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Тема / Topic	**Улучшенное управление робототехническими системами в скользящих режимах на основе выпуклой оптимизации / Enhanced Convex Optimization Driven Sliding Mode Control over Robotic Systems**

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Abstract

Controlling Remotely Operated Vehicles (ROVs) in underwater environments is challenging due to their nonlinear and unpredictable nature. This thesis explores enhancing sliding mode control (SMC) for ROVs using convex optimization to improve stability and performance while reducing chattering. A comprehensive mathematical model of an ROV is developed, addressing dynamic and kinematic properties along with environmental uncertainties. An optimized SMC scheme is designed, integrating convex optimization to adjust control parameters dynamically, thus minimizing chattering and enhancing robustness. Validation through simulations and experiments demonstrates significant improvements in trajectory tracking and disturbance rejection. The enhanced SMC scheme proves effective in simulation, confirming its practical applicability. This research advances underwater robotics by providing a robust control strategy that enhances ROV stability and efficiency, supporting complex underwater missions like pipeline inspection and environmental monitoring.

Chapter 1

Introduction

I Background

Robotic systems, particularly underwater vehicles such as Remotely Operated Vehicles (ROVs), have garnered significant attention in recent years due to their wide-ranging applications in industrial, scientific, and military domains. ROVs are employed for diverse purposes, including underwater exploration, pipeline inspection, and environmental monitoring. However, controlling these vehicles is challenging due to the highly nonlinear and unpredictable nature of the underwater environment. Therefore, effective control strategies are essential to ensure the stability and performance of ROVs in such conditions.

Control strategies for ROVs have evolved, with robust control methods like Sliding Mode Control (SMC) gaining prominence due to their effectiveness in handling system nonlinearities and uncertainties. Despite these advancements, existing control methods for ROVs face several challenges. Traditional SMC techniques, while robust, often lead to a phenomenon known as "chattering," which can cause wear and tear on mechanical components and reduce the overall efficiency of the control system. Additionally, many control algorithms struggle to adapt to real-time changes in the underwater environment, such as varying current strengths and unforeseen obstacles. These limitations underscore the need for improved control strategies that can offer both robustness and adaptability.

II Research Objective

The primary objective of this study is to enhance the control of remotely operated vehicles (ROVs) by developing an improved Sliding Mode Control (SMC) scheme that leverages convex optimization techniques. This thesis aims to address the existing issues of chattering and adaptability by dynamically adjusting control parameters through convex optimization.

The research question: How can sliding mode control for robotic systems be improved using convex optimization to enhance stability and performance while mitigating the chattering effect?

The research objectives are as follows:

1. Develop a comprehensive mathematical model for ROVs that captures the essential dynamics and uncertainties

2. Design a sliding mode control scheme enhanced by convex optimization

3. Evaluate the performance of the proposed control scheme through simulation

III Structure of the Thesis

The remainder of the work is structured as follows: Chapter 2 provides an overview of existing research on ROVs, focusing on their modeling and control strategies. In Chapter 3, the development of the mathematical model for the ROV is explored. Further, Chapter 4 describes the design of the enhanced sliding mode control scheme. Chapter 5 presents the results of the simulation and experimental validation, and discusses the findings. Finally, Chapter 6 summarizes the research contributions and suggests directions for future work.

Chapter 2

Literature Review

This chapter presents a comprehensive overview of existing research on remotely operated vehicles (ROVs), focusing on modeling and control design. This literature review aims to investigate and synthesize control strategies, addressing uncertainties from actuator dynamics and environmental factors, which correspond to the proposed research question.

Section 1 gives a brief description of ROVs, including their types and general attributes. Section 2 explains the commonly accepted assumptions and mathematical models used in the field. Section 3 offers an extensive review of various studies on the application of robust control design for underwater systems. In conclusion, Section 4 summarizes the key insights from the review and suggests a control methodology.

I Overview on ROVs

#TODO Add more sorces and get more info about ROVs.

This section provides an overview of ROVs, from their classification and applications to their inherent characteristics and challenges. The obtained knowledge forms a basis for ROV modeling and control design.

ROV physics -> complex model design

According to [1], an ROV's mechanical structure consists of a monitoring camera, a sensor for gathering navigation data, and actuators for directional control. A comparative study [2] found that the physical aspects that affect ROV functions are the accuracy of sensor systems and the thruster designs. The unpredictable nature of underwater currents, drag, and buoyancy dynamics can also have a serious impact on a ROV's performance, complicating model design.

Small ROVs are more preferrable -> wide range applications

A recent systematic review [3] concluded that there are two primary classifications for ROVs based on their functions and intended use: observation class and work class. Observation class vehicles are typically small and limited to shallow waters with propulsion power up to a few kilowatts. Work class vehicles can perform heavy-duty work, requiring significant hardware system complexity. Thus, when the functionality of these large ROVs is not necessary, a smaller ROV is preferred for a wide range of applications.

Applications of ROV -> control objectives

Several studies [1], [2] have identified that ROVs have become crucial for industrial applications, offshore oil and gas exploration, patrolling, and surveillance. Therefore, its control system should focus on position tracking and station keeping in the presence of parameter and environmental uncertainties, addressing the following issues.

#TODO Add connection to modelling, wrap it up

II Mathematical Modelling

Remotely operated vehicles require mathematical models for various purposes, including control system design, simulation, and performance analysis.

SRB

Thor I. Fossen [4] provided a complete description of the fundamentals of mathematical modeling for marine vehicles. In his book, ROV was represented as a single rigid body (SRB) by considering it as a solid mass with no internal movement or deformation. The SRB model has drastically simplified the modeling while capturing the essential dynamics of the system.

Kinematics and dynamics

From a kinematic point of view, ROVs have six degrees of freedom (DoFs). However, the orientation expressed in the rotation angles could eventually lead to the singularity. To solve this issue, [5] proposed the quaternions representation. The dynamics was derived based on classical physics laws: Newtons Second Law and Euler-Lagrange equation, forming the set of nonlinear equations. The study [6] simplified the equations and represented them in a matrix form.

Parameter estimation

However, several sources have established that some aspects of the ROV dynamics require empirical estimates due to their complex, nonlinear, and coupled nature [4], [7]. For instance, the inertia of the surrounding fluid cannot be neglected when the vessel moves through a viscous medium. Additionally, water damping is another source of nonlinearity that can be approximated as a function of the velocity. Finally, aside from buoyancy and gravity, it is common practice to cancel all other forces acting on a vehicle, although it can also impact the dynamics [7].

Thruster modelling

Moreover, thruster modeling must be applied to define the desired thrust of each thruster. A recent study by [7] stated that creating an accurate thruster model can be challenging due to the influence of factors such as motor models, hydrodynamic effects, and propeller mapping.

Uncertaint model-> control objectives

By making these simplifications, the control system cannot independently provide effective control over such uncertain dynamics. As a result, a robust controller design is necessary for precise ROV position tracking.

III Control Solutions

Connect modelling and control

Controlling an ROV is a complex task that requires a set of processes to stabilize the vehicle and to make it follow the operator's instructions. To ensure the robustness of the system, it is necessary to define a control system that can handle disturbances caused by parameter and environmental changes.
According to [2], there are two main challenges associated with ROVs control:

1. Unmodeled elements like added mass and hydrodynamic coefficients.
2. Highly nonlinear dynamics of the underwater environment which cause significant disturbances to the vehicle.

Base approach -> SMC

The research on ROV control showed several schemes that can achieve robust stability under variable disturbances. The classical approach applied is sliding mode control (SMC), which was introduced by Slotine [8]. SMC is an effective way to address the issues mentioned above and is, therefore, a feasible option for controlling underwater vehicles. However, standard SMC introduces high-frequency signals, which can cause actuator switching and consequently decrease its lifetime.

Better SMC variations

#TODO Change sources and control approaches

The modern SMC interpretations keep the main advantages, thus removing the chattering effects. One of the possible approaches is Adaptive Sliding Mode Control (ASMC) design. [9] designed an effective hybrid control mechanism for the underwater system that follows a given route, adapting to dynamic disturbances. [10] improved a control system for ROVs, eliminating the need for dynamics linearization. Another way to refine SMC is to add an integral component into the controller equation. The proposed Integral Sliding Mode Control (ISMC) reduced chattering, effectively eliminating the uncertainty of the model parameters [11].

My proposal-> use optimization

#TODO Connect previous part to my proposed control scheme

The choice of control strategy should be based on the specific requirements and characteristics of the underwater vehicle. While both SMC modifications showed precise control, these approaches faced certain challenges [11], [10]. The ISMC method had limitations in adapting to rapidly changing dynamics, while the ASMC method experienced potential trade-offs in transient performance. These problems were opposite to each other. Therefore, combining them into Adaptive Integral Sliding Mode Control (ADISMC) can be a good idea to leverage the strengths of both methods and compensate for their weaknesses.

IV Summary

#TODO Change according to my proposed control scheme

This literature review comprehensively examined the general characteristics, mathematical modeling, and control solutions for underwater vehicles. This overview identified a gap in the field of robust control for remotely operated vehicles, particularly in dealing with parameter and environmental uncertainties. While the original sliding mode control scheme provided robust stability, it suffered from high-frequency output oscillations. There were several variations of SMC available, and each algorithm has its limitations. To address this research gap, a combination of adaptive and integral sliding mode modifications was proposed.

Chapter 3

Mathematical Modelling

Remotely operated vehicles (ROVs) are complex systems that require mathematical models for various purposes, including control system design, simulation, and performance analysis. With accurate mathematical models, ROVs are able to navigate through different underwater terrains and complete control tasks with a good precision. Also, the simulation, based on these models, are suitable to test different work scenarios and detect undesirable ROV's behavior before the physical experiment.

The fundamentals of the modelling for marine vehicles were fully described in Fossen [4]. Using common assumptions, ROV is treated as a single rigid body with six degrees of freedom (DOF). By considering the vehicle as a rigid body, we can simplify the mathematical modeling process while capturing the essential dynamics of the system.

In order to effectively model rigid bodies, it is crucial to consider their kinematic and dynamic properties.

I Notations

Before examining the theory underlying the modeling process, it is crucial to establish clarity on the notations.

In the context of motion with six degrees of freedom (DOF), a set of six independent coordinates is defined within the coordinate frame: three for translational directions (surge, sway, and heave) and three for rotational directions (roll, pitch, and yaw), as shown in Fig. 1.

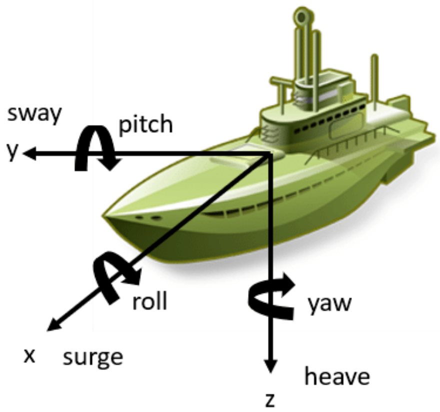

Fig. 1. Illustration of the Six Degrees of Freedom (DOF) for a Marine Vehicle

The linear position of the body in three-dimensional space is defined as $\mathbf{r} = [\mathrm{r_x}, \mathrm{r_y}, \mathrm{r_z}]^\top$, representing translations along the x, y, and z axes, respectively.

While the orientation of the body can be expressed using Euler angles, this approach can lead to singularities when the sway angle approaches $\pm 90°$. To overcome this limitation, quaternions provide a more robust representation by introducing redundancy. A quaternion, represented in scalar-first form, is defined as

$$\mathbf{q} = \mathrm{q_0} + \mathrm{q_1 i} + \mathrm{q_2 j} + \mathrm{q_3 k} = [\mathrm{q_0}, \mathrm{q_1}, \mathrm{q_2}, \mathrm{q_3}]^\top \tag{3.1}$$

This quaternion-based representation avoids the singularities associated with Euler angles and ensures a smooth and continuous description of orientation in three-dimensional space.

The velocity vectors can be defined separately: $\mathbf{v} = [\mathrm{v_x}, \mathrm{v_y}, \mathrm{v_z}]^\top$ for translation along the x, y, and z axes, and $\boldsymbol{\omega} = [\omega_\mathrm{x}, \omega_\mathrm{y}, \omega_\mathrm{z}]^\top$ for rotation around the same axes. Similarly, this applies to linear forces and torques, where $\mathbf{f}$ represents the linear forces and $\boldsymbol{\tau}$ represents the torques acting on the body.

TABLE I
Notations for Motion Parameters in ROV Modeling

Symbol	Description	Dimensionality
r	Linear position vector	$\mathbb{R}^3$
q	Angular position (orientation) vector	$\mathbb{R}^4$
v	Linear velocity vector	$\mathbb{R}^3$
ω	Angular velocity vector	$\mathbb{R}^3$
f	Vector of linear forces	$\mathbb{R}^3$
τ	Vector of torques	$\mathbb{R}^3$

For convenience, it is desirable to define combined vectors for positions, velocities, and forces as follows:

$$\bar{\mathrm{r}} = \begin{bmatrix} \mathrm{r} \\ \mathrm{q} \end{bmatrix}, \quad \bar{\mathrm{v}} = \begin{bmatrix} \mathrm{v} \\ \omega \end{bmatrix}, \quad \bar{\mathrm{f}} = \begin{bmatrix} \mathrm{f} \\ \tau \end{bmatrix}$$

To manipulate the obtained vectors, it is necessary to define cross product operators. For vectors in $\mathbb{R}^3$, the cross product is represented by multiplication with a skew-symmetric matrix $\mathrm{S}(\lambda)$:

$$\mathrm{S}(\lambda) = \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix} \tag{3.2}$$

For vectors in $\mathbb{R}^6$, the cross product operator $\bar{\times}^*$ is defined as:

$$\mathrm{x}\bar{\times}^* = \begin{bmatrix} \mathrm{S}(\mathrm{x}_2) & 0_{3\times 3} \\ \mathrm{S}(\mathrm{x}_1) & \mathrm{S}(\mathrm{x}_2) \end{bmatrix} \text{ for } \mathrm{x} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \end{bmatrix} \tag{3.3}$$

II Frames of reference

In order to determine the kinematics and dynamics of the system, it is essential to project the calculations into a consistent frame of reference. In some cases, multiple coordinate frames are established based on the system's configuration.

For ROV, it is reasonable to define two coordinate frames. These frames are the earth-fixed frame, which is inertial with fixed origin, and the body-fixed frame, which is a moving frame attached to the vehicle as depicted in Fig. 2.

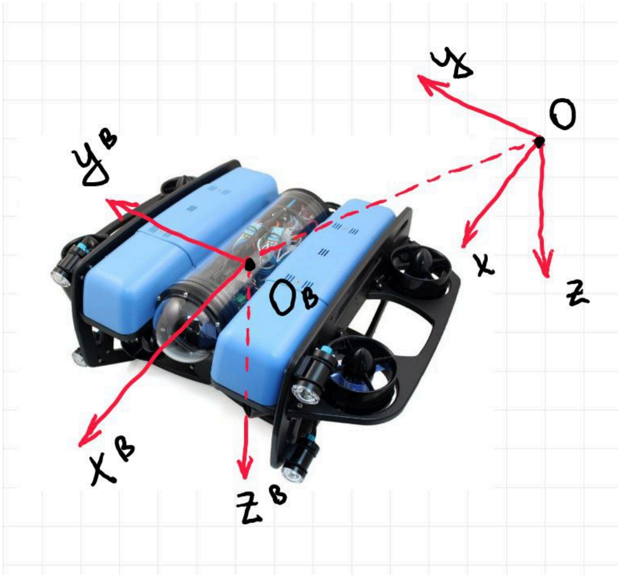

Fig. 2. Coordinate Frames Used in the Kinematic and Dynamic Modeling of ROVs

The origin of the body-fixed frame usually coincides with the vehicle's center of mass, and its axes are chosen along the vehicle's principle axes of inertia.

The state variables of the rigid body expressed in the body-fixed frame would be denoted by $^{\mathrm{B}}$ and in the earth-fixed frame by $^{\mathrm{N}}$.

III Kinematics

Kinematics describes the motion of the marine vehicle without considering the forces acting upon it. In order to describe kinematic motion of the body, it is necessary to find relation between velocities in two coordinate frames. This relation can be represented with linear transformations as:

$$\dot{\bar{\mathrm{r}}}^{\mathrm{N}} = \mathrm{J}(\bar{\mathrm{r}}^{\mathrm{N}})\bar{\mathrm{v}}^{\mathrm{B}}$$
$$\text{where } \mathrm{J}(\bar{\mathrm{r}}^{\mathrm{N}}) = \begin{bmatrix} \mathrm{R}(\bar{\mathrm{r}}^{\mathrm{N}}) & 0_{3\times3} \\ 0_{4\times3} & \mathrm{T}(\bar{\mathrm{r}}^{\mathrm{N}}) \end{bmatrix} \tag{3.4}$$

The rotational matrix R and the transformation matrix T using quaternion representation 3.1 can be expressed as follows:

$$\mathrm{R(q)} = \begin{bmatrix} 1-2\left(\mathrm{q}_2^2+\mathrm{q}_3^2\right) & 2\left(\mathrm{q}_1\mathrm{q}_2-\mathrm{q}_3\mathrm{q}_0\right) & 2\left(\mathrm{q}_1\mathrm{q}_3+\mathrm{q}_2\mathrm{q}_0\right) \\ 2\left(\mathrm{q}_1\mathrm{q}_2+\mathrm{q}_3\mathrm{q}_0\right) & 1-2\left(\mathrm{q}_1^2+\mathrm{q}_3^2\right) & 2\left(\mathrm{q}_2\mathrm{q}_3-\mathrm{q}_1\mathrm{q}_0\right) \\ 2\left(\mathrm{q}_1\mathrm{q}_3-\mathrm{q}_2\mathrm{q}_0\right) & 2\left(\mathrm{q}_2\mathrm{q}_3+\mathrm{q}_1\mathrm{q}_0\right) & 1-2\left(\mathrm{q}_1^2+\mathrm{q}_2^2\right) \end{bmatrix} \tag{3.5}$$

$$\mathrm{T(q)} = \frac{1}{2}\begin{bmatrix} -\mathrm{q}_1 & -\mathrm{q}_2 & -\mathrm{q}_3 \\ \mathrm{q}_0 & -\mathrm{q}_3 & \mathrm{q}_2 \\ \mathrm{q}_3 & \mathrm{q}_0 & -\mathrm{q}_1 \\ -\mathrm{q}_2 & \mathrm{q}_1 & \mathrm{q}_0 \end{bmatrix} \tag{3.6}$$

IV Dynamics

The Newton-Euler approach is commonly used to describe the dynamics of marine vehicles. This approach relates the applied forces and moments to the vehicle's linear and angular accelerations. The general equation of motion using the Newton-Euler approach in the body-fixed frame can be written as:

$$\mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \bar{\mathrm{v}}^{\mathrm{B}}\bar{\times}^{*}\mathrm{M}\bar{\mathrm{v}}^{\mathrm{B}} = \bar{\mathrm{f}}^{\mathrm{B}} \tag{3.7}$$

where M represents the inertia matrix of the rigid body.

The equation above can be further transformed into standard manipulator equation form:

$$\mathrm{M_B}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C_B}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} = \bar{\mathrm{f}}^{\mathrm{B}} \tag{3.8}$$

where: $\mathrm{M_B}$ = the rigid body mass matrix $\in \mathbb{R}^{6x6}$

$\mathrm{C_B}$ = the rigid body Coriolis and centripetal forces' matrix $\in \mathbb{R}^{6x6}$

Nevertheless, some additional terms should be included in the equation to determine the specifics of the ROVs model. These terms comprise added mass, which represents the inertia of the surrounding fluid, the shift of the center of buoyancy due to changes in trim and heel angles, and damping effects. By incorporating these terms into the manipulator equation derived from the Newton-Euler approach, the model becomes more accurate and reflects the natural behavior of the ROV.

A. Center of Gravity and Center of Buoyancy

Due to the robust design of the marine vehicles, the center of buoyancy (COB) is usually aligned with the center of mass (COM), but placed higher. This shift between centers causes torque acting against the capsizing (Fig. 3).

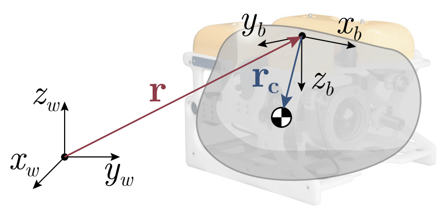

Fig. 3. Schematic representation of the marine vehicle showing the center of buoyancy (COB) and the center of mass (COM).

If we place the origin of the body frame at the center of mass, the mass matrix can be expressed as:

$$\mathrm{M_B} = \begin{bmatrix} \mathrm{mI_{3\times3}} & -\mathrm{mS}\left(\mathrm{r_G^B}\right) \\ \mathrm{mS}\left(\mathrm{r_G^B}\right) & \mathrm{I_0} \end{bmatrix} \tag{3.9}$$

where r_G^B is the vector of the gravity center in the body frame, that is eventually a zero vector.

B. Concept of added mass

Since the vehicle moves in a viscous environment, we can not neglect the inertia of the surrounding liquid. To compensate added mass effect, it is necessary to add two components into dynamics equation: the added mass and the Coriolis forces acting on the added mass.

We can define vector of dynamical parameters of our body as:

$$f_{\dot{v}} \triangleq \frac{\partial \bar{f}}{\partial \dot{\bar{v}}} \tag{3.10}$$

Consequently, the added mass matrix M_A and the Coriolis forces matrix for added mass $C_A(\bar{v}^B)$ can be expressed as:

$$M_A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = -\operatorname{diag}\{f_{\dot{v}}\}, \text{where } A_{ij} \in \mathbb{R}^{3x3} \tag{3.11}$$

$$C_A(\bar{v}^B) = \begin{bmatrix} 0_{3\times 3} & -S\left(A_{11}v^B + A_{12}\omega^B\right) \\ -S\left(A_{11}v^B + A_{12}\omega^B\right) & -S\left(A_{21}v^B + A_{22}\omega^B\right) \end{bmatrix} \tag{3.12}$$

The values of dynamical parameters are usually determined empirically. Therefore, the error on M_A and C_A can be quite large, and we will not consider these matrices in our model implementation for the control design.

C. Hydrodynamic Damping

Generally, the dynamics of underwater vehicles can be highly nonlinear and coupled. Nevertheless, during the slow non-coupled motion the damping can be approximated to linear and quadratic damping:

$$\mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}}) = -\mathrm{K}_0 - \mathrm{K}_1|\bar{\mathrm{v}}^{\mathrm{B}}| \tag{3.13}$$

The appropriate values of damping coefficients for vectors K_0 and K_1 can be discovered through several experiments.

D. Restoring forces

The common sense is to neglect all other forces acting on the vehicle except buoyancy and gravity. Although the motion of the current can also affect the dynamics, it is unpredictable and highly nonlinear, which makes it easier to compensate through control.

The weight of the body is defined as: $\mathbf{W} = \mathbf{mg}$, where m is the vehicle's mass and g is the gravity acceleration. The buoyancy force is defined as: $\mathbf{B} = \boldsymbol{\rho}\mathbf{gV}$, where ρ is the water density and V the volume of fluid displaced by the vehicle.

By transforming the weight and buoyancy force to the body-fixed frame, we get:

$$\mathrm{f}_{\mathrm{G}}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right) = \mathrm{R}^{\top}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right)[0, 0, \mathrm{W}]^{\top} \tag{3.14}$$

$$\mathrm{f}_{\mathrm{B}}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right) = -\mathrm{R}^{\top}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right)[0, 0, \mathrm{B}]^{\top} \tag{3.15}$$

Therefore, overall restoring force and moment vector is defined as:

$$g(\bar{r}^N) = -\begin{bmatrix} f_G(\bar{r}^N) + f_B(\bar{r}^N) \\ r_G^B \times f_G(\bar{r}^N) + r_B^B \times f_B(\bar{r}^N) \end{bmatrix} \tag{3.16}$$

where r_B^B is the vector of the buoyancy center in the body frame.

E. Matrix representation

Combining all the equations above, the final representation of the system dynamics is:

$$M\dot{\bar{v}}^B + C(\bar{v}^B)\bar{v}^B + D(\bar{v}^B)\bar{v}^B + g(\bar{r}^N) = \bar{f}^B \tag{3.17}$$

where: $M = M_B + M_A =$ the total mass matrix
$C(\bar{v}^B) = C_B(\bar{v}^B) + C_A(\bar{v}^B) =$ the Coriolis and centripetal forces matrix
$D(\bar{v}^B) =$ the hydrodynamic damping
$g(\bar{r}^N) =$ the restoring forces and moments

V Thrusters modelling

The force and moment vector produced by the thruster are typically represented by a complex nonlinear function $f(\bar{v}^B, V, u)$, which depends on the vehicle's velocity vector $\bar{v}^B$, the power source voltage V, and the control variable u.

However, expressing such a nonlinear relationship directly can be challenging in practical applications. As a result, some authors propose a simpler approach:

$$\bar{f}^B = T\phi(u) \tag{3.18}$$

where: T = the thrust configuration matrix, which maps body torques to truster forces $\in \mathbb{R}^{6\mathrm{xn}}$

$\phi(\mathrm{u})$ = the DC-gain transfer function, which defines relation between PWM signal and output force $\in \mathbb{R}^{\mathrm{nxn}}$

VI Summary

This chapter provides a comprehensive overview of essential concepts and equations crucial for understanding the kinematics and dynamics of ROVs.

Notation

ROVs have six degrees of freedom (DOF), which include three for translation and three for rotation. The use of quaternions to represent orientation helps to avoid singularity issues.

Frame of reference

There are two coordinate frames: the earth-fixed frame and the body-fixed frame. State variables are denoted by $_{\mathrm{B}}$ in the body-fixed frame and $_{\mathrm{N}}$ in the earth-fixed one.

Kinematics

The motion of a marine vehicle, without considering external forces, is expressed in terms of velocities for two coordinate frames (3.4):

$$\dot{\bar{\mathbf{r}}}^{\mathbf{N}} = \mathbf{J}(\bar{\mathbf{r}}^{\mathbf{N}})\bar{\mathbf{v}}^{\mathbf{B}}$$

Dynamics

The Newton-Euler approach is used to describe the dynamics of ROV by relating applied forces and moments to linear and angular accelerations (3.17):

$$\mathbf{M}\dot{\bar{\mathbf{v}}}^{\mathbf{B}} + \mathbf{C}(\bar{\mathbf{v}}^{\mathbf{B}})\bar{\mathbf{v}}^{\mathbf{B}} + \mathbf{D}(\bar{\mathbf{v}}^{\mathbf{B}})\bar{\mathbf{v}}^{\mathbf{B}} + \mathbf{g}(\bar{\mathbf{r}}^{\mathbf{N}}) = \bar{\mathbf{f}}^{\mathbf{B}}$$

Additional terms like added mass, center of buoyancy, and damping effects enhance model accuracy.

Thruster modelling

The complex relationship between thruster force and control variables is simplified using a thrust configuration matrix T and a DC-gain transfer function $\phi(\mathrm{u})$ (3.18):

$$\bar{\mathbf{f}}^{\mathbf{B}} = \mathbf{T}\boldsymbol{\phi}(\mathbf{u})$$

The mathematical models developed in this chapter lay the foundation for designing control systems. By accurately capturing the dynamics and kinematics of ROVs, these models enable precise navigation underwater. The next chapter will implement effective control strategies and enhance the performance of ROVs in various scenarios.

Chapter 4

Methodology

The development of a robust control system for underwater robots hinges on a comprehensive understanding of system dynamics and control objectives. This chapter explores control design for underwater robots, focusing on the application of sliding mode control to address challenges in marine environments. By integrating robust control strategies, the aim is to enhance the stability, accuracy, and responsiveness of underwater robotic systems operating in dynamic and unpredictable conditions.

I Design Considerations

This section provides an introduction to control systems, starting with defining objectives that serve as a foundation for understanding subsequent topics.

A. Control Objectives

The main idea of the control system design is to choose such control input u which meets the desired performance specifications while ensuring stability.

Underwater robots require precise control systems to navigate and operate effectively in challenging marine environments. These control objectives are crucial for ensuring the robot's stability, accuracy, and responsiveness:

- Position and Orientation Tracking: The robot must accurately follow a desired trajectory, maintaining its position and orientation as intended.
- Disturbance Rejection: The robot should be able to withstand external disturbances, such as ocean currents, waves, and sensor noise, to maintain stable tracking performance.
- Robustness: The control system should be robust to uncertainties in the robot's dynamics and environmental conditions, ensuring reliable operation even in unpredictable situations.
- Real-Time Implementation: The control algorithm should be computationally efficient and able to run in real-time on the robot's embedded system, enabling prompt and effective responses to changing conditions.

Next, let us discuss the challenges posed by uncertainties in control system modeling, which must be understood before diving into advanced control techniques.

B. Model Uncertainties

The estimated model dynamics may not perfectly match the actual system behavior due to imprecise parameter estimates caused by simplified dynamics and external factors. Two primary types of modeling inaccuracies are defined by J. Slotine and W. Li [12]:

1. **Structured** (or parametric) **uncertainties** : associated with errors in the terms included in the model

2. **Unstructured uncertainties** (or unmodeled dynamics) : linked to inaccuracies in the system's order

Recalling dynamics of ROV, following parameters may have some imprecision:

- **Body parameters**. The mass matrix $\mathrm{M_B}$ and the matrix of Coriolis forces $\mathrm{C_B}$ may unknown due to uncertain values of mass m and inertia matrix $\mathrm{I_0}$.

- Coefficients of **viscous damping** D. The values for linear and quadratic terms are defined empirically.

- **Restoring forces** g. Specifically, water density ρ is environment dependent and whole body volume ∇ is hard to calculate with proper accuracy.

- **Added mass parameters** $\mathrm{M_A}$ and $\mathrm{C_A}$. They cannot be calculated directly and will be excluded in future calculations.

Further, the approximated value of the parameter x is represented as $\hat{\mathrm{x}}$, while the difference between this approximation and the actual value is defined as $\tilde{\mathrm{x}} = \hat{\mathrm{x}} - \mathrm{x}$. Instead of the omitted terms, we will incorporate a common disturbance term δ.

In this way, the dynamics in equation 3.17 takes the following form:

$$\mathrm{M_B}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C_B}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) + \delta = \mathrm{T}\phi(\mathrm{u}) \tag{4.1}$$

C. Thruster Mapping Approximation

As stated before, thruster mapping is defined through the configuration matrix T and the DC-gain transfer function $\phi(\mathrm{u})$. However, $\phi(\mathrm{u})$ can be highly non-linear and voltage dependent. Let us examine the general function $\phi(\mathrm{u})$ shown in Fig. 4.

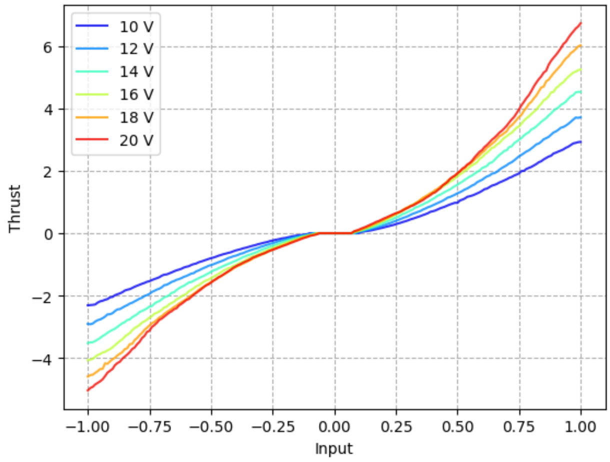

Fig. 4. Relationship Between Thrust Output and Pulse Width Modulation (PWM) Signal for ROV Thrusters

It is suggested to use a third-order polynomial to model the transfer function above. This approach is deemed suitable due to the general shape of the PWM to thrust relation.

The approximated function is expressed as

$$\hat{\phi}(\mathrm{u}) = \mathrm{k}\phi_0(\mathrm{u}) \tag{4.2}$$

where: k = the scaling coefficient

$\phi_0(\mathrm{u})$ = the nominal third-order polynomial

The value of k must be between $\mathrm{k}_{\mathrm{min}} = 0.4$ and $\mathrm{k}_{\mathrm{max}} = 1$ to cover the entire range. (Fig. 5).

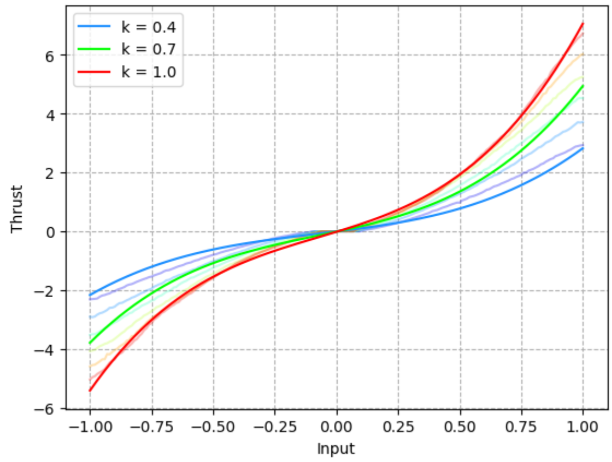

Fig. 5. Approximation Model for Thruster Mapping in ROV Control Systems

Thus, the thruster mapping takes the form:

$$\hat{\bar{\mathrm{f}}}^{\mathrm{B}} = \mathrm{T}\hat{\phi}(\mathrm{u}) = \mathrm{kT}\phi_0(\mathrm{u}) = \mathrm{B}\phi_0(\mathrm{u}) \tag{4.3}$$

D. Approximated System Dynamics

By taking into account the impact of both dynamic approximations and disturbances, we can obtain the following dynamics of the system:

$$\mathrm{M}_{\mathrm{B}}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C}_{\mathrm{B}}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) + \delta = \mathrm{B}\phi_0(\mathrm{u}) \tag{4.4}$$

For the sake of simplicity, we omit some redundant notations: $*_{\mathrm{B}}$, $\bar{*}$, $*^{\mathrm{B}}$ and $*^{\mathrm{N}}$. However, the meaning behind these notations remains valid, whereas the formulas become more comprehensible:

$$\mathrm{M}\dot{\mathrm{v}} + \mathrm{C}(\mathrm{v})\mathrm{v} + \mathrm{D}(\mathrm{v})\mathrm{v} + \mathrm{g}(\mathrm{r}) + \delta = \mathrm{B}\phi_0(\mathrm{u}) \tag{4.5}$$

Additionally, let us describe the internal and external forces acting on the body using the single nonlinear term $\mathrm{h}(\mathrm{r}, \mathrm{v})$:

$$\mathrm{h}(\mathrm{r}, \mathrm{v}) = \mathrm{C}(\mathrm{v})\mathrm{v} + \mathrm{D}(\mathrm{v})\mathrm{v} + \mathrm{g}(\mathrm{r}) \tag{4.6}$$

Finally, the approximated system dynamics is defined as:

$$\mathrm{M}\dot{\mathrm{v}} + \mathrm{h}(\mathrm{r}, \mathrm{v}) + \delta = \mathrm{B}\phi_0(\mathrm{u}) \tag{4.7}$$

E. Summary

To summarize, the main objective of the underwater control system is to accurately follow the desired trajectory, even in the presence of external disturbances and uncertainties. The ultimate goal is to make sure that the tracking error approaches zero as time progresses towards infinity.

As was previously mentioned, control systems can be negatively impacted by modeling nonlinear dynamics. Therefore, any practical design must handle them explicitly. Inverse dynamics control can be a good starting point for deriving complex nonlinear control approaches, as it addresses the nonlinearities present in the system.

II Inverse dynamics

The nonlinear control method, known as inverse dynamics, tracks a trajectory by calculating the joint actuator torques required to achieve a specific trajectory. This approach relies on exact cancellation of nonlinearities in the robot equation of motion.

The inverse dynamics control is directly related to the solution of the inverse dynamics problem. By appropriately inverting the dynamic model, a control law can cancel the nonlinear part of the dynamics, decouple the interactions between the regulated variables, and specify the time characteristics of the decay of the task errors.

A. Virtual Control

To simplify the control process, we can define a virtual control input $\boldsymbol{\nu}$, which is related to the actual control input $\mathbf{u}$ as:

$$\nu = \phi_0(\mathrm{u}) \tag{4.8}$$

It is important to note that there exists an inverse relation between the actual control input and the virtual control input, namely $\mathrm{u} = \phi_0^{-1}(\nu)$. This inverse relation will allow us to easily switch back and forth between the actual and virtual control inputs as needed.

B. Control Law Design

Recalling the approximated dynamics equation 4.7, we can design the following virtual control law to linearize the system:

$$\nu = \mathrm{B}^{+}(\mathrm{Ma} + \mathrm{h}(\mathrm{r}, \mathrm{v})) \tag{4.9}$$

while the outer-loop control a is designed as a proportional-derivative (PD) controller:

$$\mathrm{a} = \dot{\mathrm{v}}_{\mathrm{des}} - \mathrm{K}_{\mathrm{p}}\tilde{\mathrm{r}} - \mathrm{K}_{\mathrm{d}}\tilde{\mathrm{v}} \tag{4.10}$$

The equation 4.9 becomes more complex when system parameters and disturbances are unknown:

$$\nu = \hat{\mathrm{B}}^{+}(\hat{\mathrm{M}}\mathrm{a} + \hat{\mathrm{h}}(\mathrm{r}, \mathrm{v})) \tag{4.11}$$

Substitution to the dynamics yields:

$$\dot{\mathrm{v}} = \mathrm{M}^{-1}(\mathrm{B}\hat{\mathrm{B}}^{+}\mathrm{h}(\mathrm{r}, \mathrm{v}) - \hat{\mathrm{h}}(\mathrm{r}, \mathrm{v}) - \delta) + \mathrm{M}^{-1}\mathrm{B}\hat{\mathrm{B}}^{+}\hat{\mathrm{M}}\mathrm{a} \tag{4.12}$$

C. Error Analysis

In terms of tracking error $\mathrm{e} = \tilde{\mathrm{r}}$, the following system can be designed:

$$\begin{aligned}
&\mathrm{e} = \tilde{\mathrm{r}} \\
&\dot{\mathrm{e}} = \tilde{\mathrm{v}} \\
&\ddot{\mathrm{e}} = \dot{\tilde{\mathrm{v}}} = \dot{\mathrm{v}}_{\mathrm{des}} - \dot{\mathrm{v}} = \\
&\quad = \dot{\mathrm{v}}_{\mathrm{des}} - \mathrm{M}^{-1}(\mathrm{B}\hat{\mathrm{B}}^{+}\mathrm{h} - \hat{\mathrm{h}} - \delta) + \mathrm{M}^{-1}\mathrm{B}\hat{\mathrm{B}}^{+}\hat{\mathrm{M}}(\dot{\mathrm{v}}_{\mathrm{des}} - \mathrm{K}_{\mathrm{p}}\mathrm{e} - \mathrm{K}_{\mathrm{d}}\dot{\mathrm{e}})
\end{aligned} \tag{4.13}$$

Hence, the error dynamics can be represented in a form of second order differential equation as:

Add disturbance function $\mathrm{d}(\mathrm{e}, \dot{\mathrm{e}}, \mathrm{t})$

$$\begin{aligned}
\ddot{\mathrm{e}} + \mathrm{A}_1\dot{\mathrm{e}} + \mathrm{A}_0\mathrm{e} &= (\mathrm{W} - \mathrm{I})\dot{\mathrm{v}}_{\mathrm{des}} - \mathrm{M}^{-1}(\mathrm{B}\hat{\mathrm{B}}^{+}\mathrm{h} - \hat{\mathrm{h}} - \delta) \\
&= \mathrm{d}(\mathrm{e}, \dot{\mathrm{e}}, \dot{\mathrm{v}}_{\mathrm{des}})
\end{aligned} \tag{4.14}$$

with $\mathrm{W} = \mathrm{M}^{-1}\mathrm{B}\hat{\mathrm{B}}^{+}\hat{\mathrm{M}}$, $\mathrm{A}_0 = \mathrm{W}\mathrm{K}_{\mathrm{p}}$ and $\mathrm{A}_1 = \mathrm{W}\mathrm{K}_{\mathrm{d}}$

State space

$$\dot{\mathrm{x}} = \mathrm{A}\mathrm{x} + \mathrm{B}\mathrm{d} \tag{4.15}$$

$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$$

$$A = \begin{bmatrix} 0_{6\times7} & I_{6\times6} \\ -A_0 & -A_1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{6\times6} \\ I_{6\times6} \end{bmatrix}$$

Discretization(wiki), analytical solution

$$x = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bd(\tau)\tau \tag{4.16}$$

Find bounds -> define value under the integral

$$\|e\| \leq \frac{\|d\|}{\lambda_{min}(A_0)} \tag{4.17}$$

D. Summary

Add paragraph about: undefined or big bounds(no bounds on $\dot{v}_{des}$), hard to tune, not robust

As a result, the inverse dynamics technique may not be the best option for effectively controlling underwater systems. Further we introduce sliding mode control, which is a robust control technique that can achieve desired control objectives in the presence of uncertainties.

III Sliding Mode

As discussed before, there are several robust controller designs available. However, the sliding mode approach, suggested by Vadim Utkin in the late 1970s, is highly regarded as the most sophisticated and frequently implemented one [13].

Sliding mode control (SMC) is a nonlinear control method that guarantees robust control of systems with uncertainties and disturbances. This technique involves developing a sliding surface within the state space and directing the system's trajectory to slide along this surface (Fig. 6).

Compared to other nonlinear control methods, SMC is a relatively straightforward solution to implement with a basic understanding of system dynamics and sliding surface design. SMC provides a fast transient response due to the sliding dynamics, which makes it possible to track desired references or trajectories quickly.

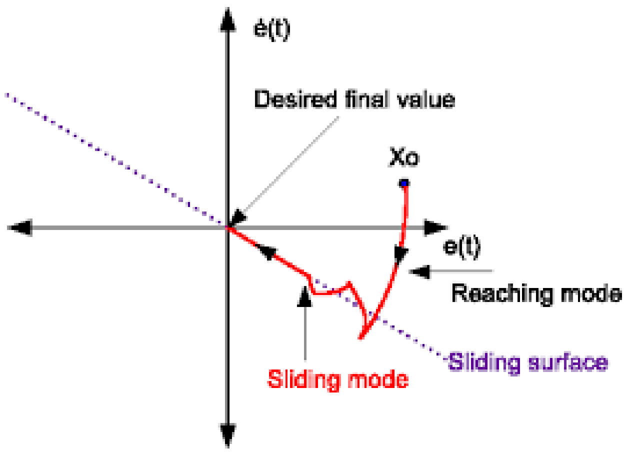

Fig. 6. General Schematic of Sliding Mode Control (SMC)

A. Sliding Surface Design

In sliding mode control, a sliding surface is a hyperplane in the state space that defines the desired system behavior. The aim of the control is to force the system's trajectory to slide along this surface. As long as the control law is in effect, the system's trajectory will stay on the sliding surface once it reaches it (Fig. 7).

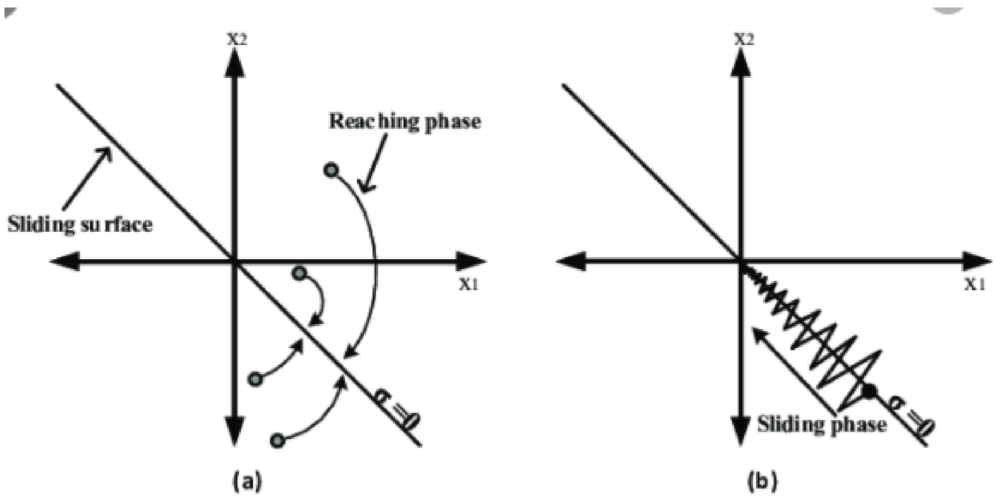

Fig. 7. Phases of Sliding Mode Control: Reaching Phase and Sliding Phase

The design of the sliding surface is critical for the performance of the SMC system. The sliding surface should be:

- Reachable: The system's trajectory should be able to reach the sliding surface in a finite amount of time.
- Invariant: The system will stay on the sliding surface as long as the control law is in effect once its trajectory reaches it.
- Attractive: The control law should attract the system's trajectory to the sliding surface and keep it there.

In order to satisfy the conditions above, the sliding surface is designed to be an invariant set. Invariant sets are sets of states in the state space that, once entered, cannot be exited under the action of the control law.

B. Sliding Condition

In the state space $\mathbb{R}^{\mathrm{n}}$, let us define the time-varying surface given by scalar equation $\mathbf{s}(\mathbf{r}, \mathbf{t})$:

$$\mathbf{s}(\mathbf{r}, \mathbf{t}) = (\frac{\mathrm{d}}{\mathrm{dt}} + \lambda)^{\mathrm{n}-1}\tilde{\mathrm{r}} \tag{4.18}$$

where n denotes the system's order and λ is a positive scalar.

In order to ensure convergence of $\mathbf{s}(\mathbf{r}, \mathbf{t})$ along all system trajectories in finite time, let us define Lyapunov candidate $\mathbf{V} = \mathbf{s^2}$ as the squared distance to the surface.

The sliding condition can then be formulated accordingly:

$$\frac{\mathrm{dV}}{\mathrm{dt}} < -\eta\sqrt{\mathrm{V}} \quad \text{or} \quad \frac{1}{2}\frac{\mathrm{d}}{\mathrm{dt}}||\mathrm{s}||^2 = \mathrm{s}^{\mathrm{T}}\dot{\mathrm{s}} < -\eta||\mathrm{s}|| \tag{4.19}$$

where $\eta > 0$ defines the rate of convergence to the sliding surface.

When the sliding condition is satisfied, the surface becomes an invariant set and implies convergence to $\tilde{\mathbf{r}}$, since the system described by the differential equation:

$$\mathrm{s} = (\frac{\mathrm{d}}{\mathrm{dt}} + \lambda)^{\mathrm{n}-1}\tilde{\mathrm{r}} = 0 \tag{4.20}$$

is inherently stable and remains at the equilibrium point $\tilde{\mathbf{r}} = \mathbf{0}$.

Applying such transformation yields a new representation of the tracking performance:

$$\mathrm{s} \to 0 \Rightarrow \tilde{\mathrm{r}} \to 0 \tag{4.21}$$

In other words, tracking $\mathbf{r}$ is the same as staying on the sliding surface. It is thus possible to replace the tracking problem of the n-dimensional vector $\mathbf{r}$ with a first order stabilization problem in s.

C. Control Law Design

The control input $\mathbf{a}$ comprises two distinct components: nominal control $\mathbf{a_n}$, and an additional robust part $\mathbf{a_s}$:

$$\mathrm{a} = \mathrm{a_n} + \mathrm{a_s} \tag{4.22}$$

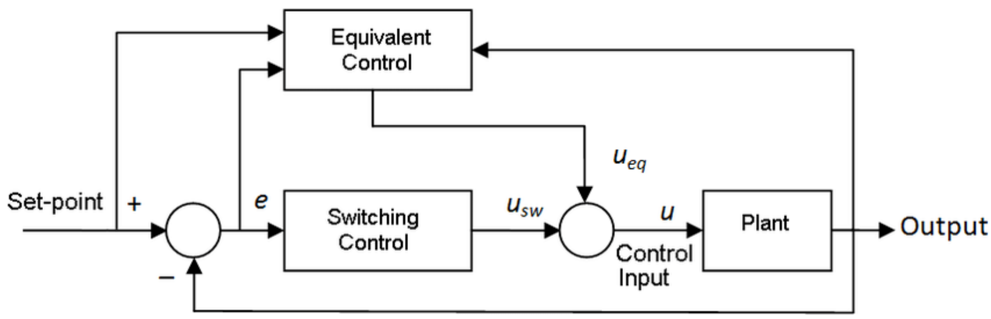

Fig. 8. Control Scheme for Sliding Mode Control (SMC) in ROVs

The nominal control $\mathbf{a_n}$ aims to compensate for the system's dynamics, while the robustifying component $\mathbf{a_s}$ is designed to enhance the controller's stability and performance by providing additional corrective action to counteract uncertainties and disturbances.

To simplify computations, we can use a virtual force $\mathrm{f_v}$:

$$\mathrm{f_v} = \mathrm{T}\nu \tag{4.23}$$

which can represent the desired behavior of the system without introducing input uncertainty.

Using the inverse dynamics approach 4.9, we can apply the outer loop controller to partially linearize the system with model estimates:

$$\hat{\mathrm{f}} = \hat{\mathrm{k}}\mathrm{f_v} = \hat{\mathrm{M}}\mathrm{a} + \hat{\mathrm{h}}(\mathrm{r},\mathrm{v}) \tag{4.24}$$

Expressing virtual control input, we obtain:

$$\mathrm{f_v} = \frac{\hat{\mathrm{M}}\mathrm{a} + \hat{\mathrm{h}}(\mathrm{r},\mathrm{v})}{\hat{\mathrm{k}}} \tag{4.25}$$

Substitution to the dynamics 4.7 yields the equation:

$$\begin{aligned} \dot{\mathrm{v}} &= \underbrace{\mathrm{M}^{-1}(\frac{\mathrm{k}}{\hat{\mathrm{k}}}\hat{\mathrm{h}}(\mathrm{r},\mathrm{v}) - \mathrm{h}(\mathrm{r},\mathrm{v}) - \delta)}_{\mathrm{F(r,v)}} + \underbrace{\frac{\mathrm{k}}{\hat{\mathrm{k}}}\mathrm{M}^{-1}\hat{\mathrm{M}}}_{\mathrm{K}}\,\mathrm{a} = \\ &= \mathrm{F}(\mathrm{r},\mathrm{v}) + \mathrm{Ka} \end{aligned} \tag{4.26}$$

where the terms $\mathrm{F}(\mathrm{r},\mathrm{v})$ and K represent dynamical and inertial uncertainties.

The time derivative of s is connected to dynamics as follows:

$$\dot{\mathrm{s}} = \dot{\tilde{\mathrm{v}}} + \lambda\,\tilde{\mathrm{v}} = \mathrm{a_n} - \dot{\mathrm{v}} = \mathrm{a_n} - \mathrm{F} - \mathrm{K}(\mathrm{a_n} + \mathrm{a_s}) = \mathrm{w} - \mathrm{Ka_s} \tag{4.27}$$

with error function $\mathrm{w}(\mathrm{r},\mathrm{v}) = (\mathrm{I} - \mathrm{K})\mathrm{a_n} - \mathrm{F}(\mathrm{r},\mathrm{v})$

Substitution to sliding condition 4.19 yields:

$$\mathrm{s^T w} - \mathrm{s^T K a_s} \le \|\mathrm{s}\|\|\mathrm{w}\| - \mathrm{s^T K a_s} \le -\eta\|\mathrm{s}\| \tag{4.28}$$

Let us recall that for any symmetric matrix P:

$$\sigma_{\min}^2 \|x\|^2 \leq \|x^T P x\| \leq \sigma_{\max}^2 \|x\|^2 \tag{4.29}$$

with $\sigma_{\min}$ and $\sigma_{\max}$ being the largest and smallest eigenvalues of matrix P.

Thus, we can use property 4.29 to choose the stabilizing control $\mathbf{a}_s$ as:

$$a_s = \frac{\alpha\hat{k}}{\sigma_{\min}^2}\hat{M}^{-1}\frac{s}{\|s\|} = \rho\frac{s}{\|s\|} \tag{4.30}$$

where $\sigma_{\min}$ is minimal singular value of M^{-1} which provide:

$$\|s\|\|w\| - s^T K a_s \leq \|s\|\|w\| - k\frac{\alpha}{\sigma_{\min}^2\|s\|}s^T M^{-1} s \leq \|s\|\|w\| - \alpha k\|s\| \tag{4.31}$$

but by definition $k_{\min} < k < k_{\max}$, therefore

$$\|s\|\|w\| - \alpha k\|s\| \leq \|s\|\|w\| - \alpha k_{\min}\|s\| < -\eta\|s\| \tag{4.32}$$

Setting gain α accordingly to:

$$\alpha > \frac{\|w\| + \eta}{k_{\min}} \tag{4.33}$$

will satisfy sliding conditions.

The final expression for sliding control:

$$a_s = \begin{cases} \rho\frac{s}{\|s\|}, & \|s\| > 0 \\ 0, & \|s\| = 0 \end{cases} \tag{4.34}$$

In order to reduce chattering, the controller above is effectively smoothed using the boundary layer:

$$\mathrm{a_s} = \begin{cases} \rho\frac{\mathrm{s}}{\|\mathrm{s}\|}, & \|\mathrm{s}\| > \epsilon \\ \rho\frac{\mathrm{s}}{\epsilon}, & \|\mathrm{s}\| \leq \epsilon \end{cases} \tag{4.35}$$

where ϵ is the boundary layer thickness.

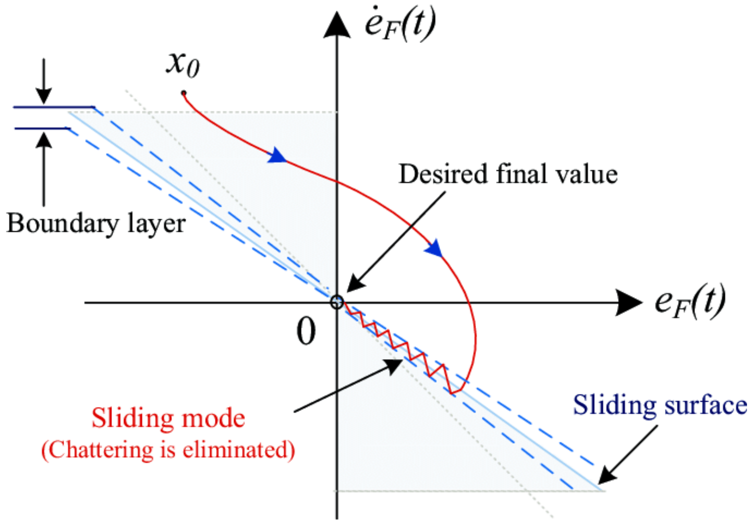

Fig. 9. Sliding Mode Control Scheme with the Addition of a Boundary Layer to Reduce Chattering

The nominal control $\mathrm{a_n}$ can be designed in a form of PD controller:

$$\mathrm{a_n} = -\mathrm{K_p}\tilde{\mathrm{r}}^{\mathrm{B}} - \mathrm{K_d}\tilde{\mathrm{v}}^{\mathrm{B}} \tag{4.36}$$

The resulting controller is then given as follows:

$$\nu = \hat{\mathrm{B}}^{+}(\hat{\mathrm{M}}(\mathrm{a_n} + \mathrm{a_s}) + \hat{\mathrm{h}}(\mathrm{r}, \mathrm{v})) \tag{4.37}$$

$$\mathrm{a_n} = -\mathrm{K_p}\tilde{\mathrm{r}} - \mathrm{K_d}\tilde{\mathrm{v}} \tag{4.38}$$

$$\mathrm{s} = \tilde{\mathrm{v}} + \lambda\,\tilde{\mathrm{r}} \tag{4.39}$$

$$\mathrm{a_s} = \begin{cases} \rho\frac{\mathrm{s}}{\|\mathrm{s}\|}, & \|\mathrm{s}\| > \epsilon \\ \rho\frac{\mathrm{s}}{\epsilon}, & \|\mathrm{s}\| \le \epsilon \end{cases} \tag{4.40}$$

D. Summary

Although sliding mode control has proven to be an effective approach, there are certain drawbacks associated with its use. These include chattering, the discontinuous nature of control, the complex design of fine-tuning process and no limits on control input. Nevertheless, ongoing advancements in control theory and implementation techniques are serving to enhance the value of this method for a range of engineering applications.

While sliding mode control offers robustness in many scenarios, optimization-based control approaches provide an alternative avenue for addressing some of its limitations.

IV Optimization-based control

Optimization-based control is a highly promising solution for complex control problems, as it endeavors to discover optimal control inputs that fulfill desired performance criteria while also taking system constraints into account.

In modern engineering applications, the pursuit of flexible and versatile control solutions necessitates the use of optimization techniques. These techniques serve to optimize system performance, incorporate constraints and objectives, and facilitate precise tuning and adaptation within complex systems.

A. Optimization problem

The core of optimization-based control lies in formulating and solving optimization problems for to the specific dynamics and constraints of the system. Recalling a sliding mode controller 4.37:

$$\hat{\mathrm{M}}(\mathrm{a_n} + \mathrm{a_s}) + \hat{\mathrm{h}}(\mathrm{r}, \mathrm{v}) = \hat{\mathrm{B}}\nu$$

This equation represents the controller dynamics, where $\hat{\mathrm{M}}$, $\hat{\mathrm{h}}$, and $\hat{\mathrm{B}}$ are approximated system matrices, $\mathrm{a_n}$ represents nominal control inputs, $\mathrm{a_s}$ denotes the sliding mode control inputs, and ν represents the virtual control input.

In sliding mode control, a key condition, known as the sliding condition, ensures the system trajectories converge to a specified surface. Mathematically, this condition can be represented as 4.28:

$$\mathrm{s^T K a_s} \leq -\eta\|\mathrm{s}\| + \|\mathrm{s}\|\|\mathrm{w}\|$$

Here, s represents the sliding surface, K is a gain matrix, η is a positive constant, and w represents disturbance.

General control optimization can be framed as a Quadratic Programming (QP) problem, aiming to minimize a cost function subject to system dynamics and constraints. The basic form of the optimization problem is as follows:

$$\begin{aligned} \min_{\mathrm{a_s},\nu} \quad & \mathrm{a_s^T R_a a_s} \\ \text{s.t.} \quad & \mathrm{s^T K a_s} \leq -\eta\|\mathrm{s}\| + \|\mathrm{s}\|\|\mathrm{w}\| \\ & \hat{\mathrm{M}}\mathrm{a_s} - \hat{\mathrm{B}}\nu = -(\mathrm{Ma_n} + \mathrm{h(r, v)}) \end{aligned} \tag{4.41}$$

However, this basic formulation lacks control bounds. To address this, additional constraints are introduced, ensuring that the control inputs remain within feasible bounds:

$$\phi_0(\mathrm{u_{min}}) \leq \nu \leq \phi_0(\mathrm{u_{max}}) \tag{4.42}$$

Moreover, to alleviate chattering - a phenomenon characterized by high-frequency oscillations in the control signal - an uncertainty term d is introduced to the sliding condition. The resulting relaxed sliding condition yields the following optimization problem:

$$\begin{aligned} \min_{\mathrm{a_s},\nu,\mathrm{d}} \quad & \mathrm{a_s^T R_a a_s} + \nu^{\mathrm{T}} \mathrm{R}_\nu \nu + \gamma^2 \mathrm{d} \\ \text{s.t.} \quad & \mathrm{s^T K a_s} \leq -\eta\|\mathrm{s}\| + \|\mathrm{s}\|\|\mathrm{w}\| + \mathrm{d} \\ & \hat{\mathrm{M}}\mathrm{a_s} - \hat{\mathrm{B}}\nu = -(\mathrm{Ma_n} + \mathrm{h(r, v)}) \\ & \phi_0(\mathrm{u_{min}}) \leq \nu \leq \phi_0(\mathrm{u_{max}}) \end{aligned} \tag{4.43}$$

Additionally, to smooth the control input and reduce abrupt changes, a penalty term is introduced to penalize large deviations between consecutive control inputs:

$$
\begin{aligned}
\min_{\mathrm{a_s},\nu,\mathrm{d}} \quad & \mathrm{a_s^T R_a a_s} + \nu^{\mathrm{T}} \mathrm{R}_\nu \nu + \gamma_0^2 \mathrm{d} + \gamma_1 \|\mathrm{a_s} - \mathrm{a_{s(prev)}}\| \\
\text{s.t.} \quad & \mathrm{s^T K a_s} \leq -\eta \|\mathrm{s}\| + \|\mathrm{s}\| \|\mathrm{w}\| + \mathrm{d} \\
& \hat{\mathrm{M}} \mathrm{a_s} - \hat{\mathrm{B}} \nu = -(\mathrm{M a_n} + \mathrm{h}(\mathrm{r}, \mathrm{v})) \\
& \phi_0(\mathrm{u_{min}}) \leq \nu \leq \phi_0(\mathrm{u_{max}})
\end{aligned}
\tag{4.44}
$$

B. Control Law Design

In optimization-based control, the control law is derived from the solution to the optimization problem outlined above. By iteratively solving the optimization problem, optimal control inputs are computed in real-time, allowing the control system to effectively adapt to changing conditions and disturbances.

C. Summary

Chapter 5

Evaluation and Discussion

I System Description

A. BlueROV Heavy

Photo

B. Input Mapping

B(inputs)

C. Simulator

mapping and plots

D. Codebase

implemented in mujoco

II Controllers

A. Inverse dynamics

Control law

PID Coefficients

Plots

B. Sliding mode

Control law

PID Coefficients + others

Plots

C. Optimization based

Control law

Plots

D. Comparison

Plot error norm

Chapter 6

Conclusion

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