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Robust Trajectory Control of Underwater Vehicles

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Abstract—**Underwater vehicles present difficult control-system design problems due to their nonlinear dynamics, uncertain models, and the presence of disturbances that are difficult to measure or estimate. In this paper, a recent extension of sliding mode control is shown to handle these problems effectively. The method deals directly with nonlinearities, is highly robust to imprecise models, explicitly accounts for the presence of high-frequency unmodeled dynamics, and produces designs that are easy to understand. Using a nonlinear vehicle simulation, the relationship between model uncertainty and performance is examined. The results show that adequate controllers can be designed using simple nonlinear models, but that performance improves as model uncertainty is decreased and the improvements can be predicted quantitatively.**

I. Introduction

MANY PROBLEMS must be solved to make robotic underwater vehicles a reality. Implicit in all such work is the ability to make the vehicle follow a trajectory precisely, whether that trajectory is preprogrammed, derived by a knowledge-based system in response to the environment, or specified by a human supervisor. High-level planning must always rely on dynamic control of the vehicle to guarantee stability and perform consistently.

The dynamics of underwater vehicles present a difficult control system design problem which traditional linear design methodologies cannot accommodate easily. The dynamics are fundamentally nonlinear in nature. Hydrodynamic coefficients often are poorly known and a variety of unmeasureable disturbances are present due to currents or a tether.

In this paper, a recently developed extension of the sliding mode control methodology [1], [2] is applied to the accurate trajectory control of underwater vehicles. The method can deal with nonlinear dynamics directly, explicitly accounts for the presence of high-frequency unmodeled modes, and is shown to be stable and behave predictably despite errors in hydrodynamic coefficients and unmodeled disturbances. Reduced order models with uncertain coefficients can be used to produce simple stable controllers, or more detailed models can be used to improve performance in an easily predictable fashion.

The method is examined in simulation using a planar model of the Experimental Autonomous Vehicle (EAVE) currently being developed by the University of New Hampshire (UNH). The influence of parameter uncertainty and the use of simplified models are demonstrated.

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II. Control Implications of the Dynamics of Underwater Vehicles

The dynamics of remotely operated or autonomous underwater vehicles are fundamentally nonlinear due to inertial, buoyancy, and hydrodynamic effects. The control of such vehicles is often more difficult in many aspects than aircraft, ships, or submarines. Underwater vehicles may not have a streamlined shape, and their dynamics can change significantly as new sensors and work packages are attached. These vehicles undergo complex multiaxis motions that accentuate nonlinearities and make linear control techniques difficult to apply.

In the range of Reynolds numbers in which vehicles typically operate, flow is turbulent and drag force is approximated by square law resistance:

$$F_d = C_d v |v|.$$

Even for one-axis motion, a fixed gain linear controller does not provide consistent performance over a wide speed range. Further, hydrodynamic coefficients are difficult to obtain and may be poorly known. Coefficients are generally derived using semi-empirical body-buildup techniques [3], or by testing the actual vehicle or a model in a tow tank [4], [5]. In either case, some uncertainty results. Even for simple shapes, drag coefficients are functions of the Reynolds number, therefore velocity, so the use of fixed coefficients is always an approximation. Any suitable control technique must have demonstrable robustness to errors in the model coefficients. Fortunately, it is usually possible to bound the uncertainty on the coefficients.

Inspection and work vehicles are generally designed to be able to move in any direction. A lack of symmetry can cause nonlinear cross-coupling effects that are very prominent during multiaxis motions and when working in a current. These effects are far more pronounced for a remotely operated vehicle than for a submarine or a torpedo, which usually moves ahead, turns, or changes depth at constant rates [5].

The disturbances that act on an underwater vehicle are difficult to model, measure, or estimate. A measurement of currents may not be available. The influence of a tether is difficult to estimate, although in principle it could be measured. Many hydrodynamic effects, particularly vortex shedding, are not included in standard models and can be quite important. Quantitative indicators of both stability and performance, given estimates of these disturbances, would be very useful.

In order to use any of the well-developed linear design methods for a nonlinear system, the models must be linearized. For a vehicle that moves predominantly straight ahead, linearizing about several values of forward speed can be a successful approach [6]. For a vehicle that can move in any direction, the system should be linearized about many combinations of speed along multiple axes in order to account for the complexity of the system. Since each linearized configuration requires that a separate design problem be solved, this approach requires a large design effort. For a vehicle able to move in n degrees of freedom (typically between four and six), linearization produces a system of order $2n$ if position is to be controlled. The designer must choose to build a controller for a system of order $2n$ or ignore coupling between axes. In addition, the stability of the resulting "gain scheduled" system may be questionable.

In summary, high-performance control of underwater vehicles presents problems not addressed easily using classical or modern linear techniques. The dynamics of underwater vehicles are described by highly nonlinear high-order systems with uncertain models and disturbances that are difficult to model or measure. In the rest of this paper these problems are addressed using a recently developed methodology that deals with nonlinear dynamics directly and can be shown to be robust to uncertain models and disturbances. Further, the method produces designs that only require modest amounts of computation. Because the techniques allow the different degrees of freedom of the control system to be decoupled without ignoring interactions, the designs produced are easier to understand than high-order linear controllers.

III. Robust Control of Nonlinear Systems Using Sliding Control

A new form of sliding control has been developed recently and shown to apply to a large class of nonlinear systems [1], [2]. The methodology is also called suction control, a name that arises from a geometric description of the resulting control action in state space rather than any hydrodynamic effects. Given a nonlinear model, the parametric uncertainty of the model, and the frequency range of unmodeled dynamics, the methodology produces a nonlinear feedback controller with guaranteed tracking precision. Robustness to the uncertainty of the model is explicitly guaranteed, often allowing simplified models to be used while preserving stability. Unlike related "Variable Structure System" approaches [7], no control chattering is present and unmodeled high-frequency dynamics are not excited.

A wide range of single-input single-output dynamic systems can be described by

$$x^{(n)}=b(X;t)[f(X;t)+U+d(t)] \tag{1}$$

where $x^{(n)}$ is the nth derivative of x, U is the input, $d(t)$ is a disturbance, and $X = [x, \dot{x}, \cdots, x^{(n-1)}]^T$ is the state. For a second-order mechanical system, $b(X;t)$ is the reciprocal of the inertia, $f(X;t)$ has units of force and accounts for the spring and damping effects or their nonlinear generalizations, and U is the applied force used to control the system.

None of the elements of the model are ever known exactly, and this control-system design technique explicitly incorporates estimates of the uncertainties to produce a robust design. The uncertainty of the nonlinear function $f(X;t)$ can be expressed as $\Delta f(X;t)$:

$$f(X;t)=\hat{f}(X;t)+\Delta f(X;t)$$

where $\hat{f}(X;t)$ is the estimate of $f(X;t)$. The magnitude of the uncertainty is bounded as

$$F(X;t)\geq|\Delta f(X;t)|.$$

Likewise $b(X;t)$ is known to be of constant sign and is bounded by a known function of time $\hat{b}(X;t)$ such that

$$1/\beta\leq\hat{b}(X;t)/b(X;t)\leq\beta.$$

The parameter β can be interpreted as the *gain margin* of the control system. Bounds on the disturbances are also known such that

$$D(t)\geq|d(t)|.$$

For a tracking or trajectory controller, the control problem is to make the state X follow a prescribed state X_d with a prescribed dynamic characteristic in the presence of dynamic uncertainty and disturbances. The tracking error is defined as

$$\tilde{x}=x-x_d$$

and the tracking error vector can be defined as

$$\tilde{X}=[\tilde{x}, \dot{\tilde{x}}, \cdots, \tilde{x}^{(n-1)}]^T.$$

The desired dynamics of the closed-loop system may be expressed as

$$s(X;t):=\left(\frac{d}{dt}+\lambda\right)^{n-1}\tilde{x}, \qquad \lambda>0. \tag{2}$$

These dynamics have a low-pass characteristic and may be visualized in the state space as a time-varying surface called a *sliding surface*. Perfect tracking is then defined as remaining or "sliding" along the surface ($s(X;t) = 0$). Note that this does not mean that all elements of $\tilde{X}$ are zero for all time, but rather that the prescribed dynamics (2) are achieved. This also implies that $\tilde{X}\rightarrow 0$ if the system originally starts on the sliding surface.

For a second-order system, the sliding surface simply becomes

$$s(X;t):=\dot{\tilde{x}}+\lambda\tilde{x} \tag{3}$$

which corresponds to a line that moves with the point (x_d, $\dot{x}d$) and has slope -λ. Fig. 1 illustrates the movement of the sliding surface and the maintenance of the sliding condition during a prescribed trajectory consisting of constant acceleration, constant velocity, and constant deceleration segments. The system state (x, $\dot{x}$) does not follow (x_d, $\dot{x}_d$) exactly, but remains on the line ($s = 0$). This indicates that the desired closed-loop dynamics

$$0=\dot{\tilde{x}}+\lambda\tilde{x}$$

are exactly achieved, and that accordingly $\tilde{x}(t) = \tilde{x}(0)e^{-\lambda t}$.

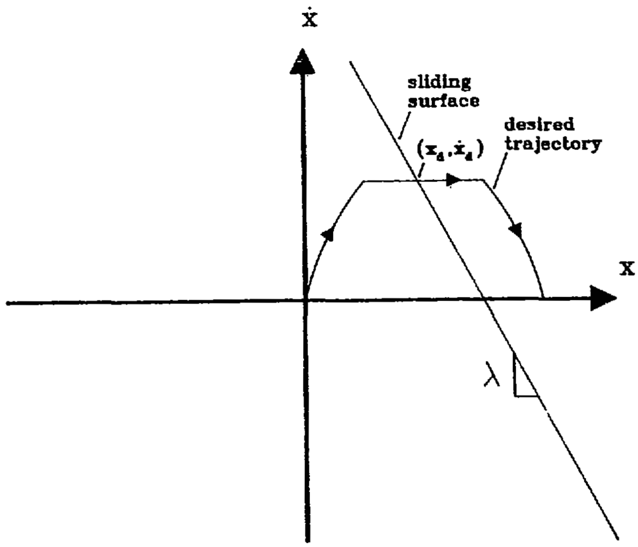

Fig. 1. The desired dynamics can be represented by a sliding surface that moves with the desired state through state space. For a second-order system, the surface is a line that passes through the desired state (x_d, x_d) with a slope of $-\lambda$.

A control law can be formulated that preserves the sliding condition despite the nonlinear uncertain nature of the plant dynamics. Equations (1) and (3) can be combined to yield the dynamics of s. For a second-order system

$$\begin{aligned} s(X;t) &:= \ddot{x} - \ddot{x}_d + \lambda\dot{\tilde{x}} \\ &= b(X;t)[f(X;t) + U] - \ddot{x}_d + \lambda\dot{\tilde{x}}. \end{aligned} \tag{4}$$

The system can be shown to be stable if s converges to zero. Because of the form of (2), the convergence of $\tilde{X}$ to 0 follows directly from convergence of s to 0. As developed in [1], convergence of s can be guaranteed by choosing a control law U that satisfies the *sliding condition*

$$\frac{1}{2}\frac{d}{dt} s^2(X;t) \leq -\eta |s| \tag{5}$$

where η is a positive constant. This stability criterion is derived from both Liapunov theory and Filipov's work on differential equations containing discontinuities [1].

Given the uncertain dynamic model, the sliding-surface definition, and the stability criteria, a suitable control law can be obtained. The control law has two parts. The first is called $\hat{U}$ and compensates directly for the known portions of the dynamics of s. In the case of no uncertainty on $b(X;t)$

$$\hat{U} = -\hat{f}(X;t) - 1/\hat{b}(X;t)\left[\sum_{p=1}^{n-1} \begin{bmatrix} n-1 \\ p \end{bmatrix} \lambda^p \tilde{x}^{(n-p)} - x_d{}^{(n)}\right]. \tag{6}$$

(6)[1]

Equation (6) is obtained by differentiating (2), substituting into (1), and solving for the control action that would keep $s(X;t)$ $= 0$ if the model were perfect. For a second-order system, the result is

$$\hat{U} = -\hat{f}(X;t) - 1/\hat{b}(X;t)[\lambda\dot{\tilde{x}} - \ddot{x}_d]. \tag{7}$$

This form of the control is similar to the computed torque techniques that have been pursued in the robotics community for several years.

A control law of the form (7) does not yet satisfy the stability criterion (5), because $f(X;t)$ and $\hat{f}(X;t)$ are not identical. In other words, there is no guarantee of robustness to uncertainties in the plant model. However, the stability criterion can be satisfied by adding a term that is discontinuous across the sliding surface

$$U = \hat{U} - K(X;\ t)\ \mathrm{sgn}\ (s) \tag{8}$$

where

$$\mathrm{sgn}\ (x) = 1, \qquad \text{for } x > 0$$

$$\mathrm{sgn}\ (x) = -1, \qquad \text{for } x \leq 0$$

and $K(X;\ t)$ is obtained directly from the time-varying bounds on parametric uncertainty and the disturbances

$$K(X;t) = F(X;t) + D(t).$$

As a result, control discontinuity across $s = 0$ grows as the model becomes less certain and as disturbances increase. This insures that s^2 is a Liapunov function of the closed-loop system, since it satisfies the stability criterion (5), and thus guarantees stability despite the uncertainty in the model and the disturbances [1].

In the case where the control gain $\hat{b}(X;\ t)$ is also uncertain, the gain must be increased:

$$U = \hat{U} - K(X;t)\ \mathrm{sgn}\ (s)$$

where

$$K(X;t) = [(\beta - 1)/\hat{b}(X;t)]|\lambda\dot{\tilde{x}} - \ddot{x}_d| + F(X;t) + D(t) \tag{9}$$

in order to compensate for the uncertainty of $\hat{b}(X;\ t)$.

This type of control can guarantee stability and perfect tracking for a large class of nonlinear systems, but is not yet very useful. The discontinuous form results in a chattering type of control action that would be very undesirable for most systems. Such chattering behavior has been one of the main reasons sliding-control techniques have not been more widely applied. This problem is solved by smoothing out the control law in a thin *boundary layer* around the sliding surface:

$$B(t) = \{X,\ |s(X;t)| \leq \phi\}.$$

A boundary layer thickness ϕ is chosen, and the discontinuous term [sgn (s)] is replaced by the saturation function [sat (s/ϕ)] where

$$\mathrm{sat}\ (x) = x, \qquad \text{for } |x| \leq 1$$

$$\mathrm{sat}\ (x) = \mathrm{sgn}\ (x), \qquad \text{for } |x| \geq 1.$$

This provides a linear interpolation across the boundary layer as illustrated in Fig. 2 for the case $n = 2$. Outside the boundary layer, U is computed as before. Further, a proper choice of boundary layer thickness ϕ essentially assigns a low-

[1] $\begin{bmatrix} n-1 \\ m \end{bmatrix} = \dfrac{n!}{m!(n-m)!}$, for $n \leq m$

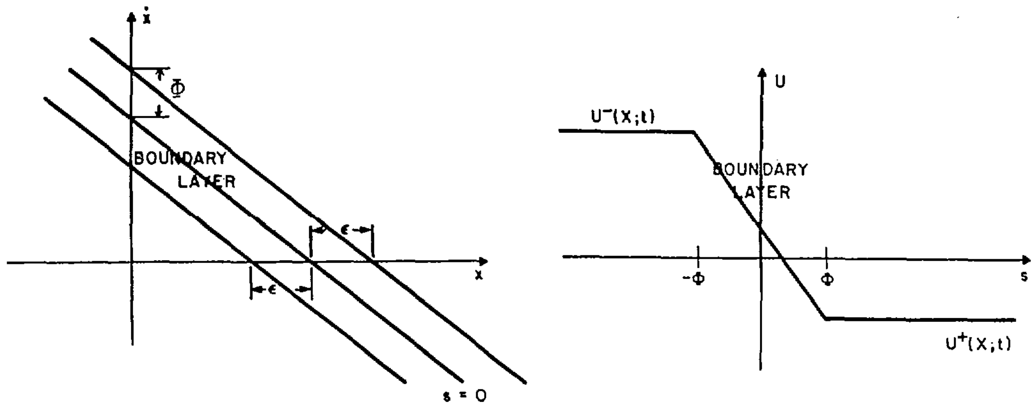

Fig. 2. The bandwidth of the control action can be limited by interpolating the control action across a thin "boundary layer" on each side of the sliding surface. The boundary layer width can be determined directly from the desired bandwidth limit and estimates of the uncertainty of the dynamics of the system to be controlled.

pass filter structure to the dynamics of s, with bandwidth λ, thus eliminating chattering provided that λ is small compared to the frequency of the first unmodeled mode in the system. Indeed, having chosen the design parameter λ to be small compared to the high-frequency unmodeled modes, the control bandwidth λ can be guaranteed by fixing the boundary layer thickness. As developed in [2], the desired thickness can be computed as

$$\phi = [\beta\hat{b}(X_d;\ t)K(X_d;\ t)]_{\max}/\lambda$$

where the maximum is based on the maximum values of the time-varying bounds on parameter uncertainty as measured along the desired trajectory. If the specified bounds on disturbances and parameter uncertainty are not exceeded, the system is guaranteed to stay within the boundary layer once inside. If a disturbance temporarily exceeds the specified bounds, the state may go outside the boundary layer. However, the sliding condition (5) implies that the system will always move back inside the boundary layer once the disturbances return to their projected levels.

This result yields a powerful design technique for linear or nonlinear systems. Boundary layer thickness ϕ follows directly from the parameter uncertainty, magnitude of unmodeled disturbances, and system bandwidth λ. Further, (3) implies that the corresponding guaranteed tracking precision is

$$\epsilon := \phi/\lambda.$$

Thus if a low-order or poorly known model is used to design a control system, stability can be guaranteed and the effect on performance can be seen directly in terms of the resulting tracking precision. This is a very straightforward view of robustness, extremely useful in a practical engineering sense.

Rather than computing the boundary layer thickness based upon the maximum uncertainty along the desired trajectory, ϕ can be made time varying, expanding when uncertainty is high and shrinking when uncertainty is low. By extending the stability criterion (5), the following dynamics of ϕ result:

$$\dot{\phi}(t) + \lambda\phi(t) = \beta\hat{b}(X_d;\ t)K(X_d; t),\quad \text{if } K(X_d;\ t) \geq \lambda\phi/[\beta\hat{b}(X_d; t)] \tag{10}$$

$$\dot{\phi}(t) + \lambda\phi(t)/\beta^2 = \hat{b}(X_d; t)K(X_d; t)/\beta,\quad \text{if } K(X_d; t) \leq \lambda\phi/[\beta\hat{b}(X_d; t)]. \tag{11}$$

The corresponding control law U is

$$U = \hat{U} - \bar{K}(X;\ t)\ \text{sat}\ (s/\phi)$$

$$\bar{K}(X;\ t) = K(X;\ t) - K(X_d;\ t) + \lambda\phi(t)/[\beta\hat{b}(X_d;\ t)]. \tag{12}$$

Changing ϕ as the uncertainty in the system varies produces a more refined tradeoff between parameter uncertainty and tracking precision, by fully exploiting the available bandwidth at all times [2]. For systems where the bandwidth of $X_d(t)$ is much less than λ, $\bar{K}(X;\ t)$ may be replaced by $K(X;\ t)$.

IV. Application of Sliding Control to Trajectory Control of Underwater Vehicles

Sliding control is now applied to the control of an underwater vehicle. The tradeoffs between uncertainty of the hydrodynamic coefficients and tracking precision are demonstrated. The effect of neglecting cross-coupling terms on tracking precision is also explored. The analysis assumes full state feedback: although the difficult measurement and state estimation problems associated with underwater vehicles are not addressed here, the robustness of sliding control to such estimation errors has been dealt with elsewhere [1].

A nonlinear model of the UNH EAVE vehicle (Fig. 3) is used to demonstrate the properties of this control methodology. The coefficients of this model are based on semi-empirical body-buildup techniques [3]. As shown in the Appendix, a version of this model for motions in the horizontal plane may be written in the form

$$\dot{u} = b_u[f_u(X) + d_u(t) + U_u]$$

$$\dot{v} = b_v[f_v(X) + d_v(t) + U_v]$$

$$\dot{r} = b_r[f_r(X) + d_r(t) + U_r].$$

The inertial velocities u, v, and r are defined in a body coordinate system as are the applied forces U_u, U_v, and U_r and the disturbance forces d_u, d_v, and d_r. Displacements x, y,

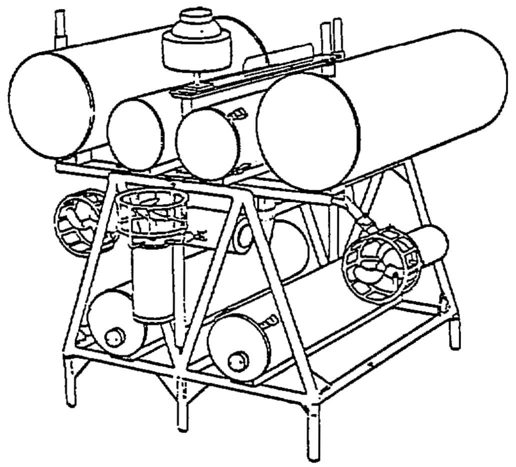

Fig. 3. The Experimental Autonomous Vehicle (EAVE) is currently under development at the University of New Hampshire. The nonhydrodynamic shape and asymmetric design result in highly nonlinear, cross-coupled dynamics typical of many underwater vehicles.

and θ represent vehicle position and orientation also expressed in the moving coordinate system. Note that if x, y, and θ are computed by directly integrating u, v, and r, care must be taken in relating these displacements to displacements defined in an inertial reference.

Modeling efforts must first focus on identifying the b and f terms and the uncertainty associated with each. The terms b_u, b_v, and b_r include both vehicle mass and added mass effects. The nonlinear functions f_u, f_v, and f_r include all velocity-dependent effects, both hydrodynamic and inertial. They are expressed as functions of the state X, where

$$X=[x,\ u,\ y,\ v,\ \theta,\ r]^T$$

which includes the three body referenced velocities and the vehicle position.

The nonlinear cross-coupling terms, detailed in the Appendix, are a major component of the dynamics at typical operating speeds, and therefore bear heavily on the control problem. The nonlinear control approach pursued here can account for this complexity in an effective and straightforward fashion. Obtaining equivalent performance using linearized models would imply a tessellation of the state space about a large number of operating points, and further would involve questionable gain scheduling and stability issues.

As presented in [1], the decoupling properties of the sliding methodology allow three controllers to be designed separately. For position and velocity control, three second-order controllers can be designed rather than a single sixth-order control system.

For each axis, the control will have the form

$$U=\hat{U}-K(X;\ t)\ \text{sat}\ [s/\phi]$$

where

$$\hat{U}=-\hat{f}(X;\ t)-(1/\hat{b})(\lambda\dot{\tilde{x}}-\ddot{x}_d).$$

Although three low-order controllers are designed, all available knowledge of interactions between axes is included in $\hat{f}(X;\ t)$. Uncertainties in the interactions contribute to $K(X;\ t)$.

The time-varying values of $K(X;\ t)$ and $\phi(t)$ can be obtained directly from the uncertainty of the model and the magnitude of the disturbances. The β terms arise from uncertainty in b, the reciprocal of the apparent mass:

$$\hat{b}=1/(m+m_{\text{added}}).$$

At this point, the influence of bandwidth, modeling uncertainty, and disturbances on tracking precision can be examined. The time-varying boundary-layer thickness is obtained from (10) and (11):

$$\dot{\phi}(t)+\lambda\phi(t)=\beta\hat{b}K(X_d), \qquad \text{if } K(X_d)\geq\lambda\phi/(\beta\hat{b})$$

$$\dot{\phi}(t)+\lambda\phi(t)/\beta^2=\hat{b}K(X_d)/\beta, \qquad \text{if } K(X_d)\leq\lambda\phi/(\beta\hat{b}).$$

The boundary-layer thickness ϕ grows proportionally with β. This indicates the rate of uncertainty of the apparent mass of the system: gain margin β must always be chosen conservatively to insure stability, but tracking errors vary linearly with β.

The choice of bandwidth can also be directly translated into tracking performance, since ϕ is inversely proportional to λ for a second-order system. Moreover, the bounds on errors in x are $\phi/\lambda = \beta\hat{b}K(X_d)/\lambda^2$. While it is important to keep λ below the first unmodeled mode, choosing λ very conservatively will greatly limit performance.

The effects of uncertainty of the velocity-dependent hydrodynamic terms are also straightforward. Errors in individual coefficients are only important in how they increase $K(X)$. It should be noted that all terms that make up $K(X)$ correspond to the bounded unmodeled forces and are added together. The consequences of including a particular model term explicitly in the controller or including it as a bounded disturbance can be obtained quantitatively. Likewise, the benefit of decreasing the uncertainty of any term (which requires either detailed analysis or testing) also can be evaluated directly. In most cases, the vehicle mass will be well known, however the added mass term will be uncertain. The uncertainty in b will be less than the uncertainty of the added mass term. Assuming the uncertainty in the vehicle mass can be neglected, the gain margin β can then be estimated as

$$\beta=1+\Delta m_{\text{added}}/(m+m_{\text{added}})$$

where Δm_{added} is the bound on uncertainty of m_{added}.

The uncertainty of the velocity-dependent functions $f(X)$ must also be estimated. For each hydrodynamic coefficient a, the uncertainty can be expressed as

$$\alpha=|\hat{a}-a|.$$

Given α for each coefficient, $\Delta f(X)$ can be computed by

$$\Delta f(X)=f^*(X)$$

$$F(X)=|\Delta f(X)|$$

where f^* has the same form as $\hat{f}$, but the additive uncertainties α are substituted for the hydrodynamic coefficients and the absolute value of the states are used.

Next, the sliding surfaces must be chosen. For second-order systems

$$s_u = \tilde{u} + \lambda_u \tilde{x}$$

$$s_v = \tilde{v} + \lambda_u \tilde{y}$$

$$s_r = \tilde{r} + \lambda_r \tilde{\theta}.$$

In each case, λ should be set well below the first unmodeled mode of the system as viewed from that axis, typically 0.4 times the first unmodeled mode or lower. Note that λ can be different for each axis.

The time-varying gain can now be computed, given the maximum value of the unmodeled disturbances D. While these quantities can also be time varying, they will be considered constants here. The gain is then

$$K(X) = (\beta - 1)|\ddot{x}_d - \lambda\dot{\tilde{x}}|/\hat{b} + F(X) + D(t). \tag{13}$$

A simulation study was performed to illustrate the effectiveness of the method and the relationship between model complexity, model uncertainty, and performance. A multiaxis motion was simulated for the UNH EAVE vehicle for three controllers based on different models: a model for which all coefficients were known with high certainty, the same model with very uncertain coefficients, and a simplified model that ignored all cross-coupling effects. In each case, a stable well-behaved closed-loop system resulted with performance within the predicted bounds and control action that was smooth and bandwidth limited. While substantial improvements result from better modeling, a simplified or uncertain model may prove sufficient depending on performance requirements.

A nonlinear model of vehicle motions in the horizontal plane was used to simulate the vehicle and as a basis for the controllers. All drag forces are dependent on the product of two velocities. The vehicle is assymetric fore and aft so yaw rate r and sway velocity v are strongly coupled in a nonlinear fashion. Details of the model are given in the Appendix.

Combined sway and yaw motions were tested to examine the ability of the controller to deal with the nonlinear coupling between these two axes. This type of motion causes forces and torques that depend on combinations of sway and yaw velocity (v and r) in a nonlinear fashion, so they would be poorly approximated by a model linearized about one value of forward speed. The closed-loop simulation was commanded to accelerate, hold constant velocity, and decelerate along both sway and yaw axes simultaneously.

The first controller used a model of the same form as the vehicle simulation, although all coefficients differed by 10 percent. While the exact values of the simulated model parameters were not known to the controller, the bounds on the uncertainties were known. In each case, the drag effects were *overestimated* in the model used to design the controller. Overestimating damping effects can tend to destabilize a system, as the controller may rely on natural damping that does not exist, so this is a worst-case assumption.

The second controller also used a model of the same form as the vehicle simulation. In this case, the uncertainty of the effective mass term was the same as in controller 1, i.e., 10 percent. However, the uncertainty of all velocity-dependent terms was 50 percent. Again, the drag effects were overestimated in the model used to design the controller.

The third controller used a very crude simplified model. Again, the effective mass was known to a precision of 10 percent. However, no model of the velocity terms was included. Instead, a static estimate of the maximum drag effect was added to the disturbance term.

For all controllers, the bandwidth limit was set at 0.5 Hz (λ = 3.14 rad/s). The digital simulation of the vehicle ran at 30 Hz and the controllers ran at 10 Hz.

Time plots of heading and yaw rate response along with the commanded values are shown in Fig. 4. All controllers are stable and perform consistently at different speeds. While performance improves as a better model is used, even the crude model used for controller 3 results in reasonable performance. As shown in Fig. 5, errors grow as the model is made less certain.

Fig. 6 shows that each controller performs within the limits predicted by the design method. The boundary layer width is plotted along with the value of s (distance off the sliding surface). In every case, the system remains inside the boundary layer. For the first two controllers, the boundary layer grows as speed increases since the model becomes less certain due to the uncertain knowledge of the hydrodynamic coefficients. For controller 3, the boundary layer width is fixed, since the model used for the controller contains a constant value for the velocity-dependent effects.

Fig. 7 shows time plots of the control action. In each case, the dynamics of the control action are within the bandwidth limit specified in the design. There is no sign of chattering, and the magnitude of the control action for the high gain controller (3) is not much higher than for the lower gain versions (1 and 2).

These results illustrate how control system design and modeling efforts can be approached together to produce a closed-loop system with desired performance. Before testing or modeling is begun, the required uncertainty bounds on the hydrodynamic coefficients can be obtained directly from the performance specifications. This will indicate that some terms, such as the added mass, should be known with reasonable confidence. Other terms can be neglected, with the effects bounded and lumped with the unmodeled disturbances, while other terms should be known with moderate precision obtainable from simple approximations rather than detailed computer analysis or tank testing. For high performance, detailed testing or analysis may be required. In any case, the modeling requirements follow directly from the desired performance.

This method can save large amounts of effort for the entire modeling and control system design cycle, especially when compared to using a large number of linearized models which are produced from a detailed precise nonlinear model.

The controller is also computationally modest. Because of the decoupling properties of the method, a series of second-order controllers can be designed rather than a single high-order controller. For each axis, 21 multiplications and 15

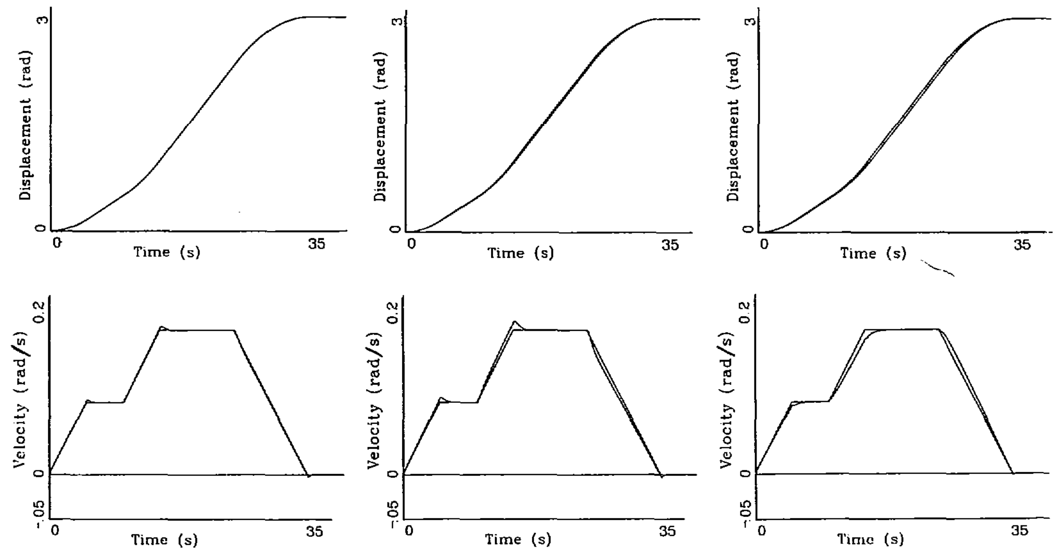

Fig. 4. Desired and actual heading and heading rate are shown for the three controllers. Controller 1 was based on a model of the same form as the vehicle simulation, but all coefficients were 10 percent in error. Controller 2 was based on the same model, but all velocity coefficients were 50 percent in error. Controller 3 used a simplified model where all velocity terms were considered as unmodeled disturbances. All controllers are stable, but performance can be seen to decrease as model uncertainty increases.

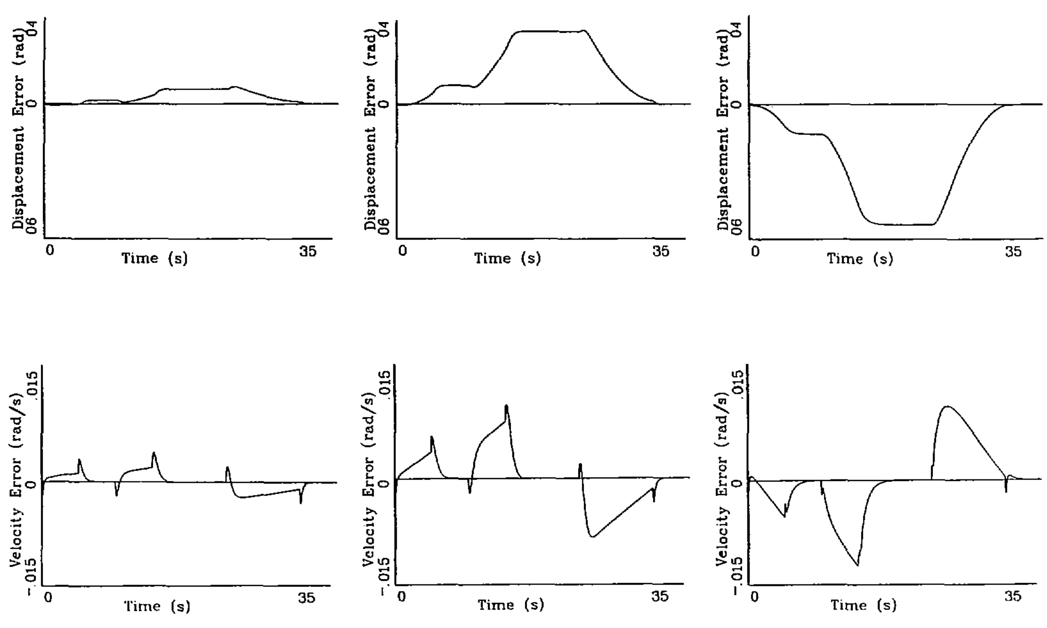

Fig. 5. Heading error and heading rate errors are plotted for the three controllers. The loss of tracking precision as the model is degraded can be seen.

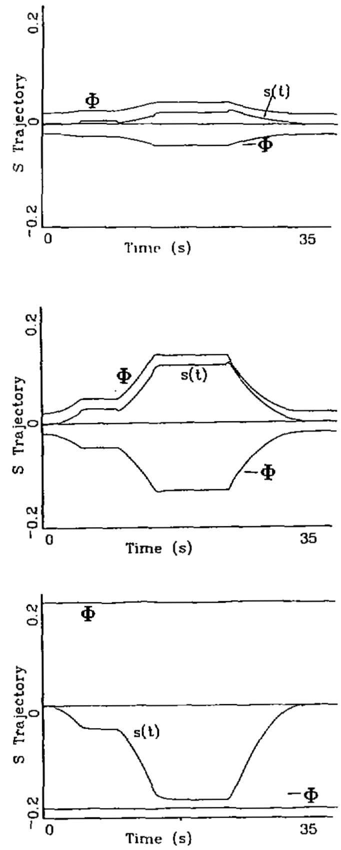

Fig. 6. S-trajectory for the three controllers. The value of s (distance off the sliding surface) is plotted versus time along with the boundary layer width ϕ. For controllers 1 and 2, the boundary layer expands as the system speeds up, since uncertainty increases with speed. For controller 3, the boundary layer width is nearly constant. In all cases, s stays within the boundary layer.

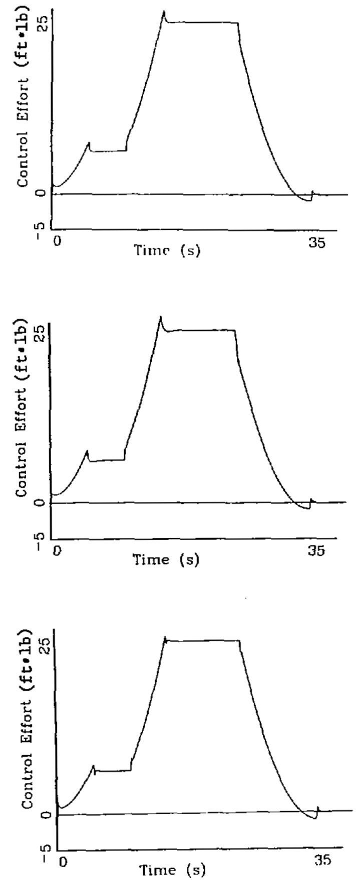

Fig. 7. Control action for the three controllers. In all cases, control action is bandwidth limited and free of chattering (1 ft·lb ≈ 0.1382 kgf·m ≈ 1.355 N·m).

additions were required at each time step using the detailed models used in controllers 1 and 2. For the simplified model used in controller 3, 11 multiplications and 10 additions were needed.

Conclusion

Sliding control has been summarized and shown to be a very attractive control system design method for underwater vehicles. The method deals with the dynamic problems of underwater vehicles, produces designs that are easy to understand, and requires modest amounts of computation.

The technique can deal with nonlinear dynamics directly, with no need for linearization. This is especially important for an underwater vehicle, which typically has highly nonlinear dynamics. Since only a single design is required over the entire operating range of the vehicle, the control system is easier to design and implement than a series of linearized controllers. This is especially important for highly maneuverable underwater vehicles that can move in all directions, since they lack clearly defined operating points about which to linearize.

The method produces extremely robust controllers that perform predictably despite the use of certain or simplified models. A clear tradeoff between model uncertainty and controller performance allows large simplifications to be made in the vehicle model used to design the control system without significantly compromising performance. Large amounts of model testing or analysis can be eliminated, the design can be made tolerant of changes in the vehicle, and the design can be easy to understand and implement.

Appendix

For the EAVE vehicle, motions in the horizontal plane may be described by the following equations, based on [3]:

$$m\dot{u} = X_u\dot{u} + X_{uu}u|u| + U_x$$

$$m\dot{v} = Y_v\dot{v} + Y_{vv}v|v| + Y_{rr}r|r| + Y_{rv}r|v| + Y_{vr}v|r| + Y_{v2}v^2 \text{ sgn } (r) + U_v$$

$$I\dot{r} = N_r\dot{r} + N_{rr}r|r| + N_{vv}v|v| + N_{rv}r|v| + N_{vr}v|r| + N_{v2}v^2 \text{ sgn } (r) + U_r.$$

The hydrodynamic forces and moments functionally represented in these equations are obtained by performing a Taylor series expansion about an equilibrium condition (all velocities equal to zero). These equations pertain to horizontal movement only. The effects of pitch, roll, and vertical movement have been removed. Inertial terms were simplified by placing the moving coordinate system at the center of mass. A more general discussion of such models can be found in [4].

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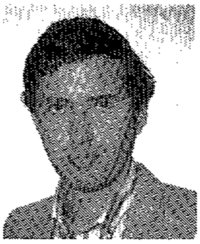

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