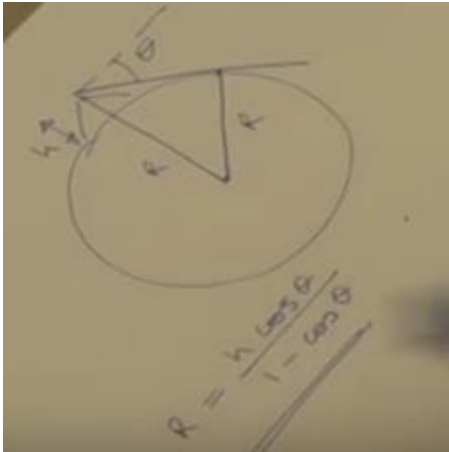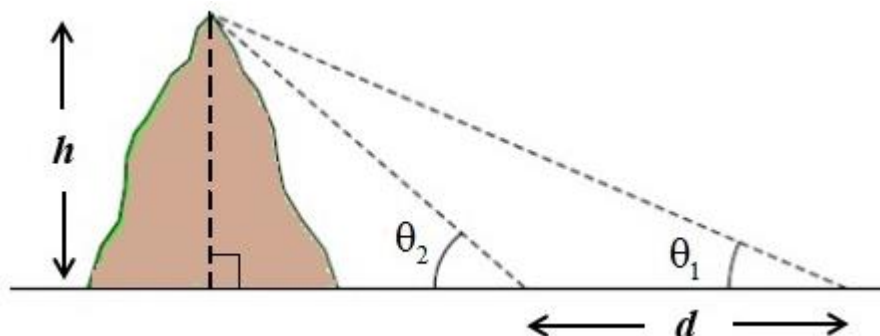# Radius of the Earth

In the early 11th century the great Islamic scientist al-Biruni (born 973, died 1048 - full name Abu Arrayhan Muhammad ibn Ahmad al-Biruni) devised a method to efficiently measure the radius of the earth. A little more than a thousand years earlier, the Greek mathematician Eratosthenes (276-194 BC) had computed the circumference of the earth but his method required measuring a distance between two cities far apart from each other by ordering soldiers to walk the distance and count their steps. Al-Biruni's method could be performed by a single person measuring three angles and a much shorter distance. But, the most important aspect of al-Biruni's clever method was the application of some algebra and trigonometry which allowed him to derive two important formulas.

Your task is to derive the two formulas and then given al-Biruni's measurements compute his value for the radius of the earth – and also the earth's circumference.
A secondary task involves considering how much al-Biruni's method (Eratosthenes' method too) relied on very accurate angle measurements.

1. Formula for the height, h, of a mountain in terms of two angles of elevation, θ_1 and θ_2, and the distance, d, between the points where θ_1 and θ_2 were measured from.
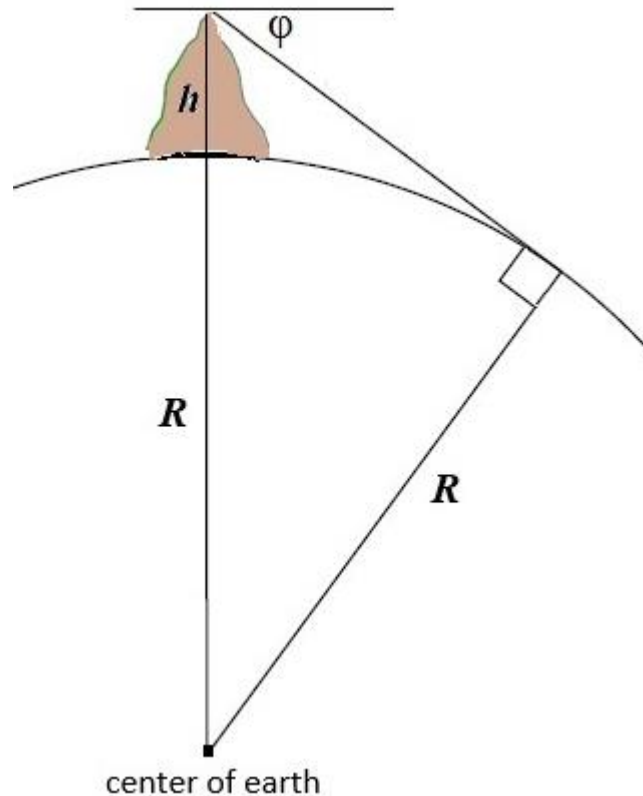

Show that $h = \dfrac{d \tan\theta_1 \tan\theta_2}{\tan\theta_2 - \tan\theta_1}$.

Radius of the Earth

2. Formula for the radius of the earth, **R**, in terms of the angle of depression, φ.

Show that $\boldsymbol{R} = \dfrac{h \cos \varphi}{1 - \cos \varphi}$

center of earth

3. For θ_1 and θ_2, al-Biruni measured angles of $5.5°$ and $7.5°$ respectively. The distance d between the points where θ_1 and θ_2 were measured was 875.11 meters. The angle of depression, φ, measured at the top of the mountain to the horizon was $0.57°$. Using the two formulas from 1 and 2, calculate the radius and circumference of the earth in kilometers accurate to four significant figures.

4. If the angle of depression, φ, measured from the top of the mountain to the horizon was off by one tenth of a degree then this would be a percentage error of approximately 17.5% (0.10 is about 17.5% of 0.57). (a) If $\varphi = 0.47°$ then by what percent would it change the value of the earth's radius obtained above for $\varphi = 0.57°$? (b) If $\varphi = 0.67°$ then by what percent would it change the value of the earth's radius obtained above for $\varphi = 0.57°$?

Radius of the Earth

ANSWERS

3. $\text{radius} \approx 6339 \, \text{km}$, $\text{circumference} \approx 39830 \, \text{km}$

4. (a) $\text{change} \approx 47.1\%$

   (a) $\text{change} \approx 27.6\%$

Radius of the Earth