Semiconductor Devices

Carrier Statistics

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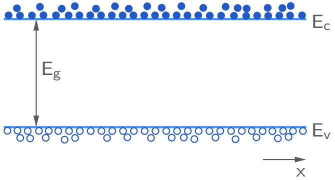
Carrier statistics in equilibrium



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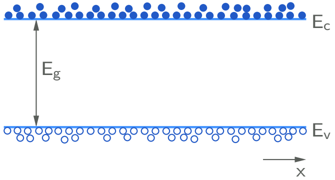

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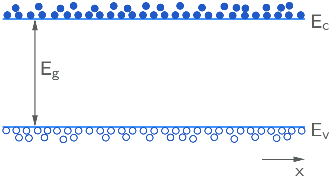

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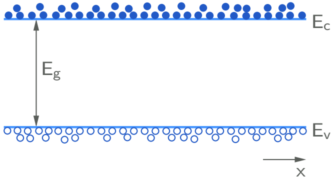

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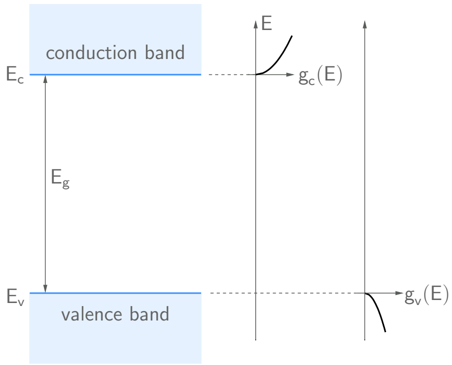
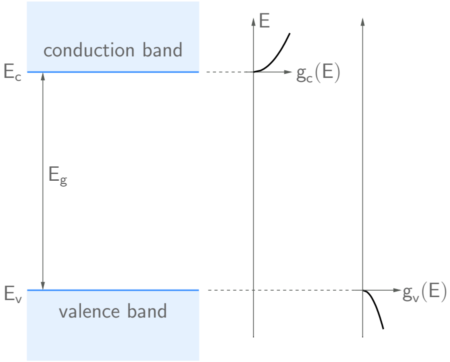
* We will first consider a semiconductor in equilibrium, i.e., without an external perturbation such as an applied voltage, a magnetic field, or optical illumination.

Electron density (n) in equilibrium



Electron density (n) in equilibrium

* The electron density depends on two factors:



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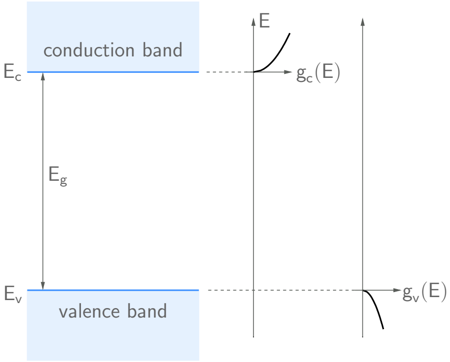

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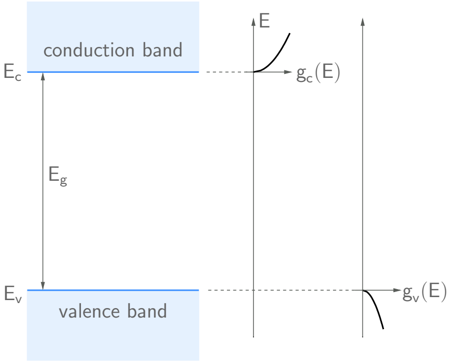

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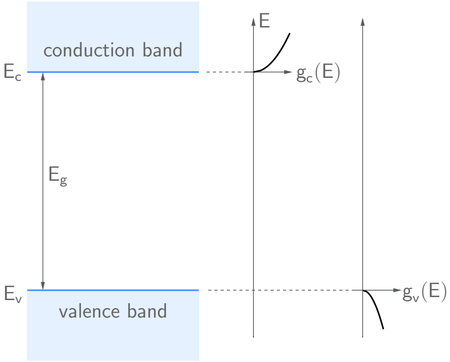

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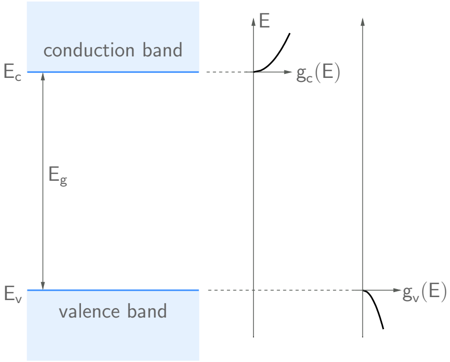
$g_c(E) = \dfrac{(m_n^*)^{3/2}\sqrt{2\,(E-E_c)}}{\pi^2\hbar^3}$, $E > E_c$, where

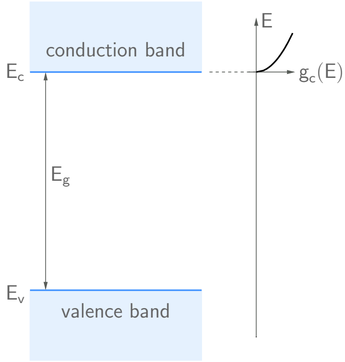
$m_n^* \equiv$ electron effective mass $= 1.18\, m_0$ for silicon at $T = 300\,$K,

$m_0 =$ free electron mass $= 9.1 \times 10^{-31}$ Kg,

$\hbar = h/2\pi$, with h (Planck constant) $= 6.63 \times 10^{-34}$ J-s.



Calculate the number of states N' between E_c and $E_c + 50\,\text{meV}$ for silicon at $T = 300\,\text{K}$.



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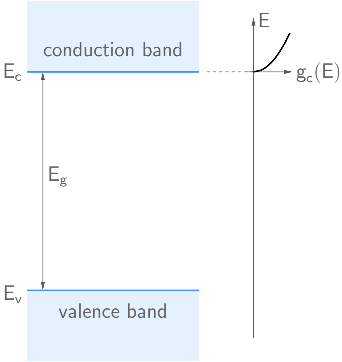

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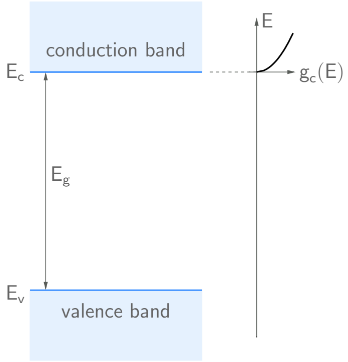

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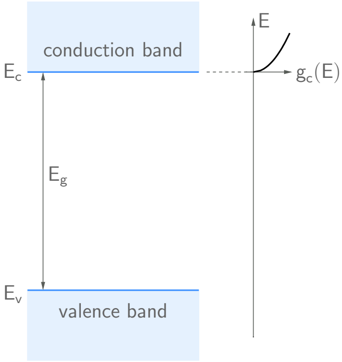

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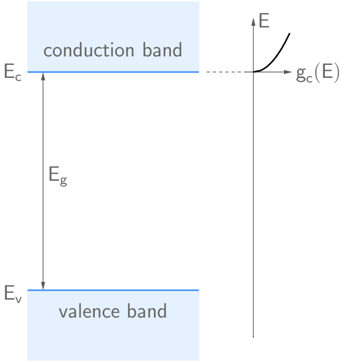

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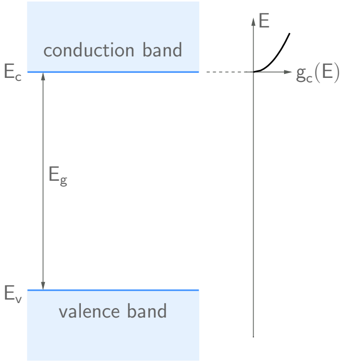

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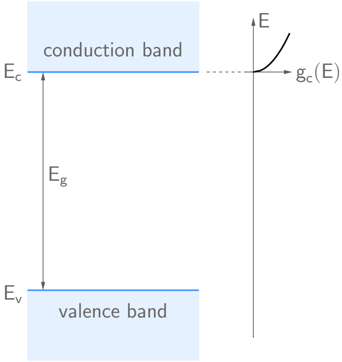
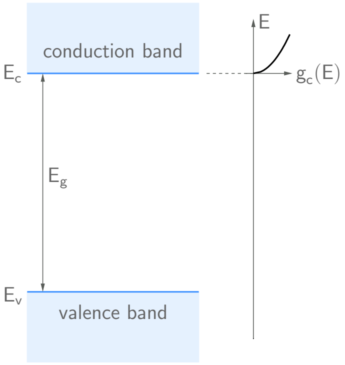
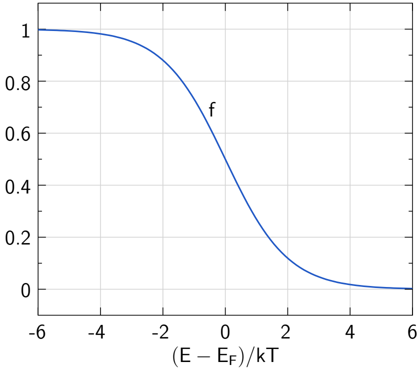


N' would be the number of electrons per unit volume (in the conduction band) if the states in the range $E_c < E < E_c + \Delta E$ were *all* occupied (and the rest of the states unoccupied). The real picture is different.

Electron density (n) in equilibrium





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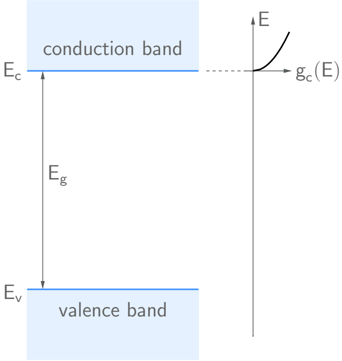

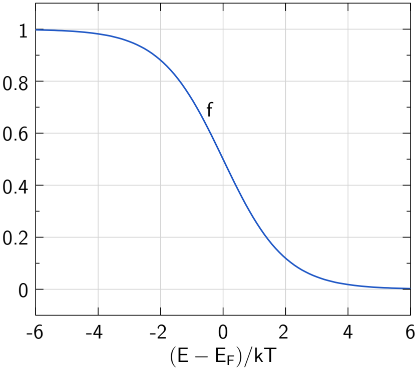

* The number of electrons in the interval E to $(E + dE)$ is not $g_c(E)dE$ but $g_c(E)f(E)dE$.

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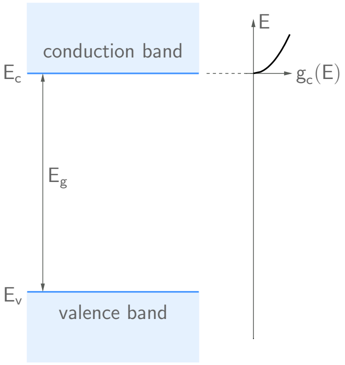

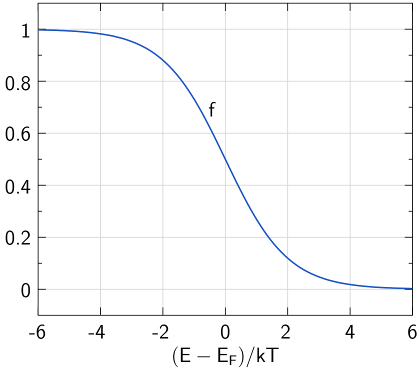

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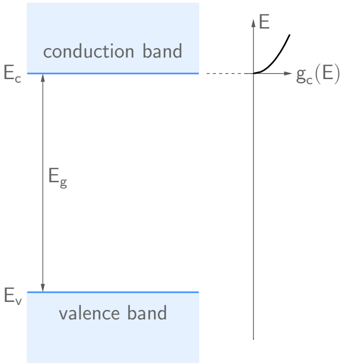

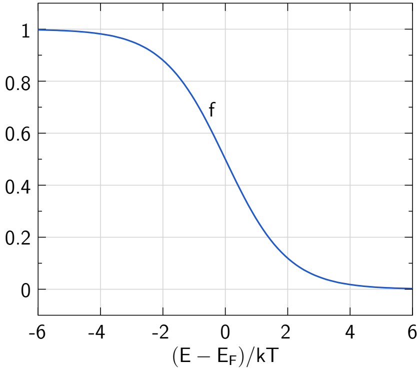

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* $f(E)$ is the probability that the state at E is occupied.
* The probability depends on the "Fermi level" E_F which typically lies in the forbidden gap.

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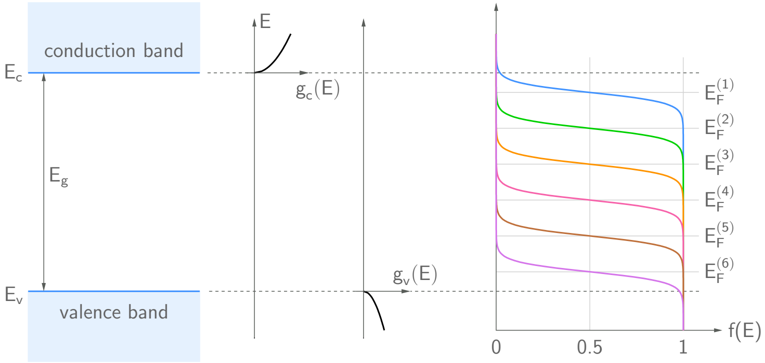

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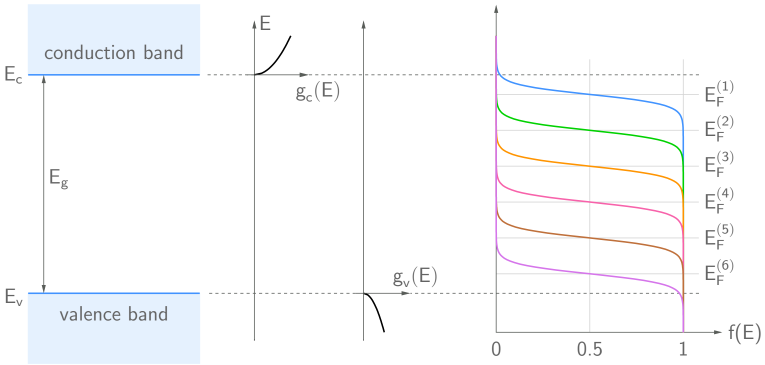

* A change in the Fermi level causes the probability function to shift, and therefore the carrier concentrations (n and p) change substantially with E_F.

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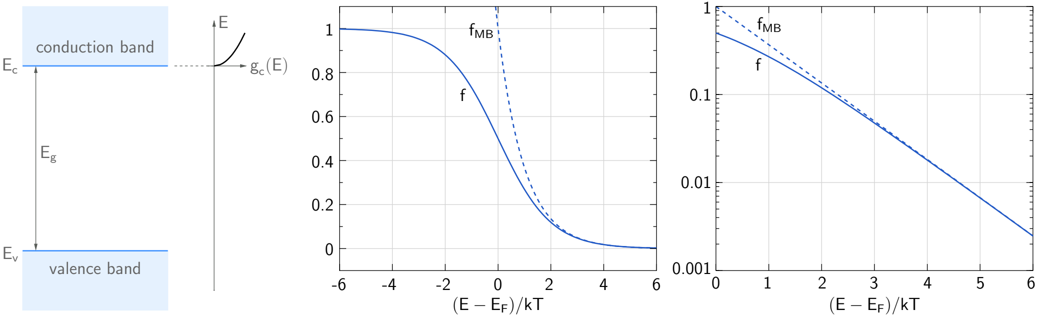

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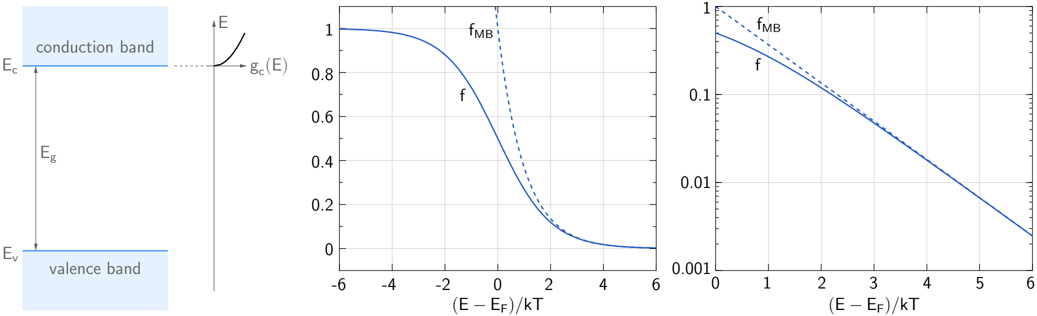

* $f(E)$ is given by the Fermi function: $f(E) = \dfrac{1}{1+e^{(E-E_F)/kT}}$ where $k = 1.38 \times 10^{-23}$ J/K (or 8.62×10^{-5} eV/K) is the Boltzmann constant.

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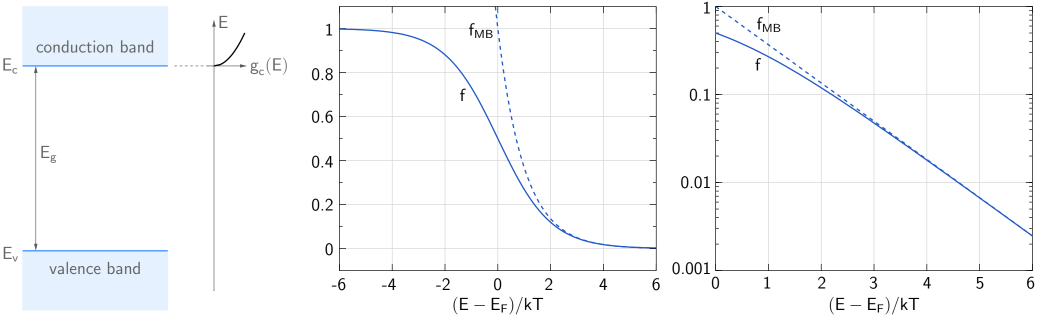

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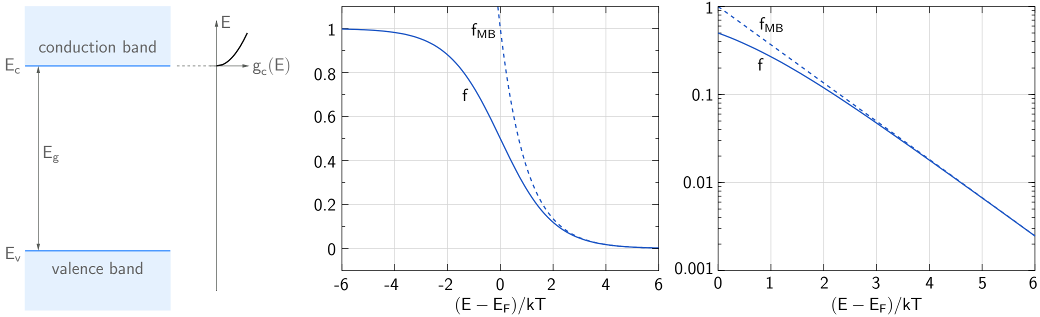

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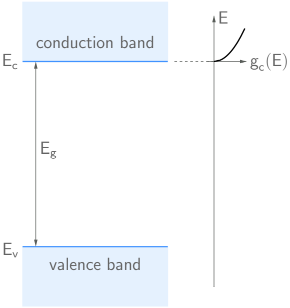

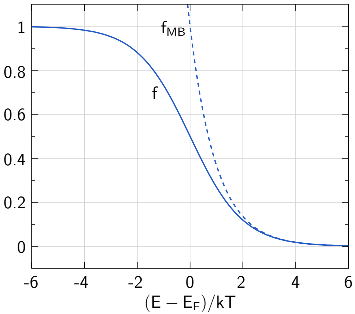

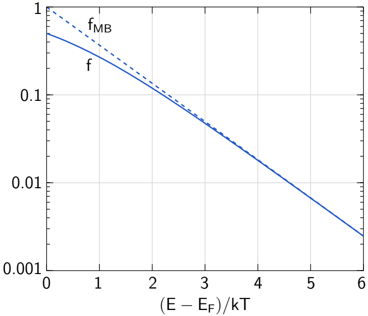

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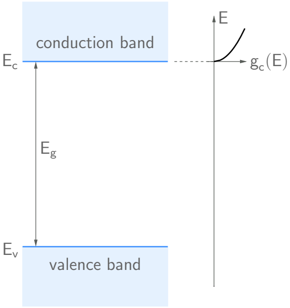

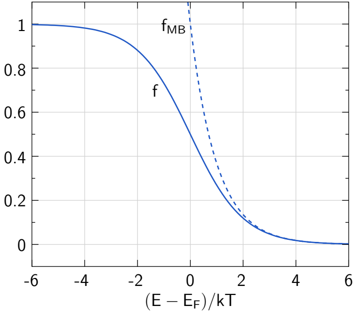

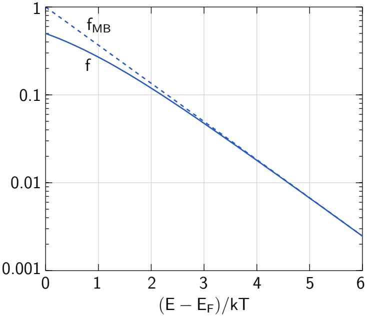

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* For $E - E_F > 3\,kT$, $e^{(E-E_F)/kT} > 20$, which is much larger than 1. We then have $f(E) \approx f_{\rm MB}(E) = e^{-(E-E_F)/kT}$, the Maxwell-Boltzmann function.

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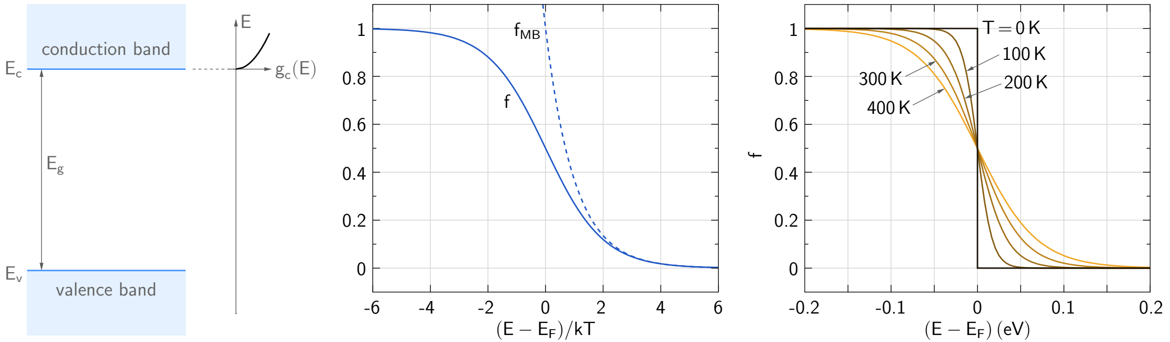

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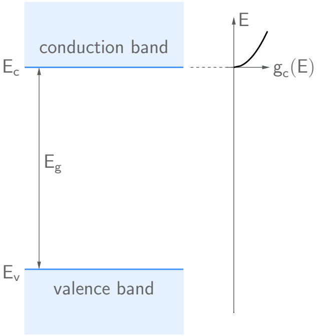

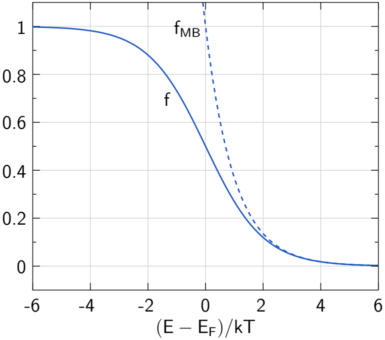

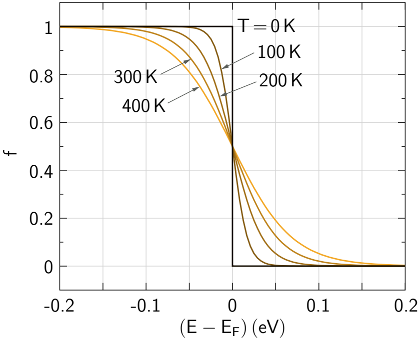

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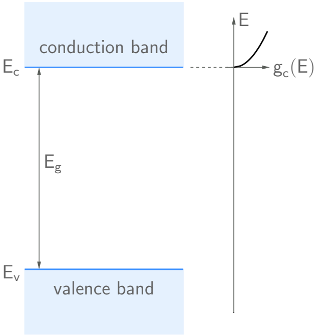

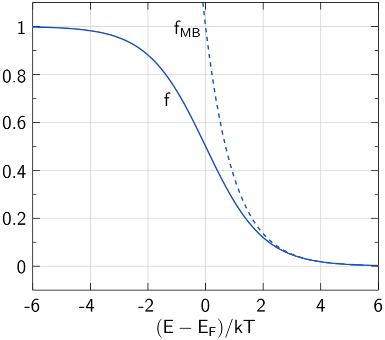

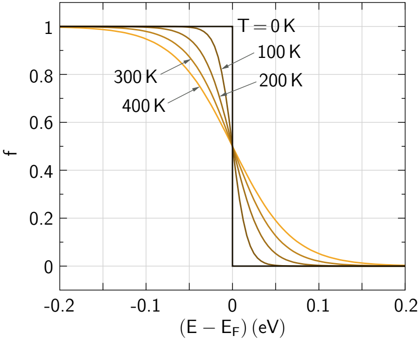

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  $\rightarrow$ $f(E)$ becomes broader as T increases.

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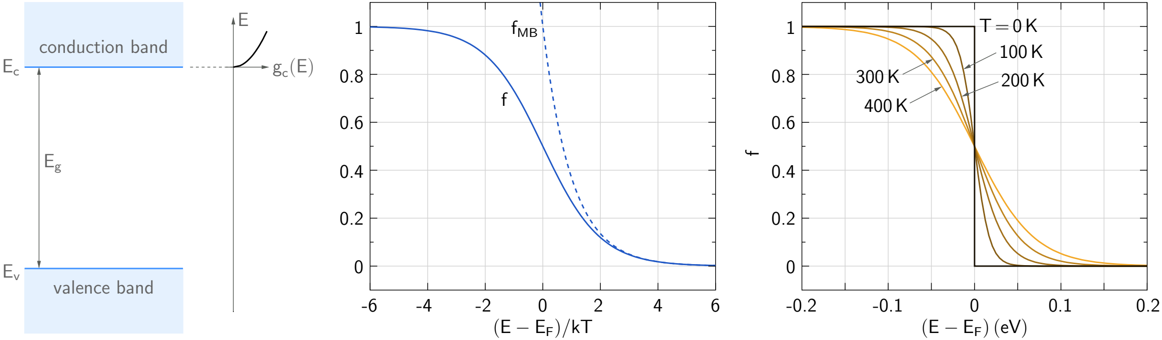

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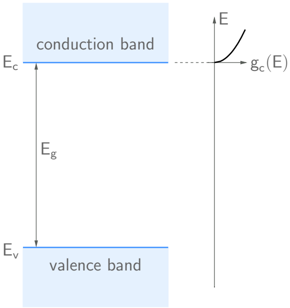
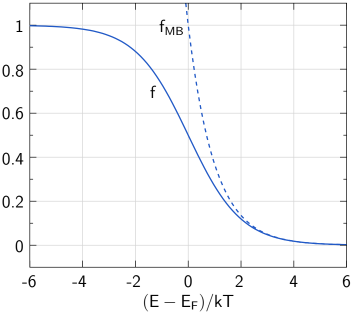
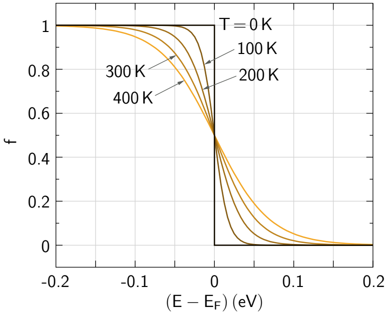
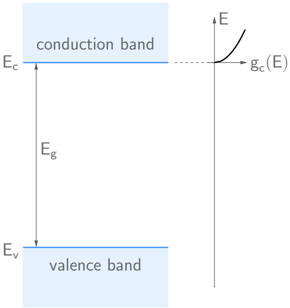
  High T: $e^{(E-E_F)/kT}$ varies slowly with E.
  $\rightarrow f(E)$ becomes broader as T increases.
- Because of the significant variation of $f(E)$ with temperature, we can expect the electron density to have a significant temperature dependence.

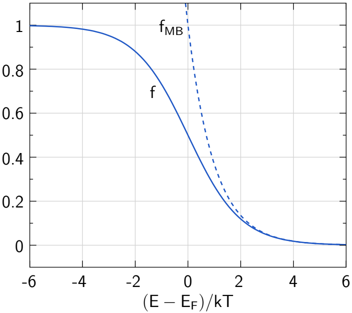


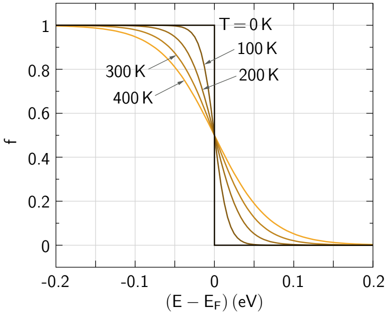




$$n = \int_{E_c}^{\infty} g_c(E)f(E)dE = \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2\hbar^3}\int_{E_c}^{\infty}\frac{\sqrt{E-E_c}}{1+e^{(E-E_F)/kT}}\,dE = \frac{(m_n^*)^{3/2}}{\pi^2\hbar^3}\,\sqrt{2}\,(kT)^{3/2}\int_0^{\infty}\frac{\eta^{1/2}}{1+e^{\eta-\eta_c}}\,d\eta$$

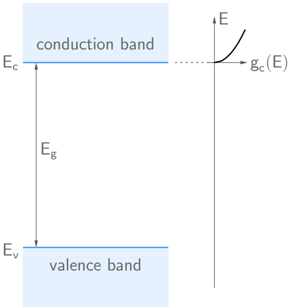


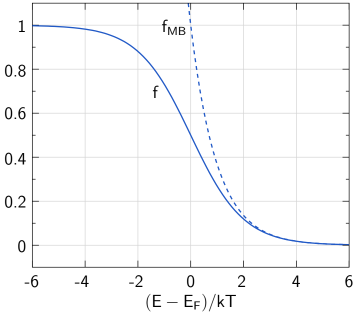


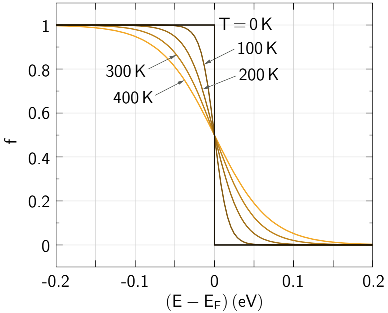


$$n = \int_{E_c}^{\infty} g_c(E) f(E) dE = \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2\hbar^3} \int_{E_c}^{\infty} \frac{\sqrt{E - E_c}}{1 + e^{(E-E_F)/kT}}\, dE = \frac{(m_n^*)^{3/2}}{\pi^2\hbar^3} \sqrt{2}\,(kT)^{3/2} \int_0^{\infty} \frac{\eta^{1/2}}{1 + e^{\eta - \eta_c}}\, d\eta$$

$$\rightarrow n = N_c \frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c) \text{ with } \eta_c = \frac{E_F - E_c}{kT}.$$



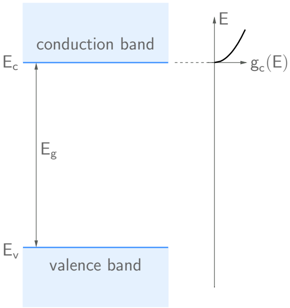


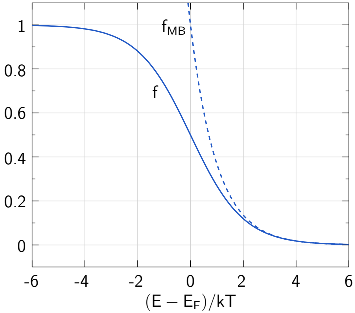


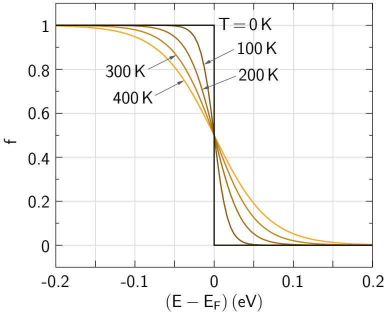
$$n = \int_{E_c}^{\infty} g_c(E) f(E) dE = \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2 \hbar^3} \int_{E_c}^{\infty} \frac{\sqrt{E - E_c}}{1 + e^{(E-E_F)/kT}} dE = \frac{(m_n^*)^{3/2}}{\pi^2 \hbar^3} \sqrt{2}\,(kT)^{3/2} \int_0^{\infty} \frac{\eta^{1/2}}{1 + e^{\eta - \eta_c}} d\eta$$

$$\rightarrow n = N_c \frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c) \text{ with } \eta_c = \frac{E_F - E_c}{kT}.$$

* $\mathcal{F}_{1/2}(\eta_c) = \int_0^{\infty} \frac{\eta^{1/2}}{1 + e^{\eta - \eta_c}} d\eta$, is called the “Fermi-Dirac integral of order 1/2.”



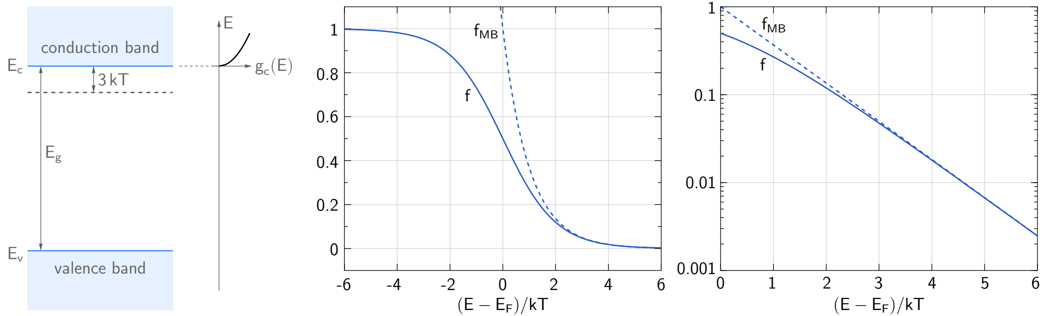




$$n = \int_{E_c}^{\infty} g_c(E) f(E) dE = \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2\hbar^3} \int_{E_c}^{\infty} \frac{\sqrt{E - E_c}}{1 + e^{(E-E_F)/kT}}\, dE = \frac{(m_n^*)^{3/2}}{\pi^2\hbar^3}\sqrt{2}\,(kT)^{3/2} \int_0^{\infty} \frac{\eta^{1/2}}{1 + e^{\eta - \eta_c}}\, d\eta$$

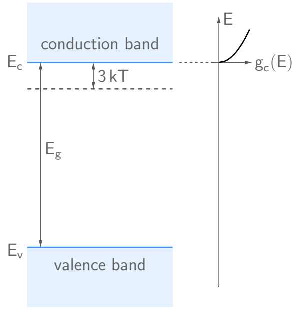
$$\rightarrow n = N_c \frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c) \text{ with } \eta_c = \frac{E_F - E_c}{kT}.$$

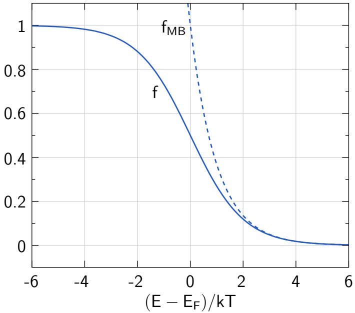
- $\mathcal{F}_{1/2}(\eta_c) = \int_0^{\infty} \frac{\eta^{1/2}}{1 + e^{\eta - \eta_c}}\, d\eta$, is called the "Fermi-Dirac integral of order 1/2."
- $N_c = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2}$ is called the "effective" density of states for the conduction band.

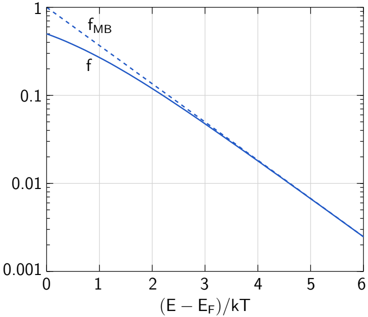


When the Fermi level is below $E_c - 3kT$, $f(E) \approx f_{\mathrm{MB}}(E)$, and

$$
\begin{aligned}
n &= \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2\hbar^3} \int_{E_c}^{\infty} \sqrt{E - E_c}\, \left(e^{-(E-E_F)/kT}\right) dE \\
&= \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2\hbar^3} (kT)^{3/2} \int_0^{\infty} \sqrt{\eta}\, e^{-\eta} \left(e^{-(E_c-E_F)/kT}\right) d\eta .
\end{aligned}
$$





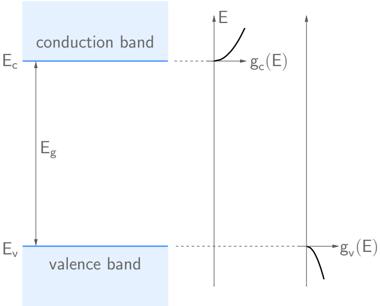


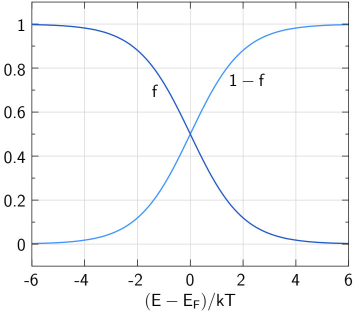
When the Fermi level is below $E_c - 3kT$, $f(E) \approx f_{\rm MB}(E)$, and

$$\begin{aligned} n &= \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2\hbar^3}\int_{E_c}^{\infty}\sqrt{E-E_c}\,\left(e^{-(E-E_F)/kT}\right)dE \\ &= \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2\hbar^3}\,(kT)^{3/2}\int_0^{\infty}\sqrt{\eta}\,e^{-\eta}\left(e^{-(E_c-E_F)/kT}\right)d\eta. \end{aligned}$$

$$\int_0^{\infty}\sqrt{\eta}\,e^{-\eta}\,d\eta = \frac{\sqrt{\pi}}{2} \rightarrow n = \frac{(m_n^*)^{3/2}\sqrt{2}}{\pi^2\hbar^3}\,(kT)^{3/2}\,\frac{\sqrt{\pi}}{2}\,e^{-(E_c-E_F)/kT} = N_c\,e^{-(E_c-E_F)/kT}.$$

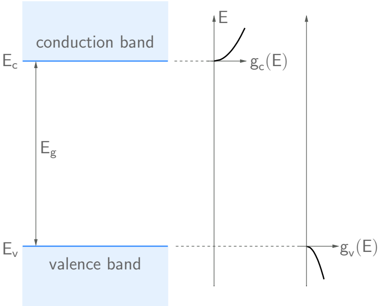
Hole density (p) in equilibrium

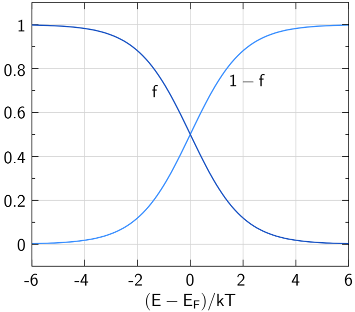




$$g_v(E) = \frac{(m_p^*)^{3/2}\sqrt{2\,(E_v - E)}}{\pi^2\hbar^3},\ \ E < E_v, \quad p = \int_{-\infty}^{E_v} g_v(E)\,(1 - f(E))\;dE.$$

Hole density (p) in equilibrium

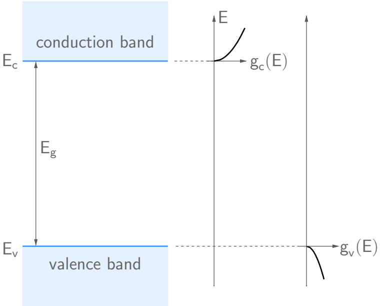


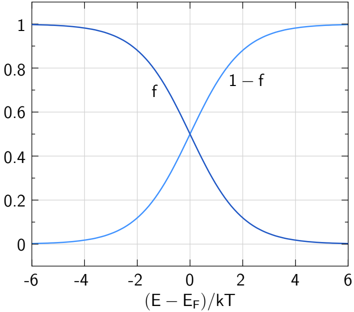


$$g_v(E) = \frac{(m_p^*)^{3/2}\sqrt{2\,(E_v - E)}}{\pi^2\hbar^3},\ \ E < E_v, \quad p = \int_{-\infty}^{E_v} g_v(E)\,(1 - f(E))\ dE.$$

$$p = \frac{(m_p^*)^{3/2}}{\pi^2\hbar^3}\int_{-\infty}^{E_v} \frac{\sqrt{2(E_v - E)}}{1 + e^{-(E-E_F)/kT}}\, dE$$

Hole density (p) in equilibrium



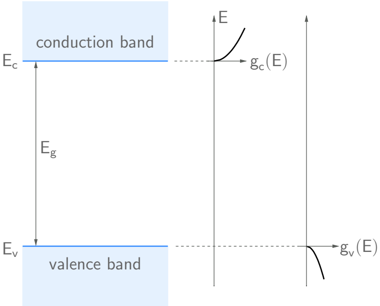


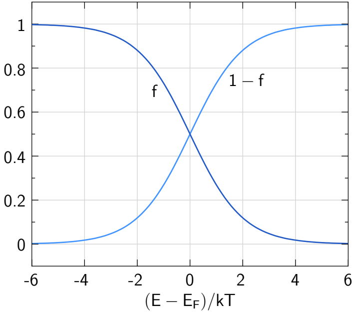
$$g_v(E) = \frac{(m_p^*)^{3/2}\sqrt{2\,(E_v - E)}}{\pi^2\hbar^3},\ \ E < E_v, \quad p = \int_{-\infty}^{E_v} g_v(E)\,(1 - f(E))\ dE.$$

$$p = \frac{(m_p^*)^{3/2}}{\pi^2\hbar^3}\int_{-\infty}^{E_v} \frac{\sqrt{2(E_v - E)}}{1 + e^{-(E-E_F)/kT}}\,dE$$

$$= \frac{(m_p^*)^{3/2}}{\pi^2\hbar^3}\,\sqrt{2}\,(kT)^{3/2}\int_0^{\infty} \frac{\eta^{1/2}}{1 + e^{\eta - \eta_v}}\,d\eta, \text{ with } \eta = \frac{E_v - E}{kT} \text{ and } \eta_v = \frac{E_v - E_F}{kT}$$

Hole density (p) in equilibrium



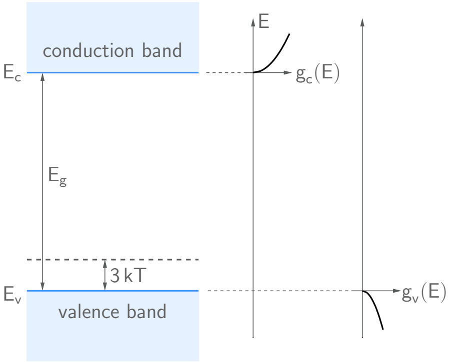


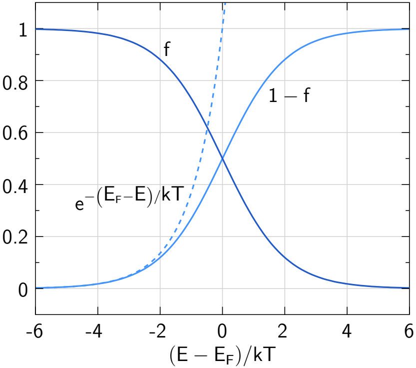
$$g_v(E) = \frac{(m_p^*)^{3/2}\sqrt{2\,(E_v - E)}}{\pi^2\hbar^3},\ E < E_v, \quad p = \int_{-\infty}^{E_v} g_v(E)\,(1 - f(E))\,dE.$$

$$p = \frac{(m_p^*)^{3/2}}{\pi^2\hbar^3}\int_{-\infty}^{E_v} \frac{\sqrt{2(E_v - E)}}{1 + e^{-(E-E_F)/kT}}\,dE$$

$$= \frac{(m_p^*)^{3/2}}{\pi^2\hbar^3}\sqrt{2}\,(kT)^{3/2}\int_0^{\infty} \frac{\eta^{1/2}}{1 + e^{\eta-\eta_v}}\,d\eta, \text{ with } \eta = \frac{E_v - E}{kT} \text{ and } \eta_v = \frac{E_v - E_F}{kT}$$

$$= N_v\,\frac{2}{\sqrt{\pi}}\,\mathcal{F}_{1/2}(\eta_v), \text{ where } N_v = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2} \text{ is the effective density of states for the valence band.}$$

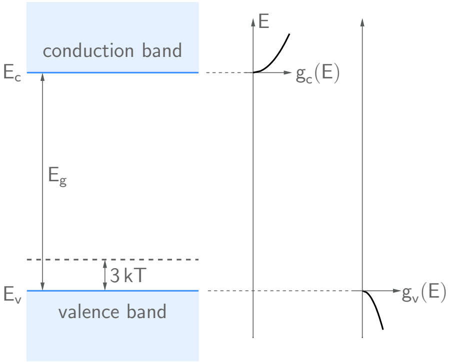


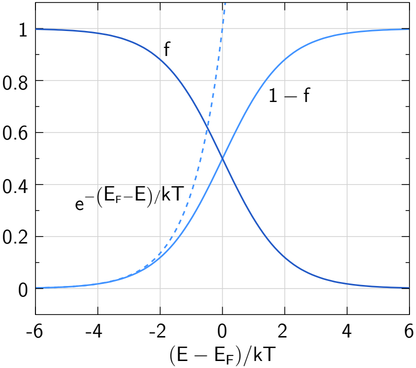


When $E_F > E_v + 3\,kT$, $1 - f(E)$ can be approximated using the Maxwell-Boltzmann function.

$1 - f(E) \approx e^{-(E_F-E)/kT} \rightarrow p = N_v\, e^{-(E_F-E_v)/kT}$.

Hole density (p) in equilibrium





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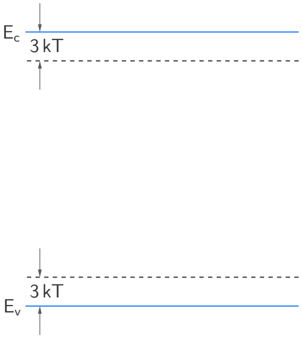


$\boxed{E_F > E_c - 3\,kT}$

(degenerate semiconductor)

$$n = N_c \frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c), \quad \eta_c = -\left(\frac{E_c - E_F}{kT}\right)$$

$$p = N_v\, e^{-(E_F - E_v)/kT}$$

$\boxed{E_v + 3\,kT < E_F < E_c - 3\,kT}$

(non-degenerate semiconductor)

$$n = N_c\, e^{-(E_c - E_F)/kT}$$

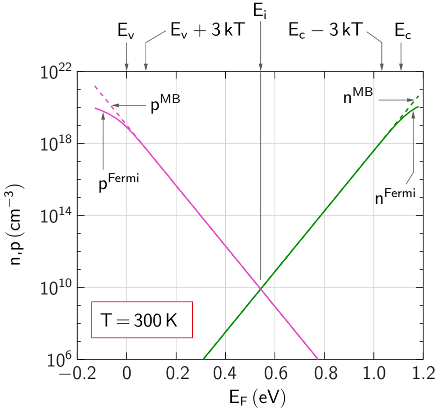
$$p = N_v\, e^{-(E_F - E_v)/kT}$$

$\boxed{E_F < E_v + 3\,kT}$

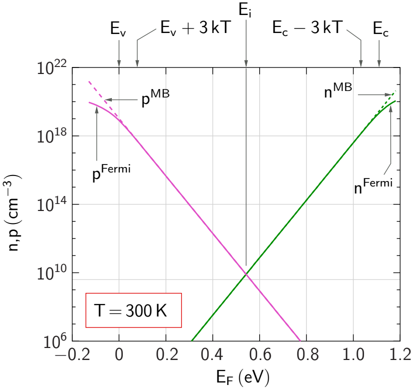
(degenerate semiconductor)

$$n = N_c\, e^{-(E_c - E_F)/kT}$$

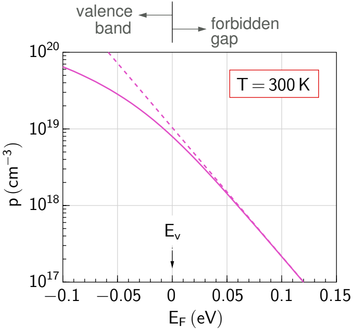
$$p = N_v \frac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_v), \quad \eta_v = -\left(\frac{E_F - E_v}{kT}\right)$$



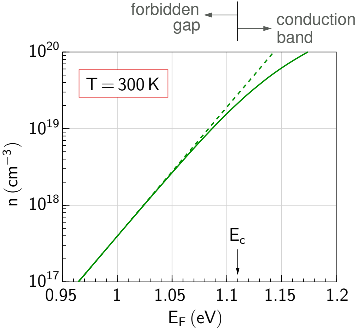
Electron and hole densities in silicon



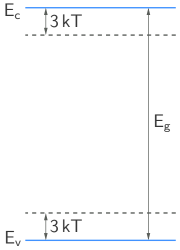
E_v
$E_v + 3\,kT$
E_i
$E_c - 3\,kT$
E_c
p^{MB}
n^{MB}
p^{Fermi}
n^{Fermi}
$T = 300\,K$
$n, p\ (cm^{-3})$
E_F (eV)
10^{22}
10^{18}
10^{14}
10^{10}
10^{6}
−0.2 0 0.2 0.4 0.6 0.8 1.0 1.2



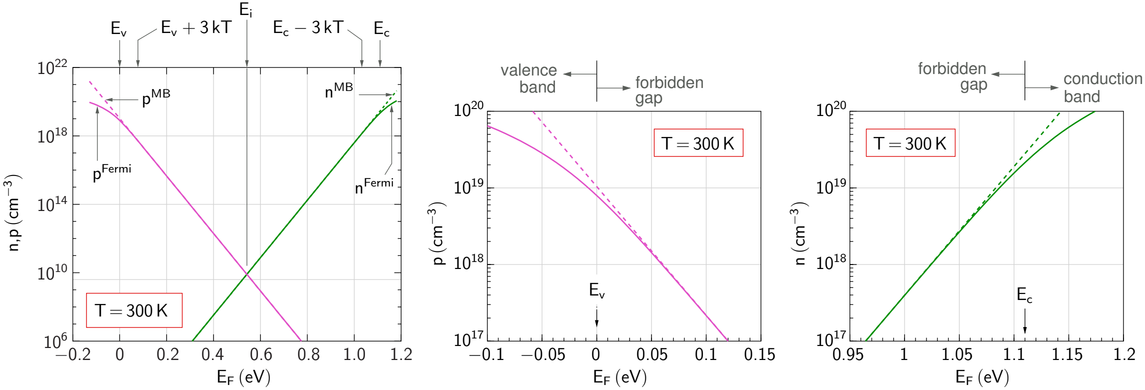
valence band
forbidden gap
$T = 300\,K$
E_v
$p\ (cm^{-3})$
E_F (eV)
10^{20}
10^{19}
10^{18}
10^{17}
−0.1 −0.05 0 0.05 0.1 0.15

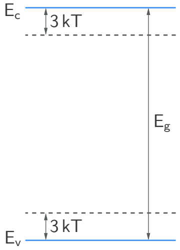


forbidden gap
conduction band
$T = 300\,K$
E_c
$n\ (cm^{-3})$
E_F (eV)
10^{20}
10^{19}
10^{18}
10^{17}
0.95 1 1.05 1.1 1.15 1.2



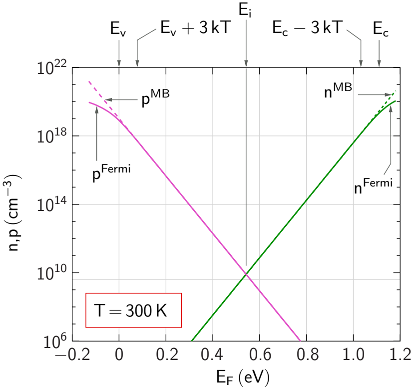
E_c
3 kT
E_g
3 kT
E_v

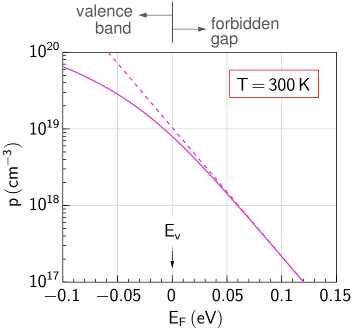


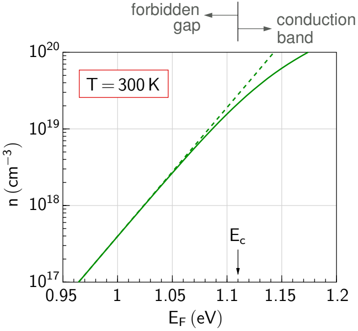


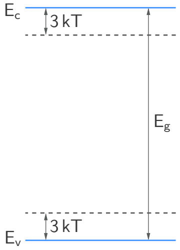
$* \quad n^{\text{Fermi}} = N_c \dfrac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c), \quad \eta_c = -\left(\dfrac{E_c - E_F}{kT}\right)$

$* \quad n^{\text{MB}} = N_c\, e^{-(E_c - E_F)/kT}$







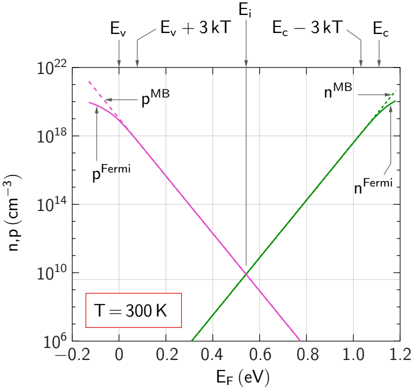


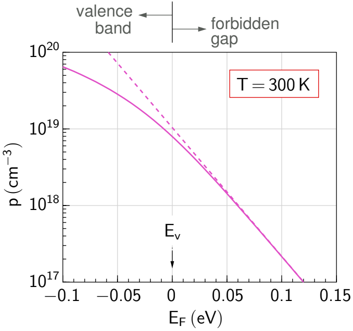
* $n^{\text{Fermi}} = N_c \dfrac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c), \quad \eta_c = -\left(\dfrac{E_c - E_F}{kT}\right)$

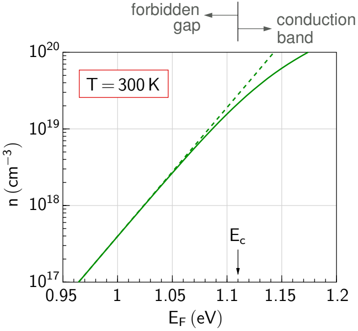
* $p^{\text{Fermi}} = N_v \dfrac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_v), \quad \eta_v = -\left(\dfrac{E_F - E_v}{kT}\right)$

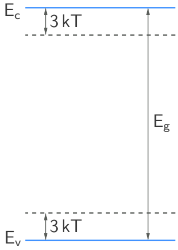
* $n^{\text{MB}} = N_c\, e^{-(E_c - E_F)/kT}$

* $p^{\text{MB}} = N_v\, e^{-(E_F - E_v)/kT}$



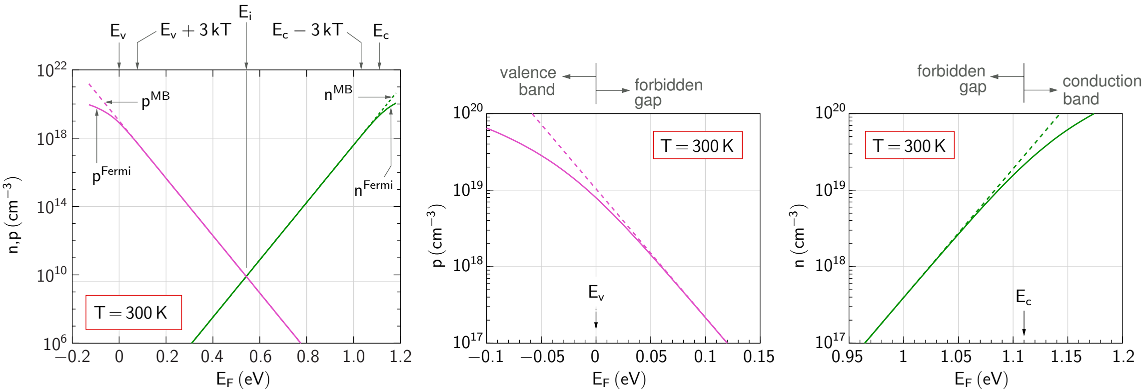


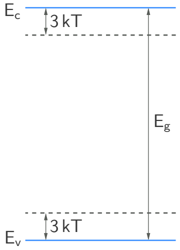




* $n^{\text{Fermi}} = N_c \dfrac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c), \;\; \eta_c = -\left(\dfrac{E_c - E_F}{kT}\right)$
* $p^{\text{Fermi}} = N_v \dfrac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_v), \;\; \eta_v = -\left(\dfrac{E_F - E_v}{kT}\right)$
* For $E_F < (E_c - 3\,kT)$, $n^{\text{MB}} \approx n^{\text{Fermi}}$

* $n^{\text{MB}} = N_c \, e^{-(E_c - E_F)/kT}$
* $p^{\text{MB}} = N_v \, e^{-(E_F - E_v)/kT}$





* $n^{\rm Fermi} = N_c \dfrac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_c), \quad \eta_c = -\left(\dfrac{E_c - E_F}{kT}\right)$

* $p^{\rm Fermi} = N_v \dfrac{2}{\sqrt{\pi}} \mathcal{F}_{1/2}(\eta_v), \quad \eta_v = -\left(\dfrac{E_F - E_v}{kT}\right)$

* For $E_F < (E_c - 3\,kT)$, $n^{\rm MB} \approx n^{\rm Fermi}$

* For $E_F > (E_v + 3\,kT)$, $p^{\rm MB} \approx p^{\rm Fermi}$

* $n^{\rm MB} = N_c e^{-(E_c - E_F)/kT}$

* $p^{\rm MB} = N_v e^{-(E_F - E_v)/kT}$

Carrier statistics in equilibrium: Example

For silicon at $T = 300\,\text{K}$ and in equilibrium (with $N_c = 2.8 \times 10^{19}\,\text{cm}^{-3}$, $N_v = 1.04 \times 10^{19}\,\text{cm}^{-3}$, $E_g = 1.12\,\text{eV}$),

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(a) Find E_F for which n and p are equal. This Fermi level is called E_i, the "intrinsic" Fermi level.

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(b) Obtain expressions for n and p in terms of $(E_i - E_F)$ (instead of $(E_c - E_F)$ and $(E_F - E_v)$). Assume non-degenerate conditions.

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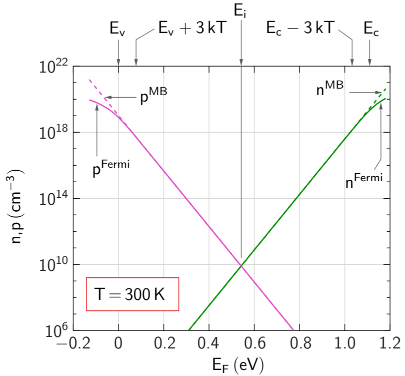
(b) Obtain expressions for n and p in terms of $(E_i - E_F)$ (instead of $(E_c - E_F)$ and $(E_F - E_v)$). Assume non-degenerate conditions.

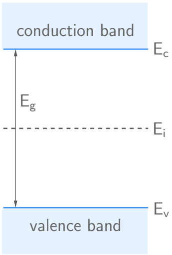
(c) Plot $g_c(E)\,f(E)$ and $g_v(E)\,[1 - f(E)]$ versus E for
$E_F = E_i + 20\,\text{meV}$,
$E_F = E_i + 10\,\text{meV}$,
$E_F = E_i$,
$E_F = E_i - 10\,\text{meV}$,
$E_F = E_i - 20\,\text{meV}$.

The condition $n = p$ is satisfied when E_F is about $(E_v + E_c)/2$.

$\rightarrow$ We can use MB statistics, i.e.,

$$N_c\, e^{-(E_c - E_F)/kT} = N_v\, e^{-(E_F - E_v)/kT}$$





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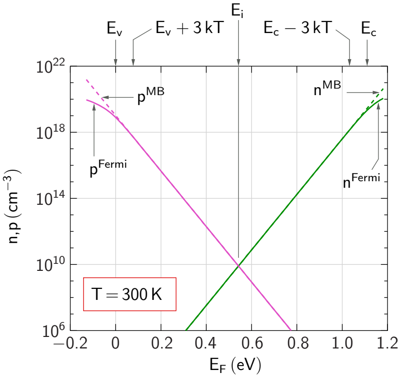

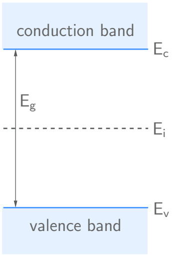

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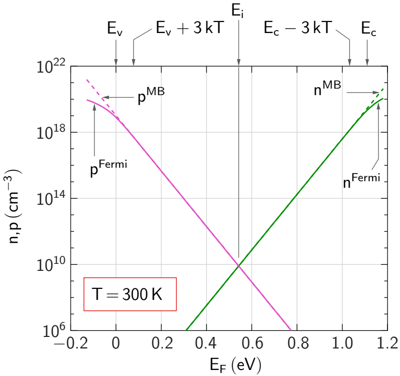

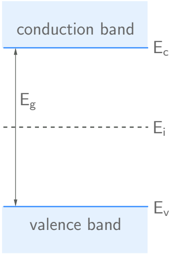

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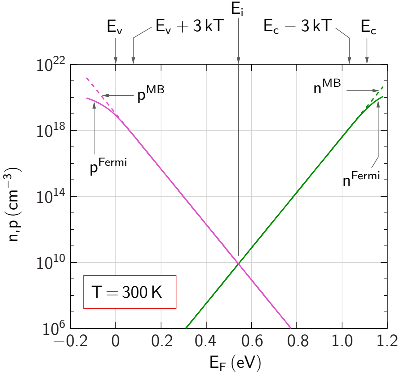
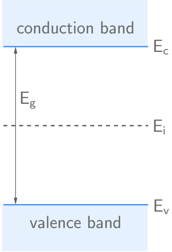
$$N_v/N_c = (m_p^*/m_n^*)^{3/2} \rightarrow E_F \equiv E_i = \frac{1}{2}(E_c + E_v) + \frac{3}{4} kT \log \frac{m_p^*}{m_n^*}.$$





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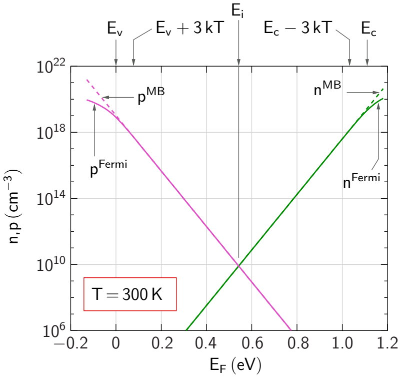
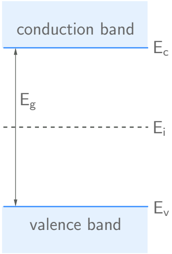
* The second term in the above equation is about -7.3 meV, i.e., the intrinsic Fermi level E_i is located 7.3 meV below the centre of the band gap.





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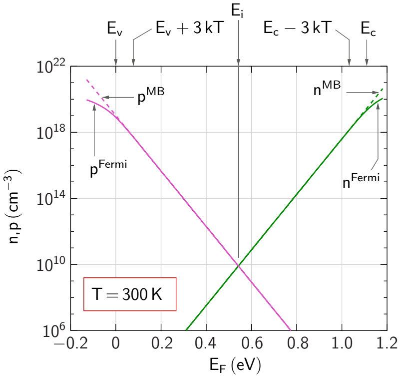
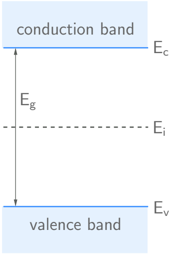
$$\rightarrow E_F = \frac{1}{2}(E_c + E_v) + \frac{kT}{2} \log \frac{N_v}{N_c}.$$

$$N_v/N_c = (m_p^*/m_n^*)^{3/2} \rightarrow E_F \equiv E_i = \frac{1}{2}(E_c + E_v) + \frac{3}{4} kT \log \frac{m_p^*}{m_n^*}.$$

* The second term in the above equation is about −7.3 meV, i.e., the intrinsic Fermi level E_i is located 7.3 meV below the centre of the band gap.
* If N_c and N_v were equal, E_i would be exactly at the centre of the band gap.





When $E_F = E_i$, we have $n = p \equiv n_i$, the "intrinsic carrier concentration."

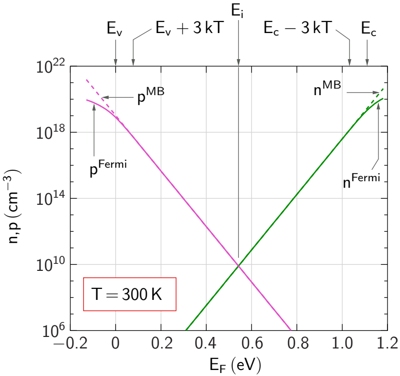
The actual electron concentration (for a given Fermi level E_F) can be written in terms of n_i as follows.

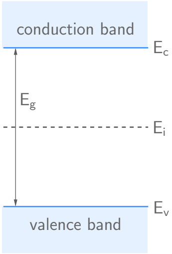
$$n = N_c\, e^{-(E_c - E_F)/kT}, \quad n_i = N_c\, e^{-(E_c - E_i)/kT}.$$

$$\rightarrow \frac{n}{n_i} = e^{(E_F - E_i)/kT} \rightarrow n = n_i\, e^{(E_F - E_i)/kT}.$$

Similarly, for the hole concentration p, we obtain

$$p = n_i\, e^{(E_i - E_F)/kT}.$$





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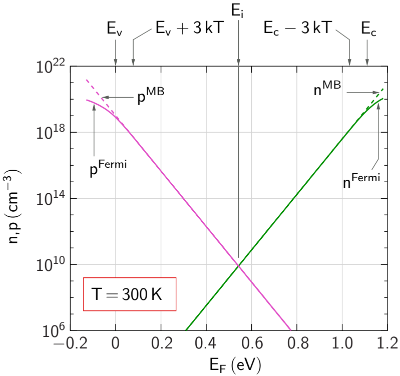

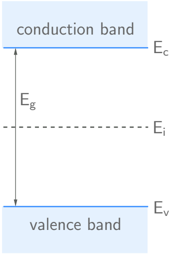

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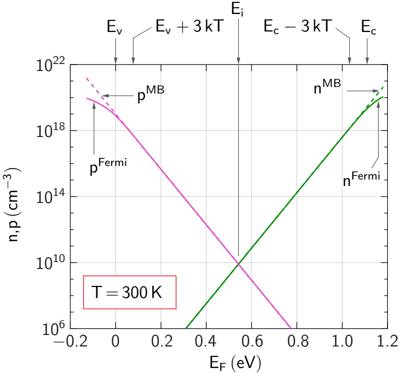

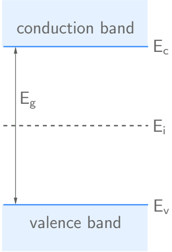

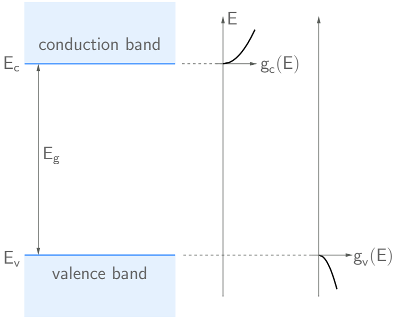

* $g_c(E) = \dfrac{(m_n^*)^{3/2}\sqrt{2\,(E - E_c)}}{\pi^2 \hbar^3}$
* $g_v(E) = \dfrac{(m_p^*)^{3/2}\sqrt{2\,(E_v - E)}}{\pi^2 \hbar^3}$
* $f(E) = \dfrac{1}{1 + e^{(E - E_F)/kT}}$
* $n = \displaystyle\int_{E_c}^{\infty} g_c(E) f(E) dE$
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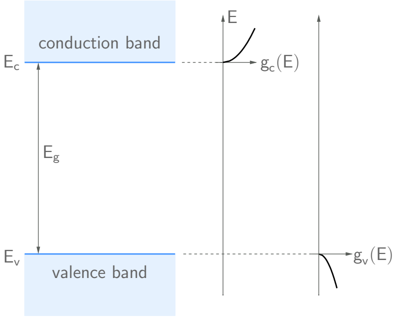

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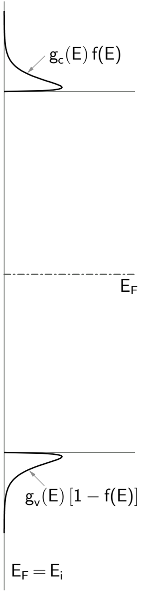

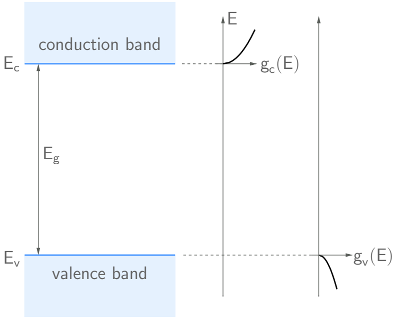

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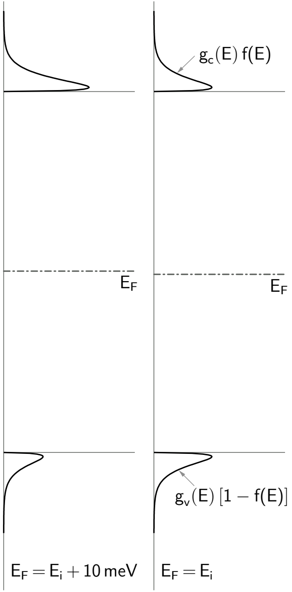

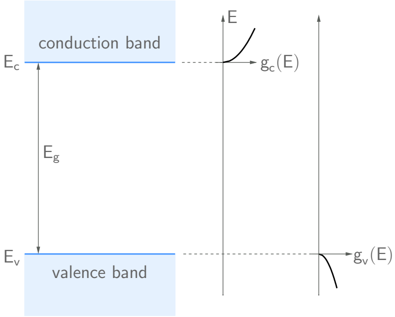

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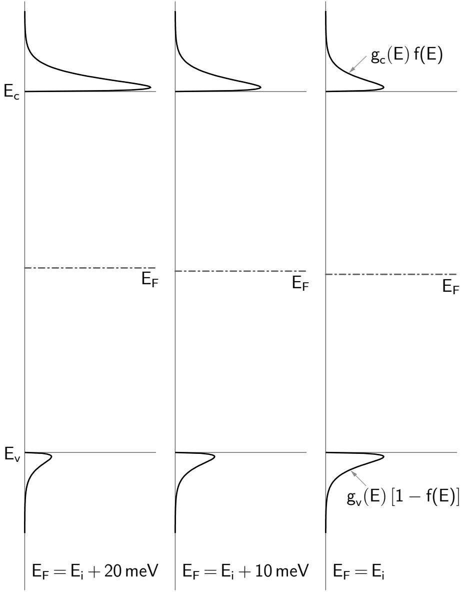

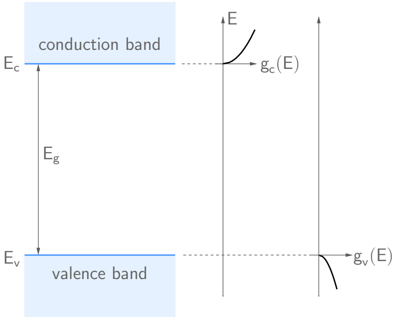

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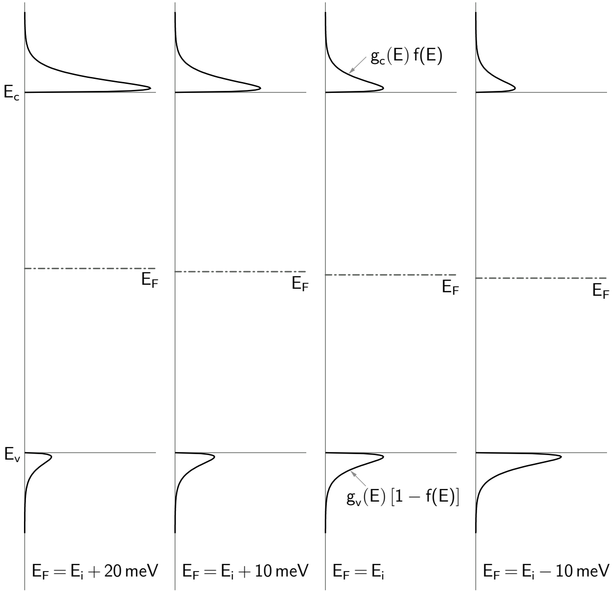

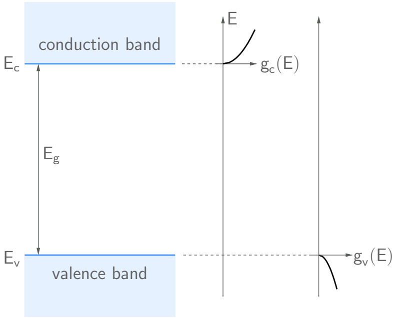

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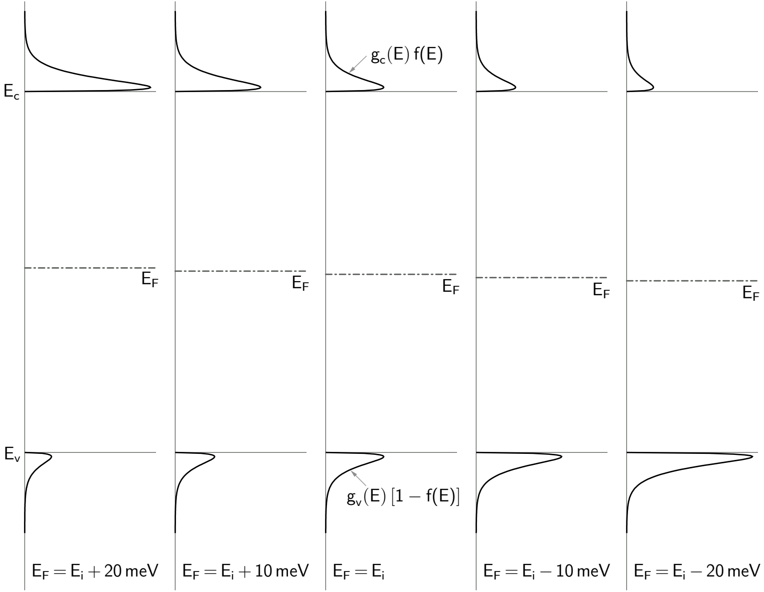

Temperature dependence of n_i

$$n = N_c\, e^{-(E_c - E_F)/kT}, \quad p = N_v\, e^{-(E_F - E_v)/kT}.$$

When $E_F = E_i$, $n = p = n_i$, i.e.,

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$$\rightarrow n\,p = n_i^2 = N_c N_v\, e^{-(E_c - E_v)/kT} = N_c N_v\, e^{-E_g/kT}$$

Temperature dependence of n_i

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$$\begin{aligned}
\rightarrow n_i &= \sqrt{N_c N_v}\, e^{-E_g/2kT} \\
&= 2\,(m_n^* m_p^*)^{3/4} \left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} e^{-E_g/2kT} \\
&\equiv K\, T^{3/2}\, e^{-E_g/2kT}
\end{aligned}$$

$$n = N_c\, e^{-(E_c - E_F)/kT}, \quad p = N_v\, e^{-(E_F - E_v)/kT}.$$

When $E_F = E_i$, $n = p = n_i$, i.e.,

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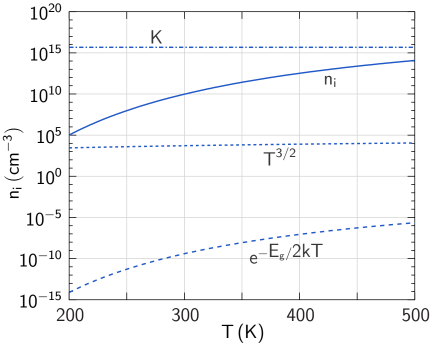
$$\rightarrow n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$
$$= 2\,(m_n^* m_p^*)^{3/4} \left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} e^{-E_g/2kT}$$
$$\equiv K\, T^{3/2}\, e^{-E_g/2kT}$$



Temperature dependence of n_i

$$n = N_c\, e^{-(E_c - E_F)/kT}, \quad p = N_v\, e^{-(E_F - E_v)/kT}.$$

When $E_F = E_i$, $n = p = n_i$, i.e.,

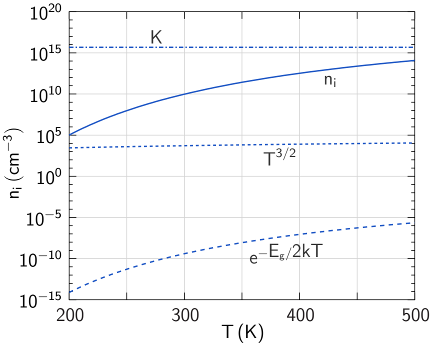
$$n = n_i = N_c\, e^{-(E_c - E_i)/kT}, \quad p = n_i = N_v\, e^{-(E_i - E_v)/kT}$$

$$\rightarrow n\, p = n_i^2 = N_c N_v\, e^{-(E_c - E_v)/kT} = N_c N_v\, e^{-E_g/kT}$$

$$\rightarrow n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$

$$= 2\,(m_n^* m_p^*)^{3/4} \left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} e^{-E_g/2kT}$$

$$\equiv K\, T^{3/2}\, e^{-E_g/2kT}$$



T (°C)	n_i (cm^{-3})
25	8.1×10^{9}
35	1.7×10^{10}
45	3.5×10^{10}
55	6.9×10^{10}
65	1.3×10^{11}
75	2.3×10^{11}

$$n = N_c\, e^{-(E_c-E_F)/kT}, \quad p = N_v\, e^{-(E_F-E_v)/kT}.$$

When $E_F = E_i$, $n = p = n_i$, i.e.,

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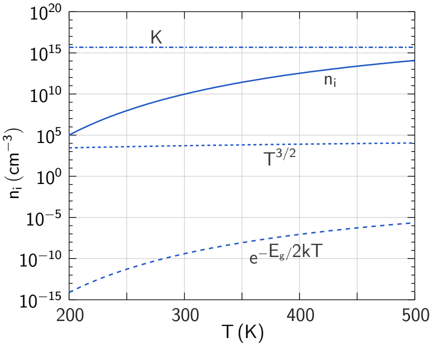
For silicon, near room temperature, n_i nearly doubles with every 10 °C rise in temperature.



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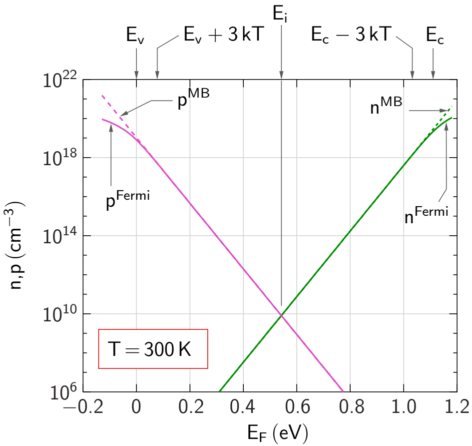
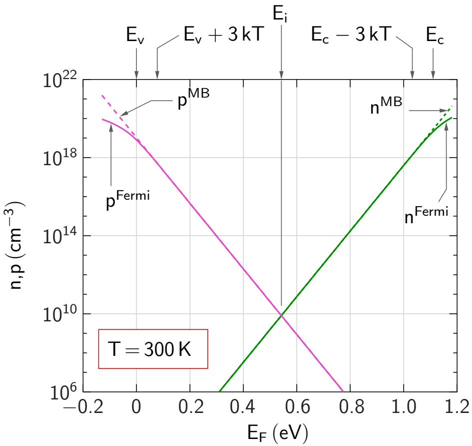
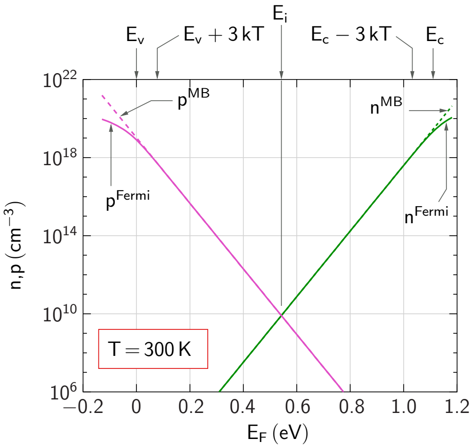


E_i
E_v
E_v + 3 kT
E_c − 3 kT
E_c
p^MB
n^MB
p^Fermi
n^Fermi
T = 300 K
10^22
10^18
10^14
10^10
10^6
−0.2
0
0.2
0.4
0.6
0.8
1.0
1.2
n,p (cm^−3)
E_F (eV)

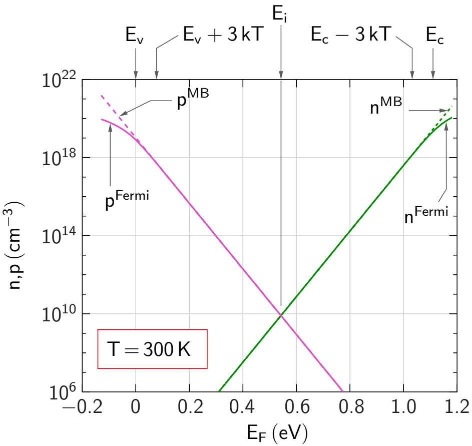
* So far, we have assumed a certain E_F (with respect to E_c and E_v) and obtained n and p.



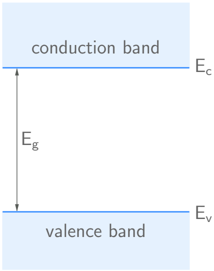
* So far, we have assumed a certain E_F (with respect to E_c and E_v) and obtained n and p.
* In practice, we only have information such as N_c, N_v, E_g, T, and doping densities (N_a and N_d).

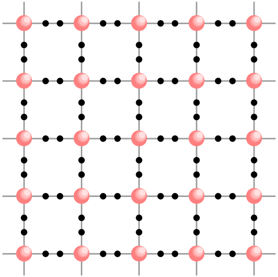


* So far, we have assumed a certain E_F (with respect to E_c and E_v) and obtained n and p.
* In practice, we only have information such as N_c, N_v, E_g, T, and doping densities (N_a and N_d).
* We now want to consider the reverse problem of finding E_F (and n, p), given the above data.



Charge considerations in equilibrium: intrinsic semiconductor

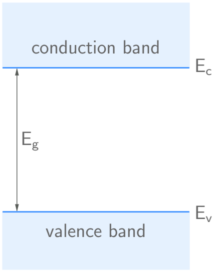


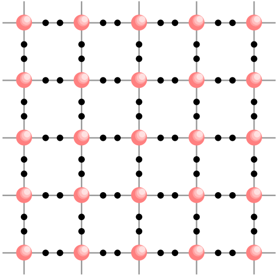


$T = 0\,\mathrm{K}$

* At 0 K, the positive charge due to the atomic cores balances the negative charge due to valence electrons.

Charge considerations in equilibrium: intrinsic semiconductor

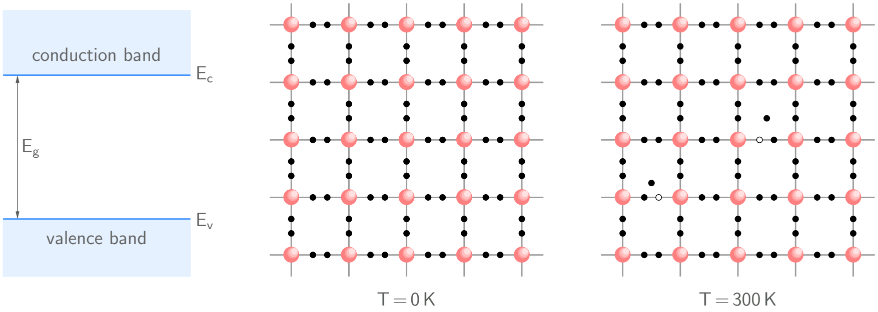




$T = 0\,K$

* At 0 K, the positive charge due to the atomic cores balances the negative charge due to valence electrons.
* As temperature increases, some of the valence electrons become free, i.e., they enter the conduction band, leaving behind positively charged holes in the valence band.

Charge considerations in equilibrium: intrinsic semiconductor



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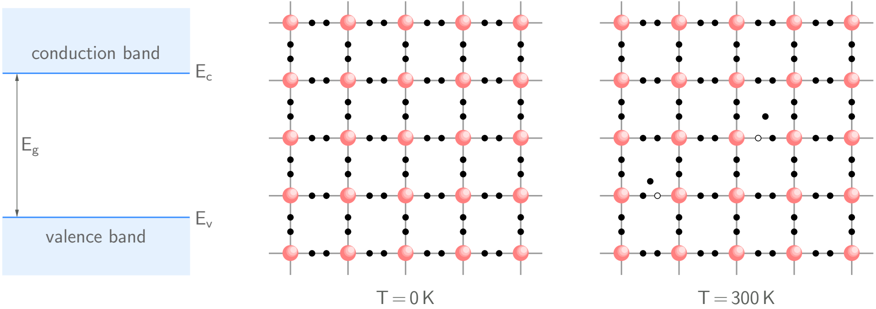
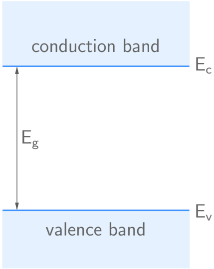
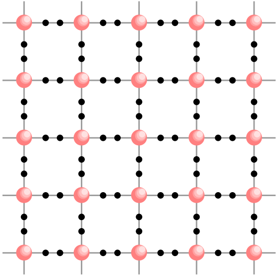


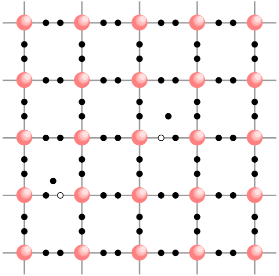
* At 0 K, the positive charge due to the atomic cores balances the negative charge due to valence electrons.
* As temperature increases, some of the valence electrons become free, i.e., they enter the conduction band, leaving behind positively charged holes in the valence band.
* The number of electrons in the conduction band is equal to the number of holes in the valence band.

Charge considerations in equilibrium: intrinsic semiconductor





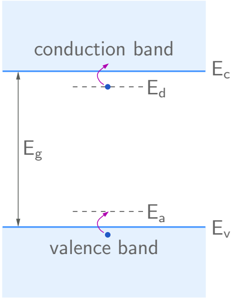
T = 0 K

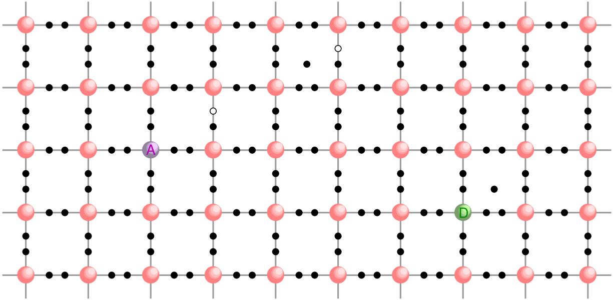


T = 300 K

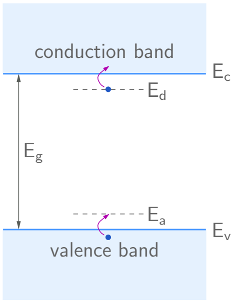
* At 0 K, the positive charge due to the atomic cores balances the negative charge due to valence electrons.
* As temperature increases, some of the valence electrons become free, i.e., they enter the conduction band, leaving behind positively charged holes in the valence band.
* The number of electrons in the conduction band is equal to the number of holes in the valence band.
* Also, their densities must be equal since the electrostatic potential is constant (no electric field) $\rightarrow n = p$.

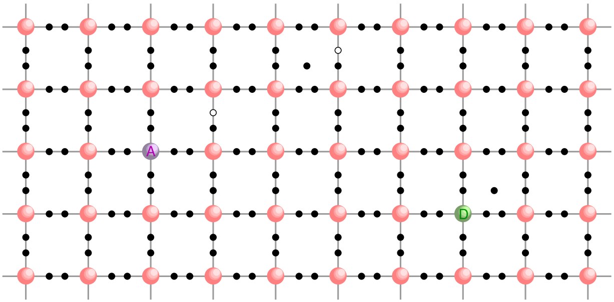
Charge considerations in equilibrium: doped semiconductor





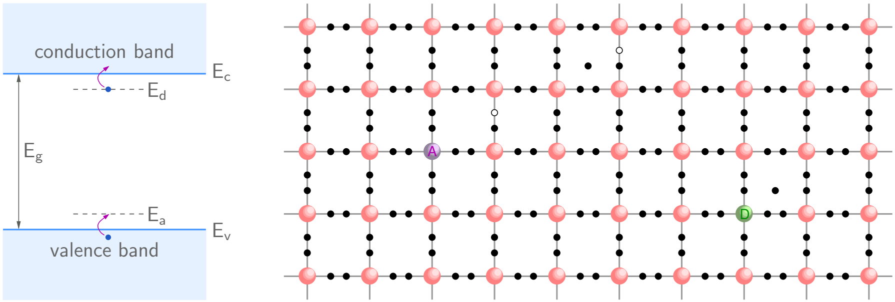
Charge considerations in equilibrium: doped semiconductor





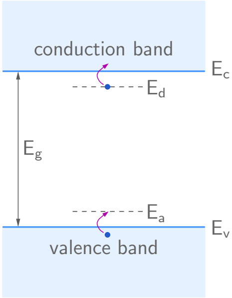
* When there are donor or acceptor atoms in the lattice, we have the following charged species.

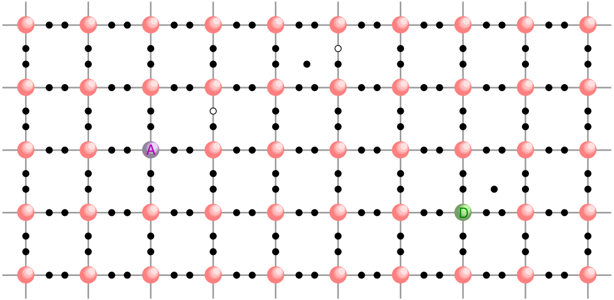
Charge considerations in equilibrium: doped semiconductor



* When there are donor or acceptor atoms in the lattice, we have the following charged species.
  - electrons in the conduction band (density n)

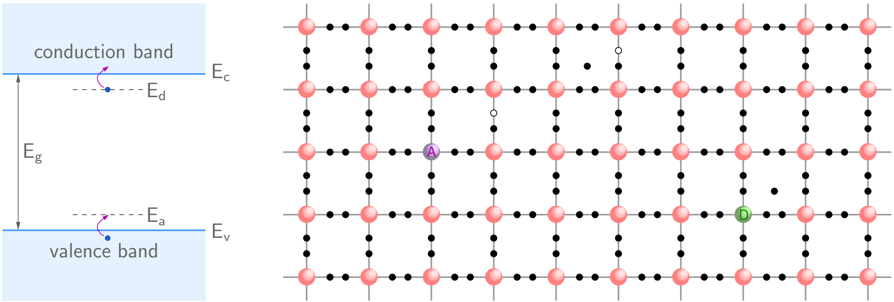
Charge considerations in equilibrium: doped semiconductor





* When there are donor or acceptor atoms in the lattice, we have the following charged species.
  - electrons in the conduction band (density n)
  - holes in the valence band (density p)

Charge considerations in equilibrium: doped semiconductor



* When there are donor or acceptor atoms in the lattice, we have the following charged species.
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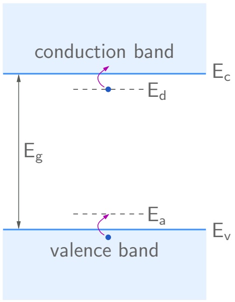
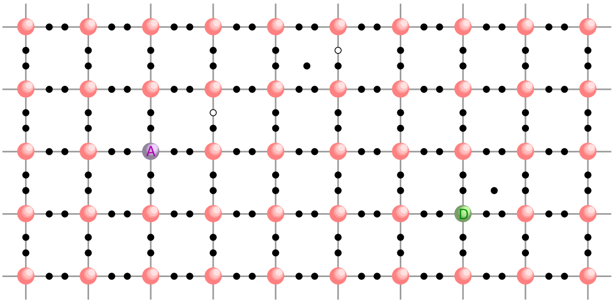
  - ionised donor atoms (density N_d^+)

Charge considerations in equilibrium: doped semiconductor





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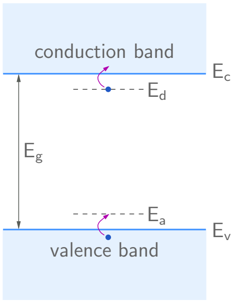
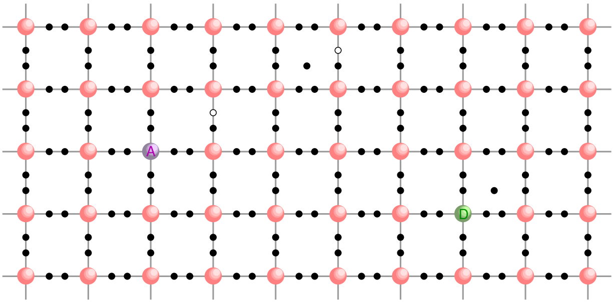
Charge considerations in equilibrium: doped semiconductor





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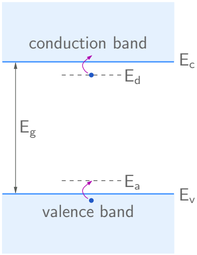
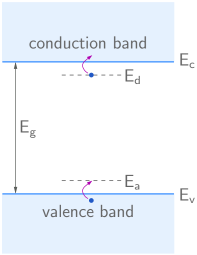
  - electrons in the conduction band (density n)
  - holes in the valence band (density p)
  - ionised donor atoms (density N_d^+)
  - ionised acceptor atoms (density N_a^-)
* If the doping densities (N_a or N_d or both) are uniform in space, charge neutrality in equilibrium requires $-qn + qp + qN_d^+ - qN_a^- = 0 \rightarrow n + N_a^- = p + N_d^+$.

Charge considerations in equilibrium



Charge considerations in equilibrium



* Let the donor density be N_d. Some of the donor atoms donate their electrons and acquire a net positive charge; the others remain neutral. $\rightarrow N_d = N_d^+ + N_d^0$.

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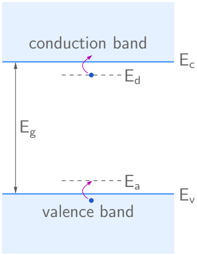

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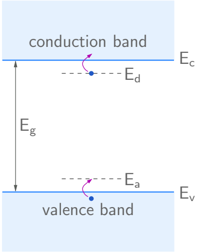

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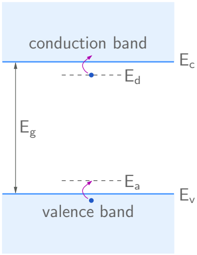
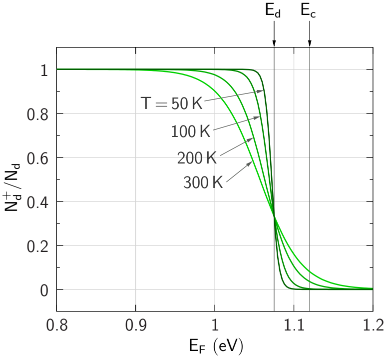
* The ratios N_d^+/N_d and N_a^-/N_a are given by $\dfrac{N_d^+}{N_d} = \dfrac{1}{1 + 2\, e^{(E_F - E_d)/kT}}$, $\dfrac{N_a^-}{N_a} = \dfrac{1}{1 + 4\, e^{(E_a - E_F)/kT}}$.

Charge considerations in equilibrium





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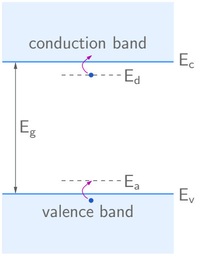
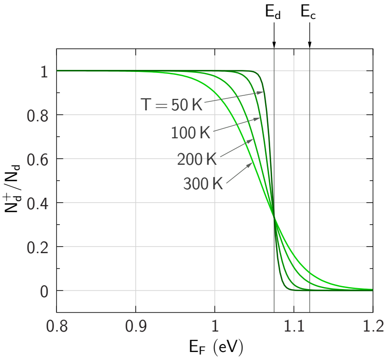
Charge considerations in equilibrium





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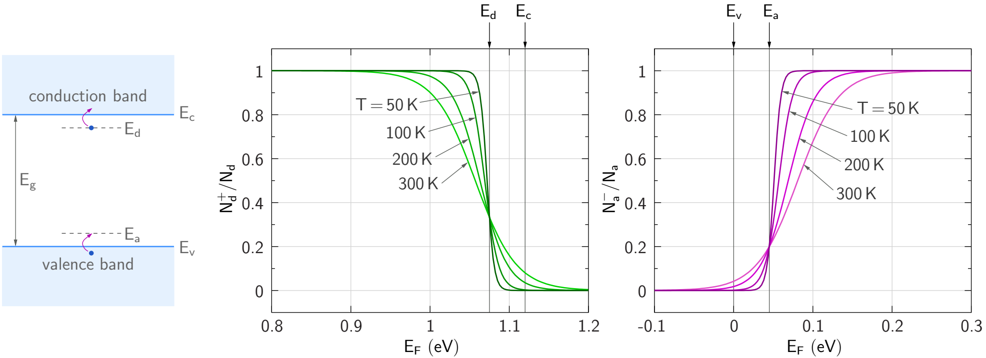
* $N_d^+/N_d \rightarrow 1$ if E_F is sufficiently below E_d.

Charge considerations in equilibrium



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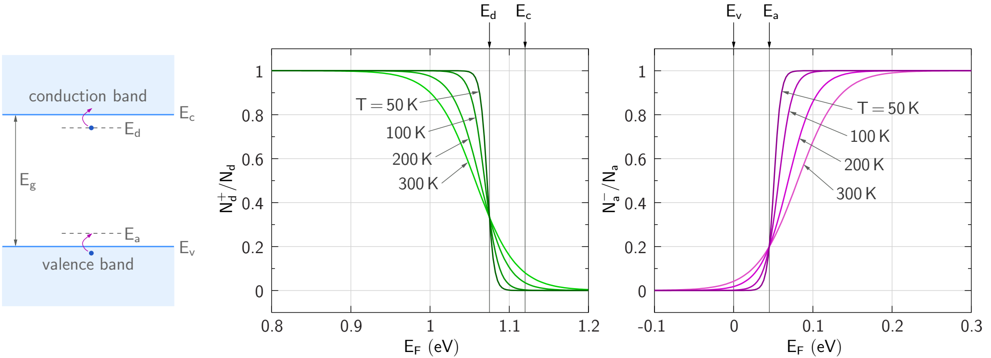
Charge considerations in equilibrium



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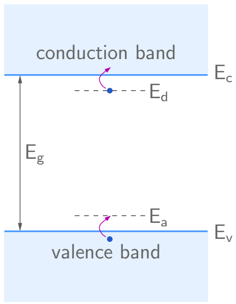
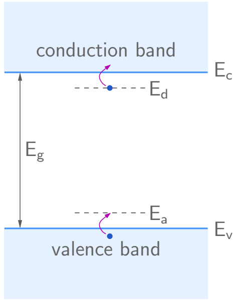
* $N_d^+/N_d \rightarrow 1$ if E_F is sufficiently below E_d.
* $N_a^-/N_a \rightarrow 1$ if E_F is sufficiently above E_a.

Calculation of n and p in equilibrium

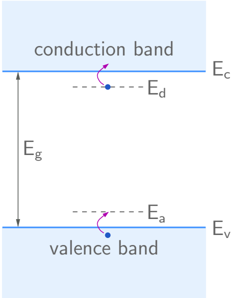


Calculation of n and p in equilibrium



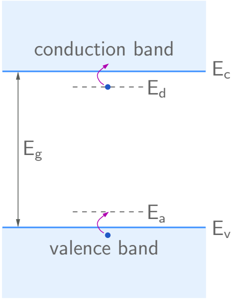
* $n + N_a^- = p + N_d^+$.

Calculation of n and p in equilibrium

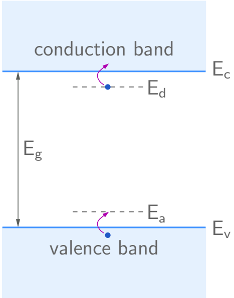


- $n + N_a^- = p + N_d^+$.
- $N_c\, e^{-(E_c - E_F)/kT} + \dfrac{N_a}{1 + 4\, e^{(E_a - E_F)/kT}} = N_v\, e^{-(E_F - E_v)/kT} + \dfrac{N_d}{1 + 2\, e^{(E_F - E_d)/kT}}$.

Calculation of n and p in equilibrium

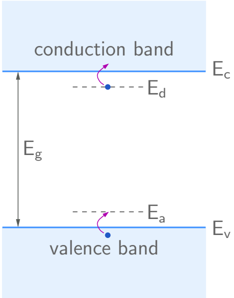


* $n + N_a^- = p + N_d^+$.
* $N_c\, e^{-(E_c - E_F)/kT} + \dfrac{N_a}{1 + 4\, e^{(E_a - E_F)/kT}} = N_v\, e^{-(E_F - E_v)/kT} + \dfrac{N_d}{1 + 2\, e^{(E_F - E_d)/kT}}$.
* We can take E_v as a reference $\rightarrow E_v = 0,\, E_c = E_g$.



* $n + N_a^- = p + N_d^+$.
* $$N_c\, e^{-(E_c - E_F)/kT} + \frac{N_a}{1 + 4\, e^{(E_a - E_F)/kT}} = N_v\, e^{-(E_F - E_v)/kT} + \frac{N_d}{1 + 2\, e^{(E_F - E_d)/kT}}.$$
* We can take E_v as a reference $\rightarrow E_v = 0,\, E_c = E_g$.
* This is a nonlinear equation in E_F and must be solved iteratively.

Calculation of n and p in equilibrium



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* $N_c\, e^{-(E_c - E_F)/kT} + \dfrac{N_a}{1 + 4\, e^{(E_a - E_F)/kT}} = N_v\, e^{-(E_F - E_v)/kT} + \dfrac{N_d}{1 + 2\, e^{(E_F - E_d)/kT}}$.
* We can take E_v as a reference $\rightarrow E_v = 0, E_c = E_g$.
* This is a nonlinear equation in E_F and must be solved iteratively.
* Note that E_g depends on the temperature. For silicon,

  $E_g(T) = E_g(0) - \alpha T^2/(\beta + T)$,

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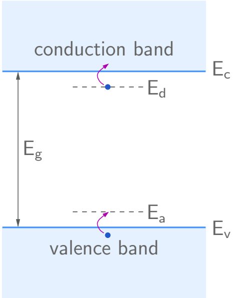
Calculation of n and p in equilibrium



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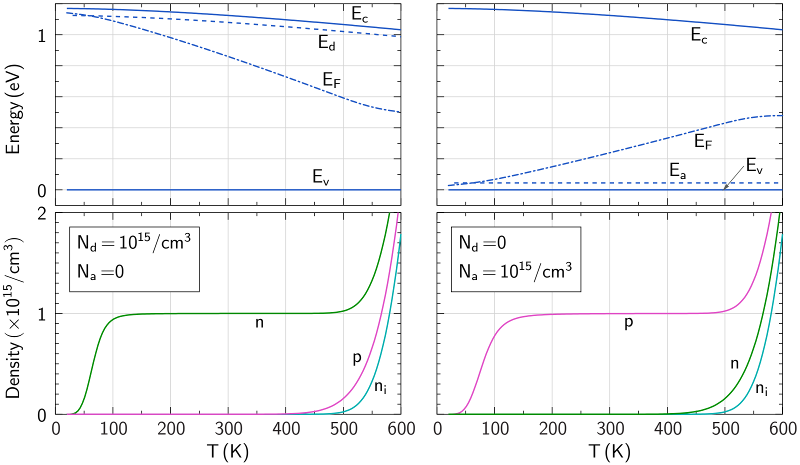
* Let us look at the results obtained for a few representative values of N_d and N_a, with $E_c - E_d = 45\,\text{meV}$, $E_a - E_v = 45\,\text{meV}$.

* With $N_d = 10^{15}\,\text{cm}^{-3}$ and $N_a = 0$,
  - At room temperature (300 K), $n \approx N_d$, and $p \ll n$.



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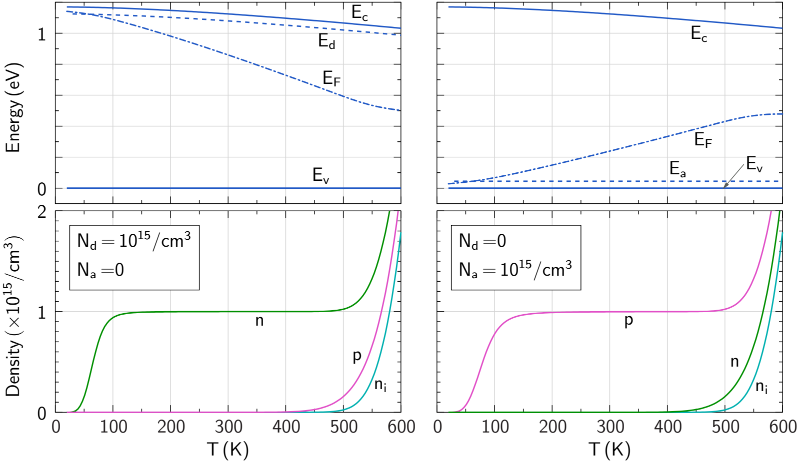

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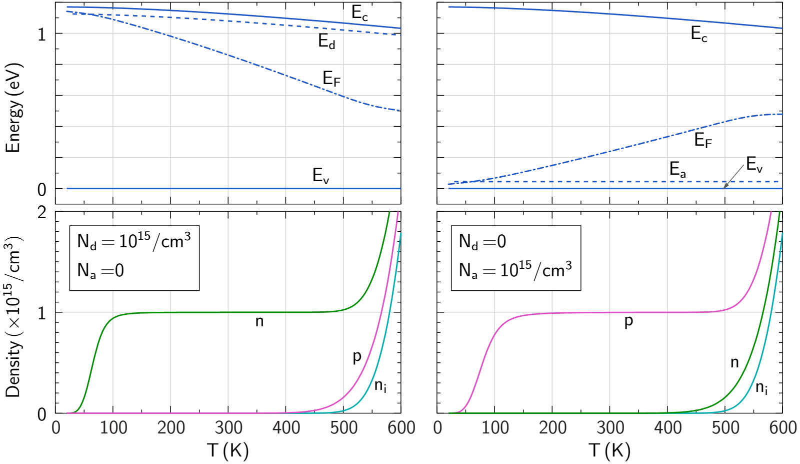

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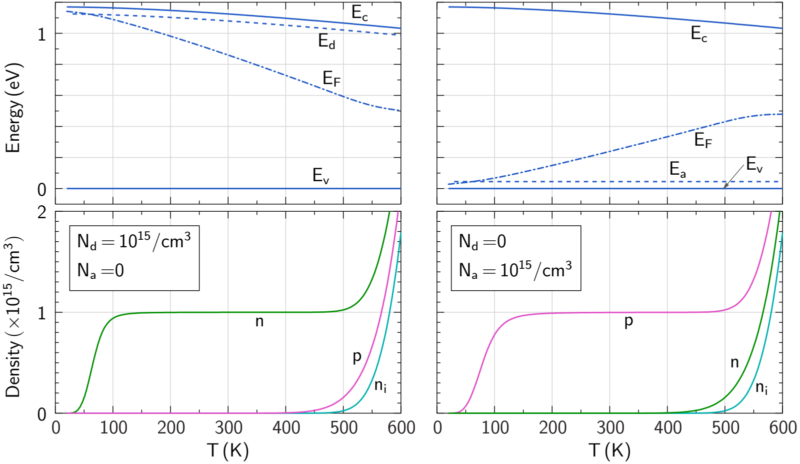

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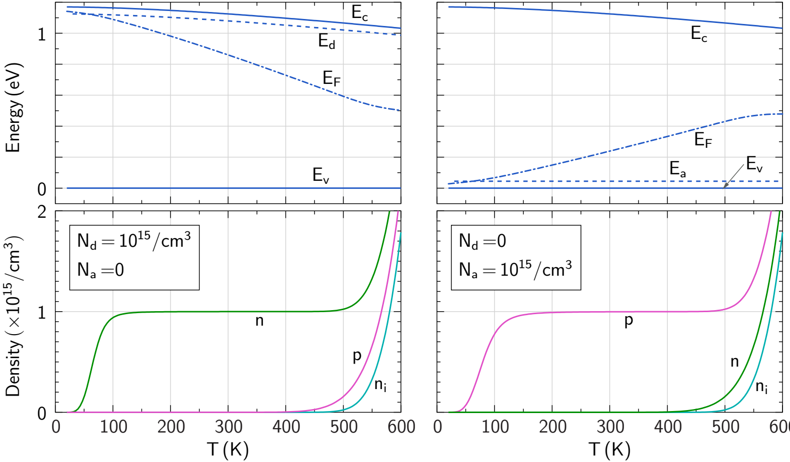
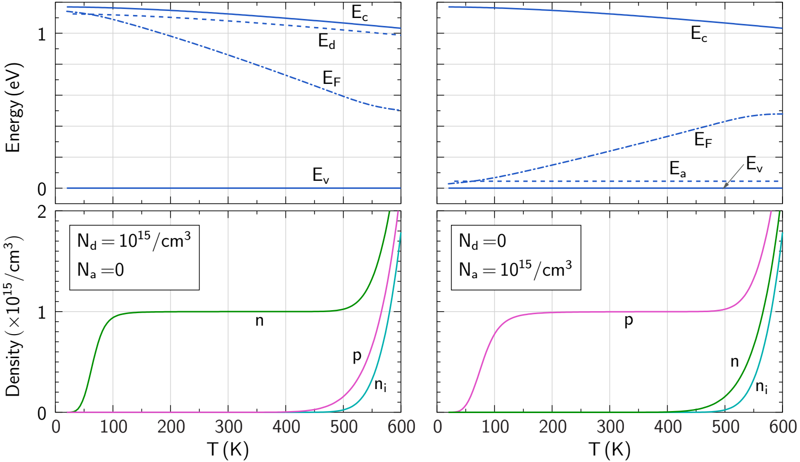
* In fact, the condition of complete ionisation is valid over a wide range of temperatures, called the "extrinsic" temperature region.



* Note that one of the carrier densities is much larger than the other (in the extrinsic region). The more abundant carrier is called the "majority carrier," and the other carrier is called the "minority carrier."



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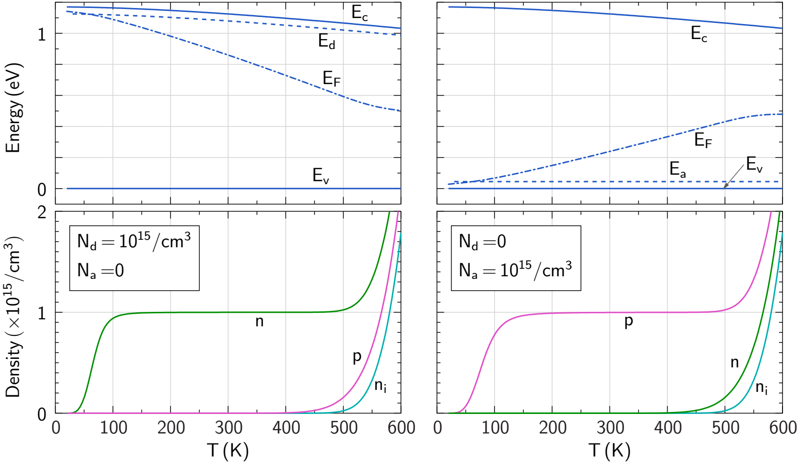

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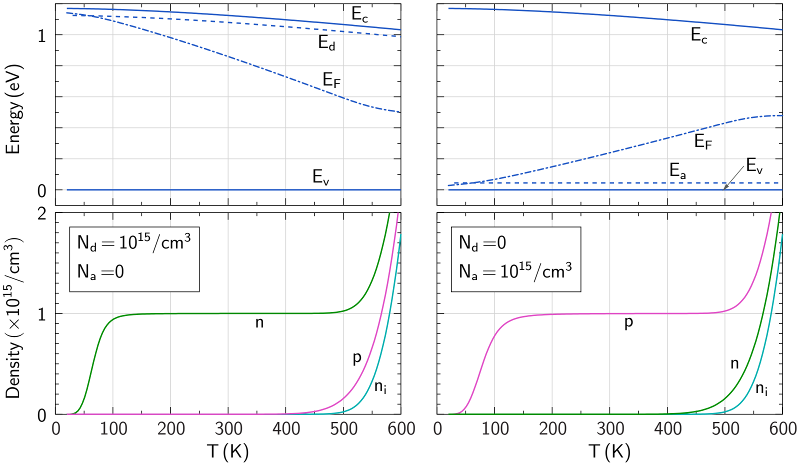
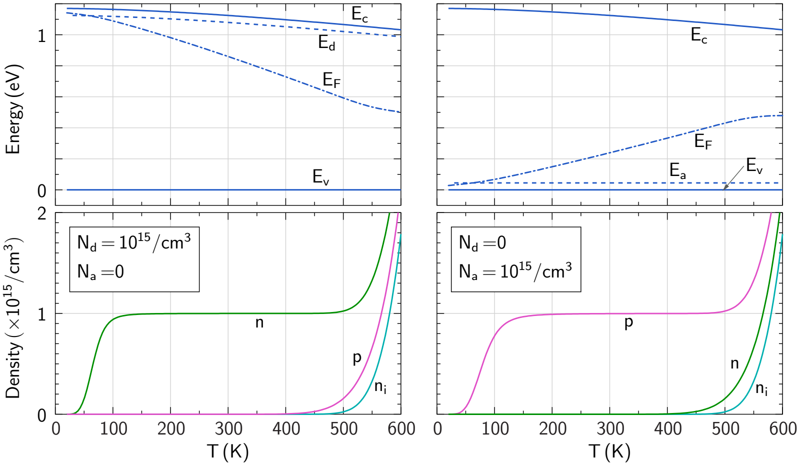
  For $N_a = 10^{15}\ \mathrm{cm}^{-3}$ and $N_d = 0$, holes are the majority carriers.
* A semiconductor with electrons as majority carriers is called an *n*-type semiconductor.
  A semiconductor with holes as majority carriers is called a *p*-type semiconductor.



* At low temperatures, a significant fraction of impurity atoms remains neutral, causing a reduction in n or p.



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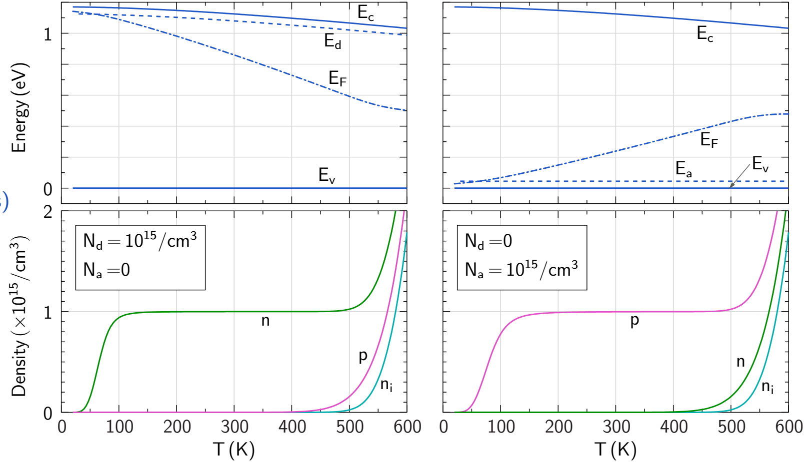

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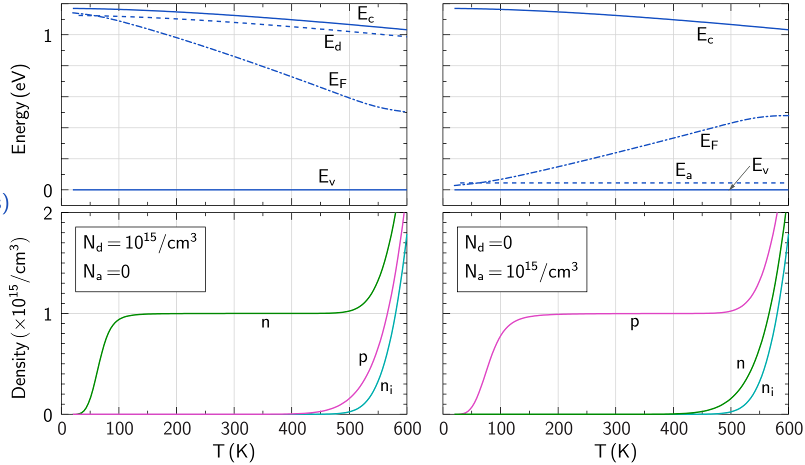
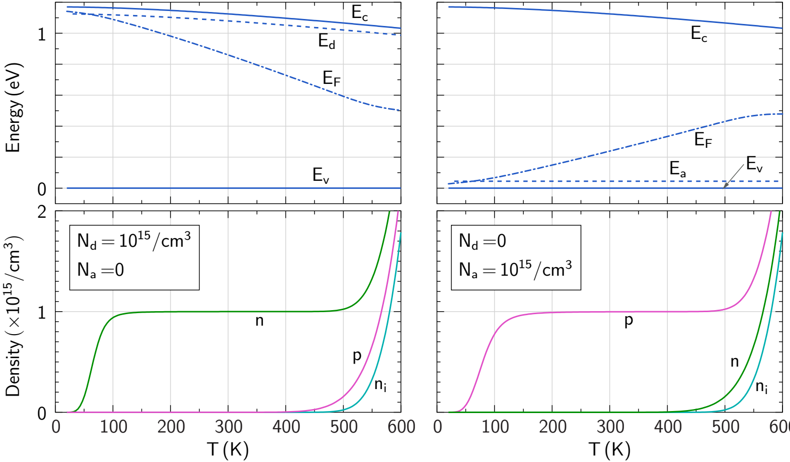
- This effect is called the carrier "freeze-out" effect, and it can be a limiting factor in low-temperature operation of semiconductor devices.



n and p in equilibrium

* At high temperatures, the intrinsic carrier concentration n_i becomes large and starts dominating.



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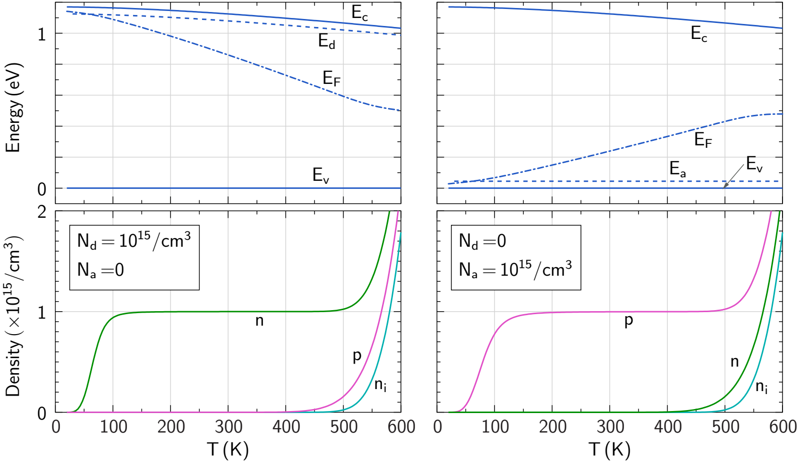

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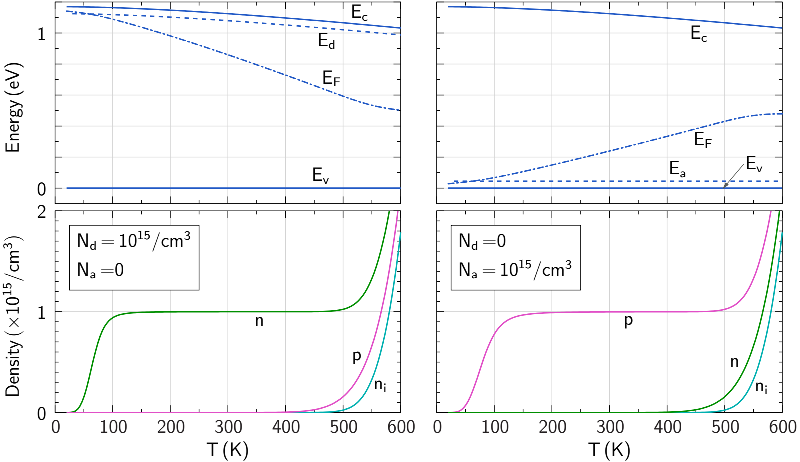
* This region is called the "intrinsic region," and it must be avoided for a semiconductor device to work as intended.



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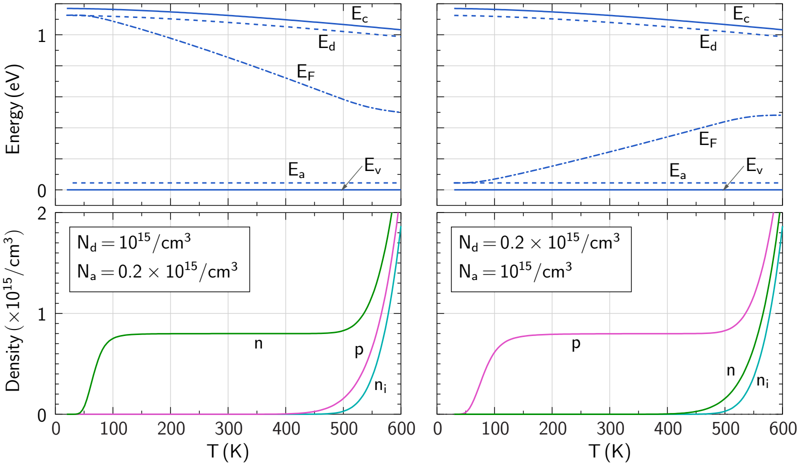

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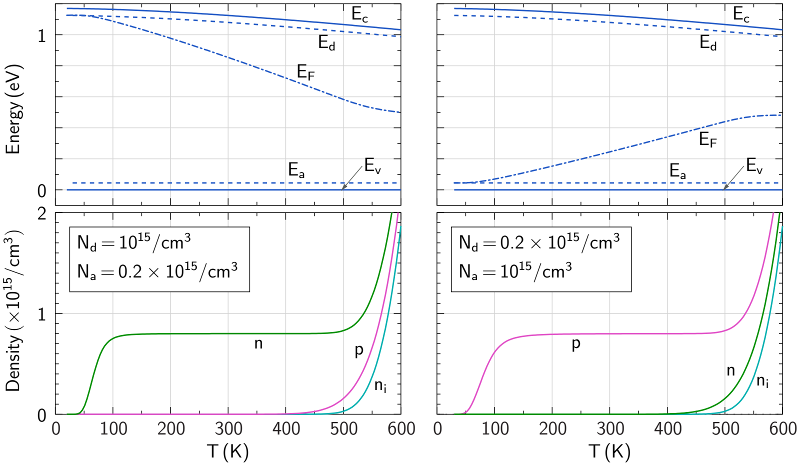

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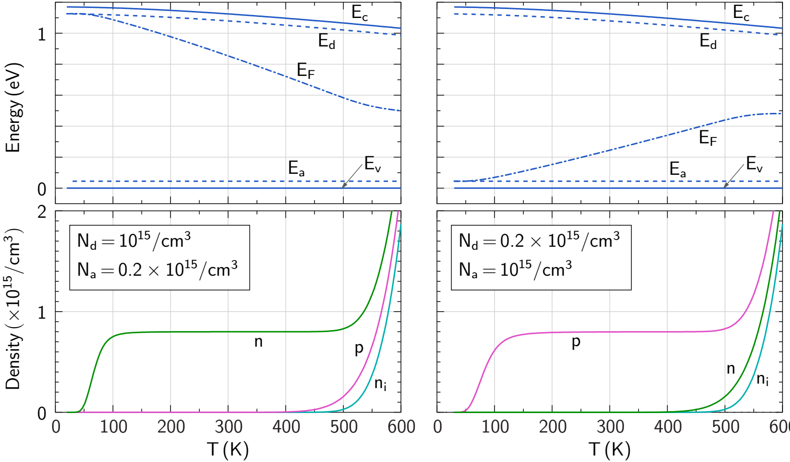

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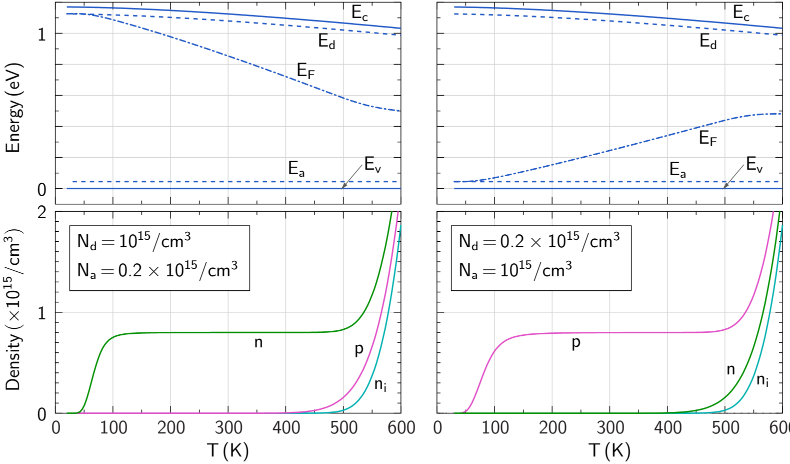
* A semiconductor with both types of dopants is called a "compensated" semiconductor.



Computation of n and p at room temperature in equilibrium

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* The above two equations can be solved to obtain n and p.

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In a silicon sample with $N_d = 5 \times 10^{16}\,\text{cm}^{-3}$, find the equilibrium electron and hole concentrations at $T = 300\,\text{K}$ ($n_i = 10^{10}\,\text{cm}^{-3}$ at $300\,\text{K}$).

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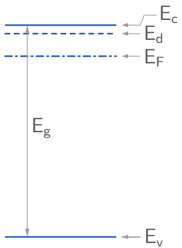
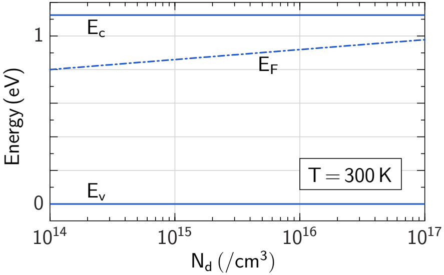
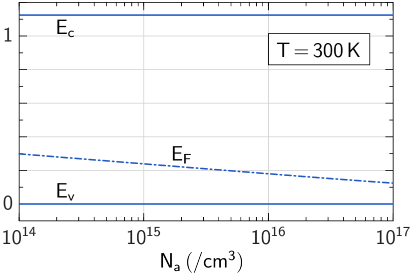


Variation of E_F with doping density (silicon, 300 K)





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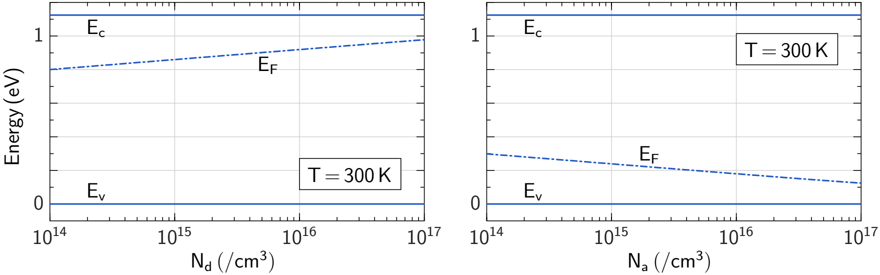

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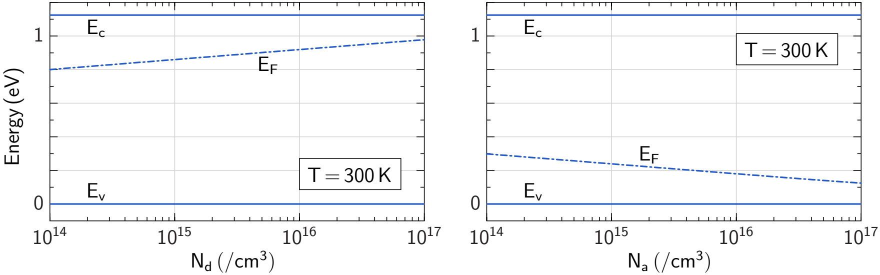

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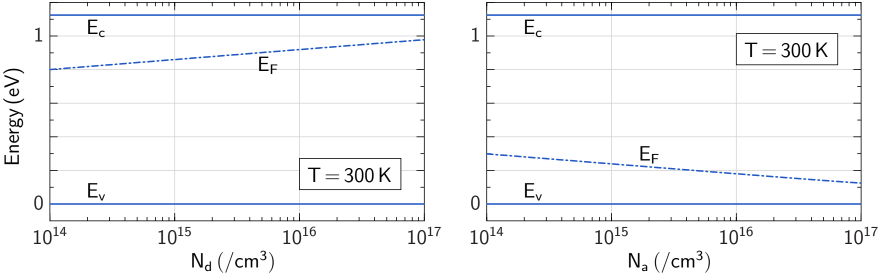
* *p*-type semiconductor: E_F gets closer to E_v as N_a is increased.

Variation of E_F with doping density (silicon, 300 K)



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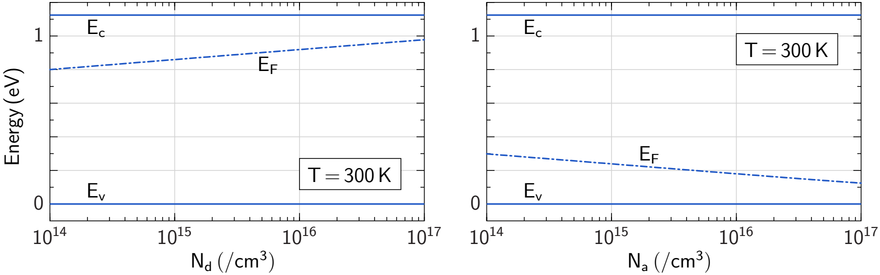

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- $p \approx N_a = N_v e^{-(E_F - E_v)/kT} \rightarrow E_F - E_v = kT \log \dfrac{N_v}{p} \approx kT\,(\log N_v - \log N_a)$.

  $\rightarrow (E_F - E_v)$ varies linearly with $\log N_a$.