Contents

Chapter 1

Literature Review

This chapter presents a comprehensive overview of existing research on remotely operated vehicles (ROVs), focusing on modeling and control design. This literature review aims to investigate and synthesize control strategies, addressing uncertainties from actuator dynamics and environmental factors, which correspond to the proposed research question.

Section 1 gives a brief description of ROVs, including their types and general attributes. Section 2 explains the commonly accepted assumptions and mathematical models used in the field. Section 3 offers an extensive review of various studies on the application of robust control design for underwater systems. In conclusion, Section 4 summarizes the key insights from the review and suggests a control methodology.

I Overview on ROVs

#TODO Add more sorces and get more info about ROVs.

This section provides an overview of ROVs, from their classification and applications to their inherent characteristics and challenges. The obtained knowledge forms a basis for ROV modeling and control design.

ROV physics -> complex model design

According to [1], an ROV's mechanical structure consists of a monitoring camera, a sensor for gathering navigation data, and actuators for directional control. A comparative study [2] found that the physical aspects that affect ROV functions are the accuracy of sensor systems and the thruster designs. The unpredictable nature of underwater currents, drag, and buoyancy dynamics can also have a serious impact on a ROV's performance, complicating model design.

Small ROVs are more preferrable -> wide range applications

A recent systematic review [3] concluded that there are two primary classifications for ROVs based on their functions and intended use: observation class and work class. Observation class vehicles are typically small and limited to shallow waters with propulsion power up to a few kilowatts. Work class vehicles can perform heavy-duty work, requiring significant hardware system complexity. Thus, when the functionality of these large ROVs is not necessary, a smaller ROV is preferred for a wide range of applications.

Applications of ROV -> control objectives

Several studies [1], [2] have identified that ROVs have become crucial for industrial applications, offshore oil and gas exploration, patrolling, and surveillance. Therefore, its control system should focus on position tracking and station keeping in the presence of parameter and environmental uncertainties, addressing the following issues.

#TODO Add connection to modelling, wrap it up

II Mathematical Modelling

Remotely operated vehicles require mathematical models for various purposes, including control system design, simulation, and performance analysis.

SRB

Thor I. Fossen [4] provided a complete description of the fundamentals of mathematical modeling for marine vehicles. In his book, ROV was represented as a single rigid body (SRB) by considering it as a solid mass with no internal movement or deformation. The SRB model has drastically simplified the modeling while capturing the essential dynamics of the system.

Kinematics and dynamics

From a kinematic point of view, ROVs have six degrees of freedom (DoFs). However, the orientation expressed in the rotation angles could eventually lead to the singularity. To solve this issue, [5] proposed the quaternions representation. The dynamics was derived based on classical physics laws: Newtons Second Law and Euler-Lagrange equation, forming the set of nonlinear equations. The study [6] simplified the equations and represented them in a matrix form.

Parameter estimation

However, several sources have established that some aspects of the ROV dynamics require empirical estimates due to their complex, nonlinear, and coupled nature [4], [7]. For instance, the inertia of the surrounding fluid cannot be neglected when the vessel moves through a viscous medium. Additionally, water damping is another source of nonlinearity that can be approximated as a function of the velocity. Finally, aside from buoyancy and gravity, it is common practice to cancel all other forces acting on a vehicle, although it can also impact the dynamics [7].

Thruster modelling

Moreover, thruster modeling must be applied to define the desired thrust of each thruster. A recent study by [7] stated that creating an accurate thruster model can be challenging due to the influence of factors such as motor models, hydrodynamic effects, and propeller mapping.

Uncertaint model-> control objectives

By making these simplifications, the control system cannot independently provide effective control over such uncertain dynamics. As a result, a robust controller design is necessary for precise ROV position tracking.

III Control Solutions

Connect modelling and control

Controlling an ROV is a complex task that requires a set of processes to stabilize the vehicle and to make it follow the operator's instructions. To ensure the robustness of the system, it is necessary to define a control system that can handle disturbances caused by parameter and environmental changes.
According to [2], there are two main challenges associated with ROVs control:

1. Unmodeled elements like added mass and hydrodynamic coefficients.

2. Highly nonlinear dynamics of the underwater environment which cause significant disturbances to the vehicle.

Base approach -> SMC

The research on ROV control showed several schemes that can achieve robust stability under variable disturbances. The classical approach applied is sliding mode control (SMC), which was introduced by Slotine [8]. SMC is an effective way to address the issues mentioned above and is, therefore, a feasible option for controlling underwater vehicles. However, standard SMC introduces high-frequency signals, which can cause actuator switching and consequently decrease its lifetime.

Better SMC variations

#TODO Change sources and control approaches

The modern SMC interpretations keep the main advantages, thus removing the chattering effects. One of the possible approaches is Adaptive Sliding Mode Control (ASMC) design. [9] designed an effective hybrid control mechanism for the underwater system that follows a given route, adapting to dynamic disturbances. [10] improved a control system for ROVs, eliminating the need for dynamics linearization. Another way to refine SMC is to add an integral component into the controller equation. The proposed Integral Sliding Mode Control (ISMC) reduced chattering, effectively eliminating the uncertainty of the model parameters [11].

My proposal-> use optimization

#TODO Connect previous part to my proposed control scheme

The choice of control strategy should be based on the specific requirements and characteristics of the underwater vehicle. While both SMC modifications showed precise control, these approaches faced certain challenges [11], [10]. The ISMC method had limitations in adapting to rapidly changing dynamics, while the ASMC method experienced potential trade-offs in transient performance. These problems were opposite to each other. Therefore, combining them into Adaptive Integral Sliding Mode Control (ADISMC) can be a good idea to leverage the strengths of both methods and compensate for their weaknesses.

IV Summary

#TODO Change according to my proposed control scheme

This literature review comprehensively examined the general characteristics, mathematical modeling, and control solutions for underwater vehicles. This overview identified a gap in the field of robust control for remotely operated vehicles, particularly in dealing with parameter and environmental uncertainties. While the original sliding mode control scheme provided robust stability, it suffered from high-frequency output oscillations. There were several variations of SMC available, and each algorithm has its limitations. To address this research gap, a combination of adaptive and integral sliding mode modifications was proposed.

Chapter 2

Mathematical Modelling

Chapter intro + motivation

Remotely operated vehicles (ROVs) are complex systems that require mathematical models for various purposes, including control system design, simulation, and performance analysis. With accurate mathematical models, ROVs are able to navigate through different underwater terrains and complete control tasks with a good precision. Also, the simulation, based on these models, are suitable to test different work scenarios and detect undesirable ROV's behavior before the physical experiment.

SRB

The fundamentals of the modelling for marine vehicles were fully described in Fossen [4]. Using common assumptions, ROV is treated as a single rigid body with six degrees of freedom (DOF). By considering the vehicle as a rigid body, we can simplify the mathematical modeling process while capturing the essential dynamics of the system.

In order to effectively model rigid bodies, it is crucial to consider their kinematic and dynamic properties.

I Notations

Before proceeding to theoretical derivations, it is necessary to clarify the notations. For the motion with six DOF, six independent coordinates are defined in the coordinate frame: three for translational directions (surge, sway, and heave) and three for rotational directions (roll, pitch, and yaw) as depicted in (Fig. **??**)

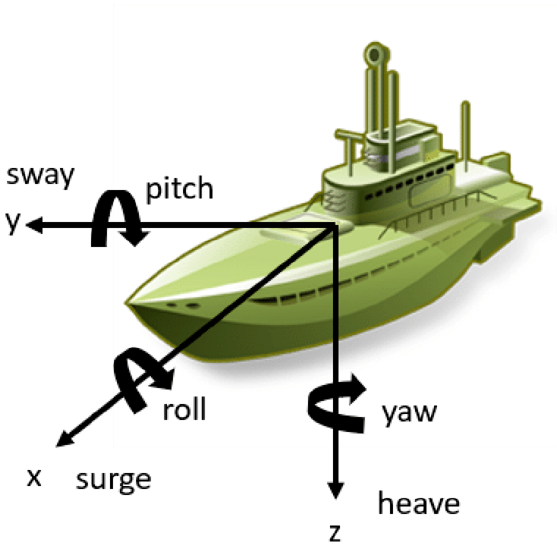


Fig. 1. 6 DOF of a marine vehicle

The linear position of the body is defined as $\mathrm{r} = [\mathrm{r_x}, \mathrm{r_y}, \mathrm{r_z}]^\top$ for translation along xyz axis respectively. The orientation can be expressed in terms of Euler angles around corresponding axis, it will eventually lead to the singularity when sway angle is $\pm 90°$. The quaternions can resolve this problem by adding redundancy into the representation. The quaternion is defined in scalar-first form as $\mathrm{q} = \mathrm{q_0} + \mathrm{q_1} \cdot \mathrm{i} + \mathrm{q_2} \cdot \mathrm{j} + \mathrm{q_3} \cdot \mathrm{k} = [\mathrm{q_0}, \mathrm{q_1}, \mathrm{q_2}, \mathrm{q_3}]^\top$

For each direction, the velocity vectors can be defined separately: $\mathrm{v} = [\mathrm{v_x}, \mathrm{v_y}, \mathrm{v_z}]^\top$ for translation along xyz axis and $\omega = [\omega_\mathrm{x}, \omega_\mathrm{y}, \omega_\mathrm{z}]^\top$ for rotation around xyz axis respectively. The same applies to linear forces f and torques τ.

To summarize, the notations look like:

Symbol	Description	Dimensionality
r	Linear position vector	$\mathbb{R}^3$
q	Angular position (orientation) vector	$\mathbb{R}^4$
v	Linear velocity vector	$\mathbb{R}^3$
ω	Angular velocity vector	$\mathbb{R}^3$
f	Vector of linear forces	$\mathbb{R}^3$
τ	Vector of torques	$\mathbb{R}^3$

Fig. 2. Notation

For the convenience, it is desirable to define combined vectors of positions, velocities and forces as: $\bar{\mathrm{r}} = \begin{bmatrix} \mathrm{r} \\ \mathrm{q} \end{bmatrix}$, $\bar{\mathrm{v}} = \begin{bmatrix} \mathrm{v} \\ \omega \end{bmatrix}$ and $\bar{\mathrm{f}} = \begin{bmatrix} \mathrm{f} \\ \tau \end{bmatrix}$

Cross product notation

In order to manipulate with obtained vectors, it is necessary to define cross product operators. For vectors in $\mathbb{R}^3$ the cross product is multiplication by skew-symmetric matrix $\mathrm{S}(\lambda)$:

$$\mathrm{S}(\lambda) = \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix}$$

While for vectors in $\mathbb{R}^6$ $\bar{\times}^*$ is the cross product operator defined as:

$$\bar{\mathrm{v}}\bar{\times}^* = \begin{bmatrix} \mathrm{S}(\omega) & 0_{3\times3} \\ \mathrm{S}(\mathrm{v}) & \mathrm{S}(\omega) \end{bmatrix}$$

II Frames of reference

In order to derive the kinematics and dynamics of the system, the calculations need to be projected into the same frame of reference. Sometimes several coordinate frames are defined based on the system configuration.

For ROV, it is reasonable to define two coordinate frames. These frames are the earth-fixed frame, which is inertial with fixed origin, and the body-fixed frame, which is a moving frame attached to the vehicle as depicted in (Fig. **??**).

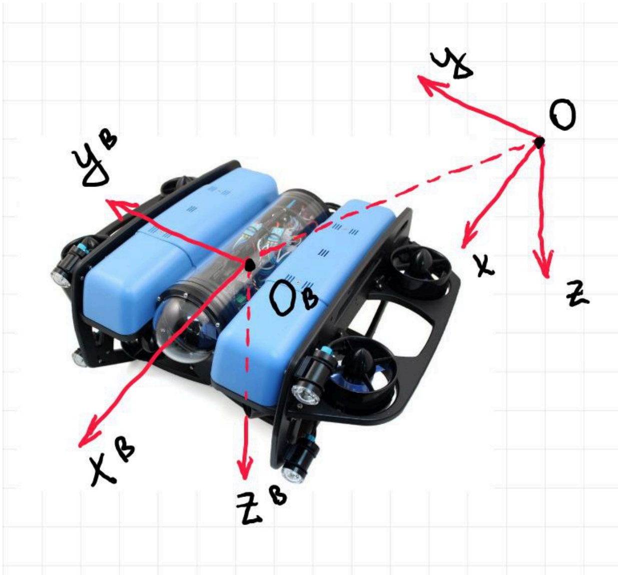


Fig. 3. The frames

The origin of the body-fixed frame usually coincides with the vehicle's center of mass, and its axes are chosen along the vehicle's principle axes of inertia.

The state variables of the rigid body expressed in the body-fixed frame would be denoted by ^{B} and in the earth-fixed frame by ^{N}.

III Kinematics

Kinematics describes the motion of the marine vehicle without considering the forces acting upon it. In order to describe kinematic motion of the body, it is necessary to find relation between velocities in two coordinate frames. This relation can be represented with linear transformations as:

$$\dot{\bar{\mathrm{r}}}^{\mathrm{N}} = \mathrm{J}(\bar{\mathrm{r}}^{\mathrm{N}})\bar{\mathrm{v}}^{\mathrm{B}}$$

$$\text{where } \mathrm{J}(\bar{\mathrm{r}}^{\mathrm{N}}) = \begin{bmatrix} \mathrm{R}(\bar{\mathrm{r}}^{\mathrm{N}}) & 0_{3\times3} \\ 0_{4\times3} & \mathrm{T}(\bar{\mathrm{r}}^{\mathrm{N}}) \end{bmatrix}$$

The rotational matrix R and the transformation matrix T using quaternions can be expressed as follows:

$$\mathrm{R}(\mathrm{q}) = \begin{bmatrix} 1-2\left(\mathrm{q}_2^2+\mathrm{q}_3^2\right) & 2\left(\mathrm{q}_1\mathrm{q}_2-\mathrm{q}_3\mathrm{q}_0\right) & 2\left(\mathrm{q}_1\mathrm{q}_3+\mathrm{q}_2\mathrm{q}_0\right) \\ 2\left(\mathrm{q}_1\mathrm{q}_2+\mathrm{q}_3\mathrm{q}_0\right) & 1-2\left(\mathrm{q}_1^2+\mathrm{q}_3^2\right) & 2\left(\mathrm{q}_2\mathrm{q}_3-\mathrm{q}_1\mathrm{q}_0\right) \\ 2\left(\mathrm{q}_1\mathrm{q}_3-\mathrm{q}_2\mathrm{q}_0\right) & 2\left(\mathrm{q}_2\mathrm{q}_3+\mathrm{q}_1\mathrm{q}_0\right) & 1-2\left(\mathrm{q}_1^2+\mathrm{q}_2^2\right) \end{bmatrix}$$

$$\mathrm{T}(\mathrm{q}) = \frac{1}{2}\begin{bmatrix} -\mathrm{q}_1 & -\mathrm{q}_2 & -\mathrm{q}_3 \\ \mathrm{q}_0 & -\mathrm{q}_3 & \mathrm{q}_2 \\ \mathrm{q}_3 & \mathrm{q}_0 & -\mathrm{q}_1 \\ -\mathrm{q}_2 & \mathrm{q}_1 & \mathrm{q}_0 \end{bmatrix}$$

IV Dynamics

The Newton-Euler approach is commonly used to describe the dynamics of marine vehicles. This approach relates the applied forces and moments to the vehicle's linear and angular accelerations. The general equation of motion using the Newton-Euler approach in the body-fixed frame can be written as:

$$\mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \bar{\mathrm{v}}^{\mathrm{B}}\bar{\times}^{*}\mathrm{M}\bar{\mathrm{v}}^{\mathrm{B}} = \bar{\mathrm{f}}^{\mathrm{B}}$$

where M represents the inertia matrix of the rigid body.

The equation above can be further transformed into standard manipulator equation form:

$$\mathrm{M_B}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C_B}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} = \bar{\mathrm{f}}^{\mathrm{B}}$$

where $\mathrm{M_B} \in \mathbb{R}^{6x6}$ is the rigid body mass matrix, $\mathrm{C_B}(\bar{\mathrm{v}}^{\mathrm{B}}) \in \mathbb{R}^{6x6}$ is the rigid body Coriolis and centripetal forces' matrix.

Nevertheless, some additional terms should be included in the equation to determine the specifics of the ROVs model. These terms comprise added mass, which represents the inertia of the surrounding fluid, the shift of the center of buoyancy due to changes in trim and heel angles, and damping effects. By incorporating these terms into the manipulator equation derived from the Newton-Euler approach, the model becomes more accurate and reflects the natural behavior of the ROV.

A. Center of Gravity and Center of Buoyancy

Due to the robust design of the marine vehicles, the center of buoyancy (COB) is usually aligned with the center of mass (COM), but placed higher. This shift between centers causes torque acting against the capsizing (Fig. **??**).

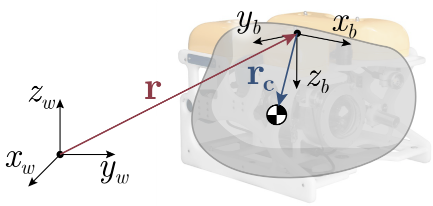


Fig. 4. The vehicle scheme

If we place the origin of the body frame at the center of mass, the mass matrix can be expressed as:

$$\mathrm{M_B} = \begin{bmatrix} \mathrm{mI_{3\times 3}} & -\mathrm{mS}\left(\mathrm{r_G^B}\right) \\ \mathrm{mS}\left(\mathrm{r_G^B}\right) & \mathrm{I_0} \end{bmatrix}$$

where $\mathrm{r_G^B}$ is the vector of the gravity center in the body frame, that is eventually a zero vector.

B. Concept of added mass

Since the vehicle moves in a viscous environment, we can not neglect the inertia of the surrounding liquid. To compensate added mass effect, it is necessary to add two components into dynamics equation: the added mass and the Coriolis forces acting on the added mass.

We can define vector of dynamical parameters of our body as:

$$\mathrm{f_{\dot{v}}} \triangleq \frac{\partial \bar{\mathrm{f}}}{\partial \dot{\mathrm{v}}}$$

Consequently, the added mass matrix $\mathrm{M_A}$ and the Coriolis forces matrix for added mass $\mathrm{C_A}(\mathrm{v^B})$ can be expressed as:

$$\mathrm{M_A} = \begin{bmatrix} \mathrm{A_{11}} & \mathrm{A_{12}} \\ \mathrm{A_{21}} & \mathrm{A_{22}} \end{bmatrix} = -\,\mathrm{diag}\left\{\mathrm{f_{\dot{v}}}\right\}, \mathrm{where}\ \mathrm{A_{ij}} \in \mathbb{R}^{3x3}$$

$$\mathrm{C_A}(\bar{\mathrm{v}}^{\mathrm{B}}) = \begin{bmatrix} 0_{3\times 3} & -\mathrm{S}\left(\mathrm{A_{11}}\mathrm{v^B} + \mathrm{A_{12}}\omega^{\mathrm{B}}\right) \\ -\mathrm{S}\left(\mathrm{A_{11}}\mathrm{v^B} + \mathrm{A_{12}}\omega^{\mathrm{B}}\right) & -\mathrm{S}\left(\mathrm{A_{21}}\mathrm{v^B} + \mathrm{A_{22}}\omega^{\mathrm{B}}\right) \end{bmatrix}$$

The values of dynamical parameters are usually determined empirically. Therefore, the error on $\mathrm{M_A}$ and $\mathrm{C_A}$ can be quite large, and we will not consider these matrices in our model implementation for the control design.

C. Hydrodynamic Damping

Generally, the dynamics of underwater vehicles can be highly nonlinear and coupled. Nevertheless, during the slow non-coupled motion the damping can be approximated to linear and quadratic damping:

$$\mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}}) = -\mathrm{K_{lin}} - \mathrm{K_{quad}}|\bar{\mathrm{v}}^{\mathrm{B}}|$$

The appropriate values of damping coefficients for vectors $\mathrm{K_{lin}}$ and $\mathrm{K_{quad}}$ can be discovered through several experiments.

D. Restoring forces

The common sense is to neglect all other forces acting on the vehicle except buoyancy and gravity. Although the motion of the current can also affect the dynamics, it is unpredictable and highly nonlinear, which makes it easier to compensate through control.

The weight of the body is defined as: $\mathrm{W} = \mathrm{mg}$, where m is the vehicle's mass and g is the gravity acceleration. The buoyancy force is defined as: $\mathrm{B} = \rho \mathrm{g} \nabla$, where ρ is the water density and ∇ the volume of fluid displaced by the vehicle.

By transforming the weight and buoyancy force to the body-fixed frame, we get:

$$\mathrm{f_G}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right) = \mathrm{R}^{\top}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right)\begin{bmatrix} 0 \\ 0 \\ \mathrm{W} \end{bmatrix} \quad \mathrm{f_B}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right) = -\mathrm{R}^{\top}\left(\bar{\mathrm{r}}^{\mathrm{N}}\right)\begin{bmatrix} 0 \\ 0 \\ \mathrm{B} \end{bmatrix}$$

Therefore, overall restoring force and moment vector is defined as:

$$\mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) = -\begin{bmatrix} \mathrm{f_G}(\bar{\mathrm{r}}^{\mathrm{N}}) + \mathrm{f_B}(\bar{\mathrm{r}}^{\mathrm{N}}) \\ \mathrm{r_G^B} \times \mathrm{f_G}(\bar{\mathrm{r}}^{\mathrm{N}}) + \mathrm{r_B^B} \times \mathrm{f_B}(\bar{\mathrm{r}}^{\mathrm{N}}) \end{bmatrix}$$

where $\mathrm{r_B^B}$ is the vector of the buoyancy center in the body frame.

E. Matrix representation

The final equation representing the system dynamics is:

$$\mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) = \bar{\mathrm{f}}^{\mathrm{B}}$$

where $\mathrm{M} = \mathrm{M_B} + \mathrm{M_A}$, $\mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}}) = \mathrm{C_B}(\bar{\mathrm{v}}^{\mathrm{B}}) + \mathrm{C_A}(\bar{\mathrm{v}}^{\mathrm{B}})$

V Thrusters modelling

The force and moment vector produced by the thruster are typically represented by a complex nonlinear function $\mathrm{f}(\bar{\mathrm{v}}^{\mathrm{B}}, \mathrm{V}, \mathrm{u})$, which depends on the vehicle's velocity vector $\bar{\mathrm{v}}^{\mathrm{B}}$, the power source voltage V, and the control variable u.

However, expressing such a nonlinear relationship directly can be challenging in practical applications. As a result, some authors propose a simpler approach:

$$\bar{\mathrm{f}}^{\mathrm{B}} = \mathrm{T}\phi(\mathrm{u})$$

where $\mathrm{T} \in \mathbb{R}^{6\mathrm{xn}}$ is the thrust configuration matrix, which maps body torques to truster forces, $\phi(\mathrm{u}) \in \mathbb{R}^{\mathrm{nxn}}$ is the DC-gain transfer function, which defines relation between PWM signal and output force.

VI Summary

This chapter provides a comprehensive overview of essential concepts and equations crucial for understanding the kinematics and dynamics of ROVs.

Notation

ROVs have six degrees of freedom (DOF), which include three for translation and three for rotation. The use of quaternions to represent orientation helps to avoid singularity issues.

Frame of reference

There are two coordinate frames: the earth-fixed frame and the body-fixed frame. State variables are denoted by $_{\mathrm{B}}$ in the body-fixed frame and $_{\mathrm{N}}$ in the earth-fixed one.

Kinematics

The motion of a marine vehicle, without considering external forces, is expressed in terms of velocities for two coordinate frames:

$$\dot{\bar{\mathrm{r}}}^{\mathrm{N}} = \mathrm{J}(\bar{\mathrm{r}}^{\mathrm{N}})\bar{\mathrm{v}}^{\mathrm{B}}$$

Dynamics

The Newton-Euler approach is used to describe the dynamics of ROV by relating applied forces and moments to linear and angular accelerations:

$$\mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) = \bar{\mathrm{f}}^{\mathrm{B}}$$

Additional terms like added mass, center of buoyancy, and damping effects enhance model accuracy.

Thruster modelling

The complex relationship between thruster force and control variables is simplified using a thrust configuration matrix T and a DC-gain transfer function $\phi(\mathrm{u})$.

$$\bar{\mathrm{f}}^{\mathrm{B}} = \mathrm{T}\phi(\mathrm{u})$$

The mathematical models developed in this chapter lay the foundation for designing control systems. By accurately capturing the dynamics and kinematics of ROVs, these models enable precise navigation underwater. The next chapter will implement effective control strategies and enhance the performance of ROVs in various scenarios.

Chapter 3

Methodology

#TODO Reformulate intro

The development of a robust control system for underwater robots hinges on a comprehensive understanding of system dynamics and control objectives. This chapter explores control design for underwater robots, focusing on the application of sliding mode control to address challenges in marine environments. By integrating robust control strategies, the aim is to enhance the stability, accuracy, and responsiveness of underwater robotic systems operating in dynamic and unpredictable conditions.

I Design Considerations

This section provides an introduction to control systems, starting with defining objectives that serve as a foundation for understanding subsequent topics.

A. Control objectives

General control design

The main idea of the control system design is to choose such control input u which meets the desired performance specifications while ensuring stability.

Underwater control design

Underwater robots require precise control systems to navigate and operate effectively in challenging marine environments. These control objectives are crucial for ensuring the robot's stability, accuracy, and responsiveness:

- Position and Orientation Tracking: The robot must accurately follow a desired trajectory, maintaining its position and orientation as intended.

- Disturbance Rejection: The robot should be able to withstand external disturbances, such as ocean currents, waves, and sensor noise, to maintain stable tracking performance.

- Robustness: The control system should be robust to uncertainties in the robot's dynamics and environmental conditions, ensuring reliable operation even in unpredictable situations.

- Real-Time Implementation: The control algorithm should be computationally efficient and able to run in real-time on the robot's embedded system, enabling prompt and effective responses to changing conditions.

Connect to the next chapter

Next, let us discuss the challenges posed by uncertainties in control system modeling, which must be understood before diving into advanced control techniques.

B. Model Uncertainties

What uncertainties?

The estimated model dynamics may not perfectly match the actual system behavior due to imprecise parameter estimates caused by simplified dynamics and external factors. From a control perspective, there exist two primary types of modeling inaccuracies:

1. structured (or parametric) uncertainties
2. unstructured uncertainties (or unmodeled dynamics)

The first kind corresponds to inaccuracies in the model's included terms, while the second kind relates to inaccuracies on the system order.

State which uncertainties ROV has

Recalling dynamics of ROV, following parameters may have some imprecision:

- Body parameters. The mass matrix $\mathrm{M_B}$ and the matrix of Coriolis forces $\mathrm{C_B}$ may unknown due to uncertain values of mass m and inertia matrix $\mathrm{I_0}$.
- Coefficients of viscous damping D. The values for linear and quadratic terms are defined empirically.
- Restoring forces g. Specifically, water density ρ is environment dependent and whole body volume ∇ is hard to calculate with proper accuracy.

Further, the approximated value of the parameter x is represented as $\hat{\mathrm{x}}$, while the difference between this approximation and the actual value is defined as $\tilde{\mathrm{x}} = \hat{\mathrm{x}} - \mathrm{x}$.

Moreover, added mass parameters $\mathrm{M_A}$ and $\mathrm{C_A}$ cannot be calculated directly and will be omitted in future calculations. Instead, we will incorporate a common disturbance term δ.

Thruster mapping approximation

As stated before, thruster mapping is defined through the configuration matrix T and the DC-gain transfer function $\phi(\mathrm{u})$. However, $\phi(\mathrm{u})$ can be highly nonlinear and voltage dependent (Fig. 1).

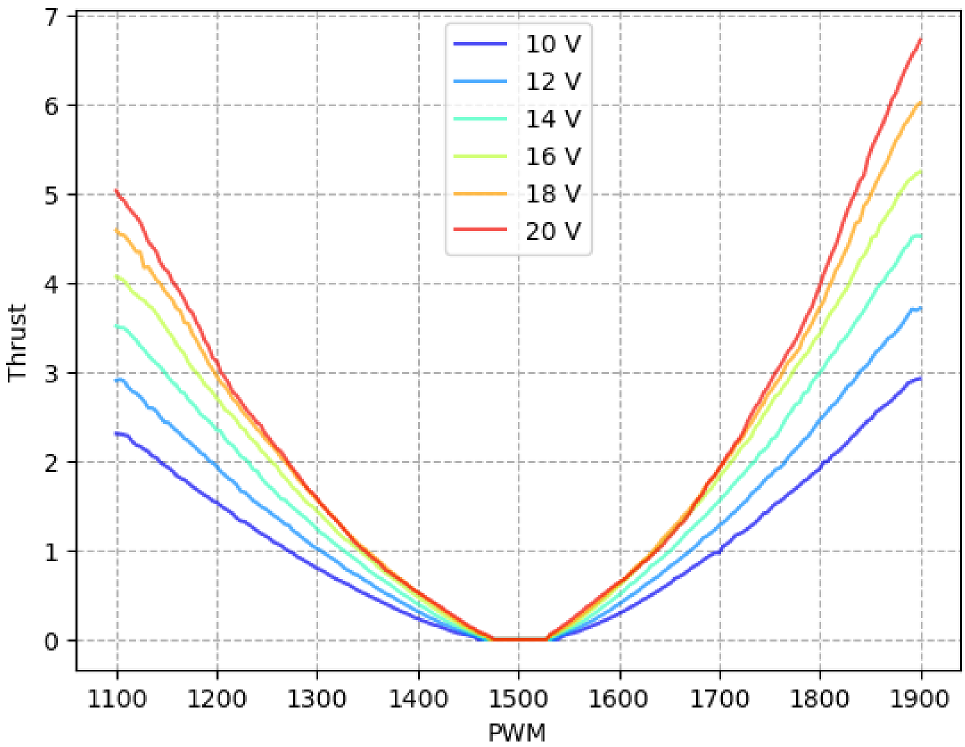


Fig. 5. The relationship between thrust and PWM.

Therefore, it is proposed to model this relation for the half of the range using two linear bounds from the inflection point. This approach is deemed appropriate due to the presence of symmetry. The nominal approximation $\mathrm{k_n}$ is defined by the middle line between the lower and upper bounds $\mathrm{k_{min}}$ and $\mathrm{k_{max}}$, which cover almost the whole possible range (Fig. 2).

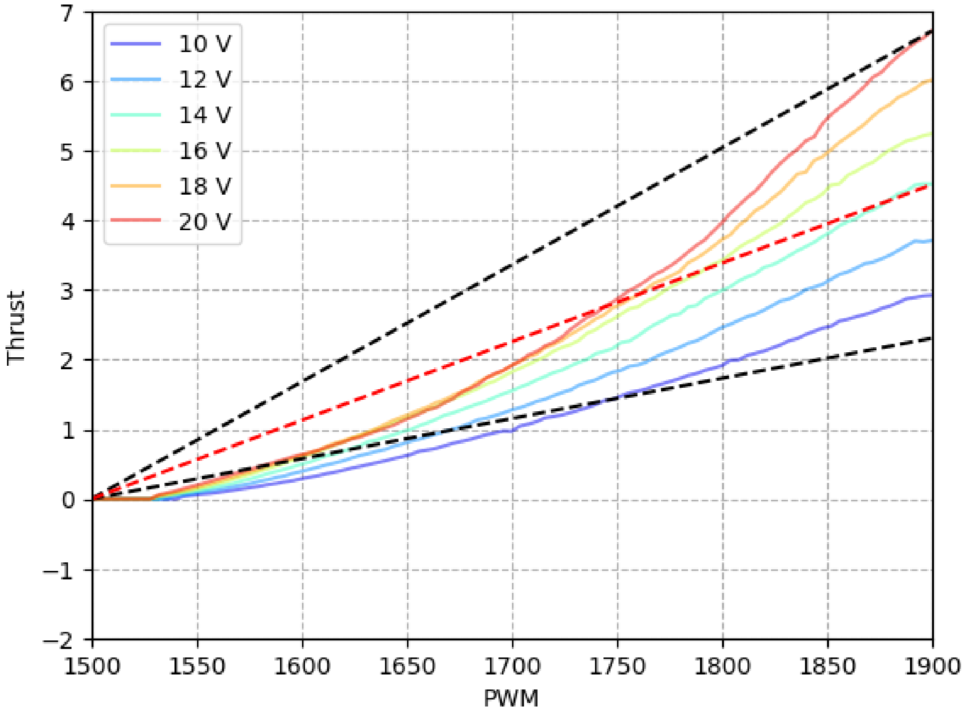


Fig. 6. The thruster mapping approximation.

The function can be expressed as $\hat{\phi}(\mathrm{u}) = \mathrm{k} \cdot \mathrm{u}$, where k is a slope coefficient. The value of k should satisfy $\mathrm{k}_{\mathrm{min}} < \mathrm{k} < \mathrm{k}_{\mathrm{max}}$, and its estimate is given by $\mathrm{k}_{\mathrm{n}} = \frac{\mathrm{k}_{\mathrm{max}} - \mathrm{k}_{\mathrm{min}}}{2}$.

Thus, the thruster mapping takes the form:

$$\hat{\bar{\mathrm{f}}}^{\mathrm{B}} = \mathrm{T}\hat{\phi}(\mathrm{u}) = \mathrm{kTu} = \mathrm{Bu}$$

Conclusion

To summarize, the ultimate goal of the underwater control system is to follow the desired trajectory $\bar{\mathrm{r}}^{\mathrm{B}}_{\mathrm{des}}$ even in the presence of external disturbances and uncertainties. By measuring the difference between the actual trajectory $\bar{\mathrm{r}}^{\mathrm{B}}$ and the desired trajectory, we obtain the tracking error, which is expressed as $\tilde{\bar{\mathrm{r}}}^{\mathrm{B}} = \bar{\mathrm{r}}^{\mathrm{B}}_{\mathrm{des}} - \bar{\mathrm{r}}^{\mathrm{B}}$.

The control objective can be reformulated to achieve $\tilde{\bar{\mathrm{r}}}^{\mathrm{B}} \to 0$ as $\mathrm{t} \to \infty$, meaning that the tracking error should approach zero as time progresses towards infinity.

Control affected

As discussed earlier, modeling inaccuracies can have strong adverse effects on nonlinear control systems. Therefore, any practical design must handle them explicitly. Inverse dynamics control can be a good starting point for deriving complex nonlinear control approaches, as it addresses the nonlinearities present in the system.

II Inverse dynamics

Intro to ID

Inverse dynamics is a nonlinear control technique that provides a trajectory tracking by calculating the required joint actuator torques to achieve a given trajectory. This approach relies on exact cancellation of nonlinearities in the robot equation of motion.

More on inverse dynamics

The inverse dynamics control is directly related to the solution of the inverse dynamics problem. By appropriately inverting the dynamic model, a control law can cancel the nonlinear part of the dynamics, decouple the interactions between the regulated variables, and specify the time characteristics of the decay of the task errors.

Control design

Recalling the system dynamics equation, we can design the following control law to linearize the system:

$$\mathrm{u} = \mathrm{B}^{+}(\mathrm{Ma} + \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}))$$

where a is outer-loop control to be designed as a proportional-derivative (PD) controller:

$$\mathrm{a} = \dot{\mathrm{v}}^{\mathrm{B}}_{\mathrm{des}} - \mathrm{K_p}\tilde{\bar{\mathrm{r}}}^{\mathrm{B}} - \mathrm{K_d}\tilde{\bar{\mathrm{v}}}^{\mathrm{B}}$$

Introduce uncertainties

In order to determine the required control inputs, it is important to have an accurate model of the system dynamics. However, this approach may not be effective when dealing with nonlinear systems that involve uncertainties and disturbances.

The equation () becomes more complex when system parameters and disturbances are unknown:

$$\hat{\mathrm{u}} = \hat{\mathrm{B}}^{+}(\hat{\mathrm{M}}\mathrm{a} + \hat{\mathrm{C}}(\mathrm{v}^{\mathrm{B}})\mathrm{v}^{\mathrm{B}} + \hat{\mathrm{D}}(\mathrm{v}^{\mathrm{B}})\mathrm{v}^{\mathrm{B}} + \hat{\mathrm{g}}(\mathrm{r}^{\mathrm{B}}))$$

Control analysis

Substitution to the dynamics yields:

$$\dot{\mathrm{v}}^{\mathrm{B}} = \mathrm{M}^{-1}(\mathrm{B}\hat{\mathrm{B}}^{+}\mathrm{f} - \hat{\mathrm{f}} - \delta) + \mathrm{M}^{-1}\mathrm{B}\hat{\mathrm{B}}^{+}\hat{\mathrm{M}}\mathrm{a}$$

with $\mathrm{f} = \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}})$

Error dynamics

In terms of tracking error $e = \tilde{r}^B$, the following system can be designed:

$$\begin{cases} e & = \tilde{r}^B \\ \dot{e} & = \tilde{v}^B \\ \ddot{e} & = \dot{\tilde{v}}^B = \dot{v}^B_{des} - \dot{v}^B = \\ & = \dot{v}^B_{des} - M^{-1}(B\hat{B}^{+}f - \hat{f} - \delta) + (M^{-1}B\hat{B}^{+}\hat{M} - I)(\dot{v}^B_{des} - K_p e - K_d \dot{e}) \end{cases}$$

#TODO Add analysis

As a result, the inverse dynamics technique may not be the best option for effectively controlling underwater systems. Further we introduce sliding mode control, which is a robust control technique that can achieve desired control objectives in the presence of uncertainties.

III Sliding Mode

Intro to SMC

As discussed before, there are several robust controller designs available. However, the sliding mode approach suggested by (Spong - ?) is highly regarded as the most sophisticated and frequently implemented one.

SMC definition

Sliding mode control (SMC) is a nonlinear control method that guarantees robust control of systems with uncertainties and disturbances. This technique involves developing a sliding surface within the state space and directing the system's trajectory to slide along this surface (Fig. 3).

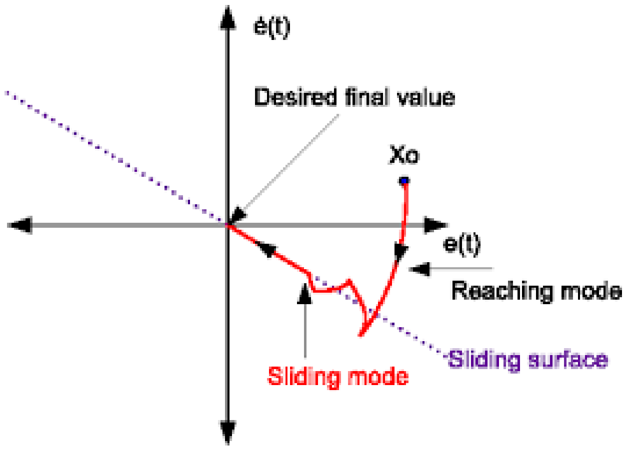


Fig. 7. The general sliding mode scheme

Advantages to use SMC

Compared to other nonlinear control methods, SMC is a relatively straightforward solution to implement with a basic understanding of system dynamics and sliding surface design. SMC provides a fast transient response due to the sliding dynamics, which makes it possible to track desired references or trajectories quickly. Additionally, the sliding surface ensures robustness to uncertainties and disturbances by making the system behavior insensitive to these factors.

In summary, SMC is a simple and effective solution for controlling nonlinear systems.

A. Sliding Surface Design

Sliding surface

In sliding mode control, a sliding surface is a hyperplane in the state space that defines the desired system behavior. The control objective is to force the system's trajectory to slide long this surface. Once the system's trajectory reaches the sliding surface, it will remain on the surface as long as the control law is applied (Fig. 4).

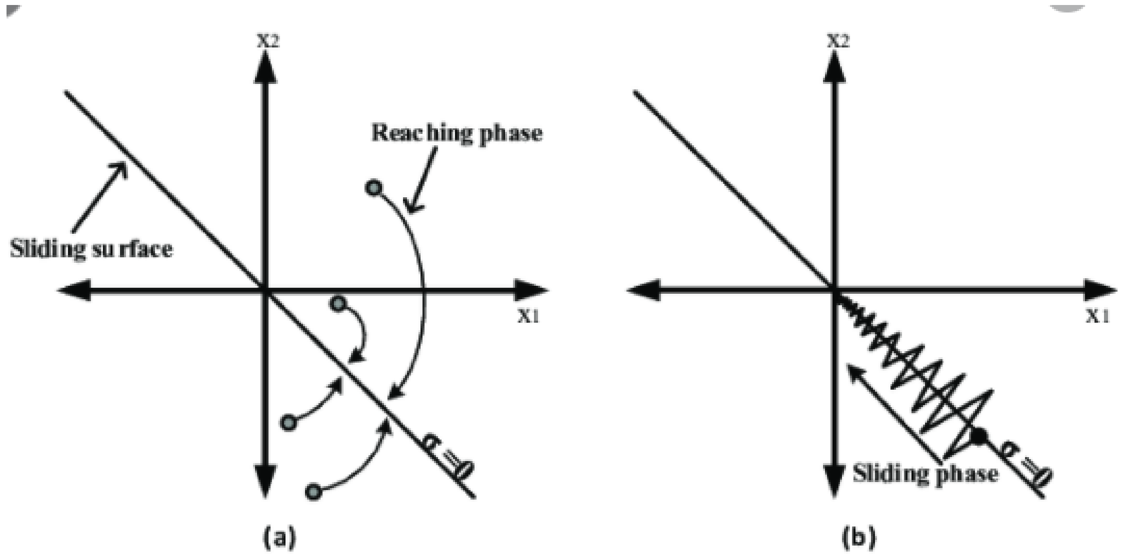


Fig. 8. The phases of sliding mode control

Properties of SMC

The design of the sliding surface is critical for the performance of the SMC system. The sliding surface should be:

- Reachable: The system's trajectory should be able to reach the sliding surface in a finite amount of time.
- Invariant: Once the system's trajectory reaches the sliding surface, it should remain on the surface for all future time.
- Attractive: The control law should attract the system's trajectory to the sliding surface and keep it there.

Invariant sets

In order to satisfy the conditions above, the sliding surface is designed to be an invariant set. Invariant sets are sets of states in the state space that, once entered, cannot be exited under the action of the control law.

General equation

Let us define time-varying surface $\mathcal{S}$ in the state space $\mathbb{R}^n$ given by scalar equation $s(\bar{r}^B, t)$:

$$s(\bar{r}^B, t) = (\frac{d}{dt} + \lambda)^{n-1}\tilde{\bar{r}}^B$$

where n is the order of the system and λ is a positive scalar.

Sliding condition

In order to ensure convergence of s along all system trajectories in finite time, let us define Lyapunov candidate $V = s^2$ as the squared distance to the surface. The sliding condition can then be formulated accordingly:

$$\frac{dV}{dt} < -\eta\sqrt{V} \quad \text{or} \quad \frac{1}{2}\frac{d}{dt}||s||^2 = s^T\dot{s} < \eta||s||$$

where $\eta > 0$ defines the rate of convergence to the sliding surface.

Invariance

Satisfying sliding condition, makes the surface an invariant set and implies convergence to $\tilde{r}^B$, since the system described by the differential equation:

$$s = (\frac{d}{dt} + \lambda)^{n-1}\tilde{r}^B = 0$$

is inherently stable and remains at the equilibrium point $\tilde{r}^B = 0$.

New control objective

Applying such transformation yields a new representation of the tracking performance:

$$s \to 0 \Rightarrow \tilde{r}^B \to 0$$

Meaning, that the problem of tracking r^B is equivalent to remaining on the sliding surface. Thus, the problem of tracking the n-dimensional vector r^B can in effect be replaced by a first order stabilization problem in s.

B. Control Law Design

Control law

The controller comprises two distinct components: nominal control $\mathrm{a_n}$, and an additional robust part $\mathrm{a_s}$ (Fig. 5):

$$\mathrm{a} = \mathrm{a_n} + \mathrm{a_s}$$

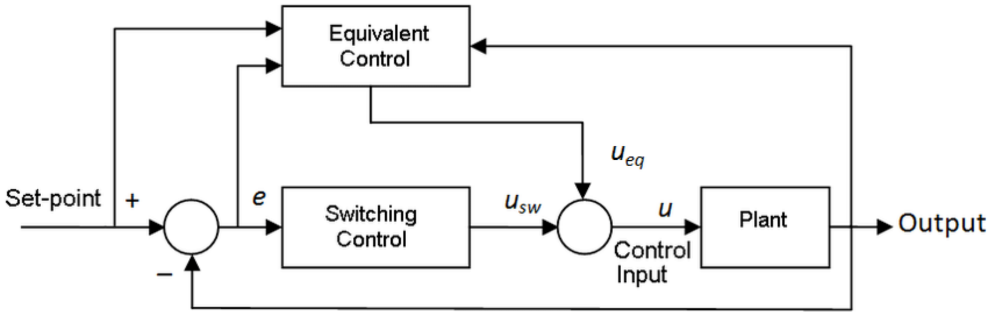


Fig. 9. The sliding mode control scheme

The nominal control $\mathrm{a_n}$ aims to compensate for the system's dynamics, while the robustifying component $\mathrm{a_s}$ is designed to enhance the controller's stability and performance by providing additional corrective action to counteract uncertainties and disturbances.

System dynamics with uncertainties

By taking into account the impact of both dynamic approximations and disturbances, we can obtain the following dynamics of the system:

$$\mathrm{M}\dot{\bar{\mathrm{v}}}^{\mathrm{B}} + \mathrm{C}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{D}(\bar{\mathrm{v}}^{\mathrm{B}})\bar{\mathrm{v}}^{\mathrm{B}} + \mathrm{g}(\bar{\mathrm{r}}^{\mathrm{N}}) + \delta = \mathrm{Bu}$$

Virtual control

To simplify computations, we can use a virtual control, $f_v = Tu$, which can represent the desired behavior of the system without introducing input uncertainty. This can make the control problem easier to formulate, leading to more efficient algorithms and computational procedures.

Inverse dynamics

Using the inverse dynamics approach, we can apply the outer loop controller to partially linearize the system with model estimates:

$$\hat{f}^B = \hat{B}u = \hat{k}Tu = \hat{k}f_v = \hat{M}a + \hat{C}(v^B)v^B + \hat{D}(v^B)v^B + \hat{g}(r^B)$$

Expressing virtual control input, we obtain:

$$f_v = \frac{\hat{M}a + \hat{C}(v^B)v^B + \hat{D}(v^B)v^B + \hat{g}(r^B)}{\hat{k}}$$

Substitution to the dynamics yields the equation:

$$\dot{\bar{v}}^B = M^{-1}(\frac{k}{\hat{k}}\hat{f}(\bar{r}^N, \bar{v}^B) - f(\bar{r}^N, \bar{v}^B) - \delta) + \frac{k}{\hat{k}}M^{-1}\hat{M}a = F(\bar{r}^N, \bar{v}^B) + Ka$$

with $f(\bar{r}^N, \bar{v}^B) = C(\bar{v}^B)\bar{v}^B + D(\bar{v}^B)\bar{v}^B + g(\bar{r}^N)$

Sliding condition

The time derivative of s is connected to dynamics as follows:

$$\dot{s} = \dot{\tilde{\bar{v}}}^B + \lambda\tilde{\bar{v}}^B = a_n - \dot{\bar{v}}^B = a_n - F - K(a_n + a_s) = w - Ka_s$$

with $w = (I - K)a_n - F$

Substitution to sliding condition yields:

$$\mathrm{s}^{\mathrm{T}}\mathrm{w} - \mathrm{s}^{\mathrm{T}}\mathrm{K}\mathrm{a}_{\mathrm{s}} \leq \|\mathrm{s}\|\|\mathrm{w}\| - \mathrm{s}^{\mathrm{T}}\mathrm{K}\mathrm{a}_{\mathrm{s}} \leq -\eta\|\mathrm{s}\|$$

Apply matrix property

Let us recall that for any symmetric matrix P:

$$\sigma_{\mathrm{min}}^2\|\mathrm{x}\|^2 \leq \|\mathrm{x}^{\mathrm{T}}\mathrm{P}\mathrm{x}\| \leq \sigma_{\mathrm{max}}^2\|\mathrm{x}\|^2$$

with σ_{min} and σ_{max} being the largest and smallest eigenvalues of matrix P.

Thus we can choose the stabilizing control a_{s} as:

$$\mathrm{a}_{\mathrm{s}} = \frac{\alpha\hat{\mathrm{k}}}{\sigma_{\mathrm{max}}^2}\hat{\mathrm{M}}^{-1}\frac{\mathrm{s}}{\|\mathrm{s}\|} = \rho\frac{\mathrm{s}}{\|\mathrm{s}\|}$$

where σ_{max} is maximal singular value of M^{-1} which provide:

$$\|\mathrm{s}\|\|\mathrm{w}\| - \mathrm{s}^{\mathrm{T}}\mathrm{K}\mathrm{a}_{\mathrm{s}} \leq \|\mathrm{s}\|\|\mathrm{w}\| - \mathrm{k}\frac{\alpha}{\sigma_{\mathrm{max}}^2\|\mathrm{s}\|}\mathrm{s}^{\mathrm{T}}\mathrm{M}^{-1}\mathrm{s} \leq \|\mathrm{s}\|\|\mathrm{w}\| - \alpha\mathrm{k}\|\mathrm{s}\|$$

but by definition $\mathrm{k}_{\mathrm{min}} < \mathrm{k} < \mathrm{k}_{\mathrm{max}}$, therefore

$$\|\mathrm{s}\|\|\mathrm{w}\| - \alpha\mathrm{k}\|\mathrm{s}\| \leq \|\mathrm{s}\|\|\mathrm{w}\| - \alpha\mathrm{k}_{\mathrm{min}}\|\mathrm{s}\| < -\eta\|\mathrm{s}\|$$

Setting gain α accordingly to:

$$\alpha > \frac{\|\mathrm{w}\| + \eta}{\mathrm{k}_{\mathrm{min}}}$$

will satisfy sliding conditions.

The final expression for sliding control:

$$\mathrm{a_s} = \begin{cases} \rho\frac{\mathrm{s}}{\|\mathrm{s}\|}, & \|\mathrm{s}\| > 0 \\ 0, & \|\mathrm{s}\| = 0 \end{cases}$$

Solve chattering problem

In order to reduce chattering, the controller above is effectively smoothed using the boundary layer:

$$\mathrm{a_s} = \begin{cases} \rho\frac{\mathrm{s}}{\|\mathrm{s}\|}, & \|\mathrm{s}\| > \epsilon \\ \rho\frac{\mathrm{s}}{\epsilon}, & \|\mathrm{s}\| \leq \epsilon \end{cases}$$

where ϵ is the boundary layer thickness.

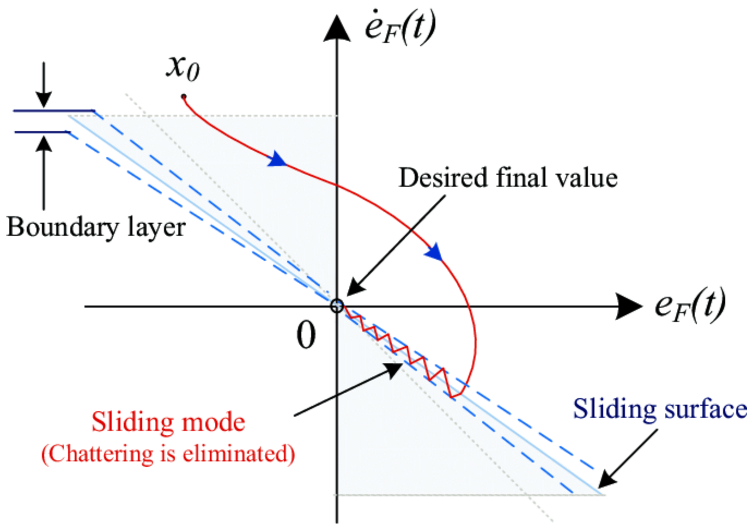


Fig. 10. The sliding mode scheme with boundary layer

Nominal torque

The nominal control $\mathrm{a_n}$ can be designed in a form of PD controller:

$$\mathrm{a_n} = -\mathrm{K_0}\tilde{\mathrm{v}}^{\mathrm{B}} - \mathrm{K_1}\tilde{\mathrm{r}}^{\mathrm{B}}$$

Final law

The resulting controller is then given as follows:

$$\hat{u} = \hat{B}^{-1}(\hat{M}a + \hat{C}(v^B)v^B + \hat{D}(v^B)v^B + \hat{g}(r^B))$$

$$a = a_n + a_s$$

$$a_n = -K_0\tilde{v}^B - K_1\tilde{r}^B$$

$$s = \tilde{\tilde{v}}^B + \lambda\tilde{\tilde{r}}^B$$

$$a_s = \begin{cases} \rho\frac{s}{\|s\|}, & \|s\| > \epsilon \\ \rho\frac{s}{\epsilon}, & \|s\| \leq \epsilon \end{cases}$$

IV Optimization control

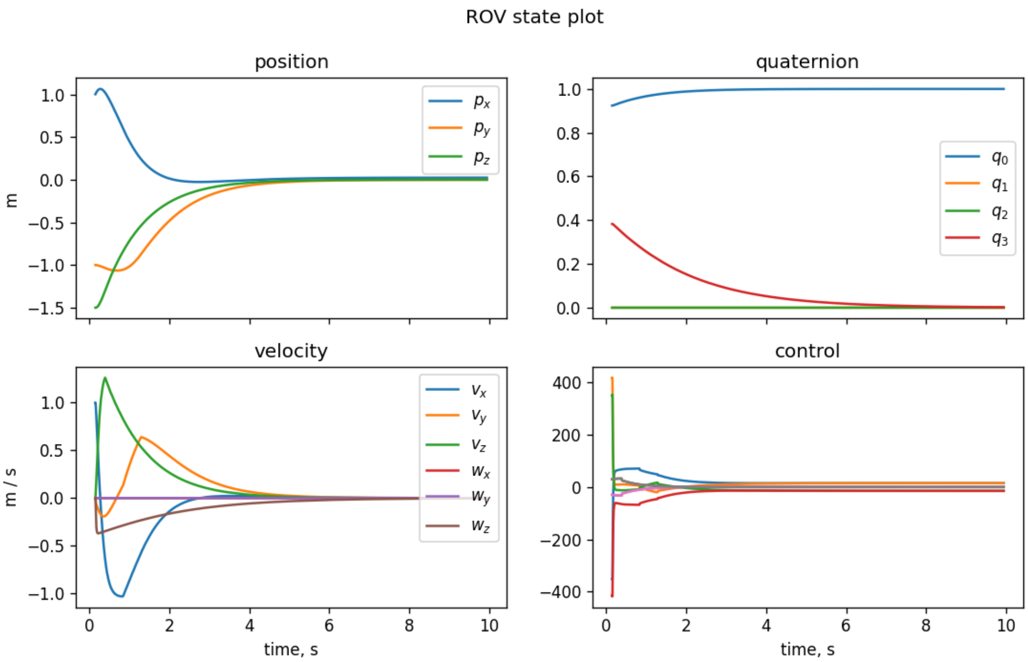
V Summary

#TODO Summarize the main findings of the chapter.

The methodology of robust control via sliding mode can be formulated in following steps:

- Define the sliding surface $s(y, t)$
- Derive nominal control $\hat{u}$ (our best guess) that may achieve $\dot{s} = 0$
- Modify control law by discontinues term that will bring system to sliding mode by satisfying the sliding condition.

Discuss the limitations and potential extensions of the SMC controller.


ROV state plot
position
m
1.0
0.5
0.0
−0.5
−1.0
−1.5
p_x
p_y
p_z
quaternion
1.0
0.8
0.6
0.4
0.2
0.0
q_0
q_1
q_2
q_3
velocity
m / s
1.0
0.5
0.0
−0.5
−1.0
v_x
v_y
v_z
w_x
w_y
w_z
0
2
4
6
8
10
time, s
control
400
200
0
−200
−400
0
2
4
6
8
10
time, s