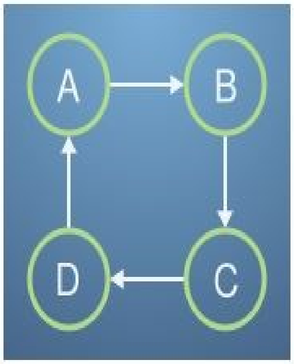
To practice, select all eigenvectors of the matrix, $A = \begin{bmatrix} 4 & -5 & 6 \\ 7 & -8 & 6 \\ 3/2 & -1/2 & -2 \end{bmatrix}$.

- [ ] $\begin{bmatrix} 1/2 \\ -1/2 \\ -1 \end{bmatrix}$
- [ ] $\begin{bmatrix} -2/\sqrt{9} \\ -2/\sqrt{9} \\ 1/\sqrt{9} \end{bmatrix}$
- [ ] None of the other options.
- [ ] $\begin{bmatrix} 1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$
- [ ] $\begin{bmatrix} -3 \\ -3 \\ -1 \end{bmatrix}$
- [ ] $\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

2. Recall from the *PageRank* notebook, that in PageRank, we care about the eigenvector of the link matrix, $L$, that has eigenvalue 1, and that we can find this using *power iteration method* as this will be the largest eigenvalue.

   PageRank can sometimes get into trouble if closed-loop structures appear. A simplified example might look like this,

   With link matrix, $L = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

   Use the calculator in Q1 to check the eigenvalues and vectors for this system.

   What might be going wrong? Select all that apply.

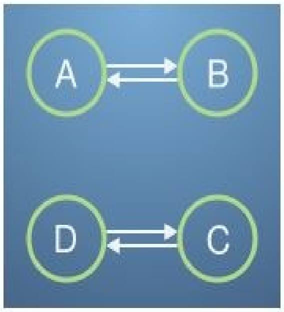
   - [ ] Some of the eigenvectors are complex.
   - [ ] Because of the loop, *Procrastinating Pat*s that are browsing will go around in a cycle rather than settling on a webpage.
   - [ ] None of the other options.
   - [ ] Other eigenvalues are not small compared to 1, and so do not decay away with each power iteration.

3. **The loop in the previous question is a situation that can be remedied by damping.**

**If we replace the link matrix with the damped,** $L' = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.7 \\ 0.7 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.7 & 0.1 \end{bmatrix}$**, how does this help?**

- ☐ The complex number disappear.
- ☐ None of the other options.
- ☐ It makes the eigenvalue we want bigger.
- ☐ There is now a probability to move to any website.
- ☐ The other eigenvalues get smaller.

4. **Another issue that may come up, is if there are disconnected parts to the internet. Take this example,**




**with link matrix,** $L = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

**This form is known as block diagonal, as it can be split into square blocks along the main diagonal, i.e.,** $L = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$, **with** $A = B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **in this case.**

**What is happening in this system?**

- [ ] There are two eigenvalues of 1.
- [ ] None of the other options.
- [ ] There are loops in the system.
- [ ] The system has zero determinant.
- [ ] There isn't a unique PageRank.

5. **By similarly applying damping to the link matrix from the previous question. What happens now?**

- [ ] The negative eigenvalues disappear.
- [ ] None of the other options.
- [ ] The system settles into a single loop.
- [ ] There becomes two eigenvalues of 1.
- [ ] Damping does not help this system.

6. **Given the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$, calculate its characteristic polynomial.**

- ○ $\lambda^2 - 2\lambda + \frac{1}{4}$
- ○ $\lambda^2 - 2\lambda - \frac{1}{4}$
- ○ $\lambda^2 + 2\lambda - \frac{1}{4}$
- ○ $\lambda^2 + 2\lambda + \frac{1}{4}$

7. By solving the characteristic polynomial above or otherwise, calculate the eigenvalues of the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$.

- ○ $\lambda_1 = -1 - \frac{\sqrt{3}}{2}, \lambda_2 = -1 + \frac{\sqrt{3}}{2}$
- ○ $\lambda_1 = 1 - \frac{\sqrt{5}}{2}, \lambda_2 = 1 + \frac{\sqrt{5}}{2}$
- ○ $\lambda_1 = -1 - \frac{\sqrt{5}}{2}, \lambda_2 = -1 + \frac{\sqrt{5}}{2}$
- ○ $\lambda_1 = 1 - \frac{\sqrt{3}}{2}, \lambda_2 = 1 + \frac{\sqrt{3}}{2}$

8. Select the two eigenvectors of the matrix $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$.

- ○ $\mathbf{v_1} = \begin{bmatrix} -1 - \sqrt{3} \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -1 + \sqrt{3} \\ 1 \end{bmatrix}$
- ○ $\mathbf{v_1} = \begin{bmatrix} 1 - \sqrt{5} \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 + \sqrt{5} \\ 1 \end{bmatrix}$
- ○ $\mathbf{v_1} = \begin{bmatrix} -1 - \sqrt{5} \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -1 + \sqrt{5} \\ 1 \end{bmatrix}$
- ○ $\mathbf{v_1} = \begin{bmatrix} 1 - \sqrt{3} \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 + \sqrt{3} \\ 1 \end{bmatrix}$

9. Form the matrix $C$ whose left column is the vector $\mathbf{v_1}$ and whose right column is $\mathbf{v_2}$ from immediately above.

By calculating $D = C^{-1}AC$ or by using another method, find the diagonal matrix $D$.

- $\begin{bmatrix} 1-\frac{\sqrt{5}}{2} & 0 \\ 0 & 1+\frac{\sqrt{5}}{2} \end{bmatrix}$
- $\begin{bmatrix} 1+\frac{\sqrt{3}}{2} & 0 \\ 0 & 1-\frac{\sqrt{3}}{2} \end{bmatrix}$
- $\begin{bmatrix} -1-\frac{\sqrt{5}}{2} & 0 \\ 0 & -1+\frac{\sqrt{5}}{2} \end{bmatrix}$
- $\begin{bmatrix} -1-\frac{\sqrt{3}}{2} & 0 \\ 0 & -1+\frac{\sqrt{3}}{2} \end{bmatrix}$

10. By using the diagonalisation above or otherwise, calculate $A^2$.

- $\begin{bmatrix} 11/4 & -1 \\ -2 & 3/4 \end{bmatrix}$
- $\begin{bmatrix} 11/4 & -2 \\ -1 & 3/4 \end{bmatrix}$
- $\begin{bmatrix} -11/4 & 2 \\ 1 & -3/4 \end{bmatrix}$
- $\begin{bmatrix} -11/4 & 1 \\ 2 & -3/4 \end{bmatrix}$