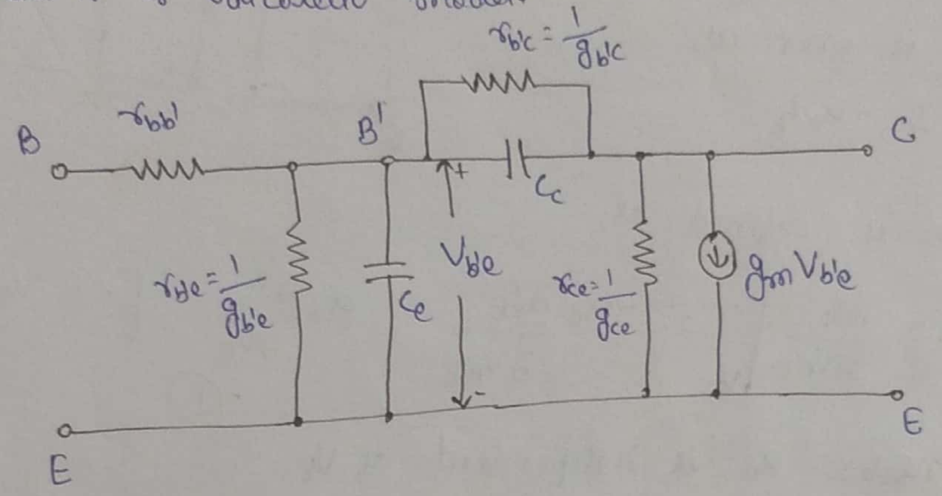
BJT at high frequencies & Multistage amplifiers

* Hybrid-π common-emitter Transistor Model:

The ckt. which will be valid at high frequencies are called hybrid-π or Giacoletto model.



The resistive components can be obtained from low frequency h-parameters. All parameters are assumed independent of frequency. It may vary over Q-pt.

The internal node B' is not physically present. Ohmic base-spreading resistance $r_{bb'}$ is lumped parameter.

The excess minority carrier injected in to base is proportional to $V_{b'e}$. So, collecter current sharted is proportional to $V_{b'e}$.

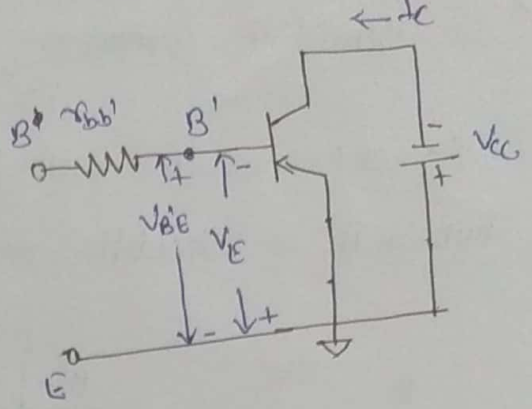
These carriers introduces $g_{b'e}$ & diffusion capacitance C_e. Further due to early effect base width modulation occur. The feedback effect is consider by $g_{b'c}$ & g_{ce}.

Barrier capacitance C_c is included. Sometimes overlap diode capacitance 'C' is used to connect b/w C & B' and C & B, in place of C_c.

* **Analysis at high frequencies :**

All resistive components in hybrid-π model can be obtained from h-parameters.

→ **Transconductance 'g_m' :**



In active region collector current is given as,

$$I_C = I_{C0} - \alpha_0 I_E$$

The 'g_m' is defined as,

$$g_m \approx \left.\frac{\partial I_C}{\partial V_{B'E}}\right|_{V_{CE}} = -\alpha_0 \frac{\partial I_E}{\partial V_{B'E}} = \alpha_0 \frac{\partial I_E}{\partial V_E} \quad [\because V_{BE} = -V_E] \quad -(1)$$

We consider α_0 is independent of V_E.

If the emitter diode resistance is 'r_e' then $r_e = \frac{\partial V_E}{\partial I_E}$

Hence $$\boxed{g_m = \frac{\alpha_0}{r_E}} \quad -(2)$$

The dynamic resistance of forward bias diode is given by V_T / I_E, where $V_T = KT/q$ [$\eta = 1$, since recombination current does not reach collector]

so, $$g_m = \frac{\alpha_0 \cdot I_E}{V_T} = \frac{I_{C0} - I_C}{V_T} \quad -(3)$$

For p.n.p transistor I_C is negative. & for n.p.n transistor I_C is positive. ($V_{B'E} = +V_E$). So, g_m is always positive. since $|I_C| \gg |I_{C0}|$

so, $$\boxed{g_m \approx \frac{|I_C|}{V_T}}$$, $V_T = T/11,600$. So at room tempt.

$$g_m = \frac{|I_C|\,mA}{26}.$$

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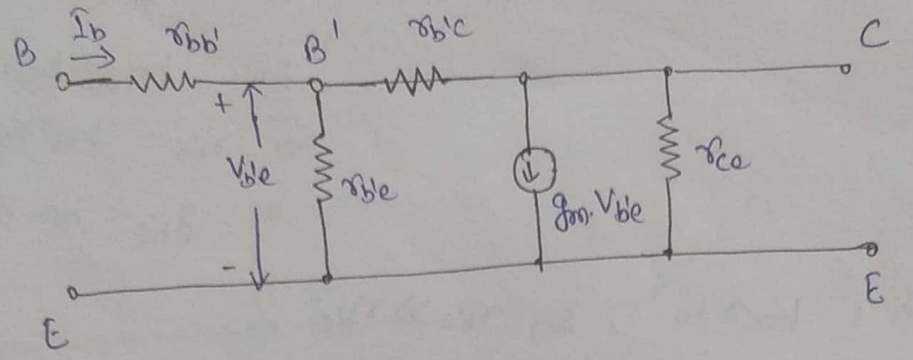
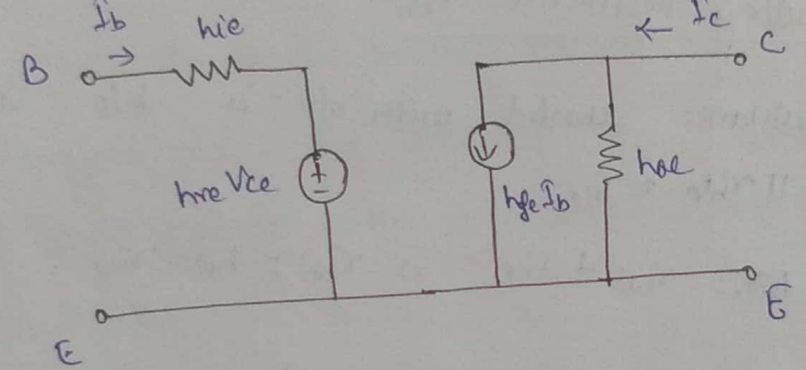
→ The input conductance '$g_{b'e}$' :

The hybrid-π model is valid at low frequencies, where all capacitances are negligible.





From the component values, we have $r_{b'c} >> r_{b'e}$. So, I_b flows into ~~…~~ $r_{b'e}$. Hence $V_{b'e} \approx I_b \cdot r_{b'e}$.

The short cut collector current is given by,

$$I_c \approx g_m \cdot V_{b'e} \approx g_m \cdot I_b \cdot r_{b'e}$$

The short ckt current gain 'h_{fe}' is defined by,

$$h_{fe} = \left.\frac{I_c}{I_b}\right|_{V_{CE}} \approx g_m \cdot r_{b'e} \quad \text{or,} \quad r_{b'e} = h_{fe}/g_m = \frac{h_{fe} V_T}{|I_C|}$$

$$\text{or,} \quad g_{b'e} = g_m / h_{fe}.$$

$$r_{b'e} \propto \text{Temperature}$$
$$\propto \frac{1}{\text{current}}$$

For some range h_{fe} remains constant.

→ Feedback conductance $g_{b'c}$:

With I/P open circuited, $I_b = 0$

So, $h_{re} = \dfrac{V_{b'e}}{V_{ce}} = \dfrac{r_{b'e}}{r_{b'e} + r_{b'c}}$ $\Rightarrow$ $h_{re}.r_{b'e} + h_{re}.r_{b'c} = r_{b'e}$

$\Rightarrow r_{b'e}(1 - h_{re}) = h_{re}.r_{b'c}$

Since $h_{re} << 1$

So, $r_{b'e} = h_{re}.r_{b'c}$

or $g_{b'c} = h_{re}.g_{b'e}$

Since $h_{re} \approx 10^{-4}$, So, $r_{b'c} >> r_{b'e}$

→ Base-Spreading Resistance '$r_{bb'}$':

The I/P resistance shorted with o/p is h_{ie}. Under this condition $r_{b'e} \parallel r_{b'c} \approx r_{b'e}$

So, $h_{ie} = r_{bb'} + r_{b'e}$ $\Rightarrow r_{bb'} = h_{ie} - r_{b'e}$.

The I/P impedance h_{ie} varies as,

$$h_{ie} = r_{bb'} + \frac{h_{fe}.V_T}{|I_c|} \approx \frac{h_{fe}.V_T}{|I_c|}$$

→ The output conductance 'g_{ce}':

With I/P open ckt, conductance is defined as h_{oe}.

For $I_b = 0$. We have,

$$I_c = \frac{V_{ce}}{r_{ce}} + \frac{V_{ce}}{r_{b'c} + r_{b'e}} + g_m V_{b'e}$$

with $I_b = 0$, we have $V_{b'e} = h_{re}.V_{ce}$, So,

$$h_{oe} = \frac{I_c}{V_{ce}} = \frac{1}{r_{ce}} + \frac{1}{r_{b'c}} + g_m h_{re}$$ $\left[\because g_m = g_{b'e}.h_{fe}\right)$

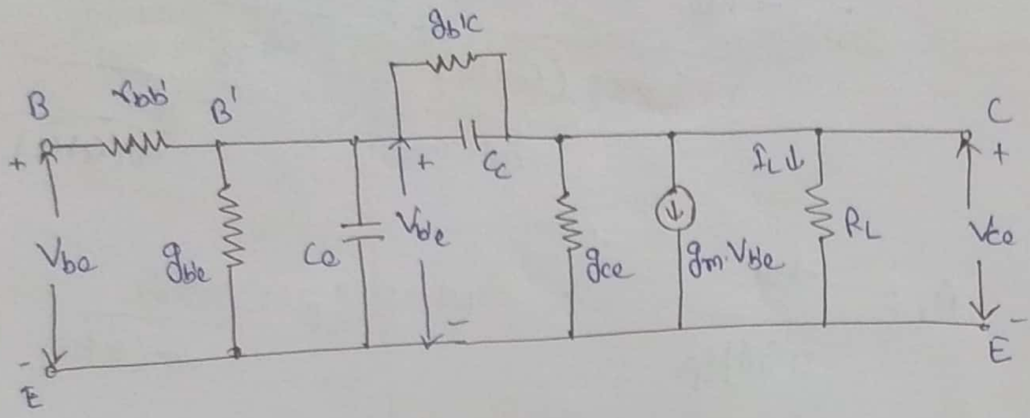
we have $r_{b'c} >> r_{b'e}$. So, $h_{oe} = g_{ce} + g_{b'c} + g_{b'e} h_{fe}.\dfrac{g_{b'c}}{g_{b'e}}$ $\left| h_{re} = \dfrac{g_{b'c}}{g_{b'e}}\right)$

or $g_{ce} = h_{oe} - (1 + h_{fe}) g_{b'c}$

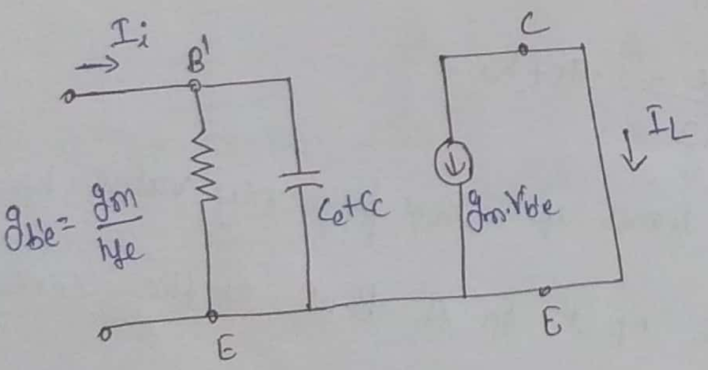
If $h_{fe} >> 1$, then $g_{ce} \approx h_{oe} - g_m h_{re}$.

* The CE short circuit current gain:

It is consider as single stage or last stage of a cascade. Here we assume $R_L = 0$.



A current source, produce I/P current of magnitude I_i & load current I_L. We neglect $g_{b'c}$, because $g_{b'c} << g_{b'e}$. g_{ce} is shunt, so it can disappear. So, equivalent circuit is given as,



The load current is given as $I_L = -g_m \cdot V_{b'e}$

where,
$$V_{b'e} = \frac{I_i}{g_{b'e} + j\omega(C_e + C_c)}$$

The current amplification under short cut condition is given as,

$$A_i = \frac{I_L}{I_i} = \frac{-g_m \cdot \cancel{V_{b'e}}}{\cancel{V_{b'e}}\{g_{b'e} + j\omega(C_e + C_c)\}} = \frac{-g_m}{g_{b'e} + j\omega(C_e + C_c)}$$

$$\Rightarrow A_i = \frac{-g_m}{g_{b'e}} \cdot \left[\frac{1}{1 + j\omega \frac{(C_c + C_e)}{g_{b'e}}} \right]$$

$$= \frac{-h_{fe}}{1 + j\,2\pi f \frac{(C_c + C_e)}{g_{b'e}}} = \frac{-h_{fe}}{1 + j\frac{f}{\frac{g_{b'e}}{2\pi(C_c+C_e)}}}$$

$$\Rightarrow A_i = \frac{-h_{fe}}{1 + j f/f_\beta} \quad ; \quad \text{where } f_\beta = \frac{g_{b'e}}{2\pi(C_c+C_e)} = \frac{g_m}{h_{fe} \cdot 2\pi(C_c+C_e)}$$

$$\Rightarrow |A_i| = \frac{h_{fe}}{\left[1 + (f/f_\beta)^2\right]^{1/2}}$$

At $f = f_\beta$

$$|A_i| = \frac{h_{fe}}{\sqrt{2}} = 0.707\, h_{fe}.$$

i.e. 0.707 times of low frequency value h_{fe}. The frequency range up to f_β is B.W. of the circuit.

The value of $A_i = -h_{fe}$ at $\omega = 0$.

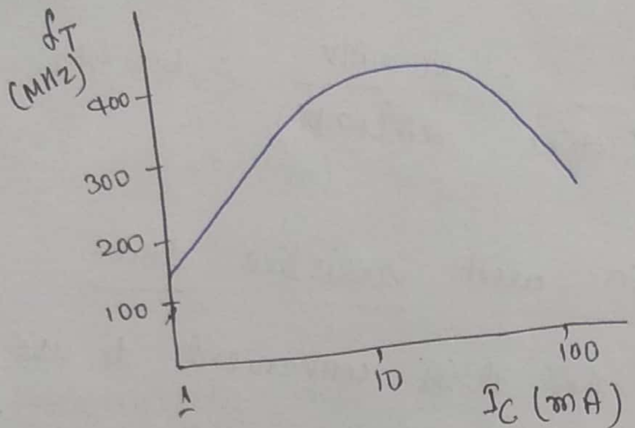
Parameter f_T:

It is the frequency at which the short circuit CE current gain attains unit magnitude, which is given as,

$$f_T \approx h_{fe} \cdot f_\beta = \frac{g_m}{2\pi(C_c+C_e)} \approx \frac{g_m}{2\pi C_e}, \text{ since } C_e >> C_c.$$

Hence $A_i = \dfrac{-h_{fe}}{1 + j f / (f_T/h_{fe})} = \dfrac{-h_{fe}}{1 + j h_{fe} (f/f_T)}$.

$f_T \rightarrow$ depends upon operating conditions of the device.



variation of V_T with I_C

For typical transistor, $f_T = 80$ MHz & $f_\beta = 1.6$ MHz. But f_T can be up to few GHz for modern BJT.

→ when $f << f_\beta$, $A_i \approx -h_{fe}$, $|A_i$ (dB)$| = 20 \log(h_{fe})$, it approaches horizontal line.

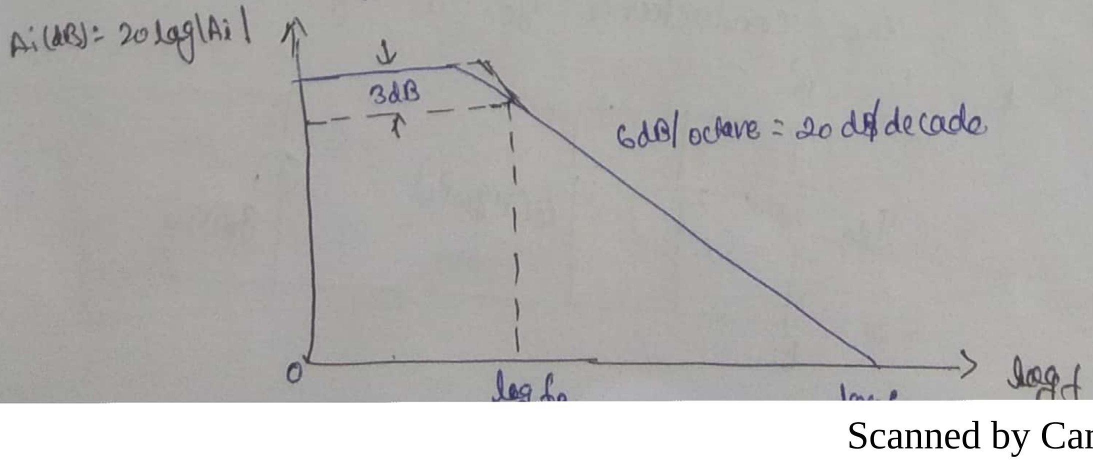
→ when $f >> f_\beta$, $|A_i| \approx h_{fe} f_\beta / f = f_T / f$,

A_i (dB) $= 20 \log f_T - 20 \log f$

at $f = f_T$, A_i(dB) $= 0$

→ for $f >> f_\beta$, plot approaches as straight line through pt $(f_T, 0)$. Slop decreases 6dB per octave or 20 dB per decade.

[$20 \log 2 = 6$ dB, f is multiplied by factor 2].



Q. With $g_m = 50\,mA/V$, $r_{b'e} = 1k$, $C_e = 1\,pF$ and $C_c = 0.2\,pF$, determine the values of f_β & f_T.

Soln.

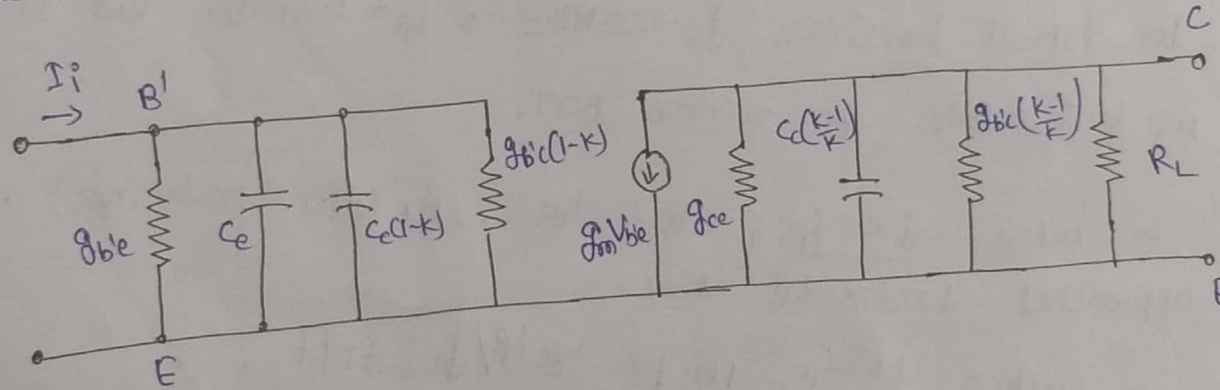
$$f_\beta = \frac{1}{2\pi r_{b'e}(C_c + C_e)} = \frac{1}{2\pi(1k)(1.2pF)} = 133\,MHz$$

$$f_T = \frac{g_m}{2\pi(C_c + C_e)} = \frac{50\,mA/V}{2\pi(1.2\,pF)} = 6.6\,GHz$$

* **Current gain with resistive load**

When $R_L \neq 0$. Then it is convenient to use Miller's theorem.

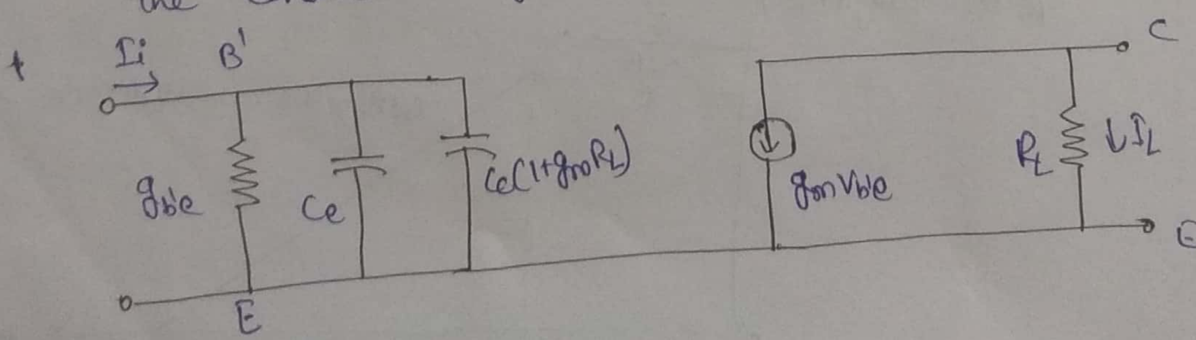
The circuit can be simplified as, where, $K = V_{ce}/V_{b'e} = -g_m R_L$



The o/p time constant is negligible in compare with i/p time constant. So, o/p capacitance $C_c(K-1/K)$ is neglected. We assume 'K' is independent of frequency.

$|K| >> 0$, hence $g_{b'c}(K-1)/K \approx g_{b'c}$, So, we neglect $g_{b'c}$ since $g_{b'c} << g_{ce}$.

The conductance g_{ce} is neglected in compare to $1/R_L$.



The lower 3 dB frequency is f_H, which is given as

$$f_H = \frac{1}{2\pi r_{b'e} C} = \frac{g_{b'e}}{2\pi C}, \quad C \approx C_e + C_c(1+g_m R_L)$$

The current gain is given as,

$$A_I = \frac{-h_{fe}}{1+j(f/f_H)} \; ; \; |A_I| = \frac{h_{fe}}{\left[1+(f/f_H)^2\right]^{1/2}}$$

Q.: The parameters are given as $g_m = 50$ mA/V, $r_{b'e} = 1$ KΩ, $C_e = 1$ pF and $C_c = 0.2$ pF, $R_L = 2$K, determine f_H.

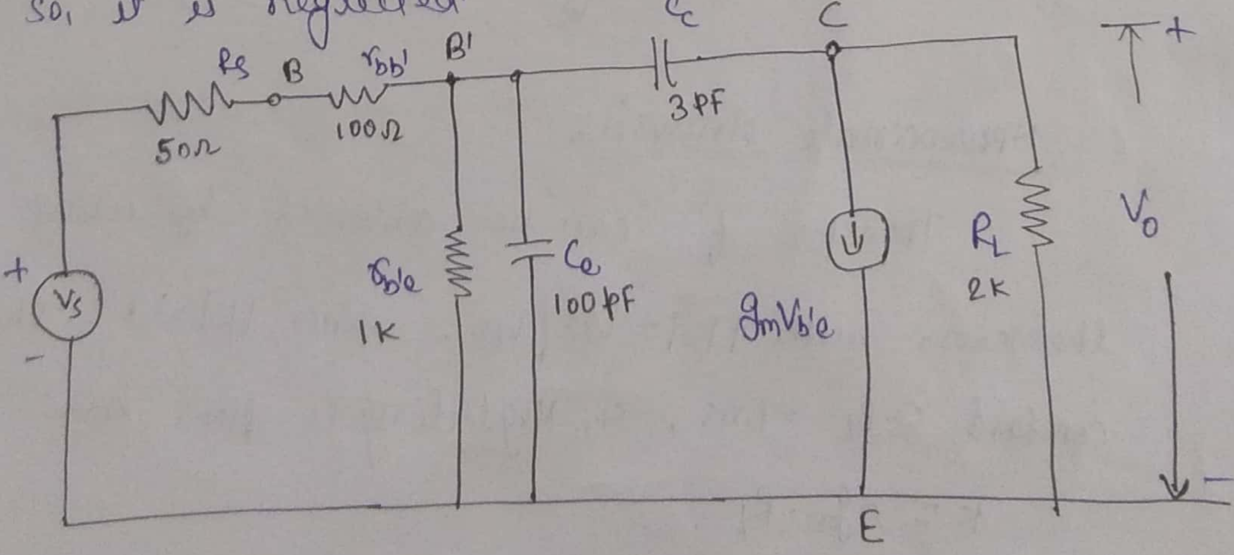
Soln.:

$$f_H = \frac{1}{2\pi r_{b'e}\left[C_e + C_c(1+g_m R_L)\right]} = \frac{1}{2\pi(1K)\left[1+0.2(1+50\times 2)\right]\times 10^{-12}}$$

$$= 7.5 \text{ MHz}.$$

* Single - stage CE transistor Amplifier Response:

Here voltage source V_s is considered with R_s (finite) resistance. $r_{ce} = 1/g_{ce}$ & $r_{b'c} = 1/g_{b'c}$ are much greater than R_L. So, it is neglected.



Transfer function: (Exact Analysis)

It is given as V_o/V_s. We consider $R_s' = R_s + r_{bb'} = 1/G_s'$

the admittance of a capacitor C is SC. Apply KCL at nodes B' and C, we get following equations.

At node B'

$$(V_s - V_{b'e}) \cdot G_s' + (V_o - V_{b'e}) SC_c = V_{b'e}(g_{b'e} + SC_e)$$

$$\Rightarrow G_s' V_s = [G_s' + g_{b'e} + s(C_e + C_c)] V_{b'e} - sC_c \cdot V_o \quad -(1)$$

At node C

$$0 = g_m V_{b'e} + V_o \cdot \frac{1}{R_L} + (V_o - V_{b'e}) SC_c$$

$$\Rightarrow 0 = (g_m - SC_c) V_{b'e} + \left(\frac{1}{R_L} + SC_c\right) \cdot V_o \quad -(2)$$

After solving eq^n (1) & (2), we get
(substitute $V_{b'e}$ value from eq^n(2) to eq^n(1))

$$\frac{V_o}{V_s} = \frac{-G_s' R_L (g_m - sC_c)}{s^2 C_e C_c R_L + s[C_e + C_c + C_c R_L(g_m + g_{b'e} + G_s')] + G_s' + g_{b'e}} \quad -(3)$$

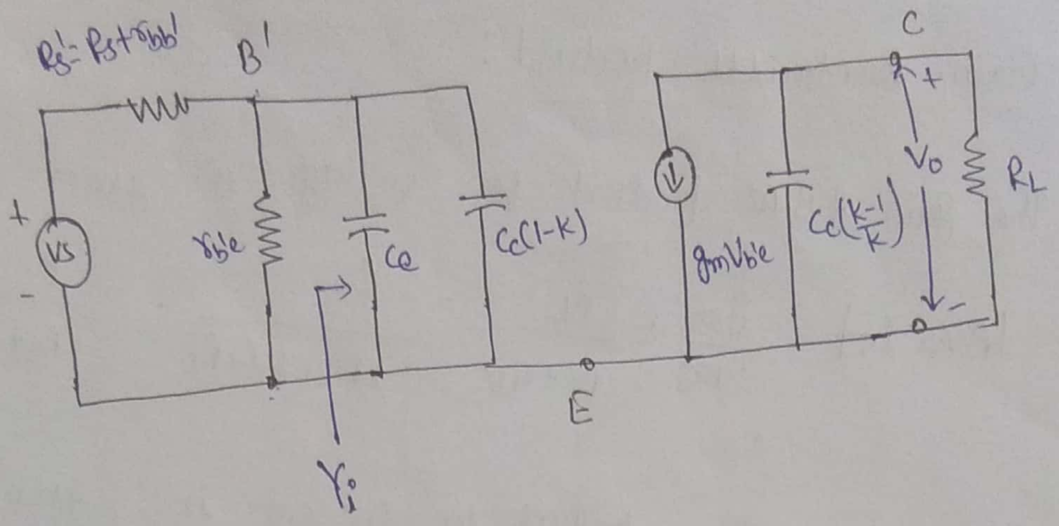
It is in the form of

$$A_{vs} \approx \frac{V_o}{V_s} = \frac{K_2(s - s_o)}{(s - s_1)(s - s_2)}$$

Approximate Analysis:

Transfer f^n can be obtained by using Miller's theorem, with $|K| = V_{ce}/V_{b'e}$. Since $|K| >> 1$, the o/p time constant $C_c R_L = 6ns$, so, Neglecting C_c from below figure,

$K = -g_m \cdot R_L$.



The I/P capacitance is given as,

$$C \approx C_e + C_c(1 + g_m R_L)$$

The I/P loop resistance is

$$R \approx R_s' \,||\, r_{b'e} = \frac{1}{G_s' + g_{b'e}}$$

The transfer f^n for the above diagram is given as

$$A_{vs} = \frac{V_o}{V_s} = \frac{-g_m \cdot R_L \cdot G_s'}{G_s' + g_{b'e} + sC}$$

i.e. $A_{vs} = \dfrac{K_2}{S - S_1}$, It is one-pole approximation.

The pole is given by $s = S_1$, where $S_1 = -\dfrac{(G_s' + g_{b'e})}{C} = -\dfrac{1}{RC}$

To find frequency response, let $s = j2\pi f$, then

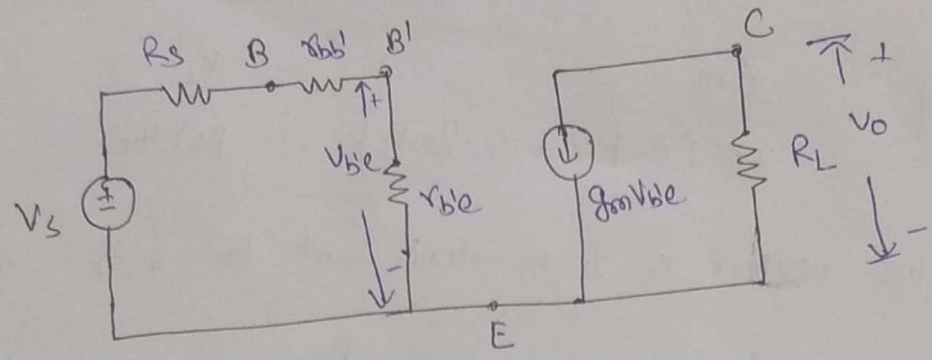
$$A_{vs} = \frac{K_2}{j2\pi f - S_1} = -\frac{K_2}{S_1} \times \frac{1}{1 - j2\pi f / S_1}$$

So, $$|A_{vs}| = \left|\frac{K_2}{S_1}\right| \frac{1}{[1 + (2\pi f / S_1)^2]^{1/2}}$$

Phase lead angle is $\varphi_1 = -\pi - \arctan \dfrac{2\pi f}{S_1}$.

Gain Bandwidth Product

In case of voltage source:



The overall voltage gain is given by,

$$A_{Vs} = \frac{V_o}{V_s} = \frac{V_o}{V_{b'e}} \cdot \frac{V_{b'e}}{V_s} = \frac{-g_m \cdot V_{b'e} \cdot R_L}{V_{b'e}} \cdot \frac{V_{b'e}}{V_s}$$

$$\Rightarrow A_{Vs} = -g_m \cdot R_L \cdot \frac{V_{b'e}}{V_s}$$

where, $\frac{V_{b'e}}{V_s} = \frac{r_{b'e}}{R_s + r_{bb'} + r_{b'e}} = \frac{r_{b'e}}{h_{ie} + R_s}$ $\left[\because r_{bb'} + r_{b'e} = h_{ie}\right]$

Hence $\boxed{A_{Vs} = \frac{-g_m \cdot R_L \cdot r_{b'e}}{h_{ie} + R_s}}$

Bandwidth $\approx f_H$ is given by, $f_H = \frac{1}{2\pi RC}$

where, $R = (R_s + r_{bb'}) \parallel r_{b'e}$ $[V_s = 0]$

$$\therefore R = \frac{(R_s + r_{bb'}) \times r_{b'e}}{R_s + r_{bb'} + r_{b'e}} = \frac{r_{b'e}(R_s + r_{bb'})}{h_{ie} + R_s}$$

$$\& \; C = C_e + C_c(1 + g_m \cdot R_L)$$

So $\boxed{f_H = \frac{1}{2\pi} \cdot \frac{(h_{ie} + R_s)}{r_{b'e}(R_s + r_{bb'}) \times \{C_e + C_c(1 + g_m R_L)\}}}$

So, $$|A_{vs}\cdot f_H| = \frac{g_m \cdot \cancel{r_{b'e}} \cdot R_L}{\cancel{(h_{ie}+R_s)}} \times \frac{\cancel{(h_{ie}+R_s)}}{2\pi \cdot \cancel{r_{b'e}}(R_s+r_{bb'}) \times \{C_e + C_c(1+g_m R_L)\}}$$

$$\Rightarrow |A_{vs}\cdot f_H| = \frac{g_m}{2\pi\{C_e + C_c + C_c \cdot g_m R_L\}} \cdot \frac{R_L}{R_s + r_{bb'}}$$

we neglect C_c term alone at low freq. So,

$$|A_{vs}\cdot f_H| = \frac{g_m}{2\pi\{C_e + C_c\, g_m R_L\}} \cdot \frac{R_L}{R_s + r_{bb'}}$$

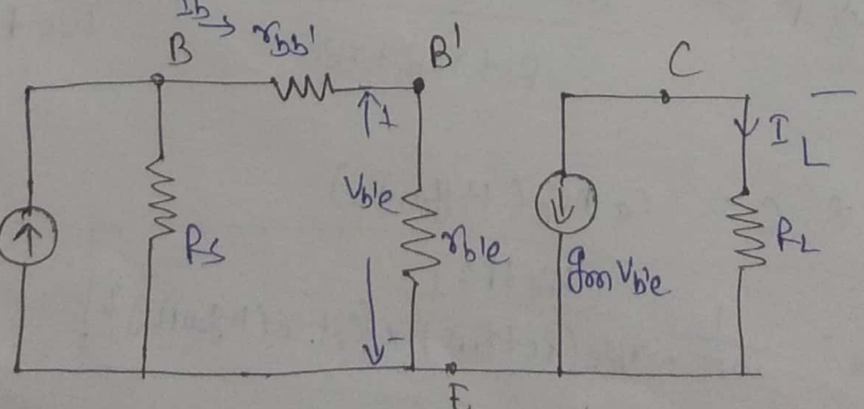
we know that $f_T = \frac{g_m}{2\pi C_e}$, so, $g_m = 2\pi f_T \cdot C_e$

Hence $$|A_{vs}\cdot f_H| = \frac{\cancel{2\pi} f_T \cdot C_e}{\cancel{2\pi}\{C_e + C_c \cdot R_L \cdot 2\pi f_T \cdot C_e\}} \cdot \frac{R_L}{(R_s + r_{bb'})}$$

$$= \frac{f_T \cancel{C_e}}{\cancel{C_e}(1 + 2\pi f_T \cdot C_c \cdot R_L)} \cdot \frac{R_L}{(R_s + r_{bb'})}$$

So, $$\boxed{|A_{vs}\cdot f_H| = \frac{f_T}{(1 + 2\pi f_T C_c \cdot R_L)} \cdot \frac{R_L}{(R_s + r_{bb'})}}$$

→ In case of current source :



The overall current gain is given as,

$$A_{Is} = \frac{I_L}{I_s} = \frac{I_L}{I_b} \cdot \frac{I_b}{I_s} = \frac{-g_m \cdot \cancel{V_{b'e}}}{\cancel{V_{b'e}}/r_{b'e}} \cdot \frac{I_b}{I_s}$$

$$\Rightarrow A_{Is} = \frac{-g_m}{1/r_{b'e}} \cdot \frac{I_b}{I_s} = -g_m \cdot r_{b'e} \cdot \frac{I_b}{I_s}.$$

where $\dfrac{I_b}{I_s} = \dfrac{R_s}{R_s + r_{bb'} + r_{b'e}} = \dfrac{R_s}{R_s + h_{ie}}$

Hence, $$\boxed{A_{Is} = \frac{-g_m \cdot r_{b'e} \times R_s}{R_s + h_{ie}}}$$

Bandwidth f_H is given by $f_H = \dfrac{1}{2\pi RC}$,

where $R = (R_s + r_{bb'}) \parallel r_{b'e} = \dfrac{(R_s + r_{bb'})\, r_{b'e}}{R_s + r_{bb'} + r_{b'e}} = \dfrac{(R_s + r_{bb'})\, r_{b'e}}{R_s + h_{ie}}$.

& $C = C_e + C_c(1 + g_m R_L)$

So, $$|A_{Is} \cdot f_H| = \frac{g_m \cdot \cancel{r_{b'e}} \cdot R_s}{\cancel{(R_s + h_{ie})}} \times \frac{1 \; \cancel{(R_s + h_{ie})}}{2\pi \cdot \{(R_s + r_{bb'})\, \cancel{r_{b'e}}\}\{C_e + C_c(1 + g_m R_L)\}}$$

$$= \frac{g_m}{2\pi\{C_e + C_c + C_c g_m R_L\}} \cdot \frac{R_s}{(R_s + r_{bb'})}$$

Only C_c term is cancelled due to $C_c \ll C_e$.

Hence $$|A_{Is} \cdot f_H| = \frac{g_m}{2\pi(C_e + C_c g_m R_L)} \cdot \frac{R_s}{(R_s + r_{bb'})}$$

we know that,

$$f_T = \frac{g_m}{2\pi C_e} \Rightarrow \text{so, } g_m = 2\pi f_T C_e$$

we can write,

$$|A_{fs} \cdot f_H| = \frac{\cancel{2\pi} f_T \cdot C_e}{\cancel{2\pi}\left(C_e + C_c \cdot 2\pi f_T \cdot C_e \cdot R_L\right)} \cdot \frac{R_s}{(R_s + r_{bb'})}$$

$$= \frac{f_T \cdot \cancel{C_e}}{\cancel{C_e}\left(1 + 2\pi f_T C_c R_L\right)} \cdot \frac{R_s}{(R_s + r_{bb'})}$$

$$\Rightarrow \boxed{|A_{fs} \cdot f_H| = \frac{f_T}{(1 + 2\pi f_T \cdot C_c R_L)} \cdot \frac{R_s}{(R_s + r_{bb'})}}$$

Multistage Amplifiers

* Frequency Response of an amplifier:

A sinusoidal signal of angular frequency 'ω' is represented by $V_m \sin(\omega t + \phi)$. If amplifier has gain 'A', and phase shift 'θ' occurs, then o/p is given as,

$$A V_m \sin(\omega t + \phi + \theta) = A V_m \sin\left[\omega\left(t + \frac{\theta}{\omega}\right) + \phi\right]$$

If 'A' is independent of freq. & $\theta \propto f$ then amplifier will preserve the form same as i/p signal.

The frequency characteristics are divided in to three regions:

(i) Midband frequencies region:

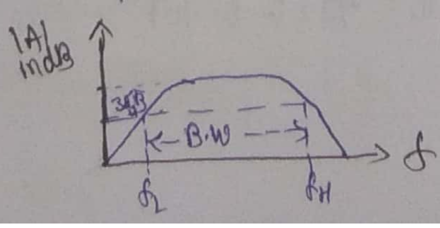
In this region amplification is constant and equal to A_0. Over, which delay is also constant.

(ii) Low-frequency region:

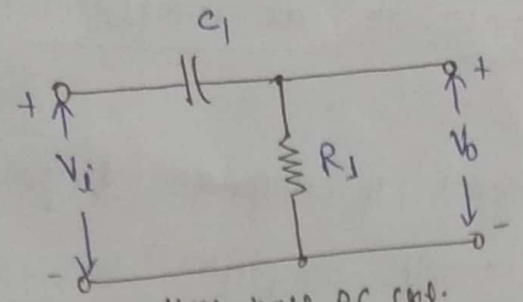
It is below midband, amplifier behave as high pass circuit. The response decreases with decreasing frequency & approaches zero at $f=0$.

(iii) High-frequency region:

It is above midband, where circuit behave as low pass network, the response decreases with increasing frequency.



→ Low frequency response:



High pass RC ckt.

By using complex variable 's', the o/p voltage is given as,

$$V_o(s) = \frac{V_i(s) \cdot R_1}{R_1 + 1/sC_1} = \frac{V_i(s) \cdot s}{s + 1/R_1C_1}$$ [multiply by s & divide by R_1 N^r & D^r]

The voltage transfer f^n is given as

$$A_L(s) = \frac{V_o(s)}{V_i(s)}$$

It has zero at $s=0$ & pole at $s = -1/R_1C_1$, where $s = j\omega$

$$\text{Hence } A_L(jf) = \frac{j2\pi f}{j2\pi f + 1/R_1C_1} = \frac{1}{1 + \frac{1}{j2\pi f R_1 C_1}} = \frac{1}{1 - \frac{j}{2\pi f R_1 C_1}}$$

$$\Rightarrow A_L(jf) = \frac{1}{1 - j(f_L/f)}, \text{ where } f_L = \frac{1}{2\pi R_1 C_1}$$

The magnitude & angle is given as,

$$|A_L(jf)| = \frac{1}{\sqrt{1 + (f_L/f)^2}}, \quad \theta_L = \arctan f_L/f .$$

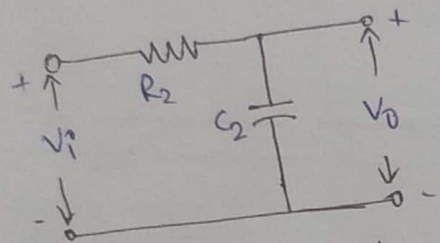
At $f = f_L$, $A_L = 1/\sqrt{2} = 0.707$. The f_L is the frequency at which gain has fallen to 0.707 times of midband value A_o.

In dB, $20 \log(1/\sqrt{2}) = 3dB$.

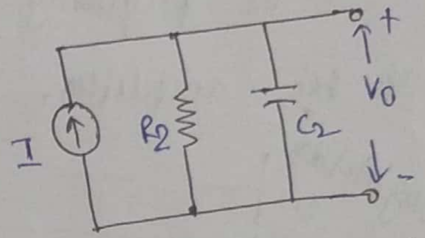
So, f_L is known as lower 3dB frequency. At this frequency

$$R_1 = \frac{1}{2\pi f_L C_1}.$$

→ High frequency Response.



Low pass RC ckt



Norton's equivalent

The o/p voltage is given as,

$$V_o(s) = \frac{1/sC_2}{R_2 + 1/sC_2} \cdot V_i(s) = \frac{1}{1 + sR_2C_2} V_i(s)$$ [∵ Multiply N^r & D^r by sC_2]

So, this transfer f^n has single pole, $s = -1/R_2C_2$.

The magnitude is given as,

$$|A_H(jf)| = |V_o(s)/V_i(s)|_{s=j2\pi f} = \frac{1}{1 + j2\pi f \cdot R_2C_2} = \frac{1}{1 + jf/\frac{1}{2\pi R_2C_2}}$$

$$= \frac{1}{1 + jf/f_H}, \text{ where } f_H = \frac{1}{2\pi R_2C_2}$$

Hence $$|A_H(jf)| = \frac{1}{\sqrt{1 + (f/f_H)^2}}$$

& $\theta_H = -\arctan f/f_H$

at $f = f_H$; $|A_H(jf)| = \frac{1}{\sqrt{2}} = 0.707$

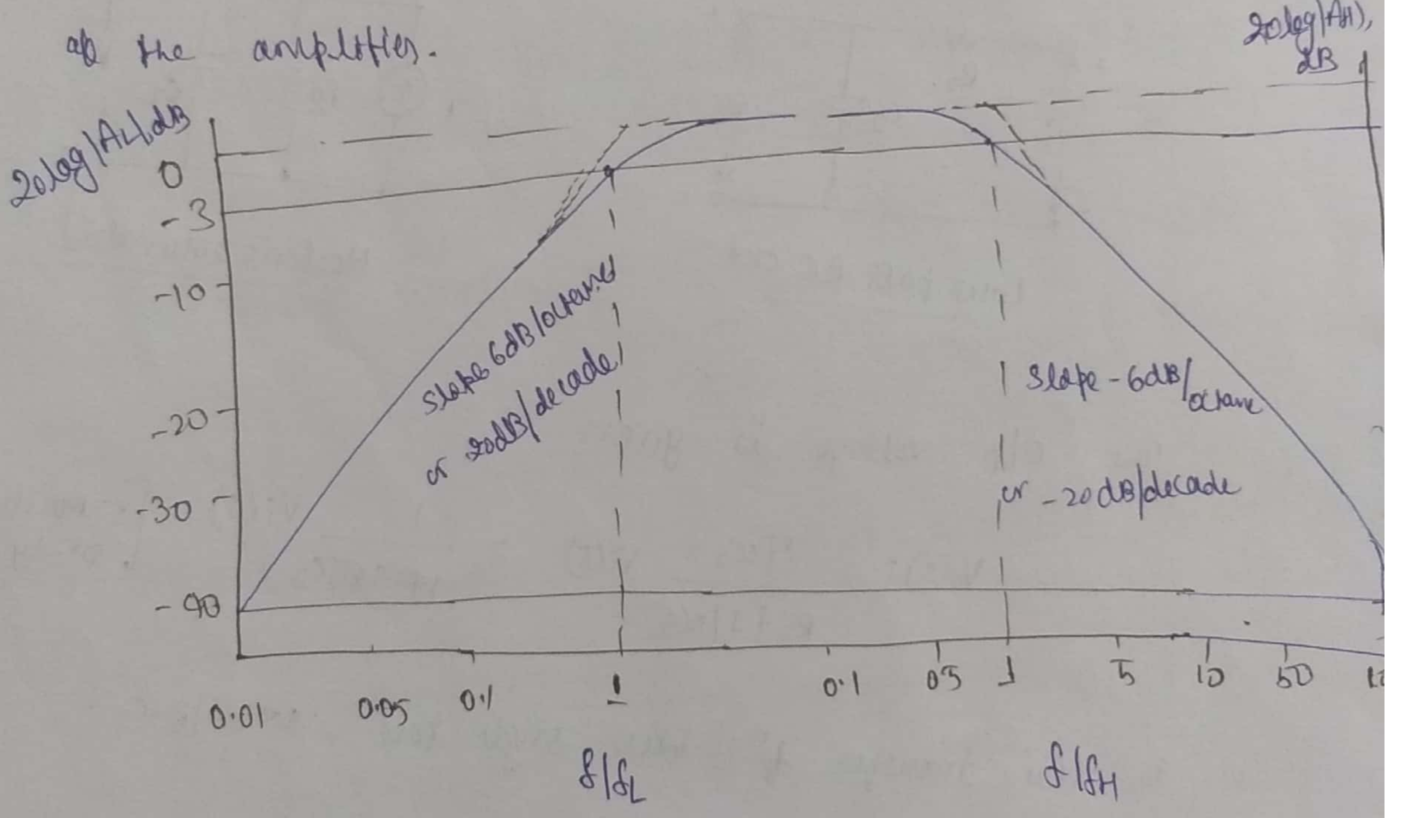
i.e. gain is reduced by $1/\sqrt{2}$ times of midband value.

f_H is called upper 3-dB frequency.

θ_L & θ_H are the angle by which the output leads the i/p neglecting initial 180° phase shift by amplifier.

→ **Bandwidth**

The frequency range from f_L to f_H is called bandwidth of the amplifier.



Q: An amplifier has a midband gain of 100, a lower 3-dB freq. of 0.1 Hz and a upper 3-dB freq. of 100, 9H.lag Hz. Determine the frequency range for which the gain is at least 99.

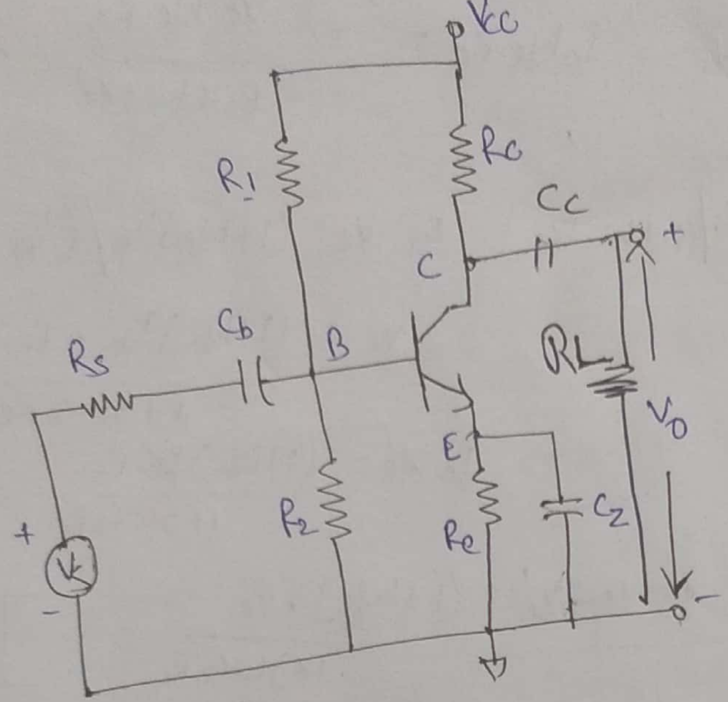
Sol^n:

$$|A(jf)| = \frac{100}{\sqrt{1+(f/f_H)^2}} \Rightarrow 99 = \frac{100}{\sqrt{1+(f_1/100)^2}}$$ Gain = 99 at f_1

$\Rightarrow f_1 = 14.2\ Hz.$

$$99 = \frac{100}{\sqrt{1+\left(\frac{0.1}{f_2}\right)^2}} \Rightarrow f_2 = 0.7\ Hz$$

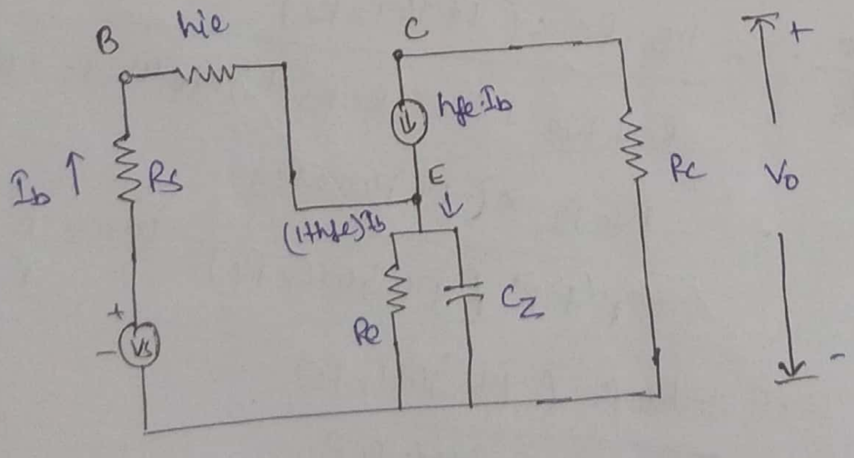
* Effect of coupling and bypass capacitors:



→ Effect of bypass capacitor

If an emitter bias R_e is used for self bias and it is desired to avoid the degeneration, Then loss of gain due to R_e can be minimized by bypassing it with large capacitance value.

We assume $R_1 \| R_2 >> R_s$ and load R_c is small enough to apply simplified model. The blocking capacitor C_b is omitted.



The o/p voltage is given by,

$$V_0 = -I_b h_{fe} \cdot R_c = -\frac{V_s \cdot h_{fe} \cdot R_c}{R_s + h_{ie} + Z_e'} \quad -(1)$$

[where $I_b = V_s/(R_s + h_{ie} + Z_e')$ & $V_e = (1+h_{fe}) I_b / (1/R_e + j\omega C_2)$

$$\Rightarrow V_e = \frac{(1+h_{fe}) I_b \cdot R_e}{1 + j\omega C_2 R_e}$$

$$\Rightarrow \cancel{I_b} Z_e' = \frac{(1+h_{fe}) \cancel{I_b} \cdot R_e}{1 + j\omega C_2 R_e}$$

Hence $Z_e' = \dfrac{(1+h_{fe}) \cdot R_e}{1 + j\omega C_2 R_e}$]

where $Z_e' = \dfrac{(1+h_{fe}) R_e}{1 + j\omega C_2 R_e} \quad -(2)$

Substitute eqn (2) in eqn (1), we get

$$V_0 = -\frac{V_s \cdot h_{fe} \cdot R_c}{R_s + h_{ie} + \frac{(1+h_{fe}) R_e}{1 + j\omega C_2 R_e}} = -\frac{V_s \cdot h_{fe} \cdot R_c \times (1 + j\omega C_2 R_e)}{(R_s + h_{ie})(1 + j\omega C_2 R_e) + (1+h_{fe}) R_e}$$

$$\Rightarrow \frac{V_0}{V_s} = -\frac{h_{fe} \cdot R_c \times (1 + j\omega C_2 R_e)}{R_s + h_{ie} + j\omega C_2 R_e R_s + j h_{ie} \omega C_2 R_e + R_e + h_{fe} R_e}$$

$$= \frac{-h_{fe} R_c \times (1 + j\omega C_2 R_e)}{R' + R(1 + j\omega C_2 R_e)} \quad \text{where } R' = (1+h_{fe}) R_e, \; R = R_s + h_{ie}$$

$$= \frac{-h_{fe} R_c (1 + j\omega C_2 R_e)}{R' + R + j\omega C_2 R_e R}$$

$$\Rightarrow A_{v_s} = \frac{V_o}{V_s} = \frac{-h_{fe}R_c\,(1+j\omega C_2 R_e)}{(R+R')\left\{1+j\omega C_2\left(\frac{R_e R}{R+R'}\right)\right\}} \quad - (3)$$

The midband gain 'A_0' is given as $\omega \to \infty$ or

$$A_0 = \frac{-h_{fe}\cdot R_c}{R} = \frac{-h_{fe}\cdot R_c}{R_s + h_{ie}}.$$

$$\left[\; V_o = -h_{fe}\cdot I_b R_c,\quad V_{in} = I_b\cdot(R_s+h_{ie}),\; \right].$$

Hence
$$\frac{A_{v_s}}{A_0} = \frac{-h_{fe}\cdot \cancel{R_c}\,(1+j\omega C_2 R_e)}{(R+R')\left\{1+j\omega C_2\left(\frac{R_e R}{R+R'}\right)\right\}}\cdot\frac{(R_s+h_{ie})}{-\cancel{h_{fe}}\cdot R_c}$$

$$= \frac{(1+j\omega C_2 R_e)R}{(R+R')\left\{1+j\omega C_2\left(\frac{R_e R}{R+R'}\right)\right\}}$$

$$= \frac{(1+j\omega C_2 R_e)\cancel{R}}{\cancel{R}(1+R'/R)\left\{1+j\omega C_2\left(\frac{R_e R}{R+R'}\right)\right\}}$$

$$\Rightarrow \frac{A_{v_s}}{A_0} = \frac{1+j2\pi f C_2 R_e}{(1+R'/R)\left\{1+j2\pi f C_2\left(\frac{R_e R}{R+R'}\right)\right\}}$$

$$= \frac{1+jf\Big/ 1/2\pi C_2 R_e}{(1+R'/R)\left\{1+jf\Big/\frac{R+R'}{2\pi C_2 R_e R}\right\}} = \frac{1+jf\Big/\frac{1}{2\pi C_2 R_e}}{(1+R'/R)\left\{1+jf\Big/\frac{(1+R'/R)}{2\pi C_2 R_e}\right\}}$$

$$\Rightarrow \frac{A_{v_s}}{A_0} = \frac{1}{(1+R'/R)}\cdot\frac{(1+jf/f_0)}{(1+jf/f_p)} \quad -(4)$$

where, $f_0 = \dfrac{1}{2\pi C_2 R_e}$ & $f_p = \dfrac{(1+R'/R)}{2\pi C_2 R_e}$ $\Big\}$ $-(5)$

$f_o \rightarrow$ determines zero and $f_p \rightarrow$ pole of the A_{vs}/A_o. Generally $R'/R >> 1$ then $f_p >> f_o$. i.e. pole and zero are widely separated.

e.g.

If $R_s = 0$, $R_e = 1\,k\Omega$, $C_z = 100\,\mu F$, $h_{fe} = 50$, $h_{ie} = 1.1\,k\Omega$, $R_c = 2\,k\Omega$.

then $f_o = 1.6$ Hz & $f_p = 76$ Hz.

Magnitude of $|A_{vs}/A_o|$ in dB is given by

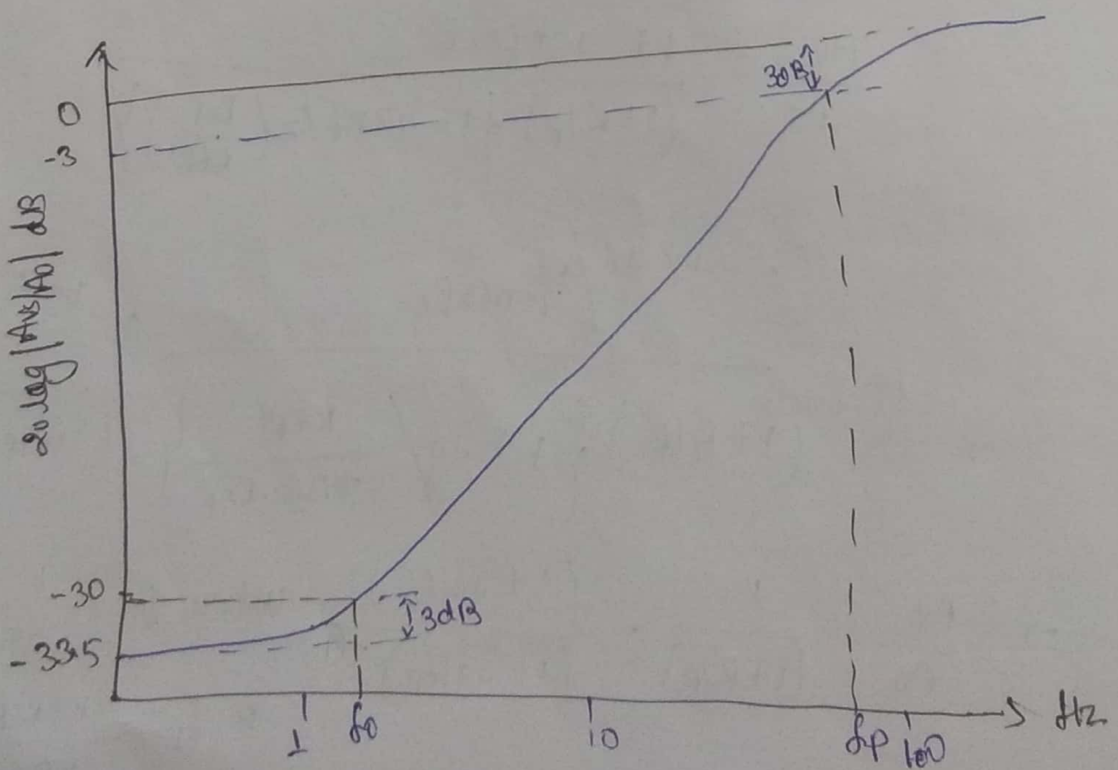
$$20\log\left|\frac{A_{vs}}{A_o}\right| = -20\log\left(1+R'/R\right) + 20\log\sqrt{1+(f/f_o)^2} - 20\log\sqrt{1+(f/f_p)^2} \quad -(6)$$

→ $f_p >> f_o$ & for $f >> f_p$ → Gain approaches to midband value A_o.

Hence $20\log|A_{vs}/A_o| = 20\log(1) = 0$ dB

→ At $f = f_p$

$|A_{vs}/A_o|$ reduced by $1/\sqrt{2}$ i.e. 3dB.

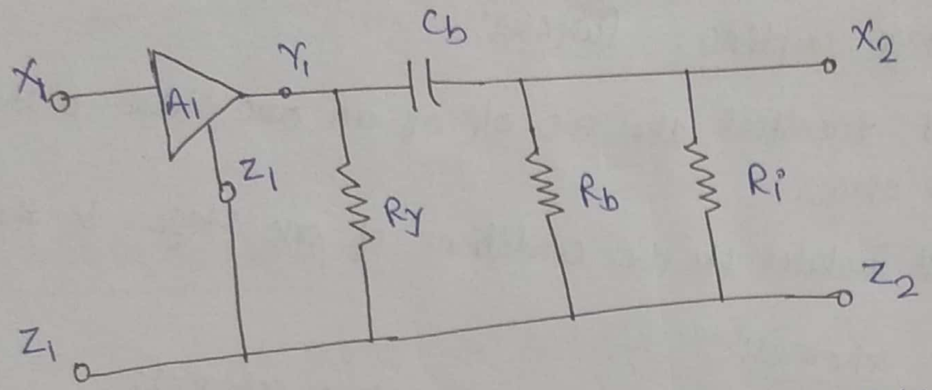


freq. response of amplifier with C_z

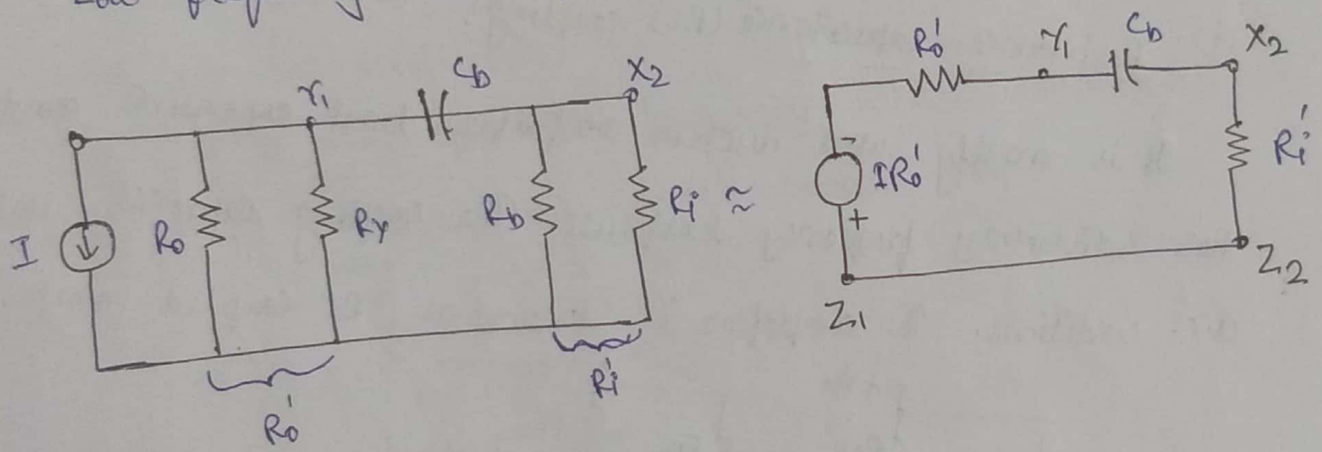
→ Effect of coupling capacitor :

The bypass capacitor is large, so, here it is neglected.

For FET :



Low frequency model for RC- coupled amplifier is given as,



The lower 3-dB frequency is given by,

$$f_L = \frac{1}{2\pi(R_o' + R_i')C_b}$$

For FET amplifier, $R_i' = R_g >> R_d$ Hence $f_L \approx 1/2\pi C_b R_g$.

* Different coupling schemes used in amplifier:

The coupling N/W is used to connect o/p of one amplifier to the i/p of another amp. in case of cascading. It is known as inter stage coupling. Purposes:

(i) It transfers the a.c. o/p of an one stage to the i/p of the next stage.

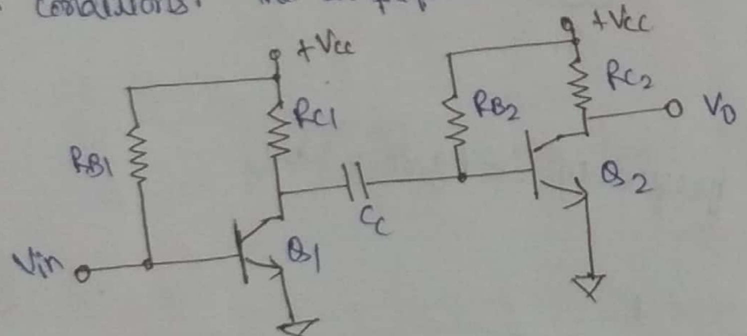
(ii) It isolates the d.c. conditions of one stage to the next.

Coupling schemes:

There are following three coupling schemes:

(i) Resistance-capacitance (RC) coupling:

It is mostly used discrete amplifier, least expensive and has satisfactory frequency response. The coupling capacitor isolates d.c. conditions. The amplifier is known as RC-coupled amp.

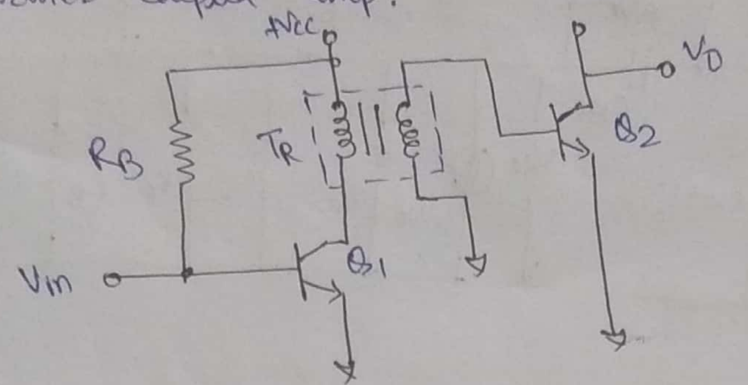


(ii) Transformer coupling:

The primary winding of the transformer act as collector load and secondary winding transfers the a.c. o/p signal directly to base of the next stage.

It increases the overall circuit gain and level of inter stage impedance matching.

But it is very expensive for broad freq. response, so it is used only in power amplifier. Amplifier is known as transformer coupled amp.

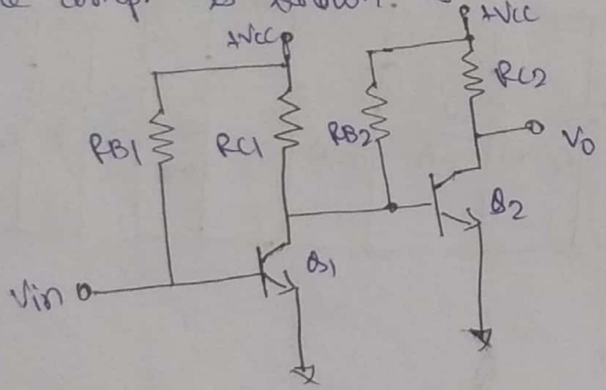


(iii) <u>Direct coupling:</u>

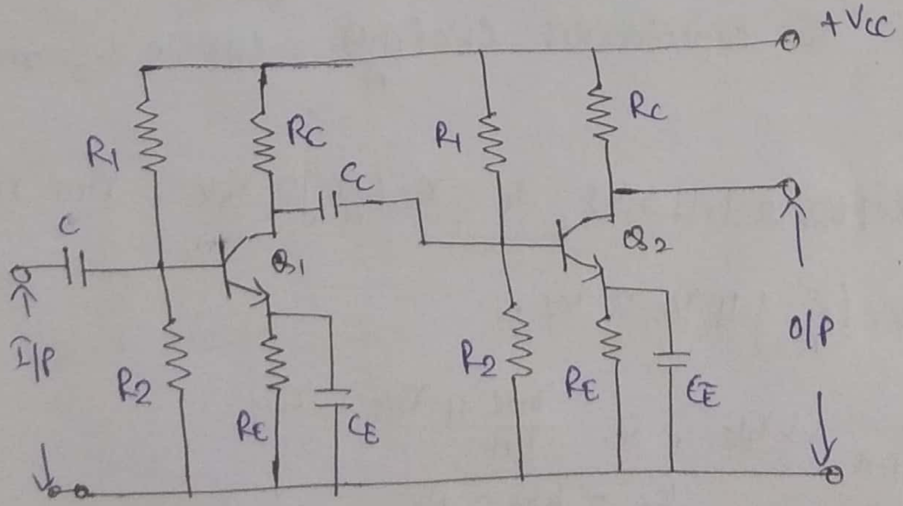
The a.c o/p signal is directly fed to the next stage, no reactance is used. Special d.c. level circuit is used to match with o/p d.c level.

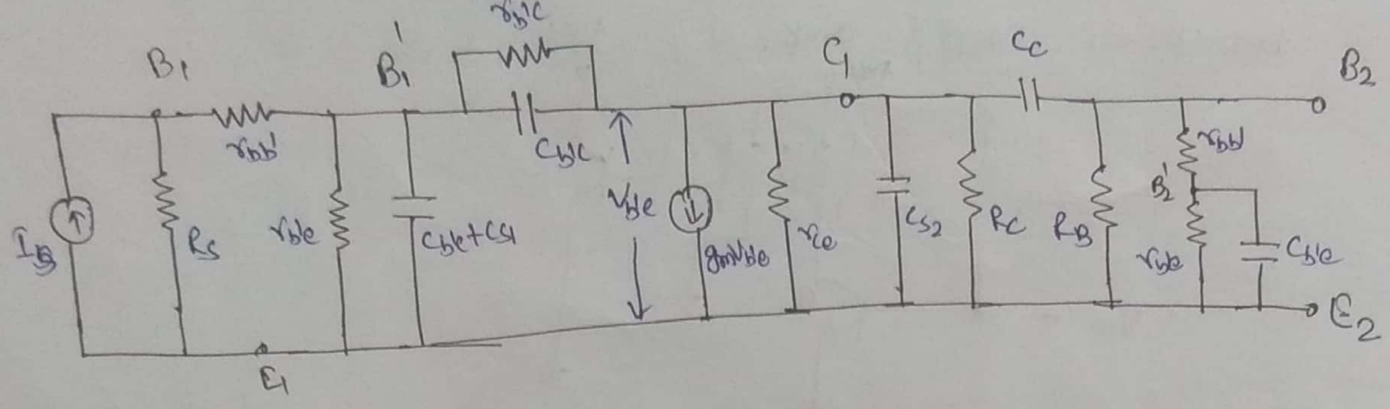
It is used for low frequency signal amplification.

The amp. is known as direct coupled amp.



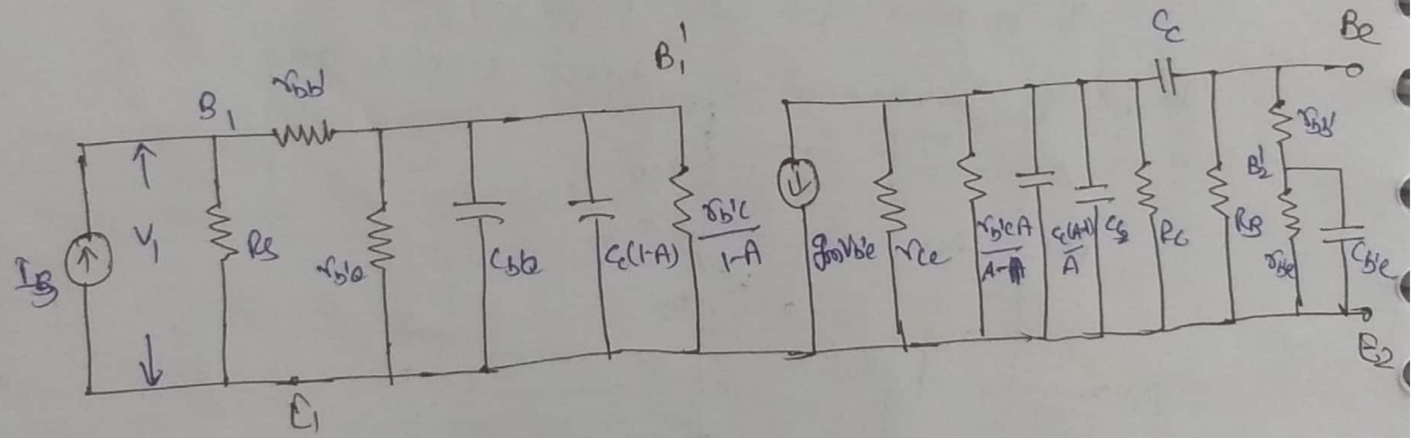
* <u>Analysis of cascaded RC coupled amplifiers</u>: (<u>At high freq.</u>).





The transistor Q_1 is replaced by high frequency - π model. The C_{s1} & C_{s2} represent stray capacitances caused by wiring. $R_B = R_1 || R_2$ for particular stage.

Miller's theorem is applied for $r_{b'c}$ & $C_{b'c}$. The equivalent circuit is given as,



This circuit is further simplified by few assumptions:

(i) The O/P time constant is negligible in compare to I/P circuit. So, capacitances $C_{b'c}\left(\frac{A-1}{A}\right)$, $C_{b'e}$ & C_{s2} may be neglected.

(ii) $A = V_{ce}/V_{b'e}$ & $|A| >> 1$, so $r_{b'c}\left(\frac{A}{A-1}\right) \simeq r_{b'c}$. But $r_{b'c} >> r_{ce}$

So, $r_{b'c}\left(\frac{A}{A-1}\right) || r_{ce} \simeq r_{ce}$.

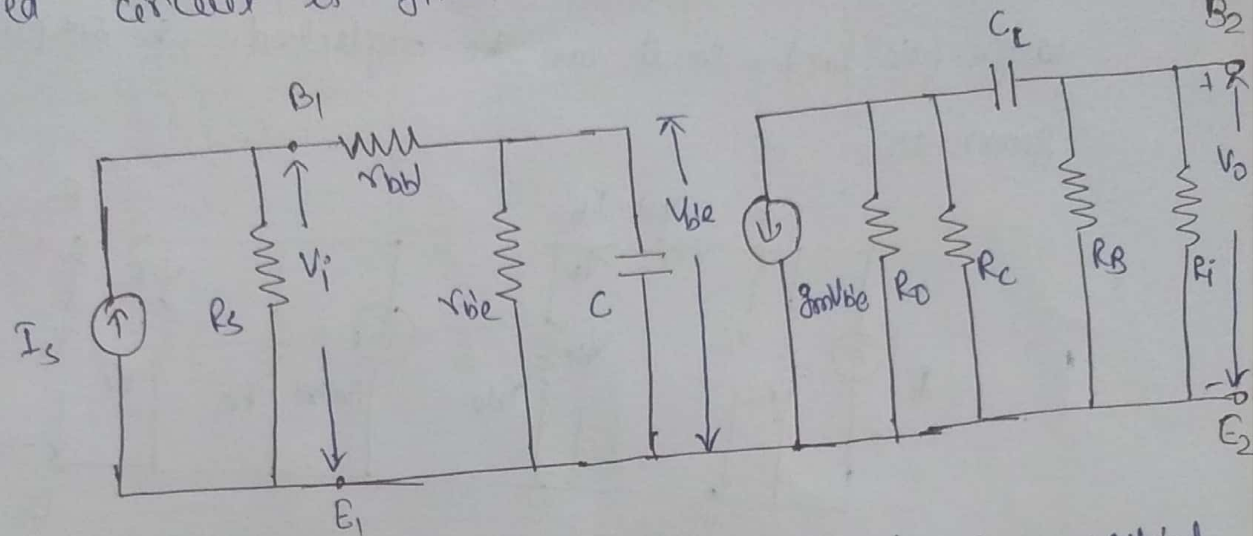
(iii) $r_{b'c}/_{1-A} >> r_{b'e}$. So, $\frac{r_{b'c}}{1-A} || r_{b'e} \simeq r_{b'e}$.

(iv) We assume $1/r_{ce} \simeq h_{oe} \simeq R_o$.

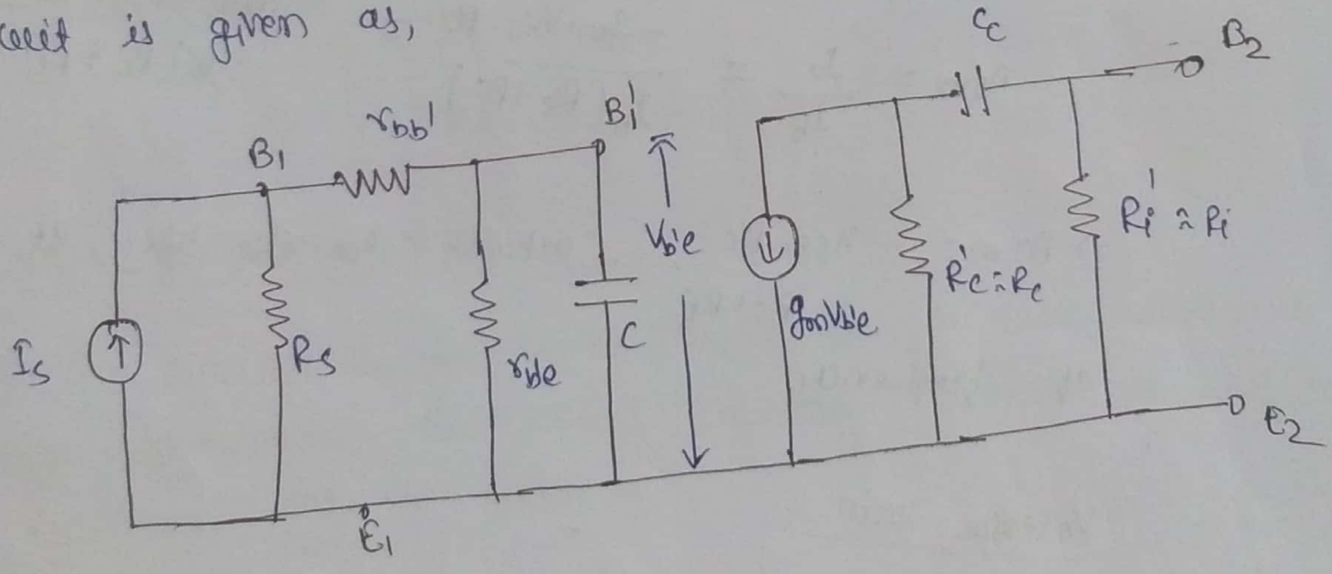
(v) $r_{bb'}$ & $r_{b'e}$ of Q_2 can be combined to form $R_i \approx h_{ie}$.

(vi) $C = C_{b'e} \| C_{s1} \| C_{b'c}(1-A)$.

Simplified circuit is given as,



Further we consider, $R_o \| R_C = R_C'$ & $R_B \| R_i = R_i'$. So, simplified circuit is given as,

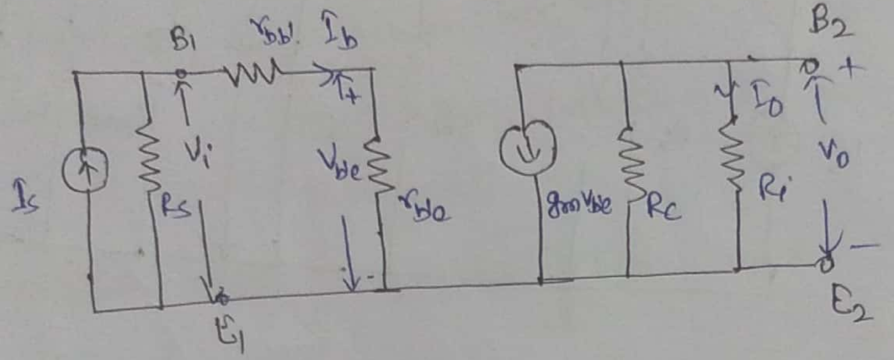


In most cases $R_o \gg R_C$, hence $R_o \| R_C \approx R_C$. Similarly $R_B \gg R_i$, so, $R_B \| R_i \approx R_i$.

The analysis of RC coupled cascaded amplifier for three frequency ranges are done by using simplified model.

→ Midband frequency range:

The reactance offered by C_c is small enough, so, it can be neglected. For small freq. shunt capacitance reactance is large ($x_c = 1/\omega c$), so it can be neglected. So simplified ckt is given as,



Current gain : A_{Im}:

$$A_{Im} = \frac{I_o}{I_b} = \frac{-g_m \cdot V_{b'e} \cdot R_c}{I_b (R_c + R_i)} = \frac{-g_m \cdot \cancel{I_b} \cdot r_{b'e} \cdot R_c}{\cancel{I_b}(R_c + R_i)} = \frac{-g_m \cdot r_{b'e} \cdot R_c}{(R_c + R_i)}$$

$\Rightarrow A_{Im} = -\frac{h_{fe} \cdot R_c}{R_c + R_i}$, where, $g_m \cdot r_{b'e} = h_{fe}$. It is independent of frequency.

Voltage gain :

$$A_{Vm} = \frac{V_o}{V_i}$$

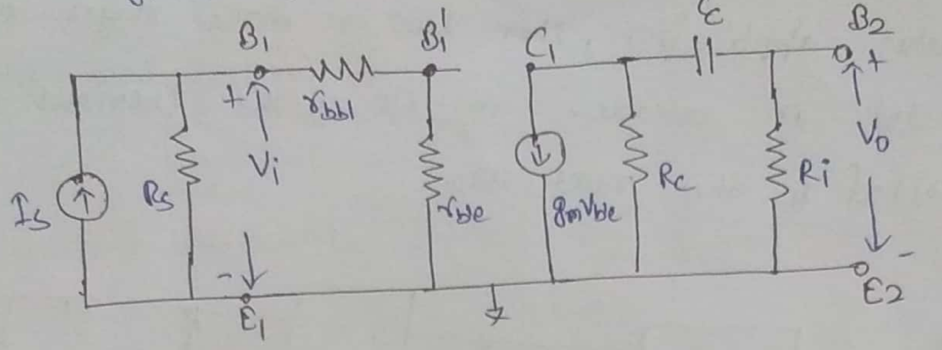
$V_o = -g_m \cdot V_{b'e} R_{ci} = -g_m \cdot I_b \cdot r_{b'e} \cdot R_{ci}$, where $R_{ci} = R_c \| R_i$

& $V_i = I_b (r_{bb'} + r_{b'e}) = I_b \cdot h_{ie}$

Hence $A_{Vm} = \frac{V_o}{V_i} = \frac{-g_m \cdot \cancel{I_b} \cdot r_{b'e} \cdot R_{ci}}{\cancel{I_b} \cdot h_{ie}} = \frac{-h_{fe} \cdot R_{ci}}{h_{ie}}$.

→ Low frequency range :

Capacitor 'C' is neglected since its reactance is extremely large as $X_C \parallel r_{b'e} \approx r_{b'e}$. But C_C can't be neglected. So, equivalent circuit is given as,

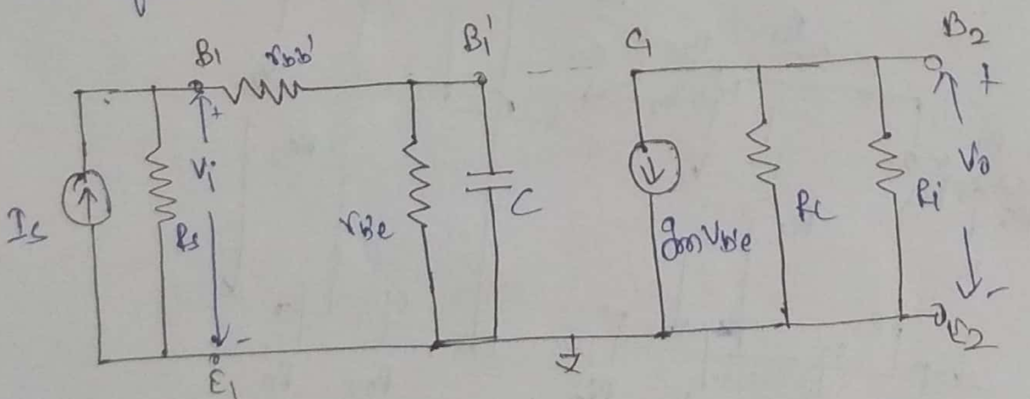


$$A_{IL} = -h_{fe} \cdot \frac{R_C}{R_C + R_i + 1/j\omega C_C} = A_{Im} \cdot \frac{R_C + R_i}{R_C + R_i + \frac{1}{j\omega C_C}}$$

$$A_{VL} = \frac{-h_{fe}}{h_{ie}} \cdot \frac{R_{Ci}}{\left[1 - \frac{j}{2\pi f C_C (R_C + R_i)}\right]}$$

→ High frequency range:

Coupling capacitor C_C can be neglected due to low reactance at high frequencies. But shunt capacitor 'C' cannot be neglected. So, equivalent ckt is given as,

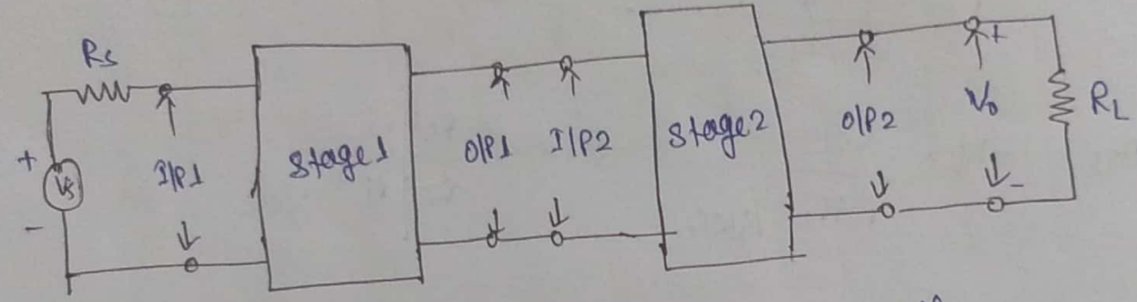


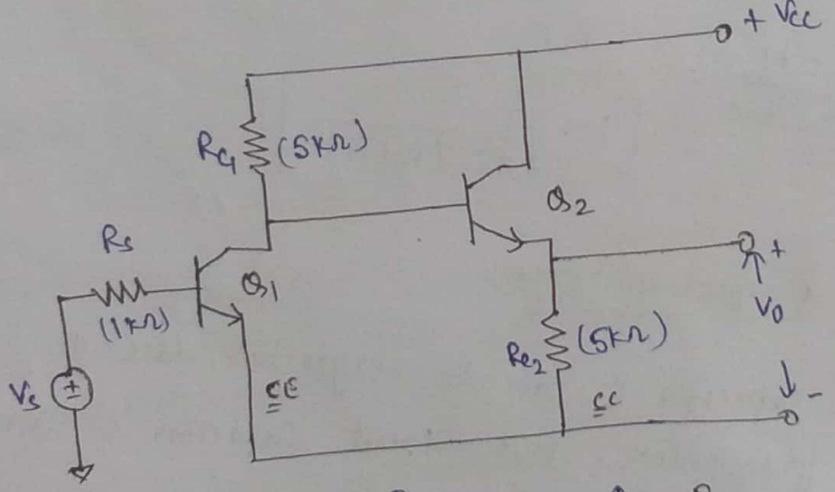
$$A_{Ih} = -g_m \, r_{b'e} \cdot \frac{R_C}{R_C + R_i} \cdot \frac{1}{1 + j\omega C r_{b'e}} = \frac{A_{Im}}{1 + j\omega C r_{b'e}}$$

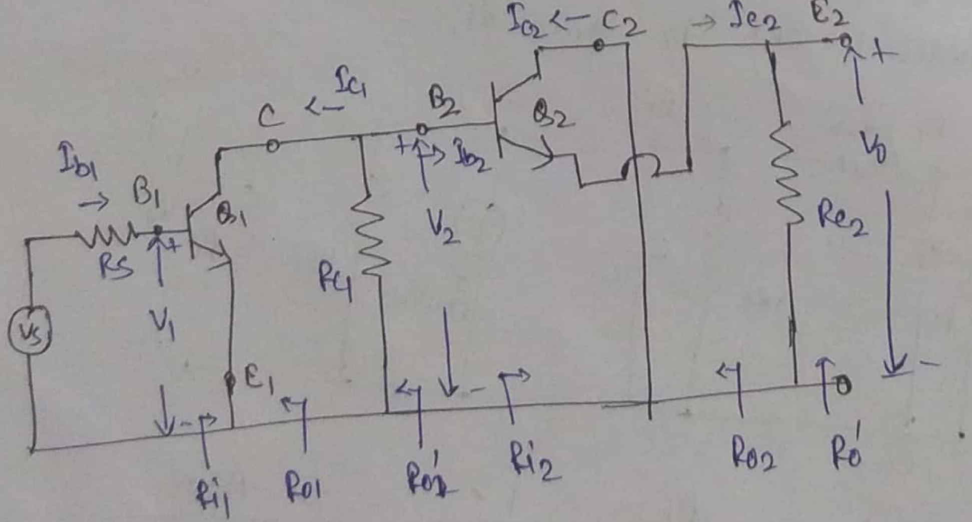
$$A_{Vh} = \frac{-h_{fe}}{h_{ie}} \cdot \frac{R_{Ci}}{1 + j\omega C r_{b'e}} = \frac{A_{Vm}}{1 + j 2\pi f C r_{b'e}}$$

Analysis of cascaded RC coupled amplifier (at low freq.):

When amplification of single stage is not sufficient or when input or output impedance is not of correct magnitude for particular application, then two or more stages may be connected in cascade. i.e. o/p of the previous stage is connected to the next stage.







The collector of 1^{st} stage is shunted with I/P impedance of 2^{nd} stage. So, we have to start from 2^{nd} stage.

2^{nd} stage: (CC)

The load resistance,

$$R_L = R_{e2}$$

→ current gain, $A_{I2} = \frac{+I_{e2}}{I_{b2}} = \frac{-h_{fc}}{1+h_{oc} \cdot R_{e2}}$

→ The i/p impedance, $R_{i2} = h_{ic} + h_{rc} \cdot A_{I2} \cdot R_{e2}$

→ The voltage gain, $A_{V2} = \frac{V_o}{V_2} = A_{I2} \cdot \frac{R_{e2}}{R_{i2}}$ or $A_{V2} = 1 - \frac{h_{ic2}}{R_{i2}}$

1^{st} stage (CE)

The load resistance of 1^{st} stage is given as

$$R_{L1} = R_{C1} \| R_{i2} = \frac{R_{C1} \cdot R_{i2}}{R_{C1}+R_{i2}}$$

The current gain is given as,

$$A_{I1} = -\frac{I_{C1}}{I_{b1}} = \frac{-h_{fe}}{1+h_{oe} \cdot R_{L1}}$$

The I/P impedance,

$$R_{i1} = h_{ie} + h_{re} \cdot A_{I1} \cdot R_{L1}$$

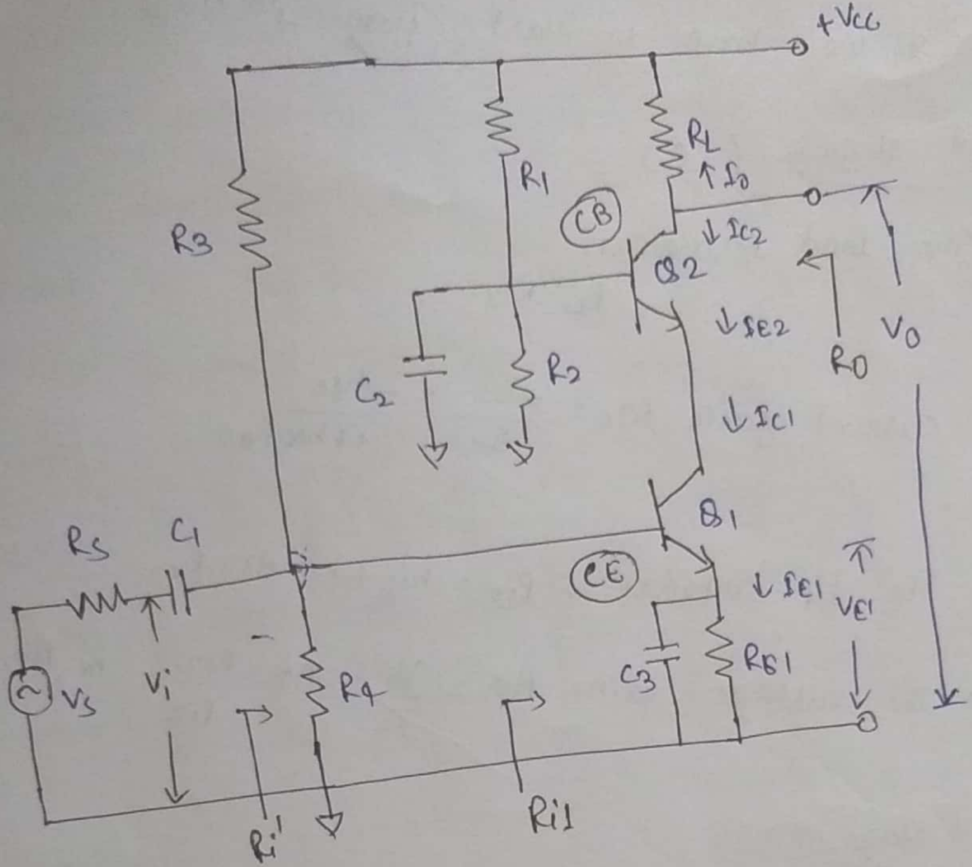
The voltage gain,

$$A_{V1} = \frac{V_2}{V_1} = \frac{A_{I1} \cdot R_{L1}}{R_{i1}}$$

Choice of configuration:

- → Intermediate stage can't be CC, since Av is less than 1.
- → Intermediate should be CE due to $h_{fe} \gg 1$, so, A_V & A_I will be high.
- → I/P stage CB or CC for impedance match. Noise consideration.
- → O/P stage on basis of impedance. CC has very low O/P resistance so it is used.

* Cascode amplifier:

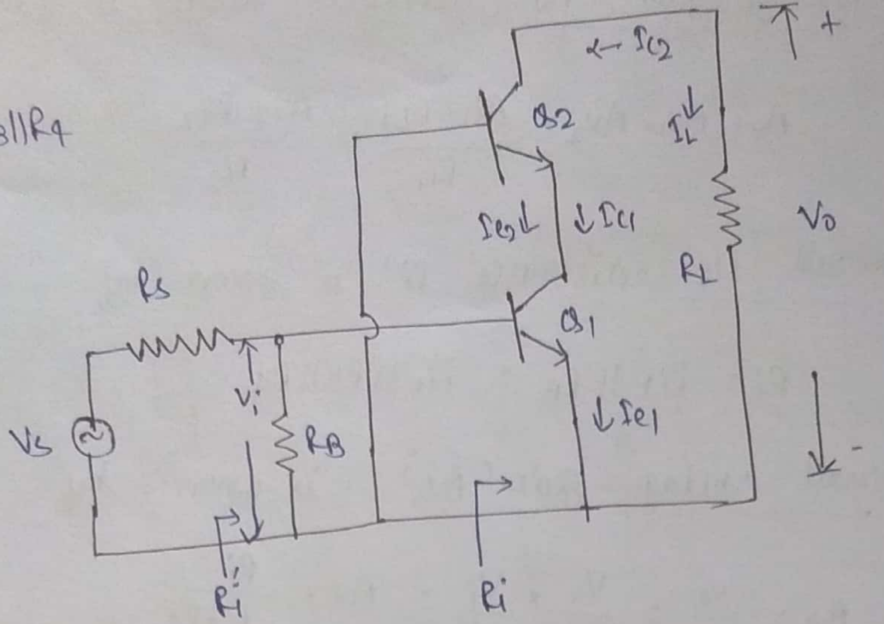


It is composite amplifier pair with a large bandwidth used for RF applications and as video amplifier. It consist of CE stage followed by CB stage directly coupled.

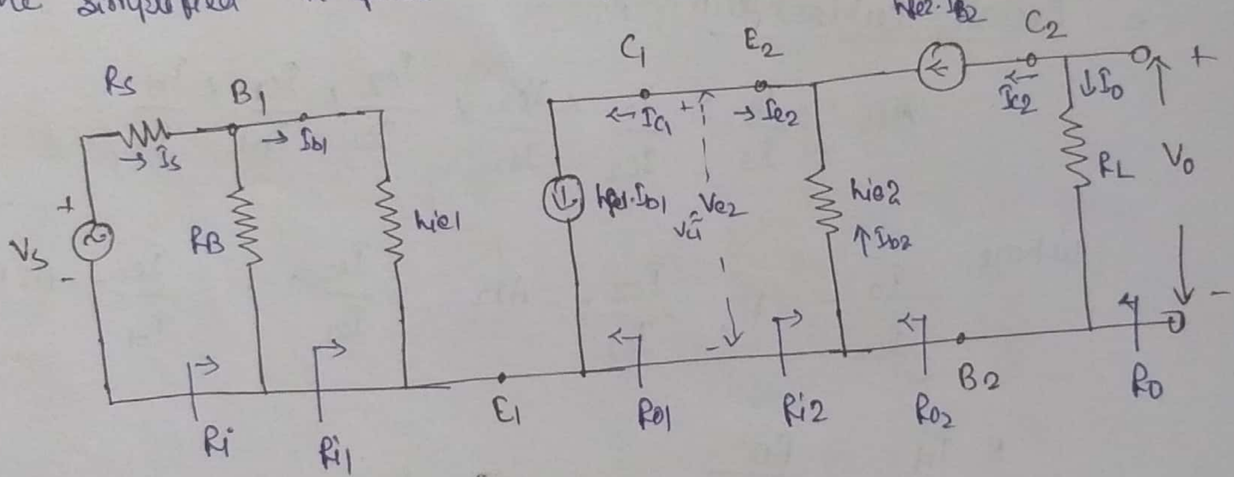
For high frequency, CB configuration has desirable characteristics. But it has low i/p impedance. So, cascode is designed to have i/p impedance of CE, A_I of CE & A_V of CB. Good isolation is required b/w i/p & o/p.

The a.c. equivalent circuit for cascode amp is drawn by shorting d.c. supply & capacitors. Later simplified h-parameter equivalent circuit is drawn.

$R_B = R_3 \| R_4$



The simplified h-parameter equivalent circuit is given as,



Analysis of 2nd stage (CB amp):

from approximate analysis, current gain is given as,

$$A_{i2} = -\frac{I_{C2}}{I_{e2}} = \frac{h_{fe}}{1+h_{fe}}, \quad R_{i2} = \frac{h_{ie}}{1+h_{fe}}, \quad A_{V2} = \frac{V_o}{V_{e2}} = \frac{h_{fe} \cdot R_L}{\frac{h_{ie} \times (1+h_{fe})}{1+h_{fe}}}$$

$$= \frac{A_{I2} \cdot R_L}{R_{i2}}$$

Analysis of 1st stage (CE amp):

$$A_{i1} = \frac{-I_{C1}}{I_{b1}} = -h_{fe}; \quad R_{i1} = h_{ie}$$

$$A_{V1} = \frac{V_{C1}}{V_{b1}} = -\frac{h_{fe} \cdot R_L}{h_{ie}} = \frac{A_{I1} \cdot R_L}{R_{i1}}; \quad R_{L1} = R_{i2}$$

→ The voltage gain 'A_v' without source is given as,

$$A_v = A_{v1} \cdot A_{v2} = \frac{A_{i1} \cdot R_{L1}}{R_{i1}} \cdot \frac{A_{i2} \cdot R_{L2}}{R_{i2}}$$

→ Overall i/p resistance 'R_i' is given by,

$$R_i = R_{i1} \parallel R_B = R_{i1} \parallel R_3 \parallel R_4$$

→ Overall voltage gain 'A_{vs}' is given by

$$A_{vs} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s} = A_v \times \frac{R_i}{R_i + R_s}$$

→ Overall current gain 'A_{Is}'

$$A_{Is} = \frac{I_o}{I_s} = \frac{I_o}{I_{c2}} \times \frac{I_{c2}}{I_{e2}} \times \frac{I_{e2}}{I_{c1}} \times \frac{I_{c1}}{I_{b1}} \times \frac{I_{b1}}{I_s}$$

where, $\frac{I_o}{I_{c2}} = -1$, $\frac{I_{c2}}{I_{e2}} = -A_{I2}$, $\frac{I_{e2}}{I_{c1}} = -1$, $\frac{I_{c1}}{I_{b1}} = -A_{I1}$

& $\frac{I_{b1}}{I_s} = \frac{R_B}{R_B + R_{i1}}$

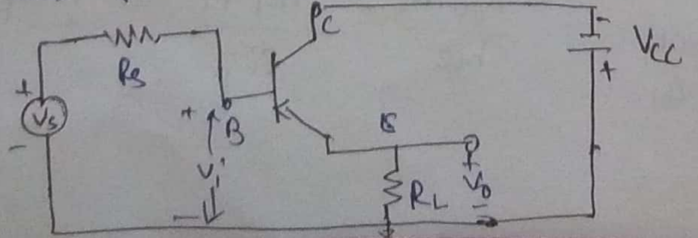
→ O/P resistance (R_o)

The $R_{o1} = R_{o2} = \infty$ for individual stage.

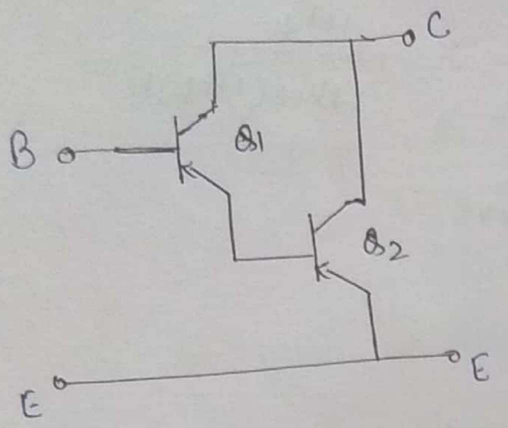
The overall o/p resistance is equal to $R_o = R_{o2} \parallel R_L \simeq R_L$.

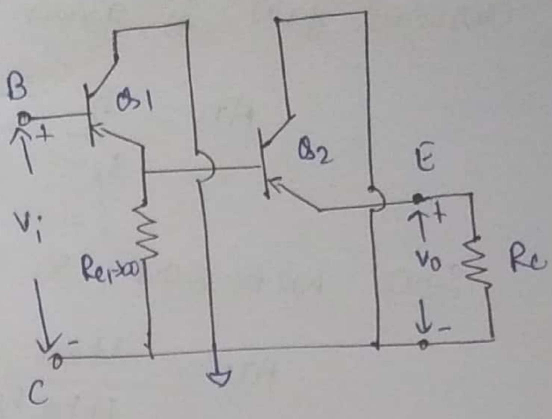
* Darlington pair:

Emitter follower provide I/P resistance lesser than 500 kΩ.

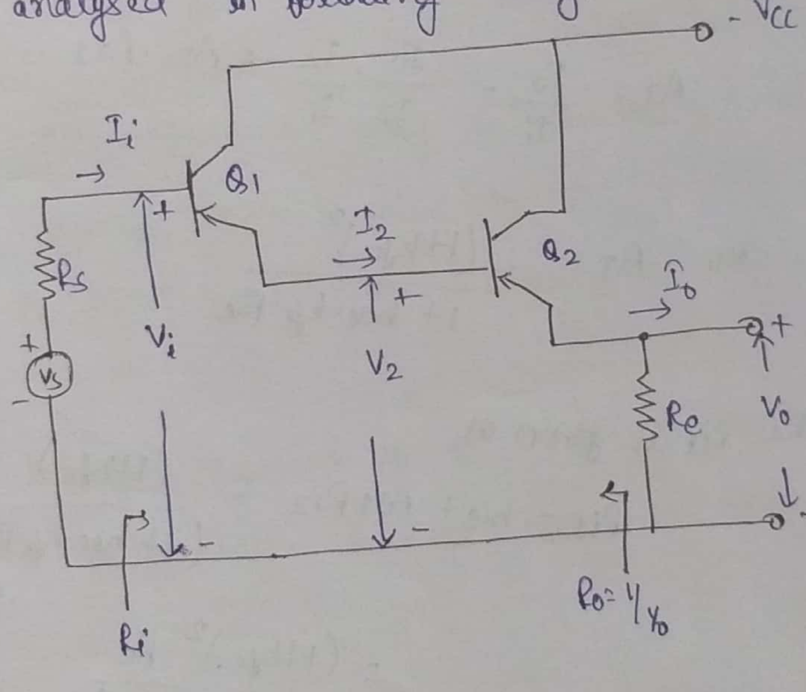


To achieve larger i/p impedances, darlington connection is used. Darlington circuit consists of two cascaded emitter followers with infinite emitter resistance in 1st stage.





It can be analysed in following way:



We assume, $h_{oe} \cdot R_e \leq 0.1$ & $h_{fe} \cdot R_e >> h_{ie}$

The current gain is given as,

$$A_{I2} = \frac{I_o}{I_2} \approx 1 + h_{fe}$$

The I/P impedance is given by,

$$R_{i2} \approx (1 + h_{fe}) R_e$$

For the first stage, load is R_{i2}. It doesn't satisfy $h_{oe} \cdot R_{i2} << 0.1$. So, we adapt for exact analysis. So current gain is given as,

$$A_{I1} = \frac{I_2}{I_i} = \frac{1+h_{fe}}{1+h_{oe} \cdot R_{i2}} = \frac{1+h_{fe}}{1+h_{oe}(1+h_{fe})R_e}$$

Since $h_{oe} \cdot R_e \leq 0.1$, so, we have,

$$A_{I1} \approx \frac{1+h_{fe}}{1+h_{oe} \cdot h_{fe} \cdot R_e}$$

The overall current gain is given by,

$$A_I = \frac{I_o}{I_i} = \frac{I_o}{I_2} \cdot \frac{I_2}{I_i} = A_{I2} \cdot A_{I1}$$

So, $$A_I \approx \frac{(1+h_{fe})^2}{1+h_{oe} \cdot h_{fe} \cdot R_e}$$

The R_{i1} is given as,

$$R_{i1} = h_{ie} + A_{I1} \cdot R_{i2} \approx \frac{(1+h_{fe})}{(1+h_{oe} \cdot h_{fe} \cdot R_e)} \times (1+h_{fe})R_e$$

$$= \frac{(1+h_{fe})^2 \cdot R_e}{(1+h_{oe} \cdot h_{fe} \cdot R_e)}$$

Both transistors are considered identical. So, we assume $h_{fe1} = h_{fe2} = h_{fe}$

The current in Q_2 is $(1+h_{fe})$ times of the current in Q_1.

So, $h_{ie1} \approx (1+h_{fe})h_{ie2}$

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The voltage gain is given as

$$A_v = 1 - \frac{h_{ie2}}{R_{i2}}\left(2 + h_{oe} \cdot h_{fe} \cdot R_e\right)$$ and

output impedance is

$$R_o \approx \frac{R_s}{(1+h_{fe})^2} + \frac{2h_{ie2}}{(1+h_{fe})}.$$

The darlington emitter follower has:

- → High i/p resistance
- → Voltage gain close to unity & less than single stage emitter follower
- → O/p impedance may be greater or smaller " depends up on value of R_s.

If $R_s = 0$, then R_o is twice of single stage.

Dis adv:

The leakage current of 1st stage is amplified by 2nd stage. So, overall leakage current may be high.