

1. Principles of Quantum Mechanics

- deals with micro particles
- ex: atoms, proton's, e^- 's, neutrons
- wave & particle
- de Broglie hypothesis

Max Planck $\Rightarrow E = hv$

$$h = 6.626 \times 10^{-34} \text{ Js}$$

Einstein's mass-energy relation

$$E = mc^2$$

de Broglie \Rightarrow Dual nature of radiation

$$\lambda_{\text{wavelength}} = \frac{h}{mv}$$

Heisenberg, Schrodinger

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{2m(E-V)}{\hbar^2} \Psi = 0$$

Difference between classical and quantum mechanics

- unlike classical mechanics is applicable for macro particles, for quantum micro particles.
- classical mechanics was developed by Newton's law whereas quantum mechanics was developed by many scientists

- Heisenberg Principle does not applicable for classical mechanics

Particle

- mass & confined to a small region.

- characteristics are mass, velocity, acceleration, momentum, energy etc.

- Two particles can move cannot exist at the same position.

Wave

Disturbance created in a medium

- characteristics are wavelength, frequency amplitude, time period (T), phase angle (ϕ)

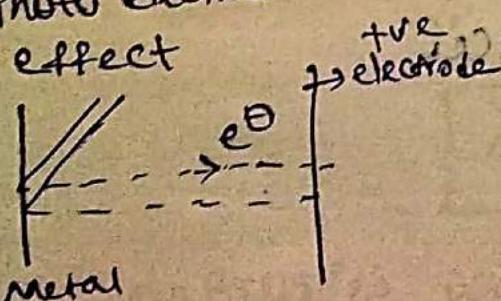
- Two more waves can superpose one another.

De Broglie hypothesis:-

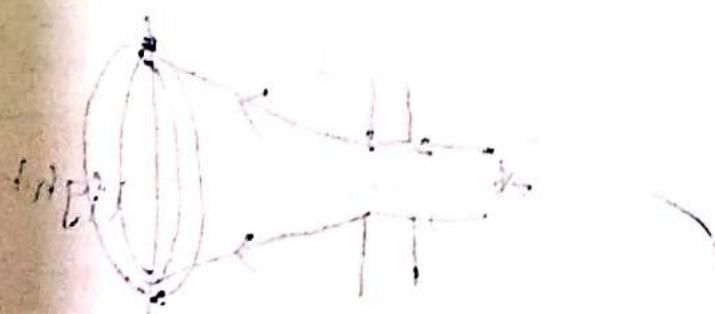
matter wave \rightarrow Dual nature of radiation (light)

Particle nature \rightarrow wave nature \rightarrow Interference
wave nature \rightarrow Diffraction
Polarisation \rightarrow One Sidedness
 \rightarrow property of light

1) Photoelectric effect



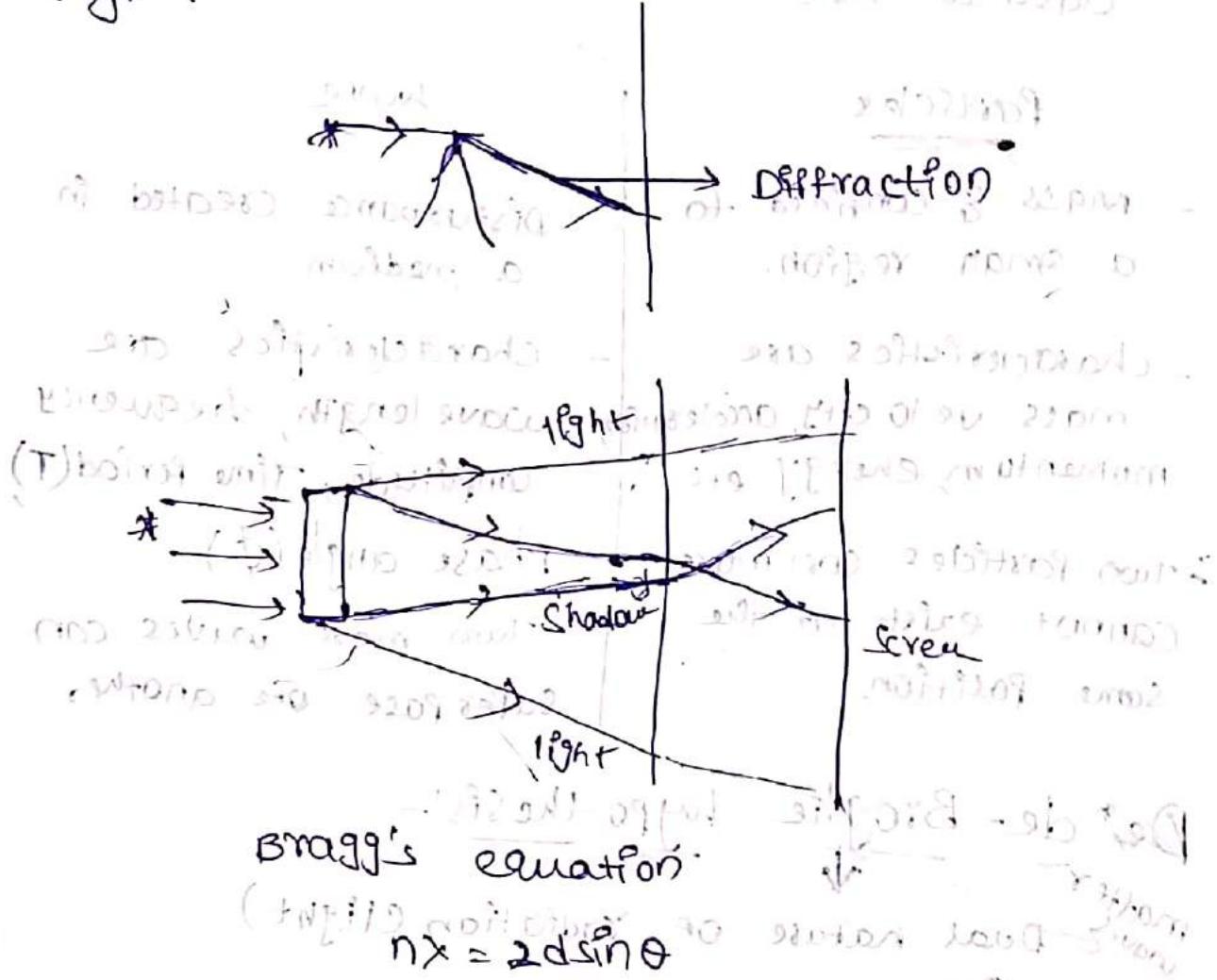
2) Compton effect



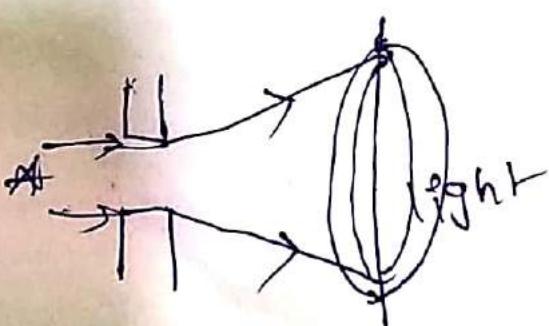
Diffraction: Bending of light at the edges and entering region is called as diffraction and

comes out or an obstacle finds the geometrical shadow

And entering into the geometrical shadow region.



If the wavelength of light should be similar to the width of obstacle size, then only diffraction takes place.



Light enters the prism and refracts at two faces. A larger angle of deviation is produced by a prism than by a lens.

DeBroglie's wavelength of matter waves

According to Planck

$$E = h\nu \quad \text{--- (1)}$$

where, h - Planck's constant

$$= 6.626 \times 10^{-34} \text{ J sec}$$

ν = Frequency of radiation

Acc to Einstein mass energy relation

$$E = mc^2 \quad \text{--- (2)}$$

where, m = mass

c = velocity of light in vacuum

$$= 3 \times 10^8 \text{ m/s}$$

From eqn (1) & (2) we get

$$h\nu = mc^2$$

$$\frac{h\nu}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} = \frac{h}{\text{momentum}} \quad \left[\because mv = p \right]$$

$$\boxed{\lambda = \frac{h}{p}} \quad \text{--- (3)}$$

deBroglie's wavelength of wave.

* deBroglie's hypothesis:-

Either radiation or matter waves exhibit only ~~one~~ ^{both} wave and particle nature.

both wave and particle nature but both can exhibit simultaneously.

Consider a particle having mass 'm' and velocity 'v'. Then the deBroglie wavelength of the particle is given by

$$\lambda = \frac{h}{mv}$$

(1) $\frac{h}{mv} = \frac{h}{\text{momentum}}$

$\lambda = \frac{h}{\text{momentum}}$
 Particle

(2) $\lambda = \frac{h}{mv}$

DeBroglie wavelength of particle

Consider a particle having energy 'E' kinetic Energy moving

'E'

$$E = \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = E$$

$$mv^2 = \frac{2E}{m}$$

$E = \frac{p^2}{2m}$

$$p^2 = 2mE$$

$$p^2 = 2mE$$

$p = \sqrt{2mE}$

∴ from deBroglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

de Broglie wavelength for moving electron:-

consider an electron having mass 'm', having velocity 'v' and this particle is applied with external potential 'V'

K.E = charge \times applied voltage

$$\frac{1}{2}mv^2 = eV$$

$$\frac{m^2v^2}{2m} = eV$$

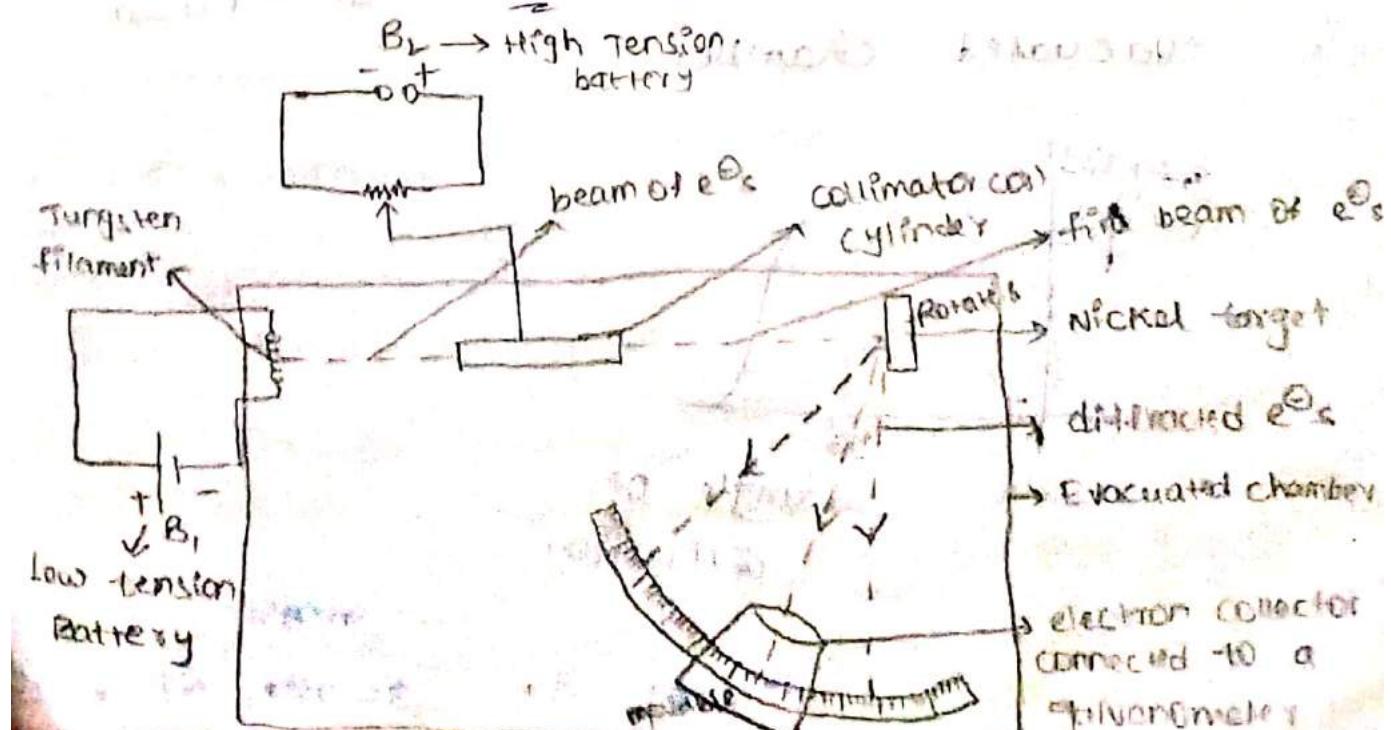
$$\frac{p^2}{2m} = eV$$

$$p^2 = 2meV$$

$$p = \sqrt{2meV}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{12.26 \text{ Å}}{\sqrt{V}}$$

* Davisson - Germer Expt :-



$$d_{Ni} = 0.091 \text{ nm} \rightarrow \text{(interplanar distance)}$$

Bragg angle $\theta = 65^\circ$

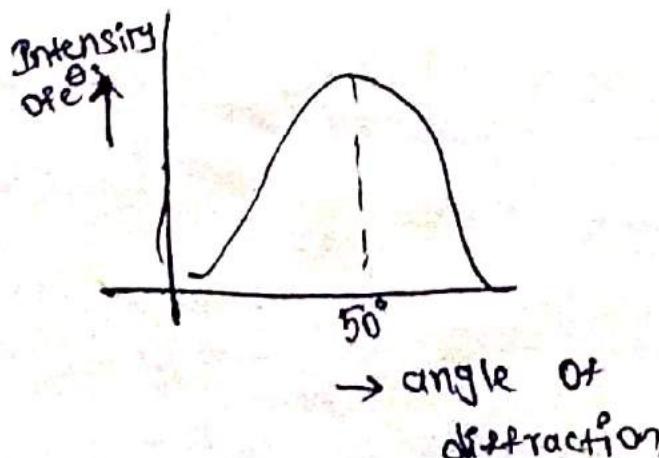
Applied voltage = 54 volts

Max no. of e^- s gets collected at an angle $= 50^\circ$

position below the X-rayed = 3.8

A low tension battery is connected to a tungsten filament, then tungsten filament ejects a beam of electrons to collimator which is connected to high tension battery with a resistance. This collimator makes the beam of e^- s into fine beam of e^- s. This fine beam of e^- s are flowed towards the Nickel target which always rotates which is known as natural diffractor. By this Nickel target the fine beam of e^- s get diffracted to an electron collector connected to a Galvanometer with a scale which is movable.

* without batteries all equipments are placed in evacuated chamber.



for $n=1$

order of diffraction

$$\lambda = 2d \sin \theta$$

$$= 2 \times 0.091 \times 10^{-9} \times \sin 65^\circ$$

$$= 0.1649 \times 10^{-9} \text{ m}$$

$$= 1.649 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda = 1.649 \text{ } \text{\AA}}$$

Acc to deBroglie

$$\lambda = \frac{12.26}{\sqrt{V}} = \frac{12.26}{\sqrt{54}} = 1.668 \text{ } \text{\AA}$$

Characteristics of Matter waves

Matter waves will have a deBroglie wavelength (λ)

$$\lambda = \frac{h}{mv}$$

1) Lighter the mass of particle, higher is wavelength associated with matter waves.

2) If $v=0 \Rightarrow$ particle is at rest, is impossible ($\lambda=\infty$)

* Matter waves are only associated with moving

particles but not at stationary.

3) Matter waves have dual nature that is particle and wave nature but both cannot exhibit simultaneously.

4) Matter waves travel faster than velocity of light.

Proof:- According to Max Planck \rightarrow relation

$$E = h\nu \quad \text{--- (1)}$$

Also to Einstein mass energy relation

$$E = mc^2 \quad \text{--- (2)}$$

From (1) & (2) equation

$$h\nu = mc^2$$

$$\nu = \frac{mc^2}{h} \quad \text{--- (3)}$$

Let us consider ' ω ' be velocity of matter waves is given by

$$\omega = \nu\lambda$$

$$= \frac{mc^2}{h} \times \frac{\lambda}{mv}$$

$$\omega = \frac{c^2}{v}$$

$$\omega \propto c^2$$

$$\boxed{\omega \gg c}$$

5) Matter waves obeys Heisenberg's uncertainty principle for a macro particle with accuracy we cannot measure simultaneously both position and velocity of a particle (or momentum)

Example

Heisenberg's Uncertainty Principle

for a micro particle with accuracy ~~possible~~
 cannot measure simultaneously both position
 and momentum of a particle.

$$\sqrt{\Delta x} \cdot \sqrt{\Delta p} \geq \frac{h}{4\pi}$$

↓ ↓

Uncertainty in Position Uncertainty in momentum

⇒ Angle and Angular Momentum

$$\sqrt{\Delta \theta} \cdot \sqrt{\Delta L} \geq \frac{h}{4\pi}$$

⇒ Energy and Time

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

who does it?

Difference between Matter waves and Electro Magnetic waves:-

Matter Waves:-

Electro magnetic Waves

- 1) Associated only with the moving particles.
- 2) deBroglie's Wavelength - 2) deBroglie's Wavelength

$$\lambda = \frac{h}{P} = \frac{h}{mv}$$

$$E = h\nu$$

$$\lambda = \frac{hc}{E}$$

- 3) Matter waves are not electro magnetic waves
- 3) In Electromagnetic wave the electric field and the magnetic field are $\perp \&$ to each other
- 4) Matter waves will have dual nature.
- 4) Electro magnetic waves which don't have dual nature
- 4) Matter waves travel faster than velocity of light
- 4) Electro magnetic waves travel along with velocity of light.

Schrodinger's Time - Independent Wave Equation

$$\Psi(x, t) = \Psi_0 \sin(\omega t - kx)$$

↓ ↓
 Max-amplitude Angular wave vector (cos)
↓ Frequency: $\omega = 2\pi f$

Position Propagation vector

(1)

Propagation vector $K = \frac{2\pi}{\lambda}$

$$= \Psi_0 \sin\left(\frac{2\pi}{\lambda}x + \omega t\right)$$

$\lambda = 2\pi / K$

partial differentiating eq. (1) twice w.r.t. x. we get

$$\frac{\partial \Psi}{\partial x} = \Psi_0 \cos(\omega t - kx) x - K$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -K\Psi_0 [-\sin(\omega t - kx) x - k]$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -K^2 \Psi_0 [\sin(\omega t - kx)]$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -K^2 \Psi$$

$$\frac{\partial^2 \Psi}{\partial x^2} + K^2 \Psi = 0$$

(2) 2nd order P.D.E.

Eq.n (2) represents second order, partial differential equation.

The solution of the equation is given by:-

$$\Psi(x) = A \sin Kx + B \cos Kx \quad (3)$$

where $A, B, K \rightarrow$ constants

In eq.n ①,

We know that $K = \frac{2\pi}{\lambda}$ (wave vector)

$$K^2 = \frac{4\pi^2}{\lambda^2} \quad \text{--- ④}$$

Substituting eq.n ④ in eq.n ② we get

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad \left[\because \lambda = \frac{h}{mv} \right]$$

$$\lambda^2 = \frac{h^2}{m^2 v^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \quad \frac{1}{\lambda^2} = \frac{m^2 v^2}{h^2}$$

Total Energy of the system

$$E = K.E + P.E \quad \text{--- ⑤}$$

$$E = \frac{1}{2} mv^2 + V$$

$$\frac{1}{2} mv^2 = E - V$$

$$\frac{m^2 v^2}{2m} = E - V$$

$$m^2 v^2 = 2m(E - V) \quad \text{--- ⑥}$$

Substitution eq. ⑥ in eq. ⑤, we get

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 2m(E - V)}{h^2} \psi = 0 \quad \text{--- ⑦}$$

$$\kappa = \frac{h}{2\pi}$$

$$\kappa^2 = \frac{h^2}{4\pi^2} \Rightarrow \frac{1}{\kappa^2} = \frac{4\pi^2}{h^2} \quad \text{--- (8)}$$

substⁿ eq. (8) in (7) we get

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)}{\kappa^2} \psi = 0 \quad \text{--- (9)}$$

Equation (9) represent Schrodinger's time-independent wave equation in One dimensional case.

In 3D case

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m(E-V)}{\kappa^2} \psi = 0 \quad \text{--- (10)}$$

Laplacian operator (∇^2)

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Eq. n (10) becomes

$$\nabla^2 \psi + \frac{2m(E-V)}{\kappa^2} \psi = 0$$

Physical significance of wave function (Ψ):-

$$\Psi = \underline{a} + \underline{i} \underline{b}$$

real imaginary

$$\Psi^* = \underline{a} - \underline{i} \underline{b}$$

complex conjugate

$$|\Psi|^2 = \Psi \cdot \Psi^*$$

Probability density of e^Θ

$|\Psi|^2 \Rightarrow$ high \Rightarrow Prob. of finding e^Θ is high
 \Rightarrow less \Rightarrow Prob. of finding e^Θ is less

To find know the Particle at this place

$$\int_{-\infty}^{\infty} |\Psi|^2 d\tau = 1$$

$d\tau = dx dy dz$

Normalised condition

here, Ψ = normalised wave function.

$$\int_{-\infty}^{\infty} |\Psi|^2 d\tau = 0$$

Orthogonal condition.

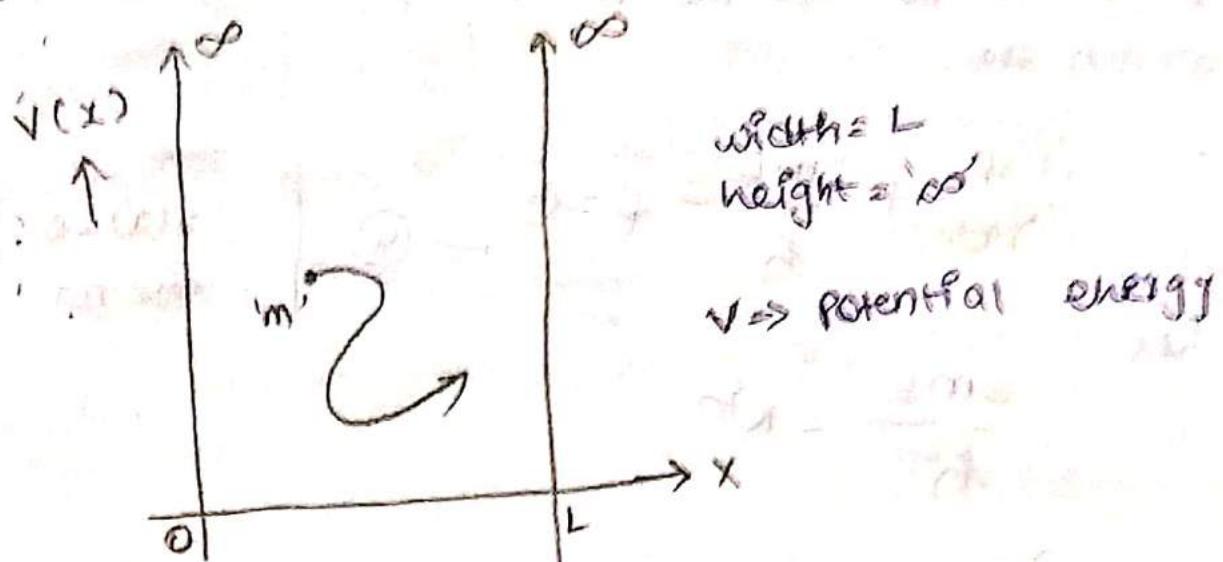
Not Normalised condition

Limitations of wave function (Ψ):-

- 1) Wave function should be finite
- 2) It should be continuous
Left limit = Right limit
- 3) It should be single valued function.

Particle in 1-D Potential box (or) Potential Well

Application of Schrodinger time-independent Eq



consider a Particle existing a one dimensional potential box of width 'L' and height = ∞ and the particle is moving freely inside the box. The particle is having some mass 'm' which can move inside freely.

$$V(x) = 0 \Rightarrow \text{inside the box}$$

$$0 < x < L \quad \} -①$$

$$V(x) = \infty \Rightarrow \text{on the box \& outside the box}$$

$$0 \geq x \geq L$$

$$\begin{aligned} \Psi(x) = 0 \text{ at } x=0 \\ \Psi(x) = 0 \text{ at } x=L \end{aligned} \quad \} -② \quad \text{boundary condition}$$

Schrodinger 1-D wave equation is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2} \psi = 0 \quad \text{--- (3)}$$

Inside the box for the particle, eqn (3) can be written as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2} \psi = 0 \quad \text{--- (4)} \quad \left[\because V(x)=0 \text{ inside the box} \right]$$

Let

$$\frac{2mE}{\hbar^2} = K^2$$

$$\frac{\partial^2 \psi}{\partial x^2} + K^2 \psi = 0 \quad \text{--- (5)}$$

Eqn (5) represents second order partial differential equation and general solution for eqn (5) is given by :-

$$\psi(x) = A \sin Kx + B \cos Kx \quad \text{--- (6)}$$

where, A, B, K are constants and which can be determined by applying boundary conditions.

$$\psi(x) = 0 \quad \text{at } x=0$$

$$\psi(x) = 0 \quad \text{at } x=L$$

By applying in eq.n ⑥ $\psi(x) = 0$ at $x=0$

At $x=0$,

$$0 = A \sin k(0) + B \cos k(0)$$

$$0 = EA(0) + B(1)$$

$$\Rightarrow B=0$$

Substituting $B=0$ in eq.n ⑥ we get

$$\Rightarrow \psi(x) = A \sin kx \quad \text{---} ⑦$$

Now, Substituting $\psi(x)=0$ at $x=L$ in eq.n ⑦ we get

$$0 = A \sin k(L)$$

$$A=0 \quad (\text{or}) \quad \sin kL \neq 0$$

$\Rightarrow A \neq 0$ because it is contradiction our assumption.

$$\sin kL = 0$$

$$\sin kL = \sin n\pi$$

$$kL = n\pi$$

$$\therefore k = \frac{n\pi}{L}$$

$$k^2 = \frac{n^2 \pi^2}{L^2}$$

$$\frac{2mE}{k^2} = \frac{n^2 \pi^2}{L^2}$$

$$E = \frac{n^2 \pi^2 \times k^2}{L^2 2m}$$

$$E = \frac{n^2 \pi^2 h^2}{2m L^2 4\pi^2}$$

$$\lambda = \frac{h}{2\pi}$$

$$\lambda^2 = \frac{h^2}{4\pi^2}$$

$$E_n = \frac{n^2 h^2}{8m L^2}; n=1, 2, 3, \dots$$

Substituting ' $\lambda = \frac{n\pi}{L}$ ' in eq.n ⑦ we get

$$\therefore \psi(x) = A \sin\left(\frac{n\pi x}{L}\right) - ⑧$$

To find A :

$$\int_0^L (\psi)^2 dx = 1$$

$$\int_0^L \left| A \sin\left(\frac{n\pi x}{L}\right) \right|^2 dx = 1$$

$$\int_0^L \left[A^2 \sin^2\left(\frac{n\pi x}{L}\right) \right] dx = 1$$

$$A^2 \int_0^L \left[\frac{1 - \cos 2\left(\frac{n\pi x}{L}\right)}{2} \right] dx = 1$$

$$\Rightarrow \frac{A^2}{2} (x)_0^L - \left(\frac{A^2}{2} \sin 2\left(\frac{n\pi x}{L}\right) \right)_0^L = 1$$

$$\Rightarrow \frac{A^2 L}{2} = 1$$

$$\Rightarrow A^2 = \frac{2}{L} \rightarrow A = \sqrt{\frac{2}{L}}$$

substituting value of λ' in eq.n ⑧ we get

$$\therefore \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad \boxed{9}$$

Interpretation of Wave function (ψ):-

corresponding to the value $n=1, 2, 3, \dots$ we get

E_1, E_2, E_3, \dots which are discrete energy levels.

1st excited energy level is $\boxed{n=2}$ which gives us

$$E_2 = \frac{4h^2}{8mL^2} \Rightarrow 4E_1 \quad \text{--- 1st}$$

zero energy of the system (or) lowest energy

of the system is when $\boxed{n=1}$ which gives us

$$E_1 = \frac{h^2}{8mL^2}$$

2nd excited energy level is $\boxed{n=3}$ which gives

$$\text{as } E_3 = \frac{9h^2}{8mL^2} = 9E_1 \quad \text{--- 2nd}$$

$$\therefore \boxed{E_n = n^2 E_1}$$

4)

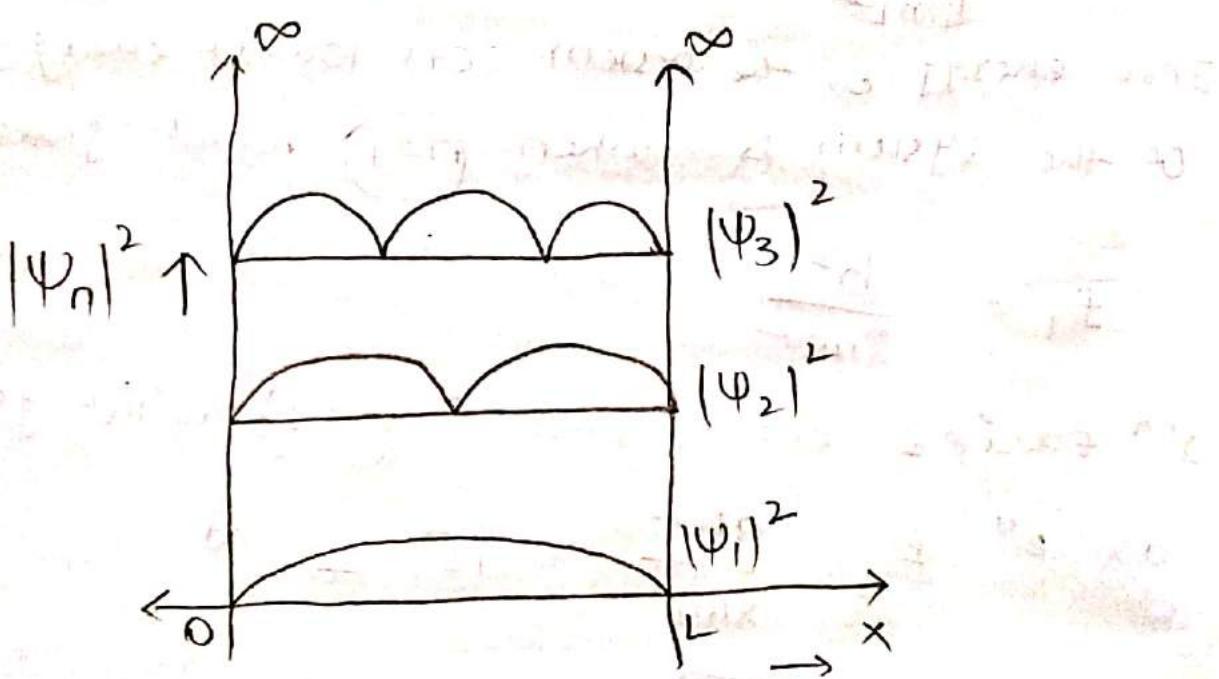
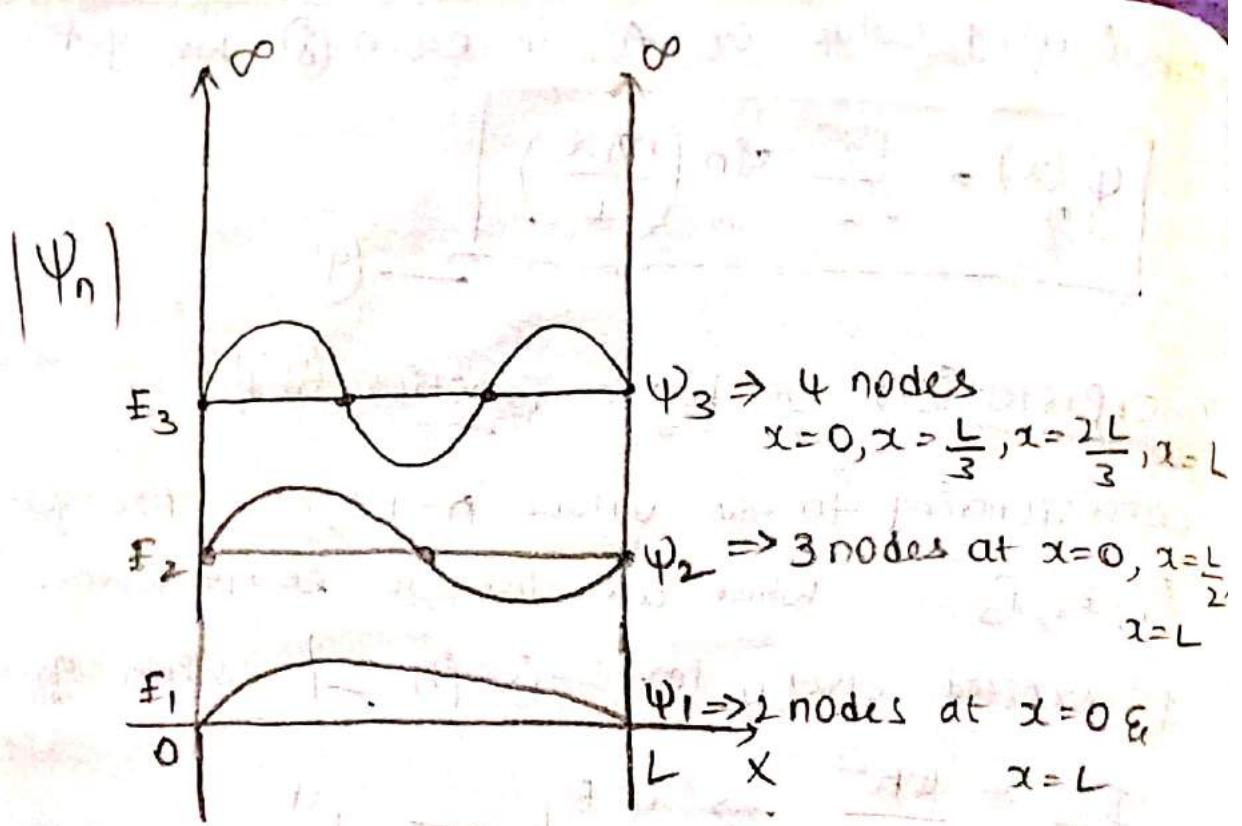
$n = 1, 2, 3, \dots$ \rightarrow Eigen Energy values \Rightarrow quantised

$E_1, E_2, E_3 \rightarrow$ Eigen Energy values

$\downarrow \downarrow$
 $\psi_1, \psi_2, \psi_3 \rightarrow$ Eigen wavefn's

$\Downarrow \Downarrow$

Normalised surely present



6) Energy Spacing

$$E_{(n+1)} - E_n = (n+1)^2 E_1 - n^2 E_1$$

$$= (n^2 + 2n + 1) E_1 - n^2 E_1$$

$$= (2n+1) E_1$$

$$1D \psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

In 3D, $\boxed{\psi_n(x, y, z) = \left(\frac{2}{L}\right)^{3/2} \cdot \sin\left(\frac{n\pi x}{L}\right) \cdot \sin\left(\frac{n\pi y}{L}\right) \cdot \sin\left(\frac{n\pi z}{L}\right)}$

Limitations of wave function (ψ):-

- 1) wave formation function should be finite
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Left limit = Right limit
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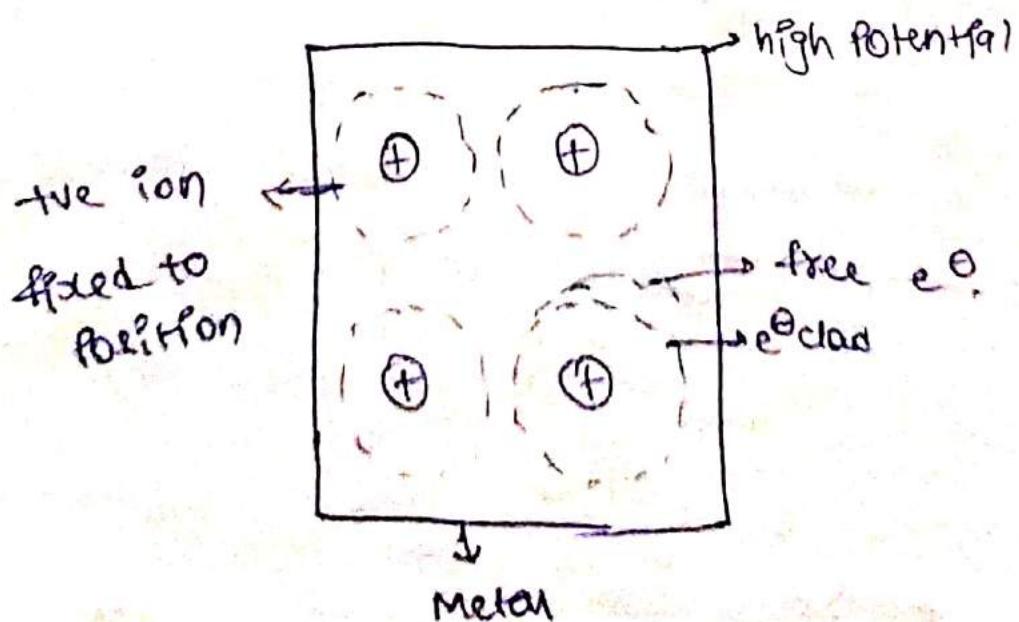
2. 1 Electron Theory of Metals

e[⊖] behaviour in metals solids

- 1) Electrical prop's of solids
- 2) Thermal properties
- 3) Magnetic properties
- 4) Optical properties

- classical free e[⊖] theory of metals 2) Quantum free e[⊖] theory 3) zone theory (or band theory of solids (or) metals)
- Drude & Lorentz Sommerfeld Bloch
- Maxwell - Boltzmann theory statistics → Bohr - Bose - Einstein statistics → Fermi - Dirac statistics

1) Classical free e[⊖] theory of Metals:-

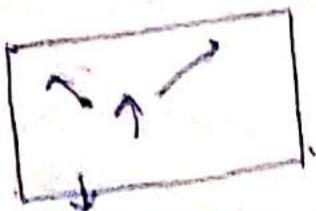


The interaction between free e^- and +ve ion (or) free e^- and free e^-

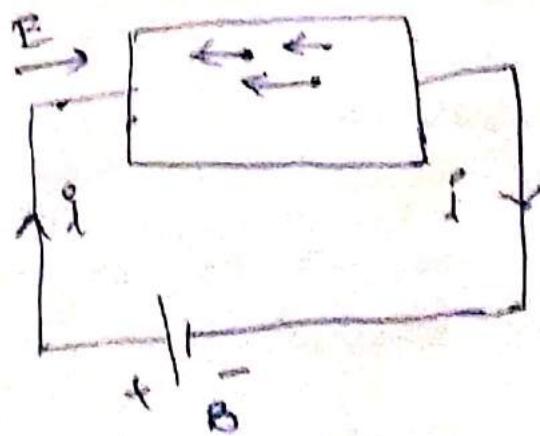
is known as Metallic bond.

Interaction (or) collisions of elastic in nature (no loss of energy)

When external thermal energy is applied to the metal, the valence e^- 's gets detached from its valence orbit. So, these valence electrons cannot come outside of the metal, because high potential is present at the ends of the metal. These removed e^- atom is known as " +ve ion ". This +ve ion is fixed to its position. The interactions (or) collisions between e^- 's are elastic in nature where there is no loss of energy.



When external field is not applied the e^- 's are in random motion.



When external field is applied the electrons move opposite to the applied field in the metal.

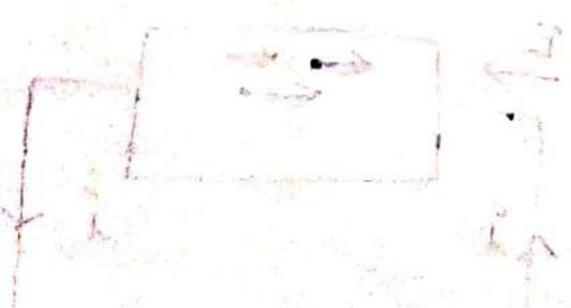
The energy required to remove an electron from the metal is known as work function.

Merits:-

- 1) Ohm's law verification
 $V = I R$
- 2) Electrical conductivity & Thermal conductivity (σ) (σ_T)
Relation is (Weidmann - Franz law)
- 3) Electrical & Thermal properties
- 4) Optical properties

Demerits:-

- 1) couldn't explain the specific conductivity (C_V)
- 2) couldn't explain the conductivity and e^{Θ} concentration



Quantum Free e^{\ominus} theory of Metals

Proposed by Sommerfeld

⇒ Interactions (or) collisions between free e^{\ominus} and a ion (or)

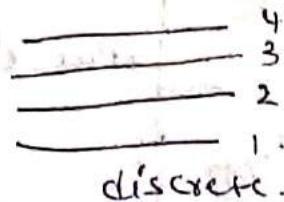
free e^{\ominus} and free e^{\oplus}

are elastic in nature. and they are negligible.

⇒ +ve ion is fixed to its position.

⇒ The free e^{\ominus} can move through out the metal but not escape from metal.

⇒ The energy levels are discrete in nature but not continuous means in particular number and they are said to be quantised



⇒ The free electrons move with constant potential energy inside the metal.

⇒ Obey's Pauli's exclusion principle means no two electrons occupies same energy level

⇒ When external field is not applied the free e^{\ominus} s moves randomly

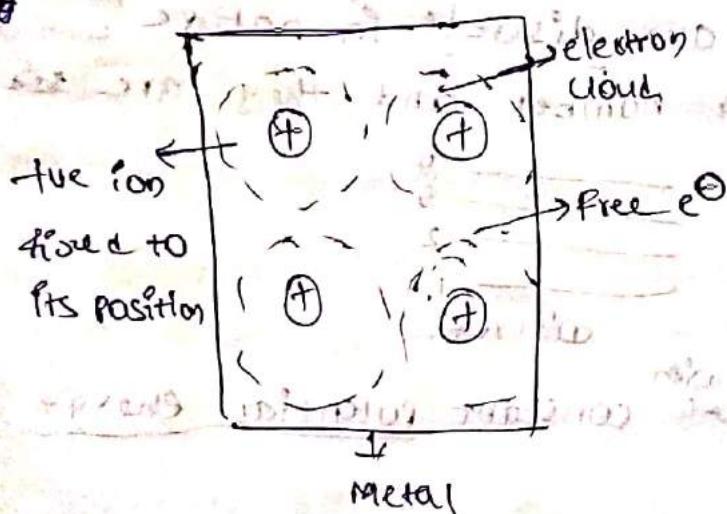
⇒ When external field applied the free e^{\ominus} s moves opposite to the applied field

Merits:-

- 1) It explained successfully specific conductance of metals.
- 2) The dependence of electrical and thermal conductivity on Temperature and electron concentration.

Drawbacks:-

- 1) It could couldn't explain why some solids behave as conductors, semi-conductors and insulators.



Maxwell - Boltzmann Statistics

Bose-Einstein Statistics

Fermi-Dirac Statistics

- It is applicable for classical laws and it is only applicable for Macro elements.

- The particles are elements are identical and distinguishable.
- The energy levels are continuous.
- There is no limit to occupy particles in the energy levels.

- It is applicable for Quantum laws and it is only applicable for Micro elements.

- The elements are Identical but indistinguishable.
- The energy levels are discrete.
- There is no limit to occupy particles in the energy levels.

- It is applicable for Quantum laws and it is applicable for micro elements.

- The elements are Identical but indistinguishable.
- The energy levels are discrete.
- There is no limit to occupy particles in the energy levels.
- There is a limit to occupy particles in the energy levels. means doesn't obey Pauli's wave principle.

- It is applicable for Quantum laws and it is applicable for micro elements.
- 1) The elements are Identical but indistinguishable.
- 2) The energy levels are discrete.

- Quantum laws and it is applicable for micro elements.
- 1) The elements are Identical but indistinguishable.
- 2) The energy levels are discrete.

- Half integral multiples of $\frac{h}{2}$

- Integral multiples of $\frac{h}{2}$

e) The no. of particles which can be arranged in different available energy levels is given by :-

$$N_p = A \cdot g_i \cdot e^{-E_i/kT}$$

A = constant

g_i = degeneracy energy levels

E_i = Energy levels

k_B = Boltzmann constant

T = Temp

f) The no. of particles can be arranged in different available energy levels is given by :-

$$N_p = \frac{g_i}{e^{(E_i + E_f)/kT} - 1}$$

g) The no. of particles can be arranged in different available energy levels is given by :-

$$n_i^e = \frac{g_i}{e^{(E_i + E_f)/kT}} + 1$$

α^e = constant

Fermi function and Fermi Energy

distribution

$$f_f$$

$$F(E)$$

$$f_f$$

$$E_F$$

occupancy probability

- Fermi function provides the probability of occupancy of an energy level, it is given by

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}$$

where, E = energy level

E_F = Fermi energy level

case (i):- when $T=0K$, $E < E_F$

$$F(E) = \frac{1}{1 + \exp\left(\frac{-E_F}{k_B(0)}\right)} = \frac{1}{1 + e^{-\infty}} = 1$$

$$F(E) = 1$$

implies that

Energy levels are completed.

$F(E) = 1$ implies that all energy levels below the Fermi energy level are completely filled with e^- at absolute '0' Temperature.

Case-(ii) - $T=0K$, $E > E_F$

$$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}} = \frac{1}{1 + e^{\infty}} = \frac{1}{\infty} = 0$$

$f(E)=0$ implies that the energy levels above fermi energy level are completely empty at absolute '0' temperature.

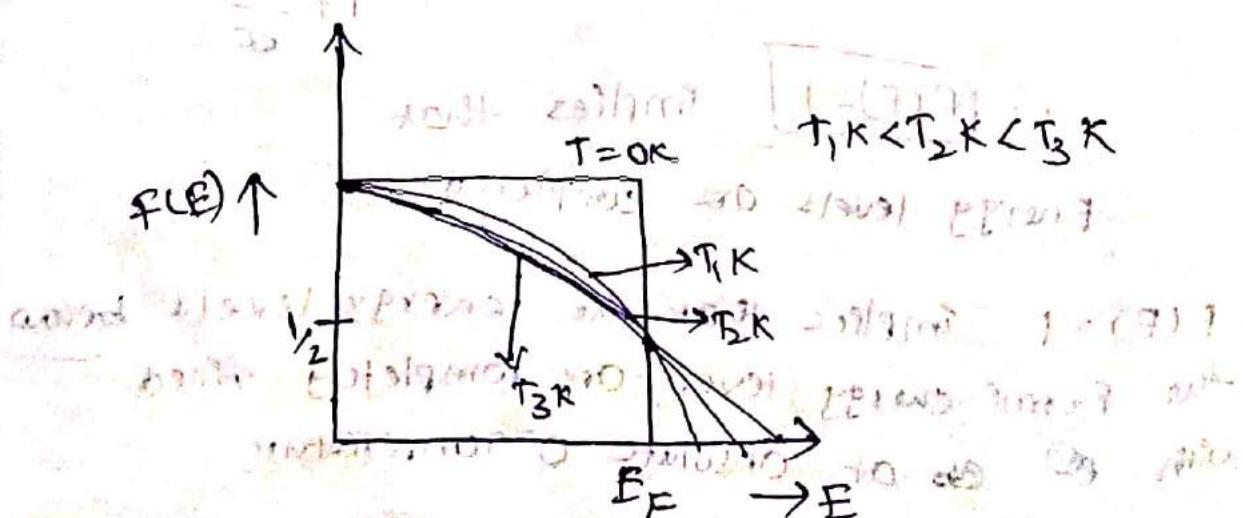
Case-(iii) - $T > 0K$, $E = E_F$

$$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}} = \frac{1}{1 + 1} = \frac{1}{2}$$

$f(E) = \frac{1}{2}$ implies that the fermi energy level is half filled with e^0 at any other temperature.

Fermi energy level -

It is the highest filled energy level with e^0 at absolute '0' temperature.



Density of Energy States! - mass per unit volume

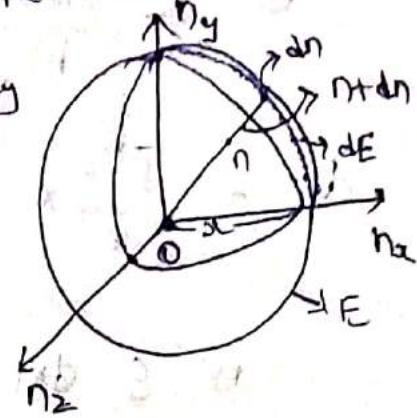
$$N_E = \int Z(E) \cdot F(E) dE$$

Occupancy Probability

Available energy levels

①

$$Z(E) = \frac{1}{8} \left[\frac{4}{3} \pi (n+dn)^3 - \frac{4}{3} \pi n^3 \right]$$



$$= \frac{1}{8} \times \frac{4}{3} \times \pi \left[(n+dn)^3 - n^3 \right]$$

$$= \frac{\pi}{6} \left[n^3 + dn^3 + 3n^2 dn + 3ndn^2 - n^3 \right]$$

$$= \frac{\pi}{6} \left[dn^3 + 3n^2 dn + 3ndn^2 \right]$$

neglecting dn^3 , $3ndn^2$ because dn^3 and dn^2 are more smaller values than dn .

$$= \frac{\pi}{6} \left[3n^2 dn \right] \quad ②$$

$$Z(E) = \frac{\pi}{2} [n^2 dn] \quad ②$$

$$E = \frac{n^2 h^2}{8mL^2}$$

$$n^2 = \frac{8mL^2 E}{h^2} \Rightarrow n = \left(\frac{8mL^2 E}{h^2} \right)^{1/2} \Rightarrow \frac{1}{n} = \left(\frac{h^2}{8mL^2 E} \right)^{1/2}$$

Integrating differentiating above equation we get

$$2ndn = \frac{8mL^2 dE}{h^2}$$

$$dn = \frac{8mL^2 dE}{2nh^2}$$

$$= \frac{1}{2} \left(\frac{8mL^2}{h^2} \right) E^{1/2} \times \left(\frac{h^2}{8mL^2} \right)^{1/2} dE$$

$$dn = \frac{E^{1/2}}{2} \times \left(\frac{8mL^2}{h^2} \right)^{1/2} dE \quad \text{--- (7)}$$

n^2 & dn in eq. n(2)

$$z(E) = \frac{\pi}{2} \times \left(\frac{8mL^2 E}{h^2} \right) \times \left(\frac{E^{1/2}}{2} \right) \left(\frac{8mL^2}{h^2} \right)^{1/2} dE$$

$$\Rightarrow \frac{\pi}{4} \left(\frac{8mL^2}{h^2} \right)^{3/2} dE \times E^{1/2} \quad \text{--- (8)}$$

$$= \frac{\pi}{4h^2} 8^{3/2} m^{3/2} L^3 E^{1/2} dE$$

here, $L^3 = 1$ because we took density for unit volume.

$$z(E) = \frac{8^{3/2} \pi m^{3/2} E^{1/2}}{4h^3} dE$$

$z(E)$

$$z(E) = 2 \times \left(\frac{8^{3/2} \pi m^{3/2} E^{1/2}}{4h^3} \right) dE$$

consider $z(E)$ or z

consider $z(E)$ be the no. of available energy levels

Let $f(E)$ be the occupancy probability of energy levels. Then electron density is given by

$$n_c = \int z(E) \cdot f(E) dE \quad \text{--- (1)}$$

and density of energy states is given by :-

and density of energy states is given by:-

$$g(E) = \frac{n_e}{\text{Volume of energy states}} \quad (2)$$

The density of energy states is the no. of available energy levels filled with e^Θ s per unit volume of the system.

Consider a sphere of radius 'n' and every point n_1, n_2 so on represents different radii of the sphere and correspondingly we have different energy levels, which are always positive. The no. of energy states in the sphere of radius 'n'

$$= \frac{4}{3} \pi n^3$$

The no. of energy states contributed by each part of the sphere is equal to $\frac{1}{8} \left(\frac{4}{3} \pi n^3 \right) = \frac{\pi}{6} n^3$.

The no. of available energy states within a small range of energy levels of E & $E+dE$ having radii n & $n+dn$ respectively is given by

$$Z(E) = \frac{\pi}{6} (n+dn)^3 - \frac{\pi}{6} n^3$$

$$Z(E) = \frac{\pi}{6} [n^2 dn]$$

We know that the energy of particle is given by:-

$$E = \frac{n^2 h^2}{8mL^2} \quad \text{where } L = \text{width of the system}$$

* n_e, n_B

Acc. to Pauli's exclusion principle, a pair of e^Θ s having opposite spin can be accommodated in each energy level so $n_e - n_B$ becomes

$$\text{Continuation} \quad (8)^{3/2} = 2^3 \cdot (2)^{3/2}$$

$$2(E) \approx 4\pi(2m)^{3/2} \frac{L^3}{h^3} E^{1/2} dE$$

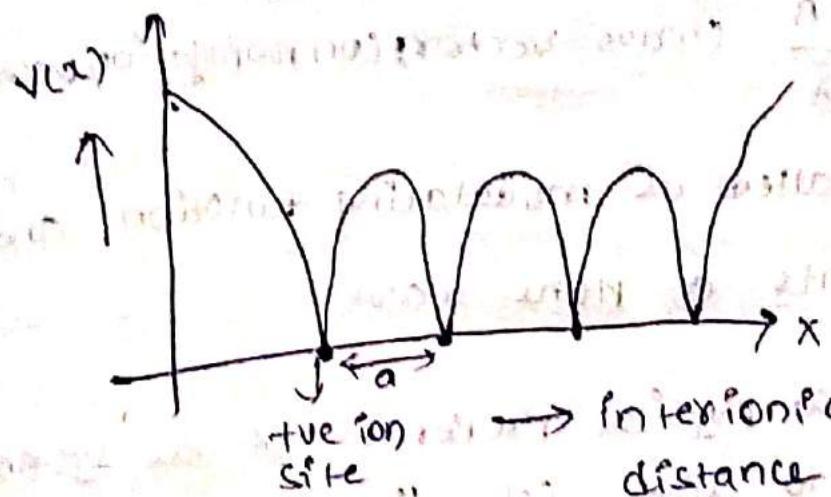
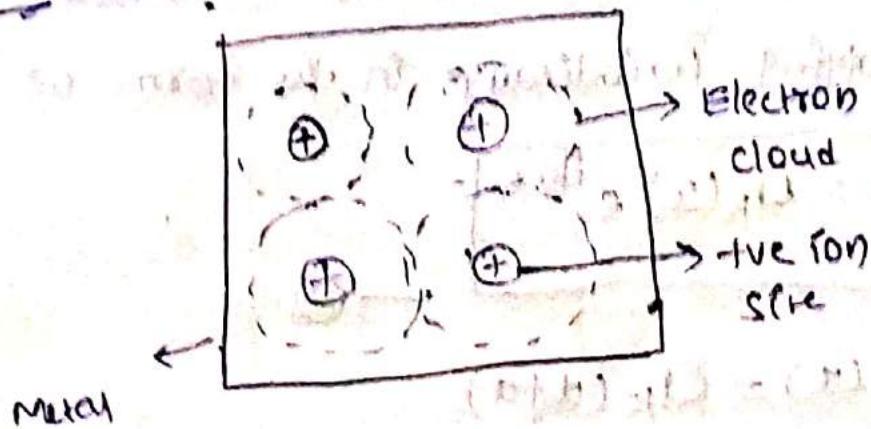
∴ The density of energy states is given by:-

$$g(E) = \frac{4\pi(2m)^{3/2}}{\sqrt{3}} \frac{L^3}{h^3} \int E^{1/2} \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)} dE$$

2. Band Theory of Solids

SSE
Bloch

Theorem:-



At +ve ion site \Rightarrow Minimum

Consider a crystal (or) metal consisting of periodic arrangement of +ve ions through which the e^- moves freely. At the +ve ion site, the potential of the e^- is minimum, and in between the +ve ion sites the potential of the e^- is maximum. Thus the potential of the e^- varies periodically. Schrodinger equation for the moving particle is given by:-

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2} \psi = 0$$

①

where, $V(x) = V(x+q)$

where, a is inter ionic distance

Bloch gave you the solution for eqn ①, where potential is varying periodically. In the form of

$$\boxed{\Psi(x) = U_k(x) e^{ikx}} \quad (2)$$

where, $U_k(x) = U_k(x+q)$

$$K = \frac{2\pi}{\lambda} \quad (\text{wave vector}) \text{ (or) propagation vector}$$

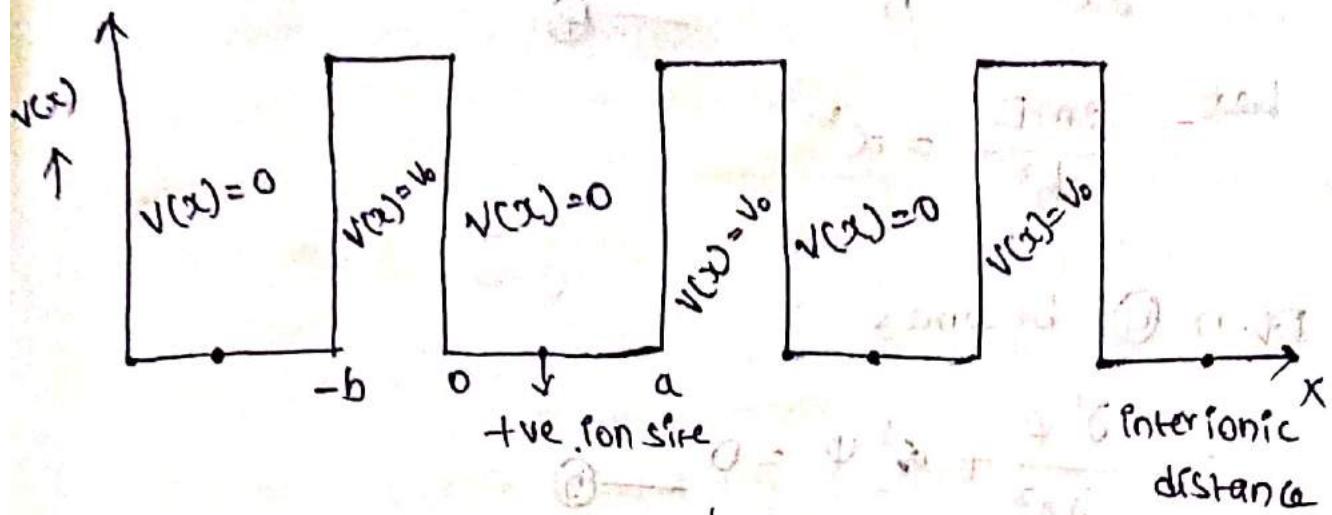
$U_k(x)$ is called as modulating function and e^{ikx} represents a plane wave.

To understand the physical properties of the system it is required to solve Schrödinger equation. However, it is extremely difficult to solve it because of the complex function in the Bloch solution. Hence Kronig-Penny model is adopted for simplification of Schrödinger equation.

Kronig Penny Model:-

(or)

Particle moving in a Periodic Potential:-



According to Zone theory, the e^Θ moves in a periodic potential which is produced by the tve ion sites and the potential of the e^Θ varies periodically, with inter ionic spacing.

The potential of the e^Θ is maximum in between the tve ion sites and it is minimum at the tve ion site.

from figure

$$V(x)=0 ; 0 < x < a \quad \text{---} ①$$

$$V(x)=V_0 ; -b < x < 0 \quad \text{---} ②$$

The Schrodinger wave equation for T-D Case is given by:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2} \psi = 0 \quad \text{---} ③$$

where, $V(x) = V(x+a)$

The solution of

when $V(x) = 0$, eq. ③ becomes

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2} \psi = 0 \quad \text{--- ④}$$

Let

$$\frac{2mE}{\hbar^2} = \alpha^2$$

Eq. n ④ becomes

$$\frac{\partial^2 \psi}{\partial x^2} + \alpha^2 \psi = 0 \quad \text{--- ⑤}$$

When $V(x) = V_0$, eq. ③ becomes

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V_0)}{\hbar^2} \psi = 0 \quad \text{--- ⑥}$$

Let

$$\frac{2m(V_0-E)}{\hbar^2} = -\beta^2$$

Eq. n ⑥ becomes

$$\frac{\partial^2 \psi}{\partial x^2} - \beta^2 \psi = 0 \quad \text{--- ⑦}$$

For eq. n ⑤ & ⑦ the solution is only of the form given by Bloch.

$$\psi(x) = u_k(x) \cdot e^{ikx} \quad \text{--- ⑧}$$

where $u_k(x) = u_k(x+a)$

$$k = \frac{2\pi}{\lambda}$$

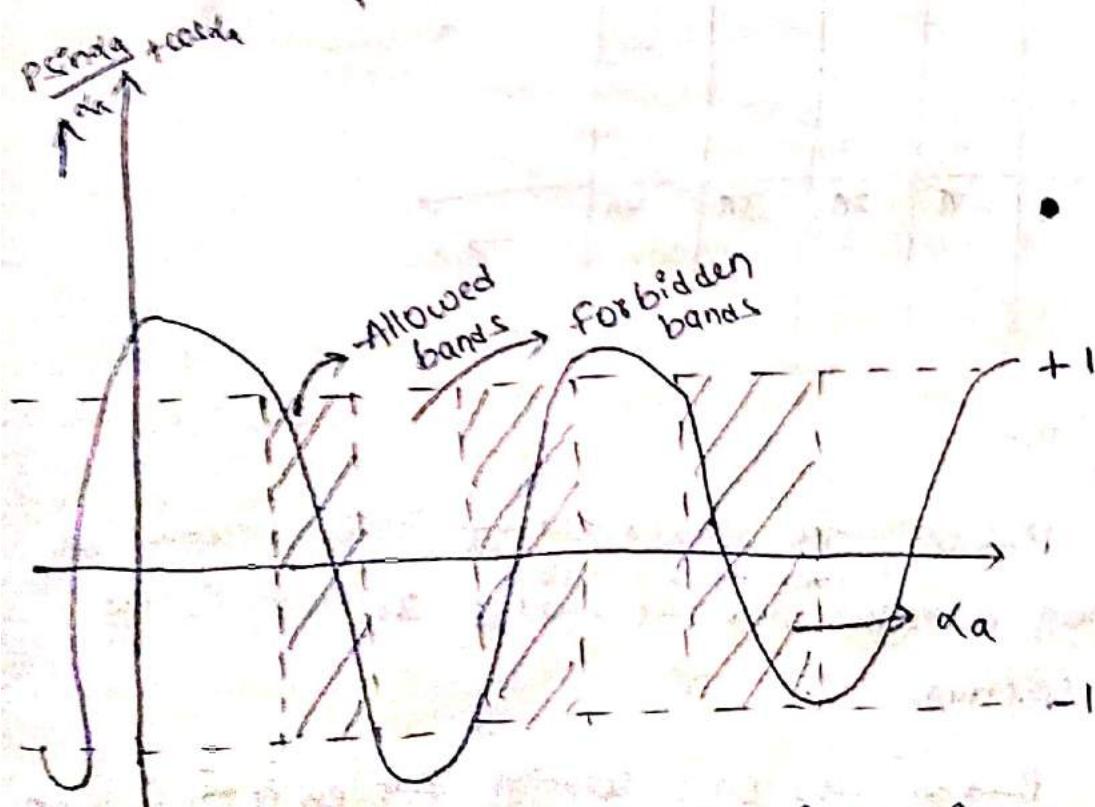
v differentiating eqn ③ twice w.r.t α and
vortex! substituting in eqn ② & eqn ④ we get
two independent second order linear differential
equations containing A, B, C, D as constants and
which can be determined by applying determinant
method as

$$\frac{P \sin \alpha}{\alpha} + \cos \alpha = \cos k \alpha$$

⑨

P - Potential barrier strength;

Graph - ①: $\frac{P \sin \alpha}{\alpha} + \cos \alpha$ Vs α



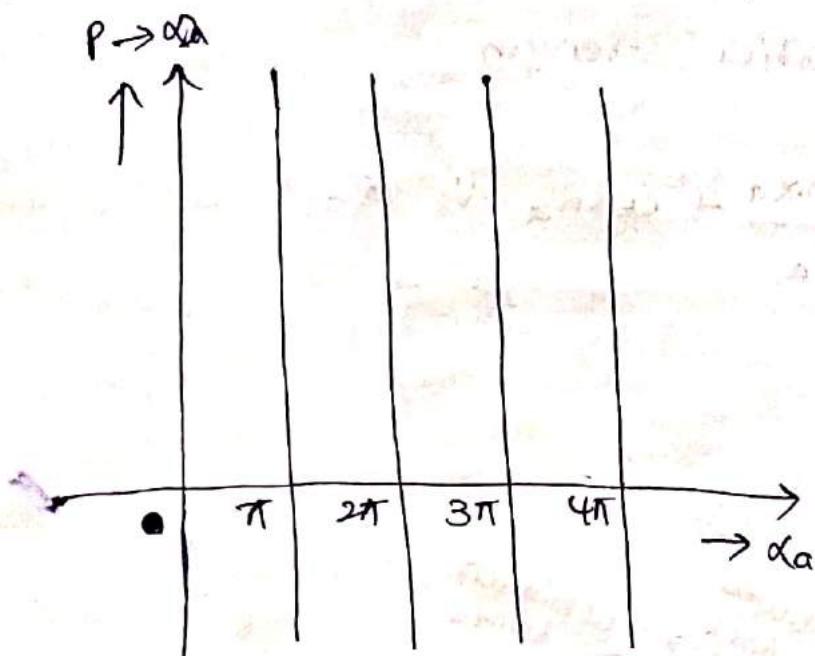
width of allowed bands increasing
width of forbidden bands decreasing.

Conclusion:

between the :

- 1) From graph, A number of allowed bands separated with a number of forbidden bands is obtained
- 2) The width of the allowed bands increases with the increase of ' α_a ' and the width of forbidden bands decreases with the increase of ' α_a '.

Graph - ② :- when $P \rightarrow \infty$ VR α_a :



Conclusion:

- 1) When $P \rightarrow \infty$, the allowed energy bands become infinitely narrow ^(means touch at a place) and the energy spectrum is the pure spectrum
- 2) When $P \rightarrow \infty$, the only solution for eqn ⑨ is given by

$$\sin \alpha_a = 0,$$

$$\sin \alpha_a = \sin n\pi$$

$$da = n\pi$$

$$d = \frac{n\pi}{a}$$

$$\alpha^2 = \frac{n^2\pi^2}{a^2}$$

$$\text{But } \alpha^2 = \frac{2mE}{h^2}$$

$$\frac{2mE}{h^2} = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2\pi^2 h^2}{2ma^2}$$

$$= \frac{n^2\pi^2 h^2}{2ma^2 4\pi^2}$$

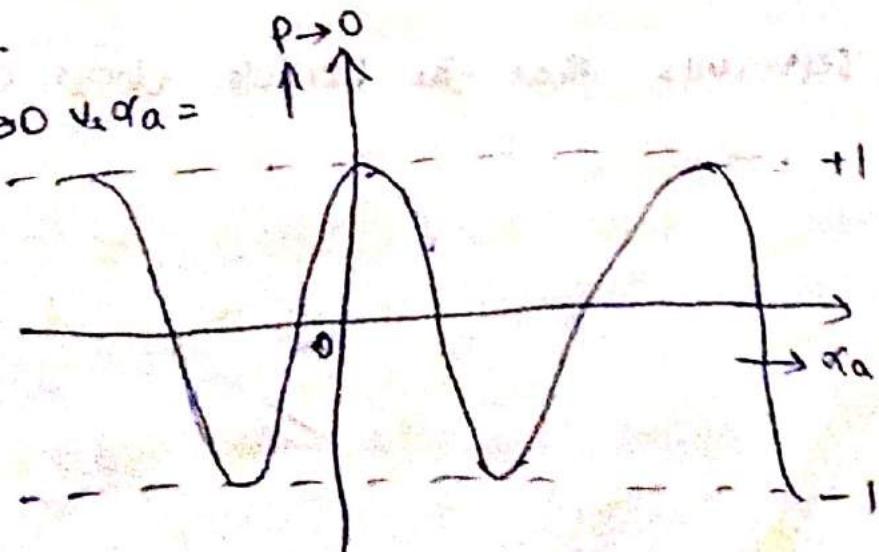
$$E = \frac{n^2 h^2}{8ma^2} \quad \textcircled{10}$$

where, $n = 1, 2, 3, \dots$

eq.n ⑩ shows that the energy spectrum of e^- contains discrete energy levels separated by forbidden bands and the particles obeys Quantum laws

Graph - ③ :-

when $P \rightarrow 0$ $\propto a =$



Conclusion!

1) From graph, we can observe that the particle will be moving within the range of $+1 \& -1$

when $P \rightarrow 0$ the solution of eqn 9 is

$$\cos \alpha = \cos k a$$

$$\alpha = k a$$

$$k = K$$

$$\alpha^2 = k^2$$

$$\frac{2mE}{k^2} = \frac{4\pi^2}{\lambda^2}$$

$$E = \frac{4\pi^2 k^2}{2m\lambda^2}$$

$$= \frac{4\pi^2 k^2}{2m \frac{k^2}{\lambda^2} \frac{4\pi^2}{\lambda^2}}$$

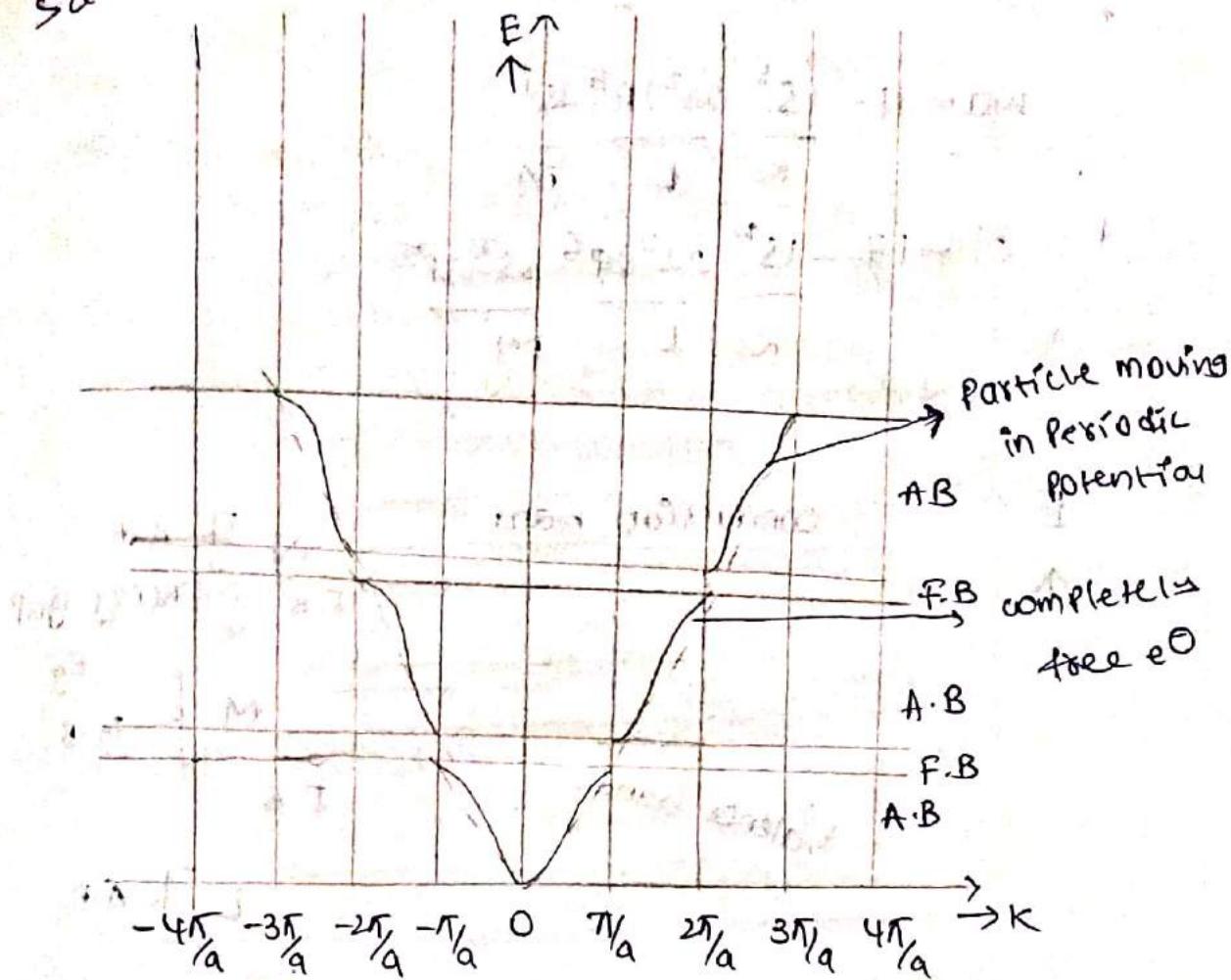
$$E = \frac{1}{2} m v^2 = K.E$$

→ 11

Eqn 11 represents that the particle obeys classical laws

Brillouin zones (or) E-K diagram,
 =
 (or)
 graph

V.V
SEE



A.B = Allowed band

$$K = \pm \frac{n\pi}{a}, n=1,2,3$$

F.B = Forbidden band.

+
discontinuities

1st Brillouin zone (or) 1st allowed band

$$K = -\frac{\pi}{a} \text{ to } +\frac{\pi}{a}$$

2nd B.Z (or) 2nd allowed band

$$K = -\frac{2\pi}{a} \text{ to } -\frac{\pi}{a} \text{ & } +\frac{\pi}{a} \text{ to } +\frac{2\pi}{a}$$

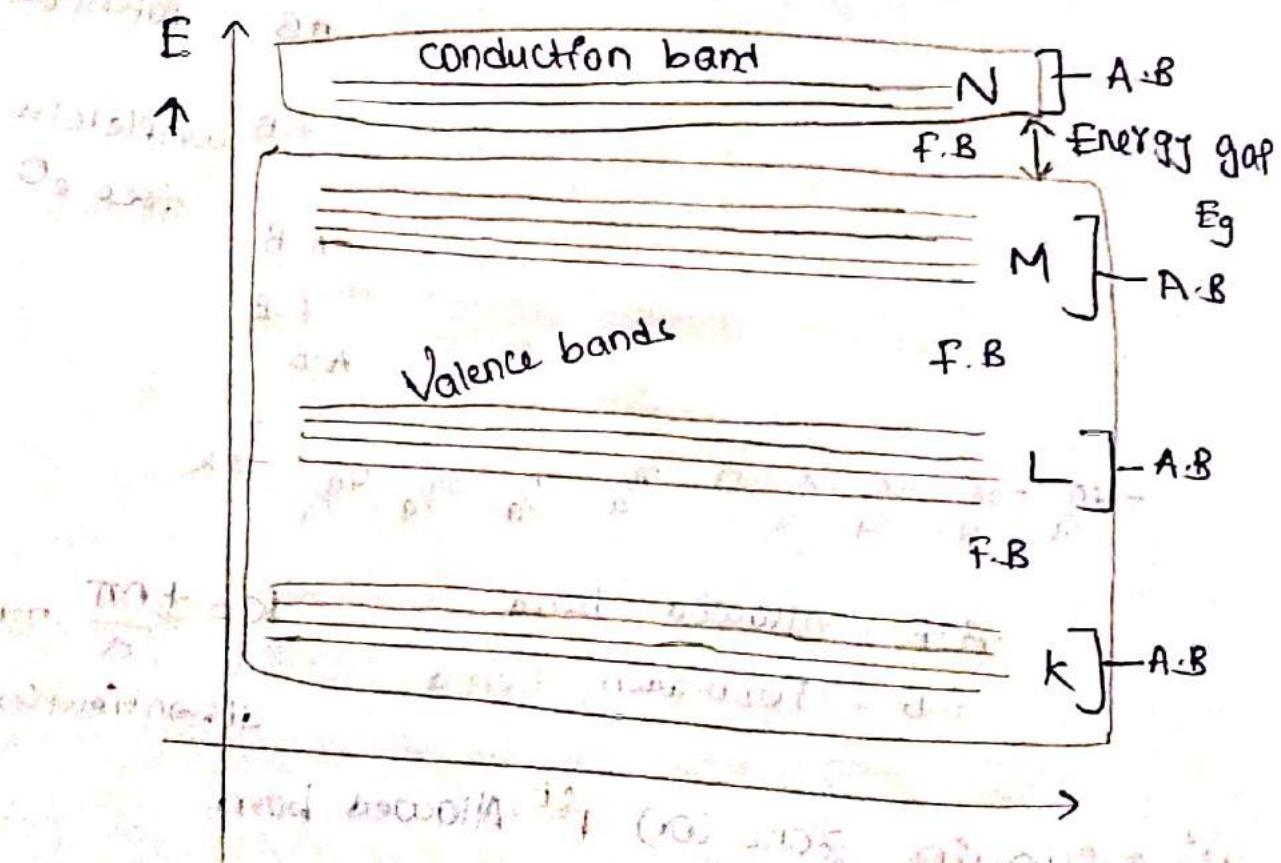
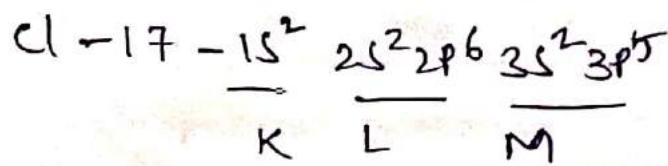
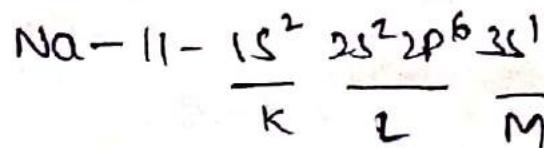
3rd B.Z (or) 3rd allowed band

$$K = -\frac{3\pi}{a} \text{ to } -\frac{2\pi}{a} \text{ & } +\frac{2\pi}{a} \text{ to } +\frac{3\pi}{a}$$

Origin of energy bands in solids :-

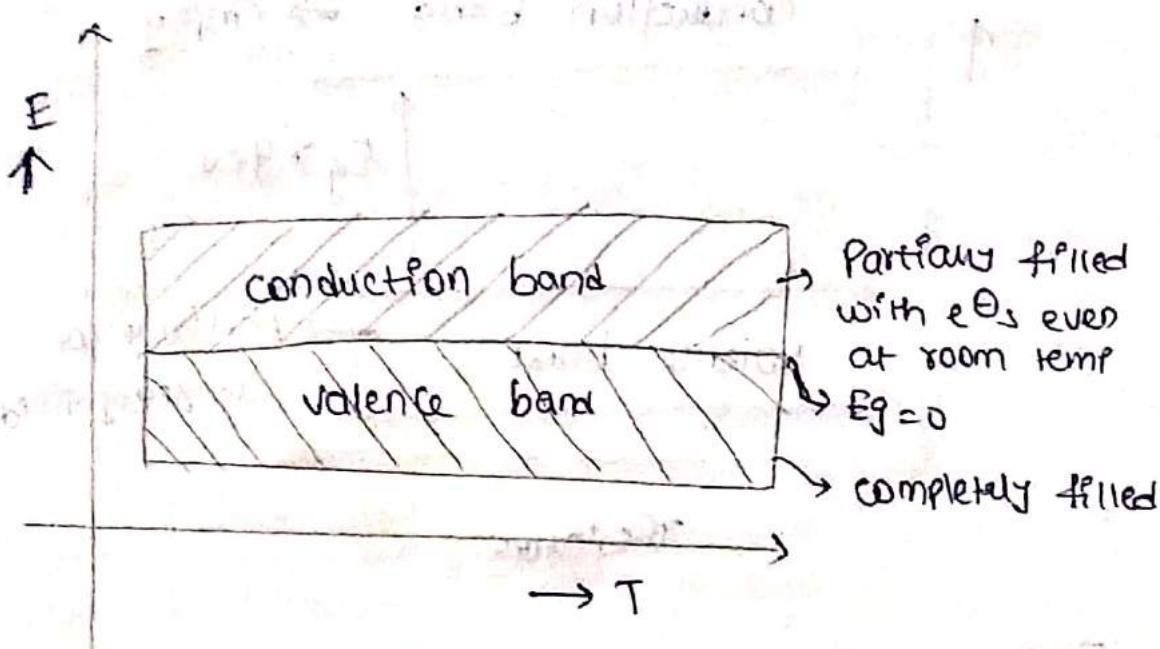
(or)

Classification of solids:-



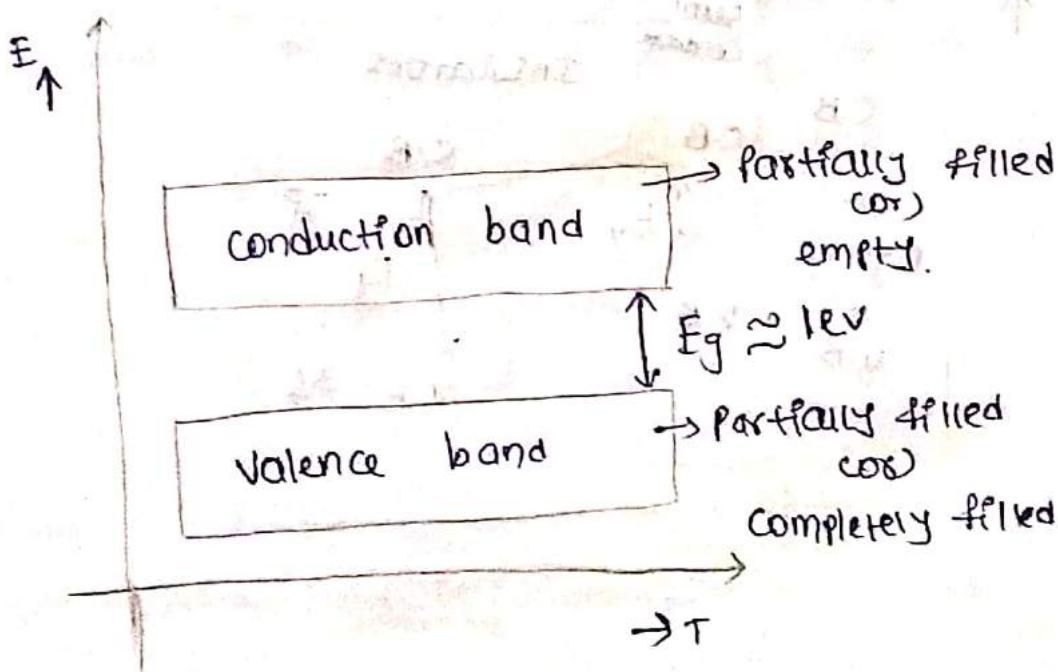
Based on Energy gap between C.B & V.B.

1) Conductors:-



Ex:- Nichrome

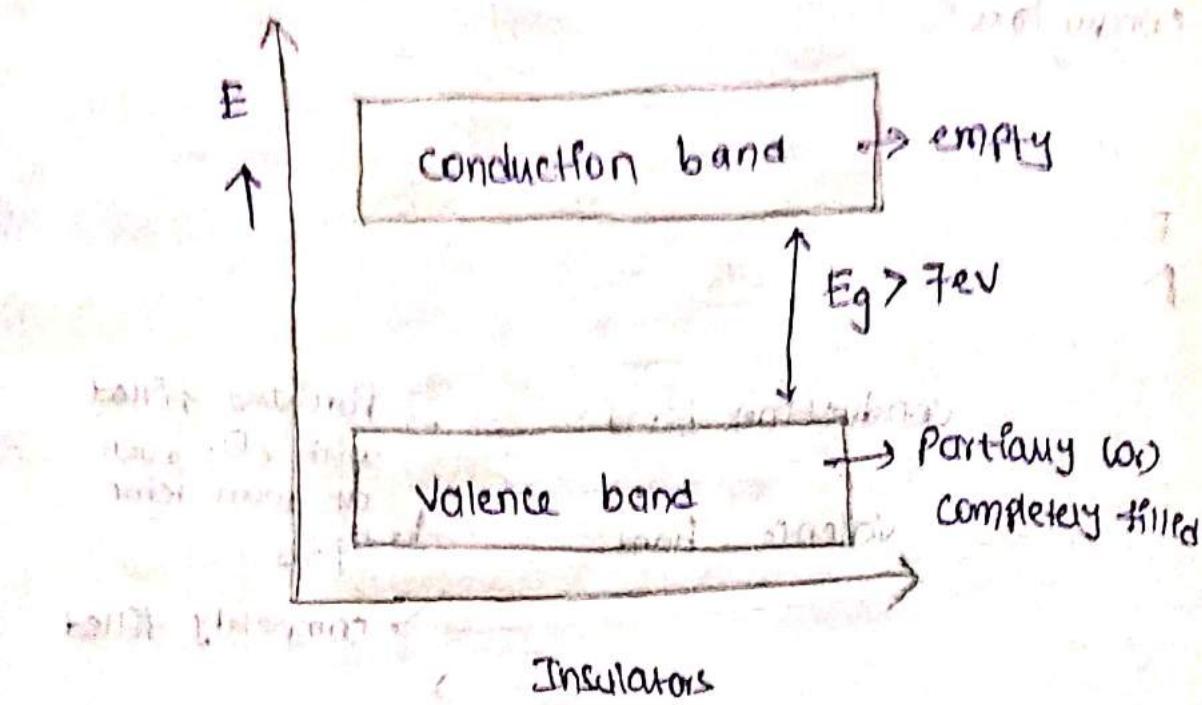
2) Semi-conductors:-



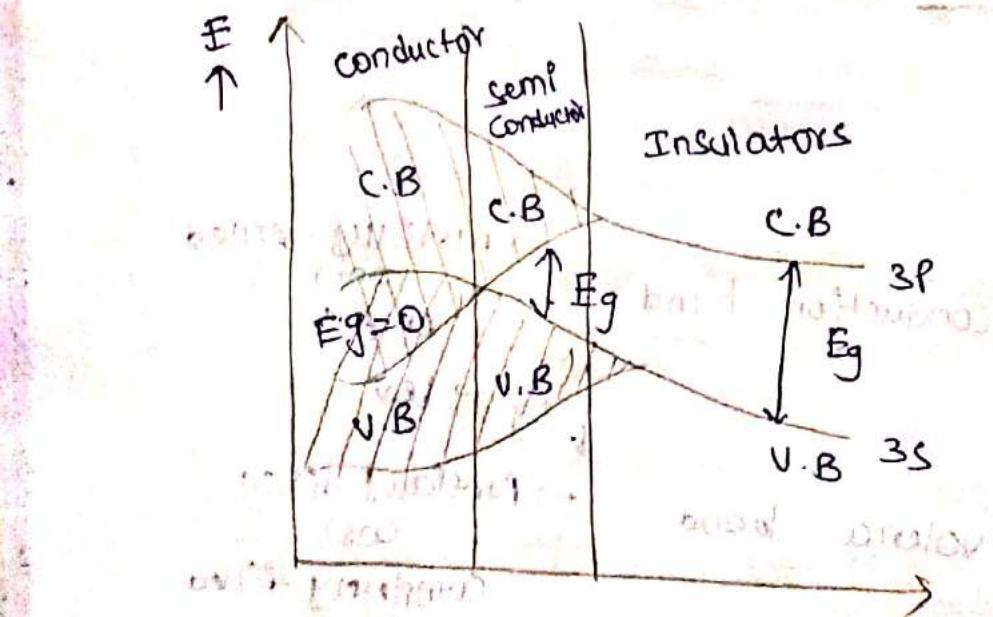
Semi conductors

Examples:- all 4th group elements

3) Insulators



Ex:- wood, rubber, plastic, felt cor. huk



Effective mass of the electron (m^*):-

It corresponds to the particle moving in a periodic potential is subjected to magnetic field (or) electric field.

We know that force $F = m^* \cdot a$ — (1)

where m^* = effective mass of the electron

a = acceleration

$$V_g = \frac{d\omega}{dk}$$

— (2)

V_g - group velocity

ω = angular momentum

$$\omega = 2\pi\vartheta$$

$$= 2\pi \frac{E}{h}$$

$$\omega = \frac{E}{\hbar}$$

$$\left[\because E = h\vartheta \quad k = \frac{\hbar}{2\pi} \right]$$

$$\vartheta = \frac{E}{\hbar} \quad \frac{1}{k} = \frac{2\pi}{\hbar}$$

$$\therefore V_g = \frac{1}{\hbar} \frac{dE}{dk}$$

Rate of change of velocity is defined as

acceleration and is given by:-

$$a = \frac{dv_g}{dt} = \frac{1}{\hbar} \frac{d^2E}{dk \times dt}$$

$$a = \frac{1}{\hbar} \frac{d^2E}{dk^2} \times \frac{dk}{dt} — (3)$$

Momentum (p) in terms of propagation vector (k) is given by

$$P = \hbar k \quad \text{--- (4)}$$

Rate of change of momentum is defined as force

$$F = \frac{dp}{dt} = \hbar \frac{dk}{dt}$$

$$\therefore \frac{dk}{dt} = \frac{\hbar}{m} F \quad \text{--- (5)}$$

Substituting (5) in (3)

$$\therefore a = \frac{1}{\hbar} \frac{d^2 E}{dk^2} \times \frac{1}{m} \cdot F$$

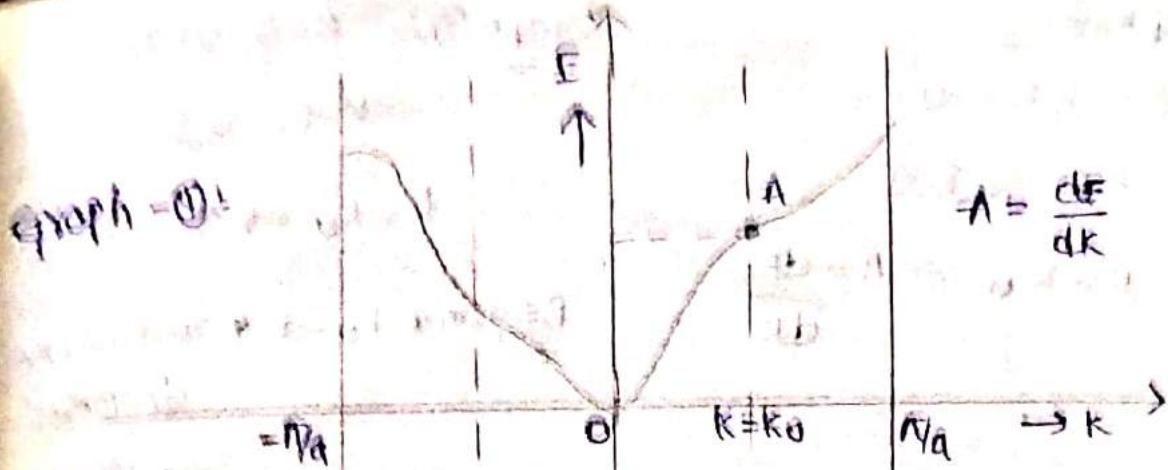
$$a = \frac{1}{\hbar^2} \times \frac{d^2 E}{dk^2} \cdot F$$

$$F = \frac{\hbar^2 \cdot a}{\left(\frac{d^2 E}{dk^2} \right)} \quad \text{--- (6)}$$

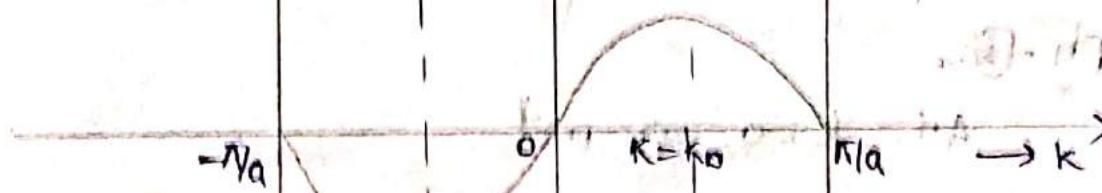
From eqn (1) & eqn (6) we get

$$m a = \frac{\hbar^2}{\left(\frac{d^2 E}{dk^2} \right)}$$

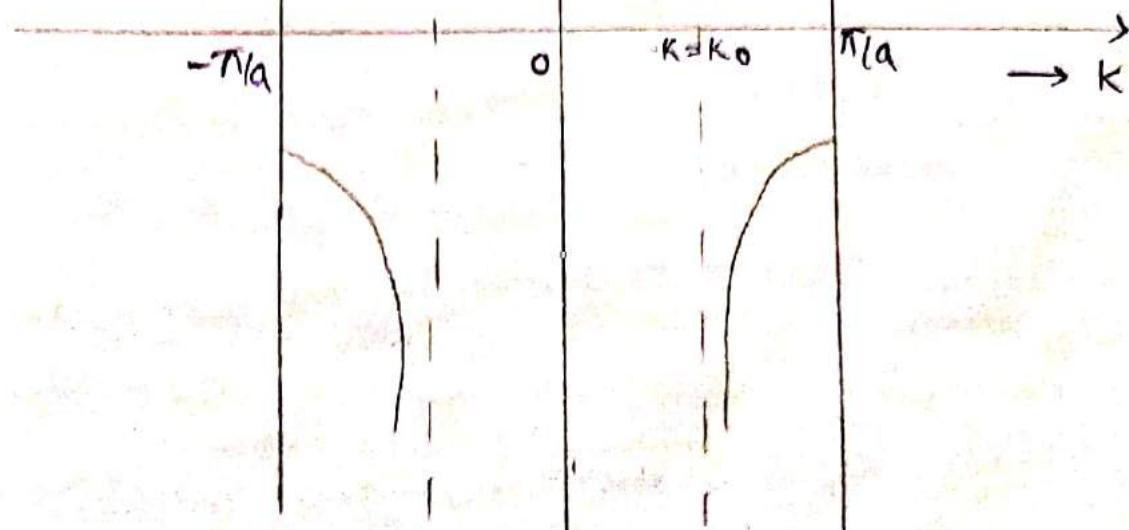
Graph - ① :-



Graph - ② :-



Graph - ③ :-



Graph-①:-

$$K=0, E=0$$

$$K \uparrow \Rightarrow E \uparrow$$

$$\text{At } K=k_0 \rightarrow A = \frac{dE}{dk}$$

Graph ②:- $K=0, V=0$

$$K \uparrow, V \uparrow$$

$$K=k_0 \rightarrow V = M_0$$

Beyond $k_0 \rightarrow V$ -decreasing
decreasing
Mode

$$K = \frac{\pi}{a} \rightarrow V=0$$

$$K=k_0$$

Point of Inflection

Graph-③:-

$$\text{At } K=0, m=m^*$$

$$K \uparrow, m \uparrow$$

$$K=k_0, m^* - \text{max}$$

Beyond $k_0, m^* \rightarrow -\infty$
 \downarrow
hole.

III 1 Semiconductor Physics

purity of material

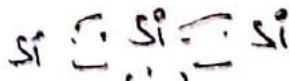
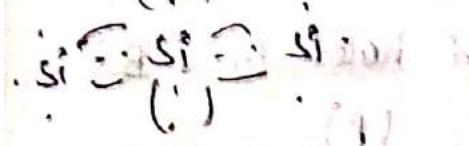
Intrinsic (Pure)

IV group no free e^-

Si, Ge, Pb etc...

Si

(:)



Si

Intrinsic

Increase the conductivity
of semiconductor

p-type

n-type

↑

↑

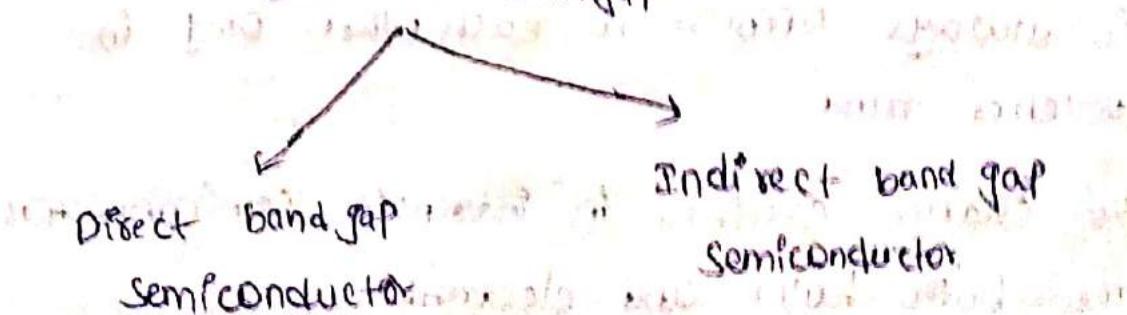
III group + Intrinsic

II group + Intrinsic

↓
impurity

(n)

Based on Band gap



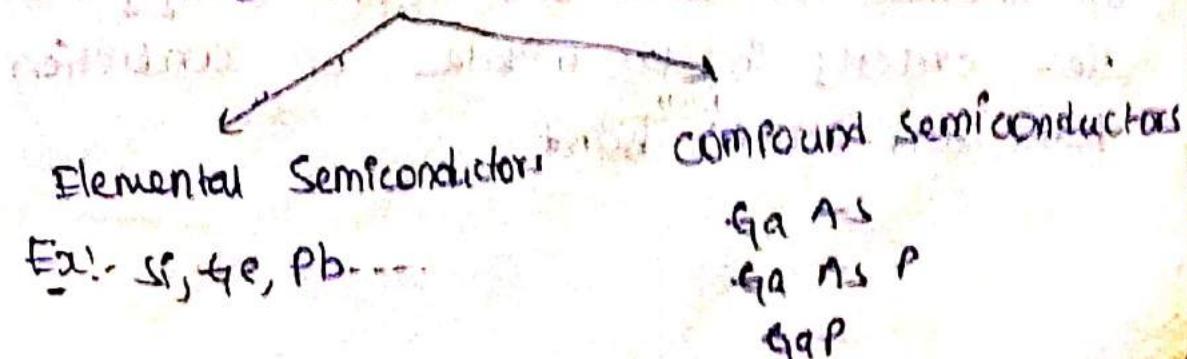
Direct band gap

Semiconductor

Indirect band gap

Semiconductor

Based on state of level



Elemental Semiconductors

Ex:- Si, Ge, Pb....

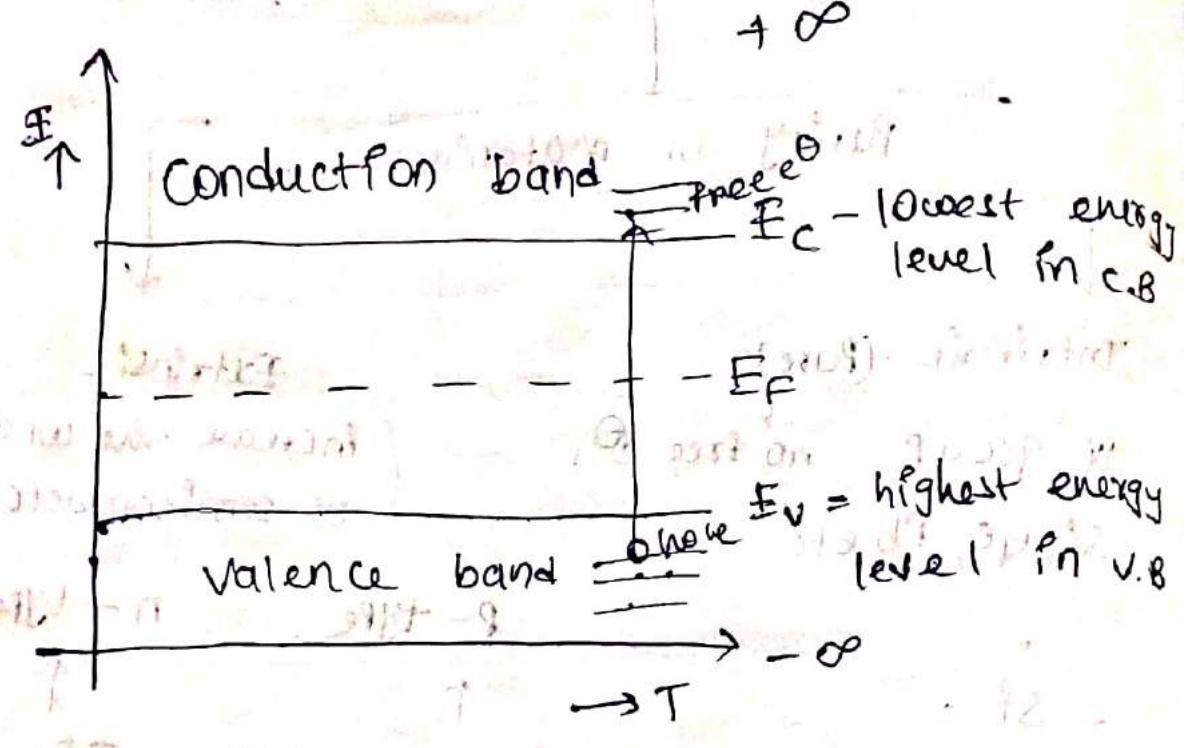
Compound Semiconductors

GaAs

GaN P

GaP

Intrinsic semiconductors:-



"The no. of e^- s in C.B = NO. of holes in V.B
(n) (p)

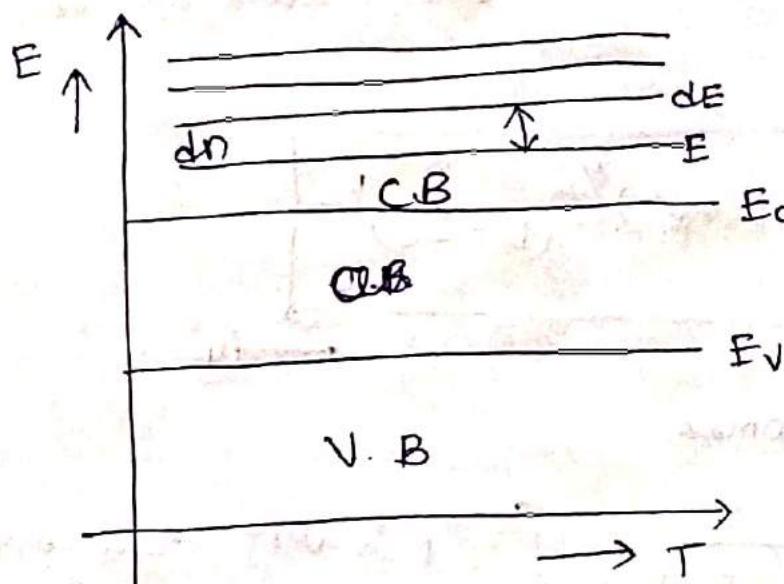
$$n = p$$

- The e^- movement and the hole movement is always opposite to each other only in valence band.
- The charge carriers in intrinsic semiconductors are both holes and electrons.
- In intrinsic semiconductors Fermi energy level lies exactly in the middle of conduction band and valence band.

- the range of conduction band is E_c to $+\infty$
- the range of valence band is E_v to $-\infty$

calculation of carrier concentration of an intrinsic semiconductor :-

Part-I:- Calculation of no. of e^{Θ} s (n) in C.B of I.S.C



Let d_n be the no. of e^{Θ} s in the range 'E' and ' $E+DE$ ' in the conduction band of an intrinsic semiconductor and it is given by:-

$$dn = Z(E) \cdot F(E) dE \quad \text{--- ①}$$

where, $Z(E)$ is called available energy levels
 $F(E)$ is called occupancy probability for e^{Θ}

$$Z(E) = \gamma \cdot E^{1/2}$$

$$\text{where, } \gamma - \text{constant} = \frac{4\pi}{h^3} (2m^*)^{3/2}$$

$E^{1/2}$ - half power of energy

$$Z(E) = \frac{4\pi}{h^3} (2m_e^*)^{3/2} E^{1/2} \quad \text{--- (2)}$$

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)} \quad \text{--- (3)}$$

In conduction band, $E > E_C$

→ because above E_C all energy levels are more

eq.n (2) becomes

$$Z(E) = \frac{4\pi}{h^3} \times (2m_e^*)^{3/2} (E - E_C)^{1/2} \quad \text{--- (4)}$$

and eq.n (3) becomes

$$F(E) = \frac{1}{\exp\left(\frac{E - E_F}{k_B T}\right)} \quad \left[\because 1 \text{ is very small compared to the exponential term we can neglect it} \right]$$

$$= e^{-\left(\frac{E - E_F}{k_B T}\right)} = e^{\left(\frac{E_F - E}{k_B T}\right)} \quad \text{--- (5)}$$

Substituting eq.n (4) & (5) in eq.n (1) we get

$$dn = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_C)^{1/2} \times e^{\left(\frac{E_F - E}{k_B T}\right)} \times dE \quad \text{--- (6)}$$

The no. of $e\theta$ s in the CB of an I.S.C is given

by:-

$$n = \int_{E_C}^{+\infty} dn$$

$$\begin{aligned} n &= \int_{E_C}^{+\infty} \frac{4\pi}{h^3} \times (2m^*_e)^{3/2} (E - E_C)^{1/2} \times e^{\left(\frac{E_F - E}{k_B T}\right)} \times dE \\ &= \frac{4\pi}{h^3} (2m^*_e)^{3/2} \cdot \exp\left(\frac{E_F}{k_B T}\right) \int_{E_C}^{+\infty} (E - E_C)^{1/2} \exp\left(\frac{-E}{k_B T}\right) dE \end{aligned} \quad \text{--- (7)}$$

Let $E - E_C = x$ Lower Limit :- $x = 0$

$E = x + E_C$ Upper Limit : $x = \infty$

$$dE = dx$$

Substituting all the above values in eq.n(7) we get

$$\begin{aligned} n &= \frac{4\pi}{h^3} (2m^*_e)^{3/2} \cdot \exp\left(\frac{E_F}{k_B T}\right) \cdot \int_0^{\infty} x^{1/2} \cdot \exp\left(\frac{-x - E_C}{k_B T}\right) dx \\ &= \frac{4\pi}{h^3} (2m^*_e)^{3/2} \cdot \exp\left(\frac{E_F - E_C}{k_B T}\right) \cdot \int_0^{\infty} x^{1/2} \cdot \exp\left(\frac{-x}{k_B T}\right) dx \end{aligned}$$

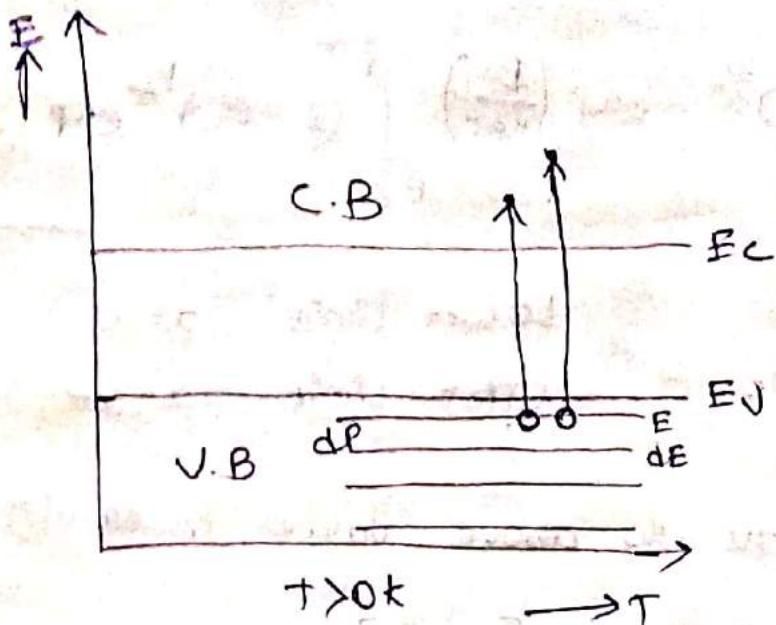
By using gamma functions we have.

$$\int_0^{\infty} x^{1/2} \exp\left(\frac{-x}{k_B T}\right) dx = \boxed{\frac{\sqrt{\pi}}{2} (k_B T)^{3/2}}$$

$$\therefore n = \frac{4\pi}{h^3} (2m^*_e)^{3/2} \cdot \exp\left(\frac{E_F - E_C}{k_B T}\right) \cdot \frac{\sqrt{\pi}}{2} (k_B T)^{3/2}$$

$$n = 2 \left(\frac{2m^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_c}{k_B T} \right)$$

Part - II - Calculation of no. of holes (p) in V.B of I.S.C.



Let dP be the number of holes present in the valence band between the range ' E ' and ' $E+DE$ ' of I.S.C and it is given by:-

$$dP = Z(E) \cdot f_h(E) \cdot dE$$

where, $Z(E)$ is called available energy levels, $f_h(E)$ is called occupancy probability of holes.

$$Z(E) = \gamma \cdot E^{1/2}$$

where, γ - constant = $\frac{4\pi}{h^3} (2m^*)^{3/2}$

$E^{1/2}$ - half power of energy

$$z(E) = \frac{4\pi}{h^3} (2m^*_h)^{3/2} \cdot E^{1/2} \quad \text{--- (2)}$$

In valence band, $E_V > E$

eq.n (2) becomes

$$z(E) = \frac{4\pi}{h^3} (2m^*_h)^{3/2} (E_V - E)^{1/2} \quad \text{--- (3)}$$

$$\therefore f_e(E) + f_h(E) = 1, \quad \text{--- (4)}$$

$$\therefore f_h(E) = 1 - f_e(E)$$

$$= 1 - \left[\frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)} \right]$$

$$= 1 - \left[1 + \exp\left(\frac{E - E_F}{k_B T}\right) \right]^{-1}$$

$$\left[\because (1+x)^{-1} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} \right]$$

$$= 1 - \left[1 - \frac{\exp\left(\frac{E - E_F}{k_B T}\right)}{1!} + \frac{\left(\exp\left(\frac{E - E_F}{k_B T}\right)\right)^2}{2!} - \dots \right]$$

$$\therefore F_h(E) = \exp\left(\frac{E - E_F}{k_B T}\right) \quad \text{--- (5)}$$

Substs eq.n (3) & (5) in eq.n (1) we get,

$$dP = \frac{4\pi}{h^3} (2m^*_h)^{3/2} (E_V - E)^{1/2} \exp\left(\frac{E - E_F}{k_B T}\right) \cdot dE$$

--- (6)

∴ No. of holes in V.B or I.S.C is given by

$$P = \int_{-\infty}^{E_V} dP = \int_{-\infty}^{E_V} \frac{4\pi}{h^3} (2m_n^*)^{3/2} (E_V - E)^{1/2} \exp\left(\frac{E - E_F}{k_B T}\right) dE$$

$$P = \frac{4\pi}{h^3} (2m_n^*)^{3/2} \exp\left(\frac{-E_F}{k_B T}\right) \int_{-\infty}^{E_V} (E_V - E)^{1/2} \cdot \exp\left(\frac{E - E_F}{k_B T}\right) dE$$

Let $E_V - E = x$

Lower $\leftarrow t \Rightarrow x \rightarrow +\infty\right)$

$$E = E_V - x \quad \text{High } t \Rightarrow x \rightarrow 0$$

$$dE = -dx$$

$$\therefore P = \frac{4\pi}{h^3} (2m_n^*)^{3/2} \exp\left(\frac{-E_F}{k_B T}\right) \int_{-\infty}^0 x^{1/2} \cdot \exp\left(\frac{E_V - x}{k_B T}\right) - dx$$

$$= \frac{4\pi}{h^3} (2m_n^*)^{3/2} \exp\left(\frac{E_V - E_F}{k_B T}\right) \int_0^\infty x^{1/2} \cdot \exp\left(\frac{x - E_F}{k_B T}\right) dx$$

$$= \frac{2}{h^2} (2m_n^*)^{3/2} \exp\left(\frac{E_V - E_F}{k_B T}\right) \frac{\sqrt{\pi}}{2} (k_B T)^{3/2}$$

$$\therefore P = 2 \left(\frac{2m_n^* \pi k_B T}{h^2} \right)^{3/2} \cdot \exp\left(\frac{E_V - E_F}{k_B T}\right)$$

Part - III :- Calculation of carrier concentration (n_i^2)
of I.S.C

for an Intrinsic S.C - the no. of e^- s in the C.B is equal to the no. of holes in the V.B.

$$\text{i.e } n = P$$

Let, $n = P = n_i^2$ then

$$n_i^2 = n \cdot P$$

①

Eq.n is called as Law of mass action

According to Law of mass action the product of no. of e^- s in the C.B and the no. of holes in the V.B of I.S.C is a constant (n_i^2)

we know that

$$n = 2 \left(\frac{2m_e^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_C}{k_B T} \right)$$

$$P = 2 \left(\frac{2m_h^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_V - E_F}{k_B T} \right)$$

$$\therefore n_i^2 = 4 \left(\frac{2\pi k_B T}{h^2} \right)^3 (m_e^* m_h^*)^{3/2} \exp \left(\frac{E_V - E_C}{k_B T} \right)$$

∴ carrier conc of I.S.C is given by

$$n_i^2 = 2 \left(\frac{2\pi k_B T}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} \exp \left(\frac{E_V - E_C}{2k_B T} \right)$$

$$E_C - E_V = E_g$$

$$E_V - E_C = -E_g$$

$$n_i = 2 \left(\frac{2\pi k_B T}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} \exp \left(\frac{-E_g}{2k_B T} \right)$$

calculation of Fermi Energy (E_F) of I.S.C.

For an I.S.C., the no. of e^Θ s in the C.B is equal to the no. of holes p_n in V.B

We know that

$$n = 2 \left(\frac{2m_e^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_C}{k_B T} \right)$$

$$p = 2 \left(\frac{2m_h^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_V - E_F}{k_B T} \right)$$

$$n = p$$

$$2 \left(\frac{2m_e^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_C}{k_B T} \right) = 2 \left(\frac{2m_h^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_V - E_F}{k_B T} \right)$$

$$(m_e^*)^{3/2} \exp \left(\frac{E_F - E_C}{k_B T} \right) = (m_h^*)^{3/2} \exp \left(\frac{E_V - E_F}{k_B T} \right)$$

Rearranging the terms, we get

$$\exp\left(\frac{2E_F}{k_B T}\right) = \left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}} \exp\left(\frac{E_V + E_C}{k_B T}\right)$$

Applying log on b.s, we get

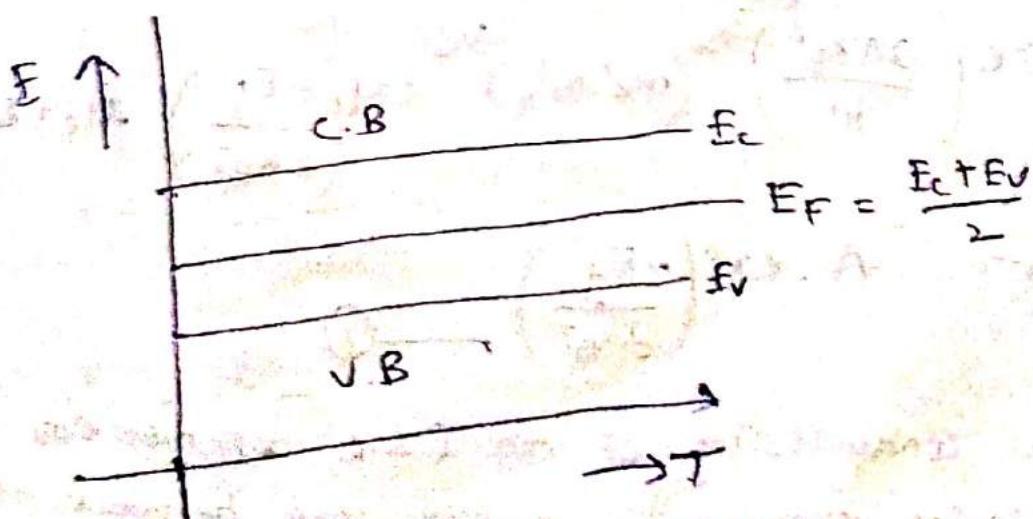
$$\frac{2E_F}{k_B T} = \frac{3}{2} \log\left(\frac{m_h^*}{m_e^*}\right) + \left(\frac{E_V + E_C}{k_B T}\right)$$

Let $m_h^* = m_e^*$; $\log 1 = 0$

$$\Rightarrow \frac{2E_F}{k_B T} = -\frac{E_V + E_C}{k_B T}$$

$$\therefore E_F = \frac{E_C + E_V}{2} \quad \text{--- (2)}$$

Eqn (2) indicates that the Fermi energy lies exactly in between conduction band and valence band of an S.I.C.



Calculation of electrical conductivity (σ_i) of I.S.C.

for an I.S.C. the electrical conductivity is given by :-

$$\sigma_i = n e \mu_e + p e \mu_h \quad \text{--- (1)}$$

where,

n = no. of e^- s in C.B

p = no. of holes in V.B

e = charge of charge carriers

μ_e = mobility of the electrons

μ_h = mobility of the holes

We know that, for an I.S.C

$$n = p = n_i$$

eq.n. (1) becomes

$$\sigma_i = n_i e \mu_e + n_i e \mu_h$$

$$= n_i e (\mu_e + \mu_h)$$

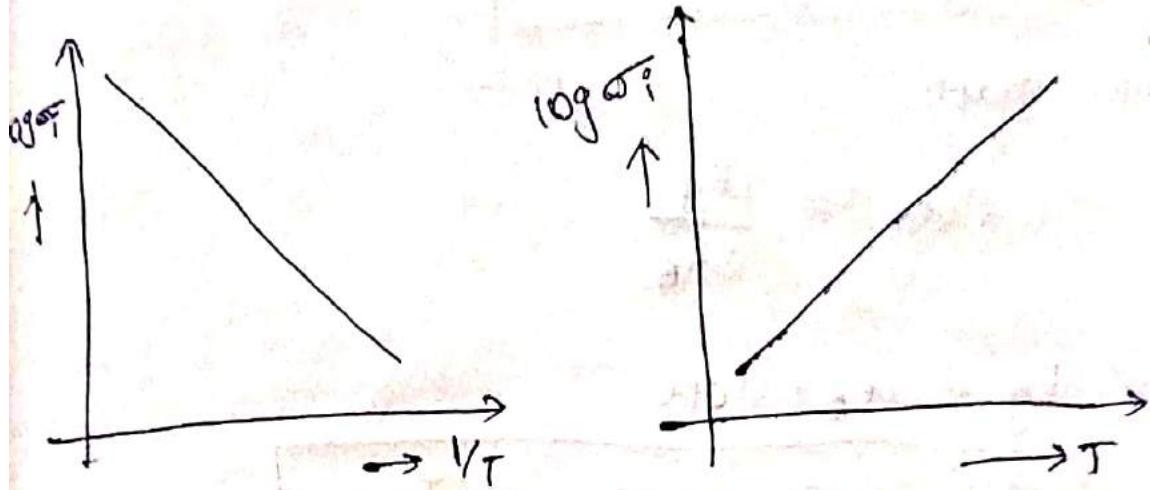
$$= 2e \left(\frac{2\pi K_B T}{h^2} \right)^{3/2} \overset{\text{negligible}}{\cancel{(m_e^* m_h^*)^{3/4}}} \exp\left(\frac{-E_g}{2K_B T}\right) (\mu_e + \mu_h)$$

$$\sigma_i = A \cdot \exp\left(\frac{-E_g}{2K_B T}\right) \quad \text{--- (2)}$$

Thus conductivity of an I.S.C depends on negative exponential energy gap between the C.B and V.B and on the inverse of temperature.

applying 'log' on both sides we get

$$\log \sigma_i = \log A - \frac{E_g}{2k_B T}$$



Calculation of Energy gap (E_g) of I.S.C:-

Resistivity of an I.S.C is given by:-

$$\rho_i = \frac{1}{A} \exp\left(\frac{E_g}{2k_B T}\right)$$

$$R = \frac{\rho l}{a} \quad \therefore \rho = \frac{Ra}{l}$$

$$\therefore \frac{Ra}{l} = \frac{1}{A} \cdot \exp\left(\frac{E_g}{2k_B T}\right)$$

$$R = \frac{l}{Aa} \exp\left(\frac{E_g}{2k_B T}\right)$$

$$R = C \exp\left(\frac{E_g}{2k_B T}\right)$$

Applying log on both sides

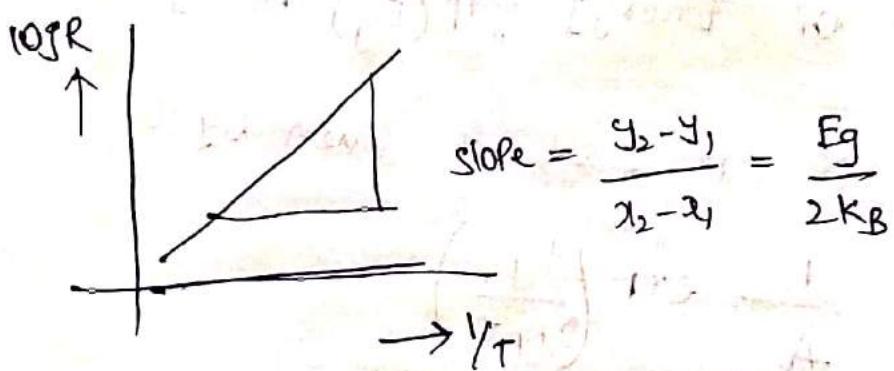
$$\log R = \log C + \frac{E_g}{2k_B T}$$

From graph,

$$\therefore \text{slope} = \frac{E_g}{2k_B}$$

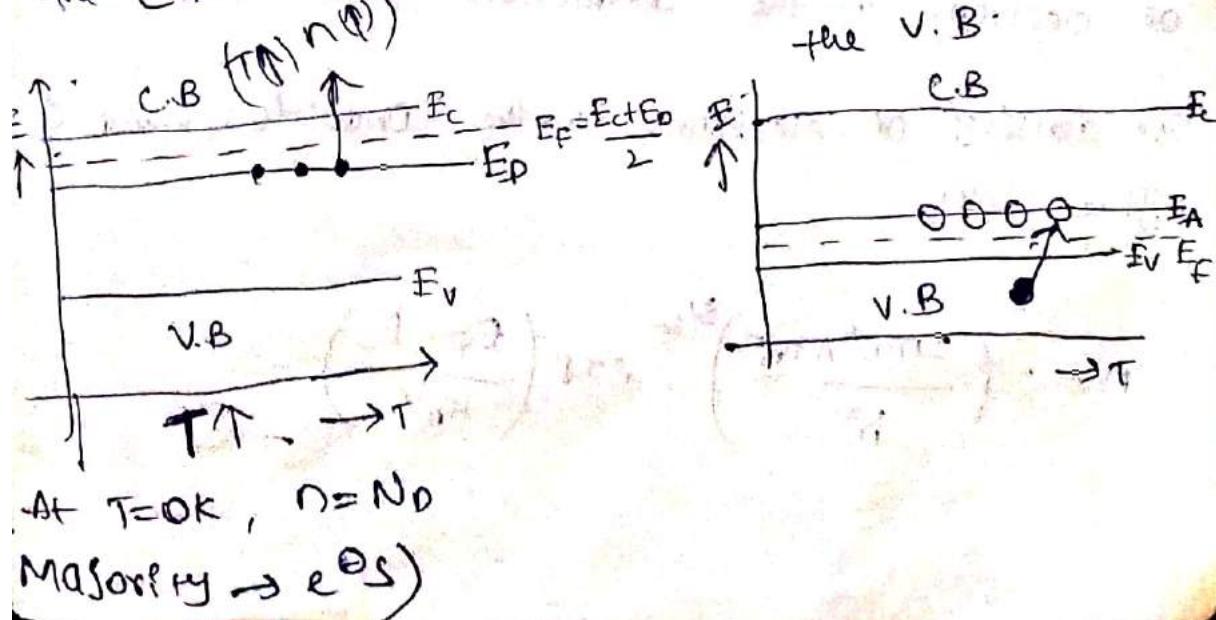
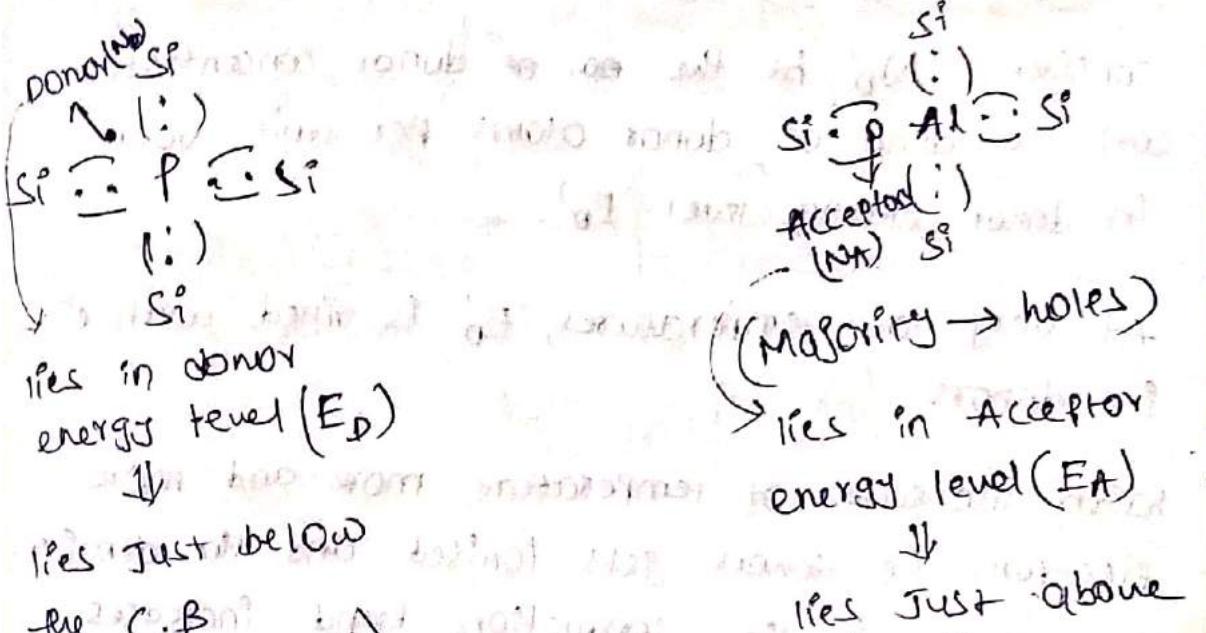
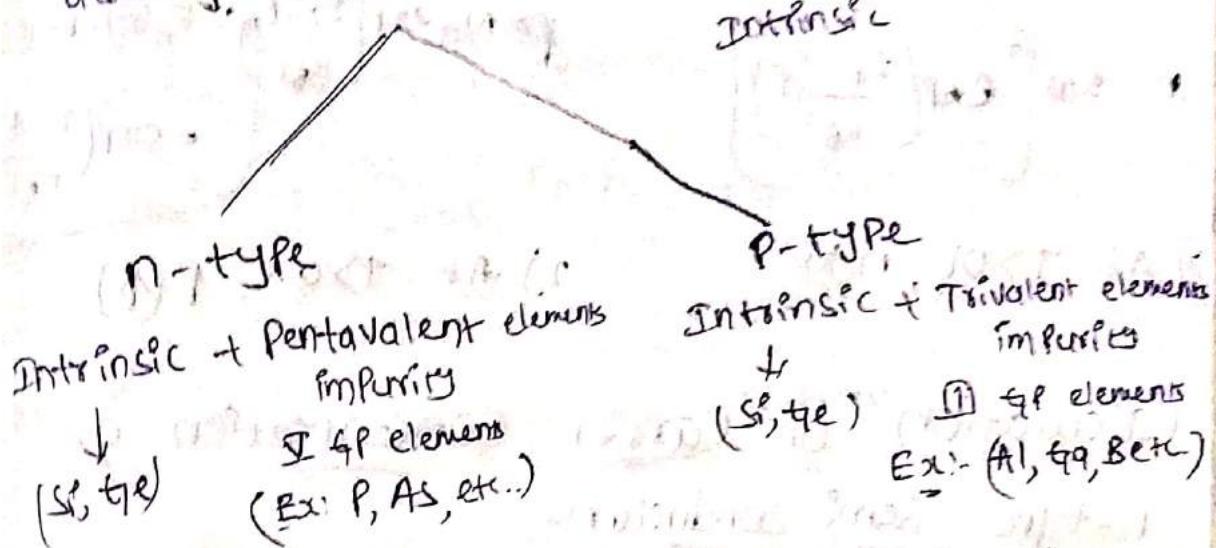
$$\therefore E_g = 2k_B \times \text{slope}$$

$$E_g = 1.9833 \times \text{slope} \times 10^{-4} \text{ e.v}$$



Extrinsic Semiconductors:-

Extrinsic semiconductor is an impure form of semiconductor which obtained by adding impurities to the semiconductor.



At $T=0K$, n -type

1) At $T=0K$,

$$n = N_D [1 - F(E_D)]$$

$$n = N_D \left[\exp\left(\frac{E_D - E_F}{k_B T}\right) \right]$$

2) At $T>0K$, $n(\uparrow)$

P-type

1) At $T=0K$

$$p = N_A [F(E_A)]$$

$$p = N_A \exp\left(\frac{E_F - E_A}{k_B T}\right) : \because f_n(E) = 1 - F(E) \\ = \exp\left(\frac{E - E_F}{k_B T}\right)$$

2) At $T>0K$, $p(\uparrow)$

Calculation of Carrier concentration of n-type Semiconductor:-

consider ' N_D ' be the ~~no.~~ donor concentration
i.e. the no. of donor atoms per unit volume
in donor energy level ' E_D '.

At very low temperatures, ' E_D ' is filled with e^- s
i.e. donors.

With increase of temperature, more and more
electrons i.e. donors gets ionised and the density
of electrons in the conduction band increases.

The density of electrons in the conduction band is
given by:-

$$n = 2 \left(\frac{2me^* \pi k_B T}{h^2} \right)^{3/2} \cdot \exp\left(\frac{E_F - E_C}{k_B T}\right) \quad \text{--- (1)}$$

the density of donors per unit volume is given

$$N_D [1 - f(E_D)] = N_D \exp\left(\frac{E_D - E_F}{k_B T}\right) \quad \text{--- (2)}$$

At $T=0K$, eq. n(1) = eq. n(2)

$$2 \left(\frac{2me^2 \pi k_B T}{h^2} \right)^{3/2} \cdot \exp\left(\frac{E_F - E_C}{k_B T}\right) = N_D \exp\left(\frac{E_D - E_F}{k_B T}\right)$$

Applying 'log' on L.S and rearranging, we get

$$\frac{2E_F}{k_B T} - \left(\frac{E_C + E_D}{k_B T} \right) = \log N_D - \log \left[2 \left(\frac{2me^2 \pi k_B T}{h^2} \right)^{3/2} \right]$$

$$\frac{2E_F}{k_B T} = \left(\frac{E_C + E_D}{k_B T} \right) + \log \left[\frac{N_D}{2 \left(\frac{2me^2 \pi k_B T}{h^2} \right)^{3/2}} \right]$$

$$E_F = \frac{E_C + E_D}{2} + \frac{k_B T}{2} \log \left[\frac{N_D^{1/2}}{2^{1/2} \left(\frac{2me^2 \pi k_B T}{h^2} \right)^{3/4}} \right] \quad \text{--- (3)}$$

At $T=0K$;

$$E_F = \frac{E_C + E_D}{2} \quad \text{--- (4)}$$

In n-type semiconductor, lies E_F in between E_C and E_D .

To calculate the density of e⁰s in the C.B
of an n-type semiconductor at any other
temperature

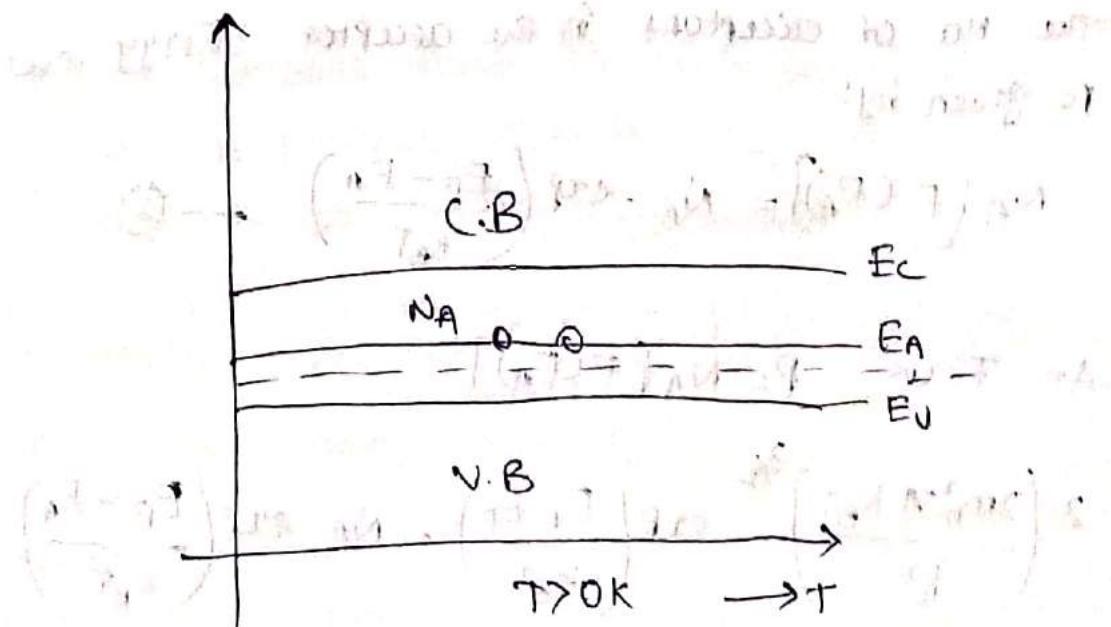
The no. of e⁰s in the C.B is given by:-

$$n = 2 \left(\frac{2m^* \pi k_B T}{h^2} \right)^{3/2} \cdot \exp\left(\frac{E_F - E_C}{k_B T}\right) \quad (5)$$

Substituting in eq.n ③ in exponential term of
eq.n ⑤, we get

$$\begin{aligned} & \exp\left(\frac{E_F - E_C}{k_B T}\right) \\ &= \exp\left\{ \frac{\frac{E_C + E_D}{2} - E_C}{k_B T} + \log\left[\frac{N_0^{1/2}}{\sqrt{2} \left(\frac{2m^* \pi k_B T}{h^2} \right)^{3/4}} \right] \right\} \end{aligned}$$

Calculation of Carrier Concentration of P-type Semiconductor:-



at and above 0 K result as follows.

Effect of Temperature on carrier concentration

At 0K temperature the no. of acceptors in the acceptor energy level is equal to the no. of holes in the V.B

As temperature increases the no. of acceptor gets more and more no. of acceptors gets atom_i
 (ionised applied with eθ_s)

and the no. of holes in the valence band increases

We know that

The no. of holes in the valence band is given by:-

$$P = 2 \left(\frac{2m_h^* \pi k_B T}{h^2} \right)^{3/2} \cdot \exp \left(\frac{E_V - E_F}{k_B T} \right) \quad \text{--- (1)}$$

The no. of acceptors in the acceptor energy level is given by:-

$$N_A [F(E_A)] = N_A \cdot \exp \left(\frac{E_F - E_A}{k_B T} \right) \quad \text{--- (2)}$$

$$\text{At } T=0\text{K}, \quad P = N_A [F(E_A)]$$

$$2 \left(\frac{2m_h^* \pi k_B T}{h^2} \right)^{3/2} \cdot \exp \left(\frac{E_V - E_F}{k_B T} \right) = N_A \exp \left(\frac{E_F - E_A}{k_B T} \right)$$

Applying log on both sides & rearranging, we get

$$\left(\frac{E_V - E_F}{k_B T} \right) - \left(\frac{E_F - E_A}{k_B T} \right) = \log N_A - \log \left\{ 2 \left(\frac{2m_h^* \pi k_B T}{h^2} \right)^{3/2} \right\}$$

$$\frac{-2EF}{KBT} + \left(\frac{E_V + E_A}{KBT} \right) = -\log \left\{ \frac{\frac{N_A}{2} \pi^{3/2}}{2 \left(\frac{2m_h^* \pi KBT}{h^2} \right)^{3/2}} \right\}$$

$$\frac{2EF}{KBT} = \left(\frac{E_V + E_A}{KBT} \right) - \log \left\{ \frac{\frac{N_A}{2} \pi^{3/2}}{2 \left(\frac{2m_h^* \pi KBT}{h^2} \right)^{3/2}} \right\}$$

$$EF = \left(\frac{E_V + E_A}{2} \right) - \frac{KBT}{2} \log \left\{ \frac{\frac{N_A^{1/2}}{\pi^{3/2}}}{2 \left(\frac{2m_h^* \pi KBT}{h^2} \right)^{3/4}} \right\} \quad \text{--- (3)}$$

At $T = 0K$

$$EF = \frac{E_V + E_A}{2} \quad \text{--- (4)}$$

Eq.n (4) indicates that the fermi energy level lies in between acceptor energy level and V.B in an P-type semiconductor.

To calculate the density of holes in the V.B of P-type Semiconductor.

We know that,

$$P = 2 \left(\frac{2m_h^* \pi KBT}{h^2} \right)^{3/2} \cdot \exp \left(\frac{E_V - EF}{KBT} \right) \quad \text{--- (5)}$$

Substituting eq.n (3) in exp term of eq.n (5), we get

$$\exp \left(\frac{E_V - EF}{KBT} \right) = \exp \left\{ \frac{E_V - \left(\frac{E_V + E_A}{2} \right)}{KBT} \right\} + \frac{KBT}{2} \log \left[\frac{\frac{N_A^{1/2}}{\pi^{3/2}}}{2 \left(\frac{2m_h^* \pi KBT}{h^2} \right)^{3/2}} \right]$$

$$= \exp\left(\frac{E_V - E_A}{2k_B T}\right) \cdot \frac{\sqrt{N_A}}{\sqrt{2} \left(\frac{2m_h^* \pi k_B T}{h^2}\right)^{3/4}} \quad \text{--- (6)}$$

Substituting eq.n(6) in eq.n(5), we get

∴ Density of holes in V.B of P-type semiconductor.

$$\therefore P = \sqrt{2} \left(\frac{2m_h^* \pi k_B T}{h^2}\right)^{3/4} \cdot \exp\left(\frac{E_V - E_A}{2k_B T}\right) \cdot \frac{\sqrt{N_A}}{\sqrt{2} \left(\frac{2m_h^* \pi k_B T}{h^2}\right)^{3/4}}$$

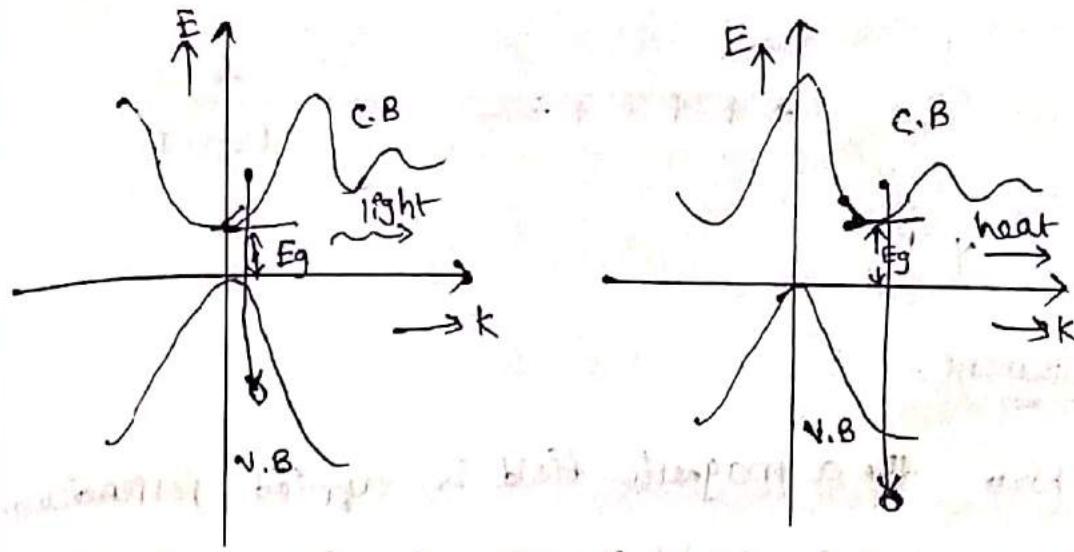
$$P = \sqrt{2} N_A \left(\frac{2m_h^* \pi k_B T}{h^2}\right)^{3/4} \exp\left(\frac{E_V - E_A}{2k_B T}\right)$$

$$P \propto \sqrt{N_A}$$

∴ The no. of holes \propto square root of no. of acceptors

Direct band gap Semiconductors and Indirect band gap Semiconductors:

Direct band gap semiconductor, Indirect band gap s.c.:



Min of C.B } Same value
Max of V.B } of k

Ex:- Diodes, transistors, amplifiers

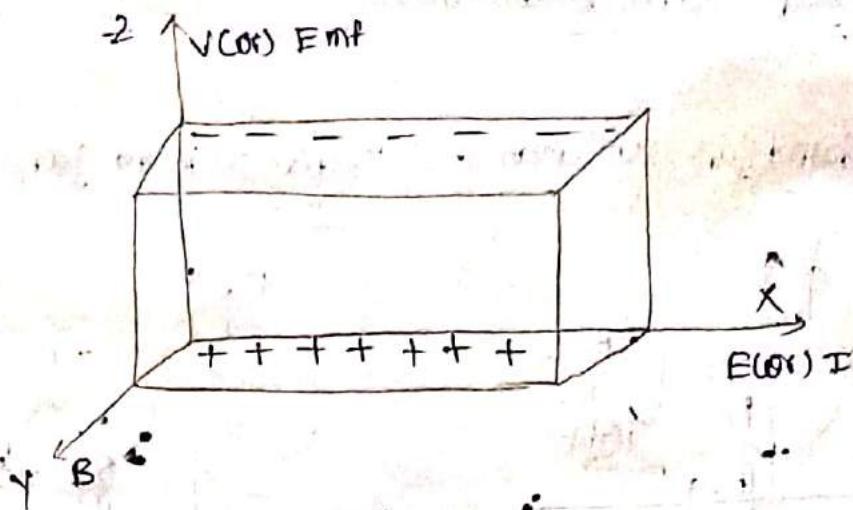
Ge Si

Ex:- Laser, LED, etc.

GaAs, GaP, GaInP

~~WTF~~

Hall Effect:-



Statement:-

When a magnetic field is applied perpendicular to the electric field (current carrying conductor), the voltage is developed across the specimen in a direction perpendicular to both current and magnetic field. This phenomenon is called as Hall effect and the voltage so developed is called as Hall e.m.f (or) Hall voltage (E_H)

$$\text{Magnetic deflecting force} = q \cdot v_d \cdot B \quad \text{--- (1)}$$

q = charge on the charge carriers

v_d = drift velocity

B = applied magnetic field.

Electric deflecting force = $q \cdot E_H$ — (2)

q - charge on the charge carriers

E_H - Hall emf

At equilibrium, the magnetic deflecting force is equal and opposite to electric deflecting force

$$\text{i.e. } q \cdot V_d \cdot B = -q \cdot E_H$$

$$\therefore E_H = -V_d \cdot B \quad \text{--- (3)}$$

current density is given by:

$$J = n \cdot q \cdot V_d \quad \text{--- (4)}$$

n - no. of charge carriers

q - charge on charge carriers

V_d - drift velocity

$$\therefore V_d = \frac{1}{nq} \cdot J$$

$$V_d = R_H \cdot J \quad \text{--- (5)}$$

$$R_H = \frac{1}{nq} \rightarrow (\text{for P-type } R_H \text{ is positive})$$

\downarrow
Hall coefficient

$$R_H = \frac{-1}{nq} \rightarrow (\text{for n-type } R_H \text{ is negative})$$

$$\therefore E_H = -R_H J \cdot B \quad (\text{P-type (P-type)})$$

$$E_H = R_H J \cdot B \quad (\text{n-type})$$

Note!

- 1) If the -ve charges gets accumulated along the +ve z-direction of this specimen (or) material, then it is n-type semiconductor.
- 2) If the +ve charges get accumulated along the +ve z-direction of this specimen (or) material, then it is p-type semiconductor.

Applications:

- 1) We can determine the type of the semi-conductor
- 2) We can calculate the carrier concentration of the semi conductor
- 3) We can calculate the mobility of semiconducto

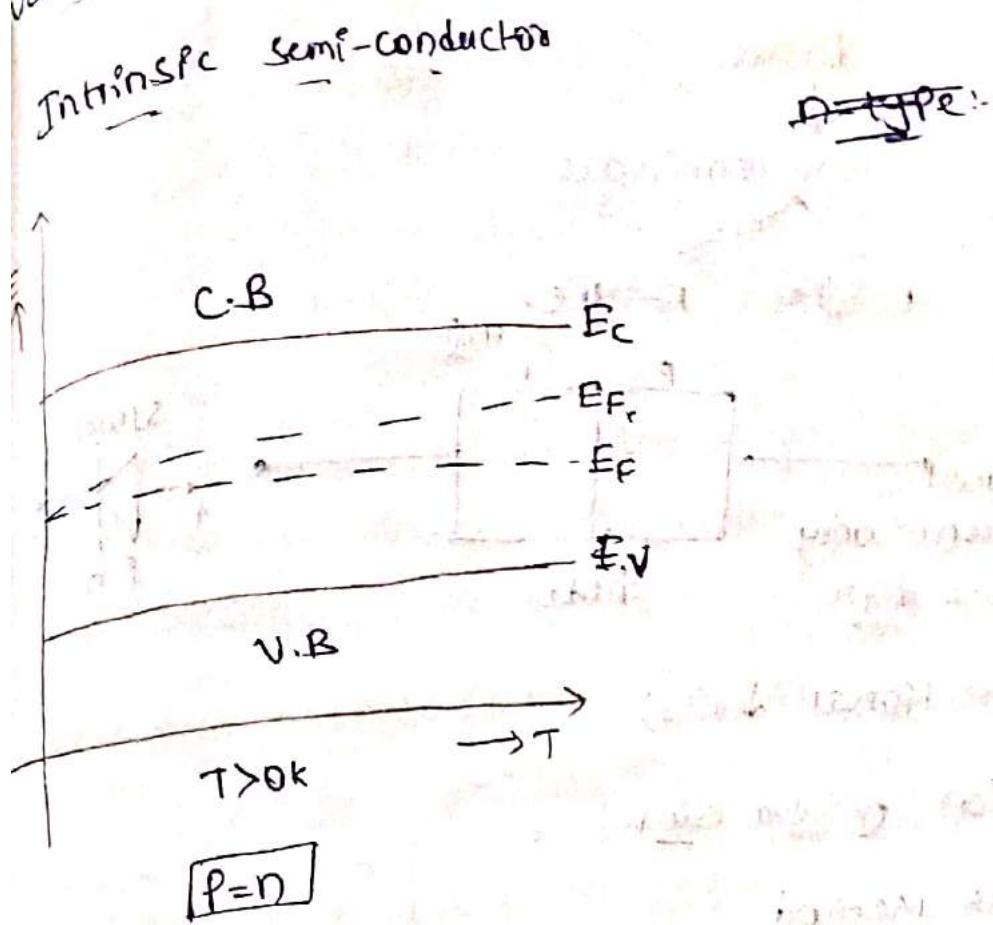
$$\sigma = n \cdot q \cdot u$$

conductivity

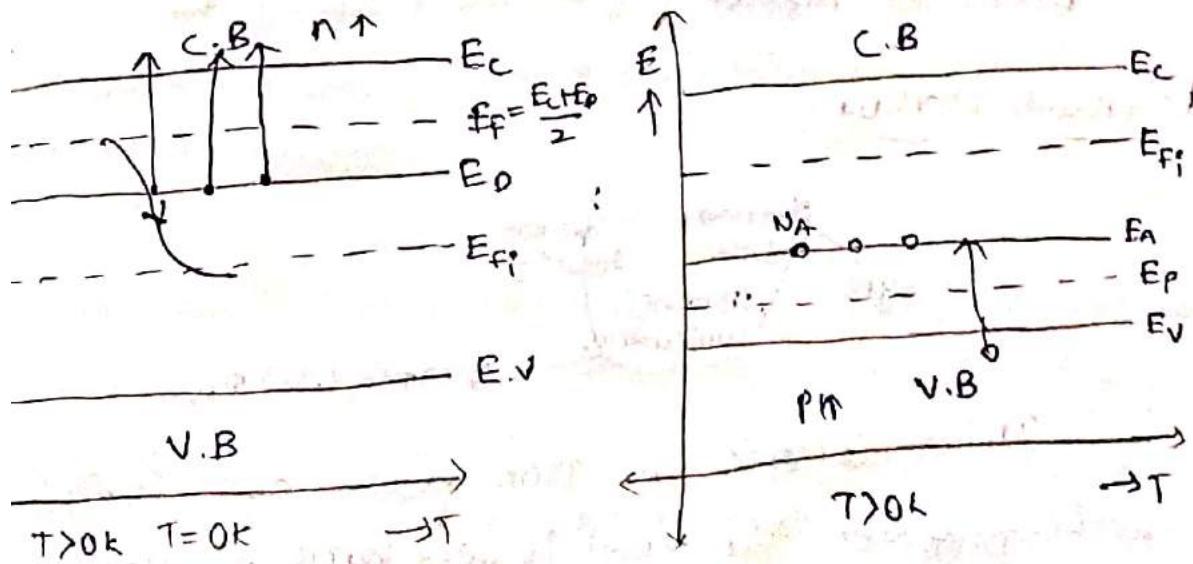
$$u = \frac{1}{n \cdot q} \cdot \sigma$$

$$u = R_H \cdot \sigma$$

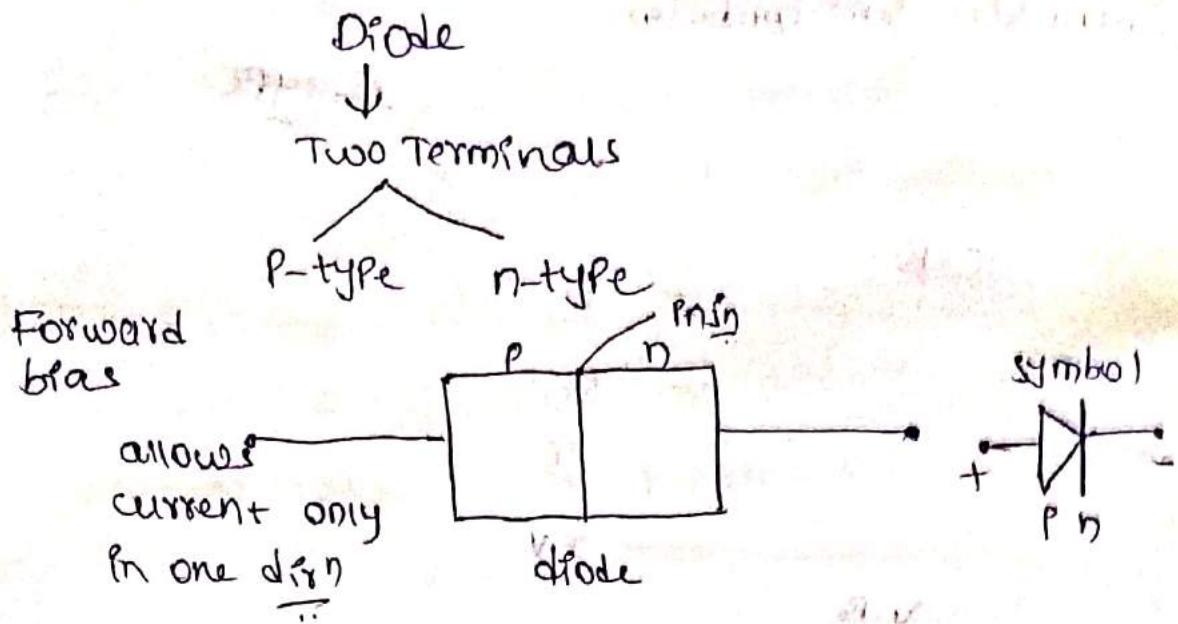
Variation of Fermi level with temperature



n-type:



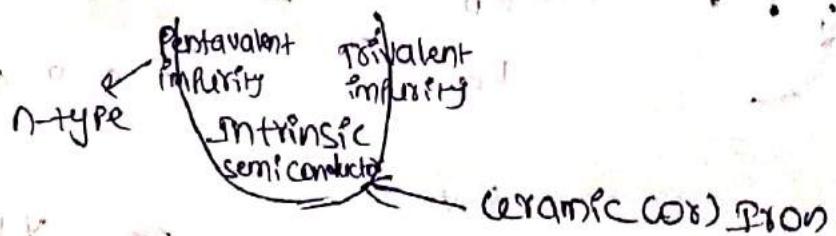
III. 2. Semiconductor Devices



Formation of the diode:-

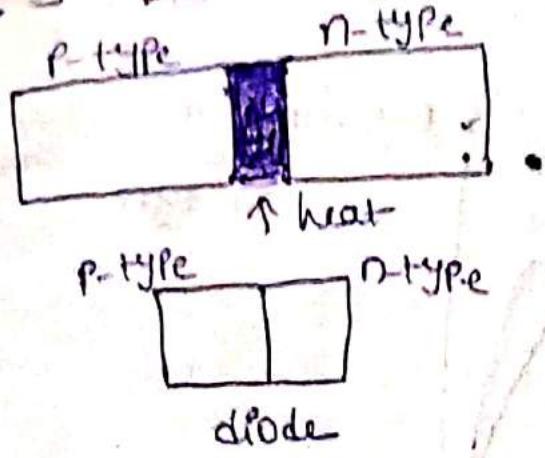
- 1) Growth Method
- 2) Alloying Method
- *3) Diffusion Method

1. Growth Method:-



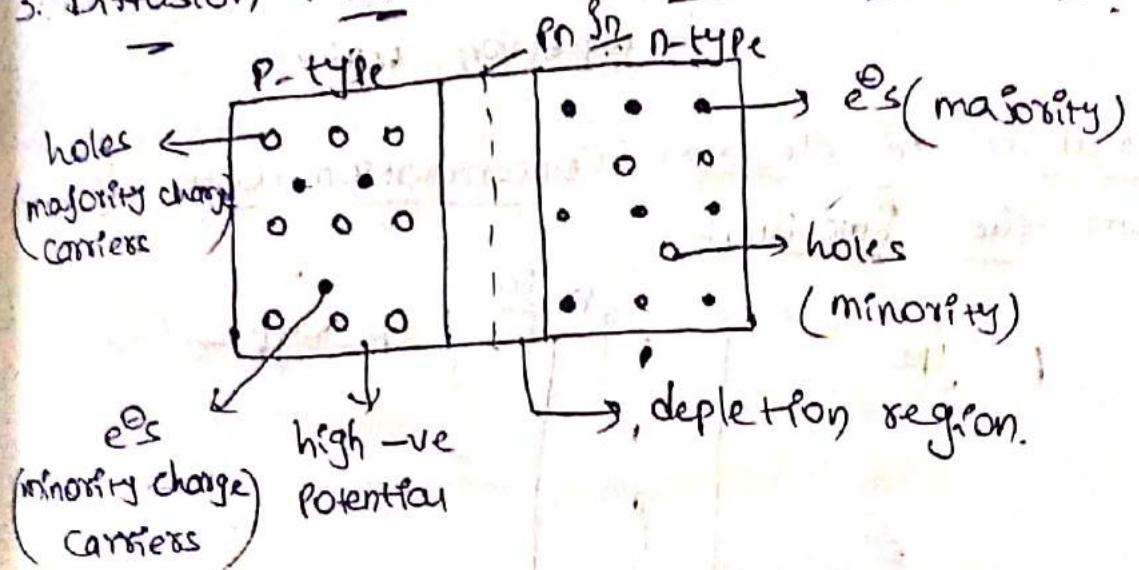
In a Ceramic cor iron bowl add intrinsic semiconductor. The bowl is very hard so more heat energy is required for many days. Then by adding Pentavalent and Trivalent impurities we get diode by making the result final product by solidifying and condensation.

Alloying Method:-



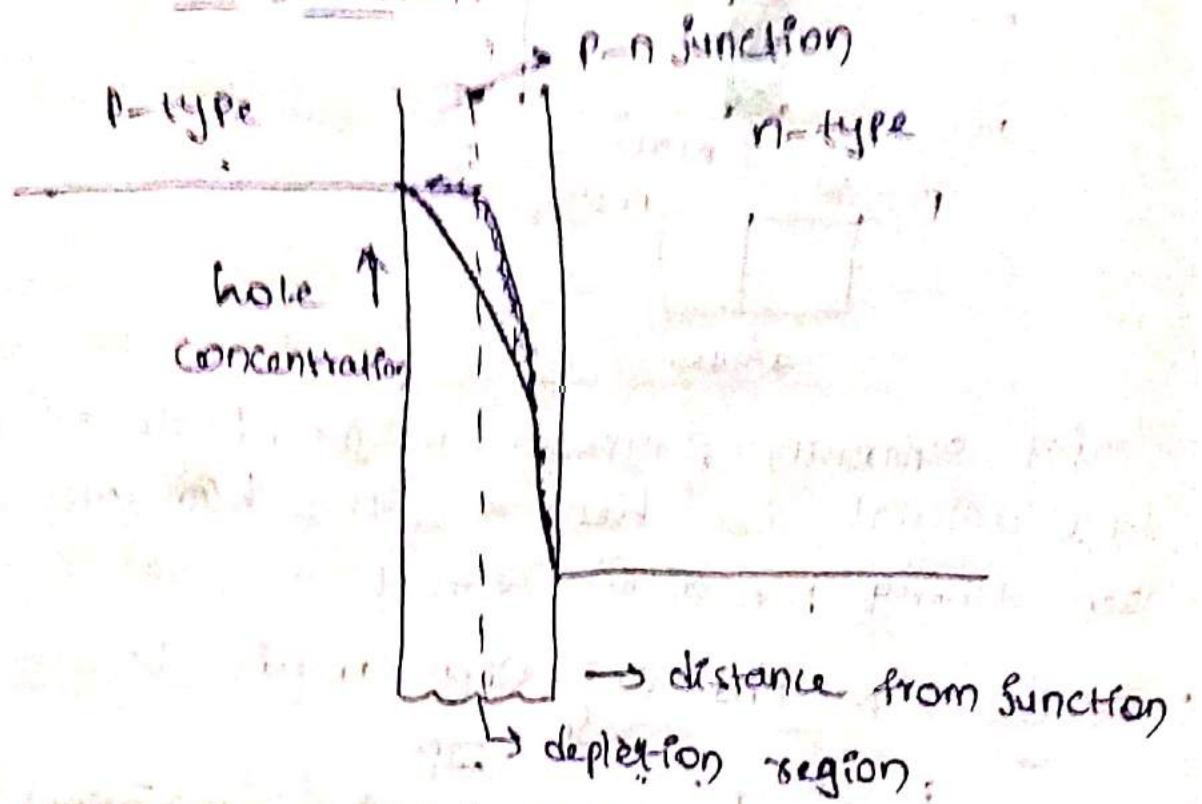
Taking separately P-type and n-type, both attaching by a material, then heating with a high temperature then taking p-n diode from it.

3. Diffusion Method (or) Open circuit braising:-

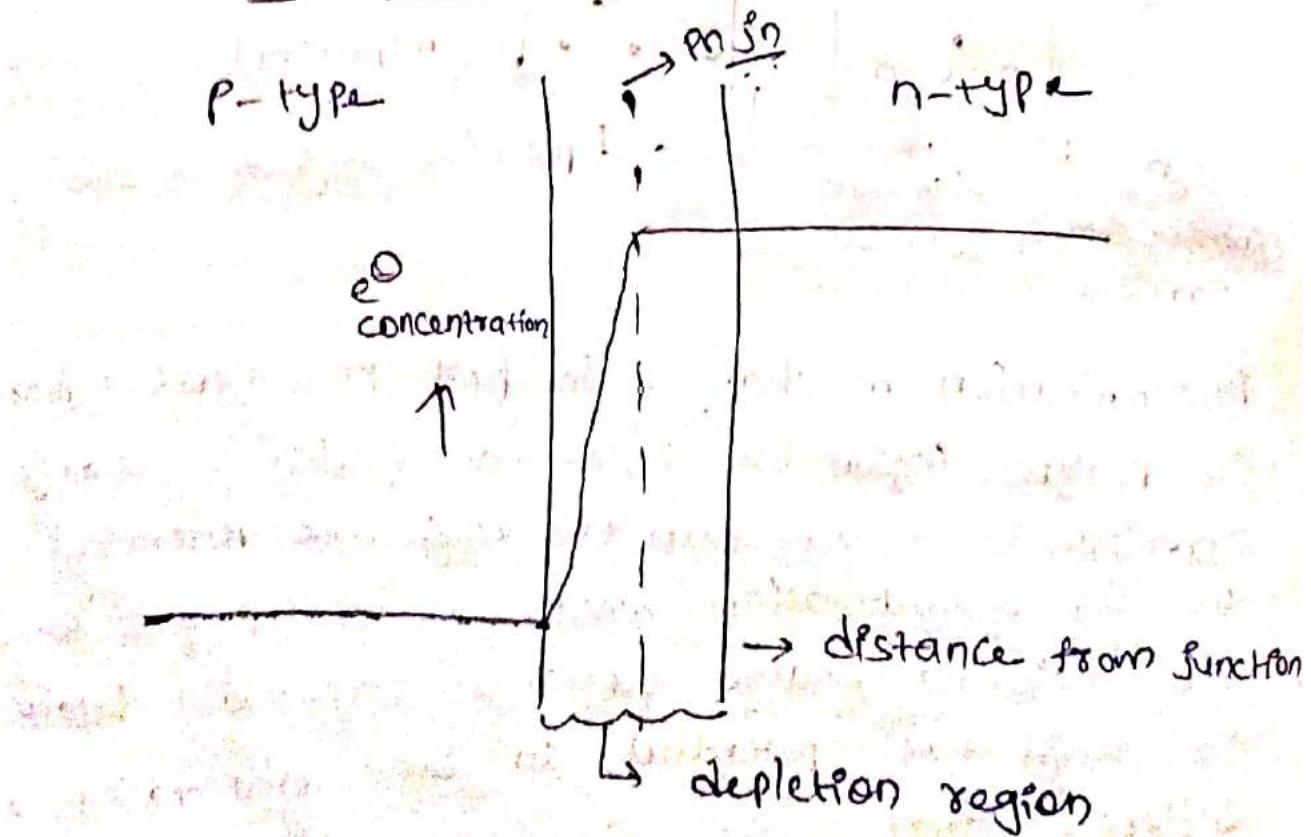


Recombination of charges in both P&n types. Where the p-type region has holes are majority charge carriers, so it will turn to high -ve Potential due to recombination. Where as n-type e^- s are majority charge carriers. so it will turn to high +ve Potential. so there will form a depletion layer, where the charges are depleted means, no charge is present in depletion layer.

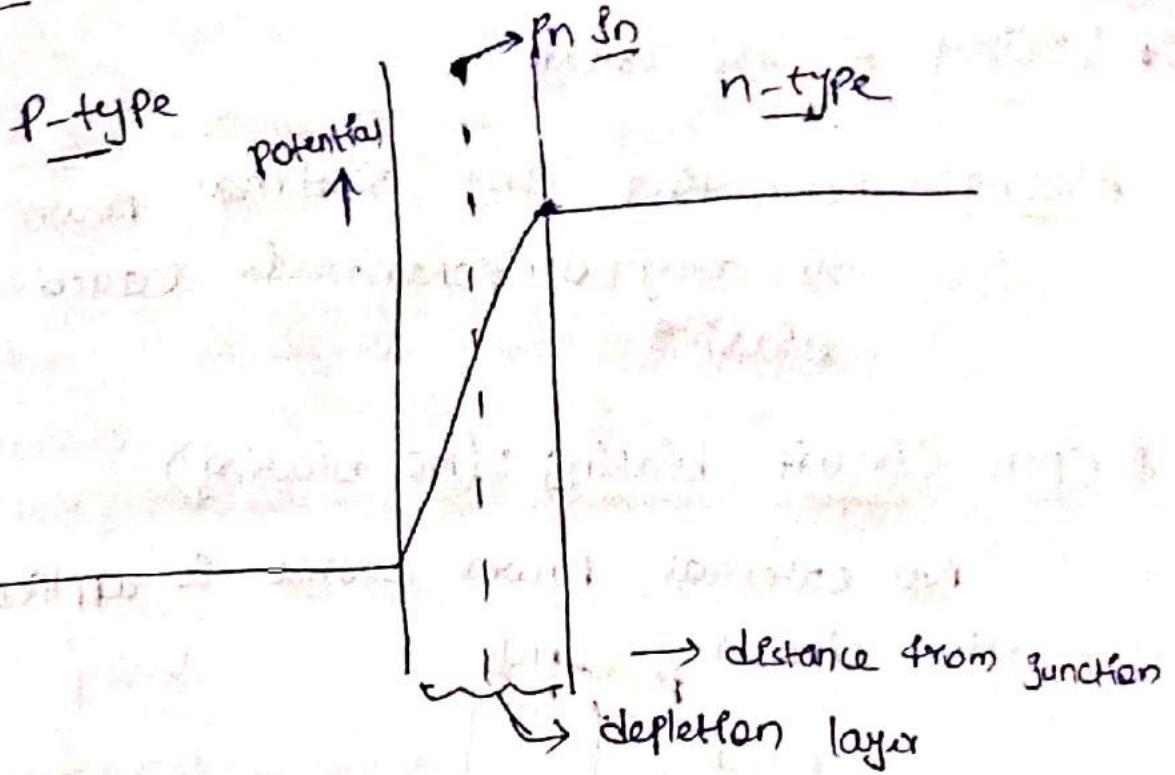
Variation of hole concentration with distance from the junction:



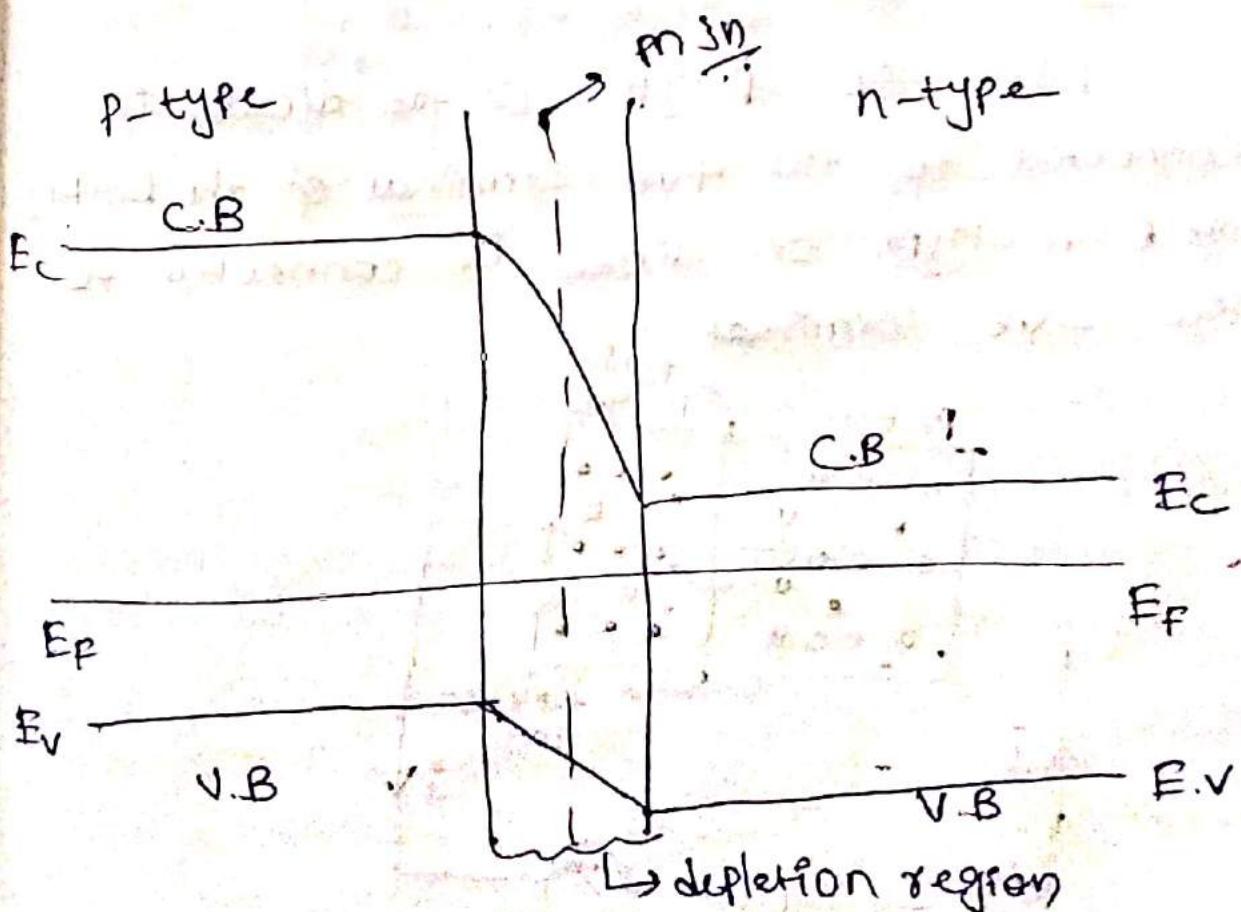
Variation of electron concentration with distance from the junction:



Variation of potential with distance from the junction:



Energy level diagram of p-n junction diode:-

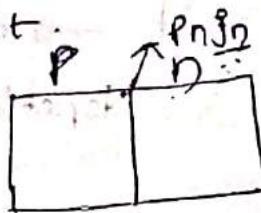


Biassing of the diode:-

Biassing:- Connecting any external power source to any component is called biassing

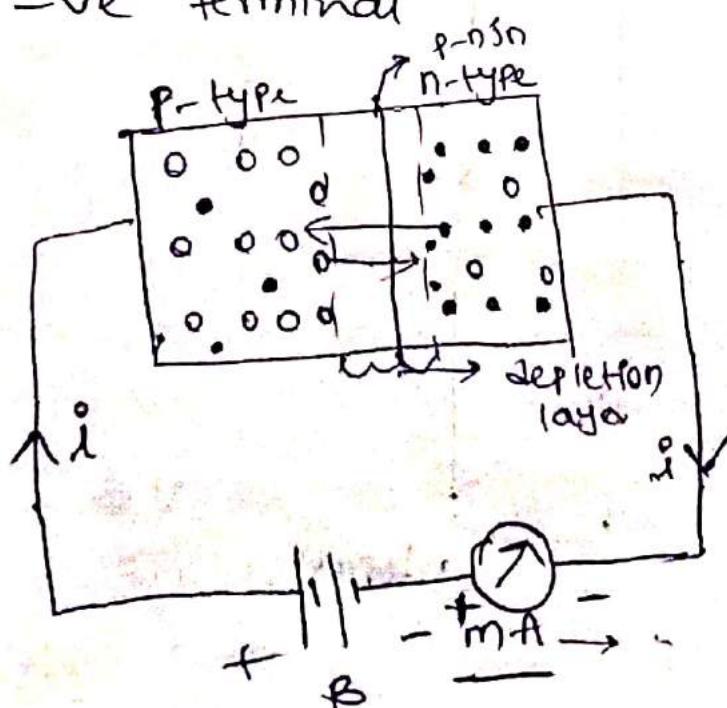
Open Circuit biassing : (No biassing)

No external power source is applied to the circuit.



i) Forward Biass:-

When the p-type of the diode is connected to the +ve terminal of the battery and n-type of diode is connected to the -ve terminal



When external battery is connected, few positive holes and holes in P-type repels each other and moves towards depletion region. The negative ions and electrons repels each other. moves towards depletion region. both these charge carriers decrease the depletion region. The movement creates the current in diode.

Mainly the current is due to majority charge carriers

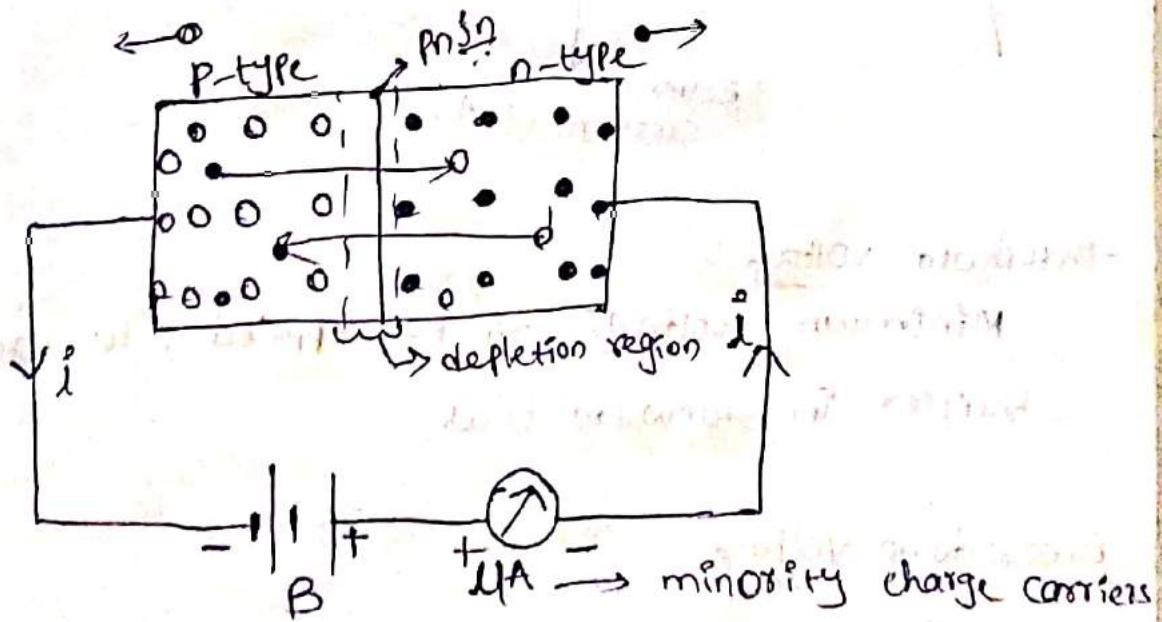
2) Reverse bias:

Diode

Battery

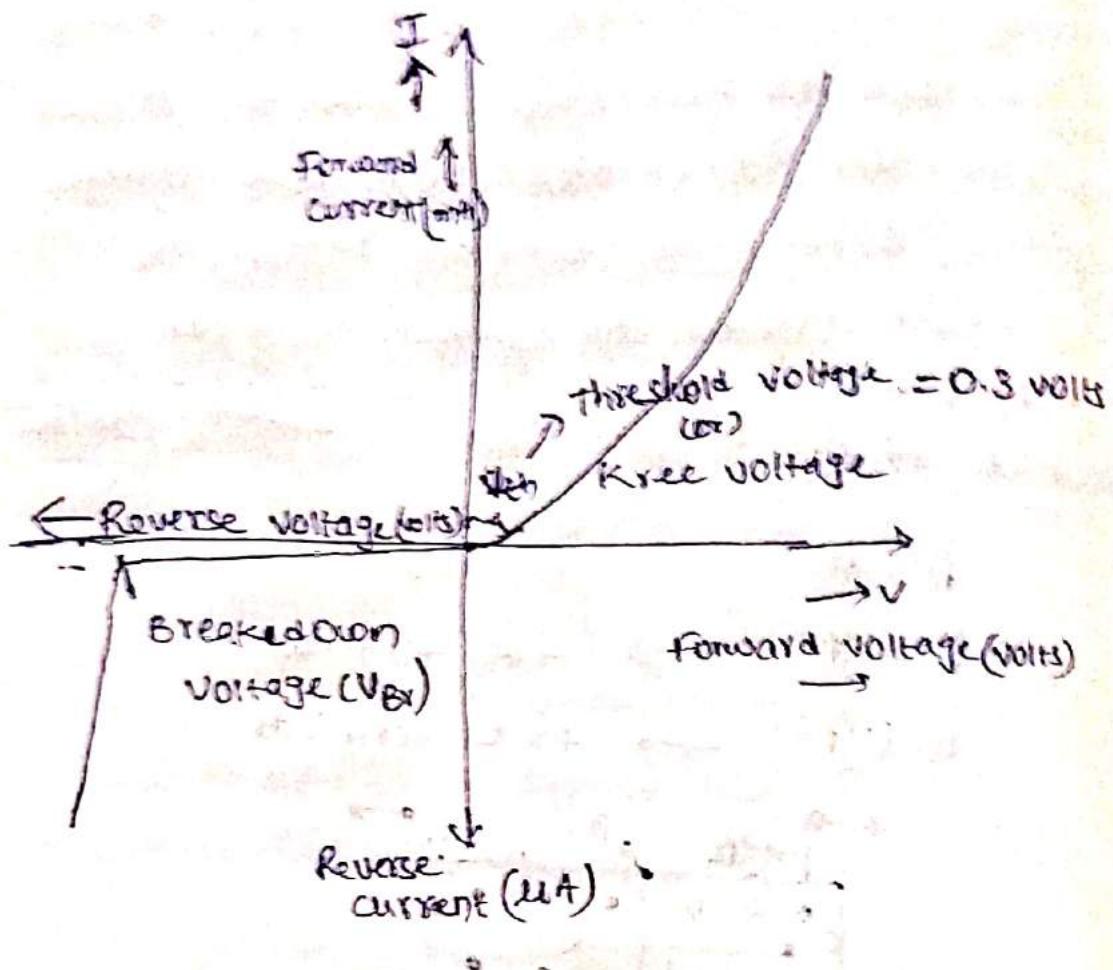
P-type \rightarrow -ve terminal

n-type \rightarrow +ve terminal



These holes in P-type, move opposite to battery.

I-V characteristics of diode



threshold voltage:-

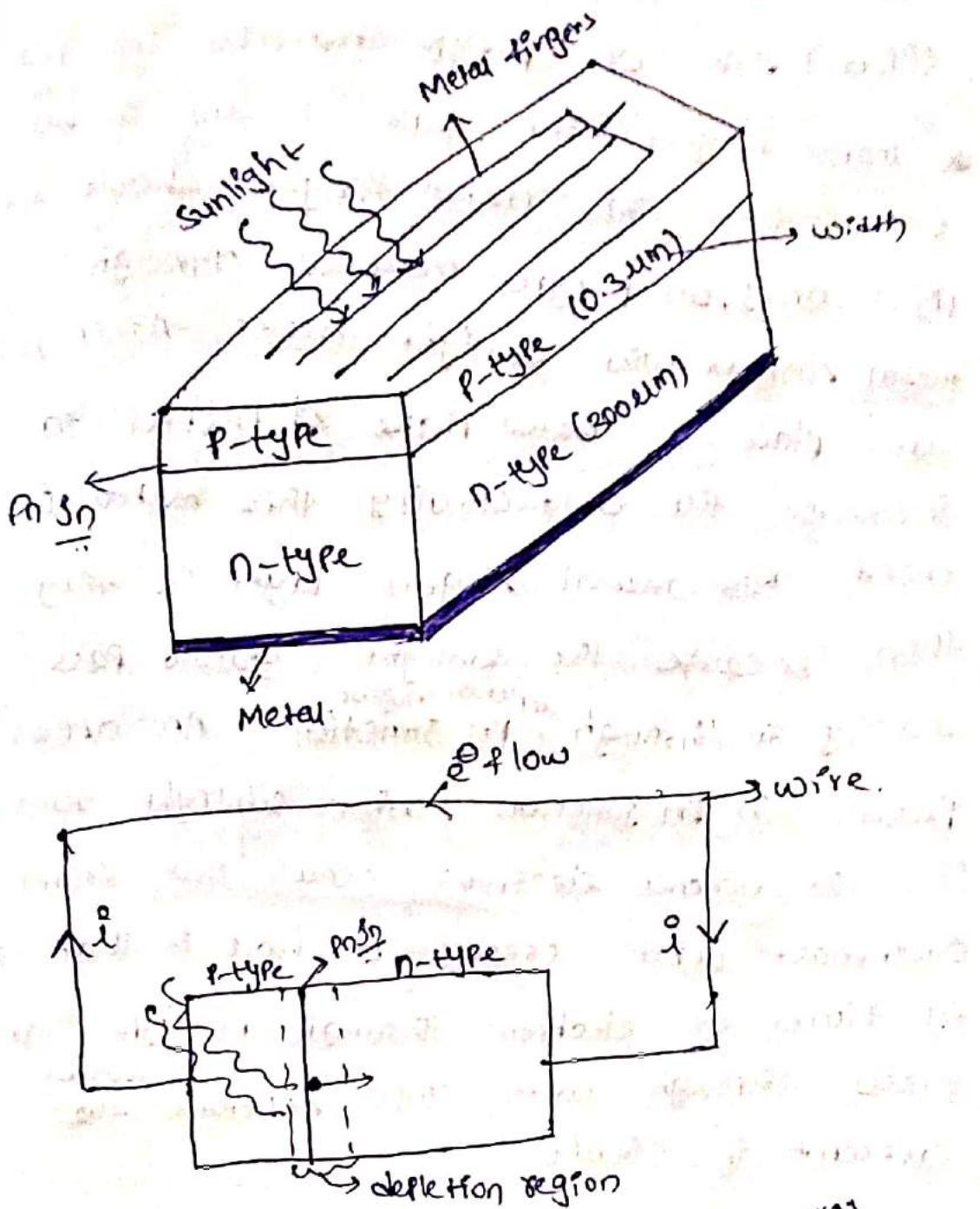
Minimum voltage to be applied, to cross the barrier in forward bias

Breakdown voltage:-

Maximum voltage, where the current increases suddenly, in reverse bias.

If the current increases suddenly, it damages the diode

Solar Cell

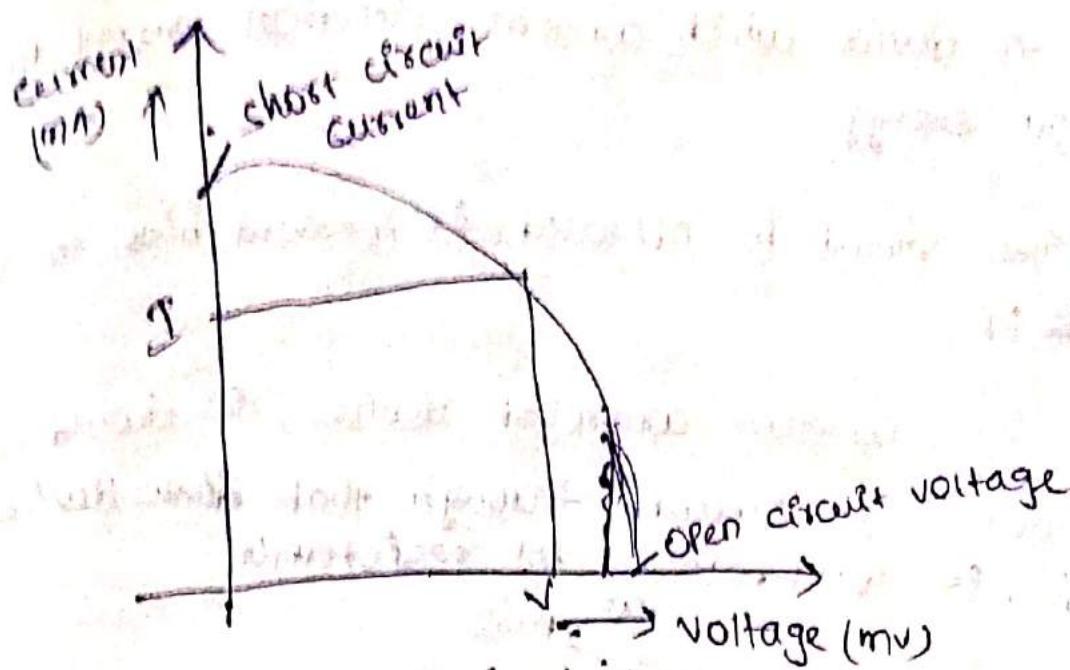


A device which converts sunlight into electrical energy. To work solar cell, we don't need any power supply. Is called as photo voltaic cell.

A device which doesn't use any power supply. P-type is very thin layer and n-type thick layer.

A solar cell is worked in photo voltage mode. Main component in solar is diode (Silicon) diode. On P-type material we place metal fingers like gaps present in between our fingers. These metal fingers should cover 15% only on P-type material. Through metal fingers the sunlight passes. After n-type we place a back plane of metal. To increase the conductivity this metal is used. This metal fingers layer is very thin because the sunlight should pass easily through a ^{depletion region} PN junction. An atom present in PN junction, when sunlight falls on it, the valence electrons comes out from outermost orbit, creates a hole in that atom. By flow of electron through n-type and passes through wire. This electron create current in circuit.

Characteristics Of Solar Cell



When voltage is '0', the maximum current is known as short circuit current.

When current is '0', the maximum voltage is known as open circuit voltage.

$$\text{Power} = V \times I$$

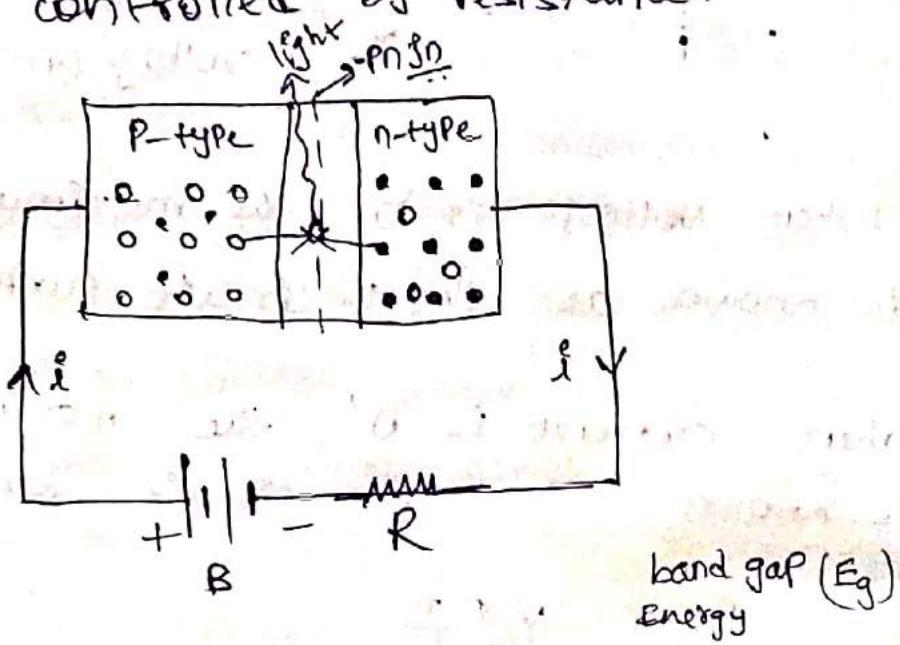
↓ ↓
Watts Volts Amps

LED: (Light Emitting diode)

A device which converts electrical energy to light energy.

Diode should be operated in forward bias to get ~~to~~ ED.

It is a current controlled device, it should not flow high current through that diode device. It is controlled by resistance.



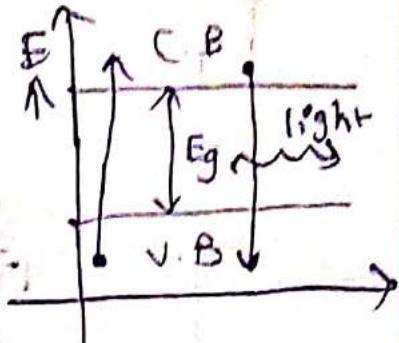
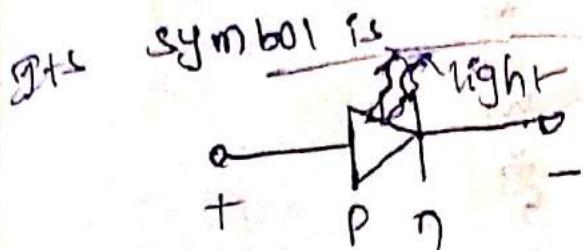
An atom present in 'depletion' region, emits its ~~electron~~ when ~~sunlight~~ is ~~passed~~. An external energy is applied to the released electron from the atom. While the electron replaces the previous hole in the atom, then emits energy in the form of light. The energy (or) band gap between conduction band, valence band if 1.8 e.v to 3 e.v it is visible range, if $< 1.8\text{ e.v}$ Infrared range, $> 3\text{ e.v}$ U.V radiation.

$v \uparrow$ VIBGYOR $v \downarrow$
 $\lambda \downarrow$ $\lambda \uparrow$

E & V

$$\uparrow E = h\nu \uparrow$$

$$\downarrow E = h\nu \downarrow$$



Ga As \rightarrow Infrared LED

Ga As P \rightarrow Red - Yellow

Ga Al P \rightarrow Green

Nitrogen \rightarrow Blue

Injecting

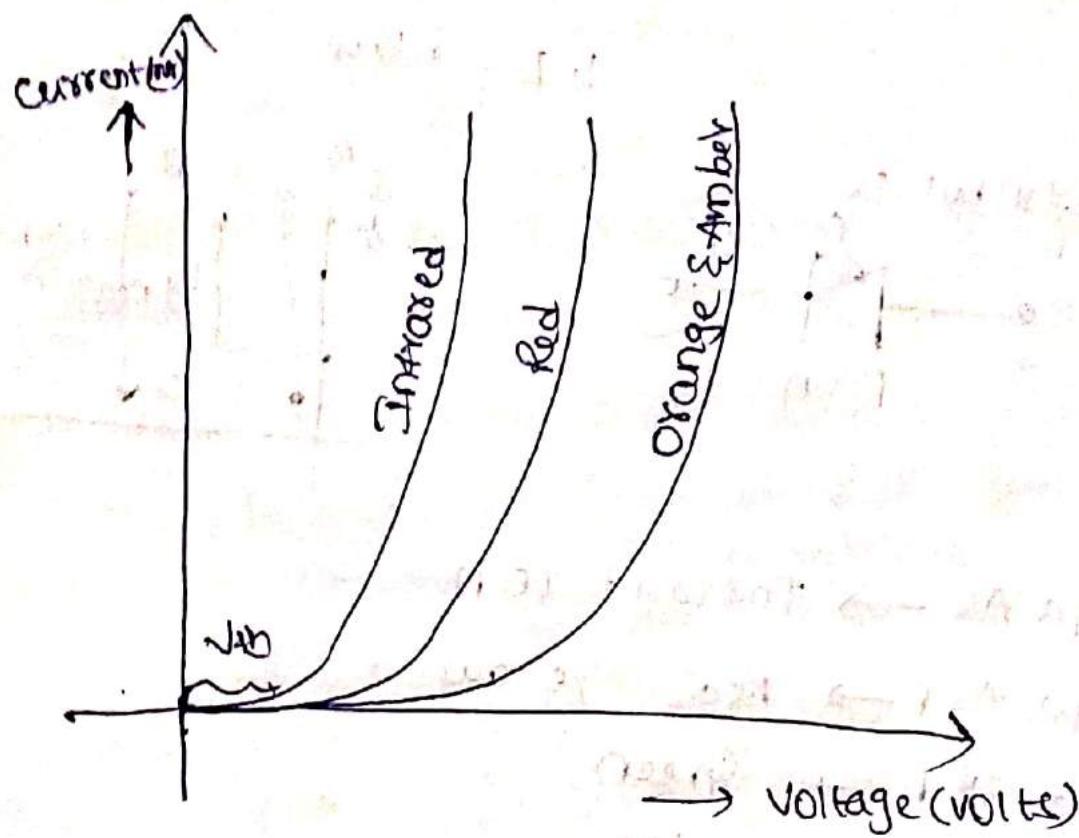
Violet \rightarrow costly
 Red \rightarrow cheaper

0 || 1 || 5 \Rightarrow 7-segment LED

2 || 1 || 3 calculators

for 8 or 4 th will Reverse biased

I-V characteristic of LED's



IV. I. LASERS

Light Amplification by Stimulated Emission of Radiation

Einstein → Theoretical

Ch. Towne's



MASER



Microwave
(GHz)

1st practical

invented

→ Maiman

laser



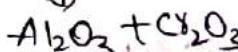
Solid state laser - Ruby lasers

Requirements for obtaining LASER (Lasing Action)

- 1) Active medium
- 2) Resonating cavity
- 3) Pumping Mechanism

1) Active medium:

Ruby rod



Ruby laser

Cr_2^{+3} ions

Ruby is red in colour, because of Cr⁺ ions

" Cr_2^{+3} " ions are the active medium

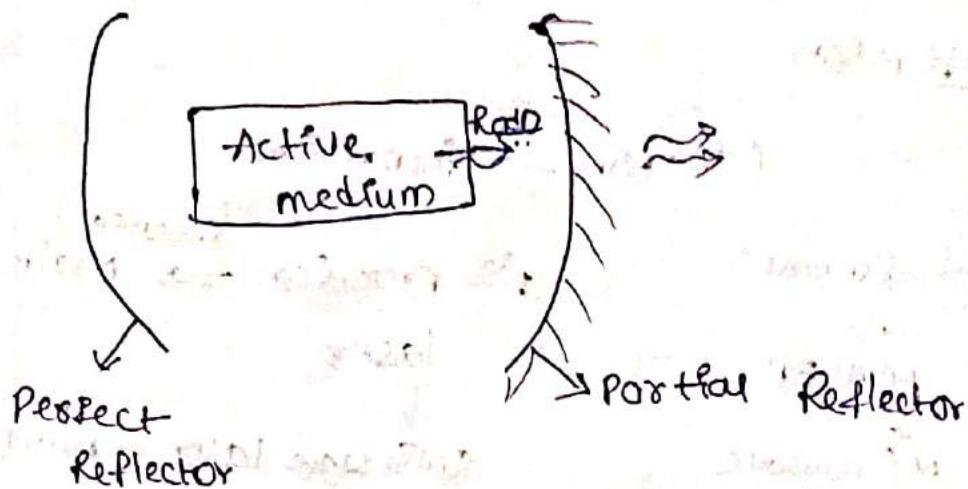
Gas laser:

- 1) He-Ne laser → only from Ne' atoms laser comes
- 2) CO₂ laser

In semiconductor $n-p-n$ laser is active medium

2) Resonating Cavity:

for proper feedback and for amplification



3) Pumping Mechanism: to maintain population inversion, condns

- 1) Optical pumping
 - 2) Electrical discharge
 - 3) Injection of current \rightarrow Direct pumping
 - 4) chemical reagent
 \Rightarrow colour, dye lasers
- $E_1 \xrightarrow{N_1} E_2 \xrightarrow{N_2}$ population
- At lower energy level it is stable, so it contains more electrons compared to higher.

Population inversion means

$$N_2 > N_1$$

External energy is supplied so, ~~some~~ electrons in E_1 goes to E_2 .

functions

xenon light: where electric electrical energy is supplied, then lower energy level

Electrons gets excited and while coming down emits light.

Characteristics of Laser

Highly Monochromatic

→ single wavelength (colour single)

Highly coherent → same frequency, same wavelength, constant phase difference

Highly directional, convergent

High Intensity

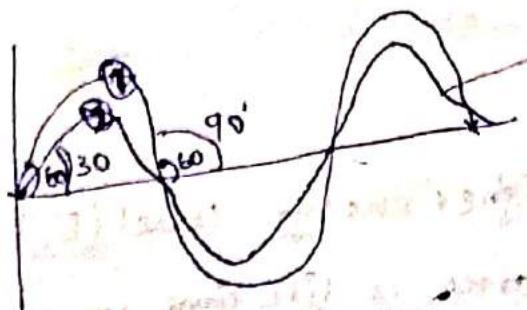
→ Brightness

frequency:- no. of cycles per second

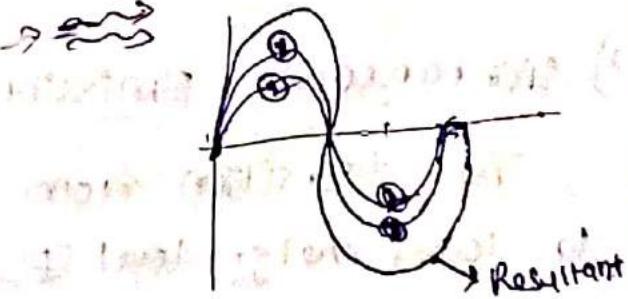
$$1A^{\circ} = 10^{-8} \text{ cm}$$

$$1A^{\circ} = 10^{-10} \text{ m}$$

constant phase difference



In phase



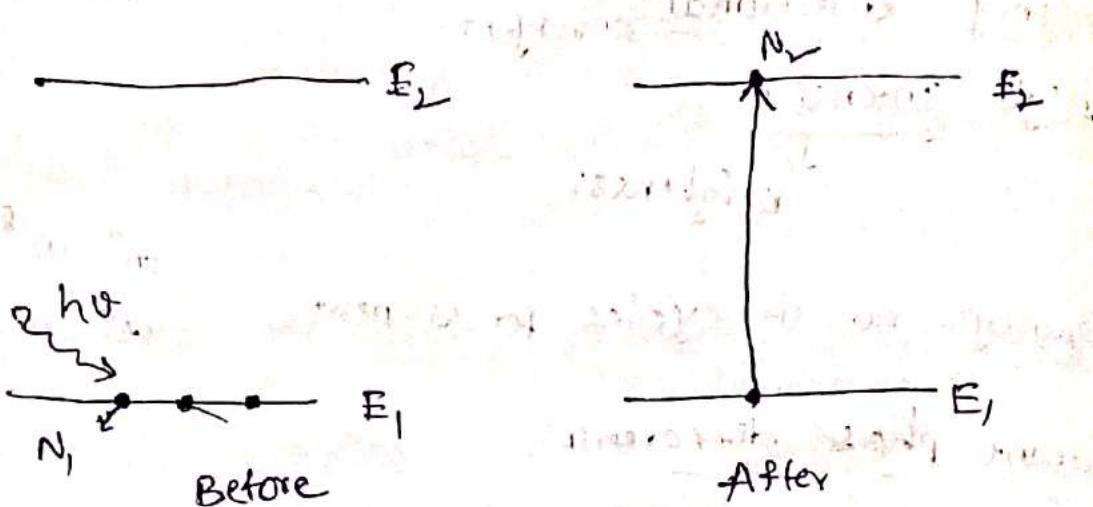
$$60^{\circ} - 30^{\circ} = 30^{\circ}$$

$$90^{\circ} - 60^{\circ} = 30^{\circ}$$

The 3 processes which occur in Laser are

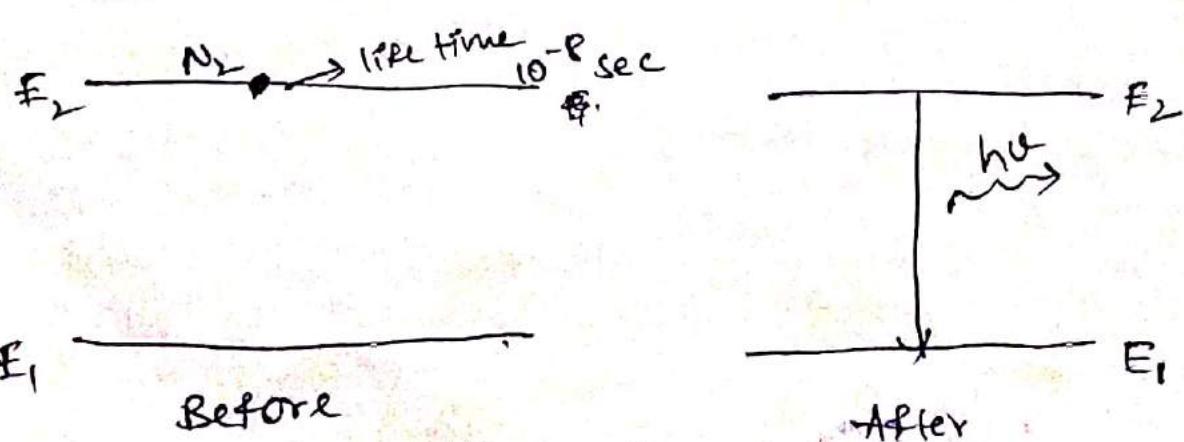
1) Stimulated Absorption:-

The process of transition from lower energy level (E_1) to higher energy level (E_2) by absorbing a photon of energy $h\nu$ is defined as Stimulated absorption.



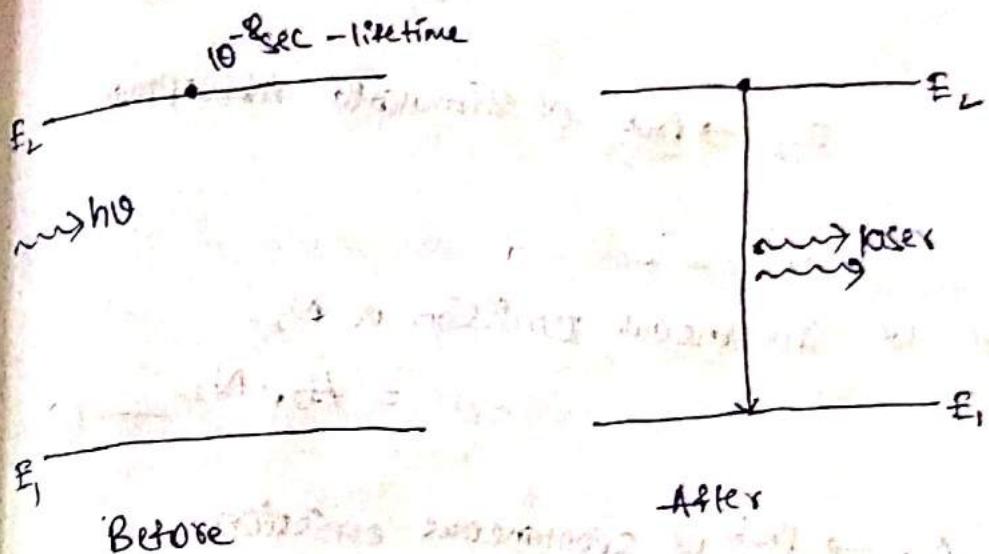
2) Spontaneous Emission:-

The transition from higher energy level (E_2) to lower energy level (E_1) after a lifetime of 10^{-8} sec of the higher energy level on its own is called as spontaneous emission. The emitted photon will not be in phase with the photon in spontaneous emission.



Stimulated Emission

The process of transition from higher energy level E_2' to lower energy level (E_1) after accepting a photon of energy $h\nu$, the atom gets de excited to the lower energy level by releasing two photons which are in phase with each other is called as Stimulated Emission.



Einstein Coefficients (or) Relations:

Let ' E_1 ' be the lower energy level

Let ' E_2' ' be the higher energy level

Let ' N_1 ' and ' N_2' ' be the no. of atoms per unit volume in E_1 and E_2' energy levels respectively.

Let $f(v)$ be the energy density of radiation in the frequency range $v & v + dv$

The rate of stimulated absorption depends on the no. of atoms per unit volume in the lower energy level N_1 and on the energy density of radiation $\rho(v)$ i.e

Rate of stimulated Absorption $\propto N_1 \cdot \rho(v)$

$$= B_{12} N_1 \cdot \rho(v) \quad \textcircled{1}$$

$B_{12} \rightarrow \underline{\text{Prob}} \text{ of stimulated Absorption}$

Rate of spontaneous emission $\propto N_2$

$$= A_{21} \cdot N_2 \quad \textcircled{2}$$

$A_{21} \rightarrow \underline{\text{Prob}} \text{ of spontaneous emission}$

The rate of stimulated emission depends on the no. of atoms per unit volume in the higher energy level N_2 and on the energy density of radiation $\rho(v)$ i.e

Rate of stimulated Emission $\propto N_2 \cdot \rho(v)$

$$= B_{21} \cdot N_2 \cdot \rho(v) \quad \textcircled{3}$$

$B_{21} \rightarrow \underline{\text{Prob}} \text{ of stimulated emission.}$

At equilibrium the rate of absorption is equal to
rate of emission.

At equilibrium, $\textcircled{1} = \textcircled{2} + \textcircled{3}$

$$B_{12} N_1 \ell(\nu) = A_{21} N_2 + B_{21} N_2 \ell(\nu)$$

$$B_{12} N_1 \ell(\nu) - B_{21} N_2 \ell(\nu) = A_{21} N_2$$

$$\ell(\nu) [B_{12} N_1 - B_{21} N_2] = A_{21} N_2$$

$$\therefore \ell(\nu) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} \quad \textcircled{4}$$

Multiply & divide the n.r and d.r of eq.n $\textcircled{4}$ with

$$B_{21} N_2$$

$$\Rightarrow \ell(\nu) = \frac{\frac{A_{21} N_2}{B_{21} N_2}}{\frac{B_{12} N_1}{B_{21} N_2} - \frac{B_{21} N_2}{B_{21} N_2}}$$

$$\therefore \ell(\nu) = \frac{A_{21}}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \quad \textcircled{5}$$

↓
Energy density
of radii

Boltzmann distribution

$$N_i = N_0 g_i \exp\left(-\frac{E_i}{k_B T}\right)$$

$$i = 1, 2, 3 \dots$$

N_0 = no. of atoms having zero energy

g_i = degeneracy factor

$$\therefore \frac{N_1}{N_2} = \frac{\text{No } g_1 \exp\left(\frac{-E_1}{k_B T}\right)}{\text{No } g_2 \exp\left(\frac{-E_2}{k_B T}\right)}$$

If $g_1 = g_2$ (these sublevels are very close),

$$\frac{N_1}{N_2} = \frac{g_1}{g_2} \exp\left(\frac{E_2 - E_1}{k_B T}\right) = \exp\left(\frac{h\nu}{k_B T}\right)$$

$$\therefore \frac{N_1}{N_2} = \exp\left(\frac{E_2 - E_1 + h\nu}{k_B T}\right)$$

$$\therefore \ell(\nu) = \frac{A_{21}/B_{21}}{\left[\frac{B_{12}}{B_{21}} \exp\left(\frac{h\nu}{k_B T}\right) - 1\right]} \quad \text{--- (6)}$$

According to Planck, the energy density of radiation is given by

$$\therefore \ell(\nu) = \frac{8\pi h\nu^3}{c^3} \left[\frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} \right] \quad \text{--- (7)}$$

By comparing eq. (6) & (7), we get

$$\frac{B_{12}}{B_{21}} = 1 \quad \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad 3 \times 10^8$$

$$B_{12} \equiv B_{21}$$

— (8)

$$\frac{A_{21}}{B_{21}} \propto \nu^{-3}$$

— (9)

In equilibrium conditions the probability of stimulated absorption is equal to probability of stimulated emission and also the ratio of probability of spontaneous emission to the probability of stimulated emission is proportional to cube of the frequency of radiation.

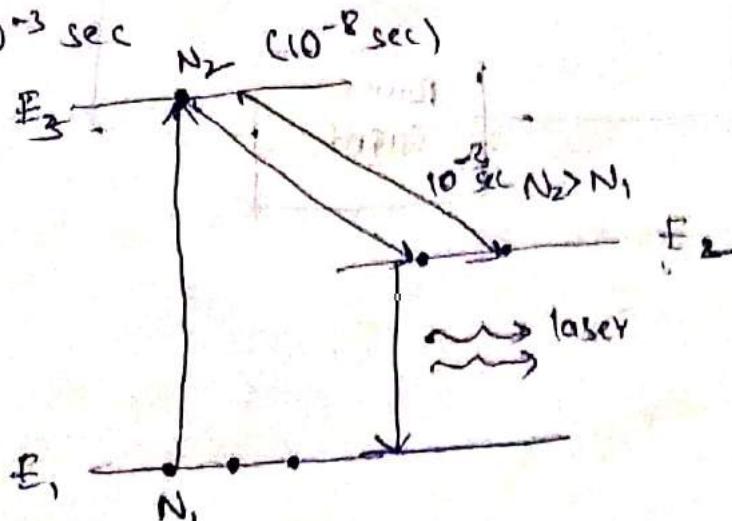
Eq.n ⑧ and ⑨ represents Einstein's relations.

Types of lasers:-

Meta stable state:-

An intermediate energy level present in between lower and higher energy level. Where the life time of atom is 10^{-3} less than higher energy level.

The higher energy level lifetime is 10^{-8} sec and it is not suitable to achieve population inversion condition. A meta stable state is an intermediate energy level whose lifetime is greater than higher energy level lifetime and it is of the order 10^{-3} sec

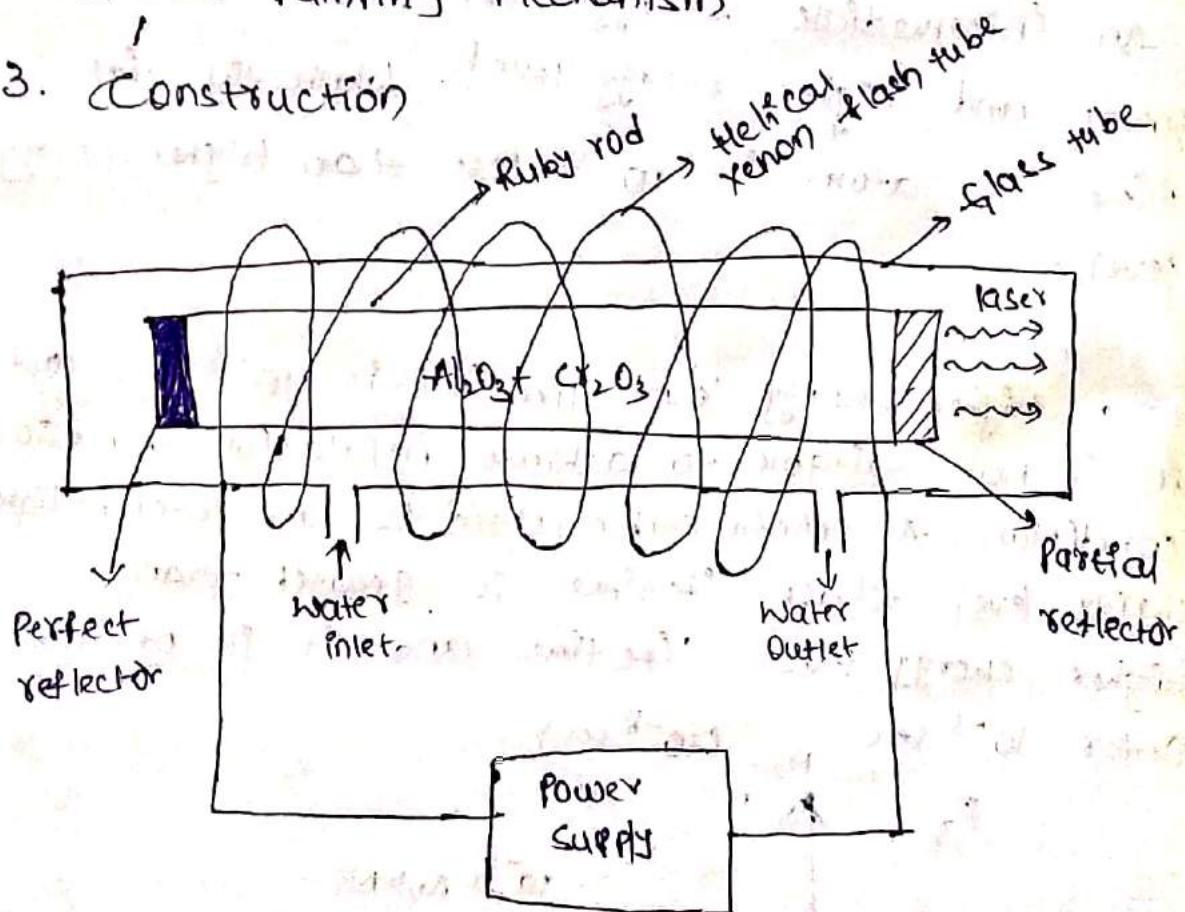


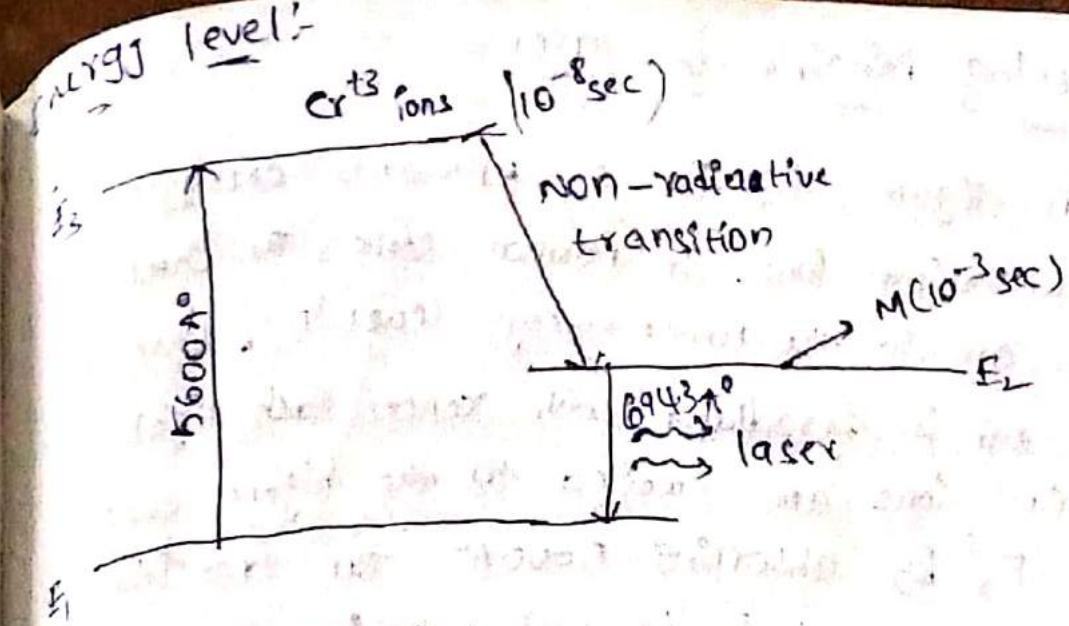
Types of Lasers

- 1) Solid State Laser — Ex: Ruby laser
- 2) Gas laser — Ex: He-Ne laser
 CO_2 laser
- 3) Semi-conductor laser
- 4) Dye laser

Ruby laser:-

1. It is a solid state laser, three level laser system
Active medium: Cr^{+3} ions
2. Optical Pumping Mechanism.
3. Construction





Construction Description:-

A ruby rod is placed in a glass tube. A ruby rod is composed of aluminium oxide and chromium oxide (Al_2O_3). The pink colour of the ruby rod is due to the presence of Cr^{+3} ions. The ends of the ruby rod are grounded and polished. Left side of the rod is completely coated with silver and it acts as a perfect reflector. The right side of the ruby rod is partially coated with silver so that it acts as a partial reflector. A xenon fast tube is surrounded helically over the glass tube. Water is sprinkled around the glass tube to avoid cracks on the glass tube. Laser created will be escaping out through the partial reflector.

5. Working Principle of laser:-

In figure, E_1 , E_2 , E_3 represents energy levels of chromium ions. In normal state, the chromium ions are in the lower energy level ' E_1 '. When the ruby rod is irradiated with xenon flash light, the Cr^{3+} ions are excited to the higher energy level ' E_3 ' by absorbing 5600\AA . The transition from ' E_1 ' to ' E_3 ' is by optical pumping.

The transition of atoms from ' E_3 ' to ' E_2 ' is in the form of non-radiative emission since the atoms can stay in the higher energy level ' E_3 ' only for a life time of 10^{-8} sec . ' E_2 ' is a meta-stable state whose life time is 10^{-3} sec which is greater than higher energy level.

The ions goes on increasing in ' E_2 ' energy level due to pumping and simultaneously the no. of ions in the ground state ' E_1 ' and goes on decreasing thus establishes population inversion condition.

The transition from ' E_2 ' to ' E_1 ' gives a photon of energy. This emitted photon if it is parallel to the reflectors, it undergoes back and forth reflections and amplifies the emitted photon. The amplified laser escapes out through the partial reflector. The de-excitation from ' E_2 ' to ' E_1 ' energy level results in stimulated emission and the amplified laser is of the order of 6943\AA .

Applications:-

- 1) In Industries for cutting and welding
- 2) Eye treatment, Ophthalmology department
- 3) Remote sensing, By using satellites. also in war fields to scan the areas.

Disadvantages:-

pulsed laser is obtained from ruby laser

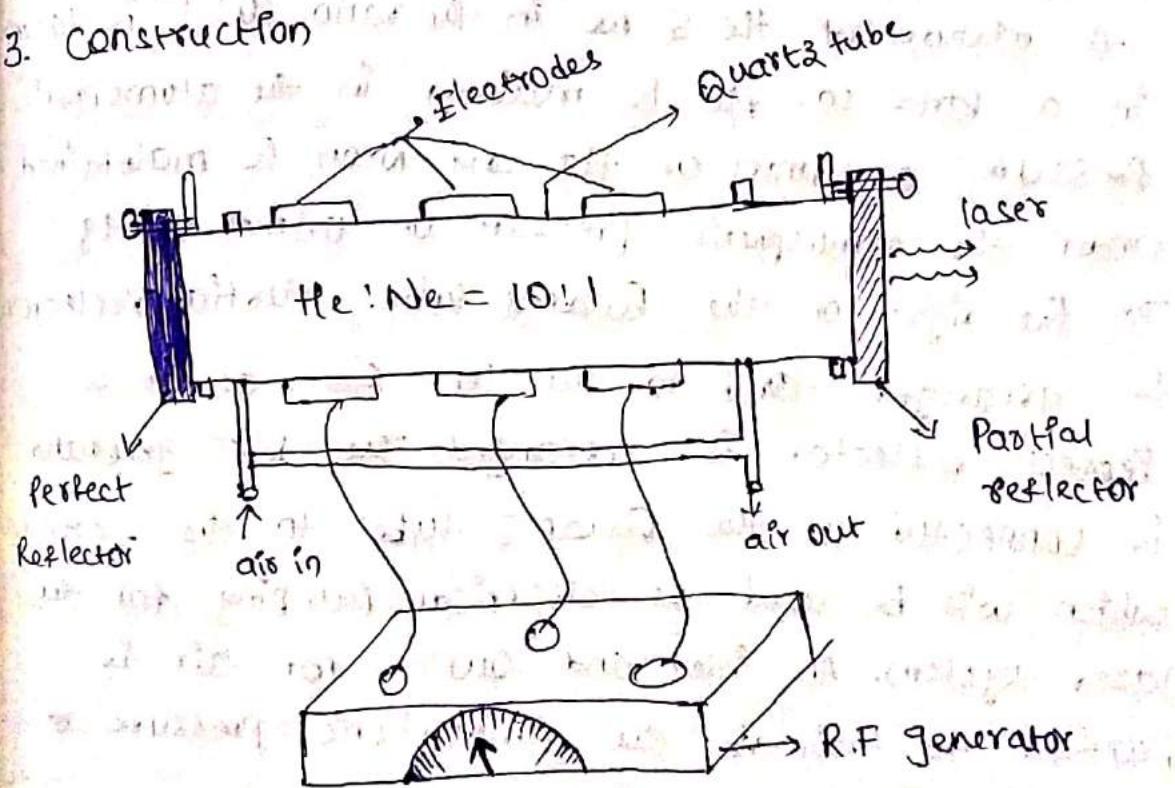
Helium - Neon Laser:-

1) It is a gas state laser, four level laser system

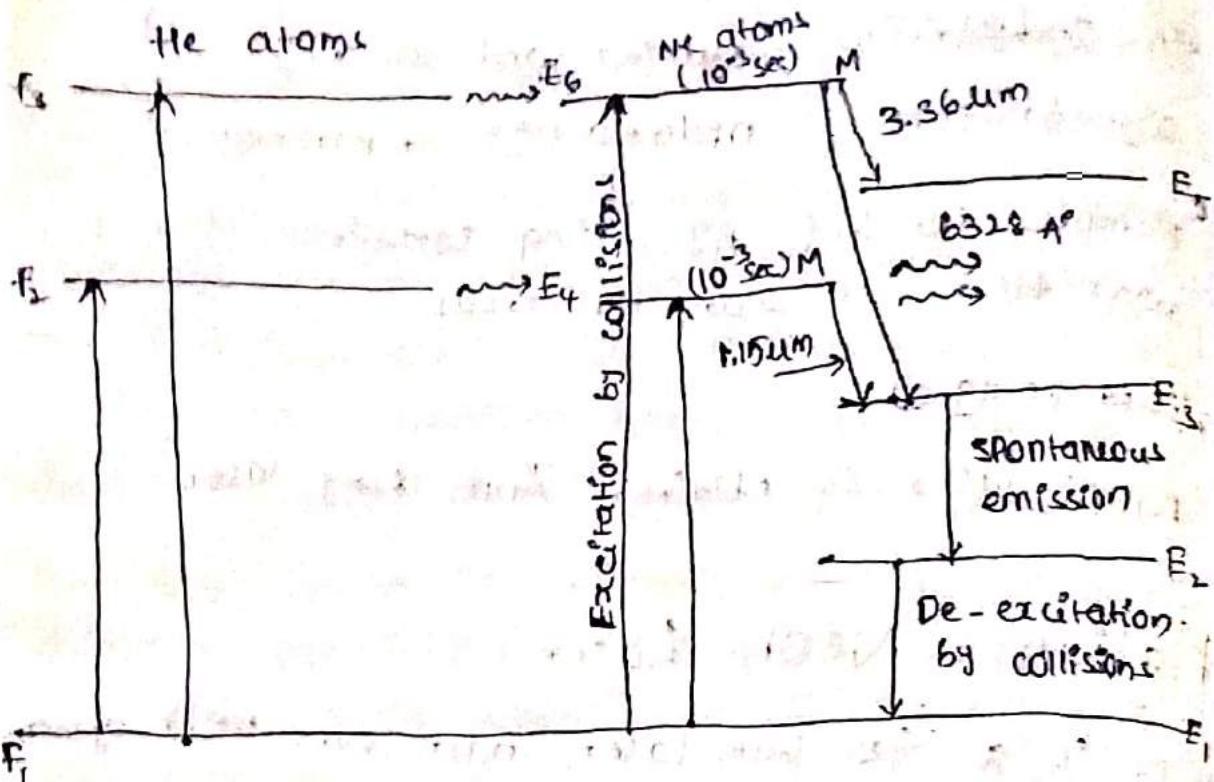
2. pumping mechanism: Electrical Method

Active medium: Neon atoms

3. construction



Energy level



4. Construction Description:

A mixture of He & Ne in the ratio 10 : 1 is taken in a parts to. He is maintained in the atmospheric pressure of 1mm of Hg and Neon is maintained under the atmospheric pressure of 0.1mm of Hg. To the right of the Quartz tube, Partial reflector is arranged "tend" to the left side. of it a perfect reflector is arranged. The R.F generator is connected to the Quartz tube to the electrodes which will be used as electrical pumping for the laser system. An inlet and outlet for air is provided to maintain the atmospheric pressure of gases in the Quartz tube.

Working Principle of He-Ne laser:

Due to the RF generator, He atoms gets excited to the higher energy levels E_2 and E_3 . The excited atoms collides with ground level neon atoms and thus energy is transferred between excited atoms and ground level 'Ne' atoms.

E_4 and E_6 are the meta stable states in 'Ne' atoms. The transitions $E_6 \rightarrow E_5$ is of the order of 3.36nm ($\approx 3360\text{\AA}$). $E_6 \rightarrow E_3$ is of the order of 6328\AA and $E_4 \rightarrow E_3$ is of the order of 1.15nm ($\approx 1150\text{\AA}$) occurs. But all these transitions will not give laser. Only $E_6 \rightarrow E_3$ transition satisfies population inversion condition and liberates a laser beam of wavelength 6328\AA .

The transition from ' E_3' to ' E_2' ' occurs by spontaneous emission and ' E_2' to ' E_1' ' transitions occurs by collisions.

The LASER beam is amplified by back forth reflection between the perfect reflector and partial reflector which is maintained at Brewster's angle.

$$\approx 57^\circ$$

6. Applications:-

Advantages: 1) We get a continuous laser

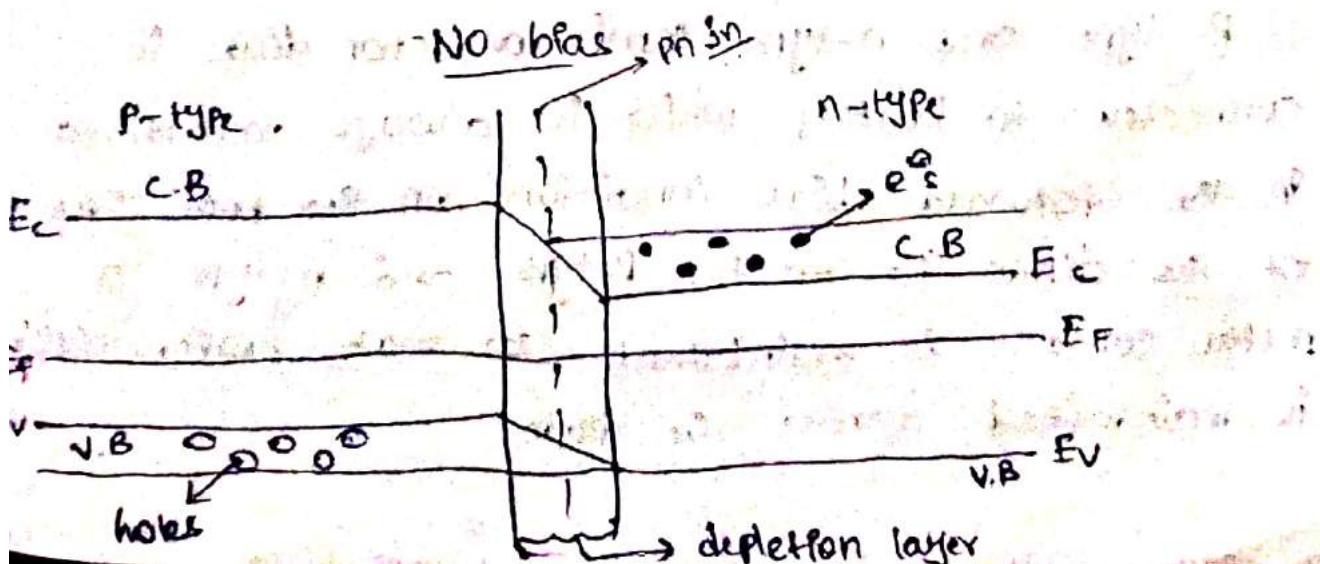
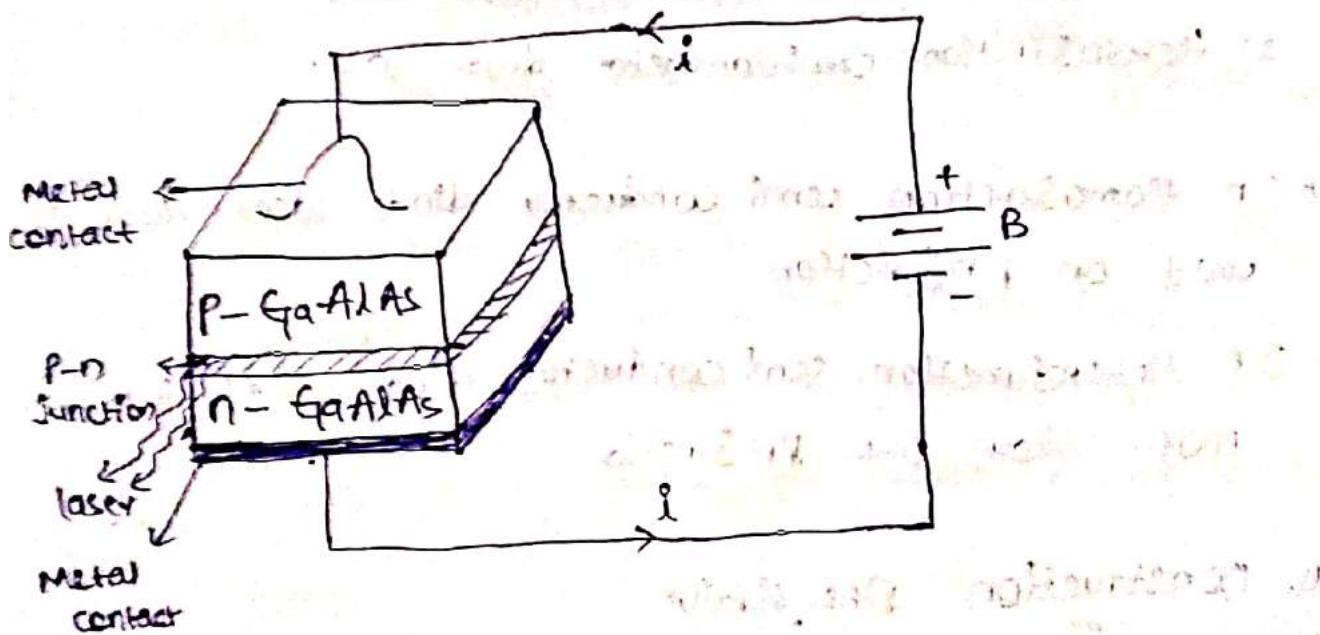
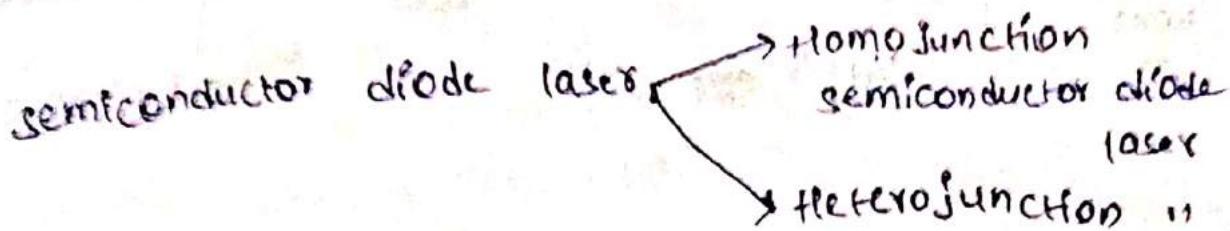
- 2) It is used to decode the bar codes
- 3) In creating holograms (a unique 3d Image)
- 4) To determine the size of the tiny particles

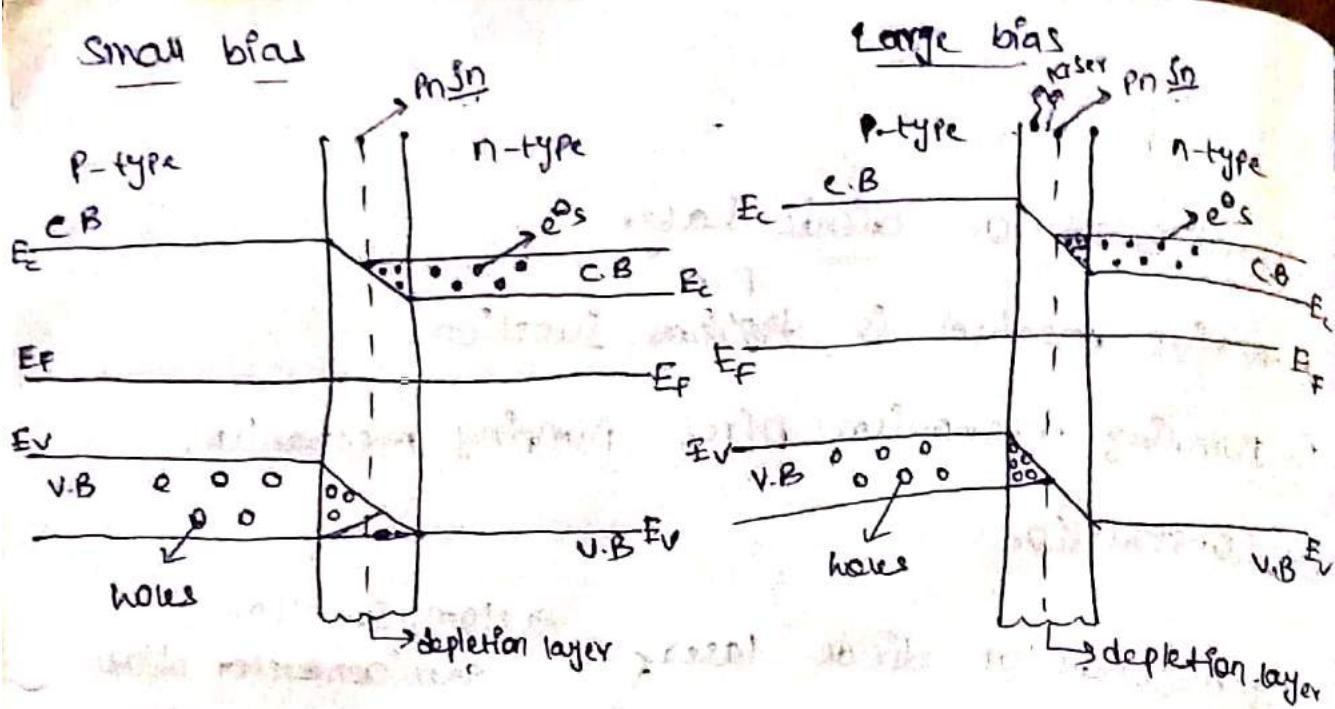
Semiconductor diode Laser:-

i) Active medium is hetero junction

ii) Pumping mechanism: Direct pumping mechanism

3. Construction:





There are two types of semi conductor lasers

- 1) Homojunction semiconductor diode laser
- 2) heterojunction semiconductor diode laser

* In homojunction semi conductor diode laser there is only one p-n junction

In heterojunction semi conductor diode laser, there is more than one p-n junction.

4. Construction Description:

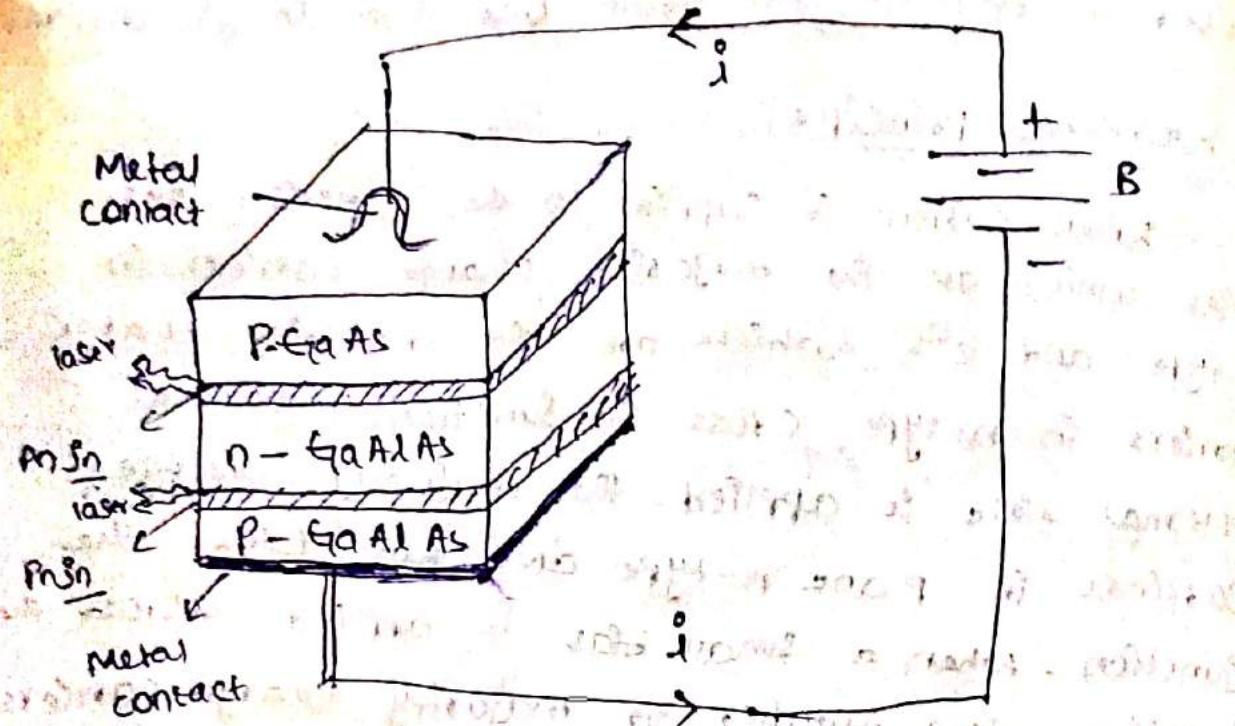
P-type and n-type semi-conductor diode is connected to battery which is always maintained in the forward bias condition. On the both sides of the diode i.e. to the P-type and n-type a metal contact is maintained, so that proper biasing is maintained across the diode.

All the three sides of the diode is roughened so that laser is emitted out from one side of the diode

5. Working Principle:

When current is supplied to the junction, the holes which are the majority charge carriers in p-type and electrons which are the majority charge carriers in n-type cross the junction. When no external field is applied the majority charge carriers in P and n-type does not cross the junction. When a small bias is applied across the junction, few number of majority charge carriers from P & n-type enter into the junction and recombination of holes and electrons takes place in the junction, but this condition will not produce laser.

When large bias is applied across the junction, a large major no. of majority charge carriers in P & n type enter into the junction and recombination of electrons and holes occurs which results in emission of laser.



Hetero - Junction semiconductor diode laser

6. Applications:-

6. Advantages:-

Homo - Junction laser gives us pulsed laser
efficiency is low (less)

Hetero - Junction laser gives continuous laser
efficiency is higher compared to
Homo - junction

Applications:-

It is used in pointers, and in various
medical fields, in computers for storage and
retrieval of information

General applications of Lasers in different fields!

In medical field to treat cancer cells, tumors, kidney stones, Blood less surgery, to remove cataract.

In Engineering field for surveying and alignment of structures.

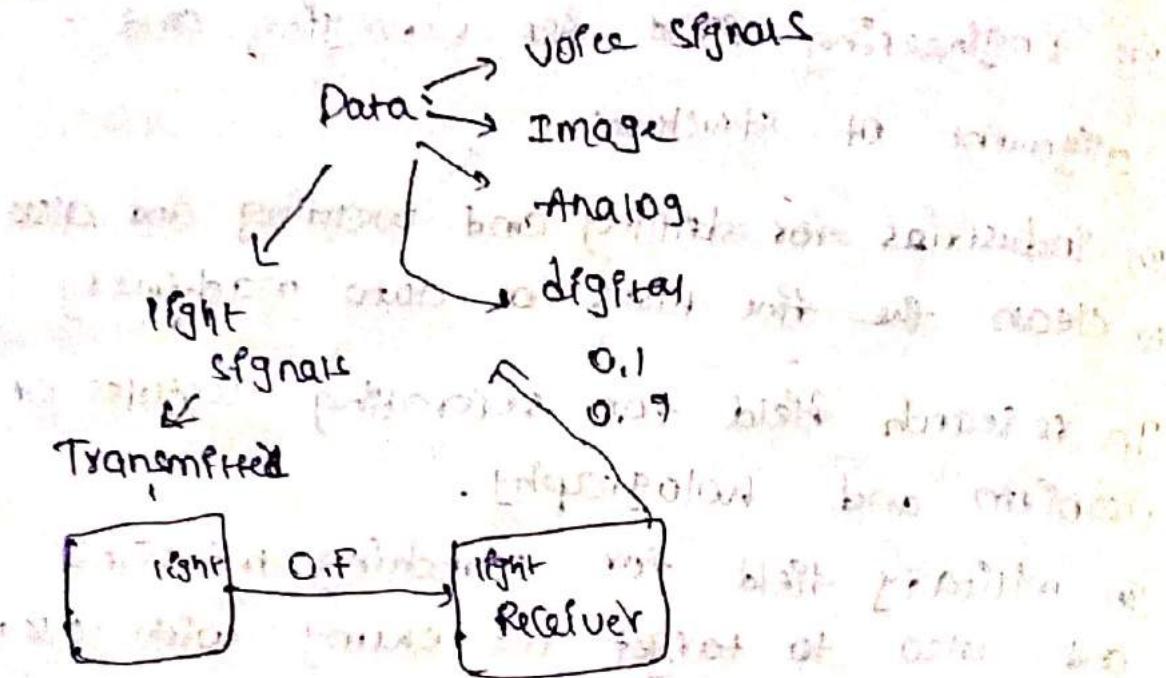
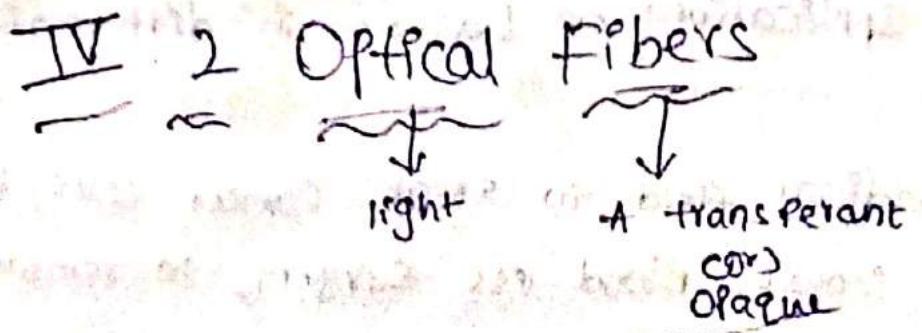
In industries for drilling and welding and also to clean the fine parts of auto machinery

In research field for separating isotopes of uranium and holography.

In military field for launching missiles and also to target the enemy with laser guns in the night

Remote sensing, for decoding bar codes (QR)

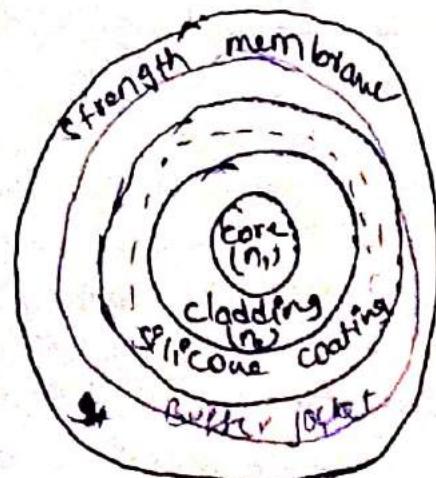
Scanners

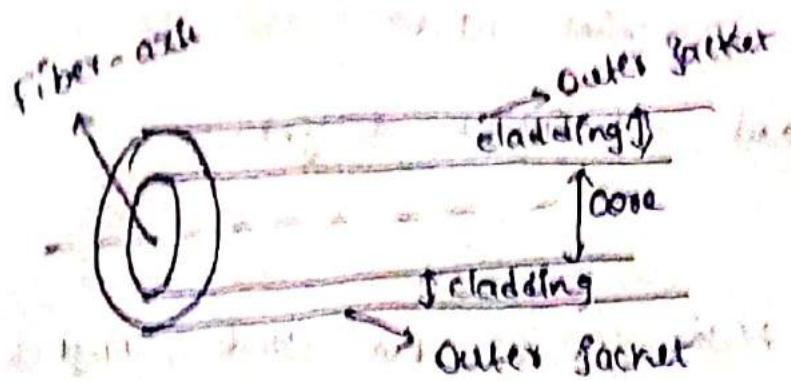


Tyrell scientist

Construction (or) parts of Optical fiber:-

- 1) Core
- 2) cladding
- 3) silicone coating
- 4) Buffer jacket
- 5) Strength membrane
- 6) Outer jacket





Core - made up of pure silica (sand) $R.F = n_1$,
 (perfect medium) has its own refractive index
 and also made up of glass core plastic

Cladding - impurities, silica, glass, plastic
 (rare medium) $R.F = (n_2)$

- Always core R.F > cladding R.F

$$n_1 > n_2$$

While passing light, it should not pass through cladding region. This light should be confined to only core region.

Silicone coating: acts as a protective layer for fracture (core) cracks in core & cladding

Buffer jacket: It will avoid the temperature changes in climate.

Strength membrane: protective layer

To safeguard all inner layers

Outer jacket: Made up of PVC cable, vinyl chloride
To avoid shocks.

N.B.P

Principle of Optical fiber(θ_0) TOTAL INTERNAL REFLECTION
(T.I.R)

Case (i): $n_1 > n_2$

$$\theta < \theta_c$$

cladding
(n_2)
Rarer

core
(n_1)
Denser

Normal

→ P.T.O

Numerical Aperture:

- } sine of maximum acceptance angle (or) [α_{max}]
- } light collecting half cone acceptance angle capacity of O.F

$$N.A = \sin \alpha_{\text{max}}$$

$$N.A = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \quad \text{--- (1)}$$

$n_0 \rightarrow$ refractive index of medium

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{(n_1 + n_2)(n_1 - n_2)}{2n_1^2}$$

Δ --- (2)

for practical $n_1 \approx n_2$

$$\Delta = \frac{n_1(n_1 - n_2)}{2n_1^2}$$

$$\Delta = \frac{n_1 - n_2}{n_1} \quad \text{--- (3)}$$

Fractional difference of
R.I of Core & cladding

From eq.(2)

$$n_1^2 - n_2^2 = 2n_1^2 \cdot \Delta$$

Applying square root on b.s

$$\sqrt{n_1^2 - n_2^2} = \sqrt{2n_1^2 \cdot \Delta}$$

$$\sqrt{n_1^2 - n_2^2} = n_1 \sqrt{2\Delta}$$

$$\therefore N.A = \frac{n_1 \sqrt{2\Delta}}{n_0}$$

for air $n_0 = 1$

$$N.A = n_1 \sqrt{2\Delta} = \sqrt{n_1^2 - n_2^2}$$

Types of Optical fibers:

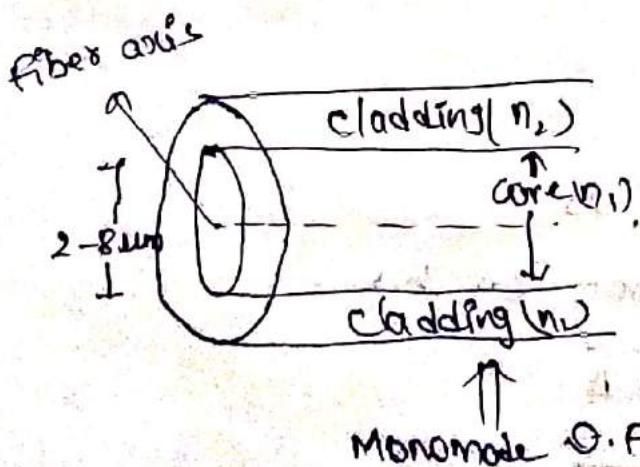
Based on

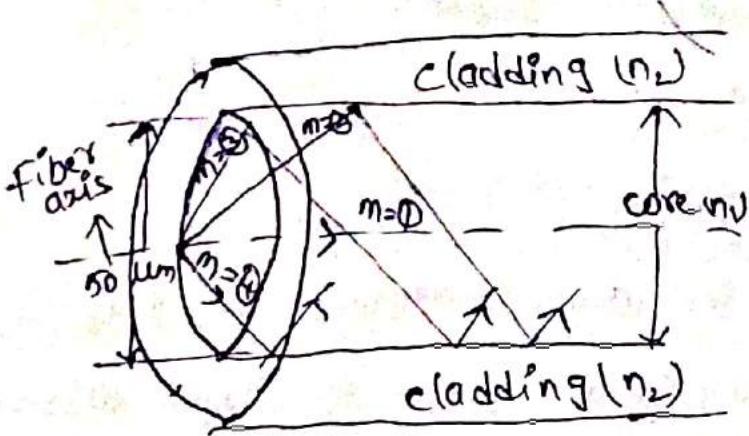
- 1) Number of modes \rightarrow Path (co) direction
- 2) Type of material
- 3) Refractive index profile

N.B
Essay

① Number of modes: Path (co) direction

- 1) Monomode
co.)
Single mode
- 2) Multi mode





↑
multimode O.F

Has multi~~no.~~ number of

② Type of material:

- 1) glass fiber
- 2) plastic fiber
- 3) glass core & plastic cladding fiber
- 4) polymer clad silica fiber (PCS)

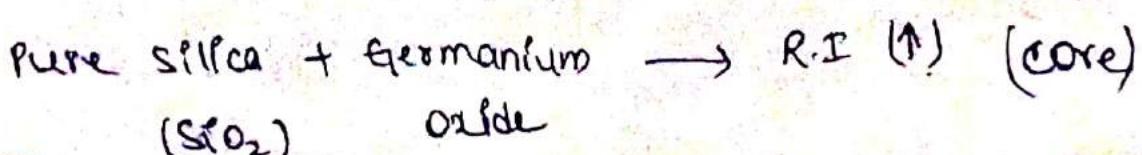
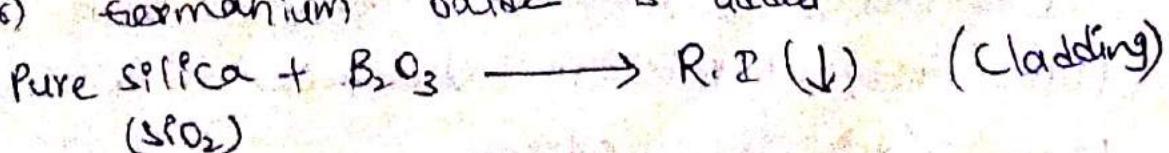
Glass fiber:

— contains ^{pure} Silica but not glass

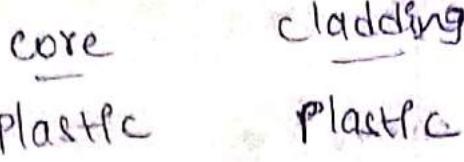
Pure silica — R.I
 1.45

using pure silica we cannot satisfy $n_1 > n_2$

so, B_2O_3 — Boron trioxide is added to pure silica
(or) Germanium oxide is added

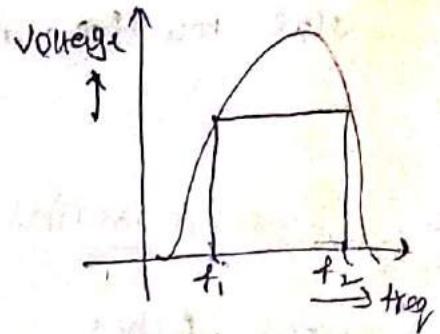


Plastic fiber: cheapest fiber



Data is lost in long distance in this fiber.
It is used to send data in short distance
and efficiency is very less. It absorbs
the transmitted data, thus less band width.

$$\text{Band width} = f_2 - f_1$$



Glass core & Plastic cladding fiber:

Efficiency is more. There is no ^{attenuation} loss in data.
It has high band width. It transmits more
data. Used to send data to long distance.

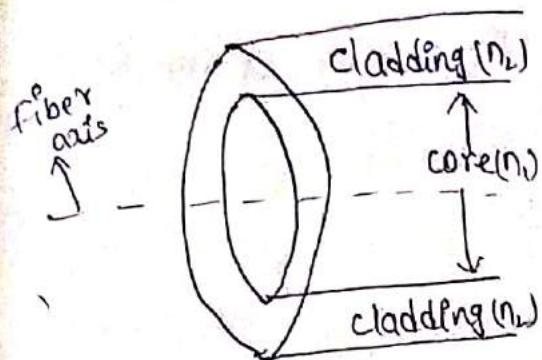
Polymer clad silica fiber: (PCF)

core cladding

Efficiency is moderate. Medium data is
transmitted. Moderate distance it is used.
Bandwidth is moderate.

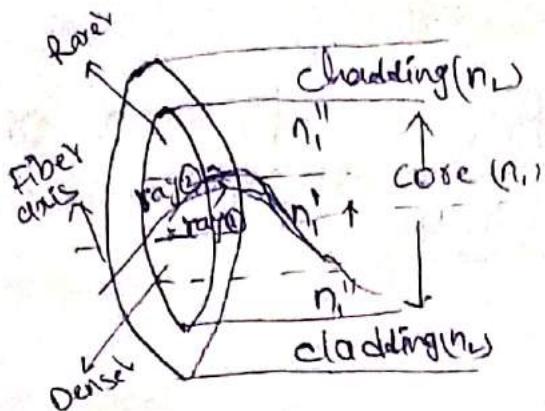
3) Jitt E Refractive index profile:

Step-index fiber (Core)



Single mode
Multi mode

Graded-index fiber
↳ Multi mode



In single mode

- Data is transmitted through fiber axis

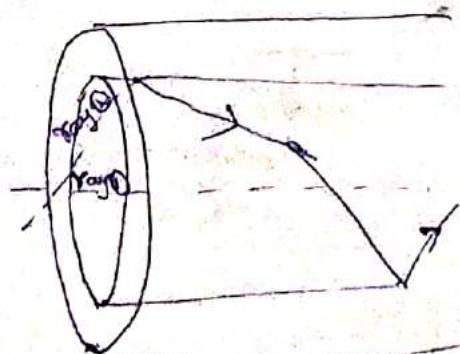
Multi mode:

Value of R.I in core is same (or) uniform

Consider two modes:

In multi mode

one is fiber axis and other.



$$\theta > \theta_c \text{ (T.I.R)}$$

Near fiber axis

Refractive index is more.

$$n_1' > n_1'' > n_2$$

Refractive index of core varies radially.

↳ changes with radius

$$n = \frac{c}{v}$$

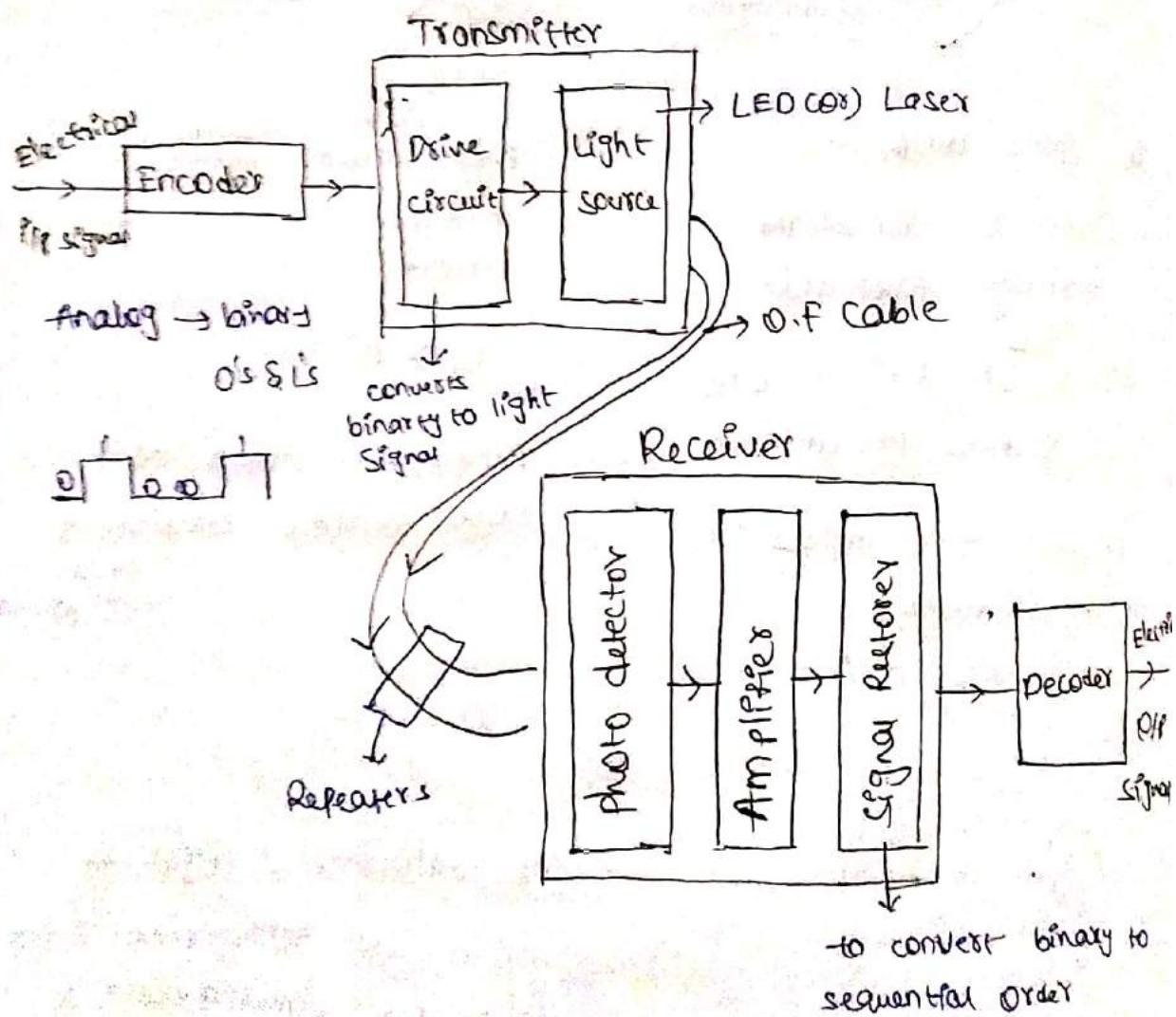
If velocity of light is more, its refractive index decreases simultaneously

so, both rays ray 1 & ray 2 travels ~~at the same time~~ gives equal output.

The time delay in transmission of data to occur is known as "Inter modal dispersion".

EJW#

Block diagram of Optical fiber communication system:



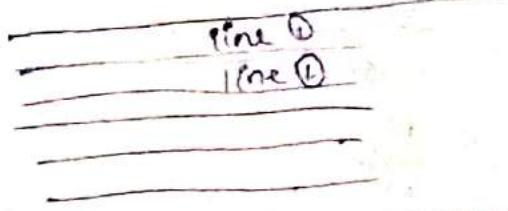
The electrical input signals are passed to the encoder which converts them into binary form. These binaries get converted into light signal

O.S. PPL11ai

Advantages of O.F.

- 1) compared to normal cable wire optical fibres are very cheap, can accommodate large data has high band width
- 2) cross talk can be avoided

Ans



there is no cross talk in O.F because light does not obey T.I.R which do not over lap.

If the signal line ① crosses its barrier it can rectified in the optical fiber.

3) Non-hazards:

If the wires gets disconnected, there is no harmfulness because O.F carries light.

Applications of O.F.

1) In medical field - Endoscopy

2) used as sensors - Detectors



Temperature, pressure changes
electron density

3) used in the telecommunication field

WTF

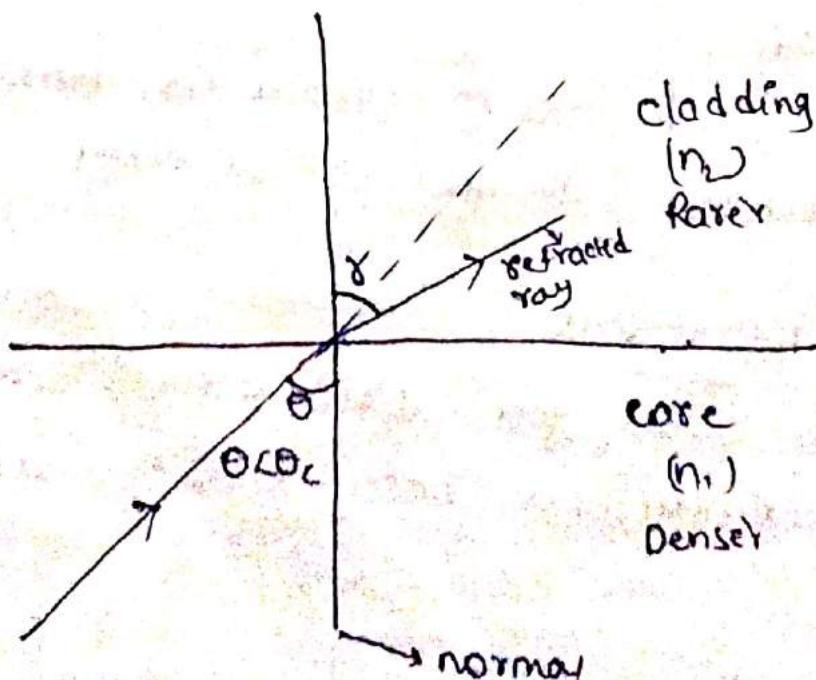
Total Internal Reflection cor principle of O.F.

Case (ii): $n_1 > n_2$, $\theta < \theta_c$

Denser ↓ Rarer ↓ Incident angle ↓ Critical angle

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$$



refraction takes place if light enters from

↳ Bending of light

Denser to rarer
g

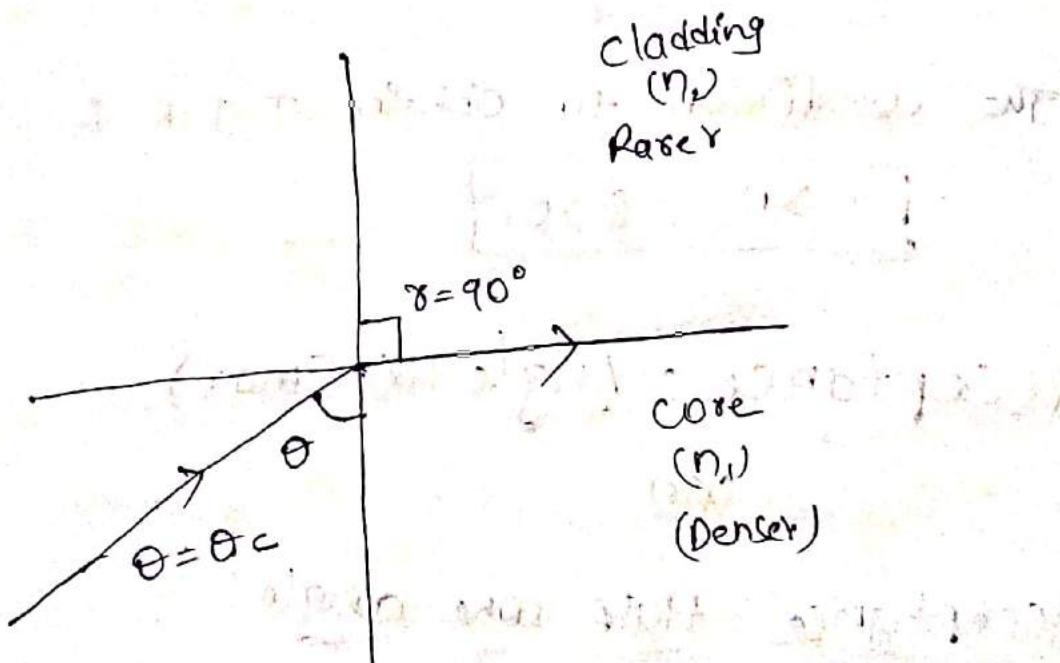
Denser - Rarer \Rightarrow Away from normal

Rarer - Denser \Rightarrow Towards the normal.

γ - refracted angle

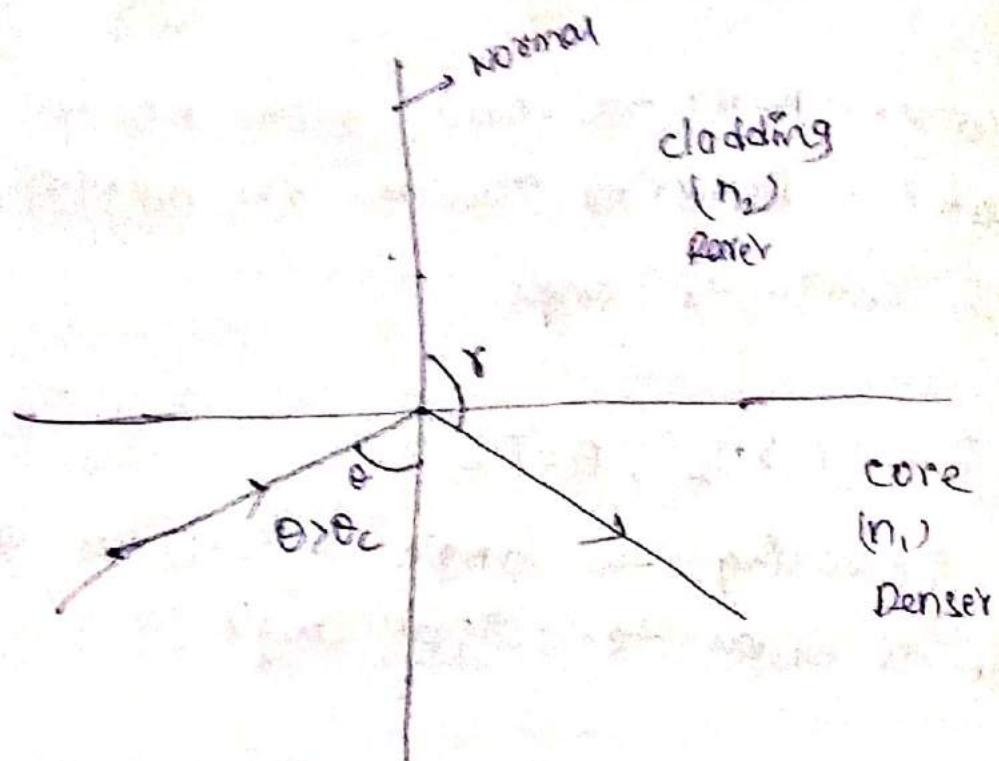
Case (ii): $n_1 > n_2$, $\theta = \theta_c$

By increasing the angle of incidence where that is equal to critical angle.



The refracted ray travels in interference or core & cladding

Case (iii) $n_1 > n_2$, $\theta > \theta_c$

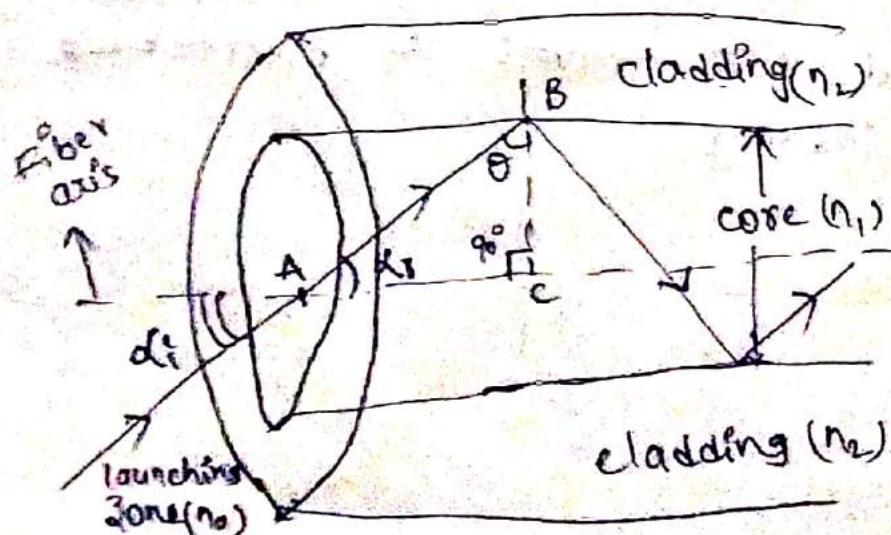


The condition to obtain T.I.R is

$$n_1 > n_2, \theta > \theta_c$$

Acceptance Angle: $\alpha_{\text{c}} (\text{max})$
(cor)

Acceptance Half cone angle!



Let light be launched from a medium of refractive index (n_0). Let the ray enter into the core of refractive index (n_1) and the ray undergoes multiple internal reflections.

Let the angle of incidence be α_i and angle of refraction be α_r .

Let the normal a is drawn at B, so that it makes an angle ' θ ' at B and angle 90° at 'C'.

According to Snell's law:

$$\frac{\sin i}{\sin r} = u$$

$$\frac{\sin \alpha_i}{\sin \alpha_r} = \frac{n_1}{n_0}$$

$$n_0 \sin \alpha_i = n_1 \sin \alpha_r$$

$$\sin \alpha_i = \frac{n_1}{n_0} \sin \alpha_r$$

From fig,

$$\alpha_r = 90^\circ - \theta$$

$$\sin \alpha_i = \frac{n_1}{n_0} \sin(90^\circ - \theta)$$

$$\Rightarrow \sin \alpha_i = \frac{n_1}{n_0} \cos \theta \quad \text{--- (1)}$$

To find $\alpha_i(\max)$!

i) $\theta < \theta_c \Rightarrow$ light enters into cladding, information is lost.

ii) $\theta \geq \theta_c \Rightarrow$ T.I.R \Rightarrow core

iii) $\theta = \theta_c \Rightarrow$ Limiting Point

\uparrow
 $\alpha_i(\max)$

For $\alpha_i(\max)$ we have three cases

i) $\theta < \theta_c$, light enters into cladding and info is lost in the cladding

ii) $\theta > \theta_c$, light undergoes T.I.R

iii) $\theta = \theta_c$, it is the limiting point (or) starting point where light enters into the core region

$$\therefore \sin \alpha_i(\max) = \frac{n_1}{n_0} \cos \theta_c$$

$$= \frac{n_1}{n_0} \sqrt{1 - \sin^2 \theta_c}$$

$$= \frac{n_1}{n_0} \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$= \frac{n_1}{n_0} \times \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \quad \left(\because \sin \theta_c = \frac{n_2}{n_1} \right)$$

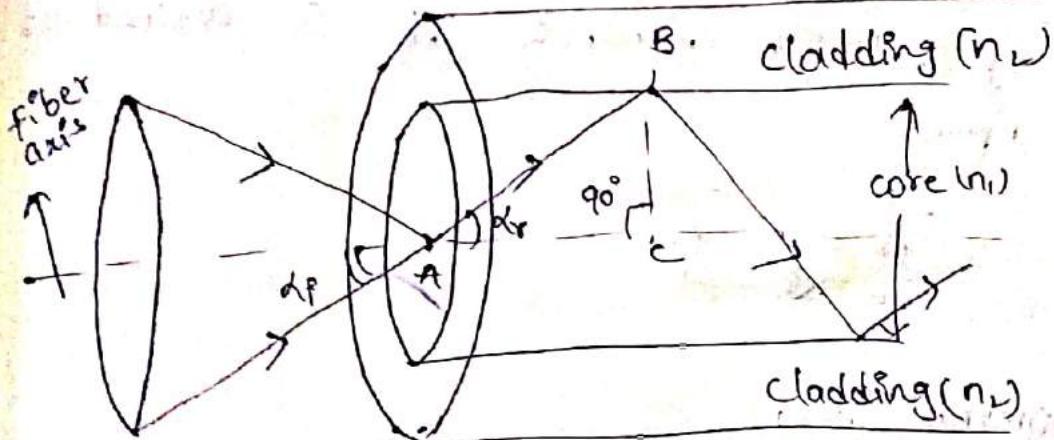
$$\sin \alpha_{\text{cmax}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

∴ Acceptance angle

$$\alpha_{\text{cmax}} = \sin^{-1} \left[\frac{(\sqrt{n_1^2 - n_2^2})}{n_0} \right]$$

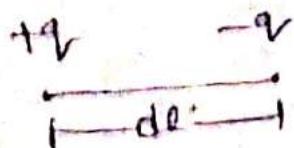
for air, $n_0 = 1$

$$\alpha_{\text{cmax}} = \sin^{-1} (\sqrt{n_1^2 - n_2^2})$$



V - I Dielectric Properties

Two equal and opposite charges



Ex: Calcite

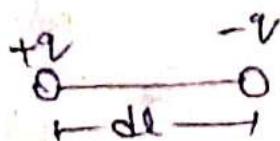
Quartz

Per Fex

Mica

Electric dipole:

Two equal and opposite charges which are separated by a small distance dl is defined as electric dipole.



Dipole Moment: (ll_d)

ll_d = charge \times distance of separation

b/w two charges

$$ll_d = q \times dl$$

units: coulomb - meter

The product of ^{magnitude} charge and distance of separation b/w two charges is defined as dipole moment

Permittivity :- (ϵ)

It gives the information about how easily we can polarise the dielectric material.

Units: farad/meter

Relative Permittivity (or) Dielectric Constant

E_r Ratio

It is defined as the ratio of permittivity of material in medium to the permittivity of material in free space.

$$E_r = \frac{\epsilon}{\epsilon_0}$$

$\epsilon \rightarrow$ permittivity of substance in medium

$\epsilon_0 \rightarrow$ permittivity of substance in free space.

$$\epsilon_0 = 8.825 \times 10^{-12} \text{ F/m}$$

$$E_r = \frac{C}{C_0}$$

$C \rightarrow$ capacitance with dielectric

$C_0 \rightarrow$ capacitance without dielectric

Polarisability :- (α)

Due to the application of external electric field, the charges in the dielectric material gets displaced from each other and dipole moment, will be varied. i.e $M \propto E$ $\Rightarrow \alpha \propto E$

$$M = \alpha E$$

where, α - polarisability
 M - Induced dipole moment

$$\therefore \boxed{\alpha = \frac{q}{E}}$$

Polarisability is defined as induced dipole moment per unit electric field.

Units: coulomb/m²

Polarisation vector! - (\vec{P})

Polarisation vector is defined as dipole moment per unit volume.

e.g. $\vec{P} = N \cdot \vec{r}$

where, N - no. of molecules per unit volume
of the dielectric material

Units: farad/m²

Electric displacement(D) \Rightarrow (F/m²)

~~NES~~ (or)

Electric flux density!

No. of lines passing through unit area from +ve charge to -ve charge across the dielectric material.

Electric flux density is given by:

$$\boxed{D = \Sigma E}$$

①

$$\text{But } \epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$\Rightarrow \epsilon = \epsilon_0 \epsilon_r$$

Substituting in ① we get

$$D = \epsilon_0 \epsilon_r \cdot E \quad \text{--- ②}$$

ALSO, electric displacement can be written in terms of Polarisation vector as

$$D = \epsilon_0 E + \vec{P} \quad \text{--- ③}$$

from ② & ③ we get

$$\epsilon_0 \epsilon_r E = \epsilon_0 E + \vec{P}$$

$$\vec{P} = \epsilon_0 \epsilon_r E - \epsilon_0 E$$

$$\vec{P} = \epsilon_0 (\epsilon_r - 1) E \quad \text{--- ④}$$

Susceptibility (χ): (NO units)

$$\chi = \frac{\vec{P}}{\epsilon_0 E} = \frac{\epsilon_0 (\epsilon_r - 1) E}{\epsilon_0 E}$$

$$\chi = \epsilon_r - 1$$

$$\epsilon_r = \chi + 1$$

Types of Polarisation:-

1) Electronic Polarisation (or) Induced Polarisation

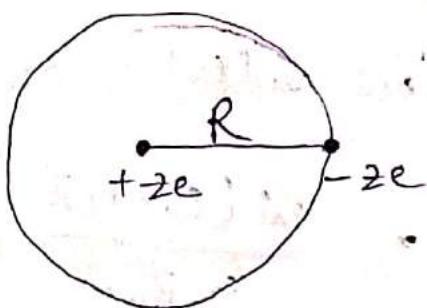
Electronic Polarisation occurs in solids consisting of atoms. The process of displacement of charges on application of external electric field is defined as electronic polarisation.

The dipole moment ' μ ' is given by:

$$\mu = \alpha_e E \quad \text{--- (1)}$$

where, α_e - electronic polarisability

consider an atom consisting of nucleus surrounded with charge ' $-ze$ ' with a radius of ' R '



Charge density (ρ) = $\frac{\text{charge}}{\text{volume}}$

$$= \frac{-ze}{\frac{4}{3}\pi R^3}$$

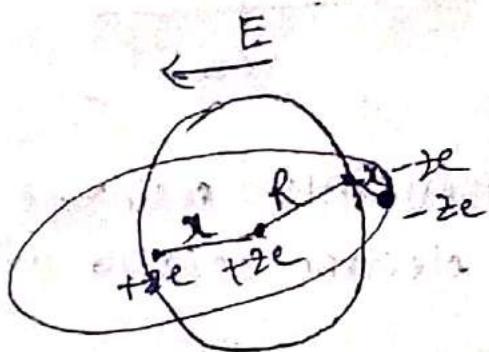
$$= \frac{-3ze}{4\pi R^3} \quad \text{--- (2)}$$

Lorentz force is the force which moves the two opposite charges, apart from each other
+ze & -ze

and it is given by:

Lorentz force = charge \times applied field

$$(F_L) = -ze \times E \quad \textcircled{3}$$



Coulomb force (F_C) = $\frac{ze \times \text{charge enclosed in a sphere}}{4\pi\epsilon_0 r^2}$

charge enclosed = volume $\times \rho$

$$= \frac{4\pi r^3}{3} \times \frac{3}{4} \frac{2e}{\pi R^3} = \frac{-ze r^3}{R^3}$$

$$F_C = ze \times -ze$$

At equilibrium two opposite forces are balanced

$$\text{I.e., } F_L = F_e$$

$$-zeE = \frac{\gamma e^2 e^2 x^2}{4\pi\epsilon_0 R^3}$$

$$E = \frac{ze}{4\pi\epsilon_0 R^3} \Rightarrow x = \frac{4\pi\epsilon_0 R^3 E}{ze}$$

Above equation represents resultant displacement made by electron cloud with respect to the nucleus.

According to induced dipole moment can be written as

$$M = ze \times x$$

$$= \cancel{ze} \times \frac{4\pi\epsilon_0 R^3 E}{\cancel{ze}}$$

$$M = 4\pi\epsilon_0 R^3 E$$

$$\frac{M}{E} = 4\pi\epsilon_0 R^3$$

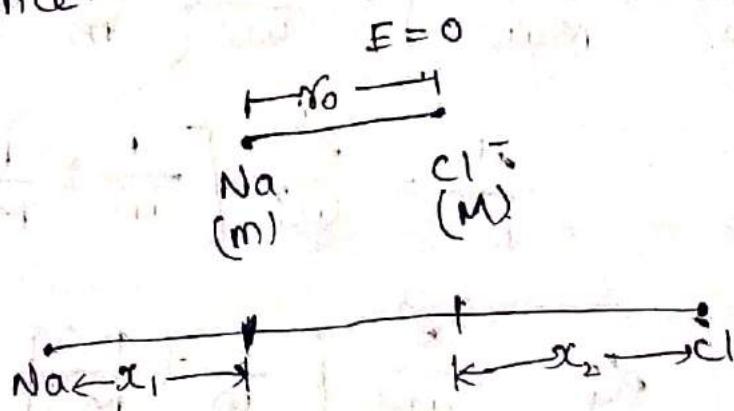
$$\boxed{de = 4\pi\epsilon_0 R^3}$$

2) Ionic polarisation (or) Atomic polarisation:

Ionic polarisation occurs due to the displacement of ions in ionic crystal.

Let us consider ionic molecule of NaCl and NaCl molecule formed at the distance (or) not equilibrium distance.

When an external electric field is applied, Na^+ cation moves by x_1 units and Cl^- anion moves by x_2 units from equilibrium distance.



$$\therefore x = x_1 + x_2$$

The resultant induced dipole moment w.r.t. the displacement made by the ions is

$$M = e \times x$$

$$M = e(x_1 + x_2) \quad \textcircled{1}$$

If β_1, β_2 are restoring force constants of Na^+ cation and Cl^- anions respectively. These are depends on mass of the ion and square of the angular frequency then

$$F = \beta x = \beta_1 x_1 = \beta_2 x_2$$

$$\beta = m\omega_0^2$$

$$F = \beta_1 x_1$$

$$F = \beta_2 x_2$$

$$x_1 = \frac{F}{\beta_1}$$

$$x_2 = \frac{F}{\beta_2}$$

$$x_1 = \frac{eE}{m\omega_0^2}$$

$$x_2 = \frac{eE}{M\omega_0^2}$$

Above displacements Sub: i, eqn ①

$$u = e \left[\frac{eE}{m\omega_0^2} + \frac{eE}{M\omega_0^2} \right] = \frac{e^2 E}{\omega_0^2} \left[\frac{1}{m} + \frac{1}{M} \right]$$

$$\frac{u}{E} = \frac{e^2}{\omega_0^2} \left[\frac{1}{m} + \frac{1}{M} \right]$$

$$d_i = \frac{e^2}{\omega_0^2} \left[\frac{1}{m} + \frac{1}{M} \right]$$

3) Orientation Polarisation (or) Dipolar Polarisation:

In ladder crystals like CH_3Cl (methyl chloride), certain centre of positive charge and centre of negative charge does not coincide. Therefore the electronegativity of the chlorine is greater than the hydrogen atom. This causes partially negative charge on chlorine and partially positive charge on hydrogen creates temporary dipole.

"When an external electric field is applied on such type of crystals like CH_3Cl they possess permanent dipole moment which are aligned themselves in the direction of the applied field called as Orientation (or) dipolar polarisation".

Orientation polarisation is dependent on temperature.

Polarisation $\propto \frac{1}{\text{Temperature}}$

$$P_0 = N\bar{\mu}$$

$$P_a = \frac{N\bar{\mu}^2 E}{3KT}$$

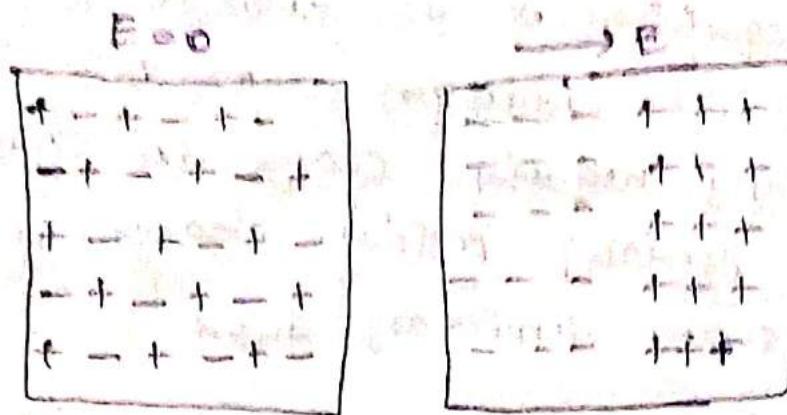
$$P_0 = N\alpha_0 E$$

In comparison

$$\alpha_0 = \frac{\bar{\mu}^2}{3KT}$$

Space charge polarization

The accumulation of charges at the interface of multiphase material or electrodes is called as space charge polarization.



In presence of external field the charge are re-distributed in the di-electric medium.

This polarization is very minute and can exist when the di-electric medium is operated at lower frequencies like 50 Hz (100Hz).

Total Polarizability

$$\alpha_{\text{total}} = \alpha_e + \alpha_g + \alpha_o$$

$$\alpha_{\text{total}} = 4\pi\epsilon_0 R^3 + \frac{e^2}{\omega^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{U^2}{3kT}$$

Total polarization

$$P_{\text{total}} = P_e + P_o + P_g + P_s$$

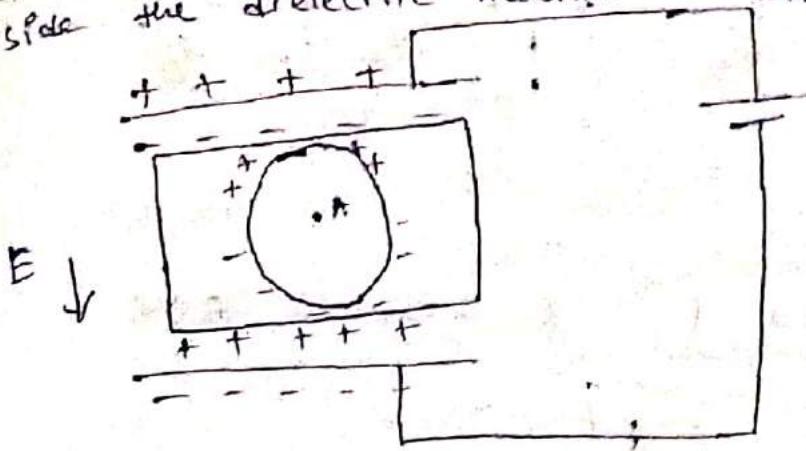
$$P_{\text{total}} = N \alpha_{\text{total}} E$$

$$P_{\text{total}} = NE \left(4\pi\epsilon_0 R^3 + \frac{e^2}{\omega^2} \left(\frac{1}{M} + \frac{1}{m} \right) + \frac{U^2}{3kT} \right)$$

This is called Langevin - Debye equation.

Internal field (or) Local field / Lorentz field

when a dielectric medium is subjected to an external electric field, the field intensity at the point inside the dielectric medium is called internal field.



$$E_i = E_1 + E_2 + E_3 + E_4 \quad \text{--- (1)}$$

E_1 → field intensity at a point 'A' due to capacitor plate charges.

E_2 → field intensity at a point 'A' due to induced charges on the dielectric surface.

E_3 → field intensity at a point 'A' due to other atoms inside spherical cavity.

E_4 → field intensity at a point 'A' due to induced charges on the

field E_1
 $\Rightarrow D = \epsilon_r \epsilon_0 E$

$$\epsilon_r = 1$$

$$D = \epsilon_0 E_1$$

$$E_1 = \frac{D}{\epsilon_0}$$

to

NKT

$$D = \epsilon_0 E + P$$

$$E_1 = \frac{\epsilon_0 E + P}{\epsilon_0} = \boxed{E + \frac{P}{\epsilon_0}} \quad \text{--- (2)}$$

Field ' E_2 ' :

From field ' E_1 ', E becomes zero

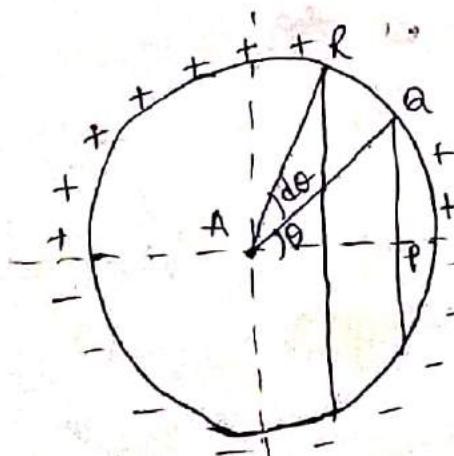
$$\therefore \boxed{E_2 = \frac{P}{\epsilon_0}} \quad \text{--- (3)}$$

Field ' E_3 ' :

the charges are having symmetrical charge distribution

$$\therefore \boxed{E_3 = 0} \quad \text{--- (4)}$$

Field ' E_4 ' :



Area of the selected portion is

$$dA = 2\pi \times PA \times QR$$

From, $\Delta P \Delta Q$

$$\sin\theta = \frac{PA}{AQ} \cdot \frac{PA}{QR} \Rightarrow PA = r \sin\theta$$

From, $\Delta Q \Delta R$

$$\sin\theta \approx d\theta = \frac{QR}{AR} \Rightarrow QR = AR d\theta$$

$$QR = r d\theta$$

$$dA = 2\pi \times r \sin\theta \times r d\theta$$

$$dA = 2\pi r^2 \sin\theta d\theta$$

The charge on the dA area is

$$dq = PA \cos\theta \times dA$$

$$dq = PA \cos\theta \times 2\pi r^2 \sin\theta d\theta$$

The field at 'A' due to dq charge is

$$dE_u = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq \cos\theta}{r^2}$$

$$dE_u = \frac{1}{4\pi\epsilon_0 \cdot r^2} \cdot PA \cos\theta \cdot 2\pi r^2 \sin\theta d\theta \times \cos\theta$$

$$dE_u = \frac{PA \cos^2\theta \sin\theta d\theta}{2\epsilon_0}$$

The total electric field due to spherical cavity surface charges is

$$E_u = \int_0^\pi dE_u$$

$$= \int_0^\pi \frac{P}{2\epsilon_0} \cos^2 \theta \sin \theta d\theta$$

$$= \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta d\theta$$

$$= \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta d(-\cos \theta) d\theta$$

$$= -\frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta d(\cos \theta) d\theta$$

$$= -\frac{P}{2\epsilon_0} \left[\frac{\cos^3 \theta}{3} \right]_0^\pi$$

$$= -\frac{P}{6\epsilon_0} [-1 - 1]$$

$$= \frac{(-P) \times (-2)}{6\epsilon_0}$$

$$E_u = \frac{P}{3\epsilon_0}$$

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$= E + \frac{P}{\epsilon_0} - \frac{P}{\epsilon_0} + 0 + \frac{P}{3\epsilon_0}$$

$$E_i = E + \frac{P}{3\epsilon_0}$$

Clausing - MOSOTTI equation:-

$$\epsilon_i = \epsilon_0 = 1$$

(for cubic crystals)

$$P = N \mu$$

$$P = N \mu e$$

$$\mu e = \alpha_e E_i$$

$$P = N \alpha_e E_i$$

We know that

$$E_i = E + \frac{P}{3\epsilon_0}$$

$$P = N \alpha_e \left(E + \frac{P}{3\epsilon_0} \right)$$

$$P = N \alpha_e E + \frac{N \alpha_e P}{3\epsilon_0}$$

$$P \left(1 - \frac{N \alpha_e}{3\epsilon_0} \right) = N \alpha_e E$$

$$P = \epsilon_0 E (\epsilon_r - 1)$$

$$\epsilon_0 E (\epsilon_r - 1) \left(1 - \frac{N \alpha_e}{3\epsilon_0} \right) = N \alpha_e E$$

$$\epsilon_0 (\epsilon_r - 1) \left(1 - \frac{N \alpha_e}{3\epsilon_0} \right) = N \alpha_e$$

$$\epsilon_0 (\epsilon_r - 1) \left(\frac{3\epsilon_0 - N \alpha_e}{3\epsilon_0} \right) = N \alpha_e$$

$$(\epsilon_r - 1) (3\epsilon_0 - N \alpha_e) = 3 N \alpha_e$$

$$3\epsilon_0(\epsilon_r - 1) - (\epsilon_r - 1)Nde = 3Nke$$

$$3\epsilon_0(\epsilon_r - 1) = Nke(3 + \epsilon_r - 1)$$

$$3\epsilon_0(\epsilon_r - 1) = Nde(2 + \epsilon_r)$$

$$\boxed{\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{Nde}{3\epsilon_0}}$$

Dielectricity:-

Piezo, pyro, Ferro electric materials are active dielectric material.

32 symmetry



II Centro symmetric

II Centro symmetric
(in combination)

20

(non-Centro symmetric)
(Piezo electric)

10 Polar axis

(Pyro electric)

10 non-Polar
axis

Hysteresis

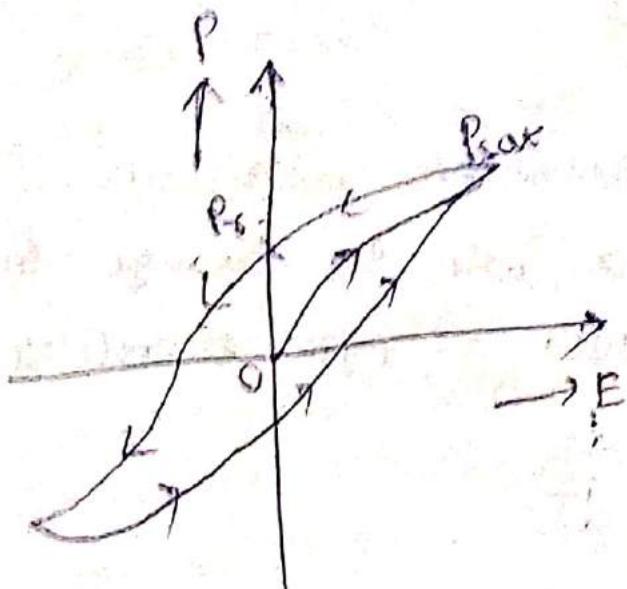
(Ferro electric)

non-Hysteresis

Piezo electricity

Mechanical stress is applied on
 $\text{Dm} \rightarrow \text{E}$ is generated

Ferro electric materials



Piezo electricity

With the application of mechanical stress (cos)
 Present on the opposite ends of the dielectric.
 material, the voltage (cos) e.m.f is produced
 (cos) created on the other opposite ends of the
 dielectric material this phenomenon is called
 as piezo electricity.

Inverse - Piezo electricity:-

On application of voltage / emf on the opposite ends of dielectric material, compression / elongations will be occurring on the other opposite ends of dielectric material.

Pyro electricity:-

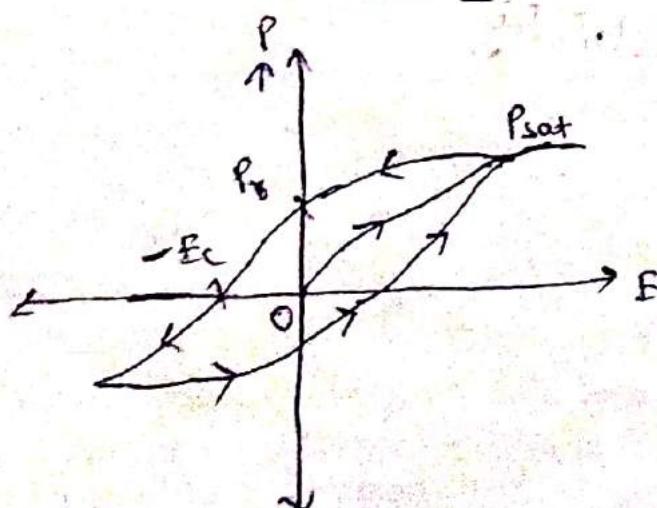
The change in spontaneous polarisation, with the change in polarisation with the change in temperature is called as pyro electricity, it is represented by $\frac{dP_s}{dT}$

Ferro electricity:-

The change in polarization with application of electric field is known as ferro electricity.

The ferro electric materials exhibit hysteresis curve.

Hysteresis curve (or) P vs E graph (or) Hysteresis loop:-



P_{sat} - Saturated Polarisation

P_r - Remanent Polarisation

$-E_c$ - coercive field

If dielectric hysteresis curve is a graph plotted b/w Polarisation vs Applied Electric Field. As the electric field is increased, the polarisation in the dielectric material also increases. At a certain point, with increase of electric field, there is no change in the polarisation i.e. saturated polarisation is attained. Beyond this point of saturation, with decrease of electric field (0^o) when electric field is removed, the curve does not retrace its original path but takes a new path.

When the external electric field is zero (0^o) removed, there is some amount of polarisation in the material which is called as remanent polarisation (P_r). To make the remanent polarisation '0', 've' electric field should be applied which is called as coercive field ($-E_c$). The loop continues but never reaches to the starting point and hence it is called as Hysteresis loop (or) curve (or)

The lag of polarisation behind the applied electric field is called as hysteresis loss (or) loop (or) curve.

Applications of dielectric materials

→ Used as transformer oil for cooling.

Used in capacitors

Used in the resonator antenna's for the producing and receiving micro waves (GHz) ($10^{12} Hz$) (giga Hz)

Used in transducers (A device which converts one form of energy to another)

Used in power generators, spark generators

V.2 Magnetic properties

Magnetic dipole-

Two equal and opposite poles separated by a small distance '2l' is called as Magnetic dipole.

Magnetic dipole moment - (μ_m)

The product of the magnetic pole strength and distance of separation b/w the two poles is called Magnetic dipole moment.

$$\mu_m = m \times 2l$$

Units : $A \cdot m^2$

Magnetic induction (or) Magnetic flux density -(B)

The no. of lines of force passing \perp through unit area of the material is defined as Magnetic induction (or) magnetic flux density.

$$B = \frac{\phi}{A}$$

Units : Tesla (or) wb/m^2

Magnetic field Intensity (H) :-

The force experienced by a unit North pole placed at any point in the magnetic field is defined as magnetic field Intensity.

Units : $A \cdot m$

Relative Permeability - (μ_r)

The ratio between permeability of the material in the medium to the permeability of the material in free space.

$$\mu_r = \frac{\mu}{\mu_0} = \frac{\text{Permeability of material in medium}}{\text{Permeability of material in free space}}$$

\downarrow
no units

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

When the external field H' is applied to bar magnet, its flux density increases

e.g. $B \propto H$

where $B = \mu H \rightarrow \mu = \frac{B}{H}$

where $\mu \rightarrow \text{permeability of material in medium.}$

Similarly, in free space

$$B = \mu_0 H$$

where, $\mu_0 \rightarrow \text{permeability of the material in free space.}$

Magnetisation (M)

The process of converting a non-magnetic material into a magnetic material is defined as Magnetisation.

$$M \propto H$$

$$M = \chi \cdot H \quad \text{units : A-m}$$

Magnetic Susceptibility :-

As magnetic field intensity is increased, magnetisation also increases

$$\text{i.e } M \propto H$$

$$M = \chi \cdot H$$

\downarrow
PSI

$$\therefore \chi = \frac{M}{H} \quad \text{units : A-m}$$

Susceptibility is defined as magnetisation per unit applied magnetic field.

VV SEAR

Prove that $M_r = \chi + 1$

We know that

$$B = M + H \quad \textcircled{1}$$

Also we know that

$$\Rightarrow B = M_0 M_r H \quad \textcircled{2}$$

To eq. ②, add ϵ_e substrat $M_0 H$

$$\begin{aligned} B &= M_0 M_r H + M_0 H - M_0 H \\ &= M_0 (M_r - 1) H + M_0 H \\ &= M_0 M + M_0 H \quad [\#(M_r - 1)H = M] \end{aligned}$$

$$B = M_0 [M + H] \quad \textcircled{3}$$

$$\Rightarrow M_0 = \frac{B}{M + H}$$

$$\therefore M_r = \frac{M}{M_0} + 1$$

$$M_r = \frac{B/H}{B/(M+H)}$$

$$M_r = \frac{B}{H} \times \frac{M+H}{B}$$

$$M_r = \frac{M}{H} + \frac{H}{H}$$

$$M_r = \chi + 1$$

Origin of Magnetic Moment in an Atom:-

The permanent magnetic moment arises in an atom due to

N.B.

① Orbital Magnetic moment due to the electron (M_{oe})

Consider an atom consisting of a moving electron with mass (m), charge ' $-e$ ', moving with angular velocity ' ω '. Let ' r ' be the radius b/w the nucleus and the electron.

Magnetic moment is given by:-
rotated orbital

$$M_{oe} = \text{current associated with the moving } e^- \times \text{Area}$$

— ①

Current associated with moving e^-

$$= \frac{\text{No. of revolutions made by } e^- \text{ with angular velocity}}{2\pi} \times \text{charge}$$

— ②

$$I = \frac{\omega}{2\pi} \times -e$$

— ③

Substituting eq. ③ in ① we get

$$M_{oe} = \frac{-e\omega}{2\pi} \times \pi r^2 = \frac{-e\omega r^2}{2}$$

— ④

Multiply & divide eqn(4) with 'm'

$$\therefore \mu_{oe} = \frac{-e m \omega r^2}{2m}$$

We know that Angular momentum $L = m \omega r^2$

$$\therefore \mu_{oe} = \frac{-e L}{2m} \quad \text{--- (5)}$$

The 've' sign indicates that always magnetic moment is always in the opposite direction to the angular momentum.

$$\gamma = \frac{\mu}{L} = \frac{-e}{2m} \quad \text{--- (6)}$$

 Gyromagnetic ratio is defined as the ratio between the magnetic moment to the angular momentum of the electron.

Angular momentum (L) can be written in terms of orbital quantum number as

$$L = \frac{l h}{2\pi} \quad \text{--- (7)}$$

where, l - orbital quantum no
 $= 0, 1, 2, \dots, (n-1)$

$$n - \text{principal quantum no}$$
$$= 1, 2, 3, 4, \dots$$

Sub eqn ④ can be set in eqn ⑤ we get

$$\therefore \mu_{oe} = -\frac{e l h}{4\pi m}$$

where,

$$\mu_B = \frac{-eh}{4\pi m} = [-9.27 \times 10^{-24} \text{ A-m}^2]$$



Bohr magneton

$$\mu_{oe} = \mu_B \times l$$

② Spin Magnetic moment due to the electron = half of the bohr magneton. (μ_{es})

$$\mu_{es} = \frac{1}{2} (\text{Bohr magneton})$$

$$= \frac{1}{2} \left(\frac{-eh}{4\pi m} \right)$$

$$= \frac{1}{2} (-9.27 \times 10^{-24}) \text{ A-m}^2$$

$$= -4.635 \times 10^{-24} \text{ A-m}^2$$

③

④ spin magnetic moment due to the nucleus (μ_n)

$$\mu_{ns} = \left(\frac{-e\hbar}{4\pi M_p} \right)$$

$$\mu_{ns} = -5.04 \times 10^{-24} \text{ A-m}^2$$