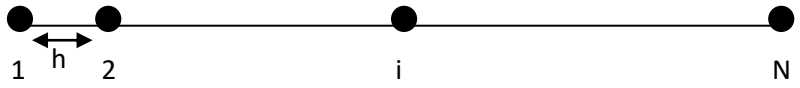
EE_735 Assignment 3: Additional material

Steps to solve poisson' s equation using newton-raphson method:

1. Descretize the given device length in N nodes:



2. Define the dopant profile.
3. Assign initial potential profile V_{old} (it is Nx1 matrix) with boundary conditions applied at nodes 1 and N.
4. Write Poisson's equation for the i_{th} node :

$$\frac{V_{i-1}-2V_i+V_{i+1}}{h^2} = -\frac{\rho_i}{\varepsilon} \qquad (1)$$

where, $\rho_i \; is \; a \; function \; of \; V_i$

a) Form the expression

$$F_i = F(V_i) = \frac{V_{i-1}-2V_i+V_{i+1}}{h^2} + \frac{\rho_i}{\varepsilon} \qquad (2)$$

b) Assume $(V_i + \Delta V_i)$ to be the root of (2), then

$$F(V_i + \Delta V_i) = 0$$

c) Using Taylor's series expansion and neglecting second and higher order terms in ΔV_j

$$F(V_i + \Delta V_i) = F(V_i) + \sum_{j=1}^{N} \frac{\partial F(V_i)}{\partial V_j} \Delta V_j = 0 \qquad (3)$$

5. Obtain equation (3) for all the nodes. Thus a set of 'N' non-linear equations is obtained which in the matrix representation can be shown as

$$\underbrace{\begin{bmatrix} F(V_1) \\ F(V_2) \\ \vdots \\ F(V_N) \end{bmatrix}}_{F(V)} + \underbrace{\begin{bmatrix} \frac{\partial F_1}{\partial V_1} & \frac{\partial F_1}{\partial V_2} & \cdots & \frac{\partial F_1}{\partial V_N} \\ \frac{\partial F_2}{\partial V_1} & \frac{\partial F_2}{\partial V_2} & \cdots & \frac{\partial F_2}{\partial V_N} \\ \vdots & \vdots & & \vdots \\ \frac{\partial F_N}{\partial V_1} & \frac{\partial F_N}{\partial V_1} & \cdots & \frac{\partial F_N}{\partial V_N} \end{bmatrix}}_{J} \times \underbrace{\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_N \end{bmatrix}}_{\Delta V} = 0$$

Nx1 NxN Nx1

where J is termed as the Jacobian matrix.

6. Obtain a solution for ΔV as

$$\Delta V = -J\backslash F(V) \quad (4)$$

7. Obtain new potential profile as

$$V_{new} = V_{old} + \Delta V \quad (5)$$

Make sure that the boundary conditions don't get changed while you update the potential profile.

8. Obtain $F(V)$ and J for V_{new} .
9. Repeat steps 6 to 8 for sufficiently large number of iterations or till ΔV goes below some predefined tolerance value.