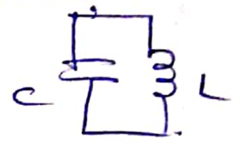
# Tuned Amplifiers

Tuned amplifier amplifies a particular frequency and rejects all the other frequencies. The load to the tuned ~~amplifier~~ is parallel resonant circuit. as shown fig. This is also known as tank circuit, or tuned circuit. The tuned circuit offers high impedance at resonant frequency. At other frequencies it has low impedance.




In Tx Rx to select a particular channel among the all other channels tuned amplifier is used

Tuned amplifiers classified into 3 types.

1. Single tuned amplifier.
2. Double tuned amplifier.
3. Stagger tuned amplifier.

Single tuned amplifier use one parallel resonant circuit as the load impedance in each stage and all the tuned circuits are tuned to the same frequency.

Double tuned amplifiers use two inductively coupled tuned circuits per stage, both the tuned circuits tuned to the same frequency

Stagger-tuned amplifier is cascading of two single tuned amplifiers, the tuned circuits in each stage tuned to slightly different frequencies. $L_1C_1 \neq L_2C_2$

① 

## Q-factor of a capacitor

$V_m \sin\omega t$


$$Q = 2\pi \times \frac{\text{Maximum Energy stored per cycle}}{\text{Energy dissipated per cycle.}}$$

$$E = \frac{1}{2} C V_{max}^2$$

$$R << \frac{1}{\omega c} \qquad V_{max} = I_m \cdot X_c = \frac{I_m}{\omega c}$$

$$E = \frac{1}{2} \cancel{C} \cdot \frac{I_m^2}{\omega^2 C^{\cancel{2}}} = \frac{1}{2} \frac{I_m^2}{\omega^2 c} \quad - ①$$

Energy dissipated per cycle = power × time

$$t = \frac{1}{f} \qquad \text{power} = \left(\frac{I_m}{\sqrt{2}}\right)^2 R$$

$$\text{Energy dissipated per cycle} = \frac{I_m^2 R}{2f} \quad - ②$$

① & ② substitute in Q factor equation

$$Q = 2\pi \left[\frac{\frac{I_m^2}{2\omega^2 C}}{\frac{I_m^2 R}{2f}}\right] = \frac{1}{\omega C R}.$$

$V_m \sin\omega t$


## Q-factor of Inductor

$$E = \frac{1}{2} L I_m^2$$

$$\text{Energy dissipated per cycle} = \frac{I_m^2 R}{2f}$$

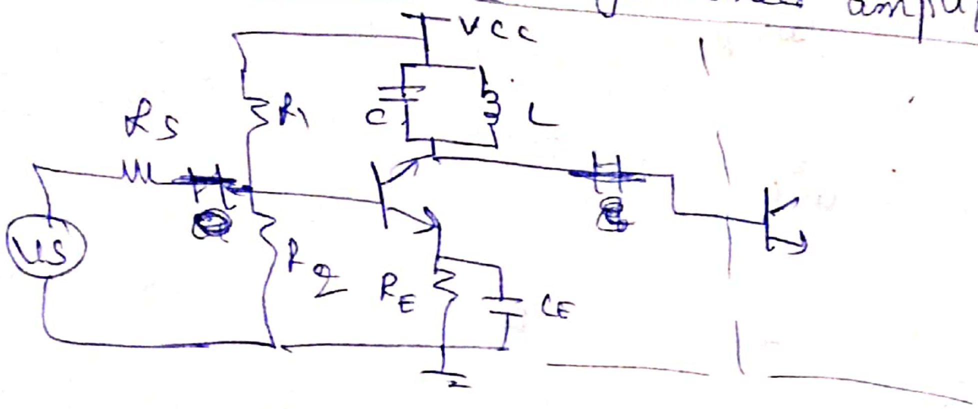
$$Q = 2\pi \frac{\frac{1}{2} L I_m^2}{\frac{I_m^2 R}{2f}} = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

(1) Parallel

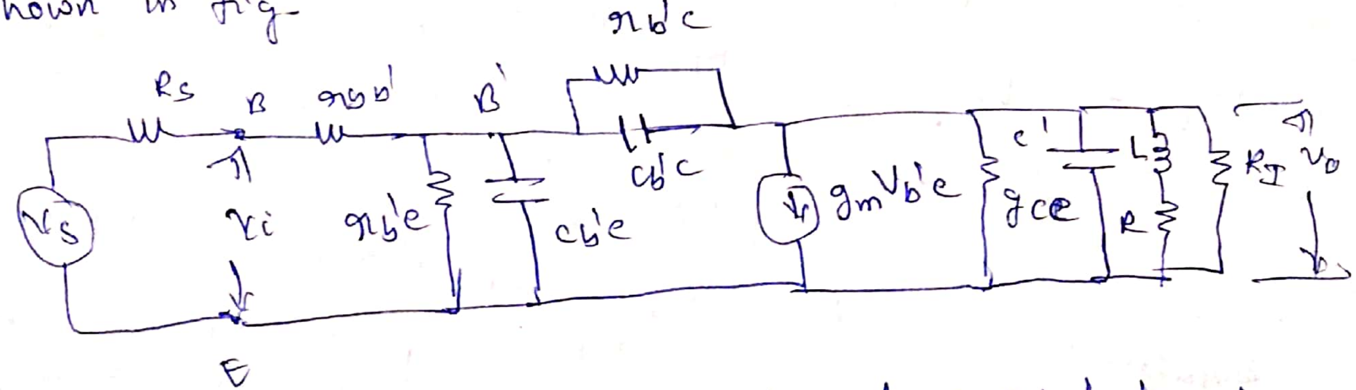
$\boxed{RL}$ $Q = \frac{R}{\omega L}$, $\boxed{RC}$ $Q = \omega C R$

(2) Series RLC $Q = \omega L / R = 1/\omega C R$, $|| ^{le}$ RLC $\frac{R}{\omega L} = \omega C R$

# Capacitance - Coupled Single tuned amplifier

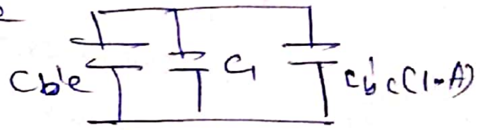



Single tuned amplifier analysed using hybrid π model shown in fig.




To find Voltage gain and Bandwidth of a single tuned amplifier is shown in fig.

Stray Capacitance at the i/p side and o/p side $C_1$ and $C_2$.




The i/p side capacitance

$$C_{in} = C_{b'e} + C_1 + C_{b'c}(1 - A_v)$$

The o/p side capacitance

$$C_{out} = C_2 + C' + C_{b'c}\left(\frac{A_v - 1}{A_v}\right) \quad (1)$$

$$g_{ce} = \frac{1}{r_{ce}} = h_{oe} - g_m h_{re} = h_{oe} = \frac{1}{R_o}$$

where $R_o$ is the o/p resistance of the current generator $g_m V_{b'e}$

Inductor in series with resistor


$$Y_i = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2}$$

$$\frac{1}{R_p} + \frac{1}{j\omega L_p} = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2} \Rightarrow R_p = \frac{R^2 + \omega^2 L^2}{R}$$

$$\omega L_p = \frac{R^2 + \omega^2 L^2}{\omega L} \Rightarrow L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L}$$

②

$$R_p = \frac{R^2+\omega^2L^2}{R} \qquad \omega L_p = \frac{R^2+\omega^2L^2}{\omega L}$$

$$L_p = \frac{R^2+\omega^2L^2}{\omega^2L} \div \omega^2L \text{ N\&D} \qquad \omega L >> R \text{ in } \frac{\omega L}{R}$$

$\omega$

$$L_p = \frac{\frac{R^2}{\omega^2L} + \frac{\omega^2L^2}{\omega^2L}}{1} = \frac{R^2}{\omega^2L} \text{ can be neglected}$$

$$\boxed{L_p = L}$$

$$R_p = \cancel{R}\left(\frac{R+\frac{\omega^2L^2}{R}}{\cancel{R}}\right) \qquad \frac{\omega^2L^2}{R} >> R$$

$R$ neglected

$$\boxed{R_p = \frac{\omega^2L^2}{R}}$$

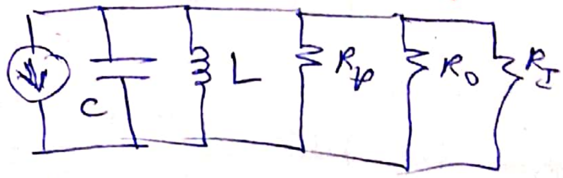
②

Equivalent circuit of the tuned amplifier shown in fig

$R_I$ - i/p impedance of the second stage

$$\frac{1}{R_T} = \frac{1}{R_p} + \frac{1}{R_o} + \frac{1}{R_I}$$

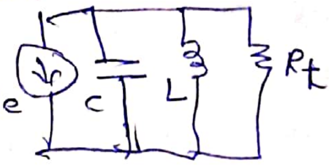
$g_mV_{be}$




$$Q_e = \frac{\text{susceptance of inductance or capacitance C}}{\text{Conductance of shunt resistance } R_t}$$

$$Q_e = \omega_o C R_t = \frac{R_t}{\omega_o L}$$

$g_mV_{be}$




$$V_o = -g_mV_{be}Z$$

$$Y = \frac{1}{Z} = \frac{1}{R_t} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1}{R_t}\left[1 + \frac{R_t}{j\omega L} + j\omega C R_t\right] \quad \text{Multiply N \& D by } \omega_o$$

$$Y = \frac{1}{R_t}\left[1 + \frac{R_t\,\omega_o}{j\omega L\cdot\omega_o} + \frac{j\omega C R_t\cdot\omega_o}{\omega_o}\right]$$

$$Y = \frac{1}{R_t}\left[1 + Q_e\frac{\omega_o}{j\omega} + jQ_e\frac{\omega}{\omega_o}\right] = \cancel{\frac{1}{R_t}\left[1+2jQ_e\right.}$$

$$= \frac{1}{R_t}\left[1 + jQ_e\left[\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right]$$

③

$$Z = \frac{1}{Y} = \frac{R_t}{1 + jQ_e\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]}$$

$$\delta = \frac{\omega}{\omega_0} - 1 \qquad 1+\delta = \frac{\omega}{\omega_0} \qquad \frac{1}{1+\delta} = \frac{\omega_0}{\omega}$$

$$Z = \frac{R_t}{1 + jQ_e\left[1+\delta - \frac{1}{1+\delta}\right]} = \frac{R_t}{1 + jQ_e\left(\frac{1+\delta^2+2\delta-1}{1+\delta}\right)}$$

$$Z = \frac{R_t}{1 + jQ_e\left[\frac{\delta^2+2\delta}{1+\delta}\right]} = \frac{R_t}{1 + j2\delta Q_e\left[\frac{\frac{\delta}{2}+1}{1+\delta}\right]}$$

$$\boxed{Z = \frac{R_t}{1 + j2\delta Q_e}} \qquad Z = R_o \| R_p \| R_I$$

$$V_{b'e} = \frac{V_i\, r_{b'e}}{r_{b'e} + r_{bb'}}$$

$$V_o = g_m V_{b'e} Z = \frac{g_m V_i\, r_{b'e}}{r_{b'e} + r_{bb'}}$$

$$A_v = \frac{V_o}{V_i} = \frac{g_m r_{b'e}}{r_{b'e} + r_{bb'}} \cdot Z = \frac{g_m r_{b'e}}{r_{b'e} + r_{bb'}} \cdot \frac{R_t}{1 + j2\delta Q_e}$$

$A_v$ at resonance $\delta = 0$ $\quad \frac{\omega}{\omega_0} - 1 = 0 \quad \frac{\omega}{\omega_0} = 1 \quad \omega = \omega_0$

$$A_{v\,res} = \frac{g_m r_{b'e}}{r_{b'e} + r_{bb'}} \cdot R_t$$

$$\frac{A_v}{A_{v\,res}} = \frac{1}{1 + j2\delta Q_e} = \frac{1}{\sqrt{1 + (2\delta Q_e)^2}}$$

$$\phi = -\tan^{-1}(2\delta Q_e)$$

At resonance

$$\frac{A_v}{A_{v\,res}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(2\delta Q_e)^2}}$$

$$1+(2\delta Q_e)^2 = 2$$

$$2\delta Q_e = 1$$

$$\delta = \frac{1}{2Q_e}$$

$$Q_e = \frac{R_t}{\omega_0 L} = \omega_0 C R_t$$ in parallel tuned ckt

$$2\delta = \frac{1}{Q_e} \Rightarrow \boxed{\delta = \frac{1}{2Q_e}}$$

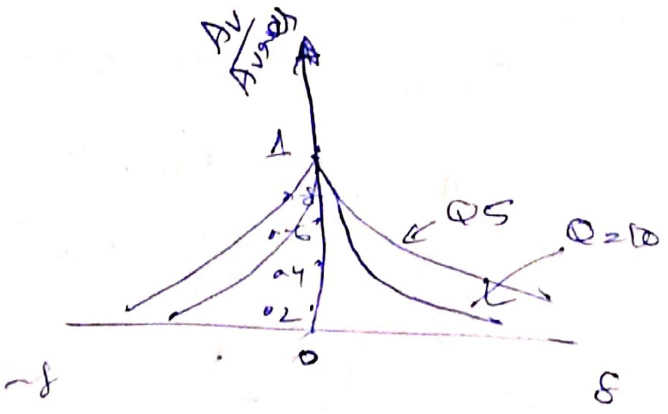
The 3dB Bandwidth is $\Delta\omega = \omega_2 - \omega_1$

$\delta + \delta$ $$\frac{\omega_2 - \omega_0}{\omega_0} + \frac{\omega_0 - \omega_1}{\omega_0} = 2\delta$$

$$\frac{\Delta\omega}{\omega_0} = 2\delta \qquad \Delta\omega = 2\delta\omega_0$$

$$\boxed{\Delta\omega = \frac{\omega_0}{Q_e}}$$

$\omega_0 \rightarrow$ resonant frequency
$Q_e \rightarrow$ Effective Quality factor



## Effect of cascading Single-tuned amplifier on Bandwidth

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$$\frac{A}{A_{res}} = \frac{1}{\sqrt{1+(2\delta Q_e)^2}}$$

$$\left[\frac{A}{A_{res}}\right]^n = \left[\frac{1}{\sqrt{1+(2\delta Q_e)^2}}\right]^n = \frac{1}{\sqrt{2}}$$

$$\left[\sqrt{1+(2\delta Q_e)^2}\right]^n = \sqrt{2}$$

Squaring on both sides.

$$\left[1+(2\delta Q_e)^2\right]^n = 2$$

$$(2\delta Q_e)^2 = 2^{1/n}-1$$

$$2\delta Q_e = \sqrt{2^{1/n}-1}$$

$$2\left[\frac{f-f_0}{f_0}\right] Q_e = \sqrt{2^{1/n}-1}$$

$$f-f_0 = \frac{f_0}{2Q_e}\sqrt{2^{1/n}-1}$$

$$f_2-f_0 = \frac{f_0}{2Q_e}\sqrt{2^{1/n}-1} \quad -(1)$$

$$f_0-f_1 = \frac{f_0}{2Q_e}\sqrt{2^{1/n}-1} \quad (2)$$

(1) + (2)

$f_2 - f_0 + f_0 - f_1$

Bandwidth $f_2 - f_1 = \frac{f_0}{2Q_e}\sqrt{2^{1/n}-1} + \frac{f_0}{2Q_e}\sqrt{2^{1/n}-1}$

$$\boxed{f_2 - f_1 = \frac{f_0}{Q_e}\sqrt{2^{1/n}-1}}$$

$\frac{f_0}{Q_e} = B_1$ Bandwidth of the single stage tuned amplifier

$n=2$

$$= \sqrt{2^{1/2}-1} = 0.643$$

$n=3$

$$= \sqrt{2^{1/3}-1} = 0.51$$

Bandwidth of n stages

$$B_{1n} = B_1\sqrt{2^{1/n}-1}$$

as n increases Bandwidth reduces

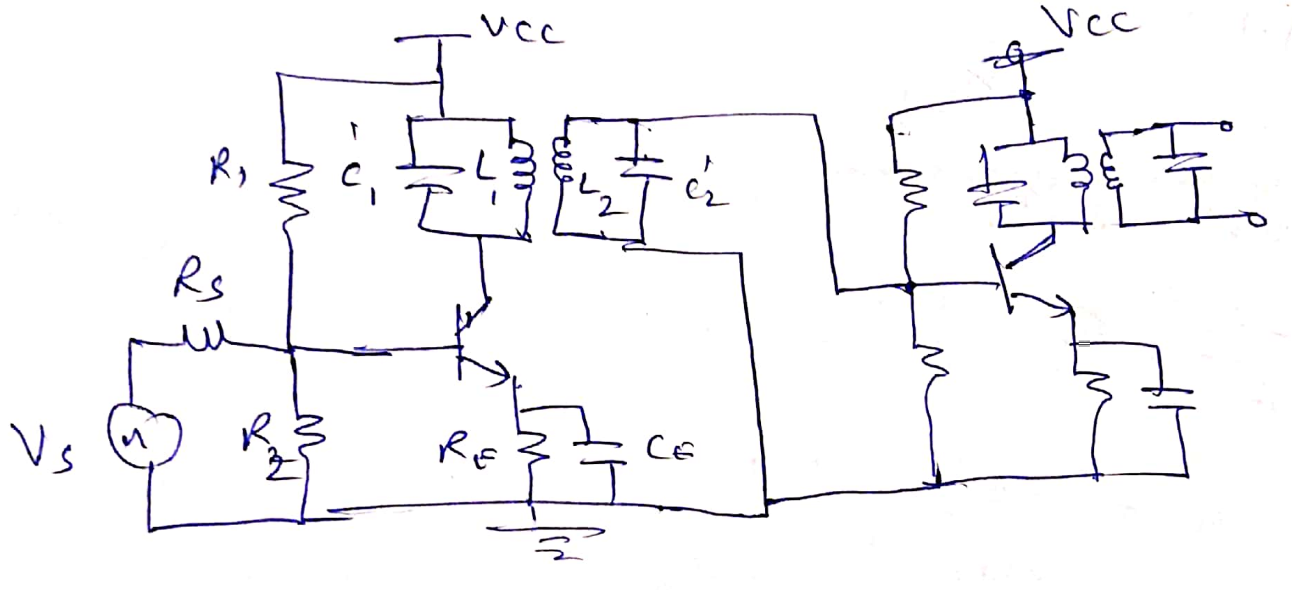
## Effect of cascading double tuned amplifiers

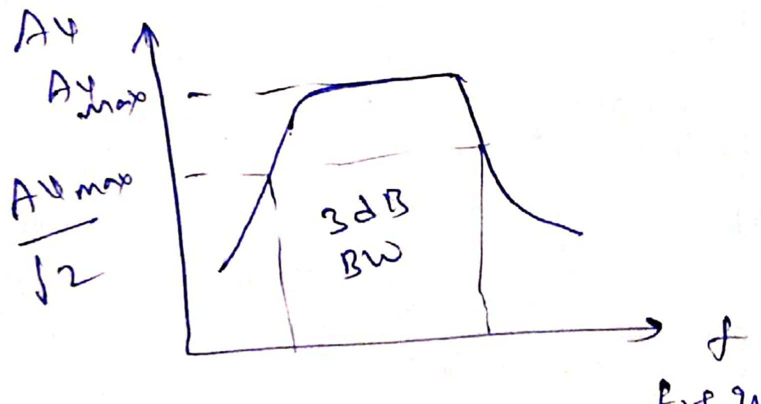
The 3db bandwidth of the cascaded double-tuned amplifier is $B_{2n} = B_2 \left[ 2^{1/n} - 1 \right]^{1/4}$.

where $B_2$ is the 3dB bandwidth of the single stage - double - tuned amplifier and $n$ is the number of identical stages connected in cascade.

### Double tuned amplifier

It uses two inductively coupled tuned circuits per stage, both the tuned circuits being tuned to the same frequency




frequency response

Frequency response is flat and has steeper ends ~~ends~~ and Gain Bandwidth product is large.

## stagger tuned amplifiers



Double tuned amplifier used to increase the Bandwidth. The alignment of double tuned amplifier is difficult. So stagger tuned amplifier is used.

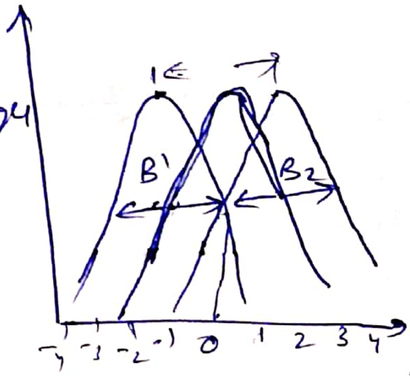
In stagger tuned amplifier has two single tuned cascade amplifiers. Each stage has certain Bandwidth. The resonant frequency of the two tuned circuits are so adjusted that they are separated by an amount equal to the bandwidth of each stage.

Since the resonant frequencies are displaced or staggered, these are known as stagger-tuned circuits.

$\Delta_1$ = Bandwidth of I stage

$\Delta_2$ = Bandwidth of II stage

$\Delta_1 = \Delta_2 = \Delta$ Bandwidth of stagger tuned amplifier.




$x = 2\delta Q_e$

the stagger tuned amplifier bandwidth $\sqrt{2}\Delta$

$$\frac{A_v}{A_{res}} = \frac{1}{1+j2\delta Q_e} = \frac{1}{1+jx}$$ where $x = 2\delta Q_e$ for single tuned amplifiers

Bandwidth $\Delta = \frac{f_0}{Q_e}$ $\quad \delta = \frac{1}{2Q_e}$

$BW = 2\delta f_0$

$\delta$' fractional deviation from resonant frequency

In stagger tuned amplifier one stage tuned to $\delta_0 f_0$ below $f_0$ and second stage tuned to $\delta_0 f_0$ above $f_0$.

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So the corresponding selectivity function of circuit is

$$\left[\frac{A}{A_{res}}\right]_1 = \frac{1}{1+j(x-1)} \quad \left[\frac{A}{A_{res}}\right]_2 = \frac{1}{1+j(x+1)}$$

$x = 2\delta Q_e$.

$$\left[\frac{A}{A_{res}}\right]_{pair} = \left(\frac{A}{A_{res}}\right)_1 \cdot \left(\frac{A}{A_{res}}\right)_2$$

$$= \frac{1}{1+j(x-1)} \cdot \frac{1}{1+j(x+1)} = \frac{1}{2-x^2+2jx}$$

$$\left|\frac{A}{A_{res}}\right|_{pair} = \frac{1}{\sqrt{(2-x^2)^2+(2x)^2}} = \frac{1}{\sqrt{4+\cancel{4x^2}-\cancel{4x^2}+x^4}}$$

$$= \frac{1}{\sqrt{4+x^4}} = \frac{1}{\sqrt{4+(2\delta_0 Q_0)^4}} = \frac{1}{2\sqrt{1+4\delta_0^4 Q_0^4}}$$

for stagger tuned

for single stage tuned amplifier

$$\frac{A_v}{A_{res}} = \frac{1}{\sqrt{1+(2\delta Q_0)^2}}$$

In stagger tuned amplifier the overall voltage gain where $\delta_0$ is the value of $\delta$ referred to new frequency and $Q_0$ is the value of $Q_e$ for each circuit referred to $\omega_0$

In stagger tuned amplifier gain decreases.