# RM 294 - Optimization

# Project 1 - Marketing Allocation Report

**Team members**

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## Introduction and Problem Scope

In today's business landscape, marketing budgets have grown to occupy a significant 11 percent of a company's overall finances. However, the pivotal question remains: How can every marketing dollar be leveraged to its fullest extent in order to maximize profitability?

The challenge at hand is the wide disparity in marketing success among companies. What works wonders for one may not yield the same results for another. Our team's mission is to transform the way companies allocate their marketing resources to make smarter, data-driven choices instead of relying on guesswork and personal opinions. To accomplish this, we have used Linear Programming modeling to come up with an optimal marketing budget strategy.

## Methodology: *Linear Programming Model*

To determine the optimal allocation of marketing budgets across multiple advertising mediums, this study formulates the problem as a linear programming problem using Python and the Gurobi optimization solver.

The primary goal is to maximize the total return on investment (ROI) derived from a marketing budget (in this case $10 million), across different advertising mediums. Mathematically, this objective can be represented as the following function:

$$\text{maximize} \sum_{i=1}^{n} (\text{ROI}_i \cdot x_i)$$

where:

- n is the total number of advertising mediums.
- $\text{ROI}_i$ is the return on investment for the advertising medium *i*
- $x_i$ denotes the budget allocation for the advertising medium *i*

This allocation process is subject to a set of specific business constraints and requirements, including considerations like the total budget limit and minimum/maximum allocation thresholds.

The optimization process also incorporates several predefined conditions:

- The budget allocation for Print and TV should not exceed that of Facebook and Email.
- The total budget allocation for social media platforms (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) must be at least twice that of SEO and AdWords.
- For each advertising platform, the budget allocation must not exceed $3 million.

To achieve the optimal allocation, we have used Gurobi's optimization framework. This involves defining the decision variables, formulating the objective function aimed at maximizing the total ROI, integrating the

constraints, and executing the optimization to determine the most advantageous budget allocation for each advertising medium.

We understand the importance of reusability in code, so we have created a dynamic Gurobi Optimization function, which we are calling for all our successive optimizations, instead of rewriting the code with changes.

In order to further expand on the implementation of the objective and conditions, we have provided a Python code snippet (Code Chunk 1) demonstrating the optimization process in Gurobi.

```
def gurobi_optimize(medium_budget_dict,marketing_budget =  10000000, n_constrs =3):
    model = gp.Model()

    #Adding the variables for Model 1, the variables being the dollars to be invested in each medium
    ojModX = model.addMVar(medium_list_len)

    #Setting the objective for Model 1, which is maximising the ROI, this is is the sum of Returns from all the mediums
    model.setObjective(gp.quicksum(medium_budget_dict[medium_list[i]]*ojModX[i] for i in range(medium_list_len)),sense=gp.GRB.MAXIMIZE)


    conlist=[0]*3
    # then come back and change the list entries one by one to represent each constraint
    conlist[0] = model.addConstr(gp.quicksum(ojModX[i] for i in range(medium_list_len)) <= marketing_budget)

    conlist[1] = model.addConstr(ojModX[medium_list.index('Print')] + ojModX[medium_list.index('TV')]
                                 <= ojModX[medium_list.index('Facebook')] + ojModX[medium_list.index('Email')] )

    conlist[2] = model.addConstr(ojModX[medium_list.index('Facebook')] + ojModX[medium_list.index('LinkedIn')] +
                                 ojModX[medium_list.index('Instagram')] + ojModX[medium_list.index('Snapchat')] +
                                 ojModX[medium_list.index('Twitter')] >= 2 *(ojModX[medium_list.index('SEO')] +
                                                                             ojModX[medium_list.index('AdWords')]))

    if n_constrs == 3:
      amountlessthan3m = model.addConstrs(ojModX[i] <= 3000000 for i in range(medium_list_len))


    model.Params.OutputFlag = 0 # tell gurobi to shut up!!

    model.optimize()

    return model,ojModX
```

*Code Chunk 1: Optimization Implementation Code*

By implementing a diverse optimization framework, we are analyzing diverse ROI scenarios and their subsequent impact on optimal budget allocation.

## Empirical Analysis I: *Initial ROI Data*

Given that the marketing budget is $10M, we have to distribute this fund amongst various advertising platforms with the objective of maximizing the company's advertising. The ROI of individual advertising platforms are given below:

| | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email |
|---|---|---|---|---|---|---|---|---|---|---|
| ROI | 3.1% | 4.9% | 2.4% | 3.9% | 1.6% | 2.4% | 4.6% | 2.6% | 3.3% | 4.4% |
| Allocation | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ |

*Table 1: ROI Proposal 1*

Let $x_i$ denote the capital allocated for $i^{th}$ advertising platform.

Our objective would be to maximize:

$$3.1\%x_0 + 4.9\%x_1 + 2.4\%x_2 + 3.9\%x_3 + 1.6\%x_4 + 2.4\%x_5 + 4.6\%x_6 + 2.6\%x_7 + 3.3\%x_8 + 4.4\%x_9$$

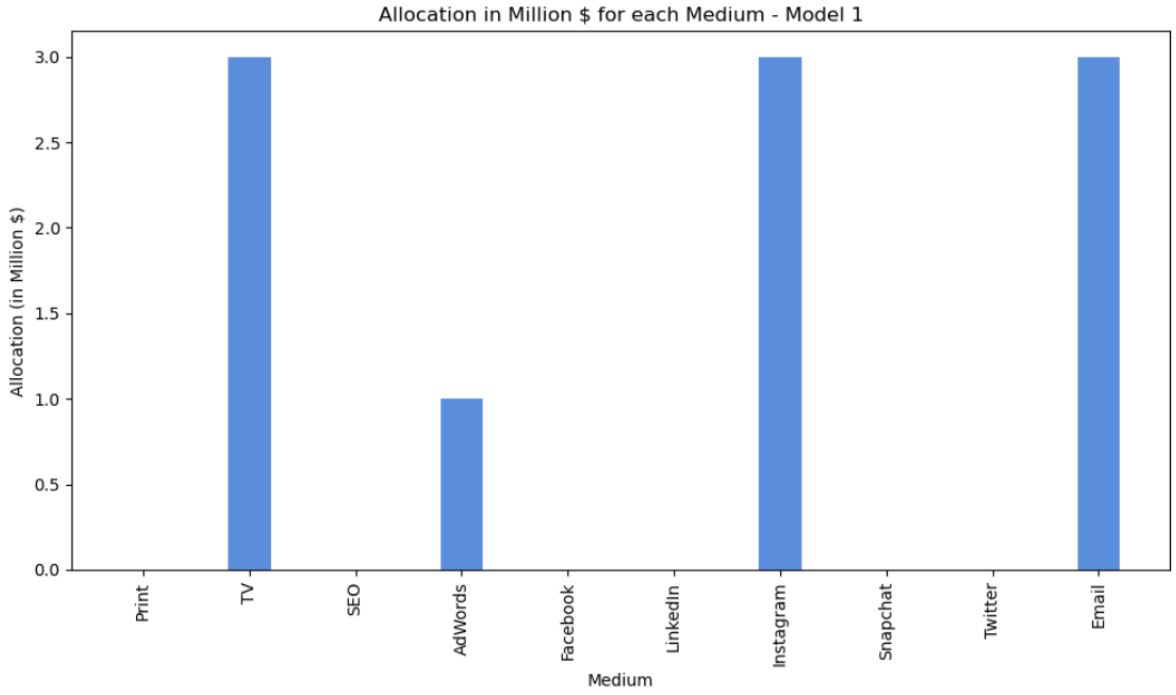


*Figure 1: Allocations for each advertising platforms based on Proposal 1*

The total return on investment with the above allocation would be $\$456,000$.

## Empirical Analysis II: *Comparative ROI Assessment*

While initial optimization results are promising, past experience highlights the need for careful validation before proceeding. To provide additional rigor, we sought a second expert opinion on the ROI estimates. This additional analysis provides a revised set of returns for ten marketing channels, capturing slightly different perceptions of expected performance. The second set of ROIs are as follows:

| | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email |
|---|---|---|---|---|---|---|---|---|---|---|
| ROI | 4.9% | 2.3% | 2.4% | 3.9% | 4.4% | 4.6% | 2.6% | 1.9% | 3.7% | 2.6% |
| Allocation | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ |

*Table 2: ROI Proposal 2*

The model works similar to the process employed in empirical analysis I, only with a different set of ROI values. After running the model with Gurobi, the new allocations are shown below.

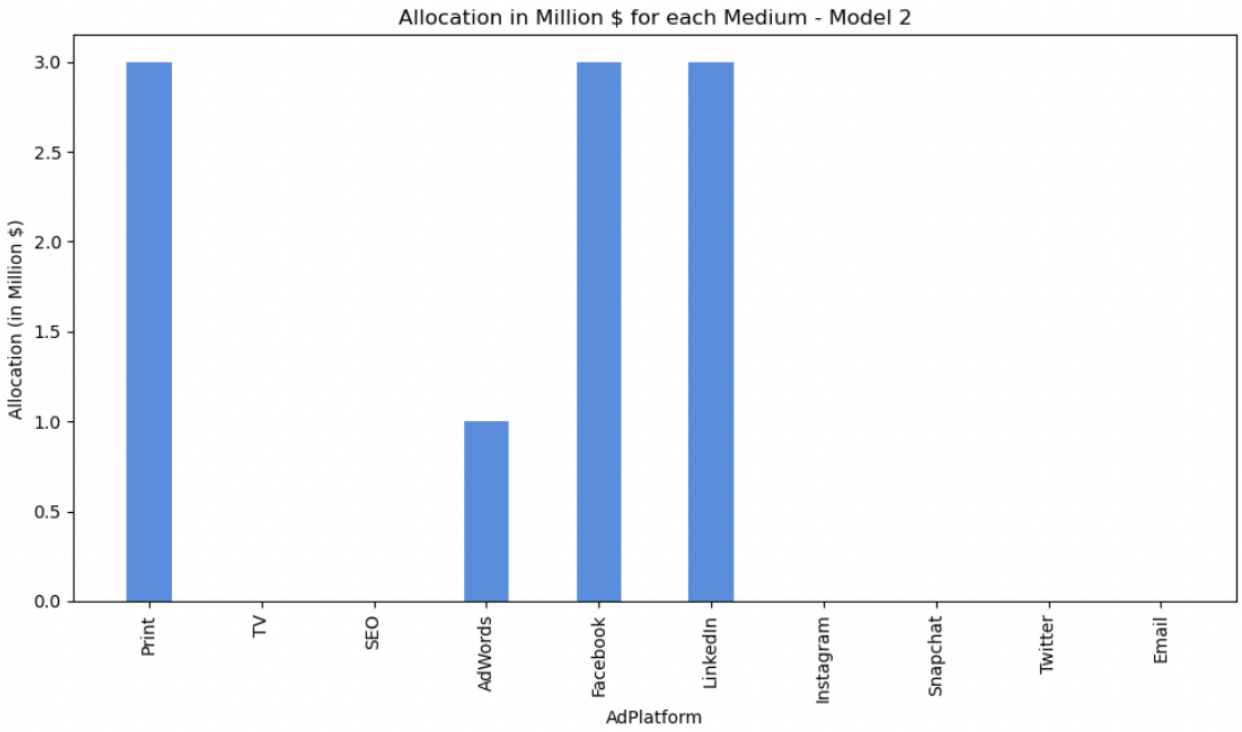


*Figure 2: Allocations for each advertising platforms based on Proposal 2*

## Assessing the allocations and the third constraint

The allocations differ between the two proposed ROIs however, the maximum ROI (optimal objective) remains the same for both choices.

| | Medium | Model 1 | Model 2 |
|---|---|---|---|
| 0 | Print | 0.0 | 3000000.0 |
| 1 | TV | 3000000.0 | 0.0 |
| 2 | SEO | 0.0 | 0.0 |
| 3 | AdWords | 1000000.0 | 1000000.0 |
| 4 | Facebook | 0.0 | 3000000.0 |
| 5 | LinkedIn | 0.0 | 3000000.0 |
| 6 | Instagram | 3000000.0 | 0.0 |
| 7 | Snapchat | 0.0 | 0.0 |
| 8 | Twitter | 0.0 | 0.0 |
| 9 | Email | 3000000.0 | 0.0 |

*Table 3: ROI comparison between Proposal 1 and 2*

Conversely, considering that the first ROI data is correct, if we had opted for the second ROI data, our return would have been $204,000 less than the return of $456,000. In the case that the second ROI data was correct and we had opted for the first ROI data, our return would have been $192,000 less than the return of $456,000.

| | Medium | Allocation 1 | Model 1 | Model 1 W/o Constraint 3 |
|---|---|---|---|---|
| 0 | Print | 0.031 | 0.0 | 0.0 |
| 1 | TV | 0.049 | 3000000.0 | 5000000.0 |
| 2 | SEO | 0.024 | 0.0 | 0.0 |
| 3 | AdWords | 0.039 | 1000000.0 | 0.0 |
| 4 | Facebook | 0.016 | 0.0 | 0.0 |
| 5 | LinkedIn | 0.024 | 0.0 | 0.0 |
| 6 | Instagram | 0.046 | 3000000.0 | 0.0 |
| 7 | Snapchat | 0.026 | 0.0 | 0.0 |
| 8 | Twitter | 0.033 | 0.0 | 0.0 |
| 9 | Email | 0.044 | 3000000.0 | 5000000.0 |

| | Medium | Allocation 2 | Model 2 | Model 2 W/o Constraint 3 |
|---|---|---|---|---|
| 0 | Print | 0.049 | 3000000.0 | 5000000.0 |
| 1 | TV | 0.023 | 0.0 | 0.0 |
| 2 | SEO | 0.024 | 0.0 | 0.0 |
| 3 | AdWords | 0.039 | 1000000.0 | 0.0 |
| 4 | Facebook | 0.044 | 3000000.0 | 5000000.0 |
| 5 | LinkedIn | 0.046 | 3000000.0 | 0.0 |
| 6 | Instagram | 0.026 | 0.0 | 0.0 |
| 7 | Snapchat | 0.019 | 0.0 | 0.0 |
| 8 | Twitter | 0.037 | 0.0 | 0.0 |
| 9 | Email | 0.026 | 0.0 | 0.0 |

*Table 4: Allocations for each advertising platforms with and without constraint 3*

Upon removing the third constraint, we can observe two things. First, the total budget of $10M is evenly split into two mediums, for both ROI datasets. Second, the optimal objective value (optimal return) increases by merely $9,000.

While it can be tempting to remove the third constraint, we must exercise caution. We must look at the increased amount in relative terms. The $9,000 increase, relative to the total optimal objective value of $456,000, holds limited significance and does not provide substantial benefit to the company. In addition, by maintaining the third constraint, we are diversifying our allocation across four advertising mediums, ensuring growth opportunities in the long run. The company will hence be able to expand its presence in multiple channels over time. Therefore, it would be beneficial for the company to maintain the third constraint and forgo the modest increment in the optimal objective value keeping diversity in mind and securing a strong foundation for the future.

## Sensitivity Analysis

```
sentivity_analysis = pd.DataFrame(
    [
        pd.Series(ojModX1.SAObjLow, index=roi_data.columns),
        pd.Series(ojModX1.SAObjUp, index=roi_data.columns)
    ], index=["Lower Bound", "Upper Bound"]
).T

sentivity_analysis["Upper Bound Fix"] = np.where(sentivity_analysis["Upper Bound"]==np.inf,1,sentivity_analysis["Upper Bound"])
sentivity_analysis
```

*Code Chunk 2: Sensitivity Analysis Code*

| | Lower Bound | Upper Bound | Upper Bound Fix |
|---|---|---|---|
| **Print** | -inf | 0.049 | 0.049 |
| **TV** | 0.039 | 0.062 | 0.062 |
| **SEO** | -inf | 0.039 | 0.039 |
| **AdWords** | 0.033 | 0.046 | 0.046 |
| **Facebook** | -inf | 0.029 | 0.029 |
| **LinkedIn** | -inf | 0.039 | 0.039 |
| **Instagram** | 0.039 | inf | 1.000 |
| **Snapchat** | -inf | 0.039 | 0.039 |
| **Twitter** | -inf | 0.039 | 0.039 |
| **Email** | 0.029 | inf | 1.000 |

*Table 5: Upper and lower bounds for the ROI of each advertising platform that would keep the optimal ROI unchanged*

We conducted an analysis to explore the flexibility of our ROI allocation strategy while maintaining the optimal ROI constant. We noticed that there were certain platforms including Print, SEO, Facebook, Linkedin, Snapchat and Twitter that could have their allocations reduced to as low as zero without affecting the optimal ROI. On the other hand, platforms such as Instagram and Email could have their allocations increased to as high of a percentage possible, so long as it doesn't exceed the maximum allocation limit of $3 million.

## Monthly Budget Allocation Strategy

Given that we have received approval to reinvest 50% of the monthly returns into the following month, we can take two approaches:

### Method 1:

This strategy is based on the premise that all investments and associated returns are realized at the end of each month. Given below is a flow chart summarizing the strategy to incorporate the same.

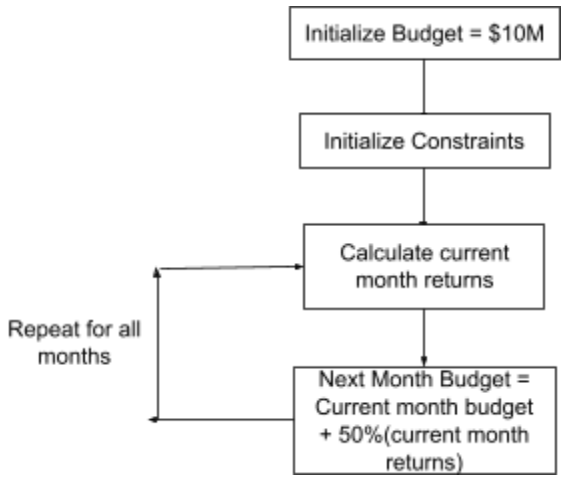


*Figure 3: Flow chart demonstrating the reinvestment strategy using Method 1*

Upon implementation, the monthly allocations are as follows:

| | Period | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email | MonthlyROI | Budget | Return percentage |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | January | 3000000.0 | 0.0 | 0.0 | 1333333.3 | 0.0 | 0.0 | 2666666.7 | 0.0 | 0.0 | 3000000.0 | 373000.0 | 10000000 | 0.0373 |
| 1 | February | 3000000.0 | 0.0 | 0.0 | 2395500.0 | 3000000.0 | 0.0 | 0.0 | 0.0 | 1791000.0 | 0.0 | 406296.0 | 10186500.0 | 0.0399 |
| 2 | March | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 1389648.0 | 0.0 | 3000000.0 | 0.0 | 414417.0 | 10389648.0 | 0.0399 |
| 3 | April | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 3000000.0 | 0.0 | 1596856.5 | 0.0 | 414486.8 | 10596856.5 | 0.0391 |
| 4 | May | 1804099.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 3000000.0 | 432143.5 | 10804099.9 | 0.04 |
| 5 | June | 3000000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 2020171.7 | 3000000.0 | 454766.5 | 11020171.7 | 0.0413 |
| 6 | July | 0.0 | 0.0 | 0.0 | 3000000.0 | 2247554.9 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 0.0 | 468654.6 | 11247554.9 | 0.0417 |
| 7 | August | 3000000.0 | 0.0 | 0.0 | 1827294.1 | 0.0 | 654588.2 | 0.0 | 0.0 | 3000000.0 | 3000000.0 | 487966.1 | 11481882.2 | 0.0425 |
| 8 | September | 1362932.6 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 0.0 | 0.0 | 3000000.0 | 1362932.6 | 459219.9 | 11725865.3 | 0.0392 |
| 9 | October | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 3000000.0 | 0.0 | 0.0 | 2955475.3 | 427575.2 | 11955475.3 | 0.0358 |
| 10 | November | 3000000.0 | 0.0 | 0.0 | 2056421.0 | 0.0 | 1112841.9 | 3000000.0 | 0.0 | 0.0 | 3000000.0 | 517375.6 | 12169262.9 | 0.0425 |
| 11 | December | 3000000.0 | 3000000.0 | 0.0 | 427950.7 | 3000000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3000000.0 | 516834.2 | 12427950.7 | 0.0416 |

*Table 6: Monthly budget for allocation for Method 1*

## Method 2:

This strategy is based on the premise that for a given month we would have $10 million and 50% of the previous month's returns. Given below is a flow chart summarizing the strategy to incorporate the same.

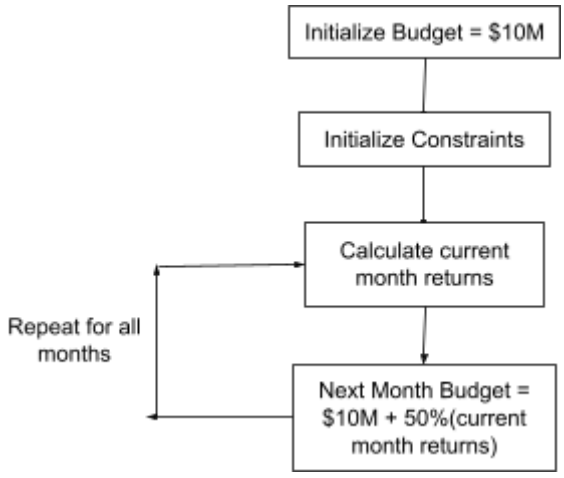


*Figure 4: Flow chart demonstrating the reinvestment strategy using Method 2*

Upon implementation, the monthly allocations are as follows:

| | Period | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email | MonthlyROI | Budget | Return percentage |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | January | 3000000.0 | 0.0 | 0.0 | 1333333.3 | 0.0 | 0.0 | 2666666.7 | 0.0 | 0.0 | 3000000.0 | 373000.0 | 10000000 | 0.0373 |
| 1 | February | 3000000.0 | 0.0 | 0.0 | 2395500.0 | 3000000.0 | 0.0 | 0.0 | 0.0 | 1791000.0 | 0.0 | 406296.0 | 10186500.0 | 0.0399 |
| 2 | March | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 1199428.7 | 0.0 | 3000000.0 | 0.0 | 407378.9 | 10199428.7 | 0.0399 |
| 3 | April | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 3000000.0 | 0.0 | 1199706.7 | 0.0 | 400189.4 | 10199706.7 | 0.0392 |
| 4 | May | 1196176.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 3000000.0 | 410866.2 | 10196176.9 | 0.0403 |
| 5 | June | 3000000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 1201480.5 | 3000000.0 | 423656.3 | 10201480.5 | 0.0415 |
| 6 | July | 0.0 | 0.0 | 0.0 | 3000000.0 | 1207644.5 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 0.0 | 428098.1 | 10207644.5 | 0.0419 |
| 7 | August | 2709694.9 | 0.0 | 0.0 | 1500000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3000000.0 | 3000000.0 | 437807.2 | 10209694.9 | 0.0429 |
| 8 | September | 607203.8 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 0.0 | 0.0 | 3000000.0 | 607203.8 | 402540.3 | 10214407.6 | 0.0394 |
| 9 | October | 0.0 | 0.0 | 0.0 | 3000000.0 | 0.0 | 3000000.0 | 3000000.0 | 0.0 | 0.0 | 1197045.3 | 371305.5 | 10197045.3 | 0.0364 |
| 10 | November | 3000000.0 | 0.0 | 0.0 | 1182065.2 | 0.0 | 0.0 | 3000000.0 | 0.0 | 0.0 | 3000000.0 | 441464.7 | 10182065.2 | 0.0434 |
| 11 | December | 3000000.0 | 2108392.7 | 0.0 | 0.0 | 3000000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 2108392.7 | 432346.2 | 10216785.4 | 0.0423 |

*Table 6: Monthly budget for allocation for Method 2*

## Our recommendation:

- Reinvesting half of the monthly ROI yields a 4% boost in the annual return. However, this also raises an intriguing prospect; if the company can find alternative avenues where reinvesting half of the monthly ROI leads to an annual return exceeding 4%, it should prioritize exploring those options.
- Conversely, if the company's strategic objective is to expand its advertising budget, compounding the return should take precedence.

## Stability Analysis

Our firm has a stable budget policy that categorizes an advertising platform as ‘Unstable’ if the monthly change in allocation of any of the advertising medium is greater than $1 million.

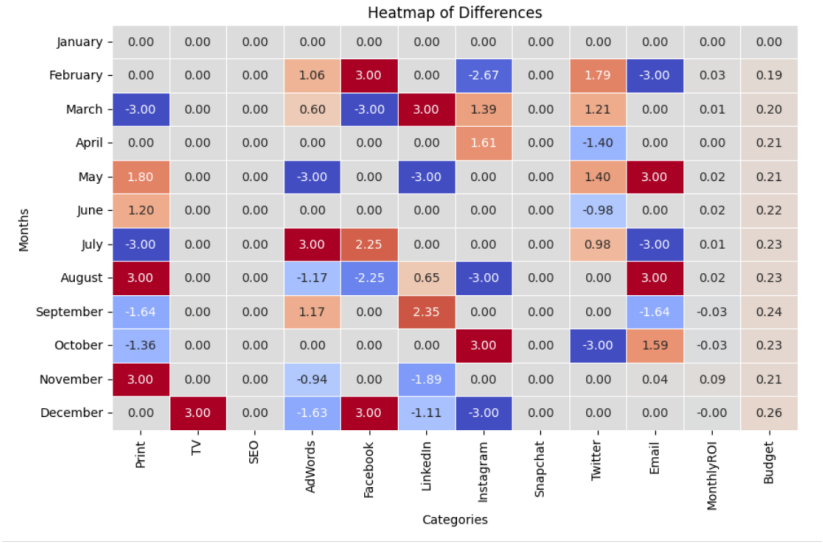


*Figure 4: Heat map representing the change in monthly allocation for each advertising medium*

It can be observed that each of the months have at least one advertising platform that sees a $1 million or higher change in allocation.

| | Period | Budget Stablility |
|---|---|---|
| 0 | January | - |
| 1 | February | Unstable |
| 2 | March | Unstable |
| 3 | April | Unstable |
| 4 | May | Unstable |
| 5 | June | Unstable |
| 6 | July | Unstable |
| 7 | August | Unstable |
| 8 | September | Unstable |
| 9 | October | Unstable |
| 10 | November | Unstable |
| 11 | December | Unstable |

*Table 7: Budget Stability*

To ensure that we can maintain a stable budget, we can add additional constraints to the model that ensures that the absolute difference between the budget for each platform is no more than $1 million. For this, we could have one variable defined for each of the decision variables which tracks the current value of the respective decision variable. When is enters the next loop we can include the below constraint:

$$x_{i\,prev} - 1 \leq x_i \leq x_{i\,prev} + 1$$

## Concluding Insights and Key Takeaways

We have developed a versatile Gurobi optimization function, which we invoke consistently whenever we seek to address our business optimization challenges. Instead of creating new instances for each specific problem, we leverage this reusable function as a standardized approach. The following were some of our key takeaways:

- We conducted a thorough analysis of ROI recommendations from two consulting firms for each advertising platform and determined that both yield an equivalent net ROI, though with distinct allocation strategies.
- A $3 million cap on each individual platform allocation serves as a crucial mechanism to diversify the company's advertising platforms, paving the way for long-term growth opportunities.
- The company has the flexibility to decide whether to reinvest half of its monthly returns into the advertising budget or allocate it to other avenues, depending on its specific business needs and strategic objectives.