What to do:

1 Copy and complete, using angles of your choice as well:

θ	$\sin 2\theta$	$2\sin\theta$	$2\sin\theta\cos\theta$	$\cos 2\theta$	$2\cos\theta$	$\cos^2 \theta - \sin^2 \theta$
0.631						
57.81°						
-3.697						

- Write down any discoveries from your table of values in 1.
- 3 In the diagram alongside, the semi-circle has radius 1 unit, and PÂB = θ.

$$\widehat{APO} = \theta$$
 { $\triangle AOP$ is isosceles}

 $\widehat{PON} = 2\theta$ {exterior angle of a triangle}

a Find in terms of θ , the lengths of:

ii [AM]

b Use △ANP and the lengths in **1** to show that:

$$\mathbf{i} \cos \theta = \frac{\sin 2\theta}{2 \sin \theta}$$

ii
$$\cos \theta = \frac{1 + \cos 2\theta}{2 \cos \theta}$$

c Hence deduce that:

$$\mathbf{i} \sin 2\theta = 2\sin\theta\cos\theta$$

ii
$$\cos 2\theta = 2\cos^2 \theta - 1$$

Hence, more conclusions:

• Since $sin^2(\theta) + cos^2(\theta) = 1$

Write down the $\cos(2\theta)$ in terms of $\cos(\theta)$ and $\sin(\theta)$:

$$cos(2\theta) =$$

Example 15

Self Tutor

Given that $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = -\frac{4}{5}$ find:

 $\sin 2\alpha$

 $b \cos 2\alpha$

$$\sin 2\alpha$$

$$= 2\sin \alpha \cos \alpha$$

$$= 2(\frac{3}{5})(-\frac{4}{5})$$

$$\begin{aligned}
\cos 2\alpha \\
&= \cos^2 \alpha - \sin^2 \alpha \\
&= \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2
\end{aligned}$$

EXERCISE 11D

- 1 If $\sin \theta = \frac{4}{5}$ and $\cos \theta = \frac{3}{5}$ find the exact values of:
 - $\sin 2\theta$
- $\cos 2\theta$
- $\cot 2\theta$
- 3 If $\sin \alpha = -\frac{2}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$, find the exact value of:
 - $a\cos \alpha$

- **b** $\sin 2\alpha$
- 4 If $\cos\beta=\frac{2}{5}$ where $270^{\circ}<\beta<360^{\circ}$, find the exact value of:
 - $a \sin \beta$

 $b \sin 2\beta$

- 5 If α is acute and $\cos 2\alpha = -\frac{7}{9}$, find without a calculator: **a** $\cos \alpha$ **b** $\sin \alpha$.
- 6 Find the exact value of $\left[\cos(\frac{\pi}{12}) + \sin(\frac{\pi}{12})\right]^2$.

7 Use an appropriate 'double angle' formula to simplify:

a $2 \sin \alpha \cos \alpha$

b $4\cos\alpha\sin\alpha$

d $2\cos^2\beta - 1$

 $e 1 - 2\cos^2 \phi$

 $9 \sin^2 M - 1$

 $\cos^2 \alpha - \sin^2 \alpha$

8 Show that:

$$(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$$

$$\cos^4 \theta - \sin^4 \theta = \cos 2\theta$$

9 Solve exactly for x where $0 \le x \le 2\pi$:

- $a \sin 2x + \sin x = 0$
- $\sin 2x + 3\sin x = 0$

10 Use the double angle formula to show that:

$$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

TRIGONOMETRIC EQUATIONS IN **QUADRATIC FORM**

EXERCISE 11E

Solve for $0 \le x \le 2\pi$ giving your answers as exact values:

a
$$2\sin^2 x + \sin x = 0$$
 b $2\cos^2 x = \cos x$

$$2\cos^2 x = \cos x$$

d
$$2\sin^2 x + 3\sin x + 1 = 0$$
 e $\sin^2 x = 2 - \cos x$

$$\sin^2 x = 2 - \cos x$$

2 Solve for $0 \le x \le 2\pi$ giving your answers as exact values:

$$\cos 2x - \cos x = 0$$

$$\cos 2x + 3\cos x = 1$$

$$\sin 4x = \sin 2x$$

$$\sin x + \cos x = \sqrt{2}$$

EXERCISE 11D

1 a
$$\frac{24}{25}$$

a
$$\frac{24}{25}$$
 b $-\frac{7}{25}$ **c** $-\frac{24}{7}$ **2 a** $-\frac{7}{9}$ **b** $\frac{1}{9}$

$$-\frac{24}{7}$$

$$-\frac{7}{9}$$

$$\frac{1}{9}$$

3 **a**
$$\cos \alpha = \frac{-\sqrt{5}}{3}$$
 b $\sin 2\alpha = \frac{4\sqrt{5}}{9}$

4 **a**
$$\sin \beta = \frac{-\sqrt{21}}{5}$$
 b $\sin 2\beta = \frac{-4\sqrt{21}}{25}$

b
$$\sin 2\beta = \frac{-4\sqrt{21}}{25}$$

5 a
$$\frac{1}{3}$$
 b $\frac{2\sqrt{2}}{3}$ 6 $\frac{3}{2}$

$$\frac{2\sqrt{2}}{3}$$

6
$$\frac{3}{2}$$

7 **a**
$$\sin 2\alpha$$
 b $2\sin 2\alpha$

$$\mathbf{b} = 2\sin 2\alpha$$

d
$$\cos 2\beta$$

$$e - \cos 2\phi$$

$$\mathbf{g} - \cos 2M$$
 $\mathbf{h} \cos 2\alpha$

$$\cos 2\alpha$$

9 **a**
$$x=0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$$

$$x = 0, \pi, 2\pi$$

EXERCISE 11E

1 **a**
$$x=0,\,\pi,\,\frac{7\pi}{6},\,\frac{11\pi}{6},\,2\pi$$
 b $x=\frac{\pi}{3},\,\frac{\pi}{2},\,\frac{3\pi}{2},\,\frac{5\pi}{3}$

d $x=\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ e no solutions

2 **a**
$$x=0,\frac{2\pi}{3},\frac{4\pi}{3},2\pi$$
 b $x=\frac{\pi}{3},\frac{5\pi}{3}$

d
$$x = 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}, 2\pi$$

$$x = \frac{\pi}{4}$$