UNIT-IV: FILTERS AND DC BRIDGES:

DC Bridges: Methods of measuring low, medium and high resistance – sensitivity of Wheat stone's bridge, Kelvin's double bridge for measuring low resistance,

If the bridge circuit can be operated with only DC voltage signal, then it is a DC bridge circuit or simply DC bridge. DC bridges are used to measure the value of unknown resistance.

MEASUREMENT OF RESISTANCE:

Resistance is one of the most basic elements encountered in electrical and electronics engineering. The value of resistance in engineering varies from very small value like, resistance of a transformer winding, to very high values like, insulation resistance of that same transformer winding. Although a multimeter works quite well if we need a rough value of resistance, but for accurate values and that too at very low and very high values we need specific methods. For the purposes of measurements, the resistances are classified into three major groups based on their numerical range of values and the resistances can be measured by various ways, depending on their range of values, as under:

1**. Low resistance :** (0 to 1 ohm): Kelvin Double Bridge method, 2. **Medium resistance:** (1 to 100 kilo-ohm): wheat stone's bridge method, 3. **High resistance** : (>100 kilo-ohm): Loss of charge method

LOW RESISTANCE: By using Kelvin Double Bridge method we can measure resistance between (0 to 1 ohm) In practice, such resistances can be found in the copper winding in armatures, ammeter shunts, contacts, switches, etc.

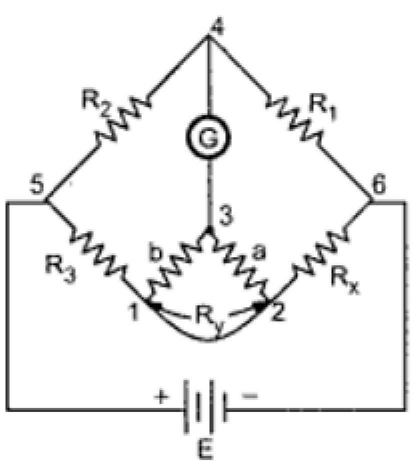


Fig. Kelvin's double bridge

This bridge consists of another set of ratio arms hence called **double bridge**. The Fig. 4.18 shows the circuit diagram of Kelvin's Double Bridge.

The second set of ratio arms is the resistances 'a' and 'b'. With the help of these resistances the galvanometer is connected to point '3'. The **galvanometer gives null indication** when the potential of the terminal '3' is same as the potential of the terminal '4'.

Thus $E_{45} = E_{513}$ ... (1)

Here E_{45} = Potential across R_2.

E_{513} = Potential across R_3 and b.

The ratio of the resistances a and b is same as the ratio of R_1 and R_2.

$$\therefore \qquad \frac{a}{b} = \frac{R_1}{R_2} \qquad \dots (2)$$

Now $$E_{45} = R_2 \cdot \frac{E}{R_1 + R_2} \qquad \dots (3)$$

Consider the path from 5-1-2-6 back to 5 through the battery E. The resistance between the terminals 1-2 is the parallel combination of R_y and $(a + b)$.

$$\therefore \qquad E = I \times [R_3 + R_y \,||\, (a+b) + R_x]$$

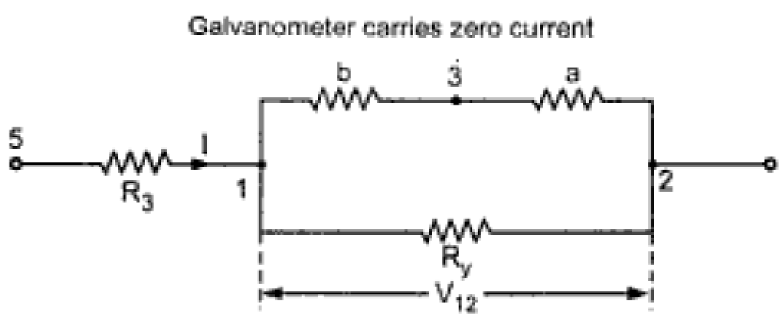
$$\therefore \qquad E = I\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \qquad \dots (4)$$

Substituting in equation (3),

$$E_{45} = \frac{R_2}{R_1 + R_2} \times I\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \qquad \dots (5)$$

For E_{513}, consider the path from the terminal 5 to 2 as shown in the Fig.

Now from the Fig. we can write,



$$V_{12} = I \times \left[\frac{R_y(a+b)}{R_y + a + b}\right]$$

and $$V_{13} = \frac{b}{a+b} \cdot V_{12}$$

$$V_{13} = \frac{b}{a+b} \cdot I\left[\frac{R_y(a+b)}{R_y + a + b}\right] \qquad \dots (6)$$

$$\therefore \quad E_{513} = I R_3 + V_{13}$$

$$\therefore \quad E_{513} = I R_3 + I \frac{b}{a+b}\left[\frac{R_y (a+b)}{R_y + a + b}\right]$$

$$\therefore \quad E_{513} = I\left[R_3 + \frac{b}{a+b}\left[\frac{R_y (a+b)}{R_y + a + b}\right]\right] \qquad \ldots (7)$$

Now $\qquad E_{45} = E_{513} \qquad$ ... For balancing

$$\therefore \frac{I R_2}{R_1 + R_2}\left[R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y}\right] = I\left[R_3 + \frac{b}{a+b}\left\{\frac{R_y (a+b)}{a+b+R_y}\right\}\right]$$

$$\therefore \quad R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} = \frac{R_1 + R_2}{R_2}\left[R_3 + \frac{b}{a+b}\left\{\frac{R_y (a+b)}{a+b+R_y}\right\}\right]$$

$$\therefore \; R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} = \left[1 + \frac{R_1}{R_2}\right]\left[R_3 + \frac{b R_y}{R_y + a+b}\right]$$

$$\therefore \; R_3 + R_x + \frac{(a+b) R_y}{a+b+R_y} = R_3 + \frac{R_1 R_3}{R_2} + \frac{b R_y}{R_y + a + b} + \frac{R_1 b R_y}{R_2 (R_y + a + b)}$$

$$\therefore \qquad R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{R_y + a + b} + \frac{R_1 b R_y}{R_2 (R_y + a + b)} - \frac{(a+b) R_y}{(R_y + a + b)}$$

$$\therefore \quad R_x = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_y}{R_2 (R_y + a + b)} - \frac{a R_y}{(a+b+R_y)}$$

$$\therefore \quad \boxed{R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{(R_y + a + b)}\left[\frac{R_1}{R_2} - \frac{a}{b}\right]} \qquad \ldots (8)$$

But $\frac{a}{b} = \frac{R_1}{R_2}$ thus $\frac{R_1}{R_2} - \frac{a}{b} = 0$

$$\therefore \quad \boxed{R_x = \frac{R_1 R_3}{R_2}} \qquad \ldots (9)$$

This is the standard equation of the bridge balance. The resistances a, b and R_y are not present in this equation. **Thus the effect of lead and contact resistances is completely eliminated.**

MEDIUM RESISTANCE:

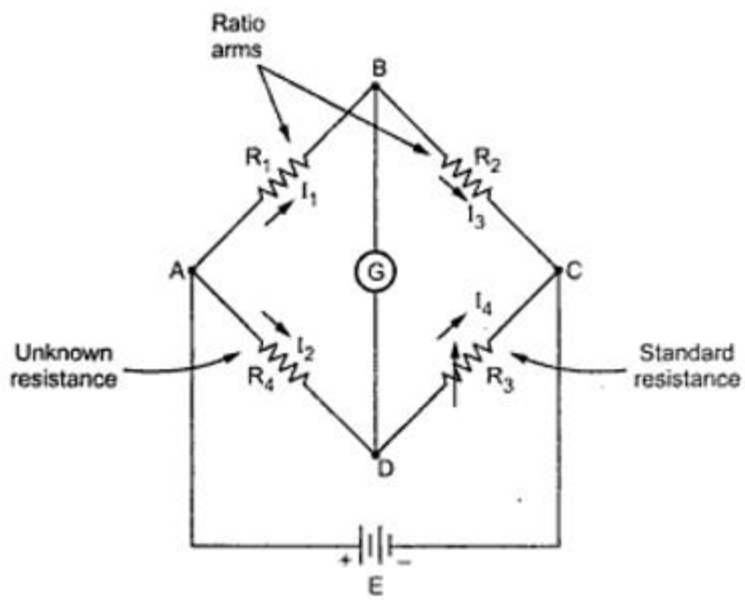
By using wheat stone's bridge method we can measure resistance Between (1 to 100 kilo-ohm) Most of the electrical apparatus used in practice, electronic circuits, carbon resistance and metal film resistors are found to have resistance values lying in this range.

WHEATSTONE BRIDGE METHOD:

The device uses for the measurement of minimum resistance with the help of comparison method is known as the Wheatstone bridge. The value of unknown resistance is determined by comparing it with the known resistance. The Wheatstone bridge works on the principle of null deflection, i.e. the ratio of their resistances are equal, and no current flows through the galvanometer. The bridge is very reliable and gives an accurate result.

The bridge consists of four resistive arms together with a source of e.m.f. and a null detector. The galvanometer is used as a null detector.

The Fig. shows the basic Wheatstone bridge circuit.



The arms consisting the resistances R_1 and R_2 are called **ratio arms.** The arm consisting the standard known resistance R_3 is called **standard arm.** The resistance R_4 is the **unknown resistance** to be measured. The battery is connected between A and C while galvanometer is connected between B and D.

Balance Condition

When the bridge is balanced, the galvanometer carries zero current and it does not show any deflection. Thus bridge works on the **principle of null deflection** or **null indication.**

To have zero current through galvanometer, the points B and D must be at the same potential. Thus potential across arm AB must be same as the potential across arm AD.

Thus $$I_1 R_1 = I_2 R_4 \quad \text{... (1)}$$

As galvanometer current is zero.

$$I_1 = I_3 \quad \text{and} \quad I_2 = I_4 \quad \text{... (2)}$$

Considering the battery path under balanced condition,

$$I_1 = I_3 = \frac{E}{R_1 + R_2} \quad \text{... (3)}$$

and $$I_2 = I_4 = \frac{E}{R_3 + R_4} \quad \text{... (4)}$$

Using equation (3) and (4) in equation (1),

$$\frac{E}{R_1 + R_2} \times R_1 = \frac{E}{R_3 + R_4} \times R_4$$

$$\therefore \quad R_1 (R_3 + R_4) = R_4 (R_1 + R_2)$$

$$\therefore \quad R_1 R_3 + R_1 R_4 = R_1 R_4 + R_2 R_4$$

$$\therefore \quad \boxed{R_4 = R_3 \frac{R_1}{R_2}} \quad \text{... (5)}$$

This is required balance condition of Wheatstone bridge.

The following points can be observed.

1. It depends on the ratio of R_1 and R_2 hence these arms are called **ratio arms**.
2. As it works on null indication, the results are not dependent on the calibration and characteristics of galvanometer.
3. The standard resistance R_3 can be varied to obtain the required balance.

SENSITIVITY OF WHEATSTONE'S BRIDGE:

When the bridge is balanced, the current through galvanometer is zero. But when bridge is not balanced current flows through the galvanometer causing the deflection. The amount of deflection depends on the **sensitivity** of the galvanometer. This sensitivity can be expressed as amount of deflection per unit current.

$$\boxed{\text{Sensitivity } S = \frac{\text{Deflection } D}{\text{Current } I}}$$

As the current is in microampere and deflection can be measured in mm, radians or degrees, the sensitivity is expressed as mm/μA, radians/μA or degrees/μA. More is the sensitivity of a galvanometer, more is its deflection for the same amount of current.

$$\therefore \quad V_{TH} = E\left\{\frac{R_4 + \Delta R}{R_3 + R_4 + \Delta R} - \frac{R_1}{R_1 + R_2}\right\} \quad \ldots (6)$$

$$\text{As} \quad \frac{R_4}{R_3} = \frac{R_1}{R_2} \quad \text{then} \quad \frac{R_1}{R_1 + R_2} = \frac{R_4}{R_4 + R_3}$$

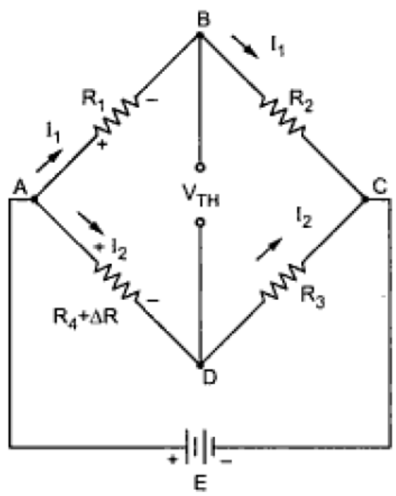


Fig. Bridge under unbalance

Using above relation in equation (6),

$$V_{TH} = E\left\{\frac{R_4+\Delta R}{R_3+R_4+\Delta R}-\frac{R_4}{R_3+R_4}\right\}$$

$$= E\left\{\frac{R_3R_4+R_3\Delta R+R_4^2+R_4\Delta R-R_3R_4-R_4^2-R_4\Delta R}{(R_3+R_4)(R_3+R_4+\Delta R)}\right\}$$

$$= \frac{E R_3 \Delta R}{(R_3+R_4)^2+(R_3+R_4)\Delta R}$$

But as ΔR is very small, $(R_3+R_4)\Delta R << (R_3+R_4)^2$

$$\therefore \quad \boxed{V_{TH} = V_g = \frac{E R_3 \Delta R}{(R_3+R_4)^2}} \quad \dots (7)$$

Now $\quad S_B = \frac{\theta}{\Delta R / R}$ = Bridge sensitivity

and $\quad \Delta R/R = \Delta R / R_4$ as there is change in R_4.

From the galvanometer sensitivity S_V,

$\theta = S_V \times e \quad$ where $\;e$ = Voltage across galvanometer = V_g

Using θ in the expression of S_B,

$$\therefore \quad S_B = \frac{S_V V_g}{\Delta R / R_4} = \frac{S_V E R_3 \Delta R R_4}{(R_3+R_4)^2} = \frac{S_V E R_3 R_4}{R_3^2+2R_3R_4+R_4^2}$$

$$\therefore \quad \boxed{S_B = \frac{S_V E}{\frac{R_3}{R_4}+2+\frac{R_4}{R_3}}} \quad \dots (8)$$

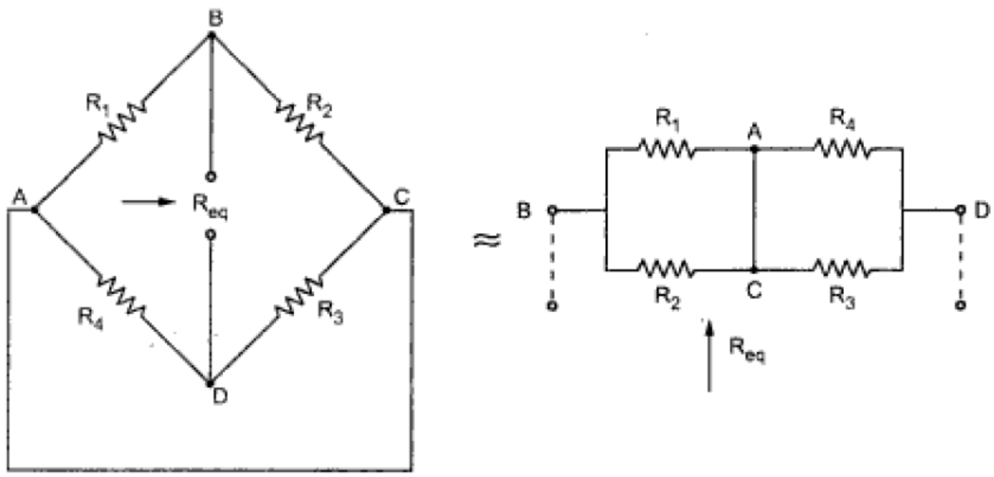
Thus the **bridge sensitivity** depends on the bridge parameters, the supply voltage and the voltage sensitivity of the galvanometer.

Key Point: *Thus maximum sensitivity occurs when* $\frac{R_3}{R_4}=1$.

For higher or lower values of R_3/R_4, the sensitivity decreases considerably.

Thevenin's Equivalent and Galvanometer Current

The Thevenin's voltage V_{TH} across the galvanometer is already obtained. Let us obtain equivalent resistance as viewed across the terminals BD, when battery E is replaced by short circuit. Thus circuit becomes,



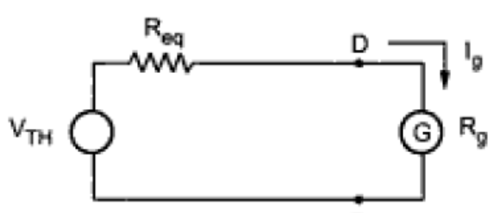
$$\therefore \quad R_{eq} = (R_1 \| R_2) + (R_3 \| R_4) \qquad \dots (9)$$

While $V_{TH} = E_{AD} - E_{AB}$ with R_4 not changed by ΔR .

$$= I_2\, R_4 - I_1\, R_1 = \frac{E}{R_3 + R_4} R_4 - \frac{E}{R_1 + R_2} R_1$$

$$\therefore \quad \boxed{V_{TH} = E\left[\frac{R_4}{R_3 + R_4} - \frac{R_1}{R_1 + R_2}\right]} \qquad \dots (10)$$

Thus Thevenin's equivalent is as shown in the Fig. 4.11.



Let R_g = Galvanometer resistance

I_g = Galvanometer current

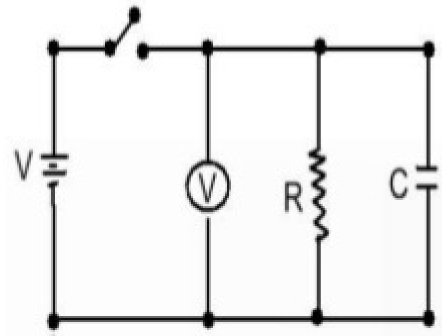
$$\therefore \quad I_g = \boxed{\frac{V_{TH}}{R_{eq} + R_g}} \qquad \dots (11)$$

where V_{TH} = Thevenin's voltage

HIGH RESISTANCE : (>100 kilo-ohm):

Resistances higher than 100kΩ are classified as high resistances. Insulation resistances in electrical equipment are expected to have resistances above this range.

Loss of charge method: This method is used for the measurement of high resistance



Loss of charge method

In this method, the resistance to be measured is connected directly across a dc voltage source in parallel with a capacitor. The capacitor is charged up to a certain voltage and then discharged through the resistance to be measured. The terminal voltage across the resistance-capacitance parallel combination is recorded for a pre-defined period of time with a help of a high-resistance voltmeter (electrostatic voltmeter or digital electrometers). Value of the unknown resistance is calculated from the discharge time constant of the circuit.

The capacitor is initially charged to some suitable voltage by means of a battery of voltage V and then allowed to discharge through the resistance. The terminal voltage is observed during discharge and it is given by,

$$v = Ve^{-t/CR}$$ OR

$$\frac{V}{v} = e^{t/CR}$$

The insulation resistance is given by,

$$R = \frac{t}{CLog_e \frac{V}{v}} = \frac{0.4343t}{CLog_{10} \frac{V}{v}}$$

The variation of voltage v with time is shown in figure,