

Unit - I

Linear Wave Shaping

→ The shape of the non-sinusoidal signal is altered by transmitting it through a linear N/W.

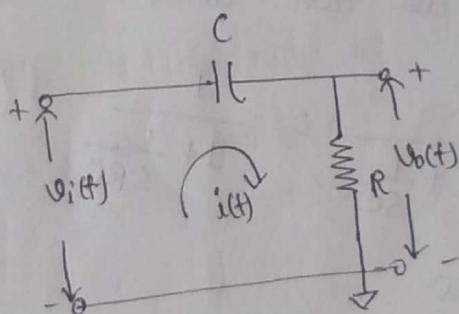
→ The linear N/W consist of R, L and C.

* Reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$
, when $f \rightarrow 0, X_C \rightarrow \infty$ (open circ)
 $f \rightarrow \infty, X_C \rightarrow 0$ (close circ)

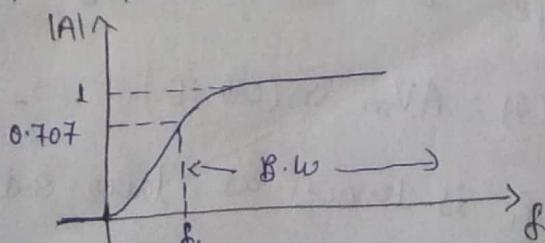
→ $X_L = \omega L = 2\pi f L$, when $f \rightarrow 0, X_L \rightarrow \infty$ (close circ)
 $f \rightarrow \infty, X_L \rightarrow 0$ (open circ)

* High pass RC circuit:



At high frequency capacitor act as short circuit and at low frequency it act as open circuit. At zero frequency, the capacitor has infinite reactance and behave as open circ, so, it is known as blocking capacitor.

This circuit attenuates low-frequency signals and allows transmission of high frequency signal with less or no attenuation. So, it is called as high pass circuit.



(3) Sinusoidal OIP:

The OIP signal is considered as,

$$v_i(t) = V_m \sin(\omega t) \quad \text{--- (1)}$$

we consider the initial voltage of the capacitor as zero.
By using KVL in terms of s domain i.e. we apply
Laplace transform, we can write,

$$V_i(s) = I(s) \left(\frac{1}{sC} + R \right) \quad \text{--- (2)}$$

The OIP voltage is given as,

$$V_o(s) = I(s) \cdot R \quad \text{--- (3)}$$

from eqn (2) & (3), we can write,

$$\frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{1}{1 + \frac{1}{sRC}} \quad \text{--- (4)}$$

we have $s = j\omega$

$$\text{so, } \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + \frac{1}{j\omega RC}} = \frac{1}{1 - \frac{1}{2} \cdot \frac{1}{\omega f RC}} \Rightarrow \boxed{A = \frac{1}{1 - \frac{j}{2} \cdot \frac{f_1}{f}}} \quad \text{--- (5)}$$

where, $f_1 = \frac{1}{2\pi RC}$

magnitude,

$$|A| = \sqrt{1 + (\frac{f_1}{f})^2} \quad \& \quad \theta = \tan^{-1} \frac{f_1}{f} \quad \text{--- (6)}$$

The OIP voltage is multiplied by 'A', so, OIP signal
is given as, $v_o(t) = AV_m \sin(\omega t + \theta) \quad \text{--- (7)}$

The frequency f_1 is known as lower 3-dB frequency.

→ Relation b/w f_1 and tilt (or sag):

The laser cut-off frequency produces a tilt.

For 10% change in capacitor voltage, pulse width is

$$\text{given as, } t = 0.1 \text{RC} = PW$$

$$\Rightarrow \frac{PW}{RC} = 0.1 = \text{fractional tilt} = \frac{PW}{RC} = 2\pi f \cdot PW.$$

When tilt is more than 10%, the voltage should be treated as exponential not linear,

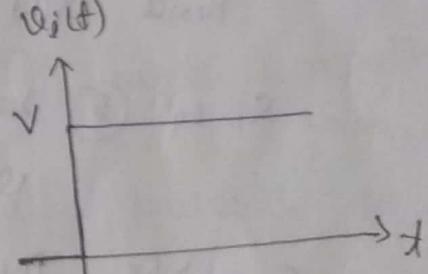
$$\text{so, } V_o = V_f - (V_f - V_i) e^{-t/RC}$$

(ii) Step input:

Step voltage is defined as,

$$v(t) = \begin{cases} 0 & \text{for } t < 0 \\ V & \text{for } t \geq 0 \end{cases}$$

-①



The instant immediately before $t=0$, is referred as $t=0^-$ and immediate after $t=0$ is referred as $t=0^+$

$$\text{i.e. } v_i(t) = 0 \text{ at } t=0^-$$

-②

$$\text{& } v_i(t) = V \text{ at } t=0^+$$

At $t=0$ capacitor will not be charged. The response at the

now is exponential with time constant $RC \approx \tau$.

$$\text{so, o/p voltage is given as, } v_o(t) = B_1 + B_2 e^{-t/RC} \quad -③$$

$B_1 \rightarrow$ steady state value of $v_o(t)$ i.e. $t \rightarrow \infty$, $B_1 = v_o(\infty)$

$B_2 \rightarrow$ depends on initial o/p voltage, $v_o(t) = v_o(0) = B_1 + B_2$

$$\text{i.e. } B_2 = v_o(0) - B_1 \approx v_o(0) - v_o(\infty)$$

So, eqⁿ ⑤ can be written as,

$$V_o(t) = V_o(0) + \{V_o(0) - V_o(\infty)\} e^{-\frac{t}{RC}} \quad - ④$$

The capacitor 'C' has d.c. component, so, $t \rightarrow 0$, $V(\infty) = 0$

i.e. we get, $V_o(t) = V_o(0) e^{-\frac{t}{RC}}$ - ⑤.

The change in voltage across the capacitor is given as,

$$V_c(t) = \frac{1}{C} \int i(t) dt \quad - ⑥$$

At $t=0$, the o/p voltage changes by an amount 'V',

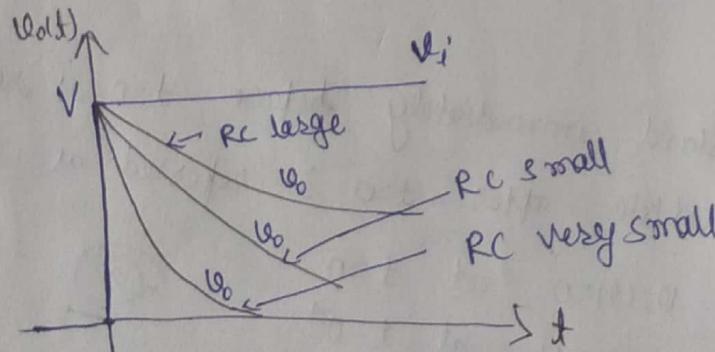
so, o/p also change abruptly by V at $t=0^+$.

Hence $V_o(0) = V$.

So, eqⁿ ⑤ becomes,

$$V_o(t) = V e^{-\frac{t}{RC}} \quad - ⑦$$

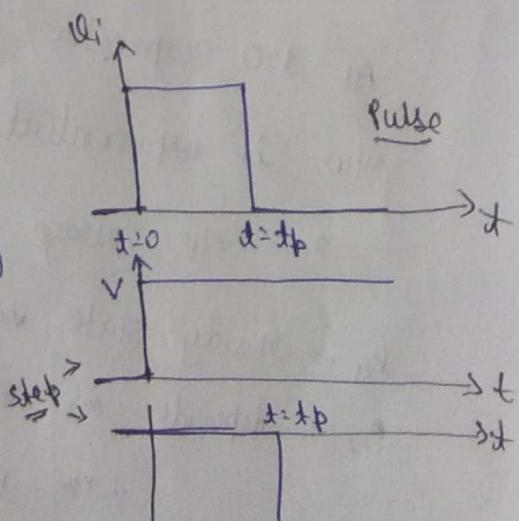
for $t > 5\tau$, the o/p will reach more than 99% of final value



(iii): Pulse input:

It is defined as,

$$V_i(t) = \begin{cases} V & ; \text{ for } 0 \leq t \leq t_p \\ 0 & ; \text{ o.w.} \end{cases} \quad - ①$$



At $t=0^+$ increased by V and at $t=t_p$ decreased by V .

The response up to $t=t_p$ is same as step voltage as O/P.

which is given as,

$$V_o(t) = V e^{-t_p/RC} \approx V_p \quad (2)$$

At $t=t_p^+$, it is given as,

$$V_o(t) = V e^{-t_p/RC} = V_p \quad (3)$$

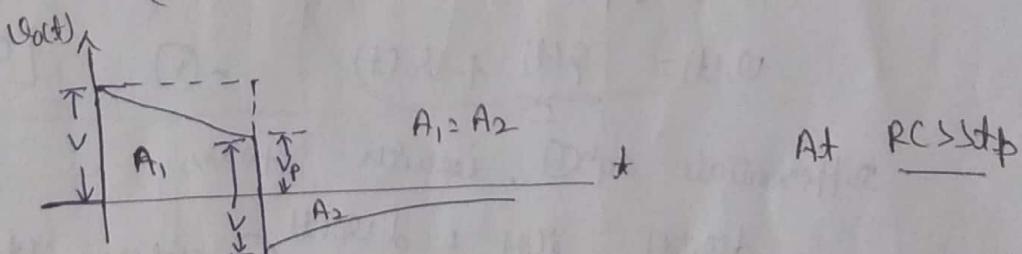
At $t=t_p$ pulse O/P falls abruptly by V . But capacitor voltage will not change instantaneously. So, O/P also drop by V .

So, $V_o(t) = V_p - V$ at $t=t_p^+$

i.e. $V_o(t) = V e^{-t_p/RC} - V \quad (4)$, which is negative value.

For $t > t_p$ the O/P rises exponentially towards zero.

i.e. $V_o(t) = (V e^{-t_p/RC} - V) e^{-(t-t_p)/RC} \quad (5)$



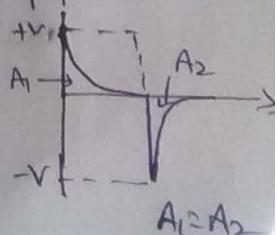
There is tilt at the top of pulse and undershoot at the end of pulse. The area below & above zero line is equal.

i.e. $A_1 = A_2$.

For $RC/t_p \gg 1$, there is slight tilt at the top and undershoot is small.

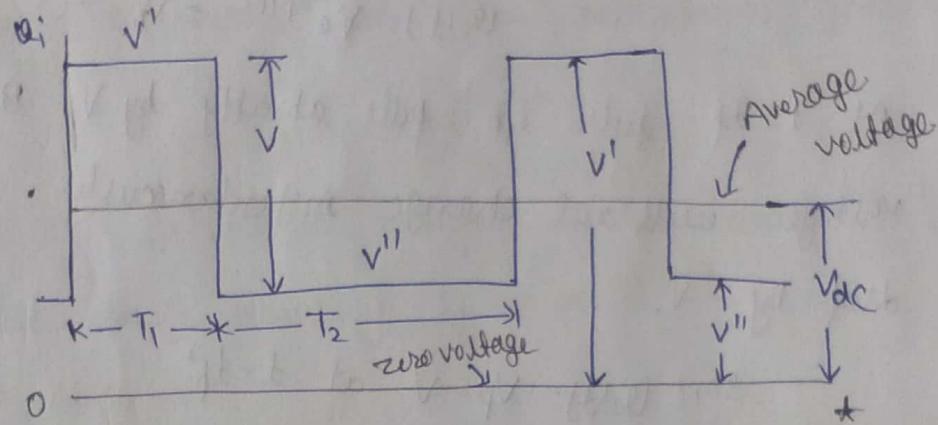
But for $RC/t_p \ll 1$, the O/P consist of spike or pip at the beginning and ending of pulse.

The process of converting pulse into spike due to less time constant is known as peaking.



(iv) Square wave input:

It is periodic waveform, which maintain itself at constant level v' for time duration T_1 and another constant level v'' for time duration T_2 . The time period is equal to $T = T_1 + T_2$.



For any periodic waveform as input, the average level of the steady state o/p signal from the circuit is always zero.

The highpass circuit now, can be written as,

$$V_o(t) = \frac{q(t)}{C} + V_{o(t)} \quad \text{--- (1)} \quad [C = \partial/q]$$

Differentiate eqn (1), which gives,

$$\frac{dV_o(t)}{dt} = \frac{i(t)}{C} + \frac{dV_{o(t)}}{dt} \quad \text{--- (2)}, \text{ where, } i(t) = \frac{dq(t)}{dt}$$

and $V_{o(t)} = i(t) \cdot R$.

So, we can write eqn (2), as,

$$\frac{dV_o(t)}{dt} = \frac{V_{o(t)}}{RC} + \frac{dV_{o(t)}}{dt}$$

Multiply by dt and integrate over time period T

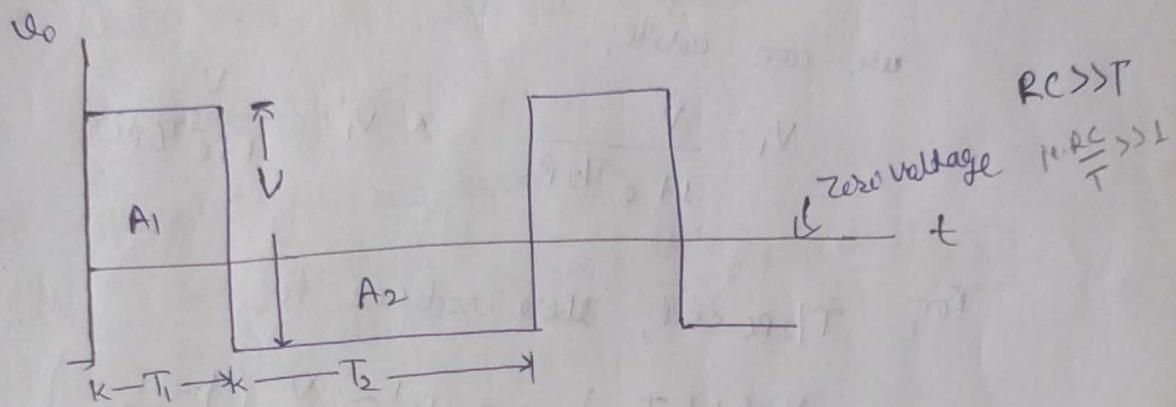
$$\int_0^T dV_o(t) = \frac{1}{RC} \int_0^T V_{o(t)} dt + \int_0^T V_{o(t)}$$

$$\Rightarrow V_o(T) - V_o(0) = \frac{1}{RC} \int_0^T V_o(t) dt + V_o(T) \quad (3)$$

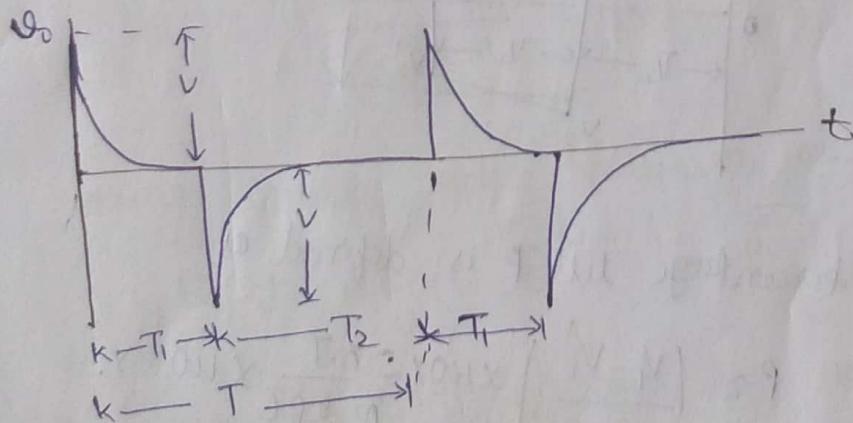
under steady state condition, $V_o(T) = V_o(0) \Rightarrow V_o(0) = V_o(T)$

Hence $\int_0^T V_o(t) dt = 0$, since it represent area under wave.

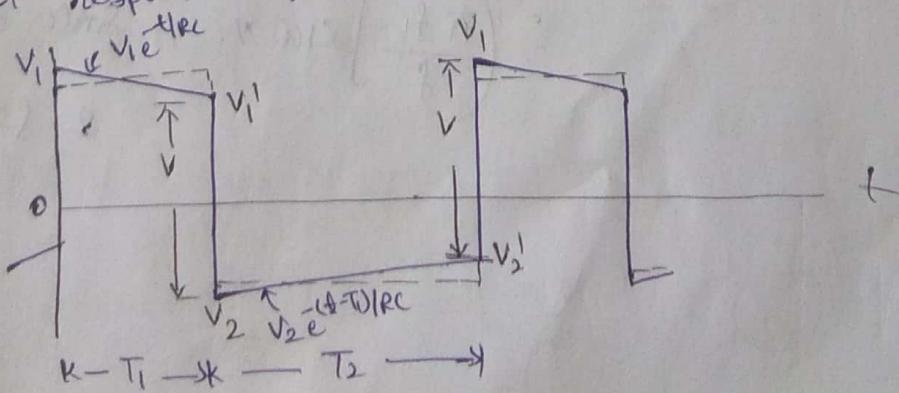
so, average level of steady state signal is always zero.



If RC/T_1 and RC/T_2 both are very small in comparison with unity, then o/p consist of alternate positive and negative peaks.



But in general response of square wave appears as shown below.



$$V_1' = V_1 e^{-T_1 RC} \quad V_1' - V_2 = V$$

$$V_2' = V_2 e^{-T_2 RC}, \quad V_1 - V_2' = V$$

In case of symmetrical square wave, $T_1 = T_2 = T/2$, ~~and~~ second

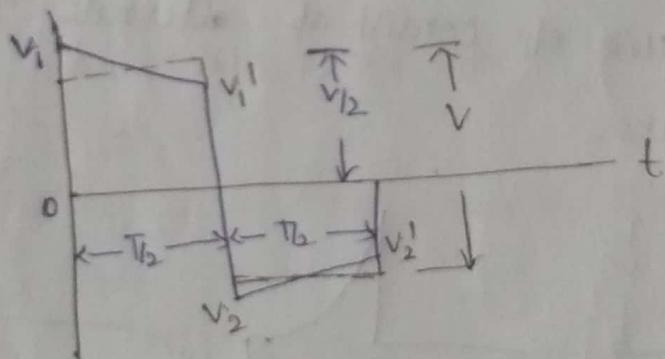
$$V_1 = -V_2 \quad \& \quad V_1' = V_2'$$

we can write,

$$V_1 = \frac{V}{1 + e^{-T_2 RC}} \quad \& \quad V_1' = \frac{V}{1 + e^{+T_2 RC}}$$

For, $T/2RC \ll 1$, these reduce to,

$$V_1 \approx \frac{V}{2} \left(1 + \frac{T}{4RC} \right) \quad \& \quad V_1' \approx \frac{V}{2} \left(1 - \frac{T}{4RC} \right)$$



The percentage tilt, P is defined as,

$$P \approx \left(\frac{V_1 - V_1'}{V_{1/2}} \right) \times 100\% = \frac{\pi \times T}{\pi \times 2RC} \times 100\%.$$

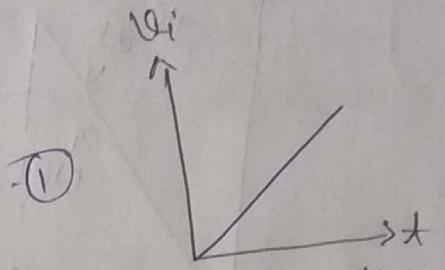
$$= \left(\frac{\pi f_1}{f} \right) \times 100\%, \quad \text{since } f_1 = \frac{1}{2\pi RC}.$$

$$\& f = \frac{1}{T}$$

(V) Ramp input:

It is defined as,

$$v_i(t) = \begin{cases} 0 & \text{for } t < 0 \\ at & \text{for } t > 0 \end{cases}$$



i.e. it is zero for $t < 0$ and increases linearly with time

for $t > 0$.

By using KVL at high pass circuit, we have,

$$v_o(t) = \frac{q(t)}{C} + v_o(t) \quad \text{--- (2)}$$

After differentiating we have,

$$\frac{d(v_i(t))}{dt} = \frac{v_o(t)}{RC} + \frac{d(v_o(t))}{dt} \Rightarrow \alpha = \frac{v_o(t)}{RC} + \frac{d(v_o(t))}{dt} \quad \text{--- (3)}$$

By taking Laplace on both sides, we have,

$$\frac{\alpha}{s} = \frac{V_o(s)}{RC} + sV_o(s) = V_o(s) \left[s + \frac{1}{RC} \right]$$

$$\Rightarrow V_o(s) = \frac{\alpha}{s(s + \frac{1}{RC})} \quad \text{--- (4)}$$

By taking inverse Laplace transform, we have

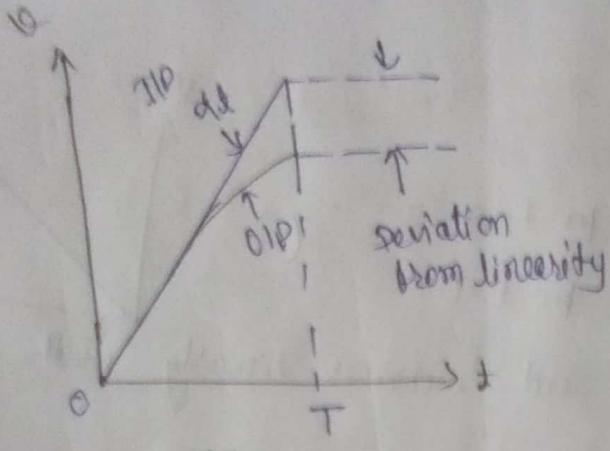
$$v_o(t) = \alpha RC \left(1 - e^{-t/RC} \right) \quad \text{--- (5)}$$

For $t \ll RC$, the eqn (5) can be written as,

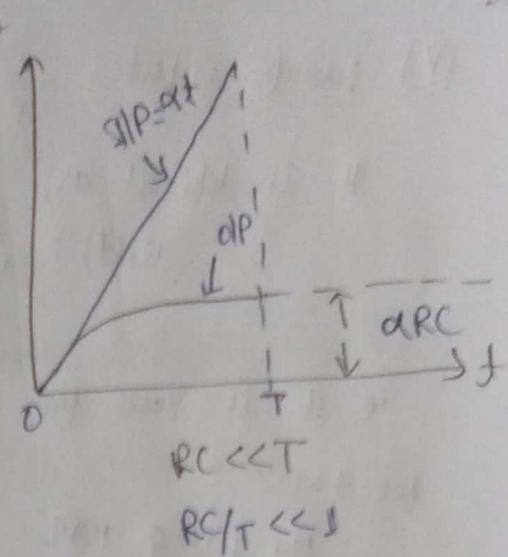
$$v_o(t) = \alpha RC \left[1 - \left\{ 1 + \left(\frac{t}{RC} \right) + \left(\frac{-t}{RC} \right)^2 \frac{1}{2!} + \dots \right\} \right]$$

$$= \alpha RC \left[\frac{t}{RC} - \frac{t^2}{2(RC)^2} + \dots \right]$$

$$= \alpha t - \frac{\alpha t^2}{2RC} = \alpha t \left(1 - \frac{t}{2RC} \right) \quad \text{--- (6)}$$



$$RC \gg T \quad \text{or} \quad \frac{RC}{T} \gg 1$$



$$RC \ll T$$

$$\frac{RC}{T} \ll 1$$

The transmission error ' e_f ' is defined as,

$$e_f = \frac{V_o(T) - V_{oL}(T)}{V_{iL}(T)} \approx \frac{T}{2RC} = \pi f_c T.$$

* High pass RC N/w as differentiator:

If time constant of high pass RC circuit is very small then capacitor charge very quickly. So, almost all i/p $V_{iL}(t)$ appears across capacitor and voltage across resistance will be negligible.

The current is determined by capacitance. The current is given as,

$$i(t) = C \frac{dV_{iL}(t)}{dt} \quad \text{--- (1)}$$

∴ o/p signal is $V_{oL}(t) = RC \frac{dV_{iL}(t)}{dt}$

∴ o/p is proportional to derivative of input.

→ The derivative of step signal is impulse of infinite amplitude at discontinuity of step. → The derivative of a rectangular pulse is a positive impulse followed by delayed negative impulse.

- The derivative of a square wave is uniformly zero except the points of discontinuity. At pt of discontinuity, it will be pulse of finite amplitude v.
- A RC differentiator converts triangular wave into square wave.
- For ramp, $v_i = at$, the value of $RC(dv/dt) = \alpha RC$, it is true except near origin. Error occurs at $t=0$, due to voltage across R , which is not negligible.
- In good differentiator if a sine wave is applied then o/p will be sine wave shifted by an angle θ .

$$\tan \theta = \frac{X_C}{R} = \frac{1}{\omega RC}$$

o/p is $\sin(\omega t + \theta)$. But in free differentiator, it must contain caswt. i.e. θ should be equal to 90° . It is possible if $R=0$ & $C=0$. However if $\omega RC = 0.01$, then $\theta = 89.4^\circ$.

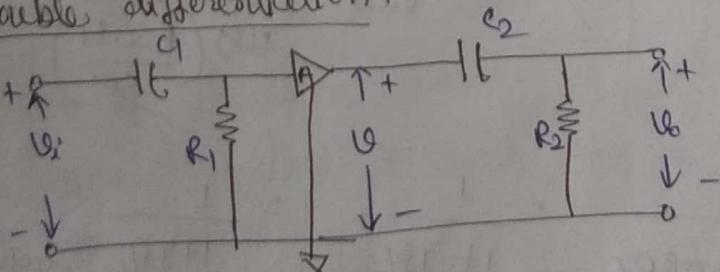
If $\omega RC = 0.1$ then $\theta = 84.3^\circ$, it is also close to 90° .

If the peak value of o/p is V_m then o/p is given as,

$$v_o = \frac{V_m R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin(\omega t + \theta)$$

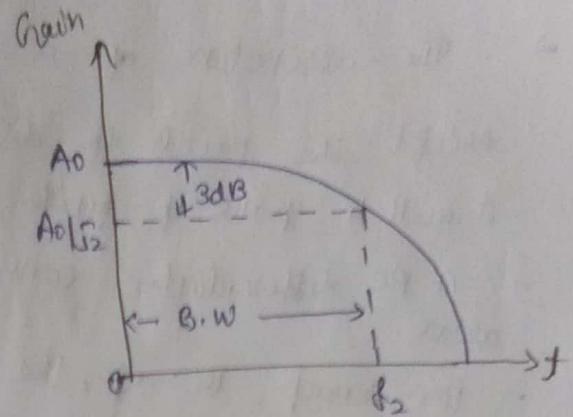
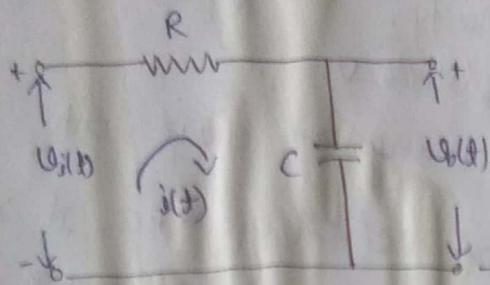
and if $\omega RC \leq 1$, then o/p is $V_m \omega RC \cos \omega t$.

- Double differentiation:



If $R_1 C_1 \times R_2 C_2$ are small relative to period of I/P waveform, this circuit can convert ramp into pulse. The initial slope of o/p waveform is multiple of gain & initial slope of I/P waveform. So, it is called as rate of rise differentiator.

* Low Pass RC circuit:



For the low frequency, the reactance of the capacitor is high. So it will act as open circuit and all S/P will appear as O/P approximately.

But for high frequency, the reactance of the capacitor is low, so, it will act as short pass and O/P is equal to zero.

(i) Sinusoidal Input:

The S/P voltage is considered as,

$$V_{in}(t) = V_m \sin \omega t \quad \text{--- (1)}$$

By using KVL & Laplace transformation, the eqn for low pass circuit is given as,

$$V_{in}(s) = I(s) \left(R + \frac{1}{sC} \right) \quad \text{--- (2)}$$

$$\text{and } V_{out}(s) = \frac{I(s)}{sC}$$

$$\text{so, gain is given as, } A = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{I(s)}{sC}}{\frac{I(s)}{sC} \left(R + \frac{1}{sC} \right)} = \frac{1}{1 + RSC}$$

$$\text{ie, } A = \frac{1}{1 + j2\pi fRC} = \frac{1}{1 + j\frac{f}{f_2}} = \frac{1}{1 + j\frac{f}{f_2}}, \text{ where } f_2 = \frac{1}{2\pi RC}$$

--- (3)

Upper 3dB freq.

$$\text{The magnitude, } |A| = \frac{1}{\sqrt{1 + (\frac{\delta}{f_2})^2}}$$

$$\text{and } \theta = -\tan^{-1}\left(\frac{\delta}{f_2}\right).$$

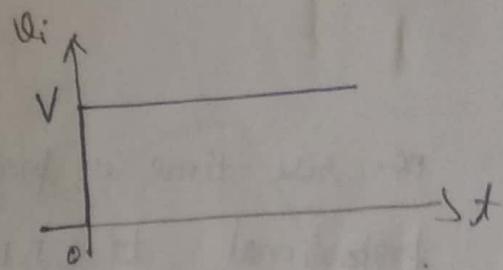
So O/P signal is given as, $V_o(t) = A V_m \sin(\omega t + \theta)$.

Here θ is negative, so, $V_o(t)$ lags behind the $V_i(t)$.

(iii) Step input:

It is defined as,

$$V_i(t) = \begin{cases} 0 & \text{for } t < 0 \\ V & \text{for } t \geq 0 \end{cases}$$



The capacitor can't charge instantaneously, so, O/P starts from zero and rise towards steady state value V .

So, O/P will be zero for $t=0$. i.e. $V_o(0)=0$.

The O/P is given as,

$$V_o(t) = B_1 + B_2 e^{-t/RC}$$

$$\Rightarrow V_o(t) = V_o(\infty) + \{V_o(0) - V_o(\infty)\} e^{-t/RC}$$

$$= V - V e^{-t/RC} \quad [\because V_o(0)=0 \text{ & } V_o(\infty)=V]$$

$$\Rightarrow \boxed{V_o(t) = V(1 - e^{-t/RC})}.$$

Rise time 'tr': It is time taken to rise the voltage from 10% to 90% of its final value.

The time required to reach $1/10$ of final value is equal to $0.1 RC$.

and time required to reach $9/10$ of final value is equal to $2.3 RC$.

$$[0.1V = V(1 - e^{-t_1/RC}) \Rightarrow e^{t_1/RC} = \frac{1}{0.9} = 1.11 \\ \Rightarrow t_1 = RC \ln(1.11) = 0.1RC]$$

$$\& 0.9V = V(1 - e^{-t_2/RC}) \Rightarrow e^{t_2/RC} = \frac{1}{0.1} = 10 \\ \Rightarrow t_2 = RC \ln 10 = 2.3RC \quad]$$

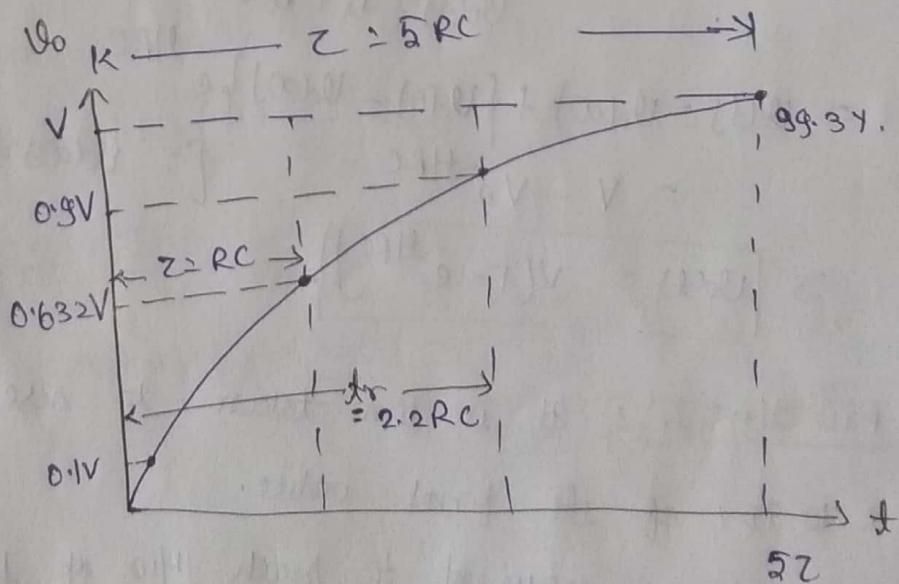
So, rise time, $t_r = t_2 - t_1 = 2.3RC - 0.1RC = 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} = \frac{0.35}{B.W.}$

$$[f_2 = \frac{1}{2\pi RC}]$$

i.e. rise time is proportional to time constant and inversely proportional to B.W.

→ Time constant ($\tau = RC$):

It is time taken by O/P to rise 63.2% of the amplitude of the input signal.



(iii) Pulse input:

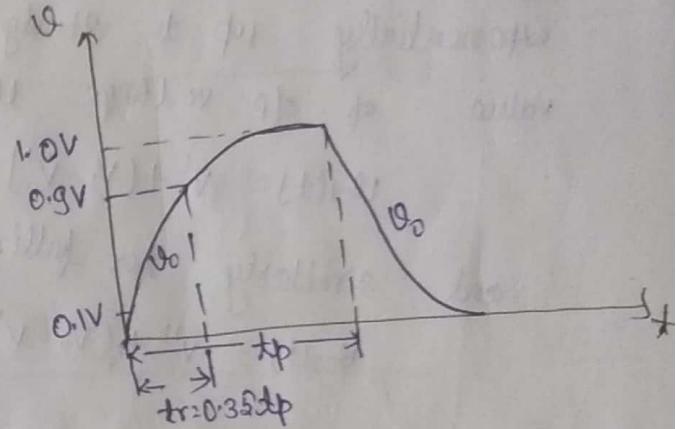
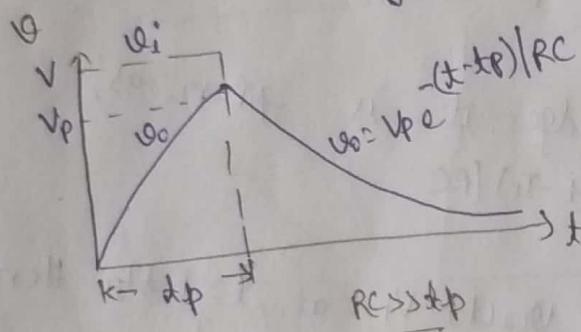
1.15.

The response to a pulse, for $t < t_p$, is same as step input, which is given as,

$$V_o(t) = V(1 - e^{-t/RC})$$

At $t = t_p$, $V_o = V_p$ and t_p must decrease to zero from V_p with time constant RC .

For $t > t_p$, the O/P approaches zero, since capacitor will not discharge suddenly.



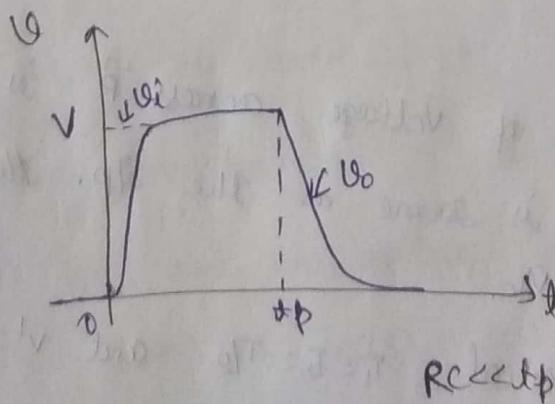
$$\text{At } f_2 = 1/t_p \quad \underline{RC < t_p}$$

To minimise distortion, the rise time must be compared with pulse width.

$$\text{If } f_2 = 1/t_p \text{ then } t_r = 0.35 t_p.$$

The pulse shape will be preserved if 3dB frequency is approximately equal to reciprocal of the pulse width.

$$\text{For } t_p = 0.5 \text{ usec, } f_2 = 2 \text{ MHz.}$$



(3v) Square wave input. L-16.

The instantaneous value is v' for a time T_1 and v'' for a time T_2 . The time period is equal to $T(T_1+T_2)$.

→ At the O/P, similar wave can be obtained if τ is small in compare with pulse width.

→ If time constant RC is comparable with period of input square wave. Then during rising portion, it will rise exponentially up to steady state value v' . If V_1 is initial value of o/p voltage then equation is given as,

$$V_{O1}(t) = V' + (V_1 - V') e^{-t/RC}$$

and similarly for falling edge, it is given as,

$$V_{O2}(t) = V'' + (V_2 - V'') e^{-(t-T_1)/RC}$$

If we set, $V_{O1}(t) = V_2$ and $V_{O2}(t) = V_1$ at $t = T_1 + T_2$ then

we can write,

$$V_2 = V' + (V_1 - V') e^{-t/RC}$$

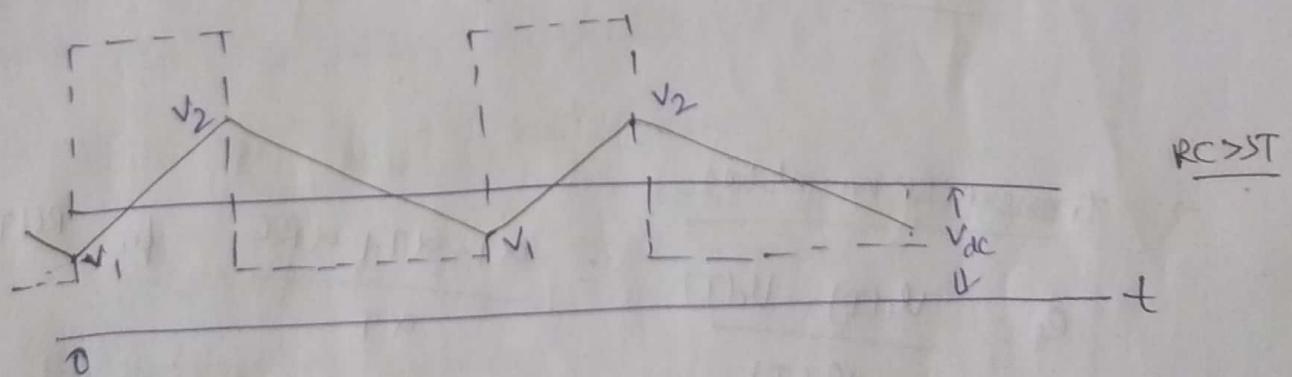
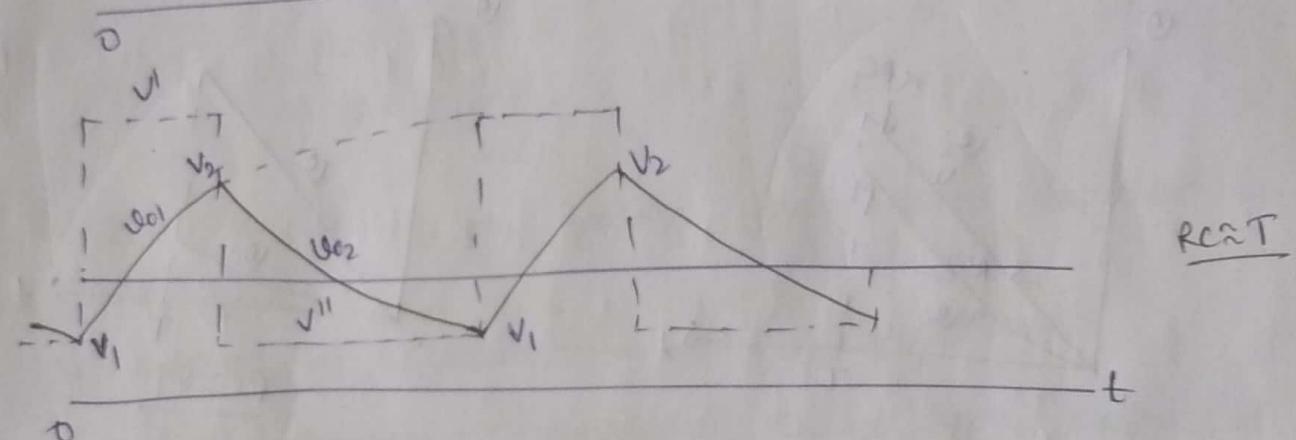
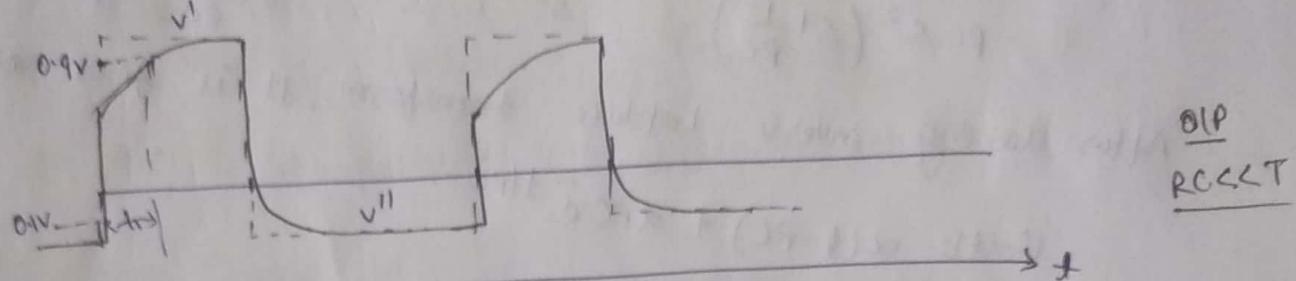
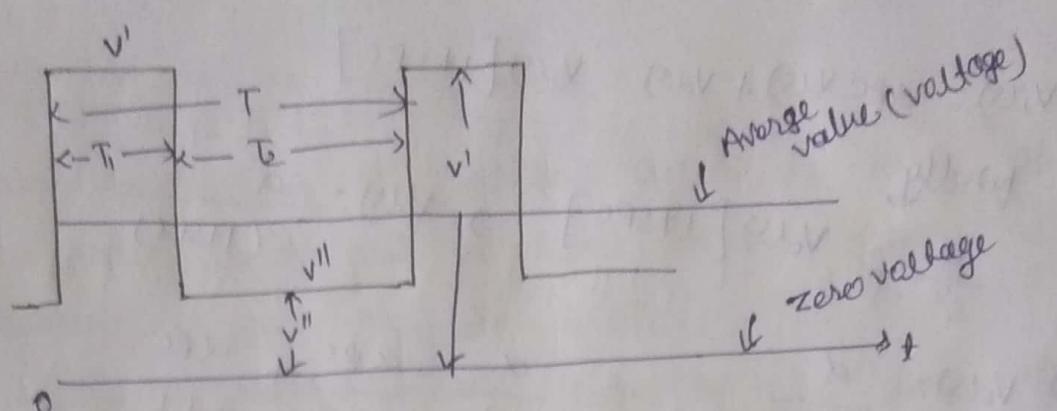
$$\& V_1 = V'' + (V_2 - V'') e^{-(t-T_1)/RC}$$

we can obtain the value of V_1 & V_2 .

→ If time constant is very large with compare to T , then o/p is consist of exponential settling, which is essentially linear.

→ Since average value of voltage across R is zero, so, dc voltage at the o/p is same as the o/p. Hence average value is indicated as V_{dc} .

→ In case of symmetrical, $T_1 = T_2 = T/2$ and $V' = V'' = V_2$.



(v) Roop Input:

It is defined as, $v_i(t) = \alpha t$; $t \geq 0$

The Laplace transform of it is given as, $V_i(s) = \frac{\alpha}{s^2}$

By using KVL, and in Laplace transform, s-domain. the equation is given as, $V_i(s) = I(s) \cdot R + \frac{I(s)}{sC}$ & $V_{ols} = \frac{I(s)}{sC}$

$$\text{So, } V_i(s) = \text{SRC } V_o(s) + V_o(s) = V_o(s)[1 + \text{SRC}]$$

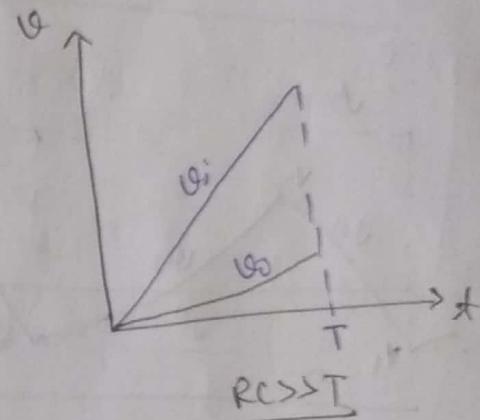
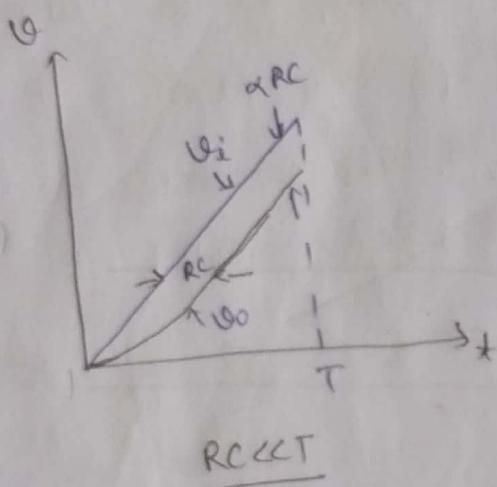
and finally,

$$\frac{\alpha}{s^2} = V_o(s)[1 + \text{SRC}] \Rightarrow V_o(s) = \frac{\alpha}{s^2(1 + \text{SRC})}$$

$$\Rightarrow V_o(s) = \frac{\alpha}{\text{RC} s^2 \left(s + \frac{1}{\text{RC}}\right)} = \frac{\alpha}{s^2} - \frac{\alpha \text{RC}}{s} + \frac{\alpha \text{RC}}{s + \frac{1}{\text{RC}}}$$

After taking inverse Laplace transform, it is given as,

$$V_o(t) = \alpha(t - RC) + \alpha RC e^{-t/RC}$$



→ Transmission errors

$$e_t = \frac{V_i(T) - V_o(T)}{V_i(T)}$$

$$= \frac{\alpha T - \alpha T + \alpha RC}{\alpha T} \quad \text{for } RC/T \ll 1$$

$$e_t = \frac{RC}{T} = \frac{1}{2\pi f_2 T}$$

* Low RC circuit as Integrator:

If time constant of RC low pass circuit is large, then capacitor charges very slowly and all input voltage appears across resistor.
So, current is given as, $i(t) = V_{in}(t)/R$.

The O/P voltage across capacitor is given as,

$$V_{out}(t) = \frac{1}{C} \int i(t) dt = \frac{1}{RC} \int V_{in}(t) dt.$$

i.e. O/P is proportional to integral of input.

→ In case of Ramp I/P,

$$V_{in}(t) = \alpha t$$

$$\text{so, } V_{out}(t) = \frac{\alpha t^2}{2RC}.$$

As time increases, the voltage drop across capacitor is not negligible.

→ The integral of a constant is a linear function.

→ In case of sinusoidal waveform, θ has been shifted at least 89.4° to satisfy integration.

$$\theta = -\tan^{-1} \frac{f_1 f_2}{f_2} = -\tan^{-1} (2\pi f R C).$$

$$\Rightarrow \tan \theta = \tan 89.4^\circ = 95.48 = 2\pi f R C.$$

For integration criteria,

$$2\pi f R C > 95.48$$

$$\Rightarrow R C > \frac{95.48}{2\pi f a} \Rightarrow R C > 15 T.$$

→ It converts square wave into triangular wave.

→ Integrator is preferred over Differentiator:

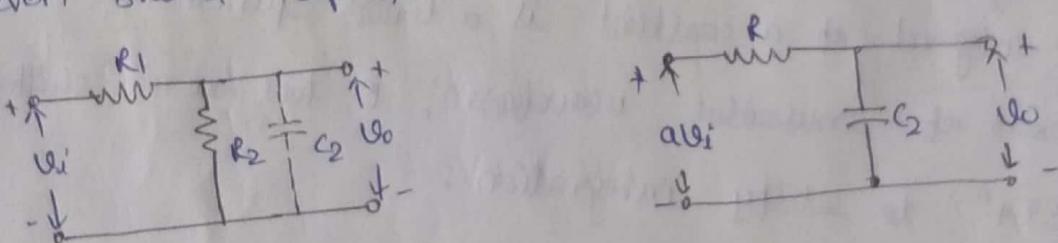
(i) The gain of the integrator decreases with freq. but in case of differentiator, it increases. so, it is easy to stabilize in case of integrator.

- (ii) An integrator is less sensitive to noise because of limited bandwidth.
- (iii) The amplifiers of differentiator may overload if ip waveform changes rapidly.
- (iv) It is convenient to introduce initial condⁿ in case of integrator.

* Attenuators:

It is resistive n/w, which is used to reduce the amplitude of a signal waveform.

For particular condition, it is possible to ensure no distortion even shunt capacitance is considered.



The simple resistor combination multiplies by ip signal by

$$a = \frac{R_2}{R_1 + R_2}, \text{ which is independent of frequency.}$$

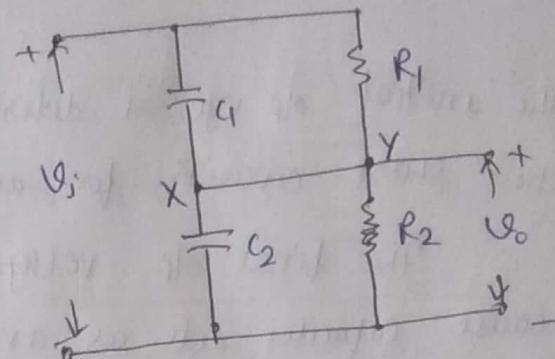
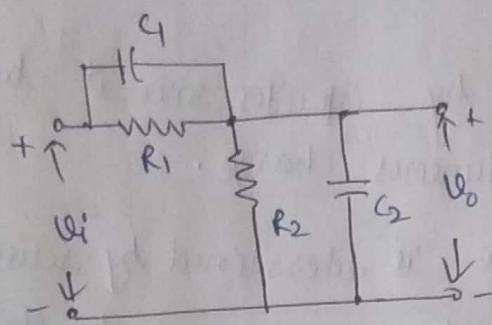
By using Thevenin's theorem, it can be drawn as,

$$R = R_1 || R_2.$$

If $R_1 = R_2 = 1 \text{ M}\Omega$ & $C_2 = 15 \text{ pF}$ then rise time $t_r = 2.2RC_2$

$\Rightarrow t_r = 16.5 \text{ usec}$. So, large rise time is not acceptable.

So, attenuator must be compensated. It is obtained by shunting R_1 with C_1 .



The same circuit is drawn in the form of bridge.

If $R_1C_1 = R_2C_2$ then bridge is balanced and no current will flow b/w X & Y. So, it can be removed.

The C is adjustable and final adjustment for compensation is made experimentally by method of square wave testing.

→ Let us consider for step I/P voltage of magnitude V. If compensation is incorrect, then voltage across C_1 and C_2 change discontinuously at $t=0$, where I/P changes abruptly. If current is finite then voltage across capacitor will not change instantaneously.

The impulse current flow in the circuit at $t=0$.

So, finite charge is given as,

$$q = \int_{t^-}^{t^+} C \frac{dV}{dt} dt$$

At $t=0^+$, by KVL, it is given as,

$$V = \frac{q}{C_1} + \frac{q}{C_2} = q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

The o/p voltage at $t=0^+$ is given as,

$$V_{o(0^+)} = \frac{q}{C_2} = \frac{C_1}{C_1 + C_2} V.$$

The initial o/p voltage is determined by capacitor, since it behave like short circuit for an instantaneous charge.

The final o/p voltage voltage is determined by resistor, because capacitor acts as an open circ under steady state condition. So, $V_o(\infty) = \frac{V_R}{R_1+R_2}$.

The perfect compensation is obtained, if $V_o(0^+) = V_o(\infty)$.

i.e. $\left(\frac{C_1}{C_1+C_2}\right)V = \left(\frac{R_2}{R_1+R_2}\right)V$, which is equivalent to $R_1 = R_2$.

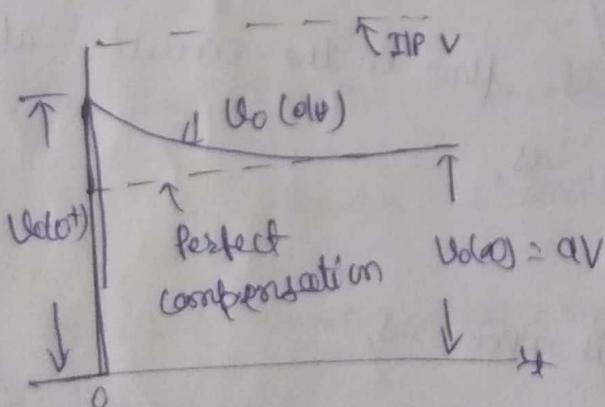
→ The value, $V_o(0^+) = 0$, for $C_1 = 0$

& $V_o(0^+) = V$, for $C_1 = \infty$.

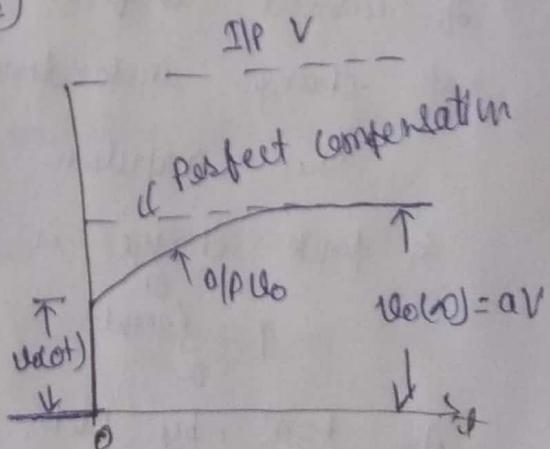
→ In practice, we can't obtain infinite current.

→ If $C_1 = \frac{C_2 R_2}{R_1} \approx C_P$; If compensation is perfect then

O/P voltage is $aV = R_2 V / (R_1 + R_2)$

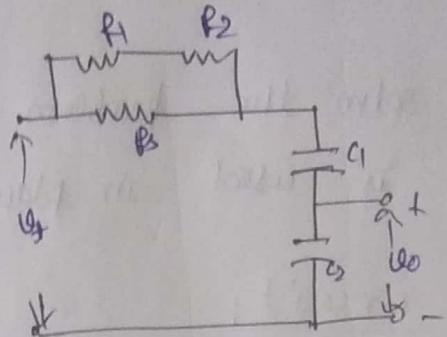
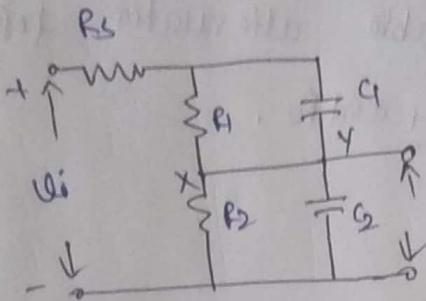


$C_1 > C_P$ over compensation.



$C_1 < C_P$ under compensation.

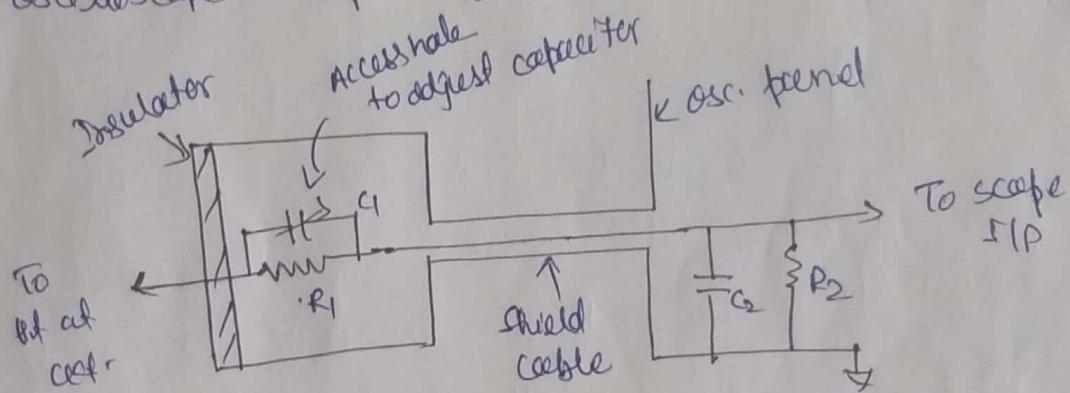
→ With step signal the attenuator is given. If $R_S \ll (R_1 R_2)$, the o/p to the attenuator will be an exponential at time constant $R_S C'$, where $C' = \frac{C_1 C_2}{C_1 + C_2}$. It is exponential rather than step.



$$\text{where } V_{\text{out}} = \frac{(R_1 + R_2)V}{R_1 + R_2 + R}.$$

→ Attenuator application as CRO probe:

To measure the signal at a point in circuit, the oscilloscope's input terminals are connected to that point.



If the pt is some distance from oscilloscope and there is chance of high impedance then shielding is required from stray field. The capacitance in several feet cable is high as 100 to 150 pF. so, combination of high I/p capacitance and high O/p impedance can't give faithful o/p.

If attenuation is 10 or 20 then input capacitance to the probe is 20 or 10 pF respectively. Now days attenuation of 100 followed by amplifier of gain 100 is available. The overall gain is 1 but probe I/p capacitance may be little 1 or 3 pF.