Contents

Chapter 1

Literature Review

This chapter presents a comprehensive overview of existing research on remotely operated vehicles (ROVs), focusing on modeling and control design. This literature review aims to investigate and synthesize control strategies, addressing uncertainties from actuator dynamics and environmental factors, which correspond to the proposed research question.

Section 1 gives a brief description of ROVs, including their types and general attributes. Section 2 explains the commonly accepted assumptions and mathematical models used in the field. Section 3 offers an extensive review of various studies on the application of robust control design for underwater systems. In conclusion, Section 4 summarizes the key insights from the review and suggests a control methodology.

I Overview on ROVs

This section provides an overview of ROVs, from their classification and applications to their inherent characteristics and challenges. The obtained knowledge forms a basis for ROV modeling and control design.

According to [1], an ROV's mechanical structure consists of a monitoring camera, a sensor for gathering navigation data, and actuators for directional control. A comparative study [2] found that the physical aspects that affect ROV functions are the accuracy of sensor systems and the thruster designs. The unpredictable nature of underwater currents, drag, and buoyancy dynamics can also have a serious impact on a ROV's performance, complicating model design.

Several studies [1], [2] have identified that ROVs have become crucial for industrial applications, offshore oil and gas exploration, patrolling, and surveillance. Therefore, its control system should focus on position tracking and station keeping in the presence of parameter and environmental uncertainties, addressing the following issues.

A recent systematic review [3] concluded that there are two primary classifications for ROVs based on their functions and intended use: observation class and work class. Observation class vehicles are typically small and limited to shallow waters with propulsion power up to a few kilowatts. Work class vehicles can perform heavy-duty work, requiring significant hardware system complexity. Thus, when the functionality of these large ROVs is not necessary, a smaller ROV is preferred for a wide range of applications.

II Mathematical Modelling

Remotely operated vehicles require mathematical models for various purposes, including control system design, simulation, and performance analysis.

Fossen [4] provided a complete description of the fundamentals of mathematical modeling for water vehicles. In this book, ROV was represented as a single rigid body. The SRB model has drastically simplified the model while capturing the essential dynamics of the system. From a kinematic point of view, ROVs have six degrees of freedom (DoFs). However, the orientation expressed in the rotation angles could eventually lead to the singularity. To solve this issue, [5] proposed the quaternions representation. The dynamics were derived based on classical physics laws: Newtons Second Law and Euler-Lagrange equation, forming the set of nonlinear equations. The study [6] simplified the similar dynamics and represented it in a matrix form.

However, several sources have established that some aspects of the ROV dynamics require empirical estimates due to their complex, nonlinear, and coupled nature [4], [7]. For instance, the inertia of the surrounding fluid cannot be neglected when the vessel moves through a viscous medium. Additionally, water damping is another source of nonlinearity that can be approximated as a function of the velocity. Furthermore, aside from buoyancy and gravity, it is common practice to cancel all other forces acting on a vehicle, although it can also impact the dynamics [7].

Moreover, thruster modeling must be applied to define the desired thrust of each thruster. A recent study by [7] stated that creating an accurate thruster model can be challenging due to the influence of factors such as motor models, hydrodynamic effects, and propeller mapping.

By making these simplifications, the control system cannot independently provide effective control over such uncertain dynamics. Therefore, a robust controller design is necessary for precise ROV position tracking.

III Control Solutions

Controlling an ROV is a complex task that requires a set of processes to stabilize the vehicle and to make it follow the operator's instructions. To ensure the robustness of the system, it is necessary to define a control system that can handle disturbances caused by parameter and environmental changes.

According to [2], there are two main challenges associated with ROVs control:

1. Unmodeled elements like added mass and hydrodynamic coefficients.
2. Highly nonlinear dynamics of the underwater environment which cause significant disturbances to the vehicle.

The recent research on ROV control showed several schemes that can achieve robust stability under variable disturbances. The classical approach applied is sliding mode control (SMC), which was introduced by Slotine [8]. SMC is an effective way to address the issues mentioned above and is, therefore, a feasible option for controlling underwater vehicles. However, standard SMC introduces high-frequency signals, which can cause actuator switching and consequently decrease its lifetime.

The modern SMC interpretations keep the main advantages, thus removing the chattering effects. One of the possible approaches is Adaptive Sliding Mode Control (ASMC) design. [9] designed an effective hybrid control mechanism for the underwater system that follows a given route, adapting to dynamic disturbances. [10] improved a control system for ROVs, eliminating the need for dynamics linearization. Another way to refine SMC is to add an integral component into the controller equation. The proposed Integral Sliding Mode Control

(ISMC) reduced chattering, effectively eliminating the uncertainty of the model parameters [11].

The choice of control strategy should be based on the specific requirements and characteristics of the underwater vehicle. While both SMC modifications showed precise control, these approaches faced certain challenges [11], [10]. The ISMC method had limitations in adapting to rapidly changing dynamics, while the ASMC method experienced potential trade-offs in transient performance. These problems were opposite to each other. Therefore, combining them into Adaptive Integral Sliding Mode Control (ADISMC) can be a good idea to leverage the strengths of both methods and compensate for their weaknesses.

IV Summary

This literature review comprehensively examined the general characteristics, mathematical modeling, and control solutions for underwater vehicles. This overview identified a gap in the field of robust control for remotely operated vehicles, particularly in dealing with parameter and environmental uncertainties. While the original sliding mode control scheme provided robust stability, it suffered from high-frequency output oscillations. There were several variations of SMC available, and each algorithm has its limitations. To address this research gap, a combination of adaptive and integral sliding mode modifications was proposed.

Chapter 2

Mathematical Modelling

Remotely operated vehicles (ROVs) are complex systems that require mathematical models for various purposes, including control system design, simulation, and performance analysis. With accurate mathematical models, ROVs are able to navigate through different underwater terrains and complete control tasks with a good precision. Also, the simulation, based on these models, are suitable to test different work scenarios and detect undesirable ROV's behaviour before the physical experiment.

The fundamentals of the modelling for marine vehicles were fully described in Fossen [4]. Using common assumptions, ROV is treated as a single rigid body with six degrees of freedom (DOF). By considering the vehicle as a rigid body, we can simplify the mathematical modeling process while capturing the essential dynamics of the system.

In order to effectively model rigid bodies, it is crucial to consider their kinematic and dynamic properties.

I Notations

Before proceeding to theoretical derivations, it is necessary to clarify the general notations. For the motion with six DOF, six independent coordinates are defined in the coordinate frame: three for translational directions (surge, sway, and heave) and three for rotational directions (roll, pitch, and yaw) as depicted in (Fig. 1)

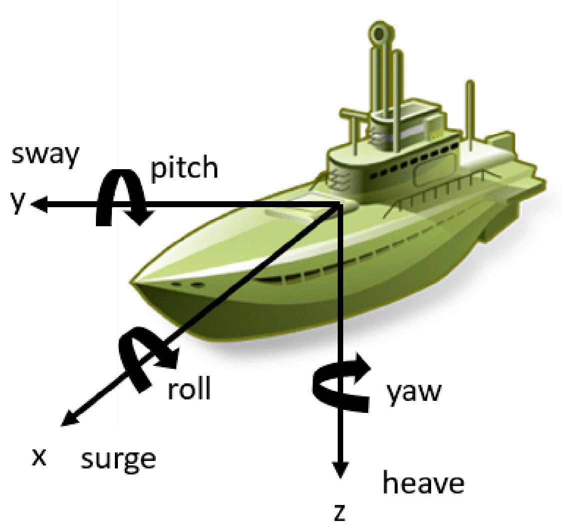

Fig. 1. 6 DOF of a marine vehicle

The linear position is defined as $\mathrm{r} = [\mathrm{r_x}, \mathrm{r_y}, \mathrm{r_z}]^\top$ for translation along xyz axis respectively, while orientation can be expressed in terms of Euler angles around corresponding axis. But Euler angles will eventually lead to the singularity when sway angle is $\pm 90°$. However, the quaternions can resolve this problem by adding redundancy into the representation. The orientation quaternion is defined in scalar-first form as $\mathrm{q} = \mathrm{q}_0 + \mathrm{q}_1 \cdot \mathrm{i} + \mathrm{q}_2 \cdot \mathrm{j} + \mathrm{q}_3 \cdot \mathrm{k} = [\mathrm{q}_0, \mathrm{q}_1, \mathrm{q}_2, \mathrm{q}_3]^\top$

For each direction, the velocity vectors can be defined separately: $\mathrm{v} = [\mathrm{v}_\mathrm{x}, \mathrm{v}_\mathrm{y}, \mathrm{v}_\mathrm{z}]^\top$ for translation along xyz axis and $\omega = [\omega_\mathrm{x}, \omega_\mathrm{y}, \omega_\mathrm{z}]^\top$ for rotation around xyz axis respectively. The same applies to linear forces f and torques τ.

To summarize, the general notations look like:

Notation	Description	Dimentionality
r	Linear position vector	$\in \mathbb{R}^3$
q	Angular position (orientation) vector	$\in \mathbb{R}^4$
v	Linear velocity vector	$\in \mathbb{R}^3$
ω	Angular velocity vector	$\in \mathbb{R}^3$
f	Vector of linear forces	$\in \mathbb{R}^3$
τ	Vector of torques	$\in \mathbb{R}^3$

Fig. 2. Notation

For the convenience, it is desirable to define combined vectors of positions, velocities and forces as: $\bar{\mathrm{q}} = \begin{bmatrix} \mathrm{r} \\ \mathrm{q} \end{bmatrix}$, $\bar{\mathrm{v}} = \begin{bmatrix} \mathrm{v} \\ \omega \end{bmatrix}$ and $\bar{\mathrm{f}} = \begin{bmatrix} \mathrm{f} \\ \tau \end{bmatrix}$

In order to manipulate with obtained vectors, it is necessary to define cross product operators:

$\mathrm{S}(\lambda)$ is a skew-symmetric matrix defined such that:

$$\mathrm{S}(\lambda) = \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix}$$

therefore $\mathrm{S}(\mathrm{u})\mathrm{v} = \mathrm{u} \times \mathrm{v}$ for vectors v,u $\in \mathbb{R}^3$

$\bar{\times}^*$ is the $\mathbb{R}^6$ cross product operator defined as:

$$\bar{\mathrm{v}}\bar{\times}^{*} = \begin{bmatrix} \mathrm{S}(\omega) & 0_{3\times3} \\ \mathrm{S}(\mathrm{v}) & \mathrm{S}(\omega) \end{bmatrix} \text{ where } \bar{\mathrm{v}} = \begin{bmatrix} \mathrm{v} \\ \omega \end{bmatrix}$$

II Frames of reference

In order to derive the kinematics and dynamics of the system, the calculations need to be projected into the same frame of reference. Sometimes several coordinate frames are defined based on the system configuration.

For ROV, it is reasonable to define two coordinate frames. These frames are the earth-fixed frame, which is inertial with fixed origin, and the body-fixed frame, which is a moving frame attached to the vehicle. as depicted in (Fig. 3).

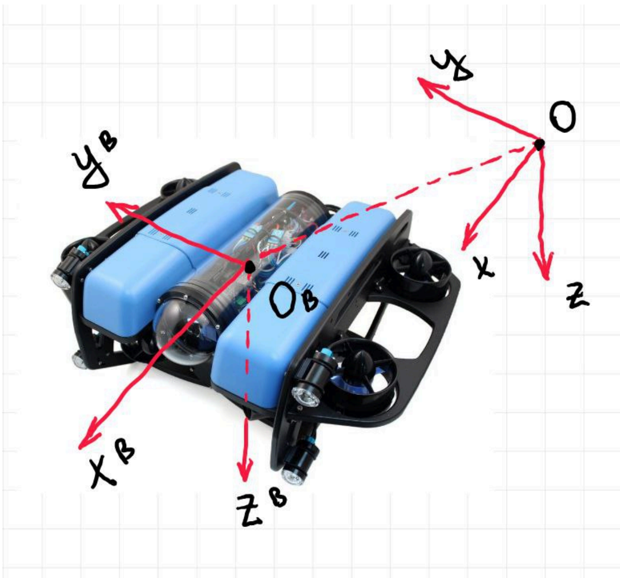

Fig. 3. The frames

The origin of the body-fixed frame usually coincides with the vehicle's center of mass, and its axes are chosen along the vehicle's principle axes of inertia.

In further derivations, the state variables of the rigid body expressed in the body-fixed frame would be noted by $^{\mathrm{B}}$ and in the earth-fixed frame by $^{\mathrm{N}}$.

III Kinematics

Kinematics describes the motion of the marine vehicle without considering the forces acting upon it. In order to describe kinematic motion of the body, it is necessary to find relation between velocities in two coordinate frames. This relation can be represented with linear transformations as:

$$\dot{\bar{\mathrm{r}}}^{\mathrm{N}} = \mathrm{J}(\bar{\mathrm{r}}^{\mathrm{N}})\bar{\mathrm{v}}^{\mathrm{B}}$$
$$\text{where } \mathrm{J}(\bar{\mathrm{r}}^{\mathrm{N}}) = \begin{bmatrix} \mathrm{R}(\bar{\mathrm{r}}^{\mathrm{N}}) & 0_{3\times3} \\ 0_{3\times3} & \mathrm{T}(\bar{\mathrm{r}}^{\mathrm{N}}) \end{bmatrix}$$

where the R is the rotational matrix and the T is the transformation matrix. The kinematic equations for the marine vehicle using quaternions can be written as follows:

$$\mathrm{R}(\mathrm{q}) = \begin{bmatrix} 1-2\left(q_2^2+q_3^2\right) & 2\left(q_1q_2-q_3q_0\right) & 2\left(q_1q_3+q_2q_0\right) \\ 2\left(q_1q_2+q_3q_0\right) & 1-2\left(q_1^2+q_3^2\right) & 2\left(q_2q_3-q_1q_0\right) \\ 2\left(q_1q_3-q_2q_0\right) & 2\left(q_2q_3+q_1q_0\right) & 1-2\left(q_1^2+q_2^2\right) \end{bmatrix}$$
$$\mathrm{T}(\mathrm{q}) = \frac{1}{2}\begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix}$$

IV Model dynamics

The Newton-Euler approach is commonly used to describe the dynamics of marine vehicles. This approach relates the applied forces and moments to the vehicle's accelerations and angular accelerations. The general equation of motion using the Newton-Euler approach in the body-fixed frame can be written as:

$$\mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \bar{\mathrm{v}}^{\mathrm{B}}\bar{\times}^{*}\mathrm{M}\bar{\mathrm{v}}^{\mathrm{B}} = \bar{\mathrm{f}}^{\mathrm{B}}$$

where M represents the inertia matrix of the rigid body and $\bar{\mathrm{f}}$ represents the total force acting on it.

The equation can be transformed into manupulator equation like:

$$\mathrm{M}_{\mathrm{B}}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C}_{\mathrm{B}}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} = \bar{\mathrm{f}}^{\mathrm{B}}$$

where $\mathrm{M}_{\mathrm{B}} \in \mathbb{R}^{6x6}$ is the rigid body mass matrix, $\mathrm{C}_{\mathrm{B}}(\bar{\mathrm{v}}^{\mathrm{B}}) \in \mathbb{R}^{6x6}$ is the rigid body Coriolis and centripetal forces matrix.

However, additional terms should be included in the equation to determine the specifics of the ROVs model. These terms comprise added mass, which represents the inertia of the surrounding fluid, the shift of the center of buoyancy due to changes in trim and heel angles, and damping effects. By incorporating these terms into the manipulator equation derived from the Newton-Euler approach, the model becomes more accurate and reflects the natural behavior of the ROV.

A. Center of Gravity and Center of Buyonancy

Due to the robust design of the marine vehicles, the center of byonancy(COB) is usually alligned with the center of mass(COM), but placed higher. This shift between centers causes torque acting against the capsize (Fig. 4).

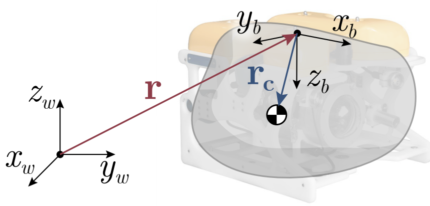

Fig. 4. The vehicle scheme

If we place the origin of the body frame at the center of mass(COM), the mass matrix can be expressed as:

$$\mathrm{M_B} = \begin{bmatrix} \mathrm{mI_{3\times3}} & -\mathrm{mS}\left(\mathrm{r_G^B}\right) \\ \mathrm{mS}\left(\mathrm{r_G^B}\right) & \mathrm{I_0} \end{bmatrix}$$

where $\mathrm{r_G^B}$ is the vector of the gravity center in the body frame, that is eventually zero vector.

The same applies to the Coriolis matrix:

$$\mathrm{C_B}(\bar{\mathrm{v}}^\mathrm{B}) = \begin{bmatrix} \mathrm{S}(\omega^\mathrm{B}) & 0_{3\times3} \\ \mathrm{S}(\mathrm{v}^\mathrm{B}) & \mathrm{S}(\omega^\mathrm{B}) \end{bmatrix} \times \mathrm{M_B}$$

B. *Concept of added mass*

Since the vehicle moves in a viscous enviroment, we can not neglect the inertia of the surrounging liquid. To compensate added mass effect, it is necessary to add two components into dynamics equation.

We can define vector of dynamical parameters of our body as:

$$\mathrm{f}_{\dot{\mathrm{v}}} \triangleq \frac{\partial \bar{\mathrm{f}}}{\partial \dot{\bar{\mathrm{v}}}}$$

Consequently the added mass matrix $\mathrm{M_A}$ and the Coriolis forces matrix for added mass $\mathrm{C_A}(\mathrm{v^B})$ can be expressed as:

$$\mathrm{M_A} = \begin{bmatrix} \mathrm{A_{11}} & \mathrm{A_{12}} \\ \mathrm{A_{21}} & \mathrm{A_{22}} \end{bmatrix} = -\,\mathrm{diag}\left\{\mathrm{f_{\dot{v}}}\right\}, \mathrm{where}\ \mathrm{A_{ij}} \in \mathbb{R}^{3x3}$$

$$\mathrm{C_A}(\bar{\mathrm{v}}^{\mathrm{B}}) = \begin{bmatrix} 0_{3\times 3} & -\mathrm{S}\left(\mathrm{A_{11}v^B} + \mathrm{A_{12}}\omega^{\mathrm{B}}\right) \\ -\mathrm{S}\left(\mathrm{A_{11}v^B} + \mathrm{A_{12}}\omega^{\mathrm{B}}\right) & -\mathrm{S}\left(\mathrm{A_{21}v^B} + \mathrm{A_{22}}\omega^{\mathrm{B}}\right) \end{bmatrix}$$

The values of dynamical parameters are usually determined empirically. The error on $\mathrm{M_A}$ and $\mathrm{C_A}$ can be quite large and we will not consider these matrices in the model for the control.

C. Hydrodynamic Damping

Generally, the dynamics of underwater vehicles can be highly nonlinear and coupled. Nevertheless, during the slow non-coupled motion the damping can be approximated to linear and quadratic damping:

$$\mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}}) = -\mathrm{K_{lin}} - \mathrm{K_{quad}}|\bar{\mathrm{v}}^{\mathrm{B}}|$$

The appropriate values of damping coefficients for vectors $\mathrm{K_{lin}}$ and $\mathrm{K_{quad}}$ can be discovered through several experiments.

D. Restoring forces

The common sense is to neglect all other forces acting on the vehicle except buoyancy and gravity. Although the motion of the current can also affect the dynamics, it is unpredictable and highly nonlinear, which makes it easier to compensate through control.

The weight of the body is defined as: $\mathrm{W} = \mathrm{mg}$, where m is the vehicle's mass and g is the gravity acceleration. The buoyancy force is defined as: $\mathrm{B} = \rho \mathrm{g} \nabla$, where ρ is the water density and ∇ the volume of fluid displaced by the vehicle.

By transforming the weight and buoyancy force to the body-fixed frame, we get:

$$\mathrm{f_G}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right) = \mathrm{R}^{\top}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right)\begin{bmatrix} 0 \\ 0 \\ \mathrm{W} \end{bmatrix} \quad \mathrm{f_B}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right) = -\mathrm{R}^{\top}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right)\begin{bmatrix} 0 \\ 0 \\ \mathrm{B} \end{bmatrix}$$

Therefore, overall restoring force and moment vector is defined as:

$$\mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) = -\begin{bmatrix} \mathrm{f_G}(\bar{\mathrm{r}}^{\mathrm{N}}) + \mathrm{f_B}(\bar{\mathrm{r}}^{\mathrm{N}}) \\ \mathrm{r_G^B} \times \mathrm{f_G}(\bar{\mathrm{r}}^{\mathrm{N}}) + \mathrm{r_B^B} \times \mathrm{f_B}(\bar{\mathrm{r}}^{\mathrm{N}}) \end{bmatrix}$$

where $\mathrm{r_B^B}$ is the vector of the buoyancy center in the body frame.

E. Matrix representation

The final system of equations for the mathematical model is:

$$\begin{cases} \mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) = \bar{\mathrm{f}}^{\mathrm{B}} \\ \dot{\bar{\mathrm{r}}}^{\mathrm{N}} = \mathrm{J}(\bar{\mathrm{r}}^{\mathrm{N}})\bar{\mathrm{v}}^{\mathrm{B}} \end{cases}$$

where $\mathrm{M} = \mathrm{M_B} + \mathrm{M_A}$, $\mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}}) = \mathrm{C_B}(\bar{\mathrm{v}}^{\mathrm{B}}) + \mathrm{C_A}(\bar{\mathrm{v}}^{\mathrm{B}})$

V Thrusters modelling

In the general case, the thruster force and moment vector will be a complicated function depending on the vehicle's velocity vector $\bar{\mathrm{v}}^{\mathrm{B}}$, voltage of the power source V and the control variable u. This relationship can be expressed as:

$$\bar{\mathrm{f}}^{\mathrm{B}} = \mathrm{T}\phi(\mathrm{u})$$

where $\mathrm{T} \in \mathbb{R}^{6\mathrm{xn}}$ is the thrust configuration matrix that maps body torques to thuster forces, $\phi(\mathrm{u}) \in \mathbb{R}^{\mathrm{nxn}}$ is the DC-gain transfer function that defines relation between PWM signal and output force, where n - number of thrusters.

VI BlueRov modelling (will be placed in a different chapter (?))

By the specification of the given thrusters, the dependency between control PWM signal and thrust is highly nonlinear (Fig. 5).

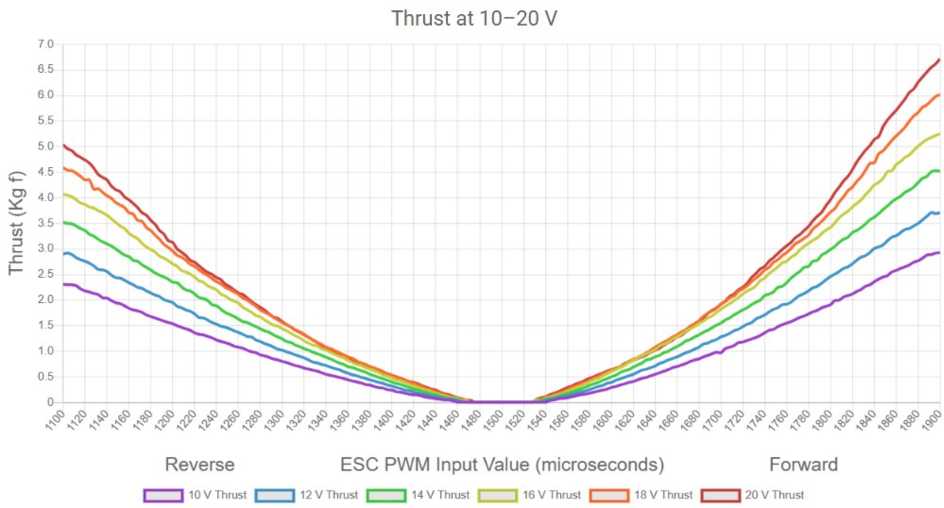

Fig. 5

In order to model this relation, the polynomial regression was applied on the normalized test data. A 5th-order approximation of the developed thrust at 16V voltage will be:

$$\phi(\mathrm{u_i}) = -0.22\mathrm{u_i^5} - 0.0135\mathrm{u_i^4} + 1.1\mathrm{u_i^3} + 0.172\mathrm{u_i^2} + 1.327\mathrm{u_i} + 0.027$$

The inverse dependency can be determined in the same way. The following expression is obtained :

$$\hat{\phi}(\mathrm{f_i}) = 0.0006\mathrm{f_i^5} - 0.0004\mathrm{f_i^4} - 0.02\mathrm{f_i^3} + 0.0006\mathrm{f_i^2} + 0.56\mathrm{f_i} - 0.0334$$

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Chapter 3

Methodology

I Control design

Introduce the concept of sliding mode control (SMC) and its advantages for controlling underwater robots.

As discussed in the previous chapter, there are several controller designs available. However, the sliding mode approach suggested by (Spong - ?) is highly regarded as the most sophisticated and frequently implemented one.

Sliding mode control (SMC) is a nonlinear control method that guarantees robust control of systems with uncertainties and disturbances. This technique involves developing a sliding surface within the state space and directing the system's trajectory to slide along this surface (Fig. 6).

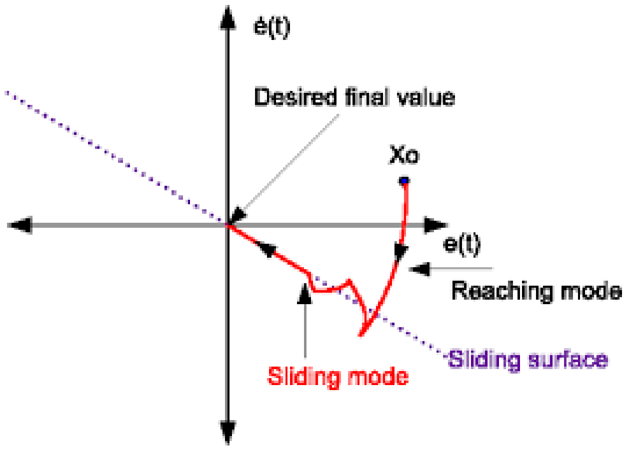

Fig. 6. The sliding mode scheme

Underwater robots face a range of challenges, including ocean currents, waves, and sensor noise. Fortunately, SMC provides a reliable and stable solution to these issues, making it an excellent choice for underwater robotics.

Additionally, SMC is known for its ability to respond quickly and accurately track desired trajectories, even in the presence of disturbances. This is particularly important for underwater robots that must navigate, dock, and manipulate objects.

Compared to other nonlinear control methods, SMC is a relatively straightforward solution to implement. With a basic understanding of system dynamics and sliding surface design.

the Leitmann approach - ?

A. Problem Formulation

Define the underwater robot system dynamics.

The final system dynamics is represented as:

$$\mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) = \bar{\mathrm{f}}^{\mathrm{B}}$$

Define control objectives

Underwater robots require precise control systems to navigate and operate effectively in challenging marine environments. These control objectives are crucial for ensuring the robot's stability, accuracy, and responsiveness:

- Position and Orientation Tracking: The robot must accurately follow a desired trajectory, maintaining its position and orientation as intended.
- Disturbance Rejection: The robot should be able to withstand external disturbances, such as ocean currents, waves, and sensor noise, to maintain stable tracking performance.
- Robustness: The control system should be robust to uncertainties in the robot's dynamics and environmental conditions, ensuring reliable operation even in unpredictable situations.
- Real-Time Implementation: The control algorithm should be computationally efficient and able to run in real-time on the robot's embedded system, enabling prompt and effective responses to changing conditions.

Derive the state-space representation of the system - ?

Identify the uncertainties and disturbances affecting the system.

The dynamics parameter estimates may be imprecise due to unmodeled dynamics and external factors. This means the estimated values might not perfectly match the actual system behavior.

$\hat{\mathrm{x}}$ indicates the approximate value of parameter x, while the estimation error is defined as $\tilde{\mathrm{x}} = \hat{\mathrm{x}} - \mathrm{x}$.

State which parameters are unknown to us - ?

B. Sliding Surface Design

Explain the concept of a sliding surface and its role in SMC.

In sliding mode control (SMC), a sliding surface is a hyperplane in the state space that defines the desired system behavior. In SMC, the sliding surface is designed to be an invariant set.

Invariant sets are sets of states in the state space that, once entered, cannot be exited under the action of the control law.

For an invariant set $\mathbb{S}$ the following is valid:

if $\mathrm{x}(\mathrm{t}_0) \in \mathbb{S}$ then $\mathrm{x}(\mathrm{t}) \in \mathbb{S}$ for $\forall\,\mathrm{t} > \mathrm{t}_0$

Sliding surface is invariant set if it satisfies the sliding condition:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{s}^2 = -\eta \mid \mathrm{s} \mid$$

The control objective is to force the system's trajectory to slide long this surface. Once the system's trajectory reaches the sliding surface, it will remain on the surface as long as the control law is applied.

The sliding surface provides robustness to uncertainties and disturbances by ensuring that the system's behavior is insensitive to these factors. This is because the control law is designed to counteract any disturbances or uncertainties that would push the trajectory off the surface. As long as the system's trajectory remains on the sliding surface, the control system will maintain stability and performance.

Therefore, the initial tracking problem $\bar{\mathrm{r}}^{\mathrm{B}} = \bar{\mathrm{r}}^{\mathrm{B}}_{\mathrm{des}}$ is equivalent to remaining on the surface $\mathbb{S}$ or $\mathrm{s}(\mathrm{t}) = 0$ for $\forall\,\mathrm{t} > \mathrm{t}_0$.

Derive the sliding surface for the underwater robot system.

General sliding surface for the system is:

$$\mathrm{s} = (\frac{\mathrm{d}}{\mathrm{dt}} + \lambda)^{\mathrm{n}-1}\tilde{\mathrm{r}}^{\mathrm{B}} = \tilde{\mathrm{v}}^{\mathrm{B}} + \lambda\,\tilde{\mathrm{r}}^{\mathrm{B}}$$

where n is the order of the system.

Analyze the properties of the sliding surface, such as reachability and invariance

The design of the sliding surface is critical for the performance of the SMC system. The sliding surface should be:

- Reachable: The system's trajectory should be able to reach the sliding surface in a finite amount of time.
- Invariant: Once the system's trajectory reaches the sliding surface, it should remain on the surface for all future time.
- Attractive: The control law should attract the system's trajectory to the sliding surface and keep it there.

Global invariant set theorem - ?

C. Control Law Design

Derive the SMC control law for the underwater robot system.

In order to find a minimum of s, which corresponds to the minimal tracking error, let us take a time derivative:

$$\dot{\mathrm{s}} = \dot{\tilde{\mathrm{v}}}^{\mathrm{B}} + \lambda\,\dot{\tilde{\mathrm{r}}}^{\mathrm{B}} = \dot{\hat{\mathrm{v}}}^{\mathrm{B}} - \dot{\mathrm{v}}^{\mathrm{B}} + \lambda\,\mathrm{v}^{\mathrm{B}} = 0$$

Substituting the system dynamics, it would give us the following equation:

$$\dot{\mathrm{s}} = -\mathrm{M}^{-1}(\mathrm{C}(\mathrm{v}^{\mathrm{B}})\mathrm{v}^{\mathrm{B}} + \mathrm{D}(\mathrm{v}^{\mathrm{B}})\mathrm{v}^{\mathrm{B}} + \mathrm{g}(\mathrm{r}^{\mathrm{N}}) - \mathrm{f}^{\mathrm{B}}) - \dot{\mathrm{v}}^{\mathrm{B}} + \lambda\,\mathrm{v}^{\mathrm{B}} = 0$$

Note that inertia matrix M is always invertible by the construction.

The final expression for control force is:

$$\mathrm{f}^{\mathrm{B}} = -\mathrm{Ma} + \mathrm{C}(\mathrm{v}^{\mathrm{B}})\mathrm{v}^{\mathrm{B}} + \mathrm{D}(\mathrm{v}^{\mathrm{B}})\mathrm{v}^{\mathrm{B}} + \mathrm{g}(\mathrm{r}^{\mathrm{N}})$$

where $\mathrm{a} = \dot{\mathrm{v}}^{\mathrm{B}} + \lambda\,\mathrm{v}^{\mathrm{B}}$ is outer-loop control.

In order to form a linear closed-loop system, we will choose control input according to:

$$\bar{\mathrm{f}}^{\mathrm{B}} = \hat{\mathrm{M}}\mathrm{a} + \hat{\mathrm{C}}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \hat{\mathrm{D}}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \hat{\mathrm{g}}(\bar{\mathrm{r}}^{\mathrm{N}})$$

where a is outer-loop control to be designed further.

If we substitute () into (), we will get double integrator system in a form:

$$\dot{\bar{\mathrm{v}}}^{\mathrm{B}} = \mathrm{a} + \eta(\bar{\mathrm{r}}^{\mathrm{N}}, \dot{\bar{\mathrm{v}}}^{\mathrm{B}}, \mathrm{a})$$

where the uncertainty function η is defined as

$$\eta(\bar{\mathrm{r}}^{\mathrm{N}}, \dot{\bar{\mathrm{v}}}^{\mathrm{B}}, \mathrm{a}) = \mathrm{M}^{-1}(\tilde{\mathrm{M}}\mathrm{a} + \tilde{\mathrm{C}}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \tilde{\mathrm{D}}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \tilde{\mathrm{g}}(\bar{\mathrm{r}}^{\mathrm{N}}))$$

In order to ensure global stability of the system, the outer loop control a designed in a way:

$$\mathrm{a} = \mathrm{a}_{\mathrm{des}}(\mathrm{t}) - \mathrm{K}_0\mathrm{v} - \mathrm{K}_1\mathrm{r} - \delta\alpha$$

where $\delta\alpha = \begin{cases} \rho\frac{\mathrm{e}}{|\mathrm{e}|}, & \text{if } \mid \mathrm{e} \mid > 0 \\ 0, & \text{if } \mid \mathrm{e} \mid = 0 \end{cases}$

Define the error e - ?

In order to reduce chattering, the boundary layer is introduced as:

$$\delta\alpha = \begin{cases} \rho\frac{\mathrm{e}}{|\mathrm{e}|}, & \text{if } \mid \mathrm{e} \mid \geq \epsilon \\ \frac{\rho}{\epsilon}\mathrm{e}, & \text{if } \mid \mathrm{e} \mid < \epsilon \end{cases}$$

where ϵ is the boundary thickness.

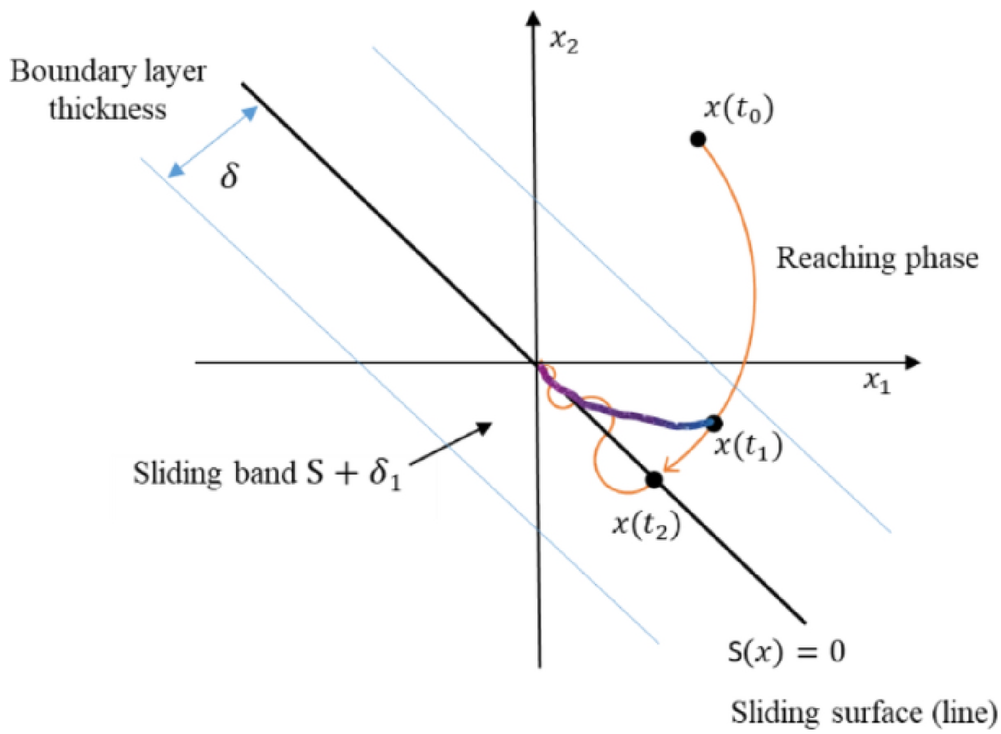

Fig. 7. The sliding mode scheme with boundary layer

Analyze the stability of the closed-loop system under SMC.

In order to prove global stability of the system, let the Lyapunov candicate will be:

$$\mathrm{V} = \mathrm{q}^{\mathrm{T}}\mathrm{Pq}$$

Let us take the time derivative, we will get

$$\dot{\mathrm{V}} = ...$$

However, we need to discover if $\dot{\mathrm{V}} = 0$ even while $\mathrm{e} \neq 0$. According to LaSalle theorem ...

D. Summary

Summarize the main findings of the chapter.

Discuss the limitations and potential extensions of the SMC controller.

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