UNIT - III

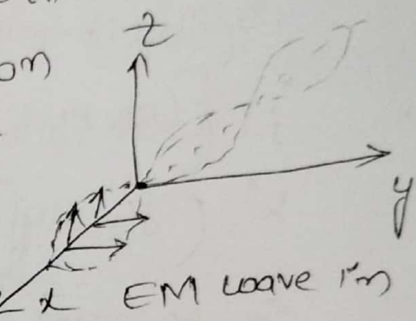
EM WAVE CHARACTERISTICS - I:-

EM WAVE:- The Combination of Time Varying and Magnetic fields produce a phenomenon which is known as Electromagnetic wave. It is a function of time and Space variables. The direction of $\bar{E}$ and $\bar{H}$ wave are perpendicular to each other.

E $\perp$ar $\perp$ $\bar{H}$ $\perp$ direction of wave propogation

EM wave travels with the speed of velocity of light $= 3\times10^{8} ms^{-1}$.

EM waves are also called as Hertzian waves.



def:- The waves are the means of transporting energy or information from source to destination. The waves consisting of electric and magnetic fields are called as Electromagnetic waves. In general, wave is a function of time an space.

Ex:- Radio waves, light rays, radar beam, T.V signals. They all follow the following properities. [They all follow the properities of waves while travelling]

a) They assume properities of waves while travelling

b) They travel with high velocity.

c) They radiate outwards from Source in all directions

Different medias:-

a) free space ($\sigma = 0, \varepsilon = \varepsilon_0, \mu = \mu_0$ (i,e, $\mu_r = 1, \varepsilon_r = 1$))

b) Perfect (lossless dielectric) ($\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_0, \mu_r = \mu_r\mu_0, \sigma << \omega\varepsilon$

c) lossy dielectric ($\sigma \neq 0, \varepsilon = \varepsilon_r\varepsilon_0, \mu = \mu_r\mu_0$)

d) Perfect or good Conductor ($\sigma = \infty, \varepsilon = \varepsilon_0, \mu = \mu_0\mu_r$, and $\sigma >> \omega\varepsilon$).

GENERAL WAVE EQUATION:-

Purpose:- We have two maxwells equations (i) $\nabla \times \bar{H} = J_c$ (ii) $\nabla \times \bar{E} = -d/dt(\bar{B})$

$\nabla \times \bar{H} = \frac{-d}{\partial t}(\bar{D})$

(i) $\nabla \times \bar{H} = \frac{-d}{\partial t}(\bar{D})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ H_x & H_y & H_z \end{vmatrix} = -d/dt(\bar{D})$$

$\hat{i}\,(d/dy\,H_z - d/dz\,H_y) = \frac{-d}{\partial t}(\bar{D})$

$\hat{j}\,(d/dx\,H_y - d/dz\,H_x) = -d/dt(\bar{D})$

$\hat{k}\,(d/dx\,H_y - d/dy\,H_x) = -d/dt(\bar{D})$

(ii) $\nabla \times \bar{E} = -d/dt(\bar{B})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ E_x & E_y & E_z \end{vmatrix} = \frac{-d\bar{B}}{\partial t}$$

$\hat{i}\,(d/dy\,E_z - \frac{d}{\partial z}E_y) = \frac{-d\bar{B}}{\partial t}$

$\hat{j}\,(d/dx\,E_z - d/dz\,E_x) = \frac{-\partial B}{\partial t}$

$\hat{k}\,(d/dx\,E_y - d/dy\,E_x) = \frac{-dB}{\partial t}$

So, in order to calculate one sample/one component either ~~of the~~ $E_x/E_y/E_z/H_x/H_y/H_z$, we need to solve six maxwells equations. These becomes very complex and difficult, Hence, inorder to calculate one component seperately easily, we go for wave Equation. Hence if we want to get individual components, in electro-magnetic problem these can be easily done with maxwell Equ Wave Equation.

General wave Equation

Consider the medium is linear, homogenic and source free region. Assume that the medium obeys ohm's law, $\bar{J} = \sigma\bar{E}$, we have maxwell's Equation. $B = \mu H$, $D = \varepsilon E$

$\nabla \times \bar{E} = -\frac{d}{\partial t}(\bar{B})$, $\nabla \times \bar{H} = J_c + J_d$, $\nabla \cdot \bar{B} = 0$, $\nabla \cdot \bar{D} = 0$

$\nabla \times \bar{E} = -d/dt(\mu\bar{H})$, $\nabla \times \bar{H} = \sigma\bar{E} + \frac{d\bar{D}}{dt}$, $\nabla \cdot (\mu\bar{H}) = 0$, $\nabla \cdot (\varepsilon\bar{E}) = 0$

$$\boxed{\nabla \times \bar{E} = -\mu \cdot d/dt(\bar{H}),\ \nabla \times \bar{H} = \sigma\bar{E} + \frac{\partial}{\partial t}\varepsilon(\bar{E}),\ \nabla \cdot \bar{H} = 0,\ \nabla \cdot \bar{E} = 0}$$

(4) wave Eq^n for Electric field

$\nabla \times \bar{E} = -\mu_0 \, d/dt \, (\bar{H})$ —(a), $\nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \frac{d}{dt}(\bar{E})$ —(b), $\nabla \cdot \bar{H} = 0$ —(c), $\nabla \cdot \bar{E} = 0$ —(d)

$\nabla \times (\nabla \times E) = -\mu_0 \nabla \times [d/dt \, (\bar{H})]$ Applying curl on both sides

$\nabla \times (\nabla \times \bar{E}) = -\mu_0 \, d/dt \, [\nabla \times \bar{H}]$ — (1),

Sub Eq^n—(2) in Eq^n—(1)

$\nabla \times (\nabla \times \bar{E}) = -\mu_0 \, d/dt \left[\sigma \bar{E} + \varepsilon \cdot d/dt \, (\bar{E})\right]$

By vector identity, $\nabla \times (\nabla \times \bar{E}) = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$

$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\mu \, d/dt \left[\sigma \bar{E} + \varepsilon \cdot d/dt \, \bar{E}\right]$

$-\nabla^2 \bar{E} = -\mu_0 \, d/dt \left[\sigma \bar{E} + \varepsilon \cdot d/dt \, \bar{E}\right]$

$\nabla^2 \bar{E} = \mu_0 \, d/dt \left[\sigma \bar{E} + \varepsilon \cdot d/dt \, \bar{E}\right]$

$$\boxed{\nabla^2 \bar{E} = \mu\sigma \cdot [d/dt \, (\bar{E})] + \mu\varepsilon \frac{d^2}{dt^2} \bar{E}}$$

wave Equation for Electric field, in uniform media

(b) wave equation for $\bar{D}$

$\nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \cdot \frac{d}{dt}(\bar{E})$ applying curl on both sides

$\nabla \times (\nabla \times \bar{H}) = \nabla \times \left[\sigma \bar{E} + \varepsilon \cdot \frac{d}{dt}(\bar{E})\right]$

$\nabla \times (\nabla \times \bar{H}) = \nabla(\nabla \cdot \bar{H}) - \nabla^2 \bar{H}.$

$\nabla(\nabla \cdot \bar{H}) - \nabla^2 \bar{H} = \sigma(\nabla \times \bar{E}) + \varepsilon \cdot \frac{d}{dt}(\nabla \times \bar{E})$

but $\nabla \times \bar{E} = -\mu \cdot \frac{d}{dt}(\bar{H})$

$\nabla(\nabla \cdot \bar{H}) - \nabla^2 \bar{H} = \sigma\left(-\mu \cdot \frac{d}{dt}(\bar{H})\right) + \varepsilon \cdot \frac{d}{dt}\left(-\mu \cdot \frac{d}{dt}(\bar{H})\right)$

$\Rightarrow -\nabla^2 \bar{H} = -\mu\sigma \cdot d/dt \, (\bar{H}) + (-\mu\varepsilon) \cdot \frac{d^2}{dt^2}(\bar{H})$

$-\nabla^2 \bar{H} = -\left[\mu\sigma \cdot \frac{d}{dt}(\bar{H}) + \mu\varepsilon \cdot \frac{d^2}{dt^2}(\bar{H})\right]$ for $\bar{H}$

$$\boxed{\nabla^2 \bar{H} = \mu\sigma \frac{d}{dt}(\bar{H}) + \mu\varepsilon \, d^2/dt^2 \, (\bar{H})}$$

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c) wave Eq^n for D

we have $\nabla^2 \bar{E} = \mu\sigma \cdot \frac{d}{dt}(\bar{E}) + \mu\varepsilon \cdot \frac{d^2}{dt^2}(\bar{E})$.

we have $D = \varepsilon\bar{E}$

$$\nabla^2(\varepsilon\bar{E}) = \mu\sigma\frac{d}{dt}(\varepsilon\bar{E}) + \mu\varepsilon \cdot \frac{d^2}{dt^2}(\varepsilon\bar{E})$$

$$\boxed{\nabla^2\bar{D} = \mu\sigma\frac{d}{dt}(\bar{D}) + \mu\varepsilon \cdot \frac{d^2}{dt^2}(\bar{D})}$$ wave Eq^n for D

(d) wave Eq^n for $\bar{B}$

$$\nabla^2\bar{H} = \mu\sigma \cdot \frac{d}{dt}(\bar{H}) + \mu\varepsilon \cdot \frac{d^2}{dt^2}(\bar{H})$$

$$\nabla^2(\mu\bar{H}) = \mu\sigma \cdot \frac{d}{dt}(\mu\bar{H}) + \mu\varepsilon \cdot \frac{d^2}{dt^2}(\mu\bar{H})$$

$$\boxed{\nabla^2\bar{B} = \mu\sigma \cdot d/dt\,(\bar{B}) + \mu\varepsilon \cdot d^2/dt^2(\bar{B})}$$ for $\bar{B}$

Hence, in general the wave equation is

$$\boxed{\nabla^2 \begin{bmatrix} \bar{E} \\ \bar{D} \\ \bar{B} \\ \bar{H} \end{bmatrix} = \mu\sigma \cdot \frac{d}{dt} \begin{bmatrix} \bar{E} \\ \bar{D} \\ \bar{B} \\ \bar{H} \end{bmatrix} + \mu\varepsilon \cdot \frac{d^2}{dt^2} \begin{bmatrix} \bar{E} \\ \bar{D} \\ \bar{B} \\ \bar{H} \end{bmatrix}}$$

This is three dimensional wave equation for all the vector fields.

In general, we have

$$\nabla \times \bar{E} = -\mu \cdot \frac{d}{dt}(\bar{H})$$

$$\nabla \times (\nabla \times \bar{E}) = -\mu \cdot d/dt\,(\nabla \times \bar{H}), \quad \nabla \times \bar{H} = \frac{d}{dt}(\bar{D}) = \frac{d}{dt}(\varepsilon\bar{E})$$

$$\underset{0}{\nabla(\nabla \cdot \bar{E})} - \nabla^2\bar{E} = -\mu \cdot \frac{d}{dt}\left[d/dt\,(\varepsilon E)\right]$$

$$\Rightarrow -\nabla^2\bar{E} = -\varepsilon\mu \cdot d^2/dt^2(\bar{E})$$

$$\nabla^2\bar{E} = \mu\varepsilon \cdot \frac{d^2}{dt^2}(\bar{E})$$

$$\boxed{\nabla^2 E - \mu\varepsilon \cdot d^2/dt^2(\bar{E}) = 0}$$

Hence, Em wave is varied w.r.t space and time

variation w.r.to space either (E_x/E_y/E_z)

variation w.r.to time.

As the above expression ...

and includes Electric and magnetic fields and hence it is called EM Wave Equation.

Wave Equation for conducting media

Conducting media is characterised by $\sigma = \infty$, $\varepsilon = \varepsilon_0$, $\mu = \mu_r \mu_0$, $\sigma >> \omega\varepsilon$.

we have Eq^ns, $\nabla^2 \bar{E} = \mu\sigma \left[\partial/\partial t (\bar{E}) \right] + \mu\varepsilon \frac{\partial^2}{\partial t^2} (\bar{E})$

$$\nabla^2 \bar{E} = \mu \left[\sigma . d/dt (\bar{E}) + \varepsilon . d^2/dt^2 (\bar{E}) \right]$$

$$\nabla^2 \bar{E} = \mu \left[\sigma . \dot{\bar{E}} + \varepsilon . \ddot{\bar{E}} \right]$$ where $\mu = \mu_0 \mu_r$, $\varepsilon = \varepsilon_0 \varepsilon_r$.

Similarly $\nabla^2 \bar{H} = \mu \left[\sigma \dot{\bar{H}} + \varepsilon . \ddot{\bar{H}} \right]$

$\nabla^2 \bar{B} = \mu \left[\sigma \dot{\bar{B}} + \varepsilon \ddot{\bar{B}} \right]$, $\nabla^2 \bar{D} = \mu \left[\sigma \dot{\bar{D}} + \varepsilon \ddot{\bar{D}} \right]$

(b) Wave Equation for Dielectric, Perfect dielectric.

we have $\nabla^2 \bar{E} = \mu (\sigma \dot{\bar{E}} + \varepsilon \ddot{\bar{E}})$

we have dielectric characteristics, ($\sigma = 0$, $\varepsilon = \varepsilon_0 \varepsilon_r$, $\mu = \mu_0 \mu_r$)

$$\nabla^2 \bar{E} = \mu \left[\underbrace{\sigma \dot{\bar{E}}}_{0} + \varepsilon \ddot{\bar{E}} \right] \Rightarrow \nabla^2 \bar{E} = \varepsilon \ddot{\bar{E}}$$

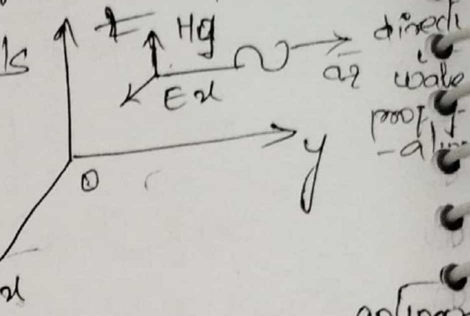
disadv →

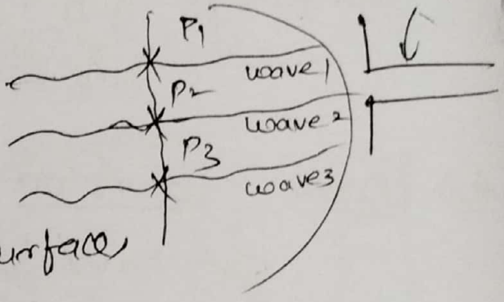
Wave Equations are Second order differential equations and difficult to solve where maxwells Equations are first order equations

UNIFORM PLANE WAVES

→ Any electromagnetic wave propogating along x direction is said to be uniform if the $\bar{E}, \bar{H}$ are independent of remaining directions (y & z) and this uniform plane waves don't have field components along the direction of wave propogation

Wavefront:– When the different signals are travelling with different phase, and we join all the equal phase points means (equal phase), then so called waveform.





plane → If the complete line joining any two points on a surface they lie on the same surface then so called plane.

If at all the phase points in wavefront, the amplitude is constant, then so called uniform plane wave.

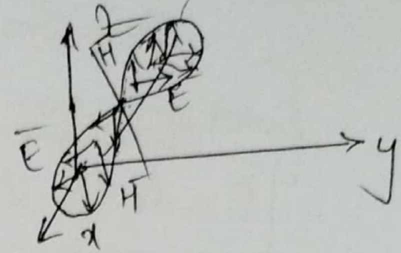
Purpose of uniform plane wave

At receiving antenna, the incident plane, if we consider normal waves then the signal strength is less because, of amplitude is different at phase points. But if we consider uniform plane waves, at receiving end, if we gather at end points, as the amplitude is constant at different phase points also, signal strength is high and such that is repeated by receiving antenna.

free space at infinite distance from the source.
we have for charge free region, Eqn is

$\nabla^2 \bar{E} = \mu_0 \varepsilon_0 \ddot{\bar{E}}$ or $\nabla^2 \bar{E} = \mu \varepsilon \frac{d^2}{dt^2}(\bar{E})$ ($\because \mu_r = 1, \varepsilon_r = 1$)

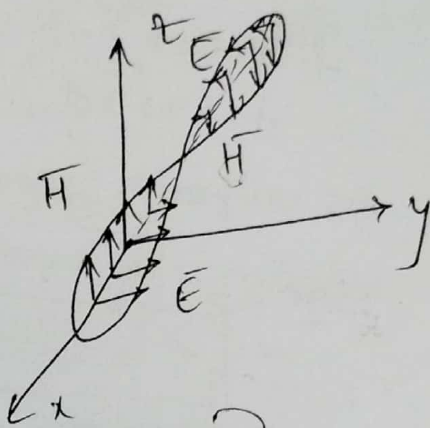
we have Laplacian operator,



$d^2/dx^2 + d^2/dy^2 + d^2/dz^2 = \nabla^2$

$\nabla^2 \bar{E} = \frac{d^2}{dx^2} \bar{E}_x + \frac{d^2}{dy^2} \bar{E}_y + \frac{d^2}{dz^2} (\bar{E}_z)$ — (1)

Considering wave is propogating in x direction
so, $\bar{E}$ doesn't along y & z direction



$\bar{E} \neq f(y)$, $E \neq f(z)$

$d/dy(\bar{E}) = 0$, $d/dz(\bar{E}) = 0$

$\bar{E} = E_x . \hat{a_x} + E_y . \hat{a_y} + E_z . \hat{a_z}$

$d^2/dx^2 (E_x) + d^2/dy^2 (E_y) + d^2/dz^2 (E_z) =$
$+ \mu \varepsilon . [d^2/dt^2 (E_x) + d^2/dt^2 (E_y) + d^2/dt^2 (E_z)]$

since, y and z components are 0.

$d^2/dx^2 (E_x) = \mu_0 \varepsilon_0 . d^2/dt^2 (E_x) \rightarrow$ indicates wave is in x-direction

$d^2/dy^2 (E_y) = \mu_0 \varepsilon_0 . d^2/dt^2 (E_y) \rightarrow$ in y-direction

$d^2/dz^2 (E_z) = \mu_0 \varepsilon_0 . d^2/dt^2 (E_z) \rightarrow$ in z-direction

$\Rightarrow$ we have Eqn, $\nabla^2 \bar{E} - \mu \varepsilon . d^2/dt^2 (\bar{E}) = 0$.

$\underbrace{d^2/dx^2}_{\text{space distance}} (E_x) - \mu \varepsilon . \underbrace{d^2/dt^2}_{\text{time}} (E_x) = 0$ Since both are not with same units.

speed = distance / time = (velocity), $v^2 = \frac{1}{\mu \varepsilon}$, $\mu \varepsilon = \frac{1}{v^2}$

$d^2/dx^2 (E_x) = \mu \varepsilon . d^2/dt^2 (E_x)$

$\mu \varepsilon . d^2/dt^2 (E_x) = \frac{1}{\mu \varepsilon} . d^2/dx^2 (E_x)$

$d^2/dt^2 (E_x) - \frac{1}{\mu \varepsilon} . d^2/dx^2 (E_x) = 0$.

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$$\partial^2/\partial t^2 (E_x) - v^2 \cdot \partial^2/\partial x^2 (E_x) = 0.$$

$$\text{or } \partial^2/\partial t^2 (\bar{E}) - v^2 \cdot \partial^2/\partial x^2 (\bar{E}) = 0 \quad -(1)$$

now let us write differential equation

$$E_x = E_m e^{j\omega t}$$

Solution for Equation (1) is

$$\boxed{\bar{E} = f_1(x - v_0 t) + f_2(x + v_0 t)}$$

$$v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, m/s$$

$x \rightarrow$ direction of propogation.

$f_1(x - v_0 t) \rightarrow$ signal in +ve x direction

$f_2(x + v_0 t) \rightarrow$ signal in backward direction.

As uniform plane wave is passing in charge free region, backward wave is 0

$$\boxed{\bar{E} = f(x - v_0 t)}$$

$x \rightarrow$ distance, $v_0 \rightarrow$ velocity, both should be of same parameter, they should convert into phase by multiplying with some constant factors

βx, where $\beta \rightarrow$ distance - phase conversion.

β for 1 unit distance $(d) = 2\pi$ radians

$$\boxed{\beta = 2\pi/d}, \text{ rad/m.}$$

$v_0 t \rightarrow$ velocity x time, where $v_0 \rightarrow$ converts time to phase

v_0 for 1 unit ~~velocity~~ (time T) $= 2\pi$ radians

$$v_0(T) = 2\pi \Rightarrow \boxed{v_0 = \omega = \frac{2\pi}{T} = \text{rad/sec.}}$$

$$\text{now velocity} = \omega/\beta = \frac{2\pi}{T} \cdot \frac{d}{2\pi} = \frac{d}{T} = m/sec.$$

If ω-β are not in linear, it indicates, that the n no. of signals travelling towards the Rx travels with different velocity. Hence, all the signals reaches at different point in end time. So, accumulation of this signals is difficult & results in phase frequency distortion

UNIFORM PLANE WAVES - ALL RELATIONS BETWEEN E & H

let us consider uniform plane wave propogating along x direction. $E = E_x \cdot \hat{a_x} + E_y \cdot \hat{a_y} + E_z \cdot \hat{a_z}$ $(E_x = 0)$

$$\nabla \times \bar{E} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ d/dx & d/dy & d/dz \\ E_x(0) & E_y & E_z \end{vmatrix}$$

since, $E_x = 0$, $d/dt(E_x) = 0$

$$\nabla \times E = \hat{a_x}\left[d/dy(E_z) - d/dz(E_y)\right] - \hat{a_y}\left[d/dx\, E_z - d/dz\, E_x\right]$$

$$+ \hat{a_z}\left[d/dx\, E_y - d/dy\, E_x\right]$$

$$\nabla \times \bar{E} = -\hat{a_y}\left[d/dx\, E_z\right] + \hat{a_z}\left[d/dx\, E_y\right] \quad (1)$$

$d/dx\,(E_x) = 0$. $\hat{a_x}$ direction is also 0

Similarly

$$\nabla \times \bar{H} = -\hat{a_y}\left[d/dx\, H_z\right] + \hat{a_z}\left[d/dx\, H_y\right]$$

we have $\nabla \times \bar{H} = d/dt\, D = d/dt\,(\varepsilon_0 \bar{E}) = \varepsilon_0 \cdot d/dt(\bar{E})$ sub in (1)

$$\nabla \times \bar{H} = -\hat{a_y}\left[d/dx\, H_z\right] + \hat{a_z}\left[d/dx\, H_y\right]$$

$$\varepsilon_0 \cdot d/dt(\bar{E}) = -\hat{a_y}\left[d/dx\, H_z\right] + \hat{a_z}\left[d/dx\, H_y\right]$$

Equating

$$\boxed{\begin{array}{l} \cancel{d/dx\, E_x = \varepsilon_0\, d/dx\, H_z} \\ d/dx\, E_y \cdot \varepsilon_0 = d/dy\, H_y \end{array}} \qquad \boxed{\begin{array}{l} -d/dx\, H_z = \varepsilon_0 \cdot d/dt\,(E_z) \\ d/dx\, H_y = \varepsilon_0 \cdot d/dt\,(E_y) \end{array}}$$

ii) Using Second maxwell's equation

$$\nabla \times \bar{E} = -d/dt\,(\bar{B})$$

$$-\hat{a_y}\left[d/dx\, E_z\right] + \hat{a_z}\left[d/dx\, E_y\right] = -d/dt\,(\bar{B})$$

$$-\hat{a_y}\left[d/dx\, E_z\right] + \hat{a_z}\left[d/dx\, E_y\right] = -\left[d/dt\,(\mu \bar{H})\right]$$

$$-\hat{a_y}\left[d/dx\, E_z\right] + \hat{a_z}\left[d/dx\, E_y\right] = -\mu\left[d/dt(\bar{H})\right]$$

$$-\hat{a_y}\left[d/dx\, E_z\right] + \hat{a_z}\left[d/dx\, E_y\right] = -\mu\left[d/dt\, H_x \cdot \hat{a_x} + d/dt\, H_y\, \hat{a_y} + d/dt \cdot H_z \cdot \hat{a_z}\right]$$

$-d/dx\,(E_z) = -\mu_0\, d/dt\, H_y$

$\Rightarrow d/dx\,(E_z) = \mu \cdot d/dt\,(H_y)$ — (3)

Next, $d/dx\,(E_y) = -d/dt(H_z)\mu$

Hence, relations are from 1st Maxwell's equation

$$\boxed{\begin{aligned} -d/dx\, H_z &= \varepsilon \cdot d/dt\,(E_y) \\ d/dx\, H_y &= \varepsilon\, d/dt\,(E_z) \\ d/dx\, E_z &= \mu_0\, d/dt(H_y) \\ d/dx\, E_y &= -\mu_0 \cdot d/dt(H_z) \end{aligned}}$$

} Maxwells - I Equation

} Maxwells - II Equation

we have, equation, $E_y = f_1(x - v_0 t)$,

$d/dt\,(E_y)\,\varepsilon_0 = -d/dx\, H_z$ — (4)

$d/dt\,(E_y) = \frac{-v_0}{v_0} \cdot f'(x_1 - v_0 t)$, $v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Sub in Eq^n — (4)

$-d/dx\, H_z = \frac{-v_0}{v_0} \cdot f'(x_1 - v_0 t)\,\varepsilon_0$.

$-d/dx\, H_z = -\frac{1}{\sqrt{\mu_0 \varepsilon_0}} \cdot \varepsilon_0 \cdot f'(x_1 - v_0 t)$

$-d/dx\, H_z = \frac{-\sqrt{\varepsilon_0}}{\sqrt{\mu_0}} \cdot f'(x_1 - v_0 t)$.

$d/dx\, H_z = -\frac{\sqrt{\varepsilon_0}}{\sqrt{\mu_0}} \cdot d/dt\, f(x_1 - v_0 t)$

$H_z = +\sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot f(x - v_0 t)$. we have $f(x - v_0 t) = E_y$

$H_z = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot E_y \Rightarrow \frac{E_y}{H_z} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

μ_0, ε_0 are constants for free space, and $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

η_0 - Intrinsic impedance $= \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{E_y}{H_z} = 120\pi$

Therefore, intrinsic impedance of free space, $\boxed{\eta_0 = 377\,\Omega}$

For any other media, $\eta = \sqrt{\mu/\varepsilon}$.

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Wave Equations in Phasor form

let us consider the wave equation $\nabla^2 \bar{E} = \mu_0 \varepsilon_0 \ddot{\bar{E}}$

let us consider the Solution expansion is

$$E_x = k_1 e^{-j\beta z} + k_2 e^{+j\beta z}$$

let the electric field representation is $\bar{E} = E_0 . e^{j\omega t}$

$$\ddot{\bar{E}} = d/dt(\dot{\bar{E}}) = d/dt\left[d/dt(\bar{E})\right] = d/dt\left[d/dt\left[E_0 . e^{j\omega t}\right]\right]$$

$$= d/dt\left[E_0 . e^{j\omega t} , j\omega t\right] = E_0 . e^{j\omega t} . (j\omega t)^2 .$$

$$= E_0 . e^{j\omega t} . (j)^2 (j\omega)^2$$

$$\nabla^2 \bar{E} = \mu_0 \varepsilon_0 \ddot{\bar{E}} = \mu_0 \varepsilon_0 \left[-E_0 . e^{j\omega t} \omega^2\right] = -\mu_0 \varepsilon_0 \omega^2 \bar{E}$$

$$\boxed{\nabla^2 \bar{E} = -\mu_0 \varepsilon_0 \omega^2 \bar{E}} \quad \text{— (1)}$$

$\nabla^2 \bar{E} = -\beta^2 \bar{E}$ — (2), By Comparison (1) and (2),

$$\beta^2 = +\mu_0 \varepsilon_0 \omega^2$$

$\boxed{\beta = \omega\sqrt{\mu_0 \varepsilon_0}}$ for free Space.

for any other media, $\boxed{\beta = \omega\sqrt{\mu\varepsilon}}$

$$\boxed{\beta = 2\pi f \sqrt{\mu\varepsilon}}$$

Wave Equation in phasor form for Conducting media

let us consider wave equation of conducting media

$$\nabla^2 \bar{E} = \sigma\mu \dot{\bar{E}} + \mu\varepsilon \ddot{\bar{E}}$$

$$\nabla^2 \bar{E} = \mu(\sigma \dot{\bar{E}} + \varepsilon \ddot{\bar{E}}) \quad , \quad E = E_0 . e^{j\omega t}$$

$$\dot{\bar{E}} = E_0 . (j\omega) . e^{j\omega t} .$$

$$\nabla^2 \bar{E} = \mu\left[j\omega\sigma + (j\omega)^2 \varepsilon\right] \bar{E} \qquad \dot{\bar{E}} = \bar{E}(j\omega)$$

$$\nabla^2 \bar{E} = \mu . j\omega\left[\sigma + j\omega \varepsilon\right] \bar{E}$$

Similarly,

$$\ddot{\bar{E}} = (j\omega)(j\omega) . \bar{E}$$

$$\nabla^2 E = \gamma^2 \bar{E}$$

where $\gamma^2 = j\omega\left[\sigma + j\omega . \varepsilon\right]\mu \qquad \ddot{\bar{E}} = (j\omega)^2 \bar{E}$

$$\boxed{\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}}$$

$\gamma = \alpha + j\beta$,

$$\gamma^2 = j^2\omega\mu\left(\sqrt{(\sigma/j) + \omega\varepsilon}\right)^2$$

$$\gamma = j\sqrt{\omega\mu(\sigma/j + \omega\varepsilon)}$$

In free space, attenuation is 0, $\alpha = 0$, $\beta = \omega\sqrt{\mu_0\varepsilon_0}$.
Because there is no reflection and obsorption of signal.

Transverse EM wave:-

If the wave propogates along x-direction, and travels E Compoment exists along y direction with the velocity, $v_0 = \omega/\beta$. H Compoments exists along z direction with the same velocity.

Generally, when wave propogates through any media its amplitude reduces and phase of it changes.

$\alpha + j\beta = \gamma$.

$\alpha \to$ Attenuation Constant,

$\beta \to$ Phase Constant.

$\gamma \to$ propogation Constant.

Attenuation Constant:-

The reduction in the amplitude of Em wave through any media is known as attenuation constant (α).

It is measured as Attenuation Constant (α).

Phase Constant (β)

Phase of EM wave changes through any media is known as phase constant (β).

$v_0 = \omega/\beta \Rightarrow \beta = \omega/v_0$.

Propogation Constant (γ)

As EM propogates through any media, the loss in amplitude (α) and phase (β) is called propogation constant. It is the combination of attenuation constant and phase constant.

$$\boxed{\gamma = \alpha + j\beta}$$

Wavelength (λ):-

Every sinusoidal signal changes its phase after 2π radians/360°, and hence the length of one complete cycle is called wavelength.

$\lambda = \frac{2\pi}{\beta}$ rad/ $\rightarrow 2\pi$

WAVE PROPAGATION IN LOSSLESS MEDIA / PERFECT DIELECTRIC / FREE SPACE

Consider wave is propogating through perfect dielectric and its properities are given by $\sigma = 0$, $\mu = \mu_r\mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$.

let us consider uniform plane wave propogating along x-direction in free space. Then wave equation in phasor form is $\nabla^2 \bar{E} = -\beta^2 \bar{E}$

$d^2/dx^2 \bar{E} = -\beta^2 \bar{E}$

$d^2/dx^2 (E_y \cdot \hat{a_y} + E_z \cdot \hat{a_z}) = -\beta^2 (E_y \cdot \hat{a_y} + E_z \cdot \hat{a_z})$

Consider only one compoment, then

$$d^2/dx^2 E_y \cdot \hat{a_y} = -\beta^2 E_y \cdot \hat{a_y}$$

$$d^2/dx^2 E_y \cdot \hat{a_y} + \beta^2 E_y \cdot \hat{a_y} = 0 \quad \text{—(1)}$$

The solution for above second order differential Equation is $\underbrace{E_y(x) = c_1 e^{-j\beta x}}_{\text{forward wave along +ve x direction}} + \underbrace{c_2 e^{j\beta x}}_{\text{backward wave in -ve x direction}}$

c_1, c_2 are arbitrary constants.

$$\hat{E_y}(x,t) = Re\left[E_y(x) e^{j\omega t}\right]$$

$$= Re\left[(c_1 e^{-j\beta x} + c_2 e^{j\beta x}) e^{j\omega t}\right]$$

$$= Re\left[c_1 e^{j\omega t - j\beta x} + c_2 e^{j\omega t + j\beta x}\right]$$

$$= Re\left[c_1 e^{j(\omega t - \beta x)} + c_2 e^{j(\omega t + \beta x)}\right]$$

$$\boxed{\hat{E_y}(x,t) = \underbrace{c_1 \cos(\omega t - \beta x)}_{\text{forward wave}} + \underbrace{c_2 \cos(\omega t + \beta x)}_{\text{Backward wave.}}}$$

$\sigma = 0$, $\mu = \mu_0$, $\epsilon = \epsilon_0$, $\epsilon_r = \mu_r = 1$, $J_c = 0$.

Characteristics of wave propogation in dielectric lossless medium

(a) Velocity (v)

$$\text{Velocity}(v) = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{\frac{1}{\sqrt{\mu_0\epsilon_0}}}{\sqrt{\mu_r\epsilon_r}}. \quad c = \frac{1}{\sqrt{\mu_0\epsilon_0}}.$$

$$v = \frac{c}{\sqrt{\mu_r\epsilon_r}} \text{ m/s.}$$

or $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{\omega}{\beta}$, hence, $\boxed{v = \omega/\beta}$

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(b) Proof for Attenuation Constant

Since, dielectric media is lossless, attenuation constant, $\boxed{\alpha = 0}$ dB/m

(c) Phase constant (β)

we have phase constant

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r \cdot \varepsilon_0\varepsilon_r} = \omega\sqrt{\varepsilon_0 \cdot \mu_0}$$

$$\boxed{\beta = \omega\sqrt{\mu_0\varepsilon_0} \text{ radians/m}}$$ as $\beta = 2\pi/\lambda$.

(d) propogation constant:- (γ)

$\gamma = \alpha + j\beta$.

but we have $\alpha = 0$, $\beta = \omega\sqrt{\mu_0\varepsilon_0}$

$\gamma = \omega j\sqrt{\mu_0\varepsilon_0} \Rightarrow \boxed{\gamma = j\omega\sqrt{\mu_0\varepsilon_0} \; m^{-1}}$

(e) Wavelength

$$\text{wavelength } (\lambda) = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} \; m.$$

$$\boxed{\lambda = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} = \frac{2\pi}{\lambda} = m}$$

(f) Intrinsic Impedance:-

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \text{ (or) } \eta = \sqrt{\mu/\varepsilon}.$$

$$\boxed{\eta = 377\Omega}$$

(g) Phase velocity (v_p)

The wave phase velocity travels with general velocity, $\boxed{v_p = v_0.}$

(1) Attenuation Constant (α) $\longrightarrow$ 0 dB/m

(2) propogation Constant (γ) $\rightarrow j\omega\sqrt{\mu_0\varepsilon_0}$ rad/m^{-1}.

(3) Phase Constant (β) $\rightarrow \omega\sqrt{\mu_0\varepsilon_0}$ rad/m

(4) Velocity (v_0) $\longrightarrow v_0 = \omega/\beta = f\lambda = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8 \, ms^{-1}$

(5) Phase velocity (v_p) $\rightarrow v_p = v_0$

(6) Intrinsic Impedance (η_s) $\rightarrow 377\Omega = \sqrt{\mu_0/\varepsilon_0}$.

(7) Wavelength (λ) $\rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}}$ m

WAVE PROPOGATION IN CONDUCTING MEDIA / LOSSY MEDI

Consider uniform plane wave Em wave is propogati along x direction in conducting media. wave Equation in phasor form is

$$\nabla^2\bar{E} = \gamma^2\bar{E}$$

$$\Rightarrow \frac{d^2}{dx^2}\bar{E} = \gamma^2\bar{E} \Rightarrow \boxed{\frac{d^2}{dx^2}\bar{E} - \gamma^2\bar{E} = 0}$$

Then above equation in time varying form

$$\bar{E}(x,t) = Re\left[c_1 e^{-\gamma x} + c_2 e^{\gamma x}\right]$$

. Since, Backward wave is neglected.

$$\bar{E}(x,t) = Re\left[c_1 e^{-\gamma x} \cdot e^{j\omega t}\right]$$

$$\bar{E}(x,t) = Re\left[c_1 e^{-\gamma x + j\omega t}\right], \quad \gamma = j\omega$$

$\gamma = \alpha + j\beta$,

$$\bar{E}(x,t) = Re\left[c_1 e^{-(\alpha+j\beta)x + j\omega t}\right]$$

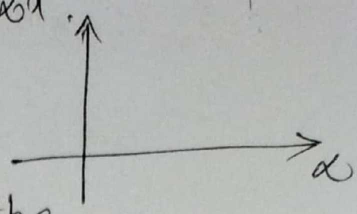
$$= Re\left[c_1 e^{-\alpha x} \cdot e^{-j\beta x + j\omega t}\right]$$

$$= Re\left[c_1 e^{-\alpha x} \cdot e^{j(\omega t - \beta x)}\right]$$

$$\boxed{\bar{E}(x,t) = c_1 e^{-\alpha x} \cdot \cos(\omega t - \beta x)}$$

$\bar{E}(x,t) = Re[c\, e^{-\alpha x} \cdot e^{j(\omega t - \beta x)}]$

$E(x,t) = c e^{-\alpha x} \cdot \cos(\omega t - \beta x)$



If $\alpha \uparrow$, $e^{-\alpha x} \downarrow$, it indicates that the Amplitude of EM is reduced factor $e^{-\alpha x}$.

o) Wave characteristics

(a) Attenuation constant (α), dB/m

Consider, $\nabla^2 E = \gamma^2 E$

$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) \Rightarrow \gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$\gamma^2 = j\omega\mu \cdot j\omega\varepsilon \left(\sigma/j\omega\varepsilon + 1\right)$

$\gamma^2 = + \omega^2\mu\varepsilon \left(\sigma/j\omega\varepsilon + 1\right)$

$\gamma^2 = +\omega^2\mu\varepsilon\left(\frac{-\sigma}{\omega\varepsilon} + 1\right) \Rightarrow \gamma^2 = \omega^2\mu\varepsilon\left(1 - \frac{\sigma}{\omega\varepsilon}\right)$

$|\gamma|^2 = \left[\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}}\right]\omega^2\mu\varepsilon$ — (a)

We have $\gamma = \alpha + j\beta$

$\gamma^2 = (\alpha + j\beta)^2 = \omega^2\mu\varepsilon\left(1 - \frac{\sigma}{\omega\varepsilon}\right)$

$|\gamma| = \sqrt{\alpha^2 + \beta^2} \Rightarrow |\gamma|^2 = \alpha^2 + \beta^2$ — (b)

$\alpha^2 + \beta^2 + j \cdot 2\alpha\beta = \omega^2\mu\varepsilon\left(1 - \sigma/\omega\varepsilon\right)$ — (a)

(or)

$\gamma^{(2 \cdot \frac{1}{2})} = (\alpha + j\beta)^{(2 \cdot \frac{1}{2})} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

Compare eqⁿ (a) & (b)

$\omega^2\mu\varepsilon\left[\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}}\right] = \alpha^2 + \beta^2$ — (c)

$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$(\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$\alpha^2 + j^2\beta^2 + 2\alpha j\beta = j\omega\mu\sigma - \omega^2\mu\varepsilon$.

$(\alpha^2 + j^2\beta^2) + j \cdot 2\alpha\beta = -\omega^2\mu\varepsilon + j\omega\mu\sigma$ — (1)

Equating both sides for Eqⁿ (1)

$(\alpha^2 - \beta^2) + j \cdot 2\alpha\beta = -\omega^2\mu\varepsilon + j\omega\mu\sigma$ — (2)

Equate Imaginary parts and real parts

$$(\alpha^2-\beta^2) = -\omega^2\mu\varepsilon,$$

$$2\alpha\beta = \sigma\mu\omega$$

we have $(\alpha+\beta^2)^2 = (\alpha-\beta)^2 + 4\alpha\beta.$

$$\alpha^2+\beta^2+2\alpha\beta = (\alpha+\beta)^2$$

$$\alpha^2-\beta^2+\alpha^2+\beta^2 = \omega^2\mu\varepsilon\sqrt{1+\frac{\sigma^2}{\omega\varepsilon^2}} - \omega^2\mu\varepsilon$$

$$2\alpha^2 = \omega^2\mu\varepsilon\left[\sqrt{1+\frac{\sigma^2}{\omega\varepsilon^2}} - 1\right]$$

$$\alpha = \frac{\omega^2}{2}\mu\varepsilon\left[\sqrt{1+\frac{\sigma^2}{\omega\varepsilon^2}} - 1\right]$$

$$\alpha = \sqrt{\frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right]}$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right]}}$$

(b) ~~Propogation~~ Phase Constant (β)

$$\alpha^2+\beta^2 = \omega^2\mu\varepsilon\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}$$

$$\alpha^2-\beta^2 = -\omega^2\mu\varepsilon$$

$$\alpha^2+\beta^2-(\alpha^2-\beta^2) = \omega^2\mu\varepsilon\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + \omega^2\mu\varepsilon$$

$$2\beta^2 = \omega^2\mu\varepsilon\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right]$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1+\sigma^2/\omega^2\varepsilon^2} + 1\right]$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\sigma^2/\omega^2\varepsilon^2} + 1\right]}}$$

(c) Propogation Constant

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$\Rightarrow \boxed{\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}\ m^{-1}}$$

(d) Intrinsil Impedance

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}},$$ for any media other than free space

(e) wavelength (λ)

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}}\ \text{rad/m}$$

(f) Velocity (v_0)

$$v_0 = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}\ \text{m/s}.$$ In general

$$v_0 = \frac{\omega}{\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}}$$

$$v_0 = \frac{1}{\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}}$$

characteristics

(a) Attenuation constant $(\alpha) = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$

(b) phase constant $(\beta) = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$

(c) $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

(d) $\lambda = \frac{2\pi}{\beta}$, (e) velocity $v_0 = \frac{1}{\sqrt{\mu\varepsilon}}$.

WAVE PROPOGATION IN DIELECTRIC MEDIA

Consider the lossy dielectric media.

Consider the maxwells equation $\nabla \times \bar{H} = J_c + \bar{J}_d$ — (1)

where according to ohm's law, $\bar{J} = \sigma \bar{E}$

$\bar{J}_d = d/dt\,(\bar{D})$

Substitute both terms in Eqn (1)

$$\nabla \times \bar{H} = \sigma \bar{E} + \frac{d}{dt}(\bar{D})$$

$$= \sigma \bar{E} + \frac{d}{dt}(\varepsilon \bar{E}) = \sigma \bar{E} + \varepsilon \cdot \frac{d}{dt}(\bar{E})$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \cdot \dot{\bar{E}}$$

$$= \sigma \bar{E} + j\omega \cdot \varepsilon \bar{E}$$

$$\left| \frac{J_c}{J_d} \right| = \frac{\sigma \cancel{E}}{\sqrt{(\varepsilon^2 \omega^2)}\cancel{E}} = \frac{\sigma}{\omega \varepsilon}$$

Case i) Consider, $\sigma/\omega\varepsilon >> 1$, $\sigma >> \omega\varepsilon$,

we know $\omega = 2\pi f$

As frequencies decreases, conductivity increase

Then at lower frequencies, the dielectric media acts as good conductor. Hence, the wave propogation is same as like in good conductor media.

Case ii) Consider, $\sigma/\omega\varepsilon << 1$, $\sigma << \omega\varepsilon$.

$\omega = 2\pi f$, As f increases, conductor decreases.

Then, at very higher frequencies, dielectric media acts as perfect dielectric media. So, the wave propogation is same as like as perfect/good dielectric media.

Wave propogation in Good ~~Cond~~ Conductor

Consider wave equation for conducting media, $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$.

$$\gamma = \sqrt{j\omega\mu\sigma\left(1 + \frac{j\omega\varepsilon}{\sigma}\right)}.$$

Case I :- When waves are travelling in good conductor ($\omega\varepsilon/\sigma \ll 1$), so, media acts as good conductor at lower frequencies.

$$\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma}\,\angle 45^\circ = \sqrt{\omega\mu\sigma}\,[\cos 45 + j\sin 45]$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}.$$

Therefore, good conducting media is always lossy media in practice.

$$\gamma = \alpha + j\beta = \sqrt{\pi f\mu\sigma} + j\sqrt{\pi f\mu\sigma}.$$

So, $\omega\varepsilon/\sigma \ll 1$, $\left(1 + \frac{j\omega\varepsilon}{\sigma}\right)$ eliminated

Skin Depth / Depth of Penetration (δ)

The distance through which the amplitude of travelling Electromagnetic wave reduces to 37% of original amplitude. It is denoted as δ.

$$\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \frac{1}{\sqrt{\frac{2\pi f\mu\sigma}{2}}} = \frac{1}{\sqrt{\pi f\mu\sigma}}.$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}\ m}$$

POLARIZATION:-

It is defined as the time varying behaviour of electric field vector at given point in space.

2) It is defined as the tip of locus of electric field vector at given point in Space as function of time

3) It defines the way in which E changes along the direction of wave propogation

Types of Polarisation.

linear, Circular and Elliptical Polarisation

(a) Linear Polarisation:-

If $\bar{E}$ oscillates along the straight line and follow the direction of propogation, then it is so called linear Polarisation.

If the direction is along E_z, z direction,

$E_x \neq 0, E_y \neq 0,$

$$\bar{E} = \sqrt{x^2+y^2},$$

then resultant angle $(\theta) = \tan^{-1}(E_y/E_x)$.

If $E_x \neq 0, E_y = 0$, then the wave is polarised along x-direction, If $E_y \neq 0, E_x = 0$, then the wave is said to be polarised along y-direction.

b) Elliptical polarisation

It is the tip of locus of $\bar{E}$ traces ellipse in Space. If $E_x \neq E_y$ and E_x & E_y are in out of phase, $\theta_y = \theta_x - \pi/2$.

c) Circular Polarisation

It is defined as the tip of locus of $\bar{E}$ traces circular or circular path in space.

If $E_x = E_y = E_0$, E_x, E_y are out of phase

i.e $\theta_y = \theta_x - \pi/2$

EM WAVE CHARACTERISTICS - II

Upto now, we considered that the waves are travelling in unbounded and non homogeneous media. But practically, very often, waves propogate in bounded regions consisting several media of different constitutive parameters $\varepsilon, \mu, \sigma, \eta$ etc.

There are two cases that practically happen when the waves are travelling and incident on boundary.

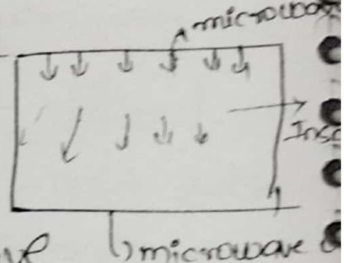
Case I :- If ; Consider a transmission line having a characteristic impedance Z_0. Assume that a line is terminated with a load impedance Z_L. If a load impedance Z_L equals to the characteristic impedance Z_0 (i.e, $Z_L = Z_0$), then line is said to be properly terminated.

If $Z_L \neq Z_0$, then there is a mismatch between the two impedances and the line is not properly terminated. Consider that the wave travelling along the line incidents at the load. The part of the wave gets obsorbed by the load, while the other part is reflected back to the generator. So, we can say that reflection occurs at the load if $Z_L \neq Z_0$. If there are two waves, are there, one is incident in forward direction and other in backward direction, then standing waves are produced along the line or transmission line.

Case 2:-

Now again consider the uniform plane wave is travelling in non-homogeneous media. When a uniform plane waves travels from one boundary one media to another media having different characteristic or Intrinsic impedances, the reflection takes place at the boundary. The part of the wave is transmitted in medium 2 and remaining part of the wave reflected back to the medium 1, depending upon the constitutive parameters of media.

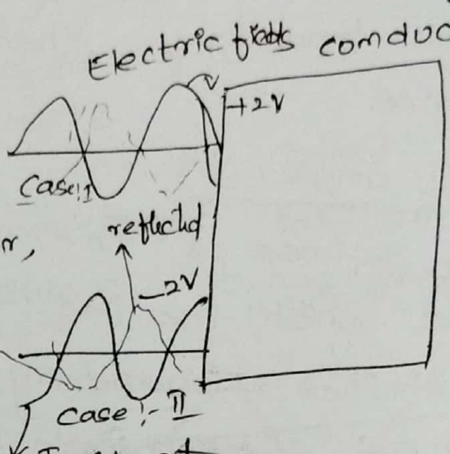
Eg:- In the microwave oven, or elevators the microwaves are will not cannot come outside. The microwaves are generated inside and reflected back to the inside it. This is because, in the inside of microwave oven, the inner boundary parts are coated with a conducting material or Conductor. These conductor does not allow the microwave frequencies to go outside. The conductivity of conductor determines the percentage of reflected waves. If the conductivity is ∞, then the waves completes reflected back. This is ideal case, where 100% reflection takes place. copper



If σ is finite, Some of the waves reflected back and Some of the waves are transmitted These products standing waves. Eg:- Iron. If $\sigma = 0$, it acts as insulator, then the waves are transmitted completely and there is no reflection.

Reason for reflection of wave by conductor:-

When the wave is incident on conductor, the waves get reflected back. Here, the conductor, the regenerates the opposite voltage equal to the incident waves and generates the wave. This wave/signal is reflected back.

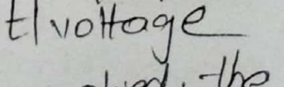


In case:-I) When the wave with Electric field 2 V/m is incident on conductor, there are free electrons in conductors and these get rearranged. These movement of ~~current~~ free electrons makes the current production, and the voltage also induced. This voltage is with opposite voltage, i.e -2V as the current direction is -ve. So, because of this, the waves are reflected back as the current/voltage produced should in opposite direction to the applied field.

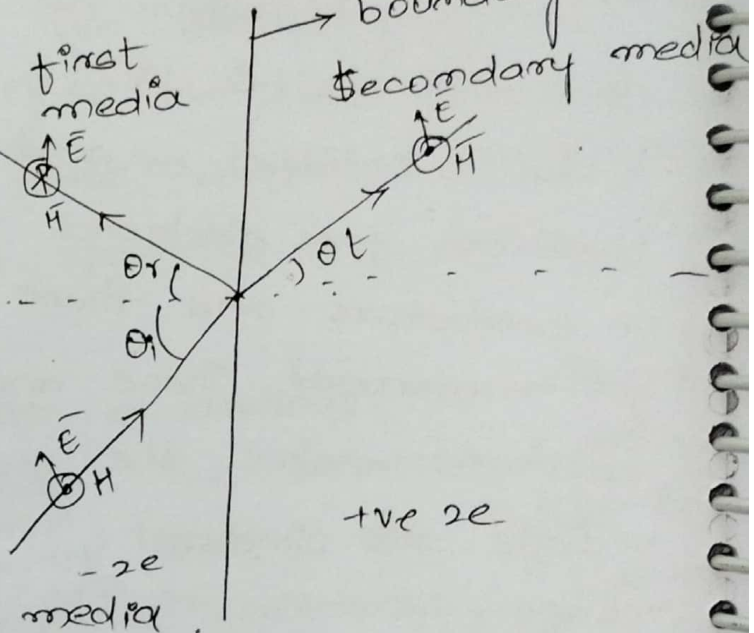
we can also conclude, the incident and reflected magnitudes are same but the signs are opposite and they cancel out each other. The net electric field is 0.

In the total process, the currents which are produced inside conductor vary with time continuously.

Another example is capacitor. If we apply field for capacitor, the current/voltage inside the capacitor. If dc voltage is applied, the steady current/voltage is produced. If ac voltage is applied time varying current/voltage is produced.



The reflection of uniform plane wave not only depends on the type of boundary. It also depends on the way in which waves are incident on the boundary or obstacle. Depending upon the manner in which the uniform plane wave is incident on the boundary, there are two types of incidence.



Incident angle (θi):-

It is defined as the angle at which the EM wave strikes a boundary ~~condit~~ at some point.

Reflected angle (θr):-

It is the angle at which the incident EM wave reflected back into same media.

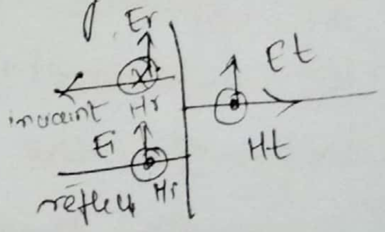
Transmission angle (θt):-

It is the angle at which incident EM wave transmission or passing or refracting into second media.

Types of incidence

→ Normal Incidence:- It is defined as the EM wave which is striking the boundary $\theta i = 90°$

In this reflected, the wave is also in the same direction



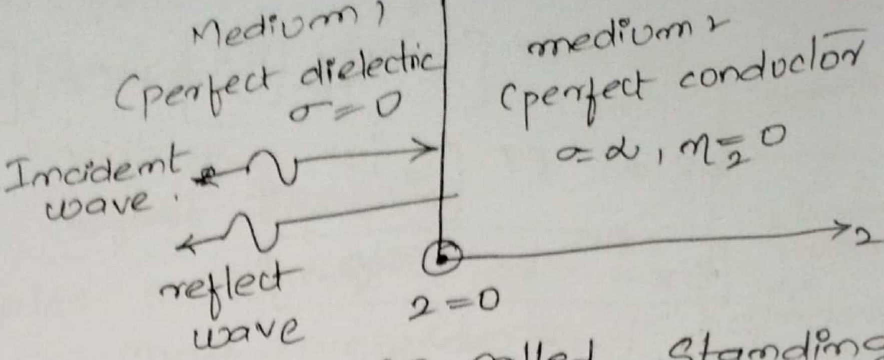
Percentage of wave transmitted is less in second media is less than oblique incidence.

→ Oblique Incidence:- It is defined as the EM wave is striking the boundary other than 90°. $\theta i \neq 90$

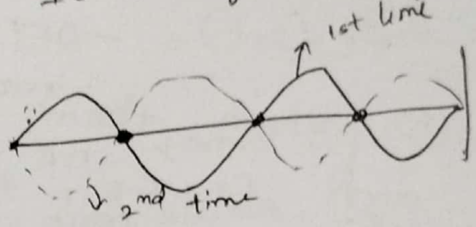
The % of wave transmission in oblique incidence is more than normal incidence.

INCIDENCE

When EM travels in air and incidenting on perfect conductor the reflection takes place and produces



standing waves. These waves are so called standing waves because there is a movement in amplitude and phase but not with position. It has fixed points.



using boundary conditions, the tangential components of E is 0.

we have, let incident wave $= E_i e^{-j\beta z}$

Reflected wave $= E_r e^{j\beta z}$.

Resultant wave = Incident wave + reflected wave

$$E_R = E_i e^{-j\beta z} + E_r \cdot e^{j\beta z}$$

Incident and reflected waves are phase reversal to satisfy the boundary condition, that the tangential componentials of E is 0 at boundary. Since, propogation is along z direction, the resultant wave along z direction is 0. i,e, $z=0$, $E_R=0$. as it a uniform plane wave.

$$\Rightarrow 0 = E_i e^{-j\beta z} + E_r e^{j\beta z} \quad (z=0)$$

$$0 = E_i + E_r \Rightarrow \boxed{E_i = -E_r}$$

This indicates that the Magnitude of incident wave and magnitude of reflected wave are equal, but they are phase reversal (i,e out of phase)

- reflected wave.

In standing wave, energy is ... as ... cavity resonator

Sub $E_i = -E_r$ in $E_R = E_i e^{-j\beta z} - E_i e^{j\beta z}$.

$$E_R = E_i \left[e^{-j\beta z} + (e^{j\beta z}) \right] = E_i \left[e^{-j\beta z} + (-e^{j\beta z}) \right]$$

$$E_R = -E_i \left[e^{j\beta z} - e^{-j\beta z} \right]$$

$$\boxed{E_R = -2E_i \sin\beta z} \, e$$

In time varying form.

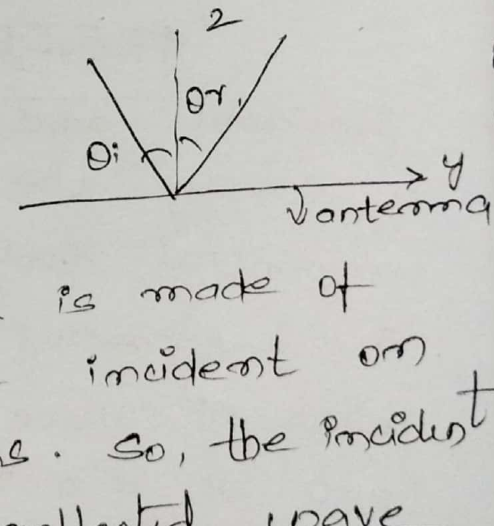
$$E_R(z,t) = Re\left[\left[-2E_i \sin\beta z \right] e^{j\omega t} \right]$$

$$E_R(z,t) = -2E_i \sin\beta z \cdot \sin\omega t .$$

(i) When there is no obsorption, that = 0, and $E_i = E_R$, then the wave is called complete standing wave,

ii) When there is standing wave and there less obsorption such that $E_i > E_R$, then the wave is called Incomplete standing wave.

Application of using Conductor as boundary

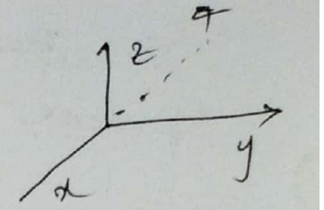


If we incidence EM wave on conductor, the waves are exactly reflected back. This happens when the incidence is oblique incidence. So, antenna is made of conductor. When the waves are incident on antenna, it exactly reflected backs. So, the incident angle is selected in which the reflected wave should reflected back exactly in desired direction

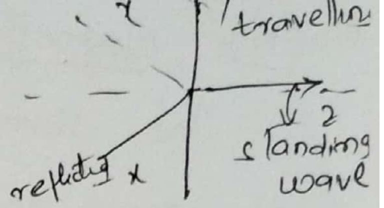
In the above figure, standing wave is produced in z direction, along y-axis travelling wave is produced.

$E_i(y,z) = E e^{-j\beta(y\sin\theta - z\cos\theta)} \cdot u_x$

$E_r(y,z) = -E e^{-j\beta(y\sin\theta + z\cos\theta)} \cdot u_x$

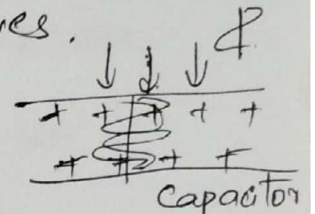


$E_{total}(y,z) = E_i(y,z) + E_r(y,z)$



Conductor for normal incidence

Cavity resonators, reflect klystrons are the best examples in which the waves are with normal incidence on boundary. Internally, it produces the standing waves.



Inspections on wave Equation

In The standing wave Equation

i) The wave Amplitude varies in time by $\sin\omega t$

ii) The wave varies in space variables by $\sin\beta z$.

$$\bar{E_R}(z,t) = -2E_i \sin\beta z \cdot \sin\omega t.$$

iii) a) Minima of wave.

As $E_R = 0$, $E_R = -2E_i\sin\beta z \Rightarrow \sin\beta z = 0 \rightarrow \sin\beta z = \sin n\pi$

$$\boxed{\beta z = n\pi}, \quad n = 0,1,2,3.$$

$\beta z = -n\pi \Rightarrow \frac{2\pi}{\lambda}\cdot z = -n\pi \quad (\because \beta = \frac{2\pi}{\lambda})$

$2z = -n\lambda \Rightarrow \boxed{z = -\frac{n\lambda}{2}}$

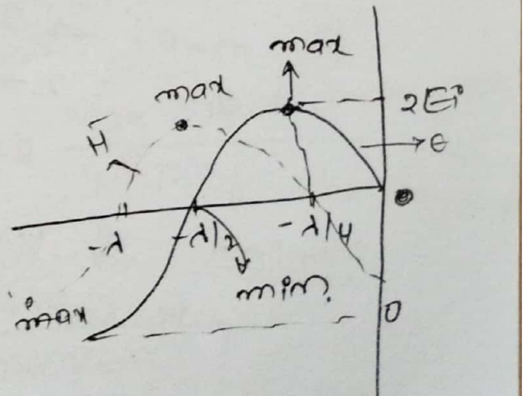
For $n=0 \Rightarrow z=0$

$n=1 \Rightarrow z = -\lambda/2$

$n=2$, $z=-\lambda$

$n=3$, $z = -3\lambda/2$

Minima are called points.



b)) Maxima of ER

$E_{R\,max} = 2E_i$,

$2E_i = -2E_i\sin\beta z \Rightarrow \sin\beta z = \sin(n\pi + \pi/2)$

$\boxed{\beta z = -\left(n\pi + \frac{\pi}{2}\right)} \Rightarrow \frac{2\pi}{\lambda} = -\pi\left(n+\frac{1}{2}\right)$

$z = -m\lambda - \lambda/4$.

If $n=0$, $z=-\lambda/4$, If $n=0$, $z=-\lambda/4$.

$n=1$, $z=-3\lambda/4$.

(iv) Magnetic Field

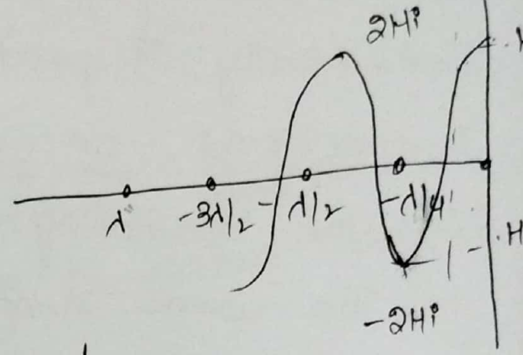
The standing wave equation for magnetic field is given as. $H_R = H_i e^{-j\beta z} + H_R e^{j\beta z}$.

As $E \perp^{lar}$ to H, if, $\boxed{E_i = -E_R}$, $\boxed{H_i = H_R}$

$$H_R = H_i e^{-j\beta z} + H_i e^{j\beta z}$$

$$H_R = 2H_i\left[e^{j\beta z} + e^{-j\beta z}\right]$$

$$\boxed{H_R = 2H_i \cos\beta z}$$



In Time Varying form.

$$H_R^{\wedge}(z,t) = 2H_i \cos\beta z . \cos\omega t .$$

Inspections:-

i) The wave amplitude varies in time by $\cos\omega t$.

ii) The wave amplitude varies in Space by $\cos\beta z$.

iii) $H_R = 2H_i \cos\beta z$.

Maxima of wave

$H_R = 2H_i$, for, $2H_i = 2H_i \cos\beta z \Rightarrow \cos\beta z = \pm 1 \rightarrow \cos\beta z = \cos m\pi$

$\beta z = -m\pi$, $m = 0,1,2,3.$, $z = -\frac{m\pi}{\beta} = -\frac{m\pi}{2\pi/\lambda} = -\frac{m\lambda}{2}$

For $m=0$, $\Rightarrow z=0$

$m=1$, $z=\lambda/2$

$m=2$, $z=-3\lambda/2$

Minima of wave

$H_R = 0$, for $2H_i = 2H_i \cos\beta z \Rightarrow \cos\beta z = 0 \rightarrow \cos\beta z = -(m\pi + \frac{\pi}{2})$

$\beta z = -(m\pi + \frac{\pi}{2}) \Rightarrow z = -\frac{m\lambda}{2} - \lambda/4$

If $m=0$, $z=-\lambda/4$

$m=1$, $z=-3\lambda/2$.

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Reflection coefficient

Reflection coefficient = $\frac{\text{Reflected wave}}{\text{incident wave}} = \Gamma$

(a) for Electric field, $\Gamma_E = \frac{E_R}{E_i}$

In conductors, reflected wave = incident wave, but with opposite magnitude/sign.

$\Gamma_E = \frac{E_R}{E_i} = \frac{E_R}{-E_i}$ $(\because E_R = -E_i)$

$\Gamma_E = \frac{-E_R}{E_i} \rightarrow \boxed{\Gamma_E = -1}$

(b) for Magnetic field, $\Gamma_H = \frac{H_R}{H_i}$

Since, $H_R = H_i$

$\Gamma_H = \frac{H_R}{H_i} = \frac{H_R}{H_R} = 1$, $\boxed{\Gamma_H = 1}$

Transmission coefficient

Transmission coefficient = $\frac{\text{Transmitted wave}}{\text{Incident wave}} = T$

(a) For Electric field, $T_E = \frac{E_t}{E_i}$

Since, $E_t = 0$,

$T_E = \frac{E_t}{E_i} = \frac{0}{E_i} = \infty$, $\boxed{T_E = \infty}$, for perfect conductor

(b) for Magnetic field

Since, $H_t = 0$

$T_H = \frac{H_t}{H_i} = \frac{0}{H_i}$, $\boxed{T_H = \infty}$

Voltage Standing Wave Ratio

Voltage Standing Wave Ratio = $\frac{\text{Max. of standing wave}}{\text{Min. of standing wave}}$

(a) for Electric field, $VSWR = \frac{E_{max}}{E_{min}}$

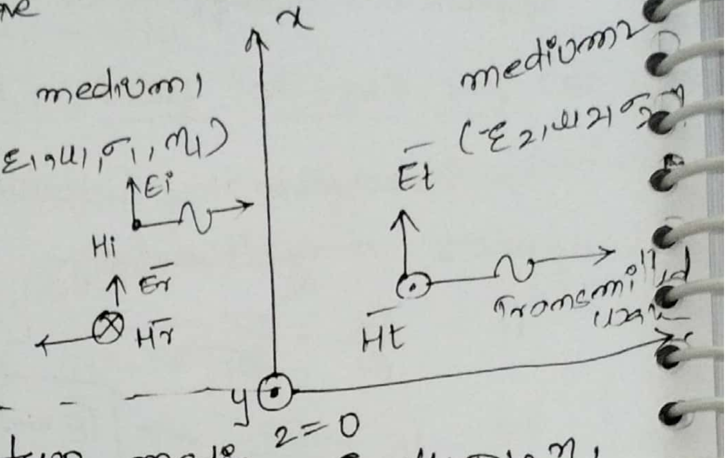
$VSWR = \frac{E_{max}}{E_{min}} = \frac{2E_i}{0} = \infty$, $\boxed{VSWR = \infty}$

(b) For Magnetic field. $VSWR = \frac{H_{max}}{H_{min}}$

$VSWR = \frac{H_{max}}{H_{min}} = \frac{2H_i}{-2H_i}$ $\boxed{VSWR = -1}$

2) NORMAL INCIDENCE ON PLANE DIELECTRIC BOUNDARY

Consider uniform plane wave is propogating along z direction and incidence at right angles at the boundary between two dielectric media, i.e $z=0$



Let the properities of two media, $\varepsilon_1, \mu_1, \sigma_1, \eta_1$, $\varepsilon_2, \mu_2, \sigma_2, \eta_2$. Depending upon the values of those two media, part of the wave will be transmitted in medium, while the other part is reflected back.

Let E_i, H_i are the electric field striking the boundary, E_t, H_t are the electric field and magnetic strengthe of transmitted in the medium 2. and E_r, H_r are the reflected Electric field and magnetic fields in media 1.

Now, find, Γ_E, Γ_H, T_E & T_H. , we have $\eta_1 = \frac{E_i}{H_i}$

$\eta_2 = \frac{E_t}{H_t}$

(a) Γ_E

consider $\eta_2 = \frac{E_t}{H_t} \rightarrow H_t = \frac{E_t}{\eta_2}$ — (2)

we have $\boxed{H_i = H_t + H_r} \Rightarrow H_i - H_r = H_t$ — (1) $\quad -\eta_1 = +\frac{E}{H}$

$\eta_1 = \frac{E_i}{H_i} \Rightarrow H_i = \frac{E_i}{\eta_1}$

Sub all the equations in Eq^n — (1)

$\frac{E_i}{\eta_1} + \left[+H_R\right] = H_t$

$H_t = H_i - H_r.$

$\Rightarrow \frac{E_i}{\eta_1} - \frac{E_R}{\eta_1} = H_t$

$\frac{E_i}{\eta_1} - \frac{E_R}{\eta_1} =$

We have $E_i = E_r + E_t$, $H_i = H_r + H_t \rightarrow H_i - H_r = H_t$ — (2)

$\eta_1 = \frac{E_i}{H_i}$; $-\eta_1 = \frac{E_r}{H_r}$; $\eta_2 = \frac{E_t}{H_t}$

$H_i = \frac{E_i}{\eta_1}$; $H_r = \frac{-E_r}{\eta_1}$; $H_t = \frac{E_t}{\eta_2}$

Then Sub in eq^n — (2) , $H_i - H_r = H_t$

$$\frac{E_i}{\eta_1} + \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$\Rightarrow \frac{E_i}{\eta_1} + \frac{E_r}{\eta_1} = \frac{E_i - E_r}{\eta_2}$$

$$\rightarrow \frac{E_i}{\eta_1} + \frac{E_r}{\eta_1} = \frac{E_i}{\eta_2} - \frac{E_r}{\eta_2}$$

$$\Rightarrow \frac{E_i}{\eta_1} - \frac{E_i}{\eta_2} = \frac{-E_r}{\eta_2} - \frac{E_r}{\eta_1}$$

$$\Rightarrow E_i\left[\frac{1}{\eta_1} - \frac{1}{\eta_2}\right] = -E_r\left[\frac{1}{\eta_1} + \frac{1}{\eta_2}\right]$$

$$\Rightarrow E_i\left[\frac{1}{\eta_2} - \frac{1}{\eta_1}\right] = E_r\left[\frac{1}{\eta_1} + \frac{1}{\eta_2}\right]$$

$$\Rightarrow E_i\left[\frac{\eta_1 - \eta_2}{\cancel{\eta_1 \eta_2}}\right] = E_r\left[\frac{\eta_1 + \eta_2}{\cancel{\eta_1 \eta_2}}\right]$$

$$\Rightarrow \frac{E_R}{E_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$\Rightarrow$ Reflection coefficient Γ_E for electric field

$$\boxed{\Gamma_E = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}}$$

For Magnetic field:-

Reflection coefficient for magnetic field $\Gamma_H = \frac{H_R}{H_i}$

we have $+\eta_1 = \frac{-E_R}{H_R}$, $\eta_1 = \frac{E_i}{H_i}$

$$\Gamma_H = \frac{H_R}{H_i} = \frac{-E_R/\eta_1}{E_i/\eta_1} = \frac{-E_R}{E_i} = -\left[\frac{E_R}{E_i}\right] = -\Gamma_E$$

$$\Gamma_H = -\Gamma_E \Rightarrow \boxed{\Gamma_H = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}}$$

b) Transmission coefficient (τ)

$$\tau_E / \tau_H = \frac{E_t}{E_i} = \frac{H_t}{H_i}$$

For electric field:-

$\tau_E = \frac{E_t}{E_i}$, we have $E_t + E_r = E_i \Rightarrow 1 = \frac{E_r}{E_i} + \frac{E_t}{E_i}$

$\frac{E_t}{E_i} = 1 - \frac{E_r}{E_i} = 1 - \Gamma_E$, we have $\Gamma_E = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$

$$\tau_E = \frac{E_t}{E_i} = 1 - \left[\frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}\right] = \frac{\cancel{\eta_1} + \eta_2 - \cancel{\eta_1} + \eta_2}{\eta_1 + \eta_2} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\boxed{\tau_E = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

For Magnetic field

$\tau_H = \frac{H_t}{H_i}$, ~~$H_t + H_r = H_i \Rightarrow 1 = \frac{H_t}{H_i} + \frac{H_r}{H_i} \Rightarrow \frac{H_r}{H_i} = 1 - \frac{H_t}{H_i}$~~

~~$\tau_H = \frac{H_t}{H_i} \Rightarrow \frac{H_t}{H_i} = 1 - \frac{H_r}{H_i} \Rightarrow \frac{H_t}{H_i} = 1 - \Gamma_H$~~

~~$\tau_H = 1$~~. In general $\tau_H = \frac{H_t}{H_i} = \frac{E_t/\eta_2}{E_i/\eta_1}$

$$= \left(\frac{E_t}{E_i}\right)\left(\frac{\frac{1}{\eta_2}}{\eta_1}\right) = \tau_E \cdot \left(\frac{\frac{1}{\eta_2}}{\eta_1}\right), \quad \tau_E = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\tau_H = \frac{2\eta_2}{\eta_1 + \eta_2}\left[\frac{\frac{1}{\eta_2}}{\eta_1}\right]$$

$$\tau_H = \frac{2\eta_2}{\eta_1 + \eta_2}\left[\frac{\eta_1}{\eta_2}\right]$$

$$\boxed{\tau_H = \frac{2\eta_1}{\eta_1 + \eta_2}}$$

In General

These are the general coefficients for all the media. Depending upon the intrinsic impedances, the above coefficients are calculated.

In General, for normal dielectric, where $\sigma \neq 0$, $\sigma =$ some finite

For Electric fields.

(1) Reflection coefficient $= \Gamma_E = \dfrac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$

(2) Transmission coefficient $= \tau_E = 1 - \Gamma_E = \dfrac{2\eta_2}{\eta_1 + \eta_2}$

For Magnetic fields:-

(1) Reflection coefficient $= \Gamma_H = -\Gamma_E = \dfrac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$

(2) Transmission coefficient $= \dfrac{2\eta_1}{\eta_1 + \eta_2} = \tau_H = \left[\dfrac{\eta_1}{\eta_2}\right]\left[\tau_E\right]$

But, for pure dielectrics, $\sigma = 0$, So, there is no reflected wave,

$E_r = 0$, $H_r = 0$, and $E_i = E_t$.

Transmission coefficient (τ)

(1) for Electric field:-

$$\tau_E = \frac{E_t}{E_i}, \text{ Since } E_i = E_t \Rightarrow \tau_E = \frac{E_i}{E_i}, \Rightarrow \boxed{\tau_E = 1}$$

So, $\boxed{\tau_E = 1}$

For Magnetic field:-

$$\tau_H = \frac{H_t}{H_i}, \text{ Since } H_i = H_t \Rightarrow \tau_H = \frac{H_i}{H_i} \Rightarrow \tau_H = 1$$

So, $\boxed{\tau_H = 1}$

Reflection Coefficient

(1) for Electric field:-

$$\Gamma_E = \frac{E_R}{E_i} = \frac{0}{E_i} = \infty, \quad \Gamma_E = \infty$$

(2) For Magnetic field:-

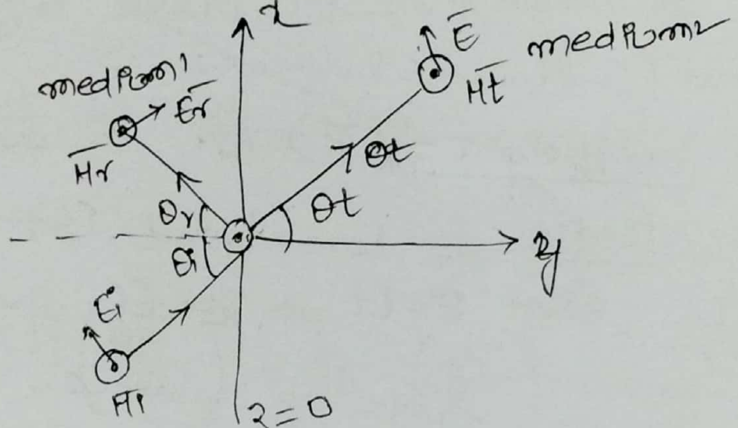
$$\Gamma_H = \frac{H_R}{H_i} = \frac{0}{H_i} = \infty, \quad \Gamma_H = \infty.$$

OBLIQUE INCIDENCE

When a uniform plane wake strikes obliquely on the surface (either conductor or dielectric), the behaviour of the reflected wave is decided by the polarization of the incident wave!

case(i) Parallel Polarisation / Vertical :-

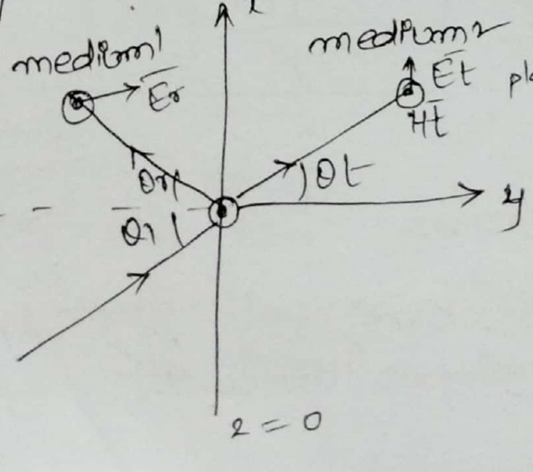
The magnetic field vector is aligned parallel to the boundary surface. In another words, the magnetic field vector is perpendicular to the plane of incidence while $\bar{E}$ is aligned parallel to the plane of incidence. $\bar{E}$ ||lal to plane of incidence,



Vertical polarization

Case ii) Horizontal / Perpendicular Polarisation:-

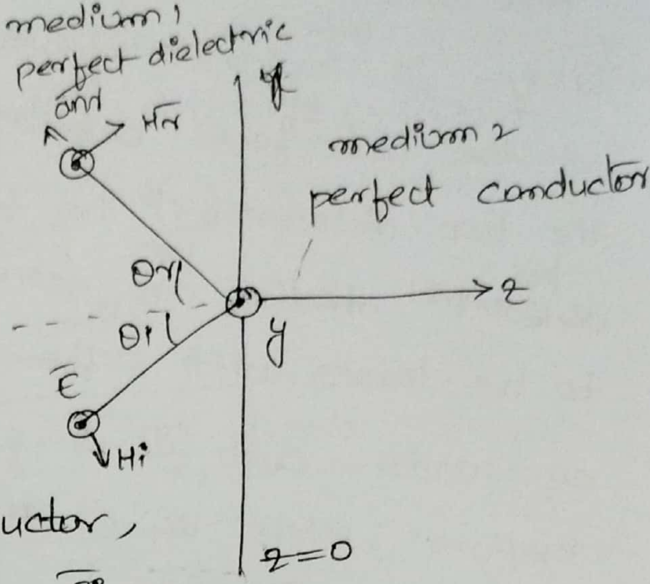
The $\bar{E}$ ⊥lar to plane of incidence. $\bar{E}$ ||lal to boundary. H



A plane of incidence: plane containing the vector in the direction of propogation of the incident wave and normal to the boundary surface

OBLIQUE INCIDENCE AT PLANE CONDUCTING BOUNDARY

Consider an interface between a perfect dielectric and perfect conductor.



Horizontal Polarization

Consider that the plane wave incident obliquely at the interface between a conductor, with the electric field vector $\bar{E}_i$ $\perp^{lar}$ to plane of incidence.

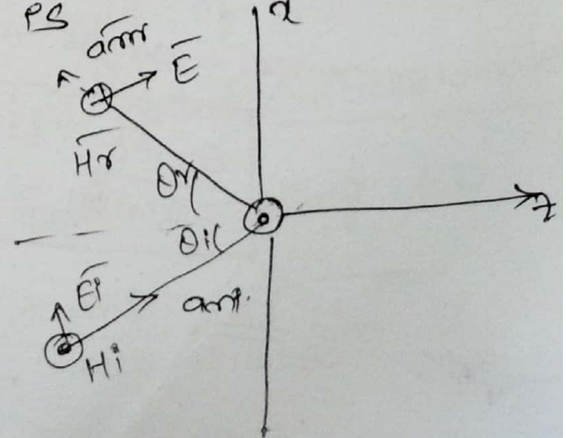
As $E_i \perp^{lar}$ to plane of incidence to this type of incidence, this type of polarization is also called perpendicular polarization or E-polarization.

The unit vector in the direction of incident wave is $\bar{a}_{ni}$. Let θ_i be the angle of incidence. Then the unit vector is given by,

$$\bar{a}_{ni} = \sin\theta_i \cdot \bar{a}_x + \cos\theta_i \, \bar{a}_z$$

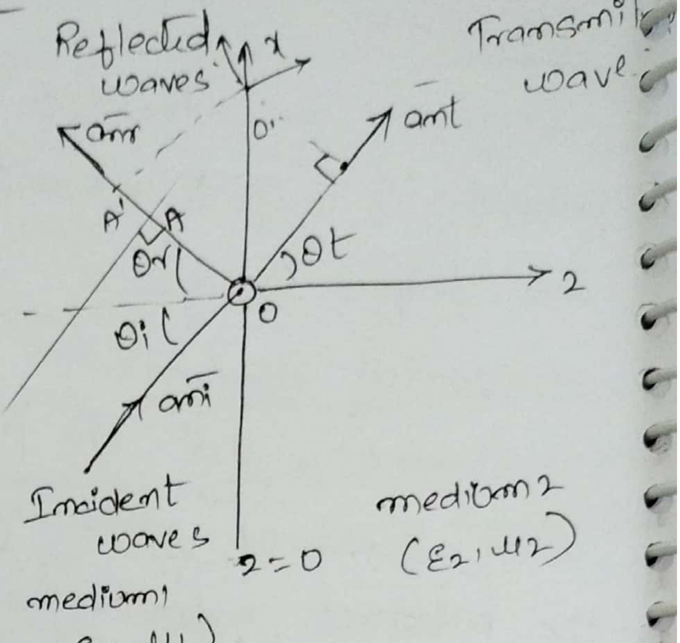
OBLIQUE INCIDENCE AT PLANE conducting boundary with vertical polarisation.

Consider that the plane is wave is obliquely incident with the $\bar{E}_i$ parallel to plane of incidence. This is also called Parallel / H-Polarization.



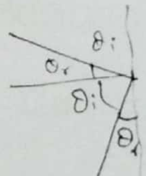
OBLIQUE INCIDENCE AT PLANE DIELECTRIC BOUNDARY

Now consider a uniform plane wave is incident obliquely at the interface between the two dielectrics. The two dielectric media are assumed to be losses with different constants. Such as μ_1, ε_1 for medium 1, while μ_2, ε_2 for medium 2. As the two media having different consecutive boundaries, at there is discontinuity. Because of this, at the interface 1 a part of the incident wave is reflected, while a part is transmitted as shown in fig.



Let the line of intersections of the line plane OA, O'A' and O'B be the plane of incident with the equiphase surfaces of the incident, reflected and transmitted waves.

From the diagram, $\sin(\theta t)$ ~~= OB~~ the distances $\overline{OA'}$ and $\overline{AO'}$ are same.



Snells law: from the diagram, angle of incidence = angle of refraction

$$\boxed{\frac{\sin\theta t}{\sin\theta_1} = \frac{n_1}{n_2}}$$, where $n = \frac{c}{v} = \frac{\text{Speed of light in free space}}{\text{Speed of light in medium}}$

$v_1 = \frac{1}{\sqrt{\mu_1 \varepsilon_1}}$, $v_2 = \frac{1}{\sqrt{\mu_2 \varepsilon_2}}$.

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{c/v_1}{c/v_2} = \frac{v_2}{v_1} = \frac{1/\sqrt{\mu_2\varepsilon_2}}{1/\sqrt{\mu_1\varepsilon_1}} = \sqrt{\varepsilon_1/\varepsilon_2} \quad \text{--- (1)}$$

For non-magnetic media, $\mu_1 = \mu_2 = \mu_0$

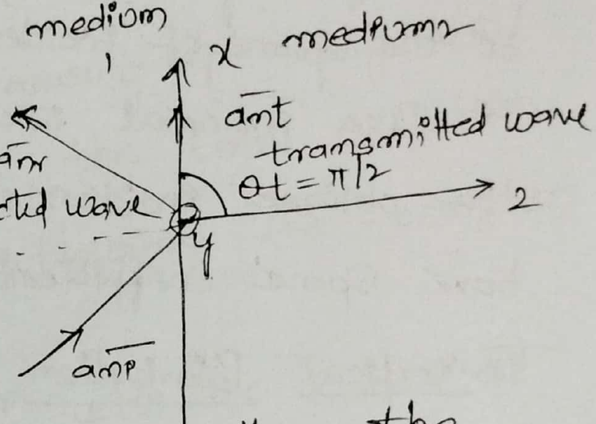
Intrinsic Impedances. $\eta_1 = \sqrt{\mu_1/\varepsilon_1} = \sqrt{\mu_0/\varepsilon_1}$, $\eta_2 = \sqrt{\mu_0/\varepsilon_2}$

$$\frac{\sin\theta_t}{\sin\theta_1} = \sqrt{\frac{\cancel{\mu_0}\,\varepsilon_1}{\cancel{\mu_0}\,\varepsilon_2}} = \sqrt{\frac{\cancel{1}\varepsilon_1}{\cancel{1}\varepsilon_2}} = \frac{\eta_2}{\eta_1}$$

Thus collectively, we can write Snells law of refraction as

$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{v_2}{v_1} = \frac{\beta_1}{\beta_2} = \frac{\eta_2}{\eta_1} = \frac{n_1}{n_2}}$$

<u>Total Reflection</u>:-



Consider that the medium 2 is denser as compared 2. i.e. $\varepsilon_1 > \varepsilon_2$. Under this condition the angle of transmission θt becomes greater than the angle of incidence. The θ_t ↑ses with θ_i. When $\theta_t = \pi/2$, the wave transmitted is aline along the interface.

If θ_i is further increases, there will be ~~no~~ total internal reflection such that no wave will be transmitted. The angle of incidence at which total reflection takes place called <u>Critical angle (θ_c)</u>. Then at $\theta_t = \pi/2$, $\theta_i = \theta_c$, Then according to Snells law.

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\varepsilon_1/\varepsilon_2} \Rightarrow \frac{\sin\pi/2}{\sin\theta_c} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

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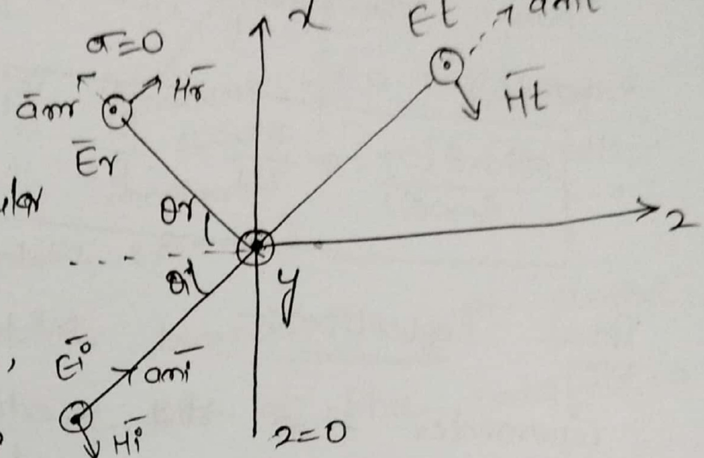
$$\frac{1}{\sin\theta_c} = \sqrt{\varepsilon_1/\varepsilon_2} \rightarrow \sin\theta_c = \sqrt{\varepsilon_2/\varepsilon_1}$$

$$\boxed{\theta_c = \sin^{-1}\left[\sqrt{\varepsilon_2/\varepsilon_1}\right] = \sin^{-1}\left[\frac{n_2}{n_1}\right]}$$

Types of polarization for oblique incidence with dielectric media.

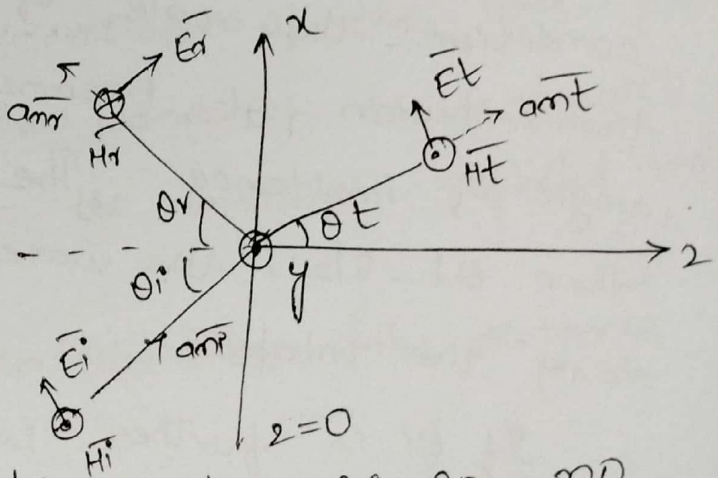
(a) Horizontal Polarization



Consider that a uniform plane wave is incident obliquely with $\bar{E}$ perpendicular to the plane of incidence

Like normal incidence, for oblique incidence also have same coefficients

b) Vertical Polarization



$\theta_i \rightarrow$ called Brewster angle.

Brewster Angle:-

The angle of incidence at which there is no reflection.

For parallel polarization, when $\theta_i = \theta_B$, no reflection takes place. Brewster angle is also called Polarizing angle.

→ condition for total Internal reflection is

$$\boxed{\theta_i \geq \theta_c}$$

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SURFACE IMPEDANCE

Surface Impedance is defined as the ratio of the tangential component of the electric field to the surface current density at the conductor surface. The surface impedance is denoted as

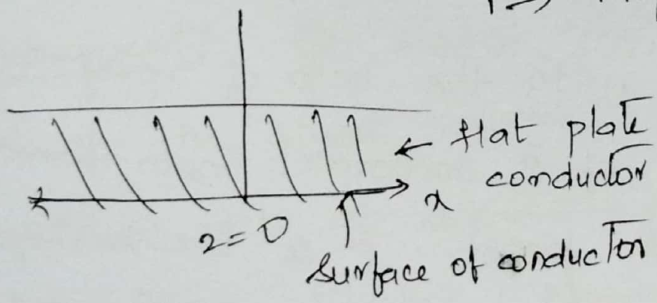
$$\boxed{Z_s = \frac{E_{tan}}{J_s}\ \Omega}$$

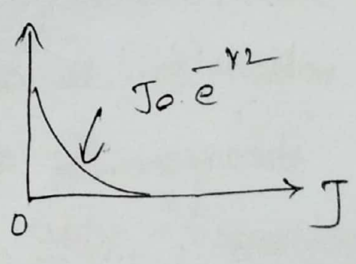
Etan is the tangential component of the electric field which is parallel to the surface of the conductor. The surface current density Js is the linear current which is the result of tangential component Etan. This indicates the total conduction current per meter width flowing in the thin sheet.

Consider a flat conductor with its surface at $z=0$ plane, then the current distribution is given below.

$J = J_0 e^{-\gamma z}$ $J_0 \rightarrow$ current density at surface

$\gamma \rightarrow$ Propogation Constant





Surface Impedance of Good Conductor

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}, \quad \boxed{R_s = \frac{1}{\delta\sigma}}$$

POYNTING VECTOR ($\bar{P}$ or $\bar{S}$)

It is the measure of rate of energy flow per unit area in the given medium.

$$\boxed{\text{Poynting vector } (\bar{P}) = \bar{E} \times \bar{H} = V/m \times A/m = \frac{W}{m^2}}$$

It is also called as power density (Power per unit density).

$\bar{P}$ shows the direction of wave propogation which is mutually perpendicular to $\bar{E}$ or $\bar{H}$. The advantage of of the above expression is to find the direction of wave propogation when both $\bar{E}$ or $\bar{H}$ are given, or any of the fields are given. This is done with the help of Maxwells Equations.

By means of EM waves, an energy can be transported from transmitter to receiver. The energy stored in an electric field and magnetic field to be transmitted at a certain rate of energy flow which can be calculated with the help of poynting theorem. The direction of $\bar{P}$ indicates instantaneous power flow at the point. $\bar{P}$ is the instantaneous power density vector associated with the electromagnetic field at a given point, To get a net power flowing out of any surface P is integrated over same closed surface.

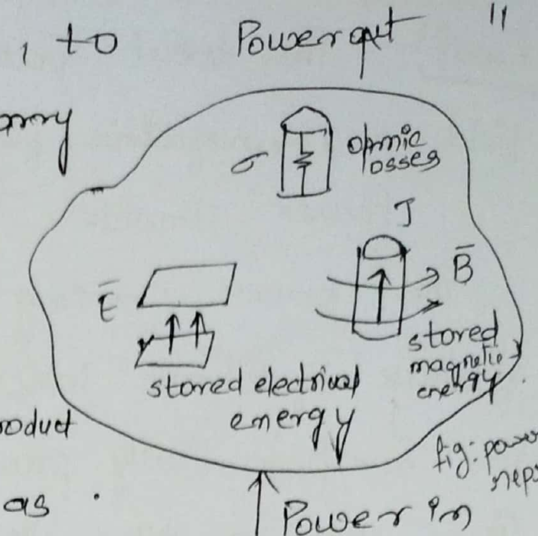


fig: power balance representation

As E and H both are vectors, to get power density, we may carry either dot product or cross product. The result of dot product is always a scalar quantity. The result of dot product cannot be performed. But as power flows in a certain direction, it is a vector quantity. To illustrate this, consider that the field is transmitted in the form of electromagnetic waves from an antenna. Both the fields are sinusoidal in nature. The power radiated from antenna has a particular direction. Hence to calculate power density, we must carry out a cross product of $\bar{E}$ and $\bar{H}$. Hence, power density $\boxed{\bar{P} = \bar{E} \times \bar{H}}$

POYNTING THEOREM

Statement:- It states that the power flowing out of the given volume is equal to the negative rate of decreasing total energy field (i.e Electric + Magnetic Energy) within it and minus the ohmic losses or conduction losses in it

$$\boxed{\oint_S (\bar{E} \times \bar{H}) \cdot ds = -\frac{\partial}{\partial t}\left[\int_v \left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right) dv\right] + \int_v (\bar{E} \cdot \bar{J})\, dv}$$

Proof:- The total power is the sum of Electric field and magnetic field. This is the power Inside.

Power Inside = Electric field + Magnetic field.

The power Coming out of the surfaces breduces because of ohmic losses inside the surface. This can be mathematically proved as shown below by the Time Varying Maxwells Equations.

we have $\nabla \times \bar{H} = \bar{J} + d/dt(\bar{D})$, $\nabla \times \bar{E} = \frac{-d}{\partial t}(\bar{B}) = \frac{-d}{\partial t}(\mu \bar{H})$

$\bar{J} = \nabla \times \bar{H} - d/dt(\bar{D})$ $\qquad = -\mu \cdot \frac{d}{\partial t} \bar{H}$

Now, we have, ohmic losses

$\bar{E} \cdot \bar{J} \Rightarrow \nabla \times \bar{H} = \bar{J} + d/dt(\bar{D})$

Dot product with $\bar{E}$ on both sides, $\bar{J} = \sigma \bar{E}$

$$\Rightarrow \bar{E} \cdot (\nabla \times \bar{H}) = \bar{E} \cdot \sigma E + \bar{E} \cdot \left[\varepsilon \cdot \frac{d\bar{E}}{\partial t}\right]$$

By using vector Identity,

$$\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A} \cdot (\nabla \times \bar{B})$$

$$\bar{H} \cdot (\nabla \times E) - \nabla \cdot (\bar{E} \times \bar{H}) = \bar{E} \cdot (\sigma \bar{E}) + \bar{E} \cdot \left[\varepsilon \cdot \frac{d\bar{E}}{\partial t}\right]$$

$$\bar{H} \cdot (\nabla \times E) - \nabla \cdot (\bar{E} \times \bar{H}) = \sigma \bar{E}^2 + \bar{E}^2 \cdot \left[\varepsilon \frac{d\bar{E}}{dt}\right]$$

, we have, $\nabla \times \bar{E} = -\mu \cdot \frac{d}{\partial t} \bar{H}$ $\left(\because \nabla \times \bar{E} = -\frac{d}{dt}(\bar{B})\right)$

$$\bar{H} \cdot [+(\nabla \times E)] = +\bar{H} \cdot \left[-\mu \cdot \frac{d\bar{H}}{\partial t}\right] = -\mu \bar{H} \cdot \frac{d\bar{H}}{\partial t}$$

Now consider term,

$$\frac{d}{\partial t}(\bar{H} \cdot \bar{H}) = \bar{H} \cdot \frac{d}{\partial t} \bar{H} + \bar{H} \cdot \frac{d\bar{H}}{dt}$$

$$\frac{d}{dt} H^2 = 2\bar{H} \cdot \frac{d\bar{H}}{dt} \Rightarrow \frac{1}{2} \frac{d}{dt}(H^2) = \bar{H} \cdot \frac{d\bar{H}}{dt}$$

Similarly, we can write,

$$\frac{1}{2}\frac{d^2}{dt^2}(\bar{E}) = \bar{E}\cdot\frac{d\bar{E}}{dt}$$

using results.

$$\bar{H}\cdot(\nabla\times\bar{E}) - \nabla\cdot(\bar{E}\times\bar{H}) = \sigma\bar{E}^2 + \bar{E}\cdot\left[\varepsilon\cdot\frac{d\bar{E}}{dt}\right]$$

(1)

$$\bar{H}\cdot(\nabla\times\bar{E}) = -\mu\bar{H}\cdot\frac{\partial\bar{H}}{\partial t}$$

$$-\mu\bar{H}\cdot\frac{\partial\bar{H}}{\partial t} - \nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + \frac{d}{dt}(E^2)\cdot\frac{1}{2}\varepsilon$$

$$-\nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + \frac{1}{2}\frac{\partial}{\partial t}\left[\varepsilon E^2\right] + \mu\bar{H}\cdot\frac{\partial\bar{H}}{\partial t}$$

$$-\nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + \frac{1}{2}\cdot\frac{d}{dt}\left[\varepsilon E^2\right] + \frac{1}{2}\frac{\partial}{\partial t}\left[\mu H^2\right]$$

$$-\nabla\cdot(\bar{E}\times\bar{H}) = \sigma E^2 + \frac{1}{2}\frac{d}{dt}\left[\mu H^2 + \varepsilon E^2\right]$$

$$P = \bar{E}\times\bar{H}$$

$$\boxed{-\nabla\cdot\bar{P} = \sigma E^2 + \frac{1}{2}\frac{d}{dt}\left[\mu H^2 + \varepsilon E^2\right]}$$

This is poynting theorem in point form.

If we integrate this power a volume, we get energy distribution as

$$-\int_v \nabla\cdot\bar{P}\,dv = \int_v \sigma E^2\,dv + \frac{d}{dt}\int_v \frac{1}{2}\left[\mu H^2\right] + \frac{\varepsilon E^2}{2}\,dv$$

Nao applying theorem in integral form

$$-\int_v \nabla\cdot\bar{P}\,dv = -\oint_s \bar{P}\cdot ds$$

$$-\oint_s \bar{P}\cdot ds = \int_v \sigma E^2\,dv + \frac{d}{dt}\int_v \frac{1}{2}\left[\mu H^2 + \varepsilon E^2\right]dv$$

$$\boxed{\oint_s \bar{P}\cdot ds = \oint_s(\bar{E}\times\bar{H})ds = -\left[\int_v(\sigma E^2)\,dv + \frac{d}{dt}\int_v \frac{1}{2}\left[\mu H^2 + \varepsilon E^2\right]dv\right]}$$

Poynting theorem in Integral form

The above Equation can also written as

$$\oint_S (\bar{E}\times\bar{H})ds = -\frac{d}{dt}\left[\int_v \left(\frac{\mu H^2+\varepsilon E^2}{2}\right]dv - \int_v \varepsilon \bar{E}^2 dv\right.$$

$$\oint_S (\bar{E}\times\bar{H})ds = -\left(+\frac{d}{dt}\left[\int_v \frac{\mu H^2+\varepsilon E^2}{2}dv + \int_v \varepsilon \bar{E}\cdot \dot{E}\, dv\right]\right.$$

$$\boxed{\oint_S (\bar{E}\times\bar{H})ds = -\left(\frac{d}{dt}\int_v \left[\frac{\mu H^2+\varepsilon E^2}{2}\right]dv + \int_v J\bar{E}\cdot dv\right.}$$

-ve' sign indicates power flowing into the volume. Power flowing out of the volume.

$$\oint_S \bar{P}\cdot d\bar{s}.$$

Appligiations;-

Used in Antennas