ANALOG AND DIGITAL COMMUNICATION

III-B.Tech.-I-Sem. **L T P C**
Subject Code: EC-PCC-311 **3 1 - 4**

Course Outcomes: Upon completion of the course, the student will be able to

1. analyze various analog modulation and demodulation schemes
2. explain various angle modulation and demodulation schemes
3. demonstrate AM, FM transmitters and receivers
4. distinguish pulse modulation and pulse code modulation schemes
5. illustrate digital modulation schemes and compute their bit error performance

CO – PO Mapping			
POs	PO1	PO2	PO3
CO1	3	3	3
CO2	3	3	3
CO3	3	3	3
CO4	3	3	3
CO5	3	3	3
3: Strong, 2: Medium, 1: Week			

Unit-I **11 hours**

Amplitude Modulation: Need for modulation, Amplitude Modulation - Time and frequency domain description, single tone modulation, power relations in AM waves, Generation of AM waves - Switching modulator, Detection of AM Waves - Envelope detector, DSBSC modulation - time and frequency domain description, Generation of DSBSC Waves - Balanced Modulators, Coherent detection of DSB-SC Modulated waves, SSB modulation - time and frequency domain description, frequency discrimination and Phase discrimination methods for generating SSB, Demodulation of SSB Waves, principle of Vestigial side band modulation.

Unit-II **8 hours**

Angle Modulation: Basic concepts of Phase Modulation, Frequency Modulation: Single tone frequency modulation, Spectrum Analysis of Sinusoidal FM Wave using Bessel functions, Narrow band FM, Wide band FM, Constant Average Power, Transmission bandwidth of FM Wave - Generation of FM Signal - Armstrong Method, Detection of FM Signal: Balanced slope detector, Phase locked loop, Comparison of FM and AM., Concept of Pre-emphasis and de-emphasis.

Unit-III **(5 + 5) 10 hours**

Part-A: Transmitters: Classification of transmitters, AM transmitters, FM transmitters.
Noise sources: Thermal noise source Arbitrary Noise Sources, Effective Noise Temperature, Noise equivalent bandwidth, Average Noise Figure, Average Noise Figure of cascaded networks, Narrow band noise, Quadrature representation of narrow band noise.

Part-B: Receivers: Radio receiver-receiver types-tuned radio frequency receiver, super heterodyne receiver, RF section and Characteristics - Frequency changing and tracking, Intermediate frequency, Image frequency, AGC, Amplitude limiting, FM Receiver, Comparison of AM and FM Receivers.

Unit-IV **10 hours**

Information Theory: Entropy information rate, Source coding: Huffman coding, Shannon Fano coding, Mutual information, Channel capacity of discrete channel, Shannon – Hartley law, Trade –off between bandwidth and SNR.
Pulse Modulation: Types of pulse modulation-PAM, PWM, PPM, comparison of FDM and TDM.
Pulse Code Modulation: PCM generation and reconstruction, non-uniform quantization and companding, DPCM, adaptive DPCM, DM and adaptive DM, noise in PCM and DM.

Unit-V **9 hours**

Digital Modulation Techniques: ASK- Modulator, Coherent ASK Detector, FSK- Modulator, Non-Coherent FSK Detector, BPSK - Modulator, Coherent BPSK Detection. Principles of QPSK, differential PSK and QAM.
Baseband Transmission and Optimal Reception of Digital Signal: A Baseband Signal Receiver, Probability of Error, Optimum Receiver, Coherent Reception, ISI, Eye Diagrams.

Textbooks:

1. Analog and Digital Communications - Simon Haykin, John Wiley, 2005.
2. Electronics Communication Systems-Fundamentals thru Advanced-Wayne Tomasi, 5[th]Ed, PHI.

References:

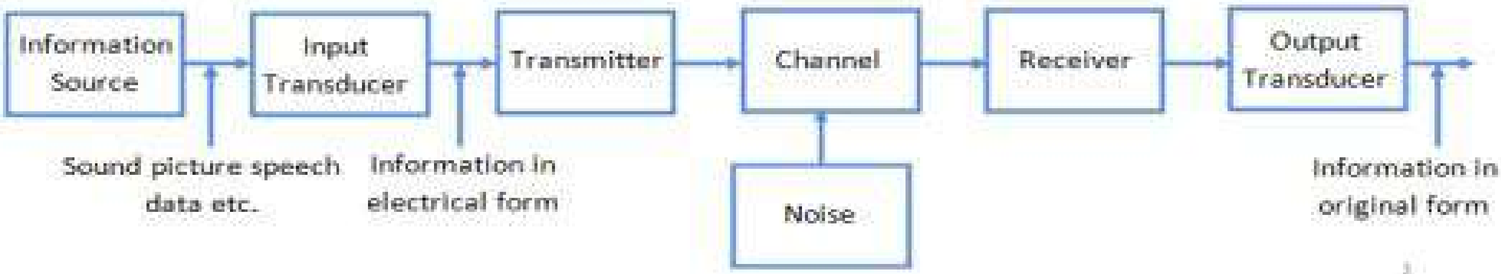
1. Communication Systems Engineering- Proakis J. G. and Salehi M., Pearson Education, 2002.
2. Electronic Communications – Dennis Roddy and John Coolean , 4th Edition , PEA, 2004
3. Electronics & Communication System – George Kennedy and Bernard Davis, TMH 2004

UNIT-1

INTRODUCTION

- The transmission of information is called communication.
- It is required that sender and receiver should understand the same language.
- we have been improving the quality of communication on behalf of growing demand for speed and complexity of information.
- The aim of this slides is to introduce the concepts of communication and the techniques of modulation sub sequent signal analysis and so on

MODEL OF COMMUNICATION SYSTEM



- Every communication has three essential elements : transmitter, channel and receiver Here the transmitter is placed at one place and receiver is placed another place and the channel is the physical medium that connect them.
- The purpose of transmitter is to convert into suitable form of signal that can transmitted through the channel.
- If the o/p of the information source is a non electric signal then a transducer convert it into electric form before it pass through the channel. Moreover, noise is introduced in channel so receiver reconstruct it and send the information to user for.
- There are two basic type of communication namely point to point and broadcast. The former take place over a link between a single transmitter and receiver, while later one have large number of receiver corresponding to a single transmitter.
- Radio and TV comes under broadcast

Communication Channel

- The channel is central to operation of a communication System.
- The information-carrying capacity of a communication system is proportional to the channel bandwidth.
- Pursuit for wider bandwidth

  - Copper wire: 1 MHz
  - Coaxial cable: 100 MHz
  - Microwave: GHz
  - Optical fibre: THz

- Uses light as the signal carrier.
- Highest capacity among all practical signals.

Amplitude Modulation

❖ **Define modulation? Explain different types of modulation.**

Modulation is the process of changing some characteristics (amplitude, frequency & phase) of a carrier wave in accordance with the instantaneous value of the modulating signal.

There are 3 types of modulations:

i) Amplitude modulation
ii) Frequency modulation and
iii) phase Modulation.

i) Amplitude modulation :-

Amplitude modulation is defined as the modulation in which the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its (carrier) frequency & phase constant.

ii) Frequency modulation :-

Frequency modulation is defined as the modulation in which the frequency of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its (carrier) amplitude & phase constant.

iii) phase modulation :-

Phase modulation is defined as the modulation in which the phase of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating Signal, Keeping its (carrier) amplitude and frequency Constant.

❖ **Explain the NEED for modulation?**

❖ **Explain the advantages of modulation?**

The advantages of modulation are

i) Reduces the height of antenna:-

Height of antenna is a function of Wavelength 'λ'. The Minimum height of antenna is given by $\lambda/4$.

i.e. height of antenna $= \frac{\lambda}{4} = \frac{c}{4f}$ $\boxed{\because \lambda = \frac{c}{f}}$

Where, $\lambda = \frac{c}{f}$,

$c = 3\times10^8$, velocity of light

f = Transmitting Frequency.

ex:- i) $f = 15\,KHz$,

height of antenna $= \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times15\times10^3} = 5000$ meters

ii) $f = 1\,MHz$,

height of antenna $= \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times1\times10^6} = 7$ meters.

From above two examples it is clear that as the transmitting frequency is increased, height of the antenna is decreased.

2) Avoids mixing of Signals:-

All audio (message) Signals ranges from 20 Hz to 20 KHz.

The transmission of message signals from various sources causes the mixing of signals and then it is difficult to separate these signals at the receiver end.

3) Increases the range of communication :-

* Low frequency signals have poor radiation and they get highly attenuated. Therefore baseband signals cannot be transmitted directly over long distances.
* Modulation increases the frequency of the signal and thus they can be transmitted over long distances.

4) Allows multiplexing of signals :-

* Modulation allows the multiplexing to be used. Multiplexing means transmission of two or more signals simultaneously over the same communication channel.

eg :-
1) Number of TV channels operating simultaneously.
2) Number of radio stations broadcasting the signals in MW & SW band simultaneously.

5) Allows adjustments in the bandwidth :-

Bandwidth of a modulated signal may be made smaller or larger.

6) Improves quality of reception :-

Modulation techniques like frequency modulation, pulse

❖ **Define standard form of amplitude modulation and explain the time and frequency domain expression of AM wave**

July 09- 6M Jan 05 - 4M

❖ **Define amplitude modulation. Derive the expression on AM by both time domain and frequency domain representation with necessary waveforms.**

July-08,12M

· Amplitude modulation is defined as the modulation in which the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating Signal keeping its (carrier) Frequency & phase constant.

Time-Domain Description :-

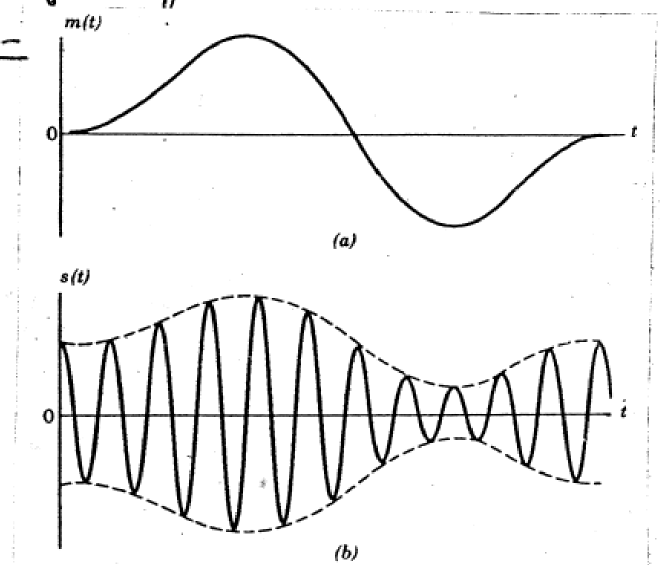


Fig ⓐ Message Signal, ⓑ AM wave s(t).

✱ The Instantaneous value of modulating Signal is given by

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (1)$$

Where, $A_m \rightarrow$ maximum amplitude of the modulating Signal

$f_m \rightarrow$ frequency of modulating Signal.

* The Instantaneous value of carrier Signal is given by

$$\boxed{c(t) = A_c \cos(2\pi f_c t)} \longrightarrow (2)$$

Where,

$A_c \rightarrow$ Maximum amplitude of the carrier Signal.

$f_c \rightarrow$ frequency of carrier Signal.

The Standard equation for AM Wave is given by

$$\boxed{S(t) = A_c\left[1+K_a m(t)\right]\cos(2\pi f_c t)} \longrightarrow (3)$$

Where,

K_a is a constant called the amplitude Sensitivity of the modulator.

Substituting eq (1) in eq (3), we get

$$S(t) = A_c\left[1+K_a A_m \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$S(t) = A_c\left[1+\mu \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

Where, $\boxed{\mu = K_a A_m}$ is called the modulation Index or modulation factor.

$$S(t) = A_c \cos(2\pi f_c t) + \mu A_c \cos(2\pi f_c t) \cdot \cos(2\pi f_m t) \longrightarrow (4)$$

equation (4) can be further expanded, by means of the trigonometric-al relation :

$$\boxed{\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)}$$

$$S(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2}\cos\left[2\pi f_c - 2\pi f_m\right]t + \frac{\mu A_c}{2}\cos\left[2\pi f_c + 2\pi f_m\right]t \rightarrow$$

equation (5) is the amplitude modulated Signal, consist of three Frequency Component

1) The first term is the carrier itself. It has a frequency 'f_c' and amplitude 'A_c'.

2) The 2nd Component is $\frac{\mu A_c}{2} \cos 2\pi(f_c - f_m)t$. It has frequency $(f_c - f_m)$ Called Lower Sideband (F_{LSB}) and having amplitude $\frac{\mu A_c}{2}$

3) Similarly 3rd Component is $\frac{\mu A_c}{2} \cos 2\pi (f_c + f_m)t$. It has frequency $(f_c + f_m)$ called upper Sideband (F_{USB}) and having - amplitude $\frac{\mu A_c}{2}$.

<u>Frequency - Domain Description :-</u>

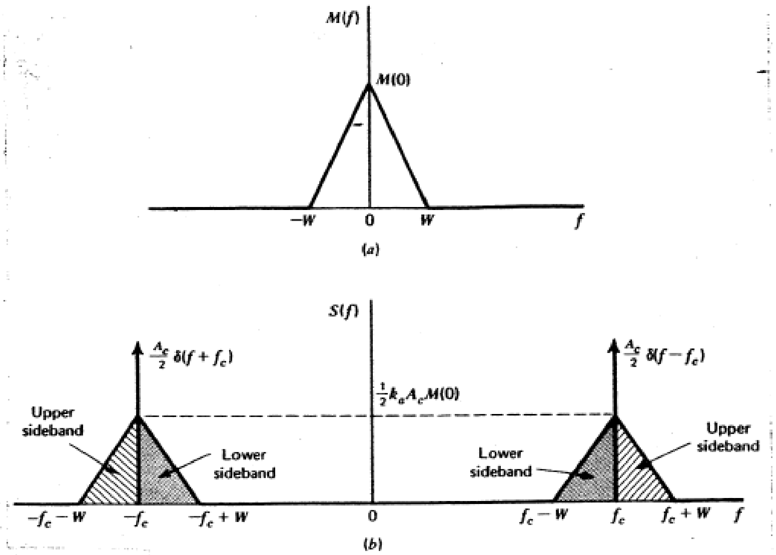
The time domain description of a conventional AM wave is given below:

$$S(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t.$$

$$S(t) = A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t \rightarrow (1)$$

Taking fourier transforms on both the Sides of eq(1), we get

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c K_a}{2}[M(f - f_c) + M(f + f_c)]$$



(a) Spectrum of baseband signal. (b) Spectrum of AM wave.

* The amplitude spectrum of the AM wave has 2 sidebands on either sides of $\pm f_c$.
* For +ve frequencies, the highest frequency component of the AM wave equals $f_c + W$, called UPPER Sideband f_{USB} and the lowest frequency component equals $f_c - W$, called LOWER Sideband f_{LSB}.

Transmission Bandwidth (B_T):-

The difference between upper sideband and lower sideband frequencies defines the transmission bandwidth 'B_T'.

$$B_T = f_{USB} - f_{LSB}$$
$$= (f_c + f_m) - (f_c - f_m)$$
$$= \cancel{f_c} + f_m - \cancel{f_c} + f_m$$
$$\boxed{B_T = 2f_m}$$

∴ Bandwidth required for transmission of an AM wave is twice the modulating signal frequency i.e. $2f_m$.

❖ **Define modulation index and percentage modulation index.**

The ratio of change in amplitude of modulating Signal to the amplitude of carrier wave is Known as modulation Index or modulation factor or modulation Co-efficient or depth of modulation or degree of modulation 'μ'.

$$\boxed{\mu = \frac{A_m}{A_c}} \quad \text{or} \quad \boxed{\mu = K_a A_m}$$

percentage modulation index

$$\boxed{\% \ \mu = \left(\frac{A_m}{A_c}\right) \times 100}$$

NOTE :-

* If A_m is greater than A_c then distortion is introduced into the System.
* The modulating Signal voltage 'A_m' must be less than carrier signal voltage 'A_c' for proper amplitude modulation.

❖ **Explain transmission efficiency of an AM wave.**

Transmission efficiency is defined as the ratio of the power carried by the Sidebands to the total transmitted power is called transmission efficiency 'η' and is given by

$$\boxed{\eta = \frac{P_{USB} + P_{LSB}}{P_T}}$$

WKT

$$P_T = P_c\left(1+\frac{\mu^2}{2}\right) \text{ and}$$

$$P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}$$

$$\eta = \frac{\frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}}{P_c\,(1+\mu^2/2)}$$

$$= \frac{\frac{\cancel{2}\,\mu^2 A_c^2}{\cancel{8}4R}}{P_c\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{2}\left[\frac{A_c^2}{2R}\right]}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{2}\cancel{P_c}}{\cancel{P_c}\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{\cancel{2}}}{\frac{2+\mu^2}{\cancel{2}}}$$

$$\boxed{\eta = \frac{\mu^2}{\mu^2+2}}$$

❖ **Obtain the expression for total transmitted power of AM wave.**

W.K.T

$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\cos 2\pi\left[f_c - f_m\right]t + \frac{\mu A_c}{2}\cos 2\pi\left[f_c + f_m\right]t$$

The AM wave has three components: unmodulated carrier, Lower Sideband and upper Sideband.

∴ The total power of AM wave is the sum of the carrier power 'P_c' and powers in the two Sidebands i.e. P_{USB} & P_{LSB}

$$P_T = P_c + P_{USB} + P_{LSB}$$

* The average carrier power

$$P_c = \frac{(A_c/\sqrt{2})^2}{R}$$

$$\boxed{P_c = \frac{A_c^2}{2R}}$$

* The average Sideband power:

$$P_{USB} = P_{LSB} = \frac{(\mu A_c/2\sqrt{2})^2}{R}$$

$$= \frac{\frac{\mu^2 A_c^2}{4\times 2}}{R}$$

$$\boxed{P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}}$$

$\therefore$ The average total power,

$$P_T = P_c + P_{USB} + P_{LSB}$$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right]$$

$$\boxed{P_T = P_c\left[1 + \frac{\mu^2}{2}\right]}$$

For 100% modulation $\mu = 1$, we have

$$P_T = P_c\left[1 + \frac{1^2}{2}\right]$$

W.K.T.

RMS value $V_{rms} = \frac{V_m}{\sqrt{2}}$

11^{ly} $A_{c_{rms}} = \frac{A_c}{\sqrt{2}}$

W.K.T

Power 'p' $= \frac{V_{rms}^2}{R}$

$$P_c = \frac{A_{c_{rms}}^2}{R}$$

$$P_c = \frac{(A_c/\sqrt{2})^2}{R}$$

$$P_{USB} = P_{LSB} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R}$$

$$= \frac{\frac{\mu^2 A_c^2}{4\times 2}}{R}$$

$$P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}$$

$$\boxed{\frac{\mu^2}{4} + \frac{\mu^2}{4} = \frac{\mu^2}{2}}$$

$$P_T = P_c\left[\frac{2+1}{2}\right]$$

$$= P_c\left[\frac{3}{2}\right]$$

$$\boxed{P_T = 1.5\,P_c}$$

NOTE :-

$$P_T = 1.5\,P_c$$

$$P_c = \frac{1}{1.5}\,P_T$$

$$\boxed{P_c = 0.666\,P_T}$$

In Amplitude modulated wave, the 66.66% of the transmitted power is used by the carrier signal and remaining 33.33% of the power is used by the sidebands (P_{USB} & P_{LSB}).

❖ **Derive the followings:**

i. **Modulation index interms of P_T and P_C.**

ii. **Current relation of AM wave.**

iii. **Modulating index interms of I_T and I_C.**

iv. **Voltage relation of AM wave.**

v. **Modulation index interms of V_T and V_C.**

i) Modulation Index interms of P_T & P_c :-

W.K.T $\quad P_T = P_c\left[1+\frac{\mu^2}{2}\right]$

$$\frac{P_T}{P_c} = 1+\frac{\mu^2}{2}$$

$$\frac{\mu^2}{2} = \frac{P_T}{P_c} - 1$$

$$\mu^2 = 2\left[\frac{P_T}{P_c} - 1\right]$$

$$\boxed{\mu = \sqrt{2\left[\frac{P_T}{P_c} - 1\right]}}$$

ii) Current relation of AM Wave :-

Let $I_T \rightarrow$ Total Current

$I_c \rightarrow$ Carrier Current

W.K.T $\quad P = I^2 R$

$\|^{ly}$ $\quad P_T = I_T^2 R$ and

$P_c = I_c^2 R$.

$$P_T = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$I_T^2 \cancel{R} = I_c^2 \cancel{R}\left[1 + \frac{\mu^2}{2}\right]$$

$$I_T^2 = I_c^2\left[1 + \frac{\mu^2}{2}\right]$$

$$I_T = \sqrt{I_c^2\left[1 + \frac{\mu^2}{2}\right]}$$

$$\boxed{I_T = I_c\sqrt{1 + \frac{\mu^2}{2}}}$$

iii) Modulation Index interms of I_T & I_c :-

W.K.T

$$I_T = I_c \sqrt{1+\mu^2/2}$$

$$I_T^2 = I_c^2 \left(1+\frac{\mu^2}{2}\right)$$

$$1+\frac{\mu^2}{2} = \left(\frac{I_T}{I_c}\right)^2$$

$$\frac{\mu^2}{2} = \left(\frac{I_T}{I_c}\right)^2 - 1$$

$$\mu^2 = 2\left(\frac{I_T}{I_c}\right)^2 - 1$$

$$\boxed{\mu^2 = \sqrt{2\left(\frac{I_T}{I_c}\right)^2 - 1}}$$

iv) Voltage relation of AM Wave :-

$\boxed{V = A}$

$A_T = V_T$

$A_c = V_c$

Let $A_T = V_T$ = Total voltage &

$A_c = V_c$ = Carrier voltage.

W.K.T $\quad P = \frac{V^2}{R}$

$\|^{ly}$ $\quad P_T = \frac{A_T^2}{R}$ &

$$P_c = \frac{A_c^2}{R}.$$

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right]$$

$$\frac{A_T^2}{\cancel{R}} = \frac{A_c^2}{\cancel{R}}\left[1+\frac{\mu^2}{2}\right]$$

$$A_T = \sqrt{A_c^2\left[1+\frac{\mu^2}{2}\right]}$$

$$\boxed{A_T = A_c\sqrt{1+\left[\mu^2/2\right]}}$$

v) Modulation Index interms of A_T & A_c.

W.K.T $\quad A_T = \sqrt{A_c^2\left[1+\mu^2/2\right]}$

$$A_T^2 = A_c^2\left[1+\mu^2/2\right]$$

$$1+\frac{\mu^2}{2} = \left(\frac{A_T}{A_c}\right)^2$$

$$\frac{\mu^2}{2} = \left[\frac{A_T}{A_c}\right]^2 - 1$$

$$\mu^2 = 2\left[\frac{A_T}{A_c}\right]^2 - 1$$

$$\boxed{\mu = \sqrt{2\left[A_T/A_c\right]^2 - 1}}$$

- **Derive modulation index using AM wave.**

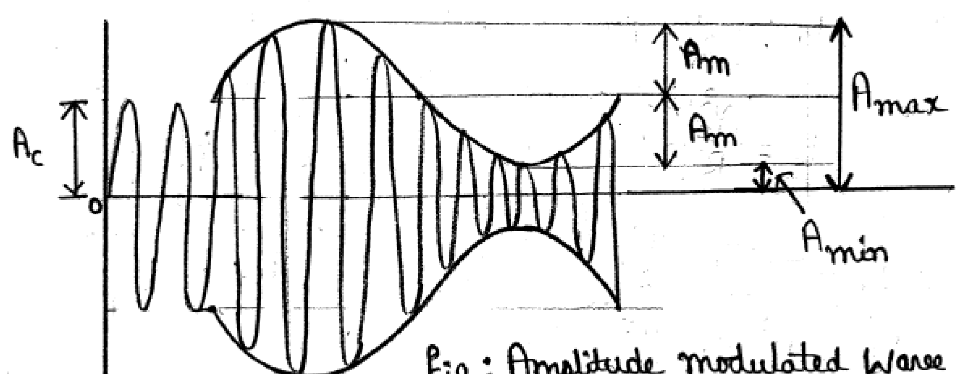


Fig : Amplitude modulated Wave

We can calculate the modulation Index from the amplitude modulate wave.

W.K.T $\boxed{\mu = \frac{A_m}{A_c}}$

From figure,

$$\boxed{A_m = \frac{A_{max} - A_{min}}{2}} \rightarrow (1)$$

$$A_c = A_{max} - A_m \rightarrow (2)$$

Substituting equation (1) in equation (2)

$$A_c = A_{max} - \left[\frac{A_{max} - A_{min}}{2}\right]$$

$$A_c = \frac{2A_{max} - A_{max} + A_{min}}{2}$$

$$\boxed{A_c = \frac{A_{max} + A_{min}}{2}}$$

$$\therefore \mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}/\cancel{2}}{A_{max} + A_{min}/\cancel{2}}$$

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

❖ **Explain amplitude modulation for single tone information.**

A Single-tone modulating Signal m(t) has a Single (tone) Frequency Component 'f_m' and is defined as follows:

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \rightarrow (1)$$

Where A_m is the amplitude of the modulating wave and f_m is the frequency of the modulating wave.

Let $\boxed{c(t) = A_c \cos(2\pi f_c t)} \rightarrow (2)$

Where A_c is the amplitude of the carrier wave and f_c is the frequency of the carrier wave.

* The time-domain expression for the Standard AM wave is

$$\boxed{S(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t} \rightarrow (3)$$

Substituting eq (1) in eq (3), we get

$$S(t) = A_c[1 + K_a A_m \cos 2\pi f_m t] \cos 2\pi f_c t$$

Since, the modulation Index $\boxed{\mu = K_a A_m}$

We get

$$S(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t. \rightarrow (4)$$

equation (4) can be further expanded, by means of the trigonometric -al relation

$$\boxed{\cos a \cdot \cos b = \frac{1}{2} \cos(a-b) + \frac{1}{2} \cos(a+b)}$$

$$S(t) = A_c \cos(2\pi f_c t) + \mu A_c \underbrace{\cos(2\pi f_c t)}_{\cos a} \cdot \underbrace{\cos(2\pi f_m t)}_{\cos b}$$

$$S(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos[2\pi f_c - 2\pi f_m]t + \frac{\mu A_c}{2} \cos[2\pi f_c + 2\pi f_m]t \rightarrow ⑤$$

Taking Fourier transform on both sides of eq ⑤, we get

$$\boxed{S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{\mu A_c}{4}\left\{\delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)]\right\} + \frac{\mu A_c}{4}\left\{\delta[f-(f_c+f_m)] + \delta[f+(f_c+f_m)]\right\}}$$

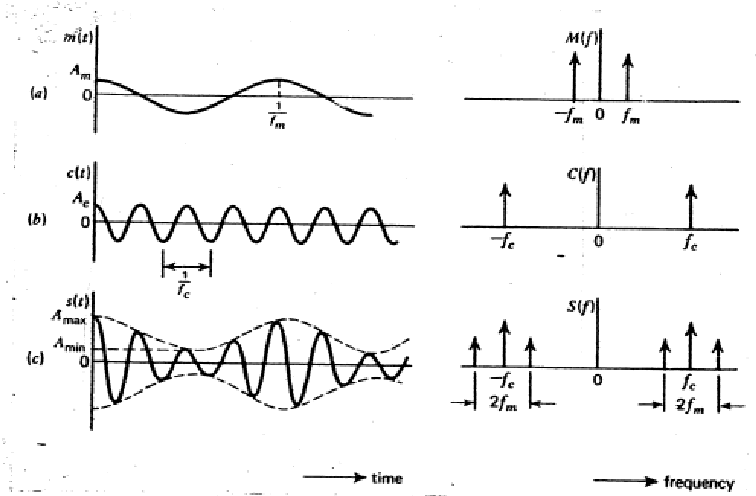


Fig ① Illustrating the time-domain (on the left) and frequency domain (on the right) characteristics of a standard amplitude modulation produced by a single tone.
ⓐ Modulating wave ⓑ Carrier wave ⓒ AM wave..

* In practice, the AM wave $s(t)$ is a voltage or current wave. The average power delivered by an AM wave to a 1-ohm resistor is calculated as follows:

Average carrier power 'P_c' $= \frac{A_c^2}{2}$

'P_{USB}', Upper Side-Frequency power $= \dfrac{\mu^2 A_c^2}{8}$

'P_{LSB}', Lower Side-Frequency power $= \dfrac{\mu^2 A_c^2}{8}$

The transmission efficiency 'η' is the ratio of the total Sideband power to the total power in the modulated wave

$$\eta = \frac{\text{Power in Sidebands}}{\text{Total power } (P_T)} = \frac{P_{USB} + P_{LSB}}{P_c\left[1+\frac{\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2 A_c^2}{8} + \frac{\mu A_c^2}{8}}{P_c\left[1+\mu^2/2\right]} = \frac{\frac{\cancel{2}\mu^2 A_c^2}{{}^{4}\cancel{8}}}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2 A_c^2}{4}}{P_c\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{2}\left[\frac{A_c^2}{2}\right]}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{2}\cancel{P_c}}{\cancel{P_c}\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{\cancel{2}}}{\frac{2+\mu^2}{\cancel{2}}}$$

$$\boxed{\eta = \frac{\mu^2}{\mu^2+2}}$$

If $\mu=1$, that is, 100 percent modulation is used, the total power in the two Side Frequencies of the resulting AM wave is only $1/3^{rd}$ of the total power in the modulated wave as Shown in fig ②.

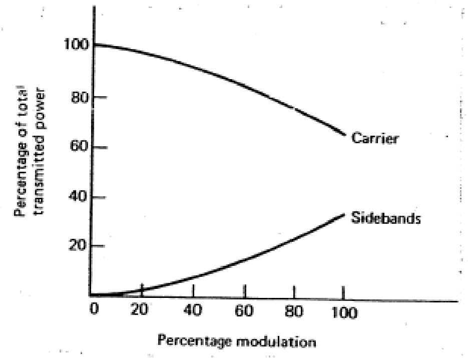


Fig ② variations of carrier power and total sideband power with percentage modulation.

❖ **A multitone modulating signal has the following time-domain form:**

$$m(t)=E_1\cos 2\pi f_1 t + E_2\cos 2\pi f_2 t + E_3\cos 2\pi f_3 t \text{ volts} \quad \text{where } E_1 > E_2 > E_3$$

$f_3 > f_2 > f_1$

i. **Give the time – domain expression for the conventional AM wave.**

ii. **Draw the amplitude spectrum for the AM wave obtained in part i. Also find the minimum transmission bandwidth.**

Sol:-

ⓐ The time-domain expression for the conventional AM wave is

$$S(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t \rightarrow ①$$

Substituting the value of m(t) in eq ①, we get

$$S(t) = A_c[1+\underline{K_a E_1}\cos 2\pi f_1 t + \underline{K_a E_2}\cos 2\pi f_2 t + \underline{K_a E_3}\cos 2\pi f_3 t]\times \cos 2\pi f_c t$$

W.K.T, $\mu_1 = K_a E_1$, $\mu_2 = K_a E_2$ & $\mu_3 = K_a E_3$

$$S(t) = A_c[1+\mu_1\cos 2\pi f_1 t + \mu_2\cos 2\pi f_2 t + \mu_3\cos 2\pi f_3 t]\cos 2\pi f_c t.$$

$$S(t) = A_c\cos 2\pi f_c t + \mu_1 A_c\cos 2\pi f_c t.\cos 2\pi f_1 t + \mu_2 A_c \cos 2\pi f_c t.\cos 2\pi f_2 t + \mu_3 A_c\cos 2\pi f_c t.\cos 2\pi f_3 t$$

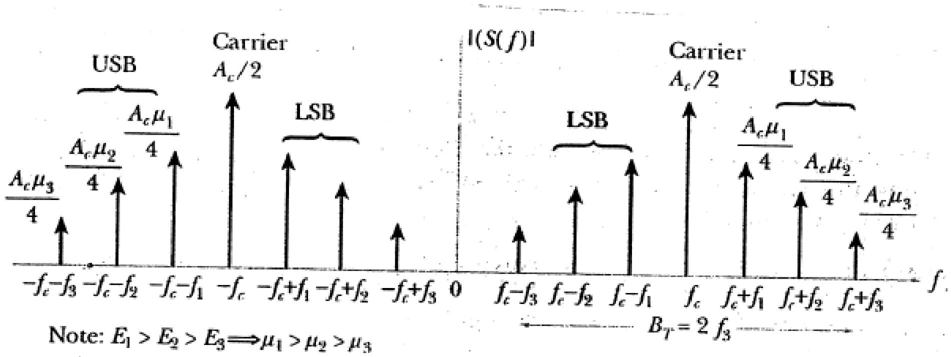
W.K.T $\boxed{\cos A.\cos B = \frac{1}{2}\cos(A+B)+\frac{1}{2}\cos(A-B)}$

$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu_1 A_c}{2} \cos 2\pi (f_c - f_1) t + \frac{\mu_1 A_c}{2} \cos 2\pi (f_c + f_1) t$$
$$+ \frac{\mu_2 A_c}{2} \cos 2\pi (f_c - f_2) t + \frac{\mu_2 A_c}{2} \cos 2\pi (f_c + f_2) t$$
$$+ \frac{\mu_3 A_c}{2} \cos 2\pi (f_c - f_3) t + \frac{\mu_3 A_c}{2} \cos 2\pi (f_c + f_3) t \rightarrow (2)$$

b) Taking Fourier transform on both sides of equation (2), we get

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{\mu_1 A_c}{4}\left\{\delta[f - (f_c - f_1)] + \delta[f + (f_c - f_1)]\right\}$$
$$+ \frac{\mu_1 A_c}{4}\left\{\delta[f - (f_c + f_1)] + \delta[f + (f_c + f_1)]\right\} + \frac{\mu_2 A_c}{4}\left\{\delta[f - (f_c - f_2)] + \delta[f + (f_c - f_2)]\right\}$$
$$+ \frac{\mu_2 A_c}{4}\left\{\delta[f - (f_c + f_2)] + \delta[f + (f_c + f_2)]\right\} + \frac{\mu_3 A_c}{4}\left\{\delta[f - (f_c - f_3)] + \delta[f + (f_c - f_3)]\right\}$$
$$+ \frac{\mu_3 A_c}{4}\left\{\delta[f - (f_c + f_3)] + \delta[f + (f_c + f_3)]\right\}$$

The amplitude spectrum is shown below



The maximum frequency is f_3.

$\therefore$ The transmission bandwidth $\boxed{B_T = 2f_3}$

❖ **Derive an expression for multitone amplitude modulation, total transmitted power and total modulation index.**

W.K.T an amplitude modulated wave is expressed as:

$$S(t) = A_c\left[1 + K_a m(t)\right] \cos 2\pi f_c t.$$

For Simplicity Consider two modulating Signal:

$$m_1(t) = A_{m1} \cos 2\pi f_{m1} t$$

$$m_2(t) = A_{m2} \cos 2\pi f_{m2} t.$$

$$\therefore \quad S(t) = A_c\left[1 + K_a\left(m_1(t) + m_2(t)\right)\right] \cos 2\pi f_c t.$$

$$= A_c\left[1 + K_a\left(A_{m1} \cos 2\pi f_{m1} t + A_{m2} \cos 2\pi f_{m2} t\right)\right] \cos 2\pi f_c t.$$

$$= A_c\left[1 + \underbrace{K_a A_{m1}}_{\mu_1} \cos 2\pi f_{m1} t + \underbrace{K_a A_{m2}}_{\mu_2} \cos 2\pi f_{m2} t\right] \cos 2\pi f_c t.$$

$$S(t) = A_c\left[1 + \mu_1 \cos 2\pi f_{m1} t + \mu_2 \cos 2\pi f_{m2} t\right] \cos 2\pi f_c t.$$

$$S(t) = A_c \cos 2\pi f_c t + \mu_1 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_{m1} t + \mu_2 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_{m2} t$$

W.K.T

$$\boxed{\cos A \cdot \cos B = \frac{1}{2} \cos(A-B) + \frac{1}{2} \cos(A+B)}$$

$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu_1 A_c}{2} \cos 2\pi\left[f_c - f_{m1}\right] t + \frac{\mu_1 A_c}{2} \cos 2\pi\left[f_c + f_{m1}\right] t$$

$$+ \frac{\mu_2 A_c}{2} \cos 2\pi\left[f_c - f_{m2}\right] t + \frac{\mu_2 A_c}{2} \cos 2\pi\left[f_c + f_{m2}\right] t \rightarrow (5)$$

From equation (5) it is Clear that, when we have two modulating frequencies, we get four additional frequencies, two upper - Sidebands (USB) $f_c + f_{m1}$, $f_c + f_{m2}$ and two lower Sidebands 'LSB' $f_c - f_{m1}$, $f_c - f_{m2}$.

Total transmitted power :-

The total power in the amplitude modulated wave is calculated as follows:

$$P_T = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

$$= \frac{(A_c/\sqrt{2})^2}{R} + \frac{\mu_1 A_c^2}{8R} + \frac{\mu_1^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R} + \cancel{2}\cdot\frac{\mu_1 A_c^2}{4\cancel{8}R} + \cancel{2}\cdot\frac{\mu_2^2 A_c}{4\cancel{8}R}$$

$$= \frac{A_c^2}{2R} + \frac{\mu_1^2 A_c^2}{4R} + \frac{\mu_2^2 A_c}{4R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$

$$\boxed{P_T = P_c\left[1 + \frac{\mu_t^2}{2}\right]}$$

$$\boxed{P_c = \frac{A_c^2}{2R}}$$

Where, $\frac{\mu_t^2}{\cancel{2}} = \frac{\mu_1^2}{\cancel{2}} + \frac{\mu_2^2}{\cancel{2}}$

$$\mu_t^2 = \mu_1^2 + \mu_2^2$$

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2}}$$

In general, Total modulation index is given by

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \dots + \mu_n^2}}$$

Generation of AM wave:-

There are two important methods of AM generation for low power applications:

1. Square Law Modulator

2. Switching Modulator.

- **Explain generation of AM wave using SQUARE- LAW modulator helps to produce AM wave. Derive the related equations and draw the waveforms** **July-05,8M**
- **Explain the generation of AM wave using SQUARE- LAW modulator along with relevant diagram & analysis.** **July-08,10M**

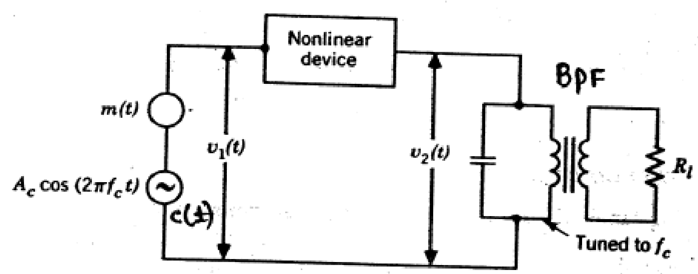


Fig ①: Square - law modulator.

The Square - law modulator consists of three elements:

1) Summer: It adds the carrier and modulating Signal.

2) Non - linear device: A device with non - linear I/p - o/p relation.

3) Band pass filter (BPF): It extract desired Signal (term) from modulator product.

* The Semiconductor diodes or transistor can be used as non-linear element and Single or double tuned circuit can be

used as the filter.

* When a non-linear element such as diode is suitably biased and the signal applied is relatively weak, it is possible to approximate the transfer characteristics as:

$$\boxed{V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)} \longrightarrow ①$$

Where a_1 and a_2 are constants.

* The I/p voltage '$V_1(t)$' is the sum of carrier signal and modulating signal.

i.e. $$\boxed{V_1(t) = A_c \cos 2\pi f_c t + m(t)} \longrightarrow ②$$

Substituting equation ② in equation ①

$$V_2(t) = a_1\left[A_c \cos 2\pi f_c t + m(t)\right] + a_2\left[A_c \cos 2\pi f_c t + m(t)\right]^2$$

W.K.T $\boxed{(a+b)^2 = a^2 + b^2 + 2ab}$

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2\left[A_c^2 \cos^2 2\pi f_c t + m^2(t) + 2m(t).A_c \cos 2\pi f_c t\right]$$

$$V_2(t) = \underline{a_1 A_c \cos 2\pi f_c t} + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t) + \underline{2a_2 m(t).A_c \cos 2\pi f_c t}.$$

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + 2a_2 m(t) A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t).$$

$$= \underbrace{a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t}_{\text{AM Wave}} + \underbrace{a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t)}_{\text{unwanted terms}} \to ③$$

* The first term of equation (3) is the desired AM wave with $K_a = \frac{2a_2}{a_1}$, amplitude Sensitivity of the AM wave.

* The remaining three terms are unwanted and are removed by appropriate filtering.

$$\therefore \quad S(t) = a_1 A_c [1 + K_a m(t)] \cos 2\pi f_c t.$$

❖ **With a neat block diagram, relevant waveforms and expressions explain generation of AM wave using SWITCHING MODULATOR**

Jan-08,10M

❖ **Explain the generation of AM wave using SWITCHING MODULATOR with relevant equations waveforms and spectrum before and after filtering process.** **Jan-07,10M Jan-05,6M July-07,10M July-08,6M July-09,8M Jan-10,10M June-107M July-09,8M**

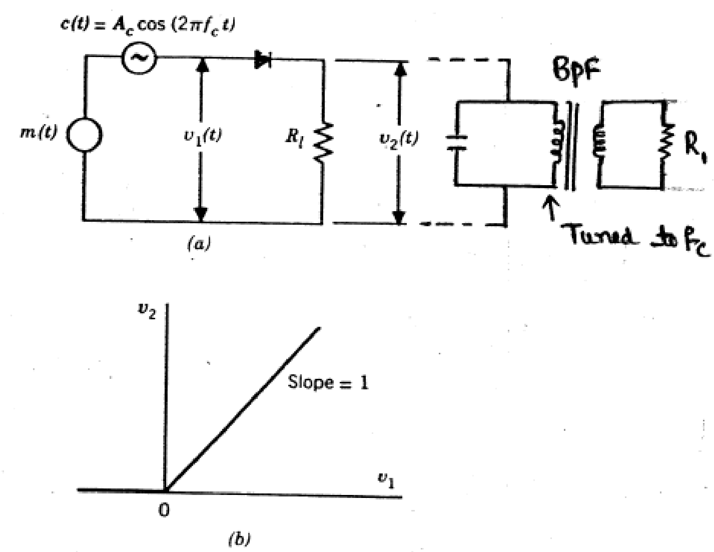


fig (1)

Switching modulator. (a) Circuit diagram. (b) Idealized input–output relation.

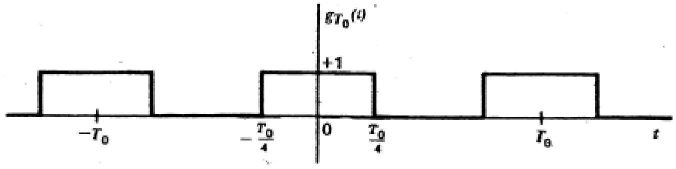


Fig ② Periodic pulse train.

* Consider a Semiconductor diode used as an ideal Switch to which a Carrier wave $C(t)$ and an message Signal $m(t)$ are Simultaneously applied as Shown in Fig ①.

* It is assumed that the Carrier wave $C(t)$ applied to the diode is large in amplitude.

The total I/p '$V_1(t)$' to the diode is given by

$$V_1(t) = m(t) + c(t)$$

$$\boxed{V_1(t) = m(t) + A_c \cos 2\pi f_c t} \longrightarrow ①$$

Where $|m(t)| \ll A_c$.

* The o/p of the diode is

$$V_2(t) = \begin{cases} V_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$

i.e. the o/p of the diode varies between 0 & V_1 at a rate equal to carrier frequency $T_0 = \frac{1}{f_c}$.

* The non-linear behavior of the diode can be replaced by assuming the weak modulating Signal compared with the carrier wave. Thus the o/p of the diode is approximately equivalent to linear-time varying operation.

Mathematically, the o/p of the diode can be written as:

$$V_2(t) = V_1(t) . g_p(t) \longrightarrow (2)$$

$$V_2(t) = [m(t) + A_c \cos 2\pi f_c t]\, g_p(t) \longrightarrow (3)$$

Where $g_p(t)$ (fig 2) is a rectangular pulse train with a period equal to $T_0 = 1/f_c$.

Representing $g_p(t)$ by its Fourier Series, we have

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c (2n-1) t]$$

$$g_p(t) = \frac{1}{2} + \underbrace{\frac{2}{\pi} \cos 2\pi f_c t}_{n=1} + \text{odd harmonic components} \rightarrow (4)$$

Substituting equation (4) in equation (3)

$$V_2(t) = [m(t) + A_c \cos 2\pi f_c t]\left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t + \cdots\right]$$

$$V_2(t) = \frac{1}{2} m(t) + \frac{2m(t)}{\pi} \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \frac{2A_c}{\pi} \underline{\cos^2 2\pi f_c t} + \cdots$$

W.K.T $\boxed{\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}}$

$$V_2(t) = \frac{m(t)}{2} + \frac{2m(t)}{\pi} \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \frac{2A_c}{\pi}\left[\frac{1}{2} + \frac{\cos 2[2\pi f_c t]}{2}\right]$$

$$V_2(t) = \frac{m(t)}{2} + \frac{2m(t)}{\pi} \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \frac{\cancel{2}A_c}{\cancel{2}\pi} + \frac{\cancel{2}A_c \cos 4\pi f_c t}{\cancel{2}}$$

$$V_2(t) = \frac{m(t)}{2} + \frac{2m(t)}{\pi} \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \frac{A_c}{\pi} + A_c \cos 4\pi f_c t + \cdots \rightarrow (5)$$

- The required AM wave centred at f_c is obtained by passing '$V_2(t)$' through an ideal 'BPF' having a centre frequency 'f_c' and bandwidth $B_T = 2W$ Hz.

* The o/p of the BPF is

$$V_2'(t) = \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t.$$

$$V_2'(t) = \frac{A_c}{2} \cos 2\pi f_c t \left[1 + \frac{2.2}{\pi A_c} m(t) \right]$$

$$= \frac{A_c}{2} \cos 2\pi f_c t \left[1 + \frac{4}{\pi A_c} m(t) \right]$$

Where $K_a = \frac{4}{\pi A_c}$ amplitude Sensitivity

$$\boxed{V_2'(t) = \frac{A_c}{2} \cos 2\pi f_c t \left[1 + K_a m(t) \right]}$$

Define Demodulation? Mention different types of AM demodulation (Detection)
OR

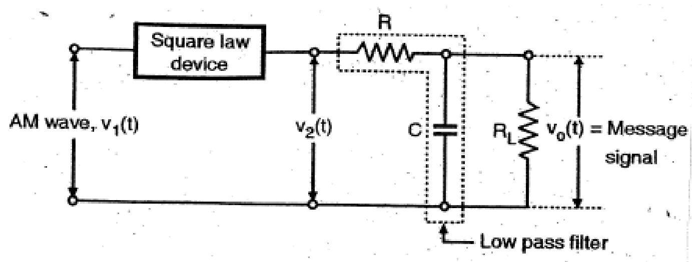
Demodulation or detection is the process of recovering the original message signal from the modulated wave at the receiver. Demodulation is the Inverse of the modulation process.

There are two types of detectors:

1) Square-law demodulator
2) Envelope detector.

❖ **Show that a SQUARE LAW device can be used for the detection of an AM wave.** **Jan-07,6M**

❖ **Show that a SQUARE LAW can be used for the detection of an AM wave.** **June-10,6M**



A square law detector

* A Square - law detector is essentially obtained by using a Square - law modulator for the purpose of detection.

* An AM Signal can be demodulated by Squaring it and then passing the Squared Signal through a Low pass filter (LPF)

The transfer Characteristics of a non- linear device is given by :

$$\boxed{V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)} \longrightarrow ①$$

Where,

$V_1(t) \rightarrow$ I/p voltage

$V_2(t) \rightarrow$ O/p voltage

a_1 and $a_2 \rightarrow$ are Constants.

* The I/p voltage of the AM wave is given by

$$\boxed{V_1(t) = A_c[1 + K_a m(t)] \cos 2\pi f_c t} \longrightarrow ②$$

Substituting equation (2) in equation (1), we get

$$V_2(t) = a_1\{A_c[1+K_a m(t)]\cos 2\pi f_c t\} + a_2\{A_c[1+K_a m(t)]\cos 2\pi f_c t\}^2$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2\{A_c^2[1+K_a m(t)]^2 \cos^2 2\pi f_c t\}$$

W.K.T

$\boxed{(a+b)^2 = a^2+b^2+2ab}$ and $\boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2 A_c^2 \cos^2 2\pi f_c t\,[1^2+K_a^2 m^2(t)+2K_a m(t)]$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2 A_c^2\left[\frac{1+\cos 2(2\pi f_c t)}{2}\right]\left[1+K_a^2 m^2(t) + 2K_a m(t)\right]$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + \frac{a_2 A_c^2}{2}\left[1+K_a^2 m^2(t)+2K_a m(t)\right](1+\cos 4\pi f_c t) \rightarrow (3)$$

* In eq (3) $\frac{a_2 A_c^2}{\cancel{2}}\cancel{2}K_a m(t)$ is the desired term which is due to the $a_2 V_1^2$ term. Hence the name of this detector is - Square law detector.

* The desired term is extracted by using a LPF. (fig) Thus the o/p of LPF is $\boxed{V_0(t) = a_2 A_c^2 K_a m(t)}$

Thus the message signal $m(t)$ is recovered at the o/p of the message signal.

Distortion in the detector o/p :-

* The other term which passes through the LPF to the load resistance R_L is as follows : $\frac{1}{2} a_2 A_c^2 K_a^2 m^2(t)$.

* This is an unwanted Signal & gives rise to a Signal distortion.
The ratio of desired Signal to the undesired one is given by:

$$D = \frac{\cancel{a_2} \cancel{A_c^2} \cancel{K_a} m(t)}{\frac{1}{2} \cancel{a_2} \cancel{A_c^2} K_a^{\cancel{2}} m^{\cancel{2}}(t)} = \frac{1}{\frac{1}{2} K_a m(t)} = \frac{2}{K_a m(t)}$$

{

We Should maximize this ratio in order to minimize the distortion. To achieve this we Should Choose $|K_a m(t)|$ Small as Compared to unity for all values of t. If K_a is Small then the AM wave is weak.

}

Envelope Detector:

❖ **How a modulating signal can be detected using a AM detector? Use a envelope detector and explain.** **July-05,8M**

❖ **Explain the detection of message signal from amplitude modulated signal using an envelope detector & bring out the significance of RC time constant**

July-09,6M July-07,5M June-09,6M July-06,5M

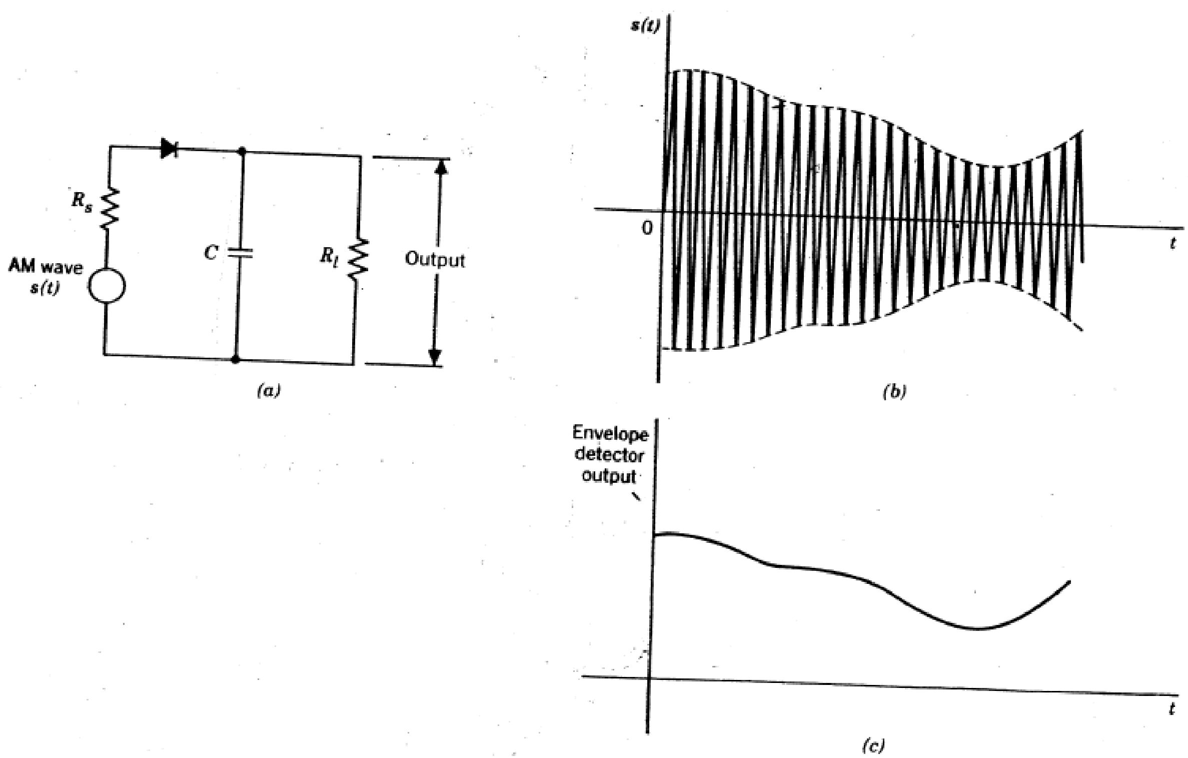


Figure
Envelope detector. (a) Circuit diagram. (b) AM wave input. (c) Envelope detector output.

* Envelope detector is a simple and highly effective device used to demodulate AM wave. It consists of a diode and a resistor capacitor (RC) filter.

operation :-

During positive half cycle of the I/p Signal, diode is forward-biased and the capacitor 'C' charges upto the peak-value of the I/p Signal. When the I/p voltage falls below this value the diode becomes reverse biased and capacitor 'C' discharges Slowly through the load resistor R_L. As a result only positive half cycle of AM wave appears across R_L.

The discharging process continues until the next positive half cycle. When the I/p Signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

Selection of the RC time Constant :-

* The capacitor charges through 'D' & R_S when the diode is 'ON' & it discharges through 'R_L' when diode is OFF.
* The charging time constant R_SC Should be Short as compared to the carrier period $1/f_c$ $\therefore$ $\boxed{R_SC \ll \frac{1}{f_c}}$ So capacitor 'c' charges rapidly.
* on the other hand the discharging time constant R_LC Should be long enough to ensure that the capacitor discharges Slowly through the load resistance 'R_L' b/w +ve peak of the carrier wave

i.e. $\boxed{\frac{1}{f_c} \ll R_LC \ll \frac{1}{W}}$, Where W = Maximum modulating frequency.

Result is that the capacitor voltage or detector o/p is very nearly the Same as the envelope of AM wave. The detector o/p usually has a small ripple at the carrier frequency. This ripple is easily removed by Low pass filter.

Advantages of AM :-

1) AM Tx's are less complex
2) AM receivers are simple, detection is easy.
3) AM receivers are cost efficient.
4) AM waves can travel a longer bandwidth.
5) Low bandwidth.

---

* Mention the disadvantages of AM wave.

(OR)

* Mention the Limitation of DSB-FC Wave (AM)

The disadvantages of AM waves are:

1) power is wasted in the transmitted signal.
2) AM needs larger bandwidth
3) AM waves gets affected due to noise.

---

* Applications of AM :-

1) Radio broadcasting
2) Picture transmission in a TV System.

Explain the disadvantages & Limitation of AM Wave (DSB-FC)

Amplitude modulation has Several disadvantages:

1) Power is wasted in the transmitted Signal

* Most of the transmitted power is in the carrier, which does not carry any information.
* For 100% modulation i.e. $\mu = 1$, only 33.33% ($1/3^{rd}$) of the total power will be in Sidebands which carries information and 66.67% ($2/3^{rd}$) of the total power will be in the carrier, which does not Contain any information

2) The DSB-FC System is Bandwidth inefficient System. The transmitted Signal requires twice the bandwidth of the message Signal i.e. $B_T = 2f_m$. This is due to the transmission of both the Sidebands, out of which only one Sideband is Sufficient to convey all the information Thus the bandwidth of DSB-FC is double than actually required.

3) AM wave gets affected due to noise:-

When the AM wave travels from the transmitter to receiver over a Communication Channel, noise gets added to it The noise will change the amplitude of the envelope of AM in a random manner. As the information is Contained in the amplitude variations of the AM wave, the noise will contaminate the information Contents in the AM. Hence the performance of AM is very poor in presence of noise.

Power Wastage in AM (DSB-FC):

* power wastage due to DSB-FC (AM) transmission.

W.K.T, the total power transmitted by an AM wave is given b

$$P_T = P_c + P_{USB} + P_{LSB} \rightarrow (1)$$

$$P_T = P_c + \frac{\mu^2}{4} P_c + \frac{\mu^2}{4} P_c \rightarrow (2)$$

In equation (2), carrier component doesnot contain any information & one sideband is redundant. So out of the total power, $P_T = P_c\left[1+\frac{\mu^2}{2}\right]$, the wasted power is given by:

$$\text{Power wastage} = P_c + \frac{\mu^2}{4} P_c$$

---

other types of Amplitude Modulation :-

Amplitude Modulation (AM)

- DSB-FC (Double Sideband Full carrier System)
- DSB-SC (Double Sideband Suppressed Carrier System)
- SSB-SC (Single Sideband Suppressed Carrier System)
- ISB (Independent Sideband System)
- VSE (vestigial Sideban Systen

❖What is DSB-SC modulation? Explain the time and frequency domain expression of DSB-SC wave.

To overcome the drawback of power wattage in AM wave (DSB-FC) an DSB-SC method is used.

* DSB-SC is a method of transmission where only the TWO Sidebands are transmitted without the Carrier (Suppressing Carrier)

OR

The Conventional AM wave in which the Carrier is Suppressed is called DSB-SC modulation.

Time domain representation of DSB-SC Wave:-

* Let $m(t)$ be the message Signal having a bandwidth equal to 'W' Hz and

$c(t) = A_c \cos 2\pi f_c t$ represents the Carrier, then the time-domain expression for DSB-SC wave is

$$S(t) = m(t)\,c(t)$$

$$\boxed{S(t) = A_c \cos(2\pi f_c t)\, m(t)} \rightarrow (1)$$

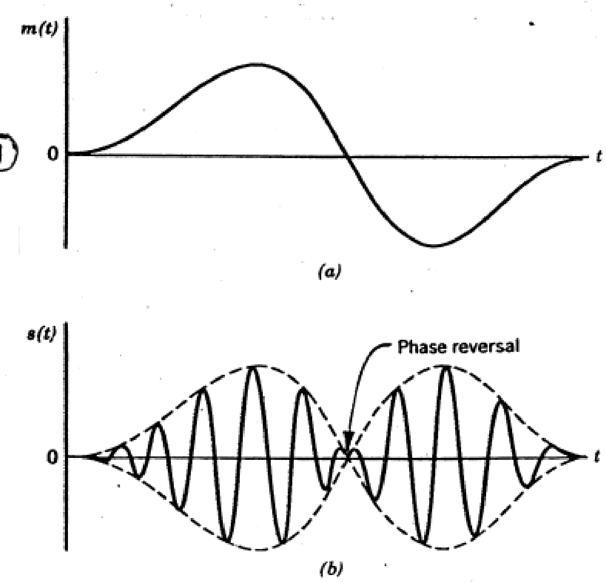


Figure
(a) *Message signal.* (b) *DSBSC-modulated wave* s(t).

* The s(t) Signal undergoes a phase reversal whenever the message Signal Crosses Zero.

Frequency - Domain Description :-

Taking Fourier transform on both Sides of equation ②, we get

$$\boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c)+M(f+f_c)\right]} \longrightarrow ③$$

Where S(f) is the Fourier transform of the modulated wave s(t)

M(f) is the Fourier transform of the message Signal m(t).

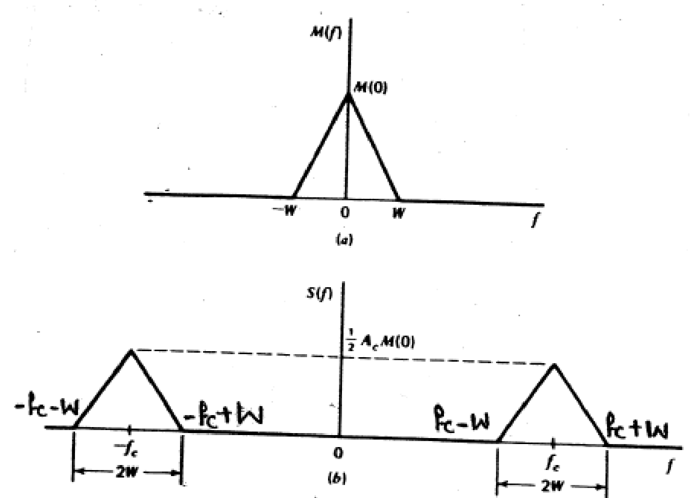


Figure
(a) Spectrum of message signal. (b) Spectrum of DSBSC modulated wave.

The amplitude Spectrum drawn above exhibits the following facts

i) on either Sides of $\pm f_c$, we have two Sidebands designated as Lower and upper Sidebands.

ii) The Impulse are absent at $\pm f_c$ in the amplitude Spectrum, Signifying the fact that the carrier term is Suppressed in the transmitted Wave.

iii) The minimum transmission bandwidth required is $2W$ i.e. twice the message bandwidth.

NOTE :- A DSB-SC Signal can be generated by a multiplier. A Carrier Signal Can be Suppressed by adding a Carrier Signal opposite in phase but equal in magnitude to the amplitude modulated Wave, So the Carrier get Cancelled Finally double Sidebands are available in the DSB-SC Wave.

❖ **Explain DSB-SC modulation for single tone information.**

* Let $m(t) = A_m \cos 2\pi f_m t$ be the Single tone modulating Signal and

$\boxed{c(t) = A_c \cos 2\pi f_c t}$

be the Carrier Signal.

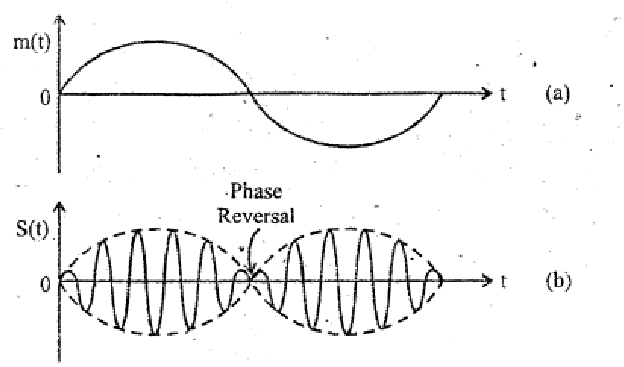


Fig. : (a) Modulating signal m(t); (b) DSBSC modulated wave s(t)

Then the time domain expression for the DSB-SC Wave is

$$S(t) = m(t) \cdot c(t)$$

$$S(t) = A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t$$

W.K.T. $$\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$\boxed{S(t) = \frac{A_m A_c}{2}\cos 2\pi (f_c - f_m) t + \frac{A_m A_c}{2}\cos 2\pi (f_c + f_m) t} \rightarrow ①$$

Taking Fourier transform on both sides of the equation ①

$$S(f) = \frac{A_m A_c}{4}\left\{ \delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)] \right\}$$
$$+ \frac{A_m A_c}{4}\left\{ \delta[f-(f_c+f_m)] + \delta[f+(f_c+f_m)] \right\}.$$

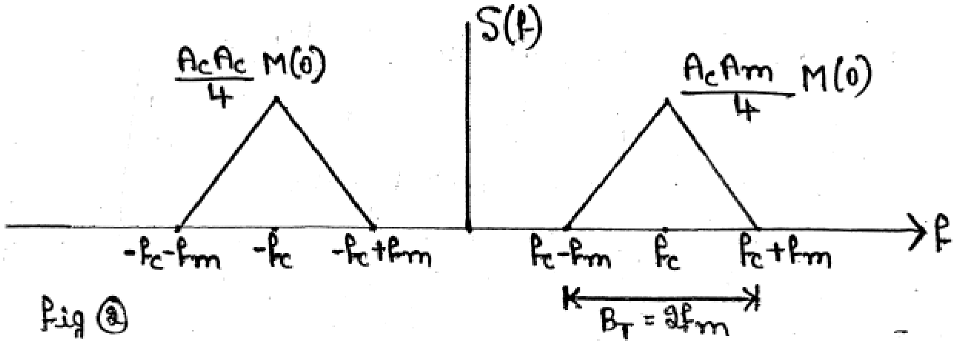
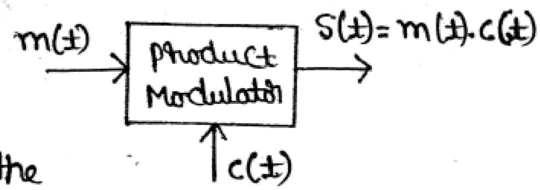


fig ②

fig ② shows amplitude spectrum of a DSB-SC signal. We observe that either side of $\pm f_c$, we have lower and upper sideband also the carrier term is suppressed in the spectrum as there are no impulses at $\pm f_c$.

* The minimum transmission bandwidth in DSB-SC is '$2f_m$'.

Generation of DSB-SC Wave:



* A DSB-SC Wave Simply Consists of the product of the modulating Signal and the carrier Signal.

* The devices used to generate DSB-SC Waves are Known as the product modulators.

  There are two types of modulators:

  1) Balanced Modulator

  2) Ring Modulator.

Balanced Modulator:

❖ **With a neat block diagram, explain the balanced modulator method of generating DSB-SC wave.** **June-10,6M**

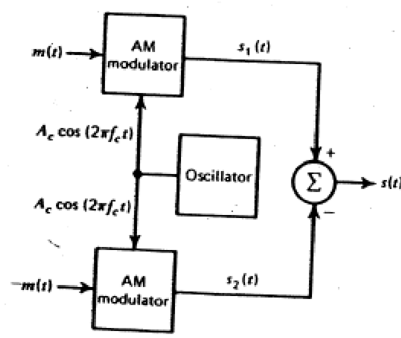


Figure
Balanced modulator.

Fig ① Shows the block diagram of a balanced modulator used for generating a DSB-SC Signal.

* It Consist of two amplitude modulators that are interconnected in Such a way as to Suppress the carrier.

* one I/p to the amplitude modulator is from an oscillator that generates a carrier wave. The Second I/p to the amplitude modulator in the top path is the modulating Signal $m(t)$ while in the bottom path is $-m(t)$.

. The o/p of the two AM modulator are as follows:

$$S_1(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t \quad \text{and}$$

$$S_2(t) = A_c[1-K_a m(t)]\cos 2\pi f_c t.$$

The o/p of the Summer is

$$S(t) = S_1(t) - S_2(t)$$

$$S(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t - \left[A_c\left(1-K_a m(t)\right)\cos 2\pi f_c t\right]$$

$$= A_c\cos 2\pi f_c t + A_c K_a m(t)\cos 2\pi f_c t - \left[A_c\cos 2\pi f_c t - A_c K_a m(t)\cos 2\pi f_c t\right]$$

$$= \cancel{A_c\cos 2\pi f_c t} + A_c K_a m(t)\cos 2\pi f_c t - \cancel{A_c\cos 2\pi f_c t} + A_c K_a m(t)\cos 2\pi f_c t.$$

$$\boxed{S(t) = 2A_c K_a m(t)\cos 2\pi f_c t} \rightarrow 1$$

* The balanced modulator o/p is equal to the product of the modulating Signal $m(t)$ & carrier $c(t)$ except the Scaling factor $2K_a$.

Taking Fourier Transform on both Side of equation 1, we get

$$S(f) = \frac{\cancel{2}A_c K_a}{\cancel{2}}\left[M(f-f_c) + M(f+f_c)\right]$$

$$\boxed{S(f) = A_c K_a[M(f-f_c) + M(f+f_c)]}$$

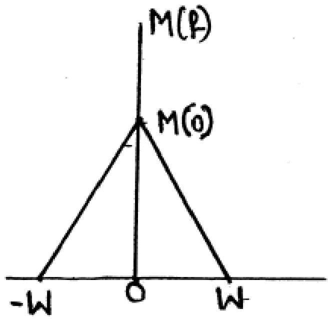


Fig ⓐ: Message Spectrum

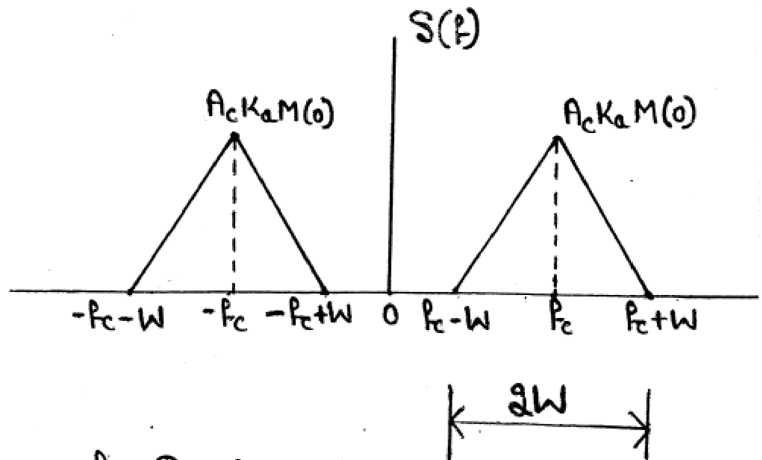


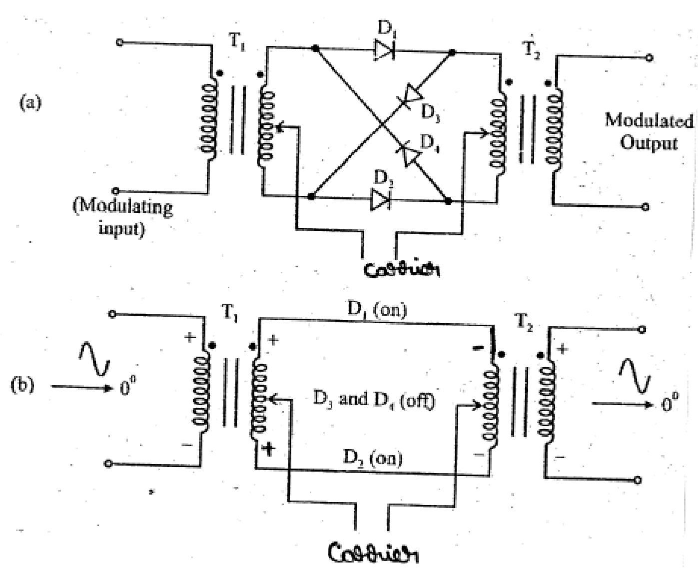
Fig ⓑ : DSB-SC Spectrum

Since the carrier component is eliminated, the o/p is called DSB-SC Signal.

❖ **Explain how RING modulator can be used to generate DSB-SC modulation** **Jan-05,9M**

❖ **Briefly explain generation of DSB-SC modulated wave using RING modulator. Give relevant mathematical expressions and waveforms.**

Jan-08,10M Jan-07,8M Jan-09,6M July-09,10M June-10,10m



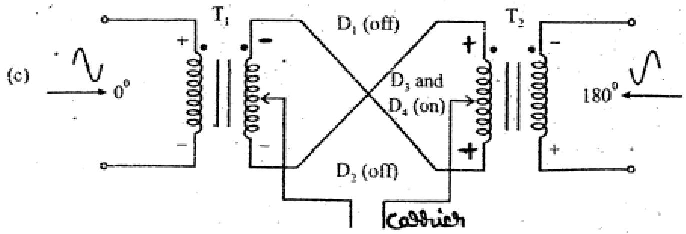


Fig. : (a) Balanced Ring Modulator
(b) Equivalent Circuit when square wave carrier positive
(c) Equivalence circuit when square wave carrier negative

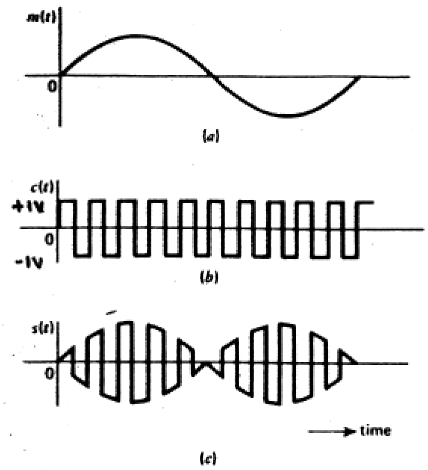


Figure
Waveforms illustrating the operation of the ring modulator for a sinusoidal modulating wave. (a) Modulating wave. (b) Square-wave carrier. (c) Modulated wave.

Ring modulator is a product modulator used for generating DSB-SC modulated wave. The ring modulator consists of :-

1) I/p transformer 'T_1'
2) O/p transformer 'T_2'
3) Four diodes connected in a bridge circuit (Ring)

The carrier amplitude 'A_c' is greater than the modulating signal amplitude 'A_m' i.e. $A_c > A_m$ and carrier frequency 'f_c' is greater than modulating signal 'f_m' $= W$ i.e. $f_c > W$.
These conditions ensures that the diode operation is controlled by $c(t)$ only.

* The diodes are controlled by a square wave carrier $c(t)$

of frequency 'f_c' which is applied by means of two center-tapped transformer.

* The modulating Signal $m(t)$ is applied to the I/p transformer 'T_1'. The o/p appears across the Secondary of the transformer 'T_2'.

operation :-

i) When the Carrier is +ve, the diodes D_1 & D_2 are forward-biased and diodes D_3 & D_4 are reverse biased. Hence the modulator multiplies the message Signal $m(t)$ by +1 i.e. $V_0(t) = m(t)$.

ii) When the Carrier is -ve, the diodes D_3 & D_4 are forward biased whereas D_1 & D_2 are reverse biased. Thus the modulator multiplies the message Signal $m(t)$ by -1 i.e. $V_0(t) = -m(t)$.

* The Square wave Carrier $c(t)$ can be represented by a Fourier Series as:

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t (2n-1)\right]$$

$$c(t) = \frac{4}{\pi}\Big[\underbrace{\cos 2\pi f_c t}_{n=1} - \underbrace{\frac{1}{3}\cos 6\pi f_c t}_{n=2} + \cdots\cdots\Big] \longrightarrow (1)$$

The ring modulator o/p is

$$s(t) = c(t) \cdot m(t) \longrightarrow (2)$$

Substituting equation (1) in equation (2), we get

$$s(t) = \left[\frac{4}{\pi}\cos 2\pi f_c t - \frac{4}{3\pi}\cos 6\pi f_c t + \cdots\cdots\right] m(t)$$

$$s(t) = \frac{4}{\pi} m(t) \cos 2\pi f_c t - \frac{4}{3\pi} m(t) \cos 6\pi f_c t + \cdots\cdots] \longrightarrow (3)$$

{ Taking Fourier transform on both sides of equation ③, we get

$$S(f) = \frac{2\cancel{4}}{\cancel{2}\pi}\left[M(f-f_c)+M(f+f_c)\right] - \frac{2\cancel{4}}{\cancel{2}\times 3\pi}\left[M(f-3f_c)+M(f+3f_c)\right]$$

$$\boxed{S(f) = \frac{2}{\pi}\left[M(f-f_c)+M(f+f_c)\right] - \frac{2}{3\pi}\left[M(f-3f_c)+M(f+3f_c)\right]}$$

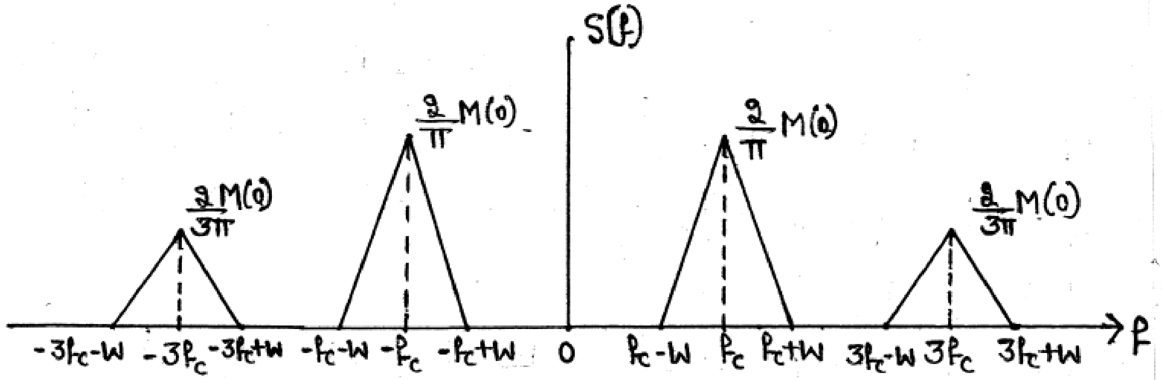


Fig : Amplitude Spectrum of S(f).

}

* The DSB-SC wave is extracted from S(t) by passing equation ③ (S(t)) through an Ideal BPF having centre frequency 'f_c' and bandwidth equal to 2W Hz.

The o/p of the BPF is

$$\boxed{S(t) = \frac{4}{\pi} m(t) \cos 2\pi f_c t}$$

COHERENT Detection of DSB-SC wave:-

❖ **With block diagram and related equations explain coherent detection of a DSB-SC wave. What are its disadvantages? Explain the synchronous receiving system(COSTAS Loop)** **June-10,8M July-08,10M**

❖ **Write a note on how coherent detection is used in DSB-SC receiver** **July-06,7M**

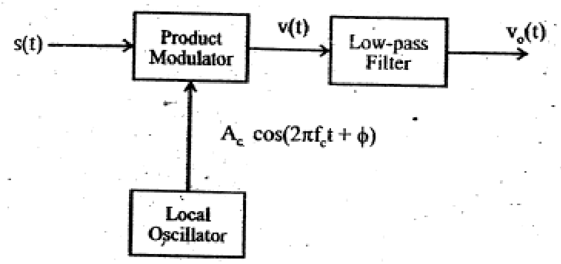


Fig. : Coherent detective for DSBSC

* The modulating Signal m(t) is recovered from a DSB-SC wave S(t) by first multiplying S(t) with a locally generated carrier wave and then low pass filtering the product as shown in fig ①.

* For faithful recovery of modulating Signal m(t), the local oscillator o/p should be exactly coherent or Synchronized in both frequency and phase with the carrier wave C(t) used in the product modulator to generate $V_o(t)$ with the local oscillator o/p equal to $\cos(2\pi f_c t + \phi)$.

The product modulator o/p can be given as:

$$V(t) = S(t) \cdot \cos(2\pi f_c t + \phi) \longrightarrow ①$$

W.K.T $\quad S(t) = A_c \cos 2\pi f_c t \cdot m(t) \longrightarrow ②$

Substituting equation ② in equation ①, we get

$$V(t) = A_c \cos(2\pi f_c t + \phi) \cos(2\pi f_c t) \cdot m(t)$$

W.K.T $\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$

$$V(t) = \frac{A_c m(t)}{2}\left[\cos(\cancel{2\pi f_c t} + \phi - \cancel{2\pi f_c t})\right] + \frac{A_c m(t)}{2}\left[\cos(2\pi f_c t + \phi + 2\pi f_c t)\right]$$

$$V(t) = \frac{A_c m(t)}{2}\cos\phi + \frac{A_c m(t)}{2}\cos(4\pi f_c t + \phi) \longrightarrow (2)$$

{ Taking Fourier transform on both sides of equation (2), we get

$$V(f) = \frac{A_c}{2} M(f)\cos\phi + \frac{A_c}{4}\left[M(f - 2f_c) + M(f + 2f_c)\right]$$

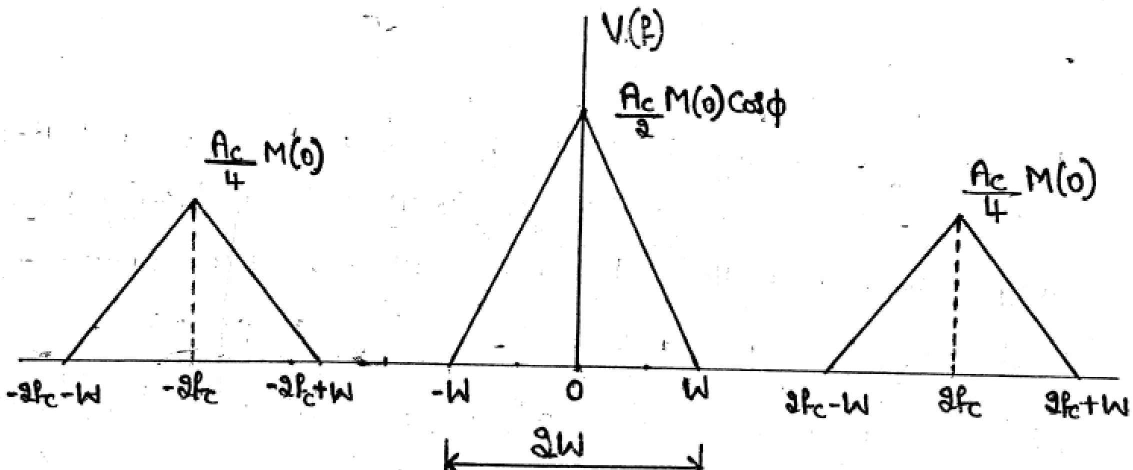


Fig (2) Amplitude Spectrum of v(f). }

* The desired message signal is obtained by passing V(t) through a LPF having the bandwidth greater than 'W' Hz but less than '$2f_c - W$' Hz.

* The o/p of the LPF is

$$\boxed{V_o(t) = \frac{A_c}{2}\cos\phi \, m(t)}$$

The demodulated signal $V_o(t)$ is therefore proportional to $m(t)$.

Where, $\phi \longrightarrow$ phase error.

When ϕ = Constant, $V_0(t)$ is proportional to $m(t)$

When $\phi = 0$, Amplitude of $V_0(t)$ is maximum.

When $\phi = \pm\pi/2$, Amplitude of $V_0(t)$ is minimum (represents the quadrature Null effect of the Coherent detector)

COSTAS LOOP:-

- **Explain the method of obtaining a practical synchronous receiving system with DSB-SC modulated waves using COSTAS loop** **June-10,8M July-08,10M**

- **With a neat block diagram explain the synchronous receiving system for receiving DSB-SC modulated waves.** **July-07,6M**

- **With neat block diagram of DSB-SC, the detection using COSTAS receiver.** **Jan-09,6M July-09, 5M**

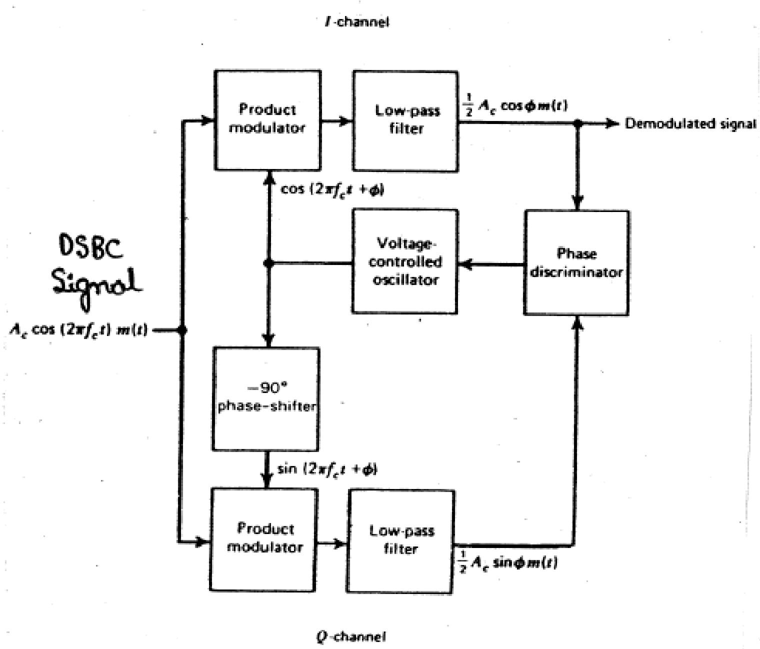


Fig : Costas loop

* The Costas loop is a method of obtaining a practical Synchronous receiver System, Suitable for demodulating - DSB-SC Waves.

* The receiver Consists of two Coherent detectors Supplied with the Same I/p Signal (DSB-SC Wave) $A_c \cos(2\pi f_c t) m(t)$, but with individual local oscillator Signals that are in-phase Quadrature with respect to each other. (i.e. the local - oscillator Signal Supplied to the product modulators are 90° out of phase).

* The frequency of the local oscillator is adjusted to be the Same as the carrier frequency 'f_c'.

* The detector in the upper path is referred to as the In-phase Coherent detector or I - Channel and that in the lower path is referred to as the Quadrature-phase Coherent detector or Q - Channel.

* These two detectors are Coupled together to form a Negative Feedback System designed in Such a way as to maintain the local oscillator Synchronous with the carrier wave.

operation:-

i) When local oscillator Signal is of the Same phase as the carrier Wave $A_c \cos(2\pi f_c t)$ used to generate the incoming DSB-SC Wave under these Conditions, the I-Channel o/p contains the - desired demodulated Signal $m(t)$, where as Q-Channel o/p is Zero.

$$V_{OI} = \frac{1}{2} A_c m(t) \cos\phi$$

i.e. Whenever the carrier is synchronized

$\phi = 0$ and $\cos\phi = \cos(0) = 1$

$$\boxed{V_{OI} = \frac{1}{2} A_c m(t)}$$ and

$$\sin\phi = \sin(0) = 0$$

$$\boxed{V_{OQ} = 0}$$

ii) When local oscillator phase changes by a small angle 'ϕ' radians, the I-channel o/p will remain unchanged, but Q-channel produces some o/p which is proportion to $\sin\phi$.

The o/p of I and Q-channels are combined in phase-discriminator (which consists of a multiplier followed by a LPF), a dc control signal is obtained that automatically corrects for local phase errors in the voltage controlled-oscillator (VCO).

Disadvantages of DSB-SC Coherent detection :-

Amplitude of the demodulated signal is maximum when $\phi = 0$ & minimum when $\phi = \pm \pi/2$ So, perfect synchronization has to be achieved for detection which inturn increases the cost of the receiver.

The received multiplexed signal $S(t)$ is applied to the two product modulator. The o/p of the Top product modulator is given by $\boxed{S_1(t) = S(t) \cos 2\pi f_c t}$

The top LPF removes the high frequency term and allows only $\frac{A_c m_1(t)}{2}$.

$$\therefore \quad \boxed{S_1(t) = \frac{A_c}{2} m_1(t)}$$

* The o/p of the bottom product modulator is given by

$$S_2(t) = S(t) \cdot \sin 2\pi f_c t.$$

* The bottom LPF removes the high frequency term & allows only $\frac{A_c m_2(t)}{2}$, Thus the o/p of LPF is

$$\boxed{S_2(t) = \frac{A_c}{2} m_2(t)}$$

* For correct operation of the Quadrature carrier multiplexing-System it is necessary to maintain the correct phase and frequency relationship between the local oscillator used in transmitter and receiver of the System.

Salient features of QAM:-

1) We can transmit more number of DSB-SC wave within the Same Channel bandwidth.

2) QAM is a bandwidth - Conservation Scheme.

3) QAM finds application in Colour Television (CTV)

* The o/p of the top product modulator is given by :

$$\boxed{S_1(t) = S(t)\cos 2\pi f_c t}$$

$$S_1(t) = \left[A_c m_1(t)\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t\right]\cos 2\pi f_c t.$$

$$S_1(t) = A_c m_1(t)\cos 2\pi f_c t.\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t.\cos 2\pi f_c t.$$

$$S_1(t) = A_c m_1(t)\cos^2 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t.\cos 2\pi f_c t.$$

W.K.T

$$\boxed{\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}} \quad \text{and} \quad \boxed{\sin A\cos B = \frac{1}{2}\sin(A-B) + \sin(A+B)}$$

$$S_1(t) = A_c m_1(t)\left[\frac{1}{2} + \frac{\cos 2(2\pi f_c t)}{2}\right] + \frac{A_c m_2(t)}{2}\sin\left[\cancel{2\pi f_c t - 2\pi f_c t}\right]^{0} + \frac{A_c m_2(t)}{2}\sin\left[2\pi f_c t + 2\pi f_c t\right]$$

$$\boxed{S_1(t) = \frac{A_c m_1(t)}{2} + \frac{A_c m_1(t)\cos 4\pi f_c t}{2} + \frac{A_c m_2(t)}{2}\sin\left[4\pi f_c t\right]}$$

* The top LPF removes the high frequency term & allows only $\frac{A_c m_1(t)}{2}$

$$\therefore \quad \boxed{S_1(t) = \frac{A_c}{2}m_1(t)}$$

* The o/p of the bottom product modulator is given by :

$$S_2(t) = S(t)\sin 2\pi f_c t$$

$$S_2(t) = \left[A_c m_1(t)\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t\right]\sin 2\pi f_c t$$

$$S_2(t) = A_c m_1(t)\sin 2\pi f_c t.\cos 2\pi f_c t + A_c m_2(t).\sin^2 2\pi f_c t.$$

W.K.T

$$\boxed{\sin^2\theta = \left[\frac{1}{2} - \frac{\cos 2\theta}{2}\right]} \quad \& \quad \boxed{\sin A.\cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)}$$

$$S_2(t) = \frac{A_c m_1(t)}{2}\sin\left[2\pi f_c t - 2\pi f_c t\right] + \frac{A_c m_1(t)}{2}\sin\left[2\pi f_c t + 2\pi f_c t\right] + \frac{A_c m_2(t)}{2} - \frac{A_c m_2(t)}{2}\cos 4\pi f_c t.$$

* The bottom LPF removes the high frequency term & allows only $\frac{A_c}{2} m_2(t)$.

Thus the o/p of LPF is

$$\boxed{s_2(t) = \frac{A_c}{2} m_2(t)}$$

Distortion in Envelope detector:-

❖ **Discuss the drawbacks of envelope detector** **Jan-10,4m**

There are two types of distortions which can occur in the detector output. They are:

1. **Diagonal Clipping and**
2. **Negative peak clipping.**

Diagonal Clipping:

This type of distortion occurs when the **RC time constant** of the load current is **too long**. Due to this the RC circuit cannot follow the fast change in the modulating envelope and is as shown in fig1.

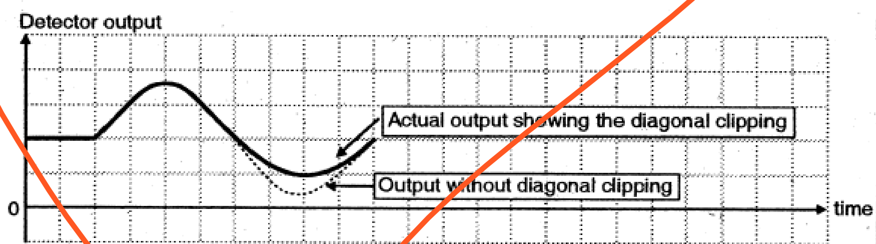


Fig. 1 : Diagonal clipping

Negative Peak Clipping:

This distortion occurs due to a fact that the modulation index on the output side of the detector is higher than that on its input side. So at higher depths of modulation of the transmitted signal, the **over modulation** may take place at the output of the detector. As a result negative peak clipping take place as shown in fig2.

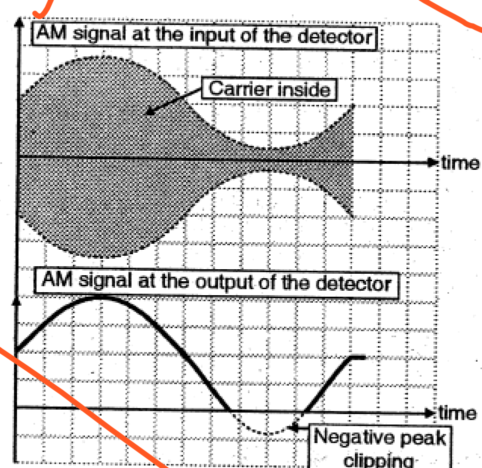


Fig. 2 : Negative peak clipping

Remedy: The distortions in detector output is reduced or eliminated by **properly choosing RC time constant.**

❖ **What is DSB-SC modulation? What are the advantages and limitations of DSB-SC as compared to standard AM?** June-09,6M Jan-05,5M

Advantages:

1. **Low Power consumption or power saving.**
2. **The modulation system is simple.**
3. **Efficiency is more than AM**
4. **Carrier wave is suppressed**
5. **Linear modulation type is required**
6. **It can be used for point to point communication**

Disadvantages:

1. **Design of receiver is complex**
2. **Bandwidth required is same as that of AM**

Application:

1. **Analogue TV systems to transmit color information.**

FORMULAE

1. Equation for AM wave : $s(t)=A_c(1+\mu\cos 2\pi f_m t)\cos 2\pi f_c t$

2. Modulation Index : $\mu = Am/Ac$

3. Amplitude of each sideband : $\mu Ac/2$

4. Upper Sideband Freq. : $f_{USB} = (fc+fm)$

5. Lower Sideband Freq. : $f_{LSB} = (fc-fm)$

6. Bandwidth of AM : $BW= 2f_m$

7. Total Txed Power : $P_t=P_c+P_{USB}+P_{LSB}$

   $P_t= Pc\ (1+ \mu^2/2)$

   $Pt=I_t^2R$

8. Power in each sideband : $P_{USB} = P_{LSB} = Pc\ (\mu/4)$

9. Total Sideband power P_{SB} : $P_{USB} + P_{LSB} = Pc\ (\mu^2/2)$

   $P_{SB} = P_T - P_c$

10. Power Wastage : $Pc + Pc\ (\mu^2/4)$

11. Transmission efficiency : $\eta = \mu^2/(2+ \mu^2)$

12. Carrier Power : $Pc=Ac^2/2R$

    $Pc=Ic^2/R$

13. Maximum Freq. in AM wave : $f_{max} = fc+fm$

14. Minimum Freq. in AM wave : $f_{min} = fc-fm$

15. Modulation index from AM Wave :

$$\mu = (Amax-Amin)/Amax+Amin)$$

Modulation by Several sinewaves

16. AM Wave with two Modulating signals :

$$s(t)=A_c(1+\mu_1\cos 2\pi f_{m1}t+\mu_2\cos 2\pi f_{m2}t)\cos 2\pi f_c t$$

17. Transmitted power : $Pt=Pc(1+ \mu^2{}_1/2 + \mu^2{}_2/2)$

18. Total Modulation index or Effective modulation index

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

19. Total Power : $Pt=Pc(1 + \mu^2{}_t / 2)$

20. Amplitudes of AM Wave : $Amax = Ac\ (1 + \mu^2)$

$Amin = Ac\ (1 - \mu^2)$

21. Peak Amplitude of carrier : $Ac = (Amax + Amin) / 2$

22. Peak Amplitude of message signal : $Ac = (Amax - Amin) / 2$

23. Modulation index from AM wave :

$$\mu = (Amax + Amin) / (Amax - Amin)$$

An amplitude modulated signal is given by

$s(t)=[10\cos(2\pi x10^6t) + 5\cos(2\pi x10^3t) + 2\cos(2\pi x10^6t) + \cos(4\pi x10^3t)]$ volts.

Find i) total modulated power ii) Sideband power and iii) net modulation index.

Jan-10,6M

Sol :-

WKT

$$\boxed{S(t) = A_c\left[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t\right] \cos 2\pi f_c t} \longrightarrow (1)$$

Given

$$S(t) = \left[10 \cos(2\pi \times 10^6 t) + 5 \cos(2\pi \times 10^6 t) \cos(2\pi \times 10^3 t) + 2 \cos(2\pi \times 10^6 t) \cos(4\pi \times 10^3 t)\right]$$

$$S(t) = 10 \cos(2\pi \times 10^6 t)\left[1 + \frac{5}{10} \cos(2\pi \times 10^3 t) + \frac{2}{10} \cos(4\pi \times 10^3 t)\right]$$

$$\boxed{S(t) = 10 \cos(2\pi \times 10^6 t)\left[1 + 0.5 \cos(2\pi \times 10^3 t) + 0.2 \cos(4\pi \times 10^3) t\right]} \rightarrow (2)$$

Comparing eq (1) & (2), we get

$A_c = 10V$, $\mu_1 = 0.5$, $\mu_2 = 0.2$, $f_1 = 1 \times 10^3$ Hz, $f_2 = 2 \times 10^3$ Hz, $f_c = 1 \times 10^6$ Hz

* Net modulation index $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.5)^2 + (0.2)^2}$

$$\boxed{\mu_t = 0.538} \longrightarrow \boxed{\text{2 Marks}}$$

$R = 1\Omega$

* Carrier power $P_c = \frac{A_c^2}{2R} = \frac{(10)^2}{2 \times 1}$

$$\boxed{P_c = 50W}$$

* Sideband power $P_{SB} = P_{USB} + P_{LSB} = \frac{\mu_t^2}{2} P_c = \frac{(0.538)^2}{2} 50$

$$\boxed{P_{SB} = 7.25W} \longrightarrow \boxed{\text{2 Marks}}$$

* Total modulated power

$$P_T = P_c\left[1+\frac{\mu_t^2}{2}\right]$$

$$= 50\left[1+\frac{0.538^2}{2}\right]$$

$\boxed{P_T = 57.25\,W}$ $\longrightarrow$ $\boxed{2\text{ Marks}}$

(OR)

$$P_T = P_c + P_{SB}$$

$$= 50W + 7.25W$$

$$\boxed{P_T = 57.25\,W}$$

NOTE :-

* Sideband power $P_{SB} = P_T - P_c = 57.25W - 50W$

$$\boxed{P_{SB} = 7.25\,W}$$

Consider a message signal m(t)=20cos(2πt)volts and a carrier signal c(t)=50cos(100πt)volts.

i. Sketch to scale resulting AM wave for 75% modulation.

ii. Find the power delivered across a load of 100Ω due to this AM wave. June-10,6M

Given :- $A_m = 20V$, $f_m = 1Hz$, $A_c = 50V$, $f_c = 50Hz$, $\mu = 0.75$ & $R = 100\Omega$

WKT AM wave is given by

$$S(t) = A_c\left[1+\mu\cos 2\pi f_m t\right]\cos 2\pi f_c t$$

$\therefore$ $\boxed{S(t) = 50\left[1+0.75\cos 2\pi(1)t\right]\cos 2\pi(50)t.}$ $\longrightarrow$ $\boxed{2\text{ Marks}}$

i) $A_{max} = A_c(1+\mu) = 50(1+0.75) = \underline{87.5V}$

$A_{min} = A_c(1-\mu) = 50(1-0.75) = \underline{12.5V}$

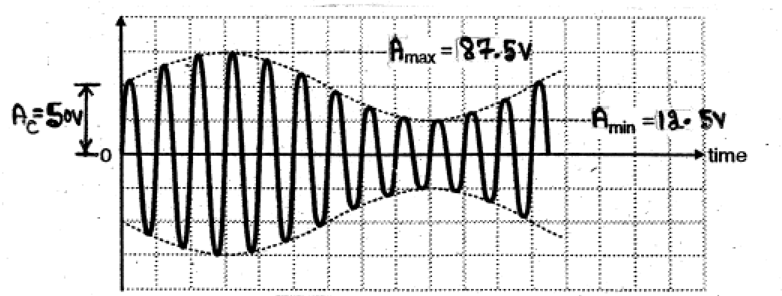


Fig. : AM wave for m = 0.75

$\longrightarrow$ 2Marks

ii) $P_T = P_c\left[1+\frac{\mu^2}{2}\right]$

$* \; P_c = \frac{A_c^2}{2R} = \frac{50^2}{2\times100} = \underline{12.5\,W}$ $\longrightarrow$ 1 Mark

$P_T = 12.5\left[1+\frac{0.75^2}{2}\right]$

$\boxed{P_T = 16.015\,W}$. $\longrightarrow$ 1 Mark

A carrier wave with amplitude 12V and frequency 10 MHz is amplitude modulated to 50% level with a modulated frequency of 1 KHz. Write down the equation for the above wave and sketch the modulated signal in frequency domain. June-10,7M

Given : $A_c = 12V$, $f_c = 10MHz$, $\mu = 0.5$, $f_m = 1KHz$

Sol :-

WKT AM wave is given by :

$S(t) = A_c[1+\mu \cos 2\pi f_m t]\cos 2\pi f_c t$

$\boxed{S(t) = 12[1+0.5\cos 2\pi(1\times10^3)t]\cos 2\pi(10\times10^6)t}$ $\longrightarrow$ 3 Marks

WKT for Single tone modulation is given by

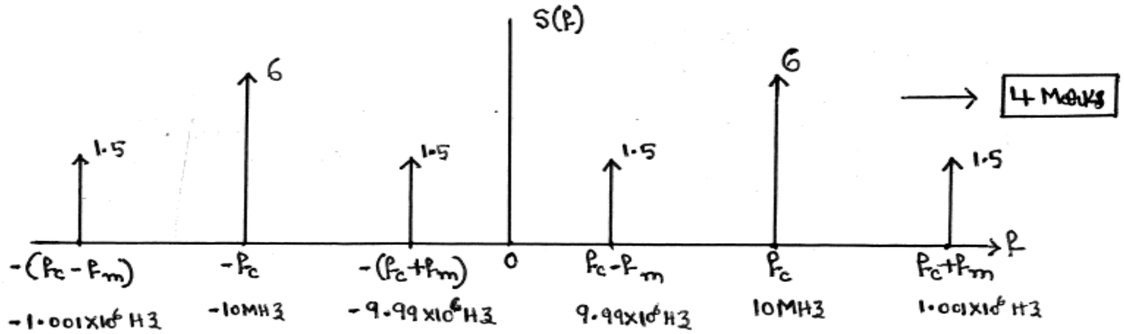
$$s(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t$$

$$s(t) = 12 \cos 2\pi (10 \times 10^6 t) + \frac{0.5 \times 12}{2} \cos 2\pi \left(10 \times 10^6 + 1 \times 10^3\right) t$$
$$+ \frac{0.5 \times 12}{2} \cos 2\pi \left(10 \times 10^6 - 1 \times 10^3\right) t$$

$$s(t) = 12 \cos 2\pi (10 \times 10^6 t) + 3 \cos 2\pi (1.001 \times 10^6 t) + 3 \cos 2\pi (9.99 \times 10^6 t) \longrightarrow (1)$$

Taking FT on both side of eq (1), we get

$$S(f) = \frac{12}{2}\left[\delta(f - 10 \times 10^6) + \delta(f + 10 \times 10^6)\right] + \frac{3}{2}\left[\delta(f - 1.001 \times 10^6) + \delta(f + 1.001 \times 10^6)\right] + \frac{3}{2}\left[\delta(f - 9.99 \times 10^6) + \delta(f + 9.99 \times 10^6)\right]$$



$\longrightarrow$ $\boxed{\text{4 Marks}}$

Consider a message signal m(t)=20cos(2πt)volts and a carrier signal c(t)=50cos(100πt)volts.

i. The resulting AM wave for 75% modulation.

ii. Sketch the Spectrum of this AM wave

iii. Find the power developed across the load of 100Ω. Jan-08, 10M June-10,10M

Given : $m(t) = 20 \cos 2\pi t \, V$, $c(t) = 50 \cos 100\pi t$ & $\mu = 0.75$

$\omega_m = 2\pi$	$\omega_c = 100\pi$
$\cancel{2\pi} f_m = \cancel{2\pi}$	$\cancel{2\pi} f_c = \cancel{100\pi}\ 50$
$f_m = 1Hz$	$f_c = 50Hz$

i) WKT

$$s(t) = A_c[1+\mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$s(t) = 50[1+0.75 \cos 2\pi(1) t] \cos 2\pi(50) t$$

$$s(t) = 50 \cos 2\pi(50) t + \underline{37.5 \cos 2\pi(50) t \cdot \cos 2\pi(1) t}$$

$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$s(t) = 50 \cos 2\pi(50) t + \frac{37.5}{2}\cos 2\pi(50-1) t + \frac{37.5}{2}\cos 2\pi(50+1) t$$

$$\boxed{s(t) = 50 \cos 2\pi(50) t + 18.75 \cos 2\pi(49) t + 18.75 \cos 2\pi(51) t} \longrightarrow (1)$$

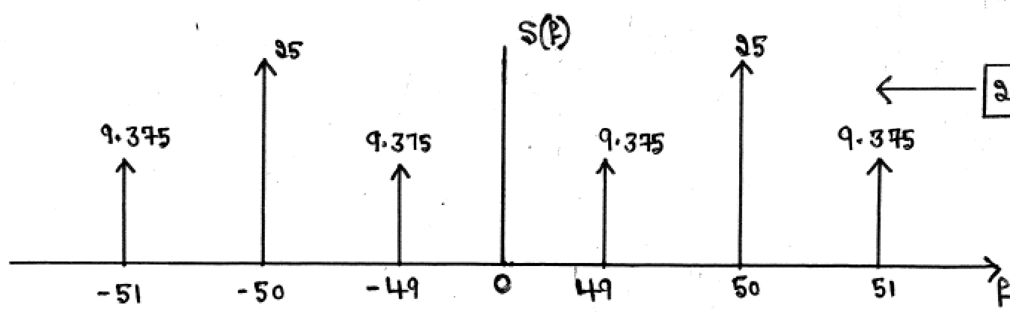
3 Marks

ii) Taking FT of eq (1), we get

$$S(f) = \frac{50}{2}[\delta(f-50)+\delta(f+50)] + \frac{18.75}{2}[\delta(f-49)+\delta(f+49)] + \frac{18.75}{2}[\delta(f-51)+\delta(f+51)]$$

$$\boxed{\begin{aligned} S(f) = 25[\delta(f-50)+\delta(f+50)] + 9.375[\delta(f-49)+\delta(f+49)] \\ + 9.375[\delta(f-51)+\delta(f+51)] \end{aligned}}$$

2 Marks



2 Marks

iii) $P_T = P_c\left(1+\frac{\mu^2}{2}\right)$

$$P_c = \frac{A_c^2}{2R} = \frac{(50)^2}{2\times 100} = \underline{12.5\,W}$$

$$P_T = 12.5\left(1+\frac{0.75^2}{2}\right) = \underline{16\,W}$$

3 Marks

The antenna current of an AM broadcast transmitter modulated to a depth of 40% by an audio sine wave is 11A. It increases to 12A as a result of sinusoidal modulation by another audio sine wave. What is the modulation index due to second wave?

OLD June-10,6M

Given :

i) $\mu_1 = 0.4$, $I_{\pm 1} = 11A$, $I_c = ?$

ii) $I_{\pm 2} = 12A$, $\mu_2 = ?$

Sol :

i) $I_{\pm 1} = I_c \sqrt{1 + \frac{\mu_1^2}{2}}$

$$I_c = \frac{I_{\pm 1}}{\sqrt{1 + \frac{\mu_1^2}{2}}} = \frac{11}{\sqrt{1 + \frac{0.4^2}{2}}}$$

$$\boxed{I_c = 10.58A}$$

ii) $I_{\pm 2} = I_c \sqrt{1 + \frac{\mu_\pm^2}{2}}$

$$I_{\pm 2}^2 = I_c^2 \left(1 + \frac{\mu_\pm^2}{2}\right)$$

$$\frac{I_{\pm 2}^2}{I_c^2} = 1 + \frac{\mu_\pm^2}{2}$$

$$\frac{\mu_\pm^2}{2} = \left(\frac{I_{\pm 2}^2}{I_c^2}\right) - 1$$

$$\frac{\mu_\pm^2}{2} = \left(\frac{12^2}{10.58^2}\right) - 1$$

$$\frac{\mu_\pm^2}{2} = 1.286 - 1$$

$$\frac{\mu_\pm^2}{2} = 0.286$$

$$\mu_\pm^2 = 0.572$$

$$\boxed{\mu_t = 0.757}$$

* WKT $\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$

$$\mu_t^2 = \mu_1^2 + \mu_2^2$$

$$\mu_2^2 = \mu_t^2 - \mu_1^2$$

$$= 0.757^2 - 0.4^2$$

$$\mu_2^2 = 0.4130$$

$$\boxed{\mu_2 = 0.642}$$

Find the ratio of maximum average power to unmodulated carrier power in AM

Sol :- Jan-07,4M

WKT $P_T = P_c\left(1 + \frac{\mu^2}{2}\right)$

When $\mu = 1$ i.e. for 100% modulation

$$P_{T(max)} = P_c\left[1 + \frac{(1)^2}{2}\right]$$

$$P_{T(max)} = 1.5\,P_c$$

$$\frac{P_{T(max)}}{P_c} = 1.5$$

$$\frac{P_{T(max)}}{P_c} = \frac{1.5}{1}$$

$\therefore$ $\boxed{P_{T(max)} : P_c \text{ is } 1.5 : 1}$

An audio frequency signal 5sin2π(1000)t is used to amplitude modulate a carrier of 100sin2π(10⁶)t. Assume modulation index of 0.4. Find

- **i. Sideband frequencies**
- **ii. Bandwidth required**
- **iii. Amplitude of each sideband**
- **iv. Total power delivered to a load of 100Ω**

Jan-05,10M

Sol:- Given :

$A_m = 5$, $A_c = 100$, $\mu = 0.4$, $f_m = 1000\,Hz$, $f_c = 1\times10^6\,Hz$.

i) Sideband frequencies:

$$f_{USB} = f_c + f_m = 1MHz + 1000\,Hz = 1.001\,MHz$$

$$f_{LSB} = f_c - f_m = 1MHz - 1000\,Hz = 999\,000\,Hz = 0.999\,MHz$$

ii) Amplitude of each Sideband frequencies:

$$\frac{\mu A_c}{2} = \frac{0.4 \times 100}{2} = 20V.$$

∴ Amplitude of upper & lower Sideband is 20V.

iii) Bandwidth required:

$$BW = 2f_m = 2 \times 1KHz = 2KHz$$

or

$$BW = f_{USB} - f_{LSB} = 1.001\,MHz - 999000\,Hz = 2KHz$$

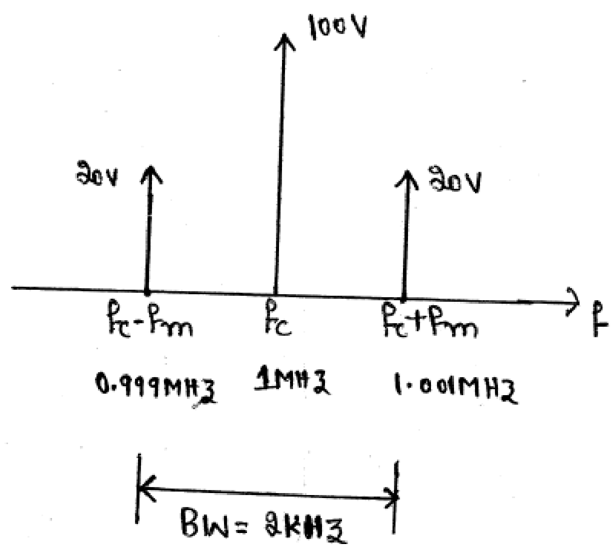
iv) Total power delivered to a load of 100 Ω:

W.K.T $$P_T = P_c\left[1+\frac{\mu^2}{2}\right] = \frac{A_c^2}{2R}\left[1+\frac{\mu^2}{2}\right]$$

$$= \frac{(100)^2}{2\times 100}\left[1+\frac{(0.4)^2}{2}\right]$$

$$\boxed{P_T = 54W}$$

v) Spectrum of AM wave:



A 1000KHz carrier is simultaneously modulated by 300Hz, 800Hz and 2KHz audio sine waves. What will be the frequency content of AM signals. **July-05,6M**

Sol: Given ; $f_c = 1000KHz$

$f_{m1} = 300Hz$, $f_{m2} = 800Hz$ & $f_{m3} = 2000Hz$

* $f_{USB1} = f_c + f_{m1} = 1000KHz + 300Hz = 1000.3KHz$
* $f_{LSB1} = f_c - f_{m1} = 1000KHz - 300Hz = 999.7KHz$.

* $f_{USB2} = f_c + f_{m2} = 1000KHz + 800Hz = 1000.8KHz$
* $f_{LSB2} = f_c - f_{m2} = 1000KHz - 800Hz = 999.2KHz$
* $f_{USB3} = f_c + f_{m3} = 1000KHz + 2KHz = 1002KHz$

* $f_{LSB3} = f_c - f_{m3} = 1000KHz - 2KHz = 998KHz$

A carrier wave $4\sin(2\pi x500x10^3t)$volts is amplitude modulated by an audio wave [0.2 sin $3(2\pi x500t)$ + 0.1 sin $5(2\pi x500t)$] volts. Determine the upper and lower sideband and sketch the complete spectrum of the modulated wave. Estimate the total power in the sideband. June-09,6M

Sol: Given : $c(t) = 4\sin(2\pi \times 500\times10^3 t) \rightarrow A_c = 4V,\ f_c = 500KHz$

$$m(t) = \underset{A_{m1}}{0.2}\sin 2\pi(\underset{f_{m1}}{1500})t + \underset{A_{m2}}{0.1}\sin 2\pi(\underset{f_{m2}}{2500})t \rightarrow$$

The message Signal Consists of two Sinewaves.

$A_{m1} = 0.2V$, $f_{m1} = 1500Hz$

$A_{m2} = 0.1V$, $f_{m2} = 2500Hz$

* USB & LSB :-

i) $USB_1 = (f_c + f_{m1}) = 500KHz + 1.5KHz = 501.5KHz$

$LSB_1 = (f_c - f_{m1}) = 500KHz - 1.5KHz = 498.5KHz$

ii) $USB_2 = (f_c + f_{m2}) = 500KHz + 2.5KHz = 502.5KHz$

$LSB_2 = (f_c - f_{m2}) = 500KHz - 2.5KHz = 497.5KHz.$

* Modulation Index of individual modulating Signals :

i) Modulation Index for 1st Signal $\mu_1 = \frac{A_{m1}}{A_c} = \frac{0.2}{4} = 0.05$

ii) Modulation Index for 2nd Signal $\mu_2 = \frac{A_{m2}}{A_c} = \frac{0.1}{4} = 0.025$

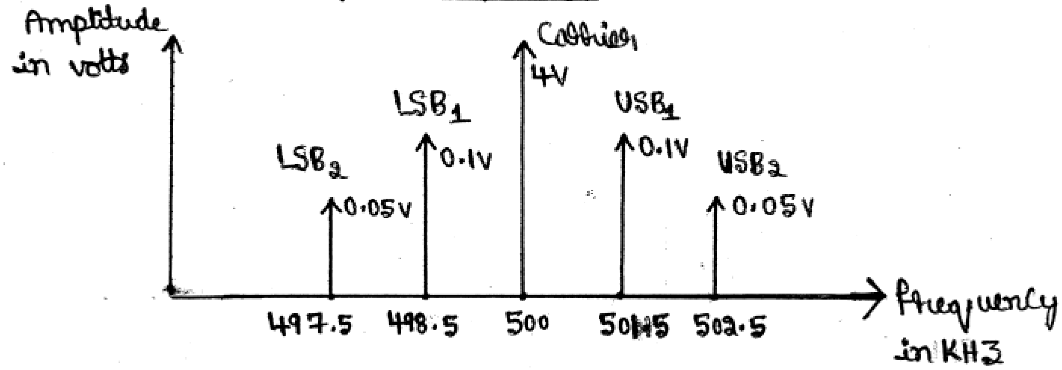
* Sideband amplitudes:

In general, amplitude of each Sideband is given by $\frac{\mu A_c}{2}$

i) Amplitude of USB_1 & LSB_1 will be: $\frac{\mu_1 A_c}{2} = \frac{0.05 \times 4}{2} = 0.1V$

ii) Amplitude of USB_2 & LSB_2 will be: $\frac{\mu_2 A_c}{2} = \frac{0.025 \times 4}{2} = 0.05V$

* Complete Spectrum of AM Signal:



* Total power in the Sidebands:-

WKT, the total power in the Sidebands is given by

$$P_{SB} = P_{USB} + P_{LSB} = P_c\left(\frac{\mu^2}{2}\right)$$

For two Signals, $P_{SB} = P_c\left(\frac{\mu_t^2}{2}\right)$

Where,

μ_t = total modulation Index =

i.e. $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.05)^2 + (0.025)^2} = 0.0559$

$$\therefore \quad P_{SB} = P_c\left[\frac{\mu_t^2}{2}\right] \qquad \text{W.K.T. } P_c = \frac{A_c^2}{2R}$$

$$= \frac{A_c^2}{2R}\left[\frac{\mu_t^2}{2}\right]$$

$$= \frac{(4)^2}{2R}\left[\frac{(0.0559)^2}{2}\right]$$

$$= \frac{\cancel{16}8}{\cancel{2}R}\left[1.56\times10^{-3}\right]$$

$$= \frac{8}{R}\left[1.56\times10^{-3}\right]$$

$$\boxed{P_{SB} = \frac{0.0125}{R}}$$

A broadcast AM transmitter radiates 50Kw of carrier power. What will be the radiated power at 85% modulation? June-08,2M

Sol:- Given: $P_c = 50KW$ & $\mu = 0.85$

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right]$$

$$= 50\times10^3\left[1+\frac{(0.85)^2}{2}\right]$$

$$\boxed{P_T = 68.0625\ KW}$$

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* Consider the message signal $m(t) = 20\cos(2\pi t)$ volts & carrier wave $c(t) = 50\cos(100\pi t)$ volts. Derive an expression for the resulting AM wave for 75% modulation. August-2002

Sol: Given: $A_m = 20V$, $f_m = 1Hz$

$A_c = 50V$, $f_c = 50Hz$ & $\mu = 0.75$

WKT $s(t) = A_c\left[1+\mu\cos 2\pi f_m t\right]\cos 2\pi f_c t.$

$$\boxed{s(t) = 50\left[1+0.75\cos 2\pi t\right]\cos(100\pi t)}$$

An audio frequency signal $10\sin 2\pi(500)t$ is used to amplitude modulate a carrier of $50\sin 2\pi(10^5)t$. Assume modulation index = 0.2. Find

i. **Sideband frequencies**
ii. **Amplitude of each sideband**
iii. **Bandwidth required**

OLD June-09,6M

Given : $\mu = 0.2$, $f_m = 500\,Hz$, $f_c = 10^5\,Hz$, $A_m = 10V$, $A_c = 50V$

Sol :-

i) Sideband frequencies :

$$f_{USB} = f_c + f_m = 10^5 + 500 = \underline{1.0005 \times 10^6\ Hz}$$

$$f_{LSB} = f_c - f_m = 10^5 - 500 = \underline{0.9995 \times 10^6\ Hz}$$

ii) Amplitude of each Sideband :

$$\frac{\mu A_c}{2} = \frac{0.2 \times 50}{2} = \underline{5V}$$

iii) Bandwidth 'B' $= 2f_m = 2 \times 500 = \underline{1000\ Hz}$

An amplitude modulated signal is given by

$S(t)=10\cos 2\pi 10^6 t + 5\cos 2\pi \times 10^6 t \cos 2\pi 10^3 t + 2\cos 2\pi 10^6 t \cos 4\pi 10^3 t$ volts.

Find various frequency components present and the corresponding modulation indices. Draw the line spectrum and find the bandwidth.

Jan-07,12M

Given :-

$$S(t) = 10\cos 2\pi 10^6 t + 5\cos 2\pi 10^6 t \cos 2\pi 10^3 t + 2\cos 2\pi 10^6 t \cos 4\pi 10^3 t$$

$$S(t) = 10\cos 2\pi(10^6)t \left[1 + \frac{5}{10}\cos 2\pi(10^3)t + \frac{2}{10}\cos 2\pi(2\times 10^3)t\right.$$

$$S(t) = 10\cos 2\pi(10^6)t\left[1 + 0.5\cos 2\pi(10^3)t + 0.2\cos 2\pi(2\times10^3)t\right] \longrightarrow (1)$$

WKT

$$s(t) = A_c \cos 2\pi f_c t \left[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t \right] \longrightarrow (2)$$

Comparing eq (1) & eq (2), we get

$A_c = 10V$, $\mu_1 = 0.5$, $\mu_2 = 0.2$, $f_1 = 10^3 Hz$, $f_2 = 2\times10^3 Hz$ & $f_c = 10^6 Hz$

Equation (1) can be rewritten as

$$s(t) = 10 \cos 2\pi(10^6)t + 5 \cos 2\pi(10^6)t \cdot \cos 2\pi(10^3)t + 2 \cos 2\pi(10^6)t \cdot \cos 2\pi(2\times10^3)t$$

By using thignometric identity

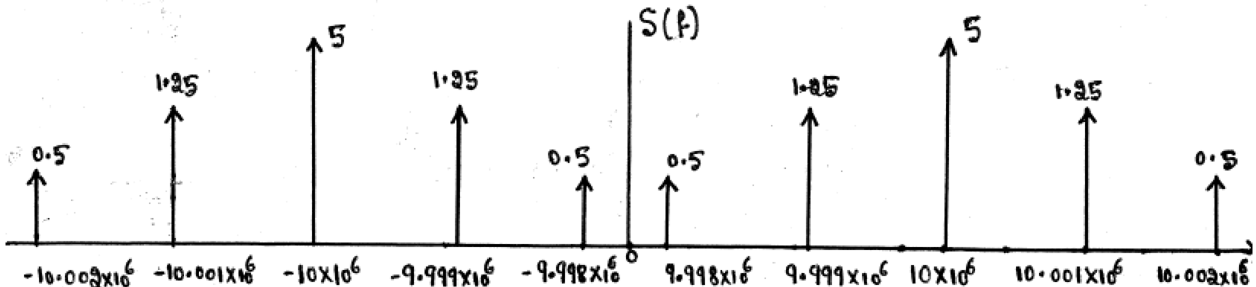
$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$s(t) = 10 \cos 2\pi(10^6)t + \frac{5}{2}\cos 2\pi(10^6 - 10^3)t + \frac{5}{2}\cos 2\pi(10^6 + 10^3)t$$
$$+ \frac{2}{2}\cos 2\pi(10^6 - 2\times10^3)t + \frac{2}{2}\cos 2\pi(10^6 + 2\times10^3)t$$

$$s(t) = 10 \cos 2\pi(10^6)t + 2.5 \cos 2\pi(9.99\times10^6)t + 2.5 \cos 2\pi(10.001\times10^6)t$$
$$+ \cos 2\pi(9.998\times10^6)t + \cos 2\pi(10.002\times10^6)t \longrightarrow (3)$$

Taking FT of eq (3), we get

$$S(f) = \frac{10}{2}\left[\delta(f-10^6) + \delta(f+10^6)\right] + \frac{2.5}{2}\left[\delta(f-9.999\times10^6) + \delta(f+9.999\times10^6)\right]$$
$$+ \frac{2.5}{2}\left[\delta(f-10.001\times10^6) + \delta(f+10.001\times10^6)\right] + \frac{1}{2}\left[\delta(f-9.998\times10^6) + \delta(f+9.998\times10^6)\right]$$
$$+ \frac{1}{2}\left[\delta(f-10.002\times10^6) + \delta(f+10.002\times10^6)\right]$$



* $BW = 2[\max(f_1, f_2)]$

$BW = 2f_2 = 2\times2KHz$

$\boxed{BW = 4KHz}$

BW = 4KHz

iii) W.K.T.

$$A_{max} = A_c(1+\mu) = 50(1+0.75) = 87.5V$$

$$A_{min} = A_c(1-\mu) = 50(1-0.75) = 12.5V$$

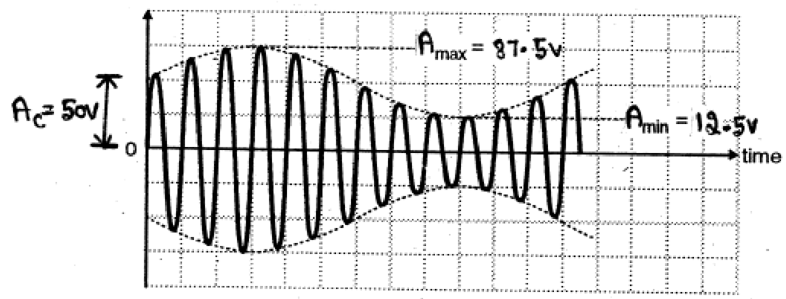


Fig. : AM wave for m = 0.75

An amplitude modulated waveform has the form

$$x_c(t) = 10[1 + 0.5 \cos 2000\pi t + 0.5 \cos 4000\pi t] \cos(20000\pi t).$$

i) Sketch the amplitude Spectrum of $x_c(t)$

ii) Find the average power content of each Spectral Component including the Carrier

iii) Modulation Index

August - 2002.

Sol: Given : $A_c = 10V$, $\mu_1 = 0.5$, $\mu_2 = 0.5$

$2\pi f_{m1} = 2000\pi$, $f_{m1} = \frac{2000\pi}{2\pi} = 1000Hz$

$2\pi f_{m2} = 4000\pi$, $f_{m2} = \frac{4000\pi}{2\pi} = 2000Hz$

$2\pi f_c = 20000\pi$, $f_c = \frac{20000\pi}{2\pi} = 10KHz.$

$\therefore f_{m1} = 1000Hz$, $f_{m2} = 2000Hz$, $f_c = 10KHz.$

* Consider a message signal $m(t) = 20\cos(2\pi t)$ V & the carrier wave $c(t) = 50\cos(100\pi t)$ V.

i) Write an expression for the resulting AM wave for 75% modulation in time domain.

ii) Draw the spectrum of AM wave.

iii) Sketch the resulting wave for 75% modulation.

July-06, 8M

Sol:- Given: $A_m = 20V$, $f_m = 1Hz$

$A_c = 50V$, $f_c = 50Hz$

$\mu = 0.75$

$\because 2\pi f_c = 100\pi$

$f_c = \frac{100\pi}{2\pi} = 50$

i) The AM signal is given by:

$$S(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t.$$

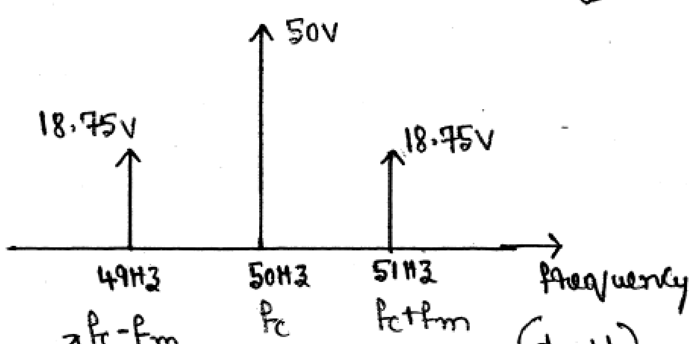
$$\boxed{S(t) = 50[1 + 0.75 \cos 2\pi t] \cos 2\pi(50)t.}$$

ii) Spectrum of AM wave:-

$f_{USB} = f_c + f_m = 50 + 1 = 51Hz$

$f_{LSB} = f_c - f_m = 50 - 1 = 49Hz.$

* Amplitude of each sideband is $\frac{\mu A_c}{2} = \frac{(0.75) \times 50}{2} = 18.75V$



NOTE:- By taking FT of S(t) plot the spectrum

* Spectrum of AM wave :-

i) $f_{USB1} = f_c + f_{m1} = 10KHz + 1KHz = 11KHz$

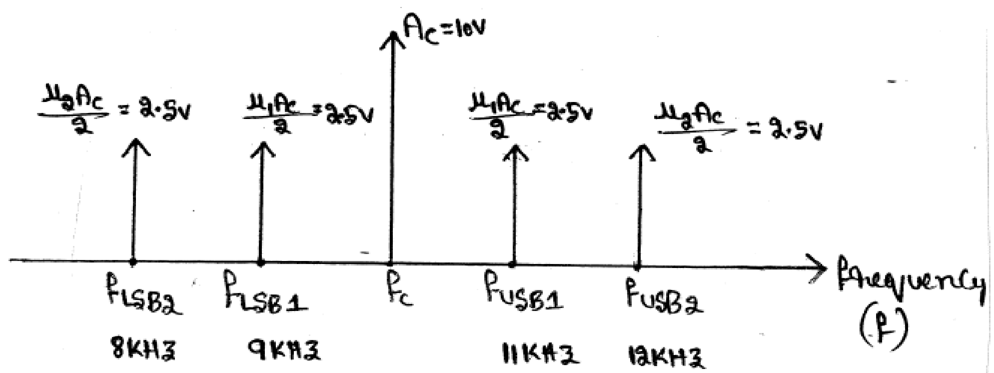
$f_{LSB1} = f_c - f_{m1} = 10KHz - 1KHz = 9KHz$

ii) $f_{USB2} = f_c + f_{m2} = 10KHz + 2KHz = 12KHz$

$f_{LSB2} = f_c - f_{m2} = 10KHz - 2KHz = 8KHz$

* Amplitude of each side band is i) $\frac{\mu_1 A_c}{2} = \frac{0.5 \times 10}{2} = 2.5V$

ii) $\frac{\mu_2 A_c}{2} = \frac{0.5 \times 10}{2} = 2.5V$



ii) Average power (P_T) :-

$$P_T = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

$$P_T = \frac{A_c^2}{2R} + \frac{\mu_1^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R} + \frac{\mu_1^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R}$$

$$= \frac{(10)^2}{2R} + \frac{(0.5)^2 (10)^2}{8R} + \frac{(0.5)^2 \times (10)^2}{8R} + \frac{(0.5)^2 \times (10)^2}{8R} + \frac{(0.5)^2 \times (10)^2}{8R}$$

$$\boxed{P_T = \frac{50}{R} + \frac{3.125}{R} + \frac{3.125}{R} + \frac{3.125}{R} + \frac{3.125}{R}}$$

iii) Modulation Index :-

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.5)^2 + (0.5)^2} = \underline{0.707}$$

Using message signal $m(t) = \frac{t}{1+t^2}$, determine & sketch the modulated wave for AM whose percentage modulation is equal to the following values.

i) $\mu = 50\%$ ii) $\mu = 100\%$ iii) $\mu = 125\%$

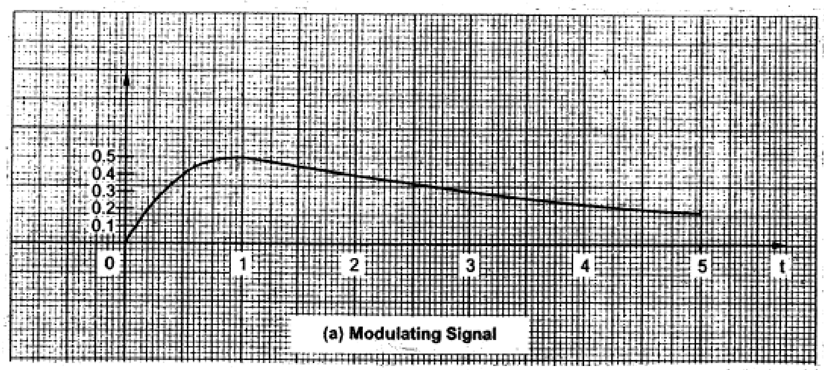
Feb, 2002, 6M

Sol: Message Signal $m(t)$:-

The modulating signal is determined from the following table & is shown in fig ①.

t	0	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0
m(t)	0	0.192	0.345	0.44	0.49	0.5	0.4	0.3	0.235	0.192

↑ Peak or maximum value of m(t)



(a) Modulating Signal

From above figure, the maximum amplitude of $m(t) = \underline{0.5V}$

* Calculate A_c for different values of μ :-

Given : $A_m = 0.5V$

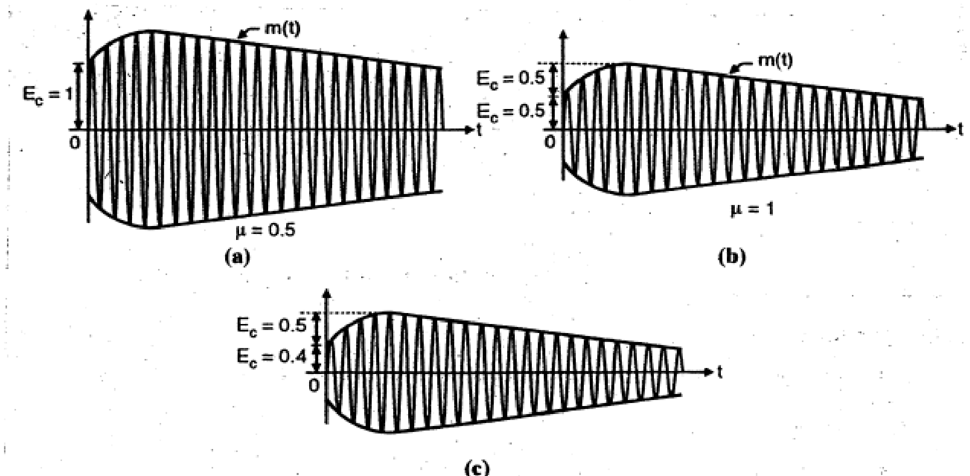
W.K.T $\boxed{\mu = \frac{A_m}{A_c}}$ & $\boxed{A_c = \frac{A_m}{\mu}}$

i) $\mu = 0.5$, $A_c = \frac{A_m}{\mu} = \frac{0.5V}{0.5} = 1V$

ii) $\mu = 1$, $A_c = \frac{A_m}{\mu} = \frac{0.5V}{1} = 0.5V$

iii) $\mu = 1.25$, $A_c = \frac{A_m}{\mu} = \frac{0.5V}{1.25} = 0.4V$

The Waveforms of the AM wave for different values of μ are Shown below



(a) AM wave for $\mu = 0.5$ (b) AM wave for $\mu = 1.0$ (c) AM wave for $\mu = 1.25$

Draw the Spectrum of an AM Signal with $c(t) = A_c \cos^2(\pi f_c)t$ & $m(t) = A_m \cos^2(\pi f_m t)$.

Sol: Given : $c(t) = A_c \cos^2(\pi f_c t)$

June-02, 6M

W.K.T $\boxed{\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}}$

$$c(t) = A_c\left[\frac{1}{2} + \frac{\cos 2\pi f_c t}{2}\right]$$

$$\therefore \quad c(t) = \frac{A_c}{2} + \frac{A_c}{2}\cos 2\pi f_c t. \longrightarrow (1)$$

Similarly

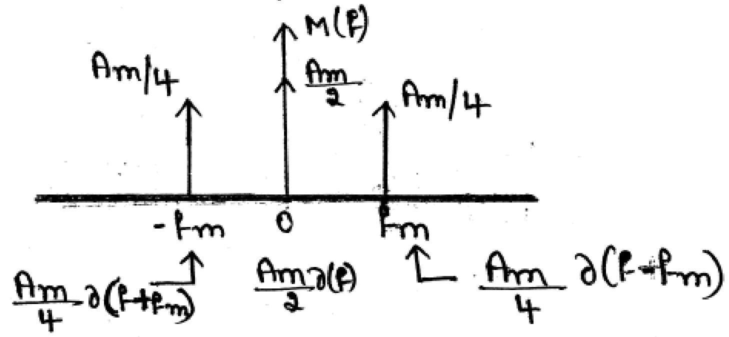
$$m(t) = A_m \cos^2(\pi f_m t)$$

$$m(t) = \frac{A_m}{2} + \frac{A_m}{2}\cos 2\pi f_m t. \longrightarrow (2)$$

* Spectrum of $m(t)$:-

We can get Spectrum of $m(t)$ by taking its Fourier transform i.e. eq ②.

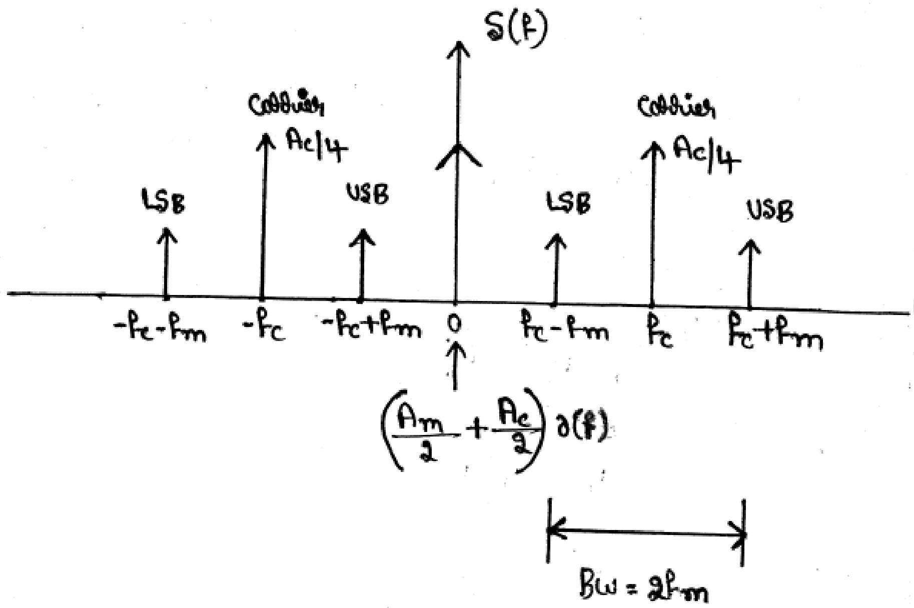
$$M(f) = \frac{A_m}{2}\delta(t) + \frac{A_m}{4}\left[\delta(f+f_m) + \delta(f-f_m)\right]$$



* Spectrum of AM Signal :-

Taking Fourier transform of $c(t)$ i.e. eq ①, we get

$$C(f) = \frac{A_c}{2}\delta(t) + \frac{A_c}{4}\left[\delta(f+f_c) + \delta(f-f_c)\right]$$



For a PN junction diode, the current 'i' through the diode and the voltage 'v' across it are related by

$$i = I_0\left[e^{-v/V_T} - 1\right]$$

Where 'I_0' is the reverse saturation current and 'V_T' is the thermal voltage. At room temperature, $V_T = 0.026$ volts.

ⓐ Expand 'i' as a power series in v, retaining terms upto v^2.

ⓑ Let $v = 0.01\cos 2\pi f_m t + 0.01\cos 2\pi f_c t$ volts, where $f_m = 1KHz$ & $f_c = 100KHz$. Sketch the spectrum of the diode current 'i'.

c) Specify the required bandpass filter to extract from 'i', an AM wave with carrier frequency 'f_c'.

d) What is the percentage modulation Index.

Jan-06, 12M

Solution :- ⓐ Given $v = 0.01\cos 2\pi f_m t + 0.01\cos 2\pi f_c t$ & $V_T = 0.026\,V$

$$i(t) = I_0\left[e^{-v/V_T} - 1\right] \longrightarrow ①$$

$$\frac{i(t)}{I_0} = \left[e^{-v/V_T} - 1\right] \longrightarrow ②$$

We can write $\boxed{e^{-x} = 1 - x + \frac{1}{2}x^2}$ Put $x = +v/V_T$

$$\boxed{e^{-v/V_T} = 1 - \frac{v}{V_T} + \frac{1}{2}\left(\frac{v}{V_T}\right)^2} \longrightarrow ③$$

Substituting equation ③ in eq ②, we get

$$\frac{i(t)}{I_0} = \cancel{1} - \frac{v}{V_T} + \frac{1}{2}\left(\frac{v}{V_T}\right)^2 - \cancel{1}$$

$$\boxed{\frac{i(t)}{I_0} = -\frac{v}{V_T} + \frac{1}{2}\left(\frac{v}{V_T}\right)^2} \longrightarrow (4)$$

b) Let $v = 0.01 \cos 2\pi f_m t + 0.01 \cos 2\pi f_c t$ volts. $\rightarrow$ (5)

divide both RHS and LHS of equation (5) by V_T

$$\frac{v}{V_T} = \frac{0.01}{V_T} \cos 2\pi f_m t + \frac{0.01}{V_T} \cos 2\pi f_c t.$$

given, $V_T = 0.026\,V$

$$\frac{v}{V_T} = \frac{0.01}{0.026} \cos 2\pi f_m t + \frac{0.01}{0.026} \cos 2\pi f_c t$$

$$\frac{v}{V_T} = 0.384 \cos 2\pi f_m t + 0.384 \cos 2\pi f_c t \longrightarrow (6)$$

Substituting equation (6) in equation (4), we get

$$\frac{i(t)}{I_0} = -0.384 \cos 2\pi f_m t - 0.384 \cos 2\pi f_c t + \frac{1}{2}\left[0.384 \cos 2\pi f_m t + 0.384 \cos 2\pi f_c t\right]^2$$

W.K.T. $(a+b)^2 = a^2 + b^2 + 2ab$

$$\frac{i(t)}{I_0} = -0.384 \cos 2\pi f_m t - 0.384 \cos 2\pi f_c t + \frac{1}{2}\left[(0.384)^2 \cos^2 2\pi f_m t + (0.384)^2 \cos^2 2\pi f_c t + 2\,(0.384) \cos 2\pi f_m t \cdot (0.384) \cos 2\pi f_c t\right]$$

$$\frac{i(t)}{I_0} = -0.384 \cos 2\pi f_m t - 0.384 \cos 2\pi f_c t + \frac{(0.384)^2}{2} \cos^2 2\pi f_m t + \frac{(0.384)^2}{2} \cos^2 2\pi f_c t + \frac{1 \cdot \cancel{2}}{\cancel{2}} (0.384)^2 \cos 2\pi f_c t \cdot \cos 2\pi f_m t.$$

$$\frac{i(t)}{I_0} = -0.384 \cos 2\pi f_m t - 0.384 \cos 2\pi f_c t + 0.073 \cos^2 2\pi f_m t + 0.073 \cos^2 2\pi f_c t + 0.147 \cos 2\pi f_c t \cdot \cos 2\pi f_m t.$$

W.K.T

$$\boxed{\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}}$$

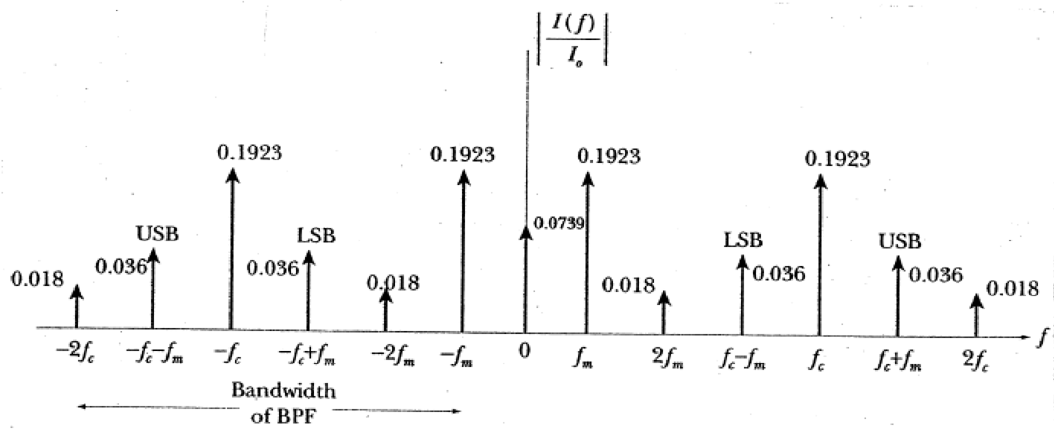
$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$\frac{i(t)}{I_o} = -0.384\cos 2\pi f_m t - 0.384\cos 2\pi f_c t + \frac{0.073}{2} + \frac{0.073}{2}\cos 4\pi f_m t$$
$$+ \frac{0.073}{2} + \frac{0.073}{2}\cos 4\pi f_c t + \frac{0.147}{2}\cos\left[2\pi(f_c - f_m)t\right]$$
$$+ \frac{0.147}{2}\cos\left[2\pi(f_c + f_m)t\right] \longrightarrow (7)$$

$\boxed{\text{NOTE:- } \frac{0.073}{2} + \frac{0.073}{2} = 0.073}$

Taking Fourier transform on both side of equation (7), we get

$$\frac{I(f)}{I_o} = -\frac{0.384}{2}\left[\delta(f - f_m) + \delta(f+f_m)\right] - \frac{0.384}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$
$$+ 0.073\,\delta(f) + \frac{0.073}{4}\left[\delta(f - 2f_c) + \delta(f + 2f_c)\right]$$
$$+ \frac{0.147}{4}\left\{\delta\left[f - (f_c - f_m)t\right] + \delta\left[f + (f_c + f_m)\right]\right\}$$
$$+ \frac{0.147}{4}\left\{\delta\left[f - (f_c + f_m)t\right] + \delta\left[f + (f_c + f_m)\right]\right\}$$



Magnitude spectrum of $\frac{I(f)}{I_o}$

c) The required AM wave centred at f_c is obtained by passing the diode current through an ideal BPF having center frequency, $f_c = 100\text{KHz}$ and $BW = 2f_m$. $f_m = 1\text{KHz}$

$$\boxed{BW = 2\text{KHz}}$$

d) W.K.T

$$\boxed{S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t}$$

$\therefore$ The time-domain expression from the o/p of the BPF is

$$S(t) = -0.3846 \cos 2\pi f_c t + \frac{0.1479}{2} \cos 2\pi (f_c + f_m) t + \frac{0.1479}{2} \cos 2\pi (f_c - f_m) t.$$

$$S(t) = -0.3846 \cos 2\pi f_c t + 0.0739 \cos 2\pi (f_c + f_m) t + 0.0739 \cos 2\pi (f_c - f_m) t \rightarrow (8)$$

From eq (8), we get

$$|A_c| = 0.3846 \;, \quad \frac{\mu A_c}{2} = 0.0739$$

$$\frac{\mu A_c}{2} = 0.0739$$

$$\mu A_c = 0.1479$$

$$\mu = \frac{0.1479}{A_c} = \frac{0.1479}{0.3846}$$

$$\mu = 0.384$$

$$\boxed{\% \; \mu = 38.4\%}$$

10> An audio frequency Signal $10 \sin 2\pi \times 500 t$ is used to amplitude modulate a carrier of $50 \sin 2\pi \times 10^5 t$. Calculate

Calculate

i> Modulation Index

ii> Sideband frequencies

iii> Amplitude of each Sideband frequencies

iv> Bandwidth required

v> Total power delivered to the load of 600Ω.

vi> plot frequency spectrum.

Sol: W.K.T modulating Signal $m(t)$ is given by

$$m(t) = A_m \cos 2\pi f_m t \text{ or } m(t) = A_m \sin 2\pi f_m t. \text{ and}$$

$$c(t) = A_c \cos 2\pi f_c t \text{ or } c(t) = A_c \sin 2\pi f_c t.$$

Given:

$$m(t) = 10 \sin 2\pi \times 500 t$$
$$c(t) = 50 \sin 2\pi \times 10^5 t.$$

$$\therefore \quad A_m = 10V, \quad A_c = 50V$$

i> Modulation Index:

$$\mu = \frac{A_m}{A_c} = \frac{10}{50} = 0.2$$

$$\% \ \mu = 0.2 \times 100 = 20$$

ii> Sideband frequencies:

W.K.T $\quad W_m = 2\pi f_m = 2\pi \times 500$

$$\therefore \boxed{f_m = 500 Hz}$$

$$W_c = 2\pi f_c = 2\pi \times 10^5$$

$$\therefore \boxed{f_c = 100 KHz}$$

$$f_{USB} = f_c + f_m = 100\,KHz + 500\,Hz = 100.5\,KHz$$

$$f_{LSB} = f_c - f_m = 100\,KHz - 500\,Hz = 99.5\,KHz$$

iii) Amplitude of each Sideband frequencies

$$\frac{\mu A_c}{2} = \frac{0.2 \times 50}{2} = 5V.$$

iv) Bandwidth required.

$$BW = 2f_m = 2 \times 500 = 1000\,Hz$$

or

$$BW = f_{USB} - f_{LSB} = 100.5\,KHz - 99.5\,KHz = 1000\,Hz$$

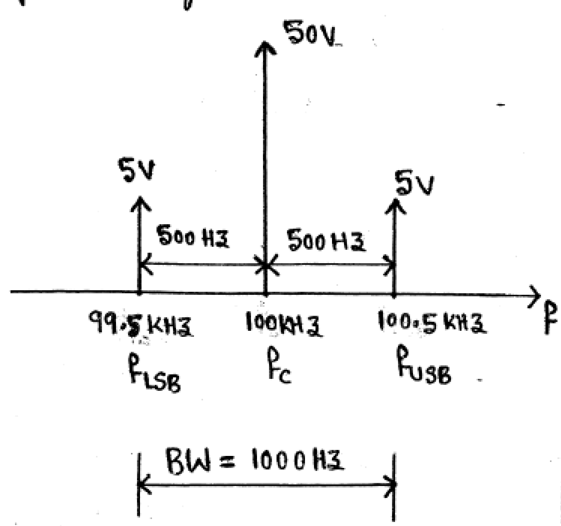
v) Total power delivered into a load of $600\,\Omega$.

$$P_T = P_c\left[1 + \frac{\mu^2}{2}\right] = \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right] \qquad \because P_c = \frac{A_c^2}{2R}$$

$$= \frac{(50)^2}{2 \times 600}\left[1 + \frac{(0.2)^2}{2}\right]$$

$$\boxed{P_T = 2.125 \text{ Watts}}$$

vi) Frequency Spectrum of AM Wave:-



A carrier wave with the amplitude 12V and frequency 10MHz is amplitude modulated to 50%. with modulating frequency 1KHz. Write down equations of the above wave and sketch the waveform in frequency domain and also find its bandwidth.

Sol:- Given: $A_c = 12V$, $f_c = 10 \times 10^6 Hz$, $\mu = 50\% = 0.5$

* Equation of carrier wave is

$$c(t) = A_c \cos 2\pi f_c t$$

$$\therefore \boxed{c(t) = 12 \cos [2\pi \times 10 \times 10^6] t}$$

* W.K.T $\mu = \frac{A_m}{A_c}$

$$A_m = \mu \times A_c = 0.5 \times 12$$

$$\boxed{A_m = 6V}$$

The Amplitude of modulating wave is 6V with the frequency 1KHZ

$\therefore$ Modulating wave is

$$m(t) = A_m \cos 2\pi f_m t$$

$$\boxed{m(t) = 6 \cos [2\pi \times 1 \times 10^3] t}$$

$\therefore$ The amplitude modulated wave is given by

$$S(t) = A_c [1 + \mu \cos (2\pi f_m) t] \cdot \cos (2\pi f_c) t$$

$$\boxed{S(t) = 12 [1 + 0.5 \cos (2\pi \times 1 \times 10^3) t] \cos (2\pi \times 10 \times 10^6) t}$$

To sketch the frequency spectrum we need f_{USB} & f_{LSB}.

$f_{USB} = f_c + f_m = 10\times10^6 + 1\times10^3$

$$\boxed{f_{USB} = 10.001\times10^6 \text{ Hz}}$$

$f_{LSB} = f_c - f_m = 10\times10^6 - 1\times10^3$

$$\boxed{f_{LSB} = 9.999\times10^6}$$

Sideband amplitude is $= \dfrac{\mu A_c}{2} \rightarrow (1)$

W.K.T $\mu = \dfrac{A_m}{A_c} \rightarrow (2)$

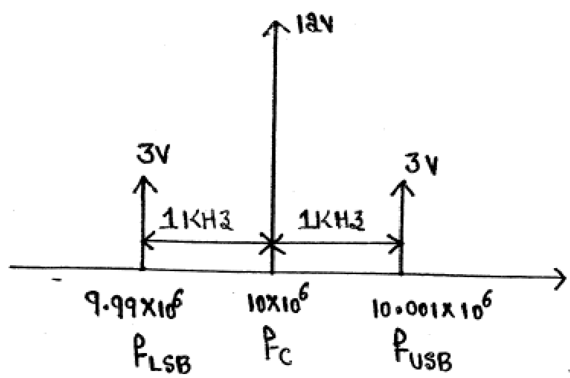
Substituting 'μ' value in eq (1)

$$= \frac{A_m}{\cancel{A_c}} \cdot \frac{\cancel{A_c}}{2}$$

$$= \frac{A_m}{2}$$

$$= \frac{6}{2}$$

$$\boxed{\text{Sideband amplitude} = 3V}$$



Bandwidth :-

$BW = 2f_m = 2\times1\times10^3$

$$\boxed{BW = 2\text{KHz}}$$

Prove that efficiency of ordinary AM is given by:

$$\eta = \frac{P_S}{P_T} \times 100 = \frac{\mu^2}{2+\mu^2} \times 100 \quad \text{for } \mu \leq 1.$$

Where,

P_S; Power carried by the sidebands

P_T; Power (total) in the AM signal.

Further i) Find η for $\mu = 0.5$ or 50% modulation

ii) Show that for a single tone AM η_{max} is 33.3% at $\mu = 1$.

* Efficiency is given by,

$$\eta = \frac{P_S}{P_T} = \frac{P_{USB} + P_{LSB}}{P_T} \rightarrow (1)$$

W.K.T

$$P_S = P_{USB} + P_{LSB}$$

i.e. $$P_{USB} = \frac{\mu^2 A_c^2}{8R}$$

$$P_{LSB} = \frac{\mu^2 A_c^2}{8R} \quad \&$$

$$P_T = P_c\left(1 + \frac{\mu^2}{2}\right)$$

Substituting the value of P_T, P_{USB} & P_{LSB} in eq (1), we get

$$\eta = \frac{\frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}}{P_c\left(1 + \mu^2/2\right)}$$

$$= \frac{\frac{\cancel{2}\mu^2 A_c^2}{4\cdot\cancel{8}R}}{P_c\left[\frac{2+\mu^2}{2}\right]} \qquad = \frac{\frac{\mu^2 A_c^2}{4R}}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{2}\left[\frac{A_c^2}{2R}\right]}{P_c\left[\frac{2+\mu^2}{2}\right]} \qquad \because P_c = \frac{A_c^2}{2R}$$

$$= \frac{\cancel{P_c}\,\frac{\mu^2}{2}}{\cancel{P_c}\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{\cancel{2}}}{\frac{2+\mu^2}{\cancel{2}}}$$

$$\boxed{\eta = \frac{\mu^2}{2+\mu^2}}$$

i) For $\mu = 0.5$

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{(0.5)^2}{2+(0.5)^2} = 0.111$$

$$\boxed{\%\,\eta = 11.11\,\%}$$

ii) For $\mu = 1$

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{(1)^2}{2+(1)^2} = 0.33$$

$$\boxed{\%\,\eta = 33.33\,\%}$$

Draw the AM waveforms for less than 100%, with 100%, more than 100% & with 0% percentage modulation.
Assume that the modulating signal is a pure sine wave.

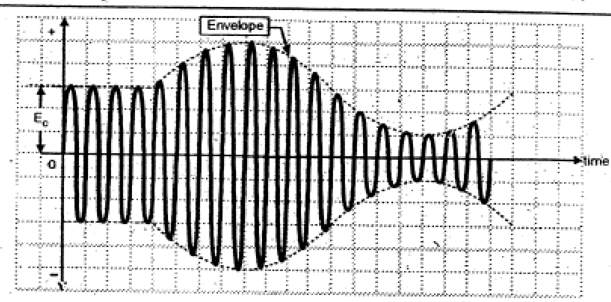
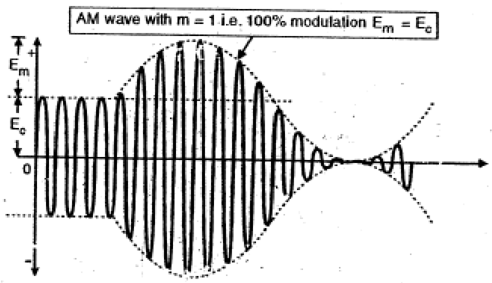
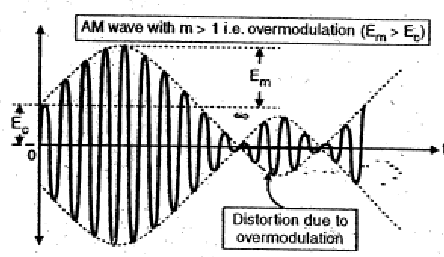


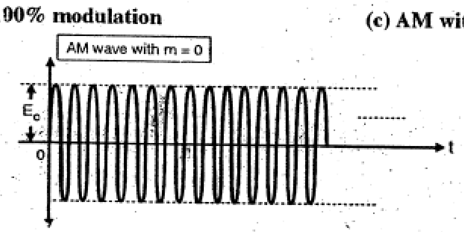
Fig. P. 2.2.1(a) : AM wave for percentage modulation less than 100 %



(b) AM wave with 100% modulation



(c) AM with over modulation



(d) AM wave with m = 0

Envelope Detector

* For minimum clippings, the time constant $R_L C_S$ should be in between the time period of I/p & o/p signal. If o/p signal m(t) ranges from 0 to W Hz & I/p is at frequency f_c,

$$\boxed{\frac{1}{f_c} < R_L C_S < \frac{1}{W}}$$

Formula :

* The condition for minimum distortion is :

$$\boxed{R_L C_S \leq \frac{1}{2\pi f_m} \cdot \frac{\sqrt{1-\mu^2}}{\mu}}$$

Show that in an envelope detector circuit the demodulator is to follow the envelope of m(t), it is required that at any time

$$\frac{1}{R_L C_S} \geqslant \frac{\omega_m \mu \sin \omega_m t}{1+\mu \cos \omega_m t}$$

Sol :- Let us assume that the capacitor discharges from the peak value 'E' at some arbitrary instant t=0. Then the voltage across the capacitor 'V_c' is given by

$$V_c = E e^{-t/R_L C_S} \rightarrow (1)$$

Using Taylor Series

$$V_c \cong E\left[1 - \frac{t}{R_L C_S}\right] \rightarrow (2)$$

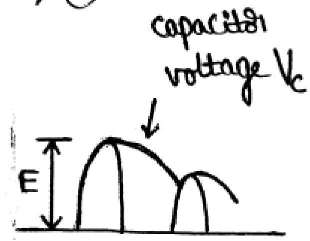
(differentiating equation ② w.r.to dt)

* The Slope of discharge is

$$\frac{dV_c}{dt} = -\frac{E}{R_L C_s} \rightarrow ③$$

* The amplitude 'E' at any instant is

$$E = A_c[1 + \mu \cos 2\pi f_m t] \rightarrow ④$$



* The Slope of this envelope is (differentiating eq ④)

$$\frac{dE}{dt} = -\mu A_c \sin 2\pi f_m t \cdot (2\pi f_m) \rightarrow ⑤$$

* In order for the capacitor to follow the envelope 'E(t)', the magnitude of the Slope of the $R_L C_s$ discharge must be greater than the magnitude of the Slope of the envelope E(t). Hence

$$\left|\frac{dV_c}{dt}\right| \geq \left|\frac{dE}{dt}\right| \rightarrow ⑥$$

Substituting eq ③ & ⑤ in eq ⑥, we get

$$\frac{E}{R_L C_s} \geq \mu A_c \sin 2\pi f_m t \cdot (2\pi f_m) \rightarrow ⑦$$

Substituting eq ④ in eq ⑦, we get

$$\frac{A_c[1 + \mu \cos 2\pi f_m t]}{R_L C_s} \geq \mu A_c \sin 2\pi f_m t \cdot (2\pi f_m)$$

$$\frac{1}{R_L C_s} \geq \frac{\mu \cancel{A_c} \sin 2\pi f_m t \cdot (2\pi f_m)}{\cancel{A_c}[1 + \mu \cos 2\pi f_m t]}$$

$$\boxed{\frac{1}{R_L C_s} \geq \frac{\omega_m \mu \sin \omega_m t}{1 + \mu \cos \omega_m t}}$$

W.K.T

$\omega_m = 2\pi f_m$

* The AM wave $10[1+0.5\cos(2\pi 500t)]\cdot\cos(2\pi 10^6 t)$ is demodulated by an envelope detector. Find the time constant τ and the resistor if capacitor used is 100 pF.

Sol:- Given : $f_m = 500\,Hz$, $f_c = 10^6\,Hz$, $\mu = 0.5$ & $C_s = 100\,pF$

The time constant $\tau = R_L C_s$ should satisfy the condition

$$\frac{1}{f_c} < R_L C_s < \frac{1}{f_m} \quad \text{and}$$

$$R_L C_s \leq \frac{1}{2\pi f_m} \frac{\sqrt{1-\mu^2}}{\mu}$$

$$\leq \frac{1}{2\pi \times 500} \cdot \frac{\sqrt{1-(0.5)^2}}{0.5} \leq \frac{\sqrt{0.75}}{500\pi}$$

$$R_L C_s \leq 5.51\times10^{-4}$$

Time Constant $\tau = 5.51\times10^{-4}$ Sec

For $C_s = 100\,pF$

$$R_L \leq \frac{5.51\times10^{-4}}{C_s} \leq \frac{5.51\times10^{-4}}{100\times10^{-12}}$$

$$\boxed{R_L = 5.51\,M\Omega}$$

Explain the detection of message signal from the amplitude modulated signal using an envelope detector and bringout the significance of the RC time constant of the circuit in detection of the message signal without distortion. Estimate this for fm=3KHz and fc=100KHz. **Jan-06,6M**

Sol :- i) Explain envelope detector

ii) Given : $f_m = 3KHz$, $f_c = 100KHz$.

W.K.T for correct demodulation, it is required that,

$$\frac{1}{f_c} \ll RC \ll \frac{1}{f_m}$$

$$\frac{1}{100\times10^3} \ll RC \ll \frac{1}{3\times10^3}$$

$$\boxed{0.01\,msec \ll RC \ll 0.33\,msec.}$$

* The time constant $\tau = RC$ should satisfy the condition

$$RC \leq \frac{1}{2\pi f_m} \quad \frac{\sqrt{1-\mu^2}}{\mu}$$

assuming $\mu = 0.5$

$$RC \leq \frac{1}{2\pi \times 3\times10^3} \cdot \frac{\sqrt{1-(0.5)^2}}{0.5}$$

$$\boxed{RC \leq 0.27\,msec}$$

* In the absence of modulation Index,

$$RC \leq \frac{1}{2\pi f_m}$$

$$RC \leq \frac{1}{2\pi \times 3\times10^3}$$

$$\boxed{RC = 0.05\,msec}$$

DSB-SC Modulator Problems

1. **A message signal m(t) with spectrum shown in <u>fig.1</u> is applied to a product modulator with a carrier wave $A_c\cos(2\pi f_c t)$ producing the DSB-SC modulated wave s(t). This modulated wave is next applied to a coherent detector. Assuming a perfect coherence between the transmitter and the receiver, determine the spectrum of the detector output when**

   i. fc=1.25KHz **Jan-06,6M**
   ii. fc=.75KHz and sketch the same
   iii. The lowest fc so that m(t) is uniquely determined from s(t)

2. **Consider a message signal m(t) with a spectrum shown in <u>fig.2.</u> The message bandwidth w=1KHz. This signal is applied to a product modulator, together with a carrier wave $A_c\cos(2\pi f_c t)$, producing the DSB-SC modulated signal s(t). The modulated signal is next applied to a coherent detector. Assuming perfect synchronism between the carrier waves in the modulator and detector. Determine the spectrum of the detector output when:**

   i. fc = 1.25KHz **ii. fc= 0.75KHz.**

   What is the lowest carrier frequency for which each component of the modulated signal s(t) is uniquely determined by m(t).

Jan-09,6M

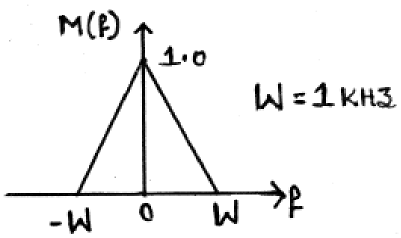


fig ①

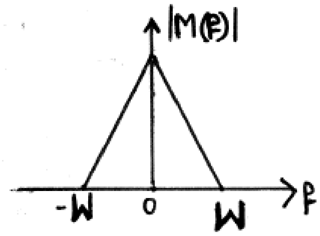


fig ②

<u>Given :-</u> $\underline{W = f_m = 1\text{KHz}}$

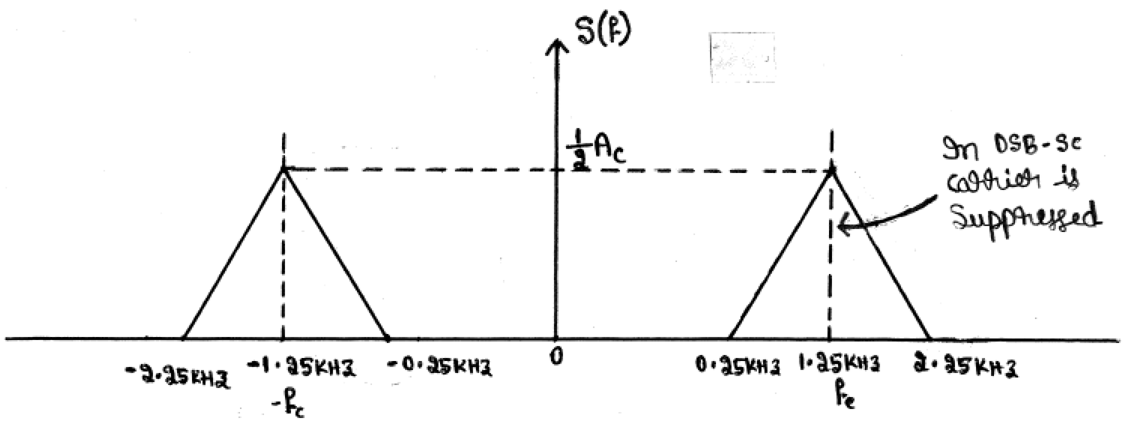
i) When $f_c = 1.25$ KHz

$f_c + W = 1.25\text{KHz} + 1\text{KHz} = 2.25\text{KHz}$

$f_c - W = 1.25\text{KHz} - 1\text{KHz} = 0.25\text{KHz}$

$-f_c + W = -1.25\text{KHz} + 1\text{KHz} = -0.25\text{KHz}$

$-f_c - W = -1.25\text{KHz} - 1\text{KHz} = -2.25\text{KHz}$



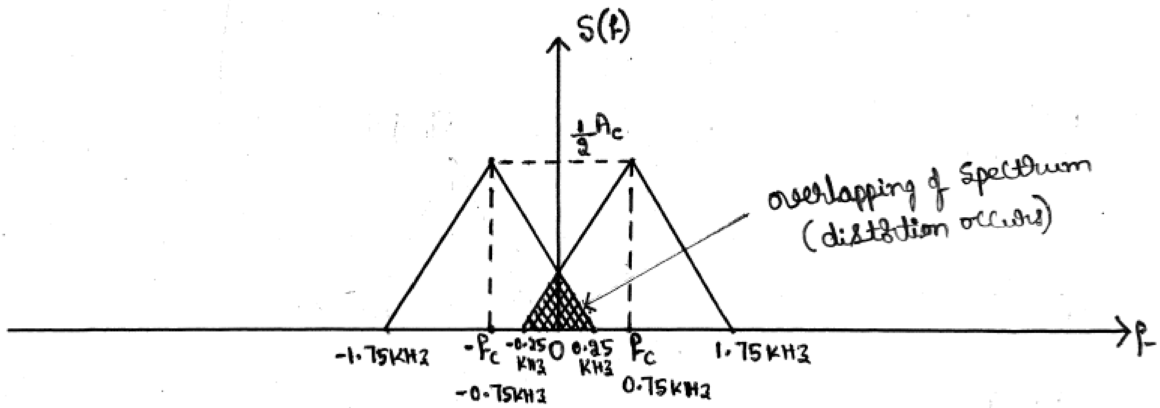
ii) When $f_c = 0.75KHz$:

$f_c + W = 0.75KHz + 1KHz = 1.75KHz$

$f_c - W = 0.75KHz - 1KHz = -0.25KHz$

$-f_c + W = -0.75KHz + 1KHz = 0.25KHz$

$-f_c - W = -0.75KHz - 1KHz = -1.75KHz$



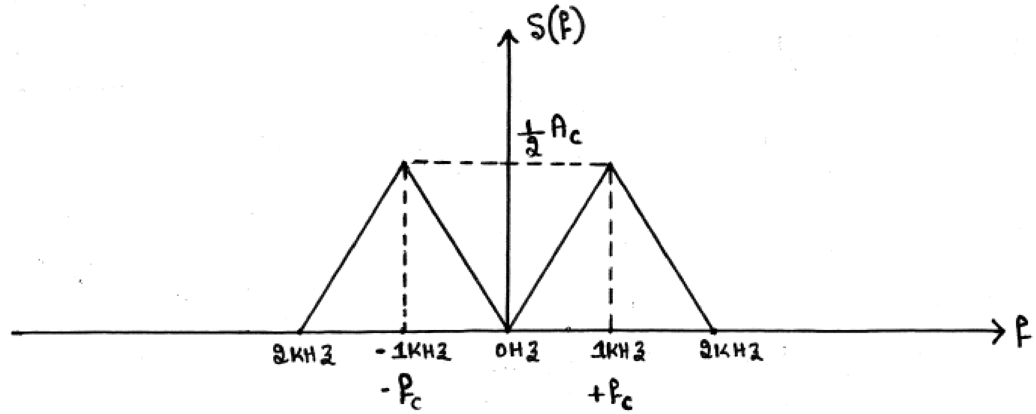
iii) The lowest f_c so that $m(t)$ is uniquely determined from $s(t)$ is 1KHz i.e. $\boxed{f_c = 1KHz}$

$f_c + W = 1KHz + 1KHz = 2KHz$

$f_c - W = 1KHz - 1KHz = 0Hz$

$-f_c + W = -1KHz + 1KHz = 0Hz$

$-f_c - W = -1KHz - 1KHz = -2KHz$



$\therefore$ Lowest f_c is $\underline{1KHz}$ so that $m(t)$ is uniquely determined from $S(t)$.

❖ **Consider a resultant wave obtained by adding a non-coherent carrier $A_c\cos(2\pi f_c t + \phi)$ to a DSB-SC wave $\cos(2\pi f_c t)\, m(t)$. This composite wave is applied to an ideal envelope detector. Find the resulting detector output. Evaluate this output for ϕ=0.**

Jan-10,8M

Sol:-

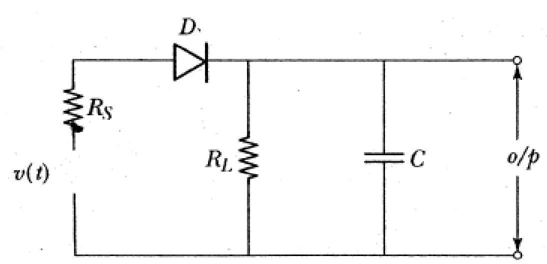


Figure P2.6 ■ Envelope detector

The I/p to the envelope detector is the Sum of non-coherent carrier and a DSBSC Wave:

$$V(t) = A_c \underline{\cos(2\pi f_c t + \phi)} + m(t) \cos 2\pi f_c t$$

WKT $\quad \cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$$V(t) = A_c \underline{\cos 2\pi f_c t} \cos\phi - A_c \sin 2\pi f_c t \cdot \sin\phi + m(t) \underline{\cos 2\pi f_c t}$$

$$V(t) = \cos 2\pi f_c t \left[A_c \cos\phi + m(t) \right] - A_c \sin 2\pi f_c t \cdot \sin\phi$$

The o/p of the envelope detector is

$$a(t) = \sqrt{(\text{Inphase Components})^2 + (\text{Quadrature Components})^2}$$

$$= \sqrt{\left[A_c \cos\phi + m(t)\right]^2 + \left[A_c \sin\phi\right]^2}$$

When $\phi = 0$, We get

$$= \sqrt{\left[A_c \cos(0) + m(t)\right]^2 + \left[A_c \sin(0)\right]^2}$$

$$= \sqrt{\left[A_c (1) + m(t)\right]^2 + \left[A_c (0)\right]^2}$$

$$= \sqrt{\left[A_c + m(t)\right]^{\cancel{2}}}$$

$$\boxed{a(t) = A_c + m(t)}$$

Hence, the o/p of the envelope detector Contains the message Signal $m(t)$.

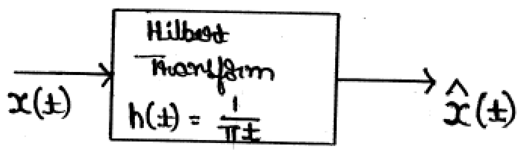
SSB MODULATION

Hilbert Transform:-

* The device which produces a phase shift of -90° for all +ve frequencies & a phase shift of $+90^\circ$ for all -ve frequencies.

The amplitude of all frequency components of the I/p signal are unaffected by transmission through device.

Such an Ideal device is called a Hilbert transform.



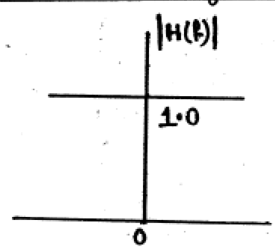


Fig ⓐ: Amplitude response

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\boxed{\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{(t-\tau)}\, d\tau}$$

Where, $\hat{x}(t)$ is the hilbert transform of $x(t)$

INVERSE Hilbert transform :-

We can recover back the original signal $x(t)$ back from $\hat{x}(t)$ by taking the inverse hilbert transform as follows:

$$\boxed{x(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{(t-\tau)}\, d\tau}$$

Interpretation of Hilbert Transform :-

The Fourier transform of $x(t)$ & $\frac{1}{\pi t}$ are as follows:

$$\boxed{\begin{array}{l} x(t) \xrightarrow{FT} X(f) \\ \frac{1}{\pi t} \xrightarrow{FT} -j\,Sgn(f) \end{array}}$$

Where Sgn is the Signum function defined as

$$Sgn = \begin{cases} 1 & , f>0 \\ 0 & , f=0 \\ -1 & , f<0 \end{cases}$$

$$\therefore \boxed{\hat{x}(t) = x(t) * \frac{1}{(\pi t)}} \longrightarrow ①$$

Taking Fourier Transform on both Side of eq①, we get

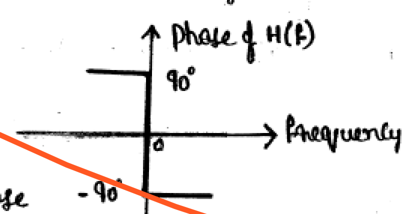
$$\hat{X}(f) = X(f)\left[-j\,Sgn(f)\right]$$

$$\therefore \boxed{\hat{X}(f) = -j\,Sgn(f).\,X(f)} \longrightarrow ②$$

Thus the hilbert transform $\hat{x}(t)$ of Signal $x(t)$ is obtained by passing $x(t)$ through a linear two port device whose transfer function is equal to $-jSgn(f)$ as Shown below.

$$x(t) \longrightarrow \boxed{H(f) = -j\,Sgn(f)} \longrightarrow \hat{x}(t)$$

Fig①: Two port device



Fig② Phase response

❖ **Define Hilbert transform. State and prove the properties of Hilbert transform.**

July-09,5M	July-07,4M	June-10,4M	Jan-10,4M	Jan-09,6M

Property 1 :-

It States that the Signal $x(t)$ & its hilbert transform $\hat{x}(t)$ have the Same amplitude Spectrum.

Proof :-

* Fourier transform of $\hat{x}(t) = \hat{X}(f) = -j\,\mathrm{Sgn}(f) \cdot X(f)$
* The magnitude of $-j\,\mathrm{sgn}(f)$ is equal to 1 for all values of 'f'.

$$\therefore \quad |\hat{X}(f)| = (1)\,|X(f)| \qquad \because \boxed{|-j\,\mathrm{Sgn}(f)| = 1}$$

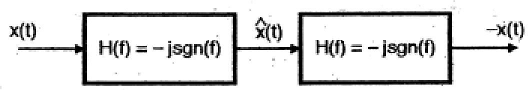
$$\boxed{|\hat{X}(f)| = |X(f)|}$$

$\therefore$ The amplitude Spectrum of $\hat{x}(t)$ is equal to $x(t)$.

Property 2 :-

This Property States that if $\hat{x}(t)$ is the hilbert transform of $x(t)$ then the hilbert transform of $\hat{x}(t)$ is $-x(t)$.

Proof :-



Cascading the ideal two port devices to obtain the double Hilbert transform

The Property-2 Suggests that the hilbert transform is taken twice as Shown in above figure.

$$H(f) * H(f) = -j\,\mathrm{Sgn}(f) \times -j\,\mathrm{Sgn}(f)$$
$$= +j^2\,\mathrm{Sgn}^2(f)$$

but $\mathrm{Sgn}^2(f) = 1$ & $j^2 = -1$

$$H'(f) = H(f) * H(f) = -1$$

$\therefore$ $H'(f) = -1$ for all the values of 'f'.

Hence the FT of o/p is

$$X(f) \times H'(f) = -X(f)$$

$$-X(f) \xrightarrow{IFT} -x(t)$$

* Thus the hilbert transform of $\hat{x}(t)$ is equal to $-x(t)$.

Property - 3 :-

This property states that the Signal $x(t)$ & its Hilbert transform $\hat{x}(t)$ are orthogonal.

Proof :- We have to prove that $\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\, dt = 0$

W.K.T
$$x(t) \xrightarrow{FT} X(f)$$
$$\hat{x}(t) \xrightarrow{FT} \hat{X}(-f)$$

$$\therefore \int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t) \cdot dt = \int_{-\infty}^{\infty} X(f) \cdot \hat{X}(-f) \cdot df$$

W.K.T
$$\boxed{\begin{aligned} \hat{X}(f) &= -j\,\mathrm{sgn}(f) \cdot X(f) \\ \hat{X}(-f) &= -j\,\mathrm{sgn}(-f) \cdot X(-f) \\ \hat{X}(-f) &= +j\,\mathrm{sgn}(f)\, X(-f) \end{aligned}}$$
$\parallel^{ly}$

$$-j\,\mathrm{sgn}(-f) = +j\,\mathrm{sgn}(f)$$

$$\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t) \cdot dt = \int_{-\infty}^{\infty} j\,\mathrm{sgn}(f)\, X(f) \cdot X(-f) \cdot df$$

but $\boxed{X(f) \cdot X(-f) = |X(f)|^2}$

$$\boxed{\int_{-\infty}^{\infty} x(t).\hat{x}(t).\,dt = \int_{-\infty}^{\infty} j\,Sgn(f)\,|X(f)|^2\,df} \longrightarrow ①$$

Eq ① is an product of odd & even function.

Where, Sgn(f) is an odd function &

$|X(f)|^2$ is an even function.

* The Integration of an odd function over the range $-\infty$ to $+\infty$ will yield to a Zero value.

$$\therefore \boxed{\int_{-\infty}^{\infty} x(t).\hat{x}(t)\,dt = 0}$$ Hence proved.

Additional properties of Hilbert transform :-

1) The magnitude Spectra of a Signal $x(t)$ & its hilbert transform $\hat{x}(t)$ are Identical.

2) The hilbert transform of an even function is odd and vice-versa.

3) The hilbert transform of a real Signal is also real.

---

pre-envelope :-

For a real valued Signal $x(t)$, its pre-envelope is defined as the Complex valued function Such that

$$\boxed{x_+(t) = x(t) + j\hat{x}(t)} \longrightarrow ①$$

Where $x(t)$ is the real part of the pre-envelope & $\hat{x}(t)$ represents the imaginary part of the pre-envelope.

* Let $X_+(f)$ represents the FT of $x_+(t)$ & is given by

$$X_+(f) = F[x(t) + j\hat{x}(t)]$$

$$X_+(f) = X(f) + j\hat{X}(f) \longrightarrow ②$$

W.K.T

$$\hat{X}(f) = -j\,\mathrm{Sgn}(f)\cdot X(f) \longrightarrow ③$$

Substituting eq ③ in eq ②, we get

$$X_+(f) = X(f) + j[-j\,\mathrm{Sgn}(f)\cdot X(f)]$$

$$= X(f) - j^2\,\mathrm{Sgn}(f)\cdot X(f)$$

$$= X(f) + \mathrm{Sgn}(f)\,X(f)$$

$j^2 = -1$

$-j^2 = 1$

$$\boxed{X_+(f) = X(f)[1 + \mathrm{Sgn}(f)]} \longrightarrow ④$$

Where

$$\mathrm{Sgn}(f) = \begin{cases} 1, & \text{for } f>0 \\ 0, & \text{for } f=0 \\ -1, & \text{for } f<0 \end{cases}$$

Substituting the values of $\mathrm{Sgn}(f)$ in eq ④, we get

i) $\mathrm{Sgn}(f) = 1$, for $f>0$

$X_+(f) = X(f)[1+1]$

$\boxed{X_+(f) = 2X(f)}$

ii) $\mathrm{Sgn}(f) = 0$, for $f=0$

$X_+(f) = X(0)[1+0]$

$\boxed{X_+(f) = X(0)}$

iii) $\mathrm{Sgn}(f) = -1$, for $f<0$

$X_+(f) = X(f)[1-1]$

$\boxed{X_+(f) = 0}$

$$\boxed{X_+(f) = \begin{cases} 2X(f), & \text{for } f>0 \\ X(0), & \text{for } f=0 \\ 0, & \text{for } f<0 \end{cases}}$$

Where $X(0)$ is the value of $X(f)$ at $f=0$.

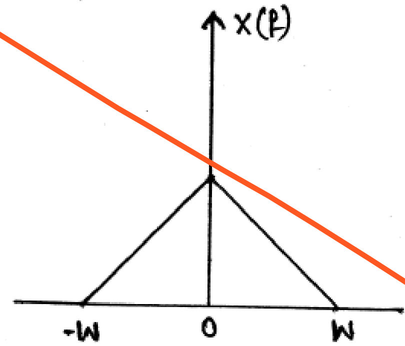


Fig ⓐ: Amplitude Spectrum of Low pass Signal $x(t)$

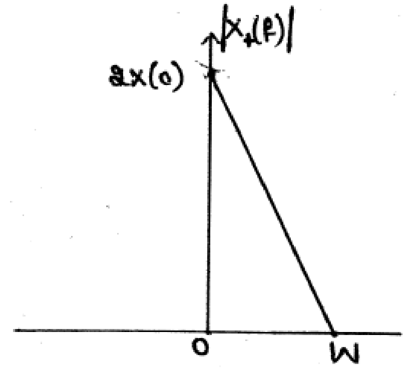


Fig ⓑ: Amplitude Spectrum of the pre-envelope $x_+(t)$.

Pre-envelope for -ve frequencies :-

* The pre-envelope for -ve frequencies is defined as:

$$\boxed{x_-(t) = x(t) - j\hat{x}(t)}$$

* The two pre-envelopes $x_+(t)$ & $x_-(t)$ are simply the complex conjugates of each other :

$$\boxed{x_-(t) = x_+^*(t)}$$

i.e. $X_-(f) = 0$, for $f>0$

$= X(0)$, for $f=0$

$= 2X(f)$, for $f<0$

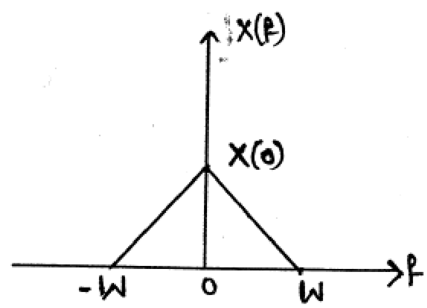


Fig ⓐ: Amplitude Spectrum of Low pass Signal $x(t)$

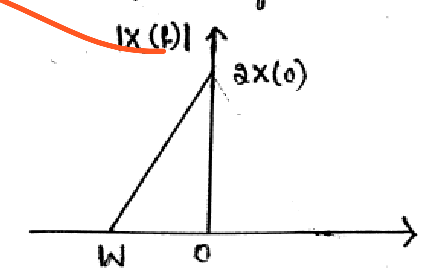


Fig ⓑ: Amplitude Spectrum of the pre-envelope $X_+(f)$.

* The Sum of $x_+(t)$ & $x_-(t)$ is given by

$$x_+(t) + x_-(t) = [x(t) + j\hat{x}(t)] + [x(t) - j\hat{x}(t)]$$

$$= x(t) + j\cancel{\hat{x}(t)} + x(t) - j\cancel{\hat{x}(t)}$$

$$\boxed{x_+(t) + x_-(t) = 2x(t)}$$

---

* What is the Indication of $(\pm)$ Sign in the Subscript of the pre-envelope?

+ Sign:-

The (+) Sign in the Subscript of pre-envelope '$x_+(t)$' indicates that the Spectrum of pre-envelope $x_+(t)$ is Non-Zero, only for the +ve Frequencies.

- Sign:-

The (-) Sign in the Subscript of pre-envelope '$x_-(t)$' indicates that the Spectrum of pre-envelope $x_-(t)$ is Non-Zero, only for the -ve Frequencies.

Canonical Representation of Band pass Signals:-

* Let the pre-envelope of a Narrow band Signal $x(t)$, with its FT $X(f)$ centered about Some Frequency $\pm f_c$, be expressed in the form

$$x_+(t) = \tilde{x}(t)\exp(j2\pi f_c t)$$

$$\boxed{x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}} \longrightarrow (1)$$

Where $\tilde{x}(t)$ is the complex envelope of the Signal.

Eq ① is the definition for the Complex envelope $\tilde{x}(t)$ in terms of the pre-envelope $x_+(t)$.

* The Spectrum of $x_+(t)$ is limited to the frequency band $f_c - W \leq f \leq f_c + W$ as Shown in fig ⓑ.

* Applying frequency-Shifting property of the Fourier transform to eq ①, then the Spectrum of the Complex envelope $\tilde{x}(t)$ is limited to the band $-W \leq f \leq W$ & Centered at the origin as Shown in fig ⓒ.

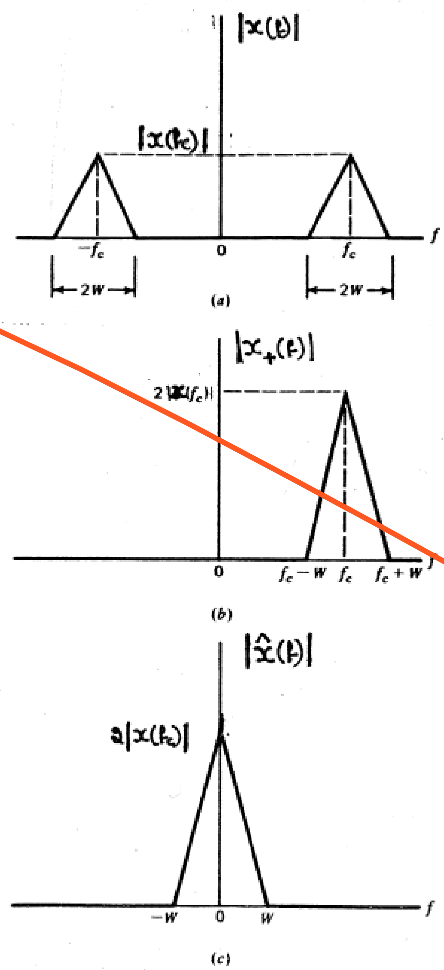


Figure . (a) Amplitude spectrum of band-pass signal $x(t)$. (b) Amplitude spectrum of pre-envelope $x_+(t)$. (c) Amplitude spectrum of complex envelope $\tilde{x}(t)$.

* The Signal $x(t)$ is the real part of the pre-envelope $x_+(t)$. Hence the given bandpass Signal $x(t)$ can be expressed interms of the Complex envelope as:

$$\boxed{x(t) = R_e[\hat{x}(t)\, e^{j2\pi f_c t}]} \longrightarrow ①$$

* In general, $\hat{x}(t)$ is a Complex quantity, we can express it as:

$$\boxed{\hat{x}(t) = x_I(t) + j x_Q(t)} \longrightarrow ②$$

Substituting eq ② in eq ①, we get

$$x(t) = R_e[x_I(t) + j x_Q(t)\, e^{j2\pi f_c t}] \longrightarrow ③$$

W.K.T $\quad e^{j\theta} = \cos\theta + j\sin\theta \qquad \theta = 2\pi f_c t$

$$\boxed{e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t)} \longrightarrow ④$$

Substituting eq ④ in eq ③, we get

$$x(t) = R_e\left\{[x_I(t) + j x_Q(t)]\,(\cos(2\pi f_c t) + j\sin(2\pi f_c t))\right\}$$

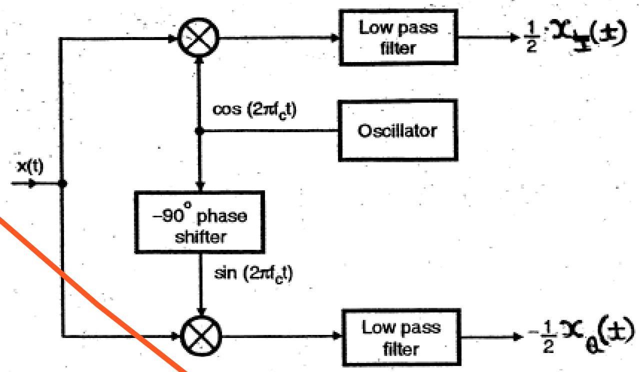
$$x(t) = R_e\left\{x_I(t)\cos(2\pi f_c t) + j x_I(t)\sin(2\pi f_c t) + j x_Q(t)\cos(2\pi f_c t) + j^2 x_Q(t)\sin(2\pi f_c t)\right\}$$

$$x(t) = R_e\left\{x_I(t)\cos(2\pi f_c t) + j x_I(t)\sin(2\pi f_c t) + j x_Q(t)\cos(2\pi f_c t) + (-1) x_Q(t)\sin(2\pi f_c t)\right\}$$

$$\boxed{x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin(2\pi f_c t)} \longrightarrow ⑤$$

* In eq ⑤ $x_I(t)$ is the in-phase Component of the band pass Signal & $x_Q(t)$ is the Quadrature of the Signal $x(t)$.

Generation of In-Phase and Quadrature phase components:-

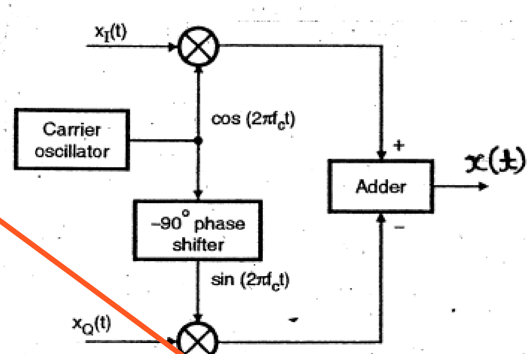


Scheme to generate the in phase and quadrature components of bandpass signal x (t)

* The $x_I(t)$ & $x_Q(t)$ are low pass signals limited to the band $-W \leq f \leq W$. The bandwidth of each filter is 'W'.

* The in-phase component $x_I(t)$ is produced by multiplying $x(t)$ with $\cos(2\pi f_c t)$ & passing the product through a LPF.

* The Quadrature component $x_Q(t)$ is obtained by multiplying $x(t)$ with $\sin(2\pi f_c t)$ & passing the product through an Identical LPF.



Reconstruction of x (t) from $x_I(t)$ and $x_Q(t)$

* The in-phase low pass signal $x_I(t)$ & $x_Q(t)$ are multiplied with the $\cos(2\pi f_c t)$ & $\sin(2\pi f_c t)$ respectively.

* The resultant product terms are then subtracted to get the bandpass signal $x(t)$.

* The multiplication process of $x_I(t)$ & $x_Q(t)$ with the carriers is a linear modulation process.

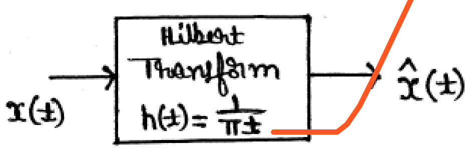
❖ **Define:**

- i. **Hilbert transform**
- ii. **Pre-envelope**
- iii. **Complex envelope**

July-09,6M July-05,6M

i) Hilbert transform :-

When phase angles of all components of a given signal are shifted by $\pm 90°$, the resulting function of time is known as Hilbert transform.



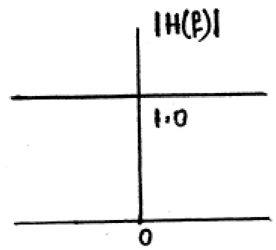


Fig ⓐ: Amplitude response.

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\boxed{\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)} \cdot d\tau} \rightarrow (1)$$

Where, $\hat{x}(t)$ is the hilbert transform of $x(t)$

* Taking Fourier transform on both sides of eq (1), we get

$$\boxed{\hat{X}(f) = -j\,\mathrm{Sgn}(f) \cdot X(f)} \rightarrow (2)$$

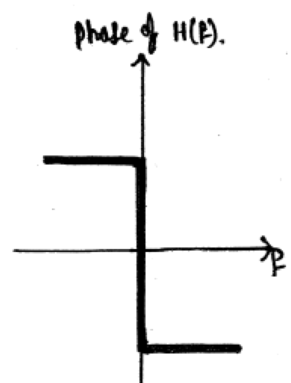


Fig ⓑ: Phase response of H(f)

ii) Pre-envelope :-

The pre-envelope of the Signal $x(t)$, is defined as the Complex-valued function can be given as:

$$\boxed{x_+(t) = x(t) + j\hat{x}(t)} \longrightarrow ①$$

Where,

$x(t)$ is the real part of the pre-envelope &

$j\hat{x}(t)$ represents the Imaginary part of the pre-envelope.

$\hat{x}(t)$ is the hilbert transform of $x(t)$.

Taking FT of $x_+(t)$ & is given by

$$X_+(f) = X(f)\,[1+\text{Sgn}(f)] \longrightarrow ②$$

Substituting $\text{Sgn}(f)$ values in eq ②, we get

$$X_+(f) = \begin{cases} 2X(f), & \text{for } f>0 \\ X(0), & \text{for } f=0 \\ 0, & \text{for } f<0 \end{cases}$$

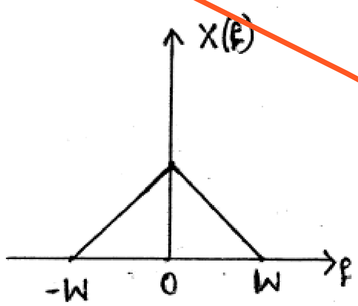


Fig ⓐ: Amplitude Spectrum of Low pass Signal

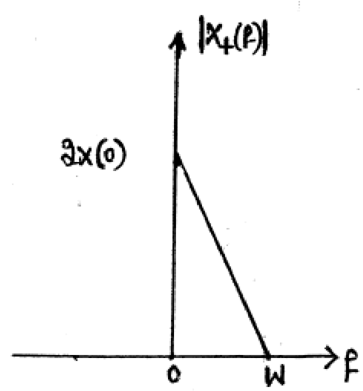


Fig ⓑ: Amplitude Spectrum of the pre-envelope $x_+(t)$.

iii) Complex envelope :-

* The pre-envelope of a Narrow band Signal $x(t)$ expressed in the form

$$\boxed{x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}} \rightarrow ①$$

Where $\tilde{x}(t)$ is the Complex envelope of the Signal.
Eq ① is the definition for the Complex envelope $\tilde{x}(t)$ interms of the pre-envelope $x_+(t)$.

* The bandpass Signal $x(t)$ can be expressed interms of the Complex envelope as:

$$\boxed{x(t) = R_e\left[\hat{x}(t)\, e^{j2\pi f_c t}\right]} \rightarrow ②$$

In general, $\hat{x}(t)$ is a Complex quantity, we can express it as:

$$\boxed{\hat{x}(t) = x_I(t) + j x_Q(t)} \rightarrow ③$$

Substituting eq ③ in eq ②, we get

$$\boxed{x(t) = R_e\left[x_I(t) + j x_Q(t)\, e^{j2\pi f_c t}\right]}$$

Single Sideband Modulation (SSB Modulation) :-

* Standard Amplitude modulation & DSB-SC modulation are - Wasteful of bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth i.e. $BW = 2f_m$.

* In both Case (AM & DSB-SC) half of the transmission bandwidth is occupied by the upper Sideband of the modulated wave, where as the other half of the transmission bandwidth is occupied by the lower Sideband of the modulated wave.

* The upper & Lower Sidebands are uniquely related to each other by Virtue of their Symmetry about the Carrier frequency 'f_c'. Thus only one Sideband is necessary for transmission of Information - & if both the carrier & the other Sideband are Suppressed at the transmitter, no Information is lost.

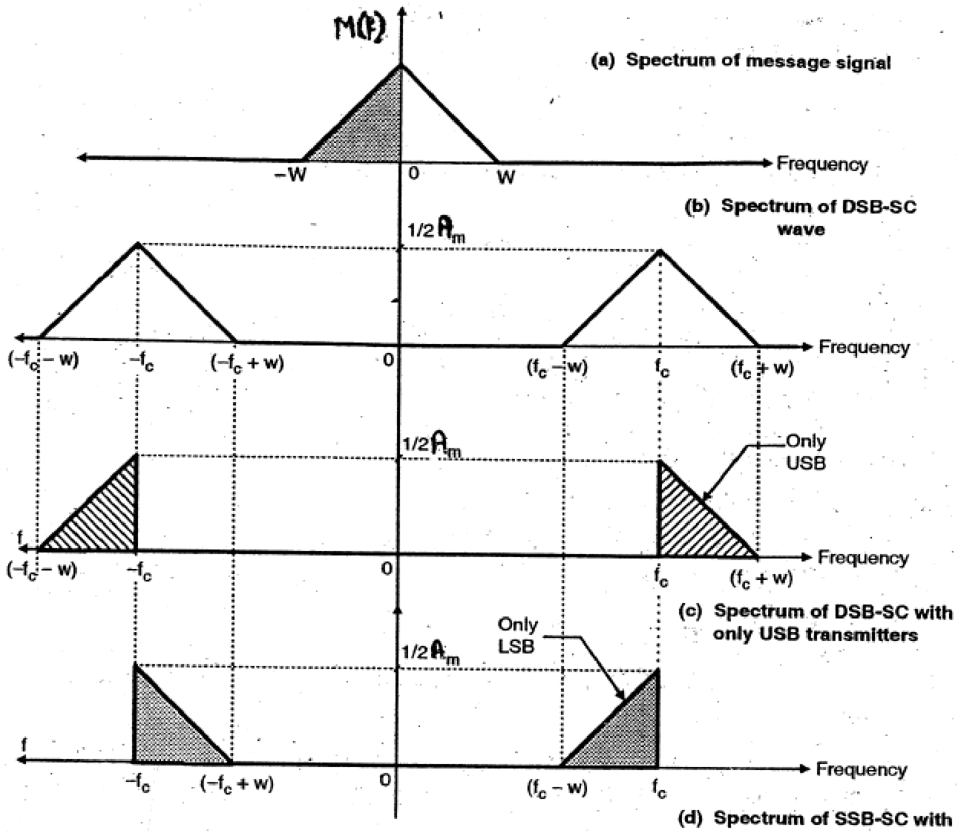
∴ Channel required the Same Bandwidth as the message Signal.

* When only one Sideband is transmitted, the modulation referred to as Single - Sideband Modulation.

Frequency-domain description of SSB :-

* The frequency - domain description of a SSB modulated Wave - depends on Which Sideband is transmitted.

fig (a) Shows Spectrum of modulating Signal '$M(f)$'. The Spectrum is limited to the band $-W \leq f \leq W$.



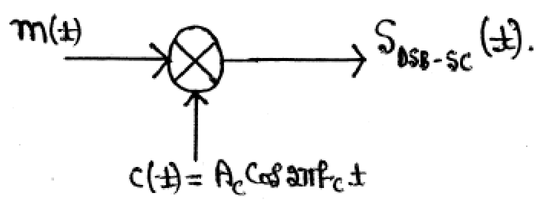
(a) Spectrum of message signal

(b) Spectrum of DSB-SC wave

(c) Spectrum of DSB-SC with only USB transmitters

(d) Spectrum of SSB-SC with only LSB transmitters

* The DSB-SC Wave can be obtained by multiplying $m(t)$ by the Carrier wave $A_c \cos(2\pi f_c t)$ as shown below.



* Fig (b) Shows the DSB-SC modulated Wave.

In SSB, When only upper Sideband is transmitted, we get frequency Spectrum as Shown in fig (c).

* When only the lower Sideband is transmitted, we get frequency Spectrum as Shown in fig (d).

Advantages of SSB :-

1) SSB required half the bandwidth required of AM & DSB-SC Signals.

2) Due to Suppression of Carrier and one Sideband, power is Saved.

3) Reduced Interference of noise. This is due to the reduced bandwidth. As the bandwidth increases, the amount of noise added to the Signal will Increase.

4) Fading does not occur in SSB transmission. (Fading means that a Signal alternately increases & decreases in Strength as it is picked up by the receiver. It occurs because the Carrier & Sideband may reach the receiver Shifted in time & phase w.r. to each other.)

Disadvantages of SSB :-

1) The generation & reception of SSB Signal is a Complex process.

2) Since carrier is absent, the SSB transmitter & Receiver need to have an excellent Frequency Stability.

3) The SSB modulation is expensive & highly Complex to Implement

---

Applications of SSB :-

1) SSB transmission is used in the applications where the power Saving is required in mobile Systems.

2) SSB is also used in applications in which bandwidth - requirements are low.

ex:- Point to point Communication, Land, air, maritime mobile communications, TV, Telemetry, military Communications, radio navigation & amateur radio.

$\boxed{\text{FORMULAE}}$

1) $S_u(t) = R_e\left[\tilde{S}_u(t)\, e^{j2\pi f_c t}\right]$

2) $\tilde{S}_u(t) = \text{IFT}\left\{\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)\right\}$

IFT :-

$$M(f) \xrightarrow{\text{IFT}} m(t)$$

$$-j\,Sgn(f)M(f) \xrightarrow{\text{IFT}} \hat{m}(t)$$

$$Sgn(f)\,M(f) \longrightarrow j\hat{m}(t)$$

---

i) $-j\,Sgn(f) \cdot M(f) \xrightarrow{\text{IFT}} \hat{m}(t)$

ii) $Sgn(f)\,M(f) \xrightarrow{\text{IFT}} j\,\hat{m}(t)$

Proof :-

$$j\hat{m}(t) = j\left[-j\,Sgn(f)\,M(f)\right]$$

$$= -j^2\,Sgn(f)M(f)$$

$$= -(-1)\,Sgn(f)\,M(f)$$

$$\boxed{j\hat{m}(t) = Sgn(f)M(f)}$$

iii) $Sgn(-f)\,M(f) \xrightarrow{\text{IFT}} -j\,\hat{m}(t)$

Proof :- $-j\,\hat{m}(t) = j\left[-j\,Sgn(f)\,M(f)\right]$

$$= +j^2\,Sgn(f) \cdot M(f)$$

$$\boxed{-j\,\hat{m}(t) = -Sgn(f)\,M(f)}$$

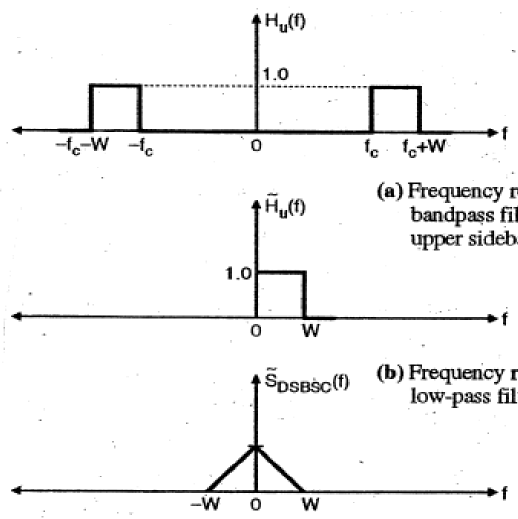
Time-domain description of SSB wave :-

❖ **Using Hilbert transform, derive the equations for SSB signals. Specify the advantages of SSB over DSB-SC.** **July-05,12M**

❖ **Derive an expression for SSB modulated wave for which upper sideband is retained.** **Jan-09,8M Jan-05,8M**

Sol :-

The SSB Signal may be generated by passing a DSB-SC modulated wave through a BPF of transfer function $H_u(f)$.



(a) Frequency response of ideal bandpass filter to select the upper sideband.

(b) Frequency response of equivalent low-pass filter.

(c) Spectrum of complex envelope of DSBSC.

* The DSB-SC modulated wave is defined mathematically as

$$\boxed{S_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)}$$

Where, $m(t) \rightarrow$ Message Signal

$A_c \cos 2\pi f_c t \rightarrow$ Carrier Signal

* The Low pass Complex envelope of the DSB-SC modulated wave is expressed as:

$$\boxed{\tilde{S}_{DSBSC}(t) = A_c m(t)}$$

* Consider the SSB modulated wave $S_u(t)$, in which only the USB is retained. It has quadrature as well as in-phase component.

Then $\tilde{S}_u(t)$ is the complex envelope of $S_u(t)$ & we can write

$$S_u(t) = Re\left[\tilde{S}_u(t) \exp(j2\pi f_c t)\right]$$

$$\boxed{S_u(t) = Re\left[\tilde{S}_u(t)\, e^{j2\pi f_c t}\right]} \longrightarrow (1)$$

Where, $R_e \longrightarrow$ real part.

* To determine $\tilde{S}_u(t)$, we proceed as follows :

i) The BPF transfer function $H_u(f)$ is replaced by an equivalent LPF of transfer function $\tilde{H}_u(f)$ as shown in fig (b).

We can express $\tilde{H}_u(f)$ as follows :

$$\tilde{H}_u(f) = \begin{cases} \frac{1}{2}[1+Sgn(f)] , & 0<f<W \\ 0 , & \text{otherwise} \end{cases} \longrightarrow (2)$$

Where, $Sgn(f)$ is the signum function.

ii) The DSB-SC modulated wave is replaced by its complex-envelope. The spectrum of this envelope is as shown in fig (c). i.e. $\boxed{\tilde{S}_{DSBSC}(f) = A_c M(f)} \longrightarrow (3)$

iii) The desired complex envelope $\tilde{S}_u(t)$ is determined by evaluating the IFT of the product $\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)$

i.e. $\tilde{S}_u(t) = IFT\left[\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)\right] \longrightarrow (4)$

Substituting eq ② & eq ③ in eq ④, we get

$$\tilde{S}_u(t) = \text{IFT}\left\{ \frac{1}{2}\left[1+\text{Sgn}(f)\right] \cdot A_c M(f) \right\}$$

$$= \text{IFT}\left\{ \frac{A_c}{2}\left[M(f) + \text{Sgn}(f) M(f)\right] \right\}$$

$$\boxed{\tilde{S}_u(t) = \frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right]} \longrightarrow ⑤$$

Substituting eq ⑤ in eq ①, we get

i.e $S_u(t) = R_e\left[\tilde{S}_u(t)\, e^{j2\pi f_c t} \right] \longrightarrow ①$

$$S_u(t) = R_e\left\{ \frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right] \cdot e^{j2\pi f_c t} \right\}$$

$$S_u(t) = R_e\left\{ \frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right] \text{Cos}(2\pi f_c t) + j\,\text{Sin}(2\pi f_c t) \right\}$$

$$= R_e\left\{ \frac{A_c}{2}\left[m(t)\text{Cos}(2\pi f_c t) + j\,m(t)\,\text{Sin}(2\pi f_c t) + j\hat{m}(t)\,\text{Cos}(2\pi f_c t) + j^2\,\hat{m}(t)\,\text{Sin}(2\pi f_c t)\right]\right\}$$

$$= R_e\left\{ \frac{A_c}{2}\left[m(t)\text{Cos}(2\pi f_c t) + j\,m(t)\,\text{Sin}(2\pi f_c t) + j\hat{m}(t)\,\text{Cos}(2\pi f_c t) - \hat{m}(t)\,\text{Sin}(2\pi f_c t)\right]\right\}$$

$$\boxed{S_u(t) = \frac{A_c}{2}\left[m(t)\,\text{Cos}(2\pi f_c t) - \hat{m}(t)\,\text{Sin}(2\pi f_c t)\right]} \longrightarrow ⑥$$

In-phase Component   Quadrature Component

Equation ⑥ Shows that the SSB modulated wave Contains only USB With an In-phase Component & a quadrature Component.

Single Tone SSB Modulation

Explain single tone modulation for transmitting only upper side (USB) frequency of SSB modulation.

* Let the modulating signal $m(t)$ is represented as

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (1)$$

* The hilbert transform of the modulating signal $m(t)$ is obtained by passing it through a -90° phase shifter. So the hilbert transform is given by

$$\boxed{\hat{m}(t) = A_m \sin(2\pi f_c t)} \longrightarrow (2)$$

* WKT the SSB wave with only USB is given by

$$S_u(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right] \longrightarrow (3)$$

Substituting eq (1) & eq (2) in eq (3), we get

$$S_u(t) = \frac{A_c}{2}\left[A_m \cos(2\pi f_m t)\cdot\cos(2\pi f_c t) - A_m \sin(2\pi f_m t)\cdot \sin(2\pi f_c t)\right]$$

W.K.T

$$\boxed{\cos(A+B) = \cos(A)\cdot\cos(B) - \sin(A)\cdot\sin(B)}$$

$$S_u(t) = \frac{A_c A_m}{2}\Big[\underset{\cos(A)}{\cos(2\pi f_c t)}\cdot\underset{\cos(B)}{\cos(2\pi f_m t)} - \underset{\sin(A)}{\sin(2\pi f_c t)}\cdot\underset{\sin(B)}{\sin(2\pi f_m t)}\Big]$$

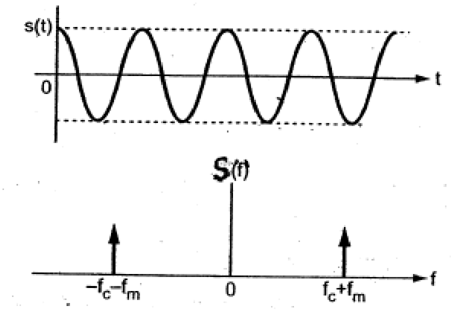
$$S_u(t) = \frac{A_c A_m}{2}\left[\cos(2\pi f_c t + 2\pi f_m t)\right]$$

$$\boxed{S_u(t) = \frac{A_c A_m}{2}\cos 2\pi\left[f_c + f_m\right]t} \longrightarrow (4)$$

Equation (4) Shows that the SSB wave consists of only the upper sideband of frequency (f_c+f_m).

* This is exactly same as the result obtained by suppressing the lower side-frequency (f_c-f_m) of the corresponding DSB-SC wave.

Spectrum of SSB with lower sideband supressed



Explain single tone modulation for transmitting only lower side (LSB) frequency of SSB modulation.

* Let the modulating signal $m(t)$ is represented as

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (1)$$

* The hilbert transform of the modulating signal $m(t)$ is obtained by passing it through a $-90°$ phase shifter. So the hilbert transform is given by:

$$\boxed{\hat{m}(t) = A_m \sin(2\pi f_m t)} \longrightarrow (2)$$

W.K.T the SSB wave with only LSB is given by:

$$\boxed{S_L(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]} \longrightarrow (3)$$

Substituting eq ① & eq ② in eq ③, we get

$$S_L(t) = \frac{A_c}{2}\left[A_m \cos(2\pi f_m t)\cos(2\pi f_c t) + A_m \sin(2\pi f_m t)\cdot\sin(2\pi f_c t)\right]$$

$$S_L(t) = \frac{A_c A_m}{2}\left[\underbrace{\cos(2\pi f_c t)}_{\cos(A)}\cdot\underbrace{\cos(2\pi f_m t)}_{\cos(B)} + \underbrace{\sin(2\pi f_c t)}_{\sin(A)}\cdot\underbrace{\sin(2\pi f_m t)}_{\sin(B)}\right]$$

W.K.T

$$\boxed{\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B}$$

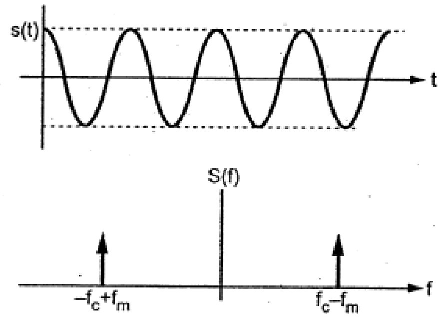
$$S_L(t) = \frac{A_c A_m}{2}\left[\cos(2\pi f_c t - 2\pi f_m t)\right]$$

$$\boxed{S_L(t) = \frac{A_c A_m}{2}\cos 2\pi\left[f_c - f_m\right]t} \longrightarrow ④$$

Equation ④ Shows that the SSB Wave Consists of only the Lower Side Frequency $(f_c - f_m)$.

* This is exactly Same as the result obtained by Suppressing the upper Side Frequency $(f_c + f_m)$ of the Corresponding DSB-SC Wave.

Spectrum of SSB with upper sideband supressed



Generation of SSB Wave:-

1. **Frequency discrimination method**
2. **Phase discrimination method or Hartley modulator**

Phase discriminator method or Hartley Modulator :-

❖ **Explain the generation of SSB-SC wave using Phase discrimination method with the help of a neat functional diagram. Bring out the merits and demerits of this.** **Jan-06,8M**

With a neat diagram, explain how SSB wave is generated using Phase shift method.

June-10,8M June-10,6M(IT) Jan-10,7M Jan-07,5M July-06,7M

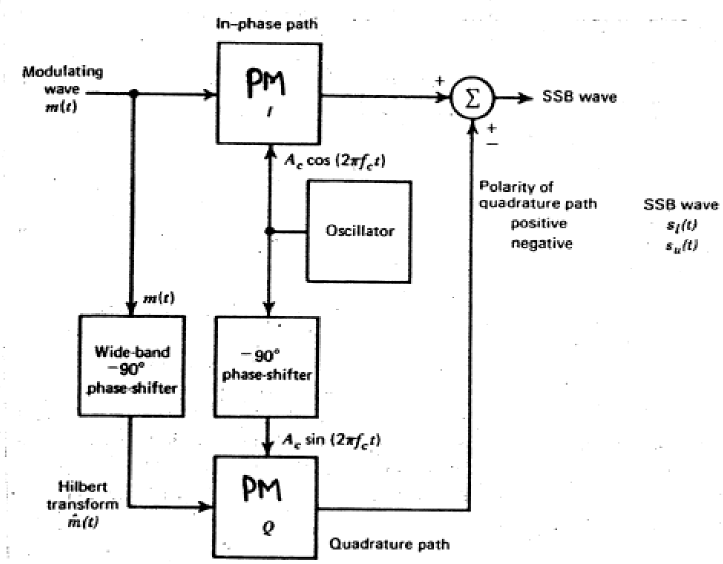


Figure
Block diagram of the phase discrimination method for generating SSB modulated waves.

Fig ① Shows the block diagram of phase discriminator method of generating SSB.

* The SSB modulator uses two product modulators I & Q, Supplied with carrier waves in phase quadrature to each other.

* The message signal $m(t)$ & a carrier signal $A_c \cos(2\pi f_c t)$ is directly applied to the product modulator I, producing a DSB-SC wave.

* The hilbert transform $\hat{m}(t)$ ($-90°$ phase shift) of $m(t)$ & carrier signal shifted by $90°$ are applied to the product modulator Q, producing DSB-SC wave.

* The o/p of product modulator 'I' is

$$\boxed{S_I(t) = m(t) A_c \cos(2\pi f_c t)}$$

* The o/p of product modulator 'Q' is

$$\boxed{S_Q(t) = \hat{m}(t) \cdot A_c \sin(2\pi f_c t)}$$

These signals $S_I(t)$ & $S_Q(t)$ are fed to a summer.

* The o/p of the summer is

$$S(t) = S_I(t) \pm S_Q(t)$$

$$\boxed{S(t) = A_c m(t) \cos 2\pi f_c t \pm A_c \hat{m}(t) \sin 2\pi f_c t}$$

* The plus sign at the summing junction yields an SSB with only the LSB i.e.

$$\boxed{S_L(t) = A_c m(t) \cos 2\pi f_c t + A_c \hat{m}(t) \sin 2\pi f_c t}$$

* Similarly the minus sign at the summing junction yields an SSB with only the USB i.e.

$$\boxed{S_u(t) = A_c m(t) \cos 2\pi f_c t - A_c \hat{m}(t) \sin 2\pi f_c t}$$

This SSB modulator is also known as the Hartley modulator.

Frequency discrimination method or Filtering method :-

Frequency discriminator method can be used for generating the SSB modulated wave if the message signal satisfies the following conditions:

i) The message signal should not have any low frequency content. (i.e the message spectrum $M(f)$ has "holes" at zero frequency)

The audio signals posses this property.

eg:- The telephone signals will have a frequency range extending from 300 Hz to 3.4 kHz. The frequencies in the range 0 - 300 Hz are absent, thereby creating an energy gap from 0 to 300 Hz

ii) The highest frequency component 'W' of the message signal $m(t)$ is much less than the carrier frequency 'f_c'.

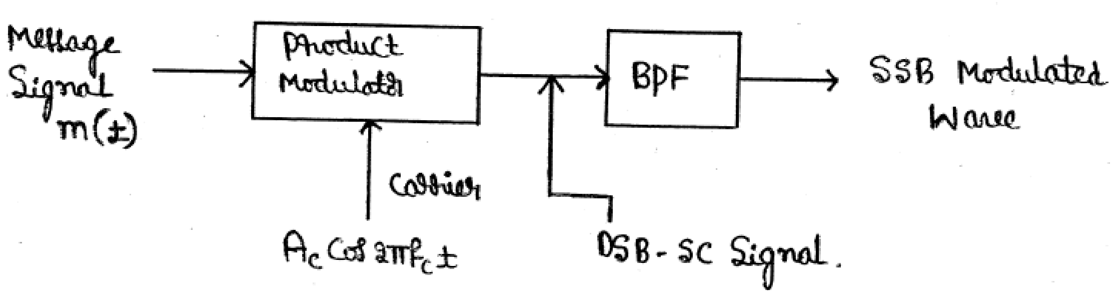


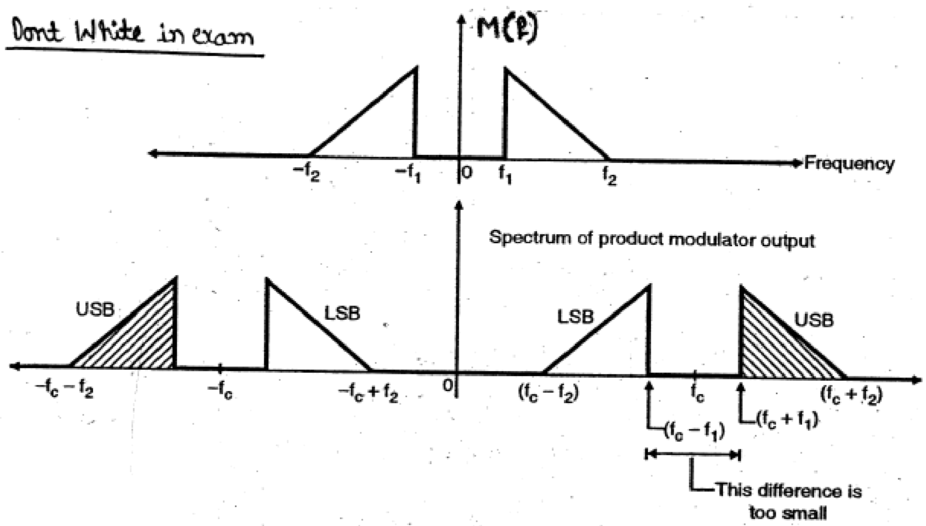
Fig ①: Block diagram of SSB modulator using frequency discriminator method

* This modulator consists of a product modulator, carrier oscillator & BPF designed to pass the desired sideband.

* At the o/p of the product modulator, we get the DSB-SC modulated wave which contains only two sidebands.

* The BPF will pass only one sideband & produce the SSB modulated wave & its o/p.

Dont Write in exam



Spectrums of message signal and product modulator output

<u>Demerits:-</u>

1) The frequency difference between the highest frequency in LSB & the lowest frequency in USB is too small as shown in fig (a):(b)

This makes the design of BPF extremely difficult, because its frequency response need to have <u>very sharp change over</u> from <u>attenuation</u> to <u>pass band</u> & vice-versa.

Design of BPF :-

The design of BPF must satisfy two basic requirements.

1) The pass band of the filter occupies the same frequency range as the spectrum of the desired SSB modulated wave.

2) The width of the <u>guard band</u> which separates the passband from stop band be twice the lowest frequency component of the message signal.

i.e. $\boxed{\text{Guard band} = 2f_1 \text{ Hz}}$

The conditions mentioned above are satisfied only by the highly selective filters using crystal resonators with a <u>high Q-factor</u> typically in the range <u>1000 to 2000.</u>

Two Stage SSB Modulator :-

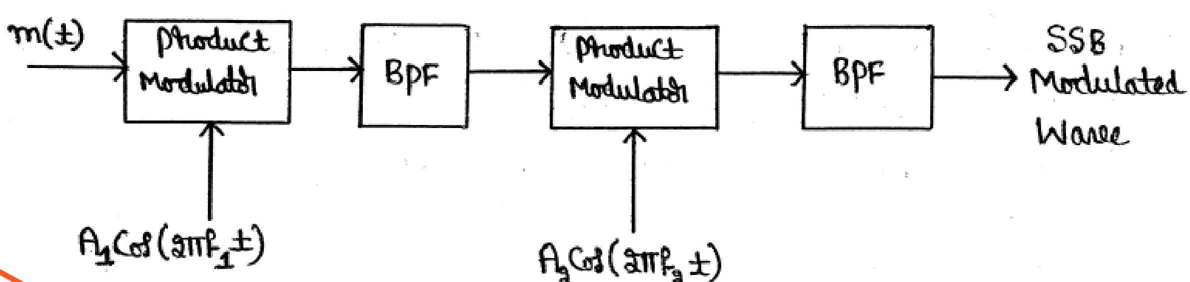


Fig ③ : A Two-Stage SSB modulator.

{ Dont Write :

* When the carrier frequency is very high as compared to the message frequency, the SSB modulated wave occupies the frequency band which is much higher than that of the message signal.

* under such operating conditions it becomes extremely difficult to design a BPF that passes the desired sideband & attenuates the unwanted sideband.

}

* The message signal m(t) modulates the carrier f_1 to produce a DSB-SC signal. This signal is passed through the 1st BPF to produce an SSB modulated signal.

* The o/p of the 1st BPF is then used to modulate another carrier 'f_2' which is higher than 'f_1'. Then the o/p of the 2nd product modulator we get another DSB-SC signal.

* Thus increases the Guard band between sideband frequency, which will make the filter design easy.

Advantages of filter method (Frequency discriminator method):-

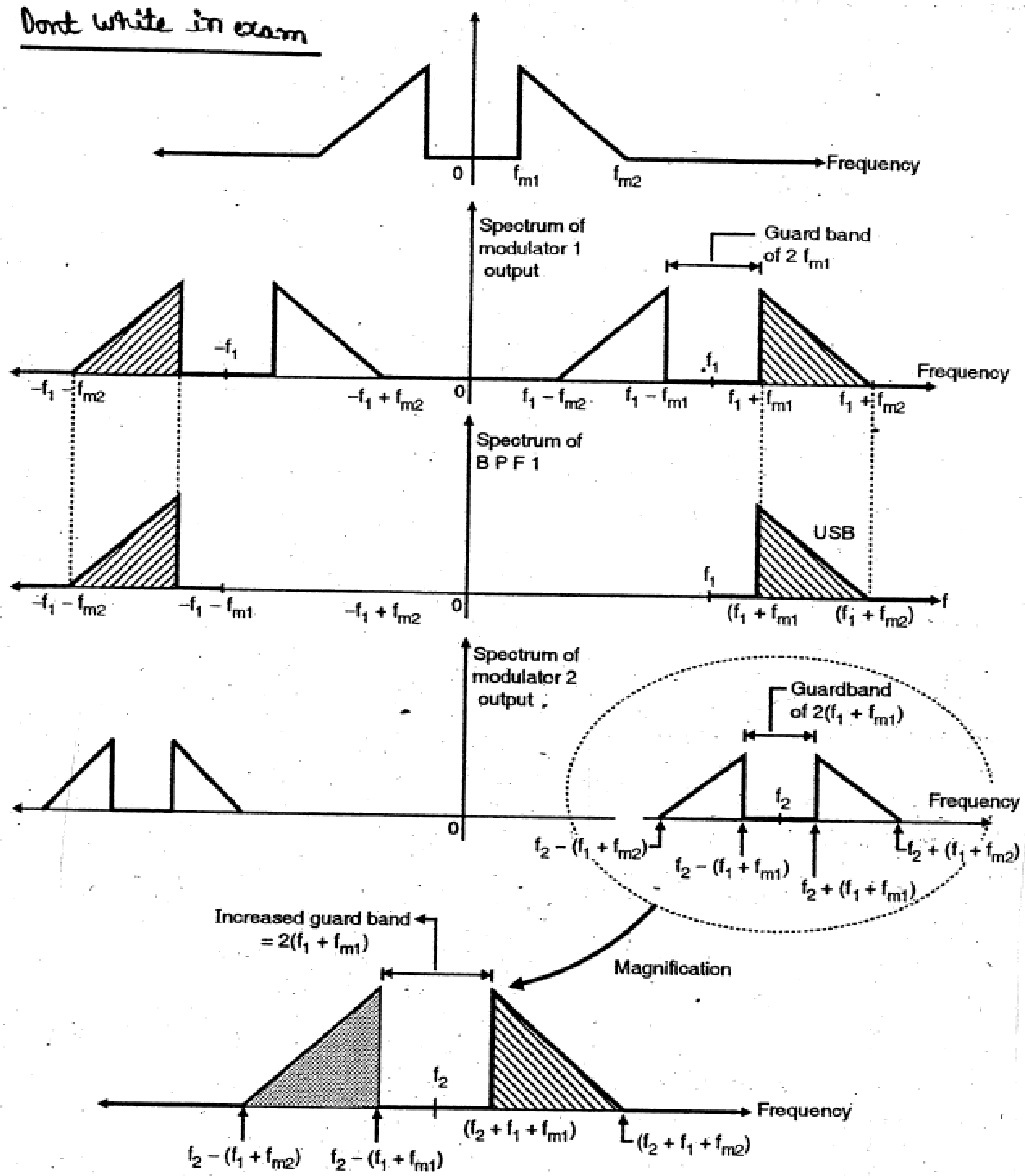
1) The filter method gives the adequate sideband suppression.
2) The sideband filters also helps to attenuate carrier if present in the o/p of balanced modulator.
3) The bandwidth is sufficiently flat & wide.

Disadvantages:-

1) They are bulky
2) Due to the inability of the system to generate SSB at high radio frequencies, the frequency up conversion is necessary.
3) Two expensive filters are to be used one for each sideband.

{ **Spectrum of two Stage SSB-modulator :-**

Dont write in exam



Spectrums of the two stage SSB modulator

}

Demodulation of SSB Wave:-

❖ **Show that the output of coherent detector of a SSB modulated wave is given by:** $V_o(t) = 1/4\, A_c m(t)\cos\phi + 1/4 A_c m(t) \sin\phi$

Where ϕ is the phase error.

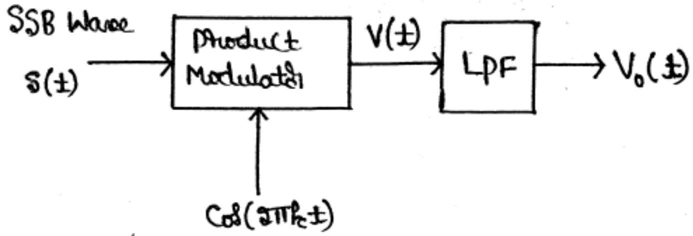


Fig ①: Coherent detection of an SSB modulated wave.

* The baseband Signal $m(t)$ can be recovered from the SSB Wave $S(t)$ by using coherent detection.

* The product modulator is having two I/p's. one I/p is the SSB modulated wave $S(t)$ & another I/p is the locally generated carrier $\cos(2\pi f_c t)$ then Low-pass filtering the modulator o/p as shown in above figure.

* Thus product modulator o/p is given by

$$\boxed{V(t) = S(t)\cos(2\pi f_c t)} \longrightarrow ①$$

WKT

$$S(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right] \longrightarrow ②$$

Substituting eq ② in eq ①, we get

$$V(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right]\cos 2\pi f_c t$$

$$V(t) = \frac{A_c}{2} m(t)\cos 2\pi f_c t \cdot \cos 2\pi f_c t \pm \frac{A_c}{2}\hat{m}(t)\cos 2\pi f_c t \cdot \sin 2\pi f_c t.$$

W.K.T

$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]}$$

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$$\boxed{\cos A \cdot \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]}$$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(2\pi f_c + 2\pi f_c)t + \cos(2\pi f_c - 2\pi f_c)t\right] \pm \frac{A_c}{4}\hat{m}(t)$$

$$\left[\sin(2\pi f_c + 2\pi f_c)t + \sin(2\pi f_c - 2\pi f_c)t\right]$$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(4\pi f_c t) + \cos(0)\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin(4\pi f_c t) + \sin(0)\right]$$

W.K.T, $\boxed{\cos(0) = 1,\ \sin(0) = 0}$

$$V(t) = \frac{A_c}{4} m(t)\left[\cos(4\pi f_c t + 1\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin(4\pi f_c t) + 0\right].$$

$$V(t) = \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t)\cos(4\pi f_c t) \pm \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)$$

$$V(t) = \underbrace{\frac{A_c}{4} m(t)}_{\text{Scaled message Signal}} + \underbrace{\frac{A_c}{4}\left[m(t)\cos(4\pi f_c t) \pm \hat{m}(t)\sin(4\pi f_c t)\right]}_{\text{Unwanted terms}}$$

* When $V(t)$ is passed through the filter, it will allow only the 1st term to pass through & will reject all other unwanted terms.

* Thus at the o/p of the filter we get the scaled message signal & the coherent SSB demodulation is achieved.

$$\therefore \boxed{V_0(t) = \frac{A_c}{4} m(t)}$$

The detection of SSB modulated waves is based on the assumption that there is perfect synchronization between local carrier & that in the transmitter both in <u>frequency</u> & <u>phase</u>.

* But in practice a phase error ϕ may arise in the locally - generated carrier wave. Thus the detector o/p is modified due to

phase error as follows:

$$\boxed{V_o(t) = \frac{A_c}{4} m(t) \cos\phi \pm \frac{A_c}{4} \hat{m}(t) \sin\phi}$$

NOTE :-

* The phase distortion is not serious with voice communication because the human ear is relatively insensitivity to phase distortion. The presence of phase distortion gives rise to what is called the Donald Duck voice effect.

* The phase distortion cannot be tolerable in the transmission of music & video signal.

Formulae

1) $\sin A \cdot \cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$

$$\sin(2\pi f_c t)\cdot\cos(2\pi f_c t) = \frac{1}{2}\sin[\cancel{2\pi f_c t - 2\pi f_c t}]^{0} + \frac{1}{2}\sin[2\pi f_c t + 2\pi f_c t]$$

$$\boxed{\sin(2\pi f_c t)\cdot\cos(2\pi f_c t) = \frac{\sin(4\pi f_c t)}{2}}$$

2) $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$\cos^2(2\pi f_c t) = \frac{1+\cos 2(2\pi f_c t)}{2}$$

$$\boxed{\cos^2(2\pi f_c t) = \frac{1}{2} + \frac{\cos(4\pi f_c t)}{2}}$$

3) $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$\sin^2(2\pi f_c t) = \frac{1-\cos 2(2\pi f_c t)}{2}$$

$$\boxed{\sin^2(2\pi f_c t) = \frac{1}{2} - \frac{\cos(4\pi f_c t)}{2}}$$

Time function	Hilbert Transform
$\cos(2\pi f_c t)$	$\sin(2\pi f_c t)$
$\sin(2\pi f_c t)$	$-\cos(2\pi f_c t)$

1) Let $S_u(t)$ denote the SSB Signal obtained by transmitting only upper Sideband & let $\hat{S}_u(t)$ denotes its Hilbert transform. Show that:

$$m(t) = \frac{2}{A_c}\left[S_u(t)\cos(2\pi f_c t) + \hat{S}_u(t)\sin(2\pi f_c t)\right] \quad \text{and}$$

$$\hat{m}(t) = \frac{2}{A_c}\left[\hat{S}_u(t)\cos(2\pi f_c t) - S_u(t)\sin(2\pi f_c t)\right]$$

Jan-2009, 6M

Sol:-

W.K.T in SSB modulation $S_u(t)$ is given by

$$\boxed{S_u(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]} \longrightarrow (1)$$

Taking hilbert transform of eq (1), we get

$$\hat{S}_u(t) = \frac{A_c}{2}\left[m(t)\widehat{\cos(2\pi f_c t)} - \hat{m}(t)\widehat{\sin(2\pi f_c t)}\right]$$

$$\hat{S}_u(t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) - \hat{m}(t)\left[-\cos(2\pi f_c t)\right]\right]$$

$$\boxed{\hat{S}_u(t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) + \hat{m}(t)\cos(2\pi f_c t)\right]} \longrightarrow (2)$$

Multiply eq (1) by $\underline{\cos(2\pi f_c t)}$

$$S_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t)$$

$$S_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}m(t)\underline{\cos^2(2\pi f_c t)} - \hat{m}(t)\underline{\sin(2\pi f_c t)\cos(2\pi f_c t)}\frac{A_c}{2}$$

$$= \frac{A_c}{2}m(t)\left[\frac{1}{2} + \frac{\cos(4\pi f_c t)}{2}\right] - \hat{m}(t)\left[\frac{\sin(4\pi f_c t)}{2}\right]\frac{A_c}{2}$$

$$\boxed{S_u(t)\cos(2\pi f_c t) = \frac{A_c}{4}m(t) + \frac{A_c}{4}m(t)\cos(4\pi f_c t) - \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)} \rightarrow (3)$$

Multiply eq (2) by $\sin(2\pi f_c t)$

$$\hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) + \hat{m}(t)\cos(2\pi f_c t)\right]\sin(2\pi f_c t)$$

$$\hat{S}_u(t)\,\sin(2\pi f_c t) = \frac{A_c}{2} m(t)\,\sin^2(2\pi f_c t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\cdot\cos(2\pi f_c t)$$

$$= \frac{A_c}{2} m(t)\left[\frac{1}{2} - \frac{\cos(4\pi f_c t)}{2}\right] + \frac{A_c}{2}\hat{m}(t)\left[\frac{\sin(4\pi f_c t)}{2}\right]$$

$$\boxed{\hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{4} m(t) - \frac{A_c}{4} m(t)\cos(4\pi f_c t) + \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)} \longrightarrow (4)$$

Add eq (3) & (4), We get

$$S_u(t)\cos(2\pi f_c t) + \hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{4} m(t) + \frac{A_c}{4}\cancel{m(t)\cos(4\pi f_c t)} - \frac{A_c}{4}\cancel{\hat{m}(t)\sin(4\pi f_c t)}$$

$$+ \frac{A_c}{4} m(t) - \frac{A_c}{4}\cancel{m(t)\cos(4\pi f_c t)} - \frac{A_c}{4}\cancel{\hat{m}(t)\sin(4\pi f_c t)}$$

$$S_u(t)\cos(2\pi f_c t) + \hat{S}_u(t)\sin(2\pi f_c t) = \frac{A_c}{2} m(t)$$

$$\boxed{m(t) = \frac{2}{A_c}\left[S_u(t)\cos(2\pi f_c t) + \hat{S}_u(t)\sin(2\pi f_c t)\right]}$$

Multiply eq (1) by $\sin(2\pi f_c t)$

$$S_u(t)\,\sin(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]\sin(2\pi f_c t)$$

$$= \frac{A_c}{2} m(t)\,\underline{\sin(2\pi f_c t)\cdot\cos(2\pi f_c t)} - \frac{A_c}{2}\hat{m}(t)\,\underline{\sin^2(2\pi f_c t)}$$

$$= \frac{A_c}{2} m(t)\left[\frac{\sin(4\pi f_c t)}{2}\right] - \frac{A_c}{2}\hat{m}(t)\left[\frac{1}{2} - \frac{\cos(4\pi f_c t)}{2}\right]$$

$$\boxed{S_u(t)\sin(2\pi f_c t) = \frac{A_c}{4} m(t)\sin(4\pi f_c t) - \frac{A_c}{4}\hat{m}(t) + \frac{A_c}{4}\hat{m}(t)\cos(4\pi f_c t)} \rightarrow (5)$$

Multiply eq (2) by $\cos(2\pi f_c t)$

$$\hat{S}_u(t)\cos(2\pi f_c t) = \frac{A_c}{2}\left[m(t)\sin(2\pi f_c t) + \hat{m}(t)\cos(2\pi f_c t)\right]\cos(2\pi f_c t)$$

$$= \frac{A_c}{2} m(t)\,\underline{\sin(2\pi f_c t).\cos(2\pi f_c t)} + \frac{A_c}{2}\hat{m}(t)\,\underline{\cos^2(2\pi f_c t)}$$

$$= \frac{A_c}{2} m(t)\left[\frac{\sin(4\pi f_c t)}{2}\right] + \frac{A_c}{2}\hat{m}(t)\left[\frac{1}{2} + \frac{\cos(4\pi f_c t)}{2}\right]$$

$$\boxed{\hat{S}_u(t)\cos(2\pi f_c t) = \frac{A_c}{4} m(t)\sin(4\pi f_c t) + \frac{A_c}{4}\hat{m}(t) + \frac{A_c}{4}\hat{m}(t)\cos(4\pi f_c t)} \rightarrow (6)$$

Subtract eq (6) - eq (5), we get

$$\hat{S}_u(t)\cos(2\pi f_c t) - S_u(t)\sin(2\pi f_c t) = \frac{A_c}{4} m(t)\cancel{\sin(4\pi f_c t)} + \frac{A_c}{4}\hat{m}(t)$$

$$+ \frac{A_c}{4}\hat{m}(t)\cancel{\cos(4\pi f_c t)} - \frac{A_c}{4} m(t)\cancel{\sin(4\pi f_c t)} + \frac{A_c}{4}\hat{m}(t)$$

$$- \frac{A_c}{4}\hat{m}(t)\cancel{\cos(4\pi f_c t)}$$

$$\hat{S}_u(t)\cdot\cos(2\pi f_c t) - S_u(t)\sin(2\pi f_c t) = \frac{A_c}{2}\hat{m}(t)$$

$$\boxed{\hat{m}(t) = \frac{2}{A_c}\left[\hat{S}_u(t)\cos(2\pi f_c t) - S_u(t)\sin(2\pi f_c t)\right]}$$

---

*) Explain the generation of SSB-SC Wave using phase discriminator method with the help of a neat functional block diagram. Bring out the merits & demerits of this.

Jan - 2006, 8M

Merits :-

1) Bulkier filters are replaced by Small filters.

2) Low audio frequencies may be used for modulation

3) It can generate SSB at any frequency

4) Easy Switching from one Sideband to other Sideband is possible.

Demerits :-

1) The o/p of two balanced modulators must be exactly Same; otherwise cancellation will be incomplete.

2) If the phase shifter provides a phase change other than 90° at any audio frequency, that particular frequency will not be completely removed from the unwanted sideband.

Hence great care in adjustment is necessary.

---

* In a coherent detection, if carrier applied is $\cos(2\pi f_c t + \phi)$, prove that there is a phase error in the o/p & o/p consists not only the message signal but also its hilbert transform.

Sol:-

W.K.T the SSB modulated wave is,

$$s(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right] \longrightarrow (1)$$

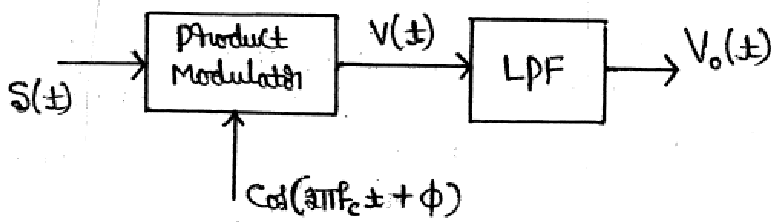


Fig (1) Coherent detector

* The o/p of the product modulator is given by

$$\boxed{V(t) = s(t)\cdot\cos(2\pi f_c t + \phi)} \longrightarrow (2)$$

Substituting eq (1) in eq (2), we get

$$V(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) \pm \hat{m}(t)\sin 2\pi f_c t\right]\cos(2\pi f_c t + \phi)$$

$$V(t) = \frac{A_c}{2}\left[m(t)\underline{\cos(2\pi f_c t + \phi)\cdot\cos(2\pi f_c t)} \pm \hat{m}(t)\underline{\cos(2\pi f_c t + \phi)\cdot\sin(2\pi f_c t)}\right]$$

{ W.K.T i) $\cos A\cdot\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$\cos(2\pi f_c t+\phi)\cdot\cos(2\pi f_c t) = \frac{1}{2}\left[\cos(\cancel{2\pi f_c t}+\phi-\cancel{2\pi f_c t})+\cos(2\pi f_c t+\phi+2\pi f_c t)\right]$$

$$\boxed{\cos(2\pi f_c t+\phi)\cdot\cos(2\pi f_c t) = \frac{1}{2}\left[\cos\phi+\cos(4\pi f_c t+\phi)\right]}$$

ii) $\sin A\cdot\cos B = \frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$

$$\sin(2\pi f_c t)\cos(2\pi f_c t+\phi) = \frac{1}{2}\left\{\sin(\cancel{2\pi f_c t}-[\cancel{2\pi f_c t}+\phi])+\sin(2\pi f_c t+[2\pi f_c t+\phi])\right\}$$

$$= \frac{1}{2}\left\{\sin(-\phi)+\sin(4\pi f_c t+\phi)\right\}$$

$$\sin(2\pi f_c t)\cos(2\pi f_c t+\phi) = \frac{1}{2}\left[-\sin\phi+\sin(4\pi f_c t+\phi)\right]$$

$\boxed{\text{NOTE: } \pm(-\sin\phi) = \mp\sin\phi}$

}

$$\therefore V(t) = \frac{A_c}{2}\left\{\frac{m(t)}{2}\left[\cos\phi+\cos(4\pi f_c t+\phi)\right]\mp\frac{\hat{m}(t)}{2}\left[\sin\phi+\sin(4\pi f_c t+\phi)\right]\right\}$$

$$= \frac{A_c}{4}m(t)\cos\phi+\frac{A_c}{4}m(t)\cos(4\pi f_c t+\phi)\mp\frac{A_c}{4}\hat{m}(t)\sin\phi$$
$$\mp\frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t+\phi)$$

$$V(t) = \underbrace{\frac{A_c}{4}\left[m(t)\cos\phi\mp\hat{m}(t)\sin\phi\right]}_{\text{Wanted Signal.}}+\underbrace{\frac{A_c}{4}\left[m(t)\cos(4\pi f_c t+\phi\mp\hat{m}(t)\sin(4\pi f_c t+\phi)\right]}_{\text{unwanted Signal}} \rightarrow (3)$$

* The o/p of the product modulator 'V(t)' is passed through a LPF to get desired Signal.

Thus the o/p of the LPF is the desired Signal & is given by:

$$\boxed{V_o(t) = \frac{A_c}{4}\left[m(t)\cos\phi\mp\hat{m}(t)\sin\phi\right]} \rightarrow (4)$$

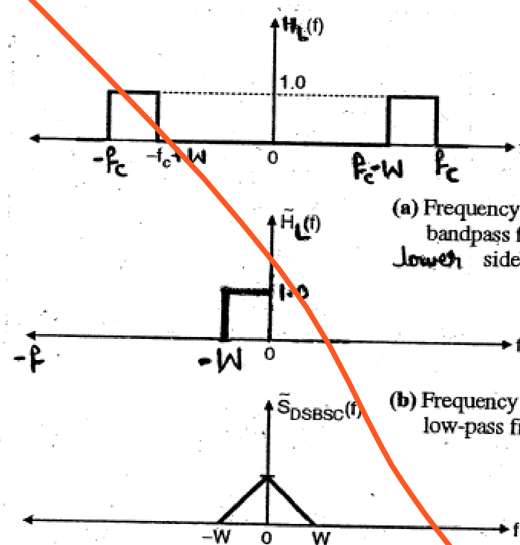
In eq (4), $\hat{m}(t)$ is hilbert transform of $m(t)$. This shows that there is a phase distortion in the o/p, due to the phase error of ϕ in the local carrier Signal.

* Derive an expression for SSB modulated wave for which Lower Sideband is retained. $\boxed{\text{Jan - 2009, 8M}}$

{ Let $S_L(t)$ denote an SSB modulated wave in which only the lower Sideband is retained.
}

Sol:-

The SSB Signal may be generated by passing a DSB-SC modulated wave through a BPF of transfer function $H_L(f)$



(a) Frequency response of ideal bandpass filter to select the lower sideband.

(b) Frequency response of equivalent low-pass filter.

(c) Spectrum of complex envelope of DSBSC.

* The DSB-SC modulated wave is defined mathematically as:

$$\boxed{S_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)}$$

Where,

$m(t) \rightarrow$ Message Signal

$A_c \cos(2\pi f_c t) \rightarrow$ Carrier Signal

* The Low pass Complex envelope of the DSB-SC modulated wave is expressed as :

$$\boxed{S_{DSBSC}(t) = A_c m(t)}$$

* Consider the SSB modulated wave $S_L(t)$, in which only the LSB is retained. It has quadrature as well as In-phase component.

Then $\tilde{S}_L(t)$ is the complex envelope of $S_L(t)$ & we can write

$$S_L(t) = R_e\left[\tilde{S}_L(t) \exp(j2\pi f_c t)\right]$$

$$\boxed{S_L(t) = R_e\left[\tilde{S}_L(t)\, e^{j2\pi f_c t}\right]}$$

Where, $R_e \rightarrow$ real part.

* To determine $\tilde{S}_L(t)$, We proceed as follows :

i) The BPF transfer function $H_L(f)$ is replaced by an equivalent LPF of transfer function $\tilde{H}_L(f)$ as shown in fig ⓑ.

We can express $\tilde{H}_L(f)$ as follows :

$$\tilde{H}_L(f) = \begin{cases} \frac{1}{2}\left[1 + \mathrm{Sgn}(-f)\right] & , \quad 0 < f < -W \\ 0 & , \quad \text{otherwise} \end{cases} \longrightarrow ②$$

Where, $\mathrm{Sgn}(-f)$ is the Signum function.

ii) The DSB-SC modulated wave is replaced by its complex envelope. The spectrum of this envelope is as shown in fig ⓒ

i.e.

$$\boxed{\tilde{S}_{DSBSC}(f) = A_c M(f)} \longrightarrow ③$$

iii) The desired complex envelope $\tilde{S}_L(t)$ is determined by evaluating the IFT of the product $\tilde{H}_L(f) \cdot \tilde{S}_{DSBSC}(f)$

i.e. $\tilde{S}_L(t) = IFT[\tilde{H}_L(f) \cdot \tilde{S}_{DSBSC}(f)] \longrightarrow$ ④

Substituting eq ② & eq ③ in eq ④, we get

$$\tilde{S}_L(t) = IFT\left\{\frac{1}{2}[1+Sgn(-f)] \cdot A_c M(f)\right\}$$

$$\tilde{S}_L(t) = IFT\left\{\frac{A_c}{2}[M(f) + Sgn(-f) M(f)]\right\}$$

$$\boxed{\tilde{S}_L(t) = \frac{A_c}{2}[m(t) - j\hat{m}(t)]} \longrightarrow ⑤$$

Substituting eq ⑤ in eq ①, we get $\left(S_L(t) = Re\left[\tilde{S}_L(t) e^{j2\pi f_c t}\right] \rightarrow ①\right)$

$$S_L(t) = Re\left\{\frac{A_c}{2}[m(t) - j\hat{m}(t)] e^{j2\pi f_c t}\right\}$$

$$S_L(t) = Re\left\{\frac{A_c}{2}[m(t) - j\hat{m}(t)] \underline{Cos(2\pi f_c t) + jSin(2\pi f_c t)}\right\}$$

$$= Re\left\{\frac{A_c}{2}\left[m(t) Cos(2\pi f_c t) + jm(t) Sin(2\pi f_c t) - j\hat{m}(t) Cos(2\pi f_c t) - j^2\hat{m}(t) Sin(2\pi f_c t)\right]\right\}$$

$$= Re\left\{\frac{A_c}{2}\left[m(t) Cos(2\pi f_c t) + jm(t) Sin(2\pi f_c t) - j\hat{m}(t) Cos(2\pi f_c t) + \hat{m}(t) Sin(2\pi f_c t)\right]\right\}$$

$\because -j^2 = +1$

$$\boxed{S_L(t) = \frac{A_c}{2}[m(t) Cos(2\pi f_c t) + \hat{m}(t) Sin(2\pi f_c t)]} \longrightarrow ⑥$$

In-phase Component — Quadrature Component.

Equation ⑥ Shows that the SSB modulated wave contains only LSB with an Inphase Component & a quadrature Component.

1. **Consider a 2-stage SSB modulator as shown in fig1. The i/p signal consists of a voice signal in a frequency range of 0.3 to 3.4 KHz. The two oscillator frequencies have values f1=100 KHz and f2=10 MHz. Specify the following:**

   i. **Sidebands of DSB-SC modulated waves appearing at the outputs of the product modulation (PM)**
   ii. **Sidebands of SSB modulated waves appearing at two BPF outputs.**
   iii. **The pass bands and guard bands of the two BPFs.**

June-10,8M

2. **Consider a two-stage product modulator with a BPF after each product modulator, where i/p signal consists of a voice signal occupying the frequency band 0.3 to 3.4 KHz. The two oscillator frequencies have values f1=100 KHz and f2=10 MHz. Specify the following:**

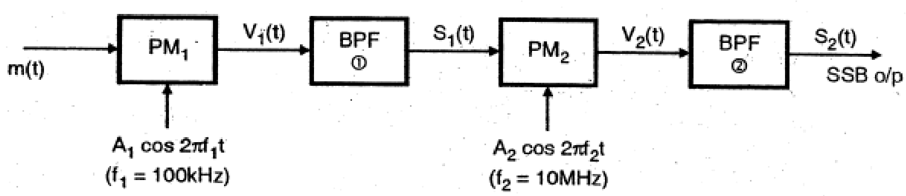
   i. **Sidebands of DSB-SC modulated waves appearing at the two product modulator output.**
   ii. **Sidebands of SSB modulated waves appearing at BPF outputs.**
   iii. **The pass bands of the two BPFs.**

June -10,9M (OLD)
March -2002, 8M

Given :-

$f_1 = 100KHz$, $f_2 = 10MHz$

$m(t) = 0.3KHz$ to $3.4KHz$



o/p of PM1 : $\boxed{f_c = f_1 = 100KHz, \quad f_m = 0.3KHz \text{ to } 3.4KHz}$

The PM1 o/p consists of two sidebands as follows

$LSB = f_c - f_m = 100KHz - (0.3KHz \text{ to } 3.4KHz)$

$\boxed{LSB = 99.7 KHz \text{ to } 99.6KHz}$

$USB = f_c + f_m = 100KHz + (0.3KHz \text{ to } 3.4KHz)$

$\boxed{USB = 100.3KHz \text{ to } 103.4KHz}$

o/p of BPF1 :- Assume that this BPF1 passes only the USB.

$$\boxed{S_1(t) = 100.3\ KHz \text{ to } 103.4\ KHz}$$

o/p of PM2 :- $\boxed{f_2 = f_c = 10MHz \ , \quad S_1(t) = f_m = 100.3\ KHz \text{ to } 103.4\ KHz}$

The PM2 o/p Consists of two Sidebands as follows:

$LSB = f_c - f_m \ = \ 10MHz - (100.3\ KHz \text{ to } 103.4\ KHz)$

$\boxed{LSB = 9.899\ MHz \text{ to } 9.8966\ MHz}$

$USB = f_c + f_m \ = \ 10MHz + (100.3\ KHz \text{ to } 103.4KHz)$

$\boxed{USB = 10.1003\ MHz \text{ to } 10.1034\ MHz}$

o/p of BPF2 :- Assume that this BPF2 passes only the USB

$$\boxed{S_2(t) = 10.1003MHz \text{ to } 10.1034\ MHz}$$

Guard band of BPF :-

Guard band is defined as the highest frequency component of LSB to the lowest frequency component of USB.

$\boxed{\text{Guard band of BPF1} = 99.7\ KHz \text{ to } 100.3KHz}$

$\boxed{\text{Guard band of BPF2} = 9.8997\ MHz \text{ to } 10.1003\ MHz}$

NOTE:-

Parameter	LSB	USB
O/P of PM1- V_1 (t)	99.7 KHz to 99.6 KHz	100.3 KHz to103.4 KHz
O/P 0f BPF1 – S_1 (t)	-	100.3 KHz to103.4 KHz
O/P of PM1- V_2 (t)	9.899 MHz to 9.8966 MHz	10.1003 MHz to 10.1034 MHz
O/P 0f BPF1 – S_2 (t)	-	10.1003 MHz to 10.1034 MHz

Guard Band of BPF-1 = 99.7 KHz to 100.3 KHz

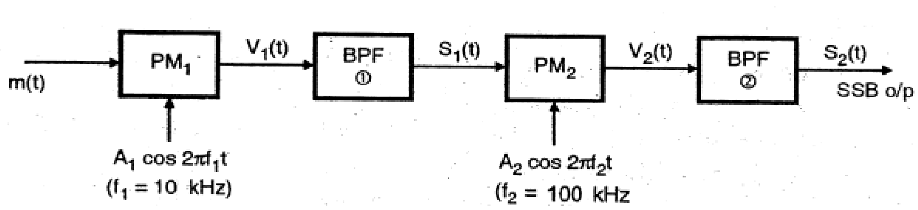
Guard Band of BPF-2 = 9.8997 MHz to10.1003 MHz

- **Consider a two-stage SSB modulator where the message signal occupies a band 0.3KHz to 4 KHz and the two carrier frequencies are f1=10 KHz and f2=100 KHz. Evaluate the following:**

  i. **Sidebands of DSB-SC modulated waves at the output of the product modulators.**
  ii. **The Sidebands of SSB the modulated waves at the outputs of BPF.**
  iii. **The pass bands and the guard bands of the two BPFs.**
  iv. **The order of the two filters assuming atleast 15dB attenuation between the passband and stop band.**
  v. **Sketch the spectrum of the signal at each stage.(Assume suitable M(f))**

Sol:-

Jan -06,12M



Given :- $m(t) = 0.3KHz$ to $4KHz$

$f_1 = 10KHz$, $f_2 = 100KHz$

o/p of PM1 :- $\boxed{f_c = f_1 = 10KHz \, , \; f_m = 0.3KHz \text{ to } 4KHz}$

$LSB = f_c - f_m = 10KHz - (0.3KHz \text{ to } 4KHz)$

$\boxed{LSB = 6KHz \text{ to } 9.7 KHz}$

$USB = f_c + f_m = 10KHz + (0.3KHz \text{ to } 4KHz)$

$\boxed{USB = 10.3KHz \text{ to } 14KHz}$

o/p of BPF1 :- Assume that BPF1 passes only the USB

$\boxed{S_1(t) = 10.3KHz \text{ to } 14KHz}$

o/p of PM2 :- $\boxed{f_c = f_2 = 100KHz \, , \; f_m = 10.3KHz \text{ to } 14KHz}$

$LSB = f_c - f_m = 100KHz - (10.3KHz \text{ to } 14KHz)$

$\boxed{LSB = 86KHz \text{ to } 89.7KHz}$

$USB = f_c + f_m$

$USB = 100KHz + (10.3KHz \text{ to } 14KHz)$

$\boxed{USB = 110.3KHz \text{ to } 114KHz}$

o/p of BPF2 :- Assume that BPF2 passes only the USB

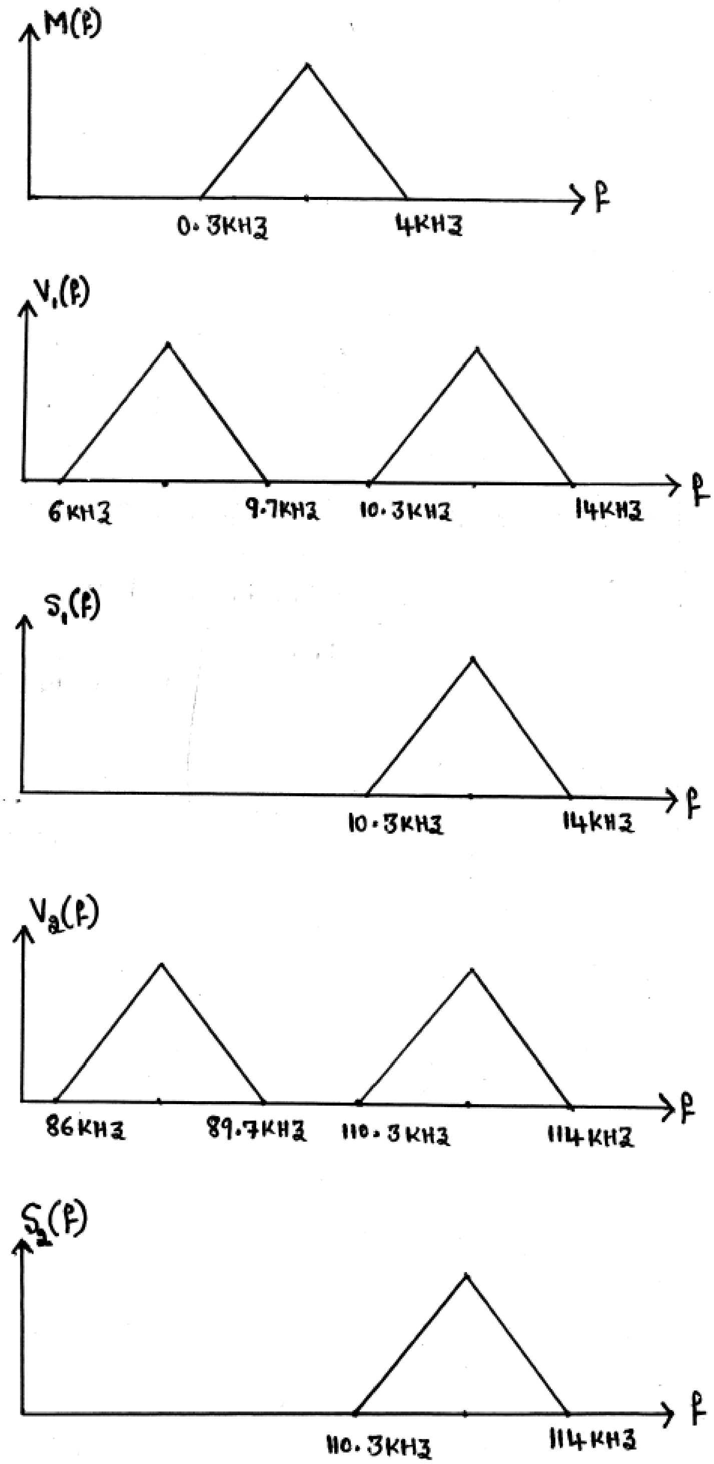
$\boxed{S_2(t) = 110.3KHz \text{ to } 114KHz}$

ii) **Guard Band of BPF's :-**

Guard band of BPF1 =	9.7KHz to 10.3KHz
Guard band of BPF2 =	89.7KHz to 110.3KHz

iv) The filter method gives Sideband Suppression upto <u>56dB</u>. Hence both BPFs are <u>1st order filters.</u>

v)



NOTE :

Parameter	LSB	USB
O/P of PM1- V_1 (t)	6 KHz to 9.7 KHz	10.3 KHz to14 KHz
O/P 0f BPF1 – S_1 (t)	-	10.3 KHz to14 KHz
O/P of PM1- V_2 (t)	86 KHz to 89.7 KHz	110.3 KHz to 114 KHz
O/P 0f BPF1 – S_2 (t)	-	110.3 KHz to 114 KHz

Guard Band of BPF-1 = 9.7 KHz to 10.3 KHz

Guard Band of BPF-2 = 89.7 KHz to110.3 KHz

---

* For the rectangular pulse shown in fig ①, evaluate its hilbert - transform.

June-10, 4M

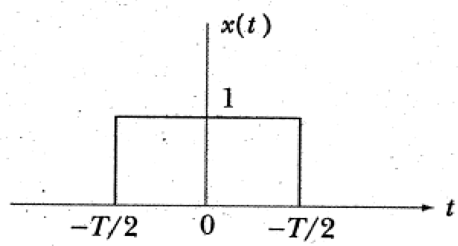


Fig ①: Rectangular pulse.

Sol :-

From fig ①, $x(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & |t| > T/2 \end{cases}$

WKT

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$
$$= \int_{-\infty}^{\infty} \frac{x(\tau)}{\pi(t-\tau)} d\tau$$
$$= \int_{-T/2}^{T/2} \frac{1}{\pi(t-\tau)} d\tau$$

$$
\begin{aligned}
&= -\frac{1}{\pi}\int_{-T/2}^{T/2}\frac{1}{\tau - t}\,d\tau \\
&= -\frac{1}{\pi}\left[\ln(\tau - t)\right]\Big|_{\tau=-T/2}^{T/2} \\
&= -\frac{1}{\pi}\left[\ln\left(\frac{T}{2} - t\right) - \ln\left(-\frac{T}{2} - t\right)\right] \\
&= \frac{1}{\pi}\ln\left[\frac{t + \frac{T}{2}}{t - \frac{T}{2}}\right]
\end{aligned}
$$

VSB Modulation:-

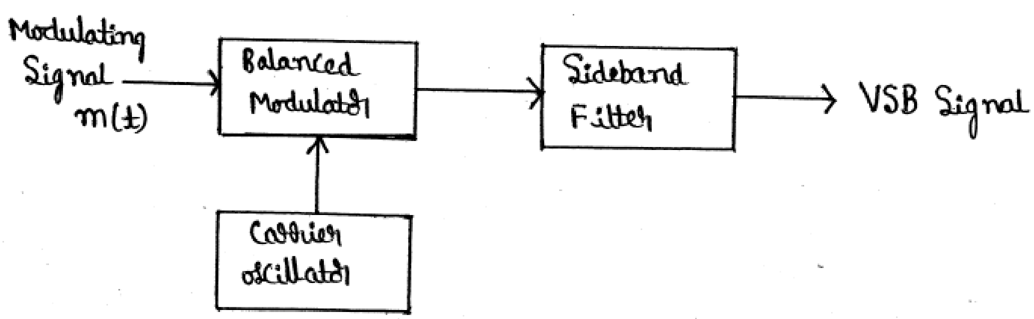
The Stringent (Very Strict Condition) frequency response requirements on the Sideband filter in SSB-SC modulation can be relaxed by allowing a part of the unwanted Sideband (called as vestige) to appear in the o/p of the modulator.

Due to this, the design of the Sideband filter is Simplified to a great extent. But the bandwidth of the System is increased Slightly.

---

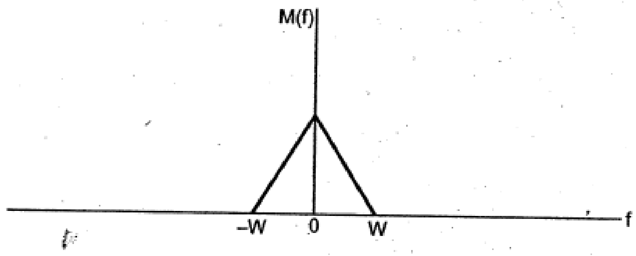
* Explain VSB modulation? Mention the advantages and - applications of VSB modulation

In VSB, one Sideband & a part of the other Sideband called as vestige is transmitted. So the bandwidth required for VSB transmission is Somewhat higher than that of SSB-modulation.

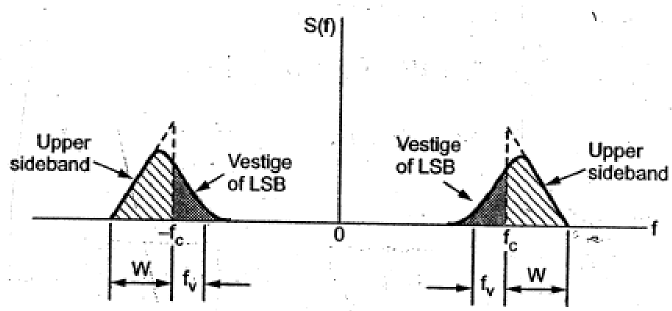


* To generate a VSB Signal, 1st we have to generate a DSB-SC Signal & then pass it through a Sideband filter as shown in figure. This filter will pass the wanted Sideband as it is along with a part of unwanted Sideband.

Frequency - domain description of VSB Wave :-



(a) Spectrum of message signal



(b) Spectrum of VSB modulated wave

* Fig shows the Spectrum of a VSB modulated Wave $S(t)$ along with the message Signal $m(t)$.

Here lower Sideband is modified into vestigial Sideband.

* Transmission bandwidth is given by

$$\boxed{B = (W + f_v)\ Hz}$$

Where,

W is message bandwidth

f_v is the width of the vestigial Sideband.

Advantages of VSB :-

The main advantage of VSB modulation is:

1) The reduction in bandwidth. It is almost as efficient as th SSB

2) Easy to design the filter.
(Due to allowance of transmitting a part of lower Sideband the Constraint on the filters have been relaxed.)

Application of VSB:-

VSB modulation has become Standard for the transmission of TV Signals. Because the video Signals need a larger-transmission bandwidth if transmitted using DSB-FC & DSB-SC techniques.

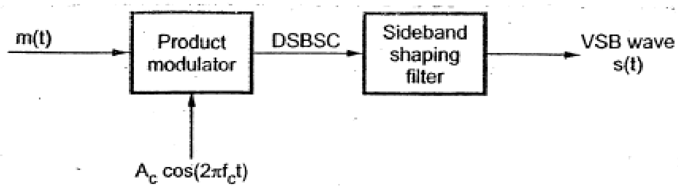
❖ **What is meant by VSB? Explain how VSB signal can be obtained from a modulating signal m(t)using a carrier $A_c\cos(2\pi f_c t)$ and later demodulated.**

July-07,8M

❖ **Explain the scheme for generation and demodulation of VSB modulated wave, with relevant spectrum of signals in the demodulation scheme. Give relevant mathematical expressions.** **June-08,10M, June-10,8M**

In VSB, one Sideband & a part of the other Sideband called as vestige is transmitted. So the bandwidth required for VSB transmission is Some what higher than that of SSB-modulation.

Generation of VSB modulated Wave:-



* The o/p of the product modulator is the DSB-SC wave & is given by:

$$S(t) = m(t) . c(t)$$

$$\boxed{S(t) = m(t) A_c \cos(2\pi f_c t)}$$

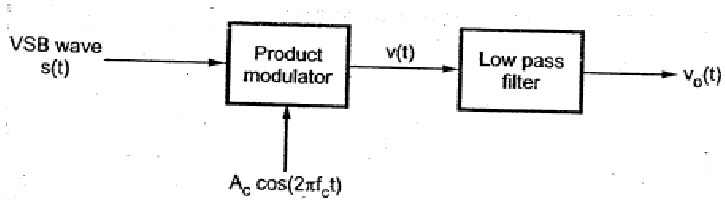
* This DSB-SC Signal is then applied to a Sideband Shaping filter. The filter will pass the wanted Sideband as it is & the vestige of the unwanted Sideband.

* Let the transfer function of the filter be $H(f)$. Hence the Spectrum of the VSB modulated Wave is given by:

$$\boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)} \longrightarrow (1)$$

Demodulation OR Detection of VSB Modulated Wave:-

❖ **Explain the coherent detection of VSB-SC wave.** **Jan-10,7M**



* The demodulation of VSB modulated wave can be achieved by passing VSB wave $S(t)$ through a Coherent detector.