# UNIT-4. Information Theory:-

**Information:-** The Information is a measure of uncertainity.

⇒ If the probability of occurrence is less than the information content (or) Vice versa. i.e the information and probability are inversely proportional to each other.

$$I(x_k) = \log \frac{1}{P(x_k)} \Rightarrow -\log P(x_k).$$

If base $= 2$, then it is called bits

base $= 10$, then it is called decimal.

$$\therefore \boxed{I(x_k) = \log_2 \frac{1}{P(x_k)}}$$

**Joint Probability of information:-**

⇒ Let us consider two event $x_i$ & $y_i$ having probabilities of occurence $P(x_i)$ & $P(y_i)$ then $P(x_i, y_i)$ is called joint Probability.

$$P(x_i, y_i) = P(x_i) \cdot P(y_i)$$

where $P(x_i)$, $P(y_i)$ are independent.

So W.K.T $I(x_i, y_i) = \log_2 \frac{1}{P(x_i, y_i)}$

$$\Rightarrow \log_2 \frac{1}{P(x_i) \cdot P(y_i)}$$

$$I(x_i, y_i) \Rightarrow \log_2 \frac{1}{P(x_i)} + \log_2 \frac{1}{P(y_i)}$$

$$\boxed{I(x_i, y_i) = I(x_i) + I(y_i)}$$

$\therefore$ Information content are also independent if Probability of occurence is independent.

## Average information (or) Entropy (H) :-

⇒ Entropy is defined as the average information for individual message.

→ Let us assume $m_1, m_2 ---- m_M$ are different message having probability of occurence $P_1, P_2, ----P_m$.

→ Let us assume that in a long Interval of time 'L' message are generated where $(L >> M)$

→ Let $N_1$ be the no of occurrence of $M_1$ out of L message then

$$P_1 = \frac{N_1}{L} \; ; \; (N_1 = P_1 L)$$

$$P_2 = \frac{N_2}{L} \; ; \; (N_2 = P_2 L)$$

$$\vdots \qquad \qquad \vdots$$

$$P_m = \frac{N_m}{L} \; ; \; (N_m = P_m L)$$

$$(N_1 + N_2 + ----- + N_m = 1)$$

Let $I(t)$ is the total amount of information of (L) messages

$$\therefore \; I(t) = N_1 \log_2\left(\frac{1}{P_1}\right) + N_2 \log_2\left(\frac{1}{P_2}\right) + ---- + N_m \log_2\left(\frac{1}{P_m}\right)$$

$$I(t) = P_1 L \log_2\left(\frac{1}{P_1}\right) + P_2 L \log_2\left(\frac{1}{P_2}\right) + --- + P_m L \log_2\left(\frac{1}{P_m}\right)$$

$$\frac{I(t)}{L} = \sum_{k=1}^{m} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$\therefore \; \boxed{H = \frac{I(t)}{L} = \sum_{k=1}^{m} P_k \log_2\left(\frac{1}{P_k}\right)} \; \text{or} \; -\sum_{k=1}^{m} P_k \log_2 (P_k).$$

Scanned with CamScanner

**Case(1):-** Show that if there is a Single message the average Information rate is equal to zero. i.e $(H=0)$. if $P_k = (0 \text{ or } 1)$

$$H = \sum_{k=1}^{m} P_k \log_2\left(\frac{1}{P_k}\right)$$

for single message $P = 1$

$$\boxed{H = P.\log_2\left(\frac{1}{P}\right) \Rightarrow 0}$$

$[\because H = 1\log_2\left(\frac{1}{1}\right) = 0]$
&
$[\;0 \log_2 \frac{1}{0} = 0]$

**Case 2:-** Show that the maximum entropy occurs at $P = \frac{1}{2}$ whenever we are transmitting two msgs & draw $P_H$ curve.

let $m_1 = P$, $m_2 = (1-P)$

$$\therefore \; H = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right).$$

$$H \Rightarrow P\log_2\left(\frac{1}{P}\right) + (1-P)\log_2\left(\frac{1}{(1-P)}\right)$$

$$\Rightarrow -\left[P\log_2 P + (1-P)\log_2(1-P)\right]$$

$$\Rightarrow \frac{-1}{\ln 2}\left[P\ln P + (1-P)\ln(1-P)\right]$$

for $\max^m$ to be occur $\frac{dH}{dP} = 0$

$$\frac{dH}{dP} \Rightarrow \frac{d}{dP}\left\{\frac{-1}{\ln 2}\left[P\ln P + (1-P)\ln(1-P)\right]\right\}$$

$$\Rightarrow \frac{-1}{\ln 2}\left[P \cdot \frac{1}{P} + \ln P \cdot 1 + (1-P)\cdot\frac{1}{(1-P)}(-1) + \ln(1-P)(-1)\right]$$

$$\Rightarrow \frac{-1}{\ln 2}\left[\cancel{1} + \ln P + \cancel{(-1)} + (-1)\ln(1-P)\right]$$

$$\left[\ln p - \ln(1-p)\right] = 0$$

$$\ln p = \ln(1-p)$$

$$p = 1-p$$

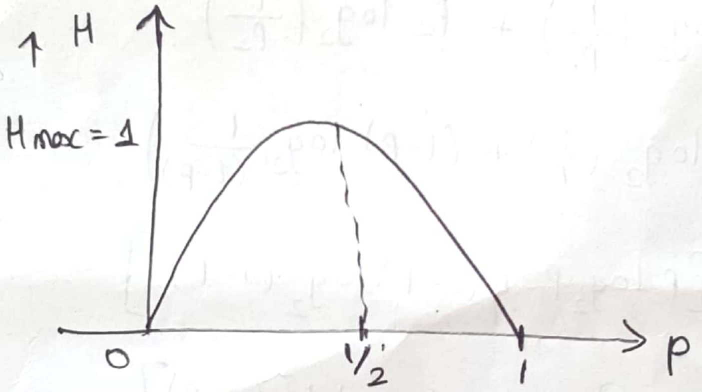
$$2p = 1$$

$$p = \frac{1}{2}.$$

$\therefore \quad H = p \log \frac{1}{p} + (1-p)\log\left(\frac{1}{1-p}\right)$

$$\boxed{H_{max} = \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2 \Rightarrow 1}$$

$\boxed{H_{max} = 1}$. (hence max$^m$ entropy occurs at $(P=\frac{1}{2})$)




Case(3) :- S.T If there are 'M' msg of equiprobable the entropy is given by $H = \log_2 M$.

Since we have 'M' msgs & all are equiprobable then

$$P_1 = P_2 = P_3 = \cdots = P_m = \frac{1}{M}$$

$$H = \sum_{k=1}^{M} P_k \log_2 \frac{1}{P_k}.$$

$$H = P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} + \cdots + P_m \log_2 \frac{1}{P_m}$$

$$H = \frac{1}{M} \log_2 M + \frac{1}{M} \log_2 M + \cdots + \frac{1}{M} \log_2 M$$

$$H = \cancel{M}\left(\frac{1}{\cancel{M}} \log_2 M\right)$$

$$\boxed{H = \log_2 M}$$

W.K.T $M = 2^N \rightarrow$ Sub in above

$$H = \log_2 2^N \Rightarrow N \log_2 2 \Rightarrow N \text{ bits/msg}$$

$$\boxed{H = N \text{ bits/msg}}$$

Information Rate (R) :-

⇒ It is defined as the no of bits transmitted per second if a source of messages transmitted at a rate of 'r' msg per second then the information rate is given by,

$$\boxed{R = rH \text{ bits/sec}}$$

It is called max^m information rate.

In PCM ~~$r_b = n f_s$~~

where ~~$f_s \geq 2f_m$ ((i.e) $f_m$ & B)~~

(6) An analog signal is Band limited to 2kHz & sampled nyquist rate & quantized into 4 levels $Q_1, Q_2, Q_3, Q_4$ with probability $P_1 = \frac{1}{8}$, $P_2 = \frac{3}{8}$, $P_3 = \frac{3}{8}$, $P_4 = \frac{1}{8}$ calculate information rate & entropy.

S: W.K.T $R = r H$.

$B = 2kHz$.

$r = 2B \Rightarrow 2(2kHz) \Rightarrow 4kHz = 4000$ messages.

Calculating $H = \sum_{k=1}^{M} P_k \log_2 \left(\frac{1}{P_k}\right)$

$$\Rightarrow \frac{1}{8}\log_2 8 + \frac{3}{8}\log_2 \frac{8}{3} + \frac{3}{8}\log_2 \frac{8}{3} + \frac{1}{8}\log_2 8$$

$$\Rightarrow \frac{1}{8}\log_2 2^3 + \frac{3}{8}\log_2 \frac{8}{3} + \frac{3}{8}\log_2 \frac{8}{3} + \frac{1}{8}\log_2 2^3$$

$$\Rightarrow \frac{3}{8}\log_2 2 + \frac{3}{8}\log_2 8/3 + \frac{3}{8}\log_2 8/3 + \frac{3}{8}\log_2 2$$

$\Rightarrow$ 2 bits / msg.

$R = r H$

$R = (4000)(2)$

$R = 8000$ bits / sec.

Scanned with CamScanner

② A discrete system of source is emitting 8 msgs with probability $\frac{1}{2}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32}$ calculate the avg information per msg, source & efficiency of the code.

Sol

$$H = \sum_{k=0}^{7} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$\Rightarrow P_0 \log_2\left(\frac{1}{P_0}\right) + P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + P_3 \log_2\left(\frac{1}{P_3}\right) + P_4 \log_2\left(\frac{1}{P_4}\right)$$

$$+ P_5 \log_2\left(\frac{1}{P_5}\right) + P_6 \log_2\left(\frac{1}{P_6}\right) + P_7 \log_2\left(\frac{1}{P_7}\right)$$

$$\Rightarrow \frac{1}{2}\log_2(2) + \frac{1}{8}\log_2 8 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + \frac{1}{16}\log_2 16 + \frac{1}{16}\log_2 16$$

$$+ \frac{1}{32}\log_2 32 + \frac{1}{32}\log_2 32$$

$$\Rightarrow \frac{1}{2} + \frac{1}{8}\log_2 2^3 + \frac{1}{8}\log_2 2^3 + \frac{1}{16}\log_2 2^4 + \frac{1}{16}\log_2 2^4 + \frac{1}{16}\log_2 2^4 +$$

$$\frac{1}{32}\log_2 2^5 + \frac{1}{32}\log_2 2^5$$

$$\Rightarrow \frac{1}{2} + \frac{3}{8} + \frac{3}{8} + \frac{4}{16} + \frac{4}{16} + \frac{4}{16} + \frac{5}{32} + \frac{5}{32}$$

$$\Rightarrow \frac{37}{16} \text{ bits/msg}$$

Avg length of code $L = \frac{37}{16}$ bits

efficiency $\eta = \frac{H}{L} \Rightarrow \frac{37/16}{37/16} = 1$ (or) 100 %

**Joint Entropy :-** The joint entropy represets entropy of Joint occurence of two or more events.

It is given as :-

$$\boxed{H(x,y) = H(y,x) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 \frac{1}{P(x_i, y_j)}}$$

here $H(Y,x)$ repersents entropy of Joint occurence of $x$ & $y$.

**Conditional Entropy :-** The conditional entropy is also known as equivocation.

The conditional entropy $H(Y/x)$ repersents uncertainity of $x$ on average, when $y$ is known.

Illy the conditional entropy $H(x/y)$ repersents uncertainity of $y$ on average when $x$ is transmitted.

⇒ Thus conditional entropy indicates the information lost across the noisy channel. The mathematically equation are given by

$$H(x/y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 \left(\frac{1}{P(x_i/y_j)}\right)$$

$$\& \quad H(Y/X) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 \left(\frac{1}{P(y_j/x_i)}\right)$$

Scanned with CamScanner

③ Marginal Entropy:- when the entropy of individual event is calculated from joint probabilities of event is called marginal entropy.

→ In Joint occurence of $x_i$ & $y_j$; from equation of probability of occurence of $P(x_i)$ & $P(y_j)$. Then the entropies of $x_i$ & $y_j$ can be calculated i.e

$$H(X) = \sum_{i=1}^{M} P(x_i)$$

Sub $P(x_i) = \sum_{j=1}^{m} P(x_i, y_j)$

then $H(x) = \sum_{i=1}^{m} \sum_{j=1}^{m} P(x_i, y_j)$

$$\boxed{H(x) \Rightarrow \sum_{i=1}^{m} \sum_{j=1}^{m} P(x_i, y_j)}$$

$$H(Y) = \sum_{j=1}^{m} P(y_j)$$

putting $P(y_j)$ from above Eq

$$P(y_j) = \sum_{i=1}^{m} P(x_i, y_j)$$

$$\boxed{H(y) = \sum_{i=1}^{m} \sum_{j=1}^{m} P(x_i, y_j)}$$

Then entropies of $H(X)$ & $H(y)$ are called marginal entropies

Relation between Joint Entropy & Conditional Entropy (8) Prove that $H(X,Y) = H(x/y) + H(Y)$

S: Consider Joint entropy $H(x,y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 \left(\frac{1}{P(x_i, y_j)}\right)$

$$\Rightarrow -\sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 P(x_i, y_j) \quad \text{—} (1)$$

from probabilty theory w.k.T

$$P(A/B) \cdot P(B) = P(AB)$$

$$P(x_i, y_j) = P(x_i/y_j)\, P(y_j) \quad \text{—} (2)$$

Sub (2) in Eq (1)

$$H(x,y) = -\sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 P(x_i/y_j) P(y_j).$$

from logarithic Properties

$$\log ab = \log a + \log b$$

$$H(x,y) = -\sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \left[\log_2 P(x_i/y_j) + \log_2 P(y_j)\right] \quad \text{—} (3)$$

hence from above Eq becomes.

$$H(y,x) = -\sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 P(x_i/y_j).$$

$$-\sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 P(y_j)$$

$$\cdot H(x,y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \left(\frac{1}{\log_2 P(x_i/y_j)}\right) + \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \left(\frac{1}{\log_2 P(y_j)}\right)$$

$$\Rightarrow \sum_{i=1}^{m}\sum_{j=1}^{n} P(x_i, y_j)\log_2\left(\frac{1}{P(x_i, y_j)}\right) + \sum_{j=1}^{n}\left\{\sum_{i=1}^{m} P(x_i, y_j)\right\}\log_2\frac{1}{P(y_j)}\Bigg\}$$

The first term in above eq is from standard probability theory.

$$H(x,y) = H(x/y) + \sum_{j=1}^{n} P(y_j)\log_2\left(\frac{1}{P(y_j)}\right)$$

$$\left[\because \sum_{i=1}^{m} P(x_i, y_j) = P(y_j)\right]$$

$$\boxed{H(x,y) = H(x/y) + H(y)}$$

Mutual Information :- The mutual information is defined as the amount of Information transferred when $x_i$ is transmitted & $y_j$ is received. It is represented as $\boxed{I(x_i, y_j) = \log \frac{P(x_i/y_j)}{P(x_i)} \text{ bit}}$

here $I(x_i, y_j)$ = mutual I/f

$P(x_i, y_j)$ = conditional Pr that $x_i$ was transmitted & $y_j$ is received

$P(x_i)$ = Probability of symbol $x_i$ for transmission.

Average mutual Information :-

→ The average mutual information is represented by $I(x, y)$. It is calculated in bit / symbol. The avg mutual information is defined as amount of source information gained per received symbol. Here the avg mutual information is different from entropy.

It is given as

$$I(X;Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j)\, I(x_i, y_j)$$

Thus $I(x_i, y_j)$ is weighted by joint probabilities $P(x_i, y_j)$ over all possible joint events putting for $I(x_i, y_j)$ in above eq we get :-

$$\boxed{I(X;Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 \frac{P(x_i/y_j)}{P(x_i)}}$$

Properties of mutual information

① The mutual information channel is symmetric ie, $\boxed{I(X;Y) = I(Y;X)}$.

② The mutual I/f can be represented in terms of entropies of channel of channel i/p & o/p and condition entropies

$$\boxed{I(X;Y) = H(X) - H(X/Y)}$$

$$\Rightarrow H(Y) - H(Y/X)$$

here $H(X/Y)$ & $H(Y/X)$ are conditional entropies

③ The mutual information is always positive ie $\boxed{I(X;Y) \geq 0}$

④ The mutual information is related to the joint entropy $H(X, Y)$ by following relation

$$\boxed{I(X;Y) = H(X) + H(Y) - H(X,Y)}$$

## Shannon's theorem channel Capacity :-

→ Given a source of "M" likely msgs with $M > 1$ which generating the information of a rate "R" & given channel Capacity "C".

→ Then if $R \leq C$ there existing a code technique, so that o/p of the source may be transmitted over a channel with Pr of error which is very very small.

→ A theorem which is complentary to shannon's theorem and applied to a channel in which noise is gaussian in nature is known as Shannon's Heartly theorem.

→ It states that channel Capacity of a white band limited gaussian channel is given by

$$\boxed{C = B \log_2 \left(1 + \frac{S}{N}\right)} \text{ bits/sec.}$$

W.K.T $N = \eta . B$

$$\therefore \boxed{C = B \log_2 \left(1 + \frac{S}{\eta B}\right)}$$

where B = Bandwidth

S = Signal power

N = Noise power Spectral density

C = channel Capacity

Scanned with CamScanner

## Trade off b/w SNR & B.W :-

According to Shannon's heartly theorem $c = B\log_2\left(1+\frac{S}{N}\right)$

Case (i) :- if $N=0$ (i.e there is no noise in the channel.

then $\frac{S}{N} = \infty$

Such channel is called noiseless channel.

Capacity of such channel will be $C = B\log_2(1+\infty)$

$$\boxed{C \Rightarrow \infty}$$

Case (ii) :- when $B.W \longrightarrow \infty$, the channel capacity will be limited.

by B.W ↑'s, noise power also ↑'s & (S/N) ↓'s.

hence B approaches $\infty$ & capacity will be limited

$$\boxed{C_\infty = \lim_{B\to\infty} C \Rightarrow 1.44\frac{S}{\eta}}$$

Scanned with CamScanner

Q. S.T when $B.W = \infty$ then $C(\infty) = 1.44 \frac{S}{\eta}$.

S. W.K.T $C = B \log_2\left(1 + \frac{S}{N}\right)$

W.K.T $N = \eta B$

$$C = B \log_2\left(1 + \frac{S}{\eta B}\right)$$

mul & div by $\frac{S}{\eta}$

$$C = \frac{S}{\eta} \cdot \frac{\eta}{S} B \log_2\left(1 + \frac{S}{\eta B}\right).$$

$$C = \frac{S}{\eta} \cdot \frac{\eta B}{S} \log_2\left(1 + \frac{S}{\eta B}\right)$$

$$C = \frac{S}{\eta} \log_2\left(1 + \frac{S}{\eta B}\right)^{\frac{\eta B}{S}} \Rightarrow \frac{S}{\eta} \log_2\left(1 + \frac{S}{\eta B}\right)^{\frac{1}{S/\eta B}}$$

let $x = \frac{S}{\eta B}$

then it is in form of $\log_2 (1+x)^{1/x}$.

$\therefore$ when $B.W \longrightarrow \infty$ then $x \rightarrow 0$

$$C_\infty = \frac{S}{\eta} \lim_{x \to 0} \log_2\left(1 + \frac{S}{\eta B}\right)^{1/S/\eta B}$$

$$\Rightarrow \frac{S}{\eta} \lim_{x \to 0} \log_2 (1+x)^{1/x}$$

$\because \boxed{\lim_{x \to 0} (1+x)^{1/x} = e}$

$$\Rightarrow \frac{S}{\eta} \log_2 e$$

$$\boxed{C_\infty \Rightarrow 1.44 \frac{S}{\eta}}$$

$\Rightarrow$ Shannon's limit.

## Source Coding :-

⇒ Source coding techniques are used to remove the redundancy present in the data. It is achieved by assigning short code words for more probable msgs, where as long code words for less probable msgs. Such coding is called Variable length coding.

⇒ In binary encoding all msgs are assigned same length code irrespective of their probability of occurrence

→ Codeword assigned to each msg is unique & the system that generate source coding is called source encoder.

→ $$\boxed{\text{Coding efficiency :- } (\eta) = \frac{H}{\bar{n}}}$$

where H is entropy.

$\bar{n}$ is avg no of bits in code word of msg.

$$\bar{n} = \sum_{k=0}^{L-1} P_k n_k .$$

**Code redundancy :-** It is the measure of bits in the encoded msg sequence. It is given as

$$\boxed{\text{Redundancy } (\gamma) = 1 - \text{code efficiency} \Rightarrow 1 - \eta}$$

There are two types of coding :-

① Shannon - Fanon coding

② Huffman coding.

**Shannon Fanon Coding:-** It is an algorithm which is a variable length coding in which each msg is represented in terms of variable bits per msg.

→ It reduce the redunancy and we can represent the msg with few no of bits so that B.W can be reduced & noise immunity can be improved.

Steps:-

① The messages are arranged according to their decreasing probabilities

② partition the msg such that probability sum in both the portions are almost equal.

③ The msgs in upper partition are assigned to bit "0" & lower partition assigned to bit "1".

④ Each partion is further subdivided into new partitions following same rule.

⑤ Partitioned is stopped when there is only one msg in partition.

⑥ Find out the code efficiency by Shannon fano code for given messages.

| message | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $m_7$ | $m_8$ |
|---|---|---|---|---|---|---|---|---|
| $P(m_i)$ | $\frac{1}{2}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{32}$ | $\frac{1}{32}$ |

| message | Probability | 1 | 2 | 3 | 4 | 5 | no of bits/msg |
|---|---|---|---|---|---|---|---|
| $m_1$ | 1/2 | 0 | | | | | 1 |
| $m_2$ | 1/8 | 1 | 0 | 0 | | | 3 |
| $m_3$ | 1/8 | 1 | 0 | 1 | | | 3 |
| $m_4$ | 1/16 | 1 | 1 | 0 | 0 | | 4 |
| $m_5$ | 1/16 | 1 | 1 | 0 | 1 | | 4 |
| $m_6$ | 1/16 | 1 | 1 | 1 | 0 | | 4 |
| $m_7$ | 1/32 | 1 | 1 | 1 | 1 | 0 | 5 |
| $m_8$ | 1/32 | 1 | 1 | 1 | 1 | 1 | 5 |

$$\eta = \frac{H}{L} \times 100\%$$

$$H = \sum_{k=1}^{8} P_k \log_2 \frac{1}{P_k} = P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} + \text{-----} + P_8 \log_2 \frac{1}{P_8}$$

$$\Rightarrow \frac{1}{2}\log_2 \frac{1}{1/2} + \frac{1}{8}\log_2 \frac{1}{1/8} + \frac{1}{8}\log_2 \frac{1}{1/8} + \frac{1}{16}\log_2 \frac{1}{1/16} + \frac{1}{16}\log_2 \frac{1}{1/16} + \frac{1}{16}\log_2 \frac{1}{1/16}$$

$$+ \frac{1}{32}\log_2 \frac{1}{1/32} + \frac{1}{32}\log_2 \frac{1}{1/32}.$$

$$\Rightarrow 37/16.$$

$$L \Rightarrow \sum_{i=1}^{8} P_k\, x(L) \Rightarrow P_1 x_1 + P_2 x_2 + P_3 x_3 + \text{-----} + P_8 x_8$$

$$\Rightarrow \frac{1}{2}\times 1 + \frac{1}{8}\times 3 + \frac{1}{8}\times 3 + \frac{1}{16}\times 4 + \frac{1}{16}\times 4 + \frac{1}{16}\times 4 + \frac{1}{32}\times 5 + \frac{1}{32}\times 5$$

$$\Rightarrow 37/16$$

$$\eta = \frac{37/16}{37/16} \Rightarrow 1\%$$

Scanned with CamScanner

② Find the code efficiency by shannon fanon coding for 4 quantization levels. $Q_1, Q_2, Q_3, Q_4$ having probability $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$.

S

| msg | Probability | 1 | 2 | 3 | bits / msg |
|---|---|---|---|---|---|
| $m_1$ | 3/8 | 0 | | | 1 |
| $m_2$ | 3/8 | 1 | 0 | | 2 |
| $m_3$ | 1/8 | 1 | 1 | 0 | 3 |
| $m_4$ | 1/8 | 1 | 1 | 1 | 3 |

$$\eta\% = \frac{H}{L}\%$$

$$H = \sum_{K=1}^{4} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$\Rightarrow \frac{3}{8}\log_2\frac{1}{3/8} + \frac{3}{8}\log_2\frac{1}{3/8} + \frac{1}{8}\log_2\frac{1}{1/8} + \frac{1}{8}\log_2\frac{1}{1/8}$$

$$\Rightarrow 1.8112$$

$$\bar{L} = \sum_{i=1}^{4} P_K L(x) \Rightarrow P_1 L_1 + P_2 L_2 + P_3 L_3 + P_4 L_4$$

$$\Rightarrow \frac{3}{8}\times 1 + \frac{3}{8}\times 2 + 1/8 \times 3 + \frac{1}{8}\times 3 = 15/8$$

$$\%\eta = \frac{H}{L}\times 100 \Rightarrow \frac{1.8112}{15/8}\times 100$$

$$\Rightarrow 96.6\%.$$

Roll No. :..................................... Signature of Invigilator (13)

**Huffman coding** :- In general coding huffman perform better than shannon fano coding for a source of given entropy which gives minimum avg word length.

Hence it is called optimum code.

**Steps** :-

① In this case the msg are arranged in decreasing order of probability are combined two msgs of lower probability is arranged then this process is continued till only two msgs remain

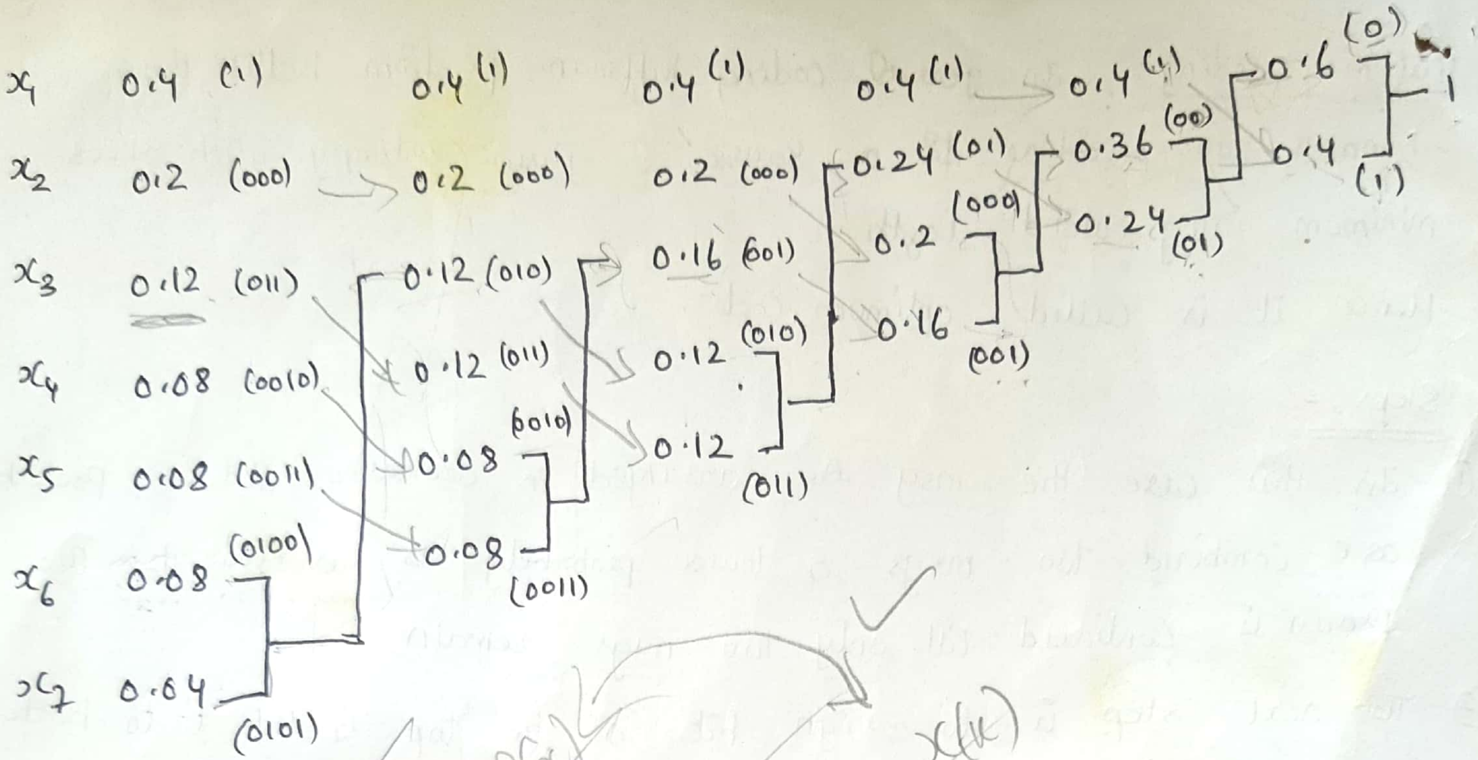
② The next step is to assign bit '0' to top & bit '1' to bottom probabilities

③ The encoded msgs is represented by the bits goes to starting from the final reduction.

⑤ Construct a huffman code & calculate the efficiency for given msg :-

| msg | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ |
|---|---|---|---|---|---|---|---|
| $P(x)$ | 0.4 | 0.2 | 0.12 | 0.08 | 0.08 | 0.08 | 0.04 |

Scanned with CamScanner




| Symbol | Probabily | Code word | lngth |
|---|---|---|---|
| $x_1$ | 0.4 | 1 | 1 |
| $x_2$ | 0.2 | 000 | 3 |
| $x_3$ | 0.12 | 011 | 3 |
| $x_4$ | 0.08 | 0010 | 4 |
| $x_5$ | 0.08 | 0011 | 4 |
| $x_6$ | 0.08 | 0100 | 4 |
| $x_7$ | 0.04 | 0101 | 4 |

$$\% \eta = \frac{H}{L} \times 100\%$$

$$L = \sum_{k=1}^{7} P_K \, L(x)$$

$$\Rightarrow 0.4\times 1 + 0.2\times 3 + 0.12\times 3 + 0.08\times 4 + 0.08\times 4 + 0.08\times 4 + 0.04\times$$

$$\boxed{L \Rightarrow 2.48}$$

Scanned with CamScanner

$$H = \sum_{k=1}^{7} P_k \log_2 \frac{1}{P_k}$$

$$\Rightarrow 0.4 \log_2 \frac{1}{0.4} + 0.2 \log_2 \frac{1}{0.2} + 0.12 \log_2 \frac{1}{0.12} + 0.08 \log_2 \frac{1}{0.08}$$

$$+ 0.08 \log_2 \frac{1}{0.08} + 0.08 \log_2 \frac{1}{0.08} + 0.04 \log_2 \frac{1}{0.64}.$$

$$\Rightarrow 2.48$$

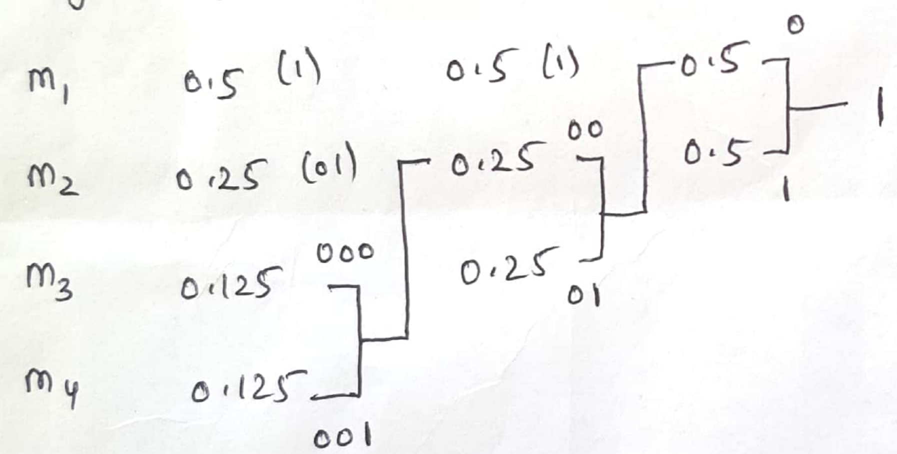
$$\eta = \frac{H}{L} \times 100 \Rightarrow \frac{2.42}{2.48} \times 100\% \Rightarrow 97.6\%$$

---

(2) There are 4 msgs & its probability are

| msg | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| Pr (msg) | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

msg Pr.




| msg | Prcodes | code word | lgth |
|---|---|---|---|
| $m_1$ | 0.5 | 1 | 1 |
| $m_2$ | 0.25 | 01 | 2 |
| $m_3$ | 0.125 | 000 | 3 |
| $m_4$ | 0.125 | 001 | 3 |

$\% \eta = \frac{H}{\bar{L}}$

$H = P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} + P_3 \log_2 \frac{1}{P_3} + P_4 \log_2 \frac{1}{P_4}$

$\Rightarrow 0.5 \log_2 \frac{1}{0.5} + 0.25 \log_2 \frac{1}{0.25} + 0.125 \log_2 \frac{1}{0.125} + 0.125 \log_2 \frac{1}{0.12}$

$\Rightarrow$ .

$\bar{L} = \sum_{i=1}^{4} P_k L(x_k)$

$\Rightarrow 0.5(1) + 0.25(2) + 0.125(3) + 0.125(3)$

$\Rightarrow$

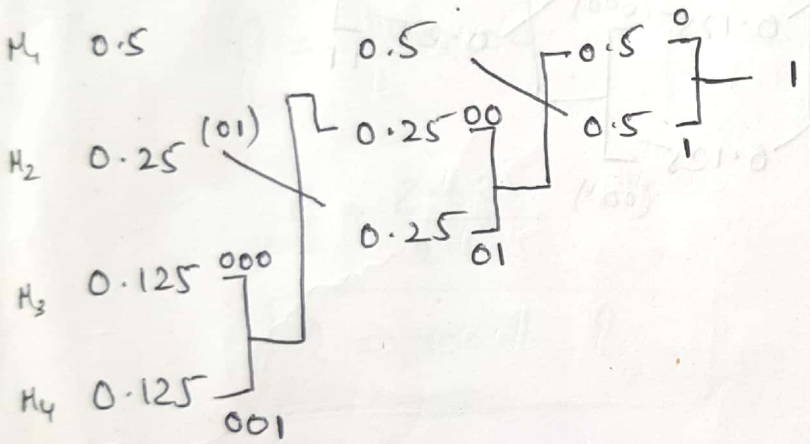
Scanned with CamScanner

Huffmann Coding :- It is generally used for data Compression.

① There are 4 Msgs & its probability are

| Msg (M) | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| Probabil $P(m)$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| | (0.5) | (0.25) | (0.125) | (0.125) |

Step ① Arrange in descending order.




Codes

$M_1 \rightarrow (1)$

$M_2 \rightarrow (01)$

$M_3 \rightarrow (000)$

$M_4 \rightarrow (001)$.

⑧ Consider a source with 7 msgs having probabilities 0.25, 0.25, 0.125, 0.125, 0.125, 0.0625, 0.0625. find avg code length, Entropy & efficiency?

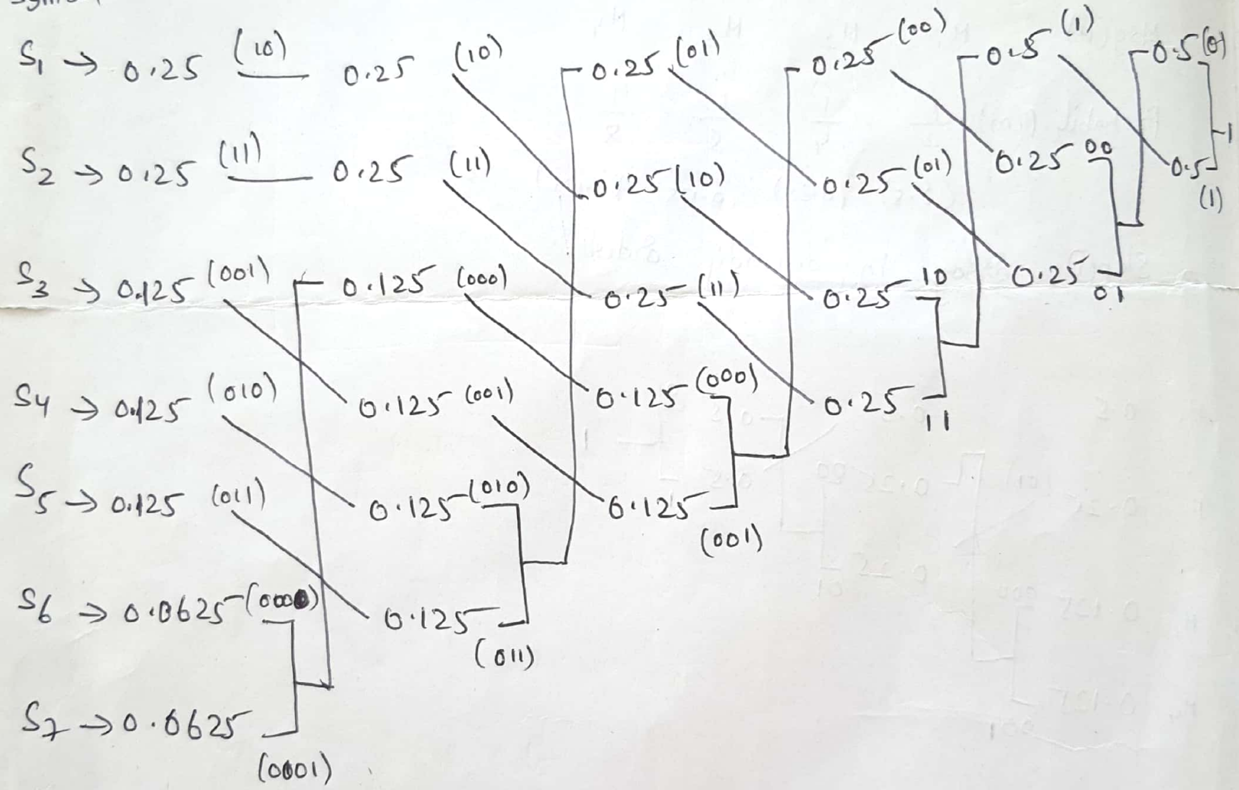
Sol. ① write probabilities in descending order.

② Avg code length $L = \sum_{k=1}^{K} P_k L_k$

③ Entropy $H(x) = \sum_{k=1}^{K} P_k \log_2 P_k$

④ Efficiency $\eta = \frac{H}{L} \times 100$.

Symbol




| Symbol | Prob | Codeword | Length |
|---|---|---|---|
| $S_1$ | 0.25 | 10 | 2 |
| $S_2$ | 0.25 | 11 | 2 |
| $S_3$ | 0.125 | (001) | 3 |
| $S_4$ | 0.125 | (010) | 3 |
| $S_5$ | 0.125 | (011) | 3 |
| $S_6$ | 0.0625 | (0000) | 4 |
| $S_7$ | 0.0625 | (0001) | 4 |

$$L = \sum_{k=1}^{n} P_k L_k .$$

$$L = P_1L_1 + P_2L_2 + P_3L_3 + P_4L_4 + P_5L_5 + P_6L_6 + P_7L_7$$

$\Rightarrow 0.25 \times 2 + 0.25 \times 2 + 0.125 \times 3 + 0.125 \times 3 + 0.125 \times 3 +$

$0.0625 \times 4 + 0.0625 \times 4$

$\Rightarrow 2.625$ bits / Symbol

$$H(x) = -\left(\sum_{k=1}^{n} P_k \log_2 P_k\right)$$

$\Rightarrow P_1 \log_2 P_1 + P_2 \log_2 P_2 + P_3 \log_2 P_3 + P_4 \log_2 P_4 + P_5 \log_2 P_5$

$+ P_6 \log_2 P_6 + P_7 \log_2 P_7$

$\Rightarrow (0.25 \log 0.25)\, 2 + (0.125 \log 0.125)\, 3 + (0.0625 \log 0.0625)\, 2$

$\Rightarrow 2.625$ bits / Symbol .

$$\eta = \frac{H}{L} \times 100$$

$$\eta = \frac{2.625}{2.625} \times 100$$

$$\boxed{\eta \Rightarrow 100\,\%}$$

① Apply shannon fano Code for following msg:-

$(x) = \quad x_1 \quad x_2 \quad x_3 \quad x_5 \quad x_5 \quad x_6$

$(P_r) = \quad 0.30 \quad 0.25 \quad 0.15 \quad 0.12 \quad 0.08 \quad 0.10$

S ① Arrange all $P_r$ in descending order.

| [X] | [$P_r$] | $S_1$ | $S_2$ | $S_3$ | code word | length |
|---|---|---|---|---|---|---|
| $x_1$ | 0.30 | 0 | 0 | | 00 | 2 |
| $x_2$ | 0.25 | 0 | 1 | | 01 | 2 |
| $x_3$ | 0.15 | 1 | 0 | 0 | 100 | 3 |
| $x_4$ | 0.12 | 1 | 0 | 1 | 101 | 3 |
| $x_6$ | 0.10 | 1 | 1 | 0 | 110 | 3 |
| $x_5$ | 0.08 | 1 | 1 | 1 | 111 | 3 |

Reduced Structure

entropy $H(x) = \sum_{i=1}^{6} P_i \cdot I_i = \sum_{i=1}^{6} P_i \log \frac{1}{P_i}$

$\Rightarrow 0.30 \log_2 \frac{1}{0.30} + 0.25 \log_2 \frac{1}{0.25} + 0.15 \log_2 \frac{1}{0.15} + - - -$

$- - - \; 0.08 \log_2 \frac{1}{0.08}$

$\Rightarrow 2.418$ bits

$L_{avg} = \sum_{i=1}^{6} P_i \times (L) = 0.30(2) + 0.25\,(2) + 0.15 \times 3 + 0.12 \times 3$
$+ 0.10 \times 3 + 0.08 \times 3$

$\Rightarrow 2.45$ bits

$\eta = \frac{H(x)}{L_{avg}} = \frac{2.418}{2.45} = 0.98.$

$\eta = 98\,\%$

(redundancy) $\gamma = 1 - \eta = 1 - 0.9869 = 0.0131$ or $1.31\,\%$

Roll No. :..........................................

| msg | Pr | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | no of bits | Code |
|---|---|---|---|---|---|---|---|---|
| $m_1$ | $\frac{1}{2}$ | 0 | | | | | 1 | 0 |
| $m_2$ | $\frac{1}{8}$ | 1 | 0 | 0 | | | 3 | 100 |
| $m_3$ | $\frac{1}{8}$ | 1 | 0 | 1 | | | 3 | 101 |
| $m_4$ | $\frac{1}{16}$ | 1 | 1 | 0 | 0 | | 4 | 1100 |
| $m_5$ | $\frac{1}{16}$ | 1 | 1 | 0 | 1 | | 4 | 1101 |
| $m_6$ | $\frac{1}{32}$ | 1 | 1 | 1 | 0 | 0 | 5 | 11100 |

(Q) A Descrete Memoryless Source has five Symbols $x_1, x_2, x_3, x_4, x_5$ with $P(x_1) = 0.4$, $P(x_2) = 0.19$ $P(x_3) = 0.16$, $P(x_4) = 0.15$ & $P(x_5) = 0.1$ ?

(1) Construct a Shannon fano Code for X, Calculate efficiency?

(2) " Huffman Code ?

(Q) Determine Huffman code for following msg with their Probabilities

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ |
|---|---|---|---|---|---|---|
| 0.05 | 0.15 | 0.2 | 0.05 | 0.15 | 0.3 | 0.1 |

# PULSE MODULATION

## Introduction:

### Pulse Modulation

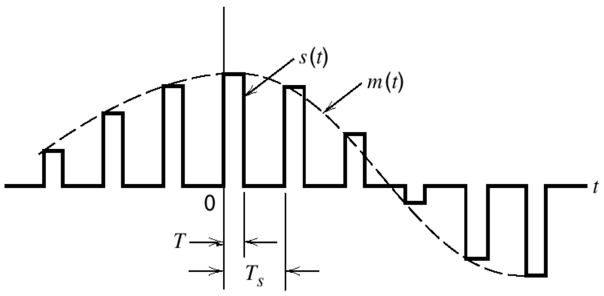
- Carrier is a train of pulses
- Example: Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) , Pulse Position Modulation (PPM)

### Types of Pulse Modulation:

- The immediate result of sampling is a pulse-amplitude modulation (PAM) signal
- PAM is an analog scheme in which the amplitude of the pulse is proportional to the amplitude of the signal at the instant of sampling
- Another analog pulse-forming technique is known as **pulse-duration modulation (PDM).** This is also known as **pulse-width modulation (PWM)**
- **Pulse-position modulation** is closely related to PDM

### Pulse Amplitude Modulation:

In PAM, amplitude of pulses is varied in accordance with instantaneous value of modulating signal.




## PAM Generation:

The carrier is in the form of narrow pulses having frequency fc. The uniform sampling takes place in multiplier to generate PAM signal. Samples are placed Ts sec away from each other.

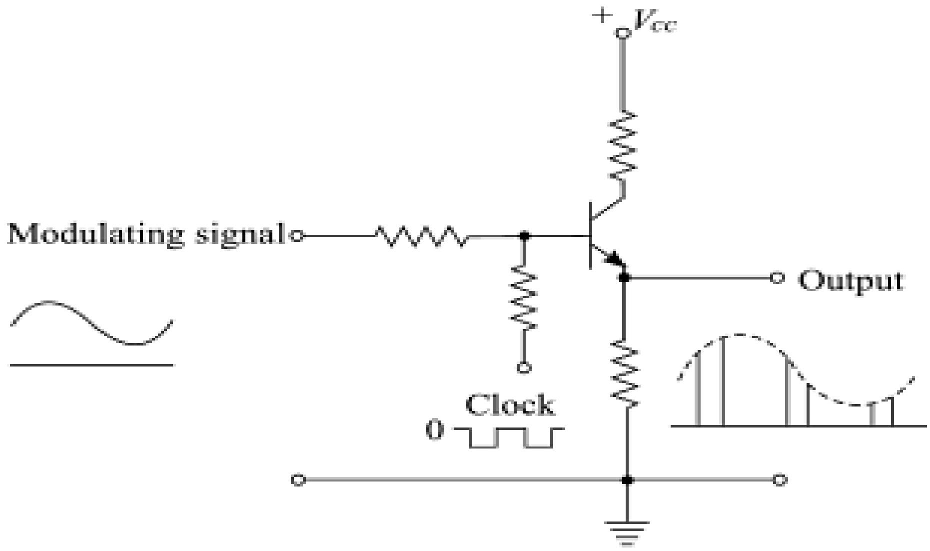



Fig.12. PAM Modulator

- The circuit is simple emitter follower.
- In the absence of the clock signal, the output follows input.
- The modulating signal is applied as the input signal.
- Another input to the base of the transistor is the clock signal.
- The frequency of the clock signal is made equal to the desired carrier pulse train frequency.
- The amplitude of the clock signal is chosen the high level is at ground level(0v) and low level at some negative voltage sufficient to bring the transistor in cutoff region.
- When clock is high, circuit operates as emitter follower and the output follows in the input modulating signal.
- When clock signal is low, transistor is cutoff and output is zero.
- Thus the output is the desired PAM signal.

**PAM Demodulator:**

- The PAM demodulator circuit which is just an envelope detector followed by a second order op-amp low pass filter (to have good filtering characteristics) is as shown below

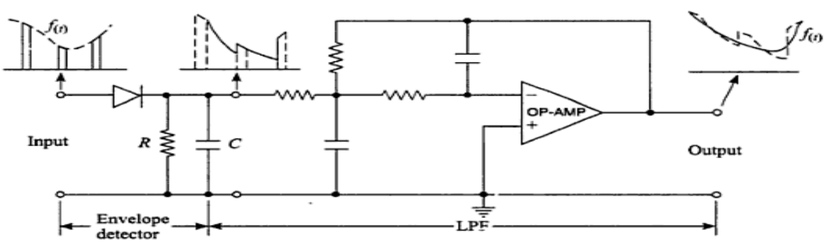
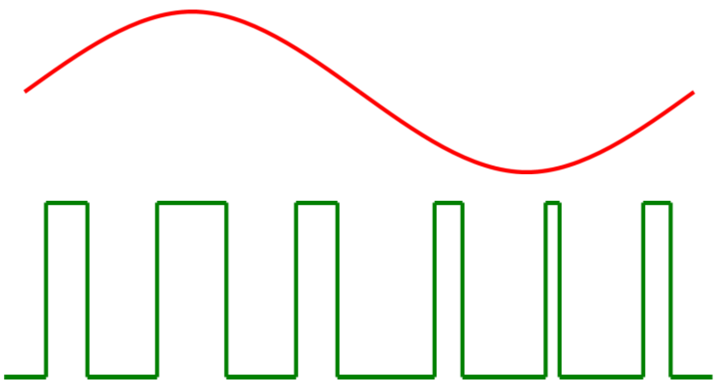



Fig.13. PAM Demodulator

**Pulse Width Modulation:**

- In this type, the amplitude is maintained constant but the width of each pulse is varied in accordance with instantaneous value of the analog signal.



- In PWM information is contained in width variation. This is similar to FM.
- In pulse width modulation (PWM), the width of each pulse is made directly proportional to the amplitude of the information signal.

**Pulse Position Modulation:**

- In this type, the sampled waveform has fixed amplitude and width whereas the position of each pulse is varied as per instantaneous value of the analog signal.
- PPM signal is further modification of a PWM signal.

**PPM & PWM Modulator:**

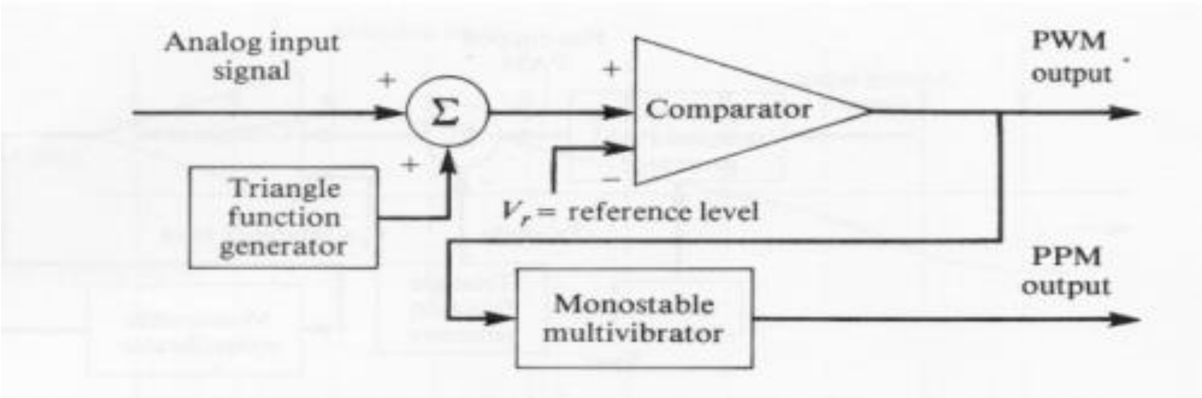



Fig.14. PWM & PPM Modulator

- The PPM signal can be generated from PWM signal.
- The PWM pulses obtained at the comparator output are applied to a mono stable multi vibrator which is negative edge triggered.

- Hence for each trailing edge of PWM signal, the monostable output goes high. It remains high for a fixed time decided by its RC components.
- Thus as the trailing edges of the PWM signal keeps shifting in proportion with the modulating signal, the PPM pulses also keep shifting.
- Therefore all the PPM pulses have the same amplitude and width. The information is conveyed via changing position of pulses.

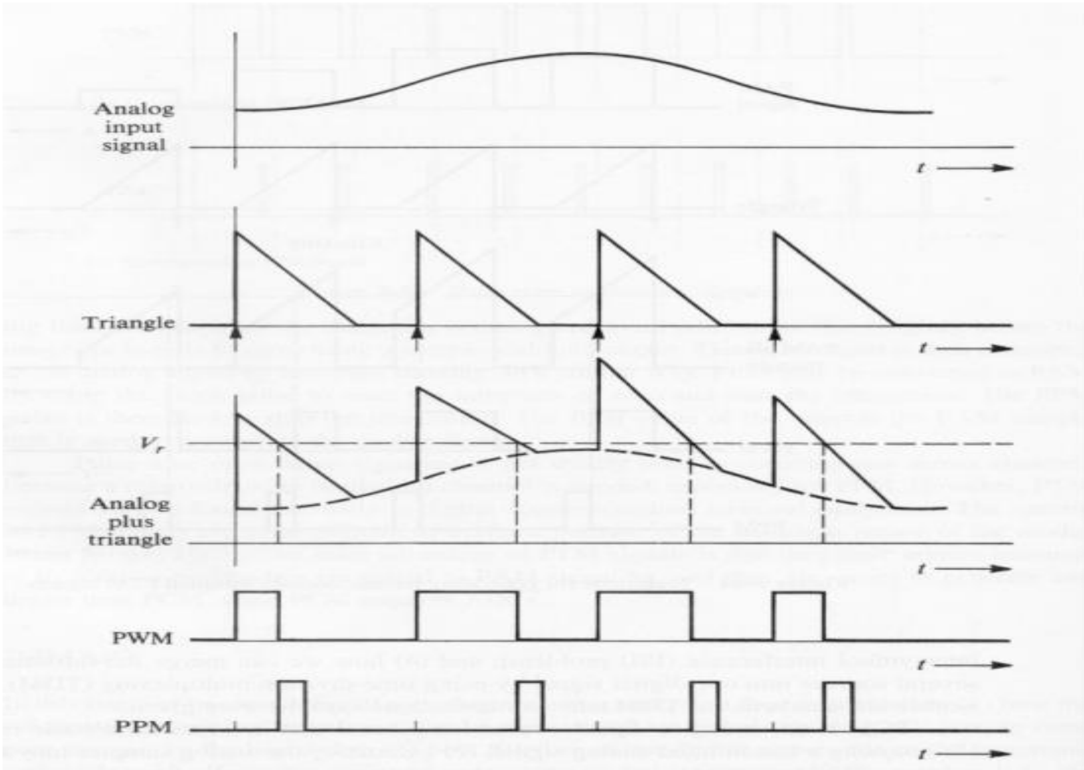



Fig.15. PWM & PPM Modulation waveforms

**PWM Demodulator:**

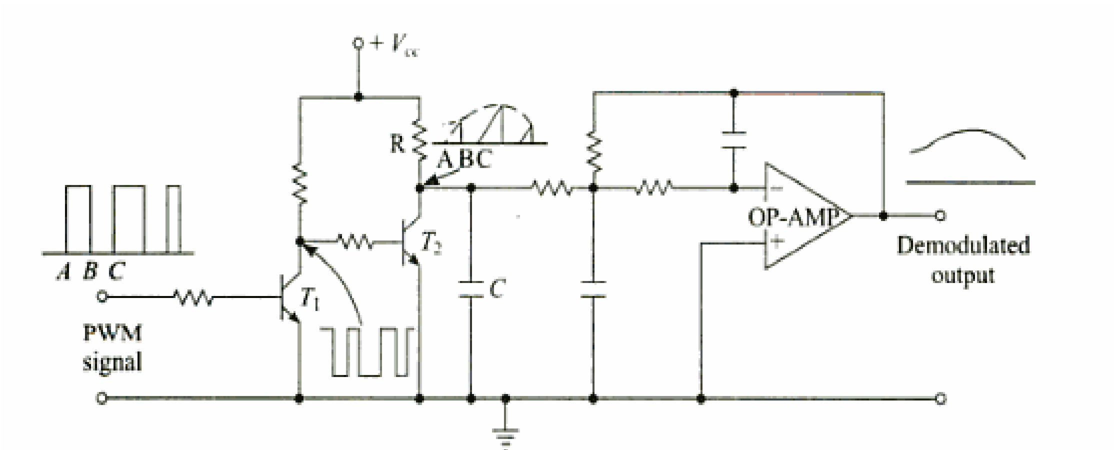



Fig.16. PWM Demodulator

- Transistor T1 works as an inverter.
- During time interval A-B when the PWM signal is high the input to transistor T2 is low.
- Therefore, during this time interval T2 is cut-off and capacitor C is charged through an R-C combination.
- During time interval B-C when PWM signal is low, the input to transistor T2 is high, and it gets saturated.
- The capacitor C discharges rapidly through T2.The collector voltage of T2 during B-C is low.
- Thus, the waveform at the collector of T2is similar to saw-tooth waveform whose envelope is the modulating signal.
- Passing it through 2nd order op-amp Low Pass Filter, gives demodulated signal.

**PPM Demodulator:**

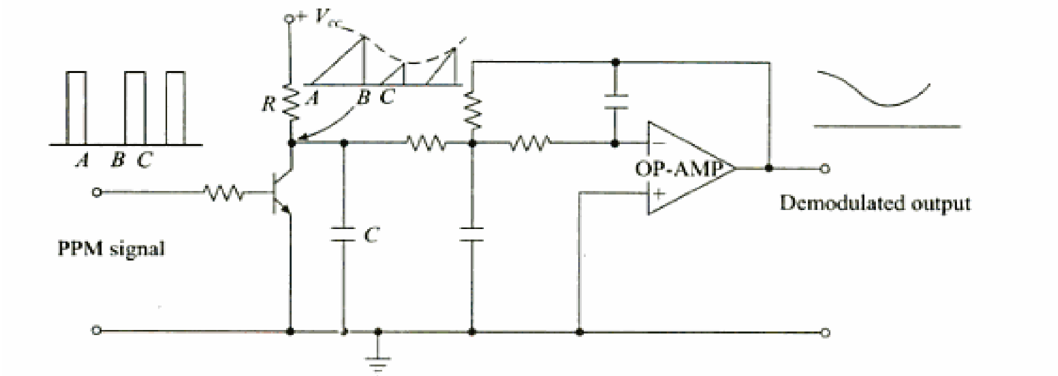
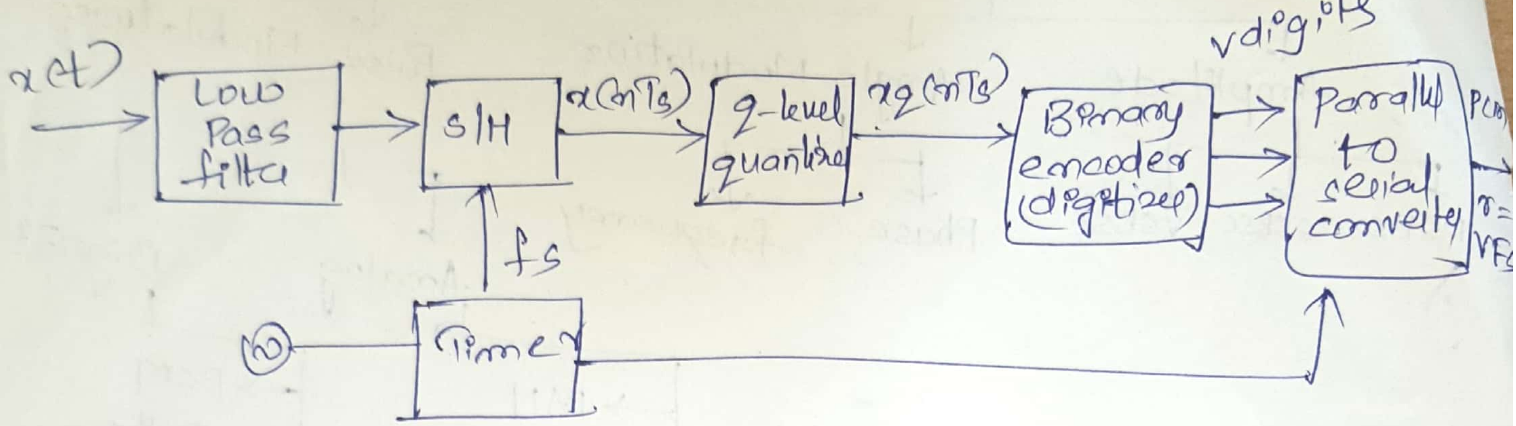



Fig.17. PPM Demodulator

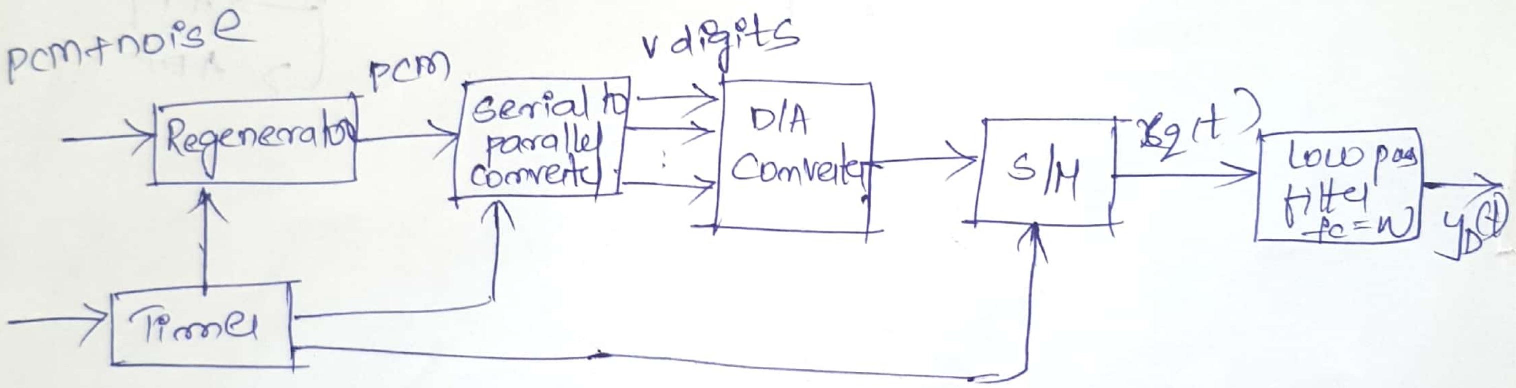
- The gaps between the pulses of a PPM signal contain the information regarding the modulating signal.
- During gap A-B between the pulses the transistor is cut-off and the capacitor C gets charged through R-C combination.
- During the pulse duration B-C the capacitor discharges through transistor and the collector voltage becomes low.
- Thus, waveform across collector is saw-tooth waveform whose envelope is the modulating signal.
- Passing it through 2nd order op-amp Low Pass Filter, gives demodulated signal.

# PCM (Pulse Code Modulation)

## PCM Generator




## PCM Receiver



PCM Transmitter Explanation

Low Pass filter:- This low pass filter blocks all the frequency components above 'W' Hz. Thus $x(t)$ is bandlimited to 'W' Hz.

Sample and Hold circuit:- The sampled and hold circuit then samples this signal at the rate of fs. Sampling frequency fs is selected sufficiently above Nyquist rate, i.e, $fs \geq 2W$ and output of sample and hold is called $x(nTs)$. $x(nTs)$ is discrete in time and continuous in Amplitude.

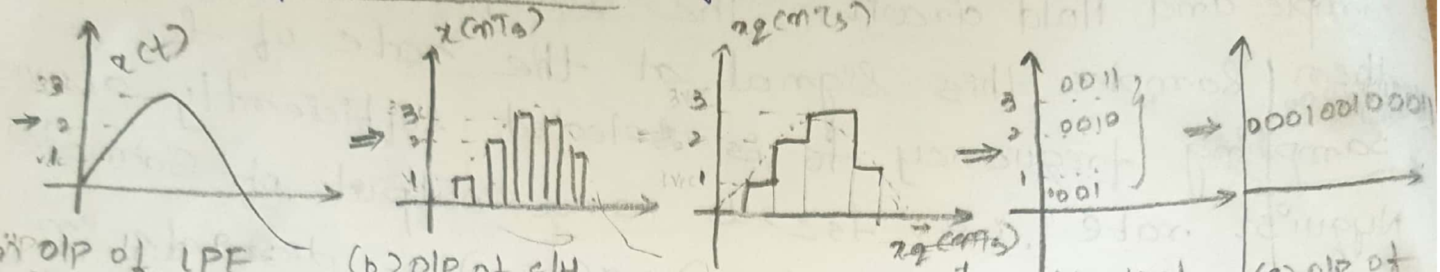
q-level quantizer:- A q level quantiser compares the input $x(nTs)$ with its fixed digital levels. It assigns any one of the digital level to $x(nTs)$ which results in minimum distorsion or error. This error is called quantisation error. This output of quantizer is a digital level called $xq(nTs)$

binary Encoder:- the quantised signal level $xq(nTs)$ is given to binary encoder. This encoder converts input signal to 'v' digits binary word. Thus $xq(nTs)$ is converted to 'v' binary bits. The encoder is called digitizer.

Parallel to Serial Converter:-

It is not possible to transmit each bit of the binary word separately on transmission line. Therefore 'v' binary bits are converted to serial bit stream to generate single baseband signal. In a parallel to serial converter, normally a shift register do this job. Thus the output of PCM generator is thus a single baseband of binary bits.

→ Thus, an oscillator/timer generates the clocks pulses for sample and hold and parallel to serial converter
→ Hence, sample and hold, Quantizer and encoder combinely form Analog to digital Converter.




a) o/p of LPF (b) o/p of S/H (c) o/p of quantiser (d) o/p of encoder (e) o/p of parallel - Serial Converter

PCM Receiver Explanation

Regenerator:- It is impossible to reconstruct exact signal x(t) because of permanent Quantisation introduced during quantisation at the transmitted. So, noise can be ↓sed by ↑ing the step size, and this can be done with the help of regenerative circuit.

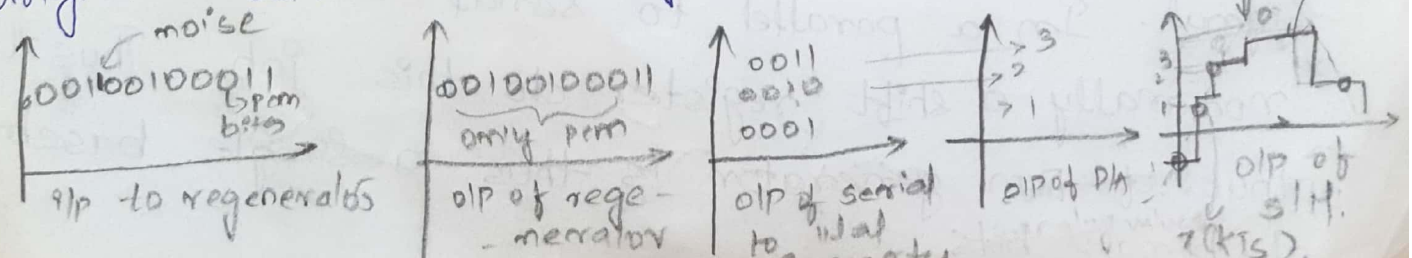
Serial - Parallel Converter:- The Serially transmitted bits are converted to parallel to reconstruct the samples

Decoder:- Decoder converts ~~Serial binary digit to~~

D/A converter:- It converts the parallely transmitted binary bits to the Analog values

S/H → sample and hold will produce the sampled output and also produces the samples Quantised output. Thus S/H performs both sampling and Quantisation o/p in the Receiver.

Low Pass filter:- Low Pass filter will reconstructed the original bandwidth w signal from S/H o/p.



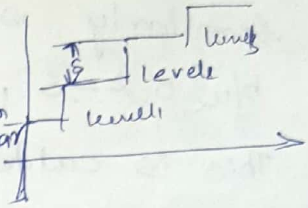

i/p to regenerator o/p of regenerator o/p of serial to parallel converter o/p of D/A o/p of S/H x(kTs)

## QUANTISATION:-

The input Sample value is quantized to nearest digital level. The difference between two quantisation levels is called step size ($\delta$).

Quantisation:-

- Uniform / linear — step size is constant over the amplitude range
  - Midtread Quantizer
  - Midriser Quantiser
  - Biased
- Non Quantisation / Non linear — step size is varying.




### (a) Midtread Quantiser:-

The transfer characteristic of the midtread quantizer as shown in figure.

$\delta \rightarrow$ step size.

For, $-\delta/2 \le x(nT_S) \le \delta/2$,

$x_q(nT_S) = 0$

$\delta/2 \le x(nT_S) \le 3\delta/2$, $x_q(nT_S) = \delta$

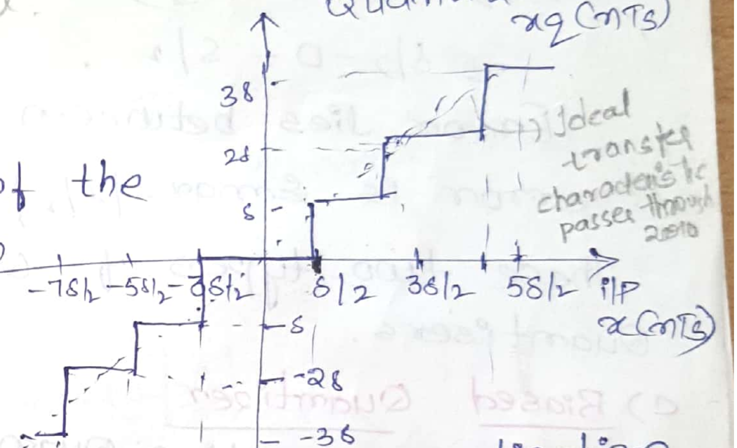



fig:- Quantization characteristic of midtread Quantizer

It is called midtread Quantizer because Quantizer o/p is zero when $x(nT_S)=0$.

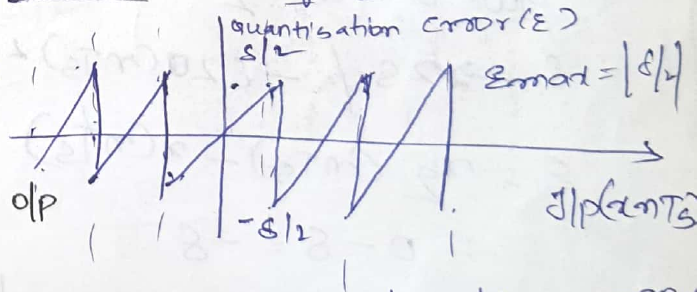
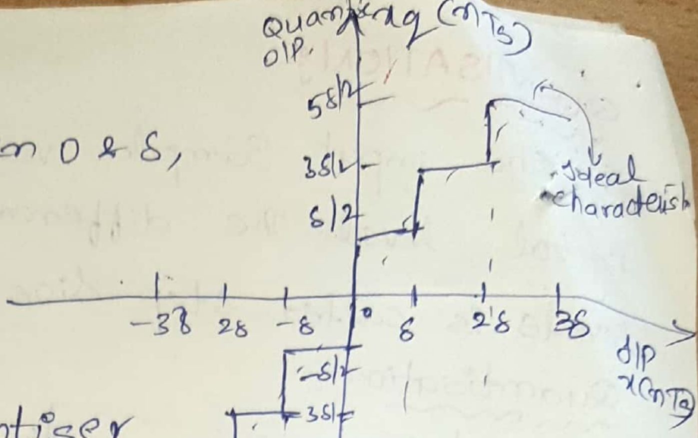



fig:- Quantization error

$\varepsilon = x_q(nT_S) - x(nT_S)$

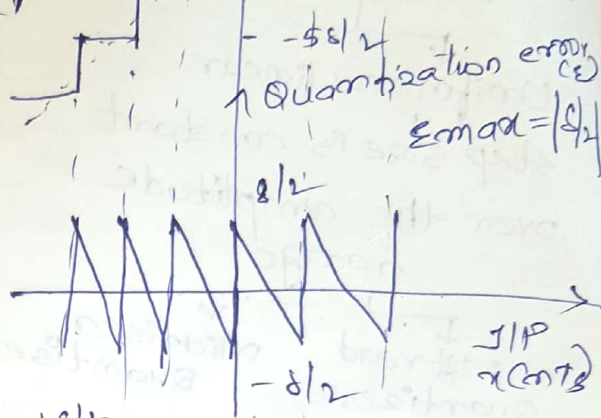
So, error lies between $-\delta/2 \le \varepsilon \le \delta/2$. So, the max. quantisation error will be given as $\varepsilon_{max} = \left|\frac{\delta}{2}\right|$

(b) Midriser Quantiser:—



When the input is between 0 & δ, the o/p is δ/2, similarly when an i/p is b/w 0 & −δ, then o −δ/2.

This is called midriser quantiser because its output is either +δ/2 or −δ/2 when input is zero



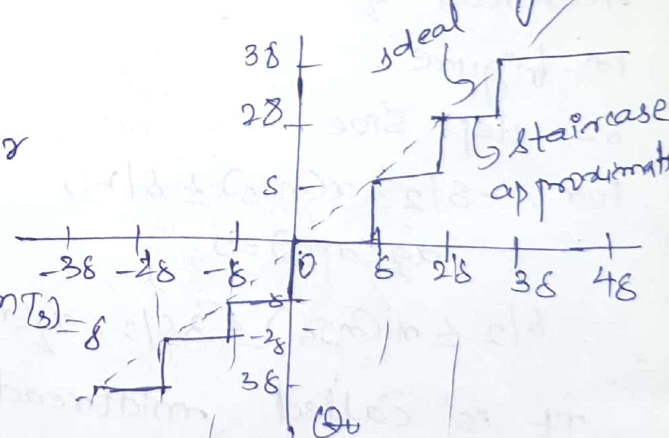
$\varepsilon = x_q(nT_s) - x(nT_s)$

$= \delta/2 - 0 = \delta/2$.

Error lies between ~~0~~ $-\delta/2$ & $+\delta/2$ so, the max. quantisation error is $\varepsilon_{max} = |\delta/2|$

These two types of Quantisers are rounding Quantisers.

c) Biased Quantizer



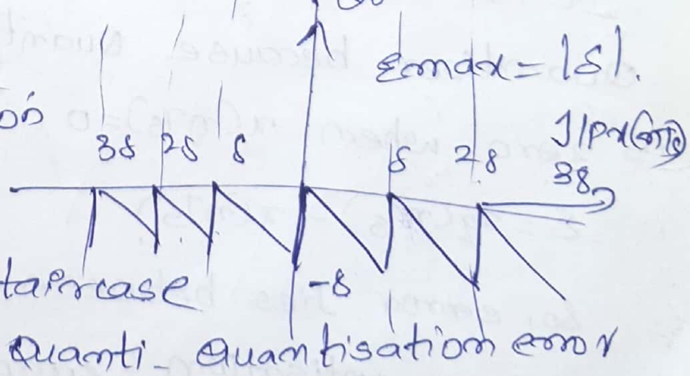
This is truncation Quantiser

b/w $0 \to \delta$, $x_q(nT_s) = 0$

$\delta \to 2\delta$ / $-\delta < x(nT_s) < 0$, $x_q(nT_s) = -\delta$

$\varepsilon = x_q(nT_s) - x(nT_s)$

$= 0 - \delta = -\delta$.



Hence, max Quantisation error is $\varepsilon_{max} = |\delta|$

The difference between staircase and dotted line is called Quanti- Quantisation error -sation error. Hence, Quantisation error is more in Biased Quantiser when compared to Midtread & Midriser Quantiser.

## Quantisation Noise

The noise which occur in Quantisation process is called Quantisation Noise.

## Noise in PCM

Quantisation error $(\varepsilon) = x_q(nT_s) - x(nT_s)$

Consider an i/p signal $x(nT_s)$ be of continuous amplitude which is in the range $-x_{max}$ to $x_{max}$.

The total amplitude range =

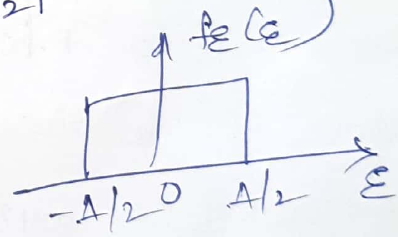
$$x_{max} - (-x_{max}) = 2x_{max}$$

If this amplitude range is divided into $q$ level quantizer, then the step size will be considered as $\Delta = \frac{2x_{max}}{q}$, if $x_{max} = 1$

$$\Delta = \frac{2}{q}.$$

Quantisation error $\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$,

$$f_\varepsilon(\varepsilon) = \begin{cases} 0, & \varepsilon < -\Delta/2 \\ 1/\Delta, & -\Delta/2 < \varepsilon < \Delta/2 \\ 0 & \varepsilon > \Delta/2 \end{cases}$$




Signal - Noise Power $= \frac{V^2_{Noise}}{R}$

$$V^2_{noise} = \overline{X^2} = E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_X(x)\,dx = \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 \cdot \left(\frac{1}{\Delta}\right) d\varepsilon$$

$$= \frac{\Delta^2}{12}.$$

Normalised $(S/N) = \frac{\text{Signal Power}}{\text{Noise Power}}$

$$q = 2^V, \quad \Delta = \frac{2x_{max}}{q} = \frac{2x_{max}}{2^V} = 2^{(1-V)} x_{max}$$

$$\boxed{S/N = \frac{3P}{x^2_{max}} \cdot 2^{2V}}$$

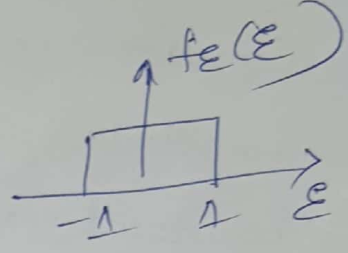
## Noise in $\delta$

Assume that slope overload distortion occurs $A_m < \frac{\Delta}{2\pi f_m T_s}$

If max. signal amplitude is there

$$A_m = \frac{\Delta}{2\pi f_m T_s}$$

$$P = \frac{V_{rms}^2}{R} = \frac{(V_m/\sqrt{2})^2}{R} = \frac{A_m^2}{2}$$



$$P = \frac{\Delta^2}{4\pi^2 f_m^2 T_s^2 (2)}$$

$$P_s = \frac{\Delta^2}{8\pi^2 f_m T_s^2}.$$

Noise Power $P_{noise} = \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 \cdot \frac{1}{2\Delta} d\varepsilon$

$$P_{noise} = \Delta^2/3$$

This noise power is uniformly distributed over $-f_s$ to $+f_s$. At the o/p of $\delta$ modulator Receiver there is low pass reconstruction filter whose cutoff frequency is $\omega$ Hz. This cutoff frequency is equal to highest sign of frequency.

$$\text{O/p noise power} = (\omega/f_s) \text{ noise power}$$

$$= (\omega/f_s)(\Delta^2/3) = \frac{\omega T_s \Delta^2}{3}$$

$$S/N = \frac{P_{signal}}{P_{noise}} = \frac{\Delta^2}{8\pi^2 f_m^2 T_s^2} \times \frac{3}{\omega T_s \Delta^2}$$

$$\boxed{S/N = \frac{3}{8\pi^2 f_m^2 \omega T_s^3}}$$

Scanned with CamScanner

## Companding in PCM



The process in which used to achieve the non uniform quantisation is called. It is used mainly inorder to improve S/N ratio of weak signals. Companding is the process of both compression & expansion. Compression is the process of amplifying the weak signal and attenuation of strong signals. Compression at the transmitter and expansion at receiver is called Companding.

**μ-law companding**

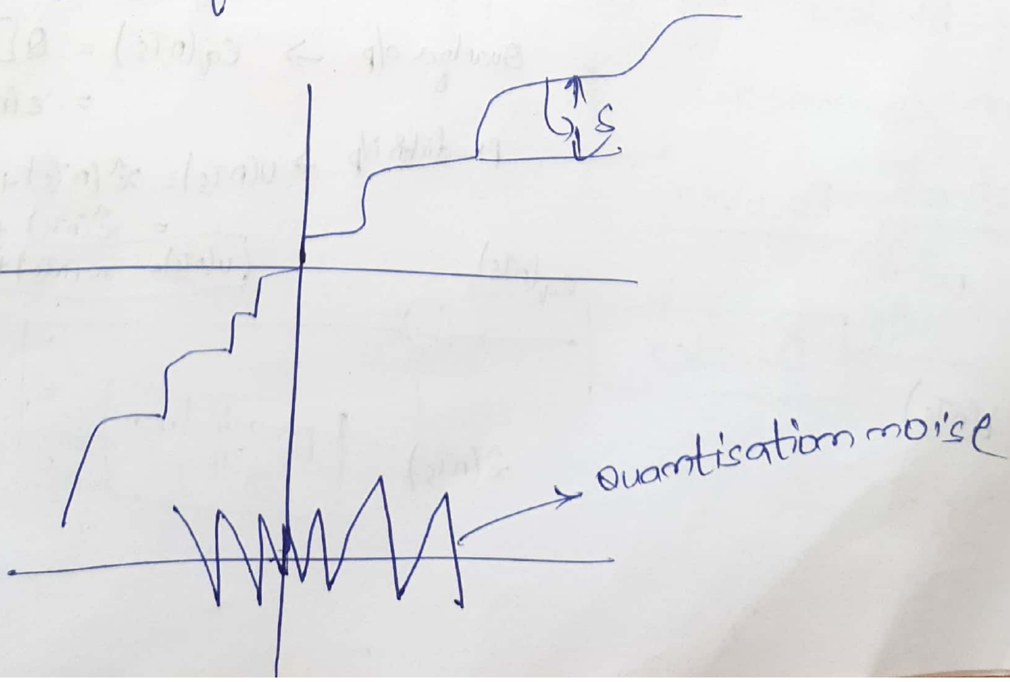
$$z(x) = \operatorname{sgn}(x) \frac{\ln(1+\mu|x|)}{\ln(1+\mu)}, \quad |x| \le 1$$

**A-law companding**

$$z(x) = \frac{A|x|}{1+\ln A} \quad \text{for } 0 \le |x| \le \frac{1}{A}.$$

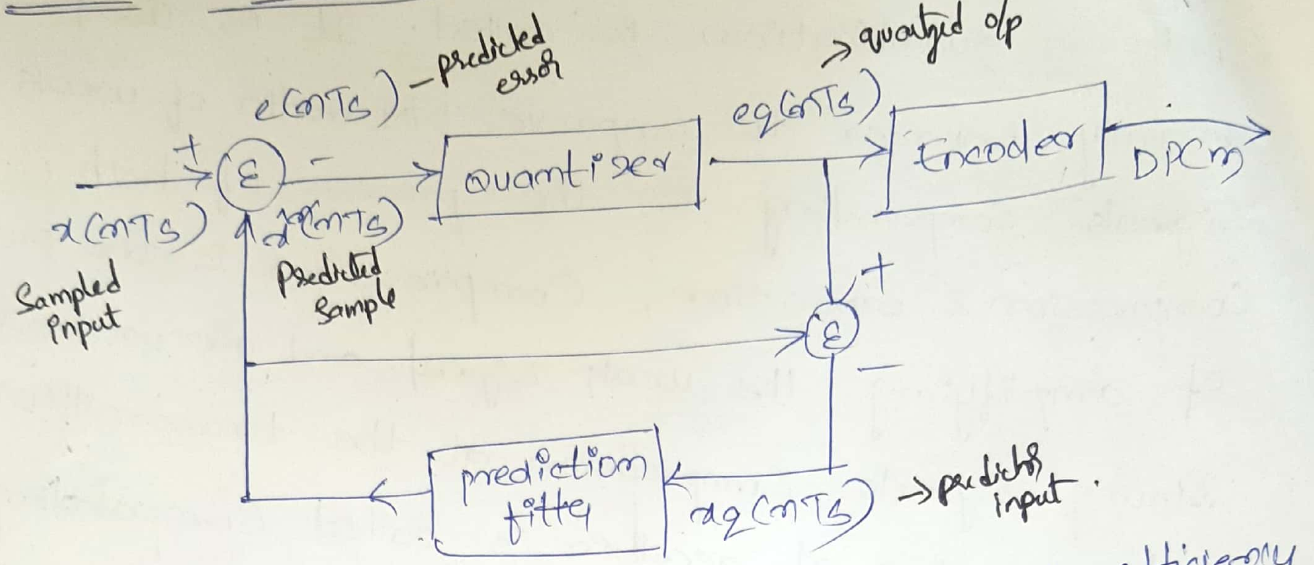
**NON UNIFORM QUANTISATION**

If step size is not constant, then it is called as non uniform Quantisation



# DPCM (Differential Pulse Code Modulation)

## DPCM Transmitter




If the redundancy is reduced in PCM, efficiency will be increased. This will done by CM. The DPCM works on the principle of prediction. The value of present sample is predicted from past sample signal which is denoted by $x(nT_s)$ and signal is denoted by $\hat{x}(nT_s)$. The comparator finds difference between present & previous sample. If both are same, that sample will be blocked by the prediction filter and later stored in summer.

Quantizer o/p → $e_q(nT_s) = Q[e(nT_s)]$

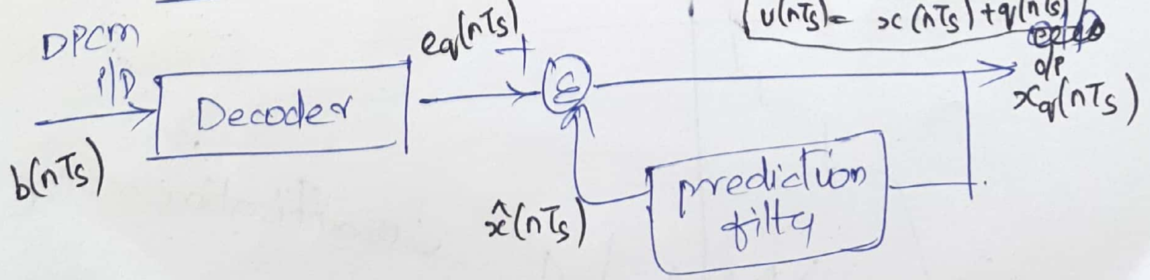
$= e(nT_s) + q(nT_s)$

Prediction i/p → $u(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$

$= \hat{x}(nT_s) + e(nT_s) + q(nT_s)$

$\boxed{u(nT_s) = x(nT_s) + q(nT_s)}$

## DPCM Receiver

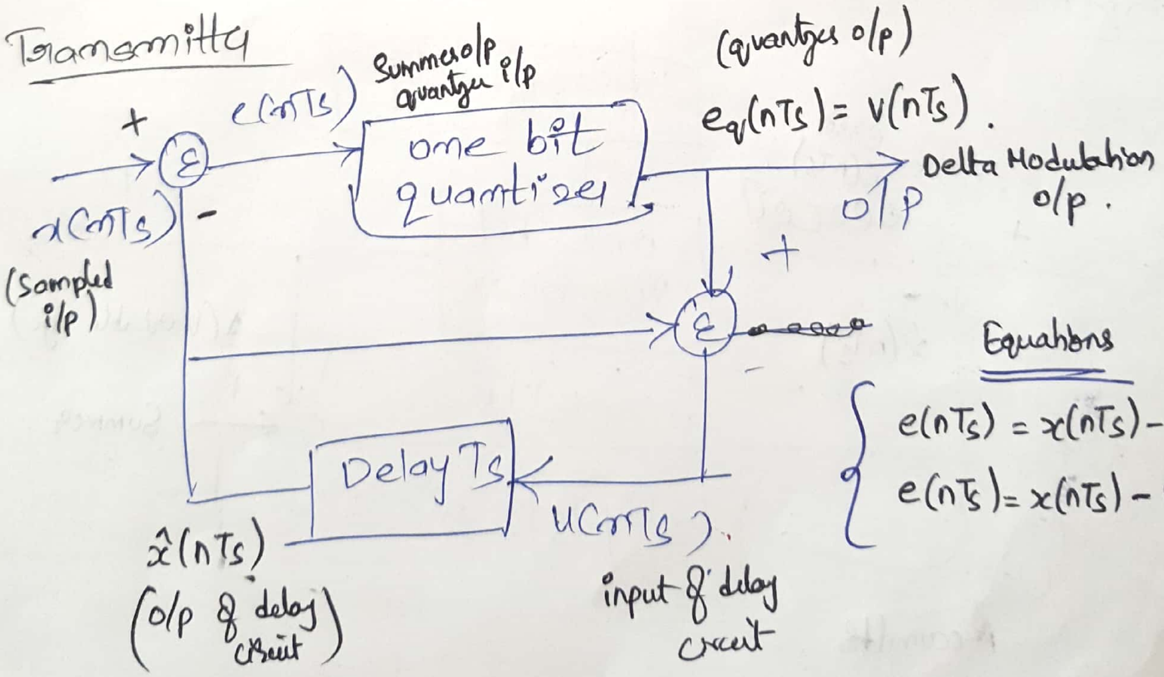


· Decoder first reconstructs the quantised error signal from incoming binary signal. The prediction filter o/p are quantised errors are summed up to give the quantised version of original. Thus signal at receiver refers from actual signal by Quantisation error $q(nT_s)$

## δ-modulation System

Drawbacks of PCM system is high signalling rate & large transmission bandwidth and this can overcome by δ modulation. In δ modulation only one bit is transmitted per one sample. The approximated signal difference i/p & staircase approximated signal confined to two levels +δ and −δ. If the step size is increased by +δ, bit '1' transmitted, otherwise bit '0' transmitted.

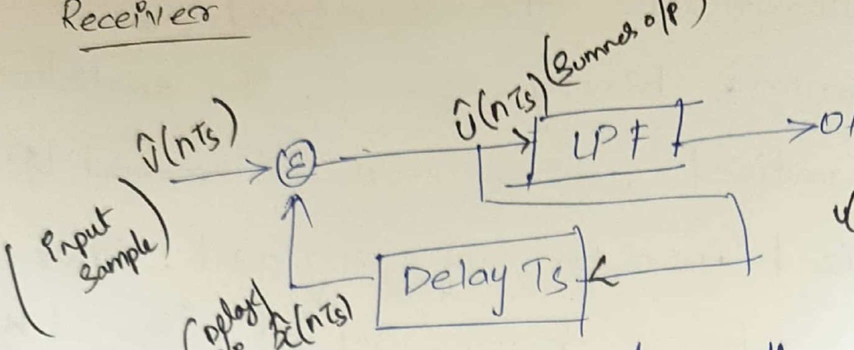
Transmitter




Equations

$$\left\{\begin{aligned} e(nT_s) &= x(nT_s) - \hat{x}(nT_s) \\ e(nT_s) &= x(nT_s) - u(nT_s - T_s) \end{aligned}\right.$$

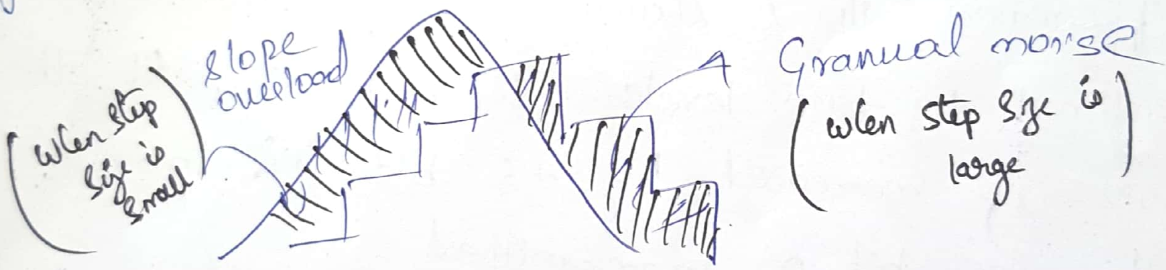
Receiver




equation:-

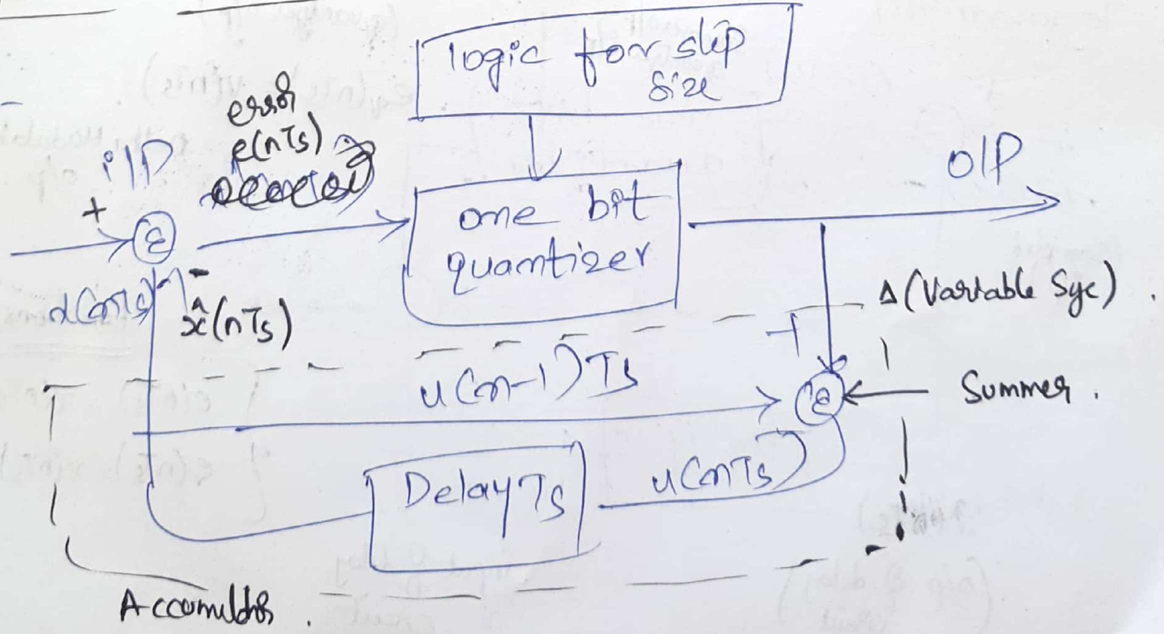
$u(nT_s) = u(nT_s - T_s) + b(nT_s)$

Accumulator generates the staircase approximated signal o/p & it is delayed by sampling period $T_s$. It is then added to the input signal. If the i/p is binary 1 then it adds $+\Delta$ step to the previous o/p. If the binary bit 0 is ~~transmit~~ then the 1 step $\Delta$ is subtracted from the delayed signal.



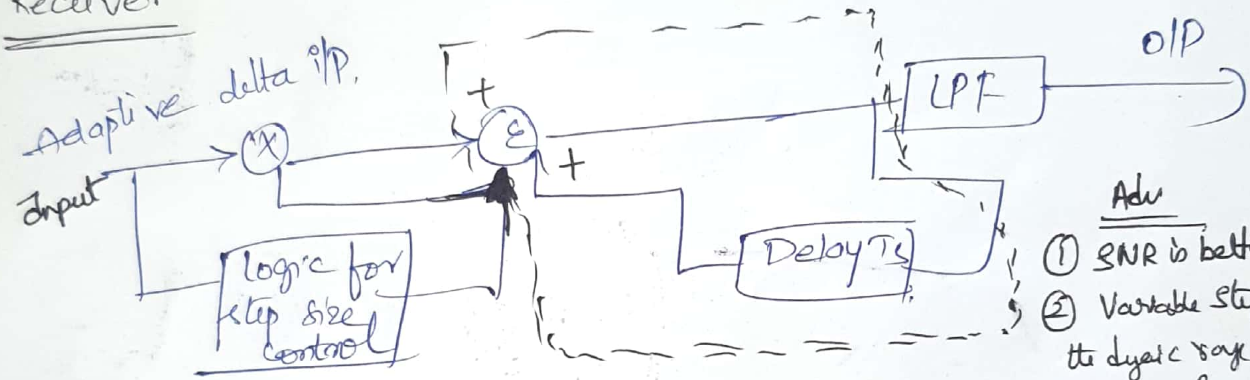

Adaptive Delta

Tx:-



In order to overcome the disadvantages of $\Delta$ modulation, we go for adaptive delta modulation. Logic for step size control is added in figure. Step size is increased or decreased depending on certain rule. For Eg, if one bit quantizer o/p is high then step size will be doubled for next sample. If one bit quantizer o/p is low, then step size will be reduced by one step

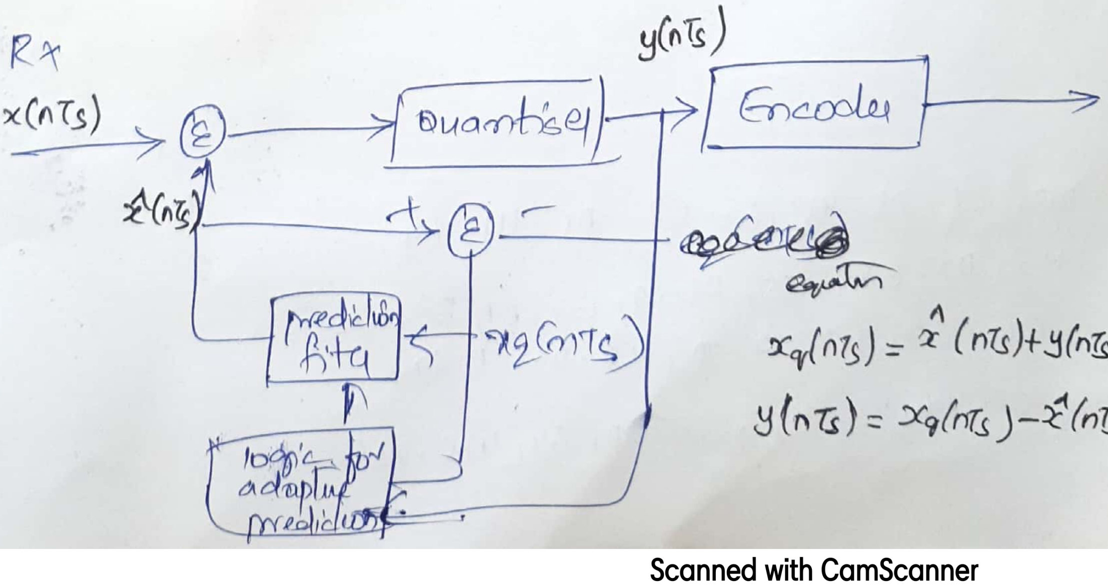
Receiver




Adv
1. SNR is better
2. Variable step size, the dynamic range of ADM is wider
3. B.W utilization is better

ADAPTIVE PCM

It is a combination of ~~Adaptive~~ PCM DPCM & $\delta$ modulation. In forward direction, the above circuit acts as Tx & reverse direction it acts as Rx




equation

$$x_q(nT_s) = \hat{x}(nT_s) + y(nT_s)$$

$$y(nT_s) = x_q(nT_s) - \hat{x}(nT_s)$$

step size depends upon amplitude level & spectrum of speech signal.