Time series Analysis & Modeling

LAB # 7 Autoregressive & Moving Average Model

DATS 6450

Using the Python program and appropriate libraries perform the following tasks:

1- Let consider an AR(2) process as

$$y(t) - 0.5y(t-1) - 0.2y(t-2) = e(t)$$

Where $e(t)$ is a WN (0,1).

   a. Find the theoretical mean and variance of y(t). (no need to use python).
   b. Using python, create a for loop that simulates above process for 1000 samples. Assume all initial conditions to be zero.
   c. Using the generated samples in part b and numpy package, find the experimental mean and variance. Compare your answer with part a. Write down your observations.
   d. Plot the y(t) with respect to number of samples.
   e. Using the python code, developed in previous labs, calculate autocorrelations for 20 lags and plot them versus number of lags. Write down your observation about the ACF of above process.
   f. Display the first 5 values of y(t) at the console.
   g. Apply the ADF-test and check if this is a stationary process. Explain your answer.

2- Using the "scipy" python package and "dlsim" command, simulate the AR(2) process in question 1.

   a. Display the first 5 values of y(t) at the console.
   b. Show that your answer to the previous part is identical to the answer in part d of previous question.

3- Write the AR(2) process in question 1, as multiple regression model and using the least square estimate (LSE), estimate the true parameters a_1 and a_2 (-0.5 and -0.2). Display the estimated parameters values at the console. What is the effect of additional samples on the accuracy of the estimate? Justify the answer by running your code for 5000 and 10000 data samples.

4- Generalized your code in the previous question, such when the code runs it asks a user the following questions:

   a. *Enter number of samples:*
   b. *Enter the mean of white noise:*
   c. *Enter the variance of white noise:*
   d. *Enter the order # of the AR process:*
   e. *Enter the corresponding parameters of AR process:*

Your code should simulate the AR process based on the entered information (a, b, c) and estimate the AR parameters accordingly. The estimated parameters must be close to the entered numbers in part c. Display the estimated parameters and the true values at the console.

f. Increase the number of samples to 5000 and display the estimated parameters.
g. Increase the number of samples to 10000 and display the estimated parameters.
h. Write down your observation on the effect of the additional samples on the accuracy of the estimation.

5- Let consider an MA(2) process as

$$y(t) = e(t) + 0.5e(t-1) + 0.2e(t-2)$$

Where $e(t)$ is a WN (1,2).

a. Find the theoretical mean and variance of y(t). (no need to use python).
b. Using python, create a for loop simulate above process for 1000 samples. Assume all initial conditions to be zero.
c. Plot the y(t) with respect to number of samples.
d. Using the python code, developed in previous labs, calculate autocorrelations for 20 lags and plot them versus number of lags. Write down your observation about the ACF of above process.
e. Increase the data samples to 10000 and then 100000. Observe the ACF. Write down your observation. There is a difference between the ACF of an AR process and MA process. What is the main difference?
f. Display the first 5 values of y(t) at the console.
g. Apply the ADF-test and check if this is a stationary process. Explain your answer.

6- Using the "scipy" python package and "dlsim" command, simulate the MA(2) process in question 5.

a. Display the first 5 values of y(t) at the console.
b. Show that your answer to the previous part is identical to the answer in part f of the previous question.

Upload the formal **report (as a single pdf**) plus **the .py file** through BB by the due date.