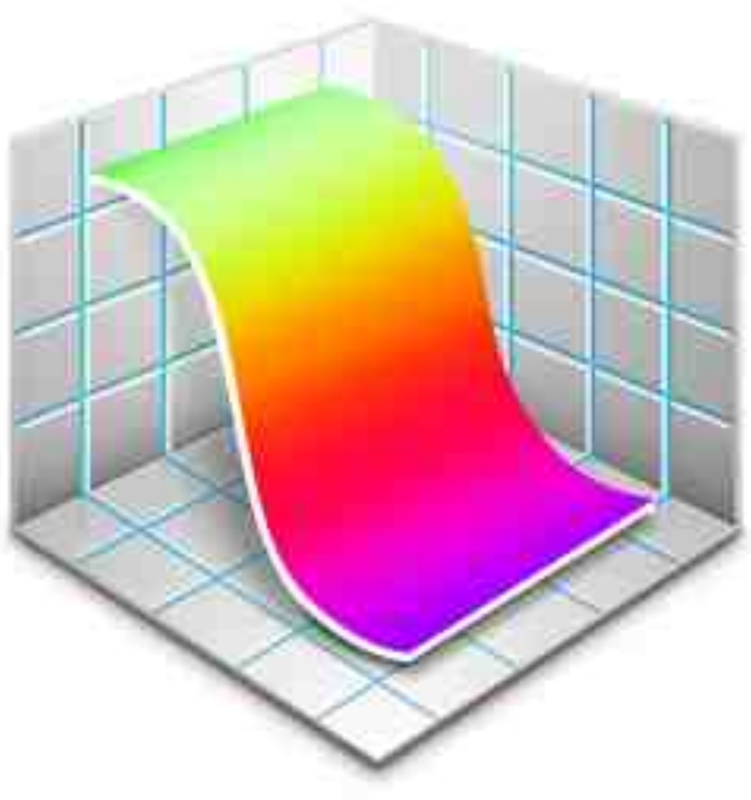
Instructions for Use



Grapher

**Versions 1.1 2.0 2.1
2.2 2.3 2.5**

(and old Curvus Pro X 1.3.2)

(including a list of bugs and their workarounds)

“ When everything else fails, read the instructions “

April 16th 2014 edition

Contents

Numerical calculations (evaluations)

Layout, appearance, saving and exporting the document

Grapher’s bugs

Appendices

Despite its bugs (with their workarounds), Grapher is an outstanding software that should please you. I wish you as much fun as I’ve experienced using it.

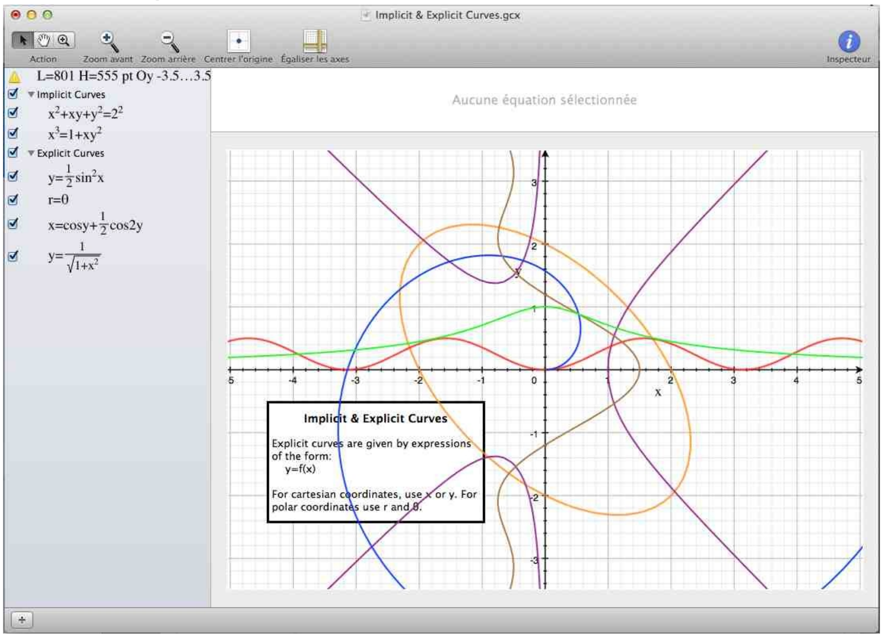
Yves Barois

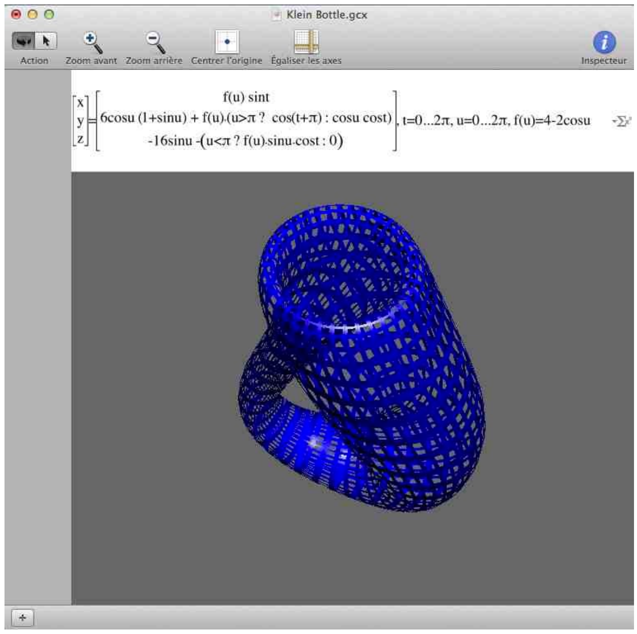
This manual (French and English versions), English documentation on Internet, various works built with Grapher, are available on :

<http://y.barois.free.fr/grapher/>

Overview of Grapher

Summary.





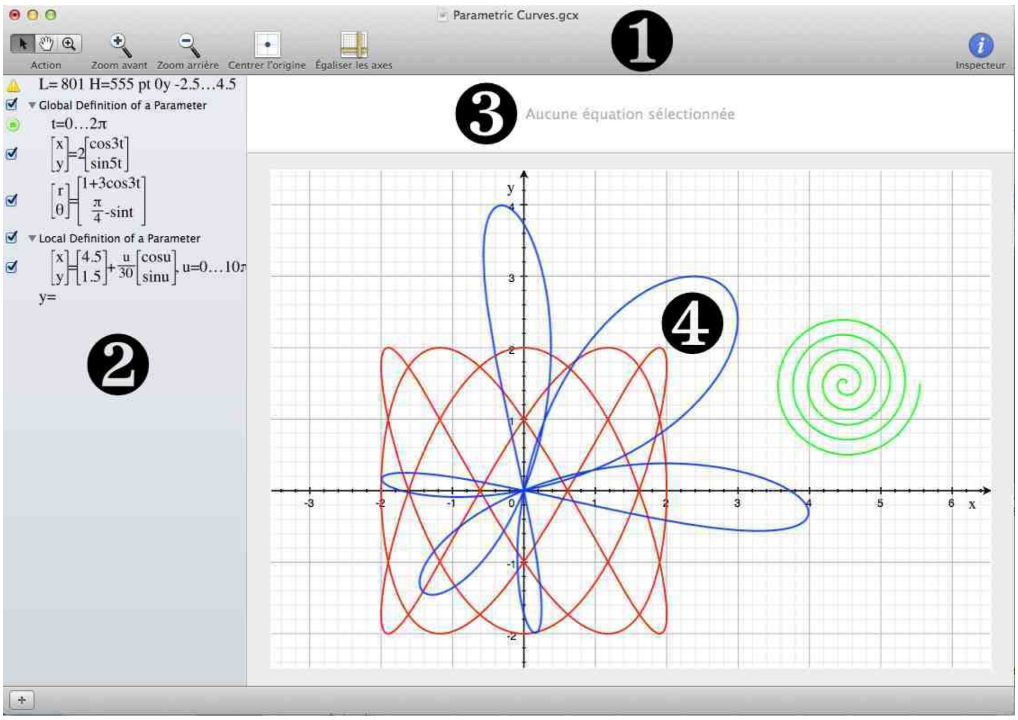
Grapher is software specifically designed for Mac OS X to show various graphic representations of equations. It can graph the following mathematical expressions :

- Explicit or implicit equations
- Parametric curves and surfaces
- Solutions of differential equations
- Discrete sequences
- Scalar and vector fields
- Inequalities and Boolean equations
- Sets of points

Grapher presents a convivial interface which allows to quickly create professional-quality graphs. Moreover, it provides additional tools such as :

- Evaluation and integration
- Animation
- Regression curves
- etc.

Intuitive user interface.



Toolbar : ❶.

Use the toolbar to select the action of the cursor, to change the limits of the frame, to show the inspectors, etc..

To customize the toolbar : **menu View > Customize Toolbar...**

List of Equations : ❷.

It shows the list of all mathematical expressions and texts entered by the equation editor, and groups and point sets.

To add a new line : **menu Equation > New...** or the ad hoc keyboard shortcuts or better, the **button [+]** at the bottom of this list.

The order of these expressions can be altered arbitrarily by simply **dragging and dropping.**

Equation editor : ❸.

Enter or change here the expression of the selected line in the list of equations ❷.

Graph : ❹ (the main part of a Grapher document).

View of the current document (graphs, margins, added objects) as it will be exported, printed or copied as TIFF ...

To insert an object : **menu Object > Insert...** You can bring up the text insertion point in the rectangles and ovals and in the box created by **Insert text.**

To insert an equation : select it in ❷ then in ❸ and transfer it by **dragging and dropping.**

Same process to display the parameter of an animation on the graph.

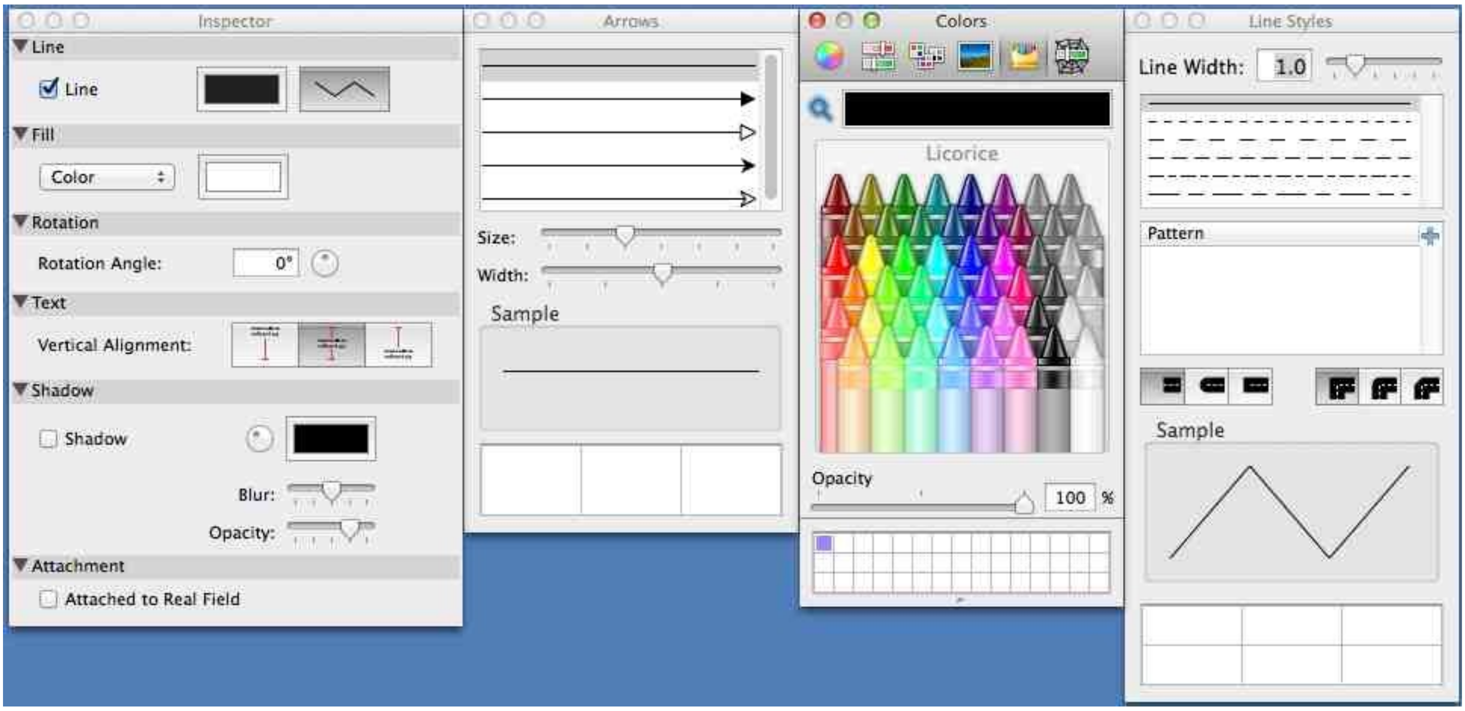
To set the graph type, its size, margins, axes, etc use : **menu File > Page Setup…** or **Print…**, **menu Format** and **menu View > all items.**

Inspectors.

To change the various properties of an element of the graph (axis, frame, curve, added object, etc.) such as :

- color
- line width
- rotation of a text object
- etc.

select it (**click the element or via menu Format > Axes & Grids…**) and use the Inspector window such as the following with the secondary windows it may use :



To display the inspector window : **button on the toolbar**, or **menu Window > Show Inspector**.

Intuitive syntax.

General.

Just enter any equation such as you might find it in books, and Grapher will graph it for you !

There is a first look at possible mathematical expressions in this chapter ; a more complete study is the subject of the chapter “Expressions“.

A few examples of expressions :

$y = \sin x$ graph defined by an explicit equation,
$x^2 + y^2 = 2^2$ implicit equation,
$r = \theta$ polar coordinates.

Operators, special characters : **menu Window > Show Equation Palette** or ⇧⌘E or **button ▾Σx²** in ❸.

Definitions of constants and functions.

To define a constant or a function, simply create the definition with the corresponding expression.

Examples : $k = 2$ definition of a constant
$f(t) = 1 + t^2$ definition of a function

These definitions can also be added after the expression of a graph equation, separated by commas :

$$y = \sin kx,\ k = 2$$

Note that in all cases the " = " sign may be replaced by " := ".

Piecewise-defined functions.

They can be defined in several ways. The simplest : (⇧⌘E) **> Equation Palette > Operators > use the button :**

$\begin{cases} \square & x<0 \\ \square & x=0 \\ \square & x>0 \end{cases}$ to enter such expressions as : $y = \begin{cases} \sqrt{1+x^2} & x<0 \\ \cos^2 x & x>0 \end{cases}$ ou $y = \begin{cases} x<-1 & -2x \\ x<1 & 1+x^2 \\ x>1 & 3-x^2 \end{cases}$

There is also a shortened syntax : $y = x > 1\ ?\ f(x)$ or $y = x > 1 : f(x)$ to define y for $x > 1$

Note that y is not defined for all x. To add an alternative condition, add a term separated by " : ", example :

$$y = x > 0\ ?\ x : -x$$

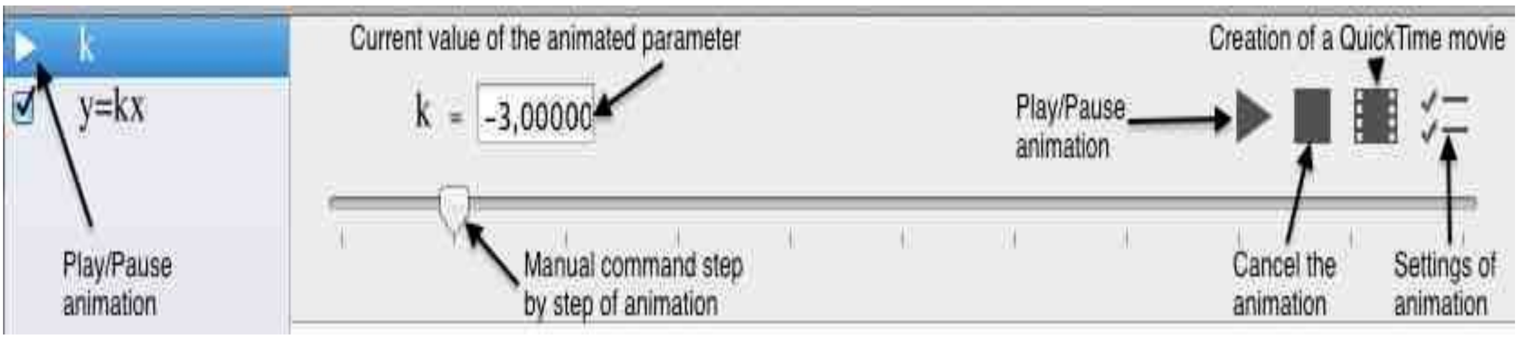
Multi-valued expressions and parameters.

You can enter a multi-valued parameter or function using the braces " { } " :
$k = \{1, 2, 3\}$; $y = \{\cos x, \sin x, \tan x\}$; $y = x^2 + kx,\ k = \{1, -1, 5, -5\}$; $y = \{-1, 1\} + e^{\{x, -x\}}$

You may use the " ... " symbol to define a domain of values :
$k = \{1…5\}$ *k* varies from 1 to 5 by increments of 1 ; $k = \{1, 1{,}2…2\}$ *k* varies from 1 to 2 by increments of 0,2.

Animations.

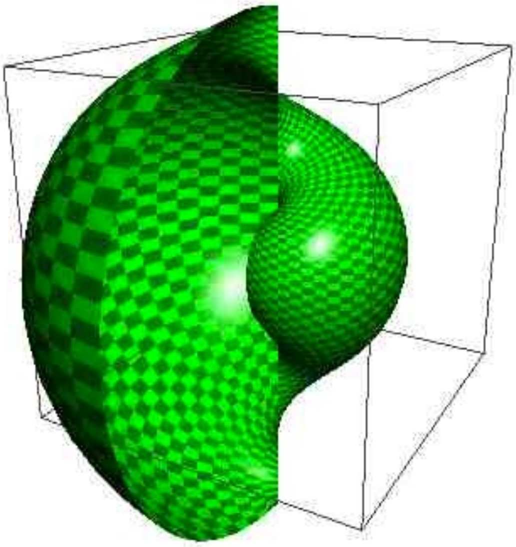
Parameter animation.
To animate the value of a parameter (defined by an expression such as " k = 1 "), select it in the list of equations and order **menu Equation > Animate Parameter**. Then appears under the toolbar :



QuickTime animation.
Check the equation chosen, do not select, then **menu Equation > Create animation… > settings > Create Animation > Save :** to create a .mov file that opens in QuickTime.

3D view.

With Grapher you can also draw 3D graphs : **menu File > New > New Graph > 3D Graph** or **menu View > Switch to 3D View** or **⌥⌘3** on the keyboard. Example :
Enter r = theta + phi that changes automatically into $r = \theta + \Phi$ to draw the following graph :



To change the viewing angle of the graph, click on the 3D view and move the mouse with one of the following keys pressed :

— none : Turn (2 rotation axes in the display plane) ;

— ⌥ (Option) : Turn (rotation axis perpendicular to the display) ;

— ⇧ (Shift) : Zoom in / Zoom out ;

— ⌘ (Command) : Move.

Export your creations.

Copy and paste.

To copy the image of the graph, choose **menu Edit > Copy as… > select TIFF or PDF or EPS** ; you may then paste it in the working window of another application.

Exporting.

To export the image of the graph, **menu File > Export > select PDF or EPS or TIFF or JPEG,** the **resolution** in the latter two cases and the **rate of compression** JPEG.

Contextual menus.

Right click or ctrl + left click on an object may open a contextual menu. For example, on an equation, it is proposed :
— Copy as TIFF or PDF or EPS or Text,
— Copy LaTeX expression.

Initiation

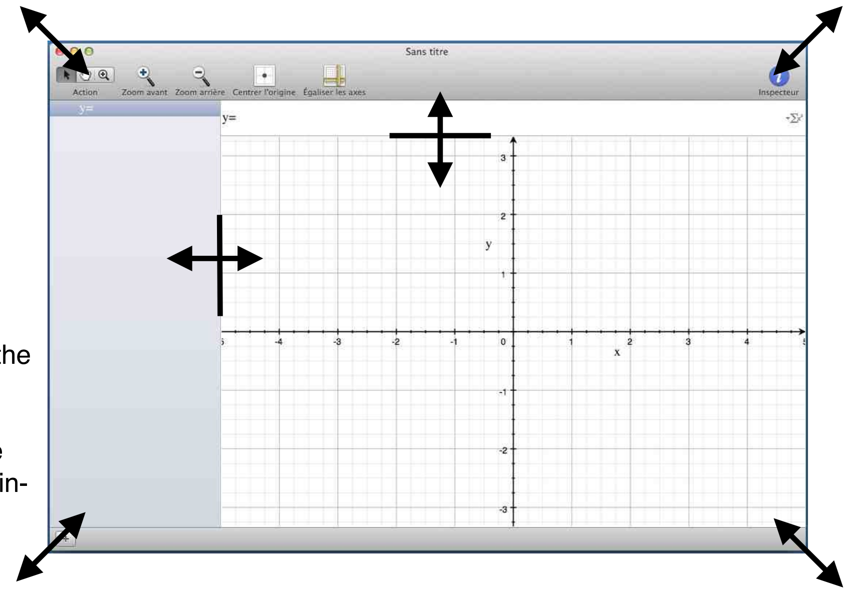
Lesson 1 : Creating a document.

In this lesson you will learn to create a new document in 2D and how to represent a series of Bessel functions. You will also learn how to include a caption and how to change the attributes of different objects.

Opening the Grapher working window.

— Method 1 : Open **Grapher > secondary window New Graph > 2D Graph > select the model > Choose** ;
— Method 2 : Grapher already opened > **menu File > New… > secondary window New Graph > 2D Graph > select the model > Choose** ; or keyboard shortcut **⌘N > secondary window New Graph > etc.** ;
— Method 3 : Grapher already opened > **menu File > Open… or Open Recent > choose etc.** ;
— Method 4 : **Double click the icon of a Grapher GCX (.gcx) file** ;
— Method 5 : **menu Examples > click a 2D example**.

We choose the method 1 here with the “default“ model



Layout of the working window.

Properly place the working window on the display.

To modify the window approximately:
— Green button top left: fullscreen / normal size;
—Control handles on the four corners of the window change only on the size of the graph;
— Internal control handle high: changes the height of the equation editor keeping constant the total height of the window;
— Internal control handle left: changes the width of the list of equations keeping constant the total width of the window.

See descriptions and workarounds of bugs nr. 1 & 2.

Precisely setting the size and margins of the graph.

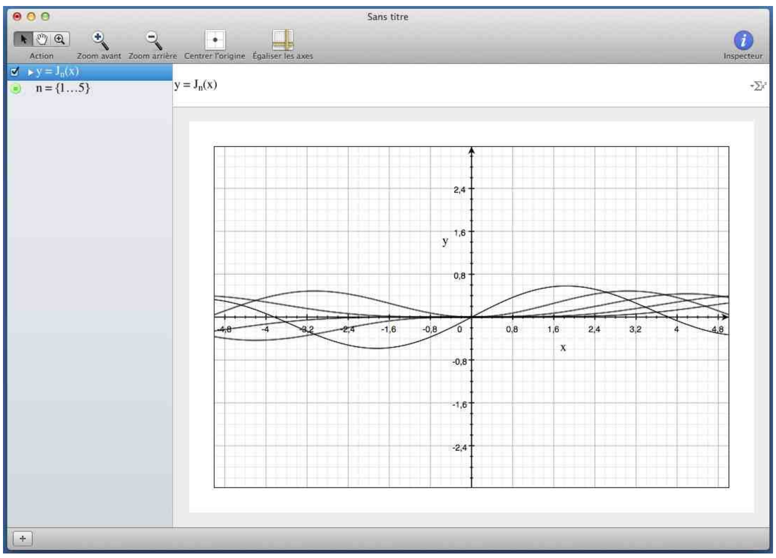
— Method 1 : **Menu Format > Layout > Size > Custom > enter length and height** with the available length units ; **enter the graph margins** which are in addition to those of the printer ;
— Method 2 **.** Maximum size on an A4 format page (to be tested with your equipment) :
- **Menu Format > Layout > Size > Custom > enter L=801 (555) pt and H= 555 (801) pt (unit : point);**
- **Menu Format > Layout > Margins > enter the margins of the graph** which are in addition to those of the printer ;
- **Menu File > Page Setup or Print… > A4, Landscape (or Portrait), 100 %.**

Note length and height of the graph : **See description and remedy of bug nr. 3.**

Here we choose here the maximum size available with Grapher 2.5 on A4 page (method 2), « 2D Graph > Margins », default values of margins.
To print : A4, Landscape, scale 100 %.

We note : L = 801 pt
H = 555 pt

Nearby : the working window of Grapher 2.5 with above settings after entering the two equations.



Entering the equations.

To plot the Bessel functions, two equations must be created : the equation of these functions and the one giving the index values.

— **Click the button [+] > menu** at the bottom of the equation list and enter this : **$y = J_n(x)$**
and to get that, type on the keyboard successively : ⇧J together, ⇧— together, n, right arrow key, left parenthesis, letter x, “y=“ should be already displayed.

Press the Return key, the equation is diplayed in the list with a “Danger“ warning to note a syntax error because the variable n of the index is not yet defined. A click on the“ Danger“ triangle open a pop-up window “Syntax Error“ : click OK and ignore.

— For the index values, call a new equation by one of these procedures :
- **Click the button [+] > menu New Equation, at the bottom of the equation list,**
- **Menu Equation > New equation,**
- Type the **keyboard shortcut ⌥⌘N,**

delete “y=“ from the equation editor, and enter one of the following formulas to define the index n (integers from 0 to 5) :

n = {0…5} or n = {0, …, 5} or n = {0, …5} or n = {0, … 5} or n = {0, 1, …5} or n = {0, 1, 2, 3, 4, 5}

Return key, the first equation does not shows anymore syntax error and the six curves are displayed on the graph.

To cancel an equation from the list **(if not used elsewhere)**, select it by clicking it, then Backspace key.

The tools : Coarse settings ot the frame limits (minima and maxima of abscissae and ordinates).

The tools are accessible by the **Toolbar**, the **menu View**, and keyboard shortcuts for five of them.

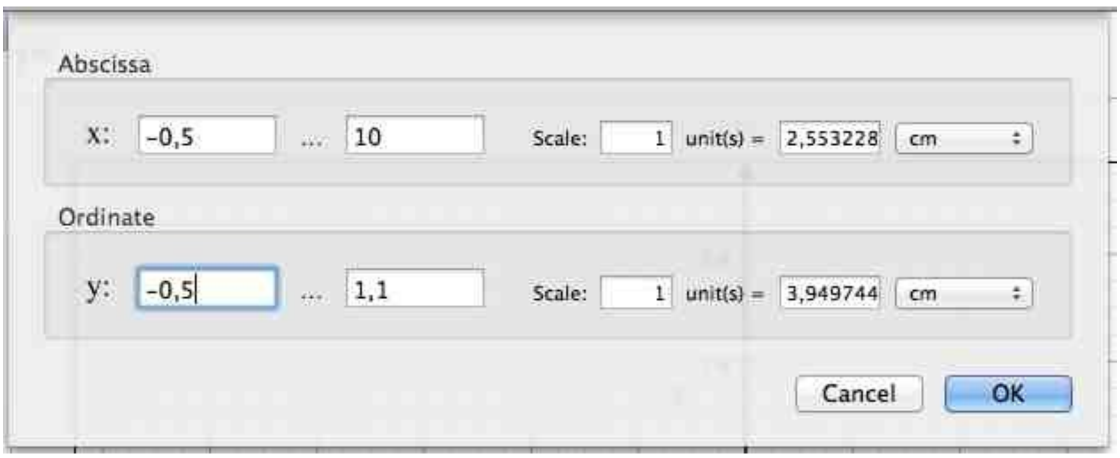
From left to right : — **Arrow or ⌘1 : selection tool ;**
— **Hand or ⌘2 : move tool** to move the graph by dragging ;
— **Loupe ⊕ or ⌘3 : reduction / enlargement tool.** Select it, place it on the graph at the point chosen to remain fixed in the graph field ; 1 click, scale multiplied by 2 ; 1 click with ⌥, scale divided by 2 ;

— **Button Zoom in (Loupe Zoom in) : 1 click or ⌘+**, graph scale multiplied by 2 ;
— **Button Zoom out (Loupe Zoom out) : 1 click or ⌘-,** graph scale divided by 2 ;
— **Button Center Origin : 1 click** brings back the origine of axes at the centre of the graph ;
— **Button Equalize Axes : 1 click** make the ordinate scale equal to the abscissa’s one,
1 click with ⌥ make the abscissa scale equal to the ordinate’s one.

Try these commands with the graph in progress in this lesson.

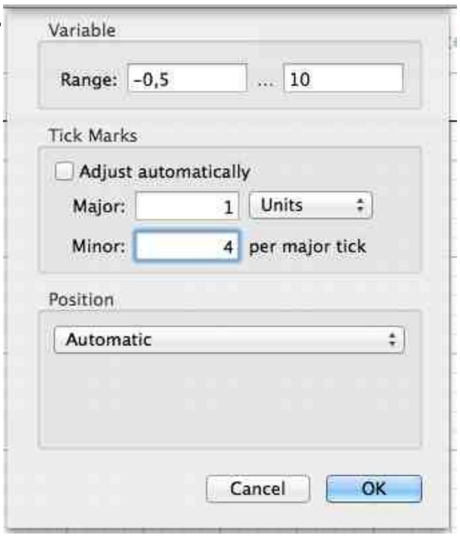
Fine adjustment of frame limits.

— Method 1 : **Menu View > Frame Limits... > enter x** from -0.5 to 10, **y** from -0.5 to 1.1 then OK in the pop-up window of the nearby figure.



— Method 2 : **Menu Format > Axes & Grids… > select Abscissa Axis > Edit… > enter the minimum and maximum values** you want for the abscissa scale,

> adjust the graduation,
> choose the axis position > OK.



Use the same procedure for the ordinate axis.

Note the minimum et maximum values of the ordinates : **see description and workaround of bugs nr. 1 et 2.**

here we note : Oy : -0.5…1.1

Changing appearance of curves.

With the Inspector :
To show or hide the Inspector : **Blue Button Inspector** of the toolbar, or **menu Window > Show Inspectors.**
Select one or several curves either on the graph or in the list of equations, and adjust in the Inspector window the thickness and color of the line, the resolution (determines the size of the saved file).

With the **menu Format > Recolor Selected Curves... :**
Select all the curves by clicking on the equation (it can be done on the graph by clicking each curve while pressing the Capitals (Shift) key > choose a built-in gradient > OK.
This method gives each curve a different color very quickly.

Adding objects to the graph.

We are going to place on the graph a legend : a short text inside a frame. To do this :

Menu Object > Insert Rectangle > place it in the chosen position **with the mouse > double-click in the rectangle to edit the text.**
To change the font or layout of the text : **menu Format > Font etc., Text etc.** and keyboard shortcut **⌘T** .
Select the rectangle > blue button Inspector > add Shadow and other settings.

More information.

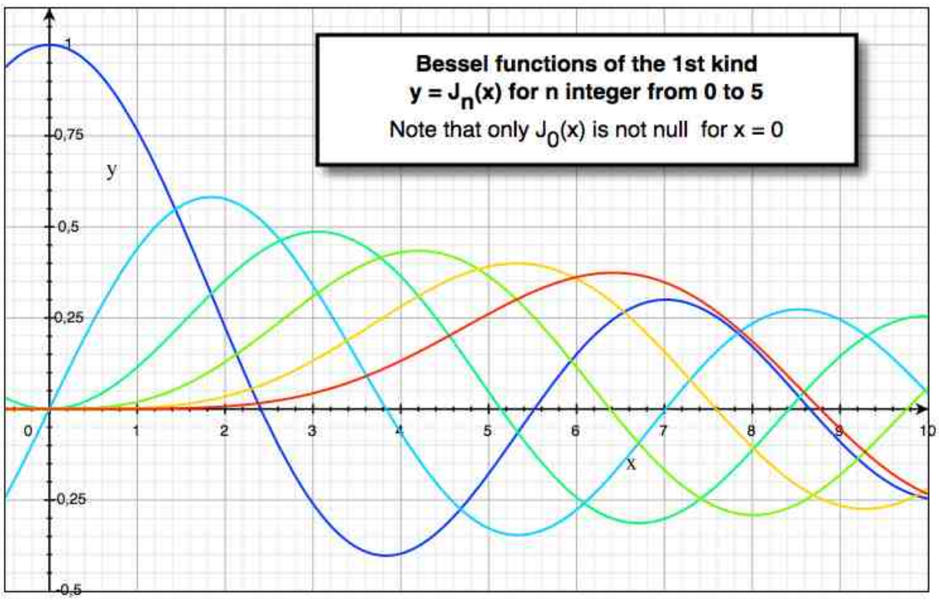
Add in the list of equations (see bugs nr. 1 & 2) this text :

“L=801 H= 555 pt Oy -0.5…1,1“

this will allow to restore the document saved as GCX (.gcx) file in its original configuration after reopening it.

Save your document :

menu > File > Save ass… > title, etc.



Congratulations ! You have just created your first document with Grapher.

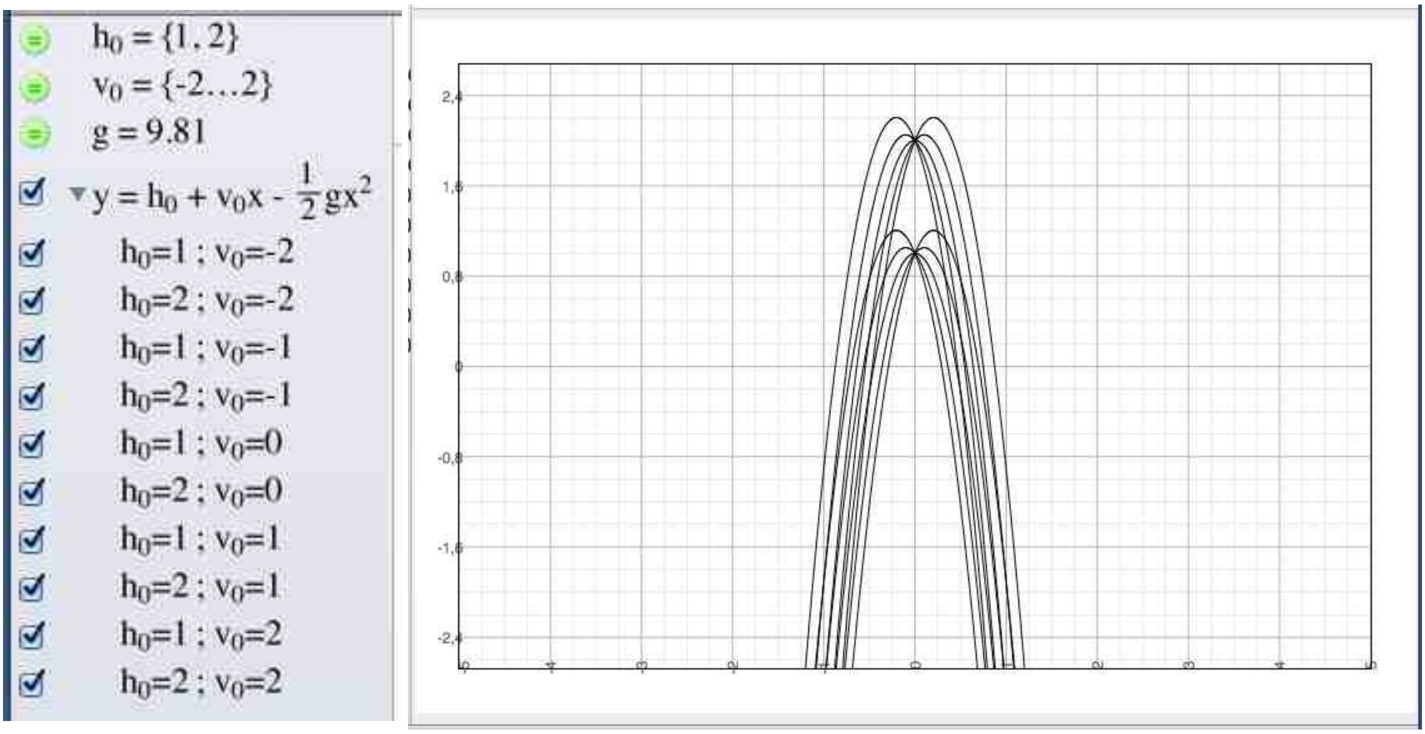
Lesson 2 : Customizing the appearance of a document.

In this lesson you will learn how to change various settings for the presentation of a document.

Consider the following problem of kinematics : at time t = 0, we throw an object from the altitude h_0 , with the initial vertical velocity v_0 ; we want to draw a graph showing the altitude of the object (ordinate) versus time t (abscissa).
t, time in s ≥ 0 ; h_0, initial altitude = 1 then 2 m ; h, altitude at time t ; v_0 , initial celerity + upward, - downward, values -2, -1, 0, 1, 2 m/s. The equation of the graph is :
$h = h_0 + v_0t - 1/2gt^2$ with $g = 9.81\ m/s^2$ Grapher allows only x, y as Cartesian coordinate names, hence the expressions to be entered :

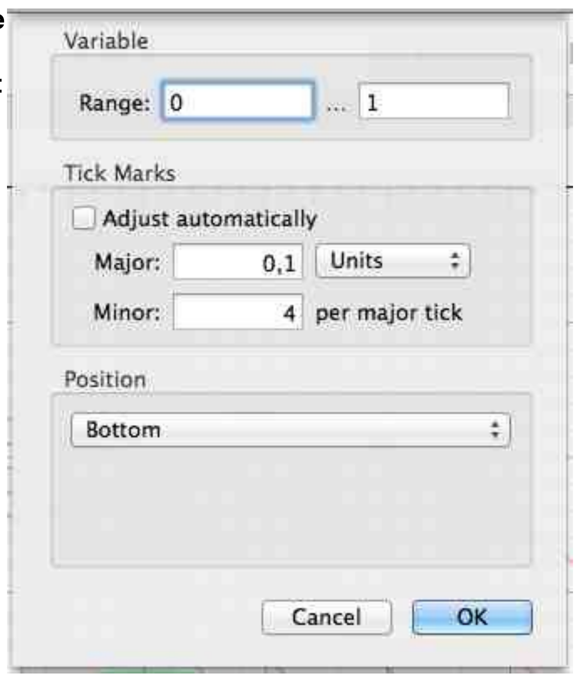
$h_0 = \{1, 2\}$
$v_0 = \{-2...2\}$
$g = 9.81$
$y = h_0 + v_0\ x - 1/2\ g\ x^2$

Open a new 2D document model “Classic“ ; set to the maximum size in A4 format Landscape ; note the dimensions, here “L = 801 H = 555 pt“ ; enter the four equations ; the result is :



Setting axes and frame limits.

Menu Format > Axes & Grids… > select Abscissa Axis > Edit… > enter the minimum and maximum values wanted for the abscissae, adjust the graduation, choose the position of the axis, as you can see in the figure nearby :



An other way is to use the menu View > Frame Limits..., but it does not show all the necessary settings.

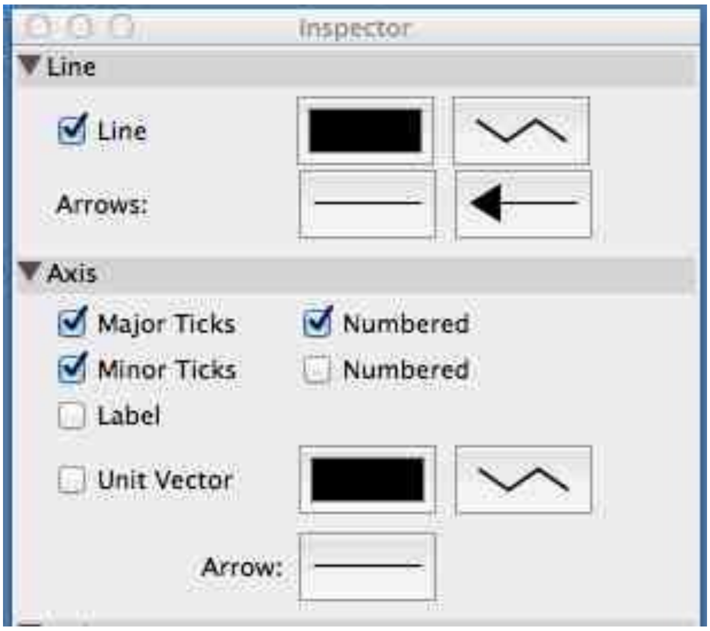
Apply the same procedure for the y-axis and note the maximum and minimum values of Oy, “Oy 0… 3“ **(see bugs nr. 1 and 2)**.

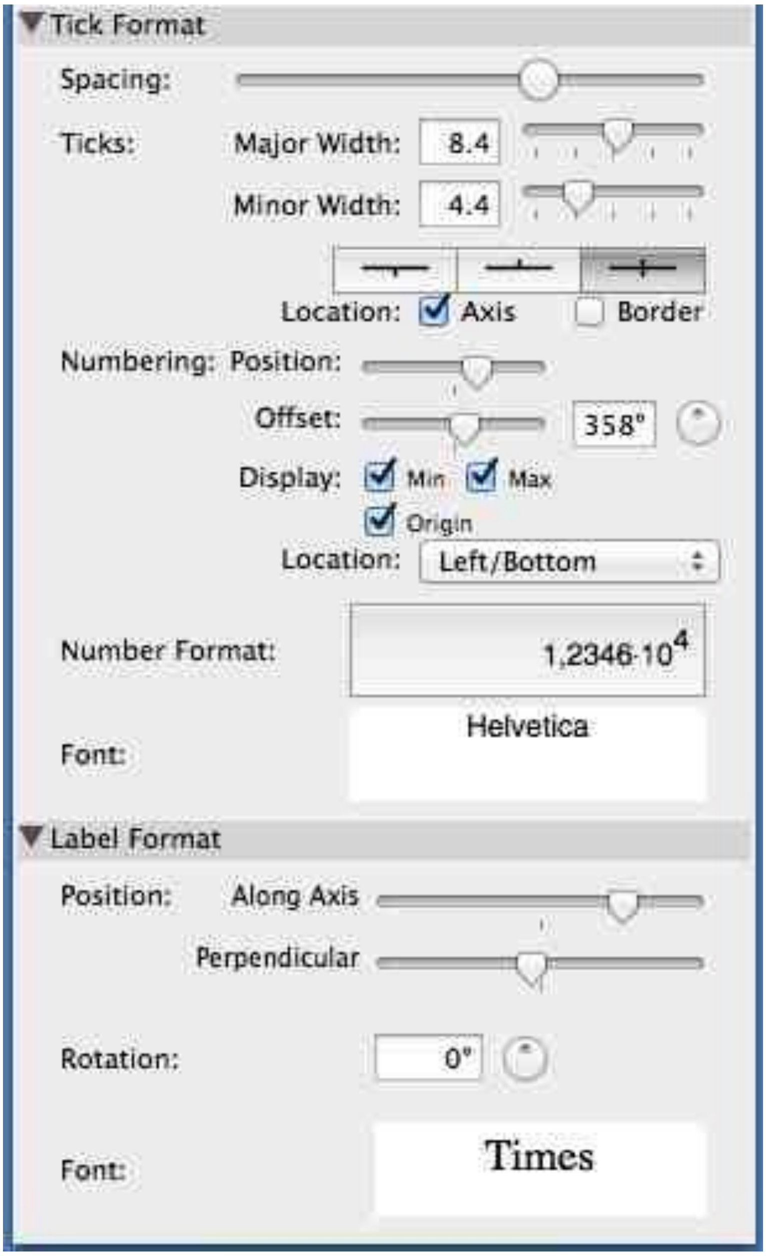
One click on an axis selects it, ready to call its Inspector.
A double-click selects it and opens a pop-up window to set its scale and position (figure nearby).

Returning to the axis Ox, after clicking OK in the window shown nearby, this axis is selected, allowing its inspector to display to choose the next options.

Customizing the axes.

Take advantage of the selection of the axis above and display its **Inspector** by clicking the **blue button of the toolbar**. Try the many options visible in the following screenshots and finish up with those shown :





Color, line width, arrows and their size, label (here x),etc.

There are many possibilities, test them, use your skills as a graph decorator !

One click on an axis will select it, ready to display its Inspector.
Two clicks will select it and open the pop-up window to set its scale and its position.

Customizing curves.

Select all equations of the curves $h_0 = 2$ while **pressing the ⌘ key** ; display the Inspector ; choose a line width of 1.9 and color “Apple Red“ ; set the resolution to the minimum compatible with a proper display and confirm by clicking the refresh button (circular arrow) (to reduce the size of .gcx files).

You should get the figure nearby :

Do the same with the curves $h_0 = 1$ but choose the color “Apple Green“.

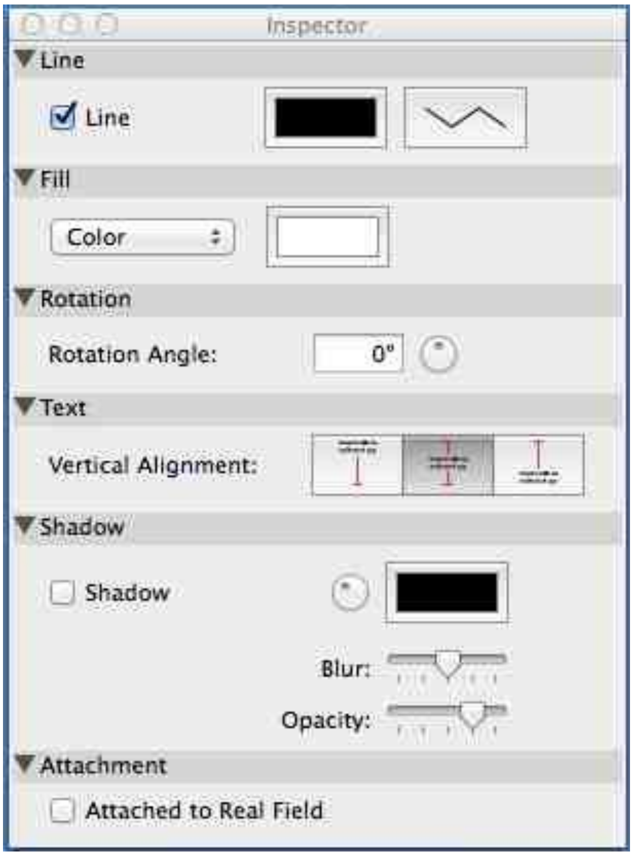
Note : you can **directly select a single curve on the plot by clicking it** and displaying the Inspector to customize it, or **several curves by clicking each one, Capitals (Shift) key pressed.**

Customizing the frame.

Select it by clicking the graph frame, display its Inspector and set the line thickness to 1.0.

Adding objects to the graph.

The menu Object > Insert Arrow, Oval, Rectangle, Text, allows you to place these objects on the graph, three of them can receive text.
Once **selected with a click,** and after **displaying its Inspector**, the various settings of the object appear.
We show here the Inspector of the object “Rectangle“ : drawing the frame or not, line width, inner and frame colors, orientation of the object, simulated shadow and its options. Attached File options :
— Attached to the real field unchecked : the object remains fixed in the graph field if we change scale and place of the axes ;
— Attached to the real field checked : the center of the object keeps constant its abscissa and ordinate.



The **selected object is deformable** by **dragging its handles** with the mouse.
If several objects added to the graph are selected, they can be **grouped, ungrouped, aligned in various ways (menu Object).**
The superposition of several objects or object-graph has settings **(the first four instructions of menu Object).**

We want to place captions near the axes “ t (s) time “ et “ h (m) altitude “ (we already unchecked “label“ (x or y) in the axis Inspectors). Do it according to the model shown at the end of this lesson.

Adding equations to the graph.

Is done by **dragging and dropping the equation selected from the equation editor onto the graph.**
These new objects are able to receive commands from the menu Object and from the inspector, and have handles to change their forms.

Add on the graph the equations of the problem defining h, h_0, v_0, variable t, set their size, their position and align them in various way (see the final model). For this, first create the new equation h = h (t), transfer it to the graph, then delete it from the equation editor ; the other two are already in the list. Do not change the equations used to plot the curves !

Other additions to the graph.

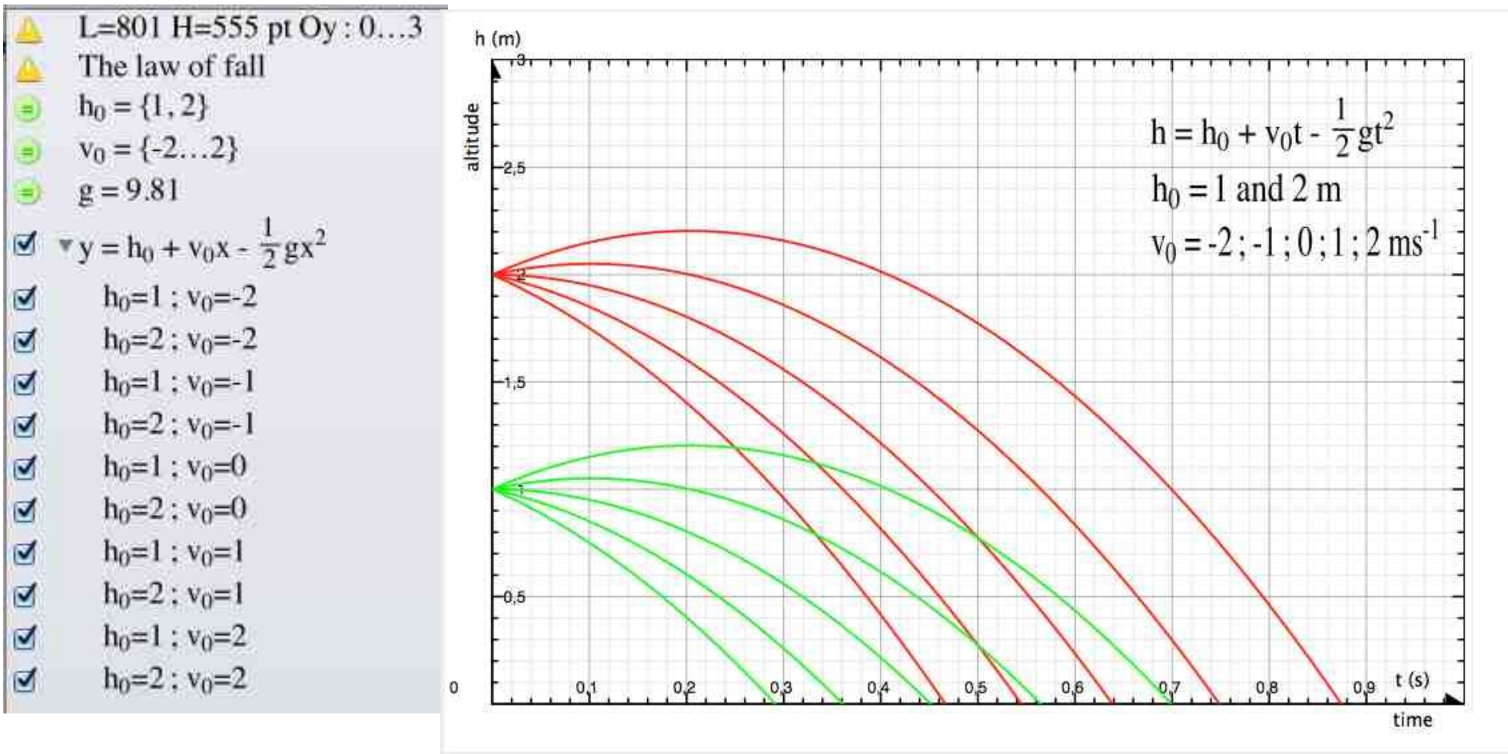
Mathematical expressions created in some equation editors (MathType, Equation Editor of Microsoft Office) can be transferred to 2D graphs of Grapher by dragging and dropping from the equation editor window onto the graph.

Last information to be entered into the document.

Still due to the **bugs nr. 1 & 2,** you add at the top of the equation list the following text :

L=801 H=555 pt Oy : 0…3

this will allow you to restore the the original document after opening the GCX file of your saved work or after modifying the dimensions of the window you are working.



Congratulations ! From now, feel free to use the Inspectors to edit the various properties of the objects.

Lesson 3 : Creating an animation (Animate Parameter).

In this lesson you will learn how to create in 2D an animation and various procedures :

— dimension the graph accurately to copy and paste it into TextEdit or another application ;
— enclose the graph in a frame with minimal margins ;
— insert text on the graph and choose font and font size ;
— drag and drop onto the graph texts, equations, parameters, from the equation editor ;
— display on the graph, in real time, parameter values calculated in Grapher ;
— animate a parameter ;
— create a QuickTime animation ;
— see again some procedures used in previous lessons.

The problem.

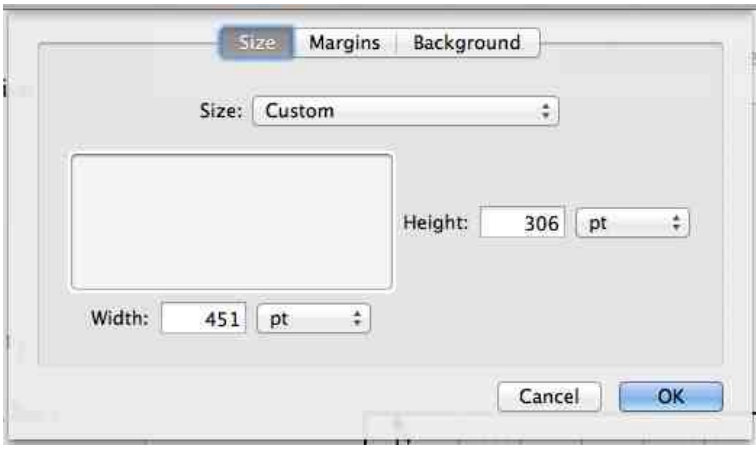
To illustrate this lesson it is intended to represent a traveling sine wave, created continuously in O and propagating along the axis Ox, increasing abscissa.

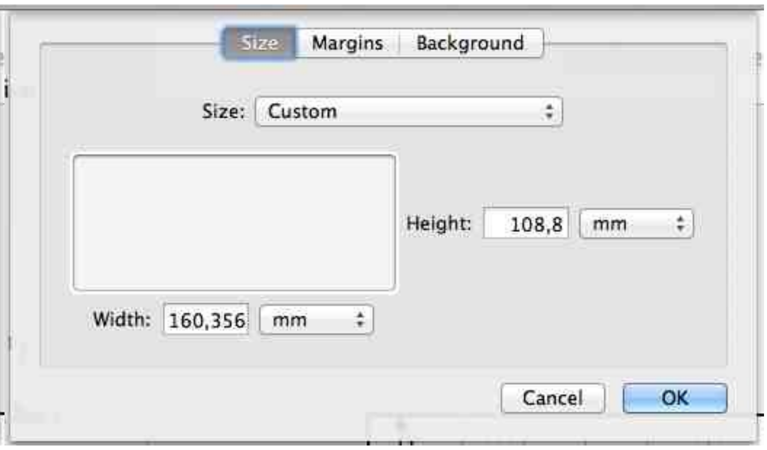
Dimensioning the graph with precision.

We are looking for the maximum length of the graph which can be placed on an A4 Portrait page in TextEdit 1.4 to 1.9 : 210 mm minus the two minimum margins of 1 inch = 210 - 2 x 25,4 = 159,2 mm = 159,2 / 25,4 x 72 = 451,3 pt (points) ; we choose the nearest integer 451 ; for the height we opt for 306 pt. Therefore the choice is :

L = 451 pt for a length of 159,10 mm (451 / 72 x 25,4),
H = 306 pt for a height of 107,95 mm.

Open a new 2D working window in Grapher : **Menu File > New… > New Graph > 2D Graph > Default > Choose**
Menu Format > Layout... > Size > Custom > enter L = 451 pt and H = 306 pt as in the view below ; change unit from points to millimeters to read L = 160,356 mm and H = 108,8 mm.





They are not the expected values ! The cause is the **bug nr. 3** by virtue of which Grapher 1.1 to 2.5 use 1 inch = 25,6 mm ; verification : 451 / 72 x 25,6 = 160,356 and 306 / 72 x 25,6 = 108,800 mm. **REMEMBER : 1 inch = 25,4 mm**
Note L = 451 H = 306 pt (**see bugs nr. 1 & 2).**

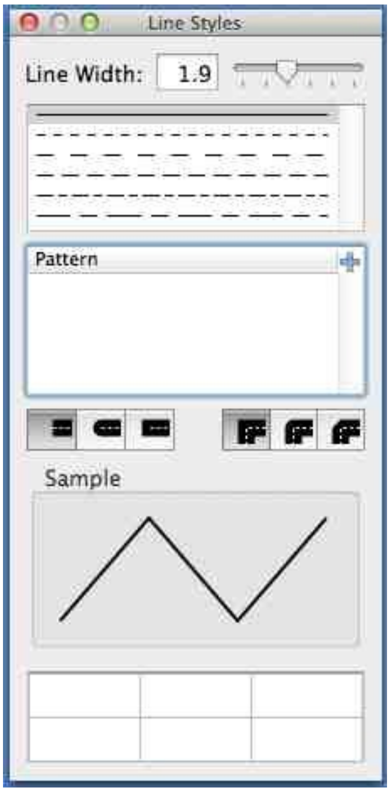
Frame for the graph and minimum size of the margins.

We need to show the limits of the graph to measure its size in the windows of other applications and after printing ; for this and for a better look of the work we wish to display the frame.

Characteristics of the frame :

— it appears by checking the box in **menu Format > Axes & Grids… > Frame ;**
— selection by clicking on its line : four markers at the corners of the graph fiefd prove the selection of the frame ;
— the frame is a line with adjustable width which enclose the set [axes + grids + curves] without encroaching on these elements ; it is placed in the margin of the graph and will be visible after saving, only if this margin has at least the same width as the frame line ;
— the **line style choice** is made after **selecting** the frame, and opening the **Inspector** ; click the **first button** for the **color**, click the **zigzag button,** that opens the **line settings** ; define here the **Line Style (continuous or dashed lines)** and its **width** measured by a **number increasing from 0.0 by increments of 0.1** ; **THE DECIMAL POINT IS REQUIRED HERE** to enter this value (which is not in points as width, height, margins of the graph) for OS X set in French.

Characteristics of the frame line and appropriate minimum margins			
Line Width		Minimum margin	
Line Styles	in pixels	in points (pt)	in mm
0.2 à 0.8	2	2	0.71
0.9 à 2.1	3	3	1.07
2.2 à 3.3	4	4	1.42
3.4 à 4.2	5	5	1.78
4.3 à 5.2	6	6	2.13
5.3 à 5.9	7	7	2.49
6.0 à 7.0	8	8	2.84



Each pixel line of the frame has a particular color, shade of the chosen one ; for black, they are mixtures from the absolute black to the palest gray. These variations in tones give the illusion of a variable thickness of the line despite a constant pixel width.

The table above is valid for lines parallel to Ox Oy axes. It is not limited in line width.

The DigitalColor Meter in Utilities allowed me to see how the lines of the framework are built with one pixel accuracy.

Continue your work : choose a black frame, a 1.9 (3 pxl) width line and enter a 3 pt (1.07 mm) value for the four margins of the graph, that were zero until now :

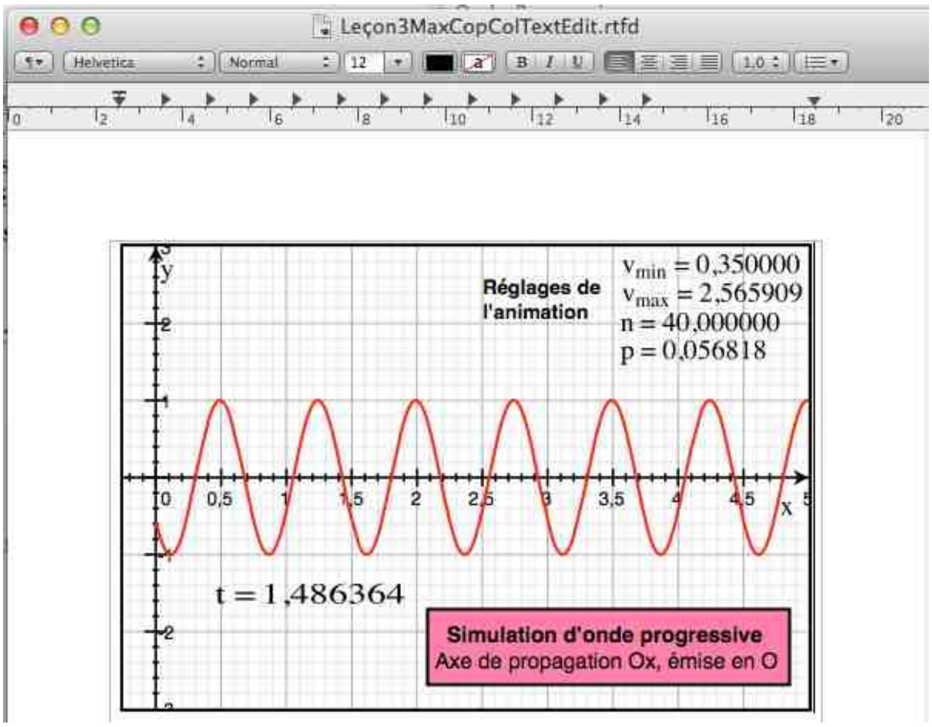
menu Format > Layout… > Margins > unit, pt ; one click in the graph deselects the frame and shows it.

Verifying and adjusting the graph dimensions.

Menu Edit > Copy as PDF (or TIFF) then Paste in TextEdit (or another application) ;
Menu File > Export > PDF > Save then Copy and Paste or drag and drop into TextEdit or other application.
Measure the dimensions of the graph in these applications, and after printing.

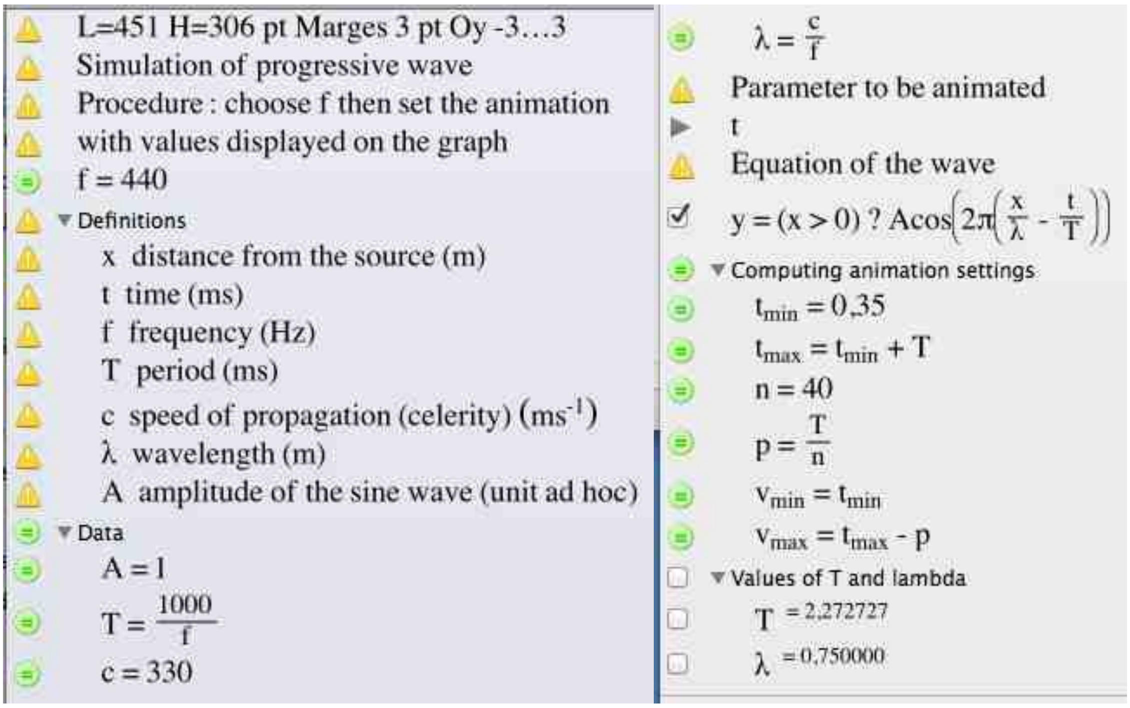
My observation (iMac, MacBook Pro, OS X 10.4.11 to 10.9, printer Canon Pixma MP610) is that the intended dimensions, entered in points (pt) in Grapher, are found precisely on the TextEdit page (the graph just enters it), it is also true with Pages.app (without moving the deformation handles) and on printed pages. Conclusion :

For very precise dimensions of graphs, enter them in points (pt) or inches (1 mm = 72 / 25.4 pt, 1“ = 25.4 mm).



The figure nearby shows that the dimensions of the graph have been calculated correctly : it enters exactly between the margins of the TextEdit page, which are shown by the left and right tab stops on the ruler.

Entering the equations.



From top to bottom and from left to right we can read :

— the graph size in pt and the minimum and maximum of the ordinate scale, again due to b**ugs nr. 1 & 2** ; here we added the margin width ;
— then the title of the document and a short manual ;
— the main parameter to enter, the frequency f ;
— now a first group that does not contain equations, only texts that specify definitions of variables and units ;
— next group,“Data“: contains two variable data and two intermediate variables calculated ;
— the parameter to animate, the time t ; we shall see how later ; at the present stage of our document, it should be preceded by the small green circle and have a value t = 1 for example ; the screenshot shows it already animated, “ t “ alone preceded by the black triangle ;
— the wave equation : note the **syntax to define y only for x > 0** ;
— group “Computing animation settings“ ; we shall study it in detail ;
— the last group for two numerical values : **the two variables are defined above by equations, to calculate and display their values, simply enter their name alone in the list of equations..**

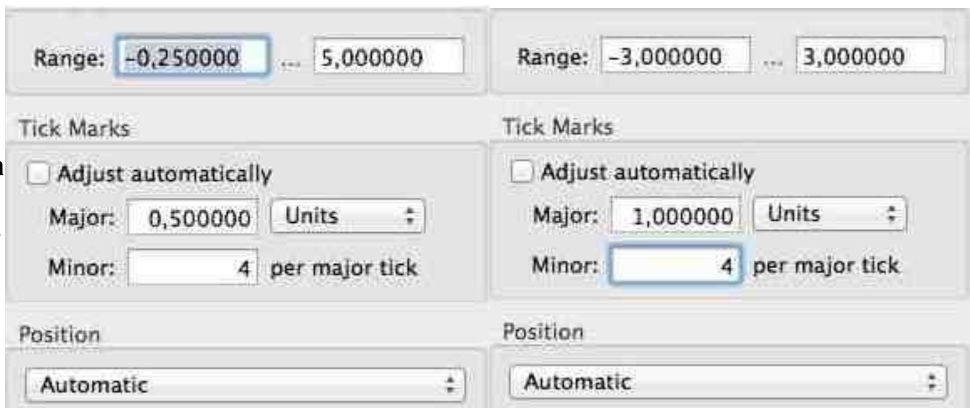
Think about using : **Menu Window > Show Equation Palette** to edit the mathematical expressions.

Comments entered via the equation editor.

— They are not equations, hence the warning (yellow triangle “danger“) in front of the line ;
— Before typing them on the keyboard, uncheck in the Preferences “Variables in italics“, otherwise your letters will be sloping (Grapher variables : x, a, etc.) or Roman (if not variables : sin, cos etc.) ;
— The equation editor is not intended for normal word processing : signs typed after a caret ^ (i.e. French accent circonflex) become exponent,, ê, ô and â, are not available, signs typed after Shift+Dash keys are subscript ; in short, the keyboard shortcuts of the equation editor are enabled (see chapter “Expressions“) ;
— An English name (no French accents) of a Greek letter, will be immediately translated... into a lower case Greek letter, except epsilon, omicron, upsilon, but xi and chi yes, ksi and khi no ; English and French words often contain Greek letters e.g. alphabet, mutation, nuclear, philodendron, etc. ; to avoid the Greek letter μ when typing “mutation“, type “m“, a space, “u“,“t“…“n“ then go back to cancel the space, the process is general ; (this Greek translation does not occur if “Use shortcuts when typing“ is unchecked) ;
— Of course, these comments may use all the usual signs and symbols of the expressions ;
— Think about using : **Menu Window > Show Equation Palette** that provides you with various symbols and also the whole Greek alphabet (upper and lower case)…

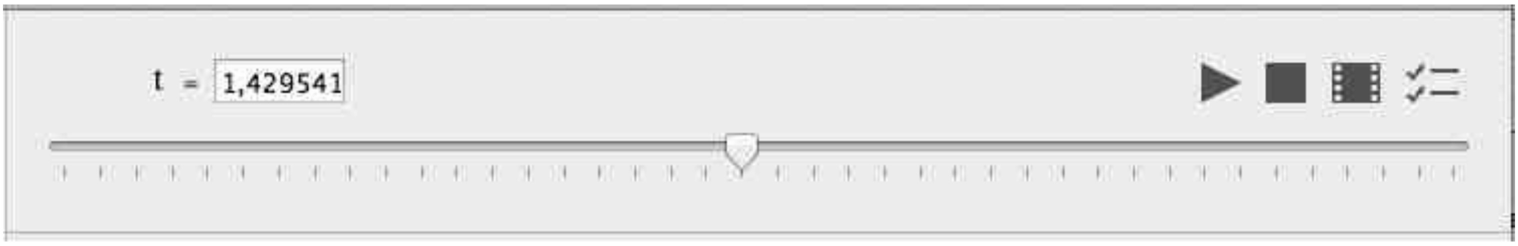
Choosing scales and graduations of the axes.

Menu Format > Axes & Grids… > Abscissa Axis > Edit… > enter the values shown in the figure nearby, do the same for the ordinate axis (second figure), finish by **clicking on the graph to deselect the axis.**



Animation of the parameter t.

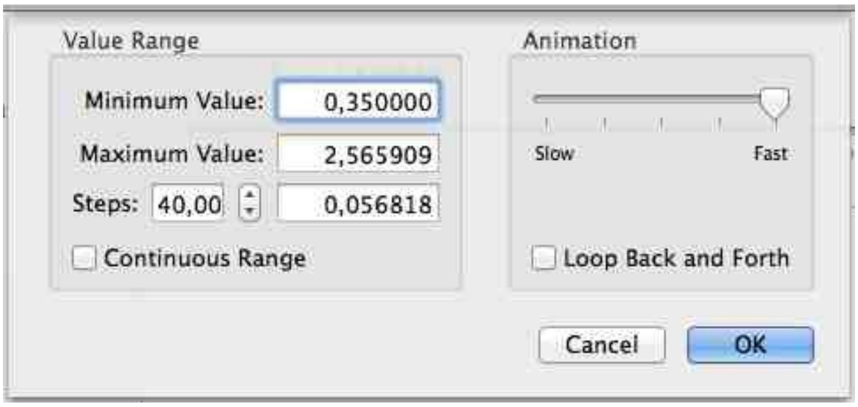
Start by **checking the correct movement of the curve** by increasing the **value of the parameter** t ; we had entered t = 1, enter successively 1.1 - 1.2 - 1.3 : the curve moves to the right, it is what we expected. Note that the parameter t is not yet animated and therefore preceded by a green circle in the list of equations. **Animate the parameter** t :
Select (click) the parameter in the list of equations > menu Equation > Animate Parameter > displays the animation banner and replaces the green circle by a black triangle before the parameter in the equation list.



Click the black square "Stop" > cancel the animation > the banner disappears, the green circle is back in front of the parameter ;
Click the right button > "Settings" of the animation > animation setting window ;
Click the black triangle > "Play/Pause" > On/Off of the animation (also applied to the triangle in the equation list) ;
Click the film button "Create a Movie" > Quick Time sequence of the animation ;
Moving the cursor with the mouse > operating "manually" the animation.

Setting the parameter animation.

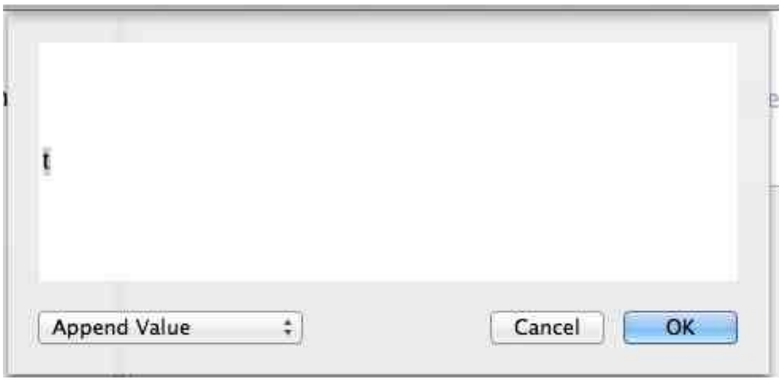
— Minimum value v_{min} , maximum value v_{max}, the parameter will pass from the first value to the second one in n steps (40 steps) p = 0,075 (if v_{max} - v_{min} = 3) ;
— Slow-Fast : choose ;
— Continuous Range : if checked, allows a smooth movement of the cursor with the mouse, otherwise movement by integer number of steps ;
— Loop Back and Forth : if checked, parameter move up and down by steps between the two extreme values ; otherwise, after reaching v_{max} it starts at v_{min} .



To give the appearance of an almost smooth movement of the traveling wave (our curve), uncheck Loop Back and Forth, take a large enough number of steps, a step equais to a small fraction of the period of the wave ; more, use the periodicity to prevent a change of the movement rhythm when returning to the minimum value, this return should be one step.
The group "Computing animation settings" in the equation list shows you the method of calculation ; we chose an arbitrary value for t_{min} = v_{min} = 0,35 ; the values calculated by Grapher take account of changes of the data. We'll display them on the graph with automatic updating when they are changing.
You have to **enter these values in the setting window of the parameter animation.**

Automatic display on the graph of values of changing parameters.

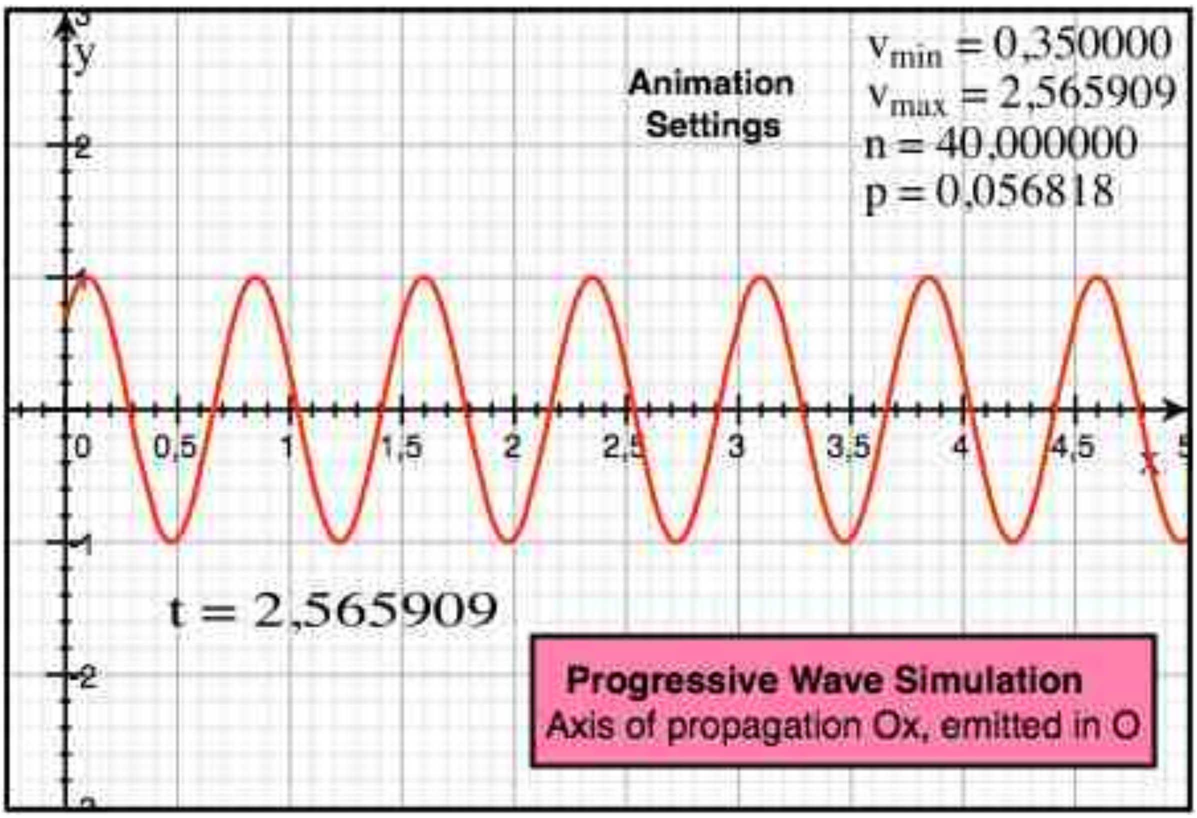
You are going to display the values of t, v_{min} , v_{max} , n, p ; the procedure is as follows :
New Equation > enter once more the name of the parameter (alone) > select it in the equation editor > drag and drop onto the graph > adjust the size with the handles > double-click it > window, selection of the parameter, Append Value (note) > the value is displayed on the graph and will be updated **> adjust the shape and size with the handles > delete from the list of equations the name of the parameter now useless, or keep it in the list of equations with its value displayed.**



Note : — No Transform : only the name of the parameter is displayed ;
— Append Value : ithe value is placed with the = sign after the parameter name ;
— Replace with Value : the value is displayed without the name.

After displaying the five parameters on the graph do not forget to enter the four setting values of the animation.

Other objects placed on the graph.



The graph nearby (scale 1) shows a legend on a colored background with a black frame and a title for the animation settings.

The following procedures were used :

— **menu Object > Align Left Edges**, after selection of the four animation settings displayed on the graph ;

— **menu Object > Insert Text, menu Format > Font > Show Fonts** > Helvetica size 13 & 14 Bold and Regular, for the inscriptions ;

— the **Inspector of these objects** for the **frame**, of width 1.9, with black solid line, and **background color** Web Safe Color FF6699 ;

— **menu Object > Group >** to group together in one block, after selection, v_{min} v_{max} n p and their title (useful to move all easily on the graph) ;

— **click the curve > Inspector > line width 2.0, color Apple Red.**

Complete your graph as shown in the figure above using these procedures.

Testing the animation.

Click parameter t in the list of equations > in the animation banner, click the right button > enter the settings displayed on the graph > with the mouse, move the cursor to the right, check the effect on the graph, verify in particular that the last step and the return to start move the curve forward the same length, without abnormal or missing step.

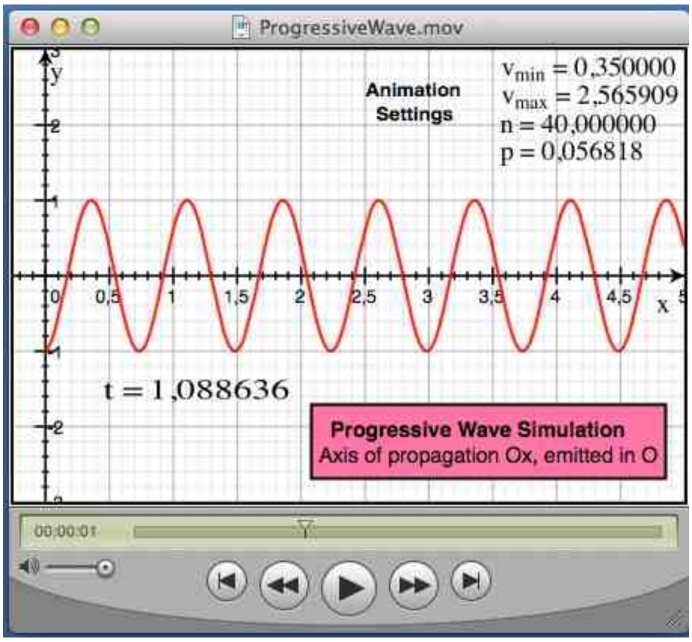
Click the triangular button in the banner to start the animation, click again to stop it.

Creating a Quick Time file.

Click the “Create a Movie“ button in the animation banner then :

• since Grapher 2.5 **> Name and save in the pop-up window the animation as a QuickTime (.mov) file** that is opened in this application.

• before Grapher 2.5 an intermediate animation opened in Grapher, to be saved or not in a QuickTime file.



Congratulations! You have shown calm and tenacity necessary to get here ; you have just created your first animation while memorizing, I hope, a lot of information about the use of Grapher in 2D. Another big thumbs up !

Lesson 4 : Differential equations. (for Grapher 2.0 see Appendix 2)

In this lesson you will learn to represent in 2D the solutions of a differential equation and other procedures :

— to save your work frequently as it progresses ;
— the anomalies due to bugs nr. 1 and 2, and their workarounds ;
— the syntax of differential equations ;
— to change the value of a parameter ;
— to use multi-valued expressions ;
— to read the coordinates of a point on a curve, those of the intersection of two curves ;
— to find the roots of a function, differentiate, integrate, calculate a definite integral .

This lesson includes reminders and repetitions that can irritate you, but remember that :

« INSTRUCTION TRIUMPHS BY REPETITION ! »

The problem.

Plot the curves representing the solutions of an second-order homogeneous linear differential equation with constant coefficients, the form is :

$$y'' + \frac{2}{\tau}y' + \left(\omega^2 + \frac{1}{\tau^2}\right)y = 0 \ \text{si} \ y = y(x) \ \text{et} \ x \in [0, \pi]$$

initial conditions for $x = 0$: $y'(0) = 0$ and $y(0) = 1$
for $\tau = 0{,}5$ and $\omega = 0$ and 5π

Compare the numerical solutions of the differential equation with algebraic solutions that are :

for $\omega \neq 0$: $y = e^{-\frac{x}{\tau}}.\left(y(0).\cos\omega x + \left(\frac{y'(0)}{\omega} + \frac{y(0)}{\tau\omega}\right).\sin\omega x\right)$ and for $\omega = 0$: $y = e^{-\frac{x}{\tau}}.\left(y(0) + \left(y'(0) + \frac{y(0)}{\tau}\right).x\right)$

Optimize numerical solutions by setting the step in the methods of Euler and Runge-Kutta.
Reserve the right to modify the parameters and initial conditions of the differential equation to observe changes in the solutions for didactic purpose.

Layout of the document.

You are going to choose a graph with half an inch (12,7 mm) margins, A4 format, layout Landscape, of maximum size compatible with your printer, and save it immediately. Here is the procedure :
Menu File > New… > New Graph > 2D Graph > Margins > Choose ;
Enlarge the window enough (green button or drag the lower right corner) ;
Menu Format > Layout… > Size > Custom > enter L = 801 H = 555 pt > OK ;
Repeat this instruction ;
Menu Format > Layout… > Margins > change unit into pt, you should read 36 pt (72 pt per inch) ;
Note L and H in pt (see bug nr. 1 & 2). In my case with Grapher 2.1 to 2.5 : **L = 801 H = 555 pt allow the graph with maximum size after printing on an A4 page (Landscape, 100 %) ; the same with L = 555 H = 801, Portrait, 100 %.**

Note. 1) You may select “Page Size“ instead of “Custom“, or use the default working window without changing its size ; choose what you think is the best for you.
2) The size of the graph on the monitor is 1 pixel per point of L and H.

Presetting axis scales :
Menu Format > Axes & Grids… > Ordinate Axis > Edit… > enter -2, 2, uncheck, 0.5, 4, Automatic > OK ;
> Abscissa Axis > Edit… > enter -0.5, 5, uncheck, 0.5, 4, Automatic > OK ;
Toolbar of the window > Alt key + click the button Equalize Axes : the abscissa scale is adjusted on the ordinate’s one (without Alt, the opposite).

In the equation editor > delete y= > type : “L = 801 H = 555 pt (or your values) **Oy : -2… 2“ > Return key to place this information at the top of the equation list.**
Note that you read the following dimensions : 28.48 19.733 1,28 cm that should be 28.258 19.578 1.27 **(see bug nr. 3).**

Menu File > Save as... > title “Equadiff“, store on Desktop > Save.

The document is saved on your desktop, quit Grapher. You are going to do two exercises to learn the workarounds for the annoying **bugs nr. 1 & 2.**

Opening a GCX file (Grapher file with extension .gcx) (workaround for bug nr. 2).

As usual, **double-click the icon** of the GCX file, Grapher starts and the document appears ; notice that the dimensions of the graph and the ordinate scale are changed.

To restore the original size of the document :

Menu Format > Layout… > Size > Custom > enter L = 801 and H = 555 pt > OK ;
Repeat this instruction ;
Menu View > Frame Limits… > enter y : -2, 2 > OK.

Keep the document in this state for the second exercise.

Changing the Grapher working window size (workaround for bug nr. 1).

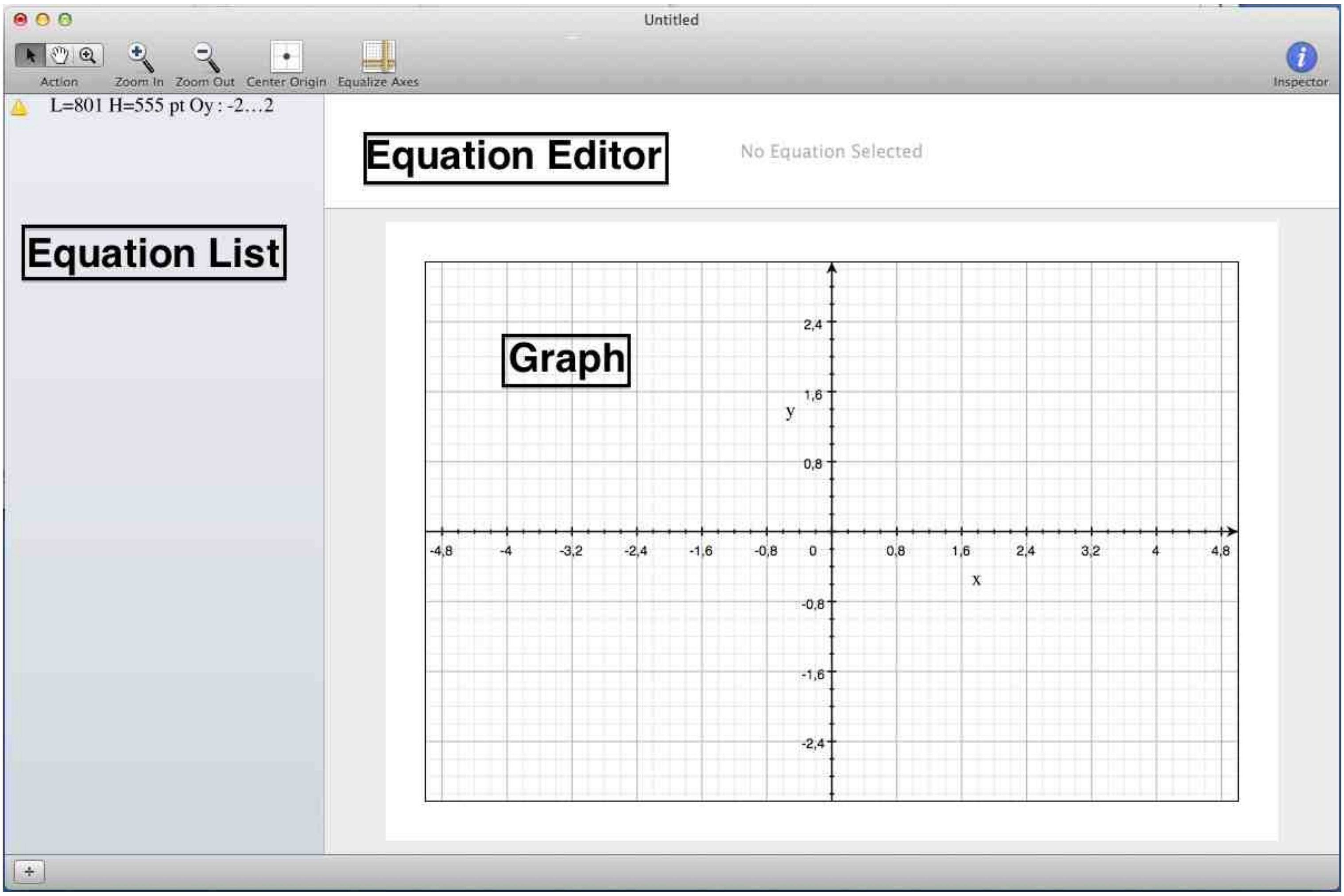
Change the size of the window by dragging its lower right corner and see the changes on the display and notice that in menu Format > Layout… > Size > Custom > L and H are modified, but the ordinate scale not.

To restore the document, use a procedure similar to that of the previous paragraph :

Menu Format > Layout… > Size > Custom > enter L = 801 and H = 555 > OK ;
Once is enough in this case..

Note : If “Page Size“ was used instead of “Custom“ for the layout, to restore the graph replace “enter L = 801 and H = 555 > OK“ with “Page Size > OK“ (to be repeated).

Here is a view of the working window of Grapher as it should appear at this stage of your work, although not in its exact dimensions this screenshot gives a good indication of relative sizes of the three fields you use, the list of equations, the equation editor, the graph :



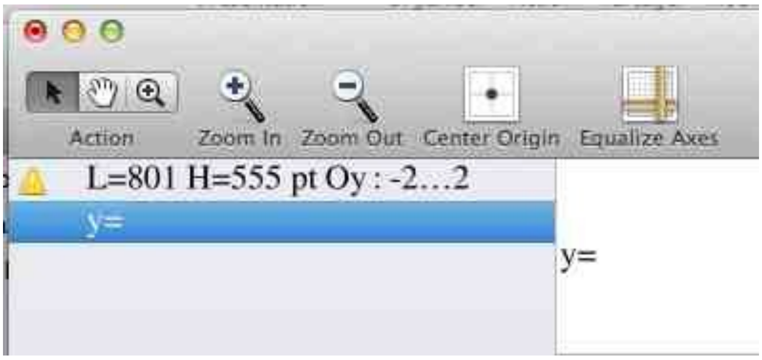
Syntax of expressions.

The syntax used in Grapher to write mathematical expressions is detailed in the "Expressions" chapter ; to give a first look of it is the purpose of this paragraph.
Configuration tested here : iMac, MacBook Pro, OS X 10.4.11 to 10.9, set in French, Language and Input Sources in System Preferences : French, Apple French AZERTY keyboard.

SOURCES OF SIGNS AND SYMBOLS :
— the **keyboard** : Grapher alters the results of some keystrokes, which are different from those of a normal word processor such as TextEdit;
— the **keyboard shortcuts** specific to Grapher ;
— the **Equation Palette** with its four tabs Standard, Operators, Greek, Symbols ;
— the **Built-in Definitions : constants and functions recognized by Grapher** ;
— the Grapher **Preferences** offer choices for signs and symbols.

Note. Convention of writing : to indicate the keyboard keystrokes, a key is represented by one of the symbols engraved on it, adjoining the symbols show the keys pressed together, if a space separates two symbols, it means successive strokes.

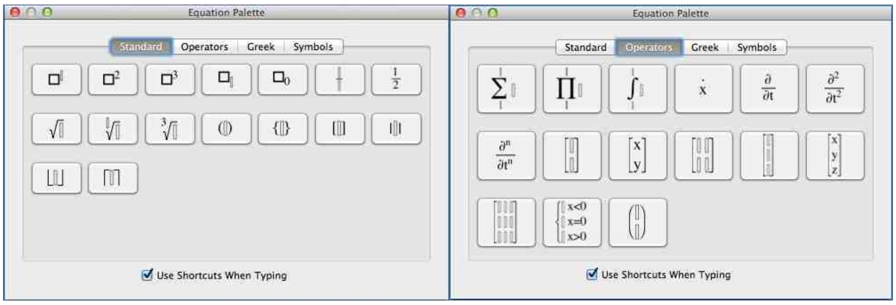
OPENING A NEW LINE OF EQUATION. Many possibilities, retain for the moment :



— Click the button **[+]** at the bottom of the equations list then contextual menu ;
— ⌥⌘**N** keyboard shortcut.
Create a new line “ y= “ selected in the equation list and the same line in the equation editor, ready to be completed or replaced.

REMOVING A LINE OF EQUATION **(NOT USED IN OTHER EQUATIONS).**
Click on the line to select, **Backspace** key ; to select **several lines press ⇧ or ⌘**.
Note: If the line to be deleted is used in other expressions (definitions of functions or variables, for example), you can make these expressions incorrect (yellow triangle "Danger"), for example by adding the = sign at the beginning of expressions, which allows you to edit later.

CREATING A GROUP : Click the button **[+]** at the bottom of the equations list then **contextual menu**.
TITLING THE GROUP : **Double-click on its line (with Grapher 1.1 see bug nr. 9)**.
The **equations** created for this group must be placed in it by **dragging and dropping**, that shifts them slightly to the right.

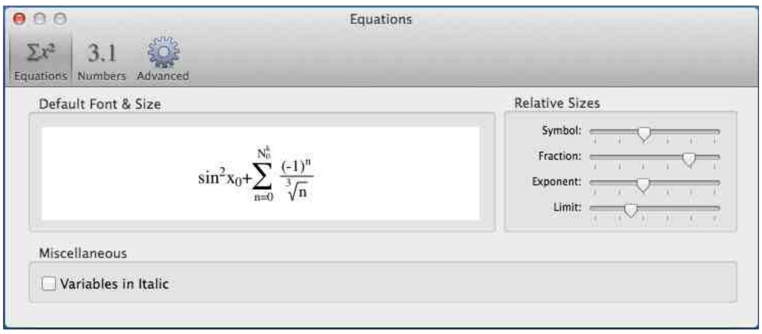


DISPLAYING THE EQUATION PALETTE :
Menu Window > Show Equation Palette.

This palette and a SIMPLIFIED ONE are accessible from the contextual menu of button ▾**Σx^2** in the equation editor.

USING THE EQUATION PALETTE.
To place a symbol at the insertion point in the equation editor : **Click on the symbol**.
Check “Use Shortcuts When Typing“, they make your work easier.

DISPLAYING BUILT-IN DEFINITIONS : **Menu Help > Show Built-in Definitions.** See list and comments included in the chapter “Using the equation editor“.



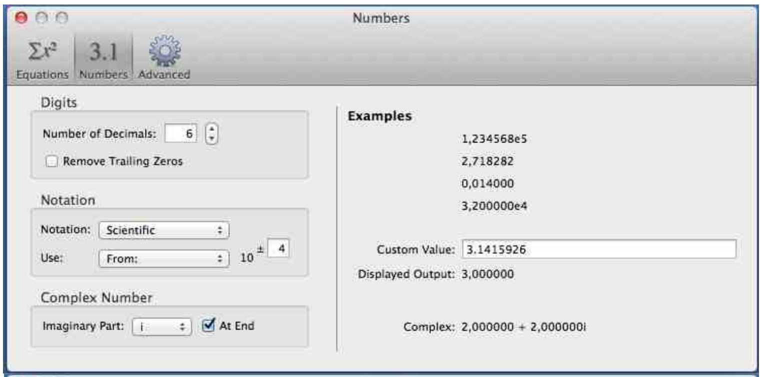
GRAPHER PREFERENCES.

Menu Grapher > Preferences...

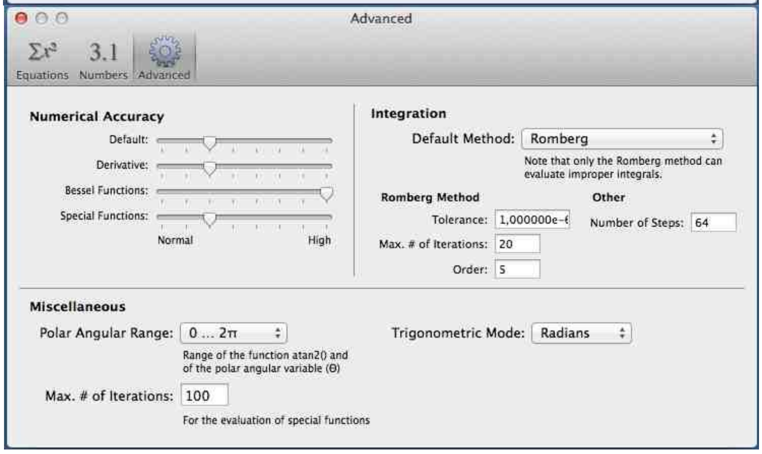
The Equations tab allows you to change the relative sizes of symbols-fractions-exponents indices-limits.

Remember to **uncheck Variables in italics** at least when you enter lines of text in the equation editor.

The Numbers tab specifies the **number of decimal places**, scientific or engineering notation, the choice **i or j** for **complex numbers.**



The Advanced tab shows settings for precision of calculations, others for numerical integration, **the choice of the angle unit (radians, decimal degrees, grades), the choice of domain [0, 2π] or [-π, π] for the polar angles and values of the function atan2(y, x).**



KEYBOARD SHORTCUTS OF THE EQUATION EDITOR.

A complete list will be given later ; here are some frequently used.To :

— insert a fraction : type : **/**

— enter an exponent : type : **^**

— enter an index : type : **⇧—**

— open parentheses : type : **(**

— open braces : type : **⌥(**

— type a lowercase Greek letter : type : **o m e g a** (on keyboard **⌥P** shows **π**)

— navigate the symbols and expressions, use : ⇠ ⇡ ⇣ ⇢

MULTI-VALUED EXPRESSIONS.

In the example a = {1,1, 2, 3,4, 5} the numerical values are 1,1 - 2 - 3,4 - 5 ; there are two uses of the comma :
— **the decimal comma : without any space separating it from the integer and decimal parts of the number ;**
— **the group « comma followed by space » : separator between each number, function, etc.**

GLOBAL AND LOCAL DEFINITIONS.

Global definition : The parameter is defined by a particular equation ; for example : **a = {1, 2.2, 3.3, 4}** ; this definition may or may not be in a group ; it applies to all expressions of the list of equations containing the parameter a. :
Local definition : the definition of the parameter is part of the equation that contains it ; for example : **y = ax, a = {0.5, 0.7}** ; this equation will graph two curves for a = 0.5 and 0.7.
A parameter, variable or function that is in a global definition cannot be also in a local definition.
A parameter, variable or function that is not in a global definition can be in a different local definition in each line of equation.
Note. Before Grapher 2.2, global and local definitions could coexist, each local definition overrides the global.

EXERCISE.

1) In the current document, enter the expressions :

$\omega_{max} = \{1.1, 2, 3.4, 5\}$ using the equation palette
$y = \omega_{max}x$ in preference to
ω^2_{max} keyboard shortcuts ;

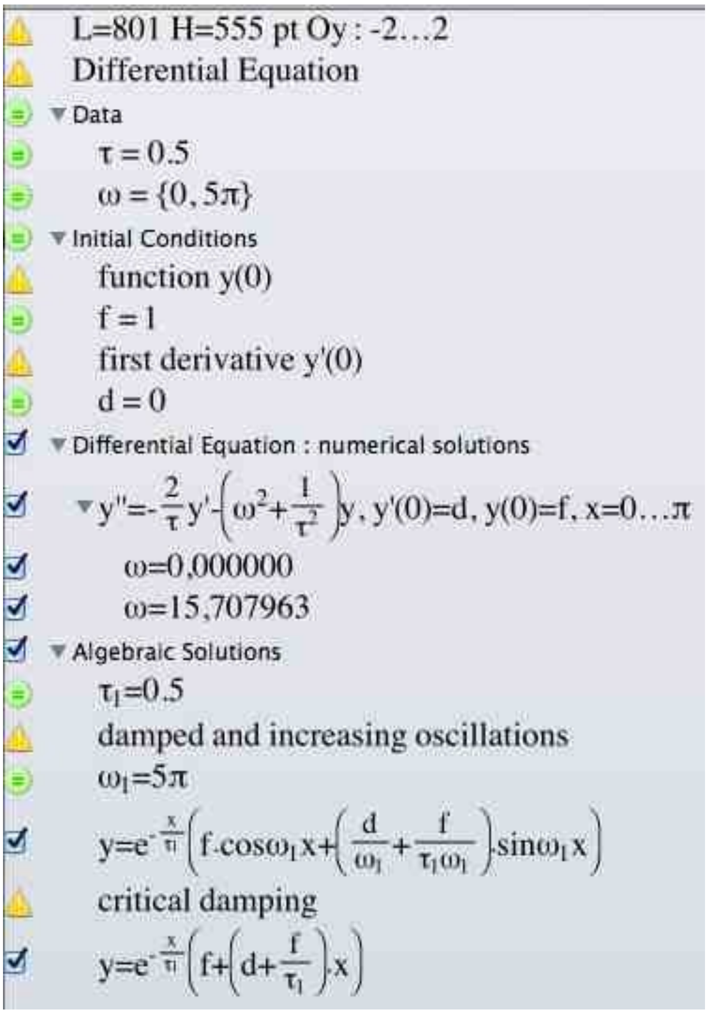
then delete these three lines of equations ;

2) Repeat all, using only the keyboard ; the keystrokes in the equation editor, after deleting “ y= “, are for the first and third expressions :

o m e g a ⇧— m a x ⇢ = ⌥(1 . 1 , space 2 , space 3 . 4 , space 5 ⇢ Return
o m e g a ⇧— m a x ⇢ ^ 2 ⇢ Return
then delete the three lines of this exercise.

You're fully ready to resume your work on differential equations.

Entering the equations of the problem.



Do it by copying the list of equations included in the figure nearby that shows :

— graph size and Oy scale (**see bug nr. 1 & 2**) ;
— title of the document ;
— four titled groups ;
— four lines of added comments ;
— four equations for the values of parameters ;
— the differential equation (the two values of ω that follow will be entered automatically by Grapher) ;
— two classical equations y = y(x).

Save (store) your work frequently.

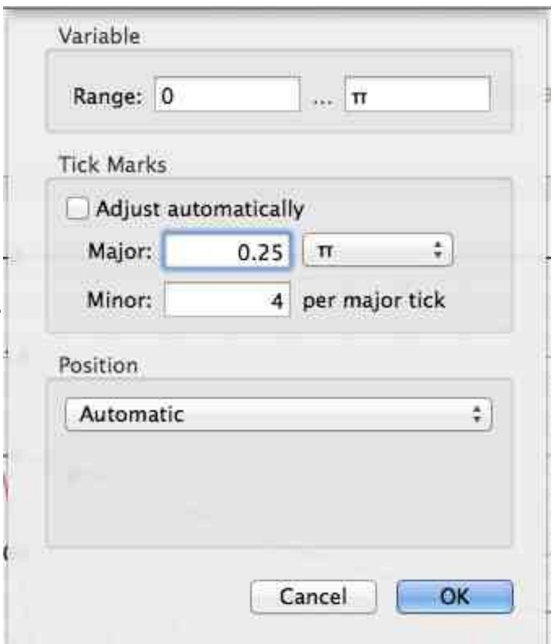
Please note :

— the **global definitions** of τ and ω, one being a **multi-valued definition** ; we had to change their name, adding “index 1“ to use new definitions at the bottom of the equation list ;
— the **decimal commas without space before nor after ;**
— the four **separators “comma space“,** one for the data ω and three of them for the differential equation ;
— y”, y’ : **prime and double prime** symbols are the **English single (apostrophe) or double quotes** of the keyboard ;
— the **optional spaces added before and after the signs =, +, -,** for more readability.

SYNTAX OF A DIFFERENTIAL EQUATION.

The expression to define it has three parts :
— the equation itself ;
— the initial conditions ;
— the domain chosen for the variable.
The form of the equation is : “ highest order derivative = function of all other terms “, the second member can contain only lower order derivatives, the function, the variable, the parameters.

Customizing the graph.



—Adjust the scale of Ox : from 0 to π, domain chosen for the variable : **menu Format > Axes & Grids…, etc.** (figure nearby).
—With the **Inspectors**, choose for the **curves** of the differential equation **color** Apple Black, **line width** 2.0, and for those of the algebraic solutions color Apple Red, line of 2.0 as well ; you may **select several equations together for common settings** ; if you **select the line of a “multiple“ equation** (here the differential equation), **all its versions are selected** (here for the two values of ω).
— Take advantage of the curve Inspector and the Apple’s **Color Palette** opened to visit the latter, which presents six tabs with seventeen options and allows you to store the colors chosen or created.
— Display the differential equation onto the graph ; for this :
Click to select it in the list of equations > the entire differential equation line is repeated in the equation editor > click and drag on the equation to select it > drag and drop onto the graph > set shape and size, finish up possibly with the Inspector (frame, colors, background, rotation angle, shadow).

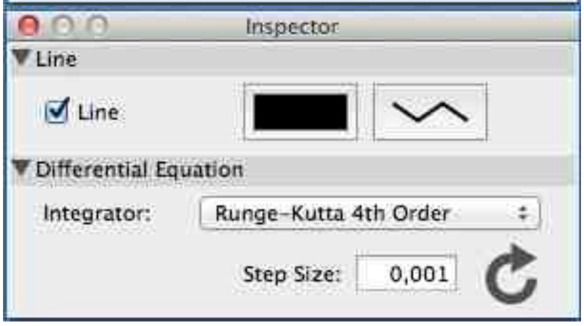
— Place a blue **legend** on the graph **in** a red **oval frame** (see the final graph) :
Use the **handles to form** the oval as you like, its **Inspector** for the frame (Apple Red, width 2.0) and the background (Apple White) ; **double-click in the figure > insertion point > menu Format > Font > Show Fonts > choose the color of the text** (blue)**, the font** (Helvetica 18)**, the shadow of the text.**
Notice that your keystrokes are placed in a rectangular frame circumscribed to the oval, and start outside the latter, which does not facilitate the layout that is easier with the inserted object Rectangle.

Remember to **save (store) your work frequently.**

Settings for differential equation solutions.

In the list of equations, check and **select with a click** the algebraic solution showing cosωx and sinωx, open the **Inspector**, choose the lowest **resolution** giving a correct curve (**click the Refresh Button, circular arrow, after each modification in the "Curve" part**) ; figure of the Inspector window nearby :

In the list of equation, check the **version ω = 5π = 15,709 of the differential equation and select it by a click** ; with a **click on the graph background**, the two curves, numerical and algebraic solutions, appear clear ; they should overlap perfectly but you can improve the calculation of the numerical solution by opening the **Inspector after again selecting the curve ω = 15,709**.
In its lower part «"Differential Equation" you may **choose the method of calculation (Euler or Runge-Kutta 4th order) and the Step Size** ; the step is 0.01 try other values 0.1 - 0.05 - 0.001- 0.0005 (**click each time the circular arrow**) ; Runge-Kutta 4th order method and a step size of 0.001 gives a good superposition of the two curves.
Do the same settings for the solution ω = 0 and the second algebraic solution ; the step size of 0.01 seems small enough for a correct calculation ; the figure nearby shows the Inspector of the differential equations.



The most suitable step size can change if you modify the domain of the variable, the axis scales, the differential equation parameters.

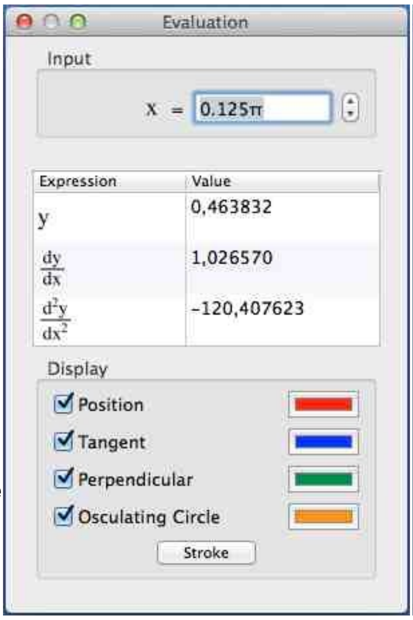
Think of **saving (storing) often your working window.**

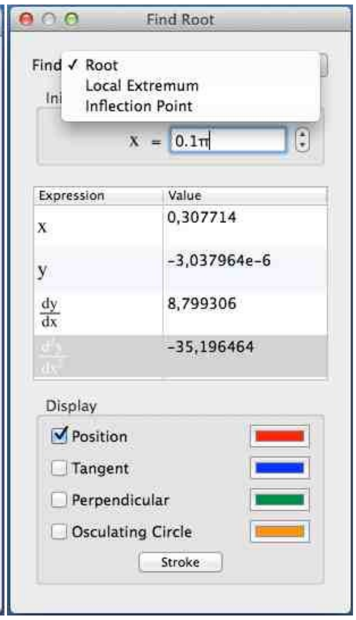
Modifying a parameter or an equation.

Click the equation > click the editor background > word processing in the equation editor.
Try to change the value of τ = 0.1 ; 0.3 ; 1 ; 0.5 to see how the numerical solution of the differential equation is modified.

Calculating coordinates of curve points.

Click on the curve or its equation > menu Equation > Evaluation... > Click the point you want on the curve, or enter a value of the variable and Return key > displays coordinates at the bottom of the window, calculation of the function, of its first and second derivatives, displays the chosen position on the graph, the tangent, perpendicular, osculating circle at this point and
click the button Stroke > displays their equations in the equation list **(Grapher 2.2 to 2.5 see bug nr. 31)**.





Finding roots, extrema, inflection points.

Click the curve or its equation > menu Equation > Find Root… > Choose the option in the contextual menu > Click an approximate point on the curve, or enter an approximate value of the variable and Return key > for the calculation of the coordinates of the particular point sought, of the first and second derivatives, which allows to verify the results : y or y' or y" very close to zero ; display the chosen position on the graph, tangent, perpendicular, osculating circle at this point and with a
click on the button Stroke > displays their equations in the list of equations **(Grapher 2.2 to 2.5 see bug nr. 31)**.

Plotting the curve of a function derivative.

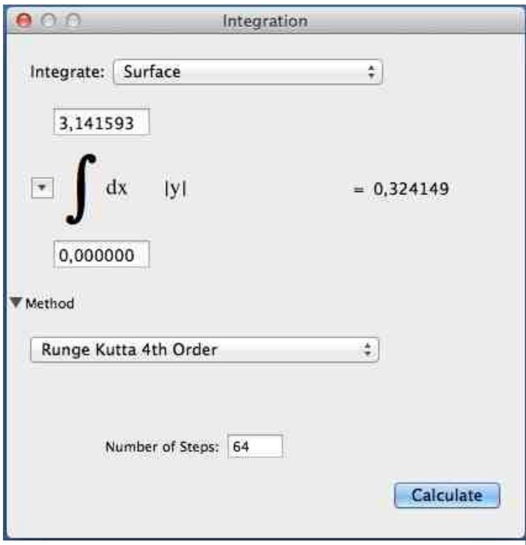
Click the equation in the list > menu Equation > Differentiate > replaces the original function and its curve with its derivative. Repeat > successive derivatives.
Menu Equation > Integrate, to return to the previous derivative, repeat until the return to the original function.

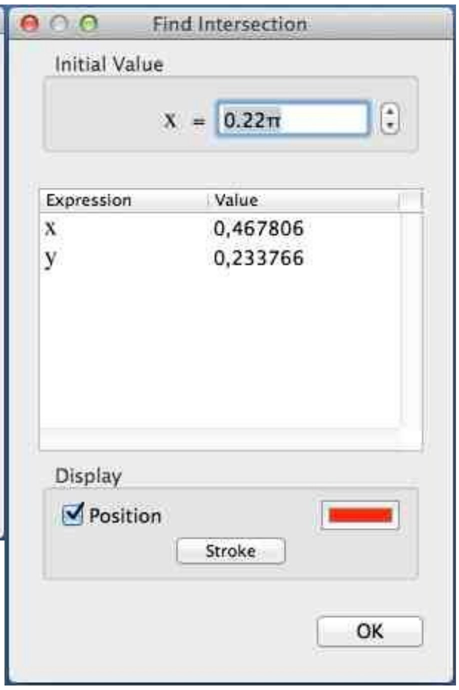
Plotting the curve of a function integral (Grapher 2.2 to 2.5 see bug nr. 29).

Click the equation in the list > menu Equation > Integrate > replaces the original function and its curve with its integral. Menu Equation > Differentiate, to return to the original function.

Calculation of a definite integral of a function.

Click the equation > menu Equation > Integration… > contextual menu > choose > enter domain of integration, method of calculation and its settings > Calculate.
To remove the integration area from the graph : menu Equation > Remove Integrated Area.





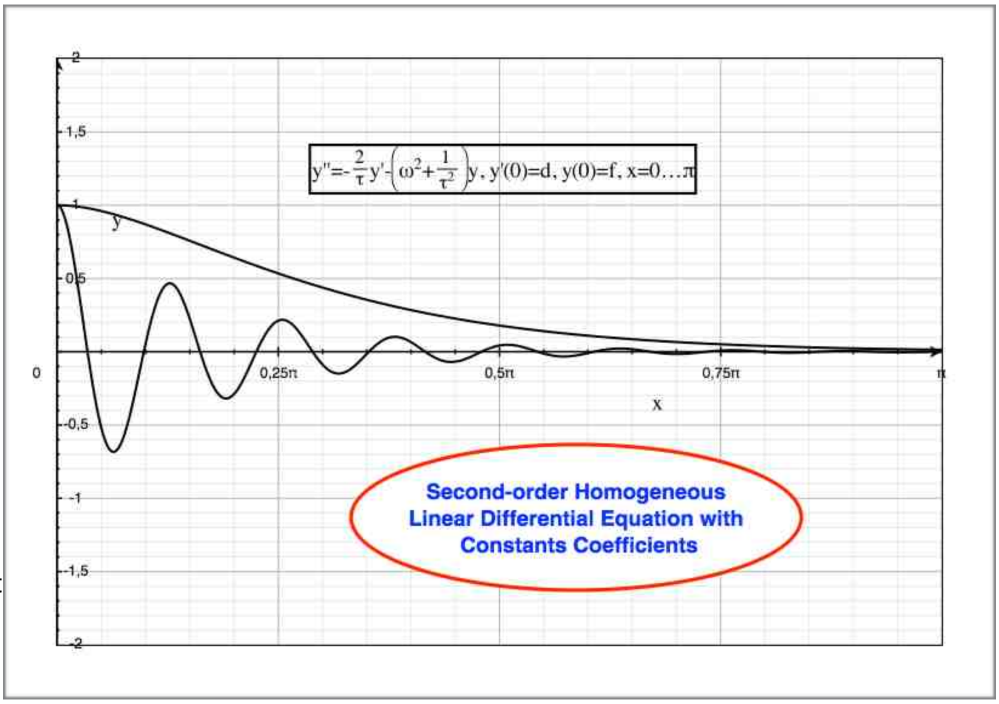
Calculation of the intersection point of two curves.

Select the two equations (clicks with ⌘ or ⇧ key) > menu Equation > Find intersection… > click on a curve near the intersection or enter an approximate value of the variable > OK.

NOTE ON THE SIX PREVIOUS CALCULATIONS : they can be possible or not, their windows, settings, options, contextual menus, may be changed, depending on the form of the equations (explicit y = f(x) or x = f(y), implicit, parametric), and the coordinate system used.

EXERCISE : enter the additional simple equation y = x into the list of equations of the current document and experiment with it some or all of these calculations ; for the intersection calculation use another equation already in the list.

Here, on the screenshot nearby, see how your final graph should appear (only solutions of the differential equation are selected in the list of equations).

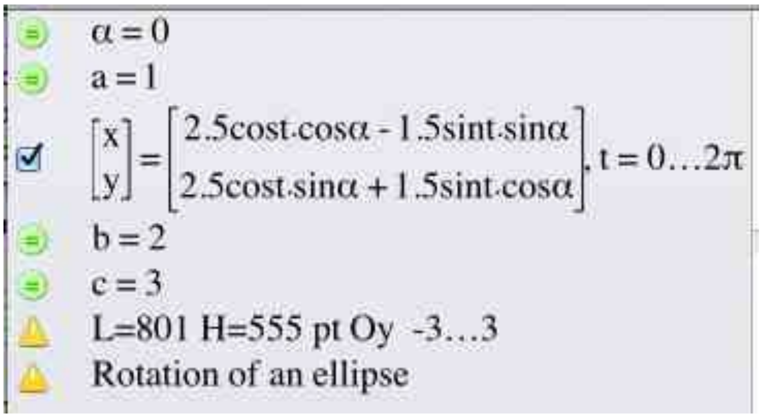


If you had the patience to get here you deserve a big applause, nay, a standing ovation ! A tribute to your memory is not too much either because you have learned a lot about Grapher.

Lesson 5 : Creating an animation (Create Animation) 2D and 3D.

In this lesson you will create animations using a method different from that of the Lesson 3 and learn :

— to create animations in a 2D graph ;
— to lay out a 3D document ;
— to create the representation of a surface ;
— to change its appearance ;
— to customize a 3D graph ;
— to animate it ;
— to save the animations in Quick Time files.



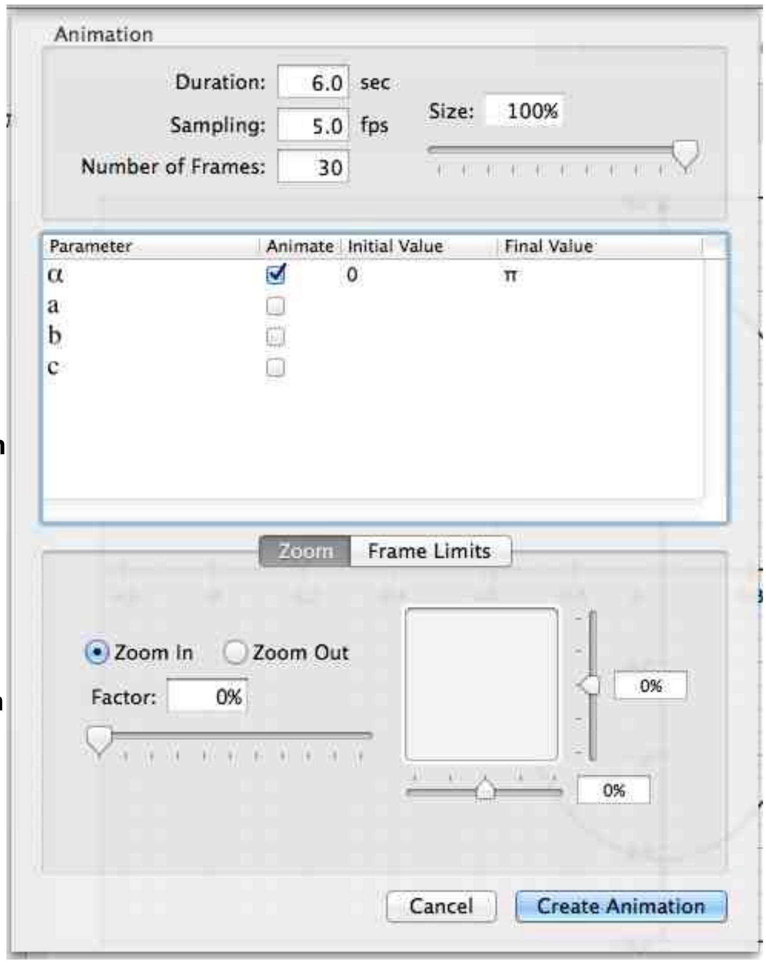
Creating the 2D graph.

Open a 2D document with the following characteristics : model Margins, size and scale Oy as shown on the figure nearby, Ox axis scale equalized to Oy axis scale, axes frame and grids visible.

Enter the equations and comments of the figure.

Customize the ellipse : minimum resolution, line width 2.0, color Apple Red.

Save your work frequently : « Lesson5-2D ».



The 2D animation pop-up window.

Open it by : **menu Equation > Create Animation...**

It shows three zones :

— the first one for the Quick Time sequence settings **(decimal point is required in this area) ;**

— the next one shows the **list of parameters of the list of equations provided that (see bug nr. 27) :**

- **they have a global definition,**
- **they are not preceded in the list by line(s) with the yellow triangle "danger",**
- **they are not preceded in the list by one or more groups ;**

— the third zone allows various types of animations.

In the list of equations we intentionally wrote the comments at the end.

There are four parameters, but only one is used in an equation of curve and therefore able to animate a graph.

Animations available in 2D.

Begin by choosing the size (of the Quick Time window), then Duration-Sampling-Number of Frames (two of these three numbers). Then you may choose one or more of the following animations :

— Check one or more parameters > enter their initial and final values ;
— Zoom in or out and factor % for both zoom ;
— Zoom on the abscissa and/or ordinate scales and their factors % ;
— Finish by **clicking the "Create Animation"** button.

- since Grapher 2.5, name and save the animation as a QuickTime (.mov) file in the pop-up window.
- before Grapher 2.5, an intermediate animation opened in Grapher to be saved or not in a QuickTime file.

Bug no. 15 : the 2D animations by Zoom have effects on the axes and grids only, the plotted curves remain perfectly stable (unless an animation by parameter is set) ; in the current state of Grapher, these animations by zoom are irrelevant.

Despite this bug try the different ways of animation with the help of data on the figure above.

Opening the 3D window.

— Method 1. **Starting from a 2D document > menu View > Switch to 3D View > remove all equations > menu Format > Graph Template…> select a model of graph > Apply ;**
— Method 2. **Menu File > New… > New Graph > 3D Graph > select a model of graph > Choose.**

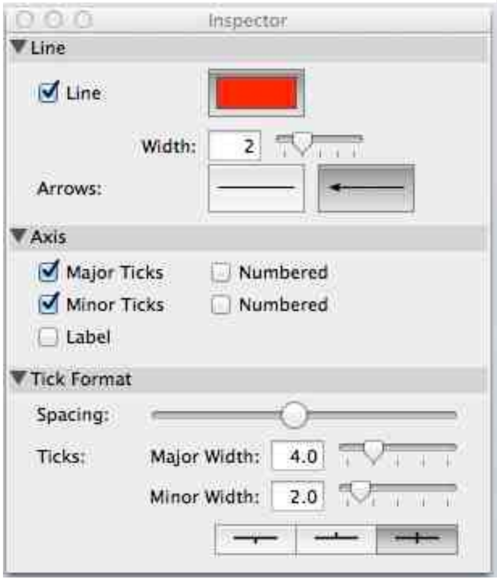
Setting size of the 3D graph.

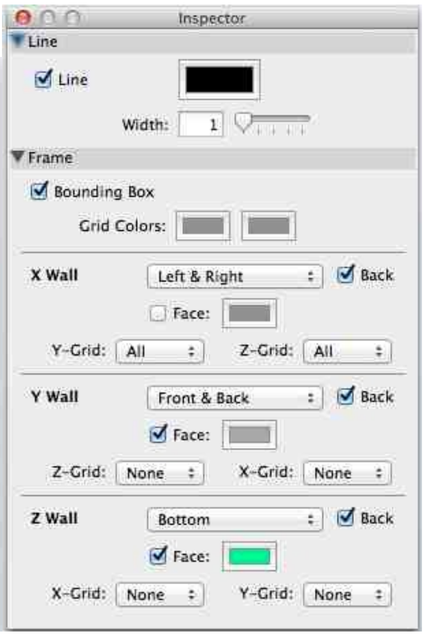
The relation between the graph size on the monitor and its dimensions after printing, exporting, transferring or copying and pasting in another application, is the same as in 2D ; that is one pixel on the screen will give 1 pt = 1/72 inch = 0,352778 mm on an A4 page after printing.
In 3D the menu Format does not allow to choose the dimensions of the figure ; the only possibility is to modify length and height of the graph field in the window by dragging the two handles in the middle of the left and upper edges, to obtain the appropriate number of pixels (pxl) or points (pt).
This requires a means of measuring the pixel dimensions of items displayed on the screen ; we may use for that the application “Grab“, menu Capture > Selection > display pixel dimensions of the rectangle selected between the two clicks, or after in Preview > menu Tools > Inspector > read L and H in the pop-up window.

Save your work : « Lesson5-3D ». The . gcx file retains the 3D window dimensions and these of the graph field.

Customizing axes, frame and background of the 3D graph.

CHANGING THE MODEL OF GRAPH : **menu Format > Graph Template… > select a model > Apply,** offers the same options as menu File > New… > New Graph > 3D Graph.
BACKGROUND, LIGHTING, SMOOTHING : **menu Format > Layout… > tabs Background (color), Lighting (of surfaces), Miscellaneous (smoothing of the lines).**
AXES AND FRAME : **menu Format > Axes & Frame… > select an axis > Edit… > settings of the scale > OK > open the Inspector > settings of line width, colors, graduation marks, arrows ;** then :
select the frame > OK > open the Inspector > line width settings, colors, of the cubical frame and its faces with grids available.





Choose a 3D graph template ; set the scales of the three axes : -5 to 5, uncheck, 1, 1, Automatic ; with their Inspectors place arrows at their positive ends.
With the Frame Inspector try the “Walls“.

There is neither label and nor numerical graduation on axes in 3D windows.

Save your work.

Entering the 3D equations.

They appear on the screenshot nearby. After this operation, be sure to **save the current document.**
No comments added about the dimensions of the graph and axis scales as in 2D : saving these informations works perfectly in 3D and the bugs nr. 1 and 2 apply only to 2D documents. The same as in 2D above, comments after the parameters.

$b=1$

$a=1$

$$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\sin t+b\cos u \\ 2t \\ b\sin u+2\cos t \end{bmatrix}, t=0\ldots2\pi, u=0\ldots2\pi$$

$c=3$

Folded Torus

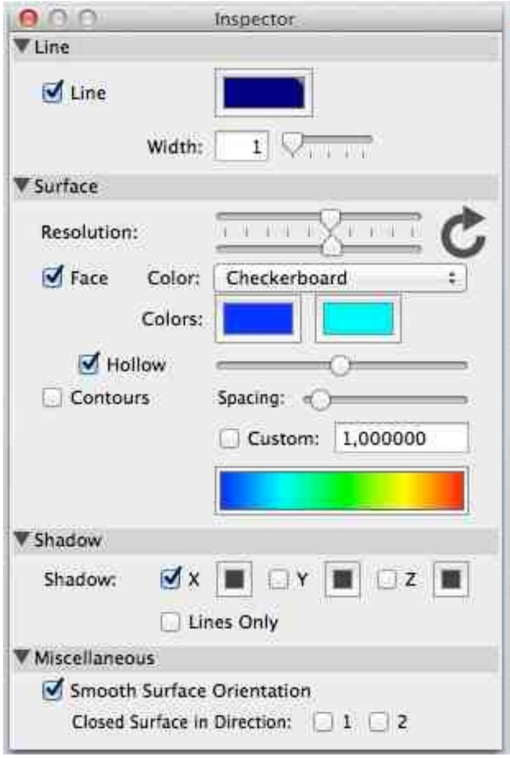
Modifying, customizing 3D curves and surfaces.

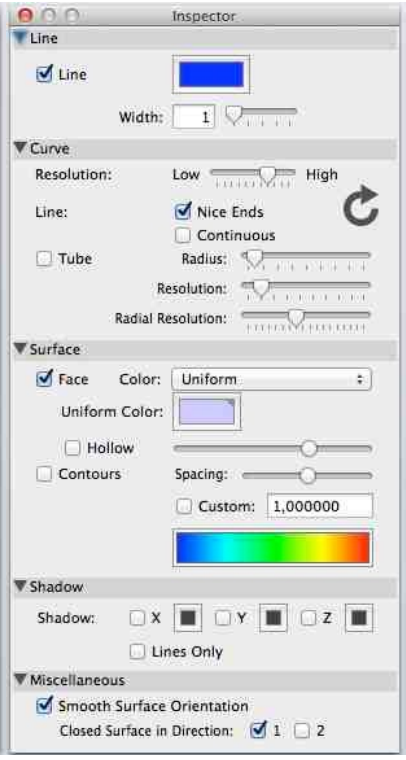
Click on the equation > open the Inspector > various settings of the surface.
The following figures show you : left side, the Inspector assigned to the work in progress (Folded Torus), right side, another type of Inspector for a 3D curve, below a picture of the final graph.

Layout of the 3D graph.

The commands at your disposal are :

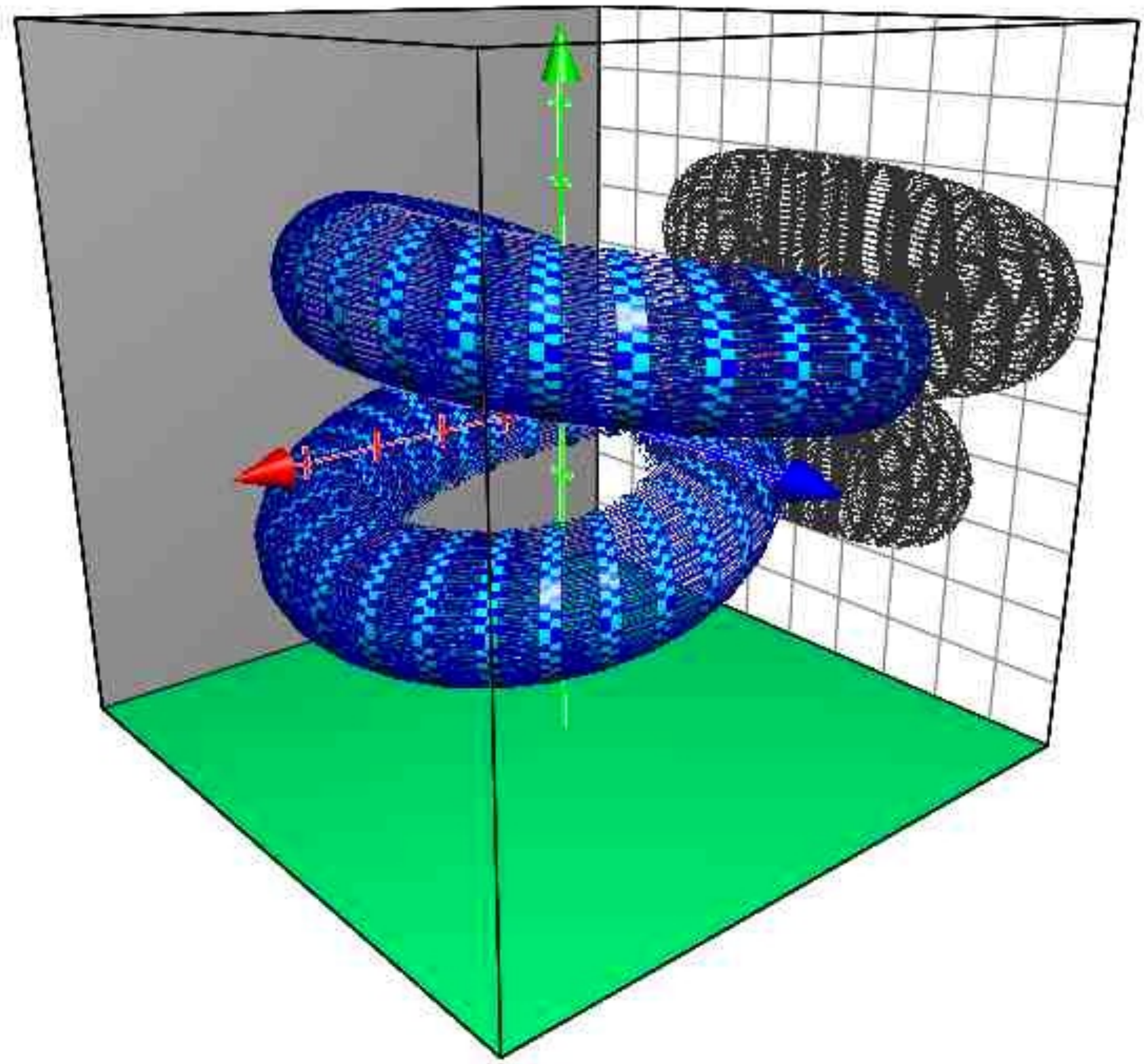
— ⇧mouse to zoom axes-frames-curves-surfaces ;
— ⌘mouse to center the whole ;
— ⌥mouse to rotate the image around an axis perpendicular to the plane of the screen and passing through the origin of axes;
— mouse for rotations around axes in the plane of the screen and passing through the origin of the axes.





Try to reproduce this aspect of the graph using the above commands, the Inspector of the frame and that of the surface (the figure nearby shows the settings adopted for the surface).

Save your work !



The 3D animation pop-up window.

Menu Equation > Create Animation… > pop-up window “Animation“ for its settings.

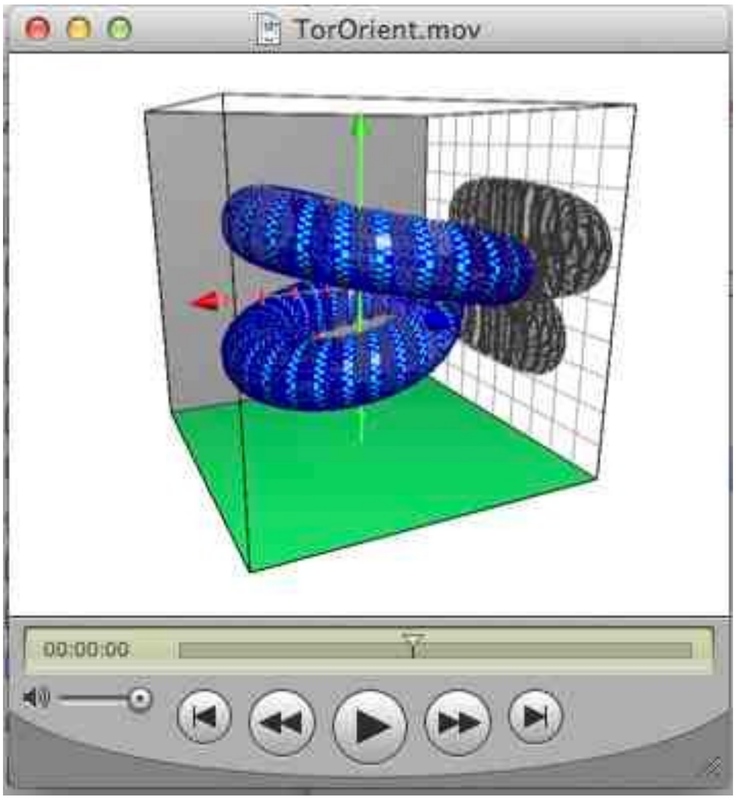
There is only one possible animation using this procedure : the **“Orientation“** tab animation of the animation window shown nearby. **(See bug nr. 16)**

The two small pictures of the graph obey the orders of layouts of the latter and allow to build an animation **by choosing two different views > Create Animation >** then **:**

- since Grapher 2.5, **name and save the animation as a QuickTime file (.mov).**
- before Grapher 2.5, **window to test the animation > possible saving in a Quick Time file (.mov).**

In 3D parameter animations are allowed using the procedure “menu Equation > Animate Parameter“ already studied in Lesson 3 for 2D graphs.

Use the Orientation tab, choose the initial and final pictures, create your animation, test it, save it and view the Quick Time file in this application as shown below :



Congratulations! You learned another way to create animations, and more important, how to use Grapher in 3D. Now you know a lot about this application, however, **do not forget that Grapher has many bugs and it is recommended that you frequently go back and reread the chapter that describes them.**

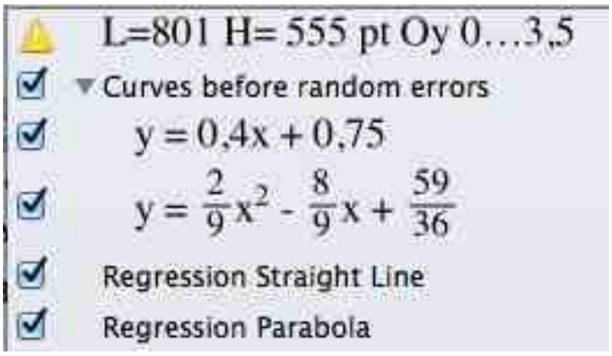
Lesson 6 : Treating a point set (Regression curve).

In this lesson you will learn how to draw graphs of points, and in more detail:

— to define sets of points ;
— to enter manually the data of a point set ;
— to import the data from a TXT file (.txt), formats for these files ;
— to export the data of a Grapher point set ;
— to operate measurements or statistical data and calculate regression curves ;
— to draw figures using the point sets.

Layout of the document.

Create a 2D document model Margins, size and axes as defined nearby, enter the equations shown.

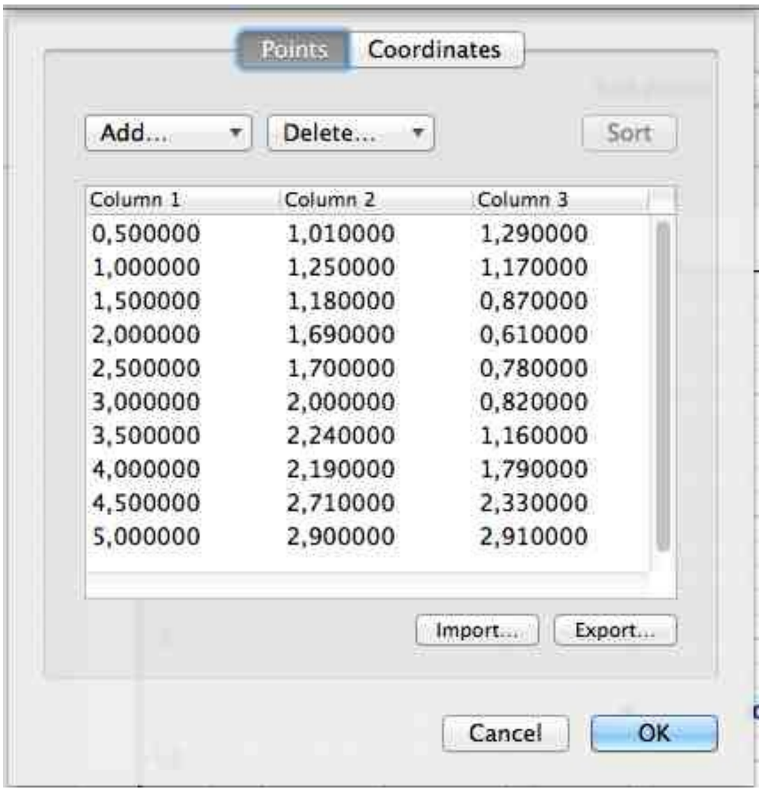


To enter the two lines **Sets of Points**, here called “Regression Straight Line“ and “Regression Parabola“ :
— either menu Equation > New Point Set ;
— or click the button [+] at the bottom of the equation list > contextual menu > New Point Set ;
— or ⌘⌥P (keyboard shortcut).

Titling a point set : double click on its line > type the name.

The Grapher point set.

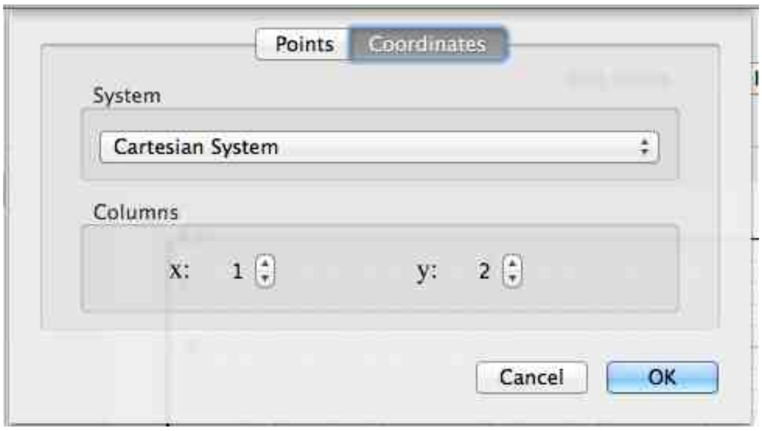
It is an array of coordinates of points of the graph in 2D or 3D : one row per point, number of columns greater than or equal to the number of coordinates, assignment of a column to each coordinate, coordinate system by choice (see figure nearby).



Entering manually the point data.

Click on the point set (the first one) **in the list of equations > Edit Points… > Points > select the existing lines (use the ⇧ or ⌘ keys) > remove them (Backspace key) > OK,** that delete the default points from the graph.
Edit Points… > Points > using the contextual menus of Add and Delete, create the number of rows (here 10) **and of columns** (here 3) **in accordance with the number of points and that of their coordinates.**
By double-clicking on each number enter the values of the table nearby > tab Coordinates > Cartesian System > assign column 1 to x and 2 to y > OK > the 10 points appear on the graph.

You entered 30 coordinates for 10 points into three columns : the first one for the abscissae, the other two to be chosen for the ordinates, that will eventually allows two clouds of points on the graph.



Importing point data from a TXT (.txt) file.

The above data were copied in TXT files (.txt) (see the next two figures) in two of the formats, in columns and on the same line, suitable for Grapher.

(Now, see APPENDIX 1 : “From the spreadsheet to Grapher“)

FORMAT IN COLUMNS (Tab Delimited Text) : one column for each coordinate, one line for each point, coordinates separated by Tabulation keystrokes. **To import : Edit points… > remove default data, set number of columns > Import… > uncheck Use Special Column Separator and Row Separator > select the TXT file .txt > Open > OK > Edit Points… > Coordinates tab > Cartesian System > assign column 1 to x and 2 to y > OK > the 10 points are plotted on the graph.**

0.50	1.01	1.29
1.00	1.25	1.17
1.50	1.18	0.87
2.00	1.69	0.61
2.50	1.70	0.78
3.00	2.00	0.82
3.50	2.24	1.16
4.00	2.19	1.79
4.50	2.71	2.33
5.00	2.90	2.91

FORMAT IN LINE : successive points separated by a semicolon, for each point coordinates in the same order separated by a comma. **To import : same as above but check Use Special Column Separator “ , “ and Use Special Row Separator “ ; “.**

Leçon6-32-2FR.txt — Modifié

```
0.50,1.01,1.29;1.00,1.25,1.17;1.50,1.18,0.87;2.00,1.69,0.61;2.50,1.70,0.78;
3.00,2.00,0.82;3.50,2.24,1.16;4.00,2.19,1.79;4.50,2.71,2.33;5.00,2.90,2.91
```

Warning ! Avoid confusion between decimal point or comma and columns or rows separators.

Exporting point data from Grapher.

Select the point set> Edit Points… > Export… > choose options of separators and number of digits (check), a name and a place (Desktop) for the file > Save.

You'll get the figure nearby with decimal commas if your computer is configured in French.

Expor

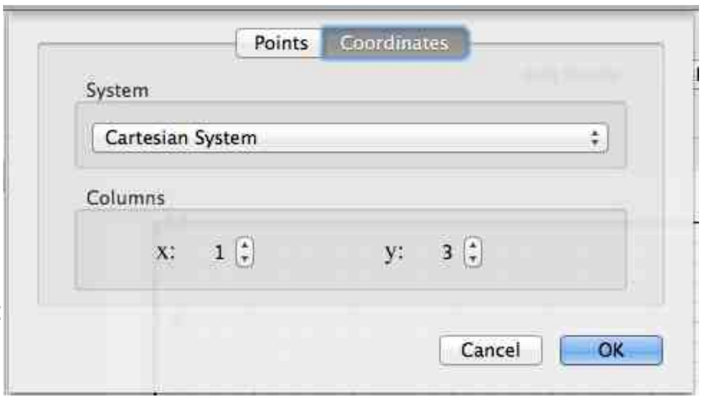
0.500000	1.010000	1.290000
1.000000	1.250000	1.170000
1.500000	1.180000	0.870000
2.000000	1.690000	0.610000
2.500000	1.700000	0.780000
3.000000	2.000000	0.820000
3.500000	2.240000	1.160000
4.000000	2.190000	1.790000
4.500000	2.710000	2.330000
5.000000	2.900000	2.910000

Tip to replace a character by an other one in a TXT file (.txt) :

Double-click > opening in TextEdit > menu Edit > Find > Find and Replace… > Find “ , “ Replace with “ . “ > Replace All > menu File > save it.

Entering the data of the second point set of your document.

Apply to the file you just saved the procedure described above in « FORMAT IN COLUMNS" ; only change : **assign column 3 to y** as shown in the figure nearby.

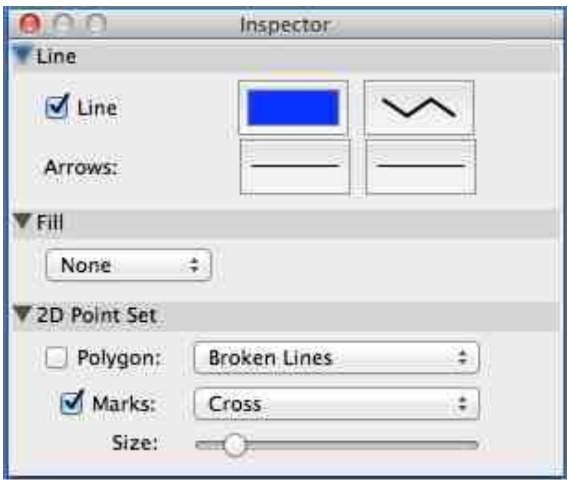


Customizing points and curves.

Select the set “Regression Straight Line“ by clicking in the list of equations or on one of its points on the graph > open its Inspector (screenshot nearby).

Uncheck Polygon (lines between the points) ; check Marks and Line, choose Cross, set their size ; click the Zigzag button, set Line Width to 2.0 ; click the Color button, choose Apple Blue ; Fill : None.

Click in the graph to deselect.



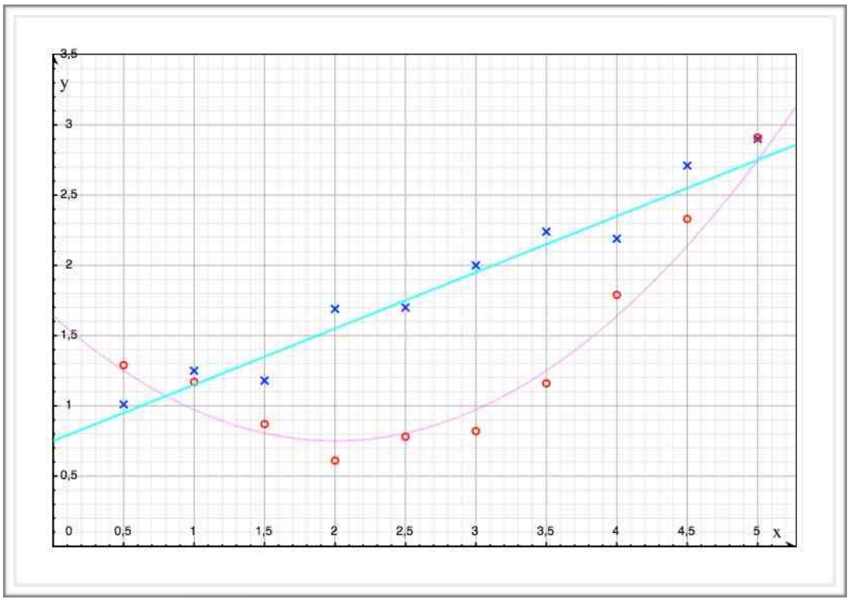
Note : the two buttons Color and Zigzag in the Line part of the Inspector work on the lines of Marks and Polygons ; Fiil is used for the interior of the marks Circle, Square, Diamond.

Do the same task with the point set “ Regression Parabola“, choose Mark “Circle“, color Apple Red.

For the line plotted from the first equation : line width 2.0, Apple Cyan ;
For the parabola from the second equation : line width 2.0, Apple Magenta.

SAVE YOUR DOCUMENT FREQUENTLY.

Your graph should look like this figure :



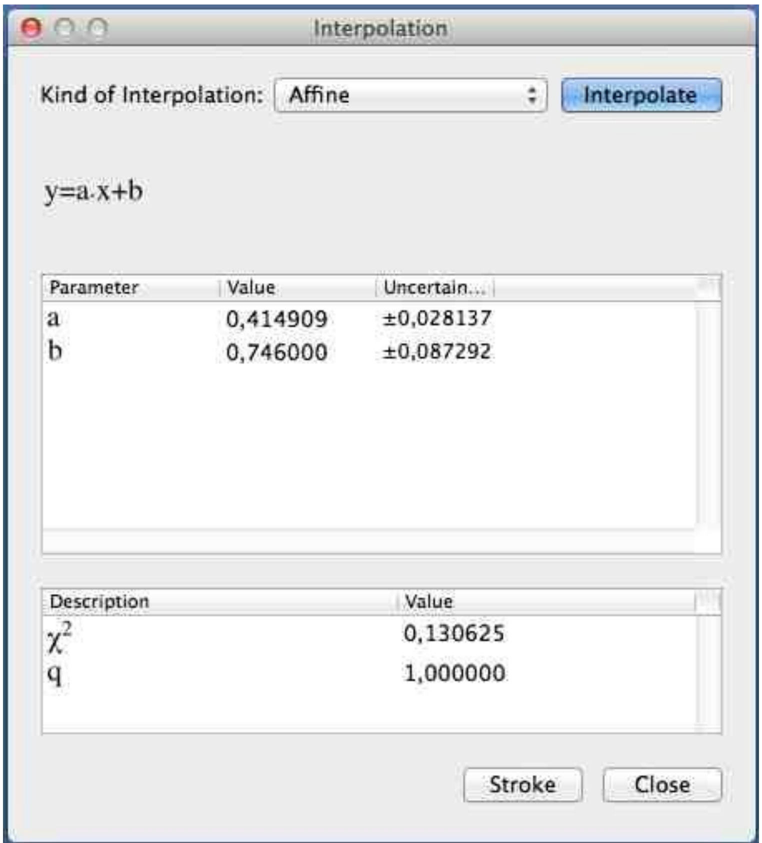
What problems to solve with the sets of points ?

First example : a physical phenomenon measured by a quantity y depends on a setting measured by the quantity x ; the devices measure x and y with errors. We search, using the graph, the function y = f (x) the most relevant.
Second example: a statistical study suggests that a characteristic measured by the quantity y seems to depend on another quantity x. We search with the graph for a suitable function y = f (x) and an assessment of its relevance.

What Grapher can do? It uses the points (x, y) and a type of curve selected by the user, and calculates the equation of the curve that minimizes the sum χ^2 of squared deviations along the ordinate axis between the measured points and this curve. In other words, Grapher determine the regression curve of y with respect to x according to the least squares criterion ; Grapher display the sum χ^2 of squared deviations ; if the curve is a straight line, a trick allows the calculation of the regression of x with respect to y, and to deduce the correlation coefficient between x and y ; and last but not least the software plots the curve on the graph.

Where do our points come from ? Starting from the equations of a line and a parabola (the two equations you entered in the list), we chose 10 values of x (number low enough that allows to fill up by hand the point set data array), programmed calculations of y values for each x by Numbers using the equations and adding random quantities from -0.25 to +0.25.

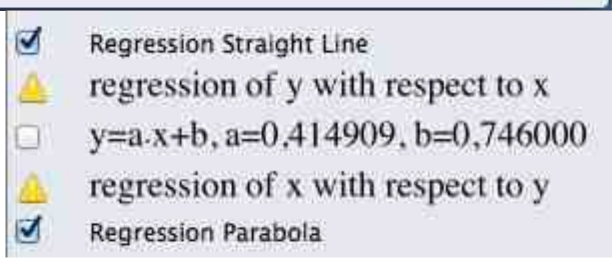
Calculating the regression line (linear regression).



Uncheck the two equations and the second set of points “Regression Parabola“ in the list to remove their curves and points on the graph. Check the set of points “Regression Straight Line“.
Looking at the cloud of points, it appears that a straight line would represent fairly well the relationship between y and x ; if you agree with me on this idea, enter a new equation similar to the first one $y = 0.4x + 0.75$ and try to adjust the two coefficients for the line to adapt best to the points.

Now let Grapher do the job for you :

Select the point set “ Regression Straight Line“ > Interpolation… > Kind of Interpolation : Affine > Interpolate > Stroke > the Interpolation window (figure nearby) shows you the results :
— a = regression coefficient of y with respect to x ;
— b = ordinate of the line when x = 0 ;
— margins of error of these two coefficients ;
— χ^2 = sum of the squared deviations along Oy between the points and the regression line.
Note : the result “ q “ is always 1 and is here for the affine interpolation only; What does it mean ?
The regression line is plotted on the graph ; customize it with the Inspector, line width 2.0, color Apple Blue, drag and drop its equation below the line of its point set.



Add two lines of comments in the list of equations as shown on the screenshot nearby.

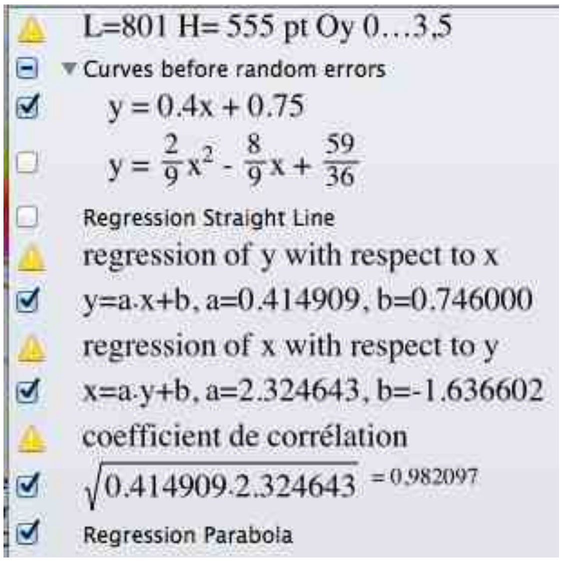
Finally : **click the button “ Close“ of the Interpolation window.**

Calculating the regression line of x with respect to y.

Grapher knows only the regression of y with respect to x (deviation along Oy) ; we want now a regression minimizing the squared deviations along Ox. Hence do the following :

Select the point set “Regression Straight Line“ > Edit points… > Coordinates > assign column 2 to x and column 1 to y > OK, the points in the graph take a position symmetrical with respect to the first bisectrix y = x **> Interpolation… > Affine > Interpolate > Stroke > Close,** new equation of line in the list, plotted on the graph.

Select the new equation > interchange y and x > Return key > drag it to the right place (see figure nearby) ; it is the equation of the regression line of x with respect to y, a is the regression coefficient of x with respect to y ; customize it : width 2.0, color Apple Green ; on the graph the two regression lines are very close, close also to the straight line of “Curves Before Random Errors“.



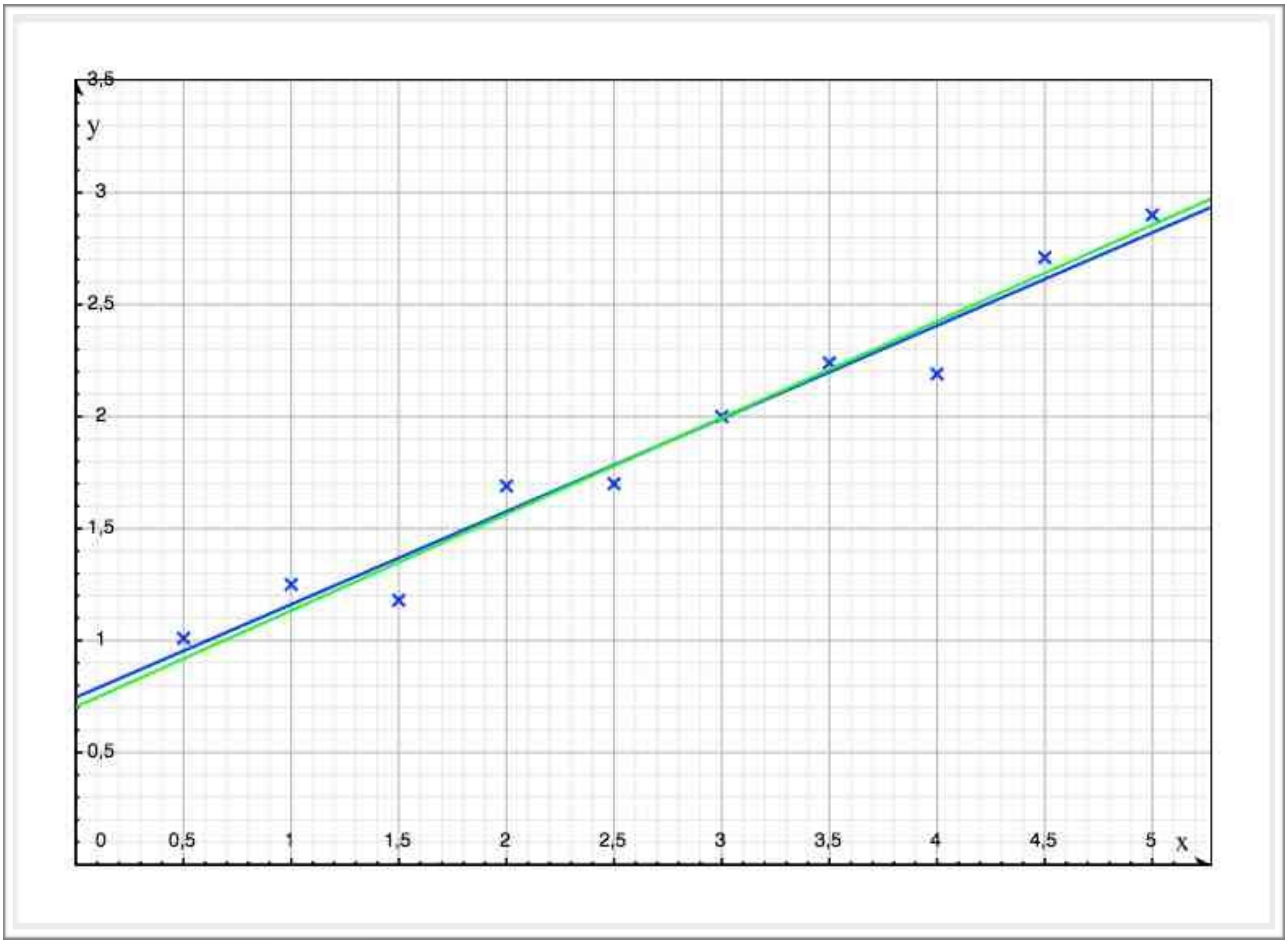
Select the point set > restore the normal assignments of the columns, 1 to x and 2 to y.

Calculating the linear correlation coefficient between x and y.

Its absolute value is the square root of the product of the regression coefficients of y with respect to x and of x with respect to y ; the closer to 1, the better the correlation. Do the calculation in Grapher inspiring you with the list of the equations above.

SAVE YOUR DOCUMENT FREQUENTLY.

Here is what your graph should look like now (with the points and the two regression lines) :



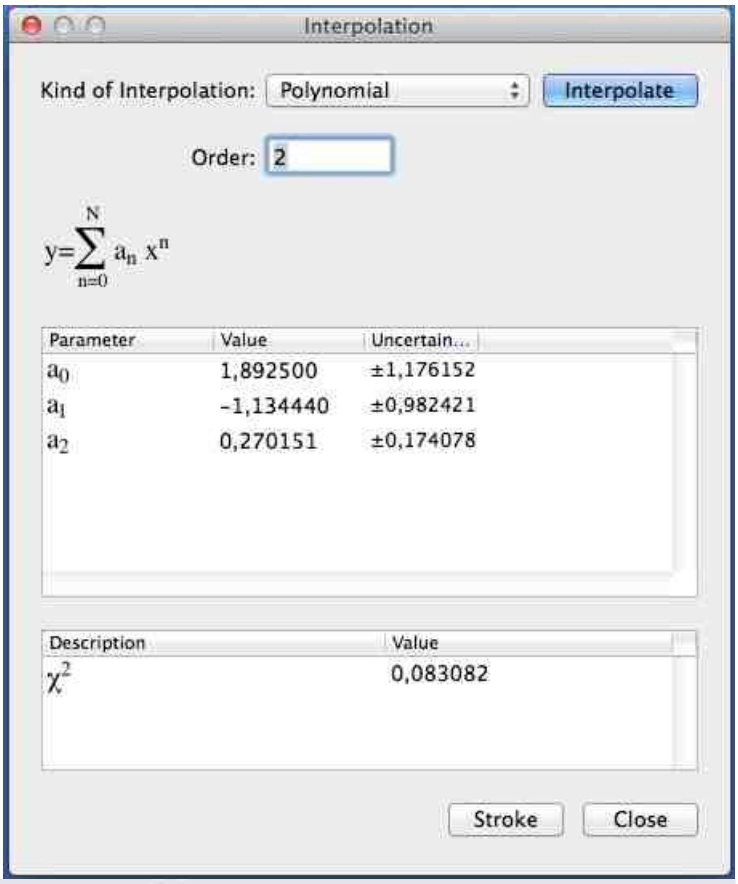
Note: the results provided by Grapher for the two regression lines were verified by manual calculation, the values of their coefficients a, b, their standard deviations, χ^2 , are exact (accurate to six significant digits).

Calculating a non linear regression curve.

You are going to use the second set of points to calculate a regression curve, other than a straight line, according to the criterion of least squares.

In the list of equations, uncheck all but the point set “Regression Parabola“, then :

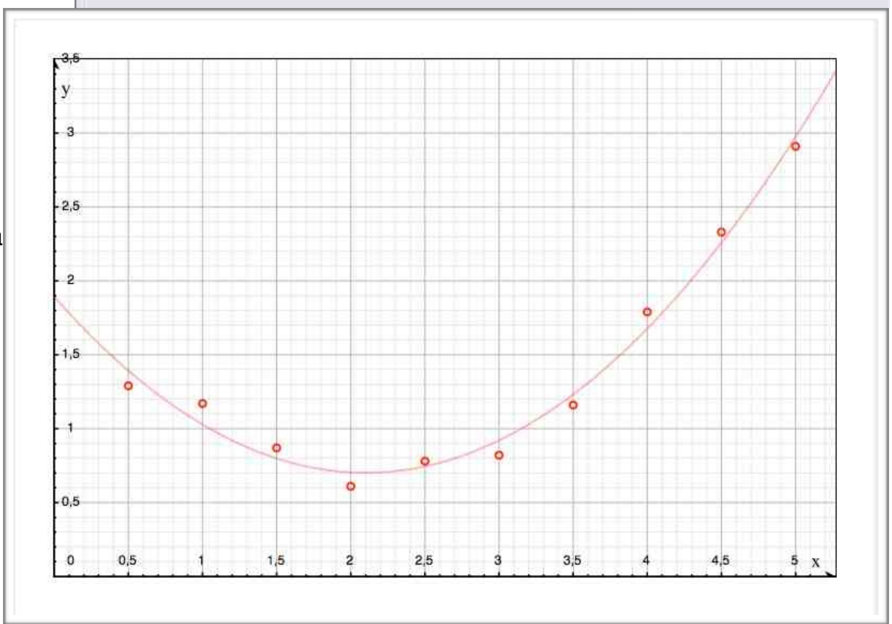
Select this point set > Interpolation… > try successively :
— Affine : χ^2 = 2,4915 and graph unsuitable ;
— Exponential : χ^2 = 0,6148 graph a little better ;
— Polynomial > Order 5 > Return key : very good graph, χ^2 = 0,0299, but complicated equation ;
— Polynomial > Order 4 > Return key : very good graph, χ^2 = 0,0279, better but still complicated ;
— Polynomial > Order 3 > Return key : very good graph, χ^2 = 0,0826, but still too much complicated ;
— Polynomial > Order 2 > Return key : graph satisfactory, χ^2 = 0,08319, rather simple equation ; this option should please you, then **> Stroke > Close,** that plots the graph and enter the regression curve equation (parabola) into the list. Results in the pop-up window nearby.



Customize the curve : width 2.0, color Apple Red.

The equation of the regression curve is now in the list of equations as seen in the figure nearby :

Here is the graph corresponding to the points and the parabola of regression :



By checking the second equation of the list you can compare this parabola to that used as a basis to calculate the point set ; they would be closer if we had a larger number of points.

It is possible to propose that Grapher calculate a custom regression equation with a list of coefficients.

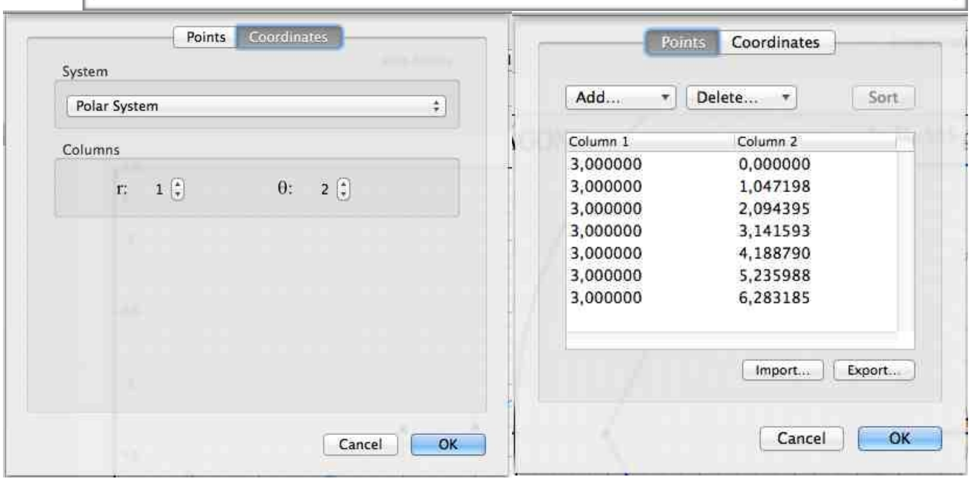
The point sets are also available in 3D with calculations of surfaces of regression.

Drawing with the point sets.

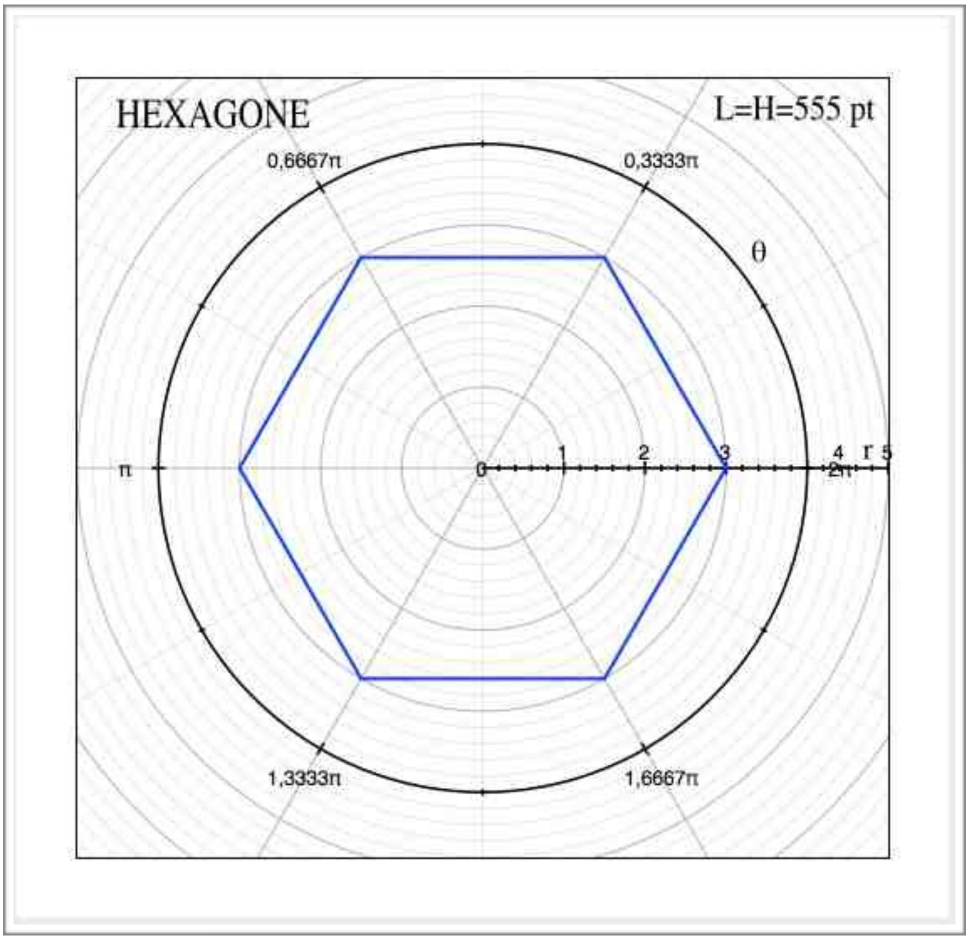
The “Polygon“ option of the point set Inspector, when checked, joins up the dots with lines, in the order of the data table.

The polar angles were entered as “ n.π / 3 “ with n integer from 0 to 6 that is 1 once more to complete the figure.

To get π with the keyboard, type “ ⌥P “.



Try to reproduce this polar graph using informations on these three screenshots, options in menu Format, Inspectors of axes and point sets, the equation editor and commands in the menu Object.



The next page, “Annex to Lesson 6“, shows the equations used by Grapher to treat the linear regression.

You have just completed this sixth and final lesson of initiation into Grapher. You have shown determination and patience, that deserves applause !

This application comes with examples (beware of **bugs nr. 6.1 et 6.2)** which will show you other tasks Grapher can do, scalar fields, vector fields, sequences, and layout tips, rules of syntax of mathematical expressions, etc.

The two **following chapters** will detail the **coordinate systems** and the **syntax of expressions**.

Annex to Lesson 6 : linear regression formulae

Definitions.

n measurement points (x_i, y_i) $\quad \overline{x} = \frac{1}{n}\sum_1^n x_i \qquad \overline{y} = \frac{1}{n}\sum_1^n y_i$

a_y regression coefficient of y with respect to x

a_x regression coefficient of x with respect to y

Regression line of y with respect to x.

$$a_y = \frac{\sum_1^n (x_i - \overline{x})(y_i - \overline{y})}{\sum_1^n (x_i - \overline{x})^2} \text{ ; } b_y = \overline{y} - a_y \cdot \overline{x} \text{ ; equation : } Y = a_y X + b_y \text{ which minimize :}$$

$$\chi_y^2 = \sum_1^n (y_i - Y_i)^2 = \sum_1^n y_i^2 - b_y \cdot \sum_1^n y_i - a_y \cdot \sum_1^n x_i y_i \text{ ; with } Y_i = a_y x_i + b_y$$

Regression line of x with respect to y.

$$a_y = \frac{\sum_1^n (x_i - \overline{x})(y_i - \overline{y})}{\sum_1^n (y_i - \overline{y})^2} \text{ ; } b_x = \overline{x} - a_x \cdot \overline{y} \text{ ; equation : } X = a_x Y + b_x \text{ which minimize :}$$

$$\chi_x^2 = \sum_1^n (x_i - X_i)^2 = \sum_1^n x_i^2 - b_x \cdot \sum_1^n x_i - a_x \cdot \sum_1^n x_i y_i \text{ ; with } X_i = a_x y_i + b_x$$

Correlation coefficient.

$$r_{xy} = \frac{\sum_1^n (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_1^n (x_i - \overline{x})^2 \cdot \sum_1^n (y_i - \overline{y})^2}} = \pm\sqrt{a_y \cdot a_x} \text{ ;}$$

if ± 1 perfect correlation between x and y

if 0 no correlation between x and y

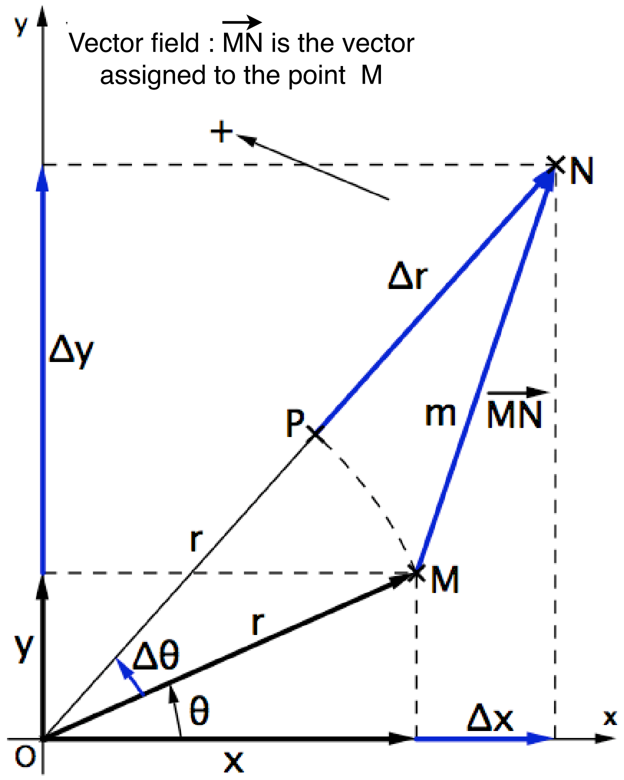
Errors, standard deviations.

Of measurements y_i : $\widehat{\sigma_y} = \sqrt{\frac{\chi_y^2}{n-2}}$; of a_y : $\widehat{\sigma_{ay}} = \frac{\widehat{\sigma_y}}{\sqrt{\sum_1^n (x_i - \overline{x})^2}}$; of b_y : $\widehat{\sigma_{by}} = \frac{\widehat{\sigma_y}}{\sqrt{\sum_1^n (x_i - \overline{x})^2}} \cdot \sqrt{\frac{1}{n}\sum_1^n x_i^2}$

Of measurements x_i : $\widehat{\sigma_x} = \sqrt{\frac{\chi_x^2}{n-2}}$; of a_x : $\widehat{\sigma_{ax}} = \frac{\widehat{\sigma_x}}{\sqrt{\sum_1^n (y_i - \overline{y})^2}}$; of b_x : $\widehat{\sigma_{bx}} = \frac{\widehat{\sigma_x}}{\sqrt{\sum_1^n (y_i - \overline{y})^2}} \cdot \sqrt{\frac{1}{n}\sum_1^n y_i^2}$

Grapher’s coordinate systems

2D coordinates of points and vectors.



2D coordinates.

Cartesian : $M\ (x, y)$; $\overrightarrow{MN}\ (\Delta x, \Delta y)$; $N\ (x+\Delta x, y+\Delta y)$

Polar : $M\ (r, \theta)$; $\overrightarrow{MN}\ (\Delta r, \Delta\theta)$; $N\ (r+\Delta r, \theta+\Delta\theta)$

Vector magnitude : $\left|\overrightarrow{MN}\right| = m$

Names and domains :

abscissa x, ordinate y :]-∞ ; +∞[

radial r :]-∞ ; +∞[input, [0 ; +∞[output

angular (polar angle) θ: [0 ; 2π] or [-π ; +π] at the option, together with atan2(y,x), in the Preferences.

Conversion of the point coordinates.

$$\begin{cases} x = r\cdot\cos\theta \\ y = r\cdot\sin\theta \end{cases} \Leftrightarrow \begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \operatorname{atan2}(y,x) \end{cases}$$

Conversion of the vector coordinates.

Approximate expressions (USED IN GRAPHER) :

$$\begin{cases} \Delta x = \Delta r\cdot\cos\theta - r\cdot\sin\theta\cdot\Delta\theta \\ \Delta y = \Delta r\cdot\sin\theta + r\cdot\cos\theta\cdot\Delta\theta \end{cases}$$

they are exact if Δx, Δy, Δr, Δθ are infinitesimals ; they will be accurate enough if Δx, Δy, Δr, Δθ are small, i.e. if m << r or Δx/x, Δy/y, Δr/r << 1 and Δθ << 1 rad .

But the exact expressions (NOT USED IN GRAPHER) should be :

$$\begin{cases} \Delta x = (r+\Delta r)\cdot\cos(\theta+\Delta\theta) - r\cdot\cos\theta \\ \Delta y = (r+\Delta r)\cdot\sin(\theta+\Delta\theta) - r\cdot\sin\theta \end{cases} \Leftrightarrow \begin{cases} \Delta r = \sqrt{(x+\Delta x)^2+(y+\Delta y)^2} + \sqrt{x^2+y^2} \\ \Delta\theta = \operatorname{atan2}(y+\Delta y,\ x+\Delta x) - \operatorname{atan2}(y,\ x) \end{cases}$$

Vector $\overline{MN}$ magnitude (exact expression) : $m = \sqrt{(\Delta x)^2+(\Delta y)^2} = \sqrt{r^2+(r+\Delta r)^2 - 2r\cdot(r+\Delta r)\cdot\cos\Delta\theta}$

Impact on the plot (graph) of vector fields.

The plots are calculated from the Δx Δy entered to define the vector field, or obtained by the approximate conversion expressions if the inputs are Δr Δθ ; in the latter case the graph will be more imprecise if the vector magnitude m is no longer negligible compared to r.

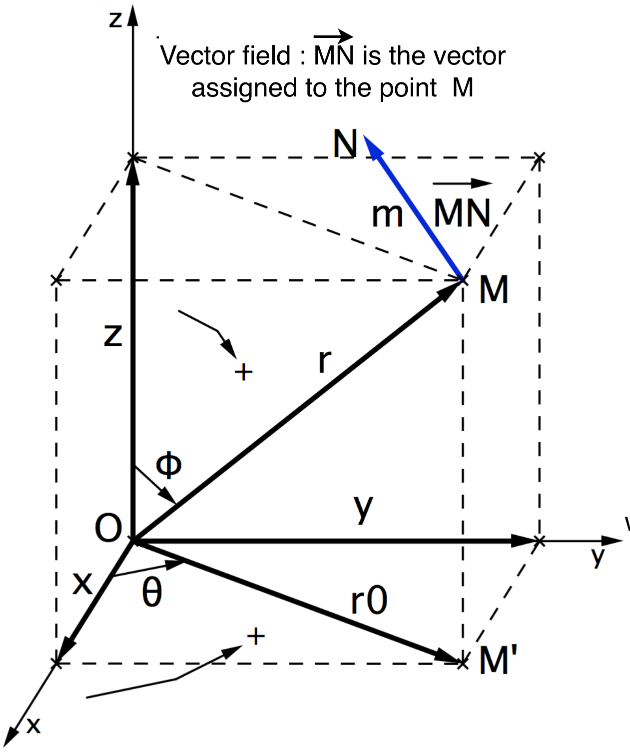
FOR EXACT VECTOR FIELD PLOTS, DEFINE THEM WITH Δx AND Δy.

On the contrary the numerical evaluations of a vector field are always exact being calculated directly with the equations entered to define it , regardless of coordinates (Δx, Δy) ou (Δr, Δθ).

Coordinate names : they are fixed in Grapher (Curvus Pro X allowed to change them) : **see bug nr. 6.1.**

Display or hide a grid or an axis : menu Format > Axes & Grids… > check or uncheck.

3D coordinates of points and vectors.



3D coordinates.

Cartesian : $M(x,y,z)$; $\overrightarrow{MN}\ (\Delta x,\Delta y,\Delta z)$; $N(x+\Delta x,y+\Delta y,z+\Delta z)$

Cylindrical : $M(r_0,\theta,z)$; $\overrightarrow{MN}\ (\Delta r_0,\Delta\theta,\Delta z)$; $N(r_0+\Delta r_0,\theta+\Delta\theta,z+\Delta z)$

Spherical : $M(r,\theta,\phi)$; $\overrightarrow{MN}\ (\Delta r,\Delta\theta,\Delta\phi)$; $N(r+\Delta r,\theta+\Delta\theta,\phi+\Delta\phi)$

Vector magnitude : $\left|\overline{MN}\right| = m$

Names and domains :

abscissa x, ordinate y, altitude z :]-∞ ; +∞[

radial r_0 in xOy plane, r in space :]-∞ ; +∞[input, [0 ; +∞[output

azimuthal θ in xOy plane : [0 ; 2π] or [-π ; +π] at the option, together with atan2(y,x), in the Preferences

polar (co-latitude) ϕ : [0 ; π]

Conversion of the point coordinates.

Cartesian ⇔ Cylindrical

$$\begin{cases} x = r_0\cdot\cos\theta \\ y = r_0\cdot\sin\theta \\ z \end{cases} \Leftrightarrow \begin{cases} r_0 = \sqrt{x^2+y^2} \\ \theta = \operatorname{atan2}(y,x) \\ z \end{cases}$$

Cylindrical ⇔ Spherical

$$\begin{cases} r_0 = r\cdot\sin\phi \\ \theta \\ z = r\cdot\cos\phi \end{cases} \Leftrightarrow \begin{cases} r = \sqrt{r_0^2+z^2} \\ \theta \\ \phi = \operatorname{Arccos}\dfrac{z}{\sqrt{r_0^2+z^2}} \end{cases}$$

$$\begin{cases} x = r\cdot\sin\phi\cdot\cos\theta \\ y = r\cdot\sin\phi\cdot\sin\theta \\ z = r\cdot\cos\phi \end{cases} \Leftrightarrow \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \operatorname{atan2}(y,x) \\ \phi = \operatorname{Arccos}\dfrac{z}{\sqrt{x^2+y^2+z^2}} \end{cases}$$

← Cartesian ⇔ Spherical

Conversion of the vector coordinates.

Approximate expressions (USED IN GRAPHER) :

Cylindrical ⇒ Cartesian

$$\begin{cases} \Delta x = \cos\theta\cdot\Delta r_0 - r_0\cdot\sin\theta\cdot\Delta\theta \\ \Delta y = \sin\theta\cdot\Delta r_0 + r_0\cdot\cos\theta\cdot\Delta\theta \\ \Delta z = \Delta z \end{cases}$$

Spherical ⇒ Cartesian

$$\begin{cases} \Delta x = (\sin\phi\cdot\Delta r + r\cdot\cos\phi\cdot\Delta\phi)\cdot\cos\theta - r\cdot\sin\phi\cdot\sin\theta\cdot\Delta\theta \\ \Delta y = (\sin\phi\cdot\Delta r + r\cdot\cos\phi\cdot\Delta\phi)\cdot\sin\theta + r\cdot\sin\phi\cdot\cos\theta\cdot\Delta\theta \\ \Delta z = \cos\phi\cdot\Delta r - r\cdot\sin\phi\cdot\Delta\phi \end{cases}$$

they are exact if Δx, Δy, Δz, Δr_0, Δr, Δθ, Δϕ are infinitesimals ; they will be accurate enough if Δx, Δy, Δz, Δr_0, Δr, Δθ, Δϕ are small, i.e. if m << r or Δx/x, Δy/y, Δz/z, $\Delta r_0/r_0$, Δr/r << 1 and Δθ, Δϕ <<1rad ..

Vector $\overline{MN}$ magnitude (exact expressions) :

$$\overline{MN} = m = \sqrt{(\Delta x)^2+(\Delta y)^2+(\Delta z)^2} = \sqrt{r_0^2+(r_0+\Delta r_0)^2-2r_0\cdot(r_0+\Delta r_0)\cdot\cos\Delta\theta+(\Delta z)^2}$$

$$= \sqrt{r^2+(r+\Delta r)^2-2r\cdot(r+\Delta r)\cdot\left(\sin(\phi+\Delta\phi)\cdot\sin\phi\cdot\cos\Delta\theta+\cos(\phi+\Delta\phi)\cdot\cos\phi\right)}$$

But the exact expressions (NOT USED IN GRAPHER) should be :

Cartesian ⇔ Cylindrical

$$\begin{cases} \Delta x = (r_0 + \Delta r_0)\cdot\cos(\theta+\Delta\theta) - r_0\cdot\cos\theta \\ \Delta y = (r_0 + \Delta r_0)\cdot\sin(\theta+\Delta\theta) - r_0\cdot\sin\theta \\ \Delta z \end{cases} \Leftrightarrow \begin{cases} \Delta r_0 = \sqrt{(x+\Delta x)^2 + (y+\Delta y)^2} - \sqrt{x^2+y^2} \\ \Delta\theta = \mathrm{atan2}(x\cdot\Delta y - y\cdot\Delta x,\, x^2+y^2+x\cdot\Delta x + y\cdot\Delta y) \\ \Delta z \end{cases}$$

Cartesian ⇔ Spherical

$$\begin{cases} \Delta x = (r+\Delta r)\cdot\sin(\phi+\Delta\phi)\cdot\cos(\theta+\Delta\theta) - r\cdot\sin\phi\cdot\cos\theta \\ \Delta y = (r+\Delta r)\cdot\sin(\phi+\Delta\phi)\cdot\sin(\theta+\Delta\theta) - r\cdot\sin\phi\cdot\sin\theta \\ \Delta z = (r+\Delta r)\cdot\cos(\phi+\Delta\phi) - r\cdot\cos\phi \end{cases} \Leftrightarrow \begin{cases} \Delta r = \sqrt{(x+\Delta x)^2 + (y+\Delta y)^2 + (z+\Delta z)^2} - \sqrt{x^2+y^2+z^2} \\ \Delta\theta = \mathrm{atan2}(x\cdot\Delta y - y\cdot\Delta x,\, x^2+y^2+x\cdot\Delta x + y\cdot\Delta y) \\ \Delta\phi = \mathrm{Arccos}\dfrac{z+\Delta z}{\sqrt{(x+\Delta x)^2 + (y+\Delta y)^2 + (z+\Delta z)^2}} - \mathrm{Arccos}\dfrac{z}{\sqrt{x^2+y^2+z^2}} \end{cases}$$

Cylindrical ⇔ Spherical

$$\begin{cases} \Delta r_0 = (r+\Delta r)\cdot\sin(\phi+\Delta\phi) - r\cdot\sin\phi \\ \Delta\theta \\ \Delta z = (r+\Delta r)\cdot\cos(\phi+\Delta\phi) - r\cdot\cos\phi \end{cases} \Leftrightarrow \begin{cases} \Delta r = \sqrt{(r_0+\Delta r_0)^2 + (z+\Delta z)^2} - \sqrt{r_0^2+z^2} \\ \Delta\theta \\ \Delta\phi = \mathrm{Arccos}\dfrac{z+\Delta z}{\sqrt{(r_0+\Delta r_0)^2 + (z+\Delta z)^2}} - \mathrm{Arccos}\dfrac{z}{\sqrt{r_0^2+z^2}} \end{cases}$$

Impact on the plot (graph) of vector fields.

The plots are calculated from the Δx Δy Δz entered to define the vector field, or obtained by the approximate conversion expressions if the inputs are Δr_0 Δθ Δz or Δr Δθ Δϕ ; in the latter case the graph will be more imprecise if the vector magnitude m is no longer negligible compared to r

FOR EXACT VECTOR FIELD PLOTS, DEFINE THEM WITH Δx Δy Δz.

On the contrary the numerical evaluations of a vector field are always exact being calculated directly with the equations entered to define it, regardless of coordinates (Δx, Δy, Δz), (Δr_0, Δθ, Δz) ou (Δr, Δθ, Δϕ).

Coordinate names : they are fixed in Grapher (Curvus Pro X allowed to change them) : **see bug nr. 6.2.**

Display or hide the frame or an axis : menu Format > Axes & Frame… > check or uncheck.
If the **frame** (cube) is on the graph, its **Inspector** allows to **display grids** on its faces.

Expressions

Expression syntax : general rules.

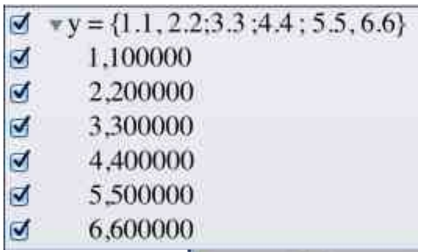
Number format.

— See “**The Display of numerical values**“ at the end of chapter **“Numerical calculations (evaluations)“**.
— Large numbers limited to 2^{1023} or 10^{308}, if the exponent is higher, Grapher display “indefinite“.
— Menu Grapher > Preferences… > number of decimal places, removing trailing zeros, standard or scientific or engineering notation, choice of i or j and of its place in complex numbers ;
— No separator every three digits, no point, no comma, no space ;
— Decimal comma in use (OS X set in French), without any space before nor after. Exceptions :
· **Decimal point (dot) required for the line width marker selected from the Inspectors,**
· **Decimal point (dot) required for the sequence settings** in the animation window of “Create Animation“,
· **Decimal point (dot) required in coordinates displayed** under the graph.
Note : it is often possible to replace the comma by the the decimal point, with OS X set in French.

Appropriate fields to receive numbers agree the **constants recognized by Grapher (e, π, etc.), parameters with values defined in the equation list, constant expressions.** For example if a = 8 is an equation, you can enter via the menu View > Frame Limits…, the abscissa scale limits as -a/3 a/4 ; these values will be calculated for a = 8, but will not follow the subsequent changes of a.

Angle units.

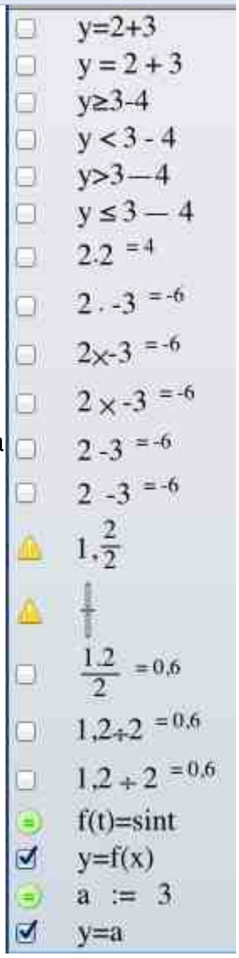
— Menu Grapher > Preferences… > Advanced > Miscellaneous > Polar Angular Range > choose **0 to 2π or -π to π** ;
— Menu Grapher > Preferences… > Advanced > Miscellaneous > Trigonometric Mode > choose **radians or decimal degrees or grades.**



Separators in a sequence of numbers, functions, expressions.

Either the couple **comma+space,** or the punctuation mark **semicolon** ; the additional spaces are allowed. Decimal commas (without spaces) and separators comma+space coexist unambiguously in a sequence of numbers.

In the figure nearby the y expression shows a very viable mixture of commas, semicolons and spaces in a multi-valued definition.



Signs of operations, equality, inequality.

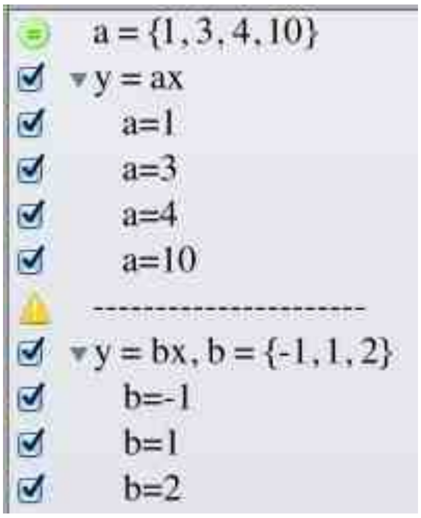
— The **signs + - . = > ≥ < ≥** may be preceded and followed **by spaces ;** one space placed before + or - indicate a multiplication, spaces before **and** after does not change the operation ;
— The **minus** sign is the dash either short or long : **- or —** obtained by **⌥- ;**
— **In definitions of constants and functions, the equal sign** is either **= or :=** (colon punctuation mark followed by the equal sign) ;
— The sign of **division / :** • typed **after a space,** places a fraction to be completed,
• typed **after a number or an expression,** places a **fraction bar** and a denominator field **under the number or the expression,**
• a decimal number with a decimal comma followed by the / division sign is not recognized and is displayed as “integer part-comma-decimal part alone as numerator of a fraction“ ; the same number with a decimal point is displayed correctly as numerator of a fraction ; **create the fraction with / before typing the dividend (numerator) if it’s a decimal number with decimal comma**, you’ll have to complete with the denominator (divisor),
• the equation palette proposes also the **÷ sign ;**
— **Multiplication** signs, they can be :
• the **point without any space before nor after**, placed between two numbers, and typed on the **keyboard (asterisk)**, or the **point** from the Symbols **of the equation palette**,
• the **classical x** proposed in this palette,
• **one or more spaces between two numbers or expressions or the juxtaposition of expressions in parentheses and of variables,**
• the **punctuation point (dot) followed by at least one space and placed between two integers** (not recognized as mathematical sign if one at least of the two numbers has decimals).

The figure nearby illustrates these many variants.

Punctuation point (dot) vs. multiplication point (dot).

— In a sequence of digits, without any space before nor after, the punctuation point is a **decimal point** ;
— Having the same look, between two numbers, without any space before nor after, **the multiplication point** is obtained in Grapher by typing the **keyboard multiplication sign (asterisk)** or clicking the **equation palette point** (Symbols subwindow). It is a little bit higher than the decimal dot.

Global and local definitions.

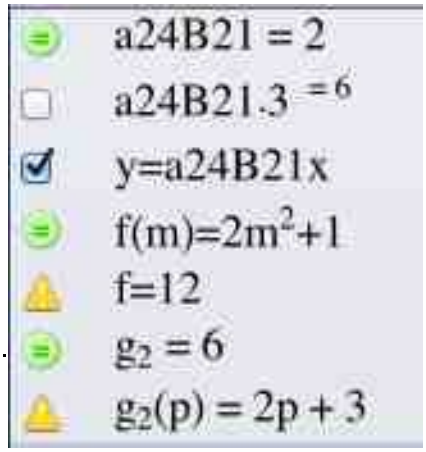


Global definition : The parameter is defined by a particular equation ; for example : a = {1, 3, 4,10} ; this definition may or may not be in a group ; it applies to all expressions of the list of equations containing the parameter **a**.
Local definition : the definition of the parameter is part of the equation that contains it ; for example : y = bx, b = {-1, 1, 2} ; this equation will graph three curves for b = -1 ; 1 and 2.

A parameter, variable or function that is in a global definition cannot be also in a local definition.
A parameter, variable or function that is not in a global definition can be in a different local definition in each line of equation.

Note. Before Grapher 2.2, global and local definitions can coexist, each local definition overrides the global.

Names of constants, variables, parameters and functions.

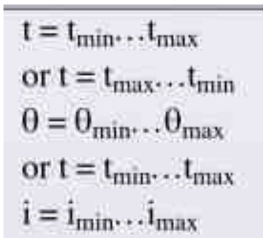


— The letters **x, y, z, r_0, r, θ, ϕ,** are assigned to the **2D & 3D coordinates** ;
— The **built-in definition names** (menu Help) are reserved for **constant and functions** of this list ;
— Are available for parameter and function names, the other letters, the other letters of the Greek alphabet (except ∂ et Δ), upper or lower cases, we can add indices and use words beginning with a letter including letters and digits ;
— The same name must not be used for a constant and a function.

This screenshot of a list of equations shows a few examples of names accepted or rejected.
(Grapher 2.2 & 2.5 : see bug nr. 28 for names of variables)

Defining the domains of the variables.

— The domain must be specified for :
- the parameter(s) of parametric equations (continuous domain) (maximum value, minimum value, in any order), ➽
- the variable of a differential equation (optionnal for coordinates) (continuous domain) (minimum value first), ➽
- the index of a sequence (successive integers) (minimum value first) ; ➽



— In other expressions, the domains of the variables (which are the coordinates of the graph) are chosen by Grapher :

- by the limits of the frame for x, y, z, r_0, r,
- 0 to 2π for θ,
- 0 to π for ϕ.

Multi-valued expressions.

It is possible to place several expressions in a single formula of the equation list ; the method is applicable to the definitions of constants, functions, equations of graphs, etc. : just place the different variants in braces.
Examples : y = 0.5{cosx, sinx, x} (3 curves) ; k = {1.1, 2.2, 3.3, 4, 6} (5 values for k) ; two ways for the same result in the following equations :

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2+\sin t \\ \{\cos 3t, \cos 5t\} \end{bmatrix}, t = 0\ldots 2\pi \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2+\sin t \\ \cos(\{3,5\}t) \end{bmatrix}, t = 0\ldots 2\pi$$

Note the separators between the values, and the absence of braces for the index of a sequence, exception shown in the figure above.

Syntax	Description	Examples
Curves		
Explicit		
$y = f(x)$ or $x = g(y)$	2D Cartesian coordinates	$y=\sin x$ or $x=\sin y$ or $y=x^2-2x+1$
$r = f(\theta)$ or $\theta = g(r)$	2D polar coordinates	$r = \sin(3\theta)$ or $\theta = r$
Implicit		
$f(x, y, r, \theta) = g(x, y, r, \theta)$	mixture of 2D coordinates	$x^2+y^2=3^2$ or $\sin 2x+\sin 4y=0.1r$
Parametric (any parameter name but coordinate name, coordinates in any order)		
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}, t = t_1 \ldots t_2$	2D Cartesian coordinates	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 3t \\ \sin 5t \end{bmatrix}, t = 0 \ldots 2\pi$
$\begin{bmatrix} r \\ \theta \end{bmatrix} = \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}, t = t_1 \ldots t_2$	2D polar coordinates	$\begin{bmatrix} r \\ \theta \end{bmatrix} = \begin{bmatrix} 2 + \sin t \\ \cos 3t \end{bmatrix}, t = 2\pi \ldots 0$
$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix}, t = t_1 \ldots t_2$	3D Cartesian coordinates (or cylindrical or spherical)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos t \\ \sin t \\ t \div 10 \end{bmatrix}, t = -10\pi \ldots 10\pi$
Complex		
$x + iy = f(t) + ig(t), t = t_1 \ldots t_2$	complex equation of a 2D curve	$x + iy = e^{it}, t = 2\pi \ldots 0$
Surfaces		
Explicit (any coordinate before the sign =)		
$z = f(x, y)$	Cartesian coordinates	$z = \sin x \sin y$ or $z = x^2 - y^2$
$z = f(r_0, \theta)$	cylindrical coordinates	$z = r_0 + \sin 3\theta$
$r = f(\theta, \phi)$	spherical coordinates	$r = 1$ or $r = \theta + \phi$
Implicit		
$f(x, y, z, r_0, r, \theta, \phi) = g(x, y, z, r_0, r, \theta, \phi)$	mixture of 3D coordinates	$x^3 + xy^2 - yz^2 = 2^2$
Parametric (any parameter name but coordinate name, coordinates in any order)		
$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f(t, u) \\ g(t, u) \\ h(t, u) \end{bmatrix}, t=t_1 \ldots t_2, u=u_1 \ldots u_2$	Cartesian coordinates	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin t \cdot \cos u \\ \sin t \cdot \cos u \\ \cos t \end{bmatrix}, t=0 \ldots \pi, u=2\pi \ldots 0$
$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} f(t, u) \\ g(t, u) \\ h(t, u) \end{bmatrix}, t=t_1 \ldots t_2, u=u_1 \ldots u_2$	cylindrical coordinates	$\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 3+\cos t \\ u \\ \sin t \end{bmatrix}, t=2\pi \ldots 0, u=2\pi \ldots 0$
$\begin{bmatrix} r \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} f(t, u) \\ g(t, u) \\ h(t, u) \end{bmatrix}, t=t_1 \ldots t_2, u=u_1 \ldots u_2$	spherical coordinates	$\begin{bmatrix} r \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ t+u \\ t \end{bmatrix}, t=0 \ldots \pi, u=0 \ldots \pi$

Syntax	Description	Examples
Fields		
Scalar fields		
$f(x, y, r, \theta)$ $f(x, y, z, r_0, \theta, r, \phi)$	2D scalar field 3D scalar field	$\sin x \sin y$ $x^2 + y^2 + z^2 + \cos 3\theta$
Vector fields		
$\Delta x = f(x, y, r, \theta)$; $\Delta y = f(x, y, r, \theta)$ $\Delta r = f(x, y, r, \theta)$; $\Delta\theta = f(x, y, r, \theta)$ $\Delta x \ldots \Delta y \ldots \text{etc.} = f(x, y, z, r_0, \theta, r, \phi)$ $\Delta\begin{bmatrix}x\\y\end{bmatrix}..\Delta\begin{bmatrix}r\\\theta\end{bmatrix}=\begin{bmatrix}f(x,y,r,\theta)\\g(x,y,r,\theta)\end{bmatrix}$ $\Delta\begin{bmatrix}x\\y\\z\end{bmatrix}.\begin{bmatrix}r_0\\\theta\\z\end{bmatrix}.\begin{bmatrix}r\\\theta\\\phi\end{bmatrix}=\begin{bmatrix}f(x,y,z,r_0,\theta,r,\phi)\\g(x,y,z,r_0,\theta,r,\phi)\\h(x,y,z,r_0,\theta,r,\phi)\end{bmatrix}$	2D explicit definitions 3D explicit definitions 2D definitions for each coordinate 3D definitions for each coordinate system (add Δ before each group of coordinates)	$\Delta x = xy$; $\Delta r = \theta$; $\Delta\theta = 1$ $\Delta r = xy\cos\theta$ $\Delta\theta = x - r + \cos\phi$ $\Delta\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-y\\x\end{bmatrix}$; $\Delta\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\sin x\\\sin y\end{bmatrix}$ $\Delta\begin{bmatrix}r\\\theta\\\phi\end{bmatrix}=\frac{1}{\sqrt{1+r^2}}\begin{bmatrix}\theta\\z\\r_0\end{bmatrix}$
Solutions to inequalities		
$f(x, y, r, \theta) < \leq > \geq g(x, y, r, \theta)$ in 2D, and in 3D : $f(x, y, z, r_0, \theta, r, \phi) < \leq > \geq g(x, y, z, r_0, \theta, r, \phi)$; one or more inequalities related by logical operators NOT, AND, OR, XOR : $\neg \wedge \vee \otimes$	Areas of the 2D graph where the proposition is true are colored differently from the bottom ; in 3D they are filled with spheres or boxes	$y > \cos 2x \otimes \neg y < \sin x$ $r_0 < 3 \ \& \ y < -1$
Constant expressions		
Calculation of constant expressions. (**R** real and **C** complex sets)		
expressions containing only numerical values, names of built-in constants (e, π, etc.), single-value constants previously defined	The numerical value is displayed after the expression in the equation list	$e^{i\pi} + 1$ or $3.2.25^3$ or c_{light} or γ or $(2+3i)(1-5i)$ or $\ln(3+2i)$
Definitions of constants (any of both signs = or :=).		
name = f(numericalvalues, single-value constants already defined, constants from the « built-in definitions »)	Definition of one single-valued constant (**R** real and **C** complex numbers sets)	$k = 1{,}25$ or $N = 10^3$ or $N_{max} = 10$ or $a_2 = 0{,}3 \cdot c_{light}$ or $b = 2 + 3i$
name = {value1, value2, value3, etc.} (value n can be a single or multi-value constant previously defined)	Multi-valued constant (**R** real and **C** complex numbers sets)	$d = \{1.1, 2.3, 5, 6.05, G\}$ or $q_4 = \{0.5 + i, 1 + 1.5i, 1.5 + 2i\}$
name = {mini, mini+r…maxi} (maxi ≥ last term wanted)	Multi-valued constant as an arithmetic progression, common dfference r (**R** reals)	i={1.2,1.4,…2.2} or i={1.2,1.4,…2.33} give the same 6 terms sequence
name = {integer mini…integer maxi} **The above definitions of constant can be global or local(see page 42)**	Multi-valued constant : successive integers (**R** reals)	i = {120…126} gives the 7 successive integers from 120 to 126
name = integer mini…integer maxi	Exception for defining the domain of the index of a sequence : braces forbidden, parentheses accepted but useless	i = 120…126 gives the 7 successive integers from 120 to 126

Syntax	Description	Examples

Definitions of functions

Built-in functions

Syntax	Description	Examples
See menu Help > Show Built-in Definitions > Functions	can be used directly in the expressions	cos(x) or cosx or lnx or arctan(x) or $J_n(x)$ or hypot(x, y,…)

Definition of a function (general case) (global or local definitions allowed) (sign = or := any)
These functions of type f(a, b, c, …) can be used in other expressions with other arguments, for instance to plot a graph : x = 1.043. f(y, θ, 2.25, …) or y = f(x, r, 0, …). **(Grapher 2.2 & 2.5 : see bug nr. 28 and 30)**

Syntax	Description	Examples
name(arguments) = expression arguments : those of various types that appear in the expression	single function multiple function	$f(x) = 1 + \cos^2 x$ or $g(x,t) = e^{-2(x-t)}$ $f(t, u, v) = 2t - 2u - v + 1$ $f(x) = \{\cos x, \sin x, \tan x\}$

Piecewise-defined functions. (sign = or := any).**(Grapher 2.2 & 2.5 : see bug nr. 28 for the names of variables)**

Syntax	Description	Examples
$f(\ldots) = \begin{cases} \text{condition1 expression1} \\ \text{condition2 expression2} \\ \ldots \quad \ldots \end{cases}$ $f(\ldots) = \begin{cases} \text{expression1 condition1} \\ \text{expression2 condition2} \\ \ldots \quad \ldots \end{cases}$	Name of a coordinate instead of f(…) for the equation of a graph. The function is not defined where the conditions are not specified. Complicated conditions possible using the operators ¬ ∧ or & ∨ ⊗, = ≠ < ≤ > ≥ ∈ ∉ ∪ ∩ [ ] The last condition can be omitted.	$f(x) = \begin{cases} x < 0 \ \sin x \\ x \geq 0 \ x - x^3 \end{cases}$ $d(x) = x \in [-2, 1] \cup [2, 3] \ ? -2 : 2$ $g(x) = \begin{cases} 0 \ x < 0 \\ x^2 \ x \leq 1 \\ 1 \end{cases}$
h(…) = condition ? expression a(…) = condition : expression	Undefined function if the condition is not satisfied.	$a(x) = x > 0 : \cos x$
b(…)=condition ? expr1 : expr2 c(…)=condition : expr1 : expr2	Expression1 if the condition is satisfied, expression2 if not satisfied.	$b(x) = x > 1 \ ? \log x : 0$

Periodic functions (Grapher 2.2 & 2.5 : see bug nr. 28 for the names of variables)

Syntax	Description	Examples
(1) f(t) = … (2) y = f(x) (explicit) (3) x_1 T (4) x_0 (5) $g(t) = f((t - x_0) \% T + x_1)$ (6) y = g(x)	Principle : repeat ad infinitum the piece of graph placed between abscissae x_1 and $x_1 + T$ (T = period), of the function y = f(x), after placing it between abscissae x_0 et $x_0 + T$. Therefore : (1) choose the function - piecewise definition often helpful ; (2) examine the graph (explicit) ; (3) choose the piece of graph ; (4) place on the abscissae axis the initial period of the periodic function ; (5) create this function ; (6) plot its graph.	(1) $f(t) = t^2 - 1$ (2) y = f(x) (3) choice : $x_1 = -0.5$ T = 1.3 (4) choice : $x_0 = -2$ Sign = ou := any for f(t) and g(t) Sign % or \ any (5)(6) y = f((x + 2) % 1.3 - 0.5) Easy example : sawtooth wave with y = x \ T or y = x % T

Point set data (see also APPENDIX 1)

Typing directly the data

Colonne 1	Colonne 2	Colonne 3
0,5	1,01	1,29
1	1,25	1,17
1,5	1,18	0,87
2	1,69	0,61
2,5	1,7	0,78
3	2	0,82
3,5	2,24	1,16
4	2,19	1,79
4,5	2,71	2,33
5	2,9	2,91

Importing a TXT (.txt) file nr.1

```
0.50  1.01  1.29
1.00  1.25  1.17
1.50  1.18  0.87
2.00  1.69  0.61
2.50  1.70  0.78
3.00  2.00  0.82
3.50  2.24  1.16
4.00  2.19  1.79
4.50  2.71  2.33
5.00  2.90  2.91
```

Importing a TXT (.txt) file nr.2

data separators « , » comma
point separators « ; » semicolon

Le... — Modifié

```
0.50,1.01,1.29;1.00,1.25,
1.17;1.50,1.18,0.87;2.00,
1.69,0.61;2.50,1.70,0.78;
3.00,2.00,0.82;3.50,2.24,
1.16;4.00,2.19,1.79;4.50,
2.71,2.33;5.00,2.90,2.91
```

Syntax	Description	Examples
Equations to graph points		
$\begin{bmatrix}x\\y\end{bmatrix}$ or $\begin{bmatrix}r\\\theta\end{bmatrix}=\begin{bmatrix}f(k)\\g(k)\end{bmatrix}, k=\{values\}$ $x+iy=f(k)+ig(k), k=\{values\}$ $\begin{bmatrix}x\\y\\z\end{bmatrix}$ or $\begin{bmatrix}r_0\\\theta\\z\end{bmatrix}$ or $\begin{bmatrix}r\\\theta\\\phi\end{bmatrix}=\begin{bmatrix}f(k)\\g(k)\\h(k)z\end{bmatrix}, k=\{values\}$ $b(x)=x>1\ ?\ \log x : 0$	2D parametric equations of points (list of discrete values for k) 2D complex equations of points 3D parametric equations of points	$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}k\\0.5k^2\end{bmatrix}, k=\{0,0.5\ldots6\}$ $x+iy=k+i\frac{k^2}{2}, k=\{0,0.5\ldots6\}$ $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}u\\0.2u^2\\0.5u\end{bmatrix}, u=\{0,0.5\ldots5\}$
Integrals and derivatives		
Integrals (Double definite Integrals : see bug nr. 26) (Grapher 2.2 & 2.5 see bug nr. 29)		
$y=\int_{x_0}^{x} dt\,(f(t))$ $y=\int_{f(x)}^{g(x)} du\,(f(u))$ $\int_{t_0}^{t_1} dt\,(f(t))$	Graph y = F(x) - F(x integral of the function y = f(x) ; or Menu Equation > Integrate (n times), return by Menu Equation > Differentiate(n times). Note : The integration variable must not be a coordinate name (if so, modify the integral). Calculation of definite integral (x0, x1) bounded of function f(x) ; or Menu > Integration > 5 options (Area, Surface, Length, Volume of Revolution, Lateral Area of Revolution).	$y=\int_{x_0}^{x} dt\,t^2\ ,\ x_0=-1$ $y=x^2$ $\int_{t_0}^{t_1} dt\,(t^2)\ ,\ t_0=-1, t_1=2\ = 3{,}000$
Derivatives. (Grapher 2.0 : see bug nr. 23)		
$y=\frac{\partial^n}{\partial x^n}f(x)$	Select y = f(x) then Menu Equation > Differentiate ; to do n times for the nth derivative ; to return, same Menu > Integrate, to do n times. Or type the expression straight.	Differentiate three times : $y=\frac{x^4}{24}$ or type these 2 lines : $f(t)=\frac{t^4}{24}$ et $y=\frac{\partial^n}{\partial x^n}f(x), n=3$ value $n \leq 3$ if n parameter n, otherwise numerical value instead of n.
$\begin{bmatrix}x\\y\end{bmatrix}=\frac{\partial^n}{\partial t^n}\begin{bmatrix}f(t)\\g(t)\end{bmatrix}, t=t_{min}\ldots t_{max}$	Derivative of parametric equation.	$\begin{bmatrix}x\\y\end{bmatrix}=\frac{\partial^n}{\partial t^n}\begin{bmatrix}5\cos t\\2\sin t\end{bmatrix}, t=0\ldots\frac{\pi}{2}$ to try for n integer from 1 to 4
shows : $\begin{cases}x=\frac{d^n}{dt^n}f(t)\\y=\frac{d^n}{dt^n}g(t)\end{cases}$	(every coordinate system 2D, 3D)	
$f(t)=\ldots$ and $g(t)=g(f(t))$ used for : $y=\frac{d^n}{dx^n}g(x)$	Derivatives of a function of a function.	$f(t)=\frac{t^3}{6}, g(t)=\cos t, h(t)=2f(t)+(g(t))^2,$ $y=h''(x)$ or $y=\frac{\partial^2}{\partial x^2}h(x)$ i.e. $2(x-\cos 2x)$
$g'(f(t))=\frac{dg(f(t))}{dt}\ldots g'''(f(t))=\frac{d^3g(f(t))}{dt^3}$	“Partial“ derivative of a function of a function.	$y=\cos'(x^2-1)$ values $y=-\sin(x^2-1)$
Syntax for derivatives : f'(x) f''(x) f"(x) f'''(x) f"'(x)	Apostrophes and quotation marks (maximum : third derivative).	$f(t)=\frac{t^3}{6}, y=f(x), y=f'(x), y=f''(x), y=f'''(x)$
$\frac{\partial^n}{\partial t^n}f(t)$ or $\frac{d^n}{dt^n}f(t)$ or $\frac{\partial^n f(t)}{\partial t^n}$	Keyboard or Equation Palette.	$\frac{\partial^6}{\partial x^6}(-\sin x)=\frac{d^1}{dx^1}(-\cos x)=\frac{\partial^0\sin x}{\partial x^0}=\sin x$
$\dot{y}=\frac{dy}{dt}, \ddot{y}=\frac{d^2}{dt^2}, \dddot{y}=\frac{d^3}{dt^3}$, etc.	Derivatives with respect to time t (cannot be used, for abscissa can’t be t, as it was in Curvus pro X)	Editable to export ($\dot{x}$ operator)

Syntax	Description	Examples
Sum, product, factorial, binomial coefficient, rounding, modulo, iteration		
$\sum_{n=n_{min}}^{n^{\circ}max} f(n)$	Sum : Fourier series	$y=\sum_{n=0}^{30}\frac{1}{2n+1}\sin((2n+1)x)$
$n!$ or $\prod_{n=1}^{n} n$ and $\prod_{n=n_{min}}^{n_{max}} f(n)$	Factorial and product	$5!$ or $\prod_{n=1}^{n} n$ values 120
$\binom{n}{k}$ or $\frac{n!}{k!(n-k)!}$	Binomial coefficient	$\binom{6}{2}$ or $\frac{n!}{k!(n-k)!}$
$\lceil a \rceil$, $\lfloor a \rfloor$	Rounded to the top, bottom integer.	$\lceil 1{,}01 \rceil$ and $\lfloor 2{,}99 \rfloor$ value 2
$a \% d$ or $a \backslash d$ $y=2\frac{x\%T}{T}-1$	Modulo operator : calculate remainder of the Euclidean division a ÷ b Graph : linear sawtooth wave, period T, amplitude [-1, +1].	$125\%6$ and $125\backslash 6$ value 5 $y=2(x\%1)-1$
$n=\{n_1, n_2, n_3, etc.\}$ $n \% d == r_c$	In a defined list of integers n, this condition retains only the n remainder in their division by d is r The example draws the curve y = ln(x) with a point for every even value of x.	$a=\{0\ldots 20\}$ $f(a)=a \% 2 == 0$ $\begin{bmatrix}x\\y\end{bmatrix}=f(a)\ ?\begin{bmatrix}a\\ \ln(a)\end{bmatrix}$
$f^{(n)}(t)$	Iteration operator (repetition n times of the function) : f(f(f(…(f(t))…)))	$\ln^{(3)}(t)$ values $\ln(\ln(\ln(t)))$
Matrices and determinants (Use : see Appendix 9, Matrices in Grapher)		
$\begin{bmatrix}\square\\\square\end{bmatrix}$ $\begin{bmatrix}\square&\square\\\square&\square\end{bmatrix}$ $\begin{bmatrix}\square\\\square\\\square\end{bmatrix}$ $\begin{bmatrix}\square&\square&\square\\\square&\square&\square\\\square&\square&\square\end{bmatrix}$	Available matrices, enter the elements directly. Maximum size 3x3	Every parametric graph . Below : rotation of +α of an ellipse.
$\begin{bmatrix}1\\2\end{bmatrix}^T=[1\ 2]$ $\begin{bmatrix}1\\2\\3\end{bmatrix}^T=[1\ 2\ 3]$	1x2 & 1x3 matrices possible by trans-position of 2x1 & 3x1 matrices.	$\alpha=\frac{\pi}{3}$ $M_\alpha=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$
$A+B$; $A-B$; $A.B$; $x.A$; $\det(A)$ $A^T, -A, A^k, \frac{1}{\lvert\det(A)\rvert}A^{-1} \text{—>} A^{-1}$	Usual operation agreed. The operation A multiplied by the absolute value of the A determinant.	$t=0\ldots 2\pi$ $\begin{bmatrix}x\\y\end{bmatrix}=M_\alpha\begin{bmatrix}3\cos t\\ \sin t\end{bmatrix}$

Differential equations (Grapher 2.0 : see bug nr. 23)

Grapher calculates the 2D and 3D graphs of solutions of differential equations :

— ordinary (i.e. one variable only) ;

— of order (i.e. that of the derivative of highest order) not limited a priori ;

— in 2D, the function can be y(x) or x(y) or r(θ) or θ(r) or parametric x(t) & y(t), or r(t) & θ(t) ; in 3D, parametric functions in the three coordinates systems (x, y, z), (r_0,θ, z), (r, θ, ϕ) ;

— the form of the equation must be :

Derivative of the highest order = Expression of(derivatives of lower orders, function, variable)

Differential equation syntax in Grapher. On a single line :

Differential equation	Initial conditions	Domain
$\frac{d^2}{dt^2}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-(x+y)\\x-y\end{bmatrix}-2\frac{d}{dt}\begin{bmatrix}x\\y\end{bmatrix},$	$\frac{d}{dt}\begin{bmatrix}x(0)\\y(0)\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}x(0)\\y(0)\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix},$	$t=0\ldots 10$
1	**2**	**3**

— Areas 1 and 2 : The various syntaxes of the derivatives are shown above.
— Area 2 : There are (n-1) initial conditions for a nth order differential equation, order of writing is indifferent ;
All initial conditions are defined for the same initial value of the variable or parameter (here 0), it is :
- the first appearing in the initial conditions (even if several different values),
- otherwise the minimum value of the variable domain,
- otherwise the minimum value on the scale of the variable on the graph ;
This initial value of the variable can be chosen within or outside the specified domain of the variable ;
— Area 3 : The domain limits are defined in the order minimum…maximum ;
The domain of a parametric variable must always be specified ;
If the variable is a coordinate, it is possible, in some cases, not specifying its domain.

Examples. 2D :

$$\frac{\partial y}{\partial x} = -\frac{9x}{25y},\ y(0)=3,\ x=-5\ldots5$$

$$\frac{dy}{dx} \text{ or } \frac{d}{dx}y \text{ or } \frac{\partial y}{\partial x} \text{ or } \frac{\partial}{\partial x}y \text{ or } y' = -\frac{9x}{25y},\ y(0)=3,\ x=-5\ldots5$$

$$\frac{\partial}{\partial\theta}r = -\frac{1}{2}\left(\frac{1}{9}-\frac{1}{25}\right)\left(\frac{\cos^2\theta}{25}+\frac{\sin^2\theta}{9}\right)^{-\frac{3}{2}}\sin2\theta,\ r(0)=5,\ \theta=0\ldots2\pi$$

$$y'' = -\frac{(y')^2}{y} - \frac{9}{25y},\ y(0)=3,\ y'(0)=0,\ x=-5\ldots5$$

(Ellipses or semi-ellipses)

3D : (Helices)

$$\frac{\partial}{\partial t}\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0{,}1 \end{bmatrix}, \begin{bmatrix} r_0(0) \\ \theta(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, t=0\ldots30 \qquad \frac{\partial^2}{\partial t^2}\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} r_0(0) \\ \theta(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \frac{\partial}{\partial t}\begin{bmatrix} r_0(0) \\ \theta(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0{,}1 \end{bmatrix}, t=0\ldots30$$

Sequences.

This Grapher function places points on the graph with coordinates (x_i, y_i) ; x_i and y_i are the i-th terms of two sequences X_n and Y_n , for n from n_{min} to n_{max} . We can also use the polar coodinates (r, θ) in 2D, and in 3D the three Cartesian coordinates (x, y, z).

Syntax of a sequence equation. On a single line :

Sequence definition	Initial terms	Domain
$\begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} 4x_i(1-x_i) \\ y_i+1 \end{bmatrix}$,	$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 0{,}2 \\ 0 \end{bmatrix}$,	$i=0\ldots20$
1	**2**	**3**

— Area 1 : Definitions of both sequences ; no restriction for index name nor its values ;
— Area 2 : Values of initial terms of the sequences ;
— Area 3 : Domain of the index (indices of terms plotted on the graph).

Example : The Fibonacci sequence

$$\begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} x_i+1 \\ y_i+y_{i-1} \end{bmatrix}, \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ i=0\ldots10$$

“Syntax error“ signal

This **signal “Danger“** is displayed in the equation list in front of the expressions with syntax errors.
It will therefore be present before the **comments** added to this list.
It will appear before the **expressions containing constants or variables not yet defined** ; in this case, the signal will disappear as soon as the appropriate definitions will exist.
And, of course, it remains before **an incorrect expression**.

Using the equation editor

It has been used so far to create graphs from equations and to calculate expressions. It can also be used to write short notes in the list of equations, and write mathematical expressions exported to other software. In the latter work it is more limited than the usual equation editors : one line at a time, a system of equations must be represented in matrix form, the only signs that can overcome the letters are points (no arrow no bar etc.) ; in return it is very easy to use with well designed keyboard shortcuts.

Entering new data into the list of equations.

New line of equation. Many possibilities :

— ⌥⌘N keyboard shortcut ;
— Click [+] at the bottom of the equation list > contextual menu > New Equation **;**
— Menu Equation > new Equation :
— **⌥⇧⌘N keyboard shortcut > choose one of 20 templates ;**
— Click [+] and contextual menu > New equation from template… > 20 templates**;**
— Menu Equation > New equation from template… > choice of 20 templates.

In the first three cases “ y= “ is displayed in the equation editor, in the three other cases it’s an equation to be completed **(see bugs 6 & 19)**.

New point set. Two ways :

— ⌥⌘P keyboard shortcut ;
— Click [+] at the bottom of the equation list > pop-up menu > New Point Set.

Double click on the line to title the point set.
See Lesson 6 page 31 to use point sets.

New group. Two ways :

— Click [+] at the bottom of the equation list > pop-up menu > New Group ;
— Menu Equation > New Group.

Double click on the line to title the group **(see bug nr. 9)**. Don’t forget to drag and drop its equations in it. **Avoid global definitions inside groups (see bug nr. 20) and imperatively empty groups in Grapher 2.0 (see bug nr. 22).**

Delete lines from the list of equations (NOT USED IN OTHER EQUATIONS).

— Click the line to Select > Backspace key ← or ⌦ Suppr ; to select **several lines press ⇧ or ⌘ keys. (See also bug nr. 20).**

Note: If the line to be deleted is used in other expressions (definitions of functions or variables, for example), you can make these expressions incorrect (yellow triangle "Danger"), for example by adding the = sign at the beginning of expressions, which allows you to edit later

The order of the expressions in the list can be changed ad libitum by simply **dragging and dropping.**

Sources of signs and symbols.

Preferences. (Setup of signs and symbols).

— Menu Grapher > Preferences… > Equations : Font and Size settings (default Times Regular 18), Relatives Sizes, Variables in Italic ;

> Numbers : Numbers settings (Number of Decimal Places, Remove Trailing Zeros), Notation (scientific, engineering) Use (never, from, always), Complex Numbers (Imaginary Part i or j, at the end or not), Examples (impact of options) ;

> Advanced : Numeric Accuracy (four sliders), Integration (choice of method and its parameters), Micellaneous (Polar Angular Range [-π, +π] or [0, 2π], Trigonometric Mode radians degree grades, Number maximum of iterations to evaluate special functions.

The equation palette. Three ways to show it :

— Button ▾ ∑x² on the equation editor > Show Equation Palette ;

— ⇧⌘E keyboard shortcut ;
— Menu Window > Show Equation Palette.
Tabs Standard, Operators, Greek, Symbols, and **check “Use Shortcuts When Typing“.**

Simplified equation palette.

Button ▾ ∑x² on the equation editor.

Equation Templates. New Equation from Template… already seen.

Built-in definitions.

Menu Help > Show Built-in Definitions. Specifies the list, spelling and syntax of constants and functions recognized by Grapher (see below page 53 the table “ Built-in definitions“).

The keyboard.

Some signs are accessible via the keyboard as in word processing applications, specific keyboard shortcuts exist in Grapher. The list is given in next pages “Keyboard shortcuts for the equation editor“.

Comments entered by the equation editor.

These are texts that you want to include in the list of equations and/or that you want to move to the graph by dragging and dropping.

— They are not equations, hence the warning (yellow triangle “danger“) in front of the line ;
— Before typing them on the keyboard, uncheck in the Preferences “Variables in Italic“, otherwise your letters will be sloping (Grapher variables : x, a, etc.) or Roman (if not variables : sin, cos etc.) ;
— The equation editor is not intended for normal word processing : signs typed after a caret ^ (or French accent circonflex) become exponent,, ê, ô and â, are not available, signs typed after Shift+Dash keys are subscript ; in short, the keyboard shortcuts of the equation editor are enable ;
— An English name (no French accents) of a Greek letter, will be immediately translated... into a lower case Greek letter, except epsilon, omicron, upsilon, but xi and chi yes, ksi and khi no ; English and French words often contain Greek letters e.g. alphabet, mutation, nuclear, philodendron, etc. ; to avoid the Greek letter μ when typing “mutation“, type “m“, a space, “u“,t… then go back to cancel the space, the process is general. (This Greek translation does not occur if “Use Shortcuts When Typing“ is unchecked).
— Of course, these comments may use all the usual signs and symbols of the expressions.

Navigating the equation editor.

Similar to other word processors, it uses in particular :

— the mouse, pointer and cursor ;
— menu Edition : Undo/Redo, Cut/Copy/Paste, Delete ;
— selection and the Backspace ← and Suppr ⌦ keys, for amendments ;
— ⇥ et ⇧⇥ keyboard shortcuts with the Tab key to navigate inside symbols ;
— ⇠ ⇣ ⇡ ⇢ arrows to exit from symbols and navigate in expressions.

Exporting an equation.

Three procedures :

— **Selection in the equation list >** equation in the editor **> right click or Ctrl-clic in the editor > pop-up menu**
> Copy as TIFF or PDF or EPS or Text,
> Copy LaTex Expression ;
— Selection in the equation list > selection in the editor > menu Edition > Copy as TIFF or PDF or EPS ;
— Selection in the equation list > selection in the editor > menu Edition > Cut or Copy.

In the latter case and with the **TIFF, PDF et EPS formats**, **an image of the equation** is stored in the clipboard and it can be pasted into another application ; this image file cannot be imported in Grapher as operational equation.
The **Text format** is another format for writing equations, **understood by Grapher** which, after importation, will display the equation with its original mathematical symbols.
The **LaTex format** is used in many computer programs but **is not understood by Grapher.**

Keyboard shortcuts for the equation editor

To get		Type	Comments
Figure	Name		
General use			
	New equation	⌥⌘N	
	New equation from a template	⇧⌥⌘N	
	New point set	⌥⌘P	
	Cancel a line of equation	← or ⌦	
	Show the equation palette	⇧⌘E	
	Navigate inside symbols	⇥ or ⇧⇥	
	Navigate in expressions	← ↑ ↓ →	and exit from symbols
,	decimal comma	,	without spaces
,space	separator	,space	
⇧.	decimal point (dot)	.	without spaces
f' f'' f'''	signs of 1st to 3rd derivatives	' and/or "	single or double English quotes
$a_{\square}$	index	⇧-	keyboard dash
$(\square)$	parentheses	(	normal use in expressions
$\{\square\}$	braces	⌥(	multiple-values and functions
$[\square]$	brackets	⇧⌥(	domains for conditions
Greek letters and infinity			
∂	lowercase delta	⌥D	
Δ	uppercase delta	⇧⌥D	
μ	lowercase mu	⌥M	
π	lowercase pi	⌥P	
α to ω	lowercase Greek letters except epsilon, omicron, upsilon	name of the letter	without French accents, use xi and chi but not ksi and khi
∞	Infinity	⌥,	or type « infinity »
Operations signs			
+	plus	⇧+ or +	plus keyboard or numeric keypad
- or —	minus	- ou ⌥- or -	keyboard dash or minus numeric k.pad
.	times (multiplication)	✶	* from keyboard or numeric keypad
$\frac{\square}{\square}$	fraction	⇧/ or /	keyboard or numeric keypad
÷	divided by	⌥:	
$a^{\square}$	exponent	^	
% or \	modulo	⇧%or⇧⌥/	
n!	factorial	!	
Relational and Boolean operators.			
=	equal to	=	
:=	equal to (constants and functions)	:=	optional
≠	not equal to	⌥=	keyboard only or <> or !=
±	plus or minus	⇧⌥=	editable but not operational
<	less than	<	
≤	less than or equal to	⌥<	you may type also =< or <=
>	greater than	⇧>	you may type also => or >=
≥	greater than or equal to	⇧⌥>	
¬	logical NOT	⌥L	you may use also the !
$[\square]$	brackets	⇧⌥(	domains for conditions
&	logical AND	&	same as ∧ from the equation palette

To get		Type	Comments
Figure	Name		
Grapher symbols			
$\sum \square$	sum	⇧⌥S	
$\prod \square$	product	⇧⌥P	
$\sqrt{\square}$	square root	⇧⌥V	
$\sqrt[\square]{\square}$	nth root	⌥V	
$\int \square$	integral	⇧⌥B	
( )	parentheses	(or)	To place parentheses, braces or brackets around an expression already written : select it, then enter the required sign (keyboard or equation palette).
{ }	braces	⌥(or ⌥)	
[ ]	brackets	⇧⌥(or ⇧⌥)	

The keyboard shortcuts above may vary slightly with the equipment used ; they were obtained with OS X 10.4.11 to 10.9.5, Grapher 1.1 to 2.5, iMac and MacBook Pro, set in French, French AZERTY keyboard with numerical keypad.

Built-in definitions

Functions (in the real set : R, or in the complex set : ℂ)

Trigonometric

Set	Function
ℂ	sin(x)
ℂ	cos(x)
ℂ	tan(x)
ℂ	cot(x)
R	sec(x)
R	csc(x)
ℂ	asin(x)
ℂ	arcsin(x)
ℂ	acos(x)
ℂ	arccos(x)
ℂ	atan(x)
ℂ	arctan(x)
R	acot(x)
R	arccot(x)
R	deg(x)
R	rad(x)
R	atan2(Δy, Δx)

Hyperbolic

Set	Function
R	sinh(x)
R	cosh(x)
R	tanh(x)
R	coth(x)
R	asinh(x)
R	arcsinh(x)
R	acosh(x)
R	arccosh(x)
R	atanh(x)
R	arctanh(x)
R	acoth(x)
R	arccoth(x)

Standard

Set	Function	Description
ℂ	$\exp(x)$ & $e^{(x)}$	exponential
ℂ	ln(x)	Napierian logarithm
ℂ	abs(x)	absolute value
R	log(x)	decimal logarithm
R	sign(x)	+1 si x > 0, 0 si x = 0, -1 si x < 0
R	sgn(x)	
R	hypot(x, y…)	$\sqrt{x^2 + y^2 + \ldots}$
R	min(x, y…)	minimum value of the list
R	max(x, y…)	maximum value of the list

R Round to integer…

floor(x)	down
ceil(x)	up
round(x)	the nearest

ℂ Complex (x is a complex expression)

Re(x)	real part	
Im(x)	imaginary part	
abs(x)	modulus	
arg(x)	argument	
conj(x)	conjugate, form a + jb	if x is a function of a parameter, these four functions draw the curve in the complex plane
inv(x)	mult. inverse, form a + jb	
det(x)	déterminant, form a + jb	
complex(x)	if x is constant, compute it into form a + jb	

R Special functions 1/2

$J_n(x)$	Bessel	$E_n(x)$	exp. intégral
$Y_n(x)$	Bessel	$A_i(x)$	Airy
$I_n(x)$	Bessel mod.	$B_i(x)$	Airy
$K_n(x)$	Bessel mod.	$S(x)$	Fresnel integral
$j_n(x)$	Bessel spher.	$C(x)$	Fresnel integral
$k_n(x)$	Bessel spher.	$Ci(x)$	cosine integral
$\Gamma(x)$	Gamma	$Si(x)$	sine integral

R Special functions 2/2

erf(x)	error function
erfc(x)	complementary error function
$Y_{l,m}(x, y)$	spherical harmonics (3D)
$P_l^m(x)$	associated Legendre polynomials

Mathematical constants		
π = 3,141 592 653 589 79	π : Archimedes constant	
e = 2,718 281 828 459 05	Base of natural logarithms	(Napier constant)
γ = 0,577 215 7	Euler-Mascheroni constant	(γ = 0,577 215 664 901 533...)
∞ undefined	Infinity symbol	
Physical constants		
$c_{light} = 2{,}99792458e8\ m \cdot s^{-1}$	Speed of light in vacuum	$(c \text{ or } c_0 = 2{,}997\ 924\ 58e8)$
$\varepsilon_0 = 8{,}85418782e-12\ F \cdot m^{-}$	Electric constant	$(1/_{\mu_0 c^2} = 8{,}854\ 187\ 817e-12)$
$F = 9{,}648456e4\ C \cdot mol^{-1}$	Faraday constant	(F = 9,648 533 65 (21)e4)
$G = 6{,}67332e-11\ m^3 \cdot kg^{-1} \cdot$	Newtonian constant of gravitation	(G = 6,673 84 (80)e-11)
$h_{planck} = 6{,}626\ 2e-34\ J \cdot s$	Planck constant	(h = 6,626 069 57 (29)e-34)
$k_{boltz} = 1{,}3807e-23\ J \cdot K^{-1}$	Boltzmann constant	(k = 1,380 648 8 (13)e-23)
$N_a = 6{,}022e23\ mol^{-1}$	Avogadro constant	$(N_a \text{or } L = 6{,}022\ 141\ 29(27)e23)$
$R = 8{,}3144\ J \cdot mol^{-1} \cdot K^{-1}$	Molar gas constant	(R = 8,314 462 1 (75))
$U = 1{,}6605655e-27\ kg$	Atomic mass constant	$(m_u = 1{,}660\ 538\ 921\ (73)e\text{-}27)$
$\mu_0 = 1{,}25663706143592e-6$	Magnetic constant	$(4\pi 10^{-7}\ H \cdot m^{-1} \text{ ou } N \cdot A^{-2})$

Ranges of the inverse trigonométric functions			
Grapher		**Mathematics**	
Name	Range	Name	Range
acos(x) or arccos(x)	0 … +π	Arccos(x)	0 … +π
asin(x) or arcsin(x)	-π/2 … +π/2	Arcsin(x)	-π/2 … +π/2
atan(x) or arctan(x)	-π/2 … +π/2	Arctanx)	-π/2 … +π/2
acot(x) or arccot(x)	0 … +π	Arccotan(x)	0 … +π
atan2(y, x)	0 … +2π ou -π … +π (depending on choice made in the Grapher Preferences)	angle α of the inverse function of the system $y = \begin{cases} \cos\alpha = \frac{x}{\sqrt{x^2+y^2}} \\ \sin\alpha = \frac{y}{\sqrt{x^2+y^2}} \end{cases}$	0 … +2π (usualprincipal value of angle α)

Notes :

1) Modified Bessel function $K_n(x)$: **see bug nr. 10 ;**

2) Spherical Bessel function $y_n(x)$ and $j_n(x)$: **see bug nr. 12 ;**

3) Airy functions : **see bug nr. 11 ;**

4) The Bessel functions exist for n integer or fractional ;

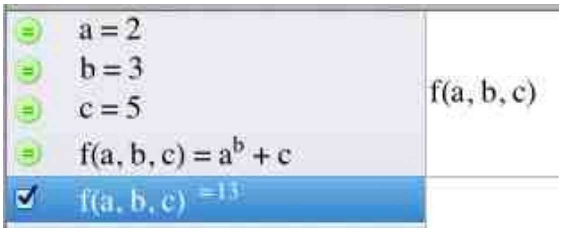
5) The spherical harmonics $Y_{l,\,m}(x, y)$ exist only if the functions are real (that is true for m = 0) ; we must have m ≥ 0 and l ≥ m ; use in 3D only, for example with spherical coordinates : $r = Y_{l,\,m}(\phi, \theta)$.

―――――――――――――――――――――――――

Numerical calculations (evaluations)

Numerical evaluation of constant expressions.

Expressions (without = sign) from the list of equations containing only constants with fixed numerical values, multi-valued excluded, in the sets of real **R** or complex ℂ numbers. If the expression is a constant or constant function defined by an equality, you must enter the name of the function alone to get its numerical value : f(a, b, c) in the figure nearby.
If the constant or the function change value, the displayed result will change too.

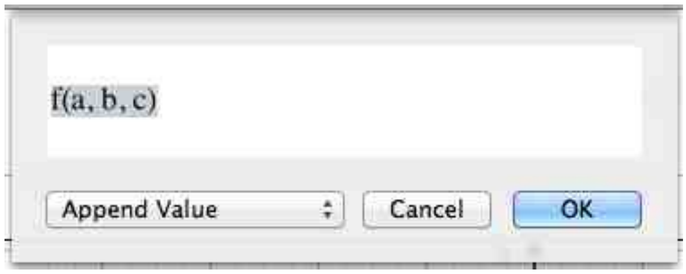


Result displayed in the equation list.

Enter the expression into the equation editor > Return key > the numerical value is displayed after the expression in the equation list.

Result displayed on the graph.

Enter the expression into the equation editor > select it in the editor > drag and drop onto the graph > adjust the size with the handles > double-click it > pop-up window, select the expression, Add the value (see note) > the value is displayed on the graph and will be updated **> adjust shape and size with the handles > delete from the list of equations the expression name no longer needed, or keep it in the list with its value displayed.**



Note : — No transform : only the expression name is displayed ;
— Replace with value : the value is displayed without the name ;
— Append value : it is placed with the = sign after the name of the expression ;
Remark : Only the real part of a complex value is displayed on the graph.

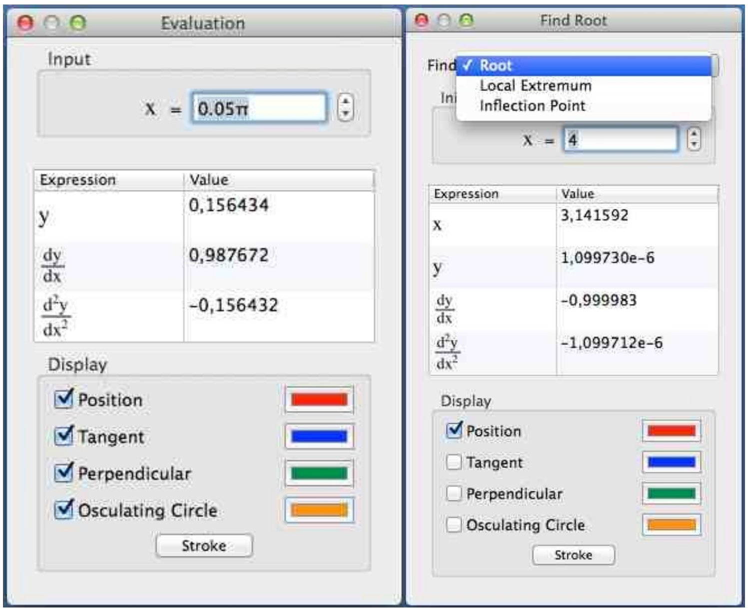
More examples : $3+4^2 \;{=19}$ $\qquad \dfrac{(1+2i).e^{0.3i}.(2+3i)}{4+i} = -1.062040 + 1.641829i$

Numerical evaluation of functions.

Conditions : Explicit and parametric functions ; functions, variables and parameters, belonging to a single coordinate system (Cartesian, polar, cylindrical or spherical) ; multi-valued functions: select only one of the values or curves of the function.

Calculation of coordinates of points of a curve.

Click on the curve or its equation > menu Equation > Evaluation... > Click the appropriate point on the curve, or enter a value of the variable then Return key > coordinates are displayed at the bottom of the window, function calculation, its first and second derivatives, the point chosen is displayed on the graph, tangent, normal, osculating circle at that point and **click the button Stroke** (at the bottom of the pop-up window Evaluation) **> display their equations in the list (Grapher 2.2 to 2.5 see bug nr. 31).**



Searching for roots, local extrema, inflection points.

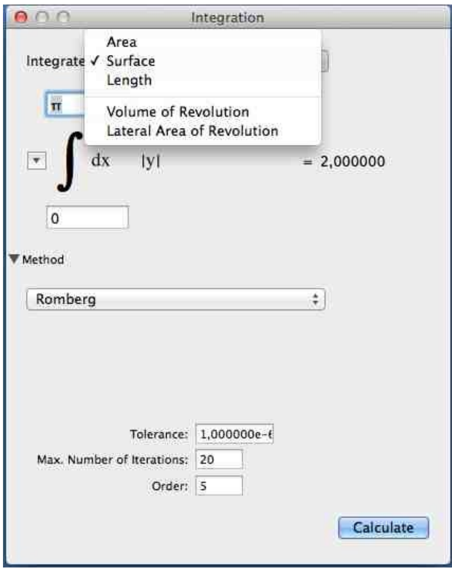
Click on the curve or its equation > menu Equation >Find Roots… > select an option in the pop-up menu > Click the approximate point on the curve, or enter an approximate value of the variable then Return key > to get the calculation of coordinates of the sought point, first and second derivatives of the function, wich allows to verify the results : y or y' or y" very close to zero ; the position, its tangent, perpendicular, osculating circle are displayed on the graph and **click the button Stroke > display their equations** in the list **(Grapher 2.2 to 2.5 see bug nr. 31).**

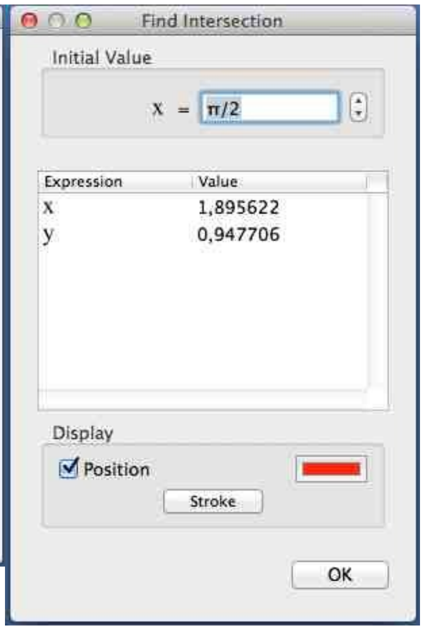
Plotting graphs of function derivatives.

Click on the equation in the list > menu Equation > Differentiate > replaces the original function and its curve by its derivative. Repeat > successive derivatives.
Menu Equation > Integrate, to go back to the previous derivative, repeat until return to the original function.
These curves allow the calculation of point coordinates, roots, local extrema, inflection points, as described above.

Plotting graphs of function integrals (Grapher 2.2 to 2.5 : see bug no. 29).

Click on the equation in the list > menu Equation > Integrate > replaces the original function and its curve by its integral. Menu Equation > Differentiate, to go back to the original function.
This curve allows the calculation of point coordinates, roots, local extrema, inflection points, as described above.





Calculating a definite integral of a function (see also bug nr. 26).

Click on the equation > menu Equation > Integration… > pop-up menu > choice > enter lower and upper limits, method of calculation with its data > Calculate.
To remove the area of integration on the graph : menu Equation > Remove Integrated Area.

Calculating the intersection point of two curves..

Select their two equations (clicks with ⌘ or ⇧ keys) > menu Equation > Find intersection… > click on a curve near the intersection or enter an approximate value of the variable > OK.

NOTE REGARDING PREVIOUS CALCULATIONS : they are possible or not, their pop-up windows, settings, options, pop-up menus, may change, depending on the form of equations (explicit y = f (x) or x = f (y), implicit, parametric) and on the coordinate system used.

Numerical evaluation of fields.

Scalar fields.

Select in the equation list > menu Equation > Evaluation… > choose inputs > Return key > read the ouputs > optional display of the position and of the gradient vector on the graph.

Whatever the coordinates used in the definition of the field, the inputs are always the Cartesian coordinates (x, y) or (x, y, z) and the outputs : $? \quad \frac{d?}{dx} \quad \frac{d?}{dy}$ and in 3D $\frac{d?}{dz}$ where ? replaces the value or the field name.

Vector fields.

— In 2D the possible definitions are (the … are functions of x, y, r, θ) :

$\Delta\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\dots\\\dots\end{bmatrix}$; $\Delta\begin{bmatrix}r\\\theta\end{bmatrix}=\begin{bmatrix}\dots\\\dots\end{bmatrix}$ and Δx = … ; Δy = … ; Δr = … ; Δθ = … ; (implies that the second coordinate is zero).

Select in the equation list > menu Equation > Evaluation… > choose inputs > Return key > read the ouputs > optional display of the vector and its position on the graph.

The inputs are the two (or the single) coordinates following the sign Δ.
The outputs are the vector coordinates used in the definition of the field, their first derivatives with respect to each coordinate input, the norm (magnitude) and the divergence (div).

— In 3D the possible definitions are similar to those in 2D : that is definition of the three coordinates in one of the three systems (Δx, Δy, Δz) or (Δr_0, Δθ, Δz) or (Δr, Δθ, Δϕ), otherwise definition of a single coordinate Δx or Δy or Δz or Δr_0 or Δr or Δθ or Δϕ. The procedure ist the same :
Select in the equation list > menu Equation > Evaluation… > choose inputs > Return key > read the ouputs > optional display of the vector and its position on the graph.

- Case 3 coordinates :

$$\Delta\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} ; \Delta\begin{bmatrix} r_0 \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} ; \Delta\begin{bmatrix} r \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} ;$$

the … are functions de x, y, z, r_0, r, θ, ϕ (mixture of coordinates allowed)

The inputs are the three coordinates after the sign Δ ; the outputs are the three coordinates of the definition, the norm (magnitude), the divergence, and the first partial derivatives of the three coordinates of the definition with respect to each of the three coordinates inputted.

- Case 1 coordinate :
The … are functions of x, y, z, r_0, r, θ, ϕ (mixture of coordinates allowed). Note that Δz alone implies the Cartesian coordinates and that Δθ alone implies the cylindrical coordinates.

$$\Delta x = \ \dots \text{ (implies } \Delta y = \Delta z = 0) \text{ ; inputs } x, y, z \text{ ; outputs } \Delta x, \text{norm, div, } \frac{d\Delta x}{dx}, \frac{d\Delta x}{dy}, \frac{d\Delta x}{dz}$$

$$\Delta y = \ \dots \text{ (implies } \Delta x = \Delta z = 0) \text{ ; inputs } x, y, z \text{ ; outputs } \Delta y, \text{norm, div, } \frac{d\Delta y}{dx}, \frac{d\Delta y}{dy}, \frac{d\Delta y}{dz}$$

$$\Delta z = \ \dots \text{ (implies } \Delta x = \Delta y = 0) \text{ ; inputs } x, y, z \text{ ; outputs } \Delta z, \text{norm, div, } \frac{d\Delta z}{dx}, \frac{d\Delta z}{dy}, \frac{d\Delta z}{dz}$$

$$\Delta r_0 = \ \dots \text{ (implies } \Delta\theta = \Delta z = 0) \text{ ; inputs } r_0, \theta, z \text{ ; outputs } \Delta r_0, \text{norm, div, } \frac{d\Delta r_0}{dr_0}, \frac{d\Delta r_0}{d\theta}, \frac{d\Delta r_0}{dz}$$

$$\Delta\theta = \ \dots \text{ (implies } \Delta r_0 = \Delta z = 0) \text{ ; inputs } r_0, \theta, z \text{ ; outputs } \Delta\theta, \text{norm, div, } \frac{d\Delta\theta}{dr_0}, \frac{d\Delta\theta}{d\theta}, \frac{d\Delta\theta}{dz}$$

$$\Delta r = \ \dots \text{ (implies } \Delta\theta = \Delta\phi = 0) \text{ ; inputs } r, \theta, \phi \text{ ; outputs } \Delta r, \text{norm, div, } \frac{d\Delta r}{dr}, \frac{d\Delta r}{d\theta}, \frac{d\Delta r}{d\phi}$$

$$\Delta\phi = \ \dots \text{ (implies } \Delta r = \Delta\theta = 0) \text{ ; inputs } r, \theta, \phi \text{ ; outputs } \Delta\phi, \text{norm, div, } \frac{d\Delta\phi}{dr}, \frac{d\Delta\phi}{d\theta}, \frac{d\Delta\phi}{d\phi}$$

Numerical evaluation of sequences.

The calculation and display of the **value(s) of the nth term(s) of a sequence** is obtained by changing the domain of the index as follows :
i = n … n > Return key ; the value(s) is(are) displayed at the end of the sequence definition in the list of equations.
If the definition of the sequence contains **multi-valued constants**, **leave them one value for each**, otherwise the calculation will select by default the last value in the definition of each constant.

Numerical evaluation of point sets.

The treatment of point sets leads to graph regression curves, to display their equations and other parameter, to calculate a correlation coeffficient.
The necessary procedures are described in detail in **Lesson 6 : Treating a point set (Regression curve) page 31 of chapter Initiation.**

Calculation accuracy.

The display of numerical values.

— Floating point with 15 significant digits (one digit, 14 decimals, e, exponent) ;
— Integers ≤ (10^{16} - 1) that is a maximum of 16 digits ;
— 2^n with n ≤ 53.

Accuracy of calculations.

For those made from exact numerical data, the accuracy is that just described above.
For calculations by other methods (integration, differentiation, differential equations, special functions, etc.). It varies and depends on the options of the "Advanced" tab in Grapher Preferences, and those selected from the pop-up of each type of operation.

Layout, appearance, saving and exporting the document

Opening a new document.

Choice of the graph model. Use one of the following ;

— **Start Grapher > pop-up window New Graph > select 2D or 3D Graph > select a model ;**
— **⌘N** or **menu File > New…** then as above ;
— **⌘O** or **menu File > Open…** a GCX file (.gcx) ;
— **Menu File > Open Recent ►** GCX file (.gcx) ;
— **Double-click on a GCX file ;**
— **Drag and drop a GCX file icon on the Grapher icon ;**
— **Menu Examples > select a 2D or 3D example (see bugs nr. 6, 19, 28, 29);**
— If a 2D document is open : **⇧⌥⌘3** or **menu View > Switch to 3D View ;**
— If a 3D document is open : **⇧⌥⌘2** or **menu View > Switch to 2D View.**

Setting the window size.

Dragging its **lower right corner** for the whole window, and dragging the **two handles** on the edges of the graph for the dimensions of the three fields (list of equations, editor, graph) **(see bugs nr. 1 & 2).**

Choosing dimensions of the graph.

— In 2D : - menu Format > Layout… > Size and Margins,
- see chapter Initiation, Lesson 3, page 15, “Dimensioning the graph with precision“,
- for the maximum size : see chapter Initiation, Lesson 4, page 20, “Layout of the document“ ;
— In 3D : see chapter Initiation, Lesson 5, page 28, “Setting size of the 3D graph“.

Restoring the graph.

Necessary after resizing the window or opening a GCX file **(see bug nr. 1 & 2)** : see chapter Initiation, Lesson 4, page 21, “Opening a GCX file …“ and “Changing the Grapher working window size“.

Formatting axes, grids and frame of the graph.

We have at our disposal:

In 2D.

— The tools : see chapter Initiation, Lesson 1, page 9, “The tools…“ ;
— Menu Format > Coordinate System… (Lin, Log, etc),
> Layout… > Margins and Background,
> Axes & Grids… (and frame) ;
— Menu View > Frame Limits… and a second way for tools ;
— **Inspectors** of each of the axes, grids and frame.

See also chapter Initiation :
— Lesson 1, page 9, “Fine adjustment of frame limits“ ;
— Lesson 2, page 11, “Setting axes and frame limits“ ; page 12, “Customizing the axes“ ; page 13, “Customizing the frame“ ;
— Lesson 3, pages 15 & 16, “Frame for the graph and minimum size of the margins“ ; page 17, “Choosing scales and graduations of the axes“ ;
— See bug nr. 2.

In 3D.

— The tools, menus Format and View, **Inspectors**, as in 2D.

See also :
— Chapter Initiation, Lesson 5, page 28, “Customizing axes, frame and background of the 3D graph“ ; page 29, “Layout of the 3D graph“ ;

Customizing curves, surfaces and points.

— Use the **Inspectors,** particularly **Resolution, Density or Increment settings,** large consumers of KB ; for example a graph with equation y = x gives a 40 KB GCX file in minimum (but adequate) resolution and 940 KB in maximum resolution ! Don't forget to click the Refresh button (circular arrow) after changing these settings.
— Menu Format > Recolor selected curves…

Also read again in **chapter Initiation :**
— Lesson 1, page 10, "Changing appearance of curves" ;
— Lesson 2, page 12, "Customizing curves" ;
— Lesson 4, pages 24 & 25, "Customizing the graph" and "Settings for differential equation solutions" ;
— Lesson 5, pages 28, "Modifying, customizing 3D curves and surfaces" ;
— Lesson 6, pages 32 & 35-36, "Customizing points and curves", "Drawing with the point sets".

Adding objects to a 2D graph.

Available processes.

You may use :
— **Menu Object** > all items ;
— **Drag and Drop equations** written with equation editors of Microsoft Office, AppleWorks, MathType, etc.

Read again in **chapter Initiation:**
— Lesson 1, page 10, "Adding objects to the graph" ;
— Lesson 2, page 13, "Adding objects to the graph", "Adding equations to the graph", "Other additions to the graph" ;
— Lesson 3, pages 18 & 19, "Automatic display on the graph of values of changing parameters", "Other objects placed on the graph" ;
— Lesson 4, page 24, "Customizing the graph".

Objects "attached to the real field" or not.

Objects added to the graph can be "attached to the real field" or not, depending on whether the ad hoc box of the Inspector is ticked or not.

— Object not attached to the real field.

This is the default situation (box unchecked).
It keeps a fixed size and its distances from the top and left edges of the graph (including margins) remain constant, when modifying axes, scales, margins, graph dimensions, size of the window, etc.
These characteristics are stored when saving as GCX file, and the objects will be at their positions after opening the .gcx file and restoring length, height and Oy scale of the graph **(see bugs nr. 2).**

— Object attached to the real field.

To do this, check the box in the Inspector.
It keeps a fixed size and a specific point of this object keeps constant its coordinates (x ; y) when moving axes and changing coordinate scales. The "specific point" of the object is the tip for an arrow, and can be selected for the others by moving with the mouse the round target in the square of the Inspector (especially edges or corners).
Changing the working window shape, the size and margins of the graph, inconsistently modifies location of the object.
After opening a saved CGX file, the objects attached to the real field have the same coordinates, but restoring size and Oy scale of the graph **(see bug nr. 2)** moves them ; therefore finally there is no possibility saving objects attached to the real field.
This procedure can be useful to name noticeable points or place comments attached to curves, they will remain properly placed and will allow copies of the graph only with different magnifications or changes of coordinate scales.

Check box "Auto" : function not yet discovered…

3D Graphs : perspective and appearance options.

Perspective.

Grapher shows the 3D graphs using real perspective view (three-point perspective), resulting in a distortion of graphs, especially the frame (cube), somewhat surprising compared to the usual isometric viee in mathematics, distortion increased since the observer is placed very close to the frame of the graph. The parameters of the Grapher perspective are not adjustable and there is no option "isometric".

Effects of zooms.

Menu View > Zoom in (out), ⌘+, ⌘- and the **buttons Zoom in and Zoom out** in the toolbar do not modify the 3D figure size, axes and frame dimensions remain unchanged, but act on the graph ones by modifying the coordinate scales.

Pop-up Window "View Angles".

The commands available via the **menu Window > Display View Angles** or by typing **⇧⌘V** affect the overall figure (axes + frame + graph) without changing the scales of coordinates. You may enter the following values :

— Δx and Δy move the figure horizontaliy and vertically in the field of graphs ;
— Δz simulates closing in or moving off by changing its size, exponential function of Δz ;
— Zoom out / Zoom in gives the same effect, but the size is proportional to the displayed parameter (> 0) ;
— ϕ, θ and ψ are three angles in degrees which allow to observe the figure from any direction with reference to the coordinate axes.

You can see in this window the words "By Default, Above, Front, Left" and buttons to create other lines. They control the seven parametrers above for standard views, as suggested by reading the preference file com.apple.grapher.plist **(see bug nr. 25)**. The mouse allows any combination ϕ θ ψ to display for example : Default 90° 45° 135°, Above 0° 0° 135°, Front 90° 90° 135°, Left 90° 90° 180° in degree in the order ϕ θ ψ.

Rotations of the 3D graph.

• If the button "Action / Rotation" is selected :

— moving the mouse East-West = rotation of the graph around Oz axis ;
— moving the mouse North-South = rotation of the graph around an East-West axis passing through the origin of axes ;
— by pressing the «option» (alt) key, moving the mouse North-South gives a rotation around an axis perpendicular to the monitor screen (zenith-nadir), which allows for example to direct the Oz axis in a position other than North-South.

• If the button "Action / Rotation" is grey : no controls on the rotation.

Other actions on the 3D graph.

• "Capitals" (Shift) key + mouse to zoom the whole figure.
• "Apple" (Command) key + mouse to move the whole figure inside the graph field.
• Buttons "Zoom in" and "Zoom out" : modify the scales on the three axes, frame and axes remaining same sizes, the plotted graph remaining limited in the same coordinates values. This figure only shows a modified size..

Other uses.

Animations.

— **Menu Equation > Animate parameter :** chapter **Initiation, Lesson 3 page 15**, detailed explanations (2D).
— **Menu Equation > Create animation… :** chapter **Initiation, Lesson 5 page 27**, detailed explanations (2D & 3D).

Point sets.

See chapter **Initiation Lesson 6 page 31** for their uses.

Scalar and vector fields.

See Menu **Examples** > 2D Examples > **Fields** and > 3D Examples > **Vector Field**. **(See bug nr. 4).**

The examples provided with this software are a source of information that should not be overlooked, once opened their GCX files are operational and editable **(See bug nr. 6 & 21).**

Information to note before saving a 2D document.

Before saving a 2D document, four numbers should be noted :

— L length (or width) of the graph (unit : point, more accurate and multilingual),
— H its height (unit : point, more accurate and multilingual),
— minimum and maximum values of the ordinate scale (Oy).

It is convenient to store these data as comments in the list of equations **(see bug nr. 27).**
These values are necessary to restore the original 2D document after opening a GCX file **(see bug nr. 2).**

Saving, exporting, templates.

Saving the document as GCX file (.gcx).

⌘S or **⇧⌘S** or **menu File > Save as…** or **Save** or **Save all**
See bug nr. 2. The document can be reopened to rework, modify, etc.

Exporting equations.

— See chapter “Using the equation editor“, page 50, “Exporting an equation“ (one at a time).
— To save the equation list alone, use screenshot with **⇧⌘4**.

Exporting the graph.

— By creating an **image file** : **menu File > Export… > choose format**, TIFF (set Resolution), PDF, EPS or JPEG (set Resolution and Compression).
— By **copying its image** in the clipboard to paste it into another application :
Ctrl-click or **right click in the graph > contextual menu Copy as TIFF, PDF or EPS**
or menu Edition > Copy as TIFF, PDF, or EPS.
— By **printing** its image : **menu File > Page Set up** then **Print…** etc., allows also to create a PDF file.

For accurate sizing of the graph : chapter Initiation, Lesson 3, pages 15 & 16, “Dimensioning the graph with precision“, “Verifying and adjusting the graph dimensions“.

Creating and storing templates.

After being created and customized, the template is saved as a GCX file **(think of bug nr. 2)** ; to make it accessible by the menu Examples, store it in a new folder placed in :
Macintosh HD > Library > Application Support > Apple > Grapher > Examples > 2D Examples, 3D Examples, **Templates.**

Grapher's bugs

A bit of history.

Grapher’s root.

Grapher 1.1, an OS X 10.4.11 Tiger Utility (Nov 2007), is the Apple’s version of Curvus Pro X version 1.3.2 (2006-2008). If a Grapher’s bug was already in Curvus Pro X this is noted after the bug title. Grapher 2.0 was released in OS X 10.5 Leopard (May 2008), then Grapher 2.1 in OS X 10.6 Snow Leopard (Oct 2009), Grapher 2.2 in OS X 10.7 Lion (Aug 2011), 2.3 in 10.7.4 (May 2012) and in Mountain Lion 10.8 (Oct 2012), Grapher 2.5 in Mavericks OS X 10.9 (Oct 2013) and Yosemite OS X 10.10 (Oct 2014).

Main changes from Curvus Pro to Grapher.

MISSING OPPORTUNITIES :

— in the Preferences : - the modules adding three functions in 2D,
- 2D : transparent background when exporting ;

— menu Format : possibility to change coordinate names ;

— toolbar : the cursor cross “Coordinates“ that allowed display their values for any point inside the 2D graph window, as well as their differences between two points ;

— menu Definition (now Equation) : to synthetize a sound according to a function form ;

— menu View > Switch to 3D view : this command remains to change the graph type from 2D to 3D (and vice versa) ; it allowed more in Curvus Pro X : to convert with one click a 2D curve into a surface of revolution, without entering its 3D equation.

I particularly regret the opportunities to choose the coordinate names, very easy way to customize equations and axes, and the sound synthesis very useful to explain its generation.

HELP AND EXAMPLES :

— The “Help“ has been reduced considerably, becoming inadequate and poorly presented, it is very damaging to this very good software ;

— the following examples are not repeated in Grapher :
- 2D : Bifurcation Diagram, Examples of Functions, Piecewise Defined functions, Series, Stationary Wave, Variable Parameter, Waves ;
- 3D : Bipolar Field, Implicit Surface, Interferences, Klein Bottle, Surface of Revolution.

These examples had a didactic interest (mathematical, syntax, animation, presentation, spectacular).

WHAT’S NEW ?

Unfortunately, there is no improvement in this area, the only news, a regrettable one, is the appearance of a large number of bugs ; their inventory and workarounds are the subjects of the next paragraph.

Grapher’s bugs and their workarounds (OS X set in French).

Bug nr. 1. (Grapher 1.1, 2.0 to 2.5). When modifying the main 2D window size, the graph length and height already formatted (menu Format) are changed : axis scale values stay the same, added objects remain same size on the same places (unit pixel) relative to the graph left upper corner ; but the graph view changes for the ratio H/L do so : **very annoying !**

Workaround : To restore the 2D graph after changing the window size : menu Format > Layout… > Size > redo the initial choice (paper size or L and H values)… or avoid having to do so by selecting a window large enough to begin with.

Proposed correction : making the graph size independent of the main window’s one.

Bug nr. 2. (Grapher 1.1, 2.0 to 2.5). Saving is incomplete in 2D (menu File > Save, Save as…). What are not saved : the last location on the screen of the main window, the graph size, the maximum and minimum values of the ordinate scale : **very very annoying !**

Note : The new windows of Grapher 1.1 (Grapher secondary window > New Graph > 2D or 3D Graph) always open in the same place on the screen(OS X 10.4.11 configured in French) depending on the language used by Grapher : if it’s English, well placed and covering most of the screen, if it is French, the windows are half length but same height and are located on the right side of the screen. This anomaly disappeared with Grapher 2.0 to 2.5 (English or French).

Workaround : to restore the 2D graph after opening a GCX files (.gcx) : menu Format > Layout… > Size > redo the initial choice (Paper size or L and H values) **to be done twice** ; then menu View > Frame limits... > enter minimum and maximum vaues of the y scale. This requires one to note four numerical values, L, H, y-min, y-max, for example at the top of the equation list (or at the bottom : see **bug nr. 27**).

Proposed correction : complete saving in 2D.

Bug nr. 3. (Curvus Pro X 1.3.2, Grapher 1.1, 2.0 to 2.5). 2D, menu Format > Layout… > Size : in this pop-up window, units conversion from inch (or points) to millimetres (or cm) is false, based on 1 inch = 25.6 mm instead of 25.4 mm. After printing, the graph dimensions on the sheet of paper are exactly those entered in points or inches in the Size window (error -0.2 % for L and - 0.5 % for H) ; using mm errors become - 1.0 % and -1.3 % ; amazing in a math software !

— in Grapher used in French language : - sizes in inches can't be used and repeat values in points ;
- inch is translated into " pouce ", please use "inch" even in French.

— in Grapher used in English language, the four units are available (with unit conversion error above).

Workaround : cm or mm sizes are to be multiplied by 25.4 / 25.6 = 0.9922.

Proposed correction : use the proper metric value of the inch, ensure that cm, mm, inches, points, are displayed in French as in English, use “inch“ instead of “pouce“ in all languages.

Bug nr. 4. (Curvus pro X 1.3.2, Grapher 1.1, 2.0 to 2.5). 2D and 3D, anomaly when saving vector fields. After opening a vector field file, we notice :

— with Grapher 1.1 and 2.0 the vector field graphs are modified : coordinates Δy or $\Delta\theta$ are zero everywhere ;

— with all versions of the software, the menu Equation > Evaluation shows a pop-up window with the first vector coordinate, Δx or Δr in 2D, Δx, Δr_0 or Δr in 3D, repeated once (2D) or twice (3D).

Workaround : do a tiny modification in the vector field equation > Return key > restore the equation > Return key > everything is in order.

Proposed correction : do what is necessary…

Bug nr. 5. (Grapher 1.1, 2.0 to 2.5). Menu View > Switch to 3D view : in 2D, this command does not play one of its roles allowed in Curvus Pro X : to create with a single click, a surface of revolution with Oz axis from a 2D generating curve $y = f(x)$, y and x becoming r_0 and z in the 3D graph.

Workaround : it’s not very difficult to enter the equation $r_0 = f(z)$ in a 3D window.

Proposed correction : if cancelling this function is not deliberate, restore it.

Bug nr. 6. (Grapher 1.1 & 2.0). 2D and 3D : coordinates θ and ϕ anomalies.

The coordinate systems available in a Grapher working window (a new one, or from an example, or from a saved .gcx file) is shown by menu Equation > New Point Set > Edit Points… > Coordinates > System choice, that reveal the names of the coordinates available in this Grapher window.

— Bug nr. 6.1. (Grapher 1.1, 2.0 to 2.5). In 2D : new windows (from the secondary window “New Graph“) use (x, y) and (r, θ) coordinates and refuse ϕ ; the same for these windows saved as .gcx files. But the “2D Examples“ (except for Polar Logarithmic) use (x, y) and (r, ϕ) and refuse θ ; that’s still true after modifying these Example files.

Workaround : Rebuild all 2D examples in new windows, so the 2D coordinate systems will be (x, y) and (r, θ) everywhere in your version of Grapher, including Examples. This will avoid confusion about the angular coordinate.

Proposed correction : same idea.

— Bug nr. 6.2. (Grapher 1.1). 3D : new windows use or show coordinates (x, y, z), (r_0, θ, z) and ... (r, θ, θ) ! So spherical coordinates are not available from Grapher new 3D windows. But “3D Examples“ use (x, y, z), (r_0, ϕ, z) and (r, ϕ, θ). Starting from an example spherical coordinates can be employed.

Workaround : Don’t cancel the 3D Grapher Examples, use them as working windows (clear up the equations), instead of new windows. This will allow you to use the three 3D coordinate systems.

Nr.6.2 bug is fixed in Grapher 2.0 to 2.5 with 3D coordinates (x, y, z), (r_0, θ, z), (r, θ, ϕ) for new windows. The Examples use the same right coordinates in **2.0**, **but ϕ and θ are permutated again in Grapher 2.1 to 2.5 examples** (except Toroid and Torus-Knot), its 3D Examples must be rebuilt in new windows .

Bug nr. 7. (Grapher 1.1). 3D : axes systems are incomplete in preview window (2D 3D curves choice) as well in main windows.

No Workaround.

Bug fixed in Grapher 2.0 to 2.5.

Bug nr. 8. (Grapher 1.1 & 2.0). 2D axes labels : in axes Inspector, label perpendicular cursor does not work properly, labels cannot be placed both sides of the axes.

The remedy : remove the axis name with the Inspector, then menu Objects > Insert Text > type the axis name > place it where you want.

Bug fixed in Grapher 2.1 to 2.5.

Bug nr. 9. (Grapher 1.1). Groups and Point Sets can't be titled.

A semi-workaround : menu Equation > New Equation > erase “y=“ > type the desired title > Return > menu Equation > New Group or Point Set (remaining without title but the line above).

Bug fixed in Grapher 2.0 to 2.5.

Bug nr. 10. (Grapher 1.1 & 2.0). Built-in-definitions : The modified Bessel function of the second kind Kn(x) exists only for n = 0 and 1.

Workaround : use the recurrence formula with K , K_{n-1} , K_{n+1} to calculate the following K_n for n ≥ 2.

Bug resolved in Grapher 2.1 to 2.5.

Bug nr. 11. (Grapher 1.1). Built-in-definitions : Grapher 1.1 quits when typing Airy functions Ai(x) or Bi(x).

Workaround : use their definitions, see <**http://mathworld.wolfram.com/AiryFunctions.html**>

Bug fixed in Grapher 2.0 to 2.5.

Bug nr. 12. (Curvus Pro X 1.3.2, Grapher 1.1, 2.0 to 2.5). Bessel spherical functions $j_n(x)$ et $y_n(x)$:

— In Curvus Pro, $j_n(x)$ is correct, but $y_n(x)$ copy $j_n(x)$ (same curve).
— In Grapher 1.1, $j_n(x)$ = 0 for all n and x, $y_n(x)$ is not recognized.
— In Grapher 2.0 to 2.5 both functions are recognized but $j_n(x) = y_n(x) = 0$ for all n and x.

Workaround : Use j_n and y_n definitions, see <**http://mathworld.wolfram.com**>. Be carefull : j, y but not J, Y.

Proposed correction : restore the proper Bessel spherical functions.

Bug nr. 13. (Grapher 2.0 French language). The icing on the cake ! In the 2D-3D graph choice window, the 2D curve called Log-Log in English should keep the same name in French instead of "Historique-historique" ! French sailors would prefer “Journal de bord-Journal de bord“ !

Bug fixed in Grapher 2.1 to 2.5.

Bug nr. 14 (Curvus Pro X 1.3.2, Grapher 1.1, 2.0 to 2.5). 2D "Animate Parameter " :

In a new 2D window (New Graph > 2D Graph), create a parameter animated curve, don't start the animation ; with the "move" tool (the hand) move the axes : the curve follows the axes. Start the animation, move axes using the move tool : the animated curve does not follows axes ; Stop animation, move axes again : the curve jumps to its right place ! Save in a .gcx file, re-open, this strange behaviour won’t hapen anymore.

Workaround : unnecessary.

Proposed correction : save this curiosity !

Bug nr. 15 (Grapher 1.1, 2.0 to 2.5). In 2D windows, menu Equation > Create Animation :

The various zoom and frame animations from the pop-up window “Animation“ move axes and grids only but not the plotted curves that remain perfectly stable. These commands are irrelevant under these conditions. It worked perfectly in Curvus Pro X 1.3.2, zooming moved axis, grid, frame and curves.

Proposed correction : restore these animations.

Bug nr. 16 (Grapher 1.1, 2.0 to 2.5). In 3D windows, menu Equation > Create Animation… : the Parameter, Zoom and Frame Limits animations from the pop-up window “Animation“ do not work. The only possible one is the Orientation tab animation.

Proposed correction : restore these animations.

Bug nr. 17. (Curvus Pro X 1.3.2, Grapher 1.1 & 2.0). Wrong unit name :

Menu Grapher > Preferences > Advanced > Trigonometric Mode : there is no angle unit called "gradient" ; the correct name is "grade" (= π / 200) (ref. <**http://www.bipm.org**>).

Workaround : unnecessary.

Bug fixed in Grapher 2.1 to 2.5.

Bug nr. 18. (Grapher 1.1). Wrong coordinate name :

Menu Grapher > Preferences > Advanced > Polar Angular Range, under 0...2π or -π...π, the explanatory note shows “ϕ“ instead of “θ“.

Bug fixed in Grapher 2.0 to 2.5.

Bug nr. 19.(Grapher 1.1). Wrong coordinates names :

Menu Equation > New Equation from Template..., the coordinates names are still default names used in Curvus Pro X 1.3.2, they were not updated after the modifications entered in Grapher.

— **Bug nr. 19.1 (Grapher 1.1).** 2D polar angle is still “ϕ“ insread of “θ“.

Workaround : replace “ϕ“ by “θ“ in the selected equation templates.

Bug fixed in Grapher 2.0 to 2.5.

— Bug nr. 19.2 (Grapher 1.1). In 3D windows, the cylindrical coordinates show "ϕ" insread of "θ", the spherical coordinates show (r, ϕ, θ) intead of (r, θ, ϕ).

Workaround : In Grapher 1.1 do not change anything **(as in bug nr. 6.2)**.

Bug fixed in Grapher 2.0 to 2.5.

Bug nr. 20 (Curvus Pro X 1.3.2, Grapher 1.1, 2.0 to 2.5). In 2D and 3D windows, the global definitions of parameters or functions included in a "Group" remain active despite they are no longer in the equation list after cancelling the group. Example : in a new 2D window, create a group and enter in it "f(x) = {sinx, cosx, tanx}", cancel the group, now there is nothing in the equation list ; enter the equation "y = f(x)" alone, the graph window display the three trigonometric curves, defined before by "f(x)" cancelled. More, the global parameter or function definition, now invisible, can't be erased and pollute indefinitely the working window ; sometimes entering new equations becomes impossible.

Workaround : Do not enter global definitions of parameters or functions into a group, or remove them from the group before cancelling it. If a group including a global definition has been cancelled, restart your work in a new window free of "ghosts" of old definitions.

Proposed correction : Knowing just the above workaround seems reasonable to me.

Bug nr. 21 (Curvus Pro X 1.3.2, Grapher 1.1, 2.0 to 2.5). In OS X set in French, the decimal comma is not accepted in the 3D examples, the decimal point (dot) is required.

Workaround : in Grapher 1.1 use the decimal point (because of bug nr. 6.2 workaround). In Grapher 2.0 to 2.5 rebuild all the examples in new windows using correct coordinate names and allowing decimal comma.

Proposed correction : rebuild 3D examples in new windows **(see bug nr. 6).**

Bug nr. 22. (Grapher 2.0). After saving a 2D or 3D document including an empty group, this .gcx file can't be opened : it's impossible to use this document.

Workaround : avoid empty groups (one letter or digit alone is enough) before saving the document..

Bug fixed in Grapher 2.1 to 2.5.

Bug nr. 23. (Grapher 2.0). Derivatives, therefore differential equations of order higher than the first one are no longer recognized (unlike Curvus Pro X 1.3.2 and Grapher 1.1 had no limit to the order of derivatives). This bug appears in Leopard working in French or in English.

No workaround known.

Bug fixed in Grapher 2.1, 2.2 and 2.3 (after OS X 10.7.2 update) to 2,5.… Great !

Bug nr. 24. (Grapher 2.0). In 3D windows, animation creation using menu Equation > Create Animation… > Orientation, becomes impossible as soon as the 3D curve or surface has been modified (complicated) with the Inspector (resolution, line, hollow, contour, shadow, etc.). After it happened once, no more animation can be created in this window. This bug was not in Grapher 1.1.

No workaround known.

Bug fixed in Grapher 2.1 to 2.5.

Bug nr. 25 (Grapher 2.0 to 2.5). In 3D, menu Window > Display View Angles : the four commands Default, Above, Front, Left, don't work.

Workaround : Use the mouse and/or direct entry of viewing angle parameters.

Proposed correction : restore these commands.

Bug nr. 26 (Grapher 2.1 to 2.5 and possibly earlier versions). The definite double integrals in 2D and 3D windows entered from the keyboard or by menu Equation / Integration give wrong results, except in special cases of constant or single variable function.

Workaround : no alternative to manual calculations.

Proposed correction : restore correct calculations.

Bug nr. 27 (Grapher 1.1, 2.0 to 2.5). The menu "Create Animation" opens a pop-up window "Animation" which lists the available parameters if 1) they have global definitions, 2) they are not preceded in the equation list by a line showing the yellow triangle "danger", 3) they are not preceded in the equation list by any group. **(In 3D see also bug nr. 16).**

Workaround : take into account these constraints, particularly in placing, if necessary, at the end of the equation list, the four numerical values mentioned in the workaround of **bug nr. 2.**

Proposed correction : unnecessary ?

Bug nr. 28 (Grapher 2.2 to 2.5) (or deliberate modification). Definitions of the functions :

— if the variable names are names of coordinates available in the working window, the function is considered an expression of fixed form. Example : if $f(x) = 3 + x^2$ we have $f(x+1) = 3 + x^2$ and not $f(x+1) = 3 + (x+1)^2$ as it was in all previous versions of Grapher and Curvus Pro X.

— if the variable names are different, the "function of a function" remains in service. Example : if $f(t) = 3 + t^2$ we have $f(t+1) = 3 + (t+1)^2$ and $f(x+1) = 3 + (x+1)^2$ as it was in all previous versions of Grapher and Curvus Pro X.

Workaround : 1) Apply in the syntax of functions. 2) The example "Periodic Functions" must be modified because of an f(x) and a g(x) not working now in Grapher 2.2 to 2.5 new windows. 3) The Grapher 2.2 to 2.5 Help don't mention this important information about the functions.

Proposed correction : Cancel this absolutely useless modification, restore the function syntax as it was in Grapher 2.1 : the use of functions showed no difficulties, only one kind of function instead of two today, the Help was right on this subject, so was the above example.

Bug nr. 29 (Grapher 2.2 to 2.5). In 2D windows, menu Equation / Integrate, or an equation of curve with indefinite integral : the curve appears as a dotted line and false for the negative values of the variable.

Workaround : Replace the name of the coordinate under the integral symbol by an other different from a coordinate name. Example : read y = integrate from 0 to x dt f(t) instead of y = integrate from 0 to x dx f(x)

Proposed correction : possibly caused by **bug nr. 28** modification ? Same proposal.

Bug nr. 30 (Grapher 2.2 to 2.5) (or deliberate modification). Local definitions no longer override global definitions.

Workaround : A parameter, variable or function with a global definition cannot also receive local definitions.

A parameter, variable or function without global definition can receive a different local definition in each line of equation.

Proposed correction : Restore the possible coexistence of both kinds of definitions ?

Bug nr. 31 (Grapher 2.2 to 2.5). Menu Equation > Evaluation… > enter value > after clicking the Stroke button or closing the Evaluation pop-up window, the osculating circle disappears from the graph. Possible explanation :

The radius of the circle is " R " and has a "local definition" on the same line as the equation of the circle. Also " R " has a "global definition" in the "Built-in Definitions", it is the "molar gas constant" ; the **bug nr. 30** renders the two definitions incompatible.

Workaround : replace R in its equation with R_0 or another available name of parameter.

Proposed correction : possibly caused by **bug nr. 30** modification ? Same proposal.

Bug nr. 32 (Grapher 2.2 and 2.3 French, German, Spanish, Italian but NOT in English). Menu Equation> Animate Parameter: two icons have disappeared from the banner, "Play / Pause" and "Stop" (see first figure page 18), the mouse click is still operational and necessary on their empty places.

Workaround : find the locations of icons using the figure on page 18.

Proposed correction : restore these two icons.

Bug fixed with Grapher 2.3 (in OS X 10.8) to 2.5.

Bug nr. 33 (Grapher 2.2 and 2.3 French, German, Spanish, Italian but NOT in English). Inspectors of graphs and differential equations : the "Refresh" icons (circular arrows) have disapeared (see first figure page 25) ; the mouse click is still operational and necessary on their empty places to "enter" the modifications.

Workaround : find the locations of icons using the figure on page 25.

Proposed correction : restore this icon.

Bug fixed with Grapher 2.3 (in OS X 10.8) to 2.5.

Bug nr. 34 (Grapher 2.5). In 3D, menu Equation > Animate Parameter : « Create Movie » does not work.

Le remède : animation running > New Screen Recording (selection) by QuickTime Player 10.3.

Proposition de correction : restore « Create Movie » which works very well in 2D.

Improving Grapher Help.

This is the purpose of this manual which I hope will facilitate the use of Grapher and which, despite its bugs, is an outstanding tool… and also free.

We wish however for a rapid removal by Apple of the 19 bugs remaining in Grapher 2.5 !

APPENDICES

APPENDIX 1

Point sets: from the spreadsheet to Grapher

A simple «Copy-Paste» is enough :

Spreadsheet layout.

— 1 coordinate per column ;
— 1 point per row ;
— number formatting : number of decimal places, minus sign for negative numbers, no thousands separator, scientific notation allowed ;
— the decimal comma is required by OS X 10.6 to 10.9 set in French ;
— the number of columns may be greater than that of the coordinates, we'll choose in Grapher the column assigned to each coordinate ;
— no blank cell, row or column within the table, and only numbers ;
— save the table selected on the worksheet.
— select and **COPY** the data block (cells) to be transferred to Grapher.

In Grapher :

— Point Set / Edit Points… / select and erase (Back Return key) default or obsolete data ;
— menu Edit / **PASTE** / OK.

You can copy-paste the data with several successive tables, they enter following one another in the set of points. Maximum number of data : unknown, one has already tried successfully several thousands points.

From the Grapher point set to the spreadsheet : same procedure **COPY-PASTE** to a block of cells selected in the spreadsheet.

Other methods and file formats.

With **Grapher 2.1 to 2.5 et OS X** set tn **French,** the **decimal comma** is required in these files.

AppleWorks 6.2.9.

— Menu File > Save as > Text ASCII > title, destination, add the extension « .txt » > Save.
Note. Decimal commas become dots.
— Open the file in TextEdit > Menu Edit > Search > Search > search , (comma) Replace by . (point) > Replace all ;
— Save the changed file with the extension «.txt ».

Excel (Microsoft Office 2004, 2008 et 2011).

— Save as > Text (Tab delimited) > title, destination > Save ;
NB. You can also : Save as… > CSV > file .csv > then change extension «.csv» into «.txt».

Numbers in iWork '08 et '09 then Numbers 3.0.

— Menu File > Export > CSV > one of the three options > title, destination > Save > file .csv > then change extension «.csv» into «.txt».

NeoOffice 3.1.2.

— Save as > Text CSV (.csv) > Save > choose a code (Unicode UTF-8), Field Separator « ; », Text Separator blank > OK > file .csv > then change extension «.csv» into «.txt».

File formats.

The resulting files can be imported into the Grapher point sets and their format is :

MarginA1TabB1TabC1TabD1.............TabZ1Return		MarginA1;B1;C1;D1;........;Z1Return
MarginA2TabB2TabC2TabD2.............TabZ2Return	or	MarginA2;B2;C2;D2;........;Z2Return
etc.		

where Ai, Bi,........,Zi are the coordinates of the point i (one line per point, one column per coordinate, no blank).

APPENDIX 2

Adaptation of lesson 4 to Grapher 2.0

Bug nr. 23 appeared with version 2.0 of Grapher ; this version can recognize only the first derivatives ; here we replace the second order differential equation in Lesson 4 by a first order equation. This bug does not exist in Curvus Pro X 1.3.2 nor in Grapher 1.1, 2.1 to 2.5.

Changes in the text of lesson 4.

Page 20 "The problem.", replace lines 3 to 7 by :

$$y' + \frac{y}{\tau} - fe^{-\frac{x}{\tau}}\left(\frac{\cos\omega x}{\tau} - \omega\sin\omega x\right) = 0 \quad \text{with} \quad y = y(x) \quad \text{and} \quad x \in [0, \pi]$$

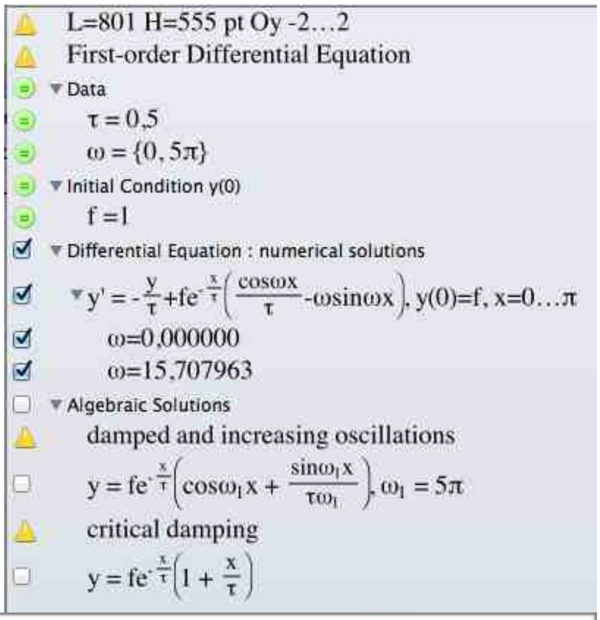
initial condition for $x = 0$: $y(0) = f$

for $\tau = 0.5$ and $\omega = 0$ then 5π

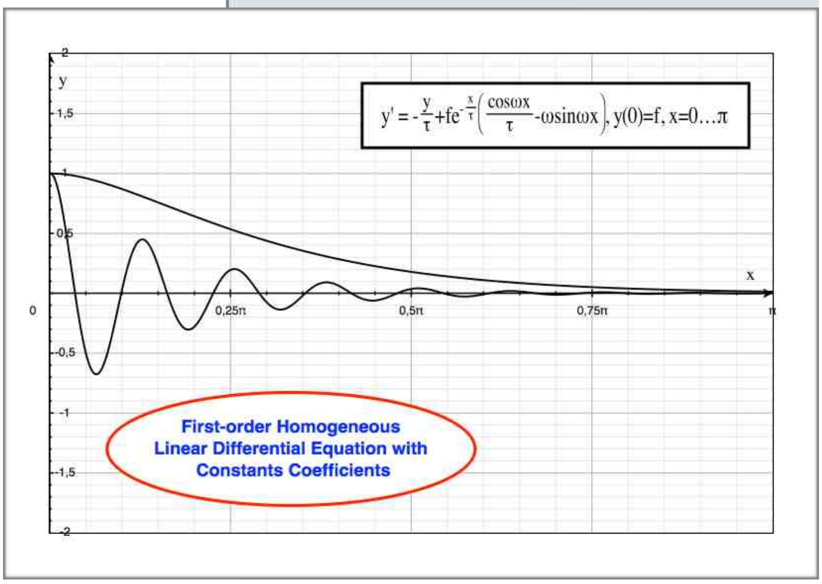
$$\text{for } \omega \neq 0 : y = fe^{-\frac{x}{\tau}}\left(\cos\omega x + \frac{\sin\omega x}{\tau\omega}\right) \text{ and for } \omega = 0 : y = fe^{-\frac{x}{\tau}}\left(1 + \frac{x}{\tau}\right)$$

Page 24 « Enter the equations of the problem. », replace this paragraph by :

Do it according to the list of the figure which includes :
- size of the graph and scale Oy (**see bug nr. 1 & 2**) ;
- title of the document ;
- four groups titled ;
- two additional comment lines ;
- three equations for the parameter values ;
- the differential equation (the two values of ω that follow will be listed by Grapher automatically) ;
- two classical equations y = y(x).



Page 26, replace the last graph by the following :



Practice of lesson 4.

The remaining text is the same, the curves obtained are the same ; you can practice it at a profit because it gives many explanations in addition to the use of differential equations.

APPENDIX 3

Calculations with complex numbers

Choice of the angle unit and its limitations.

Grapher / Preferences / Advanced / Trigonometric Mode / choose between degree, grades, radians.
The selected unit is used in direct or inverse trigonometric functions, but not in the form $\rho e^{i\alpha}$ of complex numbers nor in the angular coordinates where the angles remain always in radians.
Examples after choice of unit « degree » :

- $\theta = 30$ graph a straight line of polar angular coordinate 30 rad that is - 81.127°
- the same e^{30i} is a complex number of affix (r = 1 ; θ = 30 rad)
- but sin30 = 0.500
- and cos30 + i.sin30 is a complex number of affix (r = 1 ; θ = 30°)

The only way to enter angles in degrees in the first two cases is to display (π/180).θ°

Menu Grapher / Preferences… /

Numbers : Imaginary part (i or j) and check box « at the end » : applies only to the display of the results of constant complex expressions calculations.

Advanced : Polar angular values : apply also to the calculation of constant complex expression arguments.

Calculations of constant complex expressions.

The results are always displayed in the Cartesian form a ± ib, a ± bi, a ± jb or a ± bj. To calculate the modulus use the "absolute value" sign of the Equation Palette or the built-in function "abs(z)", for the argument the function "arg(z)".
The result of the "arg(z)" calculation is always given in radians regardless of the choice done in the preferences.

Exponents : the allowed exponents are ± n , ±1/n (n integer ≥ 0) and the radical sign √ (square root) ; the following operation signs give false results with complex : $\sqrt[2]{\square}$ $\sqrt[3]{\square}$ $\sqrt[n]{\square}$ In addition, in the case of exponent ±1/n, Grapher calculates only one of the n roots of the complex expression z.

The complex expression syntax.

Imaginary unit : you may use either i or j and both of them in one expression, provided that these letters were not defined as real numbers. This is particularly the case of the expression template of a sequence where i is the default index of the terms ; if so, use only j in complex expression terms or replace i of the sequence by another letter.

Cartesian and polar forms : Grapher supports complex expressions in the two forms, Cartesian f(t) + i.g(t) and polar $r(t).e^{i.\theta(t)}$ or r(t).(cos(θ(t)) + i.sin(θ(t))) where t is a possible parameter to draw 2D graphs.

Complex variables : in this note about complex numbers, all variables and functions mentioned so far are real : a, b, f(t), g(t), r(t), θ(t), abs(z), arg(z) with the exception of i, j, and z. Constants and variables defined in the equation editor are supposed real by default ; a variable, such as t you want to be complex must be called "complex(t)". Example :

f(t) = 1/complex(t) (definition of a function of t, complex variable)
x+iy = $f(e^{-it})$, t = 0…π/2 (quarter circle radius 1 in the first quadrant of the axes)

If we had defined the function by f(t) = 1/t only the real part of the complex expression x+iy that is 1/cos(-t) would appear on the graph, a half-line on the Ox axis from x = +1 to + ∞.

Graphical representation in the complex plane (Ox real axis, Oy imaginary axis) : the syntax must be :

x + iy = f(t), t = t1…t2 (or another parameter name, or x + jy instead of x + iy)

the parametric form is possible : $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\left(f\left(e^{-it}\right)\right) \\ \mathrm{Im}\left(f\left(e^{-it}\right)\right) \end{bmatrix}, \ t = 0\ldots\frac{\pi}{2}$$ (quarter circle seen above) or another one for the complex expression z(t) : $\begin{bmatrix} r \\ \theta \end{bmatrix} = \begin{bmatrix} \mathrm{abs}(z(t)) \\ \arg(z(t)) \end{bmatrix}, \; t = t_1 \ldots t_2$

Syntax of a complex sequence (example) :

$$\begin{bmatrix} r_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} \mathrm{abs}\left(\sum_{l=0}^{k+1} \frac{(i\pi)^l}{l!} \right) \\ \arg\left(\sum_{l=0}^{k+1} \frac{(i\pi)^l}{l!} \right) \end{bmatrix}, \begin{bmatrix} r_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} \mathrm{abs}\left(\frac{(i\pi)^0}{0!} \right) \\ \arg\left(\frac{(i\pi)^0}{0!} \right) \end{bmatrix}, \; k=0\ldots10$$

Using the built-in functions.

Accessible via the menu Help / Show Built-in Definitions. These “Instructions for use - Grapher“ lists them page 53 at the end of the chapter “Using the equation editor“ indicating for each function if the argument must be real or may be complex.

Hyperbolic functions : they admit only real arguments. But it is very easy to “build“ a version for complex argument, for example :

$$\mathrm{mycosh}(x) = \frac{e^{\mathrm{complex}(x)} + e^{-\mathrm{complex}(x)}}{2}$$

then draw the curve y = mycosh(ix) same as y = cos(x)

Complex logarithm : the natural (napierian) logarithm function “ln(x)“ admits a complex argument ; examples :
real : ln(2) = 0,693147 ; complex : $\ln(2e^{i\pi})$ = 0,693147 + i.π This is not the case of the common (decimal) logarithm “log(x)“ :
real : log(2) = 0,301030 but **beware of this anomaly : $\log(2e^{i\pi})$ = 0,693147 + i.π** as for $\ln(2e^{i\pi})$.

Example of a gain calculation in dB from the complex gain A(f) function of the signal frequency f :

$$A(f) = \frac{1}{1 + j\frac{f}{f_0}} \quad \text{and in dB : } 20\log(\mathrm{abs}(A(x))) \;\text{ or }\; 20\frac{\ln(\mathrm{abs}(A(x)))}{\ln 10} \;\text{ or }\; 20\frac{\mathrm{Re}(\ln(A(x)))}{\ln 10}$$

Menu “Equation“ : options for the curves x + iy = f(t).

“Differentiate“ and “Integrate“ : are not in use ;
“Find roots“ : is generally not possible ;
“Integration“ : generally gives wrong results ;
“Evaluation“ : generally operational and correct ;
“Find intersection“ between two curves :

- works for couples of equations x+iy=f(t) and y=g(x), x+iy=f(t) and x=g(y), and parametric pair x=f(t) y=g(t) and y=h(x) or x=h(y) ;
- refused for couples consisting of an equation x+iy=f(t) with the second of the same form or implicit or parametric pair, and of two parametric pairs.

APPENDIX 4

Indexed expressions

Finding indexed expressions in Grapher.

• Coefficients of polynomial regressions a_1 to a_n : they cannot be used elsewhere ;

• In the expression defining a sequence, there is an index (default i) applied to the 2 or 3 coordinates ; in 2D a sequence draw points on the graph : the indexed terms of the sequence cannot be used elsewhere ;

• In the matrices (maximum 3x3) : extracting an element a_{1j} is possible but not easy (see Appendix 9 pages 82-83) ;

• In the point sets : but no accessible index, we can extract neither an element nor a row nor a column ;

• In the operations defined by the ∑ et ∏ signs : no possibility to use indexed term or factor elsewhere.

Grapher having not much capacity in this area, we must find some tips…

Case of coefficients (or expressions) function of the index.

Here is an example with 2 coefficients :

a(i) = 0.5i - 2	coefficient function of i
b(i) = -3i + 1	coefficient function of i
f(i) = a(i).x + b(i)	function of the index i
y = f(0)	curve plotted : straight line y = -2x +1
y = f(2)	curve plotted : straight line y = -x - 5
etc.	

Each coefficient (or expression) is a function of a particular index.

Here is an example with 3 coefficients :

a(i) = 0.5i - 2	coefficient function of i
b(j) = -3j + 1	coefficient function of j
c(k) = k	coefficient function of k
f(i,j,k) = a(i).x + b(j) + c(k)	function of the indices i, j and k
y = f(0, 0, 0)	curve plotted : straight line y = -2x +1
y = f(2, 1, 2)	curve plotted : straight line y = -x
etc.	

The coefficients (or expressions) are not functions of indices.

Here is an example with 3 coefficients and 4 values of the index, which represent 4 circles of 4 centres and radii, animated by the index k integer ranging from 0 to 3 :

Test of index animation

$$a_1(n) = \begin{cases} 0.5 & n==0 \\ 0 & n==1 \\ -0.5 & n==2 \\ 0 & n==3 \end{cases}$$

$$a_2(n) = \begin{cases} 0 & n==0 \\ 0.5 & n==1 \\ 0 & n==2 \\ -0.5 & n==3 \end{cases}$$

$$a_3(n) = \begin{cases} 0.25 & n==0 \\ 0.5 & n==1 \\ 0.75 & n==2 \\ 1 & n==3 \end{cases}$$

k

$x+iy=a_1(k)+ia_2(k)+a_3(k).e^{it}, t=0…2\pi$

This method allows any number of indexed expressions and index values, but through many lines of writing…

With a few expressions and index values, it's faster to write the final equations without using indices.

Special case: using 2x1 or 3x1 matrices (vectors).

For a linear expression of indexed coefficients (or functions) in the form :

$y = a_i.f(x) + b_i.g(x) + c_i.h(x)$

we can use the more condensed text in two equations of the figure :

$$A = \left\{ \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}, \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}, \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}, \text{etc.} \right\}$$

$$y = A^T \begin{bmatrix} f(x) \\ g(x) \\ h(x) \end{bmatrix}$$

equivalent to these equations :

$y = a_0f(x) + b_0g(x) + c_0h(x)$

$y = a_1f(x) + b_1g(x) + c_1h(x)$

$y = a_2f(x) + b_2g(x) + c_2h(x)$

etc.

It does not really use the indices, it's limited to 3 “indexed“ coefficients (or expressions), but is very efficient at minimizing the number of signs.

APPENDIX 5

Matrices and determinants : solutions of linear equations

The size of matrices and determinants is limited to 3x3, the linear system to be resolved is limited to three equations with three unknowns.

Matrices and determinants

▼ General Data

equation to be rebuilt (coefficients a, b, c) :

$y = 0{,}5x^2 + 0{,}25x - 2$

known points coordinates :

$x+iy = -2 - 0{,}5i$ $= -2{,}000000 - 0{,}500000i$

$x+iy = 1 - 1{,}25i$ $= 1{,}000000 - 1{,}250000i$

$x+iy = 3 + 3{,}25i$ $= 3{,}000000 + 3{,}250000i$

coefficients calculation matrix :

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & 3 & 1 \end{bmatrix}$$

▼ Calculating with matrices

matrix of supposed result :

$$A_1 = \begin{bmatrix} 0{,}5 \\ 0{,}25 \\ -2 \end{bmatrix}$$

y values matrix :

$$A_3 = \begin{bmatrix} -0{,}5 \\ -1{,}25 \\ 3{,}25 \end{bmatrix}$$

checking y values calculation :

$A_4 = A \cdot A_1$

$$A_4 = \begin{bmatrix} -0{,}500000 \\ -1{,}250000 \\ 3{,}250000 \end{bmatrix}$$

coefficient calculation from the known points :

$$A_2 = \left(\frac{1}{|\det(A)|} \cdot A^{-1} \right) \cdot A_3$$

matrix of the 3 coefficients a, b, c (result) :

$$A_2 = \begin{bmatrix} 0{,}500000 \\ 0{,}250000 \\ -2{,}000000 \end{bmatrix}$$

▼ Using determinants (Cramer's method)

writing the 3 other matrices :

$$A_a = \begin{bmatrix} -0{,}5 & -2 & 1 \\ -1{,}25 & 1 & 1 \\ 3{,}25 & 3 & 1 \end{bmatrix}$$

$$A_b = \begin{bmatrix} 4 & -0{,}5 & 1 \\ 1 & -1{,}25 & 1 \\ 9 & 3{,}25 & 1 \end{bmatrix}$$

$$A_c = \begin{bmatrix} 4 & -2 & -0{,}5 \\ 1 & 1 & -1{,}25 \\ 9 & 3 & 3{,}25 \end{bmatrix}$$

coefficient calculation (Cramer's method) :

$$a = \frac{(\det(A_a))}{(\det(A))}$$

$$b = \frac{(\det(A_b))}{(\det(A))}$$

$$c = \frac{(\det(A_c))}{(\det(A))}$$

results :

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0{,}500000 \\ 0{,}250000 \\ -2{,}000000 \end{bmatrix}$$

APPENDIX 6

Expression syntax : supplements

Choice of the angle unit and its limitations.

Grapher / Preferences / Advanced / Trigonometric Mode / choose between degree, grades, radians.
The selected unit is used in direct or inverse trigonometric functions, but neither in the form $\rho e^{i\alpha}$ of complex numbers nor in the angular coordinates where the angles remain always in radians.
Examples after choice of unit "degree" :

- θ = 30 graph a straight line of polar angular coordinate 30 rad that is - 81.127°
- the same e^{30i} is a complex number of affix (r = 1 ; θ = 30 rad)
- but sin30 = 0.500
- and cos30 + i.sin30 is a complex number of affix (r = 1 ; θ = 30°)

The only way to enter angles in degrees in the first two cases is to display (π/180).θ°

The equation palette : menu Window / Show Equation Palette.

Check box "Use Shortcuts When Typing".

In addition to operate on Greek letters (see paragraph in the centre of page 50), it replaces, when the box is checked, the asterisk sign " * " (multiplication) by a point (dot) " . " in expressions.

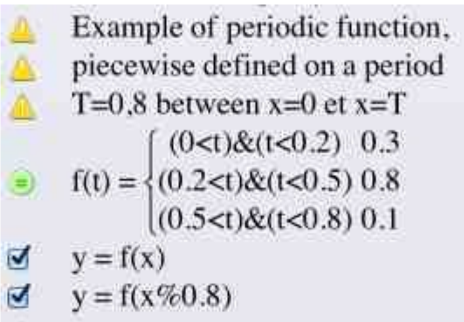
Equation Palette / Operators / Binomial Coefficient.

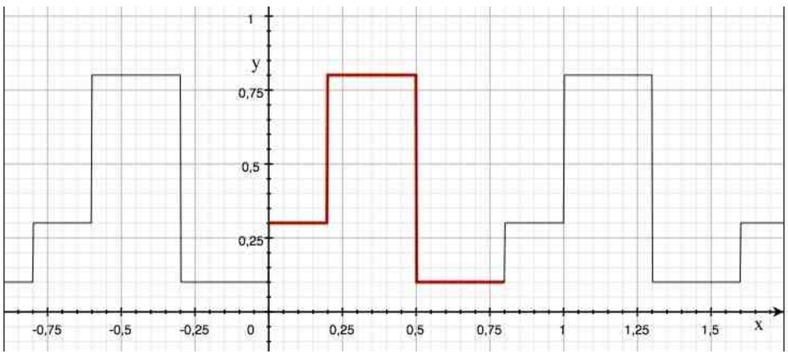
The last operator $\begin{pmatrix} \square \\ \square \end{pmatrix}$ calculate binomial coefficients $\begin{pmatrix} m \\ p \end{pmatrix} = C_m^p$

Expressions : the modulo operator.

In the example of the condition : $n \% d == r_c$ we can remember the "equality test" operator eq(a, b) also written "==" in Mathlab or Python ; the difference is that in these softwares, the formula "returns" 1 (true) or 0 (false) ; this return does not exist in Grapher in explicit form, but when used as a condition in an equation of any type (e.g. piecewise-defined functions) (n being an integer), the "answer" true/false plays its part.

Expressions : periodic piecewise-defined function (example).





Expressions : integral syntax.

Nota : The integration variable must never be a coordinate name.

Grapher allows various expressions containing the integral sign :

- constant expressions to be calculated : $\int_{t_{min}}^{t_{max}} dt\ f(t)\ ;\ \int_{t_{min}}^{t_{max}} f(t)\ ;\ \int_a^b f(t)\ ;\ \text{e.g.}: \int_{t=0}^{\infty} e^{-t}$ or other parameter or x
- functions to be plotted (all coordinate systems) :

$$y = \int_{f(x)}^{g(x)} dt\ h(t)\ ;\ y = \int_{f(x)}^{g(x)} du\ h(u)\ ;\ y = \int_{x_0}^{x} dt\ f(t)\ ;\ y = \int_{1}^{g(x)} dt\ h(t)\ ;\ y = \frac{2}{\sqrt{\pi}} \int_{t=0}^{x} e^{-t^2}$$

don't forget the space between dt and the function. **See also bugs nr. 26, 28 et 29.**

Expressions : derivative syntax.

N° 1 format : f' , f'' and f''' (but not f'''') :

This format (maximum third derivative) allows functions, numerical calculations, graphs. Examples :

f(t) = E(t)	(definition of a function of the variable t with the expression E(t))
g(t) = f'''(t)	(definition of the third derivative function of f(t))
a = 3.5	(definition of a constant)
g(a)	(numerical calculation of f'''(t) for t = a)
y = f'''(x) or y = g(x)	(graph of the third derivative of f(x))

N° 2 format : $\frac{d^n}{dt^n}$ or $\frac{\partial^n}{\partial t^n}$

This format has different possibilities from the previous :

- function definitions of this type are not allowed : $g(t)=\frac{d^n}{dt^n}f(t)$ or $g(x)=\frac{\partial^k}{\partial x^k}f(x)$
- with the order n of the derivative in literal form, we can draw the graphs :

n = {0, 1, 2, 3} (definition of the constant n)

$$y=\frac{d^n}{dx^n}f(x)$$ **limited to the third derivative : n ≤ 3**

- with the order of the derivative in numerical form, we can draw the graphs :

$$y=\frac{d}{dx}f(x)\ ,\ y=\frac{d^2}{dx^2}f(x)\ ,\ y=\frac{d^3}{dx^3}f(x)\ \ldots\ y=\frac{d^6}{dx^6}f(x)\ \ldots\ \text{with}\quad f(x)=\frac{x^7}{2520}\ \text{for example}$$

Example : Taylor series of ln(x), centred on x value a, and limited to the fourth term.

a=1

f(t)=ln(t)

$$y=f(a)+f'(a).\frac{x-a}{1!}+f''(a).\frac{(x-a)^2}{2!}+f'''(a).\frac{(x-a)^3}{3!}$$

Indeterminate forms in Grapher.

- 1) $\frac{0}{0}, \frac{\infty}{\infty}, 0.\infty, \infty-\infty$: "undefined"
- 2) $0^0=\infty^0=1^\infty=1$

APPENDIX 7 **Surfaces of revolution from 2D curves**

Surfaces de révolution obtenues par rotation d'une courbe 2D autour des axes Ox ou Oy

Surfaces of revolution obtained by rotating a 2D curve around the x- or y-axis

Équation 2D de la courbe *Curve 2D equation*		**Rotation autour de l'axe…** *Rotation around axis…*	**Équation 3D de la surface de révolution d'axe Oz** *Surface of revolution (axis Oz) 3D équation*		
explicite / implicite	**paramétrique**		**explicite / implicite**	**paramétrique cylindrique / sphérique**	**paramétrique cartésienne**
$y = f(x) \quad x \in [a ; b]$	$x = t$ $y = f(t) \quad t \in [a ; b]$	Ox	$r_0 = f(z) \quad z \in [a ; b]$	$r_0 = f(t)$ $\theta = u \quad u \in [0 ; 2\pi]$ $z = t \quad t \in [a ; b]$	$x = f(t).\cos u$ $y = f(t).\sin u \quad u \in [0 ; 2\pi]$ $z = t \quad t \in [a ; b]$
$x = f(y) \quad y \in [a ; b]$	$x = f(t)$ $y = t \quad t \in [a ; b]$	Oy	$r_0 = f(z) \quad z \in [a ; b]$		
$f(x, y) = 0$		Ox	$f(z, r_0) = 0$		
		Oy	$f(r_0, z) = 0$		
	$x = g(t)$ $y = h(t) \quad t \in [a ; b]$	Ox		$r_0 = h(t)$ $\theta = u \quad u \in [0 ; 2\pi]$ $z = g(t) \quad t \in [a ; b]$	$x = h(t).\cos u$ $y = h(t).\sin u \quad u \in [0 ; 2\pi]$ $z = g(t) \quad t \in [a ; b]$
		Oy		$r_0 = g(t)$ $\theta = u \quad u \in [0 ; 2\pi]$ $z = h(t) \quad t \in [a ; b]$	$x = g(t).\cos u$ $y = g(t).\sin u \quad u \in [0 ; 2\pi]$ $z = h(t) \quad t \in [a ; b]$
$r = f(\theta) \quad \theta \in [a ; b]$	$\theta = t \quad t \in [a ; b]$ $r = f(t)$	Ox	$r = f(\phi) \quad \phi \in [a ; b]$	$r = f(t)$ $\theta = u \quad u \in [0 ; 2\pi]$ $\phi = t \quad t \in [a ; b]$	$x = f(t).\sin t.\cos u$ $y = f(t).\sin t.\sin u \quad u \in [0 ; 2\pi]$ $z = f(t).\cos t \quad t \in [a ; b]$
		Oy	$r = f(\pi/2 - \phi)$ $\phi \in [\pi/2 - b ; \pi/2 - b]$	$r = f(t)$ $\theta = u \quad u \in [0 ; 2\pi]$ $\phi = \pi/2 - t \quad t \in [a ; b]$	$x = f(t).\cos t.\cos u$ $y = f(t).\cos t.\sin u \quad u \in [0 ; 2\pi]$ $z = f(t).\sin t \quad t \in [a ; b]$
$\theta = f(r) \quad r \in [a ; b]$	$\theta = f(t)$ $r = t \quad t \in [a ; b]$	Ox	$\phi = f(r) \quad r \in [a ; b]$	$r = t \quad t \in [a ; b]$ $\theta = u \quad u \in [0 ; 2\pi]$ $\phi = f(t)$	$x = t.\sin[f(t)].\cos u$ $y = t.\sin[f(t)].\sin u \quad u \in [0 ; 2\pi]$ $z = t.\cos[f(t)] \quad t \in [a ; b]$
		Oy	$\phi = \pi/2 - f(r) \quad r \in [a ; b]$	$r = t \quad t \in [a ; b]$ $\theta = u \quad u \in [0 ; 2\pi]$ $\phi = \pi/2 - f(t)$	$x = t.\cos[f(t)].\cos u$ $y = t.\cos[f(t)].\sin u \quad u \in [0 ; 2\pi]$ $z = t.\sin[f(t)] \quad t \in [a ; b]$
$f(r, \theta) = 0$		Ox	$f(r, \phi) = 0$		
		Oy	$f(r, \pi/2 - \phi) = 0$		
	$r = g(t)$ $\theta = h(t) \quad t \in [a ; b]$	Ox		$r = g(t) \quad t \in [a ; b]$ $\theta = u \quad u \in [0 ; 2\pi]$ $\phi = h(t)$	$x = g(t).\sin[h(t)].\cos u$ $y = g(t).\sin[h(t)].\sin u \quad u \in [0 ; 2\pi]$ $z = g(t).\cos[h(t)] \quad t \in [a ; b]$
		Oy		$r = g(t) \quad t \in [a ; b]$ $\theta = u \quad u \in [0 ; 2\pi]$ $\phi = \pi/2 - h(t)$	$x = g(t).\cos[h(t)].\cos u$ $y = g(t).\cos[h(t)].\sin u \quad u \in [0 ; 2\pi]$ $z = g(t).\sin[h(t)] \quad t \in [a ; b]$
Si f( ) est une fonction définie par morceaux *If f( ) is a piecewise defined function*		Ox / Oy	La surface construite par morceaux juxtaposés est en général mieux représentée *The surface built with several juxtaposed pieces usually show a better graph*		

APPENDIX 8

Converting files from Curvus Pro X (.cpx) to Grapher (.gcx)

Operation nr. 1.

Change the extension from “ **.cpx** “ to “ **.gcx** “ ;
Open the file in Grapher.

Other operations for 2D graphs.

- Check that the names of coordinates in the equations are the same as those imposed by Grapher ;
- Enlarge the window to the right to bring up the left edge of the graph ;
- Enlarge the equation area to a sufficient size ;
- Menu Format > Layout > Size : enter the dimensions L and H (original if possible) of the graph (to be done twice) ;
- Menu View > Frame Limits : enter the minimum and maximum values of the ordinates (see bug nr. 2) ;
- Note these values L, H, ymin and ymax in a false equation at the top or the end of the equation list (see bug nr. 27) ;
- Save the changed file.

Notes. 1) Sometimes the curves of some equations do not display after changing the extension, saving the restored file solves this problem.

2) If the coordinate names are not those imposed by Grapher, the equations have to be rewritten, that means to do again in a Grapher new window the whole work made before in Curvus ProX…

3) Beware of bugs nr. 6.1 and 23 if you use versions 1.1 or 2.0 of Grapher, the conversion of the files could be impossible.

Other operations for 3D graphs.

- Check that the names of coordinates in the equations are the same as those imposed by Grapher ;
- Enlarge the window to the right to bring up the left edge of the graph ;
- Enlarge the equation area to a sufficient size ;
- Save the changed file.

Notes. 1) If the coordinate names are not those imposed by Grapher, the equations have to be rewritten, that means to do again in a Grapher new window the whole work made before in Curvus Pro X…

2) Beware of bugs nr. 6.1 and 23 if you use versions 1.1 or 2.0 of Grapher, the conversion of the files could be impossible.

APPENDIX 9

Matrices in Grapher

Definitions.

- **nxp** : matrix dimensions ;
- **n :** number of rows ;
- **p :** number of columns ;
- **i :** row index ;
- **j :** column index ;
- **if n = p :** square matrix of order n ;
- **if n = 1 :** row vector ;
- **if p = 1 :** column vector ;
- **a_{ij} :** element (or coefficient) of the matrix ;
- **line :** row or column.

- identity and diagonal matrices of order n are square matrices of order n.

- **Matrices availables in Grapher :**
  2x1 2x2 3x1 3x3
  1x2 after transposition of 2x1
  1x3 after transposition of 3x1
  They are defined in the complex set ℂ.

examples :

$$A = A_{2x3} = \left(a_{ij}\right)_{2x3} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$$

$$\text{null (or zero) matrix} : 0_{2\times3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{identity (or unit) matrix} : I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{diagonal matrix} : B_3 = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix}$$

- **Determinant of a square matrix :**

$$\text{if } A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \det(A) = |A| = \Delta^A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\text{if } B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \quad \det(B) = |B| = \Delta^B = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{cases} +b_{11}b_{22}b_{33} - b_{11}b_{23}b_{32} \\ +b_{12}b_{23}b_{31} - b_{12}b_{21}b_{33} \\ +b_{13}b_{21}b_{32} - b_{13}b_{22}b_{31} \end{cases}$$

Addition of matrices of the same dimensions.

- **Addition of two matrices :** $A_{n\times p} + B_{n\times p} = \left(a_{ij} + b_{ij}\right)_{n\times p} \qquad A + B = B + A$

- **Opposite matrix :** $-A_{n\times p} = \left(-a_{ij}\right)_{n\times p} \qquad A_{n\times p} + \left(-A_{n\times p}\right) = \left(-A_{n\times p}\right) + A_{n\times p} = 0_{n\times p}$

Multiplication by a scalar α ou β (complex).

$$\alpha \cdot A_{n\times p} = \left(\alpha \cdot a_{ij}\right)_{n\times p} \;;\; \alpha \cdot (A + B) = \alpha \cdot A + \alpha \cdot B \;;\; (\alpha + \beta) \cdot A = \alpha \cdot A + \beta \cdot A \;;\; \alpha \cdot (\beta \cdot A) = (\alpha \cdot \beta) \cdot A$$

Transposition of a matrix.

To transpose a matrix means to exchange rows and columns.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad A^T = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \quad B^T = \begin{pmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}$$

$$C=\begin{pmatrix} c_{11} \\ c_{21} \end{pmatrix} \quad C^T=\begin{pmatrix} c_{11} & c_{21} \end{pmatrix} \quad D=\begin{pmatrix} d_{11} & d_{12} \end{pmatrix} \quad D^T=\begin{pmatrix} d_{11} \\ c_{12} \end{pmatrix}$$

$$\left(A^T\right)^T=A$$

$$\left(\alpha\cdot A^T\right)^T=\alpha\cdot A^T$$

$$E=\begin{pmatrix} e_{11} \\ e_{21} \\ e_{31} \end{pmatrix} \quad E^T=\begin{pmatrix} e_{11} & e_{21} & e_{31} \end{pmatrix} \quad F=\begin{pmatrix} f_{11} & f_{12} & f_{13} \end{pmatrix} \quad F^T=\begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \end{pmatrix}$$

$$(A+B)^T=A^T+B^T$$

$$(A\times B)^T=B^T\times A^T$$

A is symmetric if $A^T=A$, antisymmetric if $A^T=-A$

Multiplication of matrices.

- **matrix (n x p) X matrix (p x q) = matrix (n x q)** hence the only available cases in Grapher **:**

matrix (2 x 2) X matrix (2 x 2) = matrix (2 x 2)
matrix (2 x 2) X matrix (2 x 1) = matrix (2 x 1)
matrix (1 x 2) X matrix (2 x 2) = matrix (1 x 2)
matrix (3 x 3) X matrix (3 x 1) = matrix (3 x 1)
matrix (1 x 3) X matrix (3 x 3) = matrix (1 x 3)
matrix (1 x 2) X matrix (2 x 1) = matrix (1 x 1)
matrix (2 x 1) X matrix (1 x 2) = matrix (2 x 2)
matrix (1 x 3) X matrix (3 x 1) = matrix (1 x 1)
matrix (3 x 1) X matrix (1 x 3) = matrix (3 x 3)
matrix (2 x 1) X matrix (1 x 3) = matrix (2 x 3)
matrix (3 x 1) X matric (1 x 2) = matrix (3 x 2)

A x B ≠ B x A in general
the matrix multiplication
is not commutative.

- **General formula :** $C_{n\times q}=A_{n\times p}\times B_{p\times q}=\left(c_{ij}\right)_{n\times q}$ with $c_{ij}=\sum_{k=1}^{p}a_{ik}b_{kj}$ for all i from 1 to n, for all j from 1 to q

- **Available cases in Grapher :**

$$C_{2\times2}=A_{2\times2}\times B_{2\times2}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\times\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{pmatrix}$$

$$C_{2\times1}=A_{2\times2}\times B_{2\times1}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\times\begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} \\ a_{21}b_{11}+a_{22}b_{21} \end{pmatrix}$$

$$C_{1\times2}=A_{1\times2}\times B_{2\times2}=\begin{pmatrix} a_{11} & a_{12} \end{pmatrix}\times\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \end{pmatrix}$$

$$\begin{cases} C_{3\times3}=A_{3\times3}\times B_{3\times3}=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\times\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \\ =\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{pmatrix} \end{cases}$$

$$C_{3\times1} = A_{3\times3} \times B_{3\times1} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} \end{pmatrix}$$

$$\begin{cases} C_{1\times3} = A_{1\times3} \times B_{3\times3} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \\ = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \end{pmatrix} \end{cases}$$

$$C_{1\times1} = A_{1\times2} \times B_{2\times1} = \begin{pmatrix} a_{11} & a_{12} \end{pmatrix} \times \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} = a_{11}b_{11} + a_{12}b_{21}$$

$$C_{2\times2} = A_{2\times1} \times B_{1\times2} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} \\ a_{21}b_{11} & a_{21}b_{12} \end{pmatrix}$$

$$C_{1\times1} = A_{1\times3} \times B_{3\times1} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix} \times \begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \end{pmatrix} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$$

$$C_{3\times3} = A_{3\times1} \times B_{1\times3} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & b_{13} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} \\ a_{21}b_{11} & a_{21}b_{12} & a_{21}b_{13} \\ a_{31}b_{11} & a_{31}b_{12} & a_{31}b_{13} \end{pmatrix}$$

$$C_{2\times3} = A_{2\times1} \times B_{1\times3} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & b_{13} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} \\ a_{21}b_{11} & a_{21}b_{12} & a_{21}b_{13} \end{pmatrix}$$

$$C_{3\times2} = A_{3\times1} \times B_{1\times2} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} & a_{11}b_{12} \\ a_{21}b_{11} & a_{21}b_{12} \\ a_{31}b_{11} & a_{31}b_{12} \end{pmatrix}$$

Power of a square matrix of order n.

$$A_n^K = A \times A \times A \times \ldots \times A \ \text{ (K times A) with by definition } \ A_n^0 = I_n$$

$$A_n^K \times A_n^{K'} = A_n^{K+K'} \quad \left(A_n^K\right)^{K'} = A_n^{K\times K'} \quad \left(\alpha \cdot A_n^K\right) = \alpha^K \cdot A_n^K \quad \text{with } K \in N$$

Inversion of a nonsingular square matrix of order n.

- **Definitions.**

A square matrix A is nonsingular (regular) if det(A) ≠ 0, singular if det(A) = 0

$$\text{matrix inverse of } A_n \text{ nonsingular : } A_n^{-1} \text{ such that } A_n^{-1}\times A_n = A_n \times A_n^{-1} = I_n$$

$$\left(A^{-1}\right)^{-1} = A \quad (A\times B)^{-1} = B^{-1}\times A^{-1} \quad \left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T} \quad \det\left(A^{-1}\right) = \frac{1}{\det(A)}$$

- **Available cases in Grapher.**

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \quad B^{-1} = \frac{1}{\det(B)}\begin{pmatrix} b_{22}b_{33}-b_{23}b_{32} & b_{13}b_{32}-b_{12}b_{33} & b_{12}b_{23}-b_{13}b_{22} \\ b_{23}b_{31}-b_{21}b_{33} & b_{11}b_{33}-b_{13}b_{31} & b_{13}b_{21}-b_{11}b_{23} \\ b_{21}b_{32}-b_{22}b_{31} & b_{12}b_{31}-b_{11}b_{32} & b_{11}b_{22}-b_{12}b_{21} \end{pmatrix}$$

Expressions of the elements as function of their matrix.

In Grapher, constant matrices are displayed with their elements which are readable ; matrix calculation is possible with variable matrices but their elements are not visible ; in both cases it is not possible to select an element to copy and paste it in an equation. The expressions below will allow to use these matrix elements.

- **Case of a matrix 2x2.**

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad N_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad N_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{matrix} a_{11} = N_1^T A N_1 & a_{12} = N_1^T A N_2 \\ a_{21} = N_2^T A N_1 & a_{22} = N_2^T A N_2 \end{matrix}$$

- **Case of a matrix 3x3.**

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, N_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, N_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, N_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} a_{11} = N_1^T A N_1 & a_{12} = N_1^T A N_2 & a_{13} = N_1^T A N_3 \\ a_{21} = N_2^T A N_1 & a_{22} = N_2^T A N_2 & a_{23} = N_2^T A N_3 \\ a_{31} = N_3^T A N_1 & a_{32} = N_3^T A N_2 & a_{33} = N_3^T A N_3 \end{matrix}$$

- **Case of a matrix 3x1.**

$$A_{3\times 1} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \quad N_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad N_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad N_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{matrix} a_{11} = N_1^T A \\ a_{21} = N_2^T A \\ a_{31} = N_3^T A \end{matrix}$$

- **Case of a matrix 2x1.**

$$A_{2\times 1} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \quad N_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad N_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{matrix} a_{11} = N_1^T A \\ a_{21} = N_2^T A \end{matrix}$$

- **Case of a matrix 1x3.**

$$A_{1\times 3} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix} \quad N_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad N_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad N_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{matrix} a_{11} = AN_1 \\ a_{12} = AN_2 \\ a_{13} = AN_3 \end{matrix}$$

- **Case of a matrix 1x2.**

$$A_{1\times 2} = \begin{pmatrix} a_{11} & a_{12} \end{pmatrix} \quad N_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad N_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{matrix} a_{11} = AN_1 \\ a_{12} = AN_2 \end{matrix}$$

- **Case of a matrix 3x2.**

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \quad N_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad N_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad N_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad N_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad N_5 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$a_{11} = N_3^T A N_1 \quad a_{12} = N_3^T A N_2$$
$$a_{21} = N_4^T A N_1 \quad a_{22} = N_4^T A N_2$$
$$a_{31} = N_5^T A N_1 \quad a_{32} = N_5^T A N_2$$

• **Case of a matrix 2x3.**

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \quad N_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad N_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad N_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad N_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad N_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_{11} = N_4^T A N_1 \quad a_{12} = N_4^T A N_2 \quad a_{13} = N_4^T A N_3$$
$$a_{21} = N_5^T A N_1 \quad a_{22} = N_5^T A N_2 \quad a_{23} = N_5^T A N_3$$

Syntax of matrices in Grapher.

• **Creating matrices.**

— matrices 2x2, 3x3, 2x1, 3x1 : operators of the equation palette to be completed ; the elements can be real or complex, numeral or literal, constant or variable, simple or complicated expressions.

— matrices 1x2, 1x3 : by transposition of matrices 2x1 and 3x1 ;

— matrices 2x3, 3x2 : by multiplication of matrices (2x1)X(1x3) and (3x1)X(1x2) (to be used as examples only because it is not possible to choose all the elements : 5 variables to find 6 element values).

• **How to use the elements of matrices.**

— in matrix operators from the equation palette : they can be selected and used for «copy-paste».

— in constant matrices : the matrix can be displayed with the numerical values of the elements by entering the matrix name ; the elements are readable but can't be selected ; they can be expressed in terms of their matrix as described above and then normaliy used in new expressions.

— in variable matrices resulting from calculations or transformations : they can't be displayed ; to be used, the elements must be expressed in terms of their matrix.

• **Syntax.**

— obvious in most cases : A -A A^T A+B A-B AxB A.B AB A^K αxA α.A αA ;

— determinants (square matrices of dimension ≥ 1) : det(A) ;

— syntax of matrix inverse :

- entering A^{-1} gives the result I det(A) I x A^{-1} (matrix inverse x absolute value of its determinant),
- for the matrix inverse A^{-1} of A, enter :

$$\frac{1}{\left|\det(A)\right|} A^{-1} \quad \text{or} \quad A^{-1} \frac{1}{\left|\det(A)\right|}$$

Some applications.

• Conversions between 2D or 3D coordinates systems ;

• Rotations and translations of 2D and 3D graphs;

• Electronics : calculations of quadripoles ; we can use complex admittances and impedances, all matrices which characterize them, and use their elements to plot response curves.

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