Unit - III: Resonance & Magnetic circuits

- **Part-A: Resonance**
  - ✓ Series and parallel resonance circuits
  - ✓ Resonance frequency
  - ✓ Quality factor and band width determination.
- **Part-B: Magnetic circuits**
  - ✓ Magnetic circuits-Faraday's laws of electromagnetic induction
  - ✓ Concept of self and mutual inductance-coefficient of coupling.

Part-A: Resonance

Resonance

Any system having at least a pair of complex conjugate poles has a natural frequency of oscillation. If the frequency of the system driving force coincides with the natural frequency of oscillation, the system resonates and the system response become maximum. The phenomenon is known as resonance and the frequency at which this phenomenon occurs is known as ‘ **resonant frequency**’.

In electrical systems, resonance occurs when the system contains at least one inductor and one capacitor.

In the system, the phenomenon of cancellation of reactance's when the inductor and capacitor are in series or cancellation of susceptances when they are in parallel, is termed resonance. The circuit under resonance is purely resistive in nature and termed as ‘**resonant circuit**’ or ‘**tuned circuit**’.

Electrical resonance is classified into two categories:

1. Series resonance 2.Parallel resonance

✓Series resonance circuits

Consider a series circuit consists of resistor, inductor and capacitor as shown in Figure 1.1. The impedance of the circuit is

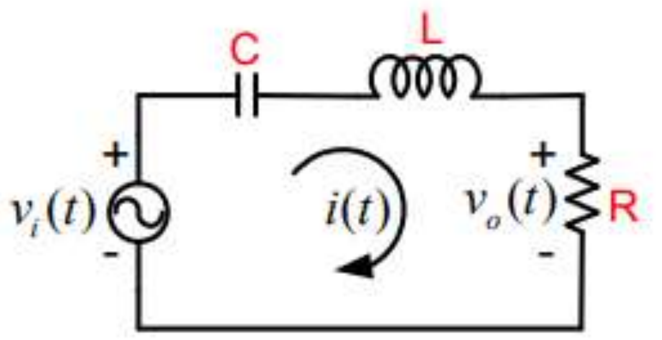


Figure 1.1: Series resonance circuit

$$Z = R + j(X_L - X_C)$$

At resonance the imaginary part is zero

$$X_L - X_C = 0$$

$$\omega_r L - 1/\omega_r C = 0$$

$$\omega_r L = 1/\omega_r C$$

$$\omega^2_r = 1/LC$$

$$\omega_r = 1/\sqrt{LC} \text{ radians/sec}$$

$$f_r = 1/2\pi\sqrt{LC} \text{ Hz}$$

The plot of the frequency response of series circuit is as shown in Figure 1.2 . At resonant frequency ω_r the current is maximum.

At resonance frequency f_r

$$Z = R \text{ and current is } I_m .$$

At half power frequencies f_1 and f_2 the current is $I_m / \surd 2$.

$$Z = \surd 2R$$

$$Z = R + jX_L - JX_C = \surd R^2 + (X_L - X_C)^2$$

$$\surd R^2 + (X_L - X_C)^2 = \surd 2R$$

$$R^2 + (X_L - X_C)^2 = 2R^2$$

$$(X_L - X_C)^2 = R^2$$

$$X_L - X_C = R$$

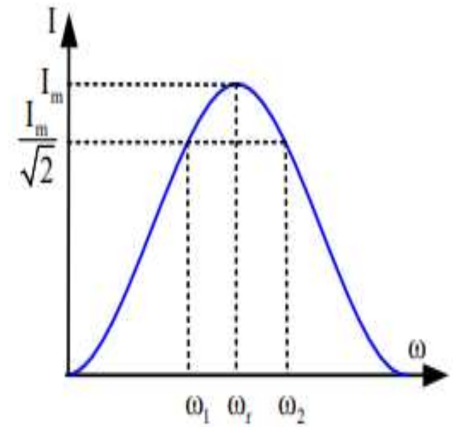


Figure 1.2: Frequency response of series circuit

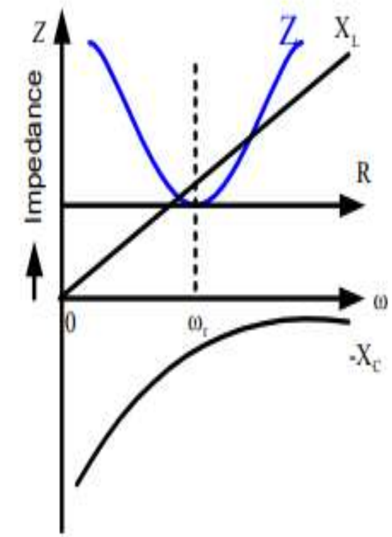


Figure 1.3: Frequency response of impedance of series circuit

At frequency ω_1 the circuit impedance $X_C > X_L$

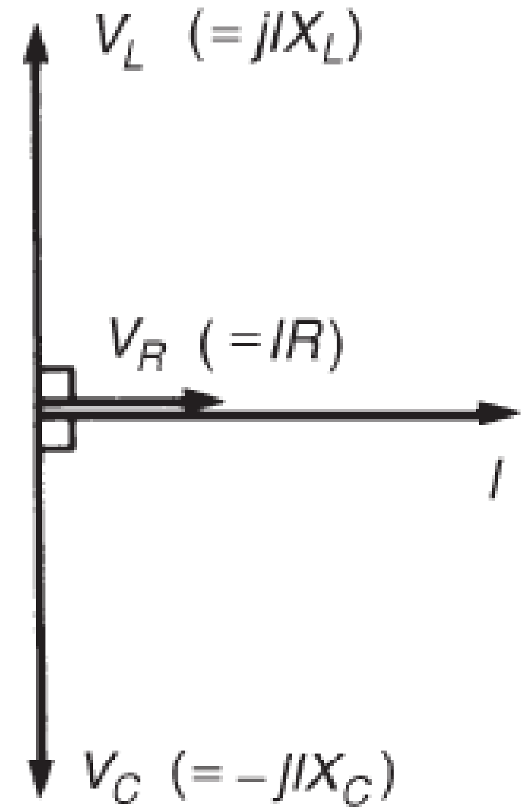
$$\begin{aligned} X_C - X_L &= R \\ \frac{1}{\omega_1 C} - \omega_1 L &= R \\ \frac{1-\omega_1^2 LC}{\omega_1 C} &= R \\ R\omega_1 C - 1 + \omega_1^2 LC &= 0 \\ \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} &= 0 \end{aligned}$$

$$a = 1, \;\; b = \frac{R}{L}, \;\; c = -\frac{1}{LC}$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



Phasor diagram

In terms of frequency f_1

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\,\right]$$

At frequency ω_2 the circuit impedance $X_L > X_C$

$$\begin{aligned}
X_L - X_C &= R \\
\omega_2 L - \frac{1}{\omega_2 C} - &= R \\
\frac{\omega_2^2 LC - 1}{\omega_2 C} &= R \\
\omega_2^2 LC - R\omega_2 C - 1 &= 0 \\
\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} &= 0
\end{aligned}$$

$$a = 1, \quad b = -\frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

Frequency is always positive

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

In terms of frequency f_2

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

Relation between ω_r , ω_1 and ω_2

$\omega_1 \times \omega_2 =$

$$= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \times \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$= \frac{1}{LC}$$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

$$\omega_r^2 = \frac{1}{LC} = \omega_1.\omega_2$$

$$\omega_r = \sqrt{\omega_1.\omega_2}$$

$$f_r = \sqrt{f_1 f_2}$$

Relation between Bandwidth & Quality factor

Bandwidth is $B = \omega_2 - \omega_1$

$$\begin{aligned} B &= \omega_2 - \omega_1 \\ &= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] - \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{R}{L} \ radians \end{aligned}$$

$$B = \frac{1}{2\pi}.\frac{R}{L} = \frac{R}{2\pi L}\ Hz$$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

$$\omega_1 = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}$$

$$\omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}$$

Quality factor Q

The quality factor Q is defined as the ratio of resonant frequency to the bandwidth

$$Q = \frac{\omega_r}{B} = \frac{\sqrt{\frac{1}{LC}}}{\frac{R}{L}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

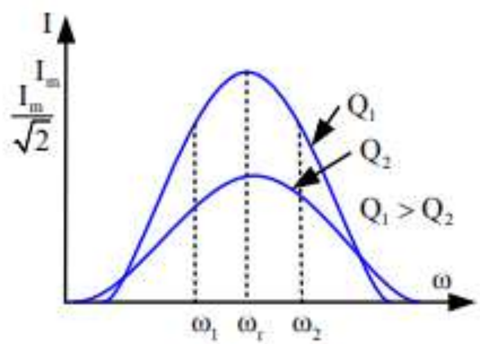
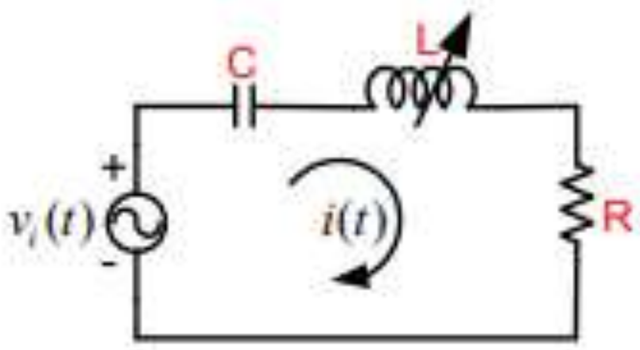


Figure 1.4: Plot of frequency verses Q

Resonance by varying circuit inductance

Consider a series RLC circuit as shown in Figure is become resonant by varying inductance of the circuit.



Let L_1 is the inductance at ω

$$\begin{aligned} X_C - X_L &= R \\ \frac{1}{\omega C} - \omega L_1 &= R \\ L_1 &= \frac{1}{\omega^2 C} - \frac{R}{\omega} \end{aligned}$$

Let L_2 is the inductance at ω

$$\begin{aligned} X_L - X_C &= R \\ \omega L_2 - \frac{1}{\omega C} &= R \\ L_2 &= \frac{1}{\omega^2 C} + \frac{R}{\omega} \end{aligned}$$

Resonance by varying circuit capacitance

Consider a series RLC circuit as shown in Figure. is become resonant by varying capacitance of the circuit.

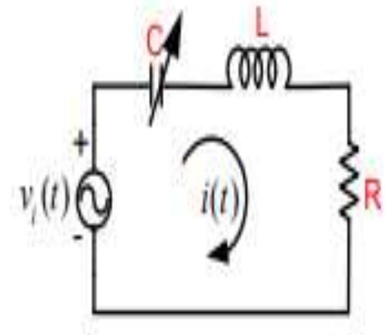


Figure 1.6: Resonance by varying capacitance

$$X_C - X_L = R \Rightarrow \frac{1}{\omega_1 C_1} - \omega_1 L = R$$

$$\frac{1}{\omega_1 C_1} = R + \omega_1 L$$

$$C_1 = \frac{1}{\omega_1^2 L + \omega_1 R}$$

Let C_2 is the capacitance at ω_2

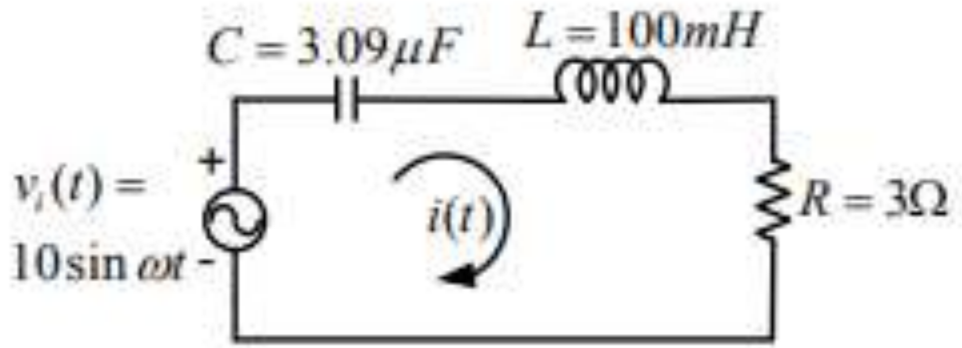
$$X_L - X_C = R \Rightarrow \omega_2 L - \frac{1}{\omega_2 C_2} = R$$

$$\frac{1}{\omega_2 C_2} = \omega_2 L - R$$

$$C_2 = \frac{1}{\omega_2^2 L - \omega_2 R}$$

Selectivity: is property of circuit in which the circuit is allowed to select a band of frequencies between f1 and

1: For the circuit shown in Figure Find (a) The resonant and half power frequencies (b) Calculate the quality factor and bandwidth (c) Determine the amplitude of the current at ω_o, ω_1, ω_2



Solution:

$$LC = 100 \times 10^{-3} \times 3.09 \times 10^{-6} = 3.09 \times 10^{-7}$$

The resonant frequency ω_o is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.09 \times 10^{-7}}} = 1800\ rad/s$$

The half power frequency ω_1, ω_2 is

$$\frac{R}{2L} = \frac{3}{2 \times 100 \times 10^{-3}} = 15$$

$$
\begin{aligned}
\omega_1 &= -\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}} \\
&= -15+\sqrt{225+\frac{1}{3.09\times10^{-7}}} \\
&= -15+\sqrt{225+3.236\times10^{6}} \\
&= -15+1798.96 = 1784\ rad/s \\
\omega_2 &= \frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}} \\
&= 15+1798.96 = 1814\ rad/s
\end{aligned}
$$

Frequency in Hz is

$$
\begin{aligned}
f_1 &= \frac{\omega_1}{2\pi}=\frac{1784}{2\pi}=284\ Hz \\
f_2 &= \frac{\omega_2}{2\pi}=\frac{1814}{2\pi}=289\ Hz
\end{aligned}
$$

Bandwidth B is

$$B=\omega_2-\omega_1=1814-1784=30\ rad/s$$

Also B is

$$B=\frac{R}{L}=\frac{3}{100\times10^{-3}}=30\ rad/s$$

Quality factor Q is

$$Q = \frac{\omega_o}{B} = \frac{1800}{30} = 60$$

The amplitude of the current at ω_o is

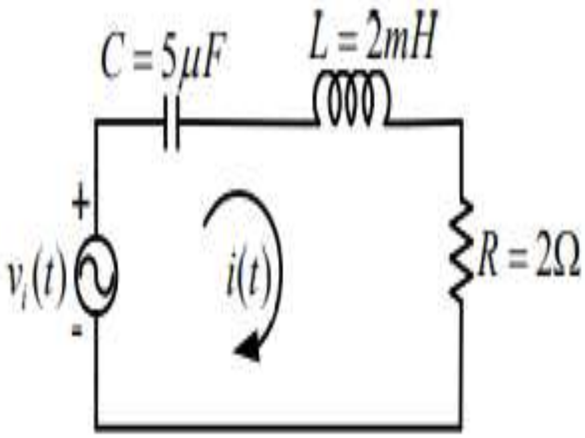
$$I = \frac{V}{R} = \frac{10}{3} = 3.33A$$

The amplitude of the current at ω_1, ω_2 is

$$I = \frac{V}{\sqrt{2}R} = \frac{10}{\sqrt{2} \times 3} = 2.36A$$

2: For the circuit shown in Figure find the resonant frequency, quality factor and bandwidth for the circuit. Determine the change in Q and the bandwidth if R is changed from R = 2 Ω to R = 0.4 Ω

Solution:



$LC = 2 \times 10^{-3} \times 5 \times 10^{-6} = 10 \times 10^{-9}$

The resonant frequency ω_o is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 10^{-9}}} = 10 \times 10^3 \ rad/s$$

B is

$$B = \frac{R}{L} = \frac{2}{2 \times 10^{-3}} = 1000 \ rad/s$$

Quality factor Q is

$$Q = \frac{\omega_o}{B} = \frac{10 \times 10^3}{1000} = 10$$

When $R = 0.4\Omega$ B is

$$B = \frac{R}{L} = \frac{0.4}{2 \times 10^{-3}} = 200 \ rad/s$$

Quality factor Q is

$$Q = \frac{\omega_o}{B} = \frac{10 \times 10^3}{200} = 50$$

✓Parallel Resonance

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure.

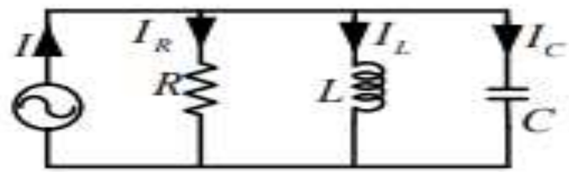


Figure 1.12: General parallel resonant circuit

The total admittance of the circuit is

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

When the circuit is at resonance the imaginary part is zero

$$\omega_r C - \frac{1}{\omega_r L} = 0$$

$$\omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Practical Parallel Resonant circuit

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure 1.13.

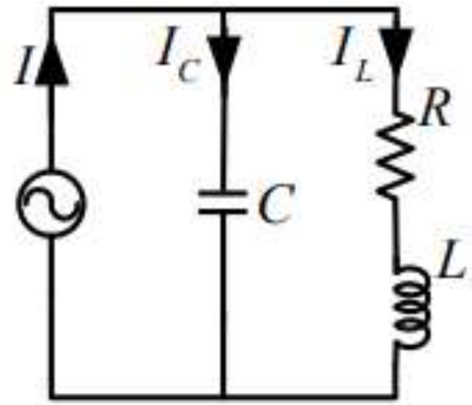


Figure 1.13: General parallel resonant circuit

The impedance of the inductor branch is

$$Z_L = R + j\omega L$$

The admittance of the inductor branch is

$$Y_L = \frac{1}{Z_L} = \frac{1}{R + j\omega L}$$

$$
\begin{aligned}
Y_L &= \frac{1}{Z_L} = \frac{1}{R + j\omega L} \\
&= \frac{1}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}
\end{aligned}
$$

Similarly the impedance of the capacitor branch is

$$Z_C = \frac{1}{j\omega C}$$

The admittance of the capacitor branch is

$$Y_C = \frac{1}{Z_C} = j\omega C$$

Total admittance of the circuit is

$$Y = Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

Separating real and imaginary parts

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\left[\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right]$$

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\left[\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right]$$

$$\omega_r C - \frac{\omega_r L}{R^2 + \omega_r^2 L^2} = 0$$

$$\omega_r C = \frac{\omega_r L}{R^2 + \omega_r^2 L^2}$$

$$R^2 + \omega_r^2 L^2 = \frac{\omega_r L}{\omega_r C} = \frac{L}{C}$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

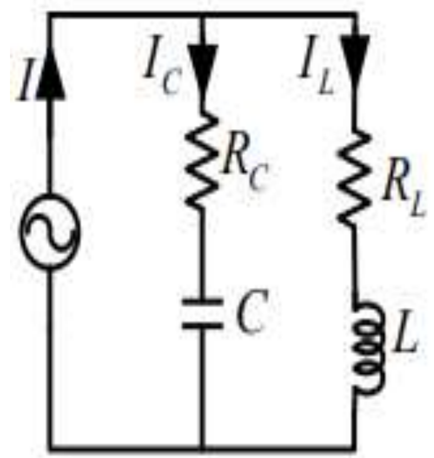
$$\omega_r^2 = \frac{\frac{L}{C} - R^2}{L^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Parallel Resonant circuit by considering capacitance resistance

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure



General parallel resonant circuit

The impedance of the inductor branch is

$$Z_L = R_L + j\omega L$$

The admittance of the inductor branch is

$$Y_L = \frac{1}{Z_L} = \frac{1}{R_L + j\omega L}$$

$$\begin{aligned} Y_L &= \frac{1}{Z_L} = \frac{1}{R_L + j\omega L} \\ &= \frac{1}{R_L + j\omega L} \times \frac{R_L - j\omega L}{R_L - j\omega L} = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} \end{aligned}$$

Similarly the impedance of the capacitor branch is

$$Z_C = R_C - j\frac{1}{\omega C}$$

The admittance of the inductor branch is

$$
\begin{aligned}
Y_C &= \frac{1}{Z_C} = \frac{1}{R_C - j\frac{1}{\omega C}} \\
&= \frac{1}{R_C - j\frac{1}{\omega C}} \times \frac{R_C + j\frac{1}{\omega C}}{R_C + j\frac{1}{\omega C}} = \frac{R_C + j\frac{1}{\omega C}}{R_c^2 + \frac{1}{\omega^2 C^2}}
\end{aligned}
$$

Total admittance of the circuit is

$$
Y = Y_L + Y_C = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j\frac{1}{\omega C}}{R_c^2 + \frac{1}{\omega^2 C^2}}
$$

Separating real and imaginary parts

Real part is

$$
\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_c^2 + \frac{1}{\omega^2 C^2}}
$$

Imaginary part is

$$\frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2}$$

$$\frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2} = 0$$

$$\frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} = \frac{\omega L}{R_L^2 + \omega^2 L^2}$$

$$\frac{1}{\omega_r C}\left(R_L^2 + \omega_r^2 L^2\right) = \omega_r L\left(R_C^2 + \frac{1}{\omega_r^2 C^2}\right)$$

$$\frac{1}{LC}\left(R_L^2 + \omega_r^2 L^2\right) = \omega_r^2\left(R_C^2 + \frac{1}{\omega_r^2 C^2}\right)$$

$$\frac{R_L^2}{LC} + \omega_r^2 \frac{L}{C} = \omega_r^2 R_C^2 + \frac{1}{C^2}$$

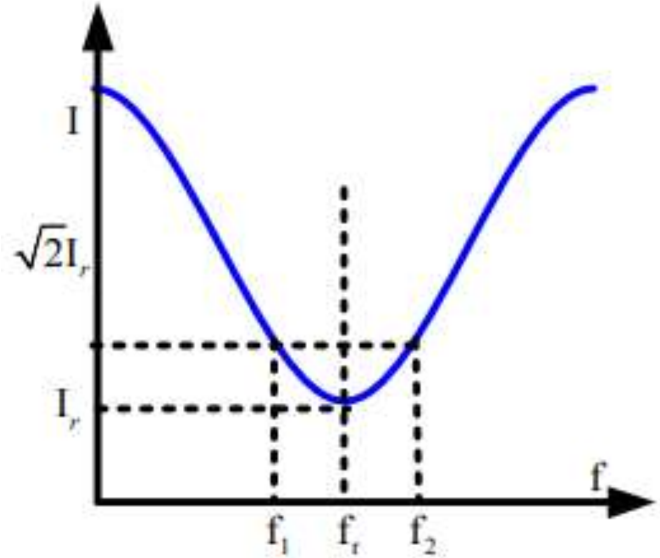
$$\omega_r^2\left(R_C^2 - \frac{L}{C}\right) = \frac{R_L^2}{LC} - \frac{1}{C^2}$$

$$\omega_r^2\left(R_C^2-\frac{L}{C}\right)=\frac{1}{LC}\left(R_L^2-\frac{L}{C}\right)$$

$$\omega_r^2=\frac{1}{LC}\times\frac{(R_L^2-\frac{L}{C})}{(R_C^2-\frac{L}{C})}$$

$$\omega_r=\sqrt{\frac{1}{LC}}\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}}$$

$$f_r=\frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}}$$



.15: plot of parallel resonant circuit

The frequency response of parallel graph is as shown in Figure 1.15. From the figure it is observed that the current is minimum at resonance. The parallel circuit is called as a rejector circuit. The circuit impedance is maximum at the resonance. The half power frequencies are at $\sqrt{2}I_r$

Calculate the resonant frequency of the circuit

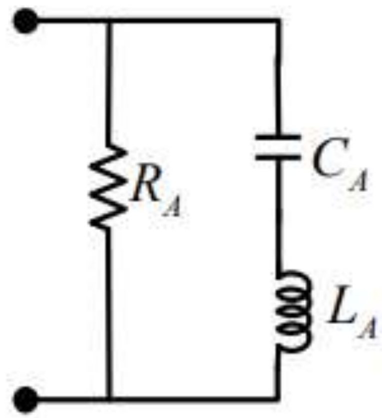


Figure 1.16: General parallel resonant circuit

$$Z_{LC} = jX_L + jX_C = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C}$$

$$Z_{LC} = \frac{j(\omega^2 LC - 1)}{\omega C}$$

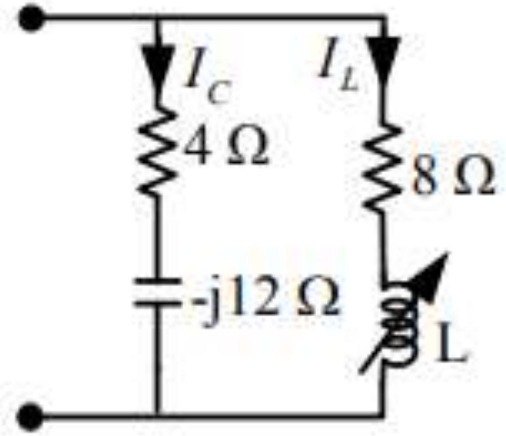
$$\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{\sqrt{LC}}$$

1. Find the value of L for which the circuit as shown in Figure is resonance at $\omega = 5000$ rad/sec.

Solution: The admittance of circuit is given by

$$Y = \frac{1}{4+jX_L} + \frac{1}{8-j12}$$
$$= \frac{4+jX_L}{4^2+X_L^2} + \frac{8-12}{8^2+12^2}$$



At resonance

$$\frac{X_L}{4^2+X_L^2} = \frac{12}{8^2+12^2}$$
$$12(4^2+X_L^2) = X_L(8^2+12^2)$$
$$192+12X_L^2 = 208X_L$$
$$12X_L^2 - 208X_L + 192 = 0$$
$$3X_L^2 - 52X_L + 48 = 0$$

$$X_{L1} = \frac{52 \pm \sqrt{52^2-(4\times3\times38)}}{2\times3}$$

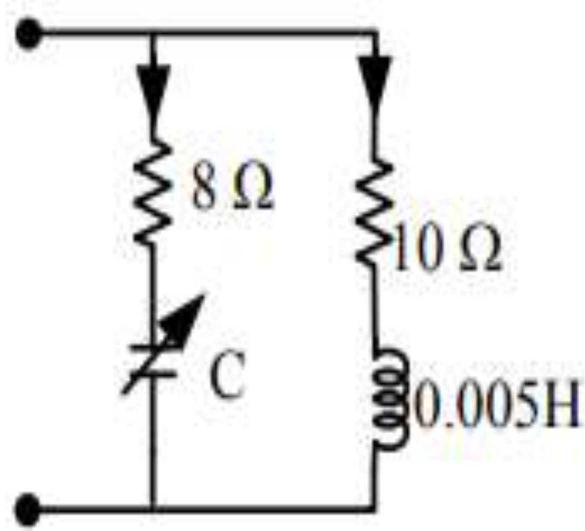
$$X_{L1} = 16.36\ \Omega$$

$$X_{L2} = 0.978\ \Omega$$

$$L1 = \frac{X_{L1}}{\omega} = \frac{16.36}{5000} = 3.27mH$$

$$L2 = \frac{X_{L2}}{\omega} = \frac{0.978}{5000} = 0.196mH$$

2. Find the value of C for which the circuit as shown in Figure is resonance at 3000 rad/sec.



$$X_L = \omega \times L = 3000 \times 0.005 = 15\ \Omega$$

The admittance of circuit is given by

$$\begin{aligned} Y &= \frac{1}{10+j15} + \frac{1}{8-jX_C} \\ &= \frac{10-j8}{10^2+15^2} + \frac{8+jX_C}{8^2+X_C^2} \\ &= \left(\frac{10}{325} + \frac{8}{64+X_C^2}\right) + j\left(\frac{X_C}{64+X_C^2} - \frac{8}{325}\right) \end{aligned}$$

At resonance

$$\begin{aligned} \frac{X_C}{64+X_C^2} - \frac{8}{325} &= 0 \\ 325X_C &= 512 + 8X_C^2 \\ 8X_C^2 - 325X_C + 512 &= 0 \end{aligned}$$

$$
\begin{aligned}
325X_C &= \frac{325 \pm \sqrt{325^2 - (4 \times 8 \times 512)}}{2 \times 8} \\
&= \frac{325 \pm \sqrt{105625 - 16384}}{16} \\
&= \frac{325 \pm 298.7}{16}
\end{aligned}
$$

$$X_{C1} = 38.98\ \Omega$$

$$X_{C2} = 1.64\ \Omega$$

$$C_1 = \frac{1}{\omega C} = \frac{1}{3000 \times 38.98} = 8.55\ \mu F$$

$$C_2 = \frac{1}{\omega C} = \frac{1}{3000 \times 1.64} = 203\ \mu F$$

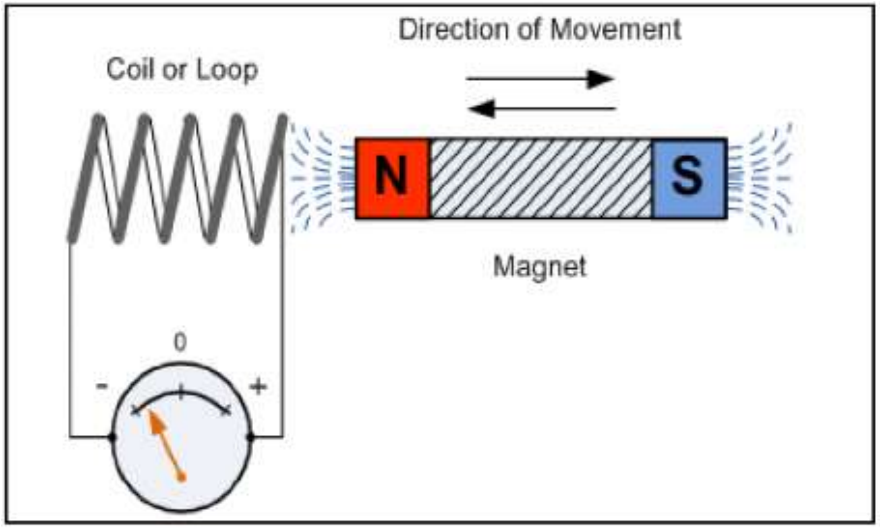
Part-B: Magnetic circuits

✓ **Magnetic circuits-Faraday's laws of electromagnetic induction**

What is Faraday's Law ?

Faraday's law of electromagnetic induction (referred to as **Faraday's law**) is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF). This phenomenon is known as **electromagnetic induction.**

Faraday's law states that a current will be induced in a conductor which is exposed to a changing magnetic field.



Laws of electromagnetic induction

Faraday's laws of electromagnetic induction state:

Faraday's First Law

'An induced e.m.f. is set up whenever the magnetic field linking that circuit changes.'

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.

Method to change the magnetic field:

- By moving a magnet towards or away from the coil
- By moving the coil into or out of the magnetic field
- By changing the area of a coil placed in the magnetic field
- By rotating the coil relative to the magnet

Faraday's Second Law

(ii) 'The magnitude of the induced e.m.f. in any circuit is proportional to the rate of change of the magnetic flux linking the circuit''

It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of the number of turns in the coil and flux associated with the coil.

Problems

1. A coil of 500 turns is linked by a flux of 0.4mwb. If the flux is reversed in 0.01 second, find the e.m.f induced in the coil.

Sol Induced e.m.f is

$$e = N\frac{d\phi}{dt} \qquad N = 500 ; \quad \text{flux} = 0.4\,\text{mwb}.$$

$dt = 0.01\,\text{second};$

$d\phi = 0.4 - (-0.4) = 0.8\,\text{mwb} = 8 \times 10^{-4}\,\text{wb}$

$$e = 500 \times \frac{8 \times 10^{-4}}{0.01} = 40V$$

2. The field of a 6 pole d-c generator each having 500 turns, are connected in series. when the field is excited, there is a magnetic flux of 0.02 wb/pole. If the field circuit is opened in 0.02 seconds and the residual magnetism is 0.002 wb/pole, calculate the average voltage which is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current?

Sol Total number of turns

$$N = 6 \times 500 = 3000$$

Total initial flux $= 6 \times 0.02 = 0.12$ wb

Total residual flux $= 6 \times 0.002 = 0.012$ wb.

Change in flux, $d\phi = 0.12 - 0.012$

$= 0.108$ wb

Time of opening the circuit

$dt = 0.02$ seconds

induced e.m.f $= N \dfrac{d\phi}{dt} = 3000 \times \dfrac{0.108}{0.02}$

$= 16{,}200$ V

The direction of this induced e.m.f is the same as that of the original direction of the exciting current.

Applications of Faraday's Law

✓Faraday law is one of the most basic and important laws of electromagnetism. This law finds its application in most of the electrical machines, industries, and the medical field, etc.

✓ Power transformers function based on Faraday's law

✓The basic working principle of the electrical generator is Faraday's law of mutual induction. The Induction cooker is the fastest way of cooking. It also works on the principle of mutual induction. When current flows through the coil of copper wire placed below a cooking container, it produces a changing magnetic field. This alternating or changing magnetic field induces an emf and hence the current in the conductive container, and we know that the flow of current always produces heat in it.

Self-inductance basics

When current passes along a wire, and especially when it passes through a coil or inductor, a magnetic field is induced. This extends outwards from the wire or inductor and could couple with other circuits. However it also couples with the circuit from which it is set up.

The magnetic field can be envisaged as concentric loops of magnetic flux that surround the wire, and larger ones that join up with others from other loops of the coil enabling self-coupling within the coil.

When the current in the coil changes, this causes a voltage to be induced the different loops of the coil - the result of self-inductance.

In terms of quantifying the effect of the inductance, the basic formula below quantifies the effect.

$$V_L = -N\frac{d\phi}{dt}$$

Where:

V_L = induced voltage in volts

N = number of turns in the coil

$d\varphi/dt$ = rate of change of magnetic flux in webers / second

The induced voltage in an inductor may also be expressed in terms of the inductance (in henries) and the rate of change of current.

$$V_L = -L\frac{di}{dt}$$

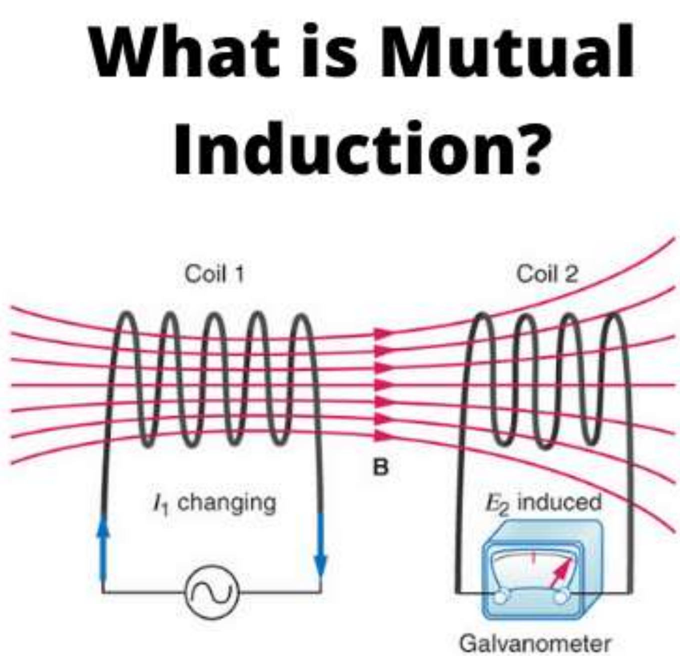
Self induction is the way in which single coils and chokes operate. A choke is used in radio frequency circuits because it opposes any change, i.e. the radio frequency signal, but allows any steady, i.e. DC current to flow.

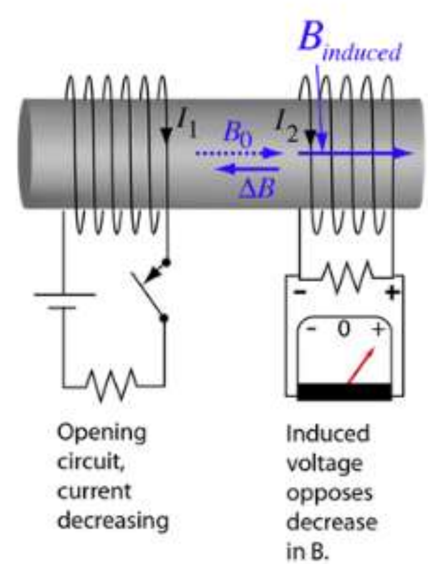
Mutual Induction

Mutual induction is a phenomenon when a coil gets induced in EMF across it due to rate of change current in adjacent coil in such a way that the flux of one coil current gets linkage of another coil.

Definition of Mutual Inductance

Mutual Inductance is the ratio between induced emf across a coil to the rate of change of current of another adjacent coil in such a way that two coils are in possibility of flux linkage.





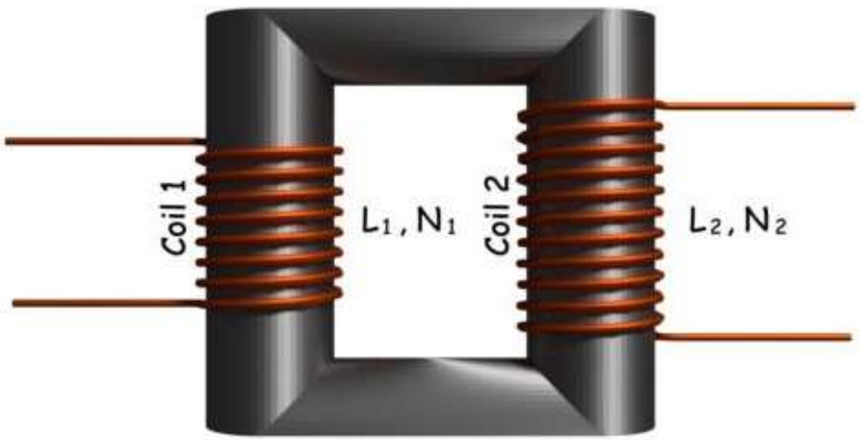
$$M(of\ a\ coil) = \frac{(emf)_{induced\ in\ that\ coil}}{\left(\frac{di(t)}{dt}\right)_{of\ other\ coil}}$$

Whenever there is a time varying current in a coil, the time varying flux will link with the coil itself and will cause self induced emf across the coil. This emf is viewed as a voltage drop across the coil or inductor.

But it is not practical that a coil gets linked only with its own changing flux. When a time varying current flows in another coil placed nearby the first one then the flux produced by the second coil may also link the first one. This varying flux linkage from the second coil will also induce emf across the first coil. This phenomenon is called **mutual induction** and the emf induced in one coil due to time varying current flowing in any other coil is called **mutually induced emf**. If the first coil is also connected to the time varying source, the net emf of the first coil is the resultant of self induced and mutually induced emf.

Coefficient of Mutual Induction or Mutual Inductance

Let us consider one coil of self inductance L_1 and another coil of self inductance L_2. Now we will also consider that there is a low reluctance magnetic core which couples these both coils in such a way that entire flux created by one coil will link the other coil. That means there will be no leakage of flux in the system.

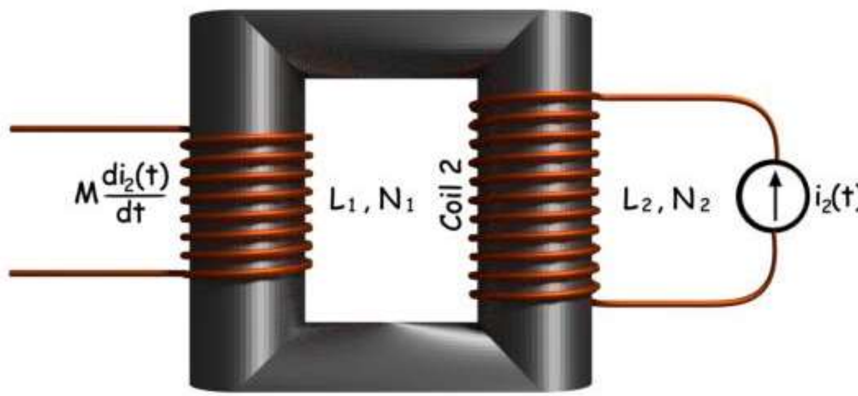


Now we will apply a time varying current at coil 1 keeping the coil 2 open circuited. The voltage induced across the coil 1 will be

$$L_1 \frac{di_1(t)}{dt}$$

Now we will keep the first coil open and apply time varying current in coil 2. Now the flux produced by coil 2 will link coil 1 through the magnetic core and as a result, the emf induced in the coil 1 will be

$$M\frac{di_2(t)}{dt}$$



Here, M is the coefficient of mutual induction or in short mutual inductance. Now without disturbing the source at coil 2, we connect a time varying current source across the coil 1. In that situation, there will be a self induced emf across the coil 1 due to its own current and also mutually induced emf across the coil 1 for the current in coil 2. So the resultant emf induced in the coil 1 is

$$L_1 \frac{di_1(t)}{dt} \pm M \frac{di_2(t)}{dt}$$

Mutually induced emf may be either additive or subtractive depending on the polarity of the coil. The expression of M is

$$M = \sqrt{L_1 L_2}$$

This expression is justified only when entire flux created by one coil will link another coil but practically it not always possible to link entire flux of one coil to another. The value of actual mutual inductance depends on the actual amount of flux of one coil links another. Here k is a coefficient which must be multiplied with M to derive actual value of mutual inductance.

$$M = k\sqrt{L_1 L_2}$$