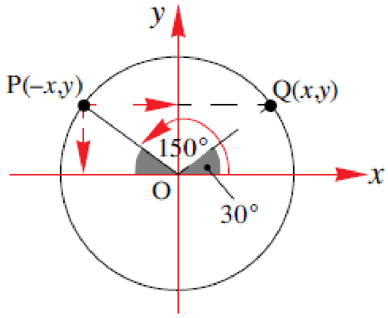
Group (1)



Consider the value of $\sin 150°$. Using the unit circle we have: By symmetry we see that the y-coordinate of Q and the y-coordinate of P are the same and so, $\sin 150° = \sin 30°$.

Therefore, $\sin 150° = \frac{1}{2}$

Note that $150° = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$ and $30° = \frac{\pi}{6}$, so that in radian form we have, $\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$.

In other words, we were able to express the sine of an angle in the second quadrant in terms of the sine of an angle in the first quadrant. In particular, we have that

$$\text{If } 0° < \theta < 90°, \sin(180 - \theta) = \sin\theta$$

Or,

$$\text{If } 0^c < \theta < \frac{\pi^c}{2}, \sin(\pi^c - \theta) = \sin\theta$$

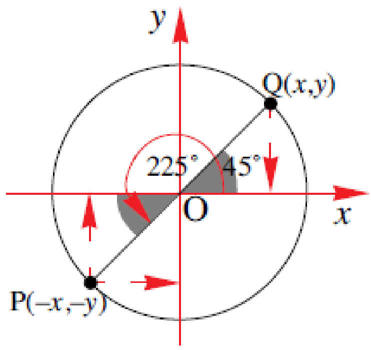
Similarly:

1. We list a number of these formulae in the table below, where $0 < \theta < \frac{\pi}{2}$ $(= 90°)$.

Quadrant	θ in degrees	θ in radians
2	$\sin(180° - \theta) = \sin\theta$ $\cos(180° - \theta) = -\cos\theta$ $\tan(180° - \theta) = -\tan\theta$	$\sin(\pi - \theta) = \sin\theta$ $\cos(\pi - \theta) = -\cos\theta$ $\tan(\pi - \theta) = -\tan\theta$

Group (2)

Next, consider the value of $\cos 225°$. Using the unit circle we have:



By symmetry we see that the x-coordinate of P has the same magnitude as the x-coordinate of Q but is of the opposite sign.
So, we have that $\cos 225° = -\cos 45°$.

Therefore, $\cos 225° = -\frac{1}{\sqrt{2}}$.

Similarly, as $225° = \frac{3\pi^c}{4}$ and $45° = \frac{\pi^c}{4}$, $\cos\frac{3\pi^c}{4} = -\cos\frac{\pi^c}{4} = -\frac{1}{\sqrt{2}}$.

In other words, we were able to express the cosine of an angle in the third quadrant in terms of the cosine of an angle in the first quadrant. In particular, we have that

If $0° < \theta < 90°$, $\cos(180 + \theta) = -\cos\theta$

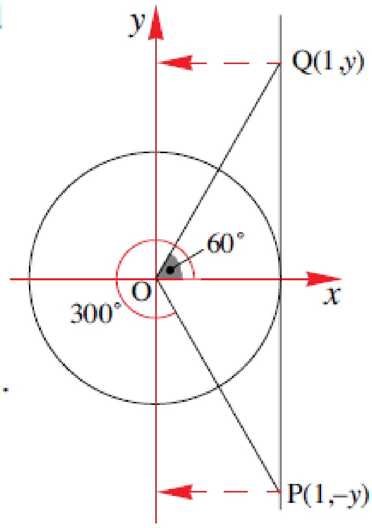
Or, If $0^c < \theta < \frac{\pi^c}{2}$, $\cos(\pi^c + \theta) = -\cos\theta$

Similarly:

Quadrant	θ in degrees	θ in radians
3	$\sin(180° + \theta) = -\sin\theta$ $\cos(180° + \theta) = -\cos\theta$ $\tan(180° + \theta) = \tan\theta$	$\sin(\pi + \theta) = -\sin\theta$ $\cos(\pi + \theta) = -\cos\theta$ $\tan(\pi + \theta) = \tan\theta$

Group (3)

As a last example we consider the value of $\tan 300°$. This time we need to add a tangent to the unit circle cutting the positive x–axis:



By symmetry we see that the y-coordinate of P has the same magnitude as the y-coordinate of Q but is of the opposite sign.

So, we have that $\tan 300° = -\tan 60°$.

Therefore, $\tan 300° = -\sqrt{3}$.

Similarly, as $300° = \frac{5\pi^c}{3}$ and $60° = \frac{\pi^c}{3}$, $\tan\frac{5\pi^c}{3} = -\tan\frac{\pi^c}{3} = -\sqrt{3}$.

In other words, we were able to express the tangent of an angle in the fourth quadrant in terms of the tangent of an angle in the first quadrant. In particular, we have that

$$\text{If } 0° < \theta < 90°,\ \tan(360-\theta) = -\tan\theta$$

Or,

$$\text{If } 0^c < \theta < \frac{\pi^c}{2},\ \tan(2\pi^c-\theta) = -\tan\theta$$

Similarly:

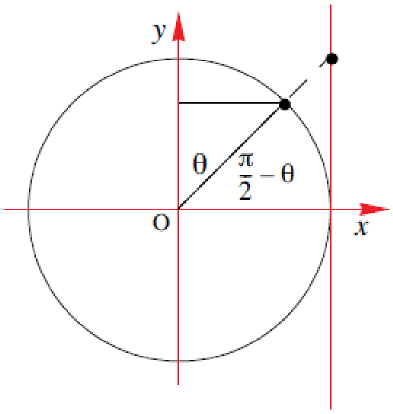
Quadrant	θ in degrees	θ in radians
4	$\sin(360°-\theta) = -\sin\theta$ $\cos(360°-\theta) = \cos\theta$ $\tan(360°-\theta) = -\tan\theta$	$\sin(2\pi-\theta) = -\sin\theta$ $\cos(2\pi-\theta) = \cos\theta$ $\tan(2\pi-\theta) = -\tan\theta$

There is another set of results that is suggested by symmetry through the fourth quadrant:

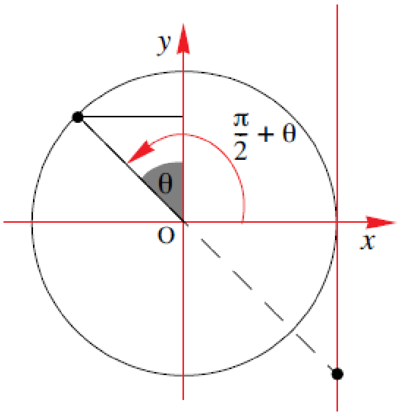
Quadrant	θ in degrees or θ in radians
4	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

Group (4)

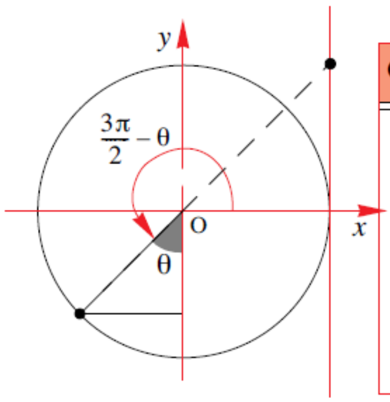
There are other trigonometric reduction formulae, where $0 < \theta < \frac{\pi}{2}$ or $0 < \theta < 90°$. These formulae however, are expressed in terms of their variation from the vertical axis.
That is:



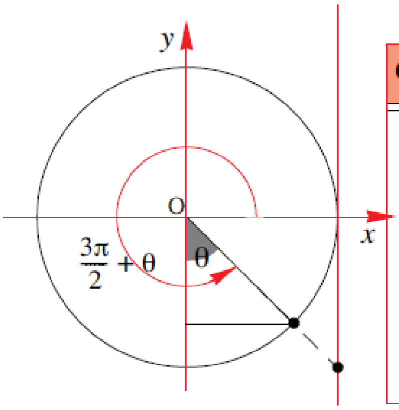
Quadrant	θ in radians or	θ in degrees
	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$,	$\sin(90° - \theta) = \cos\theta$
1	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$,	$\cos(90° - \theta) = \sin\theta$
	$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$,	$\tan(90° - \theta) = \cot\theta$



Quadrant	θ in radians or	θ in degrees
	$\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$,	$\sin(90° + \theta) = \cos\theta$
2	$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$,	$\cos(90° + \theta) = -\sin\theta$
	$\tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta$,	$\tan(90° + \theta) = -\cot\theta$



Quadrant	θ in radians or	θ in degrees
	$\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos\theta$,	$\sin(270° - \theta) = -\cos\theta$
3	$\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta$,	$\cos(270° - \theta) = -\sin\theta$
	$\tan\left(\frac{3\pi}{2} - \theta\right) = \cot\theta$,	$\tan(270° - \theta) = \cot\theta$



Quadrant	θ in radians or	θ in degrees
	$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$,	$\sin(270° + \theta) = -\cos\theta$
4	$\cos\left(\frac{3\pi}{2} + \theta\right) = \sin\theta$,	$\cos(270° + \theta) = \sin\theta$
	$\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot\theta$,	$\tan(270° + \theta) = -\cot\theta$