An Algorithm for Nonlinear Knapsack Problems

The knapsack problem is a problem in combinatorial optimization: Given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible. It derives its name from the problem faced by someone who is constrained by a fixed-size knapsack and must fill it with the most valuable items.

This paper studies the algorithm which recursively generates the complete family of undominated feasible solutions to separable nonlinear multidimensional knapsack problems is developed by exploiting discontinuity preserving properties of the maximal convolution. The "curse of dimensionality", which is usually associated with dynamic programming algorithms, is successfully mitigated by reducing an M-dimensional dynamic program to a 1-dimensional dynamic program through the use of the imbedded state space approach. Computational experience with the algorithm on problems with as many as 10 state variables is also reported and several interesting extensions are discussed.