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Question: Is set of odd numbers with binary operation (+) i.e. <0, +> an abelian group? If not explain the reasons with necessary notations.

Answer: No, it is not an abelian group.

Solution:

A group is a set of G with a binary operation that satisfies 4 rules:

1. Closure: If we take two elements from the set and apply the operation, the result must still be in the set.
Example with integers under +: $3+(-7)=-4$, still an integer.

2. Identity element: There must be a special element that does nothing under the operation.
For addition, the identity is 0 because $x+0=x$.

3. Inverse: Every element must have an opposite that brings you back to the identity.

For addition : inverse of 3 is -3, since $3+(-3)=0$

4. Associativity: The way we group things doesnt matter.

For addition, $(a+b)+c = a+(b+c)$

This is always true for integers.

Now we need to check the set of odd integers $O = \{\ldots, -3, -1, 1, 3, 5\ldots\}$ is follows the 4 rules or not.

1. closure:

If we take odd numbers : $1, 3 \in O$

$1+3=4$

4 is even but $4 \notin O$.

closure fails.

2. Identity:

The additive identity is 0. $0 \notin O$

(0 does not exist in the set off odd integers).

No identity element exist in O.

3. Inverse:

Inverse of 1 is $-1 \in O$.

Both are odd, so inverses do exist.

4. Associativity : Addition is always associative.

~~But~~ since closure and identity fails, the set of odd intergers is not even a group.

Abelian means the operation is commutative ($a+b = b+a$).

Addition is commutative. But since O is not a group at all, it's not abelian.