Ocean Engineering 105 (2015) 54–63

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Short Communication

# Dynamical sliding mode control for the trajectory tracking of underactuated unmanned underwater vehicles

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ARTICLE INFO

*Article history:*
Received 1 March 2014
Accepted 16 June 2015

*Keywords:*
Backstepping
Sliding mode control
Underactuated system
Lyapunov stability theory
Unmanned underwater vehicles

ABSTRACT

This paper proposes a novel adaptive dynamical sliding mode control based methodology to design control algorithms for the trajectory tracking of underactuated unmanned underwater vehicles (UUVs). The main advantage of the approach is that the combination of backstepping and sliding mode control enhances the robustness of an UUV in the presence of systematical uncertainty and environmental disturbances. The position and attitude dynamical equations of an underactuated UUV are first represented and analyzed using coordinate transformation with the aid of backstepping technique. Subsequently, the output feedback problem is tackled by employing adaptive sliding mode control to estimate the systematical uncertain states required by the stable velocity tracking controller. The final controlled system can be proved to be globally asymptotically stable based on Lyapunov stability theory. Simulations performed on an underactuated UUV demonstrate the effectiveness of the proposed method.



## 1. Introduction

Over the last few years, it has witnessed a great amount of research in the area of motion control of underactuated unmanned underwater vehicles (UUVs) (Aguiar and Joao, 2007). However, a control problem of UUVs continues to pose challenges to system designers, since most of them are underactuated, i.e., they have fewer actuators than the number of degrees of freedom, imposing nonintegrable acceleration constraints (Yuh, 2000). In addition, UUVs' kinematic and dynamic models are highly nonlinear and coupled, and the hydrodynamic parameters are often uncertain, especially when the vehicle may subject to unknown disturbances from ocean currents (Fossen, 1994), making trajectory tracking control design a hard work.

Focusing on the motion control of UUVs, trajectory tracking control has received relatively more attention than path following problem, since it is concerned with the design of control laws that force the vehicle to reach and follow a time-varying parameterized trajectory. Currently, different control strategies that are available for the trajectory tracking and path following of UUVs are proposed in the literature.

In Kaminer et al. (1998) and Khac and Jie (2009), a linearization method was proposed to solve the trajectory tracking control, respectively. However, the basic limitation of the approach is that the stability is only guaranteed in a neighborhood of the selected operating points. Moreover, performance can suffer significantly when the vehicle executes maneuvers that emphasize its nonlinearity and cross coupling. On the basis of backstepping and associated Lyapunov functions, some research results were proposed to tackle the control for underactuated UUVs; see for example (Pettersen and Egeland, 1999; Lapierre and Soetanto, 2007; Repoulias and Papadopoulos, 2007). Besides, a current observer was presented based on the Lyapunov stability theory and by using the backstepping technique to estimate the unknown constant ocean current (Bi et al., 2010). However, in most existing backstepping based techniques, these results require a very restrictive assumption that yaw reference velocity must satisfy persistent excitation conditions and thus, it does not converge to zero (Serrano et al., 2014). Consequently, the approach suffers from the drawback that a vehicle cannot track straight-line reference trajectories.

Compared with backstepping, adaptive control is considered to be better for plants with uncertainties because it can improve its performance with little or no information of the bounds on uncertainties. Hence, global output-feedback tracking control (Do et al., 2004a, 2004b; Bidyadhar et al., 2013), model-based output feedback control (Refsnes et al., 2008), and adaptive output feedback control based on DRFNN (Ge and Wang, 2002; Zhang et al., 2009) for underactuated UUVs were proposed, respectively. However, a drawback of these adaptive control approaches is computationally intensive for higher order systems and effective only for constant and slowly varying parameters. Unlike above adaptive

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http://dx.doi.org/10.1016/j.oceaneng.2015.06.022
0029-8018/

control, robust adaptive control technique shows the special characteristics in trajectory tracking of underactuated UUVs with unknown parameters and environmental disturbances. A novel method of time delay control that can be thought of as "instantaneous learning" of the system dynamics was proposed in Kumar et al. (2007). Implementation of the control law requires the derivative of the system states to be known, which generally considered as a drawback when it has to be calculated based on the state measurement. In Do et al. (2004a, 2004b), a nonlinear robust adaptive control strategy was proposed by using Lyapunov's direct method, the popular backstepping and parameter projection techniques.

In this paper, a methodology of the combination of backstepping and adaptive dynamical sliding mode control is proposed for underactuated UUVs to deal with the planar trajectory tracking control problem. In such a way, the proposed controller is robust and adaptive to the systematical uncertainty and environmental disturbances. Moreover, the proposed control law adopts virtual velocity error dynamics to represent attitude errors, and thus simplifies the representation of the controller. The position and attitude dynamical equations of an underactuated UUV are first derived and analyzed using coordinate transformation with the aid of backstepping technique. Subsequently, the output feedback problem is tackled by employing adaptive sliding mode control to estimate the systematical uncertain states required by the stable velocity tracking controller. Also, to demonstrate the effectiveness and performance of the developed control strategy, simulation results for the following three scenarios are presented as a circular trajectory with constant velocity, Dubins paths, and a sinusoidal trajectory with time-varying velocity. The main contributions of this paper can be summarized as follows:

(i) A methodology of the combination of backstepping and adaptive sliding mode control is proposed to design control algorithms for the trajectory tracking of underacutated UUVs. Global uniform asymptotic stability of the overall control system is proved in the paper.
(ii) Different from the traditional approach to construct Lyapunov functions, the paper utilizes a virtual velocity variable to represent the attitude error, which can avoid the occurrence of representation singularities and simplify the analytical expression of the controller.
(iii) To enhance the robustness of an underactuated UUV against to model parameter uncertainties and unknown environmental disturbances, the novel sliding surfaces are designed in terms of the velocity errors, position errors and approximate errors. In comparison with the previous work, the controller cannot only realize tracking the trajectories with constant velocity, but also satisfy the constraints with time-varying velocity.

## 2. Problem formulation

The section describes the kinematic and dynamic models of an underactuated UUV moving in the horizontal plane, and then formulates the problem of trajectory tracking control. The notation is standard.

### 2.1. UUV modeling

To study the planar motion, the general kinematic and dynamic equations of an UUV moving in the horizontal plane can be developed using an earth-fixed reference frame {E} and a body-fixed reference frame {B} as shown in Fig. 1. Here, we briefly present the mathematical model of a neutrally buoyant UUV under the following assumptions that (i) the center of mass (CM) coincides with the center of buoyancy (CB), (ii) the mass distribution is homogeneous, (iii) the hydrodynamic drag terms of order higher than two are negligible, and (iv) the heave, pitch, and roll motions can be neglected. Then, as by Repoulias and Papadopoulos (2007), the kinematics and dynamics of an underactuated UUV can be expressed by the following differential equations:

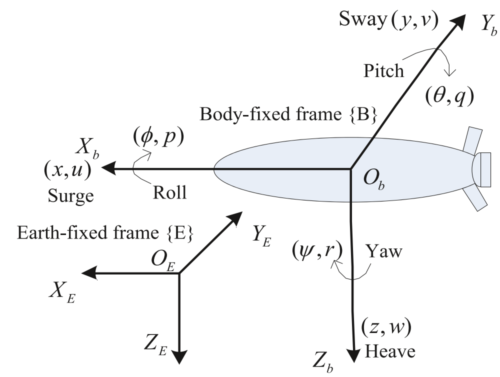

**Fig. 1.** Reference frames of an unmanned underwater vehicle.

$$\begin{cases} \dot{x} = \cos(\psi)u - \sin(\psi)v, \\ \dot{y} = \sin(\psi)u + \cos(\psi)v, \\ \dot{\psi} = r, \\ \dot{u} = \frac{m_{22}}{m_{11}}vr - \frac{d_{11}}{m_{11}}u + \frac{\tau_u + \tau_{w1}}{m_{11}}, \\ \dot{v} = -\frac{m_{11}}{m_{22}}ur - \frac{d_{22}}{m_{22}}v + \frac{\tau_{w2}}{m_{22}}, \\ \dot{r} = \frac{m_{11} - m_{22}}{m_{33}}uv - \frac{d_{33}}{m_{33}}r + \frac{\tau_r + \tau_{w3}}{m_{33}} \end{cases} \tag{1}$$

where $(x, y)$ denotes the position coordinates of an UUV in the earth-fixed frame, $\psi$ is the yaw angle of the vehicle, and $u$, $v$ and $r$ are the surge, sway, and yaw velocities, respectively. The surge force $\tau_u$ and the yaw torque $\tau_r$ are considered as the available control inputs. Parameters $m_{ii}$ and $d_{ii}$ are assumed to be positive constants and are given by the vehicle inertia and damping matrices. Clearly, the UUV is underactuated because the sway force is missing in Eq. (1).

### 2.2. Control objectives

In order to facilitate the formulation, we first define the actual variables $\mathbf{p} = [x(t), y(t), \psi(t)]^T$ and $\mathbf{v} = [u(t), v(t), r(t)]^T$. Let $\mathbf{p_d} = [x_d(t), y_d(t), \psi_d(t)]^T$ be a given sufficiently smooth time-varying desired trajectory with $\mathbf{v_d} = [u_d(t), v_d(t), r_d(t)]^T$ the reference velocity, and its derivatives with respect to time are bounded. Considering the underactuated UUVs represented in (1), we shall design a controller to render all the tracking errors $||\mathbf{p} - \mathbf{p_d}||$ and $||\mathbf{v} - \mathbf{v_d}||$ converge to a neighborhood of the origin that can be made arbitrarily small. Therefore, in comparison with path-following problem, the control objective to force the underactuated vehicle given in (1) to asymptotically track a smooth time parameterized trajectory by designing the control inputs $\tau_u$ and $\tau_r$ is considered in the paper. In addition, the control laws should be robust against to model parameter uncertainties and unknown environmental disturbances while guaranteeing a satisfactory performance of the vehicle, so it is desired to design the controller that globally uniformly asymptotically stabilize the tracking errors of an underactuated vehicle with systematical parametric uncertainties and unknown disturbances from the ocean environment (Fig. 2).

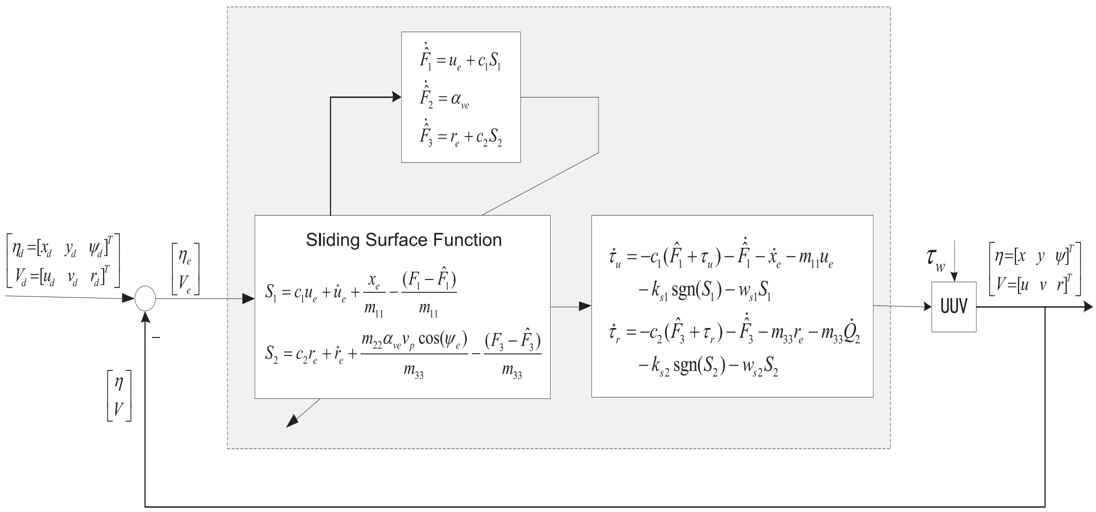


**Fig. 2.** Dynamical sliding mode control scheme of an underactuated UUV.

## 3. Controller design

This section presents Lyapunov-based control laws using backstepping technique and dynamical sliding mode control method for the underactuated UUVs with the dynamic parameters that are uncertainty in the underwater environment. Before stating the main result of the note, we first do the coordinate transformation in a form that is easier amenable for stabilization.

### 3.1. Coordinate transformation

To facilitate the design of the controller, we introduced three error variables $x_e, y_e, \psi_e$ by the coordinate transformation. And the desirable heading angle of the vehicle only relies on the reference trajectory, that is

$$\psi_d = \text{arc}\ \tan\left(\frac{\dot{y}_d}{\dot{x}_d}\right), \tag{2}$$

Then the position and attitude errors $x_e$, $y_e$, $\psi_e$ are defined in the body-fixed frame as

$$\begin{bmatrix} x_e \\ y_e \\ \psi_e \end{bmatrix} = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_d \\ y - y_d \\ \psi - \psi_d \end{bmatrix} \tag{3}$$

where $(x_d, y_d, \psi_d)$ represent the position and orientation of the vehicle in earth-fixed frame. Then, the derivatives of the position tracking error variables along (1) can be obtained

$$\begin{cases} \dot{x}_e = u - v_p \cos(\psi_e) + r y_e, \\ \dot{y}_e = v + v_p \sin(\psi_e) - r x_e, \end{cases} \tag{4}$$

where $v_p = \sqrt{\dot{x}_d^2 + \dot{y}_d^2}$.

**Assumption 1.** The reference signals $u_d$, $r_d$, $q_d$, $\dot{u}_d$, $\dot{r}_d$, $\dot{q}_d$ are bounded. There exists a strictly positive constant $u_{d\ \min}$, such that$|u_d(t)| \geq u_{d\ \min}$, $\forall t \geq 0$. The reference sway velocity satisfies: $|v_d(t)| < |u_d(t)|$, $\forall t \geq 0$.

Remark: The condition $|u_d(t)| \geq u_{d\ \min}$, $\forall t \geq 0$, covers both forward and backward tracking since the reference surge velocity is always nonzero but can be either positive or negative. Indeed, the condition $|u_d(t)| \geq u_{d\ \min}$, $\forall t \geq 0$, is much less restrictive than a persistently exciting condition on the yaw reference velocity. The condition $|v_d(t)| < |u_d(t)|$, $\forall t \geq 0$ implies that the underactuated underwater vehicle cannot track a helix with an arbitrarily large curvature and twist due to the vehicle's high inertia and no control input in the sway directions.

### 3.2. Backstepping adaptive dynamical sliding mode controller design

This section presents a trajectory tracking controller based on the combination of backstepping technique and sliding mode control method (Cheng et al., 2007; Liao et al., 2011), and analyzes the stability based on Lyapunov stability theory.

**Step 1**: Considering the **s**ubsystem of Eq. (4), we chose a Lyapunov function candidate as follows:

$$V_1 = \frac{1}{2}(x_e^2 + y_e^2) \tag{5}$$

Its derivatives along the solutions of the system (4) can be obtained

$$\begin{aligned} \dot{V}_1 &= x_e(u - v_p \cos(\psi_e) + r y_e) + y_e(v + v_p \sin(\psi_e) - r x_e) \\ &= x_e(u - v_p \cos(\psi_e)) + y_e(v + v_p \sin(\psi_e)) \end{aligned} \tag{6}$$

The traditional design method is to choose $\psi_e$ to make $y_e$ converge to a neighborhood of the origin that can be made arbitrarily small. It is, however, well known that the use of minimal attitude descriptions, such as Euler angles, can subject to the occurrence of representation singularities (Gianluca et al., 2001). It can be avoided by introducing a virtual velocity variable, which is defined as follows:

$$\alpha_v = v_p \sin(\psi_e) \tag{7}$$

From the expression of the virtual velocity, it makes the control of $\psi_e$ transform to the control of $\alpha_v$, which is a novelty in the control design.

In order to make $\dot{V}_1$ negative, $u$ and $\alpha_v$ are considered as the virtual controls, and their desired values $u_d$ and $\alpha_{vd}$ are chosen as

$$u_d = v_p \cos(\psi_e) - k_1 x_e / E \tag{8}$$

$$\alpha_{vd} = -v - k_2 y_e / E \tag{9}$$

where $E = \sqrt{1 + x_e^2 + y_e^2}$. It avoids the occurrence of the velocities $u_d$, $\alpha_{vd}$ beyond the vehicle can reach when the initial conditions $x_e$, $y_e$ are too large. And $k_1$, $k_2$ are positive constants needed to be chosen later.

**Step 2:** Considering that $u_d$ and $\alpha_{vd}$ are not true controls, thus we have to introduce error variables $u_e$ and$\alpha_{ve}$, described by

$$u_e = u - u_d, \alpha_{ve} = \alpha_v - \alpha_{vd} \tag{10}$$

To this end, the derivatives of the errors $x_e$ and $y_e$ are arranged as

$$\begin{cases} \dot{x}_e = u_e - k_1 x_e / E + r y_e \\ \dot{y}_e = \alpha_{ve} - k_2 y_e / E - r x_e \end{cases} \tag{11}$$

Thus, Eq. (6) can be rewritten as

$$\dot{V}_1 = -(k_1 x_e^2 + k_2 y_e^2)/E + u_e x_e + \alpha_{ve} y_e \tag{12}$$

The task now is to stabilize the error variables $u_e$ and $\alpha_{ve}$. In this step, we first design controller to stabilize the component $u_e$. Differentiating $u_e$ with respect to time, the surge velocity equation, using the new variables, yields

$$\begin{aligned} \dot{u}_e &= \dot{u} - \dot{u}_d \\ &= \frac{m_{22} v r - d_{11} u + \tau_{w1} - m_{11} \dot{u}_d + \tau_u}{m_{11}} \\ &= \frac{(F_1 + \tau_u)}{m_{11}} \end{aligned} \tag{13}$$

where $F_1 = m_{22} v r - d_{11} u + \tau_{w1} - m_{11} \dot{u}_d$.

Then, consider the following Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2} m_{11} u_e^2 + \frac{1}{2} (F_1 - \hat{F}_1)^2 \tag{14}$$

where $\hat{F}_1$ is an approximate value of the uncertain term $F_1$.

One of the contributions in the paper is to resolve control problems of underactuated UUVs with the systematical uncertainty and unknown environmental disturbances. According to the currently popular nonlinear techniques, dynamical sliding mode control method can be introduced and the novel sliding manifold is designed in terms of tracking errors and approximate errors. First, the sliding manifold $S_1$ yields

$$\begin{aligned} S_1 &= c_1 u_e + \dot{u}_e + \frac{x_e}{m_{11}} - \frac{(F_1 - \hat{F}_1)}{m_{11}} \\ &= c_1 u_e + \frac{(\hat{F}_1 + \tau_u) + x_e}{m_{11}} \end{aligned} \tag{15}$$

where $c_1$ is a positive constant. Then, the time derivative of $u_e$ yields

$$\dot{u}_e = S_1 - c_1 u_e - \frac{x_e}{m_{11}} + \frac{(F_1 - \hat{F}_1)}{m_{11}} \tag{16}$$

Differentiating $V_2$ with respect to time along Eq. (16), we can obtain its time derivative easily

$$\begin{aligned} \dot{V}_2 = &-(k_1 x_e^2 + k_2 y_e^2)/E + \alpha_{ve} y_e + m_{11} u_e S_1 - c_1 m_{11} u_e^2 \\ &+ (u_e - \dot{\hat{F}}_1)(F_1 - \hat{F}_1) \end{aligned} \tag{17}$$

If we select $T_1 = \dot{\tau}_u$, then $\dot{S}_1$ becomes

$$\dot{S}_1 = \frac{c_1 (F_1 + \tau_u) + T_1 + \dot{\hat{F}}_1 + \dot{x}_e}{m_{11}} \tag{18}$$

To this end, it is a rule to find a control law to move toward the sliding manifold. Consider the Lyapunov function candidate

$$V_3 = V_2 + \frac{1}{2} m_{11} S_1^2 \tag{19}$$

Its derivative becomes

$$\begin{aligned} \dot{V}_3 = &-(k_1 x_e^2 + k_2 y_e^2)/E + \alpha_{ve} y_e + m_{11} u_e S_1 - c_1 m_{11} u_e^2 \\ &+ (u_e - \dot{\hat{F}}_1)(F_1 - \hat{F}_1) + S_1 [c_1 (F_1 + \tau_u) + T_1 + \dot{\hat{F}}_1 + \dot{x}_e] \end{aligned} \tag{20}$$

If we choose dynamical sliding mode control law $T_1$ as

$$T_1 = -c_1 (\hat{F}_1 + \tau_u) - \dot{\hat{F}}_1 - \dot{x}_e - m_{11} u_e - k_{s1} \operatorname{sgn}(S_1) - w_{s1} S_1 \tag{21}$$

where $k_{s1}$ and $w_{s1}$ are positive constants. Then, yields

$$\begin{aligned} \dot{V}_3 = &-(k_1 x_e^2 + k_2 y_e^2)/E + \alpha_{ve} y_e - c_1 m_{11} u_e^2 - k_{s1} |S_1| - w_{s1} S_1^2 \\ &+ (u_e - \dot{\hat{F}}_1 + c_1 S_1)(F_1 - \hat{F}_1) \end{aligned} \tag{22}$$

Designing the adaptive law of the uncertain term $F_1$ as follows:

$$\dot{\hat{F}}_1 = u_e + c_1 S_1 \tag{23}$$

Then the time derivative of $V_3$ yields

$$\dot{V}_3 = -(k_1 x_e^2 + k_2 y_e^2)/E - c_1 m_{11} u_e^2 - k_{s1} |S_1| - w_{s1} S_1^2 + \alpha_{ve} y_e \tag{24}$$

**Step 3:** In this step, the component $\alpha_{ve}$ will be considered to be stabilized. Differentiating $\alpha_{ve}$ with respect to time, the virtual velocity equation, using the new variables, yields

$$\begin{aligned} \dot{\alpha}_{ve} &= \dot{\alpha}_v - \dot{\alpha}_{vd} \\ &= \dot{v}_p \sin(\psi_e) + v_p \cos(\psi_e)(r - \dot{\psi}_d) + \dot{v} + k_2 (E^{-1} - y_e^2 E^{-3}) \dot{y}_e \\ &\quad - k_2 x_e y_e E^{-3} \dot{x}_e \\ &= \dot{v}_p \sin(\psi_e) + v_p \cos(\psi_e)(r - \dot{\psi}_d) + \frac{F_2}{m_{22}} + Q_1 \end{aligned} \tag{25}$$

where $F_2 = -m_{11} u r - d_{22} v + \tau_{w2}$, $Q_1 = k_2 (E^{-1} - y_e^2 E^{-3}) \dot{y}_e - k_2 x_e y_e E^{-3} \dot{x}_e$.

Before proceeding to the next step of the design, some manipulations on the virtual error dynamics equation can be performed. In order to make $\dot{V}_3$ negative, $r$ is considered as a virtual control, and its desired value $r_d$ is chosen as

$$r_d = \dot{\psi}_d + \frac{-\dot{v}_p \sin(\psi_e) - \hat{F}_2 / m_{22} - Q_1 - k_3 \alpha_{ve} - y_e / m_{22}}{v_p \cos(\psi_e)} \tag{26}$$

where $\hat{F}_2$ is an approximate value of the uncertain term$F_2$. And $k_3$ is a positive constant to be chosen later.

Considering that $r_d$ is not a true control, we have to introduce error variable $r_e$ as follows:

$$r_e = r - r_d \tag{27}$$

Then, the time derivative of $\alpha_{ve}$ yields

$$\dot{\alpha}_{ve} = r_e v_p \cos(\psi_e) + (F_2 - \hat{F}_2)/m_{22} - k_3 \alpha_{ve} - y_e / m_{22} \tag{28}$$

In a similar way, consider the following Lyapunov function:

$$V_4 = V_3 + \frac{1}{2} m_{22} \alpha_{ve}^2 + \frac{1}{2} (F_2 - \hat{F}_2)^2 \tag{29}$$

Its derivative becomes

$$\dot{V}_4 = -(k_1 x_e^2 + k_2 y_e^2)/E - c_1 m_{11} u_e^2 - k_3 m_{22} \alpha_{ve}^2 - k_{s1} |S_1|$$

$$-w_{s1}S_1^2+(\alpha_{ve}-\dot{\hat{F}}_2)(F_2-\hat{F}_2)+m_{22}\alpha_{ve}r_e v_p\ \cos(\psi_e) \tag{30}$$

Designing the adaptive law of the uncertain term $F_2$ as follows:

$$\dot{\hat{F}}_2=\alpha_{ve} \tag{31}$$

Then, the derivative of $V_4$ yields

$$\begin{aligned}\dot{V}_4 = &-(k_1x_e^2+k_2y_e^2)/E-c_1m_{11}u_e^2-k_{s1}|S_1|-w_{s1}S_1^2-k_3m_{22}\alpha_{ve}^2\\ &+m_{22}\alpha_{ve}r_e v_p\ \cos(\psi_e)\end{aligned} \tag{32}$$

**Step 4:** Considering the error $r_e$ as an auxiliary control, hence, in this step, we shall design the control laws to stabilize the component$r_e$. Differentiating $r_e$ with respect to time, the angular velocity equation, using the new variables, yields

$$\begin{aligned}\dot{r}_e &= \dot{r}-\dot{r}_d\\ &=\frac{(m_{11}-m_{22})uv-d_{33}r+\tau_{w3}-m_{33}\dot{r}_d+\tau_r}{m_{33}}\\ &=\frac{(F_3+\tau_r)}{m_{33}}\end{aligned} \tag{33}$$

where $F_3=(m_{11}-m_{22})uv-d_{33}r+\tau_{w3}-m_{33}\dot{r}_d$.

As a similar way, we consider the following Lyapunov function candidate:

$$V_5=V_4+\frac{1}{2}m_{33}r_e^2+\frac{1}{2}(F_3-\hat{F}_3)^2 \tag{34}$$

where $\hat{F}_3$is estimate value of the uncertain term $F_3$.

Then, choosing dynamical sliding mode function as follows:

$$\begin{aligned}S_2 &= c_2r_e+\dot{r}_e+\frac{m_{22}\alpha_{ve}v_p\ \cos(\psi_e)}{m_{33}}-\frac{(F_3-\hat{F}_3)}{m_{33}}\\ &=c_2r_e+\frac{(\hat{F}_3+\tau_r)+m_{22}\alpha_{ve}v_p\ \cos(\psi_e)}{m_{33}}\\ &=c_2r_e+\frac{(\hat{F}_3+\tau_r)}{m_{33}}+Q_2\end{aligned} \tag{35}$$

where $Q_2=(m_{22}\alpha_{ve}v_p\ \cos(\psi_e)/m_{33})$, and $c_2$ is a positive constant. Then

$$\dot{r}_e=S_2-c_2r_e-Q_2+\frac{(F_3-\hat{F}_3)}{m_{33}} \tag{36}$$

Differentiating $V_5$ with respect to time, and substituting (36) into $\dot{V}_5$, then its derivative becomes

$$\begin{aligned}\dot{V}_5 = &-(k_1x_e^2+k_2y_e^2)/E-c_1m_{11}u_e^2-k_{s1}|S_1|-w_{s1}S_1^2-k_3m_{22}\alpha_{ve}^2\\ &+m_{33}r_eS_2-c_2m_{33}r_e^2+(r_e-\dot{\hat{F}}_3)(F_3-\hat{F}_3)\end{aligned} \tag{37}$$

Differentiating (35) with respect to time, if we select $T_2=\dot{\tau}_r$, then $\dot{S}_2$ becomes

$$\dot{S}_2=\frac{c_2(F_3+\tau_r)+T_2+\dot{\hat{F}}_3}{m_{33}}+\dot{Q}_2 \tag{38}$$

where
$\dot{Q}_2=(m_{22}/m_{33})[\dot{\alpha}_{ve}v_p\ \cos(\psi_e)+\alpha_{ve}\dot{v}_p\ \cos(\psi_e)-\alpha_{ve}v_p\ \sin(r-\dot{\psi}_d)]$.

Consider the Lyapunov function candidate as follows:

$$V_6=V_5+\frac{1}{2}m_{33}S_2^2 \tag{39}$$

Its derivative becomes

$$\begin{aligned}\dot{V}_6 = &-(k_1x_e^2+k_2y_e^2)/E-c_1m_{11}u_e^2-k_{s1}|S_1|-w_{s1}S_1^2-k_3m_{22}\alpha_{ve}^2\\ &-c_2m_{33}r_e^2+m_{33}r_eS_2+(r_e-\dot{\hat{F}}_3)(F_3-\hat{F}_3)\\ &+m_{33}S_2(\frac{c_2(F_3+\tau_r)+T_2+\dot{\hat{F}}_3}{m_{33}}+\dot{Q}_2)\end{aligned} \tag{40}$$

If we choose dynamical sliding mode control law $T_2$ as

$$T_2=-c_2(\hat{F}_3+\tau_r)-\dot{\hat{F}}_3-m_{33}r_e-m_{33}\dot{Q}_2-k_{s2}\mathrm{sgn}(S_2)-w_{s2}S_2 \tag{41}$$

where $k_{s2}$ and $w_{s2}$ are positive constants. Then, substituting (41) into (40), the equation can be rewritten as

$$\begin{aligned}\dot{V}_6 = &-(k_1x_e^2+k_2y_e^2)/E-c_1m_{11}u_e^2-k_{s1}|S_1|-w_{s1}S_1^2-k_3m_{22}\alpha_{ve}^2\\ &-c_2m_{33}r_e^2-k_{s2}|S_2|-w_{s2}S_2^2+(r_e-\dot{\hat{F}}_3+c_2S_2)(F_3-\hat{F}_3)\end{aligned} \tag{42}$$

Designing the adaptive law of the uncertain term $F_3$ as follows:

$$\dot{\hat{F}}_3=r_e+c_2S_2 \tag{43}$$

Then, the time derivative of $V_6$ yields

$$\begin{aligned}\dot{V}_6 = &-(k_1x_e^2+k_2y_e^2)/E-k_3m_{22}\alpha_{ve}^2-c_1m_{11}u_e^2-c_2m_{33}r_e^2-k_{s1}|S_1|\\ &-w_{s1}S_1^2-k_{s2}|S_2|-w_{s2}S_2^2\le 0\end{aligned} \tag{44}$$

**Theorem 1.** *Consider an underactuated UUV moving in the horizontal plane with dynamics (1) and satisfying Assumption 1, adaptive laws (23), (31) and (43), under the action of control laws (21) and (41), then the tracking error* $\mathbf{z}=[x_e,y_e,u_e,\alpha_{ve},r_e,\tilde{F}_1,\tilde{F}_2,\tilde{F}_3]$ *converges*

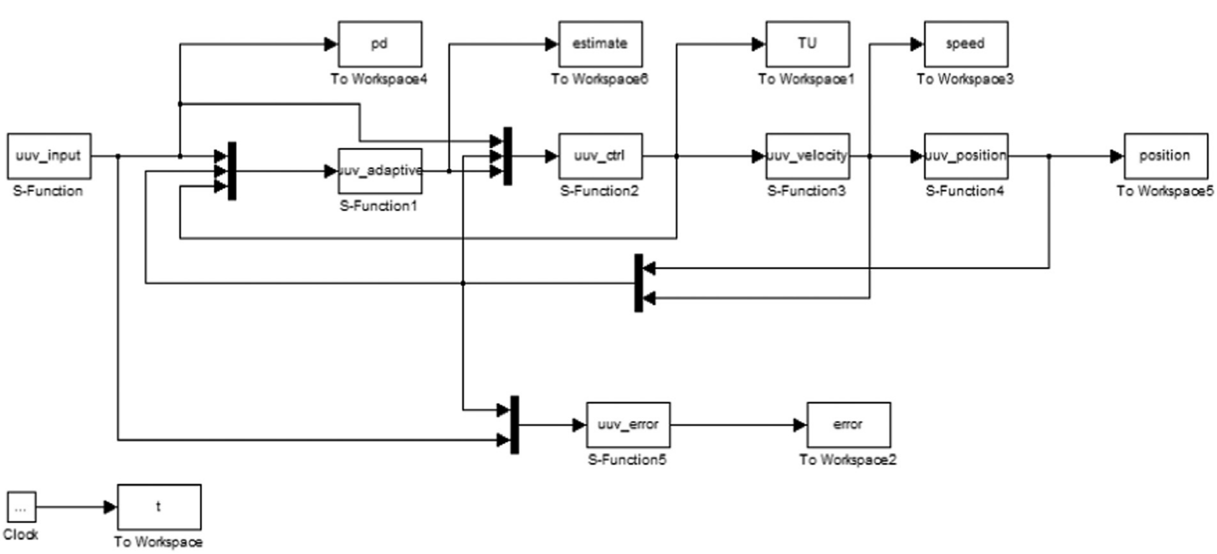


**Fig. 3.** The MATLAB/Simulink framework for the trajectory tracking controller.

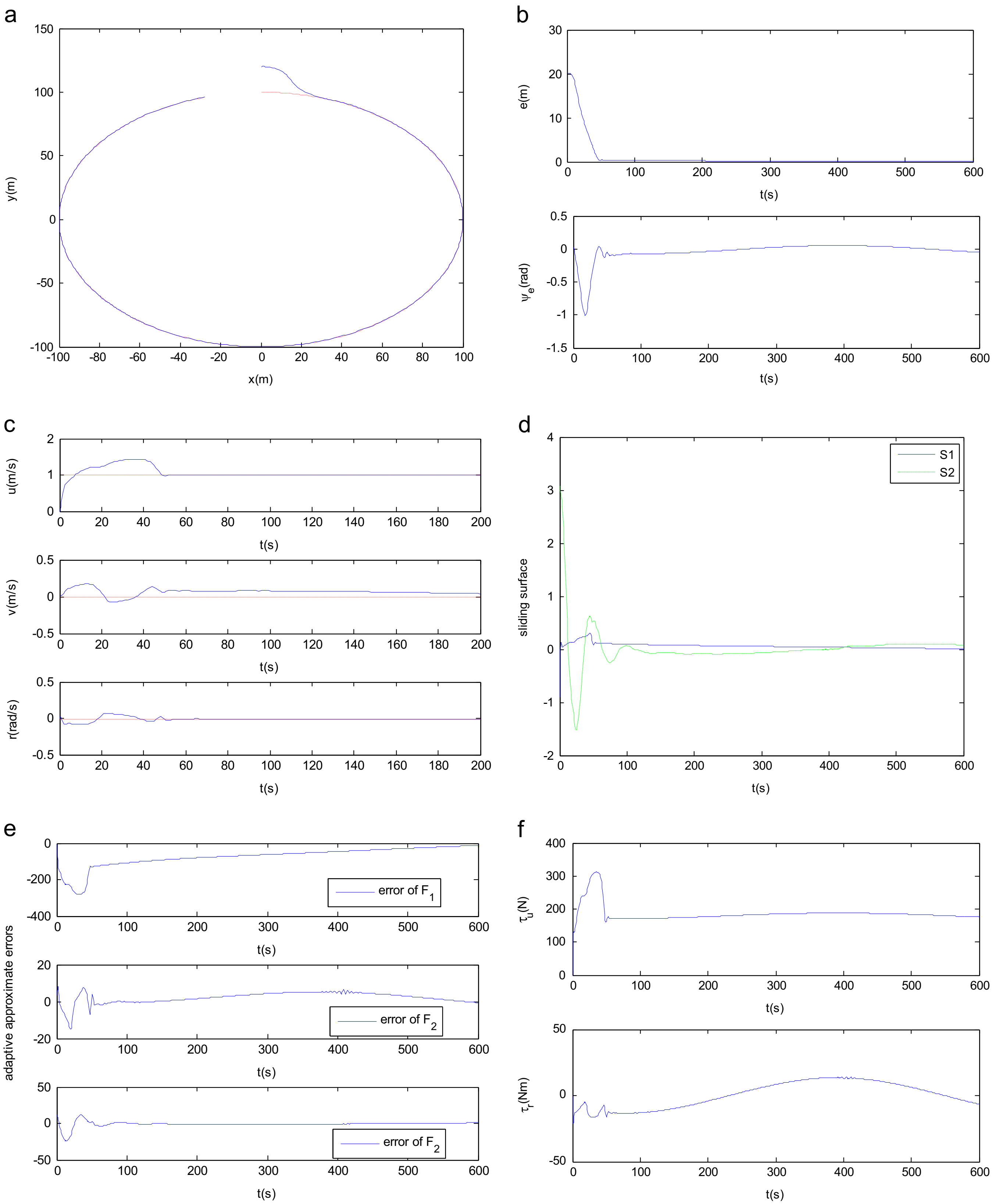


**Fig. 4.** Simulation results of circular tracking (red dotted line: desired reference; blue solid line: the actual result). (a) Circular trajectory. (b) Tracking errors of position and yaw angle. (c) Velocity-tracking results. (d) Sliding surfaces. (e) Adaptive approximate errors. (f) Control inputs. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

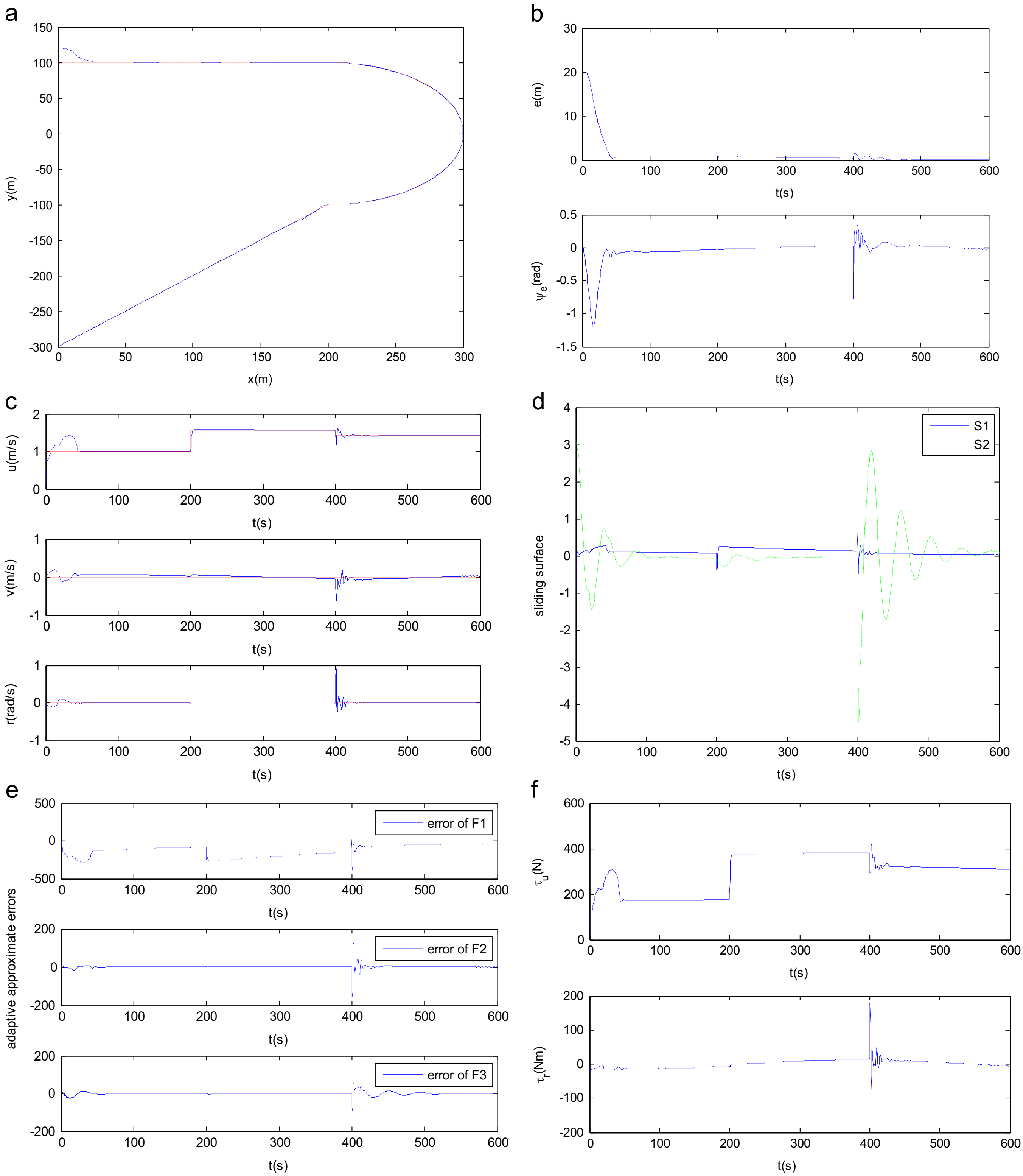


**Fig. 5.** Simulation results of Dubins paths tracking (red dotted line: desired reference; blue solid line: the actual result). (a) Dubins paths. (b) Tracking errors of position and yaw angle. (c) Velocity-tracking results. (d) Sliding surfaces. (e) Adaptive approximate errors. (f) Control inputs. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

*to zero, where $\tilde{F}_i = F_i - \hat{F}_i$, $i = 1, 2, 3$. Therefore, the final controlled system is globally uniformly asymptotically stable.*

**Proof** If we define $\boldsymbol{\rho} = [x_e, y_e, \sqrt{m_{11}}u_e, \sqrt{m_{22}}\alpha_{ve}, \sqrt{m_{33}}r_e, \tilde{F}_1, \tilde{F}_2, \tilde{F}_3, \sqrt{m_{11}}S_1, \sqrt{m_{22}}S_2]$, then, considering the Lyapunov function (39), one can obtain $2V_6 = ||\boldsymbol{\rho}||^2$. It is straightforward to prove that $\boldsymbol{\rho}$ converges to zero using a similar approach as that found in Repoulias (2007). Taking $\gamma = \min[\frac{k_1}{e_{\max}}, \frac{k_2}{e_{\max}}, c_1, k_3, c_2, 1, 1, 1, \frac{w_{s1}}{m_{11}}, \frac{w_{s2}}{m_{22}}]$, then, $\dot{V}_6 \le -2\gamma V_6$, which employing the comparison Lemma, yields$V_6(t) \le V_6(0)e^{-2\gamma t}$ for $t \in [0, +\infty)$. Doing the algebra, we conclude that $||\boldsymbol{\rho}(t)|| \le ||\boldsymbol{\rho}(0)||e^{-\gamma t}$, $t \in [0, +\infty)$. Hence the tracking error $\mathbf{z}$ converges to zero and the overall system is globally uniformly asymptotically stable.

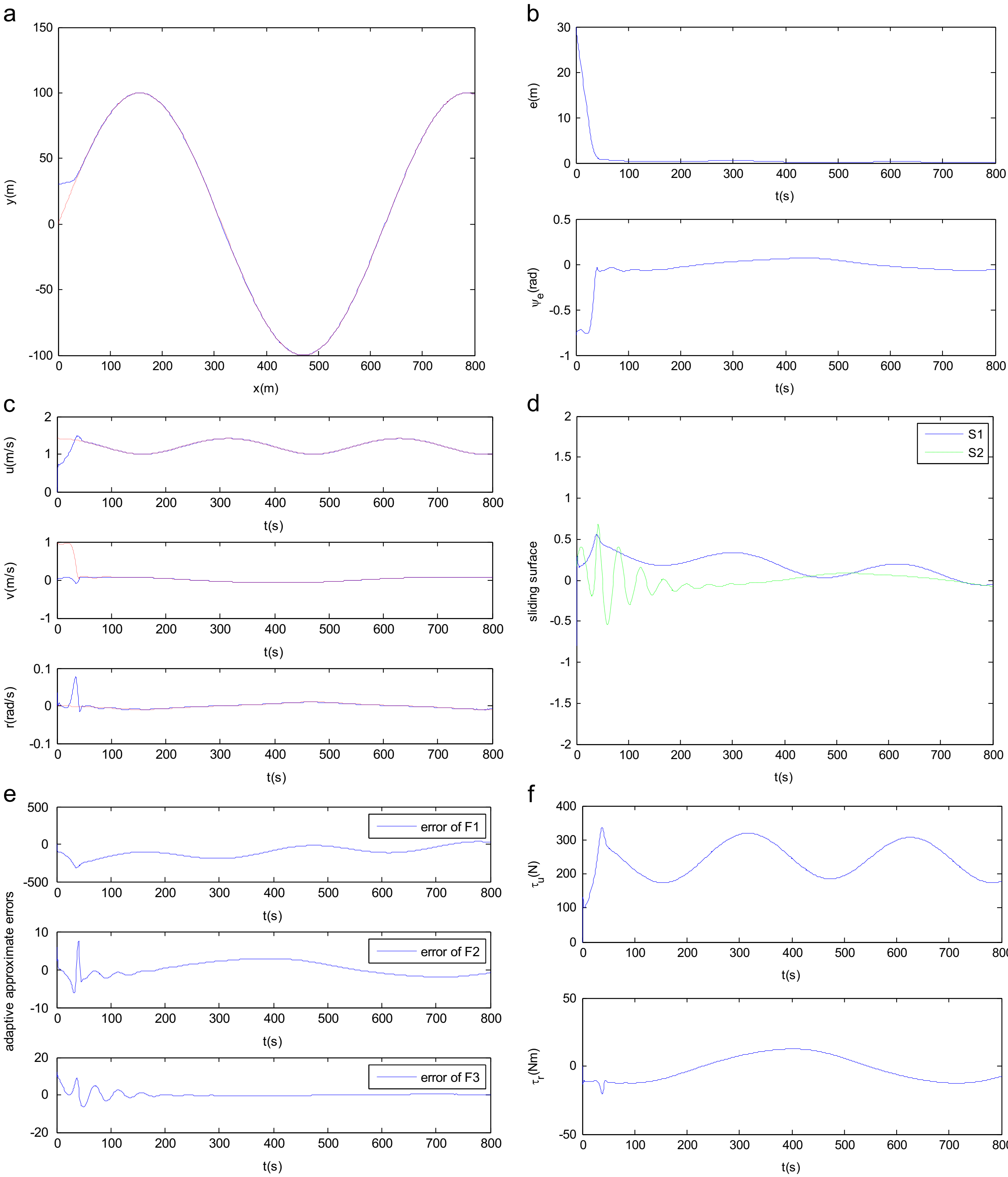


**Fig. 6.** Simulation results of sinusoidal tracking (red dotted line: desired reference; blue solid line: the actual result). (a) Sinusoidal trajectory. (b) Tracking errors of position and yaw angle. (c) Velocity-tracking results. (d) Sliding surfaces. (e) Adaptive approximate errors. (f) Control inputs. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

## 4. Simulation and discussion

In this section, three simulation examples are included to illustrate the effectiveness and efficiency of the proposed control scheme. The simulation studies are performed in MATLAB/Simulink, and the framework of the system is shown in Fig. 3. The parameters of the kinematics and dynamics (1) are $m=185$ kg, $I_z=50\ \text{kgm}^2$, $X_{\dot{u}}=-30$ kg, $Y_{\dot{v}}=-80$ kg, $N_{\dot{r}}=-30$ kg, $X_u=70$ kg/$s$, $X_{u|u|}=100$ kg/$m$, $Y_v=100$ kg/$s$, $Y_{v|v|}=200$ kg/$m$, $N_r=50\ \text{kg m}^2/\text{s}$, $N_{r|r|}=100\ \text{kg m}^2$ as in Pettersen (2009). Also, $m_{11}=m-X_{\dot{u}}$, $m_{22}=m-Y_{\dot{v}}$, $m_{33}=I_z-Z_{\dot{r}}$, $d_{11}=X_u+X_{u|u|}|u|$, $d_{22}=Y_v+Y_{v|v|}|v|$, $d_{33}=N_r+N_{r|r|}|r|$. Here, the desired reference trajectories of three scenarios are as follows: (1) a circular trajectory with constant velocity $x_d = 100\ \sin(0.01t)$, $y_d = 100\ \cos(0.01t)$; (2) so called Dubins paths as in Fossen et al. (2015) that are constructed by the interconnection of two straight lines with a circle arc; and (3) a sinusoidal trajectory with time-varying velocity $x_d = t$, $y_d = 100\ \sin(0.01t)$. Initial position and yaw angle tracking errors exist in all scenarios and initial velocities are $(u, v, r) = (0, 0, 0)$. To guarantee global uniform asymptotic stability of the overall system, the control gains are chosen as $k_1 = 1.7$, $k_2 = 1$, $k_3 = 0.1$, $c_1 = 0.8$, $c_2 = 1.3$, $k_{s1} = k_{s2} = 0.5$, $w_{s1} = w_{s2} = 1$, which can be easily obtained by using the MATLAB by trial and error. Without loss of generality, the time-varying disturbances are generated similar to that of Ghommam et al. (2010), which are given by

$$\begin{cases} \tau_{w1} = 10^{-1} m_{11} + \lambda(\sin(0.01t) - 1)(N) \\ \tau_{w2} = 10^{-1} m_{22} + \lambda(\sin(0.01t) - 1)(N) \\ \tau_{w3} = 10^{-1} m_{33} + \lambda(\sin(0.01t) - 1)(Nm) \end{cases} \tag{45}$$

where $\lambda = r$ and $(\cdot)$ is the Gaussian random noise with a magnitude of one and zero lower bound. In order to demonstrate the robustness of the proposed controller against to systematical parametric uncertainties and unknown disturbances, all simulations results are presented with aforementioned environmental disturbances and errors of the order of 10% on all system parameters. Specially, we assume that the system parameters will simultaneously increase 10% to the actual model. It will be shown below that even with these uncertainties, satisfactory performance of the underactuated UUV can be maintained. To illustrate the following simulation results clearer, we define $e = \sqrt{x_e^2 + y_e^2}$, where $e$ is the total position-tracking error that represents the planar distance between the actual vehicle and the desired.

Fig. 4 shows the simulation results for tracking a circular trajectory with constant velocity. From Fig. 4(a)–(c), it is clearly to see that all of the tracking errors are sufficiently small and the good performance can be maintained even in the presence of systematical parametric uncertainties and unknown environmental disturbances. It is noted that the proposed method in this paper does not directly design $\psi_e$ to make all of the tracking errors in sway direction converge to a neighborhood of the origin that can be made arbitrarily small as in Do et al. (2004a, 2004b). To simplify the expression of the controller, a virtual velocity variation is constructed to represent attitude errors, thus avoids differentiating the errors of orientation. In order to verify the robustness of the proposed controller, the novel dynamical sliding surfaces and adaptive approximate errors are presented in Fig. 4(d) and (e). Fig. 4(f) shows that the actual control inputs of the underactuated UUV under aforementioned conditions.

Fig. 5 shows the simulation results for Scenario 2. The trajectory shown in Fig. 5(a) is constructed by the interconnection of two straight lines with a circle arc, which is motivated by the results of so-called Dubins paths in Fossen et al. (2015). It is clearly to see that the proposed method can track not only straight lines but also curve trajectories. To present the time constraints of trajectory tracking, we can clearly see that three obviously different velocities are required in the tracking of Dubins paths from Fig.5(c). It is noted that the dynamic responses of the vehicle are sufficiently fast when yaw angle are suddenly changed due to the transition from circle arc to straight line at time 400 s. As noticed before, all of the tracking results sufficiently demonstrate the efficiency and effectiveness of the proposed method.

Fig. 6 shows the tracking results for Scenario 3. Different from the circular trajectory tracking aforementioned, here the controller has to deal with time-varying velocity that results in time-varying nonlinear dynamical couplings, see Eq. (1). Fig. 6(a) and (b) shows the desired and actual sinusoidal trajectory, including tracking errors, where very good tracking ability is observed in the case of large initial errors and abovementioned constraints. Then, the satisfactory performance of the underactuated system, when the speed of the desired trajectory changes continually, is duly analyzed in Fig. 6. The sinusoidal trajectory is generated with a time-varying linear velocity $u$ and a yaw velocity $r$. And from the sway velocity $v$, the tracking results do not present the disadvantage in previous work that imposes the yaw velocity to be nonzero. In order to obtain clear and readable results of tracking performance, some other important variables, such as the dynamic sliding surface and adaptive approximate errors are also displayed in Fig. 6(d) and (e), respectively. Values taken by the control actions so that the underwater vehicle can track the desired reference trajectory are shown in Fig. 6(f). In all of above simulation results, the validity and advantages of the proposed method are sufficiently demonstrated.

## 5. Conclusion

In this work, the planar trajectory tracking control problem for underactuated UUVs in the presence of possibly large systematical modeling uncertainty and unknown environmental disturbances is addressed. And a methodology of the combination of backstepping technique and adaptive dynamical sliding mode control method is proposed to resolve the environmental disturbances and systematical parametric uncertainties. Global uniform asymptotic stability of the overall system is demonstrated based on Lyapunov stability theory. From the simulation results, the proposed controller can remain satisfactory performance of an underactuated UUV inspite of the cases of a circular trajectory tracking with constant surge velocity, Dubins paths or even the sinusoidal trajectory tracking with time-varying velocity. In comparison with the previous published control laws (Yu et al., 2012), a novel dynamical sliding surface is designed in terms of velocity tracking errors, position tracking errors and approximate errors as a new form to guarantee the convergence of all trajectory tracking errors. Different from the control design in Do et al. (2004a, 2004b), the method proposed here adopts virtual velocity dynamics to represent attitude errors, thus avoids differentiating the errors of orientation such that it simplifies the representation of the control laws with the aid of backstepping technique. Further work can extend the results to the case of three-dimensional trajectory tracking control for underactuated UUVs with non-diagonal inertia and damping matrices, high-order damping terms. In addition, the developed methodology for the controller design can be also applied for other types of underactuated systems with nonholonomic velocity constraints.

## Acknowledgment

This work was supported by the National Natural Science Foundation of China under Grant 51179038, and partly supported by the Fundamental Research Funds for the Central Universities under Grant HEUCFX041401 and Harbin Engineering University Science and Technology on Underwater Vehicle Laboratory under Grant 9140C270208140C27004.

## Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.oceaneng.2015.06.022.

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