SHOULD BE REPLACED ON REQUIRED TITLE PAGE

Instruction

1. Open needed docx template (folder "title"/<your department or bach if bachelor student>.docx).

2. Put Thesis topic, supervisor's and your name in appropriate places on both English and Russian languages.

3. Put current year (last row).

4. Convert it to "title.pdf," replace the existing one in the root folder.

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Abstract

This is an abstract.

Chapter 1

Introduction

Plan:

1. topic area - control of underwater vehicles
2. research problem - optimized ROV control (what's the approach?)
3. research question - ...
4. hypothesis - ...

Chapter 2

Literature Review

Plan:

1. control over general SRB (Spong, Slotine)
2. control over general water vehicle (Fossen)
    (a) nonlinearities
    (b) unknown parameters(added mass)
    (c) unknown enviroment(streams)
3. basic control techniques
    (a) adaptive control
    (b) robust control
4. modern control solutions (CLF?)

Chapter 3

Mathematical Model

I Modelling

Remotely operated vehicles (ROVs) are complex systems that require mathematical models for various purposes, including control system design, simulation, and performance analysis. With accurate mathematical models, ROVs are able to navigate through different underwater terrains and complete control tasks with a good precision. Also, the simulation, based on these models, are suitable to test different work scenarios and detect undesirable ROV's behaviour before the physical experiment.

The fundamentals of the modelling for marine vehicles were fully described in Fossen(). Using common assumptions, ROV is treated as a single rigid body with six degrees of freedom (DOF). By considering the vehicle as a rigid body, we can simplify the mathematical modeling process while capturing the essential dynamics of the system.

In order to effectively model rigid bodies, it is crucial to consider their kinematic and dynamic properties.

A. Notations

Before proceeding to theoretical derivations, it is necessary to define the general notations. For the motion with six DOF, six independent coordinates will be used: three for translational directions (surge, sway, and heave) and three for rotational directions (roll, pitch, and yaw). This work will be based on the notation of The Society of Naval Architects and Marine Engineers (SNAME) introduced the 1950 for marine vessels (Fig. 1), which is widely used in marine engineering.

DOF		Forces and Moments	Linear and Angular velocity	Positions and Euler Angles
1	Surge	X	u	x
2	Sway	Y	v	y
3	Heave	Z	w	z
4	Roll	K	p	ϕ
5	Pitch	M	q	θ
6	Yaw	N	r	ψ

Fig. 1. SNAME notation

B. Frames of reference

In order to derive the kinematics and dynamics of the system, the calculations need to be projected into the same frame of reference. Sometimes several coordinate frames are defined based on the system configuration. For ROV, it is reasonable to define two coordinate frames. These frames are the earth-fixed frame, which is inertial with fixed origin, and the body-fixed frame, which is a moving frame attached to the vehicle. as depicted in (Fig. 2).

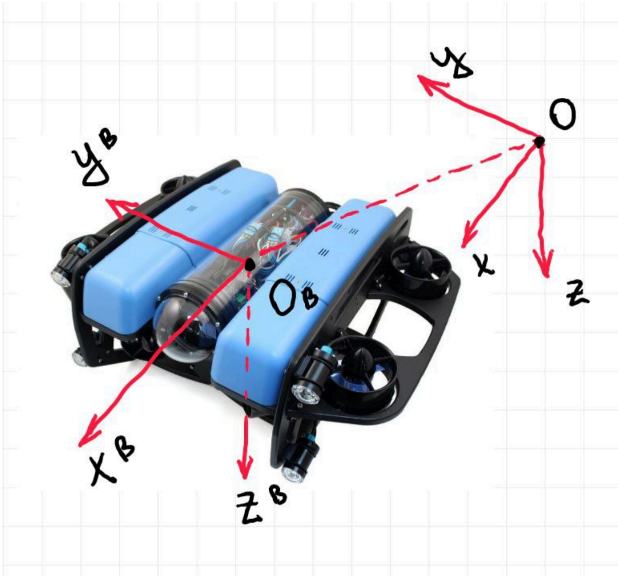

Fig. 2. The frames

The origin of the body-fixed frame usually coincides with the vehicle's center of mass, and its axes are chosen along the vehicle's principle axes of inertia.

Using the SNAME notation we could define the position and the velocity vectors:

$$\eta = \begin{bmatrix} \mathrm{x} & \mathrm{y} & \mathrm{z} & \phi & \theta & \psi \end{bmatrix}^\top$$

$$\nu = \begin{bmatrix} \mathrm{u} & \mathrm{v} & \mathrm{w} & \mathrm{p} & \mathrm{q} & \mathrm{r} \end{bmatrix}^\top$$

Here η denotes the position and orientation vector in the earth-fixed frame, ν denotes the linear and angular velocity vector in the body-fixed frame.

In order to deal further with the vector transformations between these two frames it is necessary to define cross product operators:

$S(\lambda)$ is a skew-symmetric matrix defined such that:

$$S(\lambda) = \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix}$$

therefore $S(u)v = u \times v$ for vectors $v, u \in \mathbb{R}^3$

$\bar{\times}^*$ is the $\mathbb{R}^6$ cross product operator defined as:

$$\nu \bar{\times}^* = \begin{bmatrix} S(\omega) & 0_{3\times 3} \\ S(v) & S(\omega) \end{bmatrix} \text{ where } \nu = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

C. Kinematics

Kinematics describes the motion of the marine vehicle without considering the forces acting upon it. In order to describe kinematic motion of the body, it is necessary find relation between velocities in two coordinate frames. This relation can be represented with linear transformations as:

$$\dot{\eta} = J(\eta)\nu$$
$$\text{where } J(\eta) = \begin{bmatrix} R(\eta)^\top & 0_{3\times 3} \\ 0_{3\times 3} & T(\eta) \end{bmatrix}$$

where the R matrix is the rotational matrix and the T matrix is the rate transformation matrix. These two matrices can be expressed in terms of Euler angles, but it will eventually cause the singularity. Quaternion representation can resolve the singularity problem. The kinematic equations for the marine vehicle using quaternions can be written as follows:

$$\mathbf{R}(\mathbf{q}) = \begin{bmatrix} 1-2\left(\mathrm{q}_2^2+\mathrm{q}_3^2\right) & 2\left(\mathrm{q}_1\mathrm{q}_2-\mathrm{q}_3\mathrm{q}_0\right) & 2\left(\mathrm{q}_1\mathrm{q}_3+\mathrm{q}_2\mathrm{q}_0\right) \\ 2\left(\mathrm{q}_1\mathrm{q}_2+\mathrm{q}_3\mathrm{q}_0\right) & 1-2\left(\mathrm{q}_1^2+\mathrm{q}_3^2\right) & 2\left(\mathrm{q}_2\mathrm{q}_3-\mathrm{q}_1\mathrm{q}_0\right) \\ 2\left(\mathrm{q}_1\mathrm{q}_3-\mathrm{q}_2\mathrm{q}_0\right) & 2\left(\mathrm{q}_2\mathrm{q}_3+\mathrm{q}_1\mathrm{q}_0\right) & 1-2\left(\mathrm{q}_1^2+\mathrm{q}_2^2\right) \end{bmatrix}$$

$$\mathbf{T}(\mathbf{q}) = \frac{1}{2}\begin{bmatrix} -\mathrm{q}_1 & -\mathrm{q}_2 & -\mathrm{q}_3 \\ \mathrm{q}_0 & -\mathrm{q}_3 & \mathrm{q}_2 \\ \mathrm{q}_3 & \mathrm{q}_0 & -\mathrm{q}_1 \\ -\mathrm{q}_2 & \mathrm{q}_1 & \mathrm{q}_0 \end{bmatrix}$$

while state quaternion q is expressed in scalar-first form as $\mathrm{q} = \mathrm{q}_0 + \mathrm{q}_1 \cdot \mathrm{i} + \mathrm{q}_2 \cdot \mathrm{j} + \mathrm{q}_3 \cdot \mathrm{k}$

D. Model dynamics

The Newton-Euler approach is commonly used to describe the dynamics of marine vehicles. This approach relates the applied forces and moments to the vehicle's accelerations and angular accelerations. The general equation of motion using the Newton-Euler approach in the body-fixed frame can be written as:

$$\mathrm{M}\dot{\nu} + \nu \bar{\times}^* \mathrm{M}\nu = \mathrm{f}$$

where M represents the inertia matrix of the rigid body and f represents the total force acting on it.
The equation can be transformed into manupulator equation like:

$$\mathbf{M}_{\mathrm{RB}}\dot{\nu} + \mathbf{C}_{\mathrm{RB}}(\nu)\nu = \tau_{\mathrm{RB}}$$

where $\mathrm{M_{RB}} \in \mathbb{R}^{6x6}$ is the rigid body mass matrix, $\mathrm{C_{RB}}(\nu) \in \mathbb{R}^{6x6}$ is the rigid body Coriolis and centripetal forces matrix, $\tau_{\mathrm{RB}} \in \mathbb{R}^{6x1}$ is the generalized vector of external forces and torques.

However, additional terms should be included in the equation to determine the specifics of the ROVs model. These terms comprise added mass, which represents the inertia of the surrounding fluid, the shift of the center of buoyancy due to changes in trim and heel angles, and damping effects. By incorporating these terms into the manipulator equation derived from the Newton-Euler approach, the model becomes more accurate and reflects the natural behavior of the ROV.

E. Center of Gravity and Center of Buyonancy

Due to the robust design of the marine vehicles, the center of mass(COM) is placed lower than the center of byonancy(COB). This shift betweent centers causes torque acting against capsize (Fig. 3).

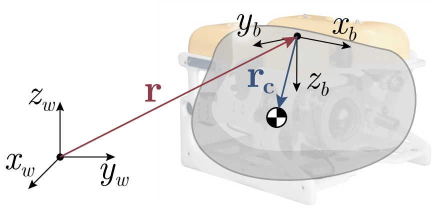

Fig. 3. The vehicle scheme

If we place the origin of the body frame at byonancy center, the mass matrix can be expressed as:

$$M_{RB} = \begin{bmatrix} mI_{3\times3} & -mS\left(r_G\right) \\ mS\left(r_G\right) & I_0 \end{bmatrix}$$

where r_G is the vector of the gravity center in the body frame.
The same applies to the Coriolis matrix:

$$C_{RB}(\nu) = \begin{bmatrix} S(\omega) & 0_{3\times3} \\ S(v) & S(\omega) \end{bmatrix} \begin{bmatrix} mI_{3\times3} & -mS\left(r_G\right) \\ mS\left(r_G\right) & I_0 \end{bmatrix}$$

These matrices can be further simplified, using an assumption that x and y shifts equal zero due to the symmetry.

F. Concept of added mass

Since the vehicle moves in a viscous enviroment, we can not neglect the inertia of the surrounging liquid. To compensate added mass effect, it is necessary to add two components into dynamics equation.
Using SNAME(1950) notation, we can define dynamical parameters of our body as:

$$X_{\dot{u}} \triangleq \frac{\partial X}{\partial \dot{u}}$$

Consequently the added mass matrix M_A and the Coriolis forces matrix for added mass $C_A(\nu)$ can be expressed as:

$$M_A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = -\,\mathbf{diag}\left\{X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\right\}$$

$$C_A(\nu) = \begin{bmatrix} 0_{3\times3} & -S\left(A_{11}v + A_{12}\omega\right) \\ -S\left(A_{11}v + A_{12}\omega\right) & -S\left(A_{21}v + A_{22}\omega\right) \end{bmatrix}$$

The dynamical parameters should be measured experimentally.

G. Hydrodynamic Damping

Generally, the dynamics of underwater vehicles can be highly nonlinear and coupled. Nevertheless, during the slow non-coupled motion the damping can be approximated to linear and quadratic damping:

$$\begin{aligned} \mathrm{D}(\nu) = &- \mathbf{diag}\{\mathrm{X_u}, \mathrm{Y_v}, \mathrm{Z_w}, \mathrm{K_p}, \mathrm{M_q}, \mathrm{N_r}\} \\ &- \mathbf{diag}\left\{\mathrm{X_{u|u|}} \mid \mathrm{u} \mid, \mathrm{Y_{v|v|}} \mid \mathrm{v} \mid, \mathrm{Z_{w|w|}} \mid \mathrm{w} \mid, \mathrm{K_{p|p|}} \mid \mathrm{p} \mid, \mathrm{M_{q|q|}} \mid \mathrm{q} \mid, \mathrm{N_{r|r|}} \mid \mathrm{r} \mid\right\} \end{aligned}$$

Again, appropriate values of dynamical parameters can be discovered through several tests.

H. Restoring forces

Common sense is to neglect all other forces acting on the vehicle except buoyancy and gravity. Although the motion of the current can also affect the dynamics, it is unpredictable and highly nonlinear, which makes it easier to compensate through control. The weight of the body is defined as: $\mathrm{W} = \mathrm{mg}$, while the buoyancy force is defined as: $\mathrm{B} = \rho \mathrm{g} \nabla$. By transforming the weight and buoyancy force to the body-fixed frame, we get:

$$\mathrm{f_G}(\eta) = \mathrm{R}^\top(\eta) \begin{bmatrix} 0 \\ 0 \\ \mathrm{W} \end{bmatrix} \quad \mathrm{f_B}(\eta) = -\mathrm{R}^\top(\eta) \begin{bmatrix} 0 \\ 0 \\ \mathrm{B} \end{bmatrix}$$

Therefore, overall restoring force and moment vector is:

$$\mathrm{g}(\eta) = -\begin{bmatrix} \mathrm{f_G}(\eta) + \mathrm{f_B}(\eta) \\ \mathrm{r_G} \times \mathrm{f_G}(\eta) + \mathrm{r_B} \times \mathrm{f_B}(\eta) \end{bmatrix}$$

I. Matrix representation

The final expression for the mathematical model is:

$$\begin{cases} \mathrm{M}\dot{\nu} + \mathrm{C}(\nu)\nu + \mathrm{D}(\nu)\nu + \mathrm{g}(\eta) = \tau \\ \dot{\eta} = \mathrm{J}(\eta)\nu \end{cases}$$

where $\mathrm{M} = \mathrm{M_{RB}} + \mathrm{M_A}$, $\mathrm{C}(\nu) = \mathrm{C_{RB}}(\nu) + \mathrm{C_A}(\nu)$

J. Thrusters modelling

In the general case, the thruster force and moment vector will be a complicated function depending on the vehicle's velocity vector ν, voltage of the power source V and the control variable u. This relationship can be expressed as:

$$\tau = \mathrm{T()K()u}$$

where $\mathrm{T} \in \mathbb{R}^{6\mathrm{x}8}$ is the control configuration matrix that maps body torques to thuster forces, $\mathrm{K} \in \mathbb{R}^{8\mathrm{x}8}$ is the thrust coefficient function that defines relation between PWM signal and output force.

K. BlueRov modelling (will be placed in a different chapter (?))

By the specification of the given thrusters, the dependency between control PWM signal and thrust is highly nonlinear (Fig. 4).

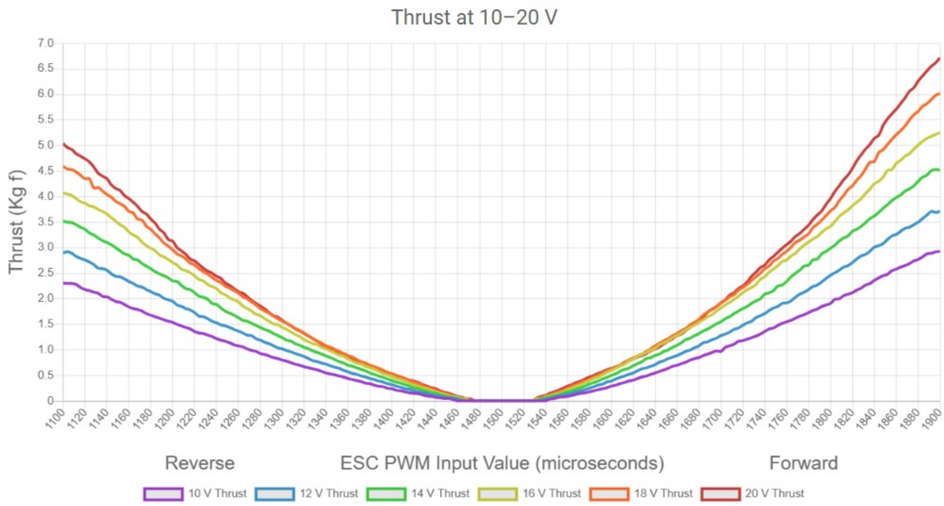

Fig. 4

In order to model this relation, the polynomial regression was applied on the normalized test data. A 5th-order approximation of the developed thrust at 16V voltage will be: $\tau = -0.22\mathrm{u}^5 - 0.0135\mathrm{u}^4 + 1.1\mathrm{u}^3 + 0.172\mathrm{u}^2 + 1.327\mathrm{u} + 0.027$ The inverse dependency can be determined in the same way. The following expression is obtained : $\mathrm{u} = 0.0006\tau^5 - 0.0004\tau^4 - 0.02\tau^3 + 0.0006\tau^2 + 0.56\tau - 0.0334$

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Chapter 4

Control

Chapter 5

Evaluation and Discussion

Chapter 6

Conclusion

Appendix A

Extra Stuff

Text.

Appendix B

Even More Extra Stuff

Text.