Sine modeling

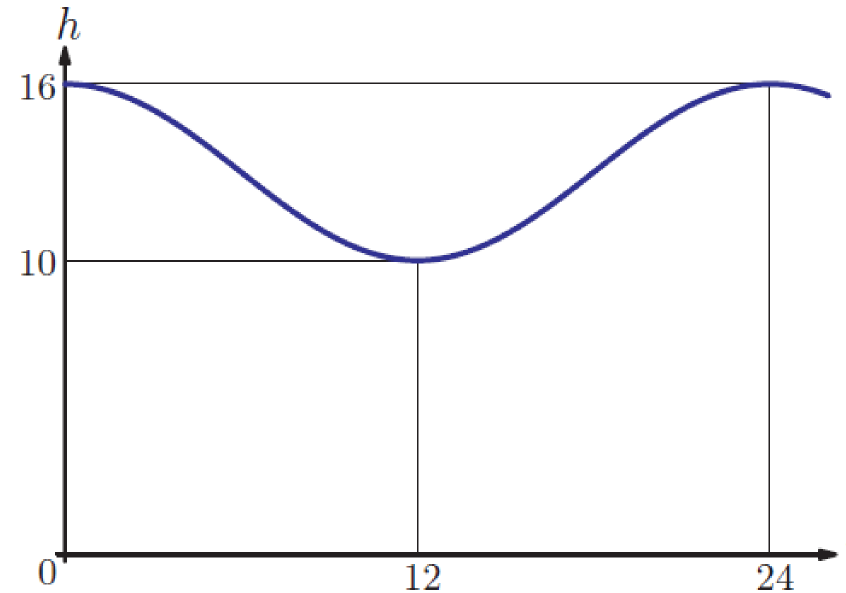
Take it easy!!!

Example 1

The height of water in the harbour is 16 m at high tide, and 12 hours later at low tide, it is 10 m. The graph below shows how the height of water changes with time over 24 hours.

(a) Find the equation for height (in metres) in terms of time (in hours) in the form $h = m + a\cos(bt)$.

(b) Find the first two times after the high tide when the height of water is 12 m.



Solution

(a) $m = \dfrac{16+10}{2} = 13$

$$m = \frac{min + max}{2}.$$

$$\text{amplitude} = \frac{16-10}{2} = 3$$
$$\therefore a = 3$$

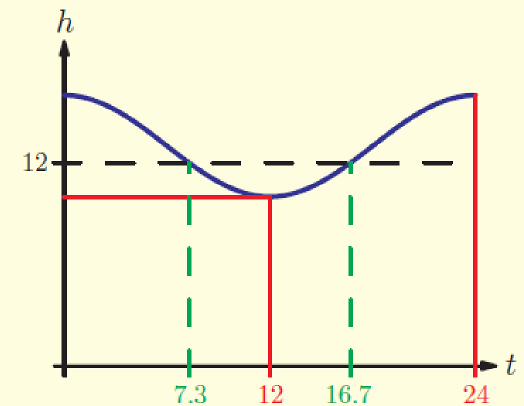
$$\text{amplitude } a = \frac{max - min}{2}.$$

$$\text{period} = 24$$
$$24 = \frac{2\pi}{b} \qquad \therefore b = \frac{\pi}{12}$$

$$\text{So } h = 13 + 3\cos\left(\frac{\pi}{12}t\right)$$

(b) $13 + 3\cos\left(\dfrac{\pi}{12}t\right) = 12$

Solve for t using GDC



The height of the water will be 12 m 7.3 hours and 16.7 hours after the high tide.

EXAMPLE 10.30

When a person is at rest, the blood pressure, P millimetres of mercury at any time t seconds can be approximately modelled by the equation

$$P(t) = -20\cos t\left(\frac{5\pi}{3}t\right) + 100,\ t \ge 0$$

(a) Determine the amplitude and period of P.

(b) What is the maximum blood pressure reading that can be recorded for this person?

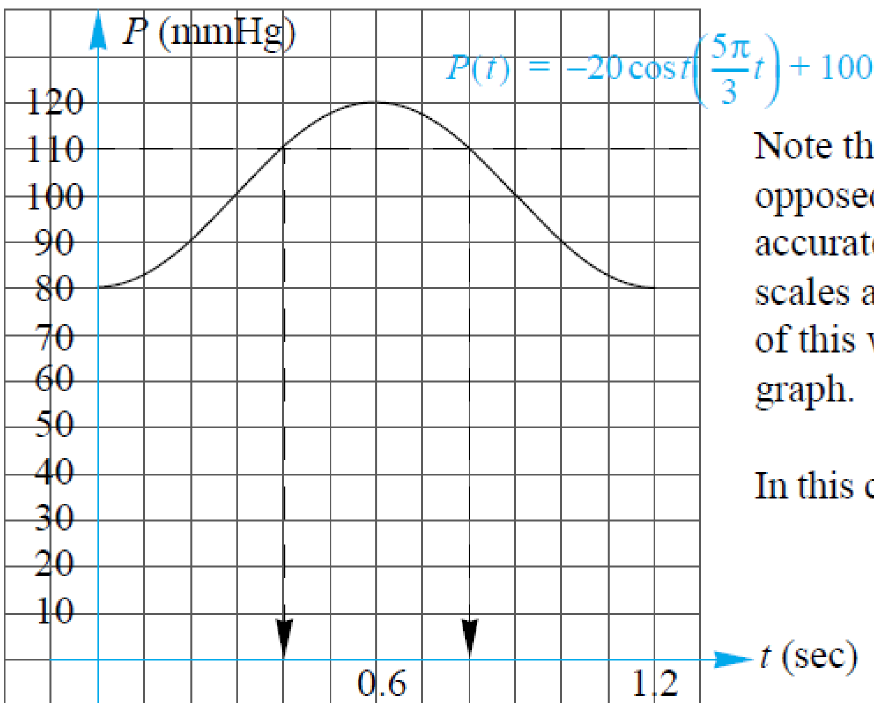
(c) Sketch the graph of $P(t)$, showing one full cycle.

(d) Find the first two times when the pressure reaches a reading of 110 mmHg.

(a) The amplitude is 20 mmHg and the period is given by $\frac{2\pi}{\frac{5\pi}{3}} = \frac{6}{5} = 1.2$ seconds.

(b) The maximum is given by (100 + amplitude) = 100 + 20 =120.

(c) One full cycle is 1.2 seconds long:



Note that the graph has been drawn as opposed to sketched. That is, it has been accurately sketched, meaning that the scales and the curve are accurate. Because of this we can read directly from the graph.

In this case, $P = 110$ when $t = 0.4$ and 0.8.

(d) Even though we have drawn the graph, we will now solve the relevant equation:

$$P(t) = 110 \Leftrightarrow 110 = -20\cos\left(\frac{5\pi}{3}t\right) + 100$$

$$\Leftrightarrow 10 = 20\cos\left(\frac{5\pi}{3}t\right)$$

$$\Leftrightarrow \cos\left(\frac{5\pi}{3}t\right) = -\frac{1}{2}$$

$$\therefore \frac{5\pi}{3}t = \pi - \mathrm{Cos}^{-1}\left(\frac{1}{2}\right), \pi + \mathrm{Cos}^{-1}\left(\frac{1}{2}\right) \quad \left[\text{Reference angle is } \mathrm{Cos}^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}\right]$$

$$\Leftrightarrow \frac{5\pi}{3}t = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\Leftrightarrow t = \frac{2}{5}, \frac{4}{5}$$