

quadcopter package

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I. INTRODUCTION

THE `quadcopter` package is filled with obtuse and often confusing code derived from various physics equations and mathematical formulas. This paper hopes to bridge the gap between quadcopter theory and the `src` files included. This paper can also be used as a reference for later expansion of this package.

II. THE MODEL NODE

While an actual quadcopter would be helpful, a simulation is the next best thing. The model node simulates the physics of a quadcopter. The physics in this section also provide insight on how motor input and sensor output behave.

A. Rotational Kinetics

The rotation matrices (orthogonal transforms between bases) and their compositions come in handy for later calculations and can be found in the appendix. Throughout this paper, I use notation of the sum of scaled unit vectors (versors), but coding is better described as `floats`. The dot product of vectors provides these scalars. The rotation matrices provide a way to move between these arrays of `floats`. For example, consider the rotation matrix from basis A to B .

$$\begin{pmatrix} \mathbf{v} \cdot \hat{\mathbf{b}}_x \\ \mathbf{v} \cdot \hat{\mathbf{b}}_y \\ \mathbf{v} \cdot \hat{\mathbf{b}}_z \end{pmatrix} = {}^B R^A \begin{pmatrix} \mathbf{v} \cdot \hat{\mathbf{a}}_x \\ \mathbf{v} \cdot \hat{\mathbf{a}}_y \\ \mathbf{v} \cdot \hat{\mathbf{a}}_z \end{pmatrix} \quad (1)$$

The rotation matrices also collect the dot products to push final calculations down the road.

$${}^B R^A = \begin{pmatrix} \hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_x & \hat{\mathbf{a}}_y \cdot \hat{\mathbf{b}}_x & \hat{\mathbf{a}}_z \cdot \hat{\mathbf{b}}_x \\ \hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_y & \hat{\mathbf{a}}_y \cdot \hat{\mathbf{b}}_y & \hat{\mathbf{a}}_z \cdot \hat{\mathbf{b}}_y \\ \hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_z & \hat{\mathbf{a}}_y \cdot \hat{\mathbf{b}}_z & \hat{\mathbf{a}}_z \cdot \hat{\mathbf{b}}_z \end{pmatrix} \quad (2)$$

The rotation of the quadcopter can be described by a composition of three rotations: from a Newtonian frame N to A , by an angle a around their common z axis (the yaw); from intermediate frames A to B , by angle b around their common y axis (the roll); and from B to the quadcopter body frame C , by angle c around their common z axis (the pitch). The angles and bases are chosen to make the angular velocities positive.

$${}_N \boldsymbol{\omega}_A = \dot{a} \hat{\mathbf{n}}_x = \dot{a} \hat{\mathbf{a}}_x \quad (3)$$

$${}_A \boldsymbol{\omega}_B = \dot{b} \hat{\mathbf{a}}_y = \dot{b} \hat{\mathbf{b}}_y \quad (4)$$

$${}_B \boldsymbol{\omega}_C = \dot{c} \hat{\mathbf{b}}_z = \dot{c} \hat{\mathbf{c}}_z \quad (5)$$

Using the notation of the rotation matrices above, the composition of rotation matrices is the matrix product.

$${}^C R^N = ({}^C R^B) ({}^B R^A) ({}^A R^N) \quad (6)$$

TABLE I
TRANSFORMS FROM THE INERTIAL FRAME N
($s_x = \sin x$, $c_x = \cos x$)

${}^U R^N$	$\hat{\mathbf{n}}_x$	$\hat{\mathbf{n}}_y$	$\hat{\mathbf{n}}_z$
$\hat{\mathbf{a}}_x$	1	0	0
$\hat{\mathbf{a}}_y$	0	c_a	s_a
$\hat{\mathbf{a}}_z$	0	$-s_a$	c_a
$\hat{\mathbf{b}}_x$	c_b	$s_a s_b$	$-s_a c_a$
$\hat{\mathbf{b}}_y$	0	c_a	s_a
$\hat{\mathbf{b}}_z$	s_b	$-s_a c_b$	$c_a c_b$
$\hat{\mathbf{c}}_x$	$c_b c_c$	$-c_a s_c + s_a s_b c_c$	$-s_a s_c - c_a s_b c_c$
$\hat{\mathbf{c}}_y$	$c_b s_c$	$c_a c_c + s_a s_b s_c$	$s_a c_c - c_a s_b s_c$
$\hat{\mathbf{c}}_z$	s_b	$-s_a c_b$	$c_a c_b$

TABLE II
TRANSFORMS FROM THE ROBOT FRAME C
($s_x = \sin x$, $c_x = \cos x$)

${}^U R^C$	$\hat{\mathbf{c}}_x$	$\hat{\mathbf{c}}_y$	$\hat{\mathbf{c}}_z$
$\hat{\mathbf{b}}_x$	c_c	s_c	0
$\hat{\mathbf{b}}_y$	$-s_c$	c_c	0
$\hat{\mathbf{b}}_z$	0	0	1
$\hat{\mathbf{a}}_x$	$c_b c_c$	$c_b s_c$	s_b
$\hat{\mathbf{a}}_y$	$-s_c$	c_c	0
$\hat{\mathbf{a}}_z$	$s_b c_c$	$-s_b s_c$	c_b
$\hat{\mathbf{n}}_x$	$c_b c_c$	$c_b s_c$	s_b
$\hat{\mathbf{n}}_y$	$-c_a s_c + s_a s_b s_c$	$c_a c_c + s_a s_b c_c$	$-s_a c_b$
$\hat{\mathbf{n}}_z$	$-s_a s_c - c_a s_b c_c$	$s_a c_c - c_a s_b s_c$	$c_a c_b$

The important combinations of dot products can be found in tables I and II.

The angular velocity is the composition of the simple rotations.

$${}_N \boldsymbol{\omega}_C = \dot{a} \hat{\mathbf{n}}_x + \dot{b} \hat{\mathbf{a}}_y + \dot{c} \hat{\mathbf{b}}_z \quad (7)$$

The angular acceleration (in the inertial frame) can be found with some manipulation of derivatives in reference frames.

$$\begin{aligned} {}_N \boldsymbol{\alpha}_C &= \frac{{}^N d}{{}^N dt} {}_N \boldsymbol{\omega}_C \\ &= \frac{{}^N d}{{}^N dt} \dot{a} \hat{\mathbf{n}}_x + \frac{{}^A d}{{}^A dt} \dot{b} \hat{\mathbf{a}}_y + {}_N \boldsymbol{\omega}_A \times \dot{b} \hat{\mathbf{a}}_y \\ &\quad + \frac{{}^B d}{{}^B dt} \dot{c} \hat{\mathbf{b}}_z + {}_N \boldsymbol{\omega}_B \times \dot{c} \hat{\mathbf{b}}_z \\ &= \ddot{a} \hat{\mathbf{n}}_x + \ddot{b} \hat{\mathbf{a}}_y + \ddot{c} \hat{\mathbf{c}}_z \\ &\quad + \dot{a} \dot{b} \hat{\mathbf{a}}_z + \dot{b} \dot{c} \hat{\mathbf{b}}_x + \dot{a} \dot{c} (\hat{\mathbf{n}}_x \times \hat{\mathbf{c}}_z) \end{aligned} \quad (8)$$

B. Linear Equations of Motion

If you choose the right reference frames the linear equations of motion are easy to express. First, the motion of the quadcopter from its origin point is defined simply:

$$\mathbf{r}_{c/o} = x \hat{\mathbf{n}}_x + y \hat{\mathbf{n}}_y + z \hat{\mathbf{n}}_z \quad (9)$$

$${}_N \mathbf{v}_c = \dot{x} \hat{\mathbf{n}}_x + \dot{y} \hat{\mathbf{n}}_y + \dot{z} \hat{\mathbf{n}}_z \quad (10)$$

$${}_N \mathbf{a}_c = \ddot{x} \hat{\mathbf{n}}_x + \ddot{y} \hat{\mathbf{n}}_y + \ddot{z} \hat{\mathbf{n}}_z \quad (11)$$

For our model we assume each motor puts out a force proportional to its square. If we let m be the total mass of the robot and k be the proportionality constant:

$$m_N \mathbf{a}_c = k (\omega_f^2 + \omega_r^2 + \omega_b^2 + \omega_l^2) \hat{\mathbf{c}}_y - mg \hat{\mathbf{n}}_y \quad (12)$$

Where f , r , b , and l stand for the front, right, back and left motors respectively. Expressed in the inertial frame N the equations of motion are:

$$\ddot{x} = \frac{k}{m} \Omega \cos b \sin c \quad (13)$$

$$\ddot{y} = \frac{k}{m} \Omega \cos a \cos c + \frac{k}{m} \Omega \sin a \sin b \sin c - g \quad (14)$$

$$\ddot{z} = \frac{k}{m} \Omega \sin a \cos c - \frac{k}{m} \Omega \cos a \sin b \sin c \quad (15)$$

where Ω is the sum of the squares of the motor inputs.