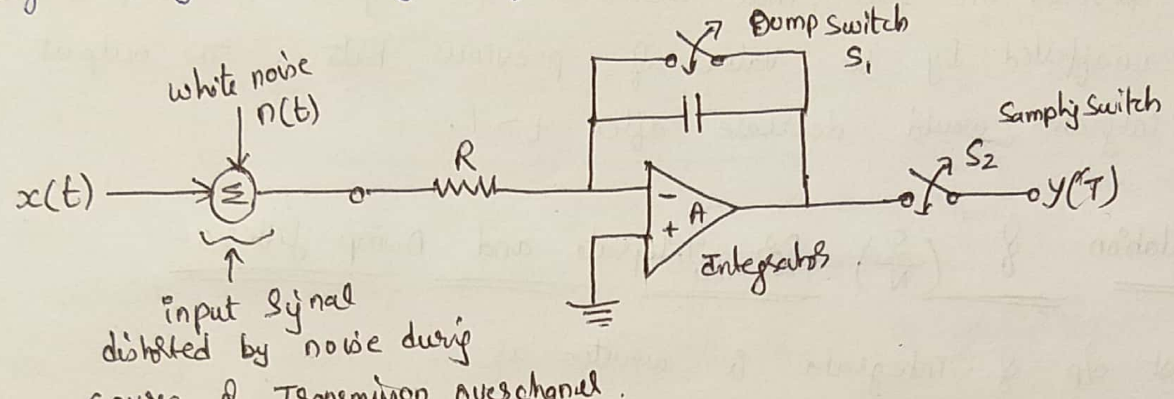
Data Transmission.

Base band Signal Receiver.

Integrate and dump Filter :-

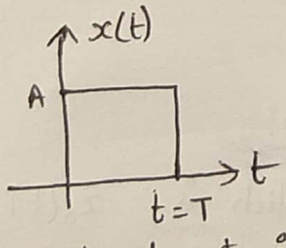
→ Let us consider a simple and basic detector circuit for detection of digital signals. such type of detector is called Integrate & dump filter.



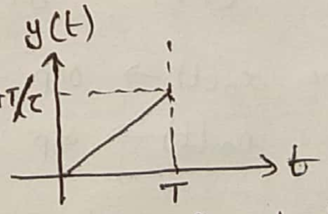
→ Here the digital signal $x(t)$ is distorted by white noise (n) during the transmission over channel. The noisy signal $[x(t)+n(t)]$ is applied as input to integrate & dump filter. The capacitor is discharged fully at beginning of bit interval. This is achieved by closing switch S_2 at beginning of bit interval.

→ The integrator then integrates noisy input signal over one bit period. & o/p is $y(t)$.

→ For square pulse input, the o/p is triangular pulse.



① Input pulse to integrator.



② output of integrator i.e $y(t) = AT/\tau$

→ At end of bit period i.e at $t=T$, the magnitude of $y(t)$ attains maximum amplitude. Hence value of $y(t)$ is sampled at end of bit period.

→ It can be proved that (S/N) is maximum at the end of bit period. Depending upon the value of $y(T)$, the decision is taken.

→ The Dump switch is then closed to discharge the capacitor to receive next bit.

→ ∴ Integrator integrates which is independent of the value of the previous bit.

→ It reveals the fact that detection in integrate & dump filter is unaffected by the values of previous bits. The output of integrator would decrease after $t > T$.

* **Calculation of $\left(\frac{S}{N}\right)$ for Integrate and Dump filter:-**

→ w.k.That o/p of integrator is written as

$$y(t) = \frac{1}{RC}\int_0^T [x(t)+n(t)]\,dt$$

→ Here integration is performed over one bit period i.e from 0 to T. The noisy signal $[x(t)+n(t)]$ is input to integrator, so above eq ① can be written as

$$y(t) = \frac{1}{RC}\int_0^T x(t)\,dt + \int_0^T n(t)\,dt \quad —②$$

(or). $y(t) = x_o(t) + n_o(t)$.

where $x_o(t) \rightarrow$ o/p signal voltage.
$n_o(t) \rightarrow$ o/p noise voltage.

Let us consider output signal voltage which is $x_o(t) = \frac{1}{RC}\int_0^T x(t)\,dt$ ③

because the value $x(t) = A$ from 0 to T Eq ③ is rewritten as

$$x_o(t) = \frac{1}{RC}\int_0^T A\cdot dt \Rightarrow \frac{A}{RC}\int_0^T 1\cdot dt \Rightarrow \frac{A}{RC}[t]_0^T$$

$$\boxed{x_o(t) \Rightarrow \frac{AT}{RC}} \quad —④$$

w.k.T $RC = \tau$
Sub in Eq ④ we get $\boxed{x_o(t) = \frac{AT}{\tau}}$ —⑤

→ The normalized signal power in standard resistance of 1Ω would be :-

output signal power :- $\frac{x_o^2(t)}{1\Omega}$.

Substituting Eq ⑤ in above Eq we get

$$\boxed{\text{output signal power} \Rightarrow \frac{A^2T^2}{\tau^2}} \quad \text{—— ⑥}$$

→ Now let us find noise power :- In order to find we have to calculate the transfer function of the integrator.

→ The transfer function over the Period of T is given by

$$H(f) = \frac{1 - e^{-j\omega T}}{j\omega RC} \quad \text{—— ⑦}$$

for $\omega = 2\pi f$ & $RC = \tau$ Sub in above eq.

$$H(f) = \frac{1 - e^{-j2\pi f T}}{j2\pi\tau}$$

Simplify above eq.

$$H(f) = \frac{1 - [\cos(2\pi f T) - j\sin(2\pi f T)]}{j2\pi f\tau}$$

Seperating the real & imaginary

$$H(f) = \frac{\sin(2\pi f T)}{2\pi f\tau} - j\frac{(1 - \cos(2\pi f T))}{2\pi f\tau}$$

Then the magnitude of above Transfer function will be

$$|H(f)|^2 = \frac{\sin^2(2\pi f T) + 1 - 2\cos(2\pi f T) + \cos^2(2\pi f T)}{(2\pi f\tau)^2}$$

Simplify the above eq we get

$$\boxed{|H(f)|^2 = \frac{\sin^2(\pi f T)}{(\pi f\tau)^2}} \quad \text{—— ⑧}$$

→ The average power of the output noise signal $n_o(t)$ may be obtained by integrating its power density spectrum.

→ It means power ⇒ $P = \int_{-\infty}^{\infty} S(f)\, df$

for standard resistance of 1Ω, the noise power would be

$$\frac{\overline{n_o^2(t)}}{1\Omega} \Rightarrow \overline{n_o^2(t)}$$

Here mean square value of noise is considered since it is a random signal i.e

$$\text{Noise power, } \overline{n_o^2(t)} = \int_{-\infty}^{\infty} S_{no}(f)\, df \quad \text{--- (9)}$$

→ we know that input & output power spectral densities are related as

$$S_{no}(f) = |H(f)|^2 \, S_{ni}(f) \quad \text{--- (10)}$$

where $H(f)$ = Transfer function of filter
$S_{no}(f)$ = PSD of o/p noise
$S_{ni}(f)$ = PSD of i/p noise.

→ Let us assume that noise is present, which is white noise,
∴ PSD of this noise would be

$$\boxed{S_{ni}(f) = \frac{N_o}{2}} \quad \text{--- (11)}$$

Sub Eq (11) in Eq (10)

$$\boxed{S_{no}(f) = |H(f)|^2 \cdot \frac{N_o}{2}} \quad \text{--- (12)}$$

Sub eq (12) in Eq (9)

$$\overline{n_o^2(t)} = \int_{-\infty}^{\infty} |H(f)|^2 \cdot \frac{N_o}{2}\, df$$

Substituting the value of $|H(f)|^2$ i.e eq (8) in above eq

$$\overline{n_o^2(t)} = \int_{-\infty}^{\infty} \frac{\sin^2(\pi f T)}{(\pi f \tau)^2} \cdot \frac{N_o}{2}\, df$$

$$\boxed{\overline{n_o^2(t)} \Rightarrow \frac{N_o}{2} \int_{-\infty}^{\infty} \frac{\sin^2(\pi f T)}{(\pi f \tau)^2}\, df} \quad \text{—— (13)}$$

let us substitute $\pi f \tau = x$

then $dx = \pi \tau\, df$ (or) $df = \frac{1}{\pi \tau} dx$.

further since $f = \frac{x}{\pi \tau}$

$\therefore \pi f T = \frac{xT}{\tau}$

Sub all these above parameters in above Eq (13)

$$\overline{n_o^2(t)} = \frac{N_o}{2} \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{xT}{\tau}\right)}{(x)^2} \cdot \frac{1}{\pi \tau}\, dx$$

rearranging the Equation, we get

$$\overline{n_o^2(t)} = \frac{N_o}{2} \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{xT}{\tau}\right)}{\left(\frac{xT}{\tau}\right)} \cdot \left(\frac{T}{\tau}\right)^2 \cdot \frac{1}{\pi \tau}\, dx$$

$$\Rightarrow \frac{N_o}{2} \cdot \frac{T^2}{\pi \tau^3} \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{xT}{\tau}\right)}{\left(\frac{xT}{\tau}\right)} \cdot dx$$

now let $\frac{xT}{\tau} = u$

So that $dx = \frac{\tau}{T} du$ & limits remained unchanged.

$\therefore$ above Eq becomes

$$\overline{n_0^2(t)} = \frac{N_0}{2} \cdot \frac{T^2}{\pi \tau^3} \int_{-\infty}^{\infty} \frac{\sin^2 u}{u^2} \cdot \frac{\tau}{T}\, du$$

$$\Rightarrow \frac{N_0 T}{2\pi \tau^2} \int_{-\infty}^{\infty} \left(\frac{\sin u}{u}\right)^2 du$$

here the function $\frac{\sin u}{u}$ is squared, the above eq is written as

$$\overline{n_0^2(t)} = \frac{N_0 T}{2\pi \tau^2} \cdot 2 \int_{0}^{\infty} \left(\frac{\sin u}{u}\right)^2 \cdot du$$

$$\Rightarrow \frac{N_0 T}{\cancel{2}\cancel{\pi}\tau^2} \cdot \cancel{2} \cdot \frac{\cancel{\pi}}{\cancel{2}}.$$

$$\boxed{\overline{n_0^2(t)} \Rightarrow \frac{N_0 T}{2\tau^2}} \quad (14).$$

$\therefore$ Signal to noise power ratio is given as

$$\left(\frac{S}{N}\right)_0 = \left(\frac{\text{Signal power}}{\text{Noise power}}\right)$$

$$\Rightarrow \frac{\frac{A^2 T^2}{\tau^2}}{\frac{N_0 T}{2\tau^2}} \Rightarrow \frac{2 A^2 T}{N_0} \Rightarrow \left(\frac{A^2 T}{N_0/2}\right)$$

$\therefore$ Signal to noise ratio of Integrate & Dump receiver is

$$\boxed{\left(\frac{S}{N}\right)_0 = \left(\frac{A^2 T}{N_0/2}\right).} \quad (15)$$

Scanned with CamScanner

* Probability of Error For integrate and Dump filter Receiver :-

Note :- Probability of error denoted by (P_e) is a good measure of performance of the detector.

→ we K.T the o/p of integrator is expressed as

$$\boxed{y(t) = x_o(t) + n_o(t)} \quad \text{—— (1)}$$

for positive pulse of amplitude A, $x_o(t)$ is given as

$$\boxed{x_o(t) = \frac{AT}{\tau} \; ; \text{ for } x(t) = A} \quad \text{—— (2)}$$

for negative pulse of amplitude $-A$, $x_o(t)$ is given by

$$\boxed{x_o(t) = \frac{-AT}{\tau} \; ; \text{ for } x(t) = -A} \quad \text{—— (3)}$$

∴ The output y(t) may be written as

$$\boxed{y(t) \Rightarrow \frac{AT}{\tau} + n_o(t) \quad ; \text{ for } x(t) \Rightarrow A} \quad \text{—— (4)}$$

Illy ;

$$\boxed{y(t) \Rightarrow \frac{-AT}{\tau} + n_o(t) \quad ; \text{ for } x(t) \Rightarrow -A} \quad \text{—— (5)}$$

S.no	Input $x(t)$	Value of $n_o(t)$ for error in output	probability of error P_e
1	$-A$	An error will be introduced if $n_o(t) > \frac{AT}{\tau}$	Calculating P_e for $n_o(t) > \frac{AT}{\tau}$
2	$+A$	An error will be introduced if $n_o(t) < -\frac{AT}{\tau}$	Calculating P_e for $n_o(t) < -\frac{AT}{\tau}$

→ The Probability density function (PDF) of the gaussian distributed function is given by

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/2\sigma^2} \quad \text{—— (6)}$$

$f_X(x)$ = Pdf of a random function x

m = mean

σ = Standard deviation.

→ here, we have to evaluate PDF for white Gaussian noise

$$\therefore \quad x = n_o(t) \quad \text{—— (7)}$$

Since noise has zero mean value i.e $(m=0)$ —(8)

Sub (7) & (8) in (6)

$$\boxed{f_X\{n_o(t)\} = \frac{1}{\sigma\sqrt{2\pi}} e^{-[n_o(t)]^2/2\sigma^2}} \quad \text{—(9)}$$

The standard deviation σ is expressed as

$$\sigma = [\text{mean square value} - \text{square of mean value}]^{1/2}$$

$$\boxed{\sigma_x = \left[\overline{X^2} - m_x^2\right]^{1/2}} \quad \text{—(10)}$$

we have mean square $\overline{X^2} = \overline{n_o^2(t)} = \frac{N_o T}{2\tau^2}$

Also mean value $m_x = 0$ for this noise

Substitute in eq (10) we get

$$\sigma = \left[\overline{n_o^2(t)} - 0\right]^{1/2} \Rightarrow \left[\overline{n_o^2(t)}\right]^{1/2}$$

$$\boxed{\sigma = \sqrt{\frac{N_o T}{2\tau^2}}} \quad \text{—(11)}$$

$\therefore$ Eq (9) can be writtn as

$$f_x\{n_o(t)\} = \frac{1}{\sqrt{\frac{N_oT}{2\tau^2}}\cdot\sqrt{2\pi}}\, e^{-[n_o(t)]^2 / 2\left(\sqrt{\frac{N_oT}{2\tau^2}}\right)^2}$$

$$f_x\{n_o(t)\} = \frac{1}{\sqrt{\frac{N_oT}{\cancel{2}\tau^2}}\cdot\sqrt{\cancel{2}\pi}}\, e^{-[n_o(t)]^2 / \cancel{2}\left(\frac{N_oT}{\cancel{2}\tau^2}\right)}$$

$$f_x\{n_o(t)\} = \frac{\tau}{\sqrt{\pi N_oT}}\, e^{-[n_o(t)]^2 / \left(\frac{N_oT}{\tau^2}\right)} \rightarrow (12)$$

The above equation defines PDF of white gaussian noise.

From property of PDF w.k.T,

$$P\left[n_o(t) > \frac{AT}{\tau}\right] = \int_{\frac{AT}{\tau}}^{\infty} f_x[n_o(t)]\, d[n_o(t)] \quad -(13)$$

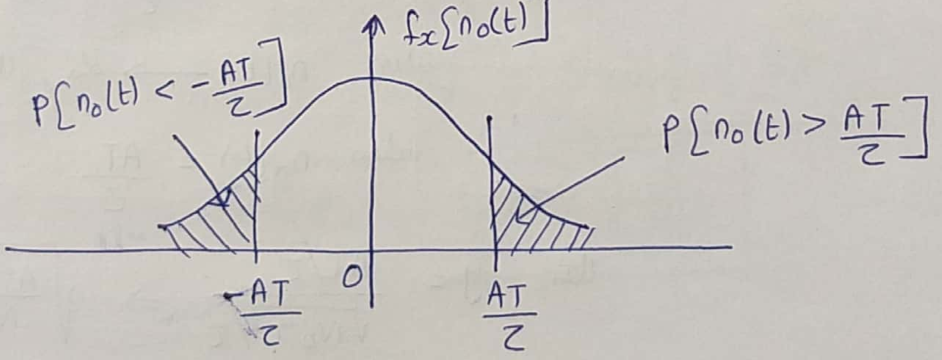


fig:- graphical repertation of PDF

Since PDF Curve is symmetric, $\therefore$ we can write

$$P\left[n_o(t) > \frac{AT}{\tau}\right] = P\left[n_o(t) < -\frac{AT}{\tau}\right] \quad -(14)$$

w.k.T Probabilities repesent error probability, bcz occurence of $-A$ & $+A$ is mutually exclusive, $\therefore$ probability of error is given by either of the two in below equation:-

$$P_e = P\left[n_o(t) > \frac{AT}{\tau}\right] = P\left[n_o(t) < -\frac{AT}{\tau}\right]$$

Subsituting Value of $P\left[n_o(t) > \frac{AT}{\tau}\right]$ in the equation

$$P_e = P\left[n_o(t) > \frac{AT}{\tau}\right] = \int_{\frac{AT}{\tau}}^{\infty} f_x\left[n_o(t)\right]\, d\left[n_o(t)\right]$$

Subsiting Value of $f_x[n_o(t)]$ in above eq.

$$P_e = \int_{\frac{AT}{\tau}}^{\infty} \frac{\tau}{\sqrt{\pi N_o T}}\, e^{-[n_o(t)]^2 / \left(\frac{N_o T}{\tau^2}\right)} \cdot d[n_o(t)] \quad (15)$$

Now let us put $\dfrac{[n_o(t)]^2}{\frac{N_o T}{\tau^2}} = y^2$

So $\dfrac{n_o(t)}{\sqrt{N_o T}/\tau} = y$

Thus $d[n_o(t)] = \dfrac{\sqrt{N_o T}}{\tau}\, dy$

when $n_o(t) \longrightarrow \infty$, then $y \longrightarrow \infty$

when $n_o(t) = \dfrac{AT}{\tau}$

then $y = \dfrac{AT/\tau}{\sqrt{N_o T}/\tau} \Rightarrow \sqrt{\dfrac{A^2 T}{N_o}}$.

with all above Subsitutions in Eq (15)

we have.

$$P_e = \int_{\sqrt{\frac{A^2T}{N_o}}}^{\infty} \frac{\tau}{\sqrt{\pi N_o T}} \cdot e^{-y^2} \cdot \frac{\sqrt{N_o T}}{\tau}\, dy$$

$$\Rightarrow \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{A^2T}{N_o}}}^{\infty} e^{-y^2}\, dy.$$

It can be rearrayed as

$$\boxed{P_e = \frac{1}{2}\left[\frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{A^2T}{N_0}}}^{\infty} e^{-y^2} \cdot dy\right]} \quad \text{--- (16)}$$

The integration inside brackets may be evaluated with help of complemtary error function. i.e

$$\boxed{\frac{2}{\sqrt{\pi}} \int_{U}^{\infty} e^{-y^2} dy = erfc(U)}$$

Thus the above equation becomes i.e (16) as

$$\boxed{P_e = \frac{1}{2} erfc \sqrt{\frac{A^2T}{N_0}}} \quad \text{--- (17)}$$

The above equation defines the probability of error P_e of the integrate & dump filter receiver.

Now since $\boxed{A^2T \Rightarrow E}$

where E is energy of bit

$$\therefore \boxed{P_e = \frac{1}{2} erfc \sqrt{\frac{E}{N_0}}} \quad \text{--- (18)}$$

OPTIMUM FILTER (i.e OPTIMUM RECEIVER).

→ Let us assume that whether the Integrate and dump filter is an optimum filter for the purpose of minimizing the probability of error P_e.

→ A generalized filter to receive binary coded signals is known as optimum filter.

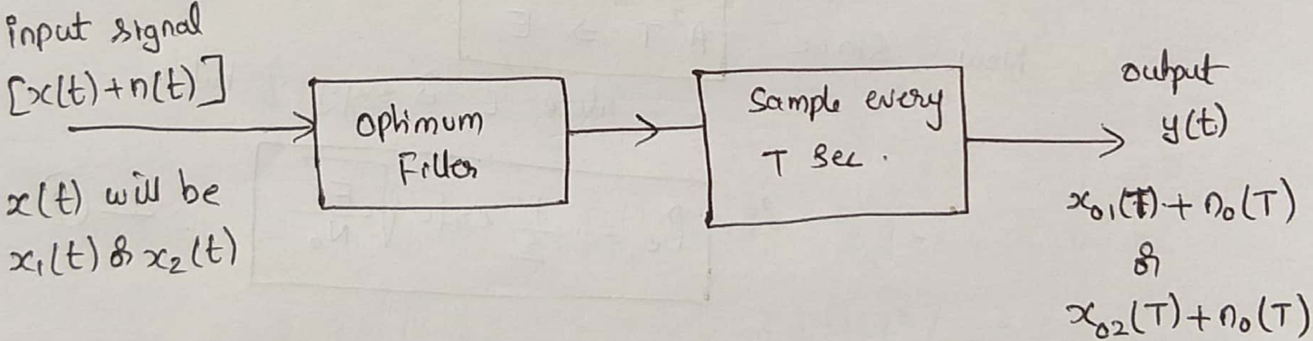
→ Let us consider the Gaussian noise which is having zero mean

→ Let the received signal be a binary waveform. which is of polar NRZ signal used to represent binary 1's & 0's.

for binary '1' ; $x_1(t) = +A$ for one bit period T

binary '0' ; $x_2(t) = -A$ for one bit period T.

→ Hence input signal $x(t)$ will be either $x_1(t)$ or $x_2(t)$.



→ The noise $n(t)$ is added to the signal $x(t)$ in the channel during transmission.

Thus the input to optimum filter will be $[x(t) + n(t)]$

output from the receiver will be ⇒ $x_{01}(T) + n_0(T)$

(or)

$x_{02}(T) + n_0(T)$

Also in the absence of noise $n(t)$, the o/p of receiver will be

$$y(T) = x_{01}(T) \quad \text{if} \quad x(t) = x_1(t)$$

$$y(T) = x_{02}(T) \quad \text{if} \quad x(t) = x_2(t).$$

$\therefore$ The decision boundary will be in middle of $x_{01}(T)$ & $x_{02}(T)$

$$\boxed{\text{Decision boundary} = \frac{x_{01}(T) + x_{02}(T)}{2}}$$

Probability of error (P_e) for optimum filter :-

→ Here we consider gaussian noise, let us consider $x_2(t)$ was transmitted but $x_{01}(T) > x_{02}(T)$.

→ If noise $n_0(T)$ is +ive & larger in magnitude compared to voltage difference $\frac{1}{2}[x_{01}(T) + x_{02}(T)] - x_{02}(T)$ then incorrect decision is taken.

→ The error will be generated if

$$n_0(T) \geq \frac{x_{01}(T) + x_{02}(T)}{2} - x_{02}(T) \geq \frac{x_{01}(T) - x_{02}(T)}{2} \quad \text{—— (1)}$$

→ we have Probability density function for $n_0(t)$. It is given as

$$f_x[n_0(t)] = \frac{1}{\sigma\sqrt{2\pi}} e^{-[n_0(t)]^2/2\sigma^2} \quad \text{—— (2)}$$

$n_0(t)$ is random function.

σ is standard deviation & function has zero mean value.

→ Hence to evaluate probability of error, we must integrate the area under PDF curve from $n_0(T) \geq \frac{x_{01}(T) - x_{02}(T)}{2}$.

$\therefore$ we have.

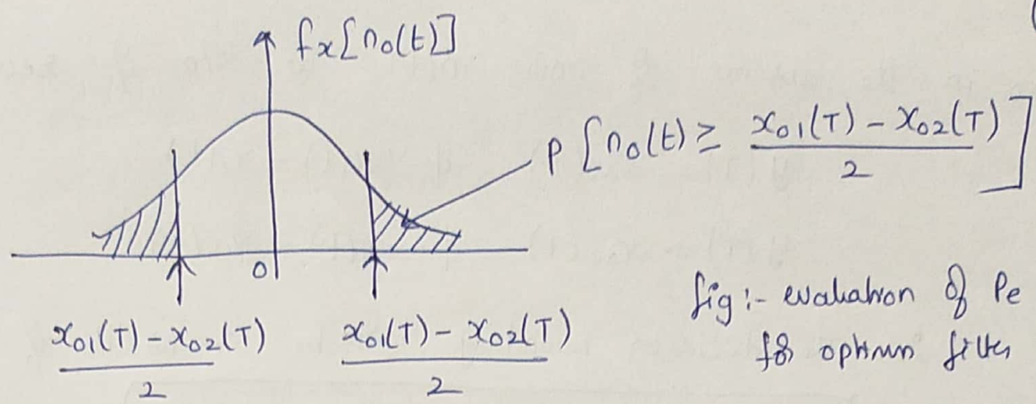


fig:- evaluation of Pe for optimum filter

$\therefore \quad P_e = P\left[n_o(T) \geq \frac{x_{01}(T) - x_{02}(T)}{2}\right]$

$$\Rightarrow \int_{\frac{x_{01}(T) - x_{02}(T)}{2}}^{\infty} f_x[n_o(t)]\, d[n_o(t)] \quad \text{—— (3)}$$

Sub (2) in (3) using

$$P_e = \int_{\frac{x_{01}(T) - x_{02}(T)}{2}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-[n_o(t)]^2/2\sigma^2} \cdot d[n_o(t)] \quad \text{—— (4)}$$

Let us Subsitute $\frac{[n_o(t)]^2}{2\sigma^2} = y^2$

$$n_o(t) = \sigma\sqrt{2}\, y$$

& $d[n_o(t)] = \sigma\sqrt{2}\, dy$

when $n_o(t) = \infty$, then $y = \infty$

& when $n_o(t) = \frac{x_{01}(T) - x_{02}(T)}{2}$

we have $\sigma\sqrt{2}\, y = \frac{x_{01}(T) - x_{02}(T)}{2}$

$$y = \frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}.$$

Subsuting all these Values.

$$P_e = \int_{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-y^2} \cdot \sigma\sqrt{2}\, dy$$

$$P_e \Rightarrow \frac{1}{\sqrt{\pi}} \int_{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}}^{\infty} e^{-y^2}\, dy .$$

Let us rearrge the equation.

$$P_e = \frac{1}{2}\left[\frac{2}{\sqrt{\pi}} \int_{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}}^{\infty} e^{-y^2} \cdot dy\right] \quad -(4)$$

To solve the above integration, let us use following standard result

$$\boxed{\frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{-y^2} \cdot dy = erfc(u)}$$

Sub in above eq/ $\therefore$

$$\boxed{P_e = \frac{1}{2}\, erfc\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right]} \quad -(5).$$

MATCHED FILTER

⇒ when the noise is white Gaussian noise, then optimum filter is Known as matched filter.

The Power Spectral density of white Gaussian noise is given by

$$\boxed{S_{ni}(f) = \frac{N_0}{2}} \quad \text{—— (1)}$$

Probability of error (Pe) for Matched Filter:-

→ To evaluate probability of error for matched filter, let us start with optimum filter. & consider special case of white Gaussian noise.

→ W.K.T error probability of optimum filter is expressed as

$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right] \quad \text{—— (2)}$$

In this Equation, we have

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)}\, df$$

W.K.T P.S.D of white noise $S_{ni}(f) = \frac{N_0}{2}$

Sub in above Equation.

Hence

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{\frac{N_0}{2}}\, df$$

$$\Rightarrow \frac{2}{N_0} \int_{-\infty}^{\infty} |X(f)|^2\, df \quad \text{—— (3)}$$

Also, Parseval's power theorem states that

$$\int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^T x^2(t)\,dt \rightarrow (4)$$

W.K.T $x(t) = x_1(t) - x_2(t)$

Sub in Eq (4)

we get $$\int_{-\infty}^{\infty} |X(f)|^2 df = \int_0^T [x_1(t) - x_2(t)]^2 dt$$

$$= \int_0^T [x_1^2(t) + x_2^2(t) - 2x_1(t)x_2(t)]\,dt$$

$$\Rightarrow \int_0^T x_1^2(t)\,dt + \int_0^T x_2^2(t)\,dt - 2\int_0^T x_1(t)x_2(t)\,dt \quad -(5)$$

where $\int_0^T x_1^2(t)\,dt \Rightarrow E_1$; i.e energy of $x_1(t)$

$\int_0^T x_2^2(t)\,dt \Rightarrow E_2$; i.e enrgy of $x_2(t)$

$\int_0^T x_1(t)x_2(t)\,dt \Rightarrow E_{12} \rightarrow$ energy due to autocorrelation between $x_1(t)$ & $x_2(t)$.

If we choose $x_1(t) = -x_2(t)$ then these enrgies will be equal.

i.e $$\boxed{E_1 = E_2 = -E_{12} = E} \text{ --- } (6)$$

Subutuing all these in eq (5)

we get $$\int_{-\infty}^{\infty} |X(f)|^2 df = [E + E - 2(-E)]$$

$$\Rightarrow 4E \longrightarrow (7)$$

Sub Eq 7 in Eq 3 we get

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} \Rightarrow \frac{2}{N_0}\cdot(4E)$$

since $\int_{-\infty}^{\infty} |x(f)|^2 df \Rightarrow 4E$

$$\therefore \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} \Rightarrow \frac{8E}{N_0} \longrightarrow (8)$$

$$\therefore \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]_{max} \Rightarrow 2\sqrt{2}\sqrt{\frac{E}{N_0}} \quad \text{— (9)}$$

Subuting the Eq (9) in Eq (2) we get

$$\boxed{P_e \Rightarrow \frac{1}{2}\,\text{erfc}\left[\sqrt{\frac{E}{N_0}}\right]} \quad \text{— (10)}$$

$\therefore$ Minimum error probabilit of a matched filter is

$$\boxed{P_e = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E}{N_0}}}$$

Inter Symbol Interference

This is a form of distortion of a signal, in which one or more symbols interfere with subsequent signals, causing noise or delivering a poor output.

Causes of ISI

The main causes of ISI are −

- Multi-path Propagation
- Non-linear frequency in channels

The ISI is unwanted and should be completely eliminated to get a clean output. The causes of ISI should also be resolved in order to lessen its effect.

To view ISI in a mathematical form present in the receiver output, we can consider the receiver output.

The receiving filter output $y(t)$. y(t) is sampled at time $t_i = iT_b$ti=iTb (with **i** taking on integer values), yielding −

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p(iT_b - kT_b)$$

$$= \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p(iT_b - kT_b)$$

In the above equation, the first term μa_iμai is produced by the $\mathbf{i^{th}}$ transmitted bit.

The second term represents the residual effect of all other transmitted bits on the decoding of the $\mathbf{i^{th}}$ bit. This residual effect is called as **Inter Symbol Interference**.

In the absence of ISI, the output will be −

$$y(t_i) = \mu a_i$$

This equation shows that the $\mathbf{i^{th}}$ bit transmitted is correctly reproduced. However, the presence of ISI introduces bit errors and distortions in the output.

While designing the transmitter or a receiver, it is important that you minimize the effects of ISI, so as to receive the output with the least possible error rate.

Correlative Coding

So far, we've discussed that ISI is an unwanted phenomenon and degrades the signal. But the same ISI if used in a controlled manner, is possible to achieve a bit rate of **2W** bits per second in a channel of bandwidth **W** Hertz. Such a scheme is called as **Correlative Coding** or **Partial response signaling schemes**.

Since the amount of ISI is known, it is easy to design the receiver according to the requirement so as to avoid the effect of ISI on the signal. The basic idea of correlative coding is achieved by considering an example of **Duo-binary Signaling**.

Duo-binary Signaling

The name duo-binary means doubling the binary system's transmission capability. To understand this, let us consider a binary input sequence $\{a_k\}$ consisting of uncorrelated binary digits each having a duration T_a seconds. In this, the signal **1** is represented by a **+1** volt and the symbol **0** by a **-1** volt.

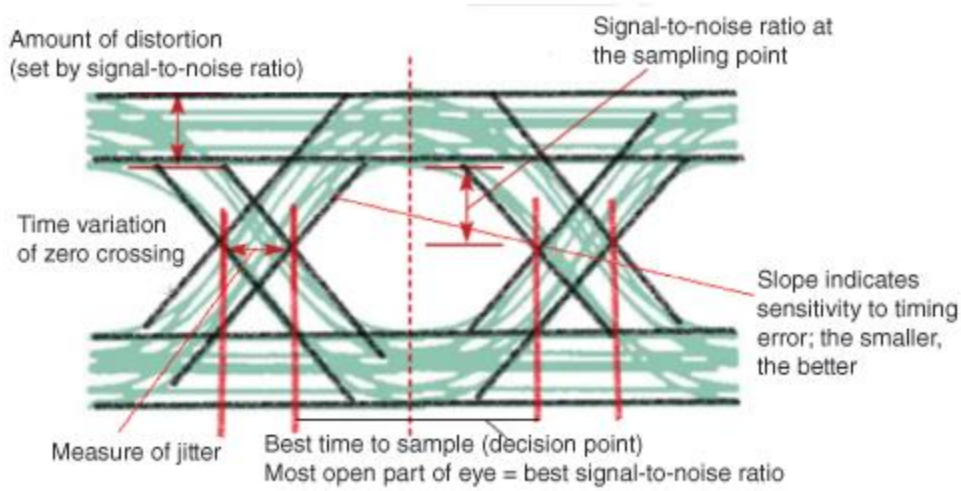
Therefore, the duo-binary coder output c_k is given as the sum of present binary digit a_k and the previous value a_{k-1} as shown in the following equation.

$$c_k = a_k + a_{k-1}$$

The above equation states that the input sequence of uncorrelated binary sequence $\{a_k\}$ is changed into a sequence of correlated three level pulses $\{c_k\}$. This correlation between the pulses may be understood as introducing ISI in the transmitted signal in an artificial manner.

Eye Pattern

An effective way to study the effects of ISI is the **Eye Pattern**. The name Eye Pattern was given from its resemblance to the human eye for binary waves. The interior region of the eye pattern is called the **eye opening**. The following figure shows the image of an eye-pattern.



Jitter is the short-term variation of the instant of digital signal, from its ideal position, which may lead to data errors.

When the effect of ISI increases, traces from the upper portion to the lower portion of the eye opening increases and the eye gets completely closed, if ISI is very high.

An eye pattern provides the following information about a particular system.

- Actual eye patterns are used to estimate the bit error rate and the signal-to-noise ratio.
- The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI.
- The instant of time when the eye opening is wide, will be the preferred time for sampling.
- The rate of the closure of the eye, according to the sampling time, determines how sensitive the system is to the timing error.
- The height of the eye opening, at a specified sampling time, defines the margin over noise.

Hence, the interpretation of eye pattern is an important consideration

UNIT-5 Digital Modulation Techniques

Digital-to-Analog signals is the next conversion we will discuss in this chapter. These techniques are also called as **Digital Modulation techniques**.

Digital Modulation provides more information capacity, high data security, quicker system availability with great quality communication. Hence, digital modulation techniques have a greater demand, for their capacity to convey larger amounts of data than analog modulation techniques.

There are many types of digital modulation techniques and also their combinations, depending upon the need. Of them all, we will discuss the prominent ones.

ASK – Amplitude Shift Keying

The amplitude of the resultant output depends upon the input data whether it should be a zero level or a variation of positive and negative, depending upon the carrier frequency.

FSK – Frequency Shift Keying

The frequency of the output signal will be either high or low, depending upon the input data applied.

PSK – Phase Shift Keying

The phase of the output signal gets shifted depending upon the input. These are mainly of two types, namely Binary Phase Shift Keying BPSKBPSK and Quadrature Phase Shift Keying QPSKQPSK, according to the number of phase shifts. The other one is Differential Phase Shift Keying DPSKDPSK which changes the phase according to the previous value.

M-ary Encoding

M-ary Encoding techniques are the methods where more than two bits are made to transmit simultaneously on a single signal. This helps in the reduction of bandwidth.

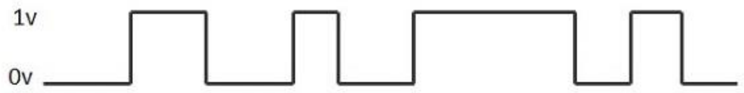
The types of M-ary techniques are −

- M-ary ASK
- M-ary FSK
- M-ary PSK

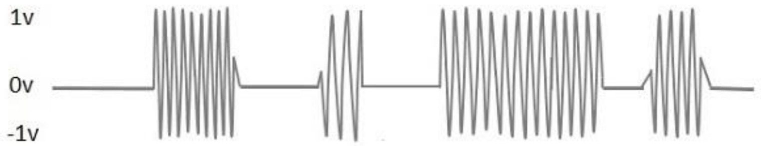
Amplitude Shift Keying ASKASK is a type of Amplitude Modulation which represents the binary data in the form of variations in the amplitude of a signal.

Any modulated signal has a high frequency carrier. The binary signal when ASK modulated, gives a **zero** value for **Low** input while it gives the **carrier output** for **High** input.

The following figure represents ASK modulated waveform along with its input.



Input binary sequence



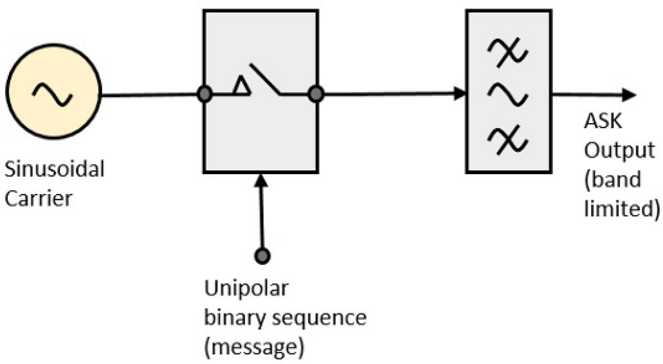
ASK Modulated output wave

To find the process of obtaining this ASK modulated wave, let us learn about the working of the ASK modulator.

ASK Modulator

The ASK modulator block diagram comprises of the carrier signal generator, the binary sequence from the message signal and the band-limited filter. Following is the block diagram of the ASK Modulator.

ASK Generation method



The carrier generator, sends a continuous high-frequency carrier. The binary sequence from the message signal makes the unipolar input to be either High or Low. The high signal closes the switch, allowing a carrier wave. Hence, the output will be the carrier signal at high input. When there is low input, the switch opens, allowing no voltage to appear. Hence, the output will be low.

The band-limiting filter, shapes the pulse depending upon the amplitude and phase characteristics of the band-limiting filter or the pulse-shaping filter.

ASK Demodulator

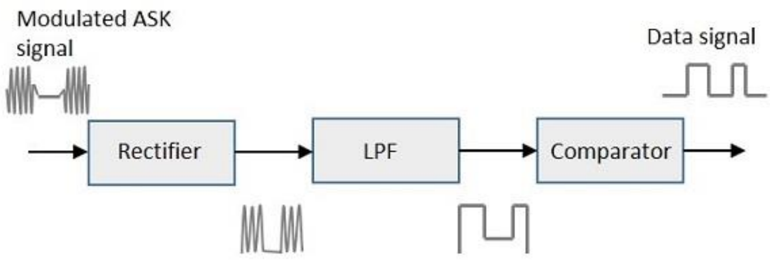
There are two types of ASK Demodulation techniques. They are −

- Asynchronous ASK Demodulation/detection
- Synchronous ASK Demodulation/detection

The clock frequency at the transmitter when matches with the clock frequency at the receiver, it is known as a **Synchronous method**, as the frequency gets synchronized. Otherwise, it is known as **Asynchronous**.

Asynchronous ASK Demodulator

The Asynchronous ASK detector consists of a half-wave rectifier, a low pass filter, and a comparator. Following is the block diagram for the same.

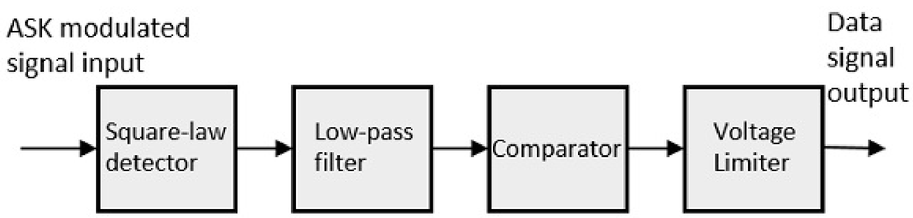


Asynchronous ASK detector

The modulated ASK signal is given to the half-wave rectifier, which delivers a positive half output. The low pass filter suppresses the higher frequencies and gives an envelope detected output from which the comparator delivers a digital output.

Synchronous ASK Demodulator

Synchronous ASK detector consists of a Square law detector, low pass filter, a comparator, and a voltage limiter. Following is the block diagram for the same.



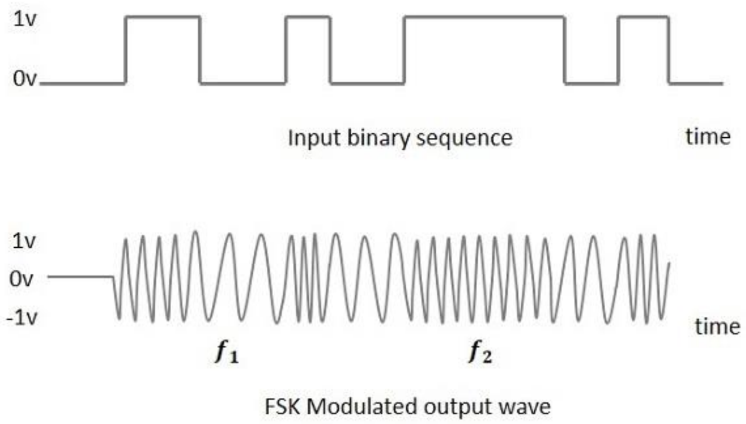
Synchronous ASK detector

The ASK modulated input signal is given to the Square law detector. A square law detector is one whose output voltage is proportional to the square of the amplitude modulated input voltage. The low pass filter minimizes the higher frequencies. The comparator and the voltage limiter help to get a clean digital output.

Frequency Shift Keying FSKFSK is the digital modulation technique in which the frequency of the carrier signal varies according to the digital signal changes. FSK is a scheme of frequency modulation.

The output of a FSK modulated wave is high in frequency for a binary High input and is low in frequency for a binary Low input. The binary **1s** and **0s** are called Mark and Space frequencies.

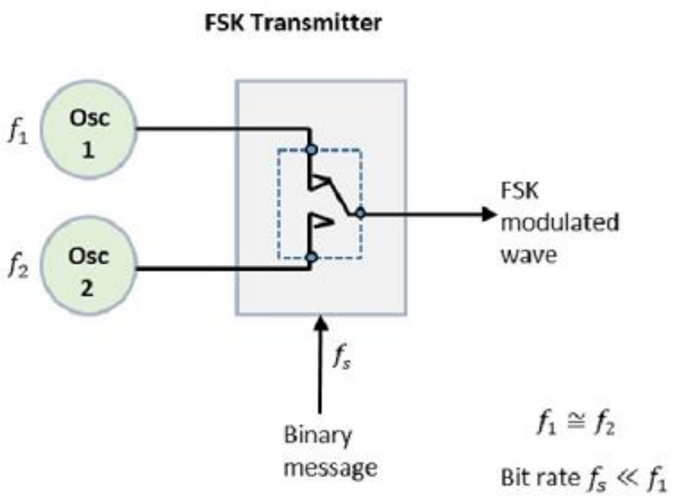
The following image is the diagrammatic representation of FSK modulated waveform along with its input.



To find the process of obtaining this FSK modulated wave, let us know about the working of a FSK modulator.

FSK Modulator

The FSK modulator block diagram comprises of two oscillators with a clock and the input binary sequence. Following is its block diagram.



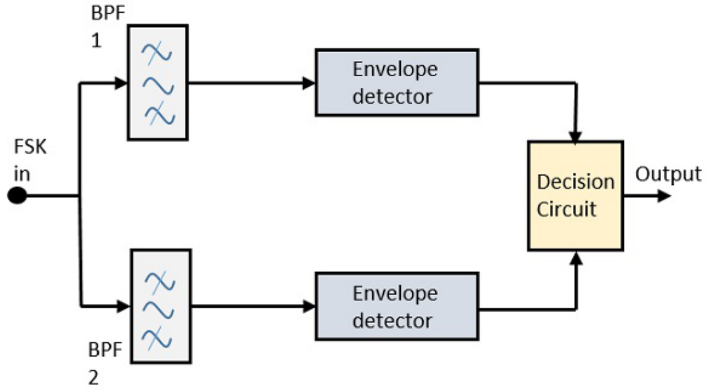
The two oscillators, producing a higher and a lower frequency signals, are connected to a switch along with an internal clock. To avoid the abrupt phase discontinuities of the output waveform during the transmission of the message, a clock is applied to both the oscillators, internally. The binary input sequence is applied to the transmitter so as to choose the frequencies according to the binary input.

FSK Demodulator

There are different methods for demodulating a FSK wave. The main methods of FSK detection are **asynchronous detector** and **synchronous detector**. The synchronous detector is a coherent one, while asynchronous detector is a non-coherent one.

Asynchronous FSK Detector

The block diagram of Asynchronous FSK detector consists of two band pass filters, two envelope detectors, and a decision circuit. Following is the diagrammatic representation.

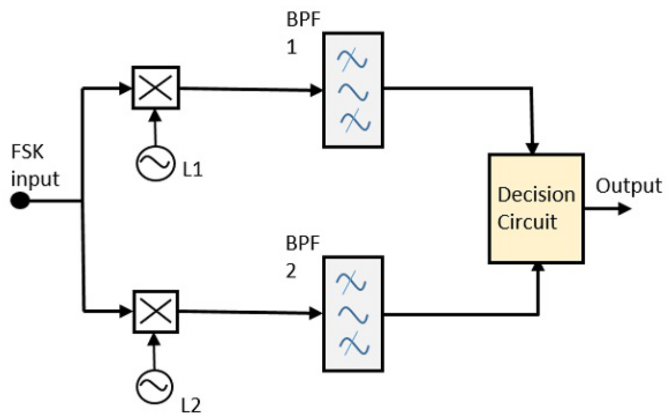


The FSK signal is passed through the two Band Pass Filters BPFsBPFs, tuned to **Space** and **Mark** frequencies. The output from these two BPFs look like ASK signal, which is given to the envelope detector. The signal in each envelope detector is modulated asynchronously.

The decision circuit chooses which output is more likely and selects it from any one of the envelope detectors. It also re-shapes the waveform to a rectangular one.

Synchronous FSK Detector

The block diagram of Synchronous FSK detector consists of two mixers with local oscillator circuits, two band pass filters and a decision circuit. Following is the diagrammatic representation.



The FSK signal input is given to the two mixers with local oscillator circuits. These two are connected to two band pass filters. These combinations act as demodulators and the decision circuit chooses which output is more likely and selects it from any one of the detectors. The two signals have a minimum frequency separation.

For both of the demodulators, the bandwidth of each of them depends on their bit rate. This synchronous demodulator is a bit complex than asynchronous type demodulators.

Phase Shift Keying PSKPSK is the digital modulation technique in which the phase of the carrier signal is changed by varying the sine and cosine inputs at a particular time. PSK technique is widely used for wireless LANs, bio-metric, contactless operations, along with RFID and Bluetooth communications.

PSK is of two types, depending upon the phases the signal gets shifted. They are −

Binary Phase Shift Keying BPSKBPSK

This is also called as 2-phase PSK or Phase Reversal Keying. In this technique, the sine wave carrier takes two phase reversals such as 0° and 180°.

BPSK is basically a Double Side Band Suppressed Carrier DSBSCDSBSC modulation scheme, for message being the digital information.

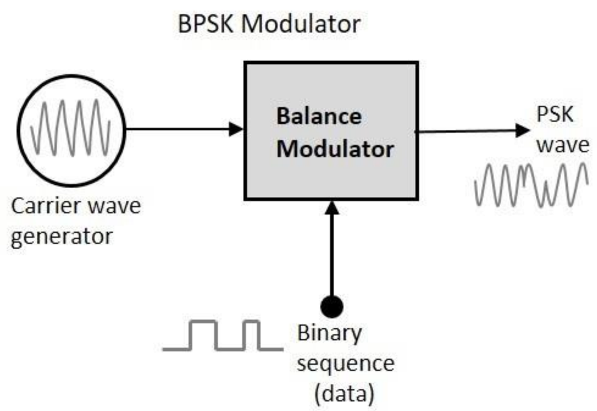
Quadrature Phase Shift Keying QPSKQPSK

This is the phase shift keying technique, in which the sine wave carrier takes four phase reversals such as 0°, 90°, 180°, and 270°.

If this kind of techniques are further extended, PSK can be done by eight or sixteen values also, depending upon the requirement.

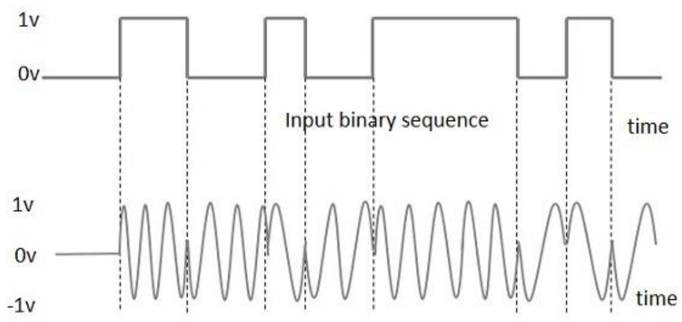
BPSK Modulator

The block diagram of Binary Phase Shift Keying consists of the balance modulator which has the carrier sine wave as one input and the binary sequence as the other input. Following is the diagrammatic representation.



The modulation of BPSK is done using a balance modulator, which multiplies the two signals applied at the input. For a zero binary input, the phase will be **0°** and for a high input, the phase reversal is of **180°**.

Following is the diagrammatic representation of BPSK Modulated output wave along with its given input.



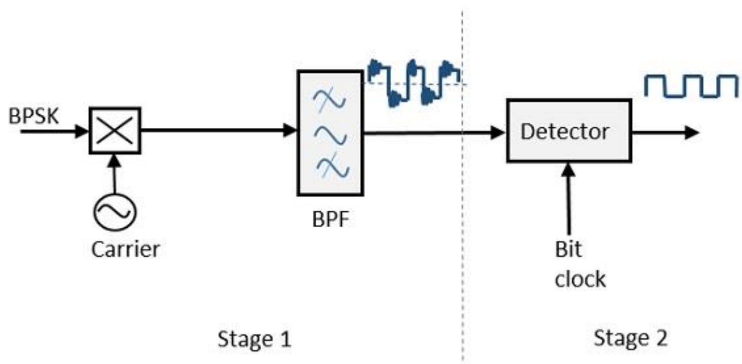
BPSK Modulated output wave

The output sine wave of the modulator will be the direct input carrier or the inverted 180°phaseshifted180°phaseshifted input carrier, which is a function of the data signal.

BPSK Demodulator

The block diagram of BPSK demodulator consists of a mixer with local oscillator circuit, a bandpass filter, a two-input detector circuit. The diagram is as follows.

BPSK Demodulator



By recovering the band-limited message signal, with the help of the mixer circuit and the band pass filter, the first stage of demodulation gets completed. The base band signal which is band limited is obtained and this signal is used to regenerate the binary message bit stream.

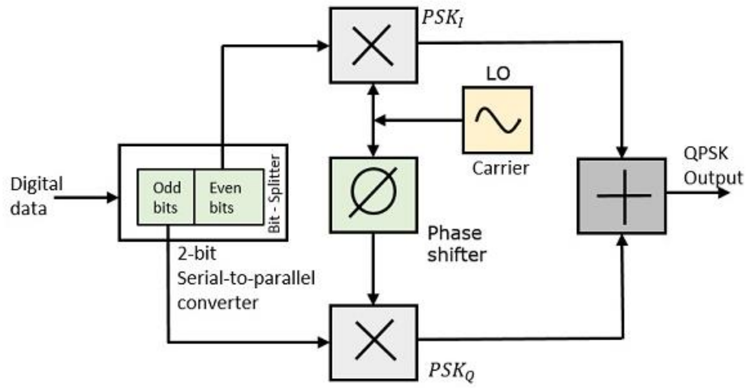
In the next stage of demodulation, the bit clock rate is needed at the detector circuit to produce the original binary message signal. If the bit rate is a sub-multiple of the carrier frequency, then the bit clock regeneration is simplified. To make the circuit easily understandable, a decision-making circuit may also be inserted at the 2nd stage of detection

The **Quadrature Phase Shift Keying** QPSKQPSK is a variation of BPSK, and it is also a Double Side Band Suppressed Carrier DSBSCDSBSC modulation scheme, which sends two bits of digital information at a time, called as **bigits**.

Instead of the conversion of digital bits into a series of digital stream, it converts them into bit pairs. This decreases the data bit rate to half, which allows space for the other users.

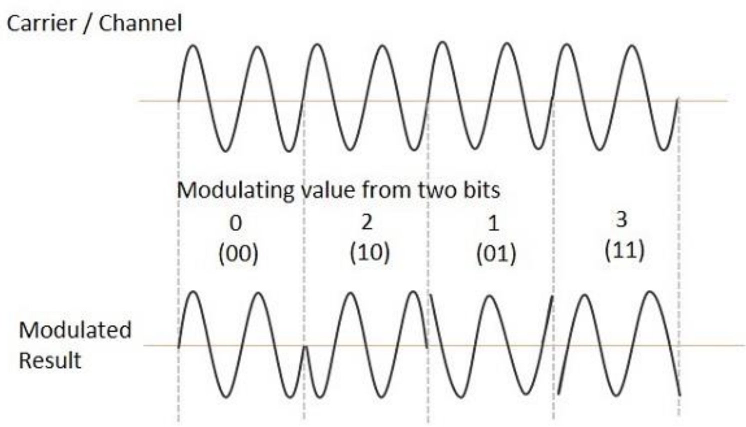
QPSK Modulator

The QPSK Modulator uses a bit-splitter, two multipliers with local oscillator, a 2-bit serial to parallel converter, and a summer circuit. Following is the block diagram for the same.



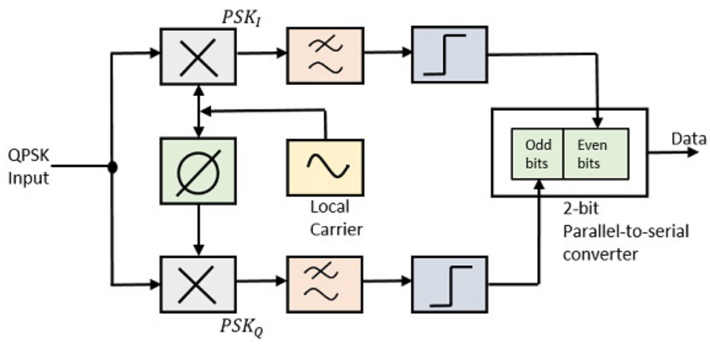
At the modulator's input, the message signal's even bits (i.e., 2nd bit, 4th bit, 6th bit, etc.) and odd bits (i.e., 1st bit, 3rd bit, 5th bit, etc.) are separated by the bits splitter and are multiplied with the same carrier to generate odd BPSK (called as **PSK**$_I$) and even BPSK (called as **PSK**$_Q$). The **PSK**$_Q$ signal is anyhow phase shifted by 90° before being modulated.

The QPSK waveform for two-bits input is as follows, which shows the modulated result for different instances of binary inputs.



QPSK Demodulator

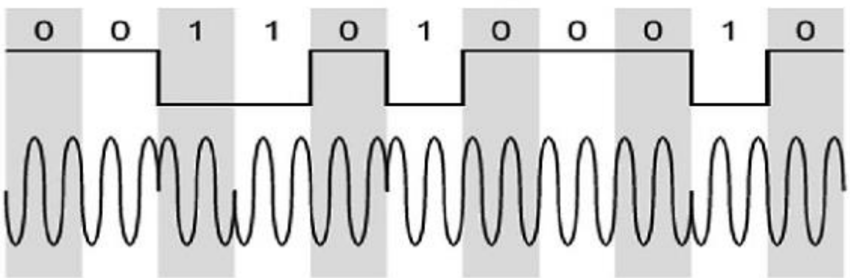
The QPSK Demodulator uses two product demodulator circuits with local oscillator, two band pass filters, two integrator circuits, and a 2-bit parallel to serial converter. Following is the diagram for the same.



The two product detectors at the input of demodulator simultaneously demodulate the two BPSK signals. The pair of bits are recovered here from the original data. These signals after processing, are passed to the parallel to serial converter.

In **Differential Phase Shift Keying** DPSKDPSK the phase of the modulated signal is shifted relative to the previous signal element. No reference signal is considered here. The signal phase follows the high or low state of the previous element. This DPSK technique doesn't need a reference oscillator.

The following figure represents the model waveform of DPSK.

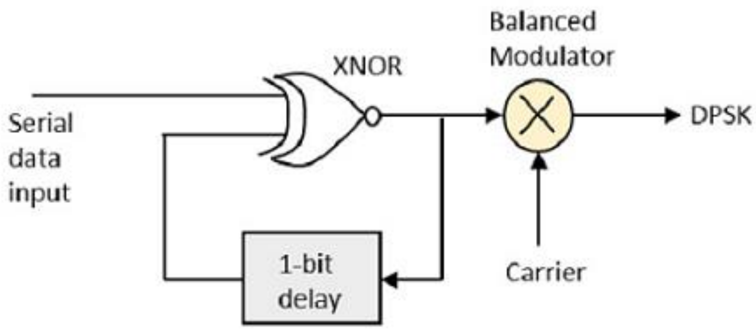


It is seen from the above figure that, if the data bit is Low i.e., 0, then the phase of the signal is not reversed, but continued as it was. If the data is a High i.e., 1, then the phase of the signal is reversed, as with NRZI, invert on 1 aformofdifferentialencodingaformofdifferentialencoding.

If we observe the above waveform, we can say that the High state represents an **M** in the modulating signal and the Low state represents a **W** in the modulating signal.

DPSK Modulator

DPSK is a technique of BPSK, in which there is no reference phase signal. Here, the transmitted signal itself can be used as a reference signal. Following is the diagram of DPSK Modulator.

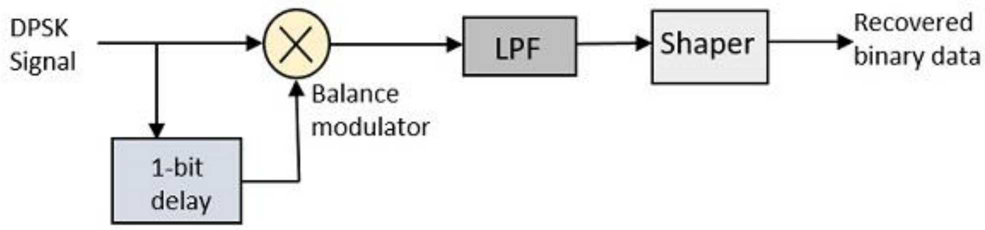


DPSK Modulator

DPSK encodes two distinct signals, i.e., the carrier and the modulating signal with 180° phase shift each. The serial data input is given to the XNOR gate and the output is again fed back to the other input through 1-bit delay. The output of the XNOR gate along with the carrier signal is given to the balance modulator, to produce the DPSK modulated signal.

DPSK Demodulator

In DPSK demodulator, the phase of the reversed bit is compared with the phase of the previous bit. Following is the block diagram of DPSK demodulator.



From the above figure, it is evident that the balance modulator is given the DPSK signal along with 1-bit delay input. That signal is made to confine to lower frequencies with the help of LPF. Then it is passed to a shaper circuit, which is a comparator or a Schmitt trigger circuit, to recover the original binary data as the output.

The word binary represents two bits. **M** represents a digit that corresponds to the number of conditions, levels, or combinations possible for a given number of binary variables.

This is the type of digital modulation technique used for data transmission in which instead of one bit, two or more bits are transmitted at a time. As a single signal is used for multiple bit transmission, the channel bandwidth is reduced.

Quadrature amplitude modulation (QAM) is modulation techniques that we can utilize in analog modulation concept and digital modulation concept. Depending upon the input signal form we can use it in either analog or digital modulation schemes. In QAM, we can modulate two individual signals and transmitted to the receiver level. And by using the two input signals, the channel bandwidth also increases. QAM can able to transmit two message signals over the same channel. This QAM technique also is known as “quadrature carrier multiplexing”.

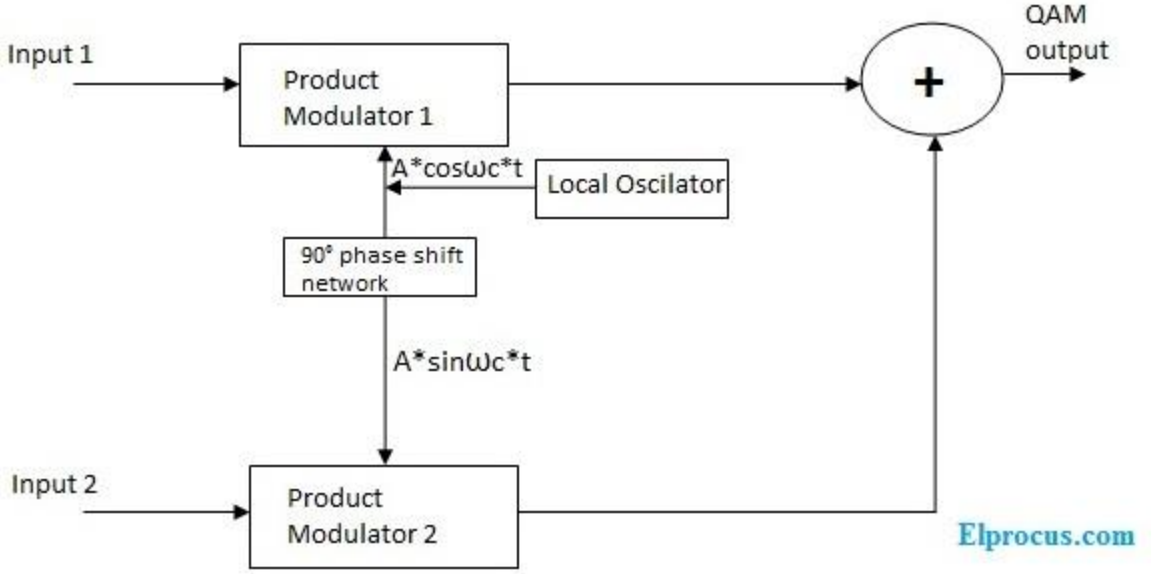
Quadrature Amplitude Modulation Definition

QAM can be defined as it is s a modulation technique that is used to combine two amplitude modulated waves into a single channel to increase the channel bandwidth.

Quadrature Amplitude Modulation Block Diagram

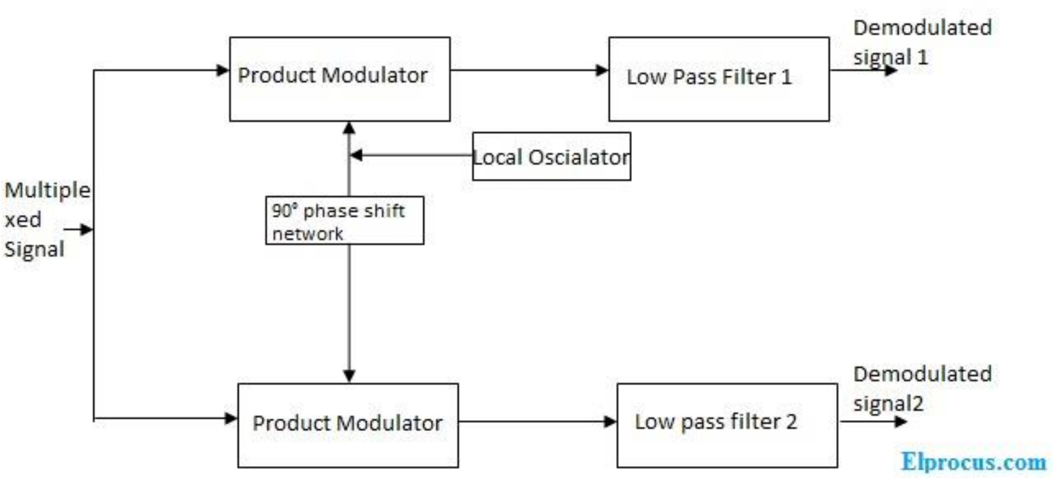
The below diagrams show the transmitter and receiver block diagram of the QAM scheme.

QAM Modulator



qam-modulator

QAM Demodulator



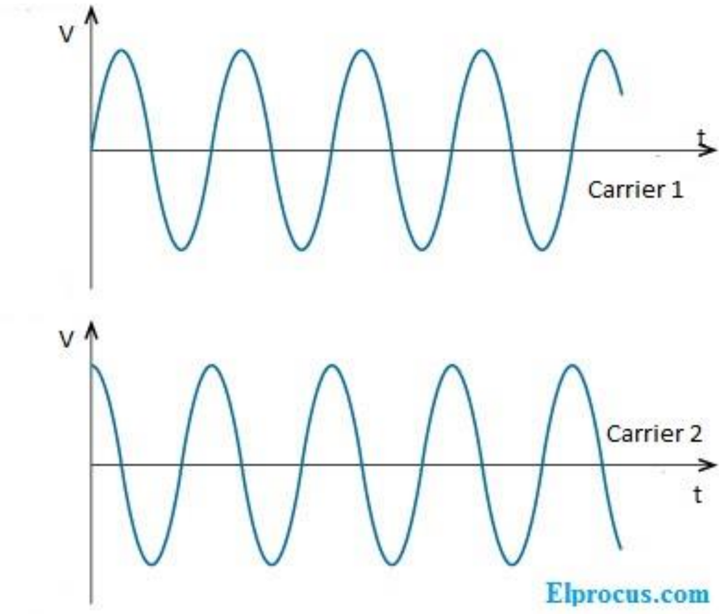
qam-demodulator

QAM Working Principle

“In the QAM transmitter, the above section i.e., product modulator1 and local oscillator are called the in-phase channel and product modulator2 and local oscillator are called a quadrature channel. Both output signals of the in-phase channel and quadrature channel are summed so the resultant output will be QAM.”

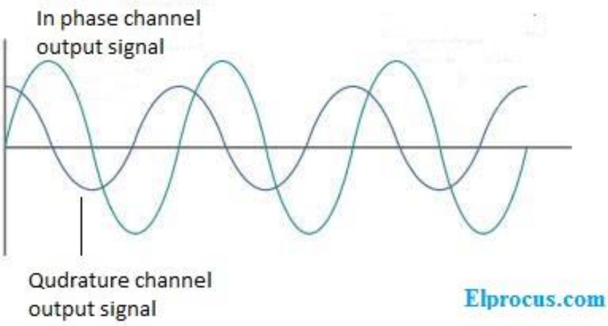
At the receiver level, the QAM signal is forwarded from the upper channel of receiver and lower channel, and the resultant signals of product modulators are forwarded from LPF1 and LPF2. These LPF’s are fixed to the cut off frequencies of input 1 and input 2 signals. Then the filtered outputs are the recovered original signals.

The below waveforms are indicating the two different carrier signals of the QAM technique.



input-carriers-of-qam

The output waveforms of QAM is shown below.



quadrature-output-signal-waveform

Advantages of QAM

The quadrature amplitude modulation advantages are listed below. They are

- One of the best advantages of QAM – supports a high data rate. So, the number of bits can be carried by the carrier signal. Because of these advantages it preferable in wireless communication networks.
- QAM's noise immunity is very high. Due to this noise interference is very less.
- It has a low probability of error value.
- QAM expertly uses channel bandwidth.

M-ary Equation

If a digital signal is given under four conditions, such as voltage levels, frequencies, phases, and amplitude, then **M = 4**.

The number of bits necessary to produce a given number of conditions is expressed mathematically as

$$N=\log_2 M$$

Where

N is the number of bits necessary

M is the number of conditions, levels, or combinations possible with **N** bits.

The above equation can be re-arranged as

$$2^N=M$$

For example, with two bits, **$2^2 = 4$** conditions are possible.

Types of M-ary Techniques

In general, Multi-level $M-ary$ modulation techniques are used in digital communications as the digital inputs with more than two modulation levels are allowed on the transmitter's input. Hence, these techniques are bandwidth efficient.

There are many M-ary modulation techniques. Some of these techniques, modulate one parameter of the carrier signal, such as amplitude, phase, and frequency.

M-ary ASK

This is called M-ary Amplitude Shift Keying $M-ASK$ or M-ary Pulse Amplitude Modulation PAM.

The **amplitude** of the carrier signal, takes on **M** different levels.

Representation of M-ary ASK

$S_m(t) = A_m \cos(2\pi f_c t) A_m \epsilon (2m-1-M)\Delta, m = 1,2....M \text{ and } 0 \le t \le T_s$ $Sm(t)=Amcos(2πfct)Amϵ(2m−1−M)Δ,m=1,2....Mand0≤t≤Ts$

Some prominent features of M-ary ASK are −

- This method is also used in PAM.
- Its implementation is simple.
- M-ary ASK is susceptible to noise and distortion.

M-ary FSK

This is called as M-ary Frequency Shift Keying $M-aryFSK$ M−aryFSK.

The **frequency** of the carrier signal, takes on **M** different levels.

Representation of M-ary FSK

$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(\frac{\pi}{T_s}(n_c+i)t\right)$ $Si(t)=2EsTscos(πTs(nc+i)t)$ $0 \le t \le T_s \text{ and } i = 1,2,3.....M$ $0≤t≤Tsandi=1,2,3.....M$

Where $f_c = \frac{n_c}{2T_s}$ $fc=nc2Ts$ for some fixed integer n.

Some prominent features of M-ary FSK are −

- Not susceptible to noise as much as ASK.
- The transmitted **M** number of signals are equal in energy and duration.
- The signals are separated by $\frac{1}{2T_s}$ $12Ts$ Hz making the signals orthogonal to each other.
- Since **M** signals are orthogonal, there is no crowding in the signal space.
- The bandwidth efficiency of M-ary FSK decreases and the power efficiency increases with the increase in **M.**

M-ary PSK

This is called as M-ary Phase Shift Keying $M-aryPSK$ M−aryPSK.

The **phase** of the carrier signal, takes on **M** different levels.

Representation of M-ary PSK

- $S_i(t) = \sqrt{\frac{2E}{T}} \cos(w_0 t + \phi_i t)$ $Si(t)=2ETcos(wot+ϕit)$
  The envelope is constant with more phase possibilities.
- This method was used during the early days of space communication.
- Better performance than ASK and FSK.
- Minimal phase estimation error at the receiver.
- The bandwidth efficiency of M-ary PSK decreases and the power efficiency increases with the increase in **M**.