$$\frac{D\, d^2 n}{dx^2} = \frac{n}{\tau}.$$

Let $n(x) = Ae^{\alpha x} + Be^{-\alpha x}$.

Substituting

$$D(A\alpha^2 e^{\alpha x} + B\alpha^2 e^{-\alpha x}) = \frac{Ae^{\alpha x} + Be^{-\alpha x}}{\tau}.$$

$$\therefore D\alpha^2 = \frac{1}{\tau} \Rightarrow \alpha = \frac{1}{\sqrt{D\tau}} = \frac{1}{\sqrt{30 \times 10^{-7}}} = 577.35$$

$n(0) = A + B = 10^{12}$.

$J = D\frac{dn}{dx} = kn \rightarrow$ at point B

$$\therefore D\alpha(Ae^{\alpha x} - Be^{-\alpha x})\Big|_{x=10^{-2}} = k(Ae^{\alpha x} + Be^{-\alpha x})\Big|_{x=10^{-2}}$$

$$1.732 \times 10^{4}\,(321.66A - (3.1088 \times 10^{-3})B)$$

$$= 10^{3}\,(321.66A + (3.1088 \times 10^{-3})B).$$

$\Rightarrow 5249.5A - 0.05695B = 0.$

$A = 1.085 \times 10^{7}$

$B = 9.99989 \times 10^{11}$

$$\therefore n(x) = 10^{7}\left[1.085e^{577.35x} + 9.99989 \times 10^{4} e^{-577.35x}\right]$$

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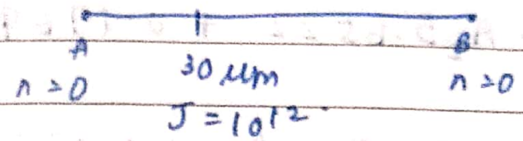
Q1(c).



$n_1(x) = A_1 e^{\alpha x} + B_1 e^{-\alpha x}$ $\qquad 0 \le x \le 30\,\mu m$

$n_2(x) = A_2 e^{\alpha x} + B_2 e^{-\alpha x}$ $\qquad 30 \le x \le 100\,\mu m.$

Boundary conditions.

① $n_1(0) \Rightarrow A_1 + B_1 = 0.$

② $n_2(100\,\mu m) \Rightarrow A_2 e^{\alpha 10^{-2}} + B_2 e^{-\alpha 10^{-2}} = 0.$

At 30 um boundary.

③ $$D \frac{\partial n_1}{\partial x}\Bigg|_{x = 30^-\mu m} - D \frac{\partial n_2}{\partial x}\Bigg|_{x = 30^+\mu m} = 10^{12}.$$

(sum of fluxes at 30^- um and 30^+ um)

④ $n_1(30\,\mu m) = n_2(30\,\mu m).$

note: $$D \frac{\partial^2 n}{\partial x^2} = \frac{n}{\tau}.$$

$$\alpha^2 D (A e^{\alpha x} + B e^{-\alpha x}) = \frac{1}{\tau} (A e^{\alpha x} + B e^{-\alpha x})$$

Hence, $$\alpha = \frac{1}{\sqrt{D\tau}} = \frac{1}{\sqrt{30 \times 10^{-7}}} = 577.35$$

Now, using boundary conditions:

⑤ $A_1 + B_1 = 0$

⑥ $321.66 A_2 + (3.1088 \times 10^{-3}) B_2 = 0$

⑦ $\alpha\,3D\left[A_1 5.6522 - 0.1769 B_1 - A_2 5.6522 + 0.1769 B_2\right] = 10^{12}$

⑧ $5.6522 A_1 + 0.1769 B_1 = 5.6522 A_2 + 0.1769 B_2$

Solving:

$A_1 = -B_1$

$A_2 = -9.6648 \times 10^{-6} B_2$.

$577.35 \times 30\left[-5.8291 B_1 + 0.176954 B_2\right] = 10^{12}$

$-5.4753 B_1 = 0.17684 B_2$.

$\Rightarrow B_2 = -30.9619 B_1$

$B_1 = -5.105 \times 10^6$.

$\Rightarrow A_1 = 5.105 \times 10^6$.

$\Rightarrow B_2 = 1.58 \times 10^8$.

$\Rightarrow A_2 = -1527.836$

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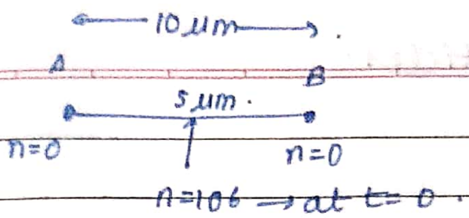
Q2.



$$\left(\frac{d^2}{dx^2} - \frac{1}{D}\frac{d}{dt}\right) n = \delta(x - x')$$

↳ source term delta function in space at time $t = 0$.

↓ Fourier transform.

$$-k^2\, c(k,t) - \frac{1}{D}\frac{dc(k,t)}{dt} = 0$$

$$c(k,t) = c(k, t=0)\exp(-k^2 D t)$$

initial condition:

$c(k,0) = c(k, t=0)$

$$c(k, t=0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(x - x')\exp(-ikx)\,dx$$

$$= \frac{\exp(-ikx')}{2\pi}$$

($x' = 5\ \mu m$ here)

Taking inverse fourier transform

$$c(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(ik(x - x'))\exp(-k^2 D t)\,dk$$

$$\Rightarrow c(x,t) = \frac{1}{(4\pi D t)^{0.5}}\exp\left(-(x - x')^2/4Dt\right)$$

$D = 10^{-4}\ cm^2/s$. ↳ (note at $t \to \infty$ $c(x,t) \to 0$).

$$\therefore c(x,t) = \frac{1}{(4\pi\, 10^{-4}\, t)^{0.5}}\exp\left(-(x - 5\times10^{-4})^2/(4\times10^{-4}\times t)\right)$$

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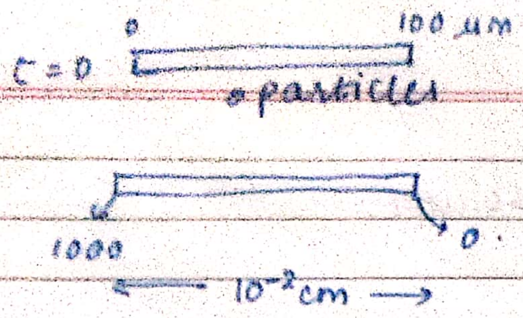
Q3.



infinite ~~time~~ source : $g(0^{+}, t) = M \quad \forall\ t > 0.$

$$c(x,t) = \frac{2}{(4\pi D t)^{0.5}} \int_0^{\infty} M \exp\left(-\frac{(x+x')^2}{4tD}\right) dx'$$

$$x + x' = p$$
$$dx' = dp$$

$$\therefore C(x,t) = \frac{2}{(4\pi D t)^{0.5}} \int_x^{\infty} M \exp\left(\frac{-p^2}{4Dt}\right) dp.$$

$$\frac{p}{\sqrt{4Dt}} = q$$
$$\frac{dp}{\sqrt{4Dt}} = dq.$$

$$c(x,t) = \frac{2}{\sqrt{\pi}} \int_{x/\sqrt{4Dt}}^{\infty} M \exp(-q^2)\, dq.$$

$$\because \operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt.$$

$$\therefore C(x,t) = M \left(\operatorname{erfc}\left(\frac{x}{\sqrt{4Dt}}\right)\right).$$

$$\therefore C(x,t) = M\left[\operatorname{erfc}\left(\frac{x}{\sqrt{4Dt}}\right)\right].$$

$M = 10^3$, $D = 30 \text{cm}^2/\text{s}$.