Bayesian Cities Model 1D

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```
library("rstan")
```

Inputs

Notation

Given

$$Y = (Y_1, \ldots, Y_i)$$

where each Y_i has its own given σ_i,assume $Y \sim N(\theta_i, \sigma_i^2)$.

We won't be looking at the individual data in any study, so we'll essentially be assuming that the sample size for every study $J = 1$. Y_i is the *point estimation*, the mean of the study, and σ_i is that study's variance.

Arbitrary Example Inputs

For now, we'll use that classic arbitrary dataset I've been using We'll hope that we can recover mu and tauSq correctly!

```
# Set input parameters
set.seed(17)
mu <- 10
tauSq <- 2

# Set general constants
C <- 2 # Number of pilots we can run
D <- 2 # Number of cities we can implement the final program in
I <- 3 # Number of studies (AKA number of cities)
J <- 1 # Number of data points per study (Currently setting this to 1 for simplicity)
num_hypothetical_draws <- 15 # Number of times you draw new pretend pilot study results

# Generate theta and sigmaSq based off of mu, tau, I, J
sigmaSq <- 10*runif(I) # For now, all sigma are ~U(0,1)
theta <- rnorm(I, mu, tauSq) # and theta ~N(mu,tauSq)
sigmaSq <- matrix(sigmaSq, I , J)
theta <- matrix(theta, I , J)

# Calculate and reshape Y
Y <- list(mu=theta, sd=sigmaSq)

# Save our input data together in a list
basic_dat_generated <- list(J=J,I=I,Y=theta,sigmaSq=sigmaSq)

# Display what we've generated
basic_dat_generated
```

```
## $J
## [1] 1
##
## $I
## [1] 3
##
## $Y
##           [,1]
## [1,] 11.522993
## [2,] 10.194808
## [3,]  8.222759
##
## $sigmaSq
##          [,1]
## [1,] 1.550508
## [2,] 9.683788
## [3,] 4.682631
```

Step 1: Calculate θ_i, μ, τ^2 using the original priors Y

We're assuming a **random effects model**, that is that

$$\theta_i \sim N(\mu, \tau^2) \text{ and } Y_i \sim N(\theta_i, \sigma_i^2)$$

To make use of this assumption, we need to estimate scalars μ and τ^2 along with the vector $\theta = (\theta_1, \ldots, \theta_N)$. We can do this using stan!

```
fit <- stan(file = 'randomEffectsModelConstrainedI.stan',
            data = basic_dat_generated,
            iter = 1000, chains = 2)
```

```
## In file included from C:/Users/Dina/Documents/R/win-library/3.3/BH/include/boost/config.hpp:39:0,
##                  from C:/Users/Dina/Documents/R/win-library/3.3/BH/include/boost/math/tools/config.hp
##                  from C:/Users/Dina/Documents/R/win-library/3.3/StanHeaders/include/stan/math/rev/co
##                  from C:/Users/Dina/Documents/R/win-library/3.3/StanHeaders/include/stan/math/rev/co
##                  from C:/Users/Dina/Documents/R/win-library/3.3/StanHeaders/include/stan/math/rev/co
##                  from C:/Users/Dina/Documents/R/win-library/3.3/StanHeaders/include/stan/math/rev/ma
##                  from C:/Users/Dina/Documents/R/win-library/3.3/StanHeaders/include/stan/math.hpp:4,
##                  from C:/Users/Dina/Documents/R/win-library/3.3/StanHeaders/include/src/stan/model/m
##                  from file93c06ba1401c.cpp:8:
## C:/Users/Dina/Documents/R/win-library/3.3/BH/include/boost/config/compiler/gcc.hpp:186:0: warning: "
##  #  define BOOST_NO_CXX11_RVALUE_REFERENCES
##  ^
## <command-line>:0:0: note: this is the location of the previous definition
##
## SAMPLING FOR MODEL 'randomEffectsModelConstrainedI' NOW (CHAIN 1).
##
## Gradient evaluation took 0 seconds
## 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Adjust your expectations accordingly!
##
##
## Iteration:   1 / 1000 [  0%]  (Warmup)
## Iteration: 100 / 1000 [ 10%]  (Warmup)
```

```
## Iteration: 200 / 1000 [ 20%]  (Warmup)
## Iteration: 300 / 1000 [ 30%]  (Warmup)
## Iteration: 400 / 1000 [ 40%]  (Warmup)
## Iteration: 500 / 1000 [ 50%]  (Warmup)
## Iteration: 501 / 1000 [ 50%]  (Sampling)
## Iteration: 600 / 1000 [ 60%]  (Sampling)
## Iteration: 700 / 1000 [ 70%]  (Sampling)
## Iteration: 800 / 1000 [ 80%]  (Sampling)
## Iteration: 900 / 1000 [ 90%]  (Sampling)
## Iteration: 1000 / 1000 [100%]  (Sampling)
##
##  Elapsed Time: 0.071 seconds (Warm-up)
##                0.066 seconds (Sampling)
##                0.137 seconds (Total)
##
##
## SAMPLING FOR MODEL 'randomEffectsModelConstrainedI' NOW (CHAIN 2).
##
## Gradient evaluation took 0 seconds
## 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Adjust your expectations accordingly!
##
##
## Iteration:   1 / 1000 [  0%]  (Warmup)
## Iteration: 100 / 1000 [ 10%]  (Warmup)
## Iteration: 200 / 1000 [ 20%]  (Warmup)
## Iteration: 300 / 1000 [ 30%]  (Warmup)
## Iteration: 400 / 1000 [ 40%]  (Warmup)
## Iteration: 500 / 1000 [ 50%]  (Warmup)
## Iteration: 501 / 1000 [ 50%]  (Sampling)
## Iteration: 600 / 1000 [ 60%]  (Sampling)
## Iteration: 700 / 1000 [ 70%]  (Sampling)
## Iteration: 800 / 1000 [ 80%]  (Sampling)
## Iteration: 900 / 1000 [ 90%]  (Sampling)
## Iteration: 1000 / 1000 [100%]  (Sampling)
##
##  Elapsed Time: 0.084 seconds (Warm-up)
##                0.098 seconds (Sampling)
##                0.182 seconds (Total)
```

```
## Warning: There were 74 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
```

```
## Warning: Examine the pairs() plot to diagnose sampling problems
```

```
fit
```

```
## Inference for Stan model: randomEffectsModelConstrainedI.
## 2 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=1000.
##
##            mean se_mean   sd   2.5%    25%    50%    75% 97.5% n_eff Rhat
## mu        10.51    0.26 6.54  -2.71   8.76  10.49  12.66 25.63   638 1.00
## tau        2.22    0.13 1.46   0.76   1.22   1.75   2.76  6.02   137 1.00
## theta[1]  11.18    0.12 1.52   8.17  10.11  11.02  12.23 14.20   168 1.00
## theta[2]  10.56    0.26 5.05  -0.65   8.15  10.50  13.01 20.91   379 1.00
```

```
## theta[3]   9.68    0.24 3.64   1.35   8.09   9.83  11.98 16.21   222 1.02
## lp__     -15.06    0.34 3.15 -21.25 -17.31 -14.71 -12.66 -9.28    85 1.01
##
## Samples were drawn using NUTS(diag_e) at Thu Jul 05 10:22:15 2018.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
# Readjust knowledge of Y based off of REM
params <- extract(fit)
for (i in 1:length(Y$mu)){
  Y$mu[i] <- mean(params$theta[,i])
}
# TODO: Do we need to update the sd at all here post basic fit?
Y
```

```
## $mu
##           [,1]
## [1,] 11.176597
## [2,] 10.555695
## [3,]  9.676882
##
## $sd
##          [,1]
## [1,] 1.550508
## [2,] 9.683788
## [3,] 4.682631
```

Step 2: Pick K studies to pilot, calculate Y_k^P

Theoretically

First, pick K studies to update through new information. For these studies, we'll assume a **fixed effects model** and draw a new sample, imagining that this is a new study Y_i^p done in the same place as study Y_i, so we can improve on our knowledge of Y_i. For now we will arbitrarily decide that $\sigma_i^P = \frac{1}{5}\sigma_i$ (AKA that these new studies done will have a fifth of the error we were encountering in the first study). So for $i \in K$,

$$Y_i^P \sim N(\theta_i, \frac{1}{5}\sigma_i)$$

Concretely

Here we're first going to go with the boring simple version where the subset K is just k=2. So we keep Y_1, Y_3 unchanged but need to make a Y_2' using the generated θ_2. We use an arbitrary 5x smaller sigma in Y_2' than in Y_2 as noted in the norm above

```
# Define set K of cities to try the pilot on and collect more data
# K <- c(2)


# for (k in K){
#   print("Theta for new pilot:")
#   print(Y$mu[k])
#   new_draw <- rnorm( J , mean = Y$mu[k] , sd = (1/5)*Y$sd[k] )
#   print("New Draw Results:")
```

```
#   print(new_draw)
# }
```

Step 3: Update all Y values to get Y'

Once those k new values get calculated, update by combining to get

$$update(Y_k, Y_k^P) = Y_k'$$

This draws on the idea that the mean can be combined with the equation

$$\mu' = \frac{\mu_2\sigma_1^2 + \sigma_2^2\mu_1}{\sigma_2^2 + \sigma_1^2}$$

and variance can be combined using

$$\sigma' = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \frac{1}{\frac{1}{\sigma_2^2} + \frac{1}{\sigma_1^2}}$$

Concretely

Now that we have the generated new information Y_k^P, we need to combine it with the old information Y_k. We can do this by combining the means weighted by their standard deviations.

```
update_Y <- function(mu1, mu2, sigSq1, sigSq2){
  update_mu <- (mu1*sigSq2 + mu2*sigSq1)/(sigSq1 + sigSq2)
  update_sigSq <- (sigSq1*sigSq2)/(sigSq1 + sigSq2)
  return(list(mu = update_mu, sd = update_sigSq))
}

# Calculate new data for all K new pilots
get_pilot_results <- function(K,Y){
  # Before update, Y_P is the same as Y
  Y_P <- Y

  # Update for each new pilot k
  for (k in K){

    # Gather New Pilot Data
    new_sigmaSq <- Y$sd[k] * (1/5)
    new_mean <- rnorm( 1 , mean = Y$mu[k] , sd = new_sigmaSq )

    # Combine the old and new data
    post_pilot <- update_Y(Y$mu[k],new_mean,Y$sd[k],new_sigmaSq)
    Y_P$mu[k] <- post_pilot$mu
    Y_P$sd[k] <- post_pilot$sd
    return(Y_P)
  }
}

# Y_P <- get_pilot_results(K,Y)
# print("updated Y")
# Y_P
```

Step 4: Use Y' to get final θ'

and once we have all the updated Y_i' we can use them to get θ_i' based off of μ', τ'^2 with $Y' \sim N(\theta_i', \sigma_i')$

```
updated_dat_generated <- list(J, I, Y=Y_P$mu, sigmaSq=Y_P$sd)
fit_updated <- stan(file = 'randomEffectsModelConstrainedI.stan',
            data = updated_dat_generated,
            iter = 100, chains = 2, verbose = FALSE)
fit_updated
```

Now check if this update was worth. How? See if you change your mind. Intuitively: - see if new prior switches which cities have positive impacts, or see if this changes which cities are in the highest C number of cities - So given we can implement in C cities, we 'change our mind' if the top C cites are different in this update than they are in the original prior set. - Remember, Y contains the original data, Y_updated has the Y' data.

```
new_ranking <- function(fit_updated,Y_P) {
  params_updated <- extract(fit_updated)
  Y_updated <- Y_P
  for (i in 1:length(Y_updated$mu)){
    Y_updated$mu[i] <- mean(params_updated$theta[,i])
    # TODO: do we update sigma as well??
  }
  new_rank <- order(Y_updated$mu, decreasing=TRUE)
  return(new_rank)
}
```

```
change_mind <- function(K,Y,original_rank){
  Y_P <- get_pilot_results(K,Y)
  updated_dat_generated <- list(J, I, Y=Y_P$mu, sigmaSq=Y_P$sd)
  fit_updated <- stan(file = 'randomEffectsModelConstrainedI.stan',
                      data = updated_dat_generated,
                      iter = 1000, chains = 2)
  new_rank <- new_ranking(fit_updated,Y_P)
  print(new_rank)
  return(!setequal(original_rank[1:D],new_rank[1:D]))
}
```

```
original_rank <- order(Y$mu, decreasing=TRUE)
original_rank
combinations <-combn(seq(I),C)
nmc <- numeric(I)
for (i in 1:ncol(combinations)){
  for (j in 1:num_hypothetical_draws){
    K <- combinations[,i]
    nmc[i] <- nmc[i] + change_mind(K,Y,original_rank)
  }
}
```

```
## Warning: There were 59 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
```

```
## Warning: Examine the pairs() plot to diagnose sampling problems
```

```
## Warning: There were 107 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
```

```
## Warning: Examine the pairs() plot to diagnose sampling problems
```

```
## Warning: There were 62 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 182 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 429 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 68 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 76 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 85 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 25 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 106 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 60 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 34 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 154 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 2 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems
```

```
## Warning: There were 121 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 58 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 299 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 99 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 55 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 63 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 47 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 152 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 103 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 56 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 176 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 318 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup
```

```
## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 49 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 124 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 64 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 134 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 32 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 36 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 112 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 43 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 50 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 45 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 57 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 262 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems
```

```
## Warning: There were 122 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 87 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 77 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 73 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 39 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 20 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 47 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## http://mc-stan.org/misc/warnings.html#bfmi-low

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: There were 32 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems
```

```
nmc
```

Brief summary of the results

```
# So what are these results exactly?
print(Y)
```

```
## $mu
##           [,1]
## [1,] 11.176597
## [2,] 10.555695
## [3,]  9.676882
##
```

```
## $sd
##          [,1]
## [1,] 1.550508
## [2,] 9.683788
## [3,] 4.682631
```

```
for (i in 1:length(nmc)){
  print(combinations[,i])
  print(paste("Number of times minds changed: ",nmc[i],"/",num_hypothetical_draws))
}
```

```
## [1] 1 2
## [1] "Number of times minds changed:  1 / 15"
## [1] 1 3
## [1] "Number of times minds changed:  0 / 15"
## [1] 2 3
## [1] "Number of times minds changed:  7 / 15"
```