UNIT - II

MAGNETOSTATICS

⇒ charges in motion with uniform velocity produces steady currents. (time invarying currents). That is called static Magnetic field.

⇒ Charges with motion with non uniform velocity produces time varying currents. Hence, it produces time varying magnetic field.

Magnetic field:-

The space or region around the current carrying conductor in which the effect is experienced is called Magnetic field.

Magnetic lines of force:-

Magnetic lines are imaginary lines which moves from north to South externally and south to north internally which forms a closed path

Magnetic flux (ϕ):-

The no. of magnetic lines of force is called magnetic flux Units: Weber.

Magnetic field Intensity:- ($\bar{H}$)

It is a vector quantity. It is denoted by the force experienced by north pole at any point in the field, is known as Magnetic field Intensity ($\bar{H}$).

$$\boxed{\bar{H} = \frac{F}{\phi} = N/Wb}$$

Magnetic flux Density:- ($\bar{B}$)

Magnitude of Magnetic lines of force is known as Magnetic flux density ($\bar{B}$). It is a vector Quantity

$$\boxed{\bar{B} = \frac{\phi}{S} = \frac{Wb}{m^2}}$$

Relation between $\bar{B}$ & $\bar{H}$

The ability of current carrying conductor with which it can have magnetic flux around the field is known as permeability. $\boxed{\bar{B} = \mu \bar{H}}$

$\mu = \mu_0 . \mu_r$, μ_r — relative permeability

$\mu_r = 1$, for non-magnetic media

$\mu_r > 1$, for magnetic media

$\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m.

Types of Currents:-

a) line Current:-

It is defined as $I \bar{dl}$ Amp, $I = \int_l \rho_l . dl$

line current density, $\boxed{\bar{I} = \rho_l \bar{v} \text{ Amp}}$

(c/m) ρ_l → line charge density, $\bar{v}$ → velocity of charge (m/sec)

$I = c/m . m/sec = c/sec$

b) Surface Current.

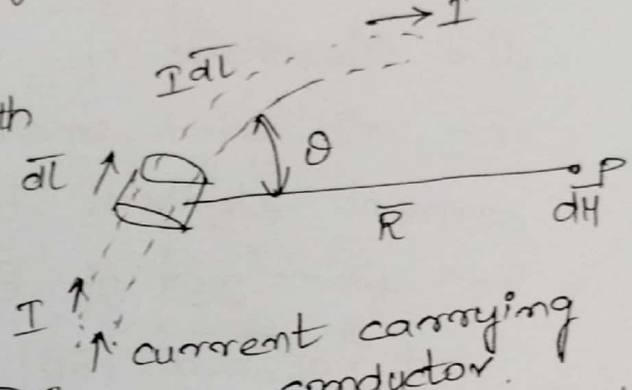
$I = \int_S \bar{K} \cdot \bar{ds}$, $\boxed{\bar{K} = \sigma \bar{v} \text{ A/m}}$

c) Volume current

$I = \int_v \bar{J} . dv \Rightarrow \boxed{\bar{J} = J \bar{v} \text{ A/m}^2}$

BIOT SAVARTS LAW:-

Consider a conductor carrying a direct current I and a steady magnetic field produced around it. The Biot Savart law allows us to obtain the differential magnetic field intensity dH, produced at a point, P, due to differential current element is Idl.



Consider a differential length dl, hence the differential current element is dl. This is small part of the current carrying conductor. The point P is at a distance R from the differential current element. The θ is the angle between the differential current element and the line joining point P to the differential current element.

Statement:- The magnetic field intensity dH produced in a point P due to differential current element Idl is,

a) Proportional to the product of the current I and the differential length dL.

b) The sine of the angle between the current element and the line joining point P to the element.

c) And inversely proportional to the square of the distance R between point P and the element.

Mathematically, the Biot-savart law can be stated as,

$$d\bar{H} \propto \frac{I\,dl\sin\theta}{R^2} \Rightarrow \boxed{d\bar{H} = \frac{k\,I\,dl\sin\theta}{R^2}}$$

k = constant of proportionality

$$k = \frac{1}{4\pi}$$

$$d\bar{H} = \frac{I\,dl\sin\theta}{4\pi R^2}$$

dl = Magnitude of vector length $\bar{dL}$ and,

$\bar{a_R}$ = Unit vector in the direction from differential current element to a point P.

Then from rule of cross product -

$$\bar{dL} \times \bar{a_R} = dL\,|\bar{a_R}|\sin\theta = dL\sin\theta.$$

$$d\bar{H} = \frac{I\,\bar{dL} \times a_R}{4\pi R^2}\ A/m, \quad \bar{a_R} = \frac{\bar{R}}{|\bar{R}|}.$$

$$\boxed{d\bar{H} = \frac{I\,\bar{dL} \times \bar{R}}{4\pi R^3}\ A/m.}$$

(a) Applications of Biot-Savarts Law:-

(1) $\bar{H}$ due to finite length of current:-

Consider a conductor of finite length placed along z axis. It carries a direct current I. The perpendicular distance of point P from z axis is r as shown in fig. The conductor is placed such that its one end is at $z = z_1$ while other at $z = z_2$.

Consider a differential element $\bar{dL}$ along z axis at a distance z from origin.

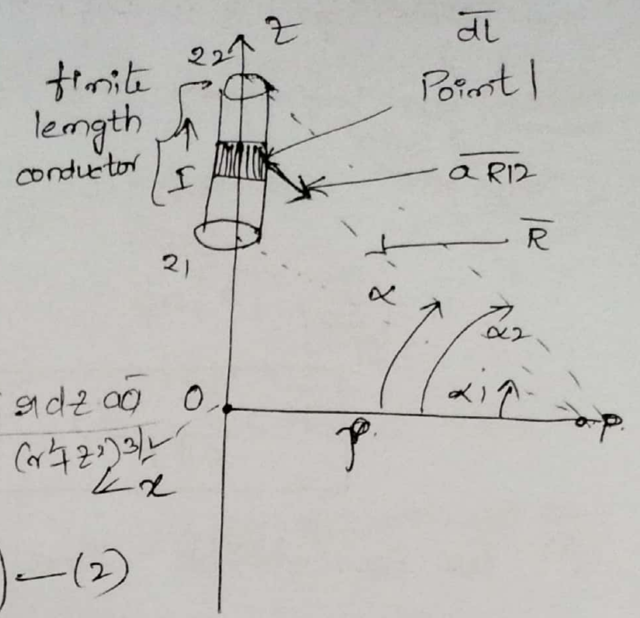
$$\therefore \bar{dL} = dz \cdot \bar{a_z}$$

from diagram $z \cdot \bar{a}_z + \bar{R} = \rho \cdot \hat{a}_\rho$

$\bar{R} = \rho\, \bar{a}_\rho - z \cdot \bar{a}_z$

$|\bar{R}| = \sqrt{\rho^2 + z^2}$

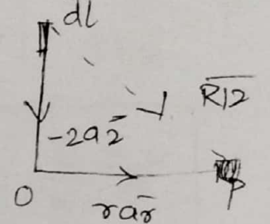
$\bar{a}_R = \dfrac{\bar{R}}{|\bar{R}|} = \dfrac{\rho \cdot \bar{a}_\rho - z\, \bar{a}_z}{\sqrt{\rho^2 + z^2}}$



$\boxed{d\bar{H} = \dfrac{I\, d\bar{l} \times \bar{a}_R}{4\pi R^3}}$ —(1) $\quad H = \displaystyle\int_{z_1}^{z_2} \dfrac{I\, \rho\, dz\, \bar{a}_\phi}{4\pi (\rho^2+z^2)^{3/2}}$ ← α

$d\bar{l} \times \bar{a}_R = (dz \cdot \bar{a}_z) \times \left(\dfrac{\rho \cdot \bar{a}_\rho - z \cdot \bar{a}_z}{\sqrt{\rho^2 + z^2}}\right)$ —(2)

$dl \times \bar{a}_R = \begin{vmatrix} \bar{a}_r & \bar{a}_\phi & \bar{a}_z \\ 0 & 0 & dz \\ \rho & 0 & -z \end{vmatrix} = \rho\, dz \cdot \bar{a}_\phi$



$d\bar{H} = \dfrac{I \cdot \rho \cdot dz \cdot \bar{a}_\phi}{4\pi (r^2+z^2)^{3/2}}$, $\cancel{\cot}\, \tan \alpha = \dfrac{z}{\rho} \Rightarrow z = \rho\, \cancel{\cot}\, \tan\alpha$.

$z = z_1 \Rightarrow$

$dz = +\rho \cdot \cancel{\text{co}}\sec^2\alpha \cdot d\alpha$

$d\bar{H} = \rho \cdot dz \cdot a_\phi \cdot \dfrac{1}{4\pi} \displaystyle\int_{\alpha_1}^{\alpha_2} \dfrac{I \cdot \rho \times +\rho\, \cancel{\text{co}}\sec^2\alpha \cdot dz\, \bar{a}_\phi}{\left(\sqrt{\rho^2 + \rho^2 \tan^2\alpha}\right)^{3/2}}$

$= \dfrac{I}{4\pi} \cdot +\rho^2 \displaystyle\int_{\alpha_1}^{\alpha_2} \dfrac{\cancel{\text{co}}\sec^2\alpha \cdot d\alpha}{\rho^3 \cdot \cancel{\text{co}}\sec^3\alpha}\, \bar{a}_\phi$

$= \dfrac{+I}{4\pi\rho} \displaystyle\int_{\alpha_1}^{\alpha_2} \dfrac{1}{\cancel{\text{co}}\sec\alpha} \cdot d\alpha \cdot \bar{a}_\phi = \dfrac{+I}{4\pi\rho} \displaystyle\int_{\alpha_1}^{\alpha_2} \cos\alpha\, \cancel{\sin\alpha} \cdot d\alpha$

$= \dfrac{+I}{4\pi\rho} (\sin\alpha)_{\alpha_1}^{\alpha_2}\, \bar{a}_\phi = \dfrac{I}{4\pi\rho} \left[\sin\alpha_2 - \sin\alpha_1\right] \bar{a}_\phi$

$$\boxed{\bar{H} = \dfrac{I}{4\pi\rho} \left[\sin\alpha_2 - \sin\alpha_1\right] \bar{a}_\phi}$$

The unit vector is $\bar{a}_\phi$ because, the direction of $\bar{H}$ is always perpendicular to the current carrying conductor, i.e, ρ or ϕ, since, ρ is not possible, the direction is along ϕ.

Note:- for infinite line current $A(0,0,-\infty)$, $B(0,0,\infty)$

$\alpha_2 = \overset{270^\circ}{\cancel{180^\circ}}$, $\alpha_1 = 90^\circ$, $\bar{H} = \frac{I}{4\pi\rho}\left[\cos 0^\circ - \cos 180^\circ\right]\bar{a\phi}$

$\sin 90^\circ - (-\sin 270^\circ)$

$= 2$

$\bar{H} = \frac{I}{4\pi\rho}[2]\bar{a\phi}$

$$\boxed{\bar{H} = \frac{I}{2\pi\rho}\cdot\bar{a\phi} \ \text{N/Wb.}}$$

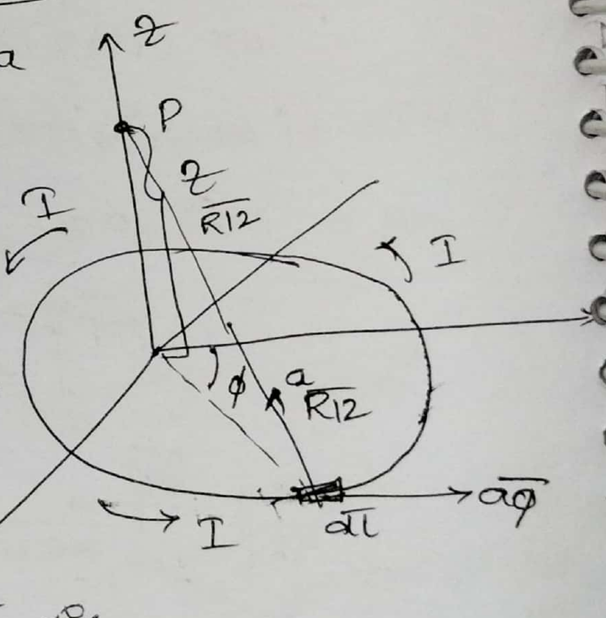
(2) for Semifinite line current, $\alpha_1 = 90^\circ$, $\alpha_2 = 0^\circ$

$\bar{H} = \frac{I}{2\pi\rho}\left[-\underset{\sin}{\cancel{\cos}}90^\circ + \underset{\sin}{\cancel{\cos}}0^\circ\right]\bar{a\phi}$

$$\boxed{\bar{H} = \frac{I}{2\pi\rho}(1)\,\bar{a\phi} \ \text{N/Wb.}}$$

$$\boxed{\bar{H} = \frac{-I}{4\pi\rho}\cdot\hat{a\phi}}$$

(3) $\bar{H}$ due to circular current carrying loop



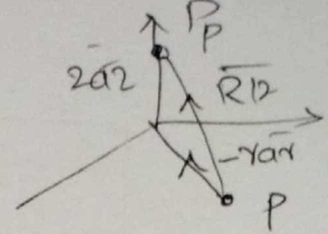
Consider a circular loop carrying a direct current I, placed in xy plane, with z axis as its axis as shown in fig. The magnetic field intensity $\bar{H}$ at point P is to be obtained. The point P is at a distance z from the plane of the circular loop, along its axis.

The radius of the circular loop is r

Consider the differential length dl of the circular loop as shown in the figure.

$\bar{dl} = dr\,\bar{ar} + r\,d\phi\,\bar{a\phi} + dz\,\bar{az}$

z = 0 = constant plane. The I dl is tangential ... in a_ϕ direction.



$$I\,dl = I\,r\,d\phi\,\bar{a}_\phi \qquad \bar{a}_{R12} = \frac{\bar{R}_{12}}{|R_{12}|}$$

$$\bar{R}_{12} = -r\bar{a}_r + z\,\bar{a}_z$$

$$|\bar{R}_{12}| = \sqrt{(-r)^2+(z)^2} = \sqrt{r^2+z^2}, \quad \bar{a}_{R12} = \frac{-r\bar{a}_r + z\,\bar{a}_z}{\sqrt{r^2+z^2}}$$

$$\bar{dl} \times \bar{a}_{R12} = \begin{vmatrix} \bar{a}_r & \bar{a}_\phi & \bar{a}_z \\ 0 & r\,d\phi & 0 \\ -r & 0 & z \end{vmatrix} = z\,r\,d\phi\,\cancel{\bar{a}_r + r^2\,d\phi}\,.\,a_z$$

$$d\bar{l} = dr\,\bar{a}_r + r\,d\phi\,\bar{a}_\phi + dz\,\bar{a}_z$$

Since, the $d\bar{l}$ direction is normal to the plane this is along ϕ direction $d\bar{l} = \rho\,d\phi\,\bar{a}_\phi$.

$$d\bar{l} \times a_{\bar{R}12} = (r\,d\phi\,\bar{a}_\phi) \times (z\,\bar{a}_z - r\,\bar{a}_r)$$

Since, vertical components are eliminated

$$d\bar{l} \times a_{\bar{R}12} = r\,d\phi\,\bar{a}_\phi \times (-r\,\bar{a}_r)$$

$$d\bar{l} \times \bar{a}_{R12} = r^2\,d\phi\,\bar{a}_z$$

$$d\bar{H} = \frac{I\,dl \times a_{\bar{R}12}}{4\pi R^3}$$

$$H = \int \frac{I}{4\pi} \cdot \frac{r^2\,d\phi\,\bar{a}_z}{(r^2+z^2)^{3/2}}$$

Since, it is a circular current carrying loop, the limits are from 0 to 2π

$$\bar{H} = \int_0^{2\pi} \frac{I}{2\times 2\pi} \cdot \frac{r^2\,d\phi\,\bar{a}_z}{(r^2+z^2)^{3/2}}$$

$$H = \frac{I}{4\pi_2} \cdot \frac{r^2\,\bar{a}_z}{(r^2+z^2)^{3/2}} \cdot (2\pi).\cancel{a_z}$$

$$\boxed{H = \frac{I}{2} \cdot \frac{r^2}{(r^2+z^2)^{3/2}}\,\bar{a}_z}$$

a) If the point of observation is at origin, i.e, $z=0$,

Then $\bar{H} = \frac{I r^2}{2(r^2)^{3/2}} \cdot \bar{a_z} = \frac{I r^2}{2r^3} \cdot \bar{a_z}$

$\boxed{\bar{H} = \frac{I}{2r} \bar{a_z}}$ at the centre of current loop

2) If there are N no. of coils then

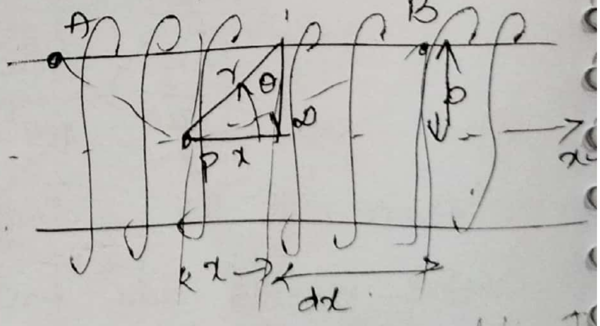
$$\boxed{\bar{H} = N \cdot \frac{I r^2}{2(r^2+z^2)^{3/2}} \cdot \bar{a_z}}$$

3) $\bar{H}$ due to Solonoid:-

Solonoid is nothing but an Electromagnet. It is formed due to few windings of circular over a iron rod.

Let us find $\bar{H}$ at the centre of Solonoid of infinite length. As we know that the magnetic field intensity due to one circular loop.

$$\bar{H} = \frac{I r^2}{2(r^2+z^2)^{3/2}} \cdot \bar{a_z}$$



Using diagram

$P^2 + x^2 = r^2$

$\bar{H} = \frac{I P^2}{2(r^2)^{3/2}} \cdot \bar{a_z} = \frac{I P^2}{2r^3} \bar{a_z}$

Therefore due to N turns the magnetic field intensity

$d\bar{H}$ $\quad d\bar{H} = \frac{N I P^2}{2r^3} \cdot dx \cdot \bar{a_z}$

From diagram, $\tan\theta = P/x \Rightarrow P = x\tan\theta$

$x = P \cdot \cot\theta \Rightarrow dx = -P(\operatorname{cosec}^2\theta)\, d\theta$

$dH = \frac{N I P^2}{2r^3} \cdot dx\, \bar{a_z} = \frac{N I P^2}{2r^3}(-P\operatorname{cosec}^2\theta)\, d\theta.$

$$dH = \frac{-NI \sin\theta}{2} \, d\theta$$

$$H = \int_{\theta=180^\circ-\alpha}^{\alpha} -\frac{NI \sin\theta}{2} \, d\theta, \quad \theta = 180^\circ - \alpha$$

$$\bar{H} = -\frac{NI}{2} (-\cos\theta)_{180^\circ-\alpha}^{\alpha} = \frac{NI}{2} \left[\cos\alpha - \cos(180^\circ-\alpha)\right]$$

$$\bar{H} = \frac{NI}{2} \left(2\cos\alpha\right) = NI\cos\alpha$$

$$\boxed{\bar{H} = NI\cos\alpha \ N/Wb}$$

At centre of Solenoid, $\alpha = 0$, $\bar{H} = NI$.

∴ For long solenoids $\bar{H}$ is uniform, i.e. H doesn't change.

(2) AMPERE'S CIRCUIT LAW:-

The line Integral of magnetic field intensity ($\bar{H}$) around a closed path is exactly equal to the direct current enclosed by that path.

$$\boxed{\oint_l \bar{H} \cdot dl = I}$$

Gauss Law is ~~Electrom~~ magnetostatics is nothing but Ampere's circuit law. This is also known as Integral form of Ampere's circuit law. This law is applicable for symmetrical distributions

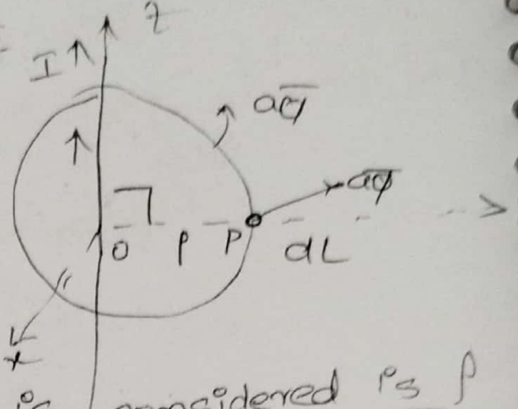
$$\oint_L \bar{H} \cdot \bar{dl} = \int_S (\nabla \times \bar{H}) \cdot ds \quad \text{—— (1)}$$

$$I_{enclosed} = \int_S \bar{J} \cdot ds \quad \text{—— (2)}, \text{ Equating (1) \& (2)}$$

$$\boxed{\nabla \times \bar{H} = \bar{J}} \rightarrow \text{Maxwell's III Eqns.}$$

This also explains non conservative nature of magnetic field intensity for static fields.

Proof:- let us consider an infinite line current is placed along z axis and closed circular path is enclosing that infinite line line current (I).



The radius of closed path is considered is ρ and at P on closed path $\bar{dl}$ is considered tangential to that point. i.e $\bar{dl} = \rho\, d\phi\, \bar{a_\phi}$

But according to Biot Savarts law, the infinite line current due to H placed along z axis.

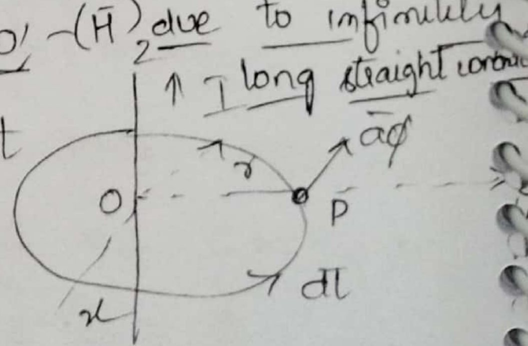
$$\bar{H} = \frac{I}{2\pi\rho}\bar{a_\phi}, \Rightarrow \oint_l \bar{H}\cdot \bar{dl} = \oint_l \frac{I}{2\pi\rho}\bar{a_\phi}\cdot \rho\, d\phi\, \bar{a_\phi}$$

$$= \frac{I}{2\pi}\oint_l d\phi = \frac{I}{2\pi}\int_0^{2\pi} d\phi = \frac{I}{2\pi}(2\pi) = I$$

$$\boxed{\oint_l \bar{H}\cdot\bar{dl} = I_{enclosed}}$$

This Amperes circuit law states that the circulation of Magnetic field Intensity is nothing but current enclosed.

Applications of Amperes circuit law: ($\bar{H}$) due to infinitely long straight conductor



Consider an infinitely long straight conductor placed along z axis, carrying a direct current I.

The magnitude of $\bar{H}$ depends on r and the direction is always tangential to the closed path, so $\bar{H}$ has only component in a_ϕ direction say H_ϕ

Consider an elementary length $\overline{dl}$ at point P in cylindrical coordinates it is $r\,d\phi$ in a_ϕ direction

$\bar{H} = H_\phi \cdot a_\phi \quad \overline{dl} = r\,d\phi\,a_\phi$

According to Ampere's Circuit law,

$$\oint \bar{H} \cdot \overline{dl} = I \Rightarrow \int_{\phi=0}^{2\pi} H_\phi \cdot a_\phi \cdot r\,d\phi \cdot a_\phi = I$$

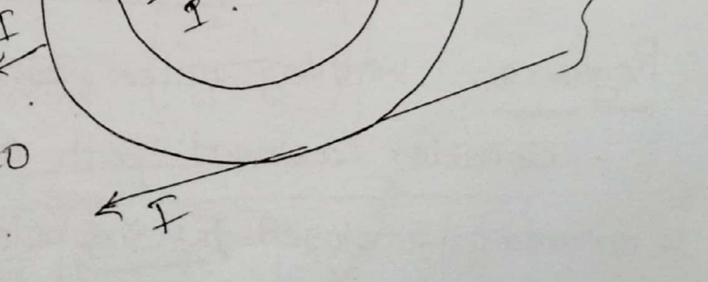
$$I = \oint_{\phi=0}^{2\pi} H_\phi \cdot d\phi \cdot r \Rightarrow I = r H_\phi \int_{\phi=0}^{2\pi} d\phi \Rightarrow I = r \cdot (2\pi) H_\phi$$

$$H_\phi = \frac{I}{2\pi r}$$

$$\boxed{\bar{H} = H_\phi \cdot a_\phi = \frac{I}{2\pi r} \cdot a_\phi \text{ A/m}}$$

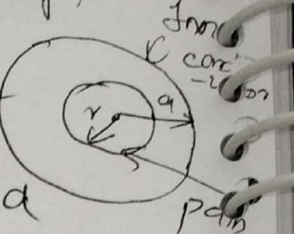
(b) $\bar{H}$ due to Coaxial Cable:-

Consider a coaxial cable as shown in figure. Its inner conductor is solid with radius a, carrying direct current I. The outer conductor is in the form of concentric cylinder whose inner radius is b and outer radius is c. This cable is placed along z axis. The current I is uniformly distributed in the inner conductor, while $-I$ is uniformly distributed in the outer conductor.



The Space between inner and outer conductor is filled with dielectric say air. The calculation of $\bar{H}$ is divided corresponding to various regions of the cable.

Region 1:- Within the conductor $0 < r < a$, consider a closed path having radius $r < a$. Hence it encloses only part of the conductor.



The area of cross section enclosed is $\pi r^2 m^2$

The total current flowing through the area πa^2 is I. Hence the current enclosed by total path

$$I' = \frac{\pi r^2}{\pi a^2} I = \frac{r^2}{a^2} I.$$

The $\bar{H}$ is again only in ϕ direction and depends only on r. $H = H_\phi . \bar{a_\phi}$.

Consider dL in a_ϕ direction which is $r\,d\phi$

$$dL = r d\phi . \bar{a_\phi} \Rightarrow H.dL = H_\phi . \bar{a_\phi} . r d\phi . \bar{a_\phi} = H_\phi . r . d\phi.$$

According to Amperes circuit law,

$$\oint \bar{H} . dl = I' \Rightarrow \oint H_\phi . r . d\phi = \frac{r^2}{a^2} I$$

$$\int_{\phi=0}^{2\pi} H_\phi . r d\phi = \frac{r^2}{a^2} I$$

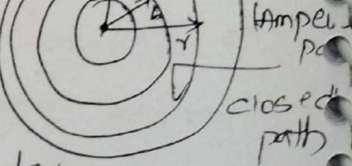
$$H_\phi . r (2\pi) = \frac{r^2}{a^2} I \Rightarrow H_\phi = \frac{r^2}{2\pi r a^2} I = \frac{r}{2\pi a^2} I$$

$$\bar{H} = \frac{Ir}{2\pi a^2} a_\phi \text{ A/m}.$$

Region 2:- Within $a < r < b$ consider a circular path which encloses the inner conductor carrying direct current I. This is the infinitely long conductor along z axis. Hence $\bar{H}$ in this region is

$$\boxed{\bar{H} = \frac{1}{2\pi r} a_\phi \text{ A/m}}$$

Region 3:- Within outer conductor, $b < r < c$.



Consider a closed path as shown. The current enclosed by the closed path is only the part of the current I, in the outer conductor. The total current $-I$ is flowing through the cross section $\pi(c^2 - b^2)$ while the closed path encloses the cross section $\pi(r^2 - b^2)$ the ...

Hence, the current enclosed by the closed path of outer conductor is

$$I' = \frac{\pi(r^2-b^2)}{\pi(c^2-b^2)}(-I) = -\frac{(r^2-b^2)}{(c^2-b^2)}I$$

Note that the closed path also encloses the inner conductor and hence current I flowing through it.

$I'' = I =$ current in inner conductor enclosed.

Total current enclosed by closed path is,

$$I_{enc} = I' + I'' = -\frac{(r^2-b^2)}{(c^2-b^2)}I + I = I\left[1-\frac{(r^2-b^2)}{(c^2-b^2)}\right]$$

$$= I\left[\frac{c^2-\cancel{b^2}-r^2+\cancel{b^2}}{c^2-b^2}\right] = I\left[\frac{c^2-r^2}{c^2-b^2}\right] \quad -(1)$$

According to Ampere's circuital law,

$\oint H \cdot dl = I_{en} \Rightarrow$ Now $\bar{H}$ is again in a_ϕ direction only and is a function of r only.

$$\bar{H} = H_\phi \cdot \bar{a_\phi}, \quad \bar{dl} = r \cdot d\phi \cdot \bar{a_\phi}$$

$$\bar{H} \cdot dl = H_\phi \cdot \bar{a_\phi} \cdot r \cdot d\phi \, \bar{a_\phi} = H_\phi \cdot r \cdot d\phi$$

$$I_{en} = \int_{\phi=0}^{2\pi} H_\phi \cdot r \cdot d\phi \quad -(2)$$ Sub Eq^n -(1) in Eq^n-(2)

$$I\left[\frac{c^2-r^2}{c^2-b^2}\right] = \int_{\phi=0}^{2\pi} H_\phi \cdot r \cdot d\phi$$

$$I\left[\frac{c^2-r^2}{c^2-b^2}\right] = H_\phi \cdot r(2\pi)$$

$$\boxed{H_\phi = \frac{1}{2\pi r}\left[\frac{c^2-r^2}{c^2-b^2}\right]}$$

$$\boxed{\bar{H} = H_\phi \cdot a_\phi = \frac{1}{2\pi r}\left[\frac{c^2-r^2}{c^2-b^2}\right] a_\phi \ A/m}$$

Region 4 :- Outside the cable, $r > c$.

Consider the closed path with $r > c$ such that it encloses both the conductors i.e, both currents $+I$ and $-I$.

Thus the total current enclosed is,

$$I_{enc} = +I - I = 0A$$

$$\boxed{\oint H \cdot dl = 0 \ A/m}$$

* MAGNETIC FLUX :-

The Magnetic flux density ($\bar{B}$) is analogous to the electric flux density ($\bar{D}$). The relation is

$\boxed{B = \mu H}$, $\mu_0 = 4\times\pi\times10^{-7}$ H/m, $\boxed{\bar{B} = \mu_0 \bar{H} \text{ for free Space}}$

Magnetic flux Density :-

The magnetic flux density has units wb/m² and hence it can be defined as the flux in webers passing through unit area in a plane at right angles to the direction flux.

If the flux passing through the unit area is not exactly at right angles to the plane consisting the area but making some angle with the plane then the flux ϕ crossing the area is given by

$$\boxed{\phi = \int_S \bar{B} \cdot d\bar{s} \ \text{webers (wb)}}$$

Now consider a closed surface which is defining a certain volume. The magnetic flux always exist in the form of closed loop. Thus for a closed surface the no. of magnetic flux lines entering must be

equal to the no. of magnetic flux lines leaving.
The single magnetic pole cannot exists like a single isolated electric charge. No magnetic flux can reside in a closed surface Hence the integral $\bar{B}\cdot\bar{ds}$ evaluated over a closed surface is always zero.

$$\boxed{\oint_S \bar{B}\cdot\bar{ds}=0}$$

This is called law of conservation of magnetic flux or Gauss law in integral form for magnetic fields. Gauss Law for Magnetostatic fields.

Applying divergence theorem

$$\oint_S \bar{B}\cdot\bar{ds}=\int_V \nabla\cdot\bar{B}\cdot dv$$

$$\int_V \nabla\cdot\bar{B}=0 \Rightarrow \boxed{\nabla\cdot\bar{B}=0} \rightarrow \text{Maxwells II Equation}$$

Maxwells Equation for static Electromagnetic fields

Differential form	Integral form.	Remarks
(1) $\nabla\cdot\bar{D}=\rho_v$	$\rightarrow \oint_S \bar{D}\cdot ds=\int_V \rho_v\, dv=Q$	$\rightarrow$ Gauss law
(2) $\nabla\times\bar{E}=0$	$\rightarrow \oint_L \bar{E}\cdot\bar{dl}=0$	$\rightarrow$ conservation of electric field
(3) $\nabla\times\bar{H}=\bar{J}$	$\rightarrow \int_L \bar{H}\cdot\bar{dl}=\int_S \bar{J}\cdot ds=I$	$\rightarrow$ Ampere's circuit law.
(4) $\nabla\cdot B=0$	$\rightarrow \oint_S B\cdot ds=0$	$\rightarrow$ Single magnetic pole cannot exist. i,e conservation of magnetic flux.

MAGNETIC SCALAR AND VECTOR POTENTIALS:-

There are two types of magnetic potentials

a) Scalar magnetic potential (V_m)

b) Vector magnetic potential ($\bar{A}$).

$\nabla \times \nabla V = 0$, $V =$ scalar.

$\nabla \cdot (\nabla \times \bar{A}) = 0$, $\bar{A} =$ vector.

a) Scalar Magnetic potential

$\nabla \times \nabla V_m = 0$ —(1), but $\bar{H} = -\nabla V_m$ —(2), sub Eqⁿ —(2) in Eqⁿ (1)

$\nabla \times (-H) = 0$ i,e $\nabla \times \bar{H} = 0$ —(3)

but $\nabla \times \bar{H} = \bar{J}$ —(4), sub Eqⁿ (3), $\bar{J} = 0$.

Thus Scalar magnetic potential V_m can be defined for Source free region where J i,e current density is zero.

$$\boxed{\bar{H} = -\nabla V_m \text{ , only for } \bar{J} = 0.}$$

Similar in to the relation between $\bar{e}$ and electric scalar potential, magnetic scalar potential can be expressed interms of $\bar{H}$ as

$$V_{m\,a,b} = -\int_a^b \bar{H} \cdot \bar{dl} \quad \text{... specified path}$$

b) ~~Vector~~ ~~Magnetic~~ ~~Potential~~ Laplace Eqⁿ

$\oint_S \bar{B} \cdot ds = 0$,

$\int \bar{B} \cdot ds = \int_V (\nabla \cdot \bar{B}) dv = 0$

$\nabla \cdot B = 0 \Rightarrow \nabla \cdot (\mu_0 H) = 0 \Rightarrow \nabla \cdot H = 0 \Rightarrow \nabla \cdot (-\nabla V_m) = 0$

$\boxed{\nabla^2 V_m = 0}$ → Laplacian Eqⁿ for Scalar magnetic potential.

b) Vector Magnetic potential

The vector magnetic potential is denoted as A and measured in wb/m. It has to satisfy that the divergence of curl of vector is always zero.

$\nabla \cdot (\nabla \times \bar{A}) = 0$

$\nabla \cdot B = 0$, $\boxed{B = \nabla \times A}$

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Thus curl of vector magnetic potential is the flux density

Now $\nabla \times H = \bar{J}$

$\nabla \times \frac{B}{\mu_0} = \bar{J}$, $B = \mu_0 H$

$\nabla \times \bar{B} = \mu_0 \bar{J} \Rightarrow \nabla \times \nabla \times \bar{A} = \mu_0 \bar{J}$

Using vector identity to express left hand side we can

$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$

$\bar{J} = \frac{1}{\mu_0}\left[\nabla \times \nabla \times \bar{A}\right] = \frac{1}{\mu_0}\left[\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}\right]$

Thus if vector magnetic potential is known then current density $\bar{J}$ can be obtained. For defining $\bar{A}$ the the current density need not to be zero.

Poisson's equation for magnetic field:-

$\bar{J} = \frac{1}{\mu_0}\left[-\nabla^2 \bar{A}\right]$

$$\boxed{\nabla^2 \bar{A} = -\mu_0 J}$$

***Forces due to Magnetic field:-**

a) force due to magnetic field on a moving charge:- (in $\bar{B}$ field)

i) charge at rest:-

Since, the charge is at rest, there are no movement of electrical charges, so there is no electric current. Therefore, there is no magnetic field. $F_e = QE$, Electric force.

If charge 'Q' at rest, $F_m = 0$. The

ii) Charge is moving with uniform velocity (u):-

When the charge is moving with uniform velocity. then $\bar{B}$ occurs. and $\bar{E}$ also occurs. Therefore, $\bar{F}$ is due to electric field and magnetic field.

$\bar{F_m} = Q(\bar{u} \times \bar{B})N$. $\rightarrow$ F_m is force experienced by a charge is moving with a velocity u in magnetic field

$$\bar{F_m} = QvB\sin\theta \; N$$

But $\bar{F_e} >> \bar{F_m}$

$$\bar{F_T} = \bar{F_e} + \bar{F_m}$$

$$\boxed{\bar{F_T} = Q\bar{E} + Q(\bar{v} \times \bar{B})}$$

↓

Lorentz force equation

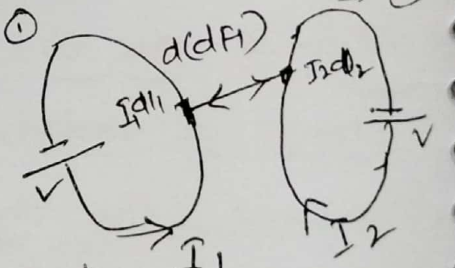
$$\bar{F} = ma = m \cdot \frac{dv}{dt} = QE + Q(\bar{v} \times \bar{B})$$

Force between two Currents:-

let us consider the force between two elements $I_1 dl_1$ and $I_2 dl_2$. Considering to Biot Savart's law both elements produce magnetic field so we may find the force $d(dF_1)$ on element $I_1 dl_1$ due to the field dl_2 produced by $I_2 dl_2$ element as shown in figure.

As we know that $dF = B \times I dl_1$ [From Lorentz force law] ②

$$d(dF_1) = I_1 dL_1 \times dB_2$$



from Biot Savarts law

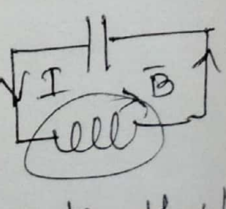
$$dB_2 = \frac{\mu_0 I_2 dl_2 \times a_{R21}}{4\pi R_{21}^2}$$

Hence

$$d(dF_1) = I_1 dl_1 \times \frac{(\mu_0 I_2 dl_2) \times a_{R21}}{4\pi R_{21}^2}$$

$$F_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{dl_1 \times dl_2 \times a_{R21}}{R_{21}^2}$$

Inductors and Inductance:-



$$\psi = \int_S \bar{B} \cdot d\bar{s} \;, \; \psi = B \cdot S.$$

λ = flux linkage = flux through 'N' turns

It medium ... $\boxed{\lambda \propto I} \Rightarrow \lambda = LI \Rightarrow \boxed{L = \frac{\lambda}{I}}$, $L = \frac{\text{Magnetic flux linkage}}{I}$

Inductance:-

It is measure of a ability that inductor can store magnetic energy. Inductance (L) is a property of physical arrangement of circuit.

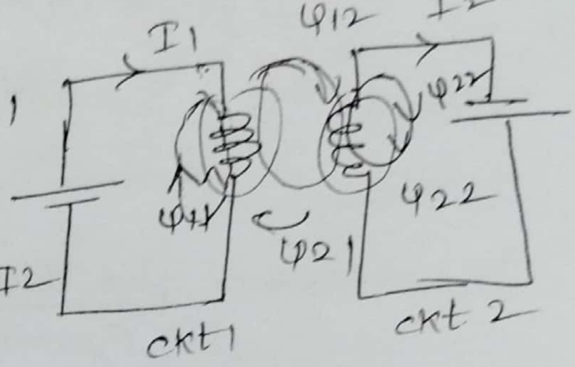
The energy stored in inductor, $W_E = \frac{LI^2}{2}$

If there are n no. of turns, then $L = \phi/I$

$$\boxed{L = \frac{n\phi}{I}\ H}$$

. A part of circuit that has inductance called Inductor.

Inductance:-



ϕ_{11} = flux on circuit due to I1

$$\phi_{11} = \int_{S_1} \bar{B_1} \cdot \bar{ds}$$

ϕ_{22} = flux on circuit due to I2

$$\phi_{22} = \int_{S_2} \bar{B_2} \cdot ds$$

ϕ_{12} = flux on circuit 2 due to I1

$$\phi_{12} = \int_{S_2} \bar{B_1} \cdot \bar{ds}$$

ϕ_{21} = flux on circuit 1 due to I2

$$\phi_{21} = \int_{S_1} \bar{B_2} \cdot ds$$

Mutual Inductance:-

$$\lambda = M_{12} = \frac{N_2 \phi_{12}}{I_1}, \quad M_{21} = \frac{N_1 \cdot \phi_{21}}{I_2}$$

Self Inductances

$$L_{11} = \frac{N_1 \cdot \phi_{11}}{I_1}, \quad L_{22} = \frac{N_2 \cdot \phi_{22}}{I_2}$$

Therefore, the total energy stored

$$\boxed{W_2 = \left(\frac{1}{2} I_1 L_1^{2} + \frac{1}{2} L_2 I_2^2\right) \pm M_{12} I_1 I_2} \text{ Joules.}$$

If I_1, I_2 are in same direction take $+M_{12}$ (M_{12}

If I_1, I_2 are in opposite direction take $-M_{12}$

($M_{12} = -M_{21}$)

a) Inductance of Solenoid:-

As we know that $\bar{H}$ due to solonoid of finite length 'l' is

$$\bar{H} = \frac{NI}{l} \hat{a}_z \text{ ; A/m}$$

but $\varphi = BS$, $B = \mu_0 H$, $\varphi = (\mu_0) H \cdot S$

$$N\varphi = N \cdot \mu_0 \cdot H \cdot S = \mu_0 N \left(\frac{NI}{l}\right) S = \frac{\mu_0 N^2 I \cdot S}{l}$$

$$\text{Inductance} = \frac{\text{flux linkage}}{\text{Current}} = \frac{N\varphi}{I} = \frac{\frac{\mu_0 N^2 I S}{l}}{I}$$

$$\boxed{\text{Inductance} = \frac{\mu_0 N^2 S}{l}}$$

b) Inductance of Toroid :-

As we know that $\bar{H}$ of Toroid is $\bar{H} = \frac{NI}{2\pi R} \hat{a}_\phi$

$$\lambda = N\varphi = N \cdot B \cdot S = N \mu_0 H S = N \mu_0 \left(\frac{NI}{2\pi R}\right) S$$

$$L = \frac{\lambda}{I} = \frac{N \mu_0 \left(\frac{NI}{2\pi R}\right) S}{I} = \frac{N^2 \mu_0 \cdot S}{2\pi R}$$

$$\boxed{L = \frac{N^2 \mu_0 S}{2\pi R} \text{ H}}$$

MAXWELL'S EQUATIONS:- (TIME VARYING FIELDS)

In case of static electromagnetic fields, the electric and magnetic fields are independent of each other. In general, the static electric fields are produced by the stationary electric charges. While the static magnetic fields are produced due to the motion of electric charges with uniform velocity or the magnetic charges. Thus we can say the source of an electrostatic field is an electric charge while that of magnetostatic field is the current through filament.

The time varying fields are produced due to the time varying currents. In case of such dynamic fields, the time varying electric field can be produced by the time varying magnetic field similarly the time varying electric field. Thus unlike static fields, in the dynamic or time varying fields, the electric and magnetic fields are interdependent.

The equations describing relationships between time varying electric and magnetic fields are known as Maxwell's equations.

FARADAY'S LAW :-

The time rate of decrease in magnetic flux is equal to the potential.

$$\boxed{V_{emf} = -N \frac{d}{dt} \varphi}$$

According to faraday's law, ~~as~~ the voltage induced around any closed conducting path is equal to the negative rate of decrement of magnetic flux within the path.

$$V_{emf} = -N \frac{d\varphi}{dt}$$

$$V_{emf} = -\oint_l \bar{E} \cdot dL \text{ —(1)}, \quad \varphi = \int_s \bar{B} \cdot ds, \quad V_{emf} = -N \frac{d}{dt} \int_s \bar{B} \cdot ds \text{ —(2)}$$

Comparing Eqn —(1) and (2)

$$V_{emf} = -N \frac{d}{dt} \int_s \bar{B} \cdot ds = \oint_l \bar{E} \cdot dL$$

The minus sign indicates that the direction of the induced e.m.f is such that to produce a current which will produce a magnetic field which oppose the original field.

The variation of the flux ϕ with respect to time t can be caused due to any one of the following conditions.

a) By having a stationary closed path in a time varying $\bar{B}$ field,

b) By having a time varying closed path in a static $\bar{B}$ field.

c) By having a time varying closed path in a time varying field $\bar{B}$.

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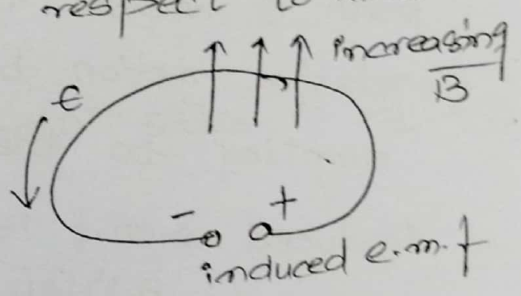
When an e.m.f is induced in a stationary closed path due to the time varying $\bar{B}$ field, the e.m.f is called statistically induced e.m.f or transformer e.m.f
When the e.m.f is induced in a time varying closed path due to the static field $\bar{B}$, then the e.m.f is called dynamically induced e.m.f or motional e.m.f.

(i) A stationary closed path in a time varying $\bar{B}$ field

Statistically induced e.m.f

The condition in which closed path is stationary and the $\bar{B}$ field is varying with respect to time

The closed circuit in which e.m.f is induced in stationary and the magnetic flux is sinusodally varying with time.



$$\oint_l \bar{E}\cdot dl = \frac{-d}{dt}\int_s \bar{B}\cdot d\bar{s} \quad \text{—(1)}$$

It is clear that the magnetic flux density is the only quantity varying with time.

we know $V_{emf} = +\int_l \bar{e}\cdot d\bar{l}$ —(2)

By stokes theorem, for eq^n (2), $\int_l \bar{e}\cdot dl = \int_s (\nabla\times\bar{e})ds$ —(3)

Comparing eq^n (1) and (3)

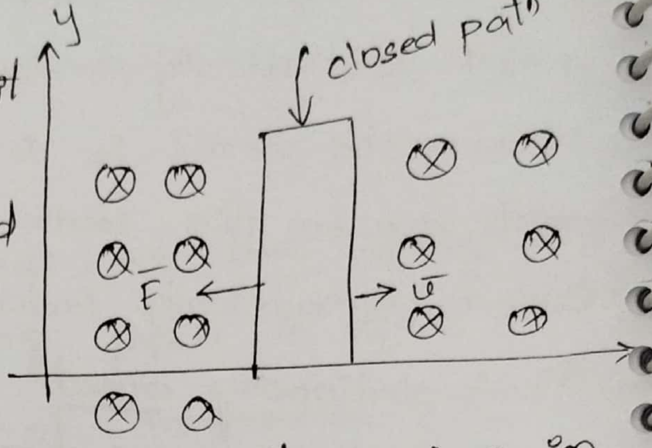
$$\int_s (\nabla\times\bar{e})ds = \frac{-d}{dt}\int \bar{B}\cdot ds$$

$$\boxed{\nabla\times\bar{e} = \frac{-d}{dt}(\bar{B})} \rightarrow \text{Maxwells eq}^n.$$

This is Maxwells Equation for time varying field which consists the relation between $\bar{E}$ and $\bar{B}$.

b) A moving closed path in static $\bar{B}$ field dynamically induced e.m.f

When a closed path or circuit is moving in a static $\bar{B}$ field, an e.m.f is induced in a closed path. This induced e.m.f is called dynamically induced e.m.f. Here the field is stationary while the closed path is moved to get a relative motion between them. This action is similar to the generator action. Hence the induced e.m.f is also called motional e.m.f or generator e.m.f



closed path moving in static $\bar{B}$ field with velocity $\bar{v}$

Consider that the charge Q is moved in a magnetic field $\bar{B}$ at a velocity $\bar{v}$. Then the force on the charge is given by

$$\bar{F} = Q\bar{v} \times B$$

Motional electric field intensity is defined as force per unit charge

$$E_m = \frac{\bar{F}}{Q} = \bar{v} \times B$$

Thus the induced e.m.f is given by

$$\oint \bar{E_m} \cdot d\bar{l} = \oint (\bar{v} \times \bar{B}) \cdot d\bar{L}$$

(c) Moving closed path in a time varying $\bar{B}$ field

A moving closed path in a time varying $\bar{B}$ field represents a general case in which both e.m.f i.e transformer e.m.f and motional e.m.f are ...

Thus the induced e.m.f in this case is the combination of these two e.m.f's. Hence the induced e.m.f for a moving closed path in a time varying magnetic field can be expressed as

Total induced e.m.f = Transformer e.m.f + Motional e.m.f

$$\oint \bar{E}\cdot d\bar{l} = \int \frac{d}{dt}\bar{B}\cdot d\bar{s} + \oint (\bar{v}\times\bar{B})\cdot d\bar{l}$$

INCONSISTENCY OF AMPERE'S CIRCUIT LAW:-

For static electromagnetic fields, the maxwell equation,

$$\nabla\times\bar{H} = \bar{J} \quad \text{—— (1)}$$

Now applying divergence on both sides to Eq^n (1)

$$\nabla\cdot(\nabla\times\bar{H}) = \nabla\cdot\bar{J} \quad \text{— (2)}$$

But, according to vector properities

$$\nabla\cdot(\nabla\times\bar{H}) = 0 \quad \text{— (3)}$$

Applying Eq^n (3) to Eq^n (2), $\boxed{\nabla\cdot\bar{J}=0}$

But, according to continuity equation,

$\nabla\cdot\bar{J} = -d/dt(\rho_v)$ for time varying fields.

But $\nabla\cdot\bar{J}=0$ is valid only for static fields. This results is not consistent with the continuity equation,

i.e $\nabla\cdot\bar{J} = -\frac{d}{dt}\rho_v$. In other words, Ampere's circuit law is not consistent and needs some modification.

Then we can let us consider some unknown term $\bar{U}$. Then we can modify Ampere's circuit law for time varying fields as

$$\nabla\times\bar{H} = \bar{J} + J_d$$

displacement current density

$\nabla \times \bar{H} = J + \bar{J_d}$

Now applying divergence property on both sides

$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} + \nabla \cdot \bar{J_d}$ — (4)

we know that $\nabla \cdot \bar{J} = \frac{-\partial}{\partial t}(\rho_v)$ from continuity eqn.

and writing $\nabla \cdot J_d = \partial/\partial t\,(\rho_v)$ for satisfying $\nabla \cdot \bar{J} = 0$

for eqn (4), $\nabla \cdot (\nabla \times H) = -\partial/\partial t\,(\rho_v) + \partial/\partial t\,(\rho_v)$

$$\boxed{\nabla \cdot (\nabla \times \bar{H}) = 0}$$

$\nabla \times \bar{H} = \bar{J} + J_d$, $\nabla \cdot \bar{D} = \rho_v \Rightarrow \nabla \cdot \frac{\partial}{\partial t} D = \frac{\partial (\rho_v)}{\partial t}$

$\nabla \cdot \frac{\partial}{\partial t}(\bar{D}) = \frac{\partial}{\partial t}(\rho_v)$, since, $\nabla \cdot J_d = \frac{\partial}{\partial t}(\rho_v)$

$\nabla \cdot \frac{\partial}{\partial t}(\bar{D}) = \nabla \cdot J_d$, $J_d = \partial/\partial t\,(\bar{D})$

So, $\boxed{\nabla \times \bar{H} = \bar{J} + \frac{\partial}{\partial t}(\bar{D})}$

where $\bar{J_d}$ = displacement current density

$\bar{J_d} = \frac{I_d}{s} \Rightarrow I_d = \int_s J_d \cdot ds$

MAXWELL'S EQUATION FOR TIME VARYING FORM

Point form	Integral form	Remarks.
1) $\nabla \cdot \bar{D} = \rho_v$	$\rightarrow \oint_s \bar{D} \cdot d\bar{s} = \int_v \rho_v \cdot dv$	$\rightarrow$ Gauss Law.
2) $\nabla \cdot \bar{B} = 0$	$\rightarrow \oint_s \bar{B} \cdot ds = 0$	$\rightarrow$ Non existence of magnetic monop
3) $\nabla \times \bar{E} = \frac{-\partial}{\partial t}(\bar{B})$	$\rightarrow \oint \bar{E} \cdot dl = -\int_s \partial/\partial t\,\bar{B} \cdot ds$	$\rightarrow$ Faraday law
4) $\nabla \times \bar{H} = \bar{J_c} + \bar{J_d}$ $= \bar{J_c} + \partial/\partial t\,\bar{D}$	$\rightarrow \oint \bar{H} \cdot dl = \int_s \bar{J_c} \cdot ds + \int_c \partial/\partial t\,\bar{D} \cdot ds$	$\rightarrow$ Ampere's circu law.

Maxwell Equations for Good Conductors, $\sigma=\infty$, $\rho_v=0$, $J_d=0$

Point form	Integral form
(1) $\nabla\cdot\bar{D}=0$ $\longrightarrow$	$\oint_S \bar{D}\cdot\bar{ds}=0$
(2) $\nabla\cdot\bar{B}=0$ $\longrightarrow$	$\oint_S \bar{B}\cdot\bar{ds}=0$
(3) $\nabla\times\bar{E}=d/dt\,\bar{B}$ $\rightarrow$	$\oint_l \bar{E}\cdot\bar{dl}=-\int_S d/dt\,\bar{B}\cdot ds$
(4) $\nabla\times\bar{H}=J_C$ $\rightarrow$	$\oint_l \bar{H}\cdot\bar{dl}=\int_S \bar{J_C}\cdot ds$

Maxwells Equation for free space

free space is non conducting medium in which

$\sigma=0$, $\rho_v=0$, $J_c=0$, $\bar{B}=$ $\bar{J_c}=0$, $\mu_r=\varepsilon_r=1$.

(1) $\nabla\cdot\bar{D}=0$ $\rightarrow$ $\oint\bar{E}\cdot\bar{dl}=-\int_S\frac{d}{dt}\bar{B}\cdot ds$

(2) $\nabla\cdot\bar{B}=0$ $\rightarrow$ $\oint_S\bar{B}\cdot ds=0$

(3) $\nabla\times\bar{E}=-d/dt\,\bar{B}$ $\rightarrow$ $\oint_S\bar{D}\cdot ds=0$

(4) $\nabla\times\bar{H}=+\frac{d\bar{D}}{dt}$ or $\rightarrow$ $\oint_l\bar{H}\cdot dl=\int_S d/dt\,\bar{D}\cdot\bar{ds}$

J_d

CONDITIONS AT BOUNDARY SURFACE / ELECTROSTATIC

BOUNDARY CONDITIONS

If the fields exists in the region consists of two different medias, that the conditions that must be satisfied by the field on the intaphase seperating two medias is known as boundary conditions.

The advantage of boundary conditions is that if the field on one media is known as the field of other media can be calculated.

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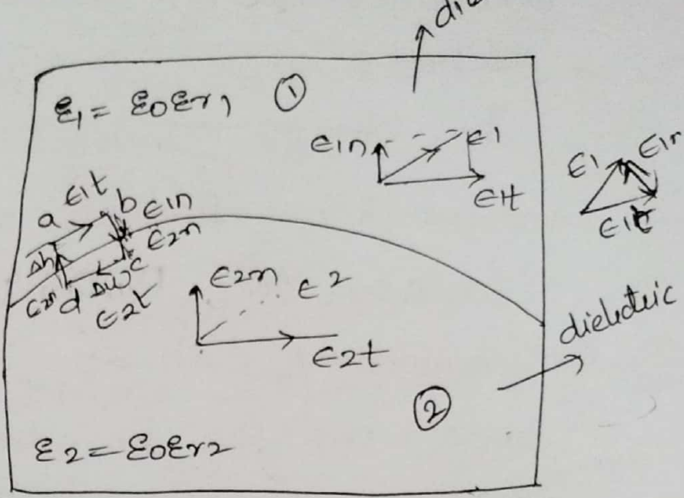
a) DIELETRIC - DIELECTRIC INTERPHASE

For calculating Boundary conditions, we use two two Maxwells Equations $\oint_l \bar{E}\cdot d\bar{l} = 0$, $\oint_s \bar{D}\cdot d\bar{s} = Q_{enclosed}$

$t \to$ tangential, $n \to$ normal.

on Intaphase a small loop ABCDA is considered.



On considering $\oint_l \bar{E}\cdot dl = 0$ to ABCDA loop.

Therefore, $E_{1t}\cdot\Delta w - E_{1n}\cdot\cancel{\frac{\Delta h}{2}} - E_{2n}\cdot\cancel{\frac{\Delta h}{2}} - E_{2t}\cdot\Delta w + E_{2n}\cdot\cancel{\frac{\Delta h}{2}} + E_{2n}\cdot\cancel{\frac{\Delta h}{2}} = 0$

$$E_1t\cdot\Delta w - E_{2t}\cdot\Delta w = 0 \Rightarrow \boxed{E_{1t} = E_{2t}} \to \text{first boundary condition}$$

The above equation is known as first boundary conditions, and the tangential components of $\bar{E}$ from one dielectric media to another dielectric media is unchanged

The tangential Components of $\bar{E}$ are continuous across boundary.

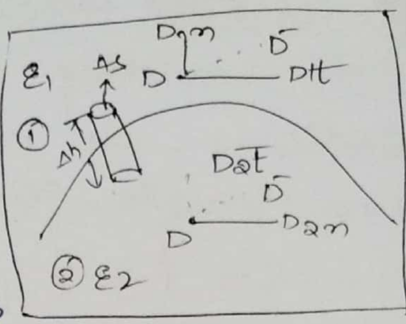
$D = \epsilon_0 E \Rightarrow E = D/\epsilon_0$

$E_{1t} = D_{t1}/\epsilon_1$, $E_{2t} = D_{2t}/\epsilon_2$

The tangential Components of $\bar{D}$ from one media to another media is ~~un~~ changed (i.e unequal)

The tangential component of $\bar{D}$ are discontinuous across boundary (or) Interphase.

Now applying $\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed}$ to a pill box :



The charge $Q_{enclosed}$, $Q_{2n} = \oint_C \bar{D} \cdot \bar{ds}$

And we know that, $\rho_s = \frac{\Delta Q}{\Delta S}$

$\rho_s \cdot \Delta S = \Delta Q$, $D_{1n} \Delta S - D_{2n} \Delta S = \rho_s \cdot \Delta S$

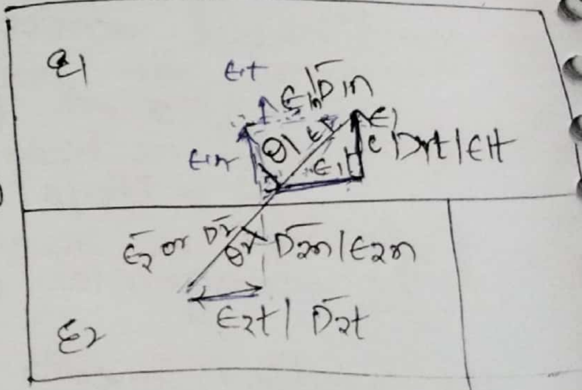
On boundary, $\Delta h \to 0$. So, the expression is $D_{1n} - D_{2n} = \rho_s$

Since, on the boundary, the no. of free charge exists on dielectric - dielectric Intaphase, $\rho_s = 0$

i.e $\boxed{D_{1n} = D_{2n}}$, $\epsilon_1 E_{1n} = \epsilon_2 . E_{2n} \Rightarrow E_{1n} = \frac{\epsilon_2}{\epsilon_1} . E_{2n}$

i.e $\boxed{E_{1n} \neq E_{2n}}$

The another advantage of Boundary conditions is that the refraction (bending) of $\bar{E}$ or $\bar{D}$ across the Intaphase can be measured. Consider following figure in which $\bar{E_1}$ or $\bar{D_1}$ in dielectric medial or $\bar{E_2}$ or $\bar{D_2}$ in dielectric media2 or making an angles. θ_1 and θ_2 respectively with normal to the intaphase



Using boundary conditions

$\bar{E_{1t}} = \bar{E_{2t}}$.

$\sin\theta_1 = \frac{E_{1t}}{E_1} \Rightarrow E_1 \cdot \sin\theta_1 = E_{1t}$ — (1)

from diagram, $E_{1t} = E_{2t}$

$E_1 \sin\theta_1 = E_2 \sin\theta_2$ — (1)

From second boundary conditions of 2 Intaphase

$D_{1n} = D_{2n}$, $\cos\theta_1 = \frac{D_{1n}}{D_1} \Rightarrow \bar{D_{1n}} = D_1 \cos\theta_1$

$\cos\theta_2 = \frac{\bar{D_{2n}}}{D_2} \Rightarrow \bar{D_{2n}} = D_2 \cos\theta_2$

$D_1 \cos\theta_1 = D_2 \cos\theta_2$ — (2)

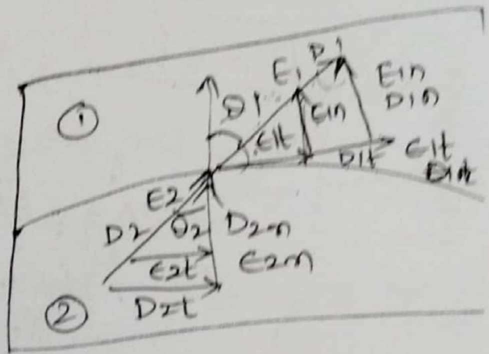
$D_1 \cos\theta_1 = D_2 \cos\theta_2 \Rightarrow \varepsilon_{r1} \cdot E_1 \cdot \cos\theta_1 = \varepsilon_{r2} \cdot E_2 \cdot \cos\theta_2$

$$\frac{E_1 \cos\theta_1}{} \quad \frac{E_1 \sin\theta_1}{\varepsilon_{r1} \cdot E_1 \cdot \cos\theta_1} = \frac{E_2 \sin\theta_2}{\varepsilon_{r2} \cdot E_2 \cdot \cos\theta_2}$$

$$\frac{\tan\theta_1}{\varepsilon_{r1}} = \frac{\tan\theta_2}{\varepsilon_{r2}} \Rightarrow \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = \frac{\varepsilon_1}{\varepsilon_2}}$$

In Magnetostatics across dielectric - dielectric intaphase the boundary conditions will be applying two maxwells equations.

$$\oint \bar{H} \cdot dl = I_{enclosed}, \quad \oint_s \bar{B} \cdot ds = 0$$



$$\bar{H_{1t}} = \bar{H_{2t}} \Rightarrow \frac{\bar{B_{1t}}}{\mu_1} = \frac{\bar{B_{2t}}}{\mu_2}, \quad B = \mu H$$

$B_{1n} = B_{2n}$

According to law of refraction

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r1}}{\mu_{r2}}}$$

(b) <u>Conductor - dielectric Intaphase</u>

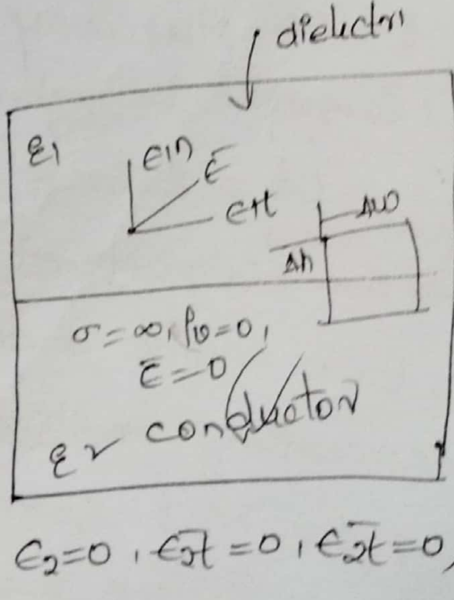
Applying $E \cdot dl$ to a closed loop,

since $E_2 = 0$, $E_{2t} = 0$, $E_{2n} = 0$

$$(\Delta w)(0) + \left(\frac{\Delta h}{2}\right) E_{1n} + \left(\frac{\Delta h}{2}\right)(0) - (\Delta w) E_t$$
$$- \left(\frac{\Delta h}{2}\right) E_{2n} - \left(\frac{\Delta h}{2}\right)(0) = 0$$

$\Delta h \to 0$

$$(\Delta w) E_t = 0 \Rightarrow \boxed{E_t = E_{2t} = 0}$$



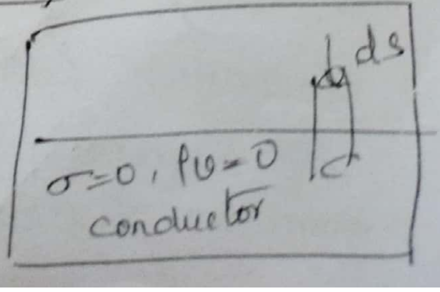
$E_2 = 0$, $E_{2t} = 0$, $E_{2t} = 0$,

since $D = \varepsilon E \Rightarrow E_t = D/\varepsilon$, $D = \varepsilon(0)$, $\boxed{D = 0}$

ii) Apply $\oint_s \bar{D} \cdot ds = 0$, $\rho_s = \frac{\Delta Q}{\Delta S}$

$\Delta Q = \rho_s \cdot \Delta S$ — (1) $\oint_s \bar{D} \cdot ds = D_n \cdot \Delta S$ — (2)

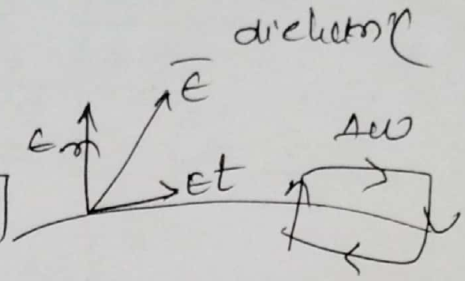
from (1) & (2), $\boxed{D_n = \rho_s}$



Conductor - dielectric Boundary Conditions

$0 = 0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} - E_t \cdot \Delta w$

$- E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2}$

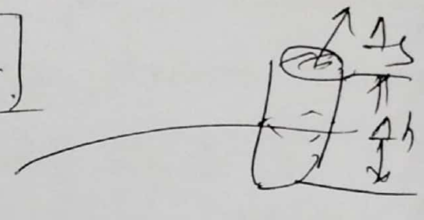


$\boxed{E_t = 0}$, $\Delta h \to 0$, So, $D_t = \varepsilon E_t \Rightarrow \boxed{D_t = 0}$

Consider cylindrical pillbox

conductor
$\bar{E} = 0$

$D = \varepsilon \bar{E} = 0$, $D_n = \frac{\Delta Q}{\Delta s} = \rho_s$



$\oint_S \bar{D} \cdot ds = Q$ $\boxed{D_n = \rho_s}$, $D_n = \varepsilon \bar{E}_n \Rightarrow \boxed{\bar{E}_n = \frac{D_n}{\varepsilon} = \frac{\rho_s}{\varepsilon}}$

$\Delta Q = D_{n_1} \cdot \Delta s - D_{n_2} \cdot \Delta s$.

$D_n \cdot \Delta s = \Delta Q \Rightarrow D_n = \frac{\Delta Q}{\Delta s} = \rho_s$.

So, $D_n = \rho_s$.

Thus under static conditions, the following conclusions can be made about perfect conductor

(1) No field may exist within a conductor. $\rho_v = 0$, $\bar{E} = 0$

(2) Since $\bar{E} = -\nabla V$, there can be no potential difference b/w any two inside conductor.

3) $\bar{E}$ must be external to conductor & normal to surface

$D_t = \varepsilon_0 \varepsilon_r E_t = 0$, $D_n = \varepsilon_0 \varepsilon_r E_n = \rho_s$.

→ An important application of the fact that $E = 0$ inside a conductor is an electrostatic screening or shielding. If conductor A kept at zero potential surrounds conductor B as shown in below figure, B is said to be electrically screened by A from other electric circuits, such as conductor C outside A. Similarly, conductor C outside A is screened from B. Thus, conductor A acts like a screen or shield, and the electrical conditions inside & outside the screen are completely independent of each other.