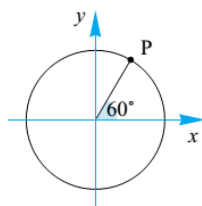


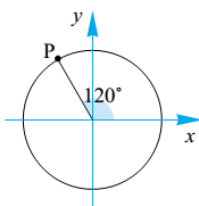
TRIGONOMETRY (UNIT CIRCLE): 03

1. Find the coordinates of the point P on the following unit circles.

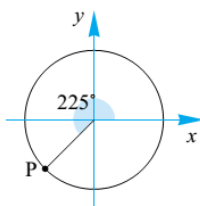
(a)



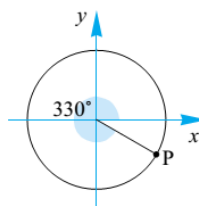
(b)



(c)



(d)



2. Find the possible exact values of $\cos \theta$ for:

a $\sin \theta = \frac{1}{2}$

b $\sin \theta = -\frac{1}{3}$

3. Find the possible exact values of $\sin \theta$ for:

a $\cos \theta = \frac{4}{5}$

b $\cos \theta = -\frac{3}{4}$

4. Find the exact values of

(a) $\cos 120^\circ$

(b) $\sin 210^\circ$

(c) $\cos \frac{7\pi}{4}$

(d) $\tan \frac{5\pi}{4}$

5. Without using a calculator, find:

a $\sin \theta$ if $\cos \theta = \frac{2}{3}$ and $0 < \theta < \frac{\pi}{2}$

b $\cos \theta$ if $\sin \theta = \frac{2}{5}$ and $\frac{\pi}{2} < \theta < \pi$

c $\cos \theta$ if $\sin \theta = -\frac{3}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$

d $\sin \theta$ if $\cos \theta = -\frac{5}{13}$ and $\pi < \theta < \frac{3\pi}{2}$.

6. Find exact values for $\sin x$ and $\cos x$ given that:

a $\tan x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$

b $\tan x = -\frac{4}{3}$ and $\frac{\pi}{2} < x < \pi$

c $\tan x = \frac{\sqrt{5}}{3}$ and $\pi < x < \frac{3\pi}{2}$

d $\tan x = -\frac{12}{5}$ and $\frac{3\pi}{2} < x < 2\pi$

7. Given that $\cos \theta = k$ and $0 < \theta < \frac{\pi}{2}$ find

(a) $\cos(\pi + \theta)$

(b) $\cos(2\pi - \theta)$

(c) $\cos\left(\frac{\pi}{2} + \theta\right)$

8. Find (a) $\sin \theta = \frac{1}{2}$, $0^\circ < \theta < 360^\circ$

(b) $\tan \theta = -\sqrt{3}$, $0 < \theta < 2\pi$

(c) $\cos \theta = 1$, $0 < \theta < 2\pi$

9. Simplify

(a) $\frac{\sin(\pi + \theta)}{\cos(2\pi - \theta)}$

(b) $\frac{\sin\left(\frac{\pi}{2} + \theta\right)\cos\left(\frac{\pi}{2} - \theta\right)}{\cos(\pi + \theta)}$, where $0 < \theta < \frac{\pi}{2}$.

10. Find the following function values. Find the exact value, if possible. Otherwise, find the approximate value accurate to 3 significant figures.

a) $\sin \frac{2\pi}{3}$

b) $\cos \frac{5\pi}{4}$

c) $\tan \frac{11\pi}{6}$

d) $\sin \frac{13\pi}{6}$

e) $\cos 3.75$

11. Find the exact value of

(a) $\sin \frac{11\pi}{6} \cos \frac{5\pi}{6} - \sin \frac{5\pi}{6} \cos \frac{11\pi}{6}$

(b) $2 \sin \frac{\pi}{6} \cos \frac{\pi}{6}$

(c) $\frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{6}}$

(d) $\cos \frac{\pi}{4} \cos \frac{\pi}{3} + \sin \frac{\pi}{4} \sin \frac{\pi}{3}$

12. Show that the following relationships are true

(a) $\sin 2\theta = 2 \sin \theta \cos \theta$, where $\theta = \frac{\pi}{3}$

(b) $\cos 2\theta = 2 \cos^2 \theta - 1$, where $\theta = \frac{\pi}{6}$.

(c) $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$, where $\theta = \frac{2\pi}{3}$.

(d) $\sin(\theta - \phi) = \sin \theta \cos \phi - \sin \phi \cos \theta$, where $\theta = \frac{2\pi}{3}$ and $\phi = -\frac{\pi}{3}$.

TRIGONOMETRIC IDENTITIES

13. Simplify the following expressions

(a) $\cos \theta + \tan \theta \sin \theta$

(b) $\frac{\cos \theta}{1 + \sin \theta} - \frac{1 - \sin \theta}{\cos \theta}$

14. Show that $\frac{1 - 2 \cos^2 \theta}{\sin \theta \cos \theta} = \tan \theta - \cot \theta$.

15. Prove the identity

a) $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{2}{\sin \theta}$

b) $\tan x + \sec x = \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$

c) $\frac{\sec \phi + \operatorname{cosec} \phi}{\tan \phi + \cot \phi} = \sin \phi + \cos \phi$

d) $\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$

16. (a) If $\tan \theta = \frac{3}{4}$, $\pi \leq \theta \leq \frac{3\pi}{2}$, find i. $\cos \theta$ ii. $\operatorname{cosec} \theta$

(b) If $\sin \theta = -\frac{3}{4}$, $\frac{3\pi}{2} \leq \theta \leq 2\pi$, find i. $\sec \theta$ ii. $\cot \theta$

ANSWER

1. **a** $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **b** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **c** $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **d** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ 2. **a** $\cos \theta = \pm \frac{\sqrt{3}}{2}$ **b** $\cos \theta = \pm \frac{2\sqrt{2}}{3}$
3. **a** $\sin \theta = \pm \frac{3}{5}$ **b** $\sin \theta = \pm \frac{\sqrt{7}}{4}$ 4. $-\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{\sqrt{2}}$: 1 5. **a** $\sin \theta = \frac{\sqrt{5}}{3}$ **b** $\cos \theta = -\frac{\sqrt{21}}{5}$ **c** $\cos \theta = \frac{4}{5}$
d $\sin \theta = -\frac{12}{13}$
6. **a** $\sin x = \frac{2}{\sqrt{13}}$, $\cos x = \frac{3}{\sqrt{13}}$ 7. $-k$ k , $-\sqrt{1-k^2}$ 8. $\theta = 30^\circ$ or 150°
b $\sin x = \frac{4}{5}$, $\cos x = -\frac{3}{5}$ $\theta = \frac{2\pi}{3}$ or $\theta = \frac{5\pi}{3}$.
c $\sin x = -\sqrt{\frac{5}{14}}$, $\cos x = -\frac{3}{\sqrt{14}}$ 2π
d $\sin x = -\frac{12}{13}$, $\cos x = \frac{5}{13}$
9. $-\tan \theta$ $-\sin \theta$ 10. $\frac{\sqrt{3}}{2}$, $-\frac{\sqrt{2}}{2}$, $-\frac{\sqrt{3}}{3}$, $\frac{1}{2}$, -0.821 . 11. **a** 0 **b** $\frac{\sqrt{3}}{2}$ **c** $\frac{1}{\sqrt{3}}$ **d** $\frac{1+\sqrt{3}}{2\sqrt{2}}$
16. **a** (i) $-\frac{4}{5}$ (ii) $-\frac{5}{3}$ **b** (i) $\frac{4}{\sqrt{7}}$ (ii) $-\frac{\sqrt{7}}{3}$