UNIT-I

Introduction to Digital Signal processing

①

Introduction:

Anything that carries some Information can be called a signal.

A signal is also defined as a physical quantity that varies with time, space or any other independent variable.

Ex: Speech signal, video signal, seismic signal, AC power supply signal, ECG (electrocardiogram) signal, EEG (electroencephalogram) signal.

ECG provides information about the health of a person's heart.

ECG provides information about brain activity of a person.

→ A signal can be a function of one or more independent variables.

One Dimensional signal:-

If a signal depends on only one independent variable, then it is known as one dimensional signal.

Ex: speech signal, Ac power supply signal.

Two Dimensional signal:-

If a signal depends on two independent variable, then the signal is known as Two dimensional signal.

Ex: picture, x-ray images, sonograms.

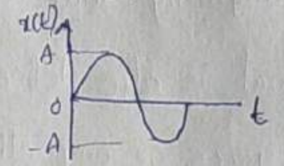
multi Dimensional signal:

If a signal depends on more than two independent variables then the signal is known as multidimensional signal.

Ex: speed of wind & air pressure are a function of latitude, longtitude, elevation and time.

Continuous time signals:-

The signals that are defined for every instant of time are known as continuous time signals.

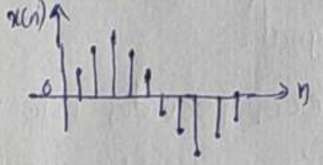


→ they are denoted by $x(t)$

→ these signals are continuous in amplitude & time.

Discrete time signals:

the signals that are defined at discrete instants of time are known as discrete time signals.

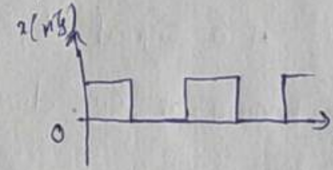


→ They are denoted by $x(n)$

→ these signals are continuous in amplitude and discrete in time.

Digital signals:-

the signals that are discrete in time and quantized in amplitude are known as Digital signals.



System:

A system is defined as a physical device that generates a response or an output signal, for a given input signal.

→ A system is an interconnection of components, performs an operation on an input signal and produces another signal as output.

→ It is a cause and effect relation between two or more signals.

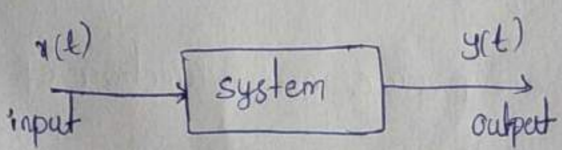


fig: A system

Ex: Amplifiers, filters (reduces noise).

The relationship between the input $x(t)$ and corresponding output $y(t)$ of a system has the form

$$y(t) = \text{operation on } x(t)$$

$$\boxed{y(t) = T[x(t)]}$$

The systems can be classified as continuous time systems and discrete time systems.

Continuous time system :-

A continuous time system is one which operates on a continuous time signal and produces a continuous time output signal.

→ If the input and output of continuous time systems are $x(t)$ & $y(t)$, then we can write

$$\boxed{y(t) = T[x(t)]}$$

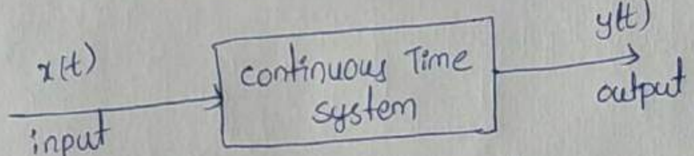


fig: Continuous Time system.

Ex: Amplifiers, filters, motor

Discrete Time system :-

A discrete time system is one which operates on a discrete time signal and produces a discrete time output signal.

→ If the input and output of discrete time systems are $x(n)$ and $y(n)$, then we can write

$$\boxed{y(n) = T[x(n)]}$$

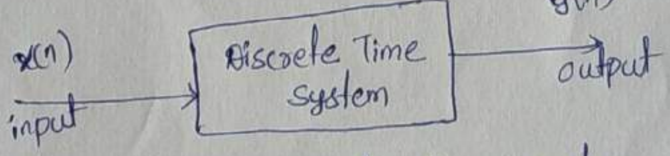


fig: A Discrete time system.

Signal processing :-

Signal processing is any operation that changes the characteristics of a signal. These characteristics include the amplitude, shape, phase and frequency content of the signal.

→ Signal processing changes the characteristics of the signal or Extract some desired information from the signal.

→ The system that processes the analog signal is known as Analog signal processing system.

fig: Analog signal processing system.

Block Diagram of Digital Signal processing System :-

Digital signal processing (DSP) refers to processing of signals by digital systems designed by using digital Integrated circuits (ICs), microprocessors and microcontrollers. the basic components of a DSP system are shown in below fig.

The DSP system involves conversion of analog signal to digital signal, then processing of the digital signal by a digital system and then conversion of the processed digital signal back to analog signal.

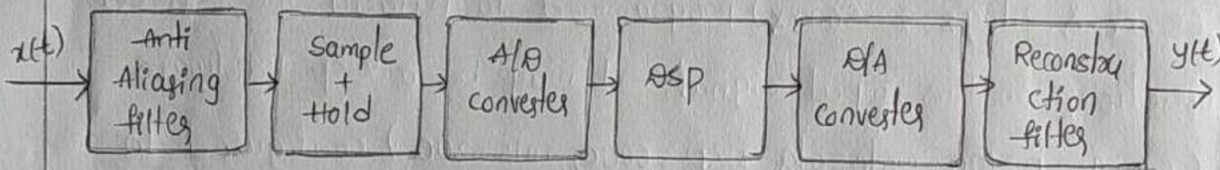


fig: Digital signal processing system

→ The source of the input signal is from a transducer or a communication signal. the signal may be an EEG or an ECG. The input signal is applied to an antialiasing filter. This is a low pass filter used to remove the high frequency noise and to bandlimit the signal. In addition to the LPF, may also be include a 50Hz notch filter, that can remove the power frequency component, which is a large part of external noise. The amplifier may be used to bring the signal upto the voltage range that is required by the input of the analog to digital conversion unit.

→ The sample and hold device provides the input to the ADC and will be required if the input signal must remain relatively constant during the conversion of the analog signal to digital format.

→ The output of the sample and hold circuit serves as the input to the ADC. The output of the ADC is an N-bit binary number depending on the value of the analog signal at its input. The ADC input signal is limited to a range of either 0 to +10V if unipolar, and -5V to +5V if bipolar (e.g AD571). The preceding Amplifier provides a signal in this range. once converted to digital form, the signal processed using digital techniques.

→ the digital signal processor may be a large programmable digital computer or a microprocessor (e.g Intel's 80xx, motorola's 68xxx & zilog's z-80xxx) programmed to perform the desired operations on the input signal. It may be a DSP Hardware (e.g ADSP2100, motorola DSP 56000, TMS 3200) configured to perform a sepecified set of operations on the input signal.

→ the digital signal from the processor is applied to the input of a DAC. The output of DAC is continuous but not smooth, the signal contains unwanted high frequency components.

→ To eliminate high frequency components, the output of DAC is applied to a reconstruction filter. the output of reconstruction filter is a smooth continuous signal, as shown in below fig.

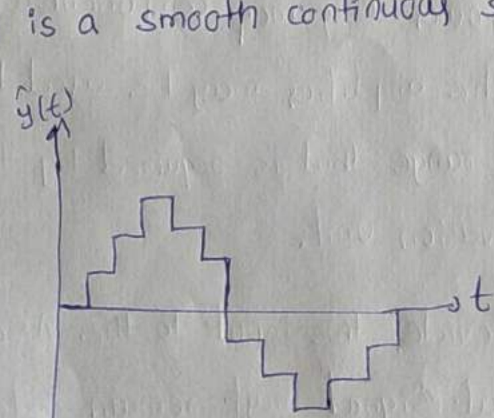


fig: DAC output

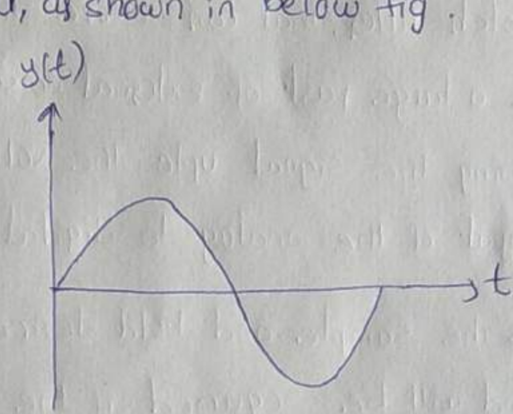


fig: Reconconstruction filter output.

Advantages of DSP:-

Digital signal processing posses several advantages over analog signal processing. They are.

1. Greater Accuracy:

the tolerances of the circuit components used to design the analog filters affects the accuracy, whereas the DSP provides superior control of accuracy.

2. cheaper:

In many applications the digital realization is comparatively cheaper than its analog counterpart.

3. Ease of Data storage:

Digital signals can be easily stored on magnetic media without loss of fidelity and can be processed off line in a remote laboratory.

4. Implementation of sophisticated algorithms:

The DSP allows us to implement sophisticated algorithms when compared to its analog counterpart.

5. Flexibility in configuration:

A DSP system can be easily reconfigured by changing the program. Reconfiguration of an analog system involves the redesign of system of system hardware.

6. Applicability to VLF signals:

The very low frequency signals such as those occuring in seismic application can be easily processed using a digital signal processor when compared to an ASP (Analog signal processing) system, where inductors and capacitors needed would be physically very large in size.

7. Time sharing :

Dsp allows the sharing of a given processor among a number of signals by time sharing thus reducing the cost of processing a signal.

Limitations of Dsp :-

1. System Complexity :

System complexity increased in the digital processing of an analog signal because of the devices such as A/D and D/A converters and their associated filters.

2. Bandwidth limited by Sampling rate :

Band limited signals can be sampled without information loss if the sampling rate is more than twice the bandwidth. Therefore the signals having Extreamly wide bandwidths requre fast sampling rate A/D converters and fast digital signal processors. But there is practical limitation in the speed of operation of A/D converters and digital signal processors.

3. Power consumption :

A variety of analog processing algorithms can be implemented using passive circuit employing inductors, capacitors and resistors that do not need any power, whereas a Dsp chip containing over 4 lakh transistors dissipates more power (1 watt).

Applications of DSP :-

1. Telecommunications :

Echo cancellation in telephone networks, Telephone dialling application, modems, line repeaters, channel multiplexing, Data Encryption video conferencing, cellular phone, FAX.

2. consumer electronics :

Digital Audio/TV, electronic music synthesizer, educational toys, FM stereo applications, sound recording applications.

3. Instrumentation and control :

Spectrum analysis, Digital filter, PLL function generator, servo control, Robot control, process control.

4. Image processing :

Image compression, Image Enhancement, Image analysis and recognition.

5. medicine :

medicine diagnostic instrumentation such as computerized Tomography (CT), X-Ray scanning, magnetic resonance imaging spectrum analysis of ECG and EEG signals to detect the various disorders in heart and brain, patient monitoring.

6. speech processing :

speech analysis methods are used in automatic speech recognition, speaker verification and speaker identification. speech synthesis techniques includes conversion of written text into speech.

7. seismology :-

DSP techniques are employed in the geophysical Exploration for oil and gas, detection of underground nuclear explosion and earthquake monitoring.

8. military :

Radar signal processing, sonar signal processing, Navigation secure communications.

elementary signals:

There are several elementary signals which play vital role in the study of signals and systems. elementary signals are used to model a large number of physical signals, which occur in nature. they are also called as standard signals.

1. Unit step signal:-

The step function is an important signal used for analysis of many systems. The step function, which exists only for positive time and is zero for negative time. If a step function has unity magnitude then it is called unit step function.

the continuous time unit step function $u(t)$ is defined as

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t<0 \end{cases}$$

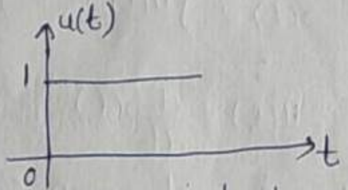


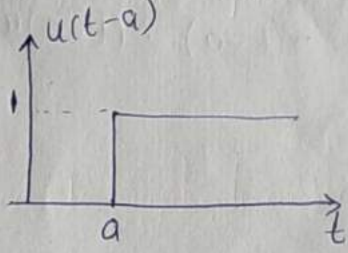
fig: unit step signal.

from the above equation for $u(t)$,

is zero if the argument $(t<0)$ and equal to 1 if the argument $t\geq 0$.

The shifted unit step signal is defined as

$$u(t-a) = \begin{cases} 1 & \text{for } t \geq a \\ 0 & \text{for } t<0 \end{cases}$$



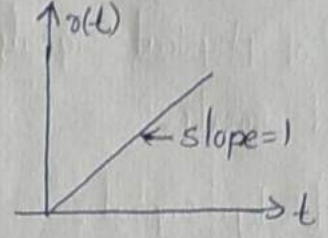
It is zero if the argument $(t-a)<0$ and equal to 1 if the argument $(t-a)\geq 0$.

fig: Delayed unit function.

Unit Ramp signal:

the continuous time unit ramp function $r(t)$ is that signal (function), which starts at $t=0$ and increases linearly with time and is defined as

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t<0 \end{cases}$$



The unit ramp signal has unit slope.

the ramp signal can be obtained by integrating the step signal. that means, a unit step signal can be obtained by differentiating the unit ramp signal.

i.e $\quad r(t) = \int u(t)dt = \int dt = t \quad$ for $t \geq 0$

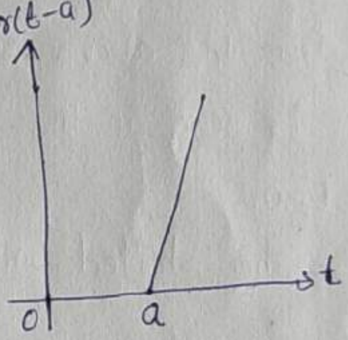
$$u(t) = \frac{d}{dt} r(t)$$

The delayed unit ramp signal $r(t-a)$ is given by

$$r(t-a) = \begin{cases} (t-a) & \text{for } t \geq a \\ 0 & \text{for } t<a \end{cases}$$

(or)

$$r(t-a) = (t-a)u(t-a)$$

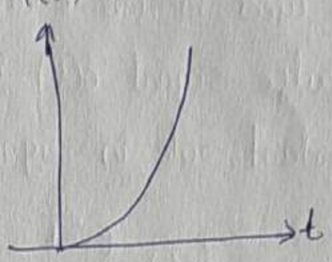


Delayed ramp s/g

Unit parabolic signal :-

The unit parabolic signal p(t), also called as unit acceleration signal starts at $t=0$, and is defined as

$$p(t) = \begin{cases} t^2/2 & \text{for } t \geq 0 \\ 0 & \text{for } t<0 \end{cases}$$

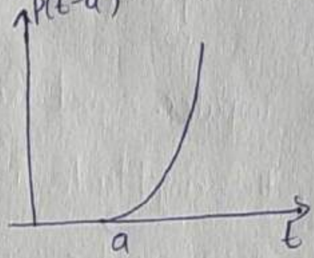


or

$$p(t) = \frac{t^2}{2} u(t).$$

The shifted version of the unit parabolic signal p(t-a) is given by

$$p(t-a) = \begin{cases} \frac{(t-a)^2}{2} & \text{for } t \geq a \\ 0 & \text{for } t<a \end{cases}$$



or

$$p(t-a) = \frac{(t-a)^2}{2} u(t-a)$$

The unit parabolic function can be obtained by integrating the unit ramp function. or double integrating the unit step function.

$$p(t) = \int\int u(t)\,dt = \int r(t)\,dt = \int t\,dt = \frac{t^2}{2} \quad \text{for } t \geq 0.$$

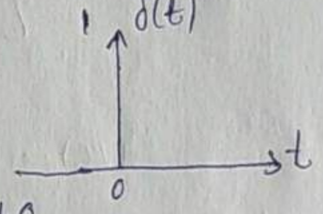
i.e
The ramp signal is derivative of parabolic signal and step signal is double derivative of parabolic function.

$$r(t) = \frac{d}{dt} r(t) \qquad u(t) = \frac{d^2}{dt^2} p(t)$$

unit impulse signal:-

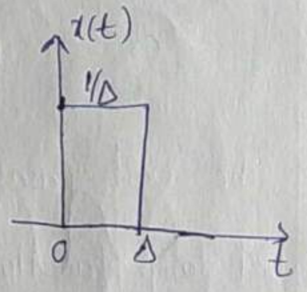
The unit impulse function is the most widely used elementary function used in the analysis of signals and systems. The continuous unit impulse signal $\delta(t)$ is also called as Dirac delta signal, plays an important role in signal analysis. It is defined as

$$\delta(t) = \begin{cases} 1 & \text{for } t=0 \\ 0 & \text{for } t \neq 0 \end{cases}$$



$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad \text{and} \quad \delta(t) = 0 \text{ for } t \neq 0.$$

the impulse signal has zero amplitude everywhere except at $t=0$. At $t=0$, the amplitude is infinity so that the area under the curve is unity. $\delta(t)$ can be represented as a limiting case of a rectugular pulse function.

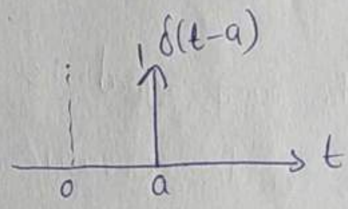


$$x(t) = \frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right]$$

$$\delta(t) = \lim_{\Delta t \to 0} x(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right]$$

A delayed unit impulse signal $\delta(t-a)$ is defined as

$$\delta(t-a) = \begin{cases} 1 & \text{for } t=a \\ 0 & \text{for } t \neq a \end{cases}$$



Note:

1. the width of the pulse is zero. this means the pulse exists only at $t=0$.
2. the height of the pulse goes to infinity.
3. the area under the pulse curve is always unity.
4. the height of arrow indicates the total area under the ~~curve~~ impulse.

→ the integral of unit impulse function is a unit step function & the derivative of unit step function is a unit impulse function.

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\, dt \qquad \delta(t) = \frac{d}{dt} u(t).$$

properties of unit impulse signal:

1. It is an even function of time 't' i.e $\delta(t) = \delta(-t)$.

2. $\int_{-\infty}^{\infty} x(t)\delta(t)dt = x(0)$; $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$.

3. $\delta(at) = \frac{1}{|a|}\delta(t)$.

4. $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0) = x(t_0)$, $x(t)\delta(t) = x(0)\delta(t) = x(0)$.

5. $x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$.

Sinusoidal signal :-

A continuous time sinusoidal signal in its most general form is given by.

$$x(t) = A\sin(\omega t + \phi)$$

$A \rightarrow$ Amplitude

$\omega \rightarrow$ Angular frequency in radians

$\phi \rightarrow$ phase angle in radians.

The time period of sinusoidal signal is given by.

$$T = \frac{2\pi}{\omega}$$

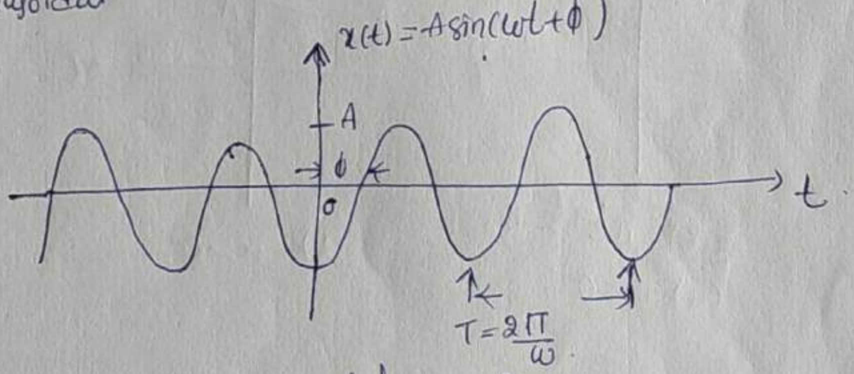


fig: Sinusoidal waveform.

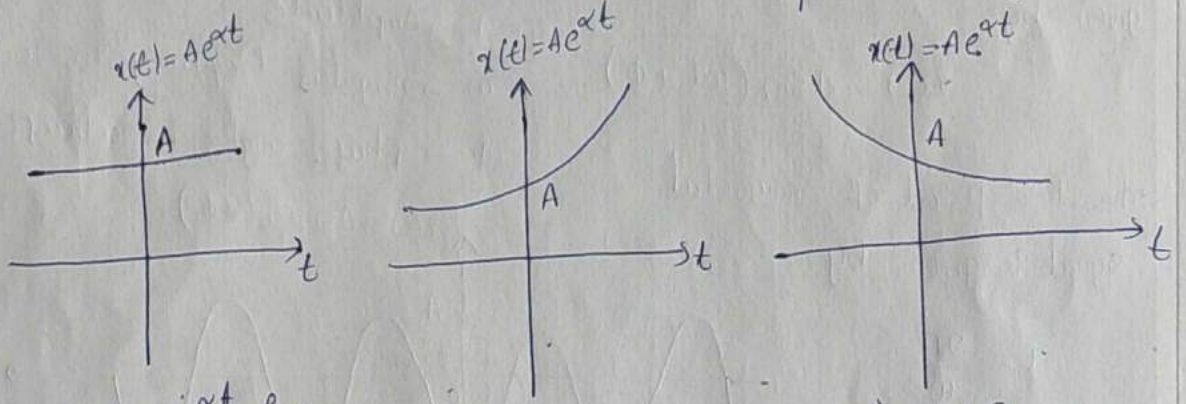
Real exponential signal:

A continuous time real exponential signal has the general form as

$x(t) = A \cdot e^{\alpha t}$ where both A & α are real.

The parameter A is the amplitude of the exponential measured at $t=0$. The parameter α can be either positive or negative. Depending on the value of α; we get different exponentials.

1. If $\alpha=0$, the signal $x(t)$ is of constant amplitude for all times.
2. If α is positive i.e $\alpha>0$, the signal $x(t)$ is a growing exponential signal.
3. If α is negative i.e $\alpha<0$, the signal $x(t)$ is a decaying exponential signal.



$x(t)=Ae^{\alpha t}$ for (a) $\alpha=0$ (b) $\alpha>0$ (c) $\alpha<0$

Complex Exponential signal:

The complex exponential signal has a general form as

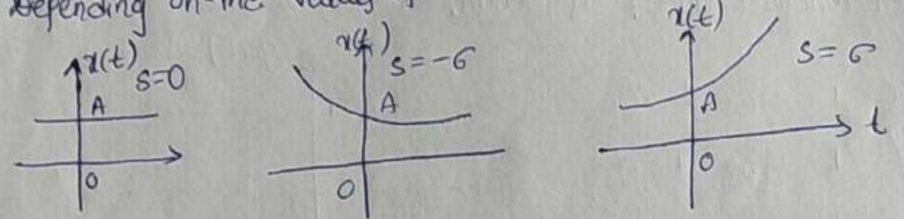
$$x(t) = Ae^{st} \qquad \text{where } A \rightarrow \text{Amplitude}$$

s is complex variable defined as $s = \sigma + j\omega$

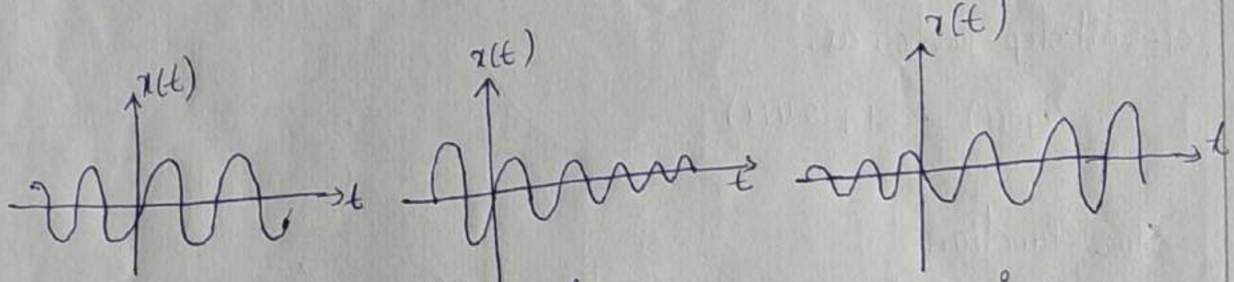
$$x(t) = Ae^{(\sigma+j\omega)t} = Ae^{\sigma t}e^{j\omega t}$$

$$x(t) = Ae^{\sigma t}[\cos\omega t + j\sin\omega t]$$

Depending on the values of σ & ω, we get different waveforms.



(a) (b)



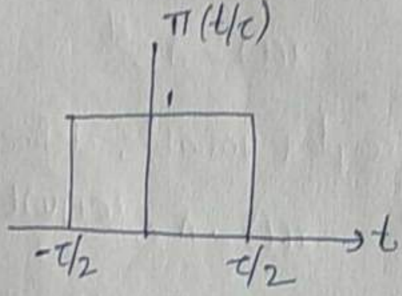
$s = j\omega$ $\qquad$ $s = \sigma \pm j\omega$, $\sigma < 0$ $\qquad$ $s = \sigma \pm j\omega$, $\sigma > 0$.

Rectangular pulse function:

The unit rectangular pulse function $\pi(t/\tau)$ is defined as,

$$\pi(t/\tau) = \begin{cases} 1 & \text{for } |t| \le \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

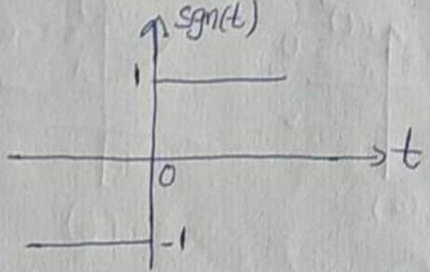
It is an even function of 't'.



Signum function:

The unit signum function sgn(t) as shown in fig and is defined as

$$sgn(t) = \begin{cases} 1 & \text{for } t>0 \\ -1 & \text{for } t<0 \end{cases}$$



The signum can be expressed interms of unitstep function as,

$$sgn(t) = -1 + 2u(t)$$

Sinc function:

The sinc function sinc(t) is defined as,

$$Sinc(t) = \frac{Sin\,t}{t} \quad \text{for } -\infty < t < \infty$$

The sinc function oscillates with time period 2π and decays with increasing 't'. Its value is zero at $n\pi$, $n = \pm1, \pm2 \ldots$ It is an even function of 't'

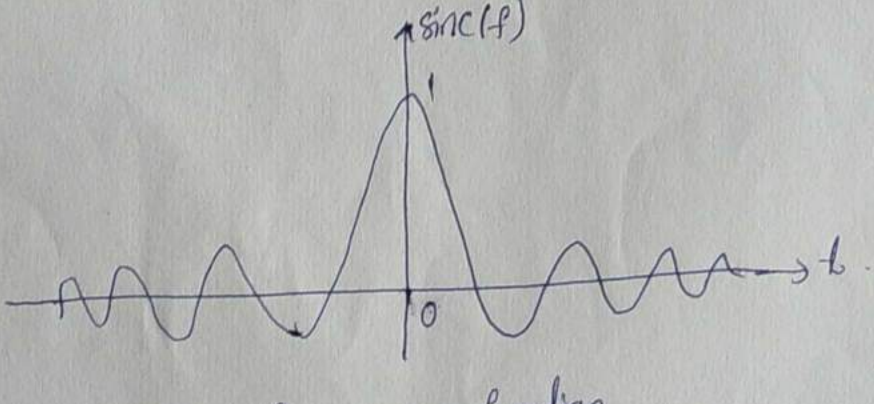


fig: sinc function.

Elementary Discrete time signals :-

There are several elementary signals which plays vital role in the study of signals and systems. These elementary signals serve as basic building blocks for the construction of more complex signals.

The standard discrete time signals are as follows

1. Unit step sequence
2. Unit ramp sequence
3. Unit parabolic sequence.
4. Unit impulse sequence.
5. Sinusoidal sequence.
6. Real Exponential sequence.
7. Complex exponential sequence.

Unit step sequence :-

The unit step sequence u(n) is defined as,

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

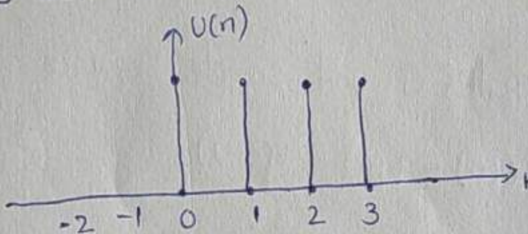


fig: unit step sequence.

It exists only for positive time and zero for negative time.

2. unit ramp sequence :-

The discrete time unit ramp sequence r(n) is that sequence which starts at n=0 and increases linearly with time and is defined as,

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

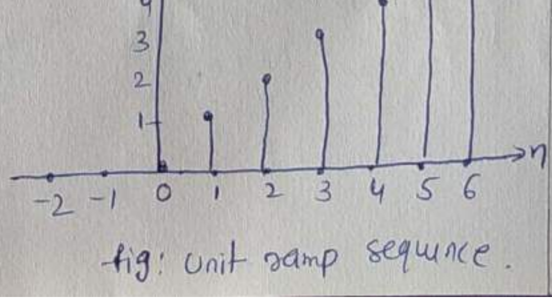


fig: unit ramp sequence.

Unit parabolic sequence :-

The discrete time unit parabolic sequence p(n) is defined as

$$p(n) = \begin{cases} \frac{n^2}{2} & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

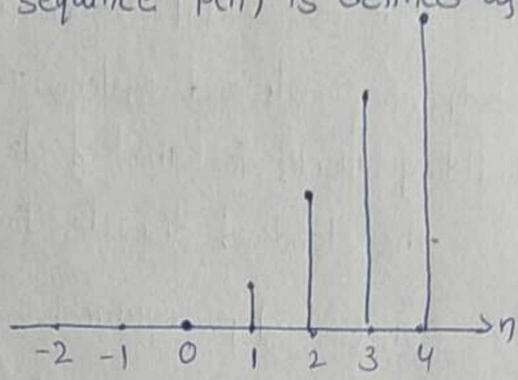


fig: unit ramp sequence.

unit Impulse sequence :-

The discrete time unit impulse sequence δ(n) is also called unit sample sequence, is defined as

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

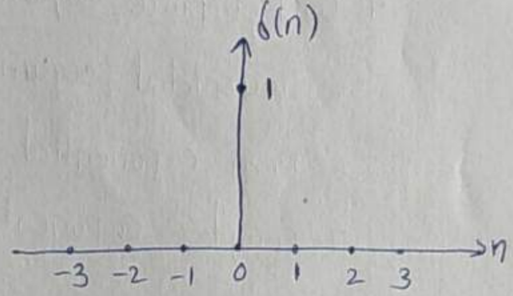


fig: unit impulse sequence.

properties of unit sample sequence.

1. $\delta(n) = u(n) - u(n-1)$.

2. $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

3. $\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k. \end{cases}$

4. $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$.

Sinusoidal sequence :-

A discrete time sinusoidal sequence is given by

$$\boxed{x(n) = A \sin(\omega n + \phi)}$$

A → Amplitude

ω → Angular frequency

ϕ → phase angle in radians.

period of the discrete time Signal is

$$N = \frac{2\pi}{\omega} m$$

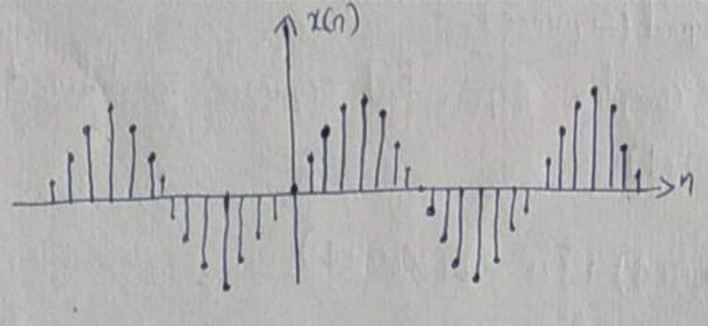


fig: sinusoidal sequence.

Real Exponential sequence :-

the discrete time real Exponential sequence is defined as

$$x(n) = a^n \text{ for all } n$$

when $a > 1$, the sequence grows Exponentially

when $0 < a < 1$, the sequence decays exponentially.

when $a < 0$, the sequence takes alternating signs.

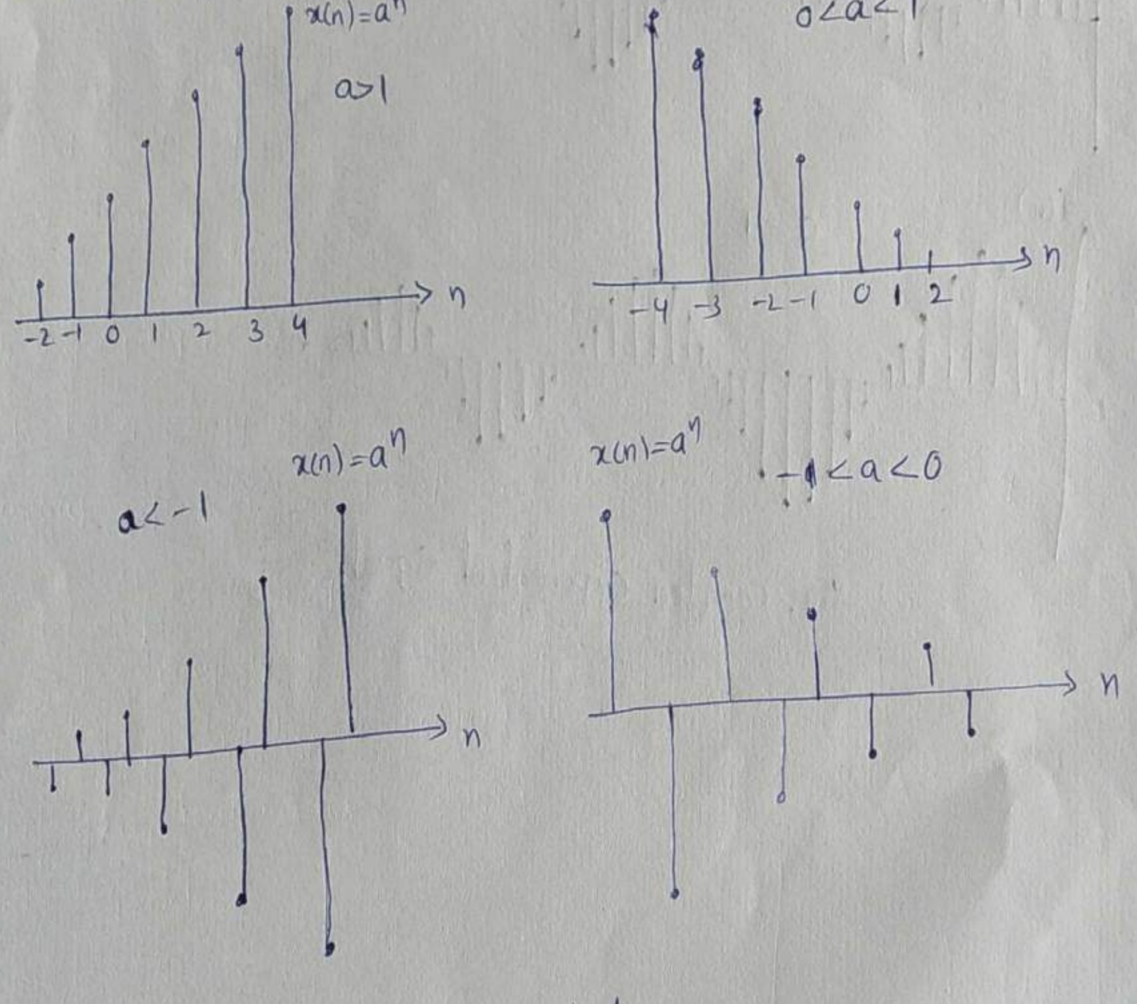


fig: Real exponential sequence.

Complex Exponential signal (sequence):-

The discrete time complex exponential sequence is defined as

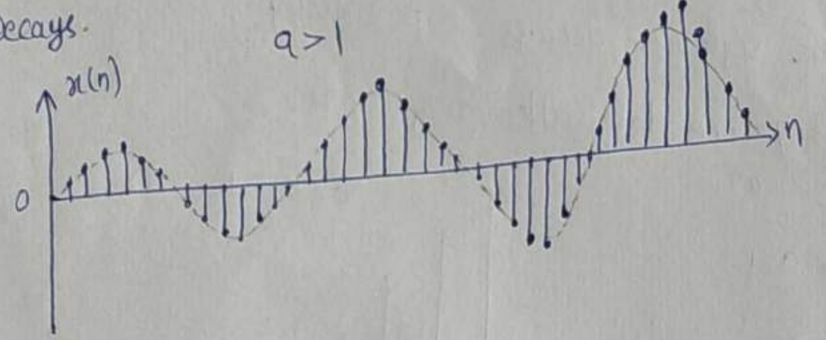
$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

$$x(n) = a^n \cos(\omega_0 n + \phi) + j a^n \sin(\omega_0 n + \phi)$$

for $|a| = 1$, the real & imaginary parts of complex exponential sequence are sinusoidal.

for $|a| > 1$, the amplitude of the sinusoidal sequence exponentially grows.

for $|a| < 1$, the amplitude of the sinusoidal sequence exponentially decays.



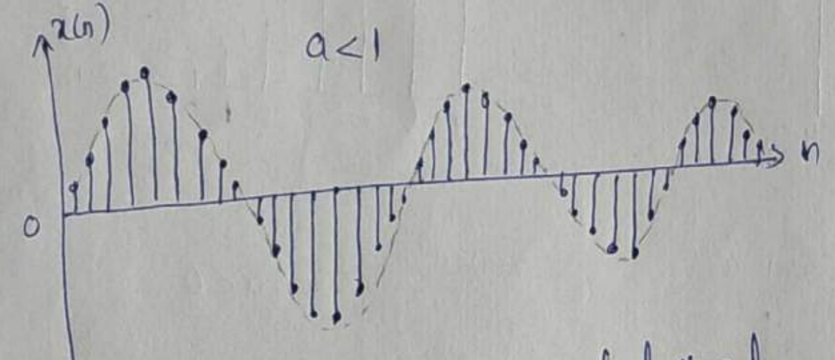


fig: Complex Exponential signal.

continuous time signals:

The signals that are defined for every instant of time are known as continuous time signals.

→ They are continuous in amplitude, as well as in time.

→ For continuous time signals, the independent variable is time.

→ They are denoted by $x(t)$

→ most of the available signals are continuous time signals.

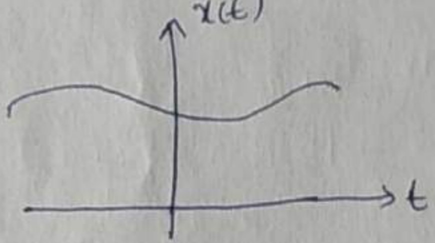


fig: continuous time signal

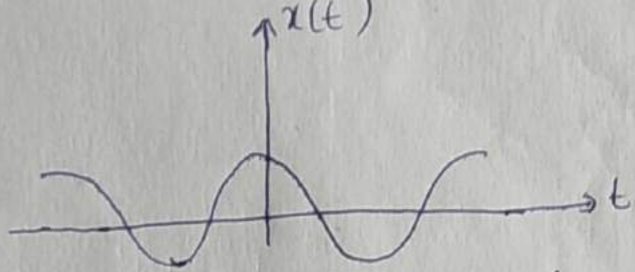


fig: continuous time signal.

Discrete time signals:

The signals that are defined only at discrete instants of time are known as discrete time signals.

→ The discrete time signals are continuous in amplitude but discrete in time.

→ For discrete time signals, the amplitude between two time instants is just not defined.

→ For discrete time signals, the independent variable is time 'n'. Since they are defined only at discrete instants of time,

→ They are denoted by a sequence $x(nT)$ or simply by $x(n)$.

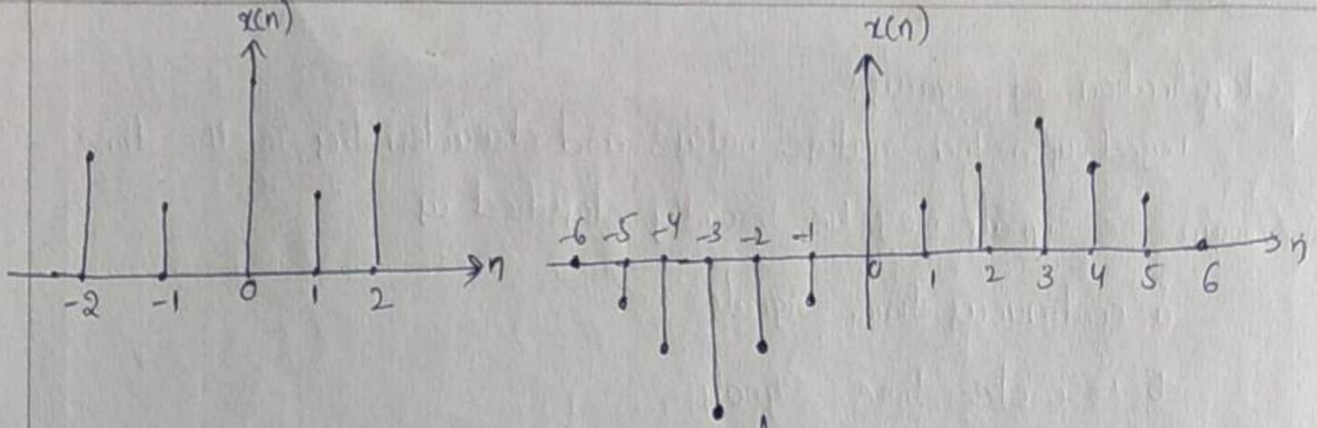


fig: Discrete time signals.

Operations on signals :-

Signal processing is a group of basic operations applied to an input signal resulting in another signal as the output. The mathematical transformation from one signal to another is represented as

$$\boxed{y(n) = T[x(n)]}$$

the basic operations are

1. shifting.
2. Time reversal
3. Time scaling
4. Scalar multiplication
5. Signal multiplier
6. Signal Addition.

shifting :-

The shift operation takes the input sequence and shifts the values by an integer increment of the independent variable. The shifting may delay or advance the sequences in time.

mathematically this can be represented as

$$y(n) = x(n-k)$$

where $x(n) \rightarrow$ input
$y(n) \rightarrow$ output

If k is positive, the shifting delays the sequence

If k is negative, the shifting advances the sequences.

Ex:

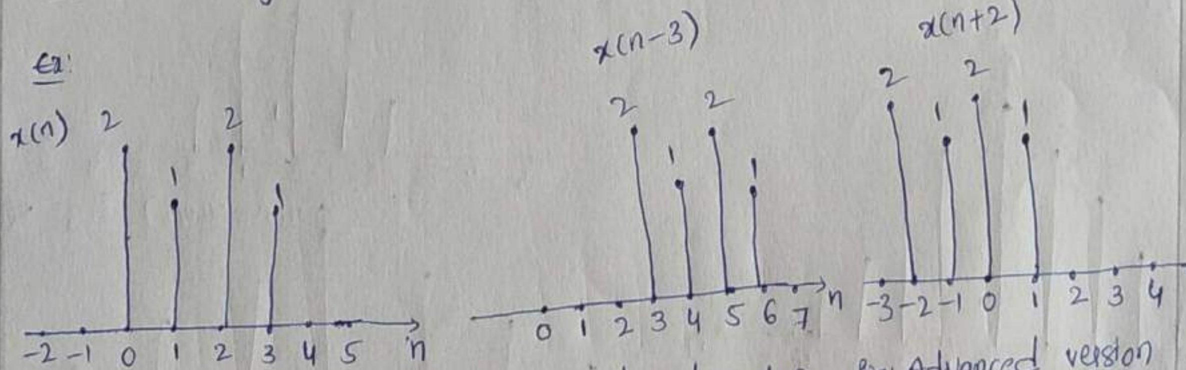


fig: Discrete time signal

fig: Delayed version signal

fig: Advanced version signal

Time Reversal :-

The time reversal of sequence $x(n)$ can be obtained by folding the sequence about $n=0$. It is denoted as $x(-n)$.

Ex:

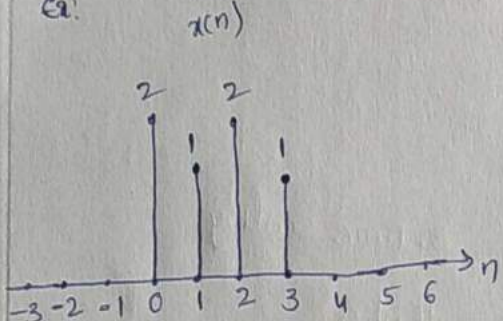


fig: Discrete time signal

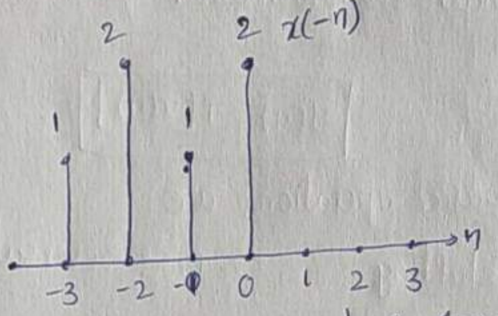


fig: Time reversal of signal $x(n)$

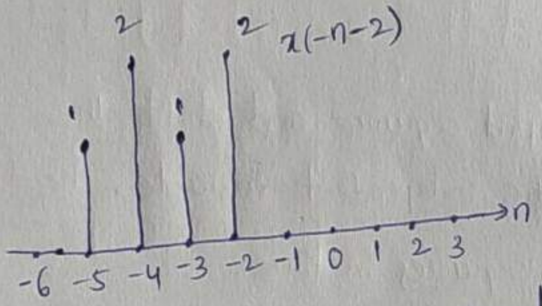
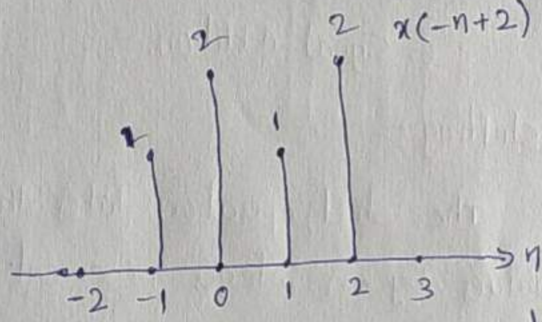


fig: shifted version of the signal $x(-n)$, advance



shifted version of the signal $x(-n)$, delay.

Time Scaling :-

This is accomplished by replacing n by αn in the sequence $x(n)$.

let $x(n)$ is a sequence, If $\alpha=2$, we get a new sequence,

$\boxed{y(n)=x(2n)}$ $\boxed{y(n)=x(\alpha n)}$

Ex:

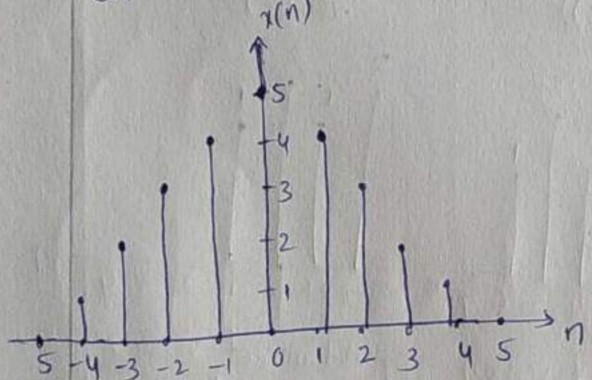
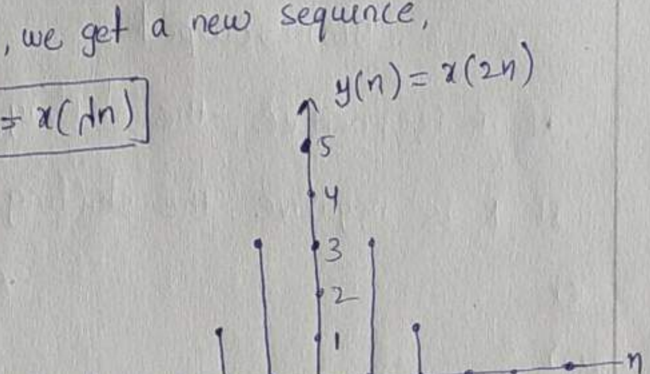


fig: Discrete time signal.



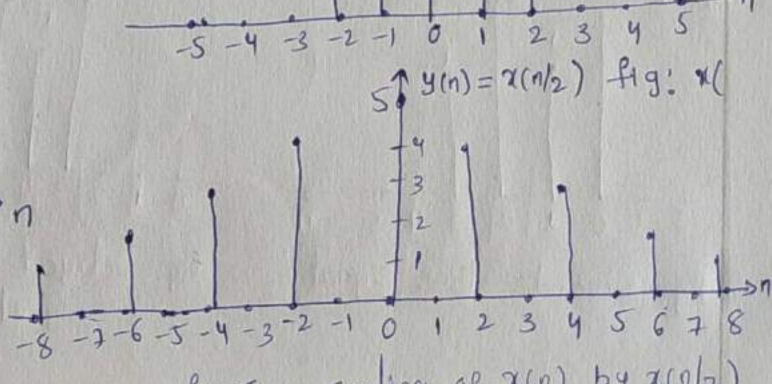


fig: Time scaling of $x(n)$ by $x(n/2)$

Scalar Multiplication / Amplitude scaling :-

The amplitude scaling of a discrete time signal can be represented by $\boxed{y(n) = a\,x(n)}$ where a is a constant

If $a>1$, the amplitudes of the samples increased i.e amplification.
If $a<1$, the amplitudes of the samples reduced i.e attenuation.

Ex: $x(n) = \{2, 1, 2\}$

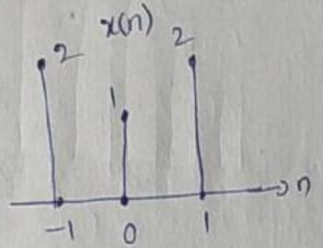


fig: Discrete time signal

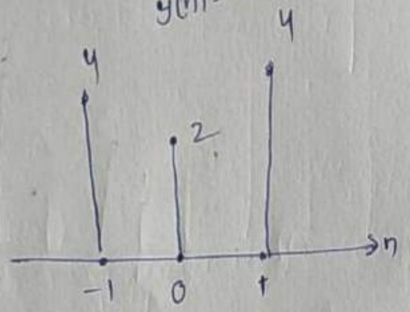


fig: $y(n) = 2x(n)$

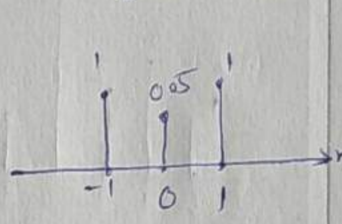


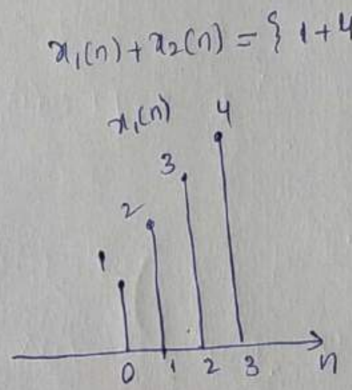
fig: $y(n) = x(n)/2$

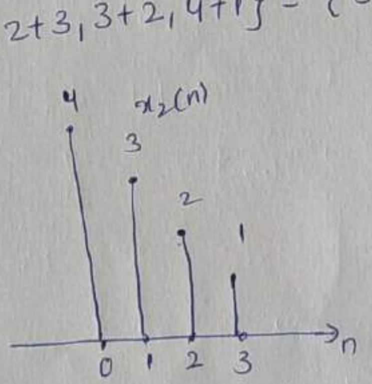
Signal Addition :-

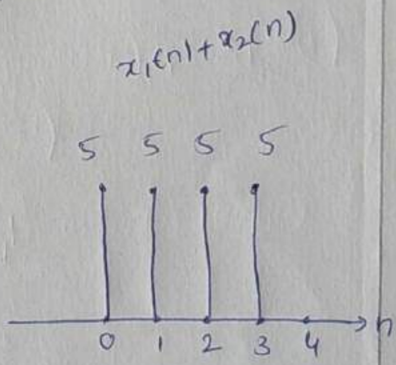
In discrete time domain, the sum of two signals $x_1(n)$ and $x_2(n)$ can be obtained by adding the corresponding sample values and the subtraction of $x_2(n)$ from $x_1(n)$ can be obtained by subtracting each sample of $x_2(n)$ from the corresponding sample of $x_1(n)$.

$x_1(n) = \{1, 2, 3, 4\}$, $x_2(n) = \{4, 3, 2, 1\}$

$x_1(n) + x_2(n) = \{1+4, 2+3, 3+2, 4+1\} = \{5, 5, 5, 5\}$







$x_1(n) - x_2(n) = \{-3, -1, 1, 3\}$

Signal multiplication :-

The multiplication of two discrete time sequences can be performed by multiplying their values at the sampling instants.

$x_1(n) = \{1, 3, 2, 4, 1\}$ $\quad$ $x_2(n) = \{2, 1, 3, 1, 5\}$.

$x_1(n)\,x_2(n) = \{1\times2, 3\times1, 2\times3, 4\times1, 1\times5\}$

$x_1(n)\,x_2(n) = \{2, 3, 6, 4, 5\}$

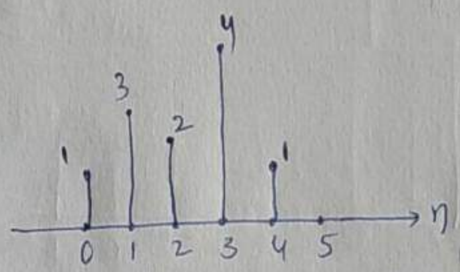


fig: Discrete time signal $x(n)$

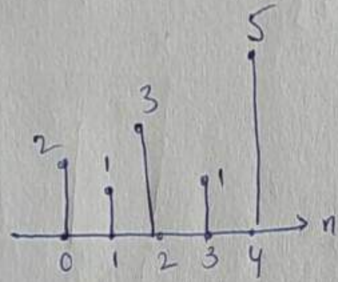


fig: $x_2(n)$

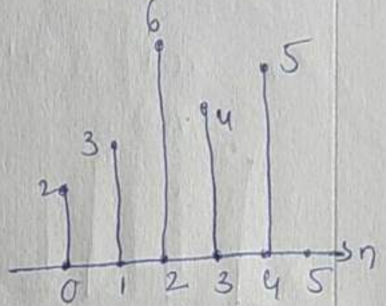


fig: $x_1(n)\,x_2(n)$

classification of Discrete time signals :-

The signals that are defined only at discrete instants of time are known as Discrete time signals.

The discrete time signals are continuous in amplitude, but discrete in time.

Discrete time signals are further classified as follows.

1. Deterministic and Random signals
2. periodic and Non periodic signals.
3. Energy and power signals.
4. causal and Non causal signals.
5. Even and odd signals.

Deterministic and Random signals :-

A signal which has no uncertainty at any given instant of time is called a Deterministic signal.

→ A deterministic signal can be completely represented by mathematical equation at any time. And its nature and amplitude at any time can be predicted.

Ex: $x(n) = \cos \omega n$, $x(n) = e^{j\omega n}$

A signal is characterised by uncertainty about its occurance is called a Non deterministic signal or Random signal.

→ A random signal cannot by represented by mathematical equation.

→ Its amplitude and phase at any instant of time cannot be predicted in advance.

→ The behaviour of a random signal is probablistic in nature and can be analyzed only stochastically. The pattern of signal is quire irregular.

Ex: Thermal noise.

periodic and Non periodic signals :-

A signal which has a definite pattern and repeats itself at regular invervals of time is called a periodic signal.

A discrete time signal $x(n)$ is said to be periodic if it satisfies the condition $x(n) = x(n+N)$ for all integers n.

→ The smallest value of 'N' which satisfies the condition $x(n)=x(n+N)$ is known as fundamental period.

fundamental period $= \frac{2\pi}{\omega_0}$ $\boxed{\omega = 2\pi\left(\frac{m}{N}\right)}$ $N \to$ fundamental period, $m \to$ integer.

A signal which does not repeat at regular intervals of time is called a Non periodic signal or Aperiodic signal.

If the condition $x(n) = x(n+N)$ is not satisfied even for one value of 'n', then the discrete time signal is Aperiodic signal.

Ex: $\left.\begin{array}{l} \sin(0.02\pi n) \\ \sin(5\pi n) \\ e^{j\pi/2 n} \end{array}\right\}$ periodic signals.

$\left.\begin{array}{l} \cos 4n \\ \cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right) \end{array}\right\}$ Non periodic signals.

Energy & power signals :-

The total energy E of a signal $x(n)$ is defined as

$$\boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$$

A signal is said to be energy signal if its total energy is finite.

The Average power of a discrete time signal $x(n)$ is defined as

$$\boxed{P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2}$$

A signal is said to be power signal if its average power is finite.

$$\left.\begin{aligned} x(n) &= \left(\tfrac{1}{2}\right)^n u(n) \\ x(n) &= u(n) - u(n-6) \end{aligned}\right\} \text{Energy signal.} \qquad \left.\begin{aligned} x(n) &= \sin\left(\tfrac{\pi}{3} n\right) \\ x(n) &= r(n) - r(n-4) \end{aligned}\right\} \text{power signal}$$

$$\boxed{\begin{aligned} &\text{For energy signal, } 0 < E < \infty \;\&\; P = 0 \\ &\text{For power signal, } 0 < P < \infty \;\&\; E = \infty \end{aligned}}$$

Causal and Non causal signals :-

A discrete time signal $x(n)$ is said to be causal if $x(n) = 0$ for $n<0$. i.e causal signal does not exist for negative time.

A signal which exists in positive as well as negative time is called non causal signal.

A discrete time signal $x(n)$ is said to be Anti causal if $x(n) = 0$ for $n>0$.

Ex: $x(n) = u(n)$, $x(n) = u(n-4)$ } causal signals.

$x(n) = u(-n)$, $x(n) = u(n+4) - u(n-2)$ } Non causal signals.

Even and odd signals :-

A Discrete time signal $x(n)$ is said to be even (symmetric) signal if it satisfies the condition

$$\boxed{x(n) = x(-n) \text{ for all } n}$$

A Discrete time signal $x(n)$ is said to be odd (Anti-symmetric) signal if it satisfies the condition

$$\boxed{x(-n) = -x(n) \text{ for all } n}$$

A signal can be represented as

$$x(n) = x_e(n) + x_o(n)$$

$$x(-n) = x_e(n) - x_o(n)$$

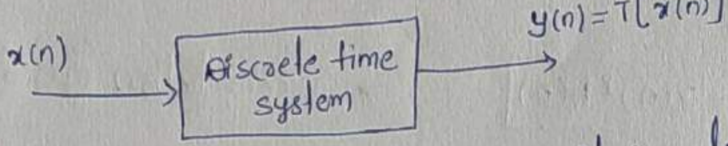
$$\boxed{\begin{aligned} x_e(n) &= \frac{1}{2}[x(n) + x(-n)] \\ x_o(n) &= \frac{1}{2}[x(n) - x(-n)] \end{aligned}}$$

Ex: $\cos wn$, $\rightarrow$ Even signal

$x(n) = \sin wn \rightarrow$ odd signal.

Impulse Response :-

A discrete time system performs an operation on an input signal based a predefined criteria to produce a modified output signal. The input signal $x(n)$ is the system excitation and $y(n)$ is the system response.



If the input to the system is a unit impulse i.e $x(n) = \delta(n)$ Then the output of the system is known as impulse response denoted by $h(n)$.

$$h(n) = T[\delta(n)] \quad (1)$$

An arbitary sequence $x(n)$ can be represented as a weighted sum of discrete impulses. Now the system response is given by

$$y(n) = T[x(n)]$$

$$y(n) = T\left[\sum_{K=-\infty}^{\infty} x(K)\delta(n-K)\right]$$

for a linear system,

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)T[\delta(n-K)] \quad (2)$$

The response to the shifted impulse sequence can be denoted by $h(n,K)$ is defined as,

$$h(n,K) = T[\delta(n-K)]$$

for a time invariant system $h(n,K) = h(n-K)$

$$\therefore T[\delta(n-K)] = h(n-K) \quad (3)$$

substitute eq(3) in equation (2), then

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)h(n-K)$$

For a linear time invariant system if the input sequence $x(n)$ and impulse response $h(n)$ is given, $y(n)$ can be determined as

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)}$$

which is known as convolution sum and can be represented as

$$\boxed{y(n) = x(n) * h(n)}$$

$* \rightarrow$ convolution operation.

Discrete Time System:-

A Discrete time system is a system, which transforms discrete time input signals into discretel time output signals. The relationship between the input $x(n)$ and output $y(n)$ of a system has form:

$$y(n) = T[x(n)]$$

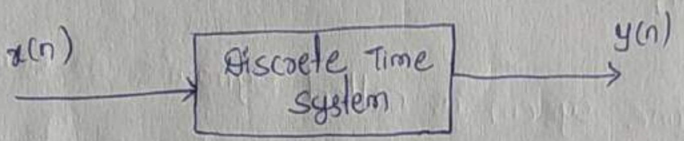


fig: Block diagram of Discrete time system

classification of Discrete time systems:

Discrete time systems are classified according to their general properties and characteristics. They are

1. Static and Dynamic systems.
2. Time Variant and time invariant systems
3. Causal and Non causal systems
4. linear and Non linear systems
5. FIR and IIR systems
6. Stable and Unstable systems.

Static and Dynamic system :-

A Discrete time system is called static or memoryless if its output at any instant n depends on the input samples at the same time, but not on past or future samples of the input, or past samples of output.

Ex: $y(n) = ax(n)$
$y(n) = 2x^2(n)$ } static systems.

A Discrete time system is called Dynamic or memory system if its output depends upon past or future inputs or past outputs.

Ex: $y(n) = x(n-1) + x(n-2)$
$y(n) = x(n) + x(n-1)$
$y(n) = x(2n)$
$y(n) + 4y(n-1) + 4y(n-2) = x(n)$ } Dynamic systems.

Ex: Find whether the following systems are static or Dynamic.

a) $y(n) = x^2(n)$.

$y(n) = x^2(n)$

$n=-1, \quad y(-1) = x^2(-1)$

$n=0, \quad y(0) = x^2(0)$

$n=1, \quad y(0) = x^2(1)$

output depends on the present value of input.

$\therefore$ $y(n) = x^2(n)$ static system.

(b) $y(n) = x(n+2)$

$n=-1, \quad y(-1) = x(1)$

$n=0, \quad y(0) = x(2)$

$n=1, \quad y(1) = x(3)$

output depends on future value of input.

$\therefore$ Given system is Dynamic system.

Time Invariant and Time variant system :-

A system is called Time invariant if its input - output characteristics do not change with time. i.e if a time shift in the input results in a corresponding time shift in the output.

let $x(n)$ be the input, delay the input sequence by 'k' samples and find output sequence, denote it as

$$y(n,k) = T[x(n-k)]$$

Delay the output sequence by 'k' samples, denote it as $y(n-k)$.

$$\boxed{\text{If} \quad y(n,k) = y(n-k)}$$

i.e If the delayed output is equal to the output due to delayed input, then system is called Time invariant.

Time variant system :

A system is called Time variant, if the delayed output is not equal to the output due to delayed input. i.e

$$\boxed{y(n,k) \neq y(n-k)}$$

where $y(n,k)$ is output due to delayed input by 'k' samples

$$y(n,k) = T[x(n-k)].$$

Ex: Determine if the following systems are time invariant or time variant.

a) $y(n) = x(n) + x(n-1)$ b) $y(n) = x(-n)$

c) $y(n) = x(n) + n\,x(n-2)$ d) $y(n) = x(n/2)$.

Sol $y(n) = x(n) + x(n-1)$

The output due to input delayed by k units

$y(n,k) = T[x(n-k)]$

$y(n,k) = x(n-k) + x(n-1-k)$ — ①

The output delayed by k units

$y(n-k) = x(n-k) + x(n-k-1)$ — ②

from ① & ②

$y(n,k) = y(n-k)$

∴ Given system is time invariant system.

(b) $y(n) = x(-n)$.

Given $y(n) = x(-n)$

The output due to input delayed by k units

$y(n,k) = T[x(n-k)]$

$y(n,k) = x(-n-k)$ — ①

The output delayed by k units

$y(n-k) = x(-(n-k))$

$y(n-k) = x(-n+k)$ — ②

from ① & ②

$y(n,k) \neq y(n-k)$

Given system is time variant system.

(c) $y(n) = x(n) + nx(n-2)$

Given $y(n) = x(n) + n\,x(n-2)$

The output due to input delayed by k units

$$y(n,k) = T[x(n-k)]$$

$$y(n,k) = x(n-k) + n\,x(n-2-k) \quad \text{—①}$$

The output due delayed by k units

$$y(n-k) = x(n-k) + (n-k)\,x(n-k-2) \quad \text{—②}$$

from equations ① & ②,

$$y(n,k) \neq y(n-k)$$

Given system is a time variant system.

(d) $y(n) = x(n/2)$

Given $y(n) = x(n/2)$

The output due to input delayed by k units

$$y(n,k) = T[x(n-k)]$$

$$y(n,k) = x\left(\frac{n}{2} - k\right) \quad \text{—①}$$

The output delayed by k units

$$y(n-k) = x\left(\frac{n-k}{2}\right) \quad \text{—②}$$

from equation ① & ②

$$y(n,k) \neq y(n-k).$$

∴ Given system is a time variant system.

Causal and Non causal system :-

A system is said to be causal if the output of the system at any time 'n' depends only on present and past inputs, but doesn't depend on future inputs.

It can be represented mathematically as

$$y(n) = F[x(n), x(n-1), x(n-2) \ldots].$$

If a system output depends not only on present and past inputs but also on future inputs then it is said to be a Non causal system.

Ex: Determine whether the systems are causal or Non causal.

a) $y(n) = x(n) + \frac{1}{x(n-1)}$

Given $y(n) = x(n) + \frac{1}{x(n-1)}$

$n=-1, \quad y(-1) = x(-1) + \frac{1}{x(-2)}$

$n=0, \quad y(0) = x(0) + \frac{1}{x(-1)}$

$n=1 \quad y(1) = x(1) + \frac{1}{x(0)}$

for all the values of n, the output depends on present and past ~~outp~~ inputs

$\therefore$ Given system is causal.

b) $y(n) = x(n^2)$

Given $y(n) = x(n^2)$

$n=-1, \quad y(-1) = x(1)$

$n=0, \quad y(0) = x(0)$

$n=1 \quad y(1) = x(1)$

for negative values of n, the system depends on future inputs. so, Given system is Non causal.

linear and Non linear systems :-

A system that satisfies the superposition principle is said to be a linear system.

Superposition principle states that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of the outputs of the system to each of the individual input signals.

A system is linear if and only if

$$\boxed{T[a_1x_1(n)+a_2x_2(n)] = a_1T_1[x_1(n)] + a_2T[x_2(n)]}$$

Ex: $y(n) = nx(n)$
$y(n) = n^2x(n)$
$y(n) = x(n)\cos wn$
} Linear systems.

Non linear system :-

A system that doesnot satisfy the superposition principle is called Non linear system.

Ex: $y(n) = x^2(n)$
$y(n) = |x(n)|$
$y(n) = 2x(n)+4$
} Non linear systems.

Ex: check whether the following systems are linear or not.

(i) $y(n) = nx(n)$.

sol $y(n) = nx(n)$ —①

outputs due to individual inputs $x_1(n)$ & $x_2(n)$

$y_1(n) = T[x_1(n)] = nx_1(n)$

$y_2(n) = T[x_2(n)] = nx_2(n)$.

weighted sum of outputs

$a_1y_1(n) + a_2y_2(n) = a_1nx_1(n) + a_2nx_2(n)$
$= n[a_1x_1(n) + a_2x_2(n)]$ —②

output due to weighted sum of inputs is

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$$

$$= a_1 n x_1(n) + a_2 n x_2(n)$$

$$y_3(n) = n[a_1x_1(n) + a_2x_2(n)] \quad \text{--- (3)}$$

from eq (2) & (3), superposition principle satisfied.

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)].$$

Hence, Given system is linear.

(ii) $y(n) = x^2(n)$.

sol $\quad y(n) = x^2(n) \quad \text{--- (1)}$

the outputs due to the signals $x_1(n)$ & $x_2(n)$ are

$$y_1(n) = T[x_1(n)] = x_1^2(n)$$

$$y_2(n) = T[x_2(n)] = x_2^2(n)$$

the weighted sum of outputs is

$$a_1y_1(n) + a_2y_2(n) = a_1x_1^2(n) + a_2x_2^2(n) \quad \text{--- (2)}$$

the output due to weighted sum of inputs

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$$

$$y_3(n) = [a_1x_1(n) + a_2x_2(n)]^2$$

$$y_3(n) = a_1^2x_1^2(n) + a_2^2x_2^2(n) + 2a_1a_2x_1(n)x_2(n) \quad \text{--- (3)}$$

from eq (2) & (3),

$$a_1y_1(n) + a_2y_2(n) \neq T[a_1x_1(n) + a_2x_2(n)]$$

superposition principle is not satisfied.

so, the given system is Non linear System.

stable and unstable systems :-

A system is said to be bounded-input, bounded-output (BIBO) stable, if and only if every bounded input yields a bounded output.

let the input signal $x(n)$ be bounded (finite) i.e

$$|x(n)| \leq M_x < \infty .$$

output signal $y(n)$ is also bounded. i.e

$$|y(n)| \leq M_y < \infty$$

where M_x & M_y are positive real numbers.

The necessary & sufficient condition for a discrete time system to be BIBO stable is given by

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty .$$

unstable system:-

A system is said to be unstable if one bounded input produces (yields) an unbounded output.

Ex: $\left.\begin{array}{l} h(n) = 3^n u(-n) \\ y(n) = x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2) \end{array}\right\}$ stable system.

$\left.\begin{array}{l} h(n) = 2^n u(n) \\ h(n) = u(n) \end{array}\right\}$ unstable systems.

FIR and IIR systems :-

A system is said to be Finite Impulse Response (FIR) system, if the impulse response consists of finite number of samples. (finite duration)

Ex:
$$h(n) = \begin{cases} -2 & n = 2,4 \\ 2 & n = 1,3 \\ 0 & \text{otherwise} \end{cases}$$

A system is said to be Infinite Impulse Response (IIR) system, if the impulse response consists of infinite number of samples. (infinite duration).

Ex: $h(n) = 2^n u(n)$.

Recursive and Non Recursive systems :

A system is said to be Recursive when the output $y(n)$ at time 'n' depends on any number of past outputs as well as present and past inputs.

The output $y(n)$ is given by

$$y(n) = F[x(n), x(n-1), x(n-2) \ldots x(n-M), y(n-1), y(n-2), \ldots y(n-N)]$$

A system is said to be Non Recursive when the output $y(n)$ depends only on the present and past input signal values.

The output $y(n)$ is given by

$$y(n) = F[x(n), x(n-1) \ldots x(n-m)].$$

Ex: $y(n) = x(n) + x(n-1) + y(n-2) \rightarrow$ Recursive system

$y(n) = x(n-1) + x(n-2) + x(n) \rightarrow$ Non Recursive system.

Investible and Non Investible system:-

A system is said to be investible if the input of the system appears at the output.

A system is investible if

$$\boxed{x(n) = T^{-1}[T[x(n)]]}$$

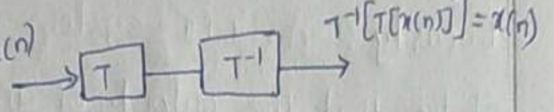


fig: Investible system.

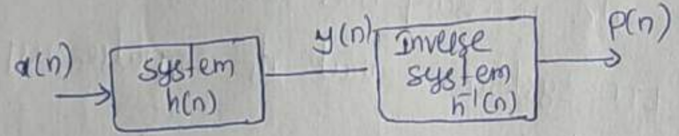
Ex: $y(n) = 3x(n)$

A system is said to be Non investible, if the output of the system is not equal to the input of the system.

Ex: $y(n) = 2x^2(n)$.

Ex: check whether the given system x(n) is investible or not

$y(n) = ax(n)$.



Sol

$$y(n) = a x(n)$$

$$P(n) = \frac{1}{a} y(n)$$

$$= \frac{1}{a}[a x(n)]$$

$$P(n) = x(n)$$

Linear Time Invariant (LTI) system :-

A system is said to be linear Time Invariant when it satisfies both linearity and time invariance.

linearity : An LTI system obeys superposition principle which states that the output of the system to a weighted sum of inputs is equal to the corresponding weighted sum of the outputs to each of the individual inputs.

Time Invariance : If the input and output relation of a system doesnot vary with time, the system is said to be time invariant.

Ex: $y(n) = x(n-2) \rightarrow$ LTI system.

stability condition for an LTI system :

The necessary and sufficient condition for the stability of an LTI system is that its impulse response is absolutely summable.

i.e $\boxed{\sum_{k=-\infty}^{\infty} |h(k)| < \infty}$

causality condition for an LTI system :-

The necessary and sufficient condition for causality of an LTI system is, its impulse response $h(n) = 0$ for negative values of n i.e

$$\boxed{h(n) = 0 \text{ for } n < 0}$$

LTI system is also known as linear shift Invariant system.

Ex: Test the stability of LTI systems.

(i) $h(n) = 0.2^n u(n)$

Sol $h(n) = (0.2)^n u(n)$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |(0.2)^n u(n)| = \sum_{n=0}^{\infty} (0.2)^n = \frac{1}{1-0.2} = \frac{1}{0.8} = 1.25 < \infty.$$

$\therefore \sum_{n=-\infty}^{\infty} |h(n)| < \infty$. Given system is stable.

(ii) $h(n) = 0.3^n u(n) + 2^n u(n)$.

Sol $h(n) = 0.3^n u(n) + 2^n u(n)$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |0.3^n u(n) + 2^n u(n)|$$

$$= \sum_{n=0}^{\infty} (0.3)^n + \sum_{n=0}^{\infty} 2^n$$

$$= \frac{1}{1-0.3} + \infty.$$

$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ if $r<1$

$\sum_{n=0}^{\infty} r^n = \infty$ if $r>1$

(iii) $y(n) = ax(n-7)$.

Sol $y(n) = a\,x(n-7)$

let $x(n) = \delta(n)$ then $y(n) = h(n)$

$h(n) = a\,\delta(n-7)$ $\quad h(n) = \begin{cases} a & \text{if } n=7 \\ 0 & \text{otherwise.} \end{cases}$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |a\,\delta(n-7)|$$

$$= a.1 = a$$

$\therefore$ Given system is stable if 'a' is finite.

(iv) $y(n) = x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$

<u>sol</u> $y(n) = x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$

let $x(n) = \delta(n)$ then $y(n) = h(n)$

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \left|\delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)\right|$$

$$= 1 + \frac{1}{2} + \frac{1}{4}$$

$$= \frac{7}{4} < \infty.$$

$$\therefore \sum_{n=-\infty}^{\infty} |h(n)| < \infty.$$

Given system is stable system.

Discrete Convolution:-

convolution is a mathematical operation used to Express the relation between input and output of an LTI system. It relates input, output and impulse response of an LTI systems as,

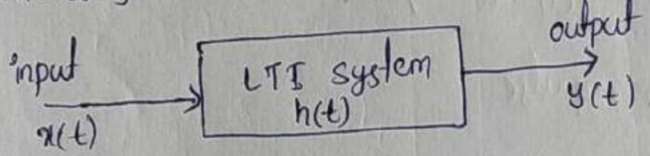
$$y(t) = x(t) * h(t)$$

$y(t) \rightarrow$ o/p of LTI
$x(t) \rightarrow$ I/p of "
$h(t) \rightarrow$ impulse response of LTI.
$* \rightarrow$ convolution operation

there are two types of convolutions.

1. continuous convolutions.
2. Discrete convolution

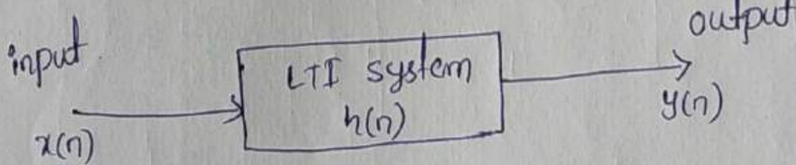
continuous convolution:



$$y(t) = x(t) * h(t)$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau}$$

Discrete ~~conda~~ convolution:



$$y(n) = x(n) * h(n)$$

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)}$$

properties of convolution:

1. commutative property.

$$x(n) * h(n) = h(n) * x(n)$$

2. Associative property

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

3. Distributive property

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

4. shifting property

if $x(n) * h(n) = y(n)$

then $x(n-k) * h(n-m) = y(n-k-m)$

5. convolution with an impulse

$$x(n) * \delta(n) = x(n).$$

Lineas Convolution Using Graphical method :-

linear convolution of two sequences can be performed by following steps.

step1: choose the starting time 'n' for evaluating the output sequence $y(n)$. If $x(n)$ starts at $n=n_1$ and $h(n)$ starts at $n=n_2$ then $n=n_1+n_2$ is a good choice.

step2: Express both the sequences interms of the index k.

step3: Fold $h(k)$ about $k=0$ to obtain $h(-k)$ and shift by 'n' to the right if 'n' is positive and to the left if 'n' is negative to obtain $h(n-k)$.

step4: multiply the two sequences $x(k)$ and $h(n-k)$ element by element and sum the product to get $y(n)$.

step5: Increment the index 'n', shift the sequence $h(n-k)$ to the right by one sample and perform step 4.

step6: Repeat step5 until the sum of products is zero for all remaining values of n.

Ex: Determine the convolution of two sequences:

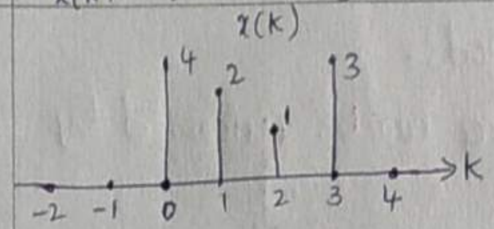
$x(n) = \{4, 2, 1, 3\}$ & $h(n) = \{1, \underset{\uparrow}{2}, 2, 1\}$

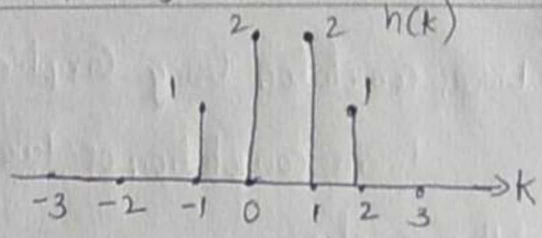
$\left.\begin{array}{l} x(n) \text{ starts at } n_1 = 0 \\ h(n) \quad " \quad " \quad n_2 = -1 \end{array}\right\}$ $y(n)$ starts at $n = n_1 + n_2 = \underline{-1}$

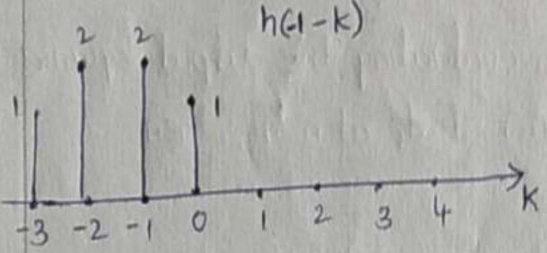
$N_1 = 4$ & $N_2 = 4$, the length of $y(n) = 4+4-1 = \underline{\underline{7}}$

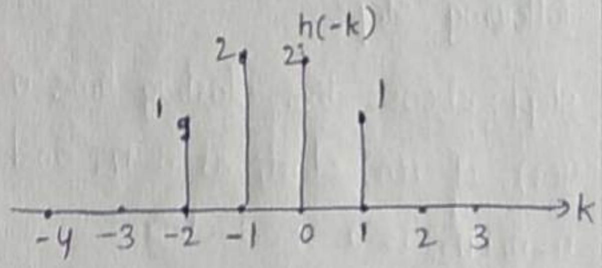
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k).$$

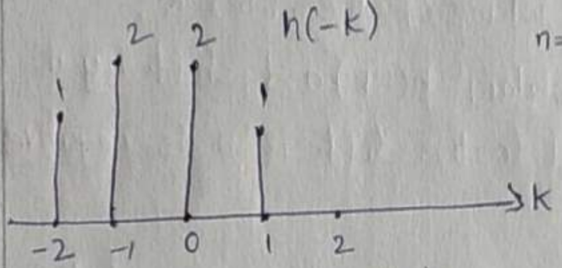
$x(k) = \{4, 2, 1, 3\}$ $\quad$ $h(k) = \{1, 2, 2, 1\}$







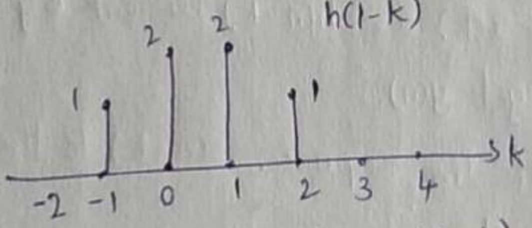




$n=-1$ $\quad y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k) = 4\times1 = 4$

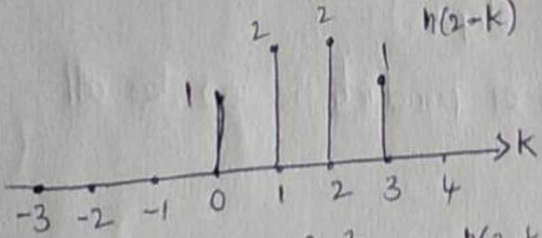
$n=0$

$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = 4\cdot2 + 2\cdot1 = 10$



$n=1$

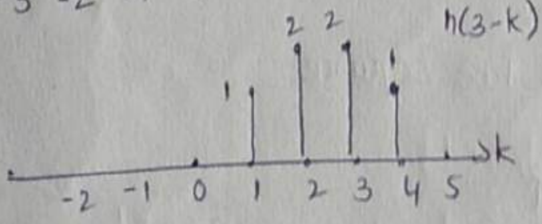
$y(1) = \sum_{k=-\infty}^{\infty} x(k)\,h(1-k) = 4\cdot2 + 2\cdot2 + 1 = 13$



$n=2$

$y(2) = \sum_{k=-\infty}^{\infty} x(k)\,h(2-k) = 4\cdot1 + 2\cdot2 + 1\cdot2 + 3\cdot1$

$= 13$

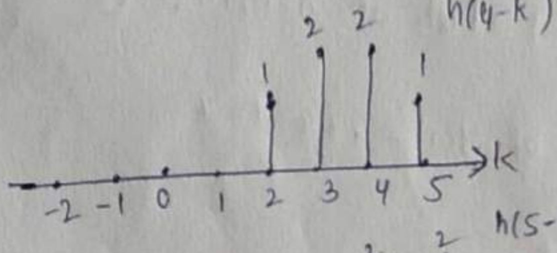


$n=3$

$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k) = 2\cdot1 + 2\cdot1 + 2\cdot3$

$= 10$

$n=4$



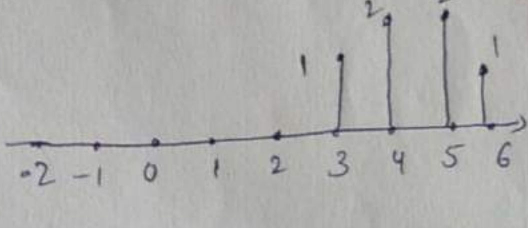
$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k) = 1\cdot1 + 2\cdot3$

$= 7$

$n=5$

$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k) = 3\cdot1$

$= 3.$



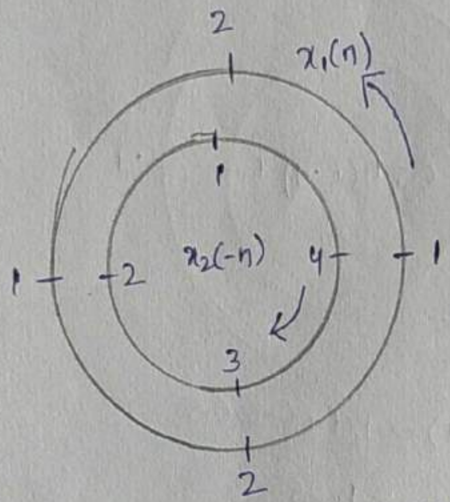
$\therefore\ y(n) = \{4, \underset{\uparrow}{10}, 13, 13, 10, 7, 3\}$

Ex: find the circular convolution of $x_1(n) = \{1, 2, 1, 2\}$ & $x_2(n) = \{4, 3, 2, 1\}$ by concentric circle method.

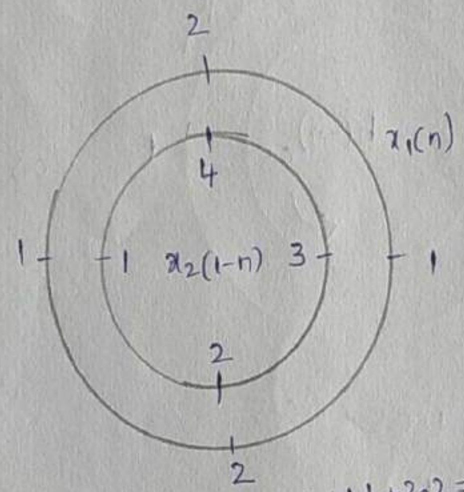
$$x_3(n) = x_1(n) \circledast x_2(n)$$

$x_1(n)$ & $x_2(n)$ are of same length. so no padding of zeros is required.

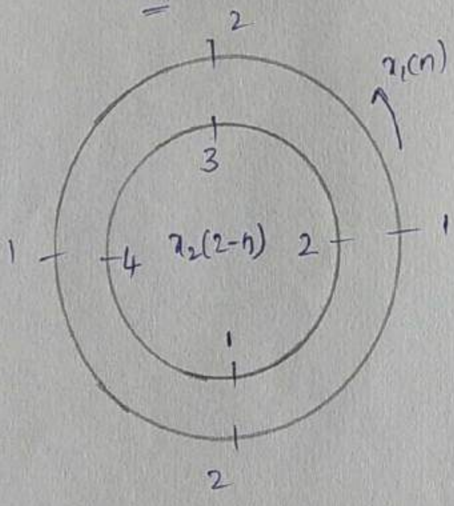
$$x_3(q) = \sum_{n=0}^{N-1} x_1(n)\, x_2(q-n)$$



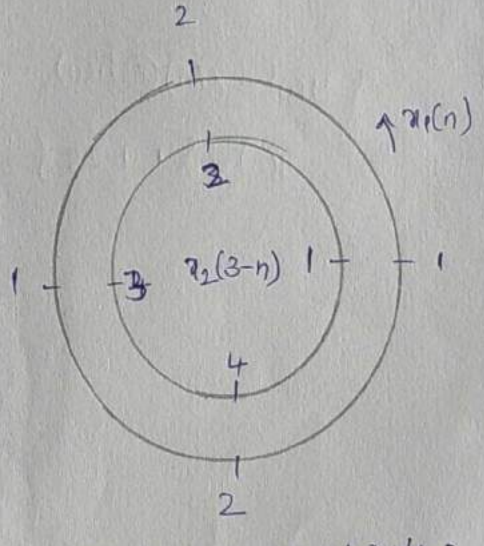
$x_3(0) = 1\times4 + 1\times2 + 1\times2 + 2\times3$
$= 14$



$x_3(1) = 1.3 + 4.2 + 1.1 + 2.2 = 16$



$x_3(2) = 1.2 + 2.3 + 1.4 + 1.2$
$= 14$



$x_3(3) = 1.1 + 1.2 + 1.3 + 4.2$
$= 16$.

$\therefore x_3(n) = x_1(n) \circledast x_2(n) = \{14, 16, 14, 16\}$.

Ex: perform the circular convolution of given two sequences.

$x_1(n) = \{2, 1, 2, -1\}$ & $x_2(n) = \{1, 2, 3, 4\}$.

matrix (NXN) of order 4x4 is formed using elements of $x_2(n)$.

column vector of order NX1 (4X1) formed using elements of $x_1(n)$

$x_1(0) = 2, \quad x_1(1) = 1, \quad x_1(2) = 2, \quad x_1(3) = -1.$

$x_2(0) = 1 \quad x_2(1) = 2 \quad x_2(2) = 3 \quad x_2(3) = 4$

$$\begin{bmatrix} \cancel{x_2(0)} \\ \cancel{x_2(1)} \\ \cancel{x_2(2)} \\ x_2(3) \end{bmatrix} \begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(4) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} = \begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2+4+6+(-2) \\ 4+1+8-3 \\ 6+2+2-4 \\ 8+3+4-1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 10 \\ 6 \\ 14 \end{bmatrix}$$

$x_3(n) = \{10, 10, 6, 14\}$.

linear Constant coefficient Difference Equations :-

The general form of an N^{th} order linear constant coefficient difference equation is,

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

where N is the order of difference equation

Solution of linear constant coefficient Difference equations can be calculated by

1. Direct method.
2. Indirect method (using z-transforms).

Direct method: The solution of the difference equation consists of two parts. i.e

$$y(n) = y_h(n) + y_p(n)$$

$y_h(n) \rightarrow$ Natural response, is known as homogeneous solution or complementary solution.

$y_p(n) \rightarrow$ forced response is called as particular solution.

To solve for $y_h(n)$, $\rightarrow$ input terms set to zero.

$$\sum_{k=0}^{N} a_k y(n-k) = 0 \quad , a_0 = 1$$

above equation is called homogenous difference equation.

To solve above equation, assume that $y_h(n) = \lambda^n$

$$\sum_{k=0}^{N} a_k \lambda^{n-k} = 0 \quad , \quad a_0 = 1.$$

$$a_0 \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \ldots a_N \lambda^{n-N} = 0.$$

$$\lambda^{n-N}\left[\lambda^N + a_1 \lambda^{N-1} + \ldots a_{N-1}\lambda + a_N\right] = 0$$

polynomial equation

$$\lambda^N + a_1 \lambda^{N-1} + \ldots a_{N-1}\lambda + a_N = 0$$

is called the characteristic polynomial of the system.

and it has 'N' roots, $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_N$. (real or complex).

If $\lambda_1, \lambda_2 \ldots \lambda_N$ are distinct, the general solution is of the form,

$$y_h(n) = C_1 \lambda_1^n + C_2 \lambda_2^n + \ldots C_N \lambda_N^n$$

where $C_1, C_2, C_3 \ldots C_N$ are weighting coeffients determined from the initial conditions.

If roots are repeated, i.e λ_1 is repeated for m times, then solution,

$$y_h(n) = \lambda_1^n (C_1 + C_2 n + C_3 n^2 + \ldots C_m n^{m-1}).$$

If char. equation has multiple complex roots, then.

if $\lambda_1, \lambda_2 = a \pm jb$ $\qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$$y_h(n) = r^n (A_1 \cos n\theta + A_2 \sin n\theta) \qquad r = \sqrt{a^2 + b^2}$$

the particular solution $y_p(n)$ is to satisfy the difference equation for the specific input signal $x(n), n \geq 0$,

$y_p(n)$ is any solution satisfying

$$1 + \sum_{k=1}^{N} a_k y_p(n-k) = \sum_{k=0}^{M} b_k x(n-k).$$

The general form of the particular solution for several inputs are shown below Table.

$x(n)$ input signal.	$y_p(n)$ particular solution.
A (constant)	K.
AM^n	KM^n
An^M	$K_0 n^M + K_1 n^{M-1} + \dots K_M$
$A^n n^M$	$A^n(K_0 n^M + K_1 n^{M-1} + \dots K_M)$
$A\cos\omega_0 n$, $A\sin\omega_0 n$	$K_1\cos\omega_0 n + K_2\sin\omega_0 n$.

To obtain total solution, add the homogeneous solution & particular solution.

$$\boxed{y(n) = y_h(n) + y_p(n)}$$

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Ex: Determine impulse Response h(n) for the system. consider a causal and stable LTI system whose input and output related through second order differential equation.

Sol: Given

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n) \quad \text{—①}$$

for impulse response, $y_p(n) = 0$.

Therefore $y(n) = y_h(n) + y_p(n) = y_h(n)$.

let $y_h(n) = \lambda^n$, substitute in equation ①.

$$\lambda^n - \frac{1}{6}\lambda^{n-1} - \frac{1}{6}\lambda^{n-2} = 0 \qquad (\because x(n) = 0 \text{ for homogeneous solution})$$

$$\lambda^{n-2}\left[\lambda^2 - \frac{1}{6}\lambda - \frac{1}{6}\right] = 0.$$

char. equation.

$$\lambda^2 - \frac{1}{6}\lambda - \frac{1}{6} = 0.$$

$$6\lambda^2 - \lambda - 1 = 0.$$

$$6\lambda^2 - 3\lambda + 2\lambda - 1 = 0$$

$$3\lambda(2\lambda - 1) + 1(2\lambda - 1) = 0.$$

$$(2\lambda - 1)(3\lambda + 1) = 0.$$

$$\lambda = 1/2, -1/3.$$

The roots are $\lambda_1 = 1/2$ $\lambda_2 = -1/3$.

$$y_h(n) = c_1\left(\frac{1}{2}\right)^n + \left(-1/3\right)^n \quad \text{—②}$$

for impulse response $x(n) = \delta(n)$

$x(n) = 0$ for $n > 0$.

$x(n) = 1$ for $n = 0$.

for $n = 0$,

$$y(0) - \frac{1}{6}y(-1) - \frac{1}{6}y(-2) = x(0).$$

i.e $y(0) = 1$ $\quad (\because x(0) = 1.$

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$$

for $n=1$,

$$y(1) - \frac{1}{6}y(0) - \frac{1}{6}y(-1) = x(1)$$

$$y(1) - \frac{1}{6}\times 1 + 0 = 0$$

$$y(1) = 1/6$$

$$y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{-1}{3}\right)^n$$

substitute $y(0)=1$ & $y(1)=1/6$

$$1 = C_1\left(\frac{1}{2}\right)^0 + C_2\left(-1/3\right)^0 \Rightarrow C_1 + C_2 = 1 \quad \text{— (3)}$$

$$\frac{1}{6} = C_1\left(\frac{1}{2}\right) + C_2\left(\frac{-1}{3}\right) \Rightarrow 3C_1 - 2C_2 = 1 \quad \text{— (4)}$$

By solving equation (3) & (4)

$$2C_1 + 2C_2 = 2$$
$$3C_1 - 2C_2 = 1$$
$$5C_1 = 3$$
$$C_1 = 3/5$$
$$\frac{3}{5} + C_2 = 1$$
$$C_2 = 2/5$$

$C_1 = 3/5$ & $C_2 = 2/5$.

substitute C_1 and C_2 in equation (2)

$$\boxed{y(n) = \frac{3}{5}\left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{-1}{3}\right)^n}$$

Ex: find the step response of the given system

$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$ — ①

sol

$y(n) = y_h(n) + y_p(n)$.

homogenous slouttion $y_h(n) = c_1(\frac{1}{2})^n + c_2(\frac{-1}{3})^n$

$y_p(n) = k$ for step input.

for $n>2$, substitute $y_p(n) = k$, $x(n) = 1$ in the given equation.

$k - \frac{1}{6}k - \frac{1}{6}k = 1$

$6k - 2k = 6 \Rightarrow 4k = 6 \Rightarrow k = 3/2$

$\therefore \boxed{y(n) = c_1(\frac{1}{2})^n + c_2(-\frac{1}{3})^n + \frac{3}{2}}$ — ②

$y(0) = 1$

$n=0, \quad y(0) = c_1 + c_2 + \frac{3}{2}$

$1 = c_1 + c_2 + 3/2$

$c_1 + c_2 = \frac{-1}{2}$ — ③

from the difference equation.

$y(0) - \frac{1}{6}y(-1) - \frac{1}{6}y(-2) = x(0)$

$y(0) = x(0) = 1,$

$x(1) = 1,$

$y(1) - \frac{1}{6}y(0) - \frac{1}{6}y(-2) = x(1)$

$y(1) - \frac{1}{6} - 0 = 1$

$y(1) = 1 + 1/6 = 7/6$

substute $y(1) = 7/6$ i.e $n=1$.

$y(1) = c_1(\frac{1}{2}) + c_2(-\frac{1}{3}) + \frac{3}{2}$

$\frac{7}{6} - \frac{3}{2} = \frac{c_1}{2} - \frac{c_2}{3}$

$7 - 9 = 3c_1 - 2c_2$

$3c_1 - 2c_2 = -2$ — ④

$2c_1 + 2\cancel{c_2} = -1$

$3c_1 - 2c_2 = -2$

$5c_1 = -3$

$c_1 = -3/5$

$c_2 = 1/10$.

solve ③ & ④ $c_1 = -3/5$, $c_2 = 1/10$. sub in eq ②

$\boxed{y(n) = -\frac{3}{5}(\frac{1}{2})^n + \frac{1}{10}(\frac{-1}{3})^n + \frac{3}{2}}$

Determine the solution of difference equation.

$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$ for $x(n) = 2^n u(n)$.

Sol

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n).$$

solution $y(n) = y_h(n) + y_p(n)$

Homogenous solution ($y_h(n)$):

substitute $y_h(n) = \lambda^n$ in eq ①.

$$\lambda^n = \frac{5}{6}\lambda^{n-1} - \frac{1}{6}\lambda^{n-2} + 0.$$

$$\lambda^n - \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2} = 0.$$

$$\lambda^{n-2}\left[\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6}\right] = 0.$$

char. equation $\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0.$

$$6\lambda^2 - 5\lambda + 1 = 0.$$

$$6\lambda^2 - 3\lambda - 2\lambda + 1 = 0.$$

$$3\lambda(2\lambda - 1) - 1(2\lambda - 1) = 0.$$

$$(3\lambda - 1)(2\lambda - 1) = 0. \Rightarrow \lambda = 1/3, 1/2$$

$$\lambda_2 = 1/3 \ \& \ \lambda_1 = 1/2.$$

so, $\boxed{y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n}$ — ②.

for the input $x(n) = 2^n u(n)$,

$$y_p(n) = K \cdot 2^n u(n).$$

substitute $x(n)$ & $y_p(n)$ in the difference equation.

$$k2^n u(n) = \frac{5}{6}k2^{n-1}u(n-1) - \frac{1}{6}k2^{n-2}u(n-2) + 2^n u(n)$$

for $n=2$,

$$k2^2 = \frac{5}{6}(2k) - \frac{k}{6} + 2^2$$

$$4k = \frac{10k-k}{6} + 4$$

$$4k - \frac{\cancel{9}^{3}k}{\cancel{6}_{2}} = 4$$

$$8k - 3k = 8 \Rightarrow k = 8/5$$

$$\therefore y_p(n) = k2^n u(n) = \frac{8}{5}2^n u(n).$$

$$\boxed{y_p(n) = \frac{8}{5}2^n u(n)} \quad (3)$$

If the

substitute (2) & (3) in equation (1)

$$y(n) = c_1\left(\tfrac{1}{2}\right)^n + c_2\left(\tfrac{1}{3}\right)^n + \frac{8}{5}2^n u(n).$$

$$y(0) = c_1 + c_2 + \frac{8}{5}$$

$$y(1) = \frac{1}{2}c_1 + \frac{1}{3}c_2 + \frac{16}{5}.$$

In the diff equation, substitute $y(-1) = y(-2) = 0$,

$$y(0) = \frac{5}{6}y(-1) - \frac{1}{6}y(-2) + 1 \Rightarrow y(0) = 1,$$

$$y(1) = \frac{5}{6}y(0) - \frac{1}{6}y(-1) + 2 \Rightarrow y(1) = \frac{5}{6} + 2 = \frac{17}{6}.$$

If we substitute $y(0)$ & $y(1)$, then will get.

$$c_1 + c_2 + 8/5 = 1 \Rightarrow c_1 + c_2 = -3/5$$

$$\frac{c_1}{2} + \frac{c_2}{3} + \frac{16}{5} = \frac{17}{6} \Rightarrow$$

$$c_1 = -1 \;\&\; c_2 = 2/5.$$

$$\boxed{y(n) = -\left(\tfrac{1}{2}\right)^n u(n) + \frac{2}{5}\left(\tfrac{1}{3}\right)^n u(n) + \frac{8}{5}2^n u(n)}$$

$1 - \frac{8}{5}$

Frequency Response:

let us consider the impulse response of a first order filter is $h(n)$ and input sequence $x(n)$ to the filter is $e^{j\omega n}$

we can find the output $y(n)$ by using convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, e^{j\omega(n-k)}$$

$$= e^{j\omega n} \sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega k}$$

$$\boxed{y(n) = \underbrace{e^{j\omega n}}_{\text{Input}}\ \underbrace{H(e^{j\omega})}_{\text{frequency Response}}}$$

$H(e^{j\omega})$ is called the frequency response of the causal LTI system whose impulse response is $h(n)$.

consider an LTI system whose impulse response $h(n) = a^n u(n)$. then the frequency response is

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

$$H(e^{j\omega}) = \sum_{n=0}^{\infty} a^n e^{-j\omega n} = \sum_{n=0}^{\infty} (a e^{-j\omega})^n$$

$$H(e^{j\omega}) = \frac{1}{1 - a\cos\omega + aj\sin\omega}$$

The magnitude response

$$|H(e^{j\omega})| = \frac{1}{\sqrt{(1-a\cos\omega)^2 + a^2\sin^2\omega}}$$

$$= \frac{1}{\sqrt{1 + a^2\cos^2\omega - 2a\cos\omega + a^2\sin^2\omega}}$$

$$\boxed{|H(e^{j\omega})| = \frac{1}{\sqrt{1+a^2-2a\cos\omega}}}$$

The phase Response

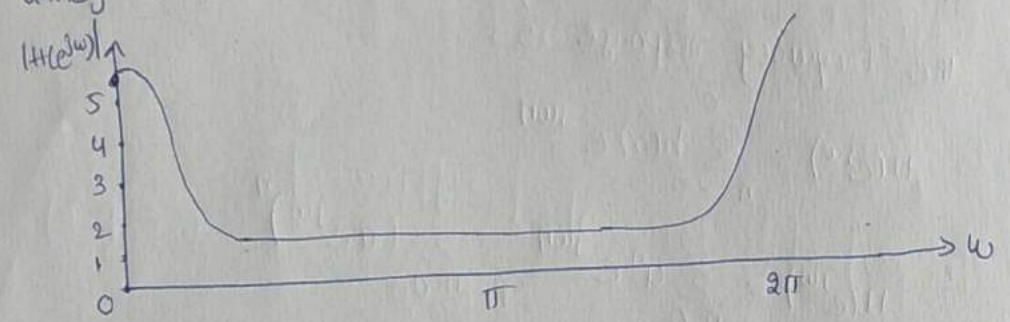
$$\boxed{\angle H(e^{j\omega}) = -\tan^{-1}\left(\frac{a\sin\omega}{1-a\cos\omega}\right)}$$

consider $a=0.8$,

By varying the values of 'ω', plot the magnitude & phase response.

ω	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\lvert H(e^{j\omega})\rvert$	5	1.402	0.78	0.6	0.55	0.6	0.78	1.402	5
$\angle H(e^{j\omega})$	0°	-52.48°	-38.66	-19.86°	0	19.86°	38.66°	52.48	0°

frequency response is a periodic function of ω with period of 2π. The magnitude of $H(e^{j\omega})$ is symmetric and the phase of $H(e^{j\omega})$ is antisymmetric over the interval $0 \le \omega \le 2\pi$.



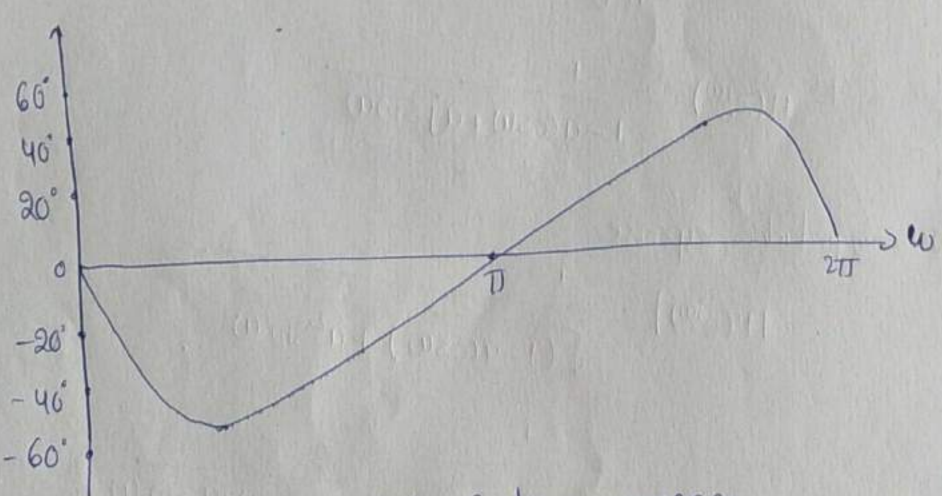


fig: magnitude & phase response.

Transfer function:-

If $H(e^{j\omega})$ is the fourier transform of the impulse response $h(n)$ and $X(e^{j\omega})$ is the fourier transform of the input sequence, we derive the relationship between $Y(e^{j\omega})$, the fourier transform of output interms of $X(e^{j\omega})$ and $H(e^{j\omega})$.

Any arbitrary sequence can be represented in the form

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \quad \text{—} ①$$

where $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

output $y(n)$ can be represented as

$$y(n) = e^{j\omega n} H(e^{j\omega})$$

where input $x(n) = e^{j\omega n}$.

we can write, $y(n)$ as follows

$$y(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} H(e^{j\omega}) d\omega \quad \text{—} ②$$

we know, $y(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(e^{j\omega}) e^{j\omega n} d\omega \quad \text{—} ③$

By comparing ② & ③, we have

$$Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$$

or

$$\boxed{H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}}$$

where $H(e^{j\omega})$ is known as transfer function of the system.

The transfer function of an LTI system is defined as the ratio of the fourier transform of the output to the fourier transform of the input.

It is also defined as the fourier transform of the impulse response h(n) of the system.

<u>Note:</u>

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega \quad \text{where } X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$y(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(e^{j\omega})e^{j\omega n}d\omega \quad \text{where } Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)e^{-j\omega n}$$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega \quad \text{where } H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n}$$

Q: find the fourier transform of the following.

a) $\delta(n-1)+\delta(n+1)$ b) $\delta(n+2)-\delta(n-2)$

Given $x(n) = \delta(n-1)+\delta(n+1)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} (\delta(n-1)+\delta(n+1)) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \delta(n-1) e^{-j\omega n} + \sum_{n=-\infty}^{\infty} \delta(n+1) e^{-j\omega n}$$

$$= e^{-j\omega} + e^{+j\omega}$$

$$\underline{X(e^{j\omega}) = 2\cos\omega}$$

(b) $x(n) = \delta(n+2)-\delta(n-2)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} [\delta(n+2)-\delta(n-2)] e^{-j\omega n}$$

$$= e^{+j2\omega} - e^{-j2\omega}$$

$$\underline{X(e^{j\omega}) = 2j\sin\omega}$$

(c) $x(n) = \left(\frac{1}{2}\right)^{n-1} u(n-1)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n} u(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^{n} = \sum_{n=0}^{\infty} \left(0.5 e^{-j\omega}\right)^{n}$$

$$= \frac{1}{1-0.5e^{-j\omega}}$$

Using time shifting property,

$$F\left[\left(\frac{1}{2}\right)^{n-1} u(n-1)\right] = \frac{e^{-j\omega}}{1-0.5e^{-j\omega}}$$

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Ex: for the sequences below, find the frequency response, plot magnitude and phase response.

$x(n) = 1$ for $n = -2, -1, 0, 1, 2$

$= 0$ ~~elsewhere~~ elsewhere.

Given $x(n) = 1$ for $n = -2, -1, 0, 1, 2$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= x(-2)\, e^{j2\omega} + x(-1)\, e^{j\omega} + x(0) + x(1)\, e^{-j\omega} + x(2)\, e^{-j2\omega}$$

$$= e^{2j\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-2j\omega} \qquad \left(\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \right)$$

$$= 1 + e^{j\omega} + e^{-j\omega} + e^{j2\omega} + e^{-j2\omega}$$

$$\boxed{X(e^{j\omega}) = 1 + 2\cos\omega + 2\cos 2\omega}$$

the magnitude response,

$$|X(e^{j\omega})| = |1 + 2\cos\omega + 2\cos 2\omega|$$

phase response, $\angle X(e^{j\omega}) = 0$ for $X(e^{j\omega}) > 0$

$= \pm\pi$ for $X(e^{j\omega}) < 0$.

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① evaluate the fourier transform of the system whose unit sample response $h(n) = 1$ for $0 \le n \le N-1$
$= 0$ elsewhere

<u>sol</u> Given $h(n) = \begin{cases} 1 & \text{for } 0 \le n \le N-1 \\ 0 & \text{elsewhere} \end{cases}$

the frequency response is given by

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} 1 \cdot e^{-j\omega n}$$

$1 + a + a^2 + \dots N$ terms $= \frac{1-a^N}{1-a}$

$$= 1 + e^{-j\omega} + e^{-j2\omega} + \dots N \text{ term}$$

$$= \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}}$$

$$= \frac{1 - (\cos\omega N - j\sin\omega N)}{1 - (\cos\omega + j\sin\omega)}$$

$$= \frac{1 - \cos\omega N + j\sin\omega N}{1 - \cos\omega + j\sin\omega}$$

$$= \frac{2\sin^2\frac{\omega N}{2} + j\,2\sin\frac{\omega N}{2}\cos\frac{\omega N}{2}}{2\sin^2\frac{\omega}{2} + j\,2\sin\frac{\omega}{2}\cos\frac{\omega}{2}}$$

$$= \frac{\cancel{2}\sin\frac{\omega N}{2}\left[\sin\frac{\omega N}{2} + j\cos\left(\frac{\omega N}{2}\right)\right]}{\cancel{2}\sin\frac{\omega}{2}\left[\sin\frac{\omega}{2} + j\cos\frac{\omega}{2}\right]}$$

$$= \frac{\sin\frac{\omega N}{2}\left[\cos\frac{\omega N}{2} - j\sin\frac{\omega N}{2}\right]}{\sin\frac{\omega}{2}\left[\cos\frac{\omega}{2} - j\sin\frac{\omega}{2}\right]}$$

$$= \frac{\sin\frac{\omega N}{2}\, e^{-j\omega N/2}}{\sin\frac{\omega}{2}\, e^{-j\omega/2}} = \frac{\sin\frac{\omega N}{2}}{\sin\frac{\omega}{2}} e^{-j(N-1)\omega/2}$$

$$\boxed{H(e^{j\omega}) = \frac{\sin\frac{\omega N}{2}}{\sin\frac{\omega}{2}}\, e^{-j(N-1)\omega/2}}$$

Ex: Determine the discrete time fourier transform of the sequence.

$x(n) = \{1, -1, 1, -1\}$.

Sol given $x(n) = \{1, -1, 1, -1\}$

$x(0)=1,\ x(1) = -1,\ x(2)=1\ , x(3)=-1.$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= x(0) + x(1)e^{-j\omega} + x(2)e^{-j2\omega} + x(3)e^{-j3\omega}$$

$$= 1 - 1.e^{-j\omega} + 1.e^{-j2\omega} + (-1)e^{-j3\omega}$$

$$= 1 - e^{-j\omega} + e^{-j2\omega} - e^{-j3\omega}$$

$$= (1-e^{-j\omega}) + e^{-j2\omega}(1-e^{-j\omega})$$

$$X(e^{j\omega}) = (1-e^{-j\omega})(1+e^{-j2\omega})$$

$$\therefore \boxed{X(e^{j\omega}) = (1-e^{-j\omega})(1+e^{-j2\omega})}$$

Ex: Determine the signal $x(n)$ for the given fourier transform

$X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \le \omega \le \pi$.

given $X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \le \omega \le \pi$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega/2} e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(n-1/2)} d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\pi(n-\frac{1}{2})} - e^{-j\pi(n-1/2)}}{j(n-\frac{1}{2})}\right] = \frac{e^{j\pi(n-1/2)} - e^{-j\pi(n-1/2)}}{2j(n-1/2)}$$

$$= \frac{\sin\pi(n-1/2)}{\pi(n-1/2)}.$$

Determine & sketch the magnitude & phase response of

$y(n) = \frac{1}{2}[x(n) + x(n-2)]$.

Given $y(n) = \frac{1}{2}[x(n) + x(n-2)]$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2}[x(n) + x(n-2)]\, e^{-j\omega n}$$

$$= \frac{1}{2}\left[\sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x(n-2) e^{-j\omega n}\right]$$

$$= \frac{1}{2}\left[X(e^{j\omega}) + e^{-2j\omega} X(e^{j\omega})\right]$$

$$Y(e^{j\omega}) = \frac{X(e^{j\omega})}{2}\left[1 + e^{-2j\omega}\right]$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + e^{-2j\omega}}{2}.$$

$$\boxed{H(e^{j\omega}) = \frac{1 + \cos 2\omega - j\sin 2\omega}{2}.}$$

magnitude response.

$$|H(e^{j\omega})| = \frac{1}{2}\sqrt{(1+\cos 2\omega)^2 + \sin^2 2\omega}$$

$$= \frac{1}{2}\sqrt{1 + 2\cos 2\omega + \cos^2 2\omega + \sin^2 2\omega}$$

$$= \frac{1}{2}\sqrt{2 + 2\cos 2\omega}$$

$$= \frac{1}{2}\cdot\sqrt{2(1+\cos 2\omega)}$$

$$= \frac{1}{2}\sqrt{2 \cdot 2\cos^2\omega}$$

$$= \frac{1}{\cancel{2}}(\cancel{2}\cos\omega)$$

$$|H(e^{j\omega})| = \cos\omega.$$

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$$H(e^{j\omega}) = \frac{1+\cos 2\omega - j\sin 2\omega}{2}$$

phase Response,

$$\angle H(e^{j\omega}) = \tan^{-1}\left[\frac{-\sin 2\omega}{1+\cos 2\omega}\right]$$

$$= \tan^{-1}\left[\frac{-\cancel{2}\sin\omega\cos\omega}{\cancel{2}\cos^2\omega}\right]$$

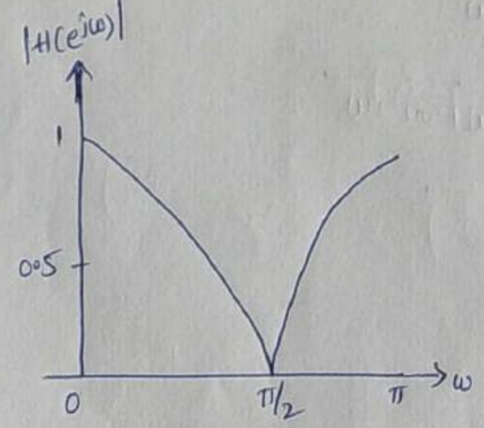
$$= \tan^{-1}[-\tan\omega]$$

$$\angle H(e^{j\omega}) = -\omega$$

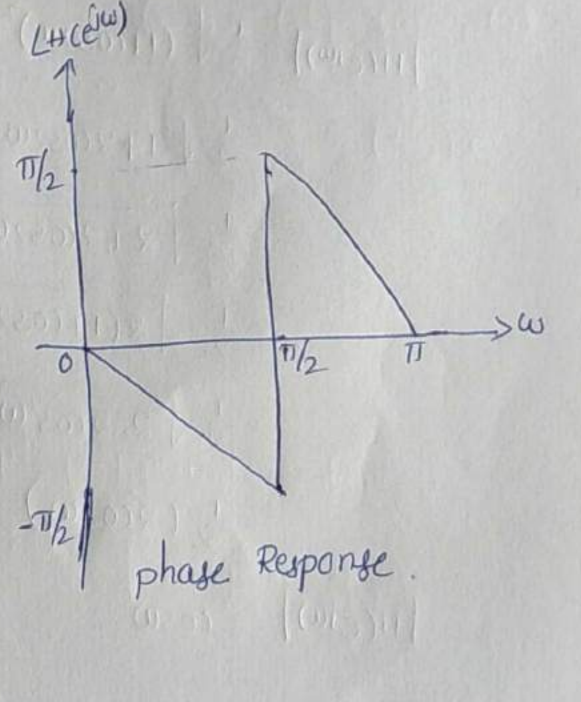
$$\therefore \angle H(e^{j\omega}) = -\omega \text{ for } H(e^{j\omega}) > 0$$

$$= -\omega + \pi \text{ for } H(e^{j\omega}) < 0$$

ω	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$H(e^{j\omega})$	1	0.812	0.707	0.5	0	-0.5	-0.707	-1
$\lvert H(e^{j\omega})\rvert$	1	0.812	0.707	0.5	0	0.5	0.707	1
$\angle H(e^{j\omega})$	0	$-\pi/6$	$-\pi/4$	$-\pi/3$	$-\pi/2$	$\pi/3$	$\pi/4$	0



magnitude Response



phase Response.

Ex: A discrete time system has a unit sample response h(n) given by $h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$. Find the system-frequency response $H(e^{j\omega})$ plot magnitude & phase response.

Sol Given $h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$

frequency Response,

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)\right] e^{-j\omega n}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} \delta(n)\, e^{-j\omega n} + \sum_{n=-\infty}^{\infty} \delta(n-1)\, e^{-j\omega n} + \frac{1}{2}\sum_{n=-\infty}^{\infty} \delta(n-2)\, e^{-j\omega n}$$

$$= \frac{1}{2}\cdot 1 + e^{-j\omega} + \frac{1}{2} e^{-2j\omega}$$

$$= e^{-j\omega}\left[\frac{e^{j\omega}}{2} + 1 + \frac{e^{-j\omega}}{2}\right]$$

$$\boxed{H(e^{j\omega}) = e^{-j\omega}\left[1 + \cos\omega\right]}$$

magnitude Response,

$$|H(e^{j\omega})| = |e^{-j\omega}(1+\cos\omega)| = |e^{-j\omega}|\,|1+\cos\omega| \qquad (\because |e^{-j\omega}| = 1)$$

$$= 1 + \cos\omega$$

phase Response,

$$\angle H(e^{-j\omega}) = -\omega \text{ for } 0 \le \omega \le \pi.$$

ω	0	$\pi/4$	$\pi/3$	$\pi/2$	$3\pi/4$	π
$\lvert H(e^{j\omega})\rvert$	2	1.707	1.5	1	0.293	0
$\angle H(e^{j\omega})$	0	$-\pi/4$	$-\pi/3$	$-\pi/2$	$-3\pi/4$	$-\pi$

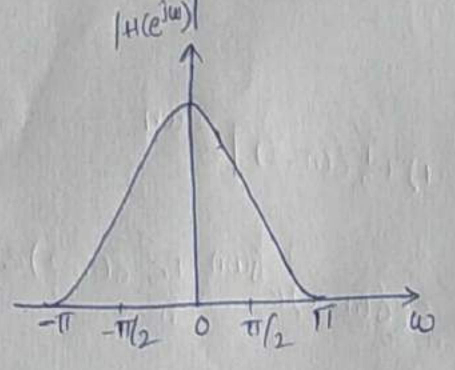


fig: plot of magnitude Response

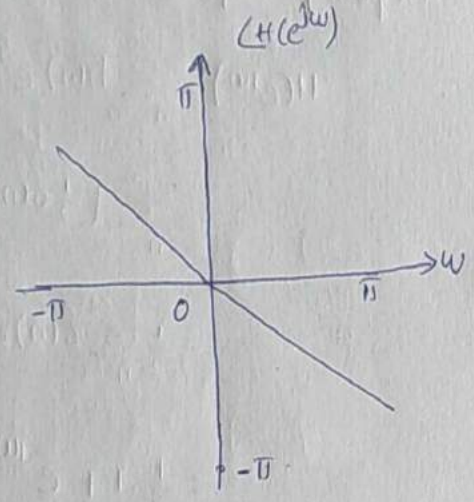


fig: plot of frequency Response