Program Structures and Algorithms
Spring 2023(SEC - 8)

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<u>TASK</u>:

1. Implementing height-weighted Quick Union with Path Compression algorithm and check that the unit tests for this class all work.
2. Generate random pairs of integers between 0 and n-1, calling connected() to determine if they are connected and union() if not. Loop until all sites are connected then print the number of connections generated.
3. Determine the relationship between the number of objects (n) and the number of pairs (m).

<u>RELATIONSHIP CONCLUSION</u>:

1. The value of ‘m’ represents the number of edges in a graph, while ‘n’ represents the number of vertices in the same graph.
2. The code keeps recursively going through all the nodes in the graph till all of them are not connected. If any two nodes are connected then that pair is ignored and a new set of random numbers is generated. This process keeps repeating till all nodes are connected to each other.
3. When the number of connected components reduces to 1, it means that all the vertices in the graph are connected, which is equivalent to having n - 1 edges in a tree.

Therefore the relationship between ‘m’ and ‘n’ can be described as follows:

$$m = n - 1$$

EVIDENCE TO SUPPORT THAT CONCLUSION

Source Code

UF_HWQUPC

```java
/**
 * Original code:
 * Copyright (c) 2000-2017, Robert Sedgewick and Kevin Wayne.
 * <p>
 * Modifications:
 * Copyright (c) 2017. Phasmid Software
 */
package edu.neu.coe.info6205.union_find;

import java.util.Arrays;

/**
 * Height-weighted Quick Union with Path Compression
 */
public class UF_HWQUPC implements UF {

    /**
     * Ensure that site p is connected to site q,
     *
     * @param p the integer representing one site
     * @param q the integer representing the other site
     */
    public void connect(int p, int q) {
        if (!isConnected(p, q)) union(p, q);
    }

    /**
     * Initializes an empty union-find data structure with {@code n} sites
     * {@code 0} through {@code n-1}. Each site is initially in its own
     * component.
     *
     * @param n               the number of sites
     * @param pathCompression whether to use path compression
     * @throws IllegalArgumentException if {@code n < 0}
     */
    public UF_HWQUPC(int n, boolean pathCompression) {
        count = n;
        parent = new int[n];
        height = new int[n];
        for (int i = 0; i < n; i++) {
            parent[i] = i;
            height[i] = 1;
        }
        this.pathCompression = pathCompression;
    }

    /**
     * Initializes an empty union-find data structure with {@code n} sites
     * {@code 0} through {@code n-1}. Each site is initially in its own
     * component.
     * This data structure uses path compression
     *
     * @param n the number of sites
     * @throws IllegalArgumentException if {@code n < 0}
     */
    public UF_HWQUPC(int n) {
        this(n, true);
    }

    public void show() {
        for (int i = 0; i < parent.length; i++) {
            System.out.printf("%d: %d, %d\n", i, parent[i], height[i]);
        }
    }

    /**
     * Returns the number of components.
     *
     * @return the number of components (between {@code 1} and {@code n})
     */
    public int components() {
        return count;
    }
    /**
     * Returns the component identifier for the component containing site
{@code p}.
     *
     * @param p the integer representing one site
     * @return the component identifier for the component containing site
```

```
{@code p}
     * @throws IllegalArgumentException unless {@code 0 <= p < n}
     */
    public int find(int p) {
        validate(p);
        int root = p;

        if (pathCompression) {
            doPathCompression(root);
        } else {
            while (root != parent[root]) {
                  root = parent[root];
            }
        }

        return parent[root];
    }
    /**
     * Returns true if the the two sites are in the same component.
     *
     * @param p the integer representing one site
     * @param q the integer representing the other site
     * @return {@code true} if the two sites {@code p} and {@code q} are in
the same component;
     * {@code false} otherwise
     * @throws IllegalArgumentException unless
     *                                 both {@code 0 <= p < n} and {@code
0 <= q < n}
     */
    public boolean connected(int p, int q) {
        return find(p) == find(q);
    }
    /**
     * Merges the component containing site {@code p} with the
     * the component containing site {@code q}.
     *
     * @param p the integer representing one site
     * @param q the integer representing the other site
     * @throws IllegalArgumentException unless
     *                                 both {@code 0 <= p < n} and {@code
0 <= q < n}
     */
    public void union(int p, int q) {
        // CONSIDER can we avoid doing find again?
        mergeComponents(find(p), find(q));
        count--;
    }
    @Override
    public int size() {
        return parent.length;
    }

    /**
     * Used only by testing code
     *
     * @param pathCompression true if you want path compression
     */
    public void setPathCompression(boolean pathCompression) {
        this.pathCompression = pathCompression;
    }
    @Override
    public String toString() {
        return "UF_HWQUPC:" + "\n  count: " + count +
                "\n  path compression? " + pathCompression +
                "\n  parents: " + Arrays.toString(parent) +
                "\n  heights: " + Arrays.toString(height);
    }

    // validate that p is a valid index
    private void validate(int p) {
        int n = parent.length;
        if (p < 0 || p >= n) {
            throw new IllegalArgumentException("index " + p + " is not
between 0 and " + (n - 1));
        }
    }

    private void updateParent(int p, int x) {
        parent[p] = x;
    }

    private void updateHeight(int p, int x) {
        height[p] += height[x];
    }
    /**
     * Used only by testing code
     *
     * @param i the component
     * @return the parent of the component
     */
    private int getParent(int i) {
        return parent[i];
    }
    private final int[] parent;   // parent[i] = parent of i
    private final int[] height;   // height[i] = height of subtree rooted
at i
    private int count;  // number of components
    private boolean pathCompression;

    private void mergeComponents(int i, int j) {
      int rootX = find(i);
      int rootY = find(j);

      if (height[rootX] < height[rootY]) {
            parent[rootX] = rootY;
            height[rootY] += height[rootX];
      } else if (height[rootX] > height[rootY]) {
            parent[rootY] = rootX;
            height[rootX] += height[rootY];
      } else {
            parent[rootY] = rootX;
            height[rootX]++;
      }
    }
    /**
     * This implements the single-pass path-halving mechanism of path
compression
     */
    private void doPathCompression(int i) {
       while (i != parent[i]) {
         parent[i] = parent[parent[i]];
         i = parent[i];
       }
    }
}
```

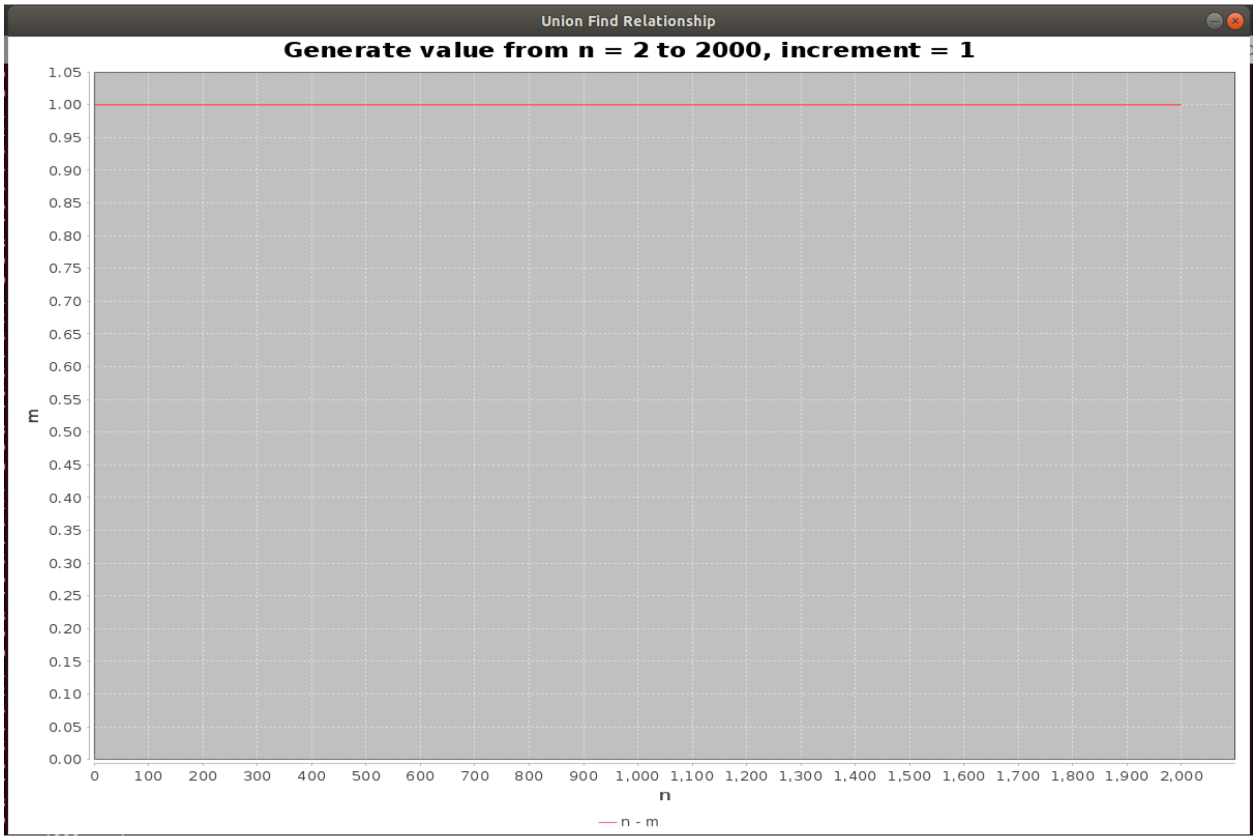
HWQUPC_Solution

```
package edu.neu.coe.info6205.union_find;

import java.util.ArrayList;
import java.util.List;
import java.util.Random;
import java.util.stream.Collectors;
import java.util.stream.IntStream;

import org.jfree.chart.ChartFactory;
import org.jfree.chart.ChartFrame;
import org.jfree.chart.JFreeChart;
import org.jfree.data.xy.XYSeries;
import org.jfree.data.xy.XYSeriesCollection;

public class HWQUPC_Solution {

	private String frameTitle;
	private String graphTitle;
	private String xAxis;
	private String yAxis;
	private int width;
	private int height;

	private List<Group> groups;

	public static class Group {
		private String name;
		private List<Pair> pairs;

		public Group(String name, List<Pair> pairs) {
			this.name = name;
			this.pairs = pairs;
		}

		public String getName() {
			return name;
		}
		public List<Pair> getPairs() {
			return pairs;
		}
	}

	public static class Pair {
		private double x;
		private double y;

		public Pair(double x, double y) {
			this.x = x;
			this.y = y;
		}

		public double getX() {
			return x;
		}
		public double getY() {
			return y;
		}
	}

	public HWQUPC_Solution(
			String frameTitle,
			String graphTitle,
			String xAxis,
			String yAxis,
			int widht,
			int height) {
		this.frameTitle = frameTitle;
		this.graphTitle = graphTitle;
		this.xAxis= xAxis;
		this.yAxis = yAxis;
		this.width = widht;
		this.height = height;

		this.groups = new ArrayList<>();
	}

	public void addGroup(Group group) {
		groups.add(group);
	}

	public void plot() {
```

```
            XYSeriesCollection collection = new XYSeriesCollection();

            for (Group group : groups) {
                  String name = group.getName();

                  XYSeries series = new XYSeries(name);

                  List<Pair> pairs = group.getPairs();
                  for (Pair pair : pairs) {
                        double x = pair.getX();
                        double y = pair.getY();

                        series.add(x, y);
                  }

                  collection.addSeries(series);
            }

            JFreeChart chart = ChartFactory.createXYLineChart(
                        graphTitle,
                        xAxis,
                        yAxis,
                        collection);

            ChartFrame chartFrame = new ChartFrame(frameTitle, chart);
            chartFrame.setVisible(true);
            chartFrame.setResizable(false);
            chartFrame.setSize(width, height);
      }

      public static void main(String[] args) {
            int min = 2, max = 2000, step = 1;

            List<Integer> inputs = IntStream.iterate(min, i -> i <= max, i
-> i + step)
                  .boxed()
                  .collect(Collectors.toList());

            List<Pair> diffPairs = new ArrayList<>(inputs.size());

            for (int input : inputs) {
                  int m = count(input);
                  int diff = input - m;

                  diffPairs.add(new Pair(m, diff));

                  System.out.println(input + " -> " + count(input) + " -> "
+ diff);
            }

            HWQUPC_Solution plotter = new HWQUPC_Solution(
                        "Union Find Relationship",
                        "Generate value from n = " + min + " to " + max +
", increment = " + step,
                        "n",
                        "m",
                        1200,
                        800);

            Group linerGroup = new Group("n - m", diffPairs);

            plotter.addGroup(linerGroup);

            plotter.plot();
      }
      public static int count(int n) {
        UF_HWQUPC uf = new UF_HWQUPC(n, true);
        Random random = new Random();
        int count = 0;
        while(uf.components() > 1) {
            int a = random.nextInt(n), b = random.nextInt(n);

            if(uf.connected(a, b)) {
                  continue;
            }

            uf.union(a, b);
            count++;
        }
        return count;
    }

}
```

GRAPHICAL REPRESENTATION

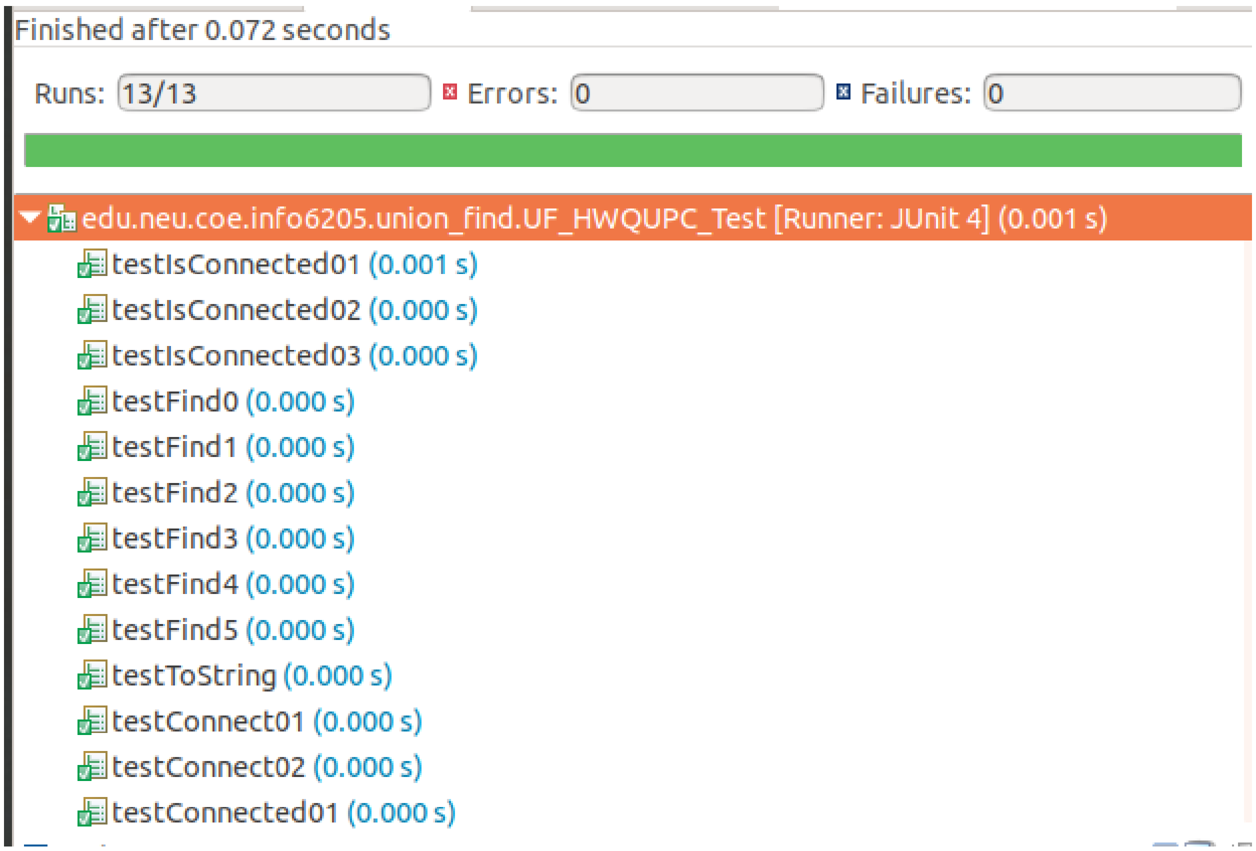
Following graph is observed when plotting the number of connections made (m) against the number of nodes in the graph (n). As it can be seen clearly that the difference between the two is constantly 1 across all values, therefore it can be concluded that it is a linear relationship.



UNIT TEST SCREENSHOTS

UF_HWQUPC_Test

Report



```
Finished after 0.072 seconds

Runs: 13/13    Errors: 0    Failures: 0

edu.neu.coe.info6205.union_find.UF_HWQUPC_Test [Runner: JUnit 4] (0.001 s)
    testIsConnected01 (0.001 s)
    testIsConnected02 (0.000 s)
    testIsConnected03 (0.000 s)
    testFind0 (0.000 s)
    testFind1 (0.000 s)
    testFind2 (0.000 s)
    testFind3 (0.000 s)
    testFind4 (0.000 s)
    testFind5 (0.000 s)
    testToString (0.000 s)
    testConnect01 (0.000 s)
    testConnect02 (0.000 s)
    testConnected01 (0.000 s)
```

Code

```
/*
 * Copyright (c) 2017. Phasmid Software
 */

package edu.neu.coe.info6205.union_find;

import edu.neu.coe.info6205.util.PrivateMethodTester;
import org.junit.Test;

import static org.junit.Assert.*;

public class UF_HWQUPC_Test {
```

```
@Test
public void testToString() {
    Connections h = new UF_HWQUPC(2);
    assertEquals("UF_HWQUPC:\n" +
            "  count: 2\n" +
            "  path compression? true\n" +
            "  parents: [0, 1]\n" +
            "  heights: [1, 1]", h.toString());
}

/**
 *
 */
@Test
public void testIsConnected01() {
    Connections h = new UF_HWQUPC(2);
    assertFalse(h.isConnected(0, 1));
}

/**
 *
 */
@Test(expected = IllegalArgumentException.class)
public void testIsConnected02() {
    Connections h = new UF_HWQUPC(1);
    assertTrue(h.isConnected(0, 1));
}

/**
 *
 */
@Test
public void testIsConnected03() {
    Connections h = new UF_HWQUPC(2);
    final PrivateMethodTester tester = new PrivateMethodTester(h);
    assertNull(tester.invokePrivate("updateParent", 0, 1));
    assertTrue(h.isConnected(0, 1));
}

/**
 *
```

```
     */
    @Test
    public void testConnect01() {
        Connections h = new UF_HWQUPC(2);
        h.connect(0, 1);
    }

    /**
     *
     */
    @Test
    public void testConnect02() {
        Connections h = new UF_HWQUPC(2);
        h.connect(0, 1);
        h.connect(0, 1);
        assertTrue(h.isConnected(0, 1));
    }

    /**
     *
     */
    @Test
    public void testFind0() {
        UF h = new UF_HWQUPC(1);
        assertEquals(0, h.find(0));
    }

    /**
     *
     */
    @Test
    public void testFind1() {
        UF h = new UF_HWQUPC(2);
        h.connect(0, 1);
        assertEquals(0, h.find(0));
        assertEquals(0, h.find(1));
    }

    /**
     *
     */
    @Test
    public void testFind2() {
        UF h = new UF_HWQUPC(3, false);
        h.connect(0, 1);
        assertEquals(0, h.find(0));
        assertEquals(0, h.find(1));
        h.connect(2, 1);
        assertEquals(0, h.find(0));
        assertEquals(0, h.find(1));
        assertEquals(0, h.find(2));
    }

    /**
     *
     */
    @Test
    public void testFind3() {
        UF h = new UF_HWQUPC(6, false);
        h.connect(0, 1);
        h.connect(0, 2);
        h.connect(3, 4);
        h.connect(3, 5);
        assertEquals(0, h.find(0));
        assertEquals(0, h.find(1));
        assertEquals(0, h.find(2));
        assertEquals(3, h.find(3));
        assertEquals(3, h.find(4));
        assertEquals(3, h.find(5));
        h.connect(0, 3);
        assertEquals(0, h.find(0));
        assertEquals(0, h.find(1));
        assertEquals(0, h.find(2));
        assertEquals(0, h.find(3));
        assertEquals(0, h.find(4));
        assertEquals(0, h.find(5));
        final PrivateMethodTester tester = new PrivateMethodTester(h);
        assertEquals(3, tester.invokePrivate("getParent", 4));
        assertEquals(3, tester.invokePrivate("getParent", 5));
    }

    /**
     *
     */
```

```java
    @Test
    public void testFind4() {
        UF h = new UF_HWQUPC(6);
        h.connect(0, 1);
        h.connect(0, 2);
        h.connect(3, 4);
        h.connect(3, 5);
        assertEquals(0, h.find(0));
        assertEquals(0, h.find(1));
        assertEquals(0, h.find(2));
        assertEquals(3, h.find(3));
        assertEquals(3, h.find(4));
        assertEquals(3, h.find(5));
        h.connect(0, 3);
        assertEquals(0, h.find(0));
        assertEquals(0, h.find(1));
        assertEquals(0, h.find(2));
        assertEquals(0, h.find(3));
        assertEquals(0, h.find(4));
        assertEquals(0, h.find(5));
        final PrivateMethodTester tester = new PrivateMethodTester(h);
        assertEquals(0, tester.invokePrivate("getParent", 4));
        assertEquals(0, tester.invokePrivate("getParent", 5));
    }
    /**
     *
     */
    @Test(expected = IllegalArgumentException.class)
    public void testFind5() {
        UF h = new UF_HWQUPC(1);
        h.find(1);
    }
    /**
     *
     */
    @Test
    public void testConnected01() {
        Connections h = new UF_HWQUPC(10);
//        h.show();
        assertFalse(h.isConnected(0, 1));
    }
}
```