EXAMPLE 10.2

Given that $\sin\theta = 0.3$, where $0 < \theta < \frac{\pi}{2}$ find

(a) $\sin(\pi + \theta)$ (b) $\sin(2\pi - \theta)$ (c) $\cos\left(\frac{\pi}{2} - \theta\right)$

EXAMPLE 10.3

Given that $\cos\theta = k$ and $0 < \theta < \frac{\pi}{2}$ find

(a) $\cos(\pi + \theta)$ (b) $\cos(2\pi - \theta)$ (c) $\cos\left(\frac{\pi}{2} + \theta\right)$

EXAMPLE 10.4

Given that $\sin\theta = k$ and $0 < \theta < \frac{\pi}{2}$ find

(a) $\tan\theta$ (b) $\operatorname{cosec}\theta$ (c) $\sec(\pi + \theta)$

EXAMPLE 10.5

Find the exact values of

(a) $\sec 45°$ (b) $\operatorname{cosec} 150°$ (c) $\cot \frac{11\pi}{6}$ (d) $\sec 0$

EXAMPLE 10.7

Simplify

(a) $\frac{\sin(\pi+\theta)}{\cos(2\pi-\theta)}$ (b) $\frac{\sin\left(\frac{\pi}{2}+\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)}{\cos(\pi+\theta)}$, where $0<\theta<\frac{\pi}{2}$.

Solutions

Solution

(a) From the reduction formulae, we have that $\sin(\pi+\theta) = -\sin\theta$.

Therefore, $\sin(\pi+\theta) = -0.3$.

(b) From the reduction formulae, we have that $\sin(2\pi-\theta) = -\sin\theta$.

Therefore, $\sin(\pi+\theta) = -0.3$.

(c) From the reduction formulae, we have that $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$.

Therefore, $\cos\left(\frac{\pi}{2}-\theta\right) = 0.3$.

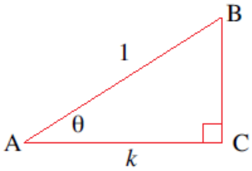
Solution

(a) $\cos(\pi+\theta) = -\cos\theta \therefore \cos(\pi+\theta) = -k$.

(b) $\cos(2\pi-\theta) = \cos\theta \therefore \cos(2\pi-\theta) = k$.

(c) $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$. However, we only have a value for $\cos\theta$.

To determine the value of $\sin\theta$ that corresponds to $\cos\theta = k$ we make use of a right-angled triangle where $\cos\theta = k$.



Construct a right-angled triangle ABC, where $\angle BAC = \theta$

so that $AC = k$ and $AB = 1$ $\left(\text{i.e., } \cos\theta = \frac{AC}{AB} = \frac{k}{1} = k\right)$.

Then, from Pythagoras's theorem, we have

$$1^2 = k^2 + BC^2 \Leftrightarrow BC = \pm\sqrt{1-k^2}$$

Therefore, as $\sin\theta = \frac{BC}{AB} \Rightarrow \sin\theta = \frac{\pm\sqrt{1-k^2}}{1} = \pm\sqrt{1-k^2}$.

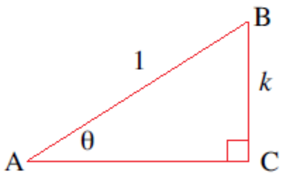
However, as $0 < \theta < \frac{\pi}{2}$, then θ is in the first quadrant and so, $\sin\theta > 0 \therefore \sin\theta = \sqrt{1-k^2}$.

Now that we have the value of $\sin\theta$ we can complete the question:

$$\sin\left(\frac{\pi}{2}+\theta\right) = -\sin\theta \therefore \sin\left(\frac{\pi}{2}+\theta\right) = -\sqrt{1-k^2}$$

Solution

As we are looking for trigonometric ratios based solely on that of the sine ratio, we start by constructing a right-angled triangle satisfying the relationship, $\sin\theta = k$



In this case, as $\sin\theta = \frac{\text{opp}}{\text{hyp}} = k \Rightarrow \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} = \frac{k}{1}$

(using the simplest ratio).

Using Pythagoras's theorem, we have

$$1^2 = k^2 + AC^2 \Leftrightarrow AC = \pm\sqrt{1-k^2}$$

(a) $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{k}{\pm\sqrt{1-k^2}}$.

However, as $0 < \theta < \frac{\pi}{2}$, $\tan\theta > 0 \therefore \tan\theta = \frac{k}{\sqrt{1-k^2}}$.

(b) $\operatorname{cosec}\theta = \frac{1}{\sin\theta} \therefore \operatorname{cosec}\theta = \frac{1}{k}$.

(c) $\sec(\pi+\theta) = \frac{1}{\cos(\pi+\theta)} = -\frac{1}{\cos\theta}$.

But, $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\pm\sqrt{1-k^2}}{1} = \pm\sqrt{1-k^2}$.

However, as $0 < \theta < \frac{\pi}{2}$, $\cos\theta > 0 \therefore \cos\theta = \sqrt{1-k^2}$.

Therefore, $\sec(\pi+\theta) = -\frac{1}{\sqrt{1-k^2}}$.

Solution

(a) $$\sec 45^\circ = \frac{1}{\cos 45^\circ} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$$

(b) $$\operatorname{cosec} 150^\circ = \frac{1}{\sin 150^\circ} = \frac{1}{\sin 30^\circ} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

(c) $$\cot\frac{11\pi}{6} = \frac{1}{\tan\left(\frac{11\pi}{6}\right)} = \frac{1}{\tan\left(-\frac{\pi}{6}\right)} = \frac{1}{-\tan\frac{\pi}{6}} = \frac{1}{-\left(\frac{1}{\sqrt{3}}\right)} = -\sqrt{3}$$

(c) $$\sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1$$

Solution

(a) $$\frac{\sin(\pi+\theta)}{\cos(2\pi-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

(b) $$\frac{\sin\left(\frac{\pi}{2}+\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)}{\cos(\pi+\theta)} = \frac{\cos\theta\sin\theta}{-\cos\theta} = -\sin\theta$$

EXERCISES 10.1

1. Convert the following angles to degrees.

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{5}$ (c) $\frac{12\pi}{10}$

2. Convert the following angles to radians.

(a) 180° (b) 270° (c) 140°

3. Find the exact value of

(a) $\sin 120^\circ$ (b) $\cos 120^\circ$ (c) $\tan 120^\circ$

(e) $\sin 210^\circ$ (f) $\cos 210^\circ$ (g) $\tan 210^\circ$

4. Find the exact value of

(a) $\sin\pi$ (b) $\cos\pi$ (c) $\tan\pi$ (d) $\sec\pi$

(e) $\sin\frac{3\pi}{4}$ (f) $\cos\frac{3\pi}{4}$ (g) $\tan\frac{3\pi}{4}$ (h) $\operatorname{cosec}\frac{3\pi}{4}$

5. Find the exact value of

(a) $\sin(-210^\circ)$ (b) $\cos(-30^\circ)$

(e) $\cot(-60^\circ)$ (f) $\sin(-150^\circ)$

EXERCISE 10.1

1. (a) 120° (b) 108° (c) 216° (d) 50° **2.** (a) π^c (b) $\frac{3\pi^c}{2}$ (c) $\frac{7\pi^c}{9}$ (d) $\frac{16\pi^c}{9}$ **3.** (a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{2}$ (c) $-\sqrt{3}$ (d) -2 (e) $-\frac{1}{2}$ (f) $-\frac{\sqrt{3}}{2}$ (g) $\frac{1}{\sqrt{3}}$ (h) $\sqrt{3}$ (i) $-\frac{1}{\sqrt{2}}$ (j) $-\frac{1}{\sqrt{2}}$ (k) 1 (l) $-\sqrt{2}$ (m) $-\frac{1}{\sqrt{2}}$ (n) $\frac{1}{\sqrt{2}}$ (o) -1 (p) $\sqrt{2}$ (q) 0 (r) 1 (s) 0 (t) undefined **4.** (a) 0 (b) -1 (c) 0 (d) -1 (e) $\frac{1}{\sqrt{2}}$ (f) $-\frac{1}{\sqrt{2}}$ (g) -1 (h) $\sqrt{2}$ (i) $-\frac{1}{2}$ (j) $-\frac{\sqrt{3}}{2}$ (k) $\frac{1}{\sqrt{3}}$ (l) $\sqrt{3}$ (m) $-\frac{\sqrt{3}}{2}$ (n) $\frac{1}{2}$ (o) $-\sqrt{3}$ (p) 2 (q) $-\frac{1}{\sqrt{2}}$ (r) $\frac{1}{\sqrt{2}}$ (s) -1 (t) $-\sqrt{2}$ **5.** (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{1}{2}$ (e) $-\frac{1}{\sqrt{3}}$ (f) $-\frac{1}{2}$ (g) $-\sqrt{2}$ (h) $-\frac{2}{\sqrt{3}}$ **6.** (a) $-\frac{1}{2}$ (b) $-\frac{1}{\sqrt{2}}$