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Robust trajectory control of underwater vehicles using time delay control law

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Abstract

A new control scheme for robust trajectory control based on direct estimation of system dynamics is proposed for underwater vehicles. The proposed controller can work satisfactorily under heavy uncertainty that is commonly encountered in the case of underwater vehicle control. The dynamics of the plant are approximately canceled through the feedback of delayed accelerations and control inputs. Knowledge of the bounds on uncertain terms is not required. It is shown that only the rigid body inertia matrix is sufficient to design the controller. The control law is conceptually simple and computationally easy to implement. The effectiveness of the controller is demonstrated through simulations and implementation issues are discussed.

Keywords: Underwater vehicle; Time delay control; Robust control; Adaptive control

1. Introduction

Underwater vehicles have become an important tool for undersea intervention in recent years. Advances in certain key technologies like energy storage, navigation, etc., are making underwater missions more economical through autonomous underwater vehicles (AUVs). Precise position and attitude control are very important for underwater surveys using AUVs since the quality of data obtained is directly related to it. AUVs are required to operate autonomously in the absence of human intelligence. Underwater vehicle dynamics is highly nonlinear, coupled and time-varying. In addition to these, the hydrodynamic parameters are often poorly known and the vehicle may be subjected to unknown forces due to ocean currents. These difficulties make the control problem challenging.

Good quality surveys can be made possible through efficient trajectory control of the vehicle. Several control techniques have been proposed in literature to deal with uncertainty. They can be broadly classified as robust approaches and adaptive approaches. Yoerger and Slotine (1985) proposed sliding mode controller for trajectory control of underwater vehicles neglecting the cross coupling terms. Healey and Lienard (1993) used multivariable sliding mode control for diving, steering and speed control of underwater vehicles with decoupled design. Fossen and Sagatun (1991) used adaptive control with online estimation of uncertain parameters. Choi and Yuh (1996) made an experimental study on underwater robots using non-regressor-based adaptive control with bound estimation. Antonelli et al. (2003) proposed an adaptive control law which takes into account the hydrodynamic parameters affecting the tracking performance. Mrad and Majdalani (2003) showed that composite adaptive control law with bounded gain forgetting converges faster than tracking error-based adaptation. Li and Lee (2005) designed an adaptive nonlinear controller for depth control of underwater vehicles with fins without restriction on the pitch angle of diving.

Compared to robust control, adaptive control is considered to be better for plants with uncertainties because it can improve its performance with adaptation with little or no information of the bounds on uncertainties

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(Slotine, 1991). A drawback of adaptive control is that it is computationally intensive for higher order systems and is effective only for constant and slowly varying parameters. Also, linear parameterization of uncertain terms is generally essential for deriving the adaptive control law. Smallwood and Whitcomb (2004) report that at present there is no uniform consensus on the exact analytical form of equations of motion of underwater vehicles. In such a case robust control approach can be used as it needs only the knowledge of the bounds on uncertain terms to guarantee stability and performance. Robust control, in general, assumes bounds based on the amount of uncertainty. Higher bounds need higher control gains to be used in the controller. Sometimes this can lead to actuator saturation. However, robust control using time delay control law can overcome this limitation.

Time delay control law was first proposed for robotic manipulators by Youcef-Toumi and Ito (1990), and Hsia and Gao (1990). This control law cancels the unknown dynamics and unexpected disturbances by the direct estimation of a function representing the combined effect of all the uncertain terms using observed information of (immediately) previous accelerations and control inputs. Stability of such a system can be maintained with very low proportional and derivative gains. Unlike traditional adaptive control, it can be thought of as "instantaneous learning" of the system dynamics. Implementation of the control law requires the derivative of the system state to be known. This is generally considered as a drawback when it has to be calculated based on the state measurement. However, in the case of motion control systems such as underwater vehicle controllers, the derivative of the system state can be obtained from the onboard navigation sensors without the need to differentiate noisy signals. This paper explores the application of time delay control law to underwater vehicle control. Except for the inertia term, this control law does not require the knowledge of either the structure of the uncertainties or the bounds on uncertainties and therefore more suitable to underwater vehicle control than the existing control laws.

In this paper, first we briefly review the equations of motion of underwater vehicles and then derive the time delay control law. It is shown that only the knowledge of the rigid body inertia matrix is sufficient to design the controller. Next, numerical simulations are presented for various situations in which the controller is expected to operate. Finally, implementation issues are discussed for the practical realization of the controller.

2. Underwater vehicle dynamics

The equations of motion of an underwater vehicle in six degrees of freedom can be written as (see Fossen, 1994)

$$\boldsymbol{M}\dot{\boldsymbol{v}} + \boldsymbol{C}(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{D}(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}, \tag{1}$$

where

$$\boldsymbol{M} = \boldsymbol{M}_{\mathrm{RB}} + \boldsymbol{M}_{\mathrm{A}}, \tag{2}$$

$$\boldsymbol{C}(\boldsymbol{v}) = \boldsymbol{C}_{\mathrm{RB}}(\boldsymbol{v}) + \boldsymbol{C}_{\mathrm{A}}(\boldsymbol{v}), \tag{3}$$

$$\boldsymbol{D}(\boldsymbol{v}) = \boldsymbol{D}_{\mathrm{L}} + \boldsymbol{D}_{\mathrm{Q}}|\boldsymbol{v}|. \tag{4}$$

$\boldsymbol{M}_{\mathrm{RB}}$ and $\boldsymbol{C}_{\mathrm{RB}}(\boldsymbol{v})$ are the rigid body mass matrix and the Coriolis and centripetal matrix, respectively. $\boldsymbol{M}_{\mathrm{A}}$ and $\boldsymbol{C}_{\mathrm{A}}(\boldsymbol{v})$ are the added mass matrix and the added Coriolis and centripetal matrix, respectively. $\boldsymbol{D}_{\mathrm{L}}$ and $\boldsymbol{D}_{\mathrm{Q}}(\boldsymbol{v})$ are the linear and quadratic drag matrices, respectively. $\boldsymbol{g}(\boldsymbol{\eta})$ is the resultant vector of gravity and buoyancy. $\boldsymbol{\tau}$ is the resultant vector of thruster forces and moments. $\boldsymbol{v} = [u\ v\ w\ p\ q\ r]^{\mathrm{T}}$ is the vector of linear and angular velocities in vehicle coordinate frame. The relationship between linear and angular velocities in vehicle frame to those in absolute frame (Fig. 1) is given by

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{v}, \tag{5}$$

where $\boldsymbol{J}(\boldsymbol{\eta})$ is the Jacobian matrix, $\boldsymbol{\eta} = [x\ y\ z\ \phi\ \theta\ \psi]^{\mathrm{T}}$, where x, y and z are absolute positions and, ϕ, θ and ψ are Euler angles in Z–Y–X convention.

Eq. (1) can be written in inertial coordinate frame as

$$\boldsymbol{M}_{\eta}\ddot{\boldsymbol{\eta}} + \boldsymbol{C}_{\eta}(\boldsymbol{v},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{D}_{\eta}(\boldsymbol{v},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{g}_{\eta}(\boldsymbol{\eta}) = \boldsymbol{\tau}_{\eta}, \tag{6}$$

where

$$\boldsymbol{M}_{\eta} = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{M}\boldsymbol{J}^{-1}, \quad \boldsymbol{C}_{\eta}(\boldsymbol{v},\boldsymbol{\eta}) = \boldsymbol{J}^{-\mathrm{T}}(\boldsymbol{C} - \boldsymbol{M}\boldsymbol{J}^{-1}\dot{\boldsymbol{J}})\boldsymbol{J}^{-1},$$

$$\boldsymbol{D}_{\eta}(\boldsymbol{v},\boldsymbol{\eta}) = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{D}\boldsymbol{J}^{-1}, \quad \boldsymbol{g}_{\eta}(\boldsymbol{\eta}) = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{g}(\boldsymbol{\eta}),$$

$$\boldsymbol{\tau}_{\eta} = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{\tau}.$$

Here, $\boldsymbol{M}_{\mathrm{RB}}$, $\boldsymbol{M}_{\mathrm{A}}$, $\boldsymbol{M}$ and $\boldsymbol{M}_{\eta}$ are all positive definite.

Though thrusters are known to influence the overall vehicle dynamics, in this paper we assume that the thruster response is fast enough for tracking the given trajectory

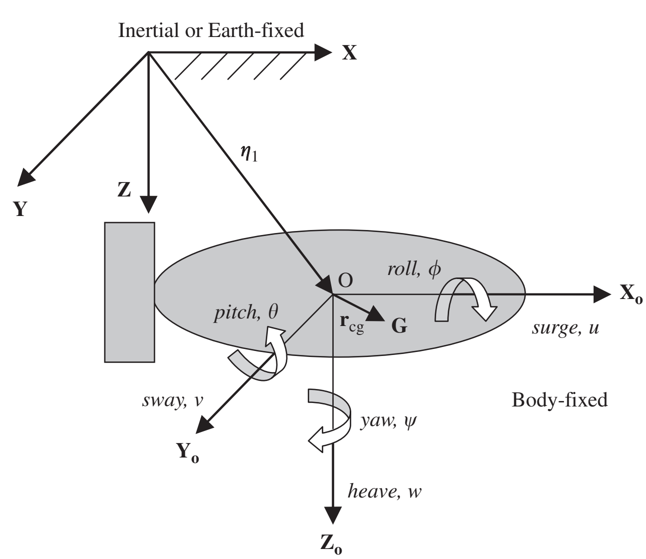

Fig. 1. Coordinate frames.

and a linear input–output relationship is applicable between desired forces/moments and thruster inputs (Antonelli et al., 2001).

3. Time delay control law

The derivation of time delay control law for underwater vehicles is similar to the one derived for robotic manipulators (Hsia and Gao, 1990). However, there is a slight deviation from the original derivation. Here, the inertia matrix $\overline{\boldsymbol{M}}_\eta$ is not a multiple of identity matrix and also not a constant matrix. Still, it will be shown that the sufficiency condition therein can be applied in this case. Eq. (6) can be written as

$$\overline{\boldsymbol{M}}_\eta \ddot{\boldsymbol{\eta}} + (\boldsymbol{M}_\eta - \overline{\boldsymbol{M}}_\eta)\ddot{\boldsymbol{\eta}} + \boldsymbol{C}_\eta(\boldsymbol{v},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{D}_\eta(\boldsymbol{v},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{g}_\eta(\boldsymbol{\eta}) = \boldsymbol{\tau}_\eta,$$

$$\overline{\boldsymbol{M}}_\eta \ddot{\boldsymbol{\eta}} + \boldsymbol{H}(\boldsymbol{\eta},\dot{\boldsymbol{\eta}},\ddot{\boldsymbol{\eta}}) = \boldsymbol{\tau}_\eta, \tag{7}$$

where

$$\boldsymbol{H}(\boldsymbol{\eta},\dot{\boldsymbol{\eta}},\ddot{\boldsymbol{\eta}}) = (\boldsymbol{M}_\eta - \overline{\boldsymbol{M}}_\eta)\ddot{\boldsymbol{\eta}} + \boldsymbol{H}_1(\boldsymbol{\eta},\dot{\boldsymbol{\eta}},\ddot{\boldsymbol{\eta}}), \tag{8}$$

and

$$\boldsymbol{H}_1(\boldsymbol{\eta},\dot{\boldsymbol{\eta}},\ddot{\boldsymbol{\eta}}) = \boldsymbol{C}_\eta(\boldsymbol{v},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{D}_\eta(\boldsymbol{v},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{g}_\eta(\boldsymbol{\eta}). \tag{9}$$

In what follows, except stated otherwise, we use $f(t)$ for any state-dependent function $f(.,\ldots)$ for convenience. The control law is given by

$$\boldsymbol{\tau}_\eta(t) = \overline{\boldsymbol{M}}_\eta(t)\boldsymbol{u}(t) + \hat{\boldsymbol{H}}(t), \tag{10}$$

where

$$\boldsymbol{u}(t) = \ddot{\boldsymbol{\eta}}_{\mathrm{d}} + K_{\mathrm{D}}\dot{\tilde{\boldsymbol{\eta}}} + K_{\mathrm{P}}\tilde{\boldsymbol{\eta}}, \tag{11}$$

and

$$\hat{\boldsymbol{H}}(t) = \boldsymbol{H}(t-\lambda). \tag{12}$$

The desired position and attitude vector is denoted by $\boldsymbol{\eta}_{\mathrm{d}}$, and the error in position and attitude by $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta}_{\mathrm{d}} - \boldsymbol{\eta}$. K_{P} and K_{D} are proportional and derivative gain matrices which are positive definite and diagonal. $\hat{\boldsymbol{H}}(t)$ is an estimate of $\boldsymbol{H}(t)$. The time delay λ is assumed to be very small such that

$$\boldsymbol{H}(t-\lambda) \approx \boldsymbol{H}(t). \tag{13}$$

From Eqs. (7), (10) and (11) we get

$$\ddot{\tilde{\boldsymbol{\eta}}} + K_{\mathrm{P}}\dot{\tilde{\boldsymbol{\eta}}} + K_{\mathrm{D}}\tilde{\boldsymbol{\eta}} = \overline{\boldsymbol{M}}_\eta(t)^{-1}(\boldsymbol{H}(t) - \boldsymbol{H}(t-\lambda)) = -\boldsymbol{\varepsilon}(t), \tag{14}$$

where $\boldsymbol{\varepsilon}(t) = \ddot{\boldsymbol{\eta}} - \boldsymbol{u}(t)$. $\boldsymbol{\varepsilon}(t)$ can be thought of as representing the approximation error in the estimate $\hat{\boldsymbol{H}}(t)$. The above equation shows that the error in the estimate $\hat{\boldsymbol{H}}(t)$ excites the trajectory error dynamics. The stability of the proposed control law can be proved by showing that $\boldsymbol{\varepsilon}(t)$ is bounded.

From Eqs. (12) and (8) we can write

$$\hat{\boldsymbol{H}}(t) = (\boldsymbol{M}_\eta(t-\lambda) - \overline{\boldsymbol{M}}_\eta(t-\lambda))\ddot{\boldsymbol{\eta}}(t-\lambda) + \boldsymbol{H}_1(t-\lambda). \tag{15}$$

Now, Eq. (10) can be written as

$$\boldsymbol{\tau}_\eta(t) = \overline{\boldsymbol{M}}_\eta(t)\boldsymbol{u}(t) + (\boldsymbol{M}_\eta(t-\lambda) - \overline{\boldsymbol{M}}_\eta(t-\lambda))\ddot{\boldsymbol{\eta}}(t-\lambda) + \boldsymbol{H}_1(t-\lambda). \tag{16}$$

Since $\boldsymbol{\varepsilon}(t) = \ddot{\boldsymbol{\eta}} - \boldsymbol{u}(t)$, we have

$$\begin{aligned}\boldsymbol{M}_\eta(t)\boldsymbol{\varepsilon}(t) &= \boldsymbol{M}_\eta(t)(\ddot{\boldsymbol{\eta}} - \boldsymbol{u}(t)),\\ &= \boldsymbol{\tau}_\eta(t) - \boldsymbol{H}_1(t) - \boldsymbol{M}_\eta(t)\boldsymbol{u}(t).\end{aligned} \tag{17}$$

Substituting $\boldsymbol{\tau}_\eta(t)$ in Eqs. (16) and (17) yields

$$\begin{aligned}\boldsymbol{M}_\eta(t)\boldsymbol{\varepsilon}(t) = {}&(\overline{\boldsymbol{M}}_\eta(t) - \boldsymbol{M}_\eta(t))\boldsymbol{u}(t) + \boldsymbol{H}_1(t-\lambda) - \boldsymbol{H}_1(t)\\ &+ (\boldsymbol{M}_\eta(t-\lambda) - \overline{\boldsymbol{M}}_\eta(t-\lambda))\ddot{\boldsymbol{\eta}}(t-\lambda).\end{aligned} \tag{18}$$

Adding and subtracting $(\boldsymbol{M}_\eta(t) - \overline{\boldsymbol{M}}_\eta(t))\ddot{\boldsymbol{\eta}}(t-\lambda)$ on the right-hand side of the above equation, we get

$$\begin{aligned}\boldsymbol{M}_\eta(t)\boldsymbol{\varepsilon}(t) = {}&(\boldsymbol{M}_\eta(t) - \overline{\boldsymbol{M}}_\eta(t))\ddot{\boldsymbol{\eta}}(t-\lambda)\\ &+ (\overline{\boldsymbol{M}}_\eta(t) - \boldsymbol{M}_\eta(t))\boldsymbol{u}(t) - (\boldsymbol{M}_\eta(t) - \boldsymbol{M}_\eta(t-\lambda)\\ &+ \overline{\boldsymbol{M}}_\eta(t-\lambda) - \overline{\boldsymbol{M}}_\eta(t))\ddot{\boldsymbol{\eta}}(t-\lambda)\\ &+ \boldsymbol{H}_1(t-\lambda) - \boldsymbol{H}_1(t).\end{aligned} \tag{19}$$

Using $\ddot{\boldsymbol{\eta}}(t-\lambda) = \boldsymbol{\varepsilon}(t-\lambda) + \boldsymbol{u}(t-\lambda)$ in Eq. (19) and rearranging yields

$$\begin{aligned}\boldsymbol{\varepsilon}(t) = {}&(I - \boldsymbol{M}_\eta(t)^{-1}\overline{\boldsymbol{M}}_\eta(t))\boldsymbol{\varepsilon}(t-\lambda) + \boldsymbol{Q}_1(t-\lambda)\\ &- (I - \boldsymbol{M}_\eta(t)^{-1}\overline{\boldsymbol{M}}_\eta(t))\boldsymbol{Q}_2(t-\lambda),\end{aligned} \tag{20}$$

where

$$\begin{aligned}\boldsymbol{Q}_1(t-\lambda) = {}&\boldsymbol{M}_\eta(t)^{-1}\{\boldsymbol{H}_1(t-\lambda) - \boldsymbol{H}_1(t)\\ &- (\boldsymbol{M}_\eta(t) - \boldsymbol{M}_\eta(t-\lambda) + \overline{\boldsymbol{M}}_\eta(t-\lambda)\\ &- \overline{\boldsymbol{M}}_\eta(t))\ddot{\boldsymbol{\eta}}(t-\lambda)\},\end{aligned} \tag{21}$$

and

$$\boldsymbol{Q}_2(t-\lambda) = \boldsymbol{u}(t-\lambda) - \boldsymbol{u}(t). \tag{22}$$

In discrete domain, Eq. (20) can be written as

$$\boldsymbol{\varepsilon}(k) = \boldsymbol{A}(k)\boldsymbol{\varepsilon}(k-1) + \boldsymbol{Q}(k-1), \tag{23}$$

where

$$\boldsymbol{Q}(k-1) = \boldsymbol{Q}_1(k-1) + (I - \boldsymbol{M}_\eta(k)^{-1}\overline{\boldsymbol{M}}_\eta(k))\boldsymbol{Q}_2(k-1), \tag{24}$$

and

$$\boldsymbol{A}(k) = I - \boldsymbol{M}_\eta(k)^{-1}\overline{\boldsymbol{M}}_\eta(k). \tag{25}$$

Here, $\boldsymbol{Q}(k-1)$ is viewed as an external disturbance. Then, for $\boldsymbol{\varepsilon}(k)$ to be convergent, the eigenvalues of $\boldsymbol{A}(k)$ should lie in the unit disc;

$$-1 < \mathrm{Eig}(\boldsymbol{A}(k)) < 1, \tag{26}$$

where the function $\mathrm{Eig}(\cdot)$ denotes eigenvalues of the argument.

Therefore, for the system to be stable for any k, $\overline{\boldsymbol{M}}_\eta(k)$ has to be chosen such that condition (26) is satisfied. We show that by choosing $\overline{\boldsymbol{M}}_\eta(k)$ as $\boldsymbol{J}^{-\mathrm{T}}(k)\boldsymbol{M}_{\mathrm{RB}}\boldsymbol{J}^{-1}(k)$, the stability condition is always satisfied. In what follows, the sample index k is dropped for simplicity

$$\begin{aligned}\mathrm{Eig}(\boldsymbol{A}) &= \mathrm{Eig}(I - \boldsymbol{M}_\eta^{-1}\overline{\boldsymbol{M}}_\eta),\\ &= \mathrm{Eig}(I - (\boldsymbol{J}^{-\mathrm{T}}\boldsymbol{M}\boldsymbol{J}^{-1})^{-1}\boldsymbol{J}^{-\mathrm{T}}\boldsymbol{M}_{\mathrm{RB}}\boldsymbol{J}^{-1}),\\ &= \mathrm{Eig}(I - \boldsymbol{J}\boldsymbol{M}^{-1}\boldsymbol{M}_{\mathrm{RB}}\boldsymbol{J}^{-1}),\\ &= \mathrm{Eig}(\boldsymbol{J}(I - \boldsymbol{M}^{-1}\boldsymbol{M}_{\mathrm{RB}})\boldsymbol{J}^{-1}),\\ \Rightarrow \mathrm{Eig}(\boldsymbol{A}) &= \mathrm{Eig}(I - \boldsymbol{M}^{-1}\boldsymbol{M}_{\mathrm{RB}}).\end{aligned} \tag{27}$$

Spangelo and Egeland (1994). The vehicle is symmetric about all the three axes and is neutrally buoyant. The following helical reference trajectory has been chosen for simulating the controller:

$$\boldsymbol{\eta}_{\mathrm{ref}} = \left[r\cos as \quad r\sin as \quad bs \quad 0 \quad 0 \quad \frac{\pi}{2} + as\right]^{\mathrm{T}},$$

where $r = 10\,\mathrm{m}$, $b = 0.3$, $a = \sqrt{(1-b^2)/r^2}$ and angles are measured in radians. The parameter s is the arc length which varies with respect to time (in seconds) as follows:

$$s(t) = \begin{cases} 0.08t^3 - 0.008t^4, & 0 \leqslant t < 5,\\ 5 + 2(t-5), & 5 \leqslant t \leqslant 27,\\ 49 + \dfrac{-100352(t-27) + 5184(t^2 - 27^2) - 118(t^3 - 27^3) + t^4 - 27^4}{125}, & 27 < t \leqslant 32.\end{cases}$$

Now, using matrix inversion lemma we can write

$$\begin{aligned}\boldsymbol{M}^{-1} &= (\boldsymbol{M}_{\mathrm{RB}} + \boldsymbol{M}_{\mathrm{A}})^{-1}\\ &= \boldsymbol{M}_{\mathrm{RB}}^{-1} - \boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}}(I + \boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}})^{-1}\boldsymbol{M}_{\mathrm{RB}}^{-1}.\end{aligned} \tag{28}$$

Post-multiplying with $\boldsymbol{M}_{\mathrm{RB}}$ on both sides, we have

$$\boldsymbol{M}^{-1}\boldsymbol{M}_{\mathrm{RB}} = I - \boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}}(I + \boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}})^{-1}, \tag{29}$$

or,

$$I - \boldsymbol{M}^{-1}\boldsymbol{M}_{\mathrm{RB}} = \boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}}(I + \boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}})^{-1}. \tag{30}$$

Since $\boldsymbol{M}_{\mathrm{RB}}$ and $\boldsymbol{M}_{\mathrm{A}}$ are both positive definite, $\boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}}$ is also positive definite. The eigenvalues of $\boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}}$ are real positive. Therefore, the eigenvalues of $\boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}}(I + \boldsymbol{M}_{\mathrm{RB}}^{-1}\boldsymbol{M}_{\mathrm{A}})^{-1}$ are real positive and less than unity. Condition (26) is satisfied. The sufficient condition for stability follows from Hsia and Gao (1990) and is given in the appendix.

Now, the control law given in Eq. (10) can be simplified as follows:

$$\boldsymbol{\tau}_\eta = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{\tau} = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{M}_{\mathrm{RB}}\boldsymbol{J}^{-1}\boldsymbol{u}(t) + \hat{\boldsymbol{H}}(t), \tag{31}$$

or,

$$\boldsymbol{\tau} = \boldsymbol{M}_{\mathrm{RB}}\boldsymbol{J}^{-1}\boldsymbol{u}(t) + \boldsymbol{J}^{\mathrm{T}}\boldsymbol{H}(t-\lambda), \tag{32}$$

$$\begin{aligned}\Rightarrow \boldsymbol{\tau} = {} & \boldsymbol{M}_{\mathrm{RB}}\boldsymbol{J}^{-1}(t)(\ddot{\boldsymbol{\eta}}_{\mathrm{d}}(t) + K_{\mathrm{D}}\dot{\tilde{\boldsymbol{\eta}}}(t) + K_{\mathrm{P}}\tilde{\boldsymbol{\eta}}(t))\\ & + \boldsymbol{\tau}(t-\lambda) - \boldsymbol{M}_{\mathrm{RB}}\boldsymbol{J}^{-1}(t-\lambda)\ddot{\boldsymbol{\eta}}(t-\lambda).\end{aligned} \tag{33}$$

Eq. (33) gives the time delay control law to be applied to the vehicle. The time delay λ is generally taken as the sample time or an integral multiple of the sample time. $\boldsymbol{\tau}(t-\lambda)$ and $\ddot{\boldsymbol{\eta}}(t-\lambda)$ are the previous control input and previous acceleration, respectively.

4. Numerical simulations

For the purpose of simulation, the hydrodynamic parameters and other vehicle parameters are adopted from Spangelo and Egeland (1994).

The desired trajectory as a function of arc length is shown in Fig. 2. The vehicle accelerates for the initial 5 s, reaches a constant speed of 2 m/s and decelerates during the last 5 s.

Simulation of the vehicle was performed using fourth order Runge–Kutta method with a fixed step size of 0.01 s. The controller takes sampled input with a sampling rate of $1/\lambda$ Hz. Simulations with different values of λ have been performed as discussed in the following. Zero order hold was used in the simulation of the sampled data system. The proportional and derivative gain matrices were chosen as $2.25I$ and $3I$, respectively for all simulations. This results in critical damping of the errors described by the linear dynamics in Eq. (14), with a pair of poles placed at $-3+0i$. The position and attitude errors for a time delay of $\lambda = 0.01$ s are shown in Figs. 3 and 4. The error is higher during the acceleration and deceleration of the vehicle corresponding to a rise in approximation error $\boldsymbol{\varepsilon}(t)$ in the estimate $\hat{\boldsymbol{H}}(t)$ during these periods.

Figs. 5 and 6 show the position and attitude tracking errors for a time delay of $\lambda = 0.1$ s. The errors with $\lambda = 0.1$ s are nearly five times the errors with $\lambda = 0.01$ s. The performance of the controller in tracking the given

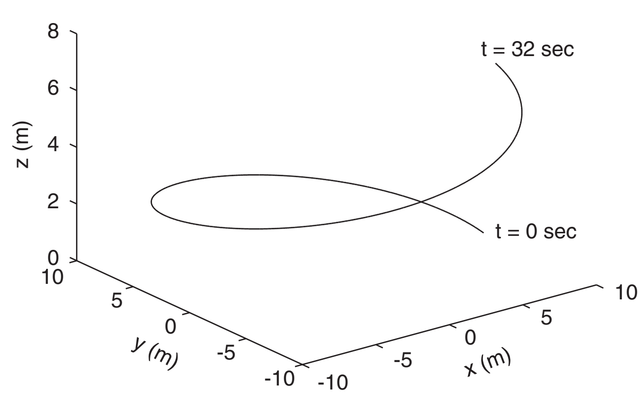

Fig. 2. Trajectory as a function of arc length s.

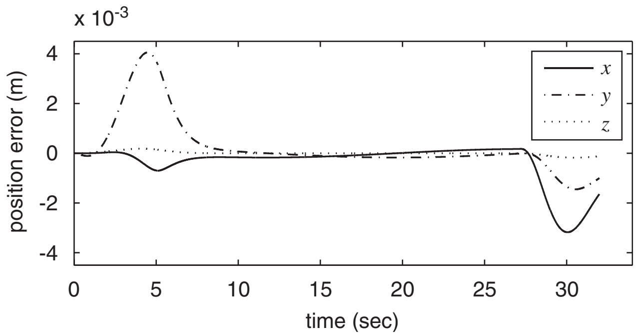

Fig. 3. Position error for $\lambda = 0.01\,\text{s}$.

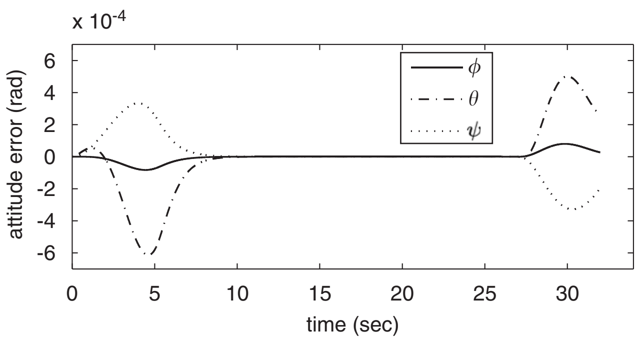

Fig. 4. Attitude error for $\lambda = 0.01\,\text{s}$.

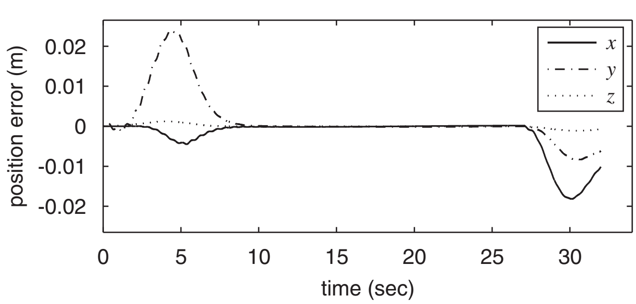

Fig. 5. Position error for $\lambda = 0.1\,\text{s}$.

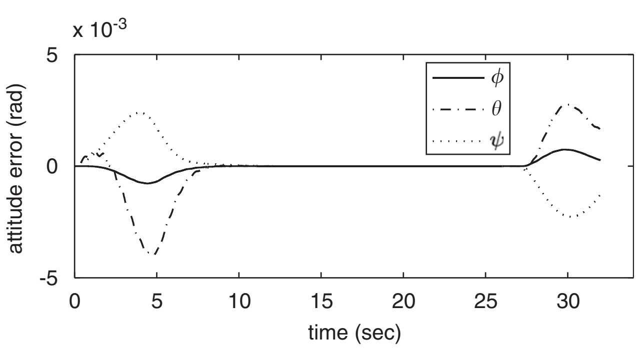

Fig. 6. Attitude error for $\lambda = 0.1\,\text{s}$.

trajectory is observed to be better with lower time delays. This behavior is expected to occur, and can be attributed to the approximation error in $\hat{\boldsymbol{H}}(t)$.

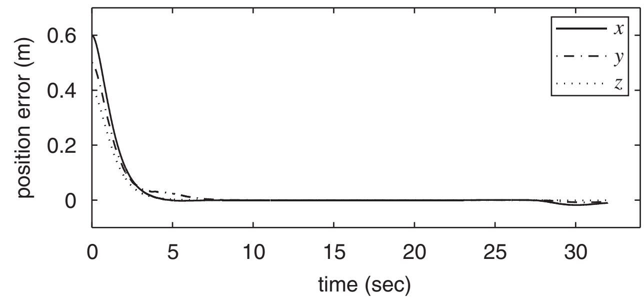

Fig. 7. Position error for $\lambda = 0.1\,\text{s}$ with non-zero initial error.

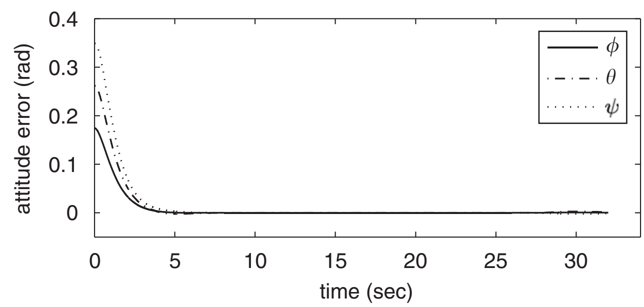

Fig. 8. Attitude error for $\lambda = 0.1\,\text{s}$ with non-zero initial error.

When underwater vehicles are launched from a ship, there can be considerable error in the initial position and orientation. Also, it is necessary to study the behavior of the controller when large errors occur. Figs. 7 and 8 show the position and attitude tracking errors for a time delay of $\lambda = 0.1\,\text{s}$ with non-zero initial error of $(0.6, 0.5, 0.4)\,\text{m}$ in x, y, z positions and $(10^\circ, 15^\circ, 20^\circ)$ in roll, pitch and yaw angles, respectively. All the errors converge to a small set, the size of which varies with time depending on the magnitude of the approximation error $\boldsymbol{\varepsilon}(t)$ as well as the proportional and derivative gains.

Underwater vehicles are often subjected to unknown disturbing forces due to ocean currents. This effect can be taken into account by considering the relative velocity $\boldsymbol{v}_\text{r}$ between the vehicle and the ocean current in the equations of motion (Fossen, 1994)

$$\boldsymbol{v}_\text{r} = \boldsymbol{v} - \boldsymbol{v}_\text{c}, \tag{34}$$

where $\boldsymbol{v}_\text{c}$ is the velocity of ocean current expressed in vehicle coordinate frame. The equations of motion can be written as

$$\boldsymbol{M}\dot{\boldsymbol{v}}_\text{r} + \boldsymbol{C}(\boldsymbol{v}_\text{r})\boldsymbol{v}_\text{r} + \boldsymbol{D}(\boldsymbol{v}_\text{r})\boldsymbol{v}_\text{r} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}. \tag{35}$$

The control law in Eq. (33) is used without modification which means that the controller does not have a direct knowledge of ocean current. Instead, the effect of ocean current is captured by the navigation sensors along with the motion of the vehicle with respect to ocean current. The vehicle is subjected to ocean current of velocity (0.7071, 0.7071, 0) m/s with a resultant of 1 m/s ($\approx$ 2 knots).

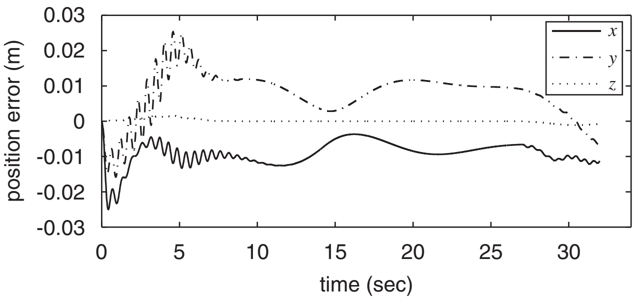

Fig. 9. Position error for $\lambda = 0.1\,\mathrm{s}$ with ocean current disturbance.

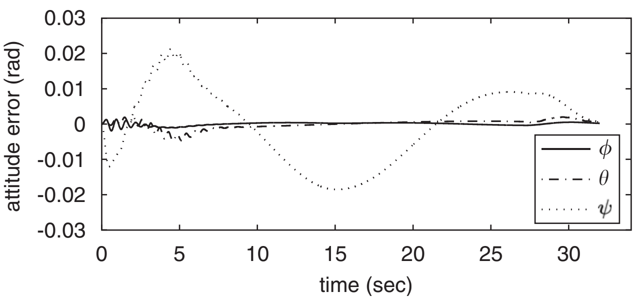

Fig. 10. Attitude error for $\lambda = 0.1\,\mathrm{s}$ with ocean current disturbance.

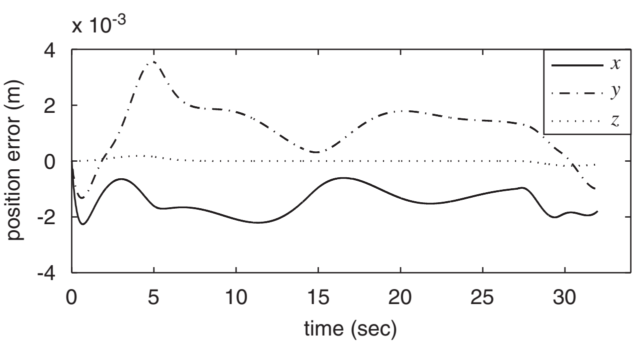

Fig. 11. Position error for $\lambda = 0.01\,\mathrm{s}$ with ocean current disturbance.

The effect of this ocean current on the trajectory tracking performance is shown in Figs. 9 and 10.

These figures show some oscillations during the initial phase of control action. Here, the initial velocity of the vehicle is assumed to be zero, i.e., the vehicle is not moving with the ocean current initially. When the control action is started, the vehicle experiences a sudden force due to ocean current trying to deviate it from its intended trajectory. The controller takes some time to cope with this unexpected disturbance which is reflected in the figure as oscillations. However, with $\lambda = 0.01\,\mathrm{s}$ as shown in Figs. 11 and 12 better performance is observed without such oscillations. The effect of ocean current on x, y and ψ is observed to be predominant since the direction of ocean current is in the horizontal plane.

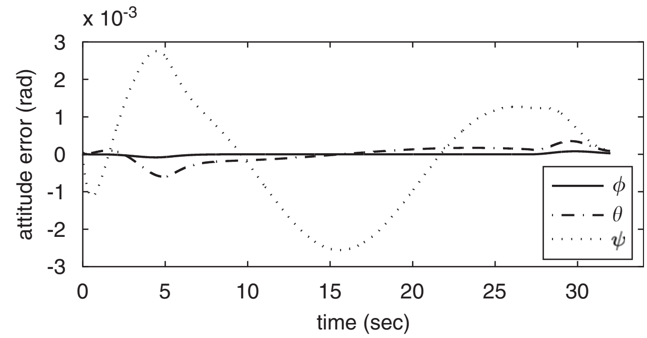

Fig. 12. Attitude error for $\lambda = 0.01\,\mathrm{s}$ with ocean current disturbance.

5. Implementation issues

The navigation system uses rate gyros and linear accelerometers with external aids to minimize the position error growth. Angular acceleration measurements are not generally available in any standard navigation systems since they are not necessary to compute positions and velocities. Angular acceleration can be provided to the controller in three ways:

(1) use angular accelerometer triad, or
(2) use additional linear accelerometer triad, or
(3) filter and differentiate angular velocity.

Angular accelerometers are already commercially available in the market. By using an additional linear accelerometer triad (apart from navigation systems's linear accelerometer triad) in a suitable configuration, it is possible to determine angular accelerations. Angular acceleration can also be determined by differentiating the angular velocity after filtering the noise as much as possible. Using linear accelerometers is preferable because the navigation system can also benefit from the additional sensors in accurately determining the positions and velocities.

The time delay used in the controller depends on the bandwidth of the actuators, the sensor suite and the processing power of the navigation and control computer. Typically, it can range from 0.01 to 0.1 s. Better performance can be achieved with lower time delays. Running the control loop at 0.01 s sample time is not difficult to achieve even with low bandwidth external aids like DVL since the basic navigation system consisting of accelerometers and rate gyros is generally capable of providing data at high sample rates and the velocities and positions determined from it are accurate for short durations of time.

The presence of inverse Jacobian in the control law should not be viewed as a disadvantage since the inverse of the rotation matrix can be obtained by transposing it and the inverse of Euler angle transformation matrix can be analytically determined. Therefore, this will not be an extra computational burden on the onboard computer.

The effect of sensor noise on the performance of time delay control law is generally not significant because the

Table 1
Zero mean Gaussian random noise for sensors

Acceleration (m/s^2 or rad/s^2) $\times 10^{-2}$	Velocity (m/s or rad/s) $\times 10^{-2}$	Position (m or rad) $\times 10^{-3}$
(5, 5, 5, 5, 5, 5)	(1, 1, 1, 1, 1, 1)	(1, 1, 1, 1.7, 1.7, 1.7)

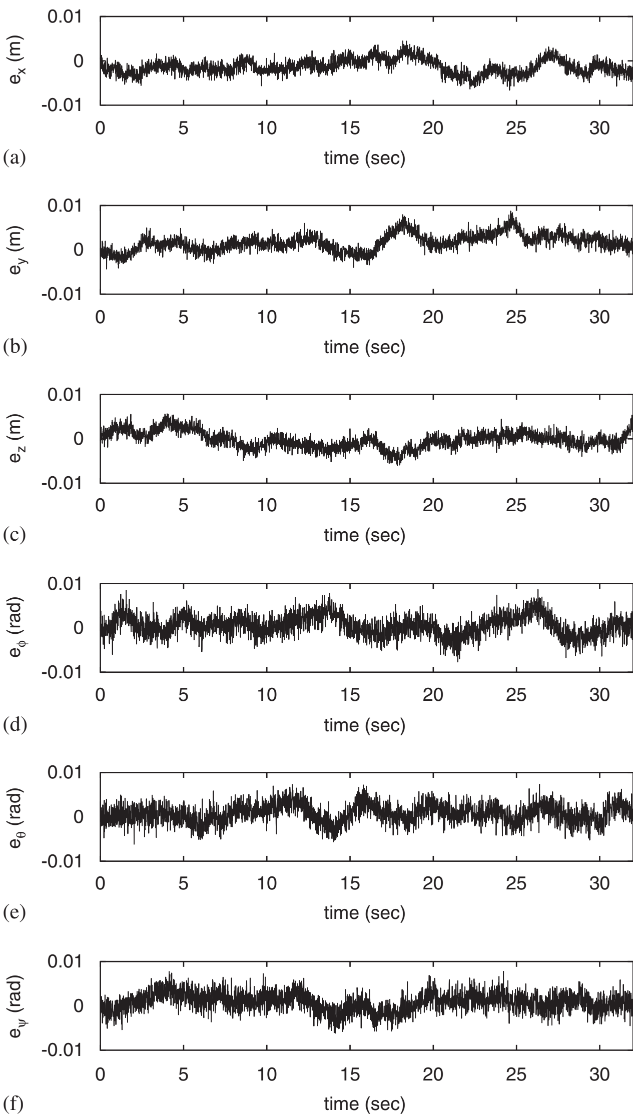

Fig. 13. Tracking errors for $\lambda = 0.01$ s with sensor noise and ocean current disturbance. (a) Error in x; (b) error in y; (c) error in z; (d) error in ϕ; (e) error in θ; (f) error is ψ.

plant acts as a low pass filter reducing the high-frequency components (Youcef-Toumi and Ito, 1990). In order to check the effect of noise on the tracking performance, the vehicle-controller system was simulated with ocean current disturbance as well as zero mean Gaussian white noise in the sensor data as given in Table 1.

The simulation results are shown in Figs. 13 and 14. For both $\lambda = 0.01$ and 0.1 s, the performance is found to be

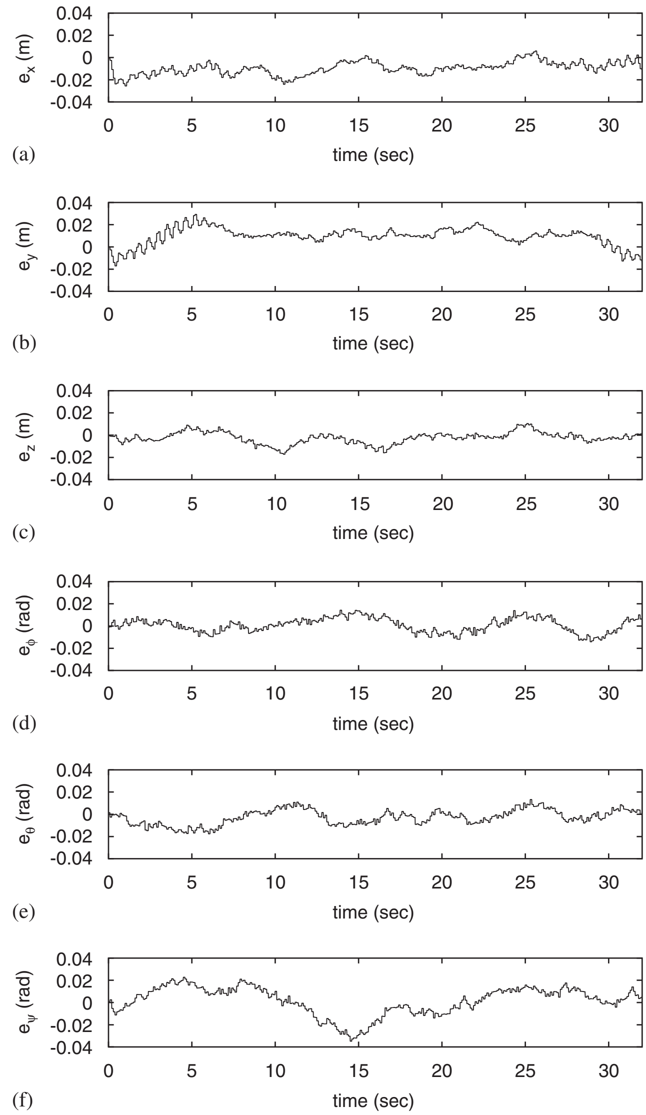

Fig. 14. Tracking errors for $\lambda = 0.1$ s with sensor noise and ocean current disturbance. (a) Error in x; (b) error in y; (c) error in z; (d) error in ϕ; (e) error in θ; (f) error is ψ.

satisfactory. However, as observed before, the performance is better for lower time delay. Though the noise model used for both these simulations was the same, Fig. 13 appears to be more noisy because of two reasons. Firstly, the range of the errors shown is of the order of the noise itself. Secondly, the sampling frequency is higher (100 Hz).

6. Conclusions

In this paper, a new robust trajectory control technique has been proposed for underwater vehicles based on direct estimation of system dynamics through delayed acceleration and control inputs. It is also shown that such a controller can be designed with the knowledge of the rigid body matrix alone. Implementation of the control law in real-world situation has been discussed. Simulation results show that the proposed controller can effectively track the given trajectory in the presence of uncertainty in hydrodynamic parameters. The computational and conceptual simplicity of the controller and its robustness to uncertainties make it very attractive for embedded applications like underwater vehicle control.

Appendix A. Matrix inversion lemma

If A, C and $C^{-1}+DA^{-1}B$ are non-singular square matrices, then $A+BCD$ is invertible, and

$$(A+BCD)^{-1}=A^{-1}-A^{-1}B(C^{-1}+DA^{-1}B)^{-1}DA^{-1}.$$

Appendix B. Sufficient condition (Hsia and Gao, 1990)

Let $\mu<1$ be the largest eigenvalue of $\boldsymbol{A}(k)$ $\forall k\geqslant 0$. Let β_1 and β_2 be positive constants such that $|\boldsymbol{Q}_{1i}|\leqslant\beta_1$, $|\boldsymbol{Q}_{2i}|\leqslant\beta_2$ $\forall k\geqslant 0$ and $i=1,2,\ldots,n$, where $\boldsymbol{Q}_{1i}$ and $\boldsymbol{Q}_{2i}$ are the ith elements of $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$, respectively. The solution of the difference equation in (23) is

$$\boldsymbol{\varepsilon}(k)=\prod_{m=1}^{k}\boldsymbol{A}(m)\boldsymbol{\varepsilon}(0)+\sum_{m=1}^{k-1}\prod_{p=m}^{k-1}\boldsymbol{A}(p+1)\boldsymbol{Q}_1(m-1)+\boldsymbol{Q}_1(k-1)-\sum_{m=1}^{k}\prod_{p=m}^{k}\boldsymbol{A}(p)\boldsymbol{Q}_2(m-1), \tag{36}$$

where $\boldsymbol{A}(k)=I-\boldsymbol{M}_\eta(k)^{-1}\overline{\boldsymbol{M}}_\eta(k)$ and $\boldsymbol{\varepsilon}(0)$ is the initial value of $\boldsymbol{\varepsilon}(k)$. Taking norm on both sides and using triangle inequality,

$$\|\boldsymbol{\varepsilon}(k)\|\leqslant\left\|\prod_{m=1}^{k}\boldsymbol{A}(m)\boldsymbol{\varepsilon}(0)\right\|+\left\|\sum_{m=1}^{k-1}\prod_{p=m}^{k-1}\boldsymbol{A}(p+1)\boldsymbol{Q}_1(m-1)\right\|+\|\boldsymbol{Q}_1(k-1)\|+\left\|\sum_{m=1}^{k}\prod_{p=m}^{k}\boldsymbol{A}(p)\boldsymbol{Q}_2(m-1)\right\|,$$

$$\begin{aligned}\|\boldsymbol{\varepsilon}(k)\|&\leqslant\mu^k\boldsymbol{\varepsilon}(0)+\sum_{m=1}^{k-1}\mu^{k-m}\|\boldsymbol{Q}_1(m-1)\|+\|\boldsymbol{Q}_1(k-1)\|+\sum_{m=1}^{k}\mu^{k-m+1}\|\boldsymbol{Q}_2(m-1)\|,\\&\leqslant\mu^k\boldsymbol{\varepsilon}(0)+n\beta_1\sum_{m=0}^{k-1}\mu^m+n\beta_2\sum_{m=1}^{k-1}\mu^m.\end{aligned} \tag{37}$$

As $k\to\infty$, $\mu^k\to 0$, and

$$\|\boldsymbol{\varepsilon}(k)\|\leqslant\frac{n(\beta_1+\mu\beta_2)}{1-\mu}. \tag{38}$$

This shows that $\boldsymbol{\varepsilon}(k)$ is asymptotically bounded.

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