Characteristic polynomials, eigenvalues and eigenvectors

1.Question 1

Given a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, recall that one can calculate its eigenvalues by solving the characteristic polynomial $\lambda^2 - (a+d)\lambda + (ad - bc) = 0$. In this quiz, you will practice calculating and solving the characteristic polynomial to find the eigenvalues of simple matrices.

For the matrix $A=\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

○

$\lambda^2 + 3\lambda + 2 = 0$

$\lambda_1 = -1, \lambda_2 = -2$

○

$\lambda^2 - 3\lambda - 2 = 0$

$\lambda_1 = 1, \lambda_2 = 2$

○

$\lambda^2 - 3\lambda - 2 = 0$
$\lambda_1 = -1, \lambda_2 = 2$

○

$\lambda^2 - 3\lambda - 2 = 0$

$\lambda_1 = 1, \lambda_2 = -2$

2.Question 2

Recall that for a matrix AA, the eigenvectors of the matrix are vectors for which applying the matrix transformation is the same as scaling by some constant.

For $A=\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ as immediately above, select all eigenvectors of this matrix.

☐

$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

□

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

□

$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

□

$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

3.Question 3

For the matrix $A=\begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}$**, what is the characteristic polynomial, and the solutions to the characteristic polynomial?**

○

$\lambda_2 + 8\lambda + 15 = 0$

$\lambda_1 = -3, \lambda_2 = -5$

○

$\lambda_2 - 8\lambda - 15 = 0$

$\lambda_1 = -3, \lambda_2 = 5$

○

$\lambda_2 + 8\lambda - 15 = 0$

$\lambda_1 = 3, \lambda_2 = -5$

○

$\lambda_2 - 8\lambda + 15 = 0$

$\lambda_1 = 3, \lambda_2 = 5$

4.Question 4

For the matrix $A=\begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}$**as immediately above, select all eigenvectors of this matrix.**

□

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

☐

$\binom{2}{1}$

☐

$\binom{3}{0}$

☐

$\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

5.Question 5

For the matrix $A=\begin{pmatrix} 1 & -1 \\ 0 & 4 \end{pmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

○

$\lambda_2 + 5\lambda + 4 = 0$

$\lambda_1 = -1, \lambda_2 = -4$

○

$\lambda_2 - 5\lambda - 4 = 0$

$\lambda_1 = -1, \lambda_2 = 4$

○

$\lambda_2 - 5\lambda + 4 = 0$

$\lambda_1 = 1, \lambda_2 = 4$

○

$\lambda_2 + 5\lambda - 4 = 0$

$\lambda_1 = 1, \lambda_2 = -4$

6.Question 6

For the matrix $A=\begin{pmatrix} 1 & -1 \\ 0 & 4 \end{pmatrix}$ as immediately above, select all eigenvectors of this matrix.

☐

$\binom{3}{-1}$

☐

$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

☐

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

☐

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

7.Question 7

For the matrix $A=\begin{pmatrix} -3 & 2 \\ 8 & 3 \end{pmatrix}$, **what is the characteristic polynomial, and the solutions to the characteristic polynomial?**

○

$\lambda_2 + 25 = 0$

$\lambda_1 = -5, \lambda_2 = 5$

○

$\lambda_2 - 25 = 0$

$\lambda_1 = -5, \lambda_2 = 5$

○

$\lambda_2 + 25 = 0$

$\lambda_1 = \lambda_2 = -5$

○

$\lambda_2 - 25 = 0$

$\lambda_1 = \lambda_2 = 5$

8.Question 8

For the matrix $A=\begin{pmatrix} -3 & 2 \\ 8 & 3 \end{pmatrix}$ as immediately above, select all eigenvectors of this matrix.

☐

$\begin{pmatrix} -1 \\ -1 \end{pmatrix}$

☐

$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

☐

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

☐

$\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

9.Question 9

For the matrix $A=\begin{pmatrix} 5 & -4 \\ 4 & -3 \end{pmatrix}$, what is the characteristic polynomial, and the solutions to the characteristic polynomial?

○

$\lambda_2 - 2\lambda + 1 = 0$

$\lambda_1 = -1, \lambda_2 = 1$

○

$\lambda_2 - 2\lambda + 1 = 0$

No real solutions.

○

$\lambda_2 - 2\lambda + 1 = 0$

$\lambda_1 = \lambda_2 = -1$

○

$\lambda_2 - 2\lambda + 1 = 0$

$\lambda_1 = \lambda_2 = 1$