① Let G be a group of order pq, where p and q are distinct primes. Prove that G is abelian.

Answer: False

Reason: Counterexample S_3. $|S_3| = 6 = 2 \cdot 3$ but S_3 is non-abelian.

② Prove that if G is a group of order p^2, where p is prime, then G is abelian if and only if it has $p+1$ subgroups of order p.

Answer: False

Reason: Every group of order p^2 is abelian. There are exactly two types: C_{p^2} and $C_p \times C_p$. The correct equivalence is: "$G \cong C_p \times C_p \Leftrightarrow G$ has $p+1$ subgroups of order p". Both groups are abelian.

③ Let G be a finite group and H be a proper subgroup of G. Prove that the union of all conjugates of H cannot be equal to G

Answer: False

Reason: For finite G the union of conjugates of a proper subgroup cannot cover G. A finite group cannot be a union of finitely many proper subgroups.

⑤ Prove that in any group G, the set of elements of finite order forms a subgroup of G.

Answer: False

Reason: In abelian group yes, but not always. Example: infinite dihedral group - two reflections (order 2) multifiply to a rotation of infinite order, so closure fails.

6 Let G be a finite group and p be the smallest prime dividing $|G|$. Prove that any subgroup of index p in G is normal.

Answer: True

Reason: Action on cosets gives homomorphism into S_p; using smallest-prime property the image ~~must~~ must force the subgroup to be normal.

9 Let G be a finite group and p be a prime number. If G has exactly one subgroup of order p^k for each $k \leq n$, where p^n divides $|G|$, prove that G has a normal Sylow p subgroup.

Answer: True

Reason: Unique subgroup of order p^n is the unique sylow p-subgroup, and uniqueness implies normality.