Pergamon

PII: S0005-1098(97)00191-X

Automatica, Vol. 34, No. 2, pp. 193–198, 1998

Printed in Great Britain
0005-1098/98 $19.00 + 0.00

Brief Paper

Robust Control of a Remotely Operated Underwater Vehicle*

GIUSEPPE CONTE † and ANDREA SERRANI

Key Words—Robust control; underwater vehicle; remotely operated; nonlinear simulation.

Abstract—A control strategy for an underwater vehicle based on a scheduling of linear H_∞ controllers has been proposed, and the overall performance of the closed-loop system have been evaluated by means of nonlinear simulation in a broad range of working conditions, with particular attention to the effects of the underwater current that acts on the vehicle. © 1998 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

The development of control systems for underwater unmanned vehicles (UUV) has received lately a great deal of attention. Typical tasks performed by UUVs include environmental data gathering as well as inspection and assembly of underwater structures. The automatic control of UUVs presents several difficulties due to the strong nonlinearity of the dynamics, the high degree of model uncertainty resulting from poor knowledge of the hydrodynamic coefficients, and the effect of external unmeasurable disturbances such as underwater current. Among the various approaches reported in the literature, which include sliding-mode in Yoerger and Slotine (1985) and Cristi *et al.* (1990), and adaptive control in Goheen and Richard (1990), we consider here the H_∞ approach that has previously been applied to UUV control in Kaminer *et al.* (1991). With respect to the latter, our work presents some basic differences: first, since state variables are not completely measurable in our setting, we deal with output feedback instead of state feedback, thus adding an observer to the complexity of the scheme. In addition, simulation results include robust stability tests with respect to the effect of depth changes and variations of the underwater current.

Specifically, we consider the model of a remotely operated vehicle (ROV) which was employed by the Italian Company SnamProgetti in the realization of underwater structures for the exploitation of combustible gas deposits. The vehicle is tethered to a supporting ship, while four thrusters provide control in the dive plane. Section 2 of the paper describes the nonlinear and the linearized models of the ROV dynamics. In Section 3 the control specifications are translated into an H_∞ optimization problem, and the choice of weighting functions are discussed. In Section 4 the robustness properties of the closed-loop system are evaluated by means of computer simulations and discussed.

Our results show that the H_∞ approach provides a computationally simple way of designing controllers for the considered ROV which, in spite of nonlinearity and parameter variations, exhibit good performances under various working conditions.

2. A MODEL FOR THE VEHICLE DYNAMICS IN THE PLANE

A description of the ROV dynamics can be derived from Newton's second law of motion. We describe the vehicle kinematics by an inertial coordinate frame $R(O_i, x, y, z)$ and a body-fixed frame $R_b(O_b, x_b, y_b, z_b)$ whose respective orientation is determined by a standard roll, pitch and yaw angles parameterization. For our purpose we restrict our attention to the problem of controlling the vehicle in the horizontal plane, the controlled variables being x, y and the yaw angle φ. Therefore, we neglect the contributions of gravity and buoyancy to the forces that act on the vehicle. The equations of motion of the ROV with horizontal plane control are given by (see Longhi and Rossolini, 1989 for details)

$$m\ddot{x} + H_x(x, \varphi, \dot{x}, \dot{y}, v_c) + R_x(\varphi, \dot{x}, \dot{y}, v_c) = T_x,$$
$$m\ddot{y} + H_y(y, \varphi, \dot{x}, \dot{y}, v_c) + R_y(\varphi, \dot{x}, \dot{y}, v_c) = T_y, \quad (1)$$
$$I_z\ddot{\varphi} + H_\varphi(\varphi, \varphi_c, v_c) + R_\varphi(\varphi, \dot{\varphi}) = T_\varphi,$$

---

* Received 18 August 1995; revised 2 July 1996; received in final form 14 May 1997. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor M. Tomizuka under the direction of Editor Yaman Arkun. *Corresponding author* G. Conte. Tel. + 39 71 2204844; Fax + 39 71 898246; E-mail gconte@anvax1.cineca.it.

†Dipartimento di Elettronica e Automatica, Università degli Studi di Ancona, Via Brecce Bianche, 60131, Ancona, Italy.

where m is the vehicle mass, I_z the inertia moment about the z-axis, $H_\bullet$ the cable traction, $R_\bullet$ the hydrodynamic forces, T_x, T_y and T_φ the applied forces along the x, y axes and torque along the z-axis, respectively, v_c and φ_c are the magnitude and the angle of incidence of the underwater current. Equations (1) are nonlinear in nature, and subject to large parameter variations as a result of the influence of the underwater environment as well as the dependence on the different load configurations at which the vehicle is expected to operate. In our tests we have considered three different working conditions, L_i, $i = 1, 2, 3$: in the first one and in the third one the vehicle is loaded with two different intervention modules; in the second one the vehicle is unloaded. For each working conditions, two different operating depths, respectively 200 and 1000 m, have been considered. Equations (1) depend on the components in the dive plane of the underwater current velocity, which has been modeled as a disturbance. The command thrusts are generated by four independent propellers, whose arrangement and characteristics are described in Longhi and Rossolini (1989). As a physical constraint on the control effort, a saturation of 5000 N on each propeller has been considered.

Our objective is to design a set of fixed linear controllers to provide robust stability and nominal performance, which include set point stabilization and disturbance rejection to counteract the effects of changes in magnitude and direction of the underwater current. Robustness is required to deal with uncertainties in the plant model, and to reduce the impact of the linear approximation. Each controller has been designed on the basis of a linearization of the plant model (1) around the origin at each load configuration L_i. In practical applications, since the payload remains unaltered during the mission, the selection of the controller is performed by a scheduler at the beginning of the task. The linearized system has the form

$$\dot{\xi} = A\xi + Bu + Lv, \qquad \eta = C\xi, \tag{2}$$

$$\xi = [\dot{x},\ x,\ \dot{y},\ y,\ \dot{\varphi},\ \varphi]^{\mathrm{T}},$$

$$u = [T_x - T_{x_0},\ T_y - T_{y_0},\ T_\varphi - T_{\varphi_0}]^{\mathrm{T}},$$

$$\eta = [x,\ y,\ \varphi]^{\mathrm{T}}, \qquad v = [v_{cx} - v_{cx_0},\ v - v_{cy_0}]^{\mathrm{T}}.$$

The input $[T_{x_0}, T_{y_0}, T_{\varphi_0}]$ is obtained via inversion of the dynamic equations at the equilibrium. A depth of 200 (m), $v_{c_0} = 0.15$ (m/s), $\varphi_{c_0} = 1.5$ (rad) have been assumed as nominal conditions.

Referring to a standard feedback configuration where $K(s)$ and $G(s)$ stand for the transfer matrices of the compensator and the plant, respectively, the problem of satisfying the performance objectives can be easily stated as a condition on the lowest singular values of the return difference matrix (Doyle and Stein, 1981)

$$\underline{\sigma}(I + G(\mathrm{j}\omega)K(\mathrm{j}\omega)) \geq |w_1(\mathrm{j}\omega)|. \tag{3}$$

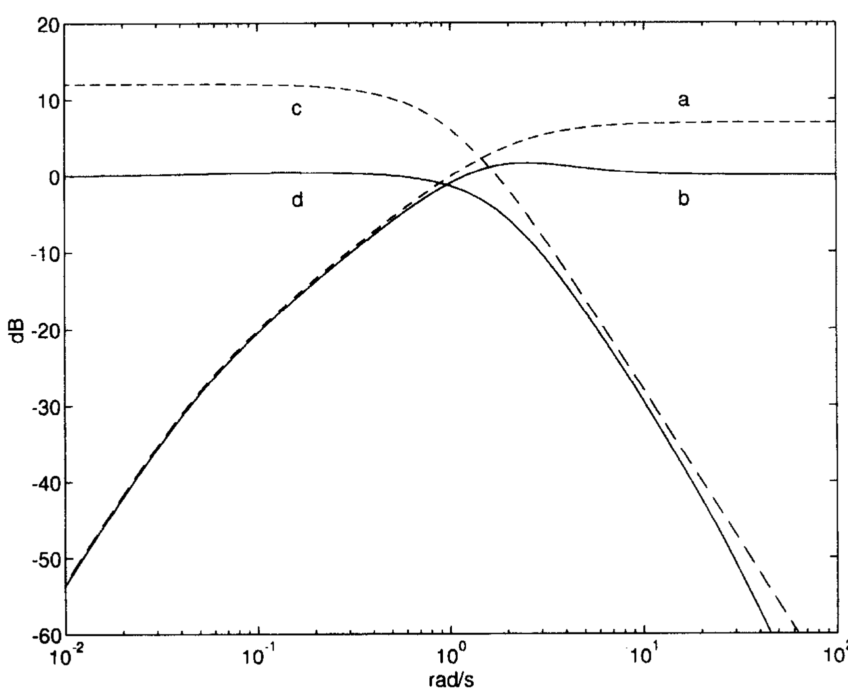

Fig. 1 Sensitivity and complementary sensitivity.

Choosing the weight $|w_1(j\omega)|$ "large" in the range of interest (typically, at low frequencies) yields disturbance rejection at the plant output and sensitivity reduction, since equation (3) is equivalent to the following condition on the sensitivity matrix:

$$\bar{\sigma}(S(j\omega)) \le \frac{1}{|w_1(j\omega)|},$$

$$S(j\omega) \doteq (I + G(j\omega)\,K(j\omega))^{-1}.$$

We assume the so-called "unstructured multiplicative model at the plant output"

$$G(s) = (I + \Delta(s))G_0(s)$$

as a representation of the uncertainties, the perturbed and the nominal model of the plant being given by $G(s)$ and $G_0(s)$, respectively. The reason behind this choice is that we are concerned with the effect of a disturbance modeled at the plant output, and that we require a very general representation of the uncertainties, since very little information on the way they affect the model is available. Since $\Delta(s)$ can be of higher order than the plant model, this kind of representation is particularly useful in treating undamped or even unstable modes that have been neglected in the model. We assume that the perturbation satisfies the bound

$$\bar{\sigma}(\Delta(j\omega)) < |w_2(j\omega)| \quad \forall \omega \in \mathbb{R}$$

which implies that the closed-loop system is robustly stable if the complementary sensitivity function

$$I - S(j\omega) = G(j\omega)K(j\omega)(I + G(j\omega)K(j\omega))^{-1}$$

satisfies

$$\bar{\sigma}(I - S(j\omega)) \le \frac{1}{|w_2(j\omega)|}. \tag{4}$$

Finding a suitable bound in equation (4) is not a trivial task to perform: the function $w_2(j\omega)$ must in principle be selected in order to take into account the effect of every destabilizing perturbation that can reasonably act on the system. The bounds may sometimes result too wide for practical use, and then too conservative, or poorly chosen, so that the controller does not provide effective robustness for the closed-loop system. However, the use of the maximum singular value allows the designer to rely on familiar frequency-domain tools as bandwidth, gain margin and peak at resonance to assess the robustness specifications of the linearized system.

3. H_∞ CONTROL DESIGN

As the control specifications are formulated in the frequency domain, they can be easily translated into an H_∞ optimization problem. Noting that, for any transfer matrix $A(j\omega) \in \mathbb{C}^{m\times n}$ and $B(j\omega) \in \mathbb{C}^{p\times n}$

$$\max\{\bar{\sigma}(A(j\omega)), \bar{\sigma}(B(j\omega))\} \le \bar{\sigma}\left(\begin{bmatrix} A(j\omega) \\ B(j\omega) \end{bmatrix}\right) \le \left\| \begin{bmatrix} A \\ B \end{bmatrix} \right\|_\infty$$

$$\forall \omega \in \mathbb{R},$$

the robustness specifications (3) and (4) are met if

$$\left\| \begin{matrix} w_1(j\omega)S(j\omega) \\ w_2(j\omega)(I - S(j\omega)) \end{matrix} \right\|_\infty \le 1. \tag{5}$$

Formulation (5) is commonly known as "mixed-sensitivity problem" (Safonov and Chiang, 1988), and can be easily implemented in the standard problem of the H_∞ optimization, which amounts in finding, among all proper real-rational controllers that stabilize the interconnected system, the one that minimizes the infinity norm of the closed-loop map T_{zw} between w and z. In our case the standard problem must be formulated so that the weighted sensitivity operators and the complementary sensitivity operator are embedded into T_{zw}. This can be done defining the augmented plant P and the interconnection structure as

$$P(j\omega) = \begin{bmatrix} w_1(j\omega)I_{3\times 3} & -w_1(j\omega)G(j\omega) \\ 0_{3\times 3} & w_2(j\omega)G(j\omega) \\ I_{3\times 3} & -G(j\omega) \end{bmatrix},$$

$$\begin{bmatrix} z_1 \\ z_2 \\ y \end{bmatrix} = P(j\omega)\begin{bmatrix} w \\ u \end{bmatrix},$$

$$u = K(j\omega)\,y,$$

which is well defined because the transfer function $G(j\omega)$ is strictly proper.

It is common practice to consider the sub-optimal formulation to find a stabilizing controller K that yields $\|T_{zw}\|_\infty \le \gamma$, γ being an upper bound for the infimum at issue. In our design methodology we consider a slightly different approach: fixing the robustness bound $w_2(j\omega)$ and starting with $\gamma = 1$ and an initial $w_1(j\omega)$ for which a stabilizing controller exists, decrease γ in order to attain the lowest value in a specified tolerance such that $\|\gamma^{-1}w_1 S\|_\infty \le 1$, under the constraint $\|w_2(I - S)\|_\infty \le 1$, which corresponds in optimizing the performance, under a guaranteed robust stability margin. A few trial-and-error iterations are needed to get the appropriate choice for the weights. The suitability of the design has to be tested first with respect to a singular values analysis on the linearized system, and then by means of nonlinear simulations. The first step is the selection of the robustness bound: we have required the system to have sufficient stability margin to tolerate

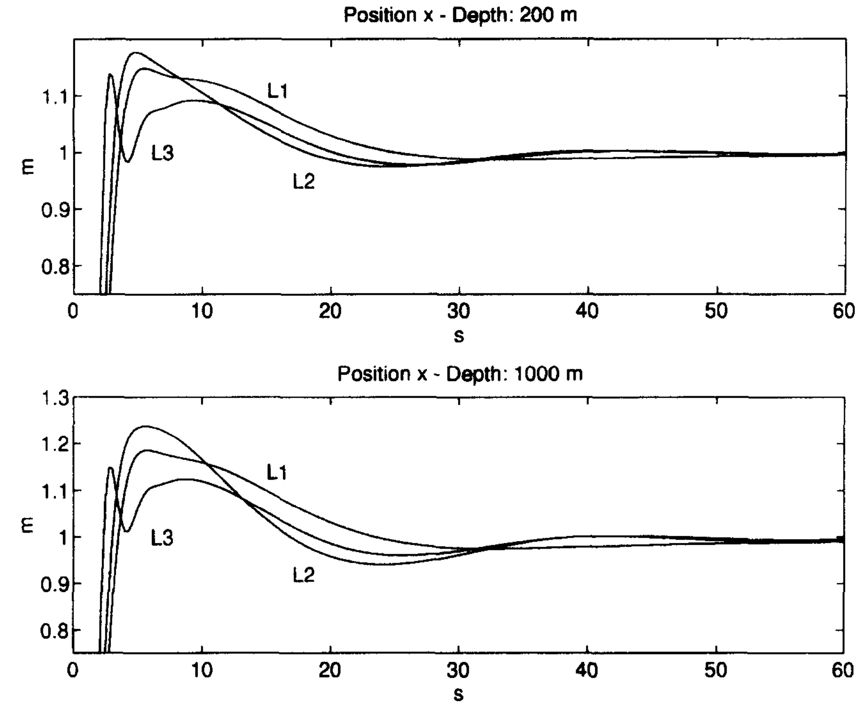

Fig. 2. Response for all load configurations.

uncertainties in the loop transfer function as large as a factor of 10 (20 dB) at 1 Hz and greater beyond this frequency. Due to the high inertia of the vehicle, and in order to limit actuators efforts, closed-loop bandwidth has been chosen no greater than 0.25 Hz (approximately 1.5 rad/s). As we need the controller to be strictly proper for the standard problem to be well posed, both the plant and the inverse of the weight must show the same roll-off (-40 dB/dec). Given these considerations, the robustness bound has been selected as $w_2(j\omega) = (2j\omega - \omega^2 + 1)/4$. For the performance weight, the first choice was to use a first order rational function, $w_1(j\omega) = k/(1 + j\omega\tau)$ to obtain the desired high gain at low frequencies. However, simulation results showed that, with such a function, a sufficiently high gain could have only be obtained at the expense of pushing the pole too close to the origin, that would have lead to an ill-conditioned state matrix in the realization of $P(j\omega)$, and a second-order proper function had to be considered. A problem that arises with second-order functions is that if the magnitude plot crosses the 0 dB with a -40 dB/dec slope, then the singular values of the sensitivity function decade too sharply near the crossover frequency, creating a peak in the complementary sensitivity function, with loss of robustness. To avoid this, $w_1(j\omega)$ has been given a zero before the crossover frequency (about 0.05 rad/s) to reduce the slope to -20 dB/dec in the crossover range: this relaxes the performance bound and increases the control activity towards robustness specifications. The weight selected as a baseline for the final design is

$$w_1(j\omega) = \frac{0.125(\omega^2 - 2.05j\omega - 0.1)}{(\omega^2 - 2\times 10^{-4}j\omega - 10^{-8})}.$$

Note that the robustness bound is improper: however, one can adopt the trick of embedding the weight into the plant model, so that the series compositon is proper and can be realized in state-space form. The state-space approach described in Glover and Doyle (1988) has been used to obtain the controllers. The resulting minimal realization of each controller has 12 states, equal in number to the order of the generalized plant. The SV Bode plot of the sensitivity and complementary sensitivity matrices and the inverse of their weighting functions are shown in Fig. 1 for the closed-loop system with respect to the configuration L_2. Similar results have been found with respect to the configurations L_1 and L_3.

4. NONLINEAR SIMULATION ANALYSIS

The performance of each controller has been evaluated in simulation with the nonlinear model of the vehicle. The behavior of the closed-loop system to step references in nominal conditions has been tested with respect to each load configuration. Then changes in depth and in magnitude and direction of the underwater current have been taken into account and the system response has

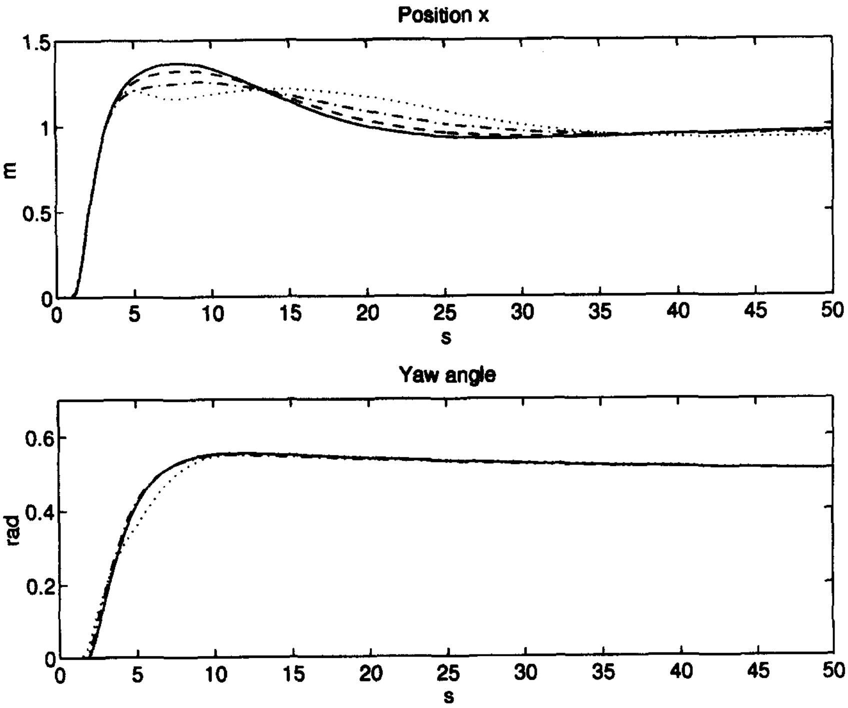

Fig. 3. Variations in magnitude.

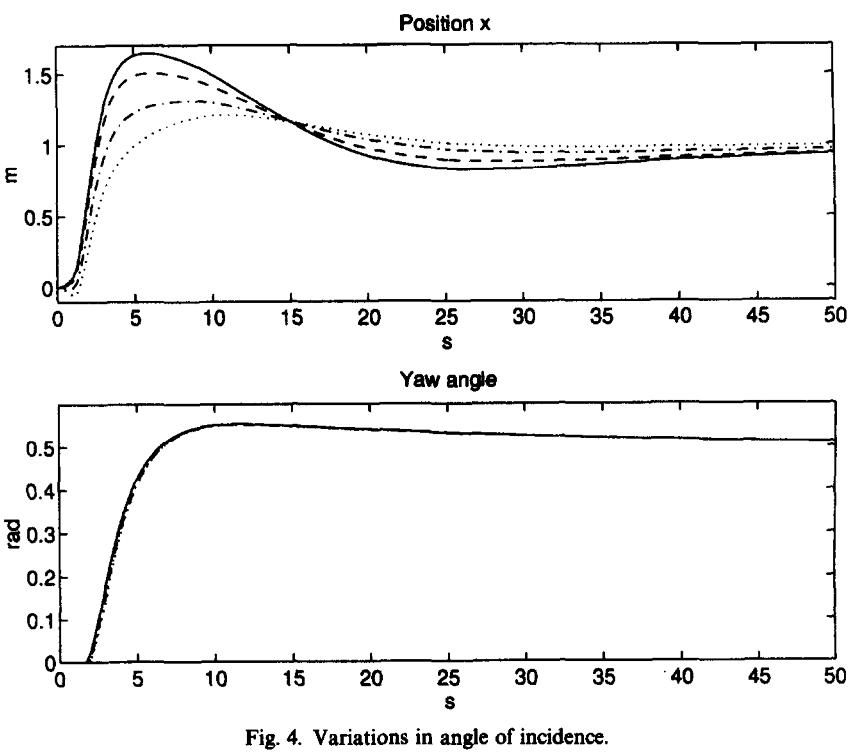

Fig. 4. Variations in angle of incidence.

been evaluated. Figure 2 compares simulations carried on with different depths, 200 and 1000 m, respectively, for the configuration L_1, the hardest to be controlled. Variations on the vehicle dynamics mainly occur as a result of an increased cable traction. It is easy to see that this has little influence on the closed-loop dynamics. The most important issue is the capability of the controllers to maintain stability and error regulation against variations of the underwater current velocity. A first test considers increasing values of the magnitude of the underwater current velocity. For this purpose, several nonlinear simulations have been performed for each configurations, starting from 0.1 (m/s). We found that above 0.3 (m/s) the saturation of the actuators imposes so severe limitations on the control activities that instability arises, especially for the configuration L_1. Below 0.3 (m/s), however, the controllers are able to maintain stability and regulation. The second test has been performed changing the current direction: the controllers have been found to provide robustness within the whole range $[0, 2\pi]$. Figures 3 and 4 summarize some results relative to the robustness test discussed above.

5. CONCLUSIONS

The H_∞ approach to robust control has been applied to the design of a control system for a remotely operated underwater vehicle (ROV). The capability of H_∞ design to guarantee robust stability has been successfully used to deal with uncertainties in the plant model, obtaining setpoint regulation and disturbance attenuation by means of an appropriate selection of the frequency weighting functions. It has been shown that the H_∞ approach provides a computationally simple way of designing controllers for the considered ROV that can be used in a practical control strategy based on scheduling.

REFERENCES

Cristi, R., F. Papoulias and A. Healey (1990). Adaptive sliding mode control of autonomous underwater vehicles in the dive plane. *IEEE J. Oceanic Engng*, **OE-15** (8), 152–160.

Doyle, J. C. and G. Stein (1981). Multivariable feedback design—concept for a classical/modern synthesis. *IEEE Trans. Automat. Control*, **AC-26**(1), 4–16.

Glover, K. and J. C. Doyle (1988). State-space formulae for all stabilizing controllers that satisfy an error bound and relation to risk sensitivity. *System Control Lett.*, **11**, 167–172.

Goheen, K. and E. Richard (1990). Multivariable self-tuning autopilots for autonomous and remotely operated underwater vehicle. *IEEE J. Oceanic Engng*, **OE-15** (8), 152–160.

Kaminer, I., A. Pascoal, C. Silvestre and P. Khargonekar (1991). Control of an underwater vehicle using H synthesis. In *Proc. 30th IEEE Conf. on Decision and Control*, Brighton, UK.

Longhi, S. and A. Rossolini (1989). Adaptive control for an underwater vehicle: simulation studies and implementation details. In *Proc. of the IFAC Workshop on Expert Systems and Signal Processing*, Copenhagen.

Safonov, M. G. and R. Chiang (1988). CACSD using the state-space L theory—a design example. *IEEE Trans. Automat. Control*, **AC-33** (3), 477–479.

Yoerger, D. and J. Slotine (1985). Robust trajectory control of underwater vehicles, *IEEE J. Oceanic Engng*, **OE-10**(4), 462–470.