18/12/19

Unit-1

Algebraic And Transcendental Equations

Introduction.

In mathematics some problems can be solved analytically and numerically

An analytical solution means framing the problem in a well understood form and calculating the exact solution

→ A numerical solution means making guesses at the solution and testing whether the problem is solved well enough to stop

→ Analytical solution gives us exact answer whereas numerical solution gives an approximate answer.

Polynomial function

A function $f(x)$ is said to be a polynomial function if $f(x)$ is a polynomial in x.

i.e $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$

where $a_0 \neq 0$ & $a_0, a_1, a_2 \dots a_n$ are constants

& n is a non-negative Integer.

Algebraic function

A function which is a sum or difference or product of two polynomial's is called an algebraic function. Otherwise the function is called non-Algebraic or transcendent Equation.

Ex:- $x^2+3x-5=0$
$x^3+4x^2+3x+2=0$ } Algebraic function

Ex:- → $f(x) = c_1e^x + c_2e^{-x} = 0$
$f(x) = 2\log x - \frac{\pi}{4} = 0$ } non-Algebraic or transcendent Equations

Root of an Equation

A number α is called a root of an Equation $f(x) = 0$ if $f(\alpha) = 0$

Ex: $f(x) = x-5$

$x=5$

Intermediate Value theorem

Let a, b be any two numbers then $f(a)$ & $f(b)$ should have Opposite signs then there Exists ($\exists$) a point c such that $c \in (a,b)$

Method-I

** Bisection method

Bisection method is a simple iterative method to solve an equation. this method is also known as Bolzano method of successive Bisection.

→ Sometimes it is referred as half interval method

Procedure

Step 1:- Let $f(x)$ be a continuous function in a closed interval (a,b)

2:- Find 'a' and 'b' values such that $f(a)$ & $f(b)$ should have opposite signs

3:- Now by Bisection method find the value of 'c' i.e => $\boxed{c = \frac{a+b}{2}}$

4:- find $f(c)$ value.

5:- Now if $f(c)$ is positive replace positive value with 'c'.

6:- If $f(c)$ is negative replace negative value with 'c'.

7: continue the process until any two successive approximations are

Equech

Problem

1) find a positive root of the Equation $x^3-4x-9=0$ using bisection method.

Sol Given $x^3-4x-9=0$

$x=0$ $f(0) = -9$ -ve

$x=1$ $f(1) = -12$ -ve

$x=2$ $f(2) = -9$ -ve

$x=3$ $f(3) = 6$ +ve

} Take successive values as a & b.

$a=2$, $b=3$

∴ The root lies between 2 and 3 i.e $a=2$ & $b=3$

a (-ve)	b (+ve)	$c = \frac{a+b}{2}$	f(c)
2	3	2.5	-ve
2.5	3	2.75	+ve
2.5	2.75	2.625	-ve
2.625	2.75	2.6875	-ve
2.6875	2.75	2.7187	+ve
2.6875	2.7187	2.7031	-ve
2.7031	2.7187	2.7109	+ve

2.7031	2.7109	2.7077	+ve
2.7031	2.7077	2.7054	

∴ The root of the given equation is

2.7054

find a positive root of the equation

$x^3 - x - 1 = 0$ correct to two decimal points

given $x^3 - x - 1 = 0$

$x = 0$ $\quad 0 - 0 - 1$

$\Rightarrow -1$

$x = 1$ $\quad 1 - 1 - 1$

-1 $\quad$ (-ve)

$x = 2$ $\quad (2)^3 - 2 - 1 = 0$

$8 - 2 - 1$

5 $\quad$ (+ve)

$a = 1$, $b = 2$

∴ The root lies between 1 and 2 i.e $a = 1$

& $b = 2$

a (-ve)	b (+ve)	$c = \frac{a+b}{2}$	f(c)
1	2	1.5	+ve
1	1.5	1.25	-ve
1.25	1.5	1.375	+ve
1.25	1.375	1.3125	-ve
1.3125	1.375	1.34375	+ve
1.3125	1.34375	1.328125	+ve

$(1.5)^3 - (1.5) - 1 = 0$

$\Rightarrow 0.875$

1.3125	1.328125	1.3203	

∴ The root of the given equation is 1.32.

19/12/19

Thursday

find a real root of a Equation x^3-9x+1 up to three decimal places lieing b/w $x=2$ and $x=4$

Sol

$a=2$ $\quad$ $a=2$ $f(a) = -9$ -ve

$b=4$ $\quad$ $b=4$ $f(b) = 29$ +ve

Given $f(x) = x^3-9x+1=0$

-ve a	+ve b	$c = \frac{a+b}{2}$	f(c)	
2	4	3	+ve	
2	3	2.5	-ve	→ -5.875
2.5	3	2.75	-ve	→ -2.95
2.75	3	2.875	-ve	→ -1.111
2.875	3	2.9375	-ve	→ -0.0
2.9375	3	2.96875	+ve	→ 0.43
2.9375	2.96875	2.953125	+ve	→ 0.15
2.9375	2.953125	2.945125	+ve	

a	b	$c = \frac{a+b}{2}$	f(c)
2.9375	2.943125	2.94171875	-ve
2.94171875	2.943125	2.94242	-ve
2.94242	2.943125	2.9427725	

∴ The root of the given equation is

2.942

find a real root of a equation $x \log_{10} x = 1.2$ which lies between 2 and 3

Sol Given $f(x) = x \log_{10} x = 1.2$

$$f(x) = x \log_{10} x - 1.2 = 0$$

$$a=2 \qquad b=3$$

$a=2$ $f(a) = -0.59$ -ve

$b=g$ $f(b) = 0.23$ +ve

-ve a	+v b	$c = \frac{a+b}{2}$	f(c)
2	3	2.5	-ve
2.5	3	2.75	+ve
2.5	2.75	2.625	-ve
2.625	2.75	2.6875	-ve
2.6875	2.75	2.7185	+ve
2.7185	2.75	2.73437	-ve

2.73437	2.75	2.7421875	+ve
2.734375	2.7421875	2.738	-ve
2.738	2.742	2.74	-ve
2.74	2.742	2.741	

The root of given equation is 2.74

find a real root of the given equation
$f(x) = x - \cos x = 0$

Sol given $f(x) = x - \cos x = 0$

when any function consisting of trigonometric equation solve the problem using Radians

$x=0$ $f(x) = 0 - \cos 0$
$= -1$ (-ve)

$x=1$ $f(x) = 1 - \cos 1 = 0.45$ (+ve)

$a=0$, $b=1$

-ve a	+ve b	$c = \frac{a+b}{2}$	f(c)
0	1	0.5	-ve
0.5	1	0.75	+ve
0.5	0.75	0.625	-ve
0.625	0.75	0.6875	-ve

0.6875	0.75	0.71875	-ve
0.71875	0.75	0.734375	-ve
0.734375	0.75	0.7421875	+ve
0.734375	0.7421875	0.73828125	-ve
0.73828125	0.7421875	0.740234375	+ve
0.73828125	0.740234375	0.739257812	+ve
0.73828125	0.739257812	0.7385	

The root of given equation is 0.73

find a real root of the given equation

$f(x) = \cos x - 3x + 1$

Sol Given $f(x) = \cos x - 3x + 1 = 0$

$x = 0 \quad f(x) = \cos 0 - 3(0) + 1 = 2 \quad$ +ve

$x = 1 \quad f(x) = \cos 1 - 3(1) + 1 = -1 \quad$ -ve

$a = 0 \quad b = 1$

a +ve	b -ve	$c = \frac{a+b}{2}$	$f(c)$
0	1	0.5	+ve
0.5	1	0.75	-ve
0.5	0.75	0.625	-ve
0.5	0.625	0.5625	+ve
0.56	0.625	0.5925	+ve
0.59	0.625	0.6075	-ve

		0.5985	-ve
0.59	0.607		
		0.594	
0.59	0.598		

20/12/19 The root of given equation is 0.59 Friday

Regular falsi method

This method is also known as false position method. In the false position method we will find the root of the equation

$f(x) = 0$

Procedure

Let $f(x)$ be a continous function on $[a, b]$

Step 1:- find 'a' and 'b' values such that $f(a)$ and $f(b)$ are of opposite signs

Step 2:- Suppose 'a' is negative and 'b' is positive.

Step 3:- find 'c' by using the formula

$$\boxed{c = \frac{a f(b) - b f(a)}{f(b) - f(a)}}$$

Step 4:- find $f(c)$ value.

Step 5:- If $f(c)$ is negative replace a by c and f(a) by f(c)

step 6:- If f(c) is positive replace 'b' by 'c' and f(b) by f(c)

step 7:- continue the process until any two successive approximations are equal upto 3 decimal Places

Problems

1) By using regular falsi method find an approximate Root of the equation $x^4 - x - 10 = 0$ that lies b/w 1.8 and 2

Sol Let $f(x) = x^4 - x - 10 = 0$

given $a = 1.8$ $= f(a) = f(1.8) = -1.3$ −ve

$b = 2$ $= f(b) = f(2) = 4$ +ve

a	b	f(a)	f(b)	$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$	f(c)
1.8	2	-1.3	4	1.849	-0.16079
1.849	2	-0.16079	4	1.854	-0.03885
1.854	2	-0.03885	4	1.855	-0.01434
1.855	2	-0.01434	4	1.855	

The root of given equation is 1.855

2) find a real root of the given equation

$x^3 - x - 4 = 0$

Sol Let $f(x) = x^3 - x - 4 = 0$

$a = 1$ $f(1) = 1 = -4$ -ve

$b = 2$ $f(2) = 2 = 2$ +ve

a	b	f(a)	f(b)	$C = \frac{af(b) - bf(a)}{f(b) - f(a)}$	f(c)
1	2	-4	+2	1.666	-1.0419
1.666	2	-1.0419	2	1.780	-0.1402
1.780	2	0.1402	2	1.794	-0.0201
1.794	2	-0.0201	2	1.796	-2.793×10^{-3} → -0.0027
~~1.796~~	~~2~~	~~-0~~			-0.0027
1.796	2	-0.0027	2	1.796	

The root of given equation is 1.796

Homework

1) $x \log_{10} x - 1.2 = 0$

2) $xe^x - 3 = 0$

3) $2x - \log_{10} x = 7$

4) $e^x \sin x = 1$

5) $\log x = \cos x$

$f(x)$

1) Let $f(x) = x\log_{10} x - 1.2 = 0$ $\qquad f(c) = x$

$a = 2 \quad f(2) = -0.59 \ (-ve)$

$b = 3 \quad f(3) = 0.23 \ (+ve$

a	b	f(a)	f(b)	$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$	f(c)
2	3	-0.59	0.23	2.719	-0.018
2.719	3	-0.018	0.23	2.739	-1.435
2.739	3	-1.435	0.23 wrong	2.963	0.197
2.739	2.963	-1.435	0.197	2.935	

Sol

a	b	f(a)	f(b)	c	f(c)
2	3	-0.59	0.23	2.719	-0.018
2.719	3	-0.018	0.23	2.739	-0.0014
2.739	3	-0.0014	0.23	2.740	-0.0005
2.740	3	-0.0005	0.23	2.740	

The root of given equation is 2.740

2) $xe^x - 3 = 0$

Let $f(x) = xe^x - 3 = 0$

$$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$x = 0$ $f(x) = 0e^0 - 3 = -3$

$x = 1$ $f(x) = 1e^1 - 3 = -0.28$

$x = 2$ $f(x) = 2e^2 - 3 = 11.77$

$\therefore$ The root lies between 1 and 2

i.e $a = 1$, $b = 2$ $f(a) = -0.28$

$f(b) = 11.77$

a	b	f(a)	f(b)	c	f(c)
1	2	-0.28	11.77	1.023	-0.154
1.023	2	-0.154	11.77	1.035	-0.086
1.035	2	-0.086	11.77	1.041	-0.051
1.041	2	-0.051	11.77	1.045	-0.028
1.045	2	-0.028	11.77	1.047	-0.017
1.047	2	-0.017	11.77	1.048	-0.011
1.048	2	-0.011	11.77	1.048	

The root of given equation is 1.048.

3) $2x - \log_{10} x = 7$

Sol let given sol $f(x) = 2x - \log_{10} x - 7 = 0$

$x = 1 \quad f(x) = -5$

$C = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

$x = 2 \quad f(x) = -3.3$

$x = 3 \quad f(x) = -1.47$

$x = 4 \quad f(x) = 0.39$

$\therefore$ The root lies between 3 and 4

i.e $a = 3$, $b = 4$ $\quad f(a) = -1.4$

$f(b) = 0.39$

a	b	f(a)	f(b)	c	f(c)
3	4	-1.47	0.39	3.790	0.0013
3	3.790	-1.47	0.0013	3.789	-0.00052
3.789	3.790	-0.0005	0.0013	3.789	

The root of given equation is 3.789

4) $e^x \sin x = 1$

Sol Let $f(x) = e^x \sin x - 1 = 0$

$x = 0 \quad f(x) = e^0 \sin(0) - 1 = -1$

$x = 1 \quad f(x) = e^1 \sin(1) - 1 = -0.95$

$x = 2 \quad f(x) = -0.742$

$x = 3 \quad f(x) = 0.051$

$x=8 \quad f(x) = -0.08$

$x=9 \quad f(x) = -0.03$

$x=10 \quad f(x) =$

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

5) $\log x = \cos x$

let $f(x) = \log x - \cos x = 0$

$x = 1 \quad f(x) = -0.54$

$x = 2 \quad f(x) = 0.717$

$\therefore$ The root lies between 1 & 2

$a = 1, \quad b = 2 \quad f(a) = -0.54$

$f(b) = 0.717$

a	b	f(a)	f(b)	c	f(c)
1	2	-0.54	0.717	1.42	0.0020
1	1.42	-0.54	0.0020	1.411	-0.010
1.411	1.42	-0.010	0.0020	1.418	-0.00052
1.418	1.42	-0.0005	0.0020	1.418	

The root of given equation is 1.418

4) $e^x \sin x = 1$

Let $f(x) = e^x \sin x - 1 = 0$

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$x = 0 \quad f(x) = -1$

$x = 1 \quad f(x) = 1.28$

$\therefore \quad a = 0 \quad b = 1 \quad f(a) = -1$

$f(b) = 1.28$

a	b	f(a)	f(b)	c	f(c)
0	1	-1	1.28	0.438	-0.342
0.438	1	-0.342	1.28	0.556	-0.079
0.556	1	-0.079	1.28	0.581	-0.018
0.581	1	-0.018	1.28	0.586	-0.0063
0.586	1	-0.0063	1.28	0.588	-0.0013
0.588	1	-0.0013	1.28	0.588	

$\therefore$ The roots of given equation is 0.588

23/12/19

Monday

Newton Raphson Method

This Newton Raphson Method is a powerful and elegant method to find the root of an Equation. This method is generally used to Improve the results obtained by the previous method's.

The formula for this method is $\boxed{c = x - \frac{f(x)}{f'(x)}}$

Merits & Demerits

i) Merits

1) In this method convergence is quite fast since the starting value is close to the required root

2) If the root is simple the convergance is quadratic

3) The accuracy of newton Raphson Method for the function f(x) which contains continous first and second derivates can be Estimated.

4) Newton Raphson Method is a single point Iteration

5) This method can be used for solving

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both algebraic and transcandent Equations.
It can also be used when the roots are complese

II) Demerits

1) In deriving the formulae for this method It is assumed that alpha 'α' is not a repeated root of $f(x)=0$.

In this case the convergence of the Iteration may not be obtained.

Hence newton Raphson method is not applicable to find the approximate values of a repeated root.

Problems

1) Apply NewtonRaphson method to find a root correct to 3 decimal Places of the Equation $x^3-3x-5=0$ which lies near $x=2$

Sol Given $f(x) = x^3-3x-5=0$

$f'(x) = 3x^2-3$.

x	$f(x)$	$f'(x)$	$c = x - \frac{f(x)}{f'(x)}$
2	-3	9	2.3333
2.333	0.7032	13.328	2.2805

x	$f(x)$	$f'(x)$	c
2.2805	0.0186	12.602	2.2790
2.2790	-0.0002	12.581	2.279

The root of given equation is 2.279

2) find a real root of the equation $x^4 - x - 9 = 0$ using newton Raphson method (NR method)

Sol: given $f(x) = x^4 - x - 9 = 0$ $\quad f'(x) = 4x^3 - 1$

$x=0 \quad f(0) = -9 \quad -ve$

$x=1 \quad f(1) = -9 \quad -ve$

$x=2 \quad f(2) = 5 \quad +ve$

if 'x' point is not given then find mid $x = \frac{a+b}{2}$

$\therefore$ The root lies b/w $a=1$ & $b=2$

$$x = \frac{a+b}{2} = \frac{1+2}{2} = \frac{3}{2} = 1.5$$

x	$f(x)$	$f'(x)$	$c = x - f(x)/f'(x)$
1.5	-5.4375	12.5	1.935
1.935	3.0842	27.98	1.8247
1.8247	0.2610	23.30	1.8134
1.8134	0.0003	22.85	1.8133

The root of given equation is 1.813

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2) find the real root of the given equation $e^x - 3x$ lieing between 0 and 1.

Sol Given $f(x) = e^x - 3x$

$f'(x) = e^x - 3$

$C = x - \frac{f(x)}{f'(x)}$

$x = \frac{0+1}{2} = \frac{1}{2} = 0.5$

x	$f(x)$	$f'(x)$	C
0.5	0.1487	-1.3512	0.6100
0.6100	0.0104	-1.1595	0.6189
0.6189	0.0001	-1.1431	0.6189

The root of given equation is 0.618

3) find the real root of the following equations by using NR method.

i) $e^x \sin x = 1$

ii) $xe^x - \cos x = 0$

iii) $x + \log_{10} x - 2 = 0$

i) $e^x \sin x = 1$

Given $f(x) = \underbrace{e^x}_{u} \underbrace{\sin x}_{v} - 1$ $\quad$ ~~$f'(x) = e^x(-\cos x)$~~ $-e^x \cos x$

$f'(x) = e^x (+\cos x) + \sin x \, e^x$ $\quad$ $d(uv) = uv' + vu'$

$= e^x(\cos x + \sin x)$

$x=0$ $f(x) = \cancel{0} -1$ (-ve)

$x=1$ $f(x) = 1.28$ (+ve)

$x = \cancel{\frac{-1+1.28}{2}} \cancel{= 0.14}$

$x = \frac{0+1}{2} = \frac{1}{2} = 0.5$

x	$f(x)$	$f'(x)$	c
~~0.5~~	~~-0.2095~~	~~1.9263~~	~~0.6087~~
0.6087	0.05099	1.0797	0.561
0.561		wrong	

sol

x	$f(x)$	$f'(x)$	c
0.5	-0.2095	2.237	0.589
0.589	~~1.1676~~ 0.0011	2.499	0.588
0.588	-0.0013	2.496	0.588

The root of given equation is 0.588

ii) $xe^x - \cos x = 0$

Given $f(x) = xe^x - \cos x = 0$

$f'(x) = xe^x + e^x \cdot 1 + \sin x$

$e^x(x+1) + \sin x$

$x=0 \quad f(0) = -1$

$x=1 \quad f(1) = 2.17$

$$x = \frac{a+b}{2} = 0.5$$

x	$f(x)$	$f'(x)$	c
0.5	-0.0532	2.9525	0.5180
0.5180	0.0007	3.0433	0.5177
0.5177	-0.0001	3.0418	0.5177

The root of given equation is 0.517

3) $x + \log_{10} x - 2 = 0$

given $f(x) = x + \log_{10} x - 2$

$f'(x) = 1 + 1/x$

$x=1 \quad f(x) = -1$

$x=2 \quad f(x) = 0.301$

$a=1, \quad b=2$

$$x = \frac{1+2}{3} = 1.5$$

x	$f(x)$	$f'(x)$	c
1.5	-0.323	1.666	1.693
1.693	-0.078	1.590	1.742
~~1.706~~ 1.742	-0.0169	1.574	1.752

1.752	-0.004	1.570	1.754
1.754	-0.001	1.570	1.754

The root of given equation is 1.754

27th dec 2019

Friday

Derivation

*** formula for finding square root of a number

By newton Raphson method we know that

$$c = x - \frac{f(x)}{f'(x)}$$

Let $f(x) = x^2 - N = 0$ (square)

$f'(x) = 2x$

$$c = x - \frac{(x^2 - N)}{2x}$$

$$= \frac{2x^2 - x^2 + N}{2x}$$

$$= \frac{x^2 + N}{2x}$$

$$= \frac{1}{2}\left(\frac{x^2}{x} + \frac{N}{x}\right) = \frac{1}{2}\left[x + \frac{N}{x}\right]$$

formula for finding cube root of a number

By newton Raphson method we know that

$$C = x - \frac{f(x)}{f'(x)}$$

$$f(x) = x^3 - N = 0 \quad \xleftarrow{} \text{cube}$$

$$f'(x) = 3x^2$$

$$C = x - \frac{(x^3 - N)}{3x^2}$$

$$= \frac{3x^3 - x^3 + N}{3x^2}$$

$$= \frac{2x^3 + N}{3x^2}$$

$$= \frac{1}{3}\left[\frac{2x^{\cancel{3}}}{\cancel{x^2}} + \frac{N}{x^2}\right] = \frac{1}{3}\left[2x + \frac{N}{x^2}\right]$$

Problem

1) Evaluate $\sqrt{28}$ to four decimal places by newton Raphson method.

Sol Given $x = \sqrt{28}$

$$f(x) = \cancel{x^2 - x} = 0$$

$$= \cancel{x^2 - \sqrt{\ }}$$

$$x^2 - N = 0$$

$$x^2 = N$$

$$x = \sqrt{28}$$

S.O.B.S

$$x^2 = 28$$

$f(x) = x^2 - 28 = 0$

$f'(x) = 2x$

$c = \frac{1}{2}\left(x + \frac{N}{x}\right)$

$x = 5 \quad f(5) = -3$

$x = 6 \quad f(6) = 8$

$\therefore$ The root lies b/w 5 & 6

$a = 5, \quad b = 6$

$x = \frac{a+b}{2} = \frac{5+6}{2} = \frac{11}{2} = 5.5$

x	$c = \frac{1}{2}(x + N/x)$
5.5	5.2954
5.2954	5.2915
5.2915	5.2915

$\therefore$ The root of $\sqrt{28}$ is 5.2915

2) find a real root of a number i) $\sqrt{10}$ & ii) $\sqrt{24}$

Sol given $x = \sqrt{10}$

S.O.B.S

$x^2 = 10$

$f(x) = x^2 - 10 = 0 \Rightarrow f'(x) = 2x$

$x=0 \quad f(x) = -10$

$x=1 \quad f(x) = -9$

$x=3 \quad f(x)=-1$

$x=4 \quad f(x)=6$

$\therefore$ The root lies between 3 & 4

$a=3, \quad b=4$

$x = \frac{3+4}{2} = 7/2 = 3.5$

x	$c = \frac{1}{2}(x + N/x)$
3.5	3.1785
3.1785	3.1623
3.1623	3.1622
3.1622	3.1622

$\therefore$ The root of $\sqrt{10}$ is 3.1622

ii) $\sqrt{24}$

Given $x = \sqrt{24}$

S.O.B.S

$x^2 = 24$

$f(x) = x^2 - 24 = 0$

$x=4 \quad f(x) = 16-24 = -8$

$x=5 \quad f(x) = 25-24 = 1$

$\therefore$ The root lies between. ~~8~~ 4 & 5

$a = \cancel{8}\ 4 \quad b=5$

$x = \frac{4+5}{2} = 9/2 = 4.5$

x	$C = \frac{1}{2}(x + N/x)$
4.5	4.9166
4.9166	4.8990
4.8990	4.8989
4.8989	4.8989

The root of $\sqrt{24}$ is 4.8989

3) find the $\sqrt[3]{15}$ using newton Raphson method

Sol given $x = \sqrt[3]{15}$

C.O.B.S

$x^3 = 15 \Rightarrow x^3 - 15 = 0$

$f'(x) = 3x^2$

$f(2) = -7$

$f(3) = 12$

$\therefore$ The root lies b/w 2 & 3

$a = 2, \quad b = 3$

$x = \frac{2+3}{2} = 5/2 = 2.5$

x	$c = \frac{1}{3}\left[2x + \frac{N}{x^2}\right]$
2.5	2.4666
2.4666	2.4662
2.4662	2.4662

The root of $\sqrt[3]{15}$ is 2.4662

ii) $\sqrt[3]{19}$

Given $x = \sqrt[3]{19}$

C.O.B.S

$x^3 = 19$

$x^3 - 19 = 0$

$x = 2 \quad f(x) = 8 - 19 = -11$

$x = 3 \quad f(x) = 27 - 19 = 8$

$x = \frac{2+3}{2} = 2.5$

x	$c = \frac{1}{3}\left[2x + \frac{N}{x^2}\right]$
2.5	2.68
2.68	2.6684
2.6684	2.6684

The root of $\sqrt[3]{19}$ is 2.6684

4) using NR - method find $x + \log_{10}^{x} = 3.375$

find up to 4 decimal places.

Sol) $f(x) = x + \log_{10} x - 3.375$

$f'(x) = 1 + \frac{1}{x}$

$x = 1 \quad f(1) = -2.375$

$x = 2 \quad f(2) = -1.073$

$x = 3 \quad f(3) = 0.102$

$x = 4 \quad f(4) = 1.22$

$\therefore$ The root lies b/w 2 & 3

$a = 2, \; b = 3$ $\qquad x = \frac{2+3}{2} = 5/2 = 2.5$

x	$f(x)$	$f'(x)$	$c = x - f(x)/f'(x)$
2.5	-0.477	1.4	2.8407
2.8407	-0.080	1.35	2.8999
2.8999	-0.012	1.34	2.9088
2.9088	-0.002	1.34	2.9102
2.9102	-0.008	1.3436	2.9107
2.9107	-0.0003	1.3435	2.9109
2.9109	-0.00007	1.3435	2.9109

The root of $x + \log_{10} x - 3.375$ is 2.9109

Iteration Method

Consider an equation $f(x)=0$ which can be taken in the form of $x = \phi(x) \rightarrow (1)$ where $\phi(x)$ satisfies the following conditions

1) For two real numbers 'a' & 'b'. $a \le x \le b$

2) For All x' lieing in the (a,b) should satisfy $0 \le m \le 1$ where 'm' is a constant

then It can be proved that Eq^n (1) has a unique root 'α' in the interval a, b

The formula for Iteration Method is

$$x_n = \phi(x_{n-1}) \quad \text{Where } n \ge 1$$

Procedure

Let $f(x)$ be a continous function in a given interval [a, b

Step1:- convert the given Equation in the form of $x = \phi(x)$

Step2:- Find $\phi'(x)$

Step3:- Now Substitute 'x' value in $|\phi' x|$ that should be less than '1' i.e $|\phi' x| < 1$

Step4:- Now find the Required root by

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using the formula $x_n = \phi(x_{n-1})$

Problems

1) By the single point Iteration method find the root of the equation $x^3-2x-5=0$ which lies near $x=2$

Sol Let $f(x) = x^3-2x-5=0$

$x = \phi(x)$

$x^3 = 2x+5$

$x = (2x+5)^{1/3}$

Here $\phi(x) = (2x+5)^{1/3}$

$\phi'(x) = \frac{1}{3}(2x+5)^{-2/3} \cdot 2$

$= \frac{2}{3(2x+5)^{2/3}}$

$|\phi(x)| < 1$

substitute $x=2$ in $\phi'(x)$

$\Rightarrow \left|\frac{2}{3(2x+5)^{2/3}}\right| = \left|\frac{2}{3(4+5)^{2/3}}\right|$

$= |0.152| < 1$

x	$\phi(x) = (2x+5)^{1/3}$
2	$(2(2)+5)^{1/3} = 2.08008$
2.08008	2.09235
2.09235	2.0941

x	$\phi(x) = (2x+5)^{1/3}$
2.09421	2.09449
2.09449	2.09454
2.09454	2.09454

The root of given Equation is 2.09454

2. find the positive root $x^4 - x - 10 = 0$ by Iteration method.

Sol Let $f(x) = x^4 - x - 10 = 0$

let $x = 0$ $\quad f(x) = -10$

$x = 1$ $\quad f(x) = -8$

$x = 2$ $\quad f(x)$

$\therefore$ The root lies b/w 1 & 2

$a = 1, \quad b = 2$ $\qquad x = \frac{a+b}{2} = \frac{1+2}{2} = 3/2 = 1.5$

$x^4 - x - 10 = 0$

$x^4 = x + 10$

$x = (x+10)^{1/4}$

Here $\phi(x) = (x+10)^{1/4}$

$\phi'(x) = \frac{1}{4}(x+10)^{1/4 - 1}$

$$\phi'(x) = \frac{1}{4}(x+10)^{-3/4} \cdot 1$$

$$= \frac{1}{4(x+10)^{3/4}}$$

Substitute $x = 1.5$ in $\phi(x)$

$$\Rightarrow \left| \frac{1}{4(1.5+10)^{3/4}} \right|$$

$$\Rightarrow 0.04003 < 1$$

x	$\phi(x) = (x+10)^{1/4}$
~~0.04~~ 1.5	1.84151
1.84151	1.85503
1.85503	1.85556
1.85556	1.85558
1.85558	1.85558

The root of given equation is 1.85558

3. $2x - \log x = 7$

Sol: let $f(x) = 2x - \log x - 7 = 0$

let $x = 1$ $f(x) = -5$

$x = 2$ $f(x) = -3.3$

$x = 3$ $f(x) = -1.4$

$x = 4$ $f(x) = 0.3$

$x = 5$ $f(x) = 2.3$

$\therefore$ The root lies b/w 3 and 4

$a=3, \quad b=4$

$$x = \frac{3+4}{2} = 7/2 = 3.5$$

$$2x - \log x - 7 = 0$$

$$2x = \log x + 7$$

$$x = \frac{\log x + 7}{2}$$

$$\phi(x) = \frac{\log x + 7}{2}$$

$$\phi'(x) = \frac{1}{2x}$$

substitute $x = 3.5$ in $\phi'(x)$

$$\left|\frac{1}{2(3.5)}\right|$$

$\Rightarrow 0.1428 < 1$

x	$\phi(x) = \frac{\log x + 7}{2}$
3.5	3.77203
3.77203	3.78828
3.78828	3.78922
3.78922	3.78927

The root of given eq^n is 3.7892

4) $\cos x - 3x + 1 = 0$

Sol let $f(x) = \cos x - 3x + 1$

let $x = 0$ $f(x) = 2$

let $x = 1$ $f(x) = -1$

$\therefore$ The root lies b/w 0 & 1

$a = 0,\ b = 1 \Rightarrow x = \frac{0+1}{2} = \frac{1}{2} = 0.5$

$\cos x - 3x + 1$

$3x = \cos x + 1$

$x = \frac{\cos x + 1}{3}$

$\phi(x) = \frac{\cos x + 1}{3} = \frac{\cos x}{3} + \frac{1}{3}$

$\phi'(x) = -\frac{\sin x}{3}$

Substitute $x = 0.5$ in $\phi'(x)$.

$\left|\frac{-\sin x}{3}\right| = \left|\frac{-\sin(0.5)}{3}\right|$

$= |-0.1598|$

→ P.T.O

x	$\phi(x) = \frac{\cos x + 1}{3}$
0.5	0.6258
0.6258	0.6034
0.6034	0.6078
0.6078	0.6069
0.6069	0.6071
0.6071	0.6071

The root of given equation is 0.6071

5) $x = \cos x$ at $x = \pi/4 = \frac{1}{\sqrt{2}} = 0.\cancel{0}707$

Let $f(x) = \cos x$

$x = 0.\cancel{0}707$

$\phi(x) = \cos x$ → Wrong

$\phi'(x) = -\sin x$

Substitute x value in $\phi'(x)$

$\phi'(x) = \left| -\sin x \right|$

$= \left| -\sin(0.707) \right| = -0.649$

5) $x = \cos x$ at $x = \pi/4$

Sol Given

$x = \cos x$

Here $\phi(x) = \cos x$

$\phi'(x) = -\sin x$

Substituting $\pi/4$ in $\phi'(x)$

$-\sin(\pi/4) = -0.7071 = |-0.7071|$

x	$\phi(x) = \cos x$
$\pi/4$	0.7071
0.7071	0.7602
0.7602	0.7246
0.7246	0.7487
0.7487	0.7325
0.7325	0.7435
0.7435	0.7361
0.7361	0.7410
0.7410	0.7377
0.7377	0.7400
0.7400	0.7384

0.7384	0.7395
0.7395	0.7388
0.7388	0.7392
0.7392	0.7390

The root of given Eqn is 0.7390

2/1/20
Thursday

Curve Fitting

Least squares curve fitting procedures

with an experimental data the data is plotted on a graph and a straight line is drawn through the plotted points. This is the usual method to fit a mathematical Equation to experimental data.

→ The method of least squares is the most systematic procedure to fit a unique curve to the given data points.

→ Let the set of data points be (x_i, y_i) $i = 1, 2, \ldots m$
Suppose the curve $y = f(x)$ is fitted to this data. Let the observed value at $x = x_i$ is

y_i and the corresponding value on the curve is $f(x_i)$. Let e_i is the error of approximation then we have $e_i = y_i - f(x_i)$.

Consider $S = (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 + \dots\dots +$

$$(y_m - f(x_m))^2 = e_1^2 + e_2^2 + \dots + e_m^2$$

The method of least squares consists of minimising 'S'.

There are four curve fitting methods

1) fitting of a straight line i.e $y = a + bx$ or $y = ax + b$

2) fitting of the second degree Polynomial parabola i.e $y = ax^2 + bx + c$ or $y = ax + bx + cx^2$

3) fitting of Exponential curve i.e $y = ae^{bx}$

4) fitting of the power curve i.e $y = ab^x$

* fitting of the straight line

Let $y = a + bx$ be a straight line

→ Then the normal Equations are $\Sigma y_i = na + b\Sigma x_i$

$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2$

Where n = no. of Observations.

Problems

1. By the method of least squares find the straight line for the following data.

x	1	2	3	4	5
y	14	27	40	55	68

Sol Let $y = a + bx$ be the straight lines → ①

The normal eqn's are $\Sigma y_i = na + b\Sigma x_i$
$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2$ } → ②

Now find $\Sigma x_i, \Sigma y_i, \Sigma x_i^2, \Sigma x_i y_i$ using the table.

Here n = no. of observations = 5

x	y	x^2	xy
1	14	1	14
2	27	4	54
3	40	9	120
4	55	16	220
5	68	25	340
$\Sigma x_i = 15$	$\Sigma y_i = 204$	$\Sigma x_i^2 = 55$	$\Sigma x_i y_i = 748$

Now substitute all the above values in Equation 2

$$\Sigma y_i = na + b\Sigma x_i$$

$$204 = 5a + 15b \rightarrow$$

$$\Sigma x_i y_i = a\,\Sigma x_i + b\,\Sigma x_i^2$$

$$748 = 15a + 55b \rightarrow$$

③

Now solve eqn ③ for getting a and b values.

$$204 = 5a + 15b$$

$$748 = 15a + 55b$$

using calci

$a = 0 \qquad b = 13.6$

now substitute 'a' and 'b' in eqn ①

i.e $y = 0 + 13.6x$

$y = 13.6x$ is the best straight line for the given data.

2 fit a straight line for the following data.

x	6	7	7	8	8	8	9	9	10
y	5	5	4	5	4	3	4	3	3

Sol n = no. of observations $= 10$

Let $y = a + bx \rightarrow$ ①

normal Eqn's $\sum y_i = na + b\sum x_i$
$\left.\sum x_i y_i = a\sum x_i + b\sum x_i^2\right\}$ ②

now find x_i, x_i^2, y_i, $x_i y_i$

n = no. of observation = 9

x_i	y_i	x_i^2	$x_i y_i$
6	5	36	30
7	5	49	35
7	4	49	28
8	5	64	40
8	4	64	32
8	3	64	24
9	4	81	36
9	3	81	27
10	3	100	30
$\sum x_i = 72$	$\sum y_i = 36$	$\sum x_i^2 = 588$	$\sum x_i y_i = 288$

i> $36 = 9a + 72b$

ii> $\left.288 = 72a + 588b\right\}$ 3rd

$a = 8$ $\quad b = -0.5$ Substitute in eqⁿ ①

$y = 8 + (-0.5)x \Rightarrow y = 8 - 0.5x$

3)

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

$y = ax + b \rightarrow (1)$

normal eqn's $\Sigma y_i = na + b\Sigma x_i$,

$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2$ (2)

find $x_i, x_i^2, y_i, x_i y_i$

number of observations = 5

x_i	y_i	x_i^2	$x_i y_i$
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2
$\Sigma x_i = 10$	$\Sigma y_i = 16.9$	$\Sigma x_i^2 = 30$	$\Sigma x_i y_i = 47.1$

1) $16.9 = 5a + 10b$

2) $47.1 = 10a + 30b$

$a = 0.72 \qquad b = 1.33$

substitute in eqn (1)

$y = 0.72 + 1.33x$ is the best straight line for given data.

4)

x	0	5	10	15	20	25
y	12	15	17	22	24	30

Sol: $y = a + bx \rightarrow (1)$

find $x_i, x_i^2, y_i, x_i y_i^2$

The normal eqn's $\Sigma y_i = na + b\Sigma x_i$

$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2$

no. of observations $= 6$

x_i	y_i	x_i^2	$x_i y_i$
0	12	0	0
5	15	25	75
10	17	100	170
15	22	225	330
20	24	400	480
25	30	625	750
$\Sigma x_i = 75$	$\Sigma y_i = 120$	$\Sigma x_i^2 = 1375$	$\Sigma x_i y_i = 1805$

1) $120 = 6a + 75b$

2) $1805 = 75a + 1375b$

$a = 11.28$ $b = 0.69$

now substitute a & b in eqn ①

$y = 11.28 + 0.69x$ is the best straightline for given data.

5 fit a straight line of the form $y = ax+b$ for the given data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Sol The given straight line is $y = ax+b$

The normal eqn's are $\Sigma y_i = a\Sigma x_i + nb$

$$\Sigma x_i y_i = a\,\Sigma x_i^2 + b\,\Sigma x_i$$

now find $x_i, x_i^2, y_i, x_i y_i$

x_i	y_i	x_i^2	$x_i y_i$
0	1	0	0
1	1.8	1	1.8
2	1.3	4	2.6
3	2.5	9	7.5
4	6.3	16	25.2
$\Sigma x_i = 10$	$\Sigma y_i = 12.9$	$\Sigma x_i^2 = 30$	$\Sigma x_i y_i = 37.1$

n = no. of observations = 5

1) $12.9 = 10a + 5b$ ③ solve

2) $37.1 = 30a + 10b$

sol

$a = 1.13$ $b = 0.32$

now substitute a & b in eqn ①

$y = (1.13)x + 0.32$ is the best straight line for given data.

3/1/20

fitting of a second degree parabola

Let $y = a + bx + cx^2$ be a parabola of second degree

The normal equations are

$$\Sigma y_i = na + b\Sigma x_i + c\Sigma x_i^2$$

$$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2 + c\Sigma x_i^3$$

$$\Sigma x_i^2 y_i = a\Sigma x_i^2 + b\Sigma x_i^3 + c\Sigma x_i^4$$

Problems

1) fit a second degree polynomial to the following

x	10	12	15	23	20
y	14	17	23	25	21

Sol Let $y = a + bx + cx^2 \rightarrow (1)$

The normal eqn's are

$$\Sigma y_i = na + b\Sigma x_i + c\Sigma x_i^2$$

$$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2 + c\Sigma x_i^3$$

$$\Sigma x_i^2 y_i = a\Sigma x_i^2 + b\Sigma x_i^3 + c\Sigma x_i^4$$

find Σx_i, Σx_i^2, Σx_i^3, Σx_i^4, $\Sigma x_i y_i$, $\Sigma x_i^2 y_i$ using the following table.

x	y	x^2	x^3	x^4	xy	x^2y
10	14	100	1000	10000	140	1400
12	17	144	1728	20736	204	2448
15	23	225	3375	50625	345	5175
23	25	529	12167	279841	575	13225
20	21	400	8000	160000	420	8400
$\Sigma x_i = 80$	$\Sigma y_i = 100$	$\Sigma x_i^2 =$ 1398	$\Sigma x_i^3 =$ 26270	$\Sigma x_i^4 =$ 521202	$\Sigma x_i y_i =$ 1684	$\Sigma x_i^2 y_i =$ 30648

$n = 5$

i) $100 = 5a + 80b + 1398c$

ii) $1684 = 80a + 1398b + 26270c$

iii) $30648 = 1398a + 26270b + 521202c$

$a = -18.72$ $\quad b = 3.00$ $\quad c = -0.069$

subsitute a,b,c values in Eqn (1)

$y = a + bx + cx^2$

$y = -8.72 + 3.00x + 0.069x^2$

$y = -8.72 + 3.00x - 0.069x^2$

2) fit a parabola for the following data.

x	1	2	3	4	5	6	7
y	2.3	5.2	9.7	16.5	29.4	35.5	54.4

Let $y = a + bx + cx^2 \rightarrow (1)$

$$\left.\begin{aligned} \Sigma y_i &= na + b\Sigma x_i + c\Sigma x_i^2 \\ \Sigma x_i y_i &= a\Sigma x_i + b\Sigma x_i^2 + c\Sigma x_i^3 \\ \Sigma x_i^2 y_i &= a\Sigma x_i^2 + b\Sigma x_i^3 + c\Sigma x_i^4 \end{aligned}\right\} (2)$$

$\Sigma x_i = 28$ $\Sigma y_i = 153$ $\Sigma xy = 848.6$

$\Sigma x_i^2 = 140$ $\Sigma x_i^3 = 784$ $\Sigma x_i^4 = 4676$ $\Sigma x_i^2 y_i = 5053$

$$\left.\begin{aligned} &\text{i)}\ 153 = 7a + 28b + 140c \\ &\text{ii)}\ 848.6 = 28a + 140b + 784c \\ &\text{iii)}\ 5053 = 140a + 784b + 4676c \end{aligned}\right\} (3)$$

Solve Eqn (3)rd

$a = 2.371$ $b = -1.09$ $c = 1.192$

substitute a,b,c values in Eqn (1)

$y = 2.371 + 1.09x + 1.192x^2$

3)

x	1	2	3	4	5
y	10	12	8	10	14

sol Let $y = a + bx + cx^2 \rightarrow ①$

The normal Eqn's are

$$\Sigma y_i = na + b\,\Sigma x_i + c\,\Sigma x_i^2$$

$$\Sigma x_i y_i = a\,\Sigma x_i + b\,\Sigma x_i^2 + c\,\Sigma x_i^3$$

$$\Sigma x_i^2 y_i = a\,\Sigma x_i^2 + b\,\Sigma x_i^3 + c\,\Sigma x_i^4$$

now find $\Sigma x_i, \Sigma x_i^2, \Sigma x_i^3, \Sigma x_i^4, \Sigma x_i y_i, \Sigma x_i^2 y_i$

$\Sigma x_i = 15 \quad \Sigma y_i = 54 \quad \Sigma x_i^2 = 55 \quad \Sigma x_i^3 = 225$

$\Sigma x_i^4 = 979, \quad \Sigma x_i y_i = 168, \quad \Sigma x_i^2 y_i = 640$

subs n = no. of observations = 5

i) $54 = 5a + 15b + 55c$

ii) $168 = 15a + 55b + 225c$

iii) $640 = 55a + 225b + 979c$

$a = 14, \quad b = -3.68, \quad c = 0.71$

substitute a, b, c values in eqn ①

$y = a + bx + cx^2$

$y = 14 + (-3.68)x + 0.71x^2$

4)

x	1	2	3	4
y	6	11	18	27

5)

sol

Sol Let $y = a + bx + cx^2 \rightarrow (1)$

normal eqn's are

$\Sigma y_i = na + b\Sigma x_i + c\Sigma x_i^2$

$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2 + c\Sigma x_i^3$

$\Sigma x_i^2 y_i = a\Sigma x_i^2 + b\Sigma x_i^3 + c\Sigma x_i^4$

now find

$\Sigma x_i = 10$ $\quad \Sigma y_i = 62$ $\quad \Sigma x_i^2 = 30$ $\quad \Sigma x_i^3 = 100$

$\Sigma x_i^4 = 354$ $\quad \Sigma x_i y_i = 190$ $\quad \Sigma x_i^2 y_i = 644$

$n = 4$

i) $62 = 4a + 10b + 30c$

ii) $190 = 10a + 30b + 100c$

iii) $644 = 30a + 100b + 354c$

$a = 3$, $b = 2$, $c = 1$

substitute a, b, c values in eq (1)

$y = a + bx + cx^2$

$y = 3 + 2x + 1x^2.$

5)

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

sol Let $y = a + bx + cx^2 \rightarrow (1)$

The normal eqn's are

$$\Sigma y_i = na + b\Sigma x_i + c\Sigma x_i^2$$

$$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2 + c\Sigma x_i^3$$

$$\Sigma x_i^2 y_i = a\Sigma x_i^2 + b\Sigma x_i^3 + c\Sigma x_i^4$$

now find

$\Sigma x_i = 30$ $\quad \Sigma y_i = 196.06$ $\quad \Sigma x_i^2 = 220$ $\quad \Sigma x_i^3 = 1800$

$\Sigma x_i^4 = 15664$ $\quad \Sigma x_i y_i = 1618.3$ $\quad \Sigma x_i^2 y_i = 14152.12$

$n = 5$

i) $196.06 = 5a + 30b + 220c$

ii) $1618.3 = 30a + 220b + 1800c$

iii) $14152.12 = 220a + 1800b + 15664c$

$a = -1591.429$ $b = 681.48$ $c = -55.86$

$a = 0.696$ $\quad b = -0.85$ $\quad c = 0.991$

substitute a, b, c values In eqⁿ ①

$y = a + bx + cx^2$

$y = 0.696 + (-0.85)x + 0.991x^2$

6/1/20

fitting of an exponential curve

Let us consider an exponential curve of the form $y = ae^{bx} \rightarrow ①$

Take log on b.s from eq ①

$\log y = \log(ae^{bx})$

$\log y = \log a + \log e^{bx}$

$\log y = \log a + bx \log e$

$\log y = \log a + bx$

Let us assume

$Y = \log y \quad A = \log a \quad B = b \quad X = x$

$Y = A + BX \rightarrow ②$

Eq ② In the form of straight line

$\Sigma Y_i = nA + B\Sigma X_i$

$\Sigma X_i Y_i = A\Sigma X_i + B\Sigma X_i^2$

Here $A = \log a$ $B = b$

$a = e^A$

Note:-

for calculating 'y' value in exponential curve we have to take $Y = \ln(y)$

Determine the constants of 'a' & 'b' by the method of least squares such that $y = ae^{bx}$ for the following data.

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

The given curve is $y = ae^{bx} \rightarrow ①$

Taking log on b.s

$\log y = \log ae^{bx}$

$\log y = \log a + \log e^{bx}$

$\log y = \log a + bx \log e$

$\log y = \log a + bx$

let $Y = \log y$ $A = \log a$ $B = b$ $X = x$

$Y = A + BX \rightarrow ②$

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Eq ② is in the form of straight line

$$\left.\begin{aligned} \Sigma y_i &= nA + B\,\Sigma x_i \\ \Sigma x_i y_i &= A\,\Sigma x_i + B\,\Sigma x_i^2 \end{aligned}\right\} ③$$

2.

$X = x$	$Y = \ln(y)$	X^2	XY
2	1.405	4	2.81
4	2.405	16	9.62
6	3.405	36	20.43
8	4.405	64	35.24
10	5.405	100	54.05
$\Sigma x_i = 30$	$\Sigma y_i = 17.02$	$\Sigma x_i^2 = 216$	$\Sigma x_i y_i = 122.15$

Sol

Sub above value in ③

$\Sigma y_i = nA + B\,\Sigma x_i$ $\qquad$ $\Sigma x_i y_i = A\,\Sigma x_i + B\,\Sigma x_i^2$

$17.02 = 5A + 30B$ $\qquad$ $122.15 = 30A + 216B$

$A = 0.405$ $\qquad$ $B = 0.5$

we have

$A = \log a$ $\qquad$ $B = b$

$a = e^A$ $\qquad$ $b = 0.5$

$a = e^{0.405}$

$= 1.499$

$y = ae^{bx}$

$y = (1.499)e^{(0.5)x}$

2. fit an exponential curve for the following data

x	1	5	7	9	12
y	10	15	12	15	21

Sol Let $y = ae^{bx} \rightarrow (1)$

log on b.s

$\log y = \log ae^{bx}$

$\log y = \log a + \log e^{bx}$

$\log y = \log a + bx \log e$

$\log y = \log a + bx$

Let $Y = \log y$ $A = \log a$ $B = b$ $X = x$

$Y = A + BX \rightarrow (2)$

$$\left.\begin{aligned} \Sigma Y_i &= nA + B\Sigma x_i \\ \Sigma x_i Y_i &= A\Sigma x_i + B\Sigma x_i^2 \end{aligned}\right\} \rightarrow (3)$$

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$\Sigma x_i = 34$, $\Sigma y_i = 1.324$ $\Sigma x_i^2 = 300$,

13.24

$\Sigma x_i y_i = 44.14$

sub in (3)

$13.24 = 5A + 34B$

$94.14 = 34A + 300B$

$A = 2.24$ $B = 0.05$

$2.24 = \log a$ $b = 0.05$

$a = e^{2.24} = 9.393$

a, b in (1)

$y = (9.393)e^{(0.05)x}$

3

x	0	1	2	3	4	5	6	7	8
y	20	30	52	77	135	211	326	550	1052

Sol let $y = ae^{bx} \rightarrow (1)$

log on b.s

$\log y = \log ae^{bx}$

$\log y = \log a + \log e^{bx}$

$\log y = \log a + bx \log e$

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$\log y = \log a + bx \log e$

$\log y = \log a + bx$

Let $Y = \log y \quad A = \log a \quad B = b \quad X = x$

$Y = A + BX \rightarrow$ ②

$\Sigma Y_i = nA + B\Sigma X_i$

$\left.\begin{array}{l} \Sigma X_i Y_i = A\Sigma X_i + B\Sigma X_i^2 \end{array}\right\} \rightarrow$ ③

$\Sigma X_i = 36 \quad \Sigma Y_i = 44.00 \quad \Sigma X_i Y_i = 205.27$

$\Sigma X_i^2 = 204$

$44.00 = 9A + 36B$

$A = 2.94 \quad B = 0.48$

$205.2 = 36A + 204B$

we have

$A = \log a \quad B = b$

$2.94 = a$

$a = e^{2.94}$

$a = 18.91 \quad b = 0.48$

a, b in ①

$y = ae^{bx}$

$y = (18.91) e^{(0.48)x}$

4.

x	77	100	185	239	285
y	2.4	3.4	7.0	11.1	19.6

Sol

Let $y = ae^{bx}$ — ①

take log on b.s

$\log y = \log ae^{bx}$

$\log y = \log a + \log e^{bx}$

$\log y = \log a + bx \log e$

$\log y = \log a + bx$

Let $Y = \log y$ $A = \log a$ $B = b$ $X = x$

$Y = A + Bx \rightarrow$ ②

$\Sigma y_i = nA + B \Sigma x_i$

$\left.\begin{array}{l} \\ \Sigma x_i y_i = A \Sigma x_i + B \Sigma x_i^2 \end{array}\right\} \rightarrow$ ③

$\Sigma x_i = 886$ $\Sigma x_i^2 = 188500$ $\Sigma y_i = 9.427$

$\Sigma x_i y_i = 1973.06$

8/1

Sub above Values in ③

$$9.427 = 5A + 886B$$

$$1973.06 = 886A + 188500B$$

$A = 0.183$ $B = 0.009$

$A = \log a$

$0.183 = \log a$

$a = e^{0.183}$

$a = 1.2008$ $b = 0.009$

a, b in ①

$y = ae^{bx}$

$y = (1.2008)e^{(0.009)x}$

8/1/20

fitting of a power curve

let $y = ab^x$ — ①

Taking log on b.s of eqn①

$\log y = \log(ab^x)$

$\log y = \log a + \log b^x$

$\log y = \log a + x \log b$

let $Y = \log y$ $A = \log a$ $B = \log b$

$X = x$

$$Y = A + BX \quad -(2)$$

Eq (2) is in the form of straight line

The normal eqⁿ are

$$\Sigma Y_i = nA + B\Sigma X_i$$

$$\Sigma X_i Y_i = A\Sigma X_i + B\Sigma X_i^2$$

Here we have

$A = \log a$ $\quad$ $B = \log b$

$a = 10^A$ $\quad$ $b = 10^B$

Note:- In this method we use $Y = \log_{10}(y)$

Problems

1) fit a power curve of the form $y = ab^x$ for the following data.

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	127.4

Sol let $y = ab^x \rightarrow (1)$

Take log on b.s of eq (1)

$$\log y = \log a + \log b^x$$

$$\log y = \log a + x \log b$$

let $Y = \log y$ $\quad$ $A = \log a$ $\quad$ $B = \log b$

$$X = x$$

$Y = A + BX$ —(2)

Eqn (2) is in the form of straight line

The normal eqn's are

$$\left.\begin{aligned} \Sigma Y_i &= nA + B\Sigma x_i \\ \Sigma x_i Y_i &= A\Sigma x_i + B\Sigma x_i^2 \end{aligned}\right\} \text{—(3)}$$

$X = x$	$Y = \log_{10}(y)$	x^2	xY
2	0.9190	4	1.838
3	1.1875	9	3.5625
4	1.5198	16	6.0792
5	1.8142	25	9.071
6	2.1051	36	12.6306
$\Sigma x = 20$	$\Sigma Y_i = 7.5456$	$\Sigma x_i^2 = 90$	$\Sigma xY = 33.1813$

(3) $\Rightarrow$ $7.5456 = 5A + 20B$

$33.1813 = 20A + 90B$

$A = 0.31$ $\quad$ $B = 0.3$

$a = 10^A$

$a = 10^{0.31}$

We have

$A = \log a$ $\quad$ $B = \log b$

$a = 10^A$ $\quad$ $b = \cancel{\log}\ 10^B$

$a = 10^{0.31}$ $\quad$ $b = 10^{0.3}$

$a = 2.041$ $\quad$ $b = 1.995$

Substitute a,b values in Eq ①

$y = (2.04)(1.995)^x$

2)

x	0	1	2	3	4	5	6	7
y	10	21	35	59	92	200	400	610

Sol let $y = ab^x \rightarrow ①$

Take log on b.s of eqⁿ ①

$\log y = \log(ab^x)$

$\log y = \log a + \log b^x$

$\log y = \log a + x \log b$

let $Y = \log y$ $A = \log a$; $B = \log b$ $X = x$

$Y = A + BX$ — (2)

Eqⁿ ② is in the form of straight line

The normal Eqn's are

$$\left.\begin{aligned} \Sigma y_i &= nA + B\Sigma x_i \\ \Sigma x_i y_i &= A\Sigma x_i + B\Sigma x_i^2 \end{aligned}\right\} \rightarrow ③$$

$\Sigma x_i = 28$, $\Sigma x_i^2 = 140$, $\Sigma y_i = 15.019$,

$\Sigma x_i y_i = 62.8415$

$15.019 = 8A + 28B$

$62.8415 = 28A + 140B$

$A = 1.021 \qquad B = 0.244$

We have

$A = \log a \qquad B = \log b$

$a = 10^A \qquad b = 10^B$

$a = 10^{1.021} \qquad b = 10^{0.244}$

$a = 10.4 \qquad b = 1.7\cancel{55}$

substitute a, b values in Eqn ①

$y = (10.4)(1.753)^x$

3)

x	1	2	3	4
y	7	11	17	27

sol Let $y = ab^x \rightarrow$ ①

Take log on b.s of Eqn ①

$\log y = \log(ab^x)$

$\log y = \log a + \log b^x$

$\log y = \log a + x \log b$

let $Y = \log y \qquad A = \log a \qquad B = \log b$

$X = x$

$Y = A + BX \rightarrow$ ②

Eqn ② is in the form of straight line.

The normal eqn's are

$$\left.\begin{aligned}\Sigma y_i &= nA + B\Sigma x_i \\ \Sigma x_i y_i &= A\Sigma x_i + B\Sigma x_i^2\end{aligned}\right\} \to ③$$

$\Sigma x_i = 10$ $\Sigma y_i = 4.548$ $\Sigma x_i^2 = 30$

$\Sigma x_i y_i = 12.344$

$4.548 = 4A + 10B$

$12.344 = 10A + 30B$

$\cancel{A} = 0.65$ $\cancel{B} = 0.1948$

$A = 0.65$, $B = 0.1948$

we have

$A = \log a$ $B = \log b$

$a = 10^A$ $b = 10^B$

$a = 10^{0.65}$ $b = 10^{0.1948}$

$a = 4.4668$ $b = 1.5660$

Substituting a, b values in ①

$y = (4.46)(1.566)^x$

4)

x	1	2	3	4
y	6	11	18	27

Sol Let $y = ab^x \rightarrow ①$

Take log on b.s of Eqn ①

$\log y = \log(ab^x)$

$\log y = \log a + \log b^x$

$\log y = \log a + x \log b$

let $Y = \log y$ $A = \log a$ $B = \log b$

$X = x$

$Y = A + Bx \rightarrow ②$

Eqn ② is in the form of straight line

The normal eqn's are

$$\left.\begin{aligned} \Sigma y_i &= nA + B\Sigma x_i \\ \Sigma x_i y_i &= A\Sigma x_i + B\Sigma x_i^2 \end{aligned}\right\} \rightarrow ③$$

$\Sigma x_i = 10$ $\Sigma y_i = 4.505$ $\Sigma x_i^2 = 30$

$\Sigma x_i y_i = 12.3515$

$4.505 = 4A + 10B$

$12.3515 = 10A + 30B$

$A = 0.58175.$ $B = 0.2178$

we have

$A = \log a$ $B = \log b$

$a = 10^A$ $b = 10^B$

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$a = 10^{0.58175}$ $\quad b = 10^{0.2178}$

$a = 3.8172$ $\quad b = 1.6512$

Substituting a, b values in ①

$$y = (3.8172)(1.6512)^x$$

fitting of a lower curve of form

$$y = ax^b$$

$$y = ax^b \rightarrow ①$$

Taking log on b.s

$$\log y = \log(ax^b)$$

$$\log y = \log a + \log x^b$$

$$\log y = \log a + b \log x$$

$\log y = Y$, $\log a = A$, $B = b$, $\log x = X$

$$Y = A + BX \quad — ②$$

this is in the form of S.T

$$\left.\begin{array}{l} \Sigma Y_i = nA + B\Sigma X_i \\ \Sigma X_i Y_i = A\Sigma X_i + B\Sigma X_i^2 \end{array}\right\} \rightarrow ③$$

$X = \log(x)$	$Y = \log(y)$	x^2	xy
$1 = 0$	$6 = 0.7781$	0	0
$2 = 0.3010$	$11 = 1.0413$	0.0906	0.3134
$3 = 0.4771$	$18 = 1.2552$	0.2276	0.5988
$4 = 0.6020$	$27 = 1.4313$	0.3624	0.8616
1.3801	4.5059	0.6806	1.7738

$$4.5059 = 4A + 1.3801B$$

$$1.7738 = 1.3801A + 0.6806B$$

$$A = 0.7566, \quad B = 1.0720$$

$$a = 5.701, \quad b = 11.803$$

$$y = (5.70)\,x^{11.8}$$