Quiz 9 of Selected Topics of Mathematical Statistics Seminar

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Quiz

Let $X_n \xrightarrow{L} X$ and *and* $Y_n \xrightarrow{P} c$ where c is a finite constant . Then it holds :

- $X_n + Y_n \xrightarrow{L} X + c$
- $X_n Y_n \xrightarrow{L} Xc$
- $X_n / Y_n \xrightarrow{L} X/c$ if c $\neq 0$

Illustrate this theorem with some examples

Example 1

Let $X_n \sim B(n, \frac{\lambda}{n})$. Then it holds $\lim\limits_{n \to +\infty} X_n \xrightarrow{L} X$ where $X \sim P(\lambda)$

Proof :

$$P(X_n = x) = \binom{n}{x} (\tfrac{\lambda}{n})^x (1 - \tfrac{\lambda}{n})^{n-x}$$

$$= \frac{n(n-1)....(n-x+1)}{x!} (\tfrac{\lambda}{n})^x \frac{(1-\frac{\lambda}{n})^n}{(1-\frac{\lambda}{n})^x}$$

$$= \frac{\lambda^x}{x!}(1 - \tfrac{1}{n})...(1 - \tfrac{x-1}{n}) \frac{(1-\frac{\lambda}{n})^n}{(1-\frac{\lambda}{n})^x}$$

$$\lim_{n \to +\infty} P(X_n = x) = \frac{\lambda^x}{x!} \lim_{n \to +\infty} (1 - \tfrac{\lambda}{n})^n = \frac{\lambda^x}{x!} e^{-\lambda}$$

Example 1

Let $X_n \sim B(n, \frac{\lambda}{n})$ and Y_n a sequence of iid RV with $E(Y_n) = \mu$ with $\mu \neq 0$ and finite variance .

$X_n \xrightarrow{L} X$ where $X \sim P(\lambda)$
$\overline{Y_n} \xrightarrow{P} \mu$ using the weak law of large numbers

According to Slutsky's theorem we have

- $X_n + \overline{Y_n} \xrightarrow{L} X + \mu$
- $X_n \overline{Y_n} \xrightarrow{L} X\mu$
- $X_n / \overline{Y_n} \xrightarrow{L} X/\mu$

Example 2

Let X defined by P(X=0) = P(X=1) =$\frac{1}{2}$ and $X_n = 1 - X$. Let Y_n a sequence of iid RV with $E(Y_n) = \mu$ with $\mu \neq 0$ and finite variance

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- $1 - X$ has the same distribution has X then $X_n \xrightarrow{L} X$
- $|X_n - X| = 1$, $P(|X_n - X| > \frac{1}{2}) = 1$ then $X_n \nrightarrow^{P} X$
- $\overline{Y_n} \xrightarrow{P} \mu$ using the weak law of large numbers

According to Slutsky's theorem we have

- $X_n + \overline{Y_n} \xrightarrow{L} X + 1$
- $X_n \overline{Y_n} \xrightarrow{L} X * 1$
- $X_n / \overline{Y_n} \xrightarrow{L} X / 1$

Example 3

Let $X_1, X_2, ...X_n$ a sequence of i.i.d RV with $E(X_i) = \mu, V(X_i) = \sigma^2 < \infty$ where μ and σ are given

Let $U_n = \sqrt{n}(\frac{\overline{X_n} - \mu}{s_n})$, $C_n = \sqrt{n}(\frac{\overline{X_n} - \mu}{\sigma})$, $D_n = \frac{\sigma}{s_n}$

$s_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X_n})^2$

We have $U_n = C_n D_n$ and since $C_n \xrightarrow{L} \mathcal{N}(0, 1)$ according to Central Limit Theorem and $D_n \xrightarrow{P} 1$ according to weak law of large numbers

According to Slutsky's theorem we have $U_n \xrightarrow{L} \mathcal{N}(0, 1)$