Chapter 4

Methodology

The development of a robust control system for underwater robots hinges on a comprehensive understanding of system dynamics and control objectives. This chapter explores control design for underwater robots, focusing on the application of sliding mode control to address challenges in marine environments. By integrating robust control strategies, the aim is to enhance the stability, accuracy, and responsiveness of underwater robotic systems operating in dynamic and unpredictable conditions.

I Design Considerations

This section provides an introduction to control systems, starting with defining objectives that serve as a foundation for understanding subsequent topics.

A. Control Objectives

The main idea of the control system design is to choose such control input u which meets the desired performance specifications while ensuring stability.

Underwater robots require precise control systems to navigate and operate effectively in challenging marine environments. These control objectives are crucial for ensuring the robot's stability, accuracy, and responsiveness:

- Position and Orientation Tracking: The robot must accurately follow a desired trajectory, maintaining its position and orientation as intended.
- Disturbance Rejection: The robot should be able to withstand external disturbances, such as ocean currents, waves, and sensor noise, to maintain stable tracking performance.
- Robustness: The control system should be robust to uncertainties in the robot's dynamics and environmental conditions, ensuring reliable operation even in unpredictable situations.
- Real-Time Implementation: The control algorithm should be computationally efficient and able to run in real-time on the robot's embedded system, enabling prompt and effective responses to changing conditions.

Next, let us discuss the challenges posed by uncertainties in control system modeling, which must be understood before diving into advanced control techniques.

B. Model Uncertainties

The estimated model dynamics may not perfectly match the actual system behavior due to imprecise parameter estimates caused by simplified dynamics and external factors. There are two primary types of modeling inaccuracies according to J. Slotine and W. Li [1]:

1. structured (or parametric) uncertainties
2. unstructured uncertainties (or unmodeled dynamics)

The first type is associated with errors in the terms included in the model, while the second type is linked to inaccuracies in the system's order.

Recalling dynamics of ROV, following parameters may have some imprecision:

- Body parameters. The mass matrix $\mathrm{M_B}$ and the matrix of Coriolis forces $\mathrm{C_B}$ may unknown due to uncertain values of mass m and inertia matrix $\mathrm{I_0}$.
- Coefficients of viscous damping D. The values for linear and quadratic terms are defined empirically.
- Restoring forces g. Specifically, water density ρ is environment dependent and whole body volume ∇ is hard to calculate with proper accuracy.
- Added mass parameters $\mathrm{M_A}$ and $\mathrm{C_A}$. They cannot be calculated directly and will be excluded in future calculations.

Further, the approximated value of the parameter x is represented as $\hat{\mathrm{x}}$, while the difference between this approximation and the actual value is defined as $\tilde{\mathrm{x}} = \hat{\mathrm{x}} - \mathrm{x}$. Instead of the omitted terms, we will incorporate a common disturbance term δ.

C. Thruster Mapping Approximation

As stated before, thruster mapping is defined through the configuration matrix T and the DC-gain transfer function $\phi(\mathrm{u})$. However, $\phi(\mathrm{u})$ can be highly nonlinear and voltage dependent (Fig. 1).

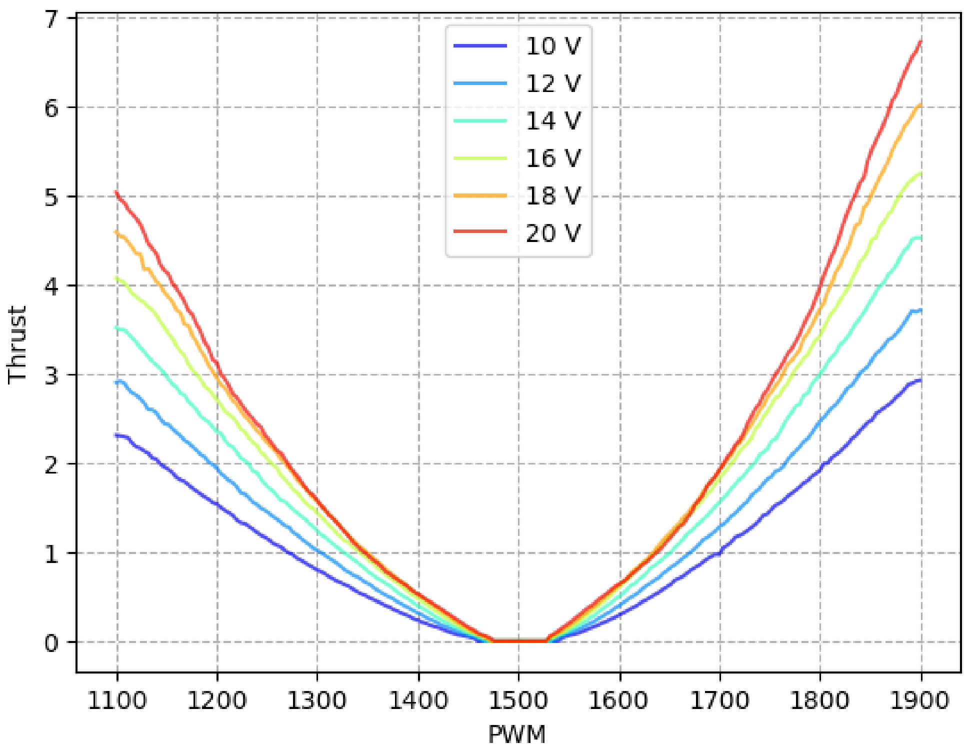

Fig. 1. The relationship between thrust and PWM.

Therefore, it is proposed to model this relation for the half of the range using two linear bounds from the inflection point. This approach is deemed appropriate due to the presence of symmetry. The nominal approximation $\mathrm{k_n}$ is defined by the middle line between the lower and upper bounds $\mathrm{k_{min}}$ and $\mathrm{k_{max}}$, which cover almost the whole possible range (Fig. 2).

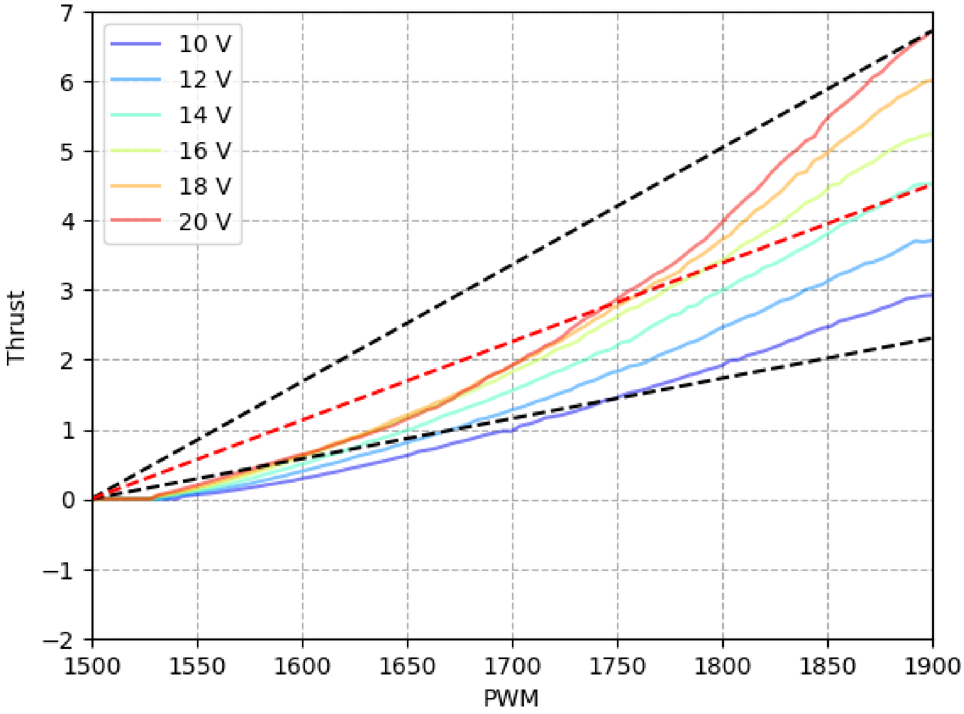

Fig. 2. The thruster mapping approximation.

The function can be expressed as $\hat{\phi}(\mathrm{u}) = \mathrm{ku}$, where k is a slope coefficient. The value of k should satisfy $\mathrm{k_{min}} < \mathrm{k} < \mathrm{k_{max}}$, and its estimate is given by $\mathrm{k_n} = \frac{\mathrm{k_{max}} - \mathrm{k_{min}}}{2}$.

Thus, the thruster mapping takes the form:

$$\hat{\bar{\mathrm{f}}}^{\mathrm{B}} = \mathrm{T}\hat{\phi}(\mathrm{u}) = \mathrm{kTu} = \mathrm{Bu} \tag{4.1}$$

By taking into account the impact of both dynamic approximations and disturbances, we can obtain the following dynamics of the system:

$$\mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) + \delta = \mathrm{Bu} \tag{4.2}$$

D. Summary

To summarize, the ultimate goal of the underwater control system is to follow the desired trajectory in the presence of external disturbances and uncertainties. By calculating the difference between the actual trajectory r^{B} and the desired trajectory $\mathrm{r_{des}}^{\mathrm{B}}$, we obtain the tracking error, which is expressed as $\tilde{\mathrm{r}}^{\mathrm{B}} = \mathrm{r}^{\mathrm{B}}_{\mathrm{des}} - \mathrm{r}^{\mathrm{B}}$. The control objective can be reformulated to ensure that the tracking error approaches zero as time progresses towards infinity.

As was previously mentioned, nonlinear control systems can be negatively impacted by modeling inaccuracies. Therefore, any practical design must handle them explicitly. Inverse dynamics control can be a good starting point for deriving complex nonlinear control approaches, as it addresses the nonlinearities present in the system.

II Inverse dynamics

The nonlinear control method, known as inverse dynamics, tracks a trajectory by calculating the joint actuator torques required to achieve a specific trajectory. This approach relies on exact cancellation of nonlinearities in the robot equation of motion.

The inverse dynamics control is directly related to the solution of the inverse dynamics problem. By appropriately inverting the dynamic model, a control law can cancel the nonlinear part of the dynamics, decouple the interactions between the regulated variables, and specify the time characteristics of the decay of the task errors.

A. Control Law Design

Recalling the system dynamics equation, we can design the following control law to linearize the system:

$$\mathrm{u} = \mathrm{B}^{+}(\mathrm{Ma} + \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}})) \tag{4.3}$$

where a is outer-loop control to be designed as a proportional-derivative (PD) controller:

$$\mathrm{a} = \dot{\mathrm{v}}^{\mathrm{B}}_{\mathrm{des}} - \mathrm{K_p}\tilde{\mathrm{r}}^{\mathrm{B}} - \mathrm{K_d}\tilde{\mathrm{v}}^{\mathrm{B}} \tag{4.4}$$

In order to determine the required control inputs, it is important to have an accurate model of the system dynamics. However, this approach may not be effective when dealing with nonlinear systems that involve uncertainties and disturbances.

The equation 4.3 becomes more complex when system parameters and disturbances are unknown:

$$\hat{\mathrm{u}} = \hat{\mathrm{B}}^{+}(\hat{\mathrm{M}}\mathrm{a} + \hat{\mathrm{C}}(\mathrm{v}^{\mathrm{B}})\mathrm{v}^{\mathrm{B}} + \hat{\mathrm{D}}(\mathrm{v}^{\mathrm{B}})\mathrm{v}^{\mathrm{B}} + \hat{\mathrm{g}}(\mathrm{r}^{\mathrm{B}})) \tag{4.5}$$

Substitution to the dynamics yields:

$$\dot{\mathrm{v}}^{\mathrm{B}} = \mathrm{M}^{-1}(\mathrm{B}\hat{\mathrm{B}}^{+}\mathrm{f} - \hat{\mathrm{f}} - \delta) + \mathrm{M}^{-1}\mathrm{B}\hat{\mathrm{B}}^{+}\hat{\mathrm{M}}\mathrm{a} \tag{4.6}$$

with $\mathrm{f} = \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}})$

B. Error Analysis

In terms of tracking error $\mathrm{e} = \tilde{\mathrm{r}}^{\mathrm{B}}$, the following system can be designed:

$$\begin{aligned}
&\mathrm{e} = \tilde{\mathrm{r}}^{\mathrm{B}} \\
&\dot{\mathrm{e}} = \tilde{\mathrm{v}}^{\mathrm{B}} \\
&\ddot{\mathrm{e}} = \dot{\tilde{\mathrm{v}}}^{\mathrm{B}} = \dot{\mathrm{v}}^{\mathrm{B}}_{\mathrm{des}} - \dot{\mathrm{v}}^{\mathrm{B}} = \\
&\quad = \dot{\mathrm{v}}^{\mathrm{B}}_{\mathrm{des}} - \mathrm{M}^{-1}(\mathrm{B}\hat{\mathrm{B}}^{+}\mathrm{f} - \hat{\mathrm{f}} - \delta) + \mathrm{M}^{-1}\mathrm{B}\hat{\mathrm{B}}^{+}\hat{\mathrm{M}}(\dot{\mathrm{v}}^{\mathrm{B}}_{\mathrm{des}} - \mathrm{K_p}\mathrm{e} - \mathrm{K_d}\dot{\mathrm{e}})
\end{aligned}$$

Hence, the error dynamics can be represented in a form of second order differential equation as:

$$\ddot{\mathrm{e}} + \mathrm{A}_1\dot{\mathrm{e}} + \mathrm{A}_0\mathrm{e} = (\mathrm{W} - \mathrm{I})\dot{\mathrm{v}}^{\mathrm{B}}_{\mathrm{des}} - \mathrm{M}^{-1}(\mathrm{B}\hat{\mathrm{B}}^{+}\mathrm{f} - \hat{\mathrm{f}} - \delta) \tag{4.7}$$

with $\mathrm{W} = \mathrm{M}^{-1}\mathrm{B}\hat{\mathrm{B}}^{+}\hat{\mathrm{M}}$, $\mathrm{A}_0 = \mathrm{W}\mathrm{K_p}$ and $\mathrm{A}_1 = \mathrm{W}\mathrm{K_d}$

As a result, the inverse dynamics technique may not be the best option for effectively controlling underwater systems. Further we introduce sliding mode control, which is a robust control technique that can achieve desired control objectives in the presence of uncertainties.

III Sliding Mode

As discussed before, there are several robust controller designs available. However, the sliding mode approach, suggested by Vadim Utkin in the late 1970s, is highly regarded as the most sophisticated and frequently implemented one [2].

Sliding mode control (SMC) is a nonlinear control method that guarantees robust control of systems with uncertainties and disturbances. This technique involves developing a sliding surface within the state space and directing the system's trajectory to slide along this surface (Fig. 3).

Compared to other nonlinear control methods, SMC is a relatively straightforward solution to implement with a basic understanding of system dynamics and sliding surface design. SMC provides a fast transient response due to the sliding dynamics, which makes it possible to track desired references or trajectories quickly.

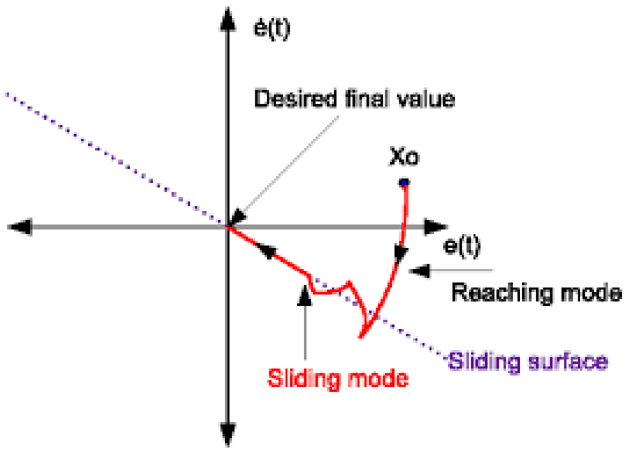

Fig. 3. A general sliding mode scheme.

Additionally, the sliding surface ensures robustness to uncertainties and disturbances by making the system behavior insensitive to these factors.

In summary, SMC is a simple and effective solution for controlling nonlinear systems.

A. Sliding Surface Design

In sliding mode control, a sliding surface is a hyperplane in the state space that defines the desired system behavior. The aim of the control is to force the system's trajectory to slide along this surface. As long as the control law is in effect, the system's trajectory will stay on the sliding surface once it reaches it

(Fig. 4).

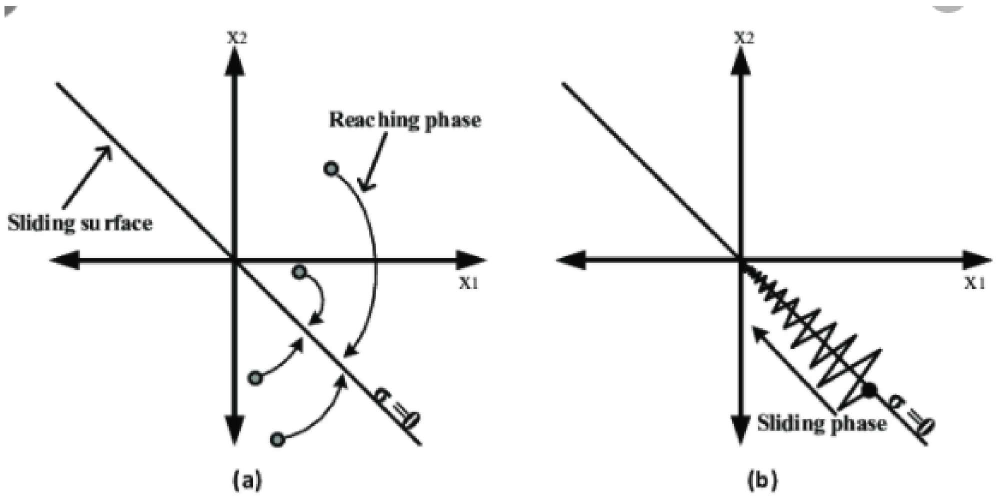

Fig. 4. The phases of sliding mode control.

The design of the sliding surface is critical for the performance of the SMC system. The sliding surface should be:

- Reachable: The system's trajectory should be able to reach the sliding surface in a finite amount of time.
- Invariant: The system will stay on the sliding surface as long as the control law is in effect once its trajectory reaches it.
- Attractive: The control law should attract the system's trajectory to the sliding surface and keep it there.

In order to satisfy the conditions above, the sliding surface is designed to be an invariant set. Invariant sets are sets of states in the state space that, once entered, cannot be exited under the action of the control law.

In the state space $\mathbb{R}^n$, let us define the time-varying surface given by scalar equation $s(\bar{r}^B, t)$:

$$s(r^B, t) = (\frac{d}{dt} + \lambda)^{n-1}\tilde{r}^B \tag{4.8}$$

where n denotes the system's order and λ is a positive scalar.

In order to ensure convergence of s along all system trajectories in finite time, let us define Lyapunov candidate $\mathrm{V} = \mathrm{s}^2$ as the squared distance to the surface. The sliding condition can then be formulated accordingly:

$$\frac{\mathrm{dV}}{\mathrm{dt}} < -\eta\sqrt{\mathrm{V}} \quad \text{or} \quad \frac{1}{2}\frac{\mathrm{d}}{\mathrm{dt}}\|\mathrm{s}\|^2 = \mathrm{s}^{\mathrm{T}}\dot{\mathrm{s}} < \eta\|\mathrm{s}\| \tag{4.9}$$

where $\eta > 0$ defines the rate of convergence to the sliding surface.

When the sliding condition is satisfied, the surface becomes an invariant set and implies convergence to $\tilde{\mathrm{r}}^{\mathrm{B}}$, since the system described by the differential equation:

$$\mathrm{s} = (\frac{\mathrm{d}}{\mathrm{dt}} + \lambda)^{\mathrm{n}-1}\tilde{\mathrm{r}}^{\mathrm{B}} = 0 \tag{4.10}$$

is inherently stable and remains at the equilibrium point $\tilde{\mathrm{r}}^{\mathrm{B}} = 0$.

Applying such transformation yields a new representation of the tracking performance:

$$\mathrm{s} \to 0 \Rightarrow \tilde{\mathrm{r}}^{\mathrm{B}} \to 0 \tag{4.11}$$

In other words, tracking r^{B} is the same as staying on the sliding surface. It is thus possible to replace the tracking problem of the n-dimensional vector r^{B} with a first order stabilization problem in s.

B. Control Law Design

The controller comprises two distinct components: nominal control $\mathrm{a_n}$, and an additional robust part $\mathrm{a_s}$:

$$\mathrm{a} = \mathrm{a_n} + \mathrm{a_s} \tag{4.12}$$

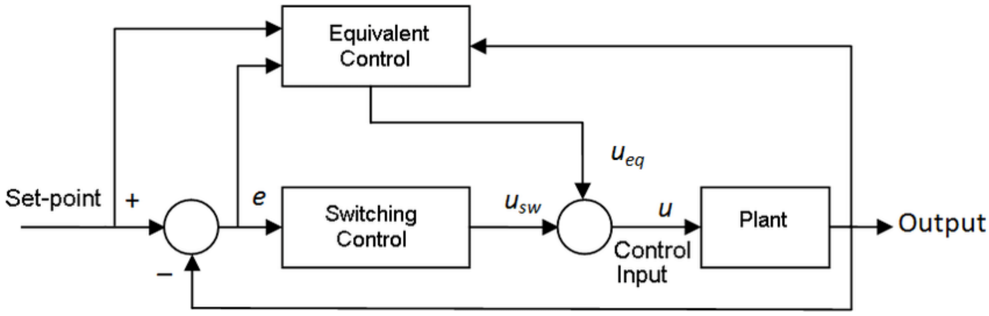

Fig. 5. The sliding mode control scheme

The nominal control $\mathrm{a_n}$ aims to compensate for the system's dynamics, while the robustifying component $\mathrm{a_s}$ is designed to enhance the controller's stability and performance by providing additional corrective action to counteract uncertainties and disturbances.

To simplify computations, we can use a virtual control, $\mathrm{f_v = Tu}$, which can represent the desired behavior of the system without introducing input uncertainty.

Using the inverse dynamics approach, we can apply the outer loop controller to partially linearize the system with model estimates:

$$\mathrm{\hat{f}^B = \hat{B}u = \hat{k}Tu = \hat{k}f_v = \hat{M}a + \hat{C}(v^B)v^B + \hat{D}(v^B)v^B + \hat{g}(r^B)} \tag{4.13}$$

Expressing virtual control input, we obtain:

$$\mathrm{f_v = \frac{\hat{M}a + \hat{C}(v^B)v^B + \hat{D}(v^B)v^B + \hat{g}(r^B)}{\hat{k}}} \tag{4.14}$$

Substitution to the dynamics yields the equation:

$$\begin{aligned} \mathrm{\dot{\bar{v}}^B} &= \mathrm{M^{-1}(\frac{k}{\hat{k}}\hat{f}(\bar{r}^N, \bar{v}^B) - f(\bar{r}^N, \bar{v}^B) - \delta) + \frac{k}{\hat{k}}M^{-1}\hat{M}a =} \\ &= \mathrm{F(\bar{r}^N, \bar{v}^B) + Ka} \end{aligned} \tag{4.15}$$

with $\mathrm{f}(\bar{\mathrm{r}}^{\mathrm{N}}, \bar{\mathrm{v}}^{\mathrm{B}}) = \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}})$

The time derivative of s is connected to dynamics as follows:

$$\dot{\mathrm{s}} = \dot{\tilde{\bar{\mathrm{v}}}}^{\mathrm{B}} + \lambda\, \tilde{\bar{\mathrm{v}}}^{\mathrm{B}} = \mathrm{a_n} - \dot{\bar{\mathrm{v}}}^{\mathrm{B}} = \mathrm{a_n} - \mathrm{F} - \mathrm{K}(\mathrm{a_n} + \mathrm{a_s}) = \mathrm{w} - \mathrm{K}\mathrm{a_s} \tag{4.16}$$

with $\mathrm{w} = (\mathrm{I} - \mathrm{K})\mathrm{a_n} - \mathrm{F}$

Substitution to sliding condition yields:

$$\mathrm{s}^{\mathrm{T}}\mathrm{w} - \mathrm{s}^{\mathrm{T}}\mathrm{K}\mathrm{a_s} \leq \|\mathrm{s}\|\|\mathrm{w}\| - \mathrm{s}^{\mathrm{T}}\mathrm{K}\mathrm{a_s} \leq -\eta\|\mathrm{s}\| \tag{4.17}$$

Let us recall that for any symmetric matrix P:

$$\sigma^2_{\mathrm{min}}\|\mathrm{x}\|^2 \leq \|\mathrm{x}^{\mathrm{T}}\mathrm{P}\mathrm{x}\| \leq \sigma^2_{\mathrm{max}}\|\mathrm{x}\|^2 \tag{4.18}$$

with σ_{min} and σ_{max} being the largest and smallest eigenvalues of matrix P.

Thus, we can use property 4.18 to choose the stabilizing control $\mathrm{a_s}$ as:

$$\mathrm{a_s} = \frac{\alpha\hat{\mathrm{k}}}{\sigma^2_{\mathrm{max}}}\hat{\mathrm{M}}^{-1}\frac{\mathrm{s}}{\|\mathrm{s}\|} = \rho\frac{\mathrm{s}}{\|\mathrm{s}\|} \tag{4.19}$$

where σ_{max} is maximal singular value of M^{-1} which provide:

$$\|\mathrm{s}\|\|\mathrm{w}\| - \mathrm{s}^{\mathrm{T}}\mathrm{K}\mathrm{a_s} \leq \|\mathrm{s}\|\|\mathrm{w}\| - \mathrm{k}\frac{\alpha}{\sigma^2_{\mathrm{max}}\|\mathrm{s}\|}\mathrm{s}^{\mathrm{T}}\mathrm{M}^{-1}\mathrm{s} \leq \|\mathrm{s}\|\|\mathrm{w}\| - \alpha\mathrm{k}\|\mathrm{s}\| \tag{4.20}$$

but by definition $\mathrm{k_{min}} < \mathrm{k} < \mathrm{k_{max}}$, therefore

$$\|\mathrm{s}\|\|\mathrm{w}\| - \alpha\mathrm{k}\|\mathrm{s}\| \leq \|\mathrm{s}\|\|\mathrm{w}\| - \alpha\mathrm{k_{min}}\|\mathrm{s}\| < -\eta\|\mathrm{s}\| \tag{4.21}$$

Setting gain α accordingly to:

$$\alpha > \frac{\|\mathrm{w}\| + \eta}{\mathrm{k_{min}}} \tag{4.22}$$

will satisfy sliding conditions.

The final expression for sliding control:

$$\mathrm{a_s} = \begin{cases} \rho\frac{\mathrm{s}}{\|\mathrm{s}\|}, & \|\mathrm{s}\| > 0 \\ 0, & \|\mathrm{s}\| = 0 \end{cases} \tag{4.23}$$

In order to reduce chattering, the controller above is effectively smoothed using the boundary layer:

$$\mathrm{a_s} = \begin{cases} \rho\frac{\mathrm{s}}{\|\mathrm{s}\|}, & \|\mathrm{s}\| > \epsilon \\ \rho\frac{\mathrm{s}}{\epsilon}, & \|\mathrm{s}\| \leq \epsilon \end{cases} \tag{4.24}$$

where ϵ is the boundary layer thickness.

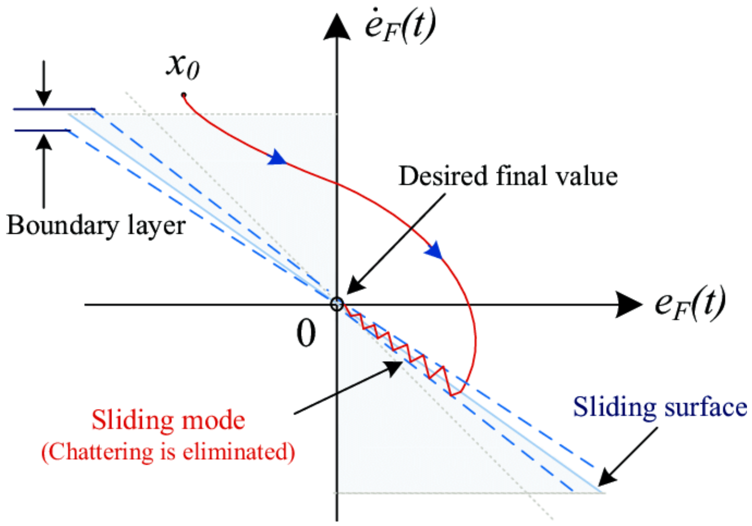

Fig. 6. The sliding mode scheme with boundary layer

The nominal control $\mathrm{a_n}$ can be designed in a form of PD controller:

$$\mathrm{a_n} = -\mathrm{K_p}\tilde{\mathrm{r}}^{\mathrm{B}} - \mathrm{K_d}\tilde{\mathrm{v}}^{\mathrm{B}} \tag{4.25}$$

The resulting controller is then given as follows:

$$\begin{aligned}
&\hat{\mathrm{u}} = \hat{\mathrm{B}}^{-1}(\hat{\mathrm{M}}\mathrm{a} + \hat{\mathrm{C}}(\mathrm{v^B})\mathrm{v^B} + \hat{\mathrm{D}}(\mathrm{v^B})\mathrm{v^B} + \hat{\mathrm{g}}(\mathrm{r^B}))\\
&\mathrm{a} = \mathrm{a_n} + \mathrm{a_s}\\
&\mathrm{a_n} = -\mathrm{K_p}\tilde{\mathrm{r}}^{\mathrm{B}} - \mathrm{K_d}\tilde{\mathrm{v}}^{\mathrm{B}}\\
&\mathrm{s} = \tilde{\tilde{\mathrm{v}}}^{\mathrm{B}} + \lambda\,\tilde{\tilde{\mathrm{r}}}^{\mathrm{B}}\\
&\mathrm{a_s} = \begin{cases} \rho\frac{\mathrm{s}}{\|\mathrm{s}\|}, & \|\mathrm{s}\| > \epsilon \\ \rho\frac{\mathrm{s}}{\epsilon}, & \|\mathrm{s}\| \le \epsilon \end{cases}
\end{aligned}$$

IV Optimization control

A. Optimization problem

B. Control Law Design

V Summary

Bibliography cited

[1] J. Slotine and W. Li, *Applied Nonlinear Control* (Prentice-Hall International Editions). Prentice-Hall, 1991, ISBN: 9780130400499.

[2] V. I. Utkin, "Variable structure systems with sliding modes," *IEEE Transactions on Automatic Control*, vol. 22, no. 2, pp. 212–222, 1977. DOI: `10.1109/TAC.1977.1101446`.