Graduate CFD-I: Project Report #3

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Contents

1 Introduction

Couette flow is the flow of a viscous fluid in the space between two surfaces, one of which is moving tangentially relative to the other. The relative motion of the surfaces imposes a shear stress on the fluid and induces flow.

2 Problem Description

There are three boundary conditions in this study that are addressed, including both a mesh study and a time step study, and the solution to the Couette flow is provided. The Laasonen Implicit method is primarily used to generate the results, which are presented in this section. However, the algorithm for the FTCS Explicit method is also provided, although it will require more effort to produce results that are satisfactory. The CFL condition is checked within each algorithm in order to determine the stability of the method.

2.1 Boundary Conditions

Three different boundary conditions are to be studied. The first boundary condition, dictates the velocity at the top layer to be equal to $1U_0$ with the velocity at the lower layer to be zero. The second boundary condition dictates the velocity at the top layer to be equal to $+1U_0$ with the velocity at the lower layer to be equal to $-0.5U_0$. Finally, the third boundary condition, dictates the velocity at the top layer to be equal to $Usin(\omega t)$ with the velocity at the lower layer to be zero.

2.2 Initial Condition

The initial condition for this problem is set so that the velocity on the top layer equals to $U(t)$ and zero otherwise.

2.3 Solution Methods

The method in this study was chosen out of the two category of explicit and implicit schemes. The results provided here are based on the implicit method, the **Laasonen Method**. The difference relation regarding the method is as follows,

$$\frac{\alpha dt}{\mathrm{dx}^2} u_{i-1}^{n+1} + \frac{\alpha dt}{\mathrm{dx}^2} u_i^{n+1} + u_i^{n+1} + \frac{\alpha dt}{\mathrm{dx}^2} u_{i+1}^{n+1} = u_i^n \tag{1}$$

The code for the FTCS Explicit method is also appended to this report which requires more debugging effort. The FTCS Explicit method is as follows,

$$u_i^{n+1} = u_i^n + \frac{\alpha \mathrm{dt}}{\mathrm{dx}^2} u_{i+1}^n - \frac{2\alpha \mathrm{dt}}{\mathrm{dx}^2} u_i^n + \frac{\alpha \mathrm{dt}}{\mathrm{dx}^2} u_{i-1}^n \tag{2}$$

3 Results and Discussion

3.1 First Boundary Condition - Implicit Method - Different Mesh Sizes

In this section, the results are shown in figure 1 for the flow with $\delta t = 0.001$, $\nu = 0.0001307$, and for three mesh sizes dividing the space in y-direction in 41, 81, and 121 segments. It is clear that the mid mesh size provide acceptable accuracy while making it more fine would increase the CFL number to become more than 0.5 which makes the solution unstable.

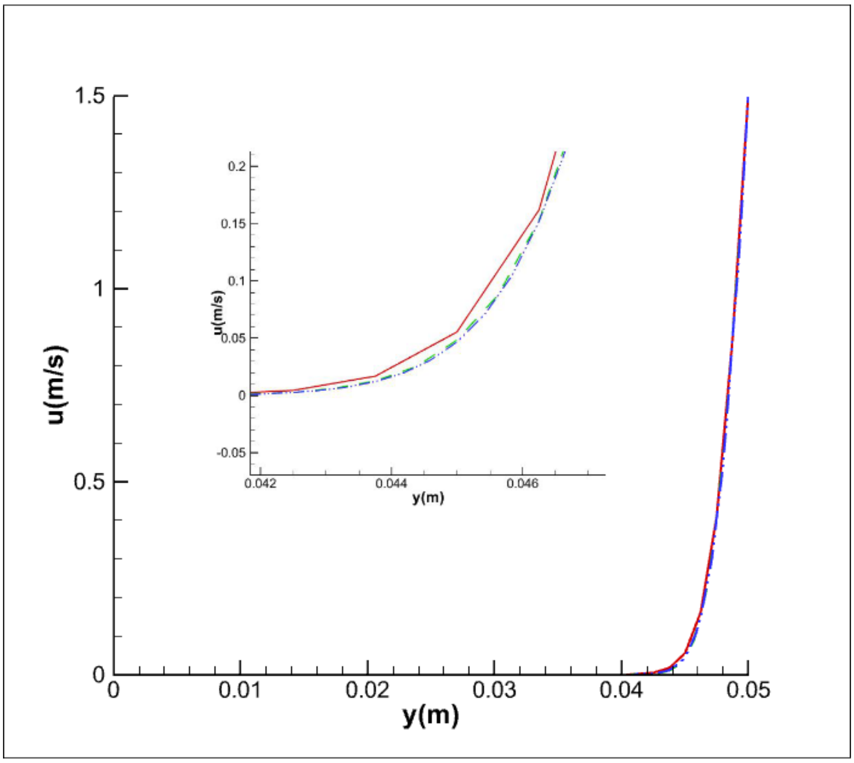

Figure 1: Couette Flow solution for different mesh sizes.

3.2 First Boundary Condition - Implicit Method - Different Time Steps

In this section, the results are shown in figure 2 for the flow with $\delta y = 0.000625$, $\nu = 0.0001307$, and for three time steps marching in time with 1001, 1051, and 2001 steps. One may observe very little difference in the accuracy in results for different time steps. Therefore, the time step was chosen for different case studies to regulate the CFL number.

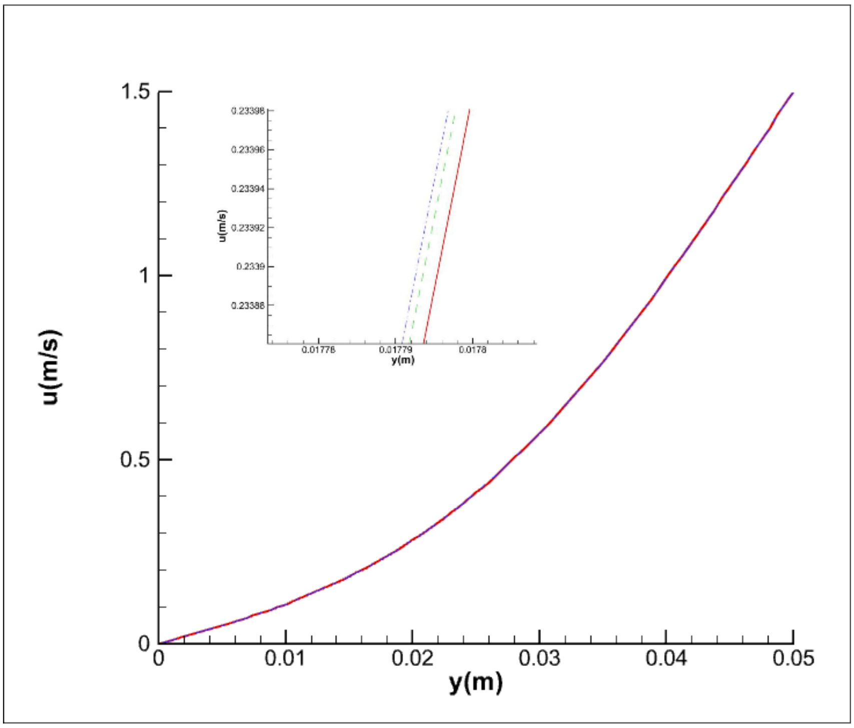

Figure 2: Couette Flow solution for different time steps.

3.3 First Boundary Condition - Implicit Method - Different Kinematic Viscosity

In this section, the results are shown in figure 3 for the flow with $\delta y = 0.000625m$, $\delta t = 0.001s$, and for three viscosity, $\nu = 0.0001307, \nu = 0.001307, \nu = 0.01307$. It is clear that the more the viscosity, the less the time for the flow to respond to the shear force between the layers.

3.4 Second Boundary Condition - Implicit Method - Optimum Mesh Size and Time Step

In this section, the results are shown in figure 4 for the flow with $\delta y = 0.000625m$, $\delta t = 0.001s$, and for the viscosity, $\nu = 0.0001307$. The boundary condition is set as the second case. The provided boundary condition has resulted in a point of inflation in the diagrams at around $y = 0.022m$.

3.5 Third Boundary Condition - Implicit Method - Optimum Mesh Size and Time Step

In this section, the results are shown in figure 5 for the flow with $\delta y = 0.000625m$, $\delta t = 0.001s$, and for the viscosity, $\nu = 0.00013077$. The boundary condition is set as the third case which is the case with oscillating velocity as the boundary condition. The boundary condition forces the flow to move forward and backward in between the two planes. One may observe the response and the evolution of the velocity profile in each direction between the two planes in time.

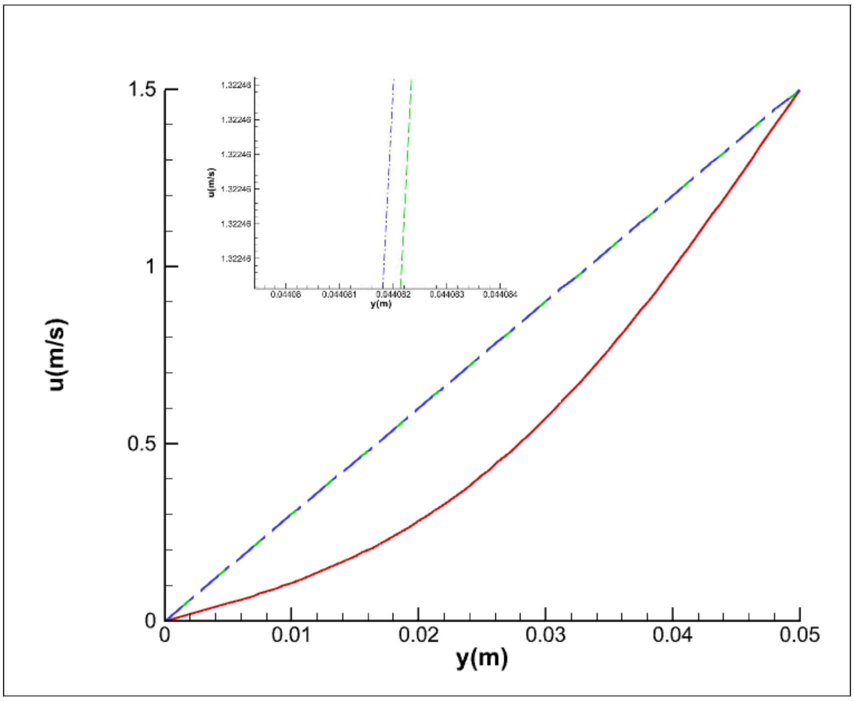

Figure 3: Couette Flow solution for different viscosity.

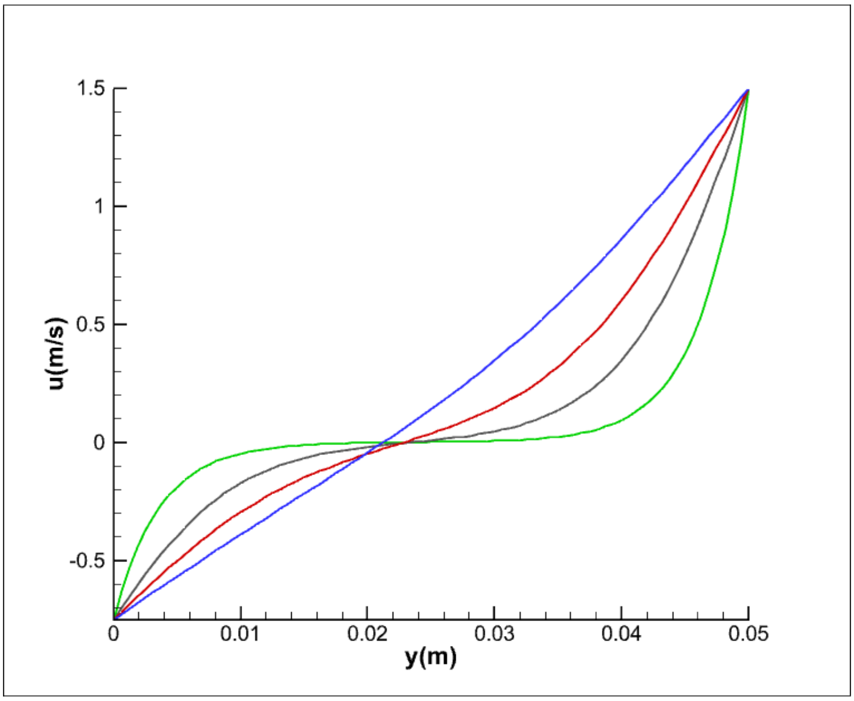

Figure 4: Couette Flow solution for the second boundary condition.

4 Conclusion

This study provides the solution to the Couette flow for three boundary conditions including both mesh study and time step study. The results are provided for based on the Laasonen Implicit method, mainly. However, the algorithm for the FTCS Explicit method is also provided which needs more effort to provide with acceptable results. To check the stability of the method, CFL condition is checked within each algorithm.

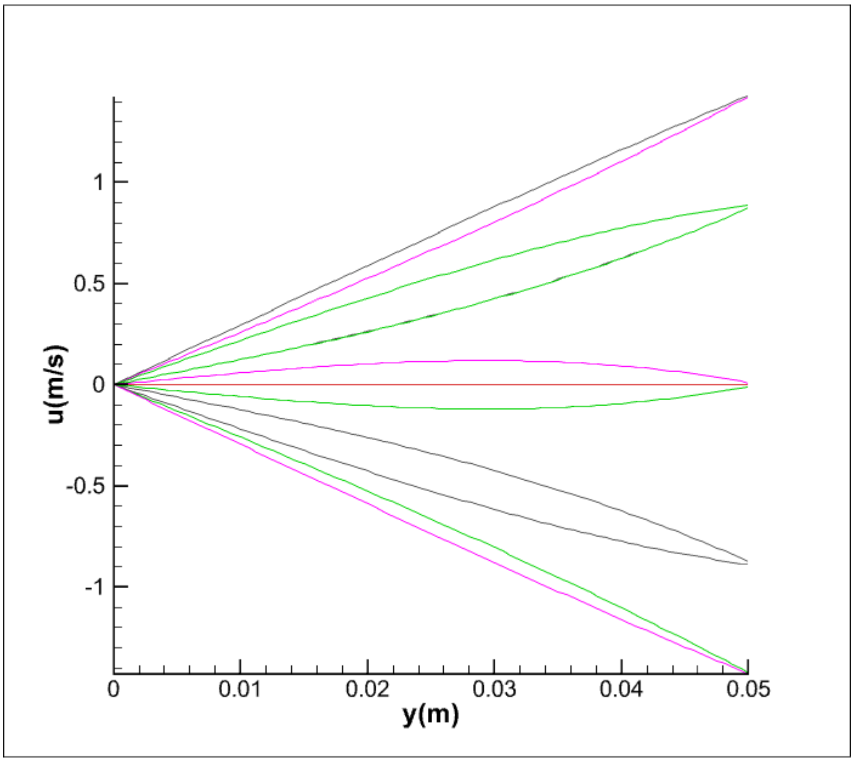

Figure 5: Couette Flow solution for the third boundary condition.

5 References

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