Math HL Worksheet on Compound Angle Identities

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angle Formulae

$$\sin 2\theta = 2 \sin\theta \cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

To prove a given identity, any one of the following approaches can be used:

1. Start with the L.H.S and then show that L.H.S = R.H.S

2. Start with the R.H.S and then show that R.H.S = L.H.S

3. Show that L.H.S = p, show that R.H.S = p $\Rightarrow$ L.H.S = R.H.S

4. Start with L.H.S = R.H.S $\Rightarrow$ L.H.S – R.H.S = 0 .

When using approaches 1., and 2., choose whichever side has more to work with.

1. Simplify $\cos 3\theta \cos\theta - \sin 3\theta \sin\theta$.

2. Without using your calculator, show that $\sin 75° = \frac{\sqrt{6}+\sqrt{2}}{4}$.

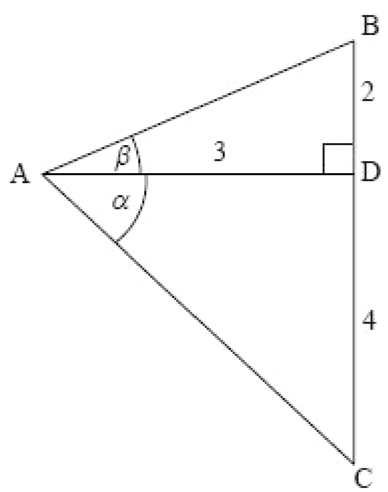
3. Suppose $\sin x - \sqrt{3}\cos x = k\cos(x+b)$ for $k > 0$ and $0 < b < 2\pi$. Find k and b.

4. Use the compound angle formulae to prove the double angle formulae:

   a $\sin 2\theta = 2\sin\theta\cos\theta$ **b** $\cos 2\theta = \cos^2\theta - \sin^2\theta$ **c** $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

5. In the diagram below, AD is perpendicular to BC.

   CD = 4, BD = 2 and AD = 3. $\hat{CAD} = \alpha$ and $\hat{BAD} = \beta$.



Find the exact value of $\cos(\alpha - \beta)$.

6. Simplify the expression: $\dfrac{\sin 3\alpha}{\sin\alpha} - \dfrac{\cos 3\alpha}{\cos\alpha}$

7. Prove the identity $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

8. Prove that $\dfrac{\sin 2\phi + \sin\phi}{\cos 2\phi + \cos\phi + 1} = \tan\phi$

9. Given that $\tan 2\theta = \dfrac{3}{4}$, find the possible values of $\tan\theta$.

10. The obtuse angle B is such that $\tan B = -\dfrac{5}{12}$. Find the values of

(a) $\sin B$; (b) $\cos B$; (c) $\sin 2B$; (d) $\cos 2B$.

11. Show that:

a $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$ **b** $\cos^4\theta - \sin^4\theta = \cos 2\theta$

12. Using a compound identity, show that $\cos\left(\dfrac{3\pi}{2} - \theta\right) = -\sin\theta$

13. Find the exact value of (a) $\cos 15°$ (b) $\tan\dfrac{5\pi}{12}$

14. Use the double angle formula to show that:

a $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ **b** $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

Answers

1. $\cos 4\theta$ 3. $k = 2$ 5. $\dfrac{17}{5\sqrt{13}}\left(= \dfrac{17\sqrt{13}}{65}\right)$ 6. **2** 9. $\tan\theta = \dfrac{1}{3}$ or -3

10. a) $\sin B = \dfrac{5}{13}$ b) $\cos B = -\dfrac{12}{13}$ c) $-\dfrac{120}{169}$ d) $\dfrac{119}{169}$ 13. a) $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$ b) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$