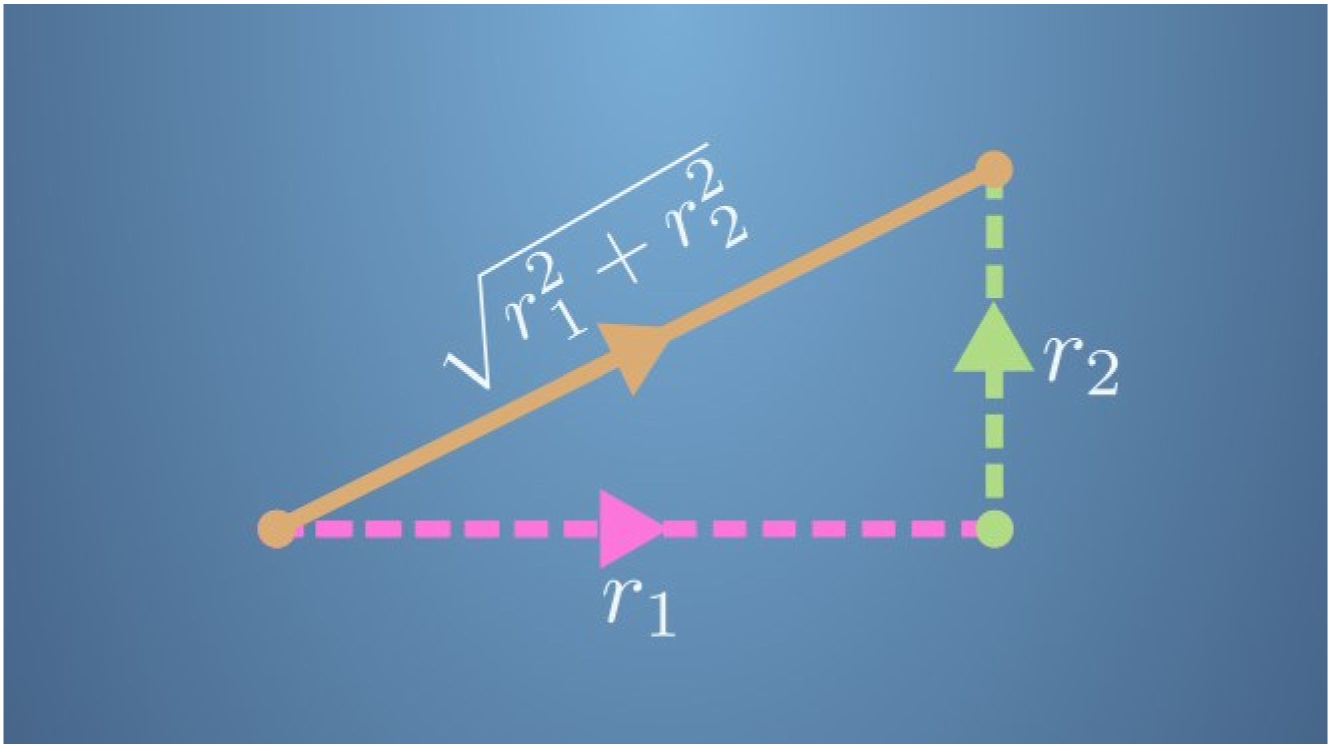
As we have seen in the lecture videos, the dot product of vectors has a lot of applications. Here, you will complete some exercises involving the dot product.

We have seen that the size of a vector with two components is calculated using Pythagoras' theorem, for example the following diagram shows how we calculate the size of the orange vector $\mathbf{r}=[r_1\ r_2]$**:**



In fact, this definition can be extended to any number of dimensions; the size of a vector is the square root of the sum of the squares of its components. Using this information, what is the size of the vector $\mathbf{s}=\begin{pmatrix}1\\3\\4\\2\end{pmatrix}$

- $|\mathbf{s}|=10$
- $|\mathbf{s}|=30$
- $|\mathbf{s}|=\sqrt{30}$

$|\mathbf{s}|=\sqrt{10}$

2.Question 2

Remember the definition of the dot product from the videos. For two $\mathbf{n}n$ component vectors, $\mathbf{a}\cdot\mathbf{b}=a_1b_1+a_2b_2+\cdots+a_nb_n$.

What is the dot product of the vectors $\mathbf{r}=\begin{pmatrix}-5\\3\\2\\8\end{pmatrix}$ and $\mathbf{s}=\begin{pmatrix}1\\2\\-1\\0\end{pmatrix}$?

$\mathbf{r}\cdot\mathbf{s}=1$

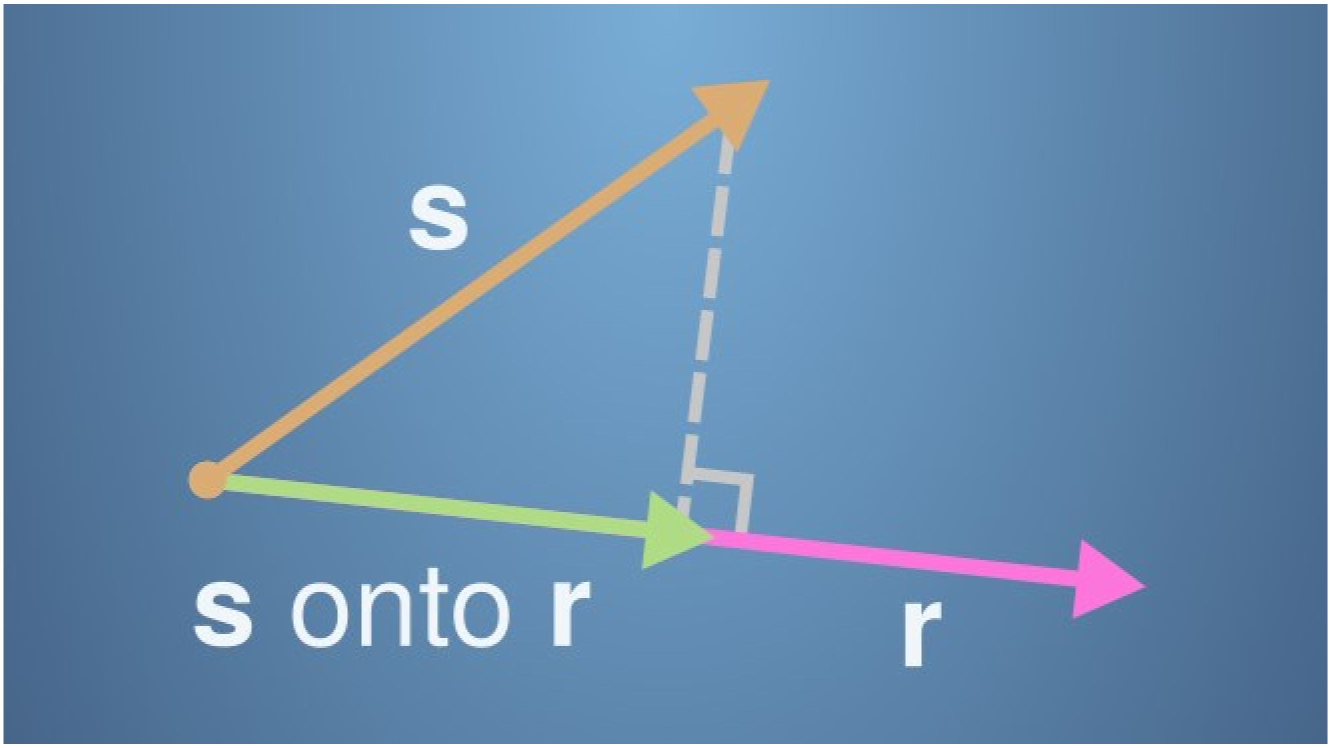
$\mathbf{r}\cdot\mathbf{s}=\begin{pmatrix}-4\\5\\4\\9\end{pmatrix}$

$\mathbf{r}\cdot\mathbf{s}=\begin{pmatrix}-5\\6\\-2\\0\end{pmatrix}$

$\mathbf{r}\cdot\mathbf{s}=-1$

3.Question 3

The lectures introduced the idea of projecting one vector onto another. The following diagram shows the projection of $\mathbf{s}$ onto $\mathbf{r}$ when the vectors are in two dimensions:



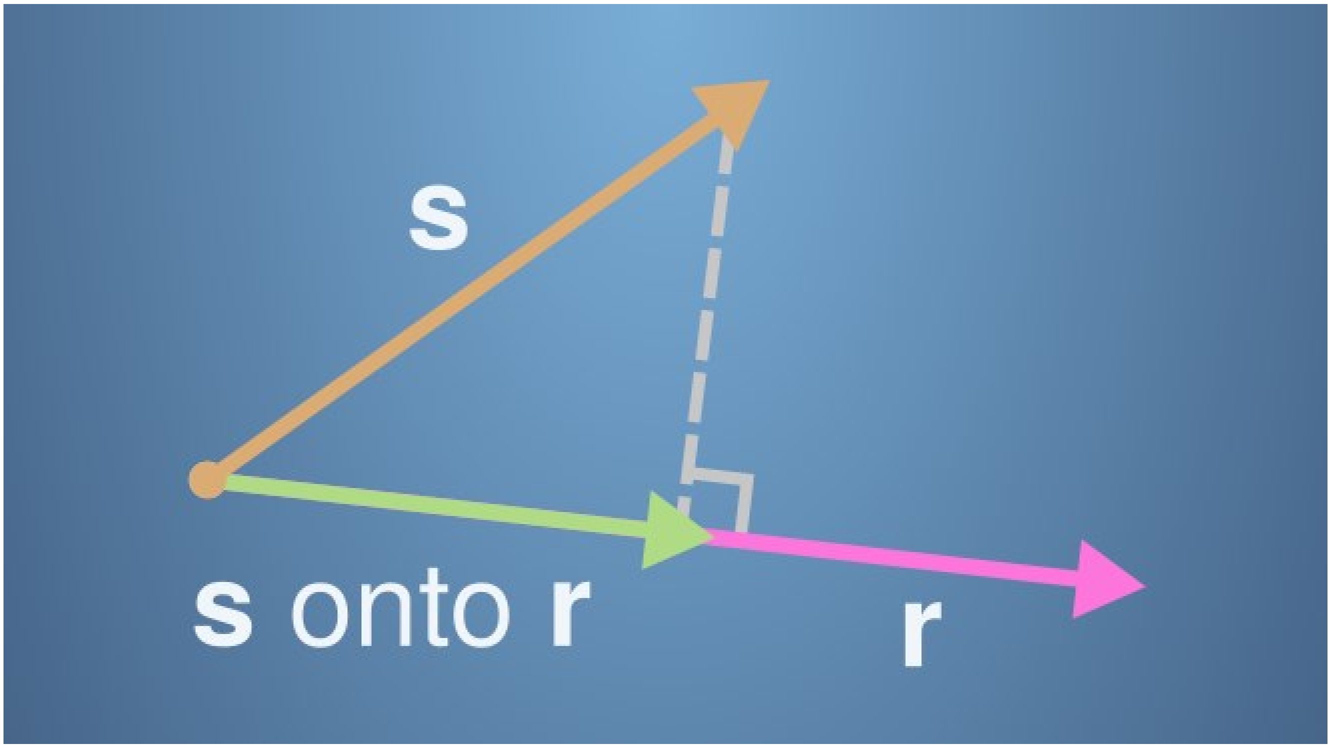
Remember that the scalar projection is the *size* of the green vector. If the angle between $\mathbf{s}$ and $\mathbf{r}$ is greater than \pi/2$\pi/2$, the projection will also have a minus sign.

We can do projection in any number of dimensions. Consider two vectors with three components, $\mathbf{r}=\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$and $\mathbf{s}=\begin{pmatrix} -5 \\ 6 \\ -2 \\ 0 \end{pmatrix}$.

What is the scalar projection of $\mathbf{s}$ onto $\mathbf{r}$?

4.Question 4

Remember that in the projection diagram, the vector projection *is* the green vector:



Let $\mathbf{r}=\begin{pmatrix}3\\-4\\0\end{pmatrix}$ **and let** $\mathbf{s}=\begin{pmatrix}1\\0\\5\\-6\end{pmatrix}$.

What is the vector projection of $\mathbf{s}$ **onto** $\mathbf{r}$**?**

- $\begin{pmatrix}6\\-8\\0\end{pmatrix}$

- $\begin{pmatrix}6\\4\\0\end{pmatrix}$

- $\begin{pmatrix}\frac{6}{5}\\\frac{8}{5}\\0\end{pmatrix}$

5.Question 5

Let $\mathbf{a}=\begin{pmatrix}3\\0\\4\end{pmatrix}$ **and** $\mathbf{b}=\begin{pmatrix}0\\5\\1\\2\end{pmatrix}$.

Which is larger, $|\mathbf{a}+\mathbf{b}|$ **or** $|\mathbf{a}|+|\mathbf{b}|$**?**

○

$|\mathbf{a}+\mathbf{b}|<|\mathbf{a}|+|\mathbf{b}|$

○

$|\mathbf{a}+\mathbf{b}|>|\mathbf{a}|+|\mathbf{b}|$

○

$|\mathbf{a}+\mathbf{b}|=|\mathbf{a}|+|\mathbf{b}|$

1 point

6.Question 6

Which of the following statements about dot products are correct?

☐

The size of a vector is equal to the square root of the dot product of the vector with itself.

☐

We can find the angle between two vectors using the dot product.

☐

The order of vectors in the dot product is important, so that $\mathbf{s}\cdot\mathbf{r}\neq\mathbf{r}\cdot\mathbf{s}$.

☐

The vector projection of $\mathbf{s}$ onto $\mathbf{r}$ is equal to the scalar projection of $\mathbf{s}$ onto $\mathbf{r}$ multiplied by a vector of unit length that points in the same direction as $\mathbf{r}$.

☐

The scalar projection of $\mathbf{s}$ onto $\mathbf{r}$ is always the same as the scalar projection of $\mathbf{r}$ onto $\mathbf{s}$.