

GATE 2021 1.CH

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Question: An ordinary differential equation Hence,

(ODE) $\frac{dy}{dx} = 2y$ with an initial condition $y(0) = 1$ has the analytical solution $y = e^{2x}$

Using Runge-Kutta second order method, numerically integrate the ODE to calculate y at $x = 0.5$ using a step size of $h = 0.5$. If the relative percentage error is defined as

$$\epsilon = \left| \frac{2.718 - 2.5}{2.718} \right| \times 100 \quad (11)$$

$$\epsilon = \frac{0.218}{2.718} \times 100 = 8\% \quad (12)$$

$$\epsilon = \left| \frac{y_{\text{analytical}} - y_{\text{numerical}}}{y_{\text{analytical}}} \right| \times 100$$

then the value of ϵ at $x = 0.5$ is

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Solution:

Analytical solution is given by:

$$y = e^{2x} \quad (1)$$

At $x = 0.5$, analytical solution is

$$y(0.5) = e^{2 \times 0.5} = 2.718 \quad (2)$$

According to question:

$$f(x, y) = 2y \quad (3)$$

By Runge-kutta 2nd order method,

$$K_1 = hf(x_o, y_o) = h(2y_o) \quad (4)$$

$$K_1 = 0.5(2 \times 1) = 1 \quad (5)$$

$$K_2 = h[f(x_o + h, y_o + K_1)] \quad (6)$$

$$K_2 = h[2(1 + 1)] \quad (7)$$

$$K_2 = 0.5 \times 4 = 2 \quad (8)$$

$$K = \frac{K_1 + K_2}{2} = \frac{1 + 2}{2} = 1.5 \quad (9)$$

$$y_1 = y_{\text{numerical}} = y_o + K = 1 + 1.5 = 2.5 \quad (10)$$