UNIT – IV
FILTERS

- Filters : classification of filters
- Constant K – low pass , High pass , Band pass and Band stop filter.
- Illustrative problems.

Ideal Filter:

An ideal filter would transmit signals under the pass band frequencies without attenuation and completely suppress the signal with attenuation band of frequencies with a sharp cutt-off profile.

A filter is an electrical network that can transmit signals within a specified frequency range. This frequency range is called 'pass band' and other frequency band where the signals are suppressed is called 'attenuation band' or 'stop band'.

Properties of filter:

a, Characterstic impedance: The characterstic impedance (Z_0) of a filter matches with the circuit to which it is connected throughout the pass band. This prevents reflection loss in the combination.

b, Pass band characterstics: The filter should have minimum attenuation in its pass band range and high attenuation in the stop band range.

c, Cutt-off frequency Charactorstics: The frequency that seperates the pass band and attenuation bands is known as cutt-off frequency.

→Applications of filters

1, T.V Broadcasting,

2, A.M detection,

3, Telephony,

4, Radio broadcasting,

5, Voice rejection.

6, audio Amplifiers.

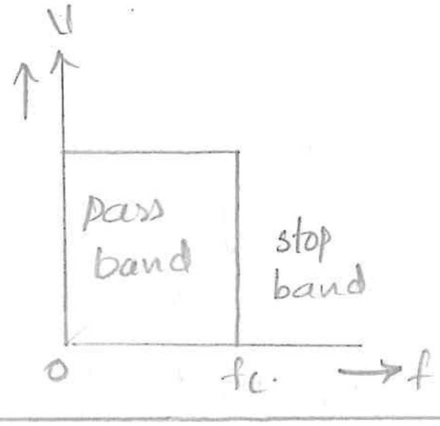
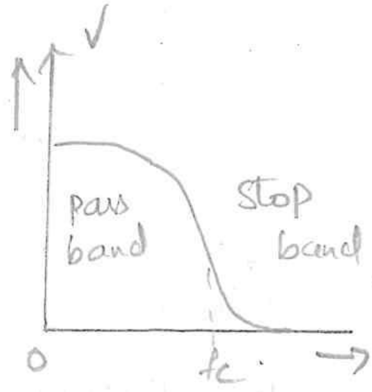
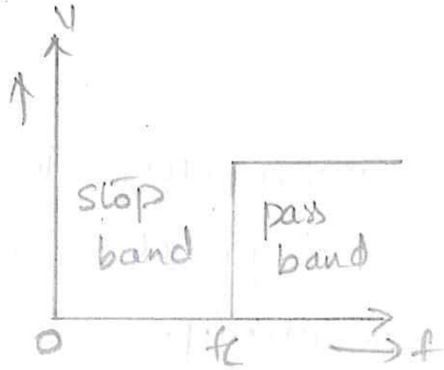
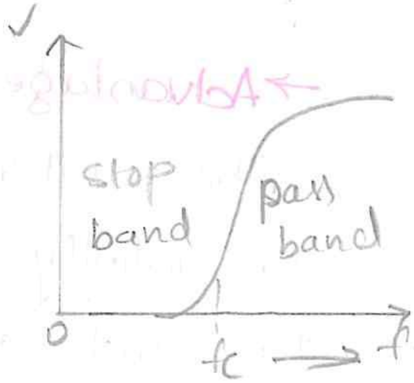
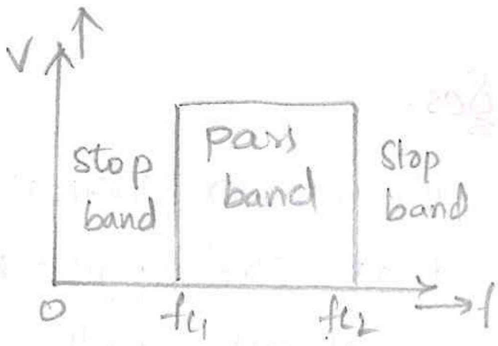
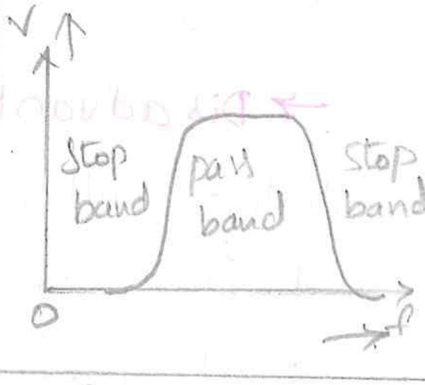
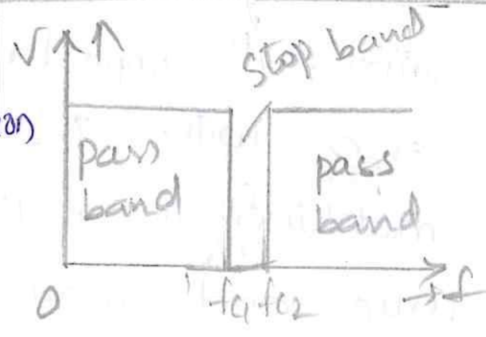
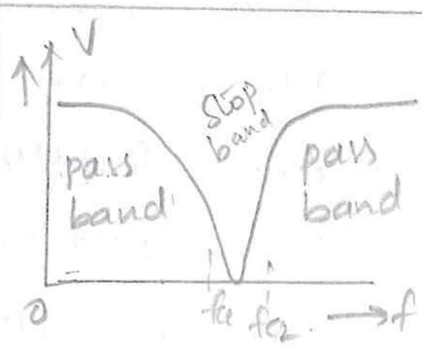
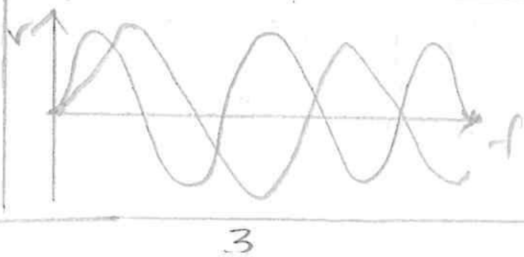
→ Classification of filters

a, Depending upon the relation between the arm impedances (i.e series arm impedance Z_1 and shunt arm impedance Z_2) the filters are classified as

(i, Constant K-filter or prototype filter.

(ii m-derived filter.

b, identifying their frequency characterstics the filters are classified as

	Type	Ideal characterstics	Practical characterstics
1	Low pass filter		
2	High pass filter.		
3	Band pass filter.		
4,	Band Stop or Band Elimaination or Notch filter		
5	All pass filter.		

→ Constant K filter (prototype Filter)

- In this filter, the series and shunt arm impedances (Z_1 and Z_2) are such that $Z_1 Z_2 = R_0^2$ $= K$ (constanst)

where R_0 = real number and independent of frequency.

- Any filter, where this relationship is maintained is known as constant K or prototype filter.

→ Advantages:

1, The filter attenuation in stop band tends to infinity at frequencies that are distant from the cutt-off frequency.

2, Easy to design

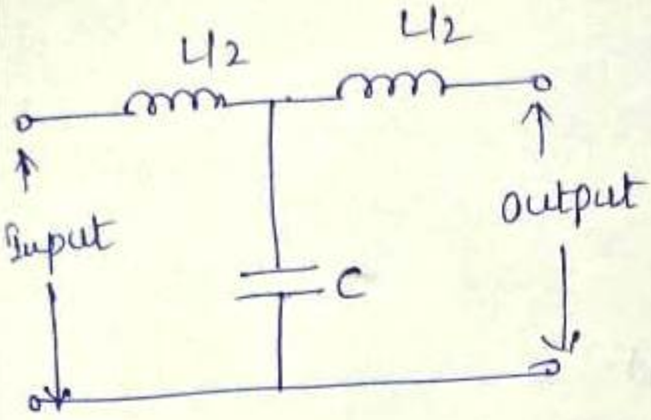
→ Disadvantages:

1, The filter attenuation increases very slowly and this means that the pass-band and stop-band are not always well defined.

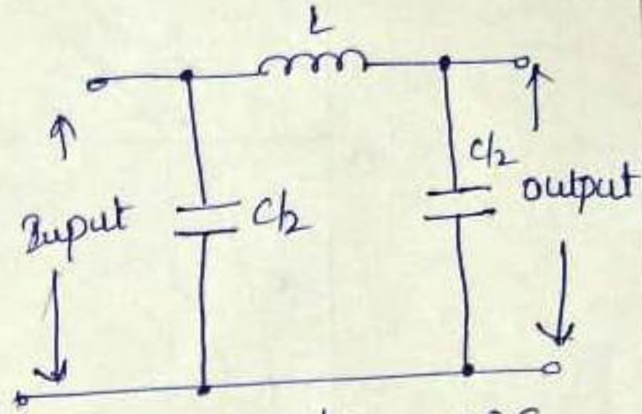
2, There are significant impedance variations with frequency even with the pass-band. If the impedance matching is required, then this can be an issue and may need to be overcome - possibly with the use of attenuators, although this reduces the signal level.

→ Constant K low pass filter (LPF)

→ It is the simplest type of filter which allows all frequencies, upto the specified cutt-off frequency, to pass through it and attenuates all other frequencies above the cutt off frequency.



T- section LPF



π - section LPF

In T or π section total series impedance

$$Z_1 = j\omega L \quad [\because X_1 = \omega L] \quad -(1)$$

and total shunt impedance

$$Z_2 = \frac{1}{j\omega C} = \frac{-j}{\omega C} \quad \left[\because X_2 = -\frac{1}{\omega C}\right] \quad -(2)$$

Multiplying eq (1) & eq (2)

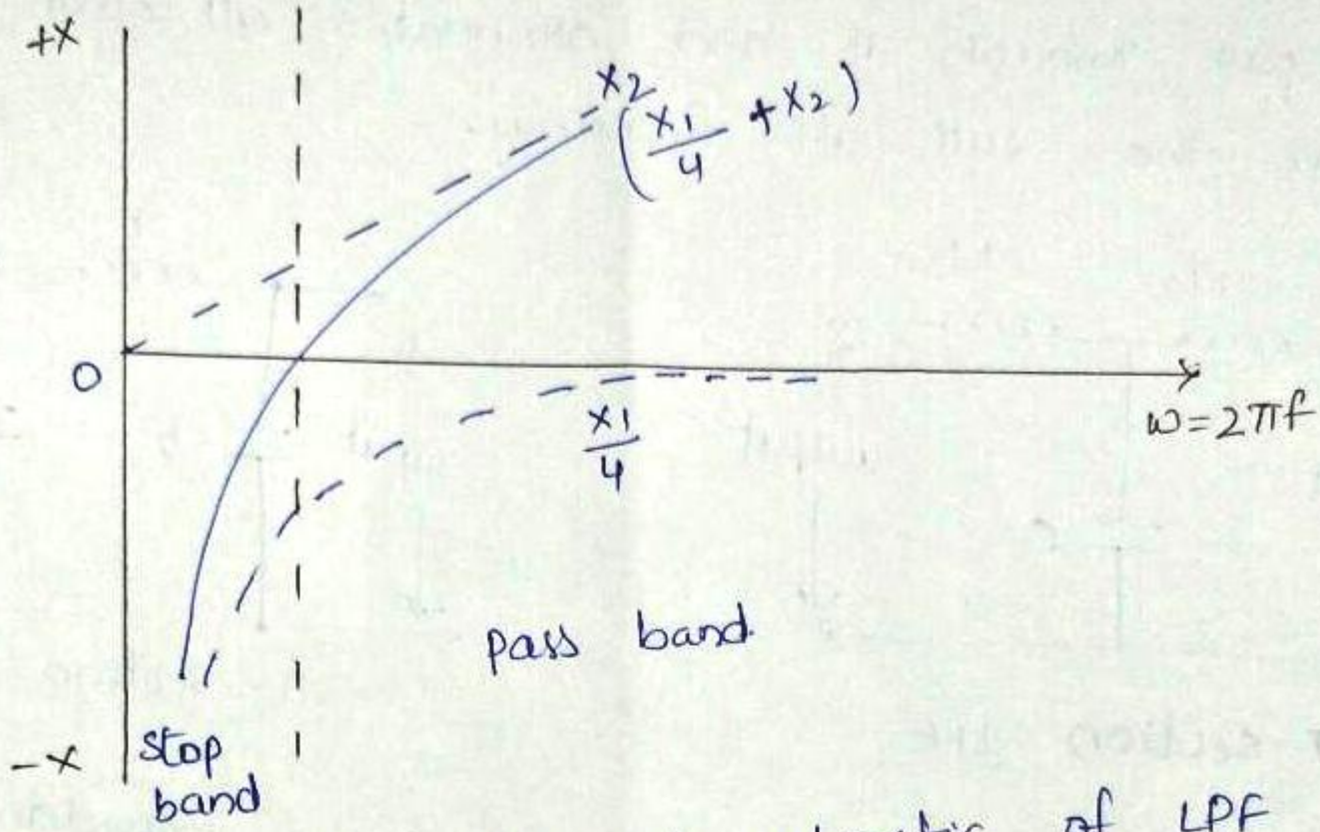
$$Z_1 Z_2 = j\omega L\left[-\frac{j}{\omega C}\right] = \frac{L}{C} = R_0^2 \quad -(3)$$

Again $\frac{Z_1}{4Z_2} = \frac{-\omega^2 LC}{4}$ — (4)

But from two port network

$$Z_{0T} = \sqrt{Z_1 Z_2\left[1 + \frac{Z_1}{4Z_2}\right]} \quad -(5)$$

where Z_{0T} = characterstic impedance of the π section of the reactance characterstic



Reactance frequency characterstic of LPF

Substituting the value of $Z_1 Z_2$ and $\frac{Z_1}{4Z_2}$ from in eq ⑤

$$Z_{0T} = \sqrt{\frac{L}{C}\left[1+\left[-\frac{\omega^2 LC}{4}\right]\right]}$$

$$= \sqrt{\frac{L}{C}} \cdot \sqrt{1-\frac{\omega^2 LC}{4}}$$

$$= R_0 \cdot \sqrt{1-\frac{\omega^2 LC}{4}} = R_0 \cdot \sqrt{1-\frac{\omega^2}{4/LC}}$$

$$Z_{0T} = R_0 \cdot \sqrt{1-\frac{\omega^2}{\omega_c^2}} \quad \because \text{where } \omega_c^2 = \frac{4}{LC}$$

$$= R_0 \cdot \sqrt{1-\left[\frac{f}{f_c}\right]^2}$$

$$Z_{OT} = \left[R_0 \sqrt{1 - \frac{\omega^2 LC}{4}} \right] \text{ is real if}$$

$$\frac{\omega^2 LC}{4} < 1 \text{ \&}$$

imaginary if $\frac{\omega^2 LC}{4} > 1$.

Hence the low pass filter possesses a cutt-off frequency given by

$$\omega_C = \frac{2}{\sqrt{LC}} \quad \text{or} \quad \boxed{f_C = \frac{1}{\pi\sqrt{LC}}} \quad \text{— (6)}$$

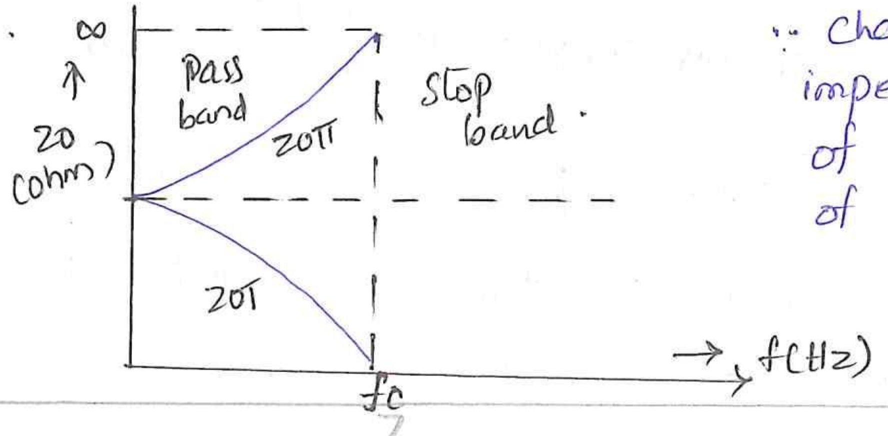
• For the π- section, the characterstic impedance is given by

$$Z_{O\pi} = \frac{Z_1 Z_2}{Z_{OT}} = \frac{R_0^2}{R_0 \sqrt{1 - \left[\frac{f}{f_c}\right]^2}}$$

$$= \frac{R_0}{\sqrt{1 - \left[\frac{f}{f_c}\right]^2}}$$

In pass band $f < f_c$: $Z_{O\pi}$ is real & positive

$f > f_c$: $Z_{O\pi}$ is imaginary



∴ Characterstic impedance profile of π & T sections of an LPF

- Attenuation (α) and phase shift (β) characterstics of LPF Section

In two port network

$$\cosh\gamma = 1+\frac{Z_1}{2Z_2} \quad —⑦$$

where $\gamma = \alpha + j\beta$

In a filter section $Z_1 = jx_1$ and $Z_2 = jx_2$, where x_1 and x_2 are the reactances of the series & shunt branch.

For an LPF section $x_1 = j\omega L$ $\quad x_2 = -\frac{j}{\omega C}$

Also, for the same LPF T section

$$\left[1+\frac{Z_1}{2Z_2}\right] = 1-\frac{\omega^2 LC}{2} \quad \left[\because \frac{Z_1}{4Z_2} = -\frac{\omega^2 LC}{4}\right]$$

$$\cosh\gamma = 1-\frac{\omega^2 LC}{2}$$

or $$\cosh(\alpha+j\beta) = 1-\frac{\omega^2 LC}{2} \quad —⑧$$

$$\cosh\alpha\cos\beta + j\sinh\alpha\sin\beta = 1-\frac{\omega^2 LC}{2} \quad —⑨$$

Equating the real & imaginary parts of above equation ⑨

$$\cosh\alpha\cos\beta = 1-\frac{\omega^2 LC}{2} \quad —⑩$$

$$\sinh\alpha\sin\beta = 0 \quad —⑪$$

- For pass band. In the pass band, there is no attenuation Hence $\alpha = 0$

$$\cos\beta = 1 - \frac{\omega^2 LC}{2} \quad -(12)$$

The value of $\cos\beta$ swings from $+1$ and -1. Hence $\cos\beta = 1$ gives, from eq (12) $\frac{\omega^2 LC}{2} = 0$ ie $\omega = 0$. This gives $f = 0$ i.e the lower cut off frequency of the filter operation.

If $\cos\beta = -1$ gives the higher cut off frequency.

From eq (12) $\frac{\omega^2 LC}{2} = 2$.

i.e $\omega_c = \frac{2}{\sqrt{LC}}$ giving $f_c = \frac{1}{\pi\sqrt{LC}}$

(i.e the higher cutt-off frequency).

from eq (12)

$$\cos\beta = 1 - \frac{\omega^2 LC}{2} = 1 - \frac{\omega^2}{2/LC} = 1 - 2\frac{\omega^2}{\omega_c^2}$$

$$\beta = \cos^{-1}\left[1 - 2\frac{\omega^2}{\omega_c^2}\right] \text{ radian} \quad -(13)$$

Also, $\cos\beta = 1 - 2\sin^2\frac{\beta}{2} = 1$

$$= 1 - \frac{\omega^2 LC}{2}.$$

$$2\sin^2\frac{\beta}{2} = 2\frac{\omega^2}{\omega_c^2}$$

$$\beta = 2\sin^{-1}\frac{\omega}{\omega_c} \text{ rad.}$$

- For attenuation band:

In attenuation band it is obvious that $\alpha \neq 0$. Hence it is justified to answer, from eq (11)

$$\beta = \pi$$

Then from eq (10) for $n = 1$

$$-\cosh\alpha = 1 - 2\frac{\omega^2}{\omega_c^2}$$

$$\alpha = \cosh^{-1}\left[2\frac{\omega^2}{\omega_c^2} - 1\right] \text{ Nepers} \quad (14)$$

Also $\quad - \cosh\alpha = 1 - 2\cosh^2\frac{\alpha}{2}$

$$1 - 2\cosh^2\frac{\alpha}{2} = 1 - 2\frac{\omega^2}{\omega_c^2}$$

which gives

$$\alpha = 2\cosh^{-1}\left[\frac{\omega}{\omega_c}\right] \text{ Nep} = 2\cosh^{-1}\left[\frac{f}{f_c}\right) \text{ Nepers}.$$

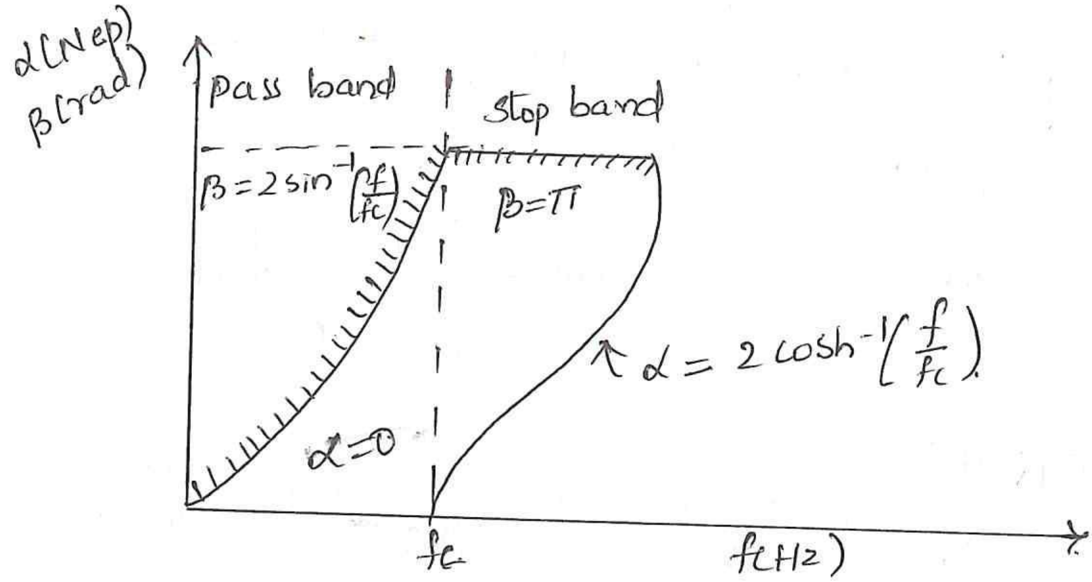


Fig: α and β characterstics of LPF section

→ Design of a prototype LPF section.

$$K = R_0 = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$L = \frac{R_0}{\pi f_c}$$

$$C = \frac{1}{\pi R_0 f_c}.$$

Problems

1, In a simple T section, a low pass filter has a design impedance R_o. Find $Z_{o\pi}$ at $0.9 f_c$.

sol

$$\frac{f}{f_c} = 0.9$$

for LPF

$$Z_{o\pi} = \frac{R_o}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} = \frac{R_o}{\sqrt{[1-(0.9)^2]}} = 2.3R_o$$

2, Design a constant K- lowpass filter having cut-off frequency 2.5 kHz and design resistance $R_o = 700\Omega$. Also find the frequency at which this filter produces attenuation of 19.1 dB. Find its characterstic impedances and phase constant at pass band and stop or attenuation band.

sol Given data

$$f_c = 2.5 KHz = 2500 Hz$$

$$R_o = 700\Omega$$

Attenuation $\alpha = 19.1 dB = \frac{19.1}{8.686} = 2.199$ nepers.

The design elements of LPF are L and C given by the relations

$$L = \frac{R_o}{\pi f_c} = \frac{700}{\pi \times 2500} = 89.127 mH$$

$$C = \frac{1}{\pi R_o f_c} = \frac{1}{\pi \times 700 \times 2500} = 0.182 \mu F.$$

Attenuation in attenuation band of a LPF is given by the relation

$$\alpha = 2\cosh^{-1}\left[\frac{f}{f_c}\right]$$

where f be the frequency at which this filter produces attenuation 19.1 dB or 2.199 nepers.

$$\frac{f}{f_c} = \cosh\left(\frac{\alpha}{2}\right)$$

$$\frac{f}{2500} = \cosh\left(\frac{2.199}{2}\right) = 1.66785$$

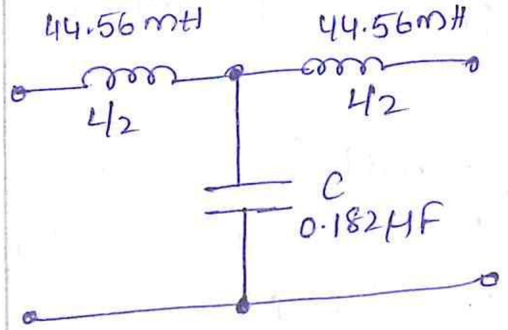
$$f = 2500 \times 1.66785 = 4.170 \text{kHz}.$$

The phase constant (β) in pass band is given by

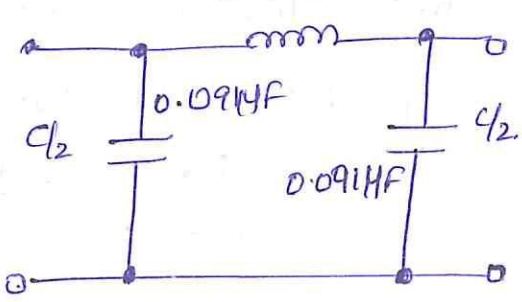
$$\beta = 2\sin^{-1}\left[\frac{f}{f_c}\right] = 2\sin^{-1}\left(\frac{4.170}{2.500}\right) \text{radians.}$$

But as $f > f_c$, β in pass band is imaginary or impracticable, but in attenuation band the phase constant (β) is given by

$$\beta = \pi \text{ radians}.$$



T-type section LPF



π-type LPF

3, A T- Section Low pass filter has series inductance 80mH and shunt capacitance 0.022μF. Determine the cutt-off frequency and nominal design impedance (Ro). Also design an equivalent π-section.

Sol From the Given data

$$L = 80\,mH \quad \text{and} \quad C = 0.022\,\mu F$$

Therefore, the cutt-off frequency f_c is given by

$$f_c = \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi\sqrt{(80\times10^{-3})(0.022\times10^{-6})}}\ Hz.$$

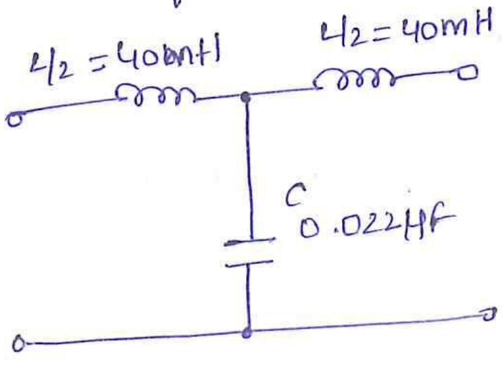
$$f_c = 7.587\ kHz.$$

The nominal design impedance (R_o) is given by

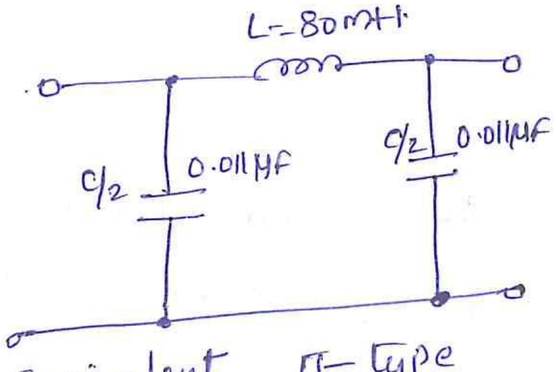
$$R_o = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{80\times10^{-3}}{0.022\times10^{-6}}}\ \Omega = 1.907\,k\Omega.$$

The required T and π - section filters are



T- type LPF



Equivalent π- type low pass filter.

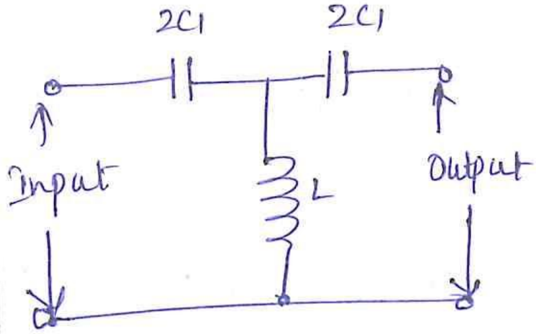
→ Constant K High pass Filter.

In both T and π sections, the total series impedance is given by

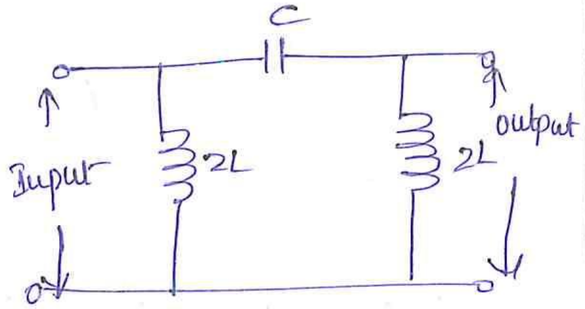
$$Z_1 = \frac{1}{j\omega C} \quad — (1)$$

and shunt impedance

$$Z_2 = j\omega L \quad — (2)$$



T-Type prototype HPF



Π-section prototype HPF.

Hence eq (1) × eq (2)

$$Z_1 Z_2 = \frac{1}{j\omega C} \times j\omega L = \boxed{\frac{L}{C} = R_0^2} \quad — (3)$$

$$\Rightarrow \boxed{R_0 = \sqrt{\frac{L}{C}}}$$

The characterstic impedance of a T section being given by

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

In case of constant - K HPF section

$$Z_{0T} = \sqrt{-\frac{1}{4\omega^2 C^2} + \frac{L}{C}} \quad — (4)$$

$$= \sqrt{\frac{L}{C}}\sqrt{1 - \frac{1}{4\omega^2 CL}}$$

$$= R_0\sqrt{1-\frac{1}{4\omega^2 LC}} \quad \text{--- (4)}$$

If $4\omega^2 LC > 1$: Z_{OT} is real and filter works in the pass band.

$4\omega^2 LC < 1$: Z_{OT} is imaginary, the filter lies in the attenuation band.

The cutt off frequency is given by

$$4\omega_c^2 LC = 1 \qquad \text{i.e} \quad \omega_c = \frac{1}{2\sqrt{LC}}$$

$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}} \quad \text{--- (5)}$$

Also $Z_{OT} = R_0\sqrt{1-\frac{1}{4\omega^2 LC}}$

$$= R_0\sqrt{1-\frac{\omega_c^2}{\omega^2}} = R_0\sqrt{1-\frac{f_c^2}{f^2}} \quad \text{--- (6)}$$

where $\omega_c^2 = 1/4LC$

Hence $Z_{O\pi} = \frac{R_0^2}{Z_{OT}} = \frac{R_0^2}{R_0\sqrt{1-\frac{\omega_c^2}{\omega^2}}} = \frac{R_0}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$

$$\boxed{Z_{O\pi} = \frac{R_0}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}}} \quad \text{--- (7)}$$

The relative frequency characterstic has been shown

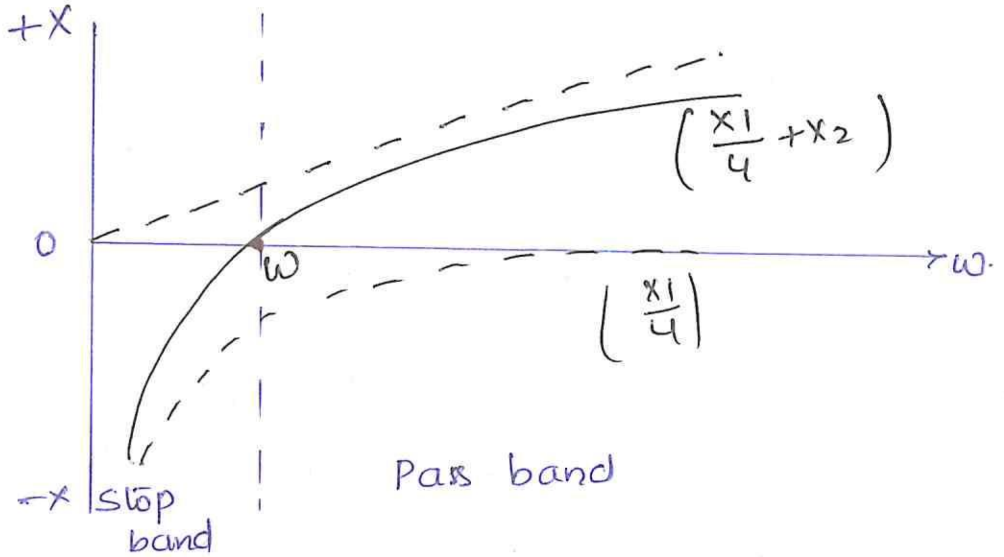
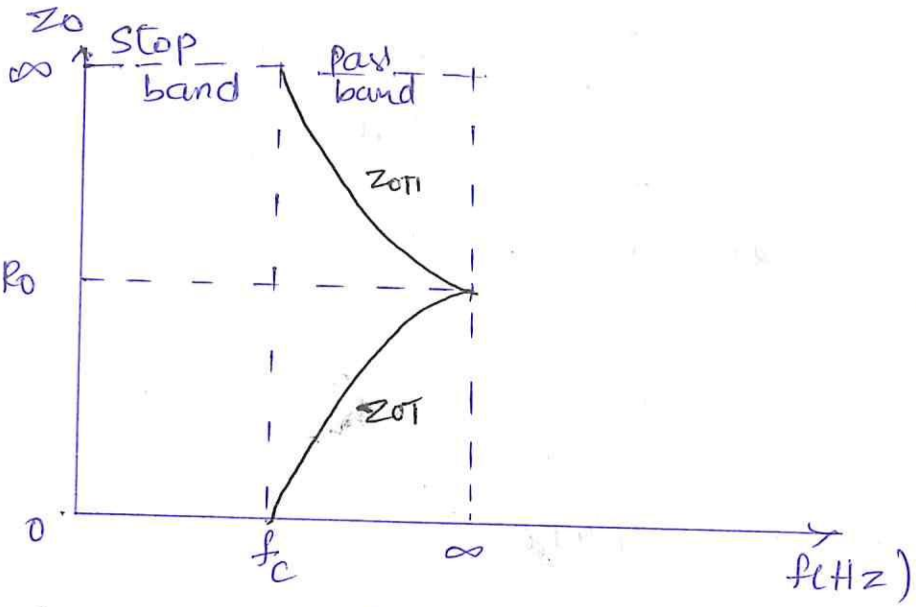


Fig: Reactance frequency characterstic of HPF



Zo profile of HPF

→ Attenuation (α) and the pase shift (β) characterstic of HPF

For a T network, the propogation constant (γ) is given by the relation

$$\cosh \gamma = 1 + \frac{Z_1}{2Z_2} \quad \text{———} \quad (8)$$

But $1 + \frac{Z_1}{2Z_2} = 1 + \frac{1/j\omega C}{2j\omega L} = 1 + \frac{1}{j\omega C} \times \frac{1}{2j\omega L}$

$$\boxed{\cosh \gamma = 1 - \frac{1}{2\omega^2 LC}} \quad — (9)$$

But $\gamma = \alpha + j\beta$

$$\cosh(\alpha + j\beta) = 1 - \frac{1}{2\omega^2 LC}$$

$$\cosh\alpha \cos\beta + j \sinh\alpha \sin\beta = 1 - \frac{1}{2\omega^2 LC} \quad — (10)$$

Equating Real and imaginary eq.

$$\cosh\alpha \cos\beta = 1 - \frac{1}{2\omega^2 LC} \quad — (10.a)$$

$$j \sinh\alpha \sin\beta = 0 \quad — (10.b)$$

from eq 10.b either $\alpha = 0$ or $\beta = n\pi$.

• For pass band

In pass band $\alpha = 0$ (as there is no attenuation. Phase shift exists giving the value of

$$\cos\beta = 1 - \frac{1}{2\omega^2 LC} \quad \text{when } \alpha = 0.$$

It is evident from $\cos\beta = 1$ gives cutt off frequency (as with $\cos\beta = 1$, from equation $\cos\beta = 1 - \frac{1}{2\omega^2 LC}$,

$$\omega = \infty \text{ i.e } f = \infty)$$

This can be refered as cutt-off frequency.

when $\cos\beta = -1$ in eq 10-a

$$-1 = 1 - \frac{1}{2\omega^2 LC}$$

$$\omega_c^2 = \frac{1}{4LC}$$ [$\because$ ω_c represents a particular value of ω when $\cos\beta = -1$].

Hence, the lower cutt - off frequency

$$\omega_c = \frac{1}{2\sqrt{LC}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

Also from $\cos\beta = 1 - \frac{1}{2\omega^2 LC}$

$$\cos\beta = 1 - \frac{2\omega_c^2}{\omega^2}$$

$$\beta = \cos^{-1}\left[1 - \frac{2\omega_c^2}{\omega_c}\right] = \cos^{-1}\left[1 - 2\left[\frac{f_c}{f_c}\right]^2\right].$$

Again $\cos\beta = 1 - 2\sin^2\frac{\beta}{2} = 1 - 2\frac{\omega_c^2}{\omega^2}$

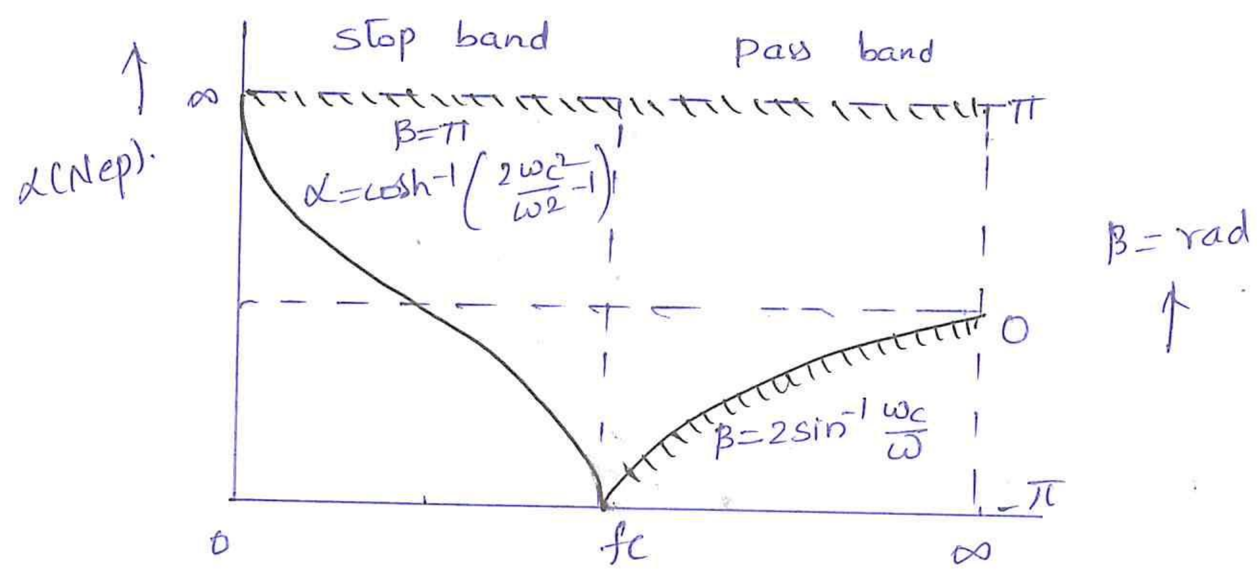
$$\boxed{\beta = 2\sin^{-1}\frac{\omega_c}{\omega}} \text{ radains} \quad \text{——} \ (11)$$

- For Attenuation Band:

utilizing $\beta = \pi$ in eq 10.b, α being not equal to zero ($\alpha \neq 0$)

$$\cosh\alpha = -1 + \frac{1}{2\omega^2 LC} = -1 + 2\frac{\omega_c^2}{\omega^2}$$

$$\alpha = \cosh^{-1}\left[2\frac{\omega_c^2}{\omega} - 1\right] \quad \text{——} \ (12)$$



α and β profile of HPF.

(1) Design a T and π section constant -K high pass filter having cutt-off frequency of 12KHz and nominal impedance $R_0 = 500\Omega$. Also find (i) its characteristic impedance and phase constant at 24KHz and (ii) attenuation at 4KHz.

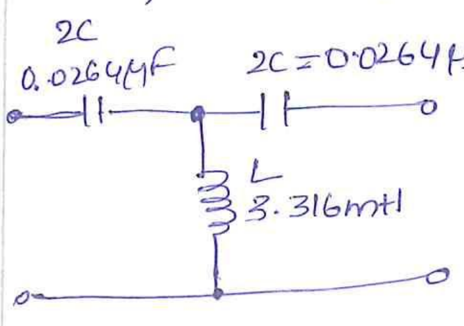
Sol The shunt arm inductance L is given by

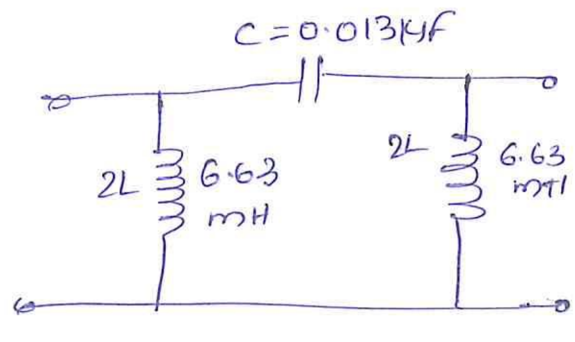
$$L = \frac{R_0}{4\pi f_c} = \frac{500}{4\times\pi\times12\times10^3} = 3.316\,mH.$$

The series arm capacitance C is given by

$$C = \frac{1}{4\pi R_0 f_c} = \frac{1}{4\times\pi\times500\times12\times10^3} = 0.0013\mu F$$

Hence, the require T and π-section high pass filter.





(i) If $f = 24KHz$, The characterstic impedances of T and π- section are given by

$$Z_{OT} = R_o\sqrt{1-\left[\frac{f_c}{f}\right]^2}$$

$$= 500\sqrt{1-\left[\frac{12\times10^3}{24\times10^3}\right]^2} = 433.013\,\Omega.$$

$$Z_{O\pi} = \frac{R_o}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$$

$$= \frac{500}{\sqrt{1-\left[\frac{12\times10^3}{24\times10^3}\right]^2}} = 577.350\,\Omega$$

The phase constant (β) in pass band is given by

$$\beta = 2\sin^{-1}\left[\frac{f_c}{f}\right]$$

$$= 2\sin^{-1}\left(\frac{12\times10^3}{24\times10^3}\right) = 60^\circ.$$

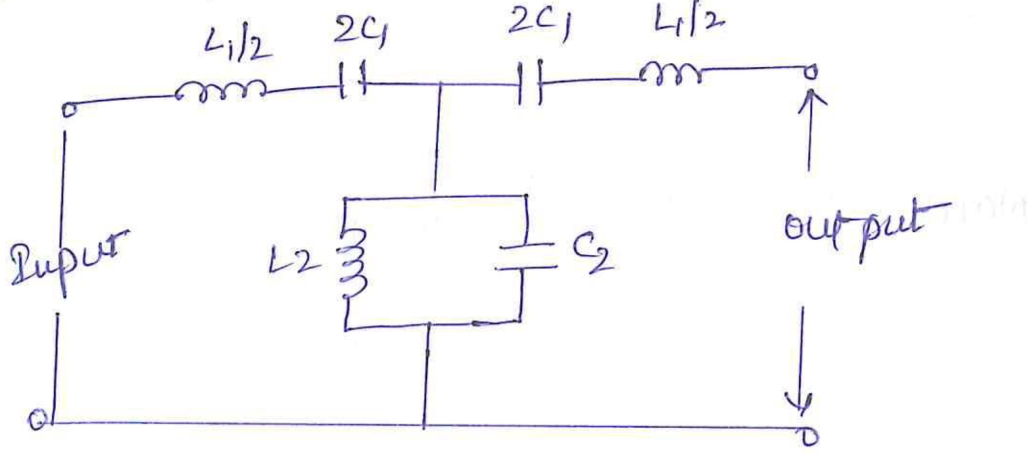
(ii) In attenuation band, α is given by at frequency 4 KHz.

$$\alpha = 2\cosh^{-1}\left[\frac{f_c}{f}\right] \text{neper}$$

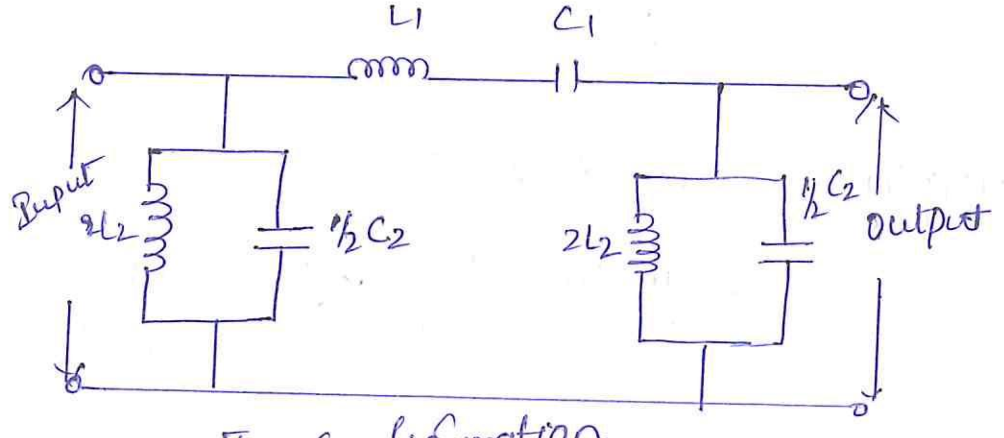
$$= 2\cosh^{-1}\left[\frac{12\times10^3}{4\times10^3}\right] \text{neper} = 3.525 \text{ nepers.}$$

→ Band Pass Filter (BPF)

- This filter is a combination of two parallel tuned circuits. At resonance, the shunt arm behaves as a rejector circuit offering a very high impedance in the shunt arm while the series arm at resonance offers least impedance (acceptor circuit); thus, any electric signal, incident at the input of the filter allows its transmission through it within the pass band.



T- configuration



π- configuration.

Hence, at resonance, for each of the series and shunt arm,

$$\omega_0^2 L_1 C_1 = 1 \quad \text{—} (1)$$

$$\omega_0^2 L_2 C_2 = 1 \quad \text{—} (2)$$

[∵ at resonance $f_0 = \frac{1}{2\pi\sqrt{LC}}$ and $\omega_0 = \frac{1}{LC}$

f_0 & ω_0 being the resonance frequency in Hz or rad/sec]

Comparing eq (1) & (2)

$$\omega_0^2 L_1 C_1 = \omega_0^2 L_2 C_2$$

$$\frac{L_1}{C_2} = \frac{L_2}{C_1} \quad \text{—} (3)$$

Now $Z_1 = j\omega L_1 + \frac{1}{j\omega C_1}$

$$= j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right] \quad \text{—} (4)$$

$$Z_2 = \frac{j\omega L_2\left(\frac{1}{j\omega C_2}\right)}{j\omega L_2 + \frac{1}{j\omega C_2}} = j\,\frac{\omega L_2}{1-\omega^2 L_2 C_2}. \quad \text{—} (5)$$

Multiplying eq (4) & (5)

$$Z_1 Z_2 = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right] \times j\,\frac{\omega L_2}{1-\omega^2 L_2 C_2}$$

$$= \frac{L_2(\omega^2 L_1 C_1 - 1)}{C_1(\omega^2 L_2 C_2 - 1)} \quad \text{—} (6)$$

finally $Z_1 Z_2 = \frac{L_2}{C_1}$ [i.e $= \frac{L_1}{C_2}$ by using eq (3)]

$= R_0^2$ (say)

$Z_1 Z_2 = R_0^2$, R_0 being a real quantity. —— (7)

The characterstic impedance Z_0 of any T section being given by

$$Z_{0T}^2 = \frac{Z_1^2}{4} + Z_1 Z_2$$

normalisation for Z_1 yields

$$\frac{Z_{0T}^2}{Z_1} = \frac{Z_1}{4} + Z_2$$

$$\frac{Z_{0T}^2}{Z_1 Z_2} = \frac{Z_1}{4Z_2} + 1 \qquad \text{—— (8)}$$

[$\because$ Dividing both sides by Z_2]

From eq (8), it can be observed that the limiting condition ($Z_{0T} = 0$) is achieved if

$$\frac{Z_1}{4Z_2} = -1$$

Thus for, $\frac{Z_1}{4Z_2} = -1$

$$Z_1 = -4Z_2.$$

$$Z_1 . Z_1 = -4Z_2 . Z_1$$

$$Z_1^2 = -4Z_1 Z_2$$

$$= -4Z_0^2 \Rightarrow -4R_0^2.$$

$$Z_1 = \pm j2R_0 \qquad \text{—— (9)}$$

Equation (9) indicates that Z_1 is either positive or negative at the limiting condition.

It shows that Z_1 is positive for a particular frequency f_1 and Z_1 is negative for another particular frequency, f_2.

- Substituting the value of Z_1 from eq (4) in eq (9)

$$j\left[\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 C_1}\right] = j2R_0$$

[taking +ve value of Z_1 and corresponding frequency at f_1]

$$\omega^2 L_1 C_1 - 1 = 2R_0\omega_1 C_1$$

$$\omega_1^2 L_1 C_1 - 2R_0\omega_1 C_1 - 1 = 0$$

$$\omega_1 = \frac{2R_0 C_1 \pm \sqrt{4R_0^2 C_1^2 + 4L_1C_1}}{2L_1C_1}$$

$$f_1 = \frac{2R_0 C_1 \pm \sqrt{4R_0^2 C_1^2 + 4L_1C_1}}{2\pi \times 2L_1C_1}$$

$$= \frac{2C_1 R_0 + \sqrt{\left[R_0^2 + \frac{L_1}{C_1}\right]4C_1^2}}{2\pi \times 2L_1C_1}$$

$$= \frac{2C_1\left[R_0 + \sqrt{R_0^2 + \frac{L_1}{C_1}}\right]}{2\pi \times 2L_1C_1}$$

$$f_1 = \frac{R_0 + \sqrt{R_0^2 + \frac{L_1}{C_1}}}{2\pi L_1} \quad \text{—— (10)-a}$$

The equation (10.a) provides the particular one of the cutt-off frequencies of the BPF when Z_1 behaves as a positive impedance.

• Assuming f_2 to be other particular frequency when Z_1 behaves as negative impedance, in a similar way

$$j\left[\frac{\omega_2^2 L_1C_1 - 1}{\omega_2 C_1}\right] = -j2R_0$$

$$j\,\omega_2^2 L_1C_1 - 1 = -j2R_0\,\omega_2 C_1$$

$$\omega_2^2 L_1C_1 - 1 = -2R_0\omega_2 C_1$$

$$\omega_2^2 L_1C_1 - 2R_0\omega_2C_1 - 1 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This gives

$$\omega_2 = \frac{-2R_0C_1 \pm \sqrt{4R_0^2C_1^2 + 4L_1C_1}}{2L_1C_1} \quad \text{—— (11)}$$

$$f_2 = \frac{-2R_0C_1 \pm \sqrt{4R_0^2C_1^2 + 4L_1C_1}}{2\pi \times 2L_1C_1}$$

$$= \frac{-R_0 \pm \sqrt{R_0^2 + \frac{L_1}{C_1}}}{2\pi L_1}$$

$$f_2 = -\left[\frac{R_0 + \sqrt{R_0^2 + \frac{L_1}{C_1}}}{2\pi L_1} \right] \quad \text{—— (11-a)}$$

Eq (11-a) gives the second cutt-off frequency of the B.P.F.

- As negative frequency does not carry any physical meaning hence the sign of the root of f_1 and f_2 both are taken to be positive.

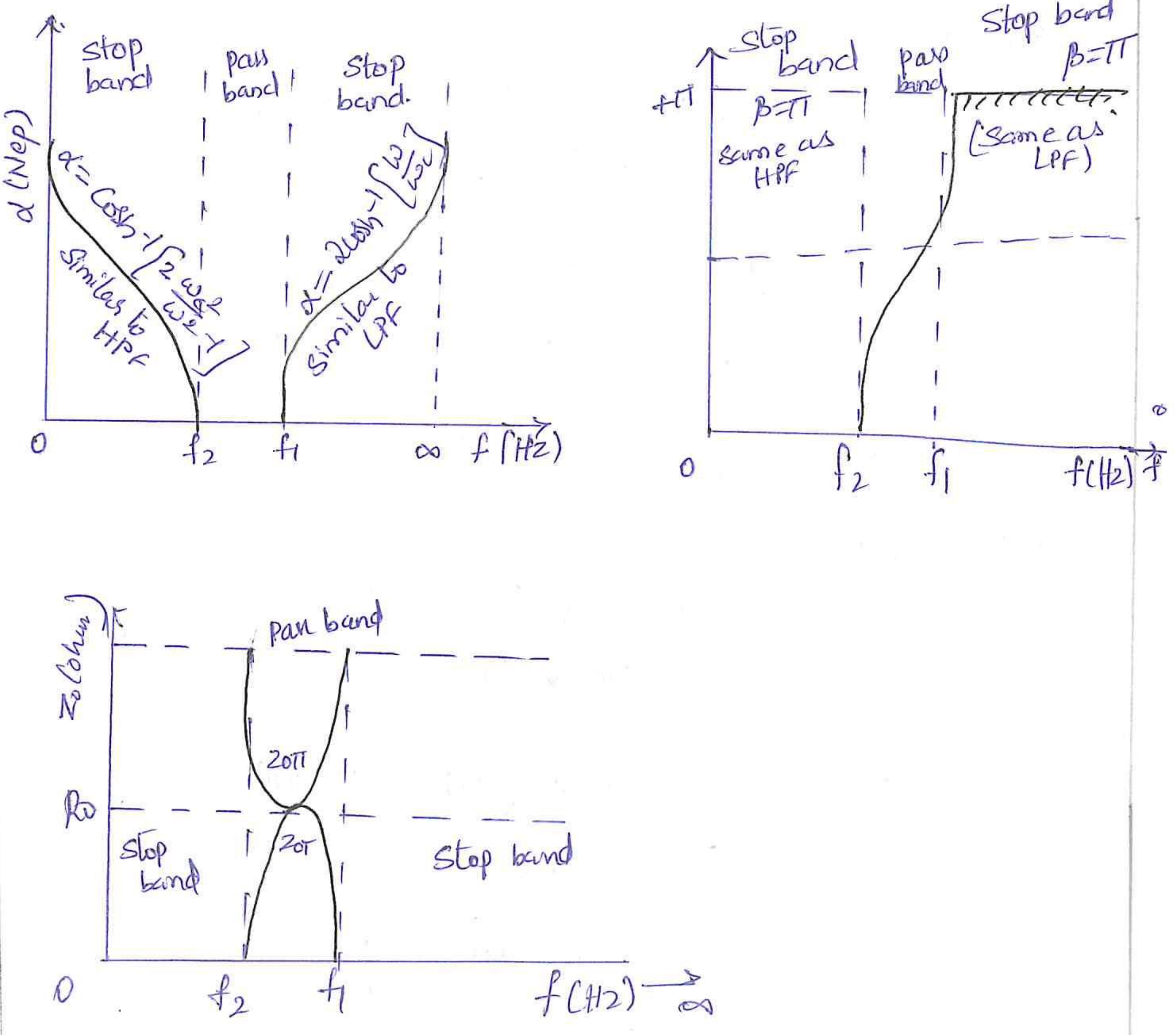


Figure: characterstic of BPF section.

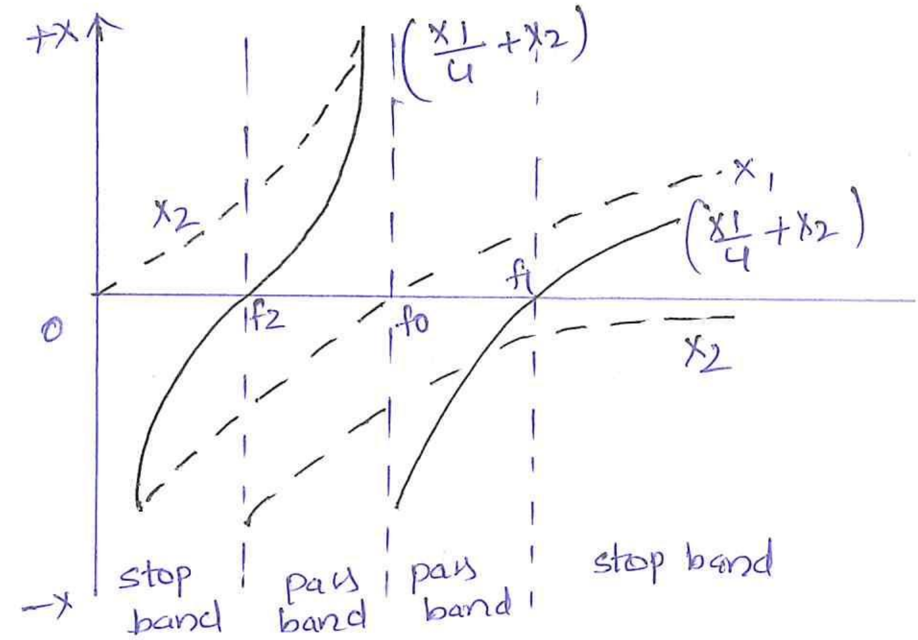


Figure: Rectance frequency characterstics of BPF

→ Design of Prototype Band Pass filter.

Taking $Z_1 = -j2R_0$ & substituting the values of Z_1

$$\frac{\omega_2^2 L_1 C_1 - 1}{\omega_2 C_1} = -2R_0$$

[∵ ω_2 being the frequency in rad/sec when $Z_1 = -j2R_0$]

$$\frac{1-\omega_2^2 L_1 C_1}{\omega_2 C_1} = 2R_0$$

$$1-\omega_2^2 L_1 C_1 = 2R_0\omega_2 C_1$$

$$1-\frac{\omega_2^2}{\omega_0^2} = 2R_0\omega_2 C_1 \qquad \left[\because L_1C_1 = L_2C_2 = \frac{1}{\omega_0^2}\right]$$

$$1-\frac{f_2^2}{f_0^2} = 2R_0 \times 2\pi f_2 \times C_1$$

[∵ f_2 in Hz corresponds to ω_2 in rad/sec]

$$1 - \frac{f_2^2}{f_1 f_2} = 4\pi f_2 R_0 C_1$$

$$[\because f_0^2 = f_1 f_2]$$

As the magnitude of Z_1 remains same at f_1 & f_2 the only difference being the change of sign between f_1 and f_2 hence it is justified to write

$$Z_1 \text{ at } f_1 = Z_1 \text{ at } f_2$$

$$\left[j\omega_1 L_1 + \frac{1}{j\omega_1 C_1} \right] = -\left[j\omega_2 L_1 + \frac{1}{j\omega_2 C_1} \right]$$

$$\left[\omega_1 L_1 - \frac{1}{\omega_1 C_1} \right] = -\left[\omega_2 L_1 - \frac{1}{\omega_2 C_1} \right]$$

$$\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 C_1} = -\frac{\omega_2^2 L_1 C_1 - 1}{\omega_2 C_1}$$

$$\left[\frac{\omega_1^2}{\omega_0^2} - 1 \right] / \omega_1 = -\left[\frac{\omega_2^2}{\omega_0^2} - 1 \right] / \omega_2$$

$$\left[\frac{f_1^2}{f_0^2} - 1 \right] / f_1 = -\left[\frac{f_2^2}{f_0^2} - 1 \right] / f_2$$

$$1 - \frac{f_1^2}{f_0^2} = \frac{f_1}{f_2} \left[\frac{f_2^2}{f_0^2} - 1 \right]$$

$$1 - \frac{f_1^2}{f_0^2} = \frac{f_1 f_2}{f_0} - \frac{f_1}{f_2}$$

$$1 + \frac{f_1}{f_2} = \frac{f_1 f_2}{f_0} + \frac{f_1^2}{f_0^2}$$

$$= \frac{f_1 f_2 + f_1^2}{f_0^2}$$

$$\frac{f_1+f_2}{f_2} = \frac{f_1 f_2 + f_1^2}{f_0^2}$$

$$f_0^2 = \frac{(f_1 f_2 + f_1^2)}{f_1+f_2} = f_1 f_2.$$

$$\boxed{f_0 = \sqrt{f_1 f_2}.} \quad \text{—— (12)}$$

$$1 - \frac{f_2}{f_1} = 4\pi f_2 R_0 C_1$$

$$f_1 - f_2 = 4\pi f_1 f_2 R_0 C_1$$

$$\boxed{C_1 = (f_1 - f_2) \times \frac{1}{4\pi f_1 f_2 R_0}} \text{ farad} \quad \text{—— (13)}$$

Again from $L_1 C_1 = \frac{1}{\omega_0^2}$

$$L_1 = \frac{1}{\omega_0^2 C_1} = \frac{1}{(2\pi f_0)^2 \times C_1}$$

$$= \frac{1}{4\pi^2 f_1 f_2 (f_1 - f_2) \frac{1}{4\pi f_1 f_2 R_0}}$$

—— (14)

$$L_1 = \frac{R_0}{\pi (f_1 - f_2)} \text{ Henry}$$

Also from

$$R_0 = \sqrt{\frac{L}{C}}, \quad L = R_0^2 C. \quad \text{i.e} \quad L_2 = R_0^2 C_1$$

$$\left[\because \frac{L_2}{C_1} = \frac{L_1}{C_2} = R_0^2\right]$$

$$L_2 = R_0^2 C_1 = (f_1 - f_2) \frac{1}{4\pi f_1 f_2 R_0} \cdot R_0^2$$

$$\boxed{L_2 = \frac{R_0 (f_1 - f_2)}{4\pi f_1 f_2} \text{ Henry}} \quad \text{—— (15)}$$

Similarly $C_2 = \frac{L_1}{R_0^2}$ gives

$$C_2 = \frac{R_0/\pi(f_1-f_2)}{R_0^2}$$

$$\boxed{C_2 = \frac{1}{\pi R_0 (f_1-f_2)} \text{ farad}}$$

1. Design a prototype bandpass filter section (T & π) having cut-off frequencies of 3000Hz and 6000 HZ and nominal characterstics impedance of 600Ω. Also find the resonant frequency of shunt arm or series arm.

Sol From the given data

$f_2 = 3000\,Hz \quad f_1 = 6000\,Hz$ and $R_0 = 600\,\Omega$

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} = \frac{600}{\pi(6000-30000)} H.$$

$$= 63.662\text{ mH}$$

Hence $\frac{L_1}{2} = \frac{63.662}{2} = 31.831\text{ mH}.$

The series arm capacitance (C_1) is given by

$$C_1 = \frac{(f_1-f_2)}{4\pi R_0 f_1 f_2}.$$

$$= \frac{6000-3000}{4\times\pi\times600\times3000\times6000}$$

$= 0.022\mu F.$

Hence, $2C_1 = 0.044\mu F.$

Again, the shunt arm ~~impedance~~ inductance (L_2) and capacitance (C_2) are given by

$$L_2 = \frac{R_0(f_1 - f_2)}{4\pi f_1 f_2}$$

$$= \frac{600(6000-3000)}{4\pi \times 6000 \times 3000} \, F$$

$$L_2 = 7.96 mH$$

Hence $2L_1 = 15.92 mH$

$$C_2 = \frac{1}{\pi R_0 (f_1 - f_2)}$$

$$= \frac{1}{\pi \times 600\,(6000-3000)} \, F$$

$$= 0.177 \mu F$$

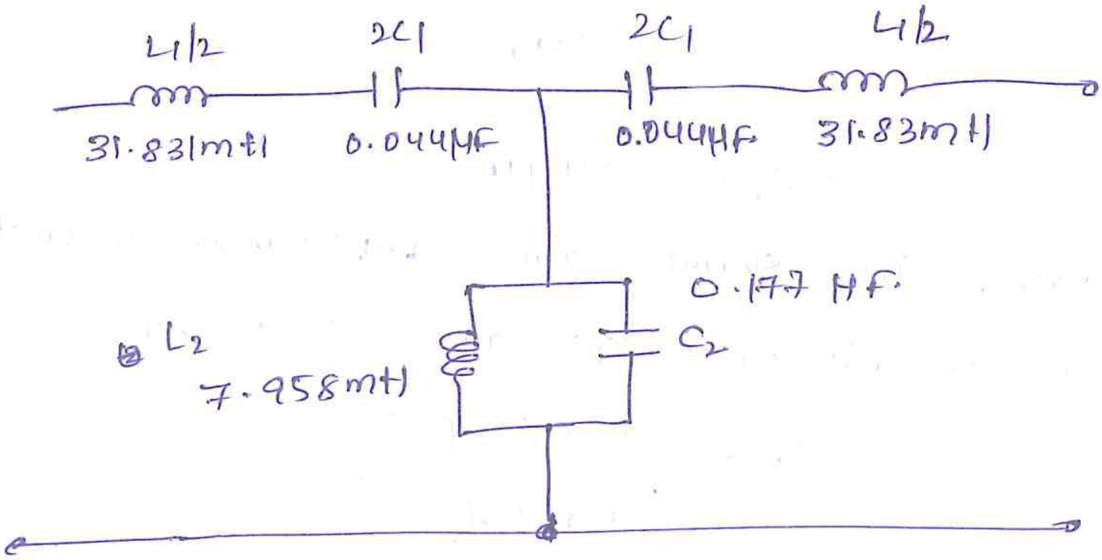
Hence $\frac{C_2}{2} = 0.0885 \mu F$

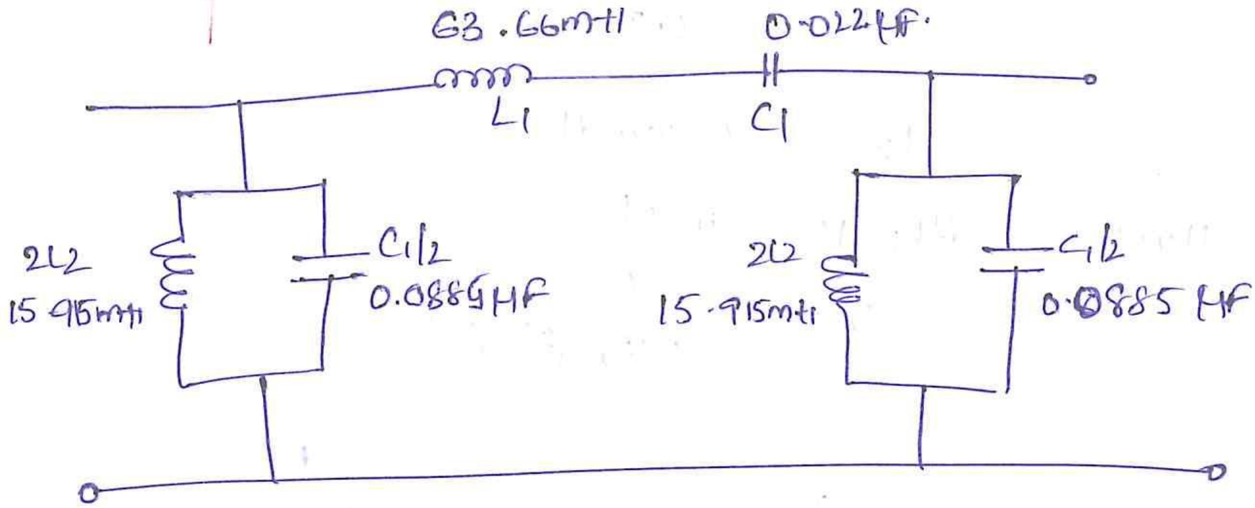
Now, the resonant frequency (f_0) of the band pass filter is given by

$$f_0 = \sqrt{f_1 f_2} = \sqrt{3000 \times 6000}$$

$$= 4242.640 Hz.$$

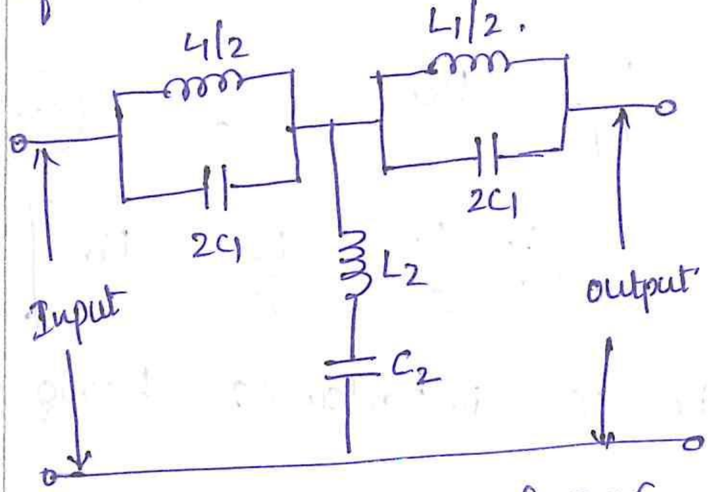
The band pass filter (T and π - section) are shown as


L1/2
2C1
2C1
L1/2
31.831mH
0.044μF
0.044μF
31.83mH
0.177 μF
L2
C2
7.958mH

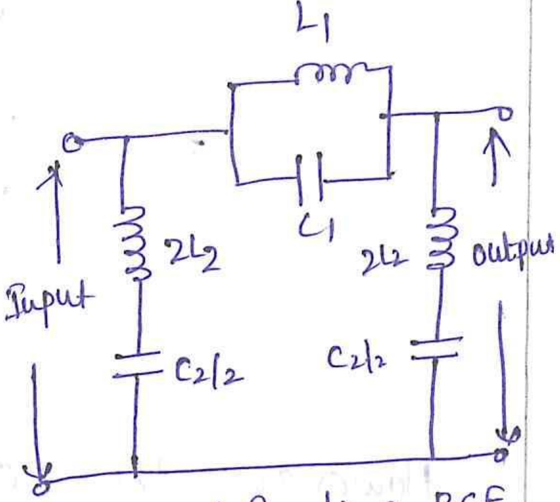

63.66mH
0.022μF
L1
C1
2L2
15.95mH
C1/2
0.0885μF
2L2
15.915mH
C1/2
0.0885 μF

→ Analysis of a Prototype Band stop filter (BSF)

- A band stop filter, in its simplest form is a tandem of a low pass filter with a high pass filter.
- Here, the cutt off frequency of the ~~low~~ high pass filter is higher than the cutt-off frequency of the low pass filter.
- The overlapping attenuation band of the two ~~band~~ filters then constitutes the stop band of the band stop filter.
- Z_1 becomes series impedance of a parallel tuned circuit and Z_2 is the shunt impedance of series tuned circuit.



T- configuration of BSF



Π - configuration BSF

- Operation is initiated when resonance occurs so that shunt impedance (Z_2) becomes minimum and series impedance (Z_1) becomes maximum. Thus incident signal is blocked at this particular condition

Thus at resonance.

$$\omega_0^2 L_1 C_1 = 1 \qquad \text{—①}$$

$$\omega_0^2 L_2 C_2 = 1 \qquad - (2)$$

$$L_1 C_1 = L_2 C_2 = \frac{1}{\omega_0^2} \qquad - (3)$$

However

$$Z_1 = 2\left[\frac{\frac{j\omega L_1}{\cancel{2}}\left[\frac{1}{2j\omega C_1}\right]}{j\frac{\omega L_1}{2} + \frac{1}{2j\omega C_1}}\right] = j\,\frac{\omega L}{1-\omega^2 L_1 C_1} \qquad - (4)$$

$$Z_2 = j\omega L_2 + \frac{1}{j\omega C_2} = j\left[\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right] \qquad - (5)$$

Therefore

$$Z_1 Z_2 = j\,\frac{\omega L_1}{1-\omega^2 L_1 C_1} \times j\left(\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right)$$

$$= \frac{L_1}{C_2} = \frac{L_2}{C_1} = R_0^2 \qquad - (6.9)$$

$$\left[\because \frac{L_1}{C_2} = \frac{L_2}{C_1} = R_0^2\right].$$

However the characterstic impedance being given by $Z_{0T}^2 = \frac{Z_1^2}{4} + Z_1 Z_2$.

normalisation by Z_1 and division by Z_2 gives.

$$\frac{Z_{0T}^2}{Z_1 Z_2} = \frac{Z_1}{4Z_2} + 1$$

$$\frac{Z_1}{4Z_2} = -1$$

gives the critical condition as with

$$\frac{Z_1}{4Z_2} = -1$$

$$Z_{OT} = 0$$

$$\frac{Z_1}{4Z_2} = -1$$

$$Z_1 = -4Z_2$$

$$Z_1^2 = -4Z_1Z_2$$

$$= -4R_0^2$$

$$\therefore \quad Z_1 = \pm j2R_0$$

Citing similar resoning as done for the BPF, it can be commented that Z_1 will be positive with f_1 and Z_2 will be negative with system frequency f_2

considering Z_1 to be positive,

$$Z_1 = +j2R_0$$

$$j\frac{\omega_1 L_1}{1-\omega_1^2 L_1 C_1} = j2R_0$$

$$\omega_1 L_1 = 2R_0(1-\omega_1^2 L_1 C_1)$$

$$\omega_1^2 L_1 C_1 \cdot 2R_0 + \omega_1 L_1 - 2R_0 = 0$$

$$\omega_1 = \frac{-L_1 \pm \sqrt{L_1^2 + 4L_1C_1 \cdot 4R_0^2}}{2L_1C_1 \cdot 2R_0}$$

$$= \frac{-L_1 \pm \sqrt{L_1^2 + 16L_1C_1R_0^2}}{2L_1C_1 \cdot 2R_0}$$

$$f_1 = \frac{-L_1 \pm \sqrt{L_1^2 + 16L_1C_1R_0^2}}{8\pi L_1C_1R_0}$$

$$= \frac{-1 \pm \sqrt{1+16\frac{C_1}{L_1}R_0^2}}{8\pi C_1 R_0} \text{ Hz.} \quad \text{— (7)}$$

Equation (7) provides the lower root of frequency of the Band stop filter.

considering the negative impedance Z_1 for f_2

$$Z_1 = -j2R_0$$

$$j\frac{\omega_2 L_1}{1-\omega_2^2 L_1 C_1} = -j2R_0$$

[$\because \omega_2$ being the frequency at which $Z_1 = -j2R_0$]

$$\frac{\omega_2 L_1}{\omega_2^2 L_1 C_1 - 1} = 2R_0$$

$$\omega_2 L_1 = 2R_0 (\omega_2^2 L_1 C_1 - 1)$$

$$2R_0 \cdot \omega_2^2 L_1 C_1 - \omega_2 L_1 - 2R_0 = 0$$

$$\omega_2 = \frac{L_1 \pm \sqrt{L_1^2 + 16R_0^2 L_1 C_1}}{4R_0 L_1 C_1}$$

$$= \frac{1 \pm \sqrt{1 + 16R_0^2 \frac{C_1}{L_1}}}{4R_0 L_1 C_1}$$

$$\boxed{f_2 = \frac{1 \pm \sqrt{1 + 16R_0^2 \frac{C_1}{L_1}}}{8\pi R_0 C_1} \text{ Hz.}} \quad \text{— (8)}$$

f_2 gives the higher cutt-off frequency.

→ Design of a Band stop filter.

From $Z_1 = j2R_0$

$$j\left[\frac{\omega_1 L_1}{1-\omega_1^2 L_1 C_1}\right] = j2R_0$$

$$1-\omega_1^2 L_1 C_1 = \frac{\omega_1 L_1}{2R_0}$$

$$1-\frac{\omega_1^2}{\omega_0^2} = \frac{\omega_1 L_1}{2R_0}$$

$$1-\frac{f_1^2}{f_0^2} = \frac{\pi f_1 L_1}{R_0}$$

$$1-\frac{f_1^2}{f_1 f_2} = \frac{\pi f_1 L_1}{R_0} \qquad [\because f_0^2 = f_1 f_2]$$

$$1-\frac{f_1}{f_2} = \frac{\pi f_1 L_1}{R_0}$$

$$\boxed{L_1 = \frac{R_0(f_2-f_1)}{\pi f_1 f_2} \text{ Henry}} \quad — (9)$$

But $\omega_0^2 L_1 C_1 = 1$

$$C_1 = \frac{1}{\omega_0^2 L_1} = \frac{1}{4\pi^2 f_0^2 L_1}$$

Sub L_1 value

$$C_1 = \frac{1}{4\pi^2 (f_1 f_2)\left(\frac{R_0(f_2-f_1)}{\pi f_1 f_2}\right)}$$

$$\boxed{C_1 = \frac{1}{4\pi R_0 (f_2-f_1)} \text{ farad.}} \quad — (10)$$

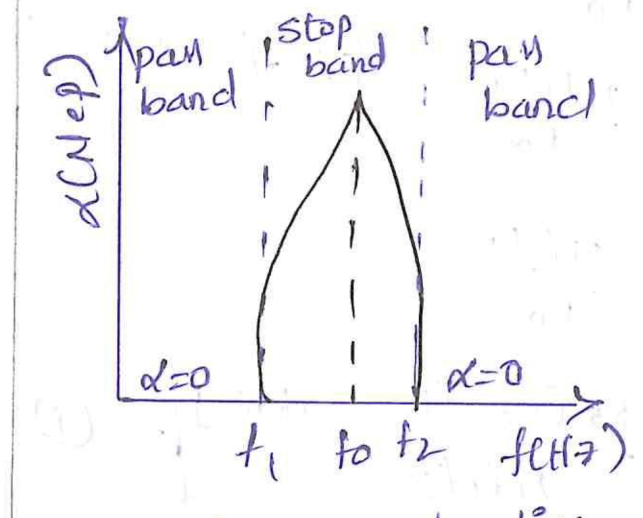
From $C_2 = \dfrac{L_1}{R_0^2}$

$$C_2 = \frac{R_0 (f_2 - f_1) / \pi f_1 f_2}{R_0^2}$$

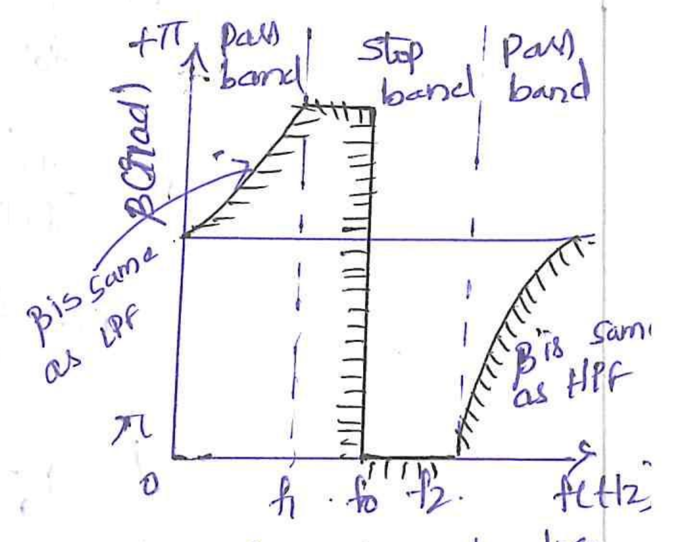
$$\boxed{C_2 = \frac{f_2 - f_1}{\pi f_1 f_2 R_0} \text{ farad}} \quad \text{— (11)}$$

$$L_2 = C_1 R_0^2$$

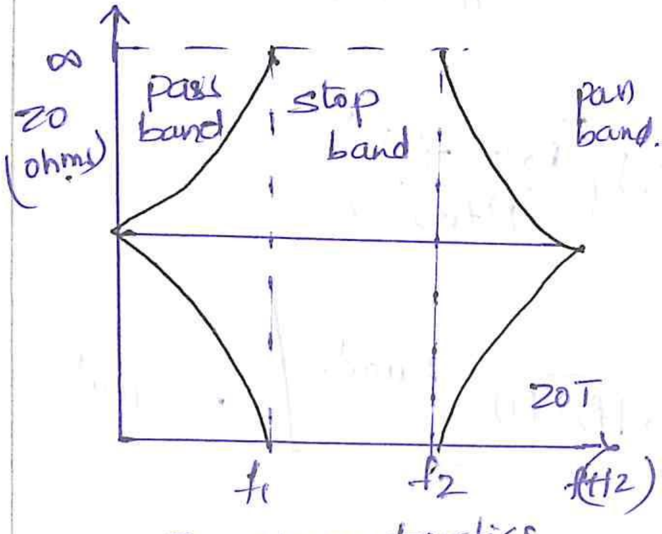
$$\boxed{L_2 = \frac{R_0}{4\pi (f_2 - f_1)}} \text{ Henry} \quad \text{— (12)}$$



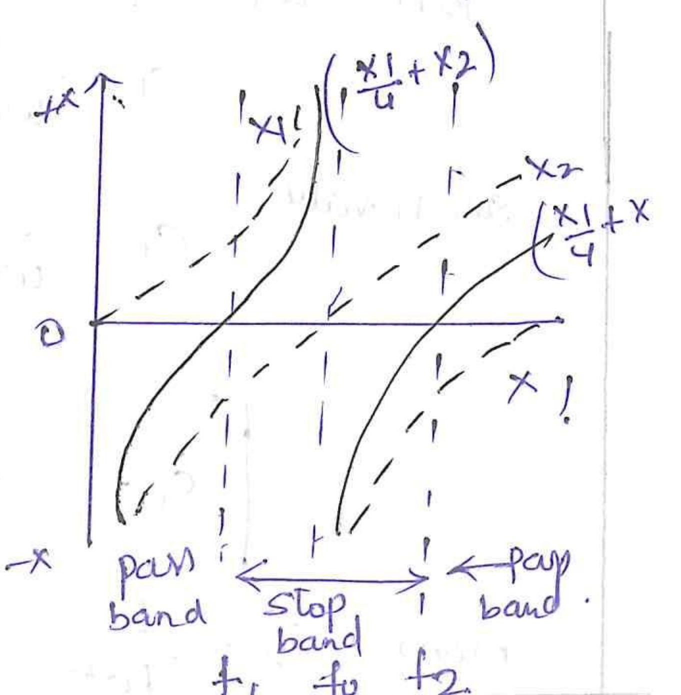
α Characterstics



β Characterstics



Zo characterstics



Reactance - frequency characteristics.

Design a prototype band stop filter section having cutt-off frequencies of 2000 Hz and 5000 Hz and design resistance of 600Ω.

Sol The series arm inductance (L_1) and capacitance (C_1) are given by

$$L_1 = \frac{R_0 (f_2 - f_1)}{\pi f_1 f_2}$$

$$= \frac{600(5000-2000)}{\pi \times 2000 \times 5000}$$

$$\boxed{L_1 = 57.32\,mH}$$

$$\frac{L_1}{2} = \frac{57.32}{2} = 28.66\,mH$$

$$C_1 = \frac{1}{4\pi R_0 (f_2 - f_1)}$$

$$= \frac{1}{4\pi \times 600 \times (5000-2000)}$$

$$= 0.044\,\mu F$$

Hence $2C_1 = 2 \times 0.044\,\mu F$

$$= 0.088\,\mu F.$$

The shunt inductance (L_2) and capacitance (C_2)

$$L_2 = \frac{R_0}{4\pi (f_2 - f_1)}$$

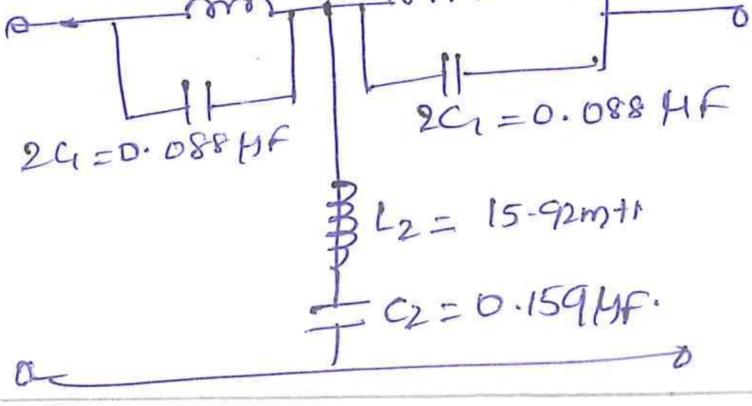
$$= \frac{600}{4\pi (5000-2000)}$$

$$\boxed{L_2 = 15.92\,mH}$$

$$C_2 = \frac{(f_2 - f_1)}{\pi R_0 f_1 f_2}$$

$$= \frac{5000-2000}{\pi \times 600 \times 5000 \times 2000}$$

$$\boxed{C_2 = 0.159\,\mu F}$$



Band stop T. section filter