DEFINITION OF SINE , COSINE & TANGENT

UNIT CIRCLE

DIPLOMA PROGRAMME

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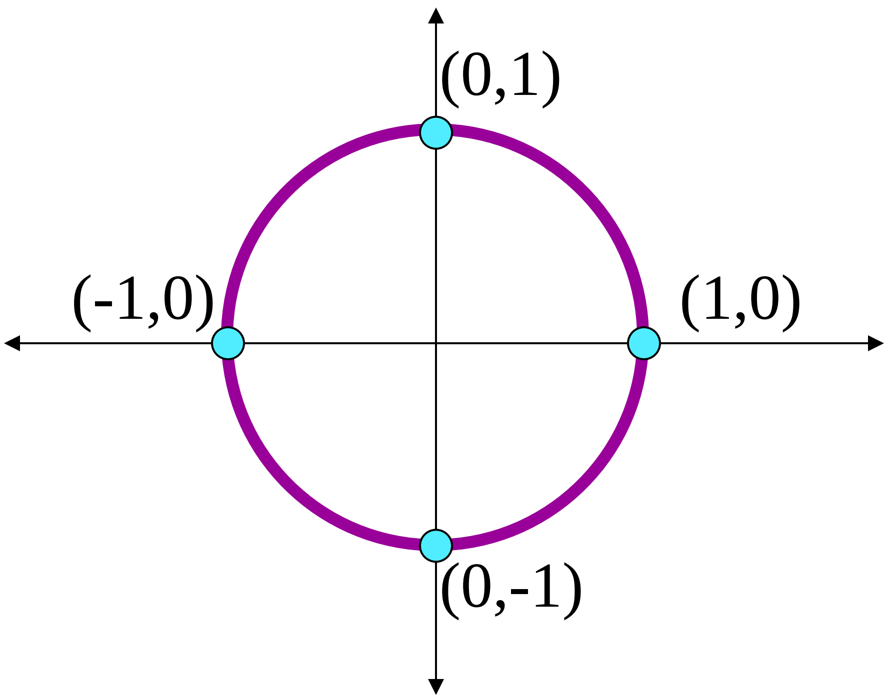
A circle with center at (0, 0) and radius 1 is called a unit circle.

The equation of this circle would be

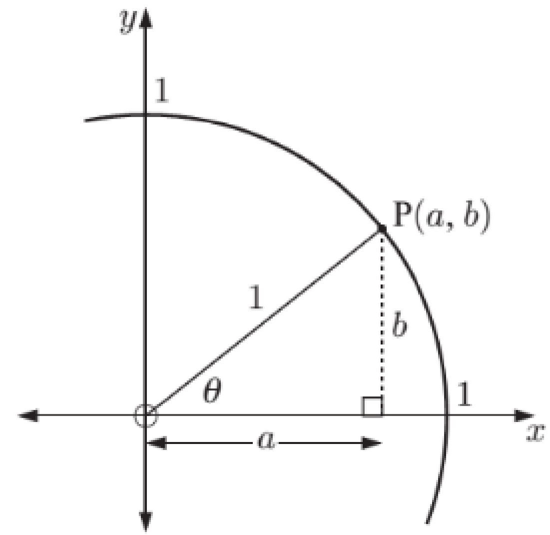
$$x^2 + y^2 = 1$$



So points on this circle must satisfy this equation.

DEFINITION OF SINE AND COSINE

Consider a point $P(a, b)$ which lies on the unit circle in the first quadrant. [OP] makes an angle θ with the x-axis as shown.



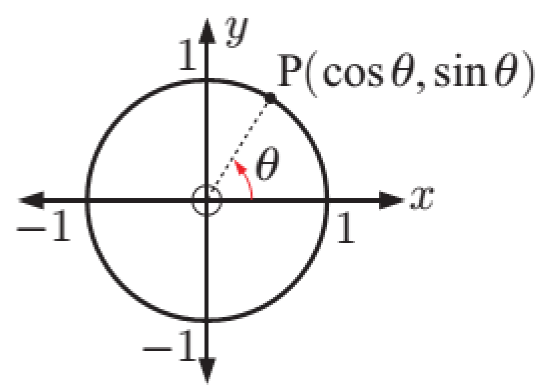
Using right angled triangle trigonometry:

$$\cos\theta = \frac{\text{ADJ}}{\text{HYP}} = \frac{a}{1} = a \qquad \tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{b}{a} = \frac{\sin\theta}{\cos\theta}$$

$$\sin\theta = \frac{\text{OPP}}{\text{HYP}} = \frac{b}{1} = b$$

In general, for a point P anywhere on the unit circle:

- $\cos\theta$ is the x-coordinate of P
- $\sin\theta$ is the y-coordinate of P

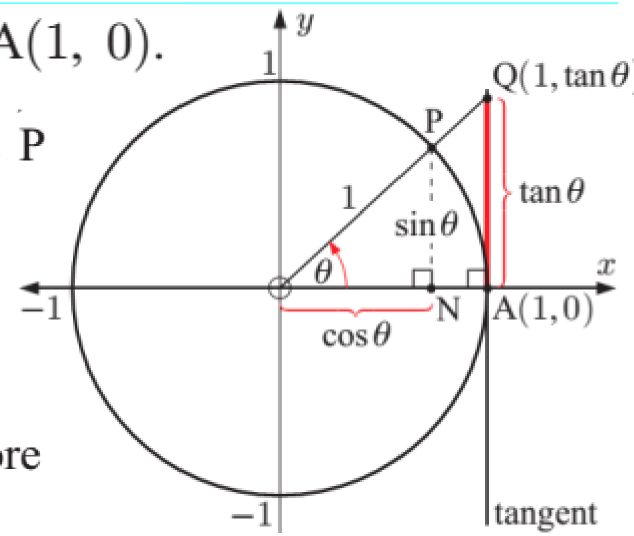


DEFINITION OF TANGENT

Suppose we extend [OP] to meet the tangent at $A(1, 0)$.

The intersection between these lines occurs at Q, and as P moves so does Q.

The position of Q relative to A is defined as the **tangent function**.



Notice that $\triangle$s ONP and OAQ are equiangular and therefore similar.

Consequently $\dfrac{AQ}{OA} = \dfrac{NP}{ON}$

Under the definition that $AQ = \tan\theta$,

hence $\dfrac{AQ}{1} = \dfrac{\sin\theta}{\cos\theta}$.

$$\boldsymbol{\tan\theta = \frac{\sin\theta}{\cos\theta}}.$$

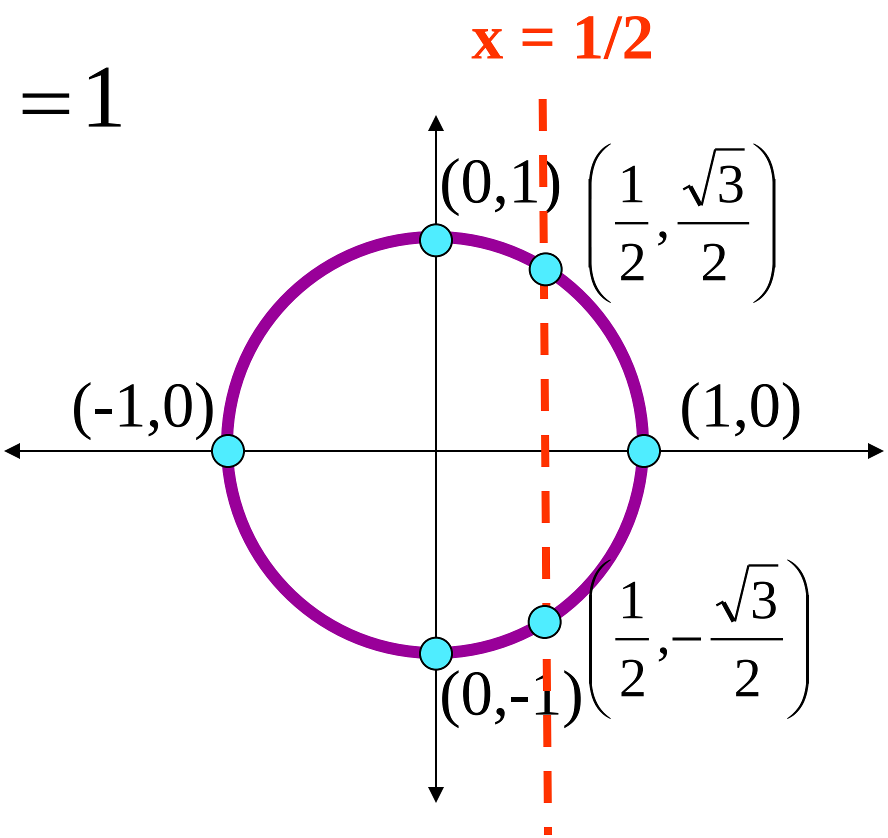
Let's pick a point on the circle. We'll choose a point where the *x* is 1/2. If the *x* is 1/2, what is the *y* value?

$$x^2 + y^2 = 1$$

$$\left(\frac{1}{2}\right)^2 + y^2 = 1$$

$$y^2 = \frac{3}{4}$$

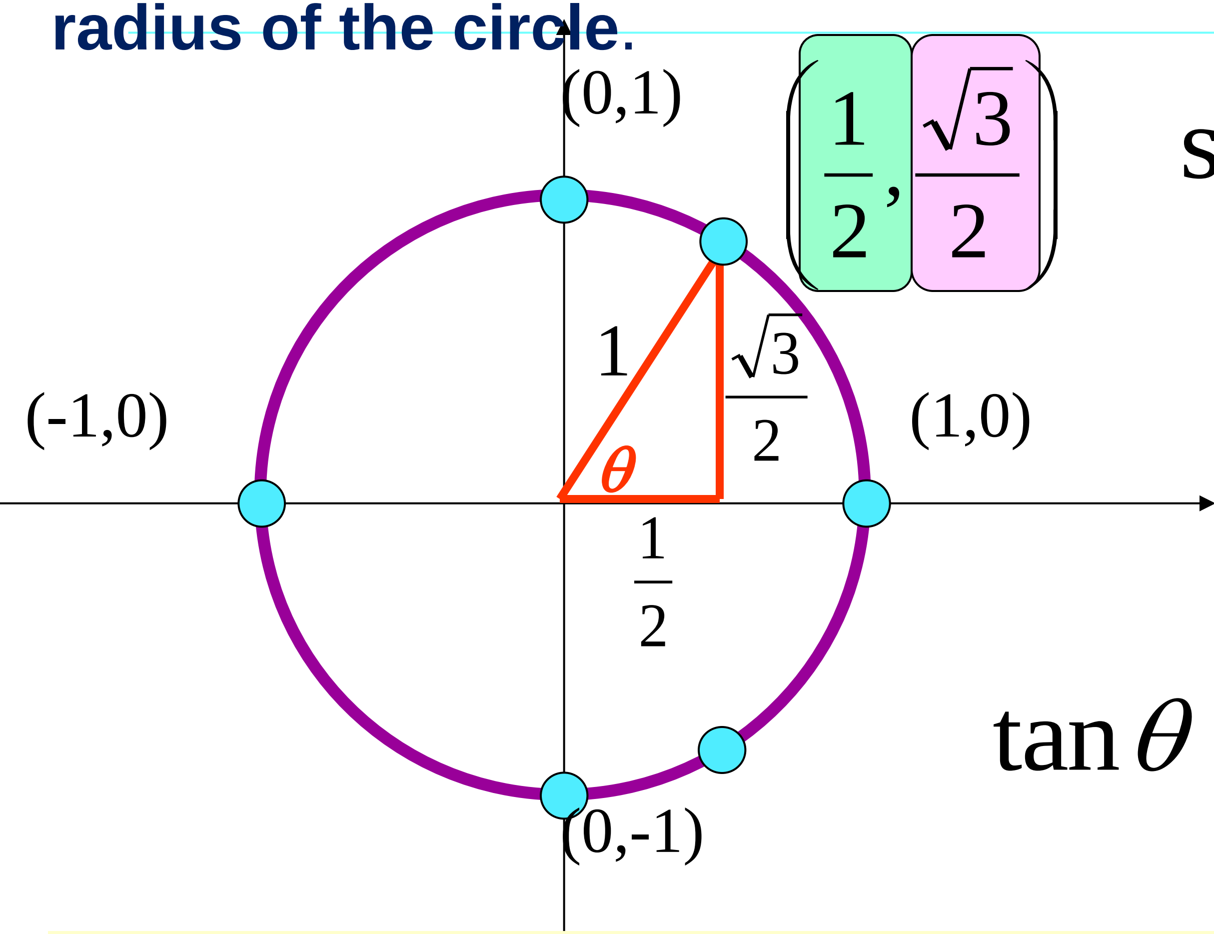
$$y = \pm\frac{\sqrt{3}}{2}$$



You can see there are two *y* values. They can be found by putting 1/2 into the equation for *x* and solving for *y*.

We'll look at a larger version of this and make a right triangle.

We know all of the sides of this triangle. The bottom leg is just the *x* value of the point, the other leg is just the *y* value and the hypotenuse is always 1 because it is a radius of the circle.



$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

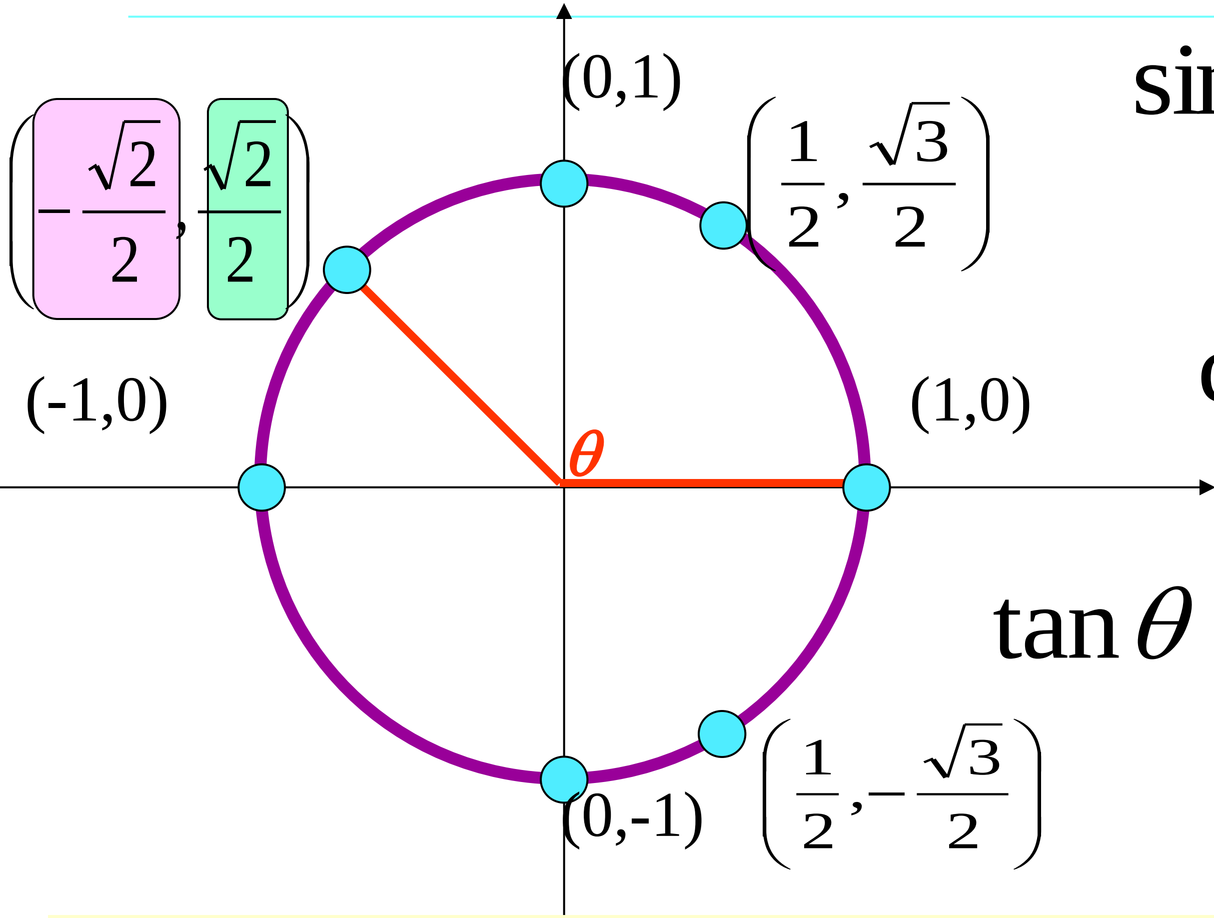
$$\sin\theta = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

$$\tan\theta = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Notice the sine is just the *y* value of the unit circle point and the cosine is just the *x* value.

So if I want a trig function for θ whose terminal side contains a point on the unit circle, the *y* value is the sine, the *x* value is the cosine and *y*/*x* is the tangent.



$$\sin\theta = \frac{\sqrt{2}}{2}$$

$$\cos\theta = -\frac{\sqrt{2}}{2}$$

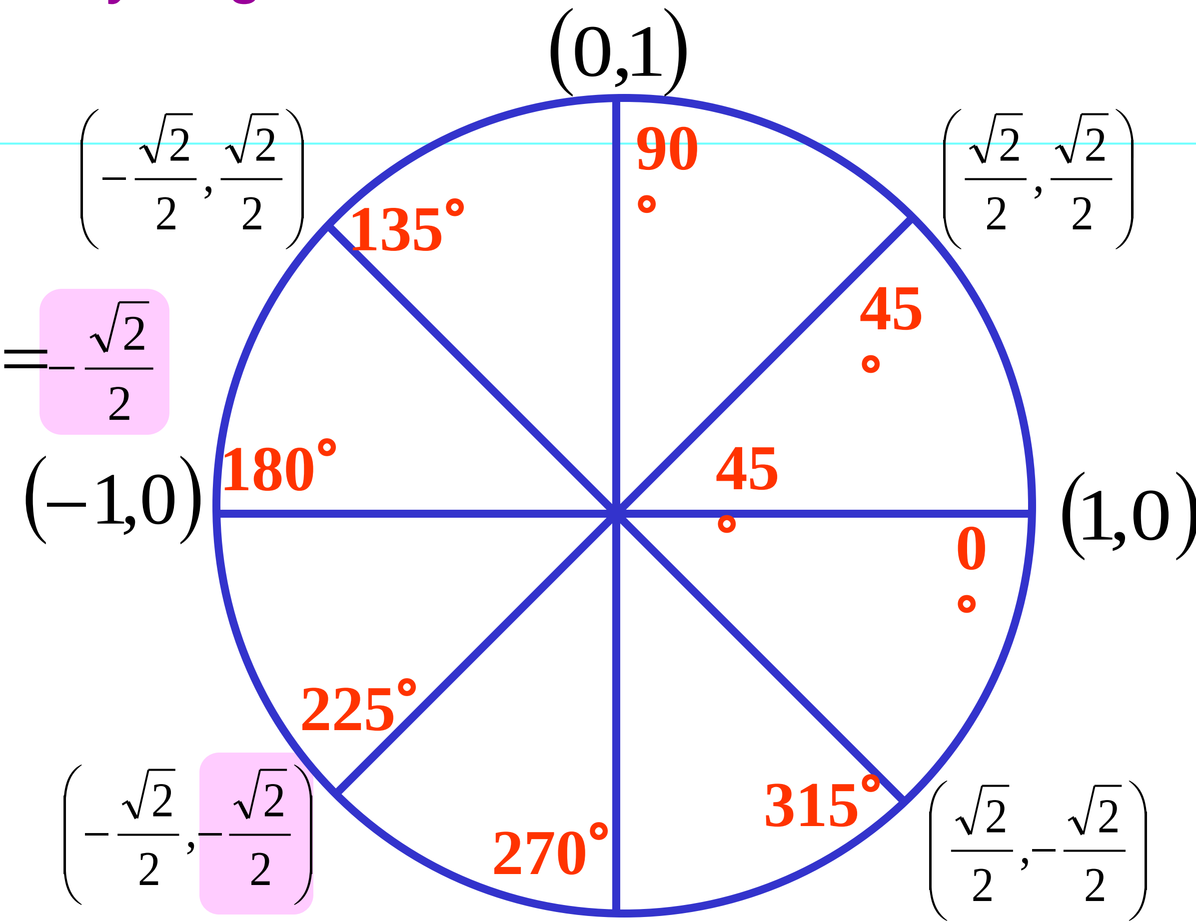
$$\tan\theta = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$$

We divide the unit circle into various pieces and learn the point values so we can then from memory find trig functions.

Here is the unit circle divided into 8 pieces. Can you figure out how many degrees are in each division?

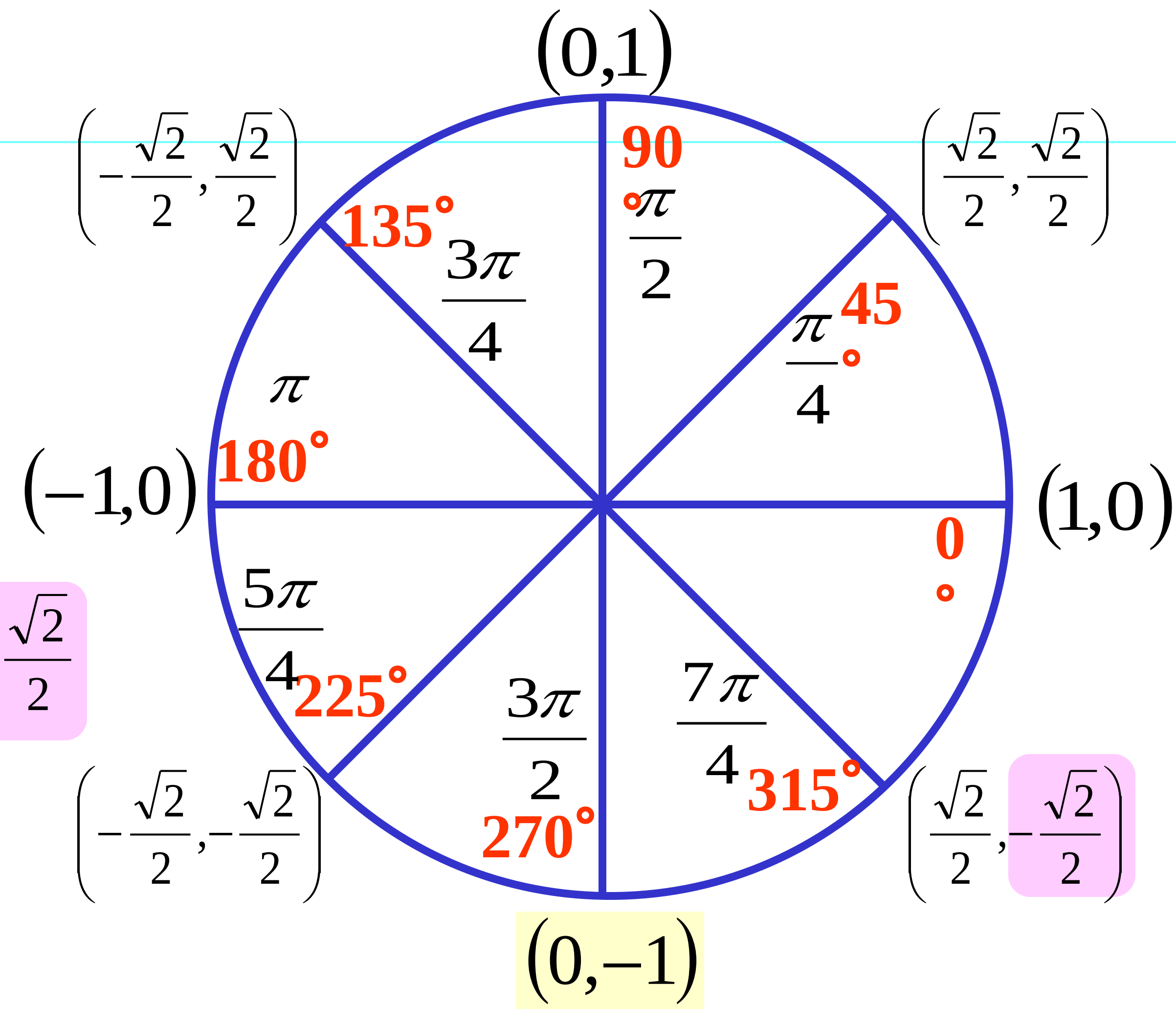
These are easy to memorize since they all have the same value with different signs depending on the quadrant.

$$\sin 225^\circ = -\frac{\sqrt{2}}{2}$$



We can label this all the way a $(0,-1)$ with how many degrees an angle would be and the point on the unit circle that corresponds with the terminal side of the angle. We could then find any of the trig functions.

Can you figure out what these angles would be in radians?



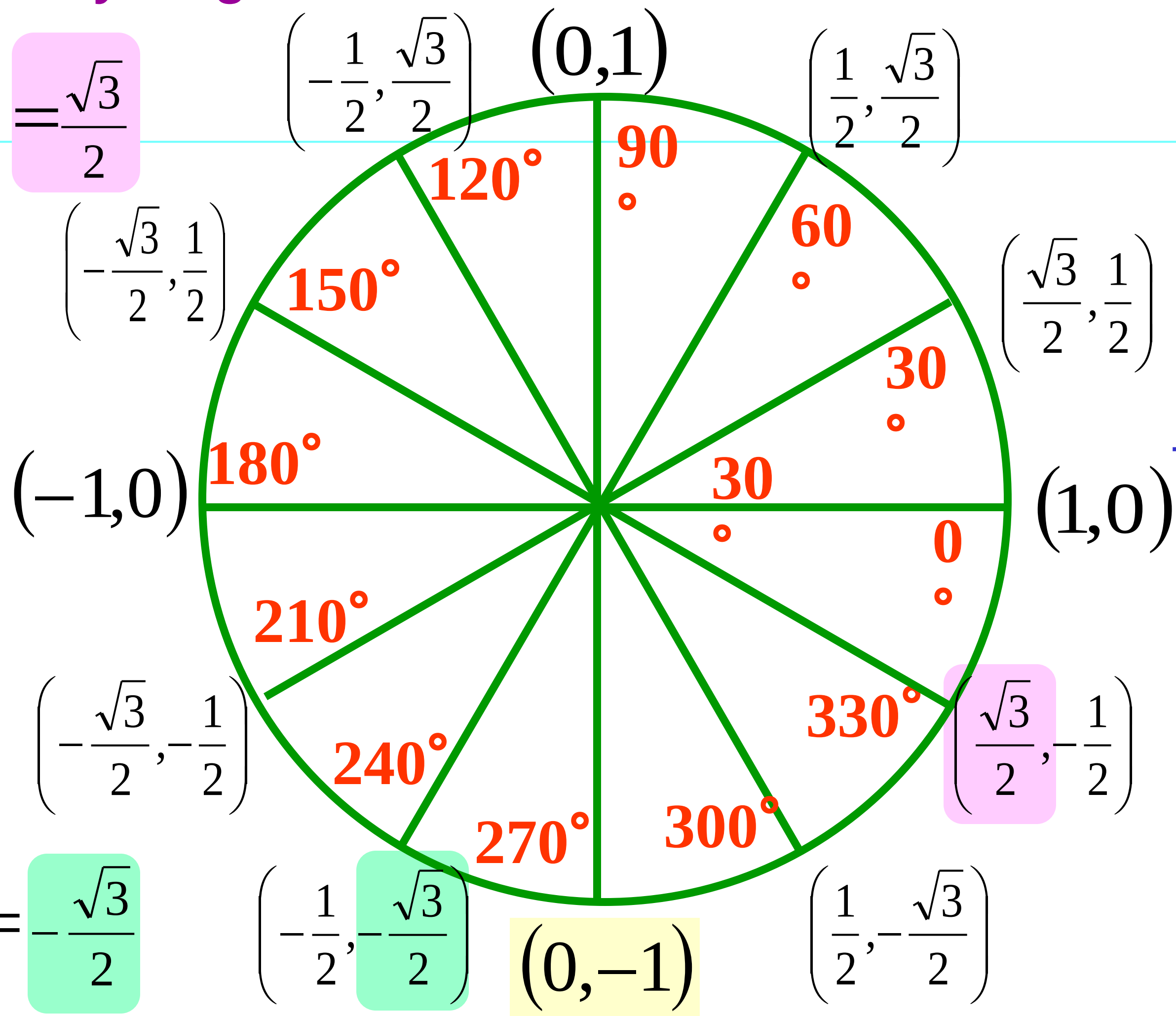
$$\sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$$

The circle is 2π all the way around so half way is π. The upper half is divided into 4 pieces so each piece is $\pi/4$.

Here is the unit circle divided into 12 pieces. Can you figure out how many degrees are in each division?

You'll need to memorize these too but you can see the pattern.

$$\cos 330^\circ = \frac{\sqrt{3}}{2}$$



$$\sin 240^\circ = -\frac{\sqrt{3}}{2}$$

We can again label the points on the circle and the sine is the *y* value, the cosine is the *x* value and the tangent is *y* over *x*.

You should memorize this. This is a great reference because you can figure out the trig functions of all these angles quickly.

The Unit Circle