A Distributed Feedback-based Online Process Optimization Framework for Optimal Resource Sharing

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Abstract

Distributed real-time optimization (RTO) enables optimal operation of large-scale process systems with common resources shared across several clusters. Typically in distributed RTO, the different subsystems are optimized locally, and a centralized master problem is used to coordinate the different subsystems in order to reach system-wide optimal operation. This is especially beneficial in industrial symbiosis, where only limited information can be shared between the different clusters. However, one of the main challenges with this approach is the need to solve numerical optimization problems online for each subsystem. With the recent surge of interest in feedback optimizing control, where the optimization problem is converted into a feedback control problem, this paper proposes a distributed feedback-based RTO (DFRTO) framework for optimal resource sharing in an industrial symbiotic setting. In this approach, a master coordinator updates the shadow price for the shared resource, and the different subsystems locally optimize their operation using feedback control for the given shadow price. The proposed framework is shown to converge to a stationary point of the system-wide optimization problem, and is demonstrated using an industrial symbiotic offshore oil and gas production system with shared resources.

Keywords: Distributed RTO, Feedback-based RTO, Industrial symbiosis

1. Introduction

In the face of growing competition, stringent emission regulations, and increased necessity for sustainable manufacturing, there is a clear need to focus on energy and resource efficiency in order to reduce waste. In the process and manufacturing industries, there is an increasing trend of *industrial symbiosis*, where different organizations come together in an industrial cluster/eco-park, and share resources and equipment in a mutually beneficial manner.

As the process industry is embracing industrial symbiosis, this creates new challenges. Finding a feasible and optimal operation for a large-scale system is challenging and typically requires information about the entire process, in the form of models, real time measurements, local constraints and the economic objective. This challenge is only amplified in an industrial symbiotic setting with shared resources, since the different companies might be reluctant to share information across the different organizations, for example due to intellectual property rights, trade secrets, and market competitiveness.

One potential solution that facilitates industrial symbiosis is the distributed optimization framework, where the different subsystems are locally modeled and optimized and a centralized master problem coordinates the subproblems. This addresses privacy and data sharing issues, since only limited information is shared between the different subsystems [1]. There are different strategies that can be used to decompose a large-scale problem into several smaller subproblems, which can be broadly categorized into primal decomposition and dual decomposition [2].

In primal decomposition, the different subproblems report the price they are willing to pay for the shared resource, and the master coordinator directly allocates the shared resource accordingly. However, this approach may require the subsystems to share additional knowledge about the local constraints to the master coordinator in order to ensure that the allocated resource is feasible for the subproblems. Therefore, this approach may not be suitable for industrial symbiosis [1].

Dual decomposition, also known as Lagrangian decomposition, on the other hand is a price-based coordination, where the master coordinator sets the price of the shared resource, which regulates the local decision making in each subsystem. Unlike primal decomposition, this approach does not require information about the local constraints to be shared with the master coordinator, which makes it a favorable approach for industrial symbiosis. Both the primal and dual decomposition strategies involve iteratively solving the subproblems and the master coordinator,

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where each subproblem solves a numerical optimization problem at each iteration.

Decomposition strategies are a popular area of research, in the context of both model predictive control (MPC) as well as real-time optimization (RTO). In this paper, we focus on steady-state real time optimization, and the reader is simply referred to [3] for a comprehensive compilation of literature on distributed MPC. Research on distributed RTO for large-scale process systems has been gaining increasing interest, with some notable works such as [4, 5, 6, 7, 1, 8] to name a few. However, the use of distributed RTO methods in practice remains rather low, if not nonexistent [1]. The main reasons for this is attributed to the computational cost of solving the numerical optimization problems online and the slow convergence rate.

Currently, there is active research to improve the rate of convergence, such that the master and subproblems converge to a feasible optimal solution in a small number of iterations/communication rounds. Some notable works in this direction include fast ADMM [9], Newton-based methods (ALADIN) [10], and quadratic approximations [1] to name a few.

Despite the algorithmic developments to improve the convergence rate, the subproblems still need to solve numerical optimization problems online at each iteration, which is a more fundamental limiting factor for practical implementation of real time optimization due to computational and numerical robustness issues [11, 12]. In addition to the computational cost of solving numerical optimization problems online, the lack of technical expertise and competence to implement and maintain such numerical optimization-based RTO is one of the major impeding factors for practical application in many industries. The expected benefits of optimization are at risk without regular maintenance and monitoring [13], which requires expert knowledge. As pointed out by the authors in [14], the performance degradation due to lack of maintenance and support often leads to the application being turned off by the operator. For this reason, many traditional process industries still prefer to optimize their operations using simple feedback control tools [15]. Therefore, there is a need to develop a distributed framework with limited information exchange and at the same time without the need to solve numerical optimization problems online. This would enable industrial symbiosis even in the case where some participating organizations prefer to use only feedback control.

Recently, there has been a surge of interest in achieving optimal operation using feedback control. This is often referred to as "*feedback optimizing control*" [16, 17] or "*direct input adaptation*" [18], where the aim is to translate the economic objectives into control objectives, thereby achieving optimal process operation by directly manipulating the input using feedback control. The concept of feedback optimizing control dates back to the 1980s [16] motivated by the industrial and academic gap. Some recent survey articles such as [18, 19, 20, 21, 22] provides a good overview of the different feedback-based RTO methods that have been developed across several research groups since then. In addition to process control, feedback-based optimization is also gaining popularity in other application domains such as power systems, see for example [23, 24].

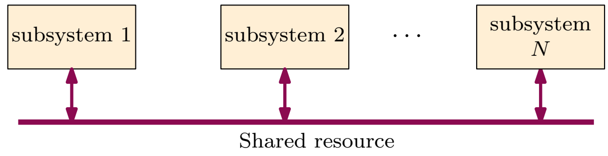


Figure 1: Schematic representation of a large-scale process system with N clusters coupled by the shared resource.

Feedback optimizing control, in general, is more suited for unit operations, or for small-scale processes. In large-scale systems, it becomes easier to design feedback optimizing control for small subgroups of processes. This leads to a decentralized control structure, where some clusters of operating units are optimized locally without any coordination. However, when the system is coupled in one form or the other, system-wide optimal operation does not result from the aggregates of individual operating units in a decentralized fashion. This was also discussed in detail in the same paper that introduced the concept of feedback optimizing control [16], where the authors quantified the relative loss due to insufficient or lack of coordination. Therefore, there is also a need for a distributed feedback optimizing control structure with a master coordinator, in order to broaden the utilization of feedback optimizing control for large-scale process systems.

To this end, the two main motivations to develop a distributed feedback-based RTO framework are 1) to enable industrial symbiosis without the need to solve numerical optimization problems online, and 2) to broaden the applicability of feedback optimizing control to large-scale systems with complex interconnections. This paper aims to address this gap and formulates a distributed feedback-based RTO (DFRTO) framework for large-scale systems with shared resources as shown in Fig. 1. In particular, this paper formulates, "*what to control*" such that the overall economic objectives are translated into control objectives for each subproblem. When used along with a centralized price-based coordination mechanism, this ensures that the closed-loop trajectories of the different subsystems converge to a stationary solution of the system-wide optimization problem, which is often also optimal in many applications.

The main contributions of this paper are, a distributed feedback-based RTO (DFRTO) framework for optimal resource sharing and convergence analysis of the proposed DFRTO scheme.

The reminder of the paper is organized as follows. The proposed method is described in Section 2 and the convergence analysis is shown in Section 3. A subsea oil and gas production optimization problem involving two companies in an industrial symbiotic setting with shared resource is used as a case study to demonstrate the effectiveness of the proposed approach in Section 4. Section 5 provides useful discussions and future research directions before concluding the paper in Section 6.

2. Proposed method

2.1. Problem formulation

Consider an optimal resource sharing problem written in the generic form

$$\min_{x_1,\ldots,x_N} \sum_{i=1}^{N} f_i(x_i) + f_s\left(\sum_{i=1}^{N} A_i x_i\right) \tag{1}$$

where $x_i \in \mathcal{X}_i \subseteq \mathbb{R}^{n_i}$ denotes the decision variables (i.e. the degrees of freedom) for the i^{th} subsystem, $f_i : \mathbb{R}^{n_i} \to \mathbb{R}$ denotes the local objective of the i^{th} subsystem, and $f_s : \mathbb{R}^m \to \mathbb{R}$ is the shared objective that couples the different subsystems with $A_i \in \mathbb{R}^{m\times n_i}$ and m is the number of shared resource constraints.

This can equivalently be written by introducing an additional variable $x_0 \in \mathbb{R}^m$

$$\min_{x_0,x_1,\ldots,x_N} \sum_{i=1}^{N} f_i(x_i) + f_s(x_0) \tag{2a}$$

$$\text{s.t.} \sum_{i=1}^{N} A_i x_i = x_0 \tag{2b}$$

which can be further condensed as

$$\min_{x_0,x_1,\ldots,x_N} \sum_{i=0}^{N} f_i(x_i) \tag{3a}$$

$$\text{s.t.} \sum_{i=0}^{N} A_i x_i = 0 \tag{3b}$$

where $f_0 = f_s$ and $A_0 = -I_m$.

Remark 1. Note that the shared resource may either be consumed or produced by the different subsystems. $x_i > 0$ implies that the shared resource is being consumed by subsystem i, and $x_i < 0$ implies that the shared resource is produced by subsystem i.

Assumption 1. *$f_i(\cdot)$ is smooth, but may be nonconvex, $\mathcal{X}_i$ is a closed convex set, and A_i has full rank.*

The Lagrangian of (3) is given by,

$$\mathcal{L}(x_0,\ldots,x_N,\lambda) = \sum_{i=0}^{N} f_i(x_i) + \lambda^\mathsf{T} \sum_{i=0}^{N} A_i x_i \tag{4}$$

where $\lambda \in \mathbb{R}^m$ is the Lagrange multiplier of the coupling constraint.

Defining $\mathbf{x} := \{x_0,\ldots,x_N\}$, the necessary conditions of optimality for this problem can be stated as

$$\nabla_\mathbf{x}\mathcal{L}(\mathbf{x},\lambda) = \sum_{i=0}^{N} \nabla_{x_i} f(x_i) + \sum_{i=0}^{N} A_i^\mathsf{T}\lambda = 0 \tag{5a}$$

$$\sum_{i=0}^{N} A_i x_i = 0 \tag{5b}$$

and a point $(\mathbf{x}^*, \lambda^*)$ that satisfies (5) is known as a KKT point, or a stationary point.

For the KKT point to be a local optimum, we further require that the Hessian of the Lagrangian $\boldsymbol{H}(\mathbf{x},\lambda)$ is positive definite, that is, $\mathbf{d}^\mathsf{T}\boldsymbol{H}(\mathbf{x},\lambda)\mathbf{d} > 0$ holds for all $\mathbf{d} \neq 0$ such that $\mathbf{A}^\mathsf{T}\mathbf{d} = 0$, where $\mathbf{A} := [A_0, A_1 \ldots, A_n]^\mathsf{T}$. If this is true, then strong second order sufficient conditions (SSOSC) is said to hold at the KKT point $(\mathbf{x}^*, \lambda^*)$.

The objective here is to drive the process to a KKT point of (3) in a distributed fashion with limited information exchange using only simple feedback controllers, such as PID control. To do this, we first decouple the subproblems and then identify self-optimizing controlled variables for each subsystem.

Notice that the cost (3a) is additively separable, but the shared resource constraint (3b) couples the different subproblems together. We see that the Lagrangian (4) is additively separable.

$$\mathcal{L}_i(x_i,\lambda) = f_i(x_i) + \lambda^\mathsf{T} A_i x_i \tag{6}$$

We can therefore decompose the problem using the Lagrangian decomposition framework by relaxing the coupling constraints [25]. This is known as dual decomposition or Lagrangian decomposition. In the standard distributed RTO framework, the different subproblems solve the unconstrained optimization problem

$$x_i^*(\lambda) = \arg\min_{x_i\in\mathcal{X}_i} \mathcal{L}_i(x_i,\lambda) \tag{7}$$

for a given λ, and the master coordinator updates λ, typically using the dual ascent step,

$$\lambda^+ = \lambda + \alpha \sum_{i=0}^{N} A_i x_i^* \tag{8}$$

where $\alpha = diag(\alpha_1,\ldots,\alpha_m)$ with $\alpha > 0$ is the step-size. Traditionally, the subproblems (7) and the master problem (8) are iteratively solved until convergence.

The Lagrangian decomposition framework has an economic interpretation, where the Lagrange multiplier λ is the shadow price of the shared resource. Here the goal of the master coordinator is to find an equilibrium price for the shared resource such that the supply matches the demand in the micro market. In other words, when the supply of the shared resource increases, the master coordinator decreases the price λ in order to encourage consumption by the subproblems. Similarly, if the demand for the shared resource increases, the master coordinator increases the price λ to find the equilibrium price. Such problems have been studied extensively in general equilibrium theory, economics, resource allocation, optimal exchange etc. [25, 26], and have also been studied in the context of process systems engineering (PSE), see for example [4, 5, 1] and the references therein.

However, in this paper, we do not want to explicitly solve (7), instead we want to translate the unconstrained optimization problem (7) into a feedback control problem. In other words, *the objective is to find a self-optimizing controlled variable for each subproblem as a function of the shadow price $c_i(\lambda)$, which when kept at a constant setpoint c_i^{sp} leads to optimal operation of the local subsystem, and when the master coordinator updates the shadow price, leads to system-wide optimal operation*[1].

Remark 2. Note that the dual ascent step (8) in the master coordinator can be seen as a simple integral controller that drives the coupling constraint $\sum_{i=0}^{N} A_i x_i$ to zero, that is the price λ is updated if the supply does not match the demand.

The ideal self-optimizing variable is the steady-state cost gradient which must be driven to a constant setpoint of zero, thereby satisfying the necessary condition of optimality (5). NCO-tracking control [27], extremum seeking control [28, 29], Feedback RTO [30], hill-climbing control [31] etc. are some of the feedback optimizing control approaches in the RTO literature that use the steady-state cost gradient as the self-optimizing variable.

Therefore, for each subproblem (7), the self-optimizing variable $c_i(\lambda) \in \mathbb{R}^{n_i}$ can be expressed as

$$c_i(\lambda) := \nabla_{x_i} \mathcal{L}_i(\lambda) = \nabla_{x_i} f(x_i) + A_i^\mathsf{T} \lambda \tag{9}$$

which must be driven to a constant setpoint of $c_i^{sp} = 0$. Note that the controlled variable is now a function of the shadow price λ, which is updated by the master coordinator using (8), just as in the traditional distributed RTO scheme. Controlling $c_i(\lambda)$ requires the online estimation of the local cost gradient $\nabla_{x_i} f(x_i)$, which can be done using any suitable model-based or model-free gradient estimation scheme, see for example [22] and the references therein.

If the objective function is nonconvex, then controlling (9) may lead to some convergence issues. In order to make the dual decomposition approach robust and yield convergence, it is common in the distributed optimization framework to use the *augmented Lagrangian* function instead. Similarly, we can also consider the augmented Lagrangian in the distributed feedback-based RTO framework to ensure the convergence properties even in the case where $f_i(x_i)$ is nonconvex.

[1]Note that for the sake of exposition, this is stated assuming that the stationary point is also the optimum point here.

The augmented Lagrangian of (3) can be expressed as

$$\mathcal{L}_\rho(\mathbf{x}, \lambda) = \sum_{i=0}^{N} f_i(x_i) + \lambda^\mathsf{T} \sum_{i=0}^{N} A_i x_i + \frac{\rho}{2} \left\| \sum_{i=0}^{N} A_i x_i \right\|^2 \tag{10}$$

where an additional regularization term is added to the Lagrangian (4). Clearly, any stationary point of the augmented Lagrangian (10) is also a stationary point of the Lagrangian (4). The different subproblems can be expressed as an unconstrained optimization problem for a given λ,

$$x_i^*(\lambda) = \arg \min_{x_i \in \mathcal{X}_i} \mathcal{L}_{\rho,i}(x_i, \lambda) \tag{11}$$

where

$$\mathcal{L}_{\rho,i}(x_i, \lambda) = f_i(x_i) + \lambda^\mathsf{T} A_i x_i + \frac{\rho}{2} \left\| \sum_{i=0}^{N} A_i x_i \right\|^2 \tag{12}$$

In the traditional distributed RTO, this is solved using the alternating directions method of multipliers (ADMM), where the i^{th} subproblem is solved by fixing x_j for all $j \neq i$, similar to one pass of a Gauss-Seidel method [25]. In this paper, instead of solving this problem using ADMM, we convert it into a feedback control problem by controlling the steady-state gradient of the augmented Lagrangian $\nabla_{x_i} \mathcal{L}_{\rho,i}(\lambda)$ to a constant setpoint of zero. In this case, the self-optimizing variable $c_i(\lambda) \in \mathbb{R}^{n_i}$ for each subsystem is expressed as,

$$c_i(\lambda) := \nabla_{x_i} f(x_i) + A_i^\mathsf{T} \lambda + \rho A_i^\mathsf{T} \underbrace{\sum_{i=0}^{N} A_i x_i}_{=r} \tag{13}$$

where $r =: \sum_{i=0}^{N} A_i x_i$ is the residual that represents the total surplus or shortage of the shared resource (i.e supply/demand). The master coordinator updates the shadow price λ using the dual ascent step (8) with $\alpha = \rho$. Note that since the residual r is a real-time measurement, the feedback-based approach does not need to be solved in an alternating directions fashion.

Remark 3. Compared to the self-optimizing controlled variable in (9), we now need r in addition. Note that we do not need to share information regarding the individual contribution/consumption by each subsystem, but we only need the overall residual r.

For a convex problem, the controlled variables (9) and (13) converges to the same stationary point, since at the optimum $r = 0$ (thanks to the integral action in the master coordinator) and (7) and (11) are equivalent. One of the main motivation for using (13) instead of (9) is to ensure convergence properties if the objective function is nonconvex, which will be shown later in Section 3.

Apart from the advantages of feedback optimizing control noted in Section 1, it also has other advantages. For example, the DFRTO approach can be implemented at

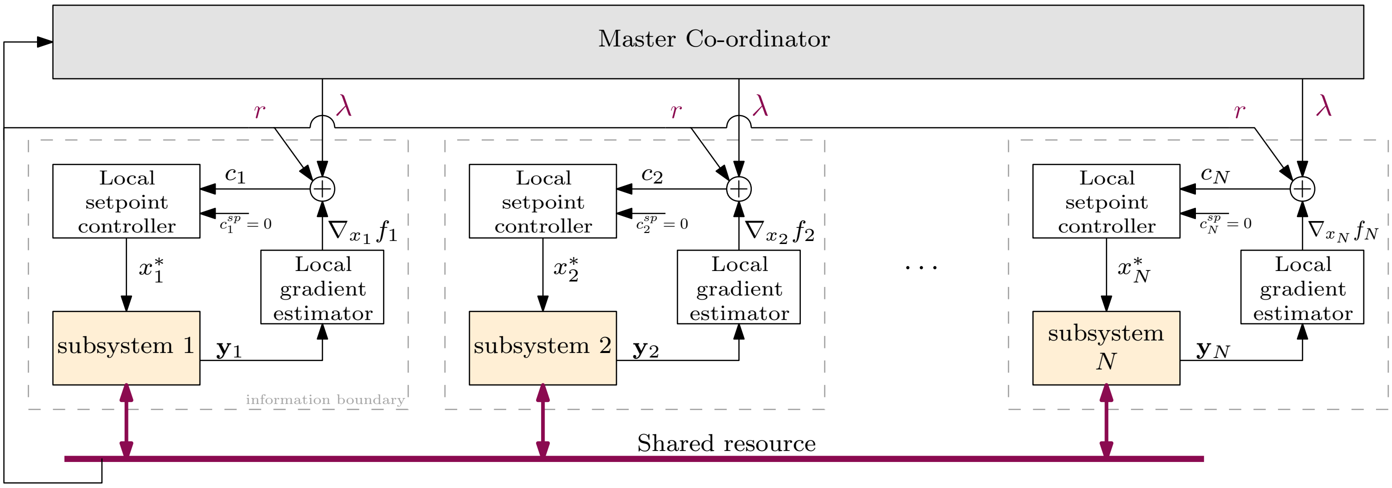


Figure 2: Schematic representation of the proposed Distributed feedback based RTO framework. Anything inside the information boundary (depicted using gray dashed lines) is contained within the subsystem. The residual r and the shadow price λ are the only variables that are shared across the different subsystems.

higher sampling rates than the traditional distributed RTO framework, since we do not need to solve numerical optimization problems online. In addition, the proposed DFRTO approach does not need to wait for the process to reach steady-state before re-optimizing, thus alleviating the steady-state wait-time issue associated with the traditional RTO framework [11, 8, 12].

Remark 4. The sampling time of the local controllers for the different subsystems may be chosen independently. The sampling time of the master coordinator (denoted by $t, t+1, \dots$) may either be the same as the local controllers, or slower.

Remark 5. In the case where $x_i \in \mathcal{X}_i$ becomes optimally active, then the feedback control problem for the subsystem simply becomes an active constraint control problem [17].

2.2. Distributed feedback-based RTO (DFRTO)

We now formulate the distributed feedback-based RTO framework, which is schematically shown in Fig. 2. The three main components of the DFRTO framework are :

1. For each subsystem $i = 0, \dots, N$, estimate $\nabla_{x_i} f_i$ using the local real time measurements [2].
2. For each subsystem $i = 0, \dots, N$, control

$$c_i(\lambda[t]) = \nabla_{x_i} f_i + A_i^\mathsf{T}\lambda[t] + \rho A_i^\mathsf{T} \underbrace{\sum_{i=0}^{N} A_i x_i}_{=r}$$

   for a given shadow price $\lambda[t]$ to a constant setpoint of $c_i^{sp} = 0$ using simple feedback controllers.

3. At every sample time of the master coordinator $t+1$, gather the residual $r[t+1] = \sum_{i=0}^{N} A_i x_i[t+1]$ and update the shadow price

$$\lambda[t+1] = \lambda[t] + \rho \sum_{i=0}^{N} A_i x_i[t+1] \tag{14}$$

   in the centralized master coordinator, and broadcast $\lambda[t+1]$ to the subsystems.

The shadow price $\lambda[t]$ and the residual r are the only information that are shared among the different subsystems. This is also clearly shown in Fig. 2, with the *information boundary* for each subsystem shown in gray dashed lines, and only λ and r crosses the information boundary of each subsystem.

The traditional distributed optimization framework typically requires several iterations between the subproblems and the master coordinator to converge to a KKT point. However, in the proposed distributed feedback-based RTO, we do not iterate between the master and subproblem, since $x_i[t]$ is a real time measurement, and not the solution to a numerical optimization problem. Therefore the "iteration" is done in real-time, and as time $t \to \infty$, $x_i[t]$ computed by the different controllers converges to a KKT-point of the original problem. This can be seen as the traditional distributed RTO with a single iteration between the master and subproblems with warm-starting at every time step.

3. Convergence analysis

In this section we analyze the convergence properties of the proposed distributed feedback-based RTO scheme for an optimization problem of the form (3), where $f_i(x_i)$ is possibly nonconvex, but smooth. We show that by using the proposed self-optimizing controlled variable with the

[2]Direct measurements of the local cost f_i is required when using model-free gradient estimation methods [22]

penalty parameter ρ chosen sufficiently large, the system converges to a feasible set of stationary solutions. Here we use the augmented Lagrangian (10) as the merit function and show that it monotonically decreases over time using the proposed framework. To show convergence of the proposed method, we follow a similar framework as in [32], where the augmented Lagrangian was used as the merit function to guide convergence of nonconvex ADMM problems.

Assumption 2 (Perfect control). *In each subsystem, we have perfect control such that $c_i(\lambda[t]) = c_i^{sp}$ for all i at each sampling time of the master coordinator. Furthermore we assume that there is no communication delay for the globally shared variables r and λ.*

Assumption 3 (Lipschitz continuous gradient). *The shared cost $f_0(\cdot)$ is smooth nonconvex, and has a Lipschitz continuous gradient with a positive constant $L_0 > 0$, i.e.*

$$\|\nabla f_0(a) - \nabla f_0(b)\| \leq L_0 \|a - b\|$$

Definition 1 (Strong convexity). *For $a, b \in \mathbb{R}^n$, any function $h : \mathbb{R}^n \to \mathbb{R}$ is said to be γ-strongly convex if*

$$h(a) - h(b) \leq \nabla h(a)^\mathsf{T}(a - b) - \frac{\gamma}{2}\|a - b\|^2$$

Assumption 4. *The penalty parameter ρ in the self-optimizing variable (13) is chosen sufficiently large such that*

(i) *The augmented Lagrangian (10) is γ-strongly convex in the sense of Definition 1*
(ii) $\rho\gamma > 2L_0^2$
(iii) $\rho > L_0$

Lemma 1 (Successive boundedness of the shadow price). *Suppose Assumptions 2 and 3 hold, then the following inequalities hold*

$$\|\lambda[t+1] - \lambda[t]\|^2 \leq L_0^2 \|x_0[t+1] - x_0[t]\|^2 \tag{15}$$

Proof. Assuming perfect control, at time $t+1$, the controlled variables for $i = 0$ is given by

$$\nabla_{x_0} f_0(x_0[t+1]) + A_0^\mathsf{T}\lambda[t] + \rho A_0^\mathsf{T}\sum_{i=0}^{N} A_i x_i[t+1] = 0$$

From the master update step (14) at time $t+1$, we have with $A_0 = -I_m$

$$\begin{aligned} \nabla_{x_0} f_0(x_0[t+1]) + A_0^\mathsf{T}\lambda[t+1] &= 0 \\ \nabla_{x_0} f_0(x_0[t+1]) - \lambda[t+1] &= 0 \\ \Rightarrow \lambda[t+1] &= \nabla_{x_0} f_0(x_0[t+1]) \end{aligned} \tag{16}$$

From Assumption 3, we have

$$\begin{aligned} \|\lambda[t+1] - \lambda[t]\| &= \|\nabla_{x_0} f_0(x_0[t+1]) - \nabla_{x_0} f_0(x_0[t])\| \\ &\leq L_0 \|x_0[t+1] - x_0[t]\| \end{aligned}$$

from which (15) follows. □

The following lemma bounds the successive difference of the overall unconstrained cost (10).

Lemma 2 (Successive boundedness of the augmented Lagrangian). *Given Assumptions 2, 3 and 4, the following holds for the distributed feedback-based RTO*

$$\begin{aligned} &\mathcal{L}_\rho(\mathbf{x}[t+1], \lambda[t+1]) - \mathcal{L}_\rho(\mathbf{x}[t], \lambda[t]) \\ &\leq \left(\frac{L_0^2}{\rho} - \frac{\gamma}{2}\right)\|x_0[t+1] - x_0[t]\|^2 \\ &\qquad - \sum_{i=1}^{N}\frac{\gamma}{2}\|x_i[t+1] - x_i[t]\|^2 \end{aligned} \tag{17}$$

Proof. We split the L.H.S. into two parts,

$$\begin{aligned} &\mathcal{L}_\rho(\mathbf{x}[t+1], \lambda[t+1]) - \mathcal{L}_\rho(\mathbf{x}[t], \lambda[t]) \\ &= \underbrace{\mathcal{L}_\rho(\mathbf{x}[t+1], \lambda[t+1]) - \mathcal{L}_\rho(\mathbf{x}[t+1], \lambda[t])}_{=\mathcal{A}} \\ &\quad + \underbrace{\mathcal{L}_\rho(\mathbf{x}[t+1], \lambda[t]) - \mathcal{L}_\rho(\mathbf{x}[t], \lambda[t])}_{=\mathcal{B}} \end{aligned}$$

We start by bounding $\mathcal{A}$

$$\begin{aligned} \mathcal{A} = &\sum_{i=0}^{N} f_i(x_i[t+1]) + \lambda^\mathsf{T}[t+1]\sum_{i=0}^{N} A_i x_i[t+1] \\ &+ \frac{\rho}{2}\left\|\sum_{i=0}^{N} A_i x_i[t+1]\right\|^2 \\ &- \sum_{i=0}^{N} f_i(x_i[t+1]) - \lambda^\mathsf{T}[t]\sum_{i=0}^{N} A_i x_i[t+1] \\ &- \frac{\rho}{2}\left\|\sum_{i=0}^{N} A_i x_i[t+1]\right\|^2 \\ = &(\lambda[t+1] - \lambda[t])\left(\sum_{i=0}^{N} A_i x_i[t+1]\right) \end{aligned}$$

From the master update step (14),

$$\mathcal{A} = \frac{1}{\rho}\|\lambda[t+1] - \lambda[t]\|^2$$

Now we consider $\mathcal{B}$. Using Assumption 4.i and the definition of strong convexity,

$$\begin{aligned} \mathcal{B} &= \mathcal{L}_\rho(\mathbf{x}[t+1], \lambda[t]) - \mathcal{L}_\rho(\mathbf{x}[t], \lambda[t]) \\ &\leq \sum_{i=0}^{N} \nabla_{x_i}\mathcal{L}_{\rho,i}(x_i[t+1])(x_i[t+1] - x_i[t]) \\ &\qquad - \sum_{i=0}^{N}\frac{\gamma}{2}\|x_i[t+1] - x_i[t]\|^2 \\ &\leq -\sum_{i=0}^{N}\frac{\gamma}{2}\|x_i[t+1] - x_i[t]\|^2 \end{aligned}$$

where the last inequality comes from Assumption 2. Adding $\mathcal{A}$ and $\mathcal{B}$,

$$\begin{aligned}\mathcal{A}+\mathcal{B} &\leq \frac{1}{\rho}\left\|\lambda[t+1]-\lambda[t]\right\|^2 - \sum_{i=0}^{N}\frac{\gamma}{2}\|x_i[t+1]-x_i[t]\|^2 \\ &\leq \frac{L_0^2}{\rho}\left\|x_0[t+1]-x_0[t]\right\|^2 - \sum_{i=0}^{N}\frac{\gamma}{2}\|x_i[t+1]-x_i[t]\|^2 \\ &\leq \left(\frac{L_0^2}{\rho}-\frac{\gamma}{2}\right)\left\|x_0[t+1]-x_0[t]\right\|^2 - \sum_{i=1}^{N}\frac{\gamma}{2}\|x_i[t+1]-x_i[t]\|^2\end{aligned}$$

where the inequality in the second line comes from Lemma 1. This proves (17). □

Lemma 2 implies that if $2L_0^2 \leq \rho\gamma$ (i.e. Assumption 4.ii holds), then the unconstrained system-wide cost function (10) will monotonically decrease since the R.H.S of (17) is always negative. Since $2L_0^2$ is a constant, one can easily find ρ such that Assumption 4.ii holds as long as $\gamma > 0$.

We now have to show that the unconstrained system-wide cost function (10) is also convergent in addition to being monotonically decreasing.

Lemma 3. *Consider the same setup as in Lemma 2, and further assume that $\sum_{i=0}^{N} f_i(x_i)$ is lower bounded, then the following limit exists and is also lower bounded*

$$\lim_{t\to\infty}\mathcal{L}_\rho(\mathbf{x}[t+1],\lambda[t+1])$$

Proof. From (16), we can write

$$\begin{aligned}\mathcal{L}_\rho(\mathbf{x},\lambda) &= \sum_{i=0}^{N} f_i(x_i[t+1]) + \lambda^{\mathsf{T}}[t+1]\sum_{i=0}^{N}A_i x_i[t+1] \\ &\quad + \frac{\rho}{2}\left\|\sum_{i=0}^{N}A_i x_i[t+1]\right\|^2 \qquad (18) \\ &= \sum_{i=0}^{N} f_i(x_i[t+1]) + \nabla_{x_0} f_0(x_0[t+1])\left(\sum_{i=0}^{N}A_i x_i[t+1]\right) \\ &\quad + \frac{\rho}{2}\left\|\sum_{i=0}^{N}A_i x_i[t+1]\right\|^2 \qquad (19)\end{aligned}$$

From Assumption 3, we have [33]

$$\begin{aligned}&f_0(x_0[t+1]) + \nabla_{x_0} f_0(x_0[t+1])\left(\sum_{i=1}^{N}A_i x_i[t+1] - x_0[t+1]\right) \\ &\geq f_0\left(\sum_{i=1}^{N}A_i x_i[t+1]\right) - \frac{L_0}{2}\left\|\sum_{i=0}^{N}A_i x_i[t+1]\right\|^2 \qquad (20)\end{aligned}$$

Substituting (20) in (19) yields,

$$\begin{aligned}\mathcal{L}_\rho(\mathbf{x},\lambda) \geq &\sum_{i=1}^{N} f_i(x_i[t+1]) + f_0\left(\sum_{i=1}^{N}A_i x_i[t+1]\right) \\ &+ \frac{\rho-L_0}{2}\left\|\sum_{i=0}^{N}A_i x_i[t+1]\right\|^2 \qquad (21)\end{aligned}$$

Since $\rho > L_0$ (Assumption 4.iii) and $\sum_{i=0}^{N} f_i(x_i)$ is assumed to be lower bounded, (21) implies that $\mathcal{L}_\rho(\mathbf{x},\lambda)$ is also lower bounded. The lower bound on $\mathcal{L}_\rho(\mathbf{x},\lambda)$ obtained above along with the monotonicity of $\mathcal{L}_\rho(\mathbf{x},\lambda)$ from Lemma 2 implies convergence. □

With this we are now ready to state the main convergence result.

Theorem 1 (Convergence of DFRTO)**.** *Consider the distributed feedback-based RTO framework as described in Section 2.2. Given Assumptions 2 – 4, we have the following,*

(i) *Primal feasibility of the coupling constraint*

$$\lim_{t\to\infty}\left\|\sum_{i=0}^{N}A_i x_i[t+1]\right\| = 0 \qquad (22)$$

(ii) *Dual feasibility*

$$\nabla_{x_i} f(x_i^*) + \lambda^* = 0 \quad \forall i = 1,\ldots,N \qquad (23)$$

where $x_i^* = \lim_{t\to\infty} x_i[t]$ *and* $\lambda^* = \lim_{t\to\infty}\lambda[t]$.

Proof. From Lemma 2 and Assumption 4, the R.H.S of (17) ≤ 0. From Lemma 3, as $t \to \infty$, the L.H.S. of (17) $\to 0$. Therefore, we have

$$\lim_{t\to\infty}\|x_i[t+1]-x_i[t]\| = 0 \quad \forall i = 1,\ldots,N \qquad (24)$$

Using Lemma 1, this implies

$$\lim_{t\to\infty}\|\lambda[t+1]-\lambda[t]\| = 0 \qquad (25)$$

Therefore, from the master update step (14) we arrive at (22). This proves primal feasibility of the coupling constraint.

From (24) and (25), let

$$\begin{aligned}&\lim_{t\to\infty} x_i[t+1] = x_i[t] = x_i^* \quad \forall i = 0,\ldots,N \\ &\lim_{t\to\infty}\lambda[t+1] = \lambda[t] = \lambda^*\end{aligned}$$

Substituting this in (16) gives

$$\nabla_{x_i} f(x_i^*) + A_i^{\mathsf{T}}\lambda^* = 0 \quad \forall i = 0,\ldots,N$$

□

To summarize, we have shown that for optimal resource sharing problems with linear constraints of the form (3), convergence of the distributed feedback-based RTO framework to a feasible set of stationary solution can be achieved by choosing a sufficiently large penalty parameter ρ in the self-optimizing controlled variable (13) and the master coordinator (14). By doing so, the proposed DFRTO framework is guaranteed to drive the system to a stationary point, which is often also optimal in many applications.

Remark 6. Equations (22) and (23) imply that the proposed distributed feedback-based RTO scheme converges to a KKT point. If the Hessian of the Lagrangian is positive definite at every point in the feasible hyperplane described by the coupling constraint, then the KKT point is also the unique minimum, and in this case, the proposed approach converges to the system-wide optimum.

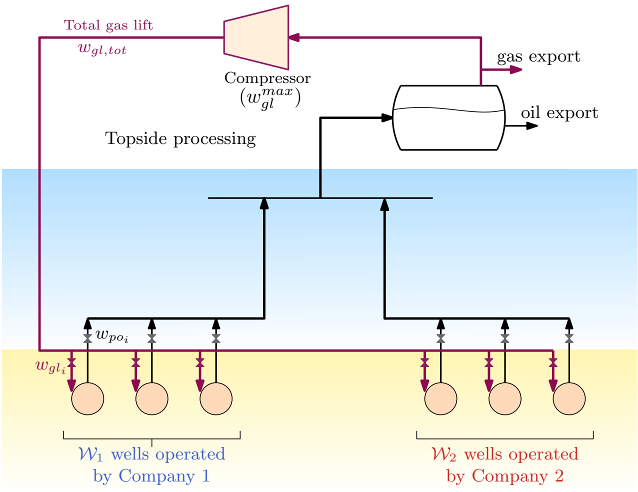


Figure 3: Schematic representation of an oil and gas production network with two subsea clusters operated by different companies, with a common processing facility. The gas-lift is a shared resource provided by the processing facility that must be optimally allocated between the two clusters.

4. Case Study: Optimal resource sharing in an oil production network

4.1. Problem formulation

As the era of easy oil is declining, offshore and subsea oil and gas production networks are becoming more complex. Often subsea wells producing from remote reservoir pockets are tied-back to an existing common processing facility since it may not be economically viable to construct dedicated processing facilities, especially for reservoirs with relatively low recoverable resources. It is not uncommon that wells producing from different reservoir sections are operated by different companies, but share a common processing facility. Resources are often limited in an offshore facility, and must be optimally allocated in order to maximize production. Distributed real time production optimization enables optimal resource sharing in such production networks[3] [8, 35].

However, the offshore production industry tends to prefer simple feedback control tools that can be implemented on the digital control system (DCS), for various reasons that are discussed at length in [36]. In such cases, the proposed distributed feedback-based RTO enables real-time production optimization of the production network using simple feedback controllers and at the same time with limited information sharing.

In this paper, we consider a subsea production network with $N = 2$ subsea clusters comprising of three gas-lifted wells each, making up a total of six gas-lifted wells. We assume that the two subsea clusters, denoted by the sets $\mathcal{W}_1$ and $\mathcal{W}_2$ respectively, are operated by two different companies that share a common processing facility, as shown in Fig. 3. Gas-lift is an artificial lift technology, where compressed gases are injected into the wells to increase production. In this case study, the lift gas which is a shared resource is compressed in the topside processing facility and must be optimally allocated between the two clusters.

The objective is to maximize the revenue from the oil production from each subsea cluster and minimize the costs associated with gas compression. Thus the system-wide optimization problem is stated as

$$\min \quad -\$_o \sum_{i\in\mathcal{W}_1} w_{po,i} - \$_o \sum_{i\in\mathcal{W}_2} w_{po,i} + \$_{gl} \sum_{i\in\mathcal{W}_1\cup\mathcal{W}_2} w_{gl,i} \tag{26a}$$

$$\text{s.t} \quad \sum_{i\in\mathcal{W}_1\cup\mathcal{W}_2} w_{gl,i} \leq w_{gl}^{max} \tag{26b}$$

where $\$_o$ is the oil price, $\$_{gl}$ is the cost of gas compression, $\sum_{j\in\mathcal{W}_1} w_{po,j}$ and $\sum_{j\in\mathcal{W}_2} w_{po,j}$ are the total oil produced by clusters 1 and 2 respectively, $w_{gl,tot} = \sum_{i\in\mathcal{W}_1\cup\mathcal{W}_2} w_{gl,i}$ is the total lift gas supplied by the compressor, which has a maximum capacity of w_{gl}^{max}. This problem can be written in the general form (3) as shown below.

Subsystem 1 - Wells operated by company 1: Company 1 operates three wells denoted by the set $\mathcal{W}_1 = \{1a, 1b, 1c\}$, and the local objective is to maximize the oil production from the three wells. Hence

$$x_1 = \begin{bmatrix} w_{gl,1a} & w_{gl,1b} & w_{gl,1c} \end{bmatrix}^\mathsf{T}$$
$$f_1 = -\$_o \sum_{i\in\mathcal{W}_1} w_{po,i}$$
$$A_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Subsystem 2 - Wells operated by company 2: Company 2 operates three wells denoted by the set $\mathcal{W}_2 = \{2a, 2b, 2c\}$, and the local objective is to maximize the oil production from the three wells. Hence

$$x_2 = \begin{bmatrix} w_{gl,2a} & w_{gl,2b} & w_{gl,2c} \end{bmatrix}^\mathsf{T}$$
$$f_2 = -\$_o \sum_{i\in\mathcal{W}_2} w_{po,i}$$
$$A_2 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Shared objective: The shared objective $f_s = f_0$ is to minimize the costs associated with gas compression in the topside processing facility. Hence

$$x_0 = w_{gl,tot}$$
$$f_0 = \$_{gl} w_{gl,tot}$$
$$A_0 = -1$$

In this work, we assume $\$_o = 1$ and $\$_{gl} = 0.25$.

4.2. Problem setup

We now solve this problem using the proposed distributed feedback-based RTO (DFRTO) scheme. The self-optimizing controlled variables for the different subsystems are given by:

[3] One such example is the Norne FPSO, where subsea wells producing from different reservoir sections operated by Equinor and Eni Norge in the Norwegian sea share the same processing facility [34].

- **Subsystem** $i = 0$

$$c_0(\lambda) = \$_{gl} - \lambda - \rho r$$

- **Subsystem** $i = 1$

$$c_1(\lambda) = \begin{bmatrix} \frac{\partial f_1}{\partial w_{gl,1a}} + \lambda + \rho r \\ \frac{\partial f_1}{\partial w_{gl,1b}} + \lambda + \rho r \\ \frac{\partial f_1}{\partial w_{gl,1c}} + \lambda + \rho r \end{bmatrix}$$

- **Subsystem** $i = 2$

$$c_2(\lambda) = \begin{bmatrix} \frac{\partial f_2}{\partial w_{gl,2a}} + \lambda + \rho r \\ \frac{\partial f_2}{\partial w_{gl,2b}} + \lambda + \rho r \\ \frac{\partial f_2}{\partial w_{gl,2c}} + \lambda + \rho r \end{bmatrix}$$

where λ is the shadow price of the lift gas, ρ is the penalty parameter that satisfies Assumption 4 and

$$r = -w_{gl,tot} + \sum_{i \in \mathcal{W}_1} w_{gl,i} + \sum_{i \in \mathcal{W}_2} w_{gl,i}$$

denotes the residual of the shared resource constraint.

In this case study, we choose to use a model-based gradient estimation scheme from [30] to estimate the steady-state cost gradient $\frac{\partial f_i}{\partial w_{gl,i}}$ for each subsystem. The gradient estimation scheme uses a nonlinear ODE model that models each subsystem individually. The use of the model-based gradient estimation scheme from [30] for gas-lifted wells has previously been demonstrated in [36] and [37]. Since the proposed DFRTO scheme is not contingent upon the particular gradient estimation scheme used here, the reader is simply referred to [30, 36, 37] for more detailed description of the gradient estimation scheme. The model equations and the model parameters for the six wells can be found in [36, Appendix A].

For each subsystem, we design three SISO PI controllers that control $c_1(\lambda)$ and $c_2(\lambda)$ to $c_1^{sp} = [0, 0, 0]^\mathsf{T}$ and $c_2^{sp} = [0, 0, 0]^\mathsf{T}$ respectively, making a total of six PI controllers. The controllers are tuned using the SIMC tuning rules as described in [36]. In this example, c_0 can be driven to its setpoint of zero by setting

$$x_0^* = \min \left[w_{gl}^{max}, \left(\frac{-\$_{gl} + \lambda}{\rho} \right) + \sum_{i \in \mathcal{W}_1} w_{gl,i} + \sum_{i \in \mathcal{W}_2} w_{gl,i} \right] \tag{27}$$

Note that a minimum selector is used in (27) to switch between the unconstrained optimum and the constraint w_{gl}^{max} [38]. The master coordinator updates the shadow price with a sampling time of 1s. The plant simulator is modeled as an Index-1 DAE model, that comprises of the entire system, which is simulated using the `IDAS` integrator [39]. The performance of the proposed DFRTO approach is benchmarked using the ideal steady-state optimum values computed by solving the system wide optimization problem (26) in a centralized manner using the `IPOPT` solver [40][4].

[4] The models were implemented in `MATLAB v.2019b` which can be found in the supplementary information or in https://github.com/dinesh-krishnamoorthy/Industrial-Symbiosis/tree/master/Feedback_DistRTO_AL

4.3. Simulation results

In this simulation, we test the performance of the distributed feedback-based RTO over a period of 12 hours. Disturbances enter the system in the form of maximum compressor capacity w_{gl}^{max} and the ratio of gas to oil entering the well from the reservoir (feed disturbance). The gas-oil-ratio (GOR) disturbance profile used in the simulation is shown in Fig. 4c, and the maximum compressor capacity w_{gl}^{max} is shown in the left subplot of Fig. 4b in black dotted lines.

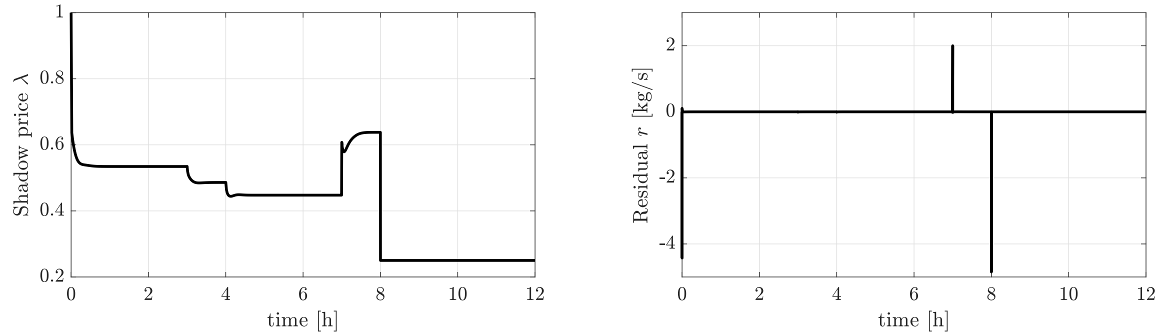
The shadow price λ and the residual r, that are shared across the different subsystems are shown in Fig. 4a. The residual r shown in the right subplot of Fig. 4a clearly indicates that the proposed DFRTO scheme attains primal feasibility of the coupling constraints.

The total gas lift rate consumed, and the total oil produced by the two clusters using the proposed method is shown along with the ideal steady-state optimum (gray dashed lines) in Fig. 4b, which indicates that the proposed method is able to drive the system to a stationary point (which is also the optimum point in this case study), in a distributed fashion without solving numerical optimization problems online.

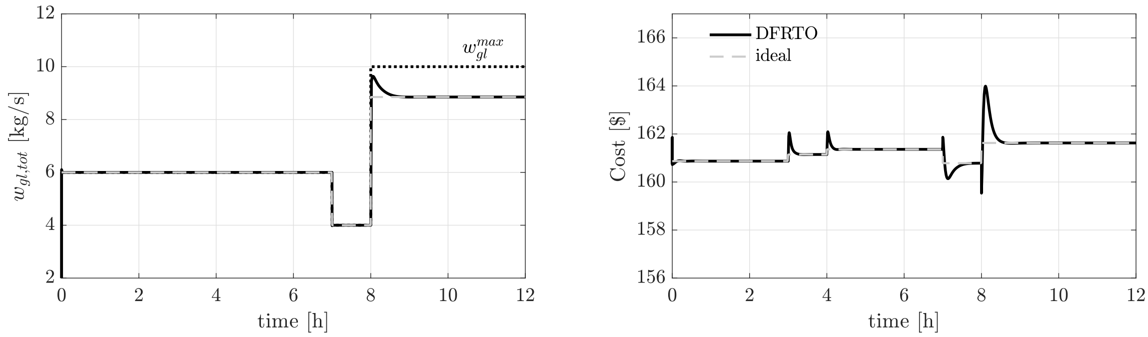
The controlled variables $c_1(\lambda)$ and $c_2(\lambda)$, along with the manipulated variables $w_{gl,i}$ for the different wells operated by company 1 and company 2 are shown in Fig. 5a and Fig. 5b respectively. To benchmark the performance of the proposed DFRTO approach, the ideal optimal gas lift rates for the different wells operated by company 1 and 2 are plotted in dashed lines in Fig. 5a and Fig. 5b respectively. This clearly shows that using the proposed DFRTO method x_i converges to same stationary solution as the overall optimization problem (26). The controlled variables $c_i(\lambda)$ shown on the left subplots in Fig. 5a and Fig. 5b indicates that the proposed DFRTO method is also able to attain dual feasibility. The simulation with noise can be found in the supplementary information.

As mentioned earlier, one of the main advantages of the proposed approach is that it does not need to solve the optimization problems online. In the example above, solving each subproblem in the traditional ADMM approach incurred a computation cost of 3 orders of magnitude[5] compared to using feedback controllers. Using ADMM also leads to solving the master and subproblems iteratively, further increasing the online computational cost. The proposed approach on the other hand does not need to iterate between the master problem and the subproblems, and does not need to solve numerical optimization problems online. The simulation results obtained using ADMM can be found in the supplementary information.

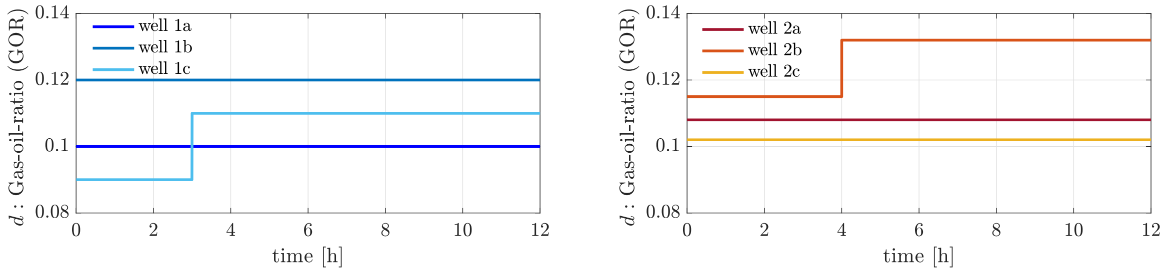
[5] $\sim$0.015s, as opposed to $\sim 0.15 \times 10^{-4}$s using a standard 2.6 GHz 16GB memory processor



(a) Public variables shared across the different subsystem.

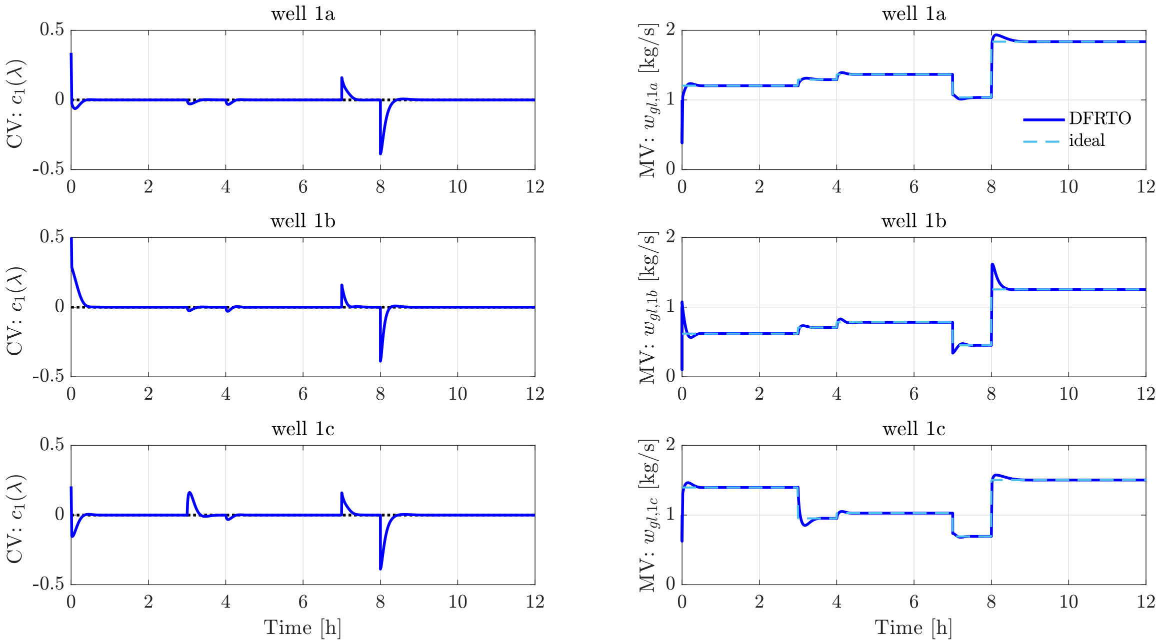


(b) System wide optimal operation

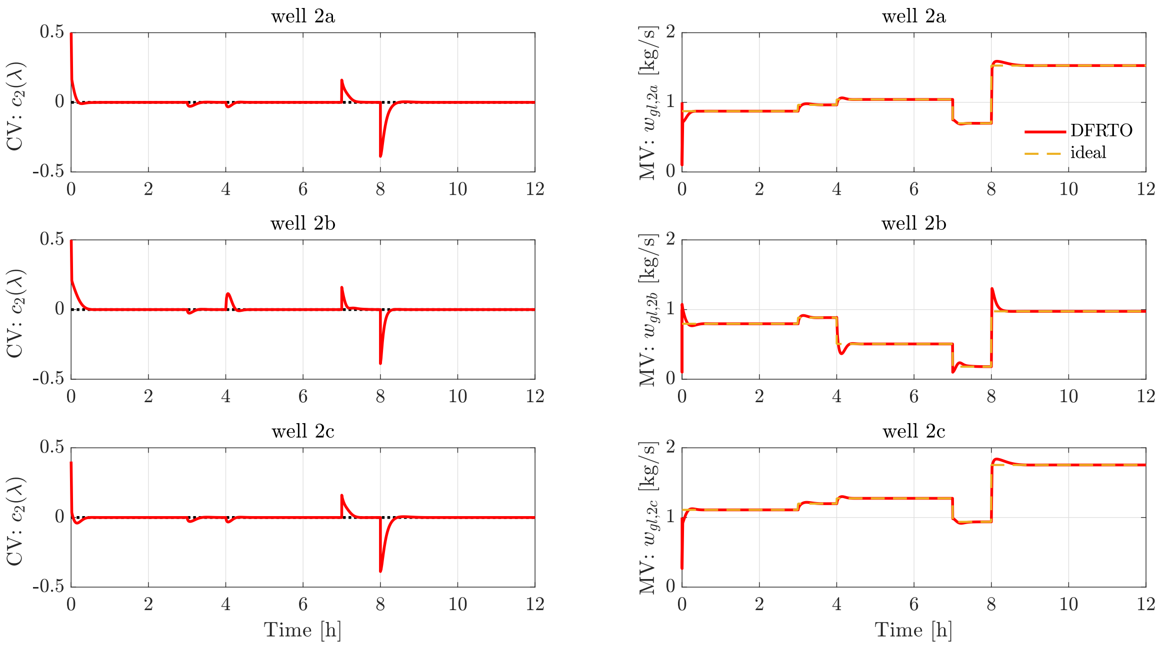


(c) Disturbance profile: Gas-oil-ratio (GOR) from the two reservoir sections

Figure 4: Simulation results of the proposed DFRTO framework. (a) The shadow price λ and the residual r that is shared across all the subsystems. (b) The total gas lift rate and the system wide optimal cost obtained using the DFRTO approach (shown in solid black) and the ideal steady-state optimum (shown in dashed gray). (c) The gas-oil-ratio (GOR) from the two reservoir sections, which acts as the feed disturbance



(a) Wells operated by company 1



(b) Wells operated by company 2

Figure 5: Controlled variables and manipulated variables for the different wells using the proposed DFRTO approach compared with the ideal steady-state optimum. (a) Wells operated by company 1 (shown in solid blue). The ideal steady-state optimal gas lift rate for each well is shown in light blue dashed lines. (b) Wells operated by company 2 (shown in solid red). The ideal steady-state optimal gas lift rate for each well is shown in yellow dashed lines.

5. Discussions

Section 2.2 presented a distributed feedback-based RTO framework with a centralized master coordinator, where the different subsystems use simple feedback control to drive the process to a stationary point of the original optimization problem (3), which was also shown using an oil production optimization case study in Section 4. The proposed approach is computationally fast, since this does not require the need to solve numerical optimization problems online. This also enables the feedback controllers to be implemented at higher sampling rates than traditional model-based RTO.

5.1. Constraint feasibility

We noted earlier that the proposed framework does not iterate between the master and the subproblems unlike the traditional distributed RTO framework, since this is done in real time. This implies that, like any feedback-based RTO method, the coupling constraint may not be feasible during the transients. This was also noted in Theorem 1, where primal feasibility is guaranteed only upon convergence. However, this is not an issue, since the focus here is steady-state real time optimization.

Even in the case of traditional distributed steady-state RTO, the optimal solution computed iteratively may be primal feasible, but this is only provided as *setpoints* to the lower level regulatory controllers. Consecutively, the actual closed-loop trajectory of the system itself may not be feasible during the transients until the process reaches steady-state.

5.2. Choice of self-optimizing controlled variables

In the proposed DFRTO framework, we considered the self-optimizing controlled variable (13). Using this controlled variable enables us to analyze under what conditions the proposed framework converges to a stationary point. Furthermore, the convergence analysis in Section 3 provide guidelines on choosing the penalty parameter ρ, which is also used as the step length in the dual ascent step in the master coordinator. Alternatively, one can also use the simpler self-optimizing variable obtained from the unaugmented Lagrangian (9). Although the convergence properties using (9) is not guaranteed, it may work well in practice.

We also considered the optimal sharing problem of the form (3), where the coupling constraints are linear. Although this may seem restrictive, a wide range of optimal resource sharing problem arising in the process industry can be expressed in this form. In the case of additively separable nonlinear coupling constraints of the form

$$\sum_{i=1}^{N} g_i(x_i) = 0$$

the self-optimizing controlled variable in the proposed DFRTO framework can be modified as

$$c_i(\lambda) = \nabla_{x_i} f_i(x_i) + \nabla_{x_i} g_i(x_i)(\lambda + \rho r)$$

where $r = \sum_i g_i(x_i)$. We now have to estimate the constraint gradient $\nabla_{x_i} g_i(x_i)$ in addition to the cost gradient.

In many industrial symbiosis systems, the different subsystems must agree upon a common variable, for example, the flow rate of a particular stream from one subsystem to another. This leads to a consensus problem, where each susbsystem has a local copy of the common variable, and the master coordinator ensures that the local copies of the common variable are equal to the optimum value. In the case of a consensus problem of the form,

$$\min_{x_0, x_1, \dots} \sum_i f_i(x_i) + f_s(x_0) \tag{28a}$$

$$\text{s.t.} \quad x_i = x_0 \quad \forall i \tag{28b}$$

the self-optimizing controlled variables can be given by driving the gradient of the augmented Lagrangian of each subproblem to a constant setpoint of zero, i.e.

$$c_i(\lambda_i) = \nabla_{x_i} f_i(x_i) + \lambda_i + \rho(x_i - x_0) \quad \forall i$$

The convergence analysis framework presented in Section 3 can also be used to provide convergence properties of the feedback-based consensus problem with suitable adjustments.

5.3. Plant-model mismatch

As mentioned earlier, any gradient estimation scheme may be used with the proposed DFRTO framework. In Section 4, we have used a model-based gradient estimation scheme [30] assuming no structural uncertainty. In the presence of structural mismatch, one can alternatively estimate the plant gradients directly from the cost measurement in a model-free fashion. However, the convergence to the stationary point is significantly slower when using a model-free gradient estimation scheme as opposed to model-based gradient estimation scheme, as noted in several works, see for example [22, 30, 19] and the references therein. In addition, it is important to understand the requirements and limitations of the different model-free gradient estimation algorithms, such as the need for direct cost measurements, persistence of excitation etc., see for example discussions in [22].

The proposed DFRTO framework also allows one to easily combine both model-based and model-free gradient estimation in different subsystems. For example, the subsystem 1 may use a model-based gradient estimation scheme, whereas subsystem 2 may use a model-free gradient estimation scheme. Moreover, each subproblem may also use a combination of model-based and model-free gradient estimation methods in a hierarchical framework as shown in Fig. 6 such that the model-based gradient estimation scheme enables fast convergence to the model optimum, where as the slow model-free gradient estimation

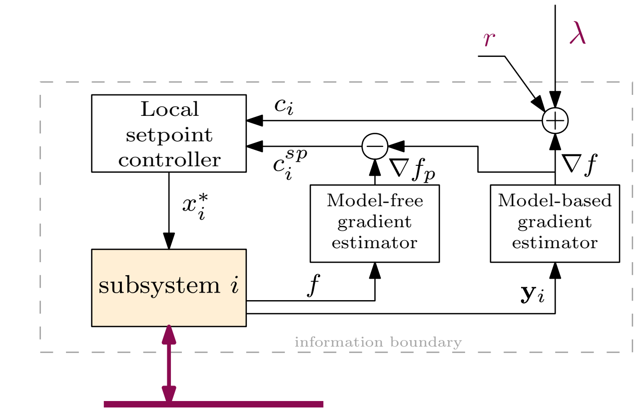


Figure 6: The proposed DFRTO framework using both model-based and model-free gradient estimation.

scheme adjusts the setpoint c_i^{sp} in order to drive the process to the plant optimum. In this case, instead of driving the controlled variable (13) to a constant setpoint of zero, the setpoint is now given by

$$c_i^{sp} = \nabla f_p - \nabla f$$

where ∇f_p is the plant gradient estimated directly from the cost measurement and ∇f is the model-gradient.

This is similar to the idea used in modifier adaptation (MA) scheme for RTO [41], where the term $(\nabla f_p - \nabla f)$ is the so-called *modifier*, and instead of using the modifier in the numerical optimization problem, it can be used to "modify" the setpoint used in the feedback controller as shown in Fig. 6.

5.4. Methodology agnostic approach

Perhaps most intriguingly, the proposed framework enables a methodology agnostic approach, where different RTO tools may be used by the different subsystems, truly enabling industrial symbiosis. For example, currently the distributed RTO framework requires that the all the subproblems are solved using the traditional model-based RTO approach. However, in an industrial symbiosis with different organizations, one organization may wish to use a traditional model-based RTO, whereas another organization may prefer to use simple feedback controllers, while another organization prefers to use a purely data-drive approach. Lack of consensus between the different organizations on the RTO tool impedes successful industrial symbiosis.

Since the centralized coordinator used in the proposed approach is the same as the one used in the traditional distributed RTO, this enables the use of traditional distributed RTO along with the feedback-based distributed RTO, such that some of the subproblems are solved numerically, while others using feedback control. Furthermore, as mentioned earlier, both model-based and model-free data-driven approaches can be used simultaneously with the proposed framework. This is a natural and interesting research direction that would enable co-ordination among the different subsystems without imposing strict requirements on the RTO methodology in order to establish a centralized coordinator.

6. Conclusion

This paper proposed a distributed feedback-based online process optimization framework for optimal resource sharing problems without the need to solve numerical optimization problems online. We proposed a local self-optimizing variable for each subsystem (13) expressed as a function of the shadow price, which can be controlled to a constant setpoint of zero using simple feedback controllers. As the centralized master coordinator (14) updates the shadow price to reach market equilibrium, this leads to a stationary point of the overall system. Theorem 1 showed that the proposed DFRTO framework is guaranteed to converge to a stationary point of the system, which is often also optimal in many applications. This proposed approach was demonstrated using a subsea oil and gas production optimization case example.

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