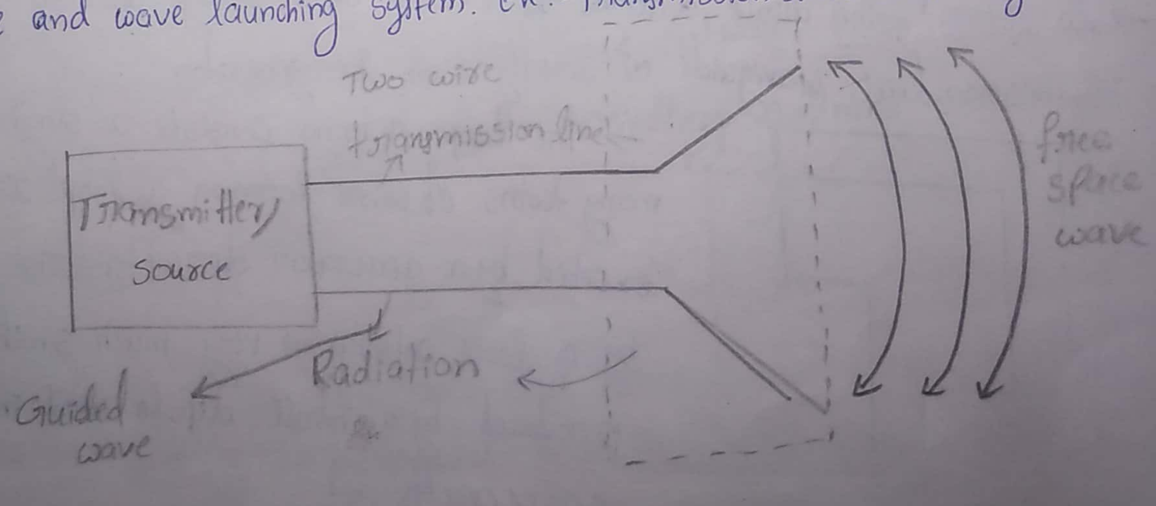
UNIT-I

Antenna Basics

* Antenna is a conductor.
* Antenna is made of copper because of low cost and it is a good conducting material.
* According to IEEE, Antenna is defined as a Radiosense device that is used to transmit (or) receive EM waves (or) radio waves.
* The electrical charges are the sources of the electromagnetic (EM) fields. When these sources are time varying, the EM wave propagate away from the sources and radiation takes place. In general, radiation to be considered as a process of transmitting energy.
* The radiation of the EM wave into the space is effectively achieved by using a conducting or dielectric structures called Antennas or radiators.
* A metallic device used for radiating or receiving radio waves is called Antenna.
* Thus, antenna is regarded as transition between the free space and the system used for launching EM waves. The system used for launching the EM wave is either transmission line or waveguide.
* Thus, Antenna acts as a matching device between transmission line or w/g the free space and wave launching system. Ex:- Transmission line or waveguide.

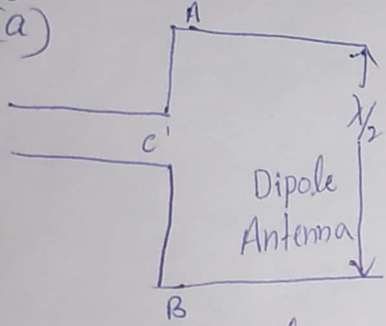


* Guided wave :- The wave that is along the transmission line.
* Free space wave :- The wave that is coming out from antenna.
* Defination :- The line structure associated with the region of transition which is used to couple the Guided wave and free space wave (or) free space wave and Guided wave.
* If the antenna is connected to guided wave, then it is so called transmitting Antenna. If the antenna is connected to free space wave, then it is called Receiving Antenna.
* Antenna is connected between free space and transmission line.

Types of Antenna's :-

The Antennas may be of wire antennas, travelling wave Antennas, slots and operature antennas, reflectors and lenses, Integrated circuit type antennas.

(a)



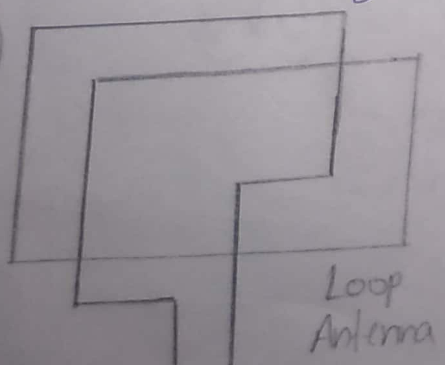
→ It consists of two equal and opposite charges seperated by a shortest distance. Hence it is so called Dipole.

→ Dipole Antenna is called practical reference antenna because it is symmetrical antenna as the two lengths CA, CB are equal.

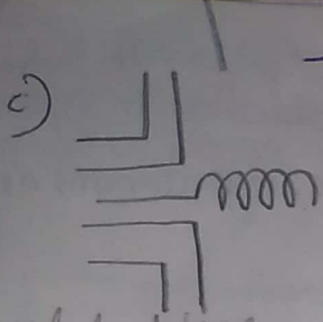
→ The exciting source is kept at the centre. with ~~dipole~~ the exciting source at the centre, a symmetrical dipole is produced. Applications, → Towers, dipole antennas.

→ In general, the dipole antenna is excited by a voltage obtained from a transmission line, waveguide or directly from the generator.

b)



→ The Loop Antenna Consists a single turn or many turns of wire forming a loop. It is generally excited by a generator directly. The field produced by a loop antenna is very much similar to that produced by a small dipole used in buidings, spacecrafts.

c) 

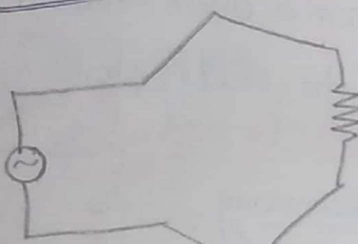
Helical Antenna

→ The antenna with a wire in the form of a helix by ground plane is called helical Antenna helix.

* All the above three antennas are called wire Antennas.

d) Travelling wave Antenna :- In travelling wave Antenna, the antenna is
designed in such a way that a travelling wave in one direction is obtained.
The velocity of this wave equals to the velocity of light and it excites the waves
in the space in the same direction strongly. Thus the maximum directivity can be
achieved. In this load is connected at the end so there is small reflections and the
wave travelling in one direction. It is also called as non-Resoning antenna

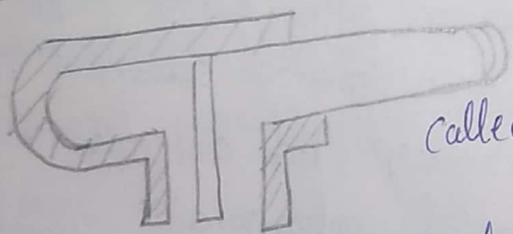
Rhombic Antenna:-



→ The rhombic antenna is an example of travelling wave antenna.

→ When only one half of the antenna is utilised, the antenna is called V-Antenna.

* Dielectric Antenna:-

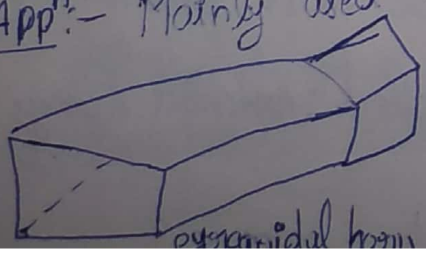


→ The travelling wave antenna in which the travelling wave is guided by dielectric is called dielectric antenna. In this near the cut-

off, the phase velocity equals to velocity of light. The fields produced
external outside a dielectric guide. These outward fields excited the
desired radiation in space. Such travelling antennas are useful
for broadband signals.

3) Aperture Antennas (Pyramidal horn, conical horn, Rectangular waveguide)

App^n:- Mainly used in spacecraft and Aircraft applications.



pyramidal horn

→ pyramidal horn antenna is larger aperture antenna.

→ It is also known as electromagnetic horn.

→ It is mostly useful for broadband signals.

→ In general, a field across an aperture excites radiation in spa[ce]. If an aperture is small, it must be resonant to excite large amount o[f] power,

→ But if an apertture is large, it need not to be resonant.

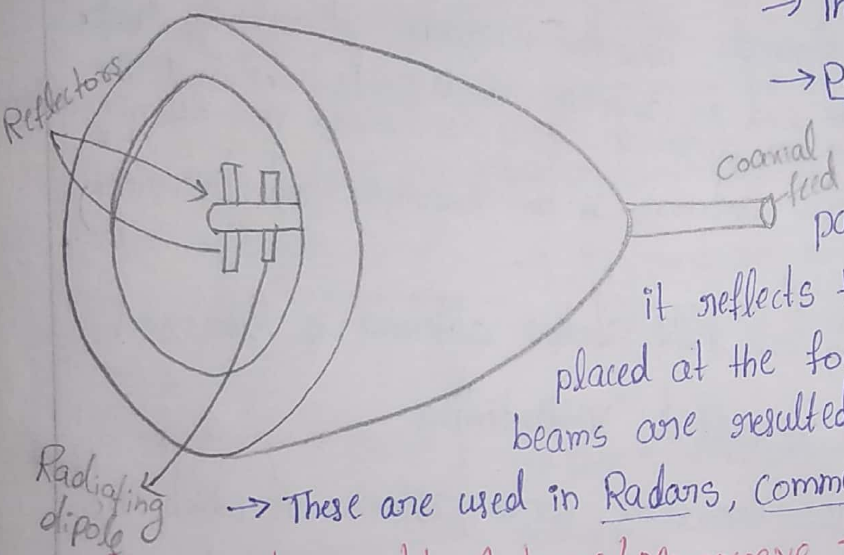
→ The horn antenna at is tapered at one end which provides a trans-tion b/w guiding and sorroundings.

4) Parabolic Reflector Antennas :-

→ Application :- used in dish antennas

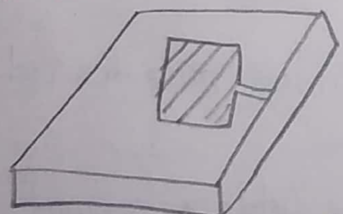
→ These are microwave radiation antenna

→ principle :- The EM waves are reflecte[d] by a conducting sheet. The dish of parabolic reflector acts as mirror and it reflects the radiation from a dipole or horn placed at the focal point. Then the exactly parallel beams are resulted.



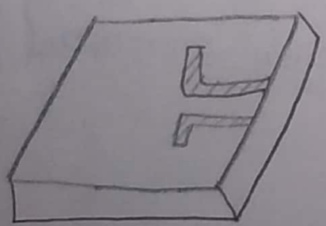
→ These are used in Radars, communications, Astronamy.

5) Microstrip patch Antenna / Microwave Integrated ckt :-



→ The antenna to be used with μ wave integrated ckt may be placed on a dielectric substrate. The best example is microstrip patch antenna.

Application.- used in mobile, Govt & commercial applications.



→ The beside figure shows horn type antenna used with an IC. As Antenna is placed horn type antenna on a dielectric substrate. the IC type antenna is called patch antenna.

LENS ANTENNA ~~.- ARRAYS~~

→ ANTENNA ARRAYS :- When a no. of antennas are connected in parallel with one source element to provide more amount of radiation in single side or double side is called Antenna Arrays.

Ex:- Broadside Antenna Array, Endfire Antenna Array.

Appl^n:- Longdistance communication

Antenna classification

- wire Antennas
  - → Dipole Antenna
  - → Loop antenna
  - → Helix/Helical antenna
  - Applications: Automobiles, buildings, spacecrafts/Aircrafts
- Travelling wave Antenna
  - → Rhombic Antenna
  - → Dielectric antenna
  - Appl^n:- for broad band signals.
- Aperture Antennas
  - → pyramidal horn antenna
  - → conical horn.
  - → Rectangular waveguide horn antenna
  - Appl^n: Space craft and Aircraft applications
- parabolic Reflector Antennas
  - ↓ dish antennas
  - ↓ TV
- micro strip patch antennas
  - ↓ μwave IC antenna mobiles, govt commercial
- Antenna Array
  - → Broadside Antenna Array
  - → End fire
  - → collinear
  - → parasitic
  - → long distance

Applications of Antenna as Resonator:-

As the two arms are of conductors, some of the radiations are reflected back because, in conducting medium loss takes place. i.e, reflections takes place, As the lengths are equipotential, many waves as reflected back and standing waves takes place. Hence dipole antenna acts as resonator circuit.

ii) a) If charge is moving with uniform velocity along straight conductor an infinite length there is no acceleration and deacceleration. no radiation

b) If conductor is irregular shape (bent, curved, terminated). So accelera- and deacceleration takes place. Hence conductor acts as antenna.

iii) When charge oscillates, along straight line conductor also, it acts as antenna because as to oscillate frequency should change ($v = f\lambda$). To change frequency, velocity changes. As velocity takes place acceleration and deacceleration takes place.

Eg:- Water in a pipe when we press a pipe, height of spreading of water changes means it depends upon structure like antenna.

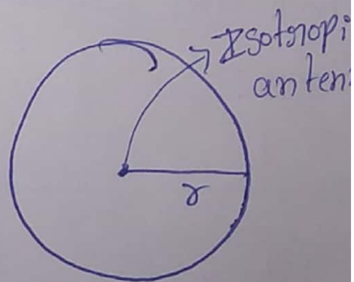
* Isotropic Radiator (or) Isotropic Source (or) Unipole (or) Lossless Antenna

Isotropic Radiator:-

The radiator which radiates electromagnetic energy in all directions uniformly is called Ideal Isotropic Radiator. This is also known as Directionless Antenna.

practically, Ideal Isotropic Antenna is not possible. practical antennas are able to radiate in particular desired direction only.

power radiated by Isotropic Radiator:-



consider Isotropic radiator at its center and it is radiating equally in all directions. Hence, radial component is taken. So, P_{avg} is taken.

Area of sphere $= 4\pi r^2$.

According to poynting theorem $\bar{P}_{rad} = \iint \bar{P}.ds$.

We have taken radial component only.

$$\bar{P}_{rad} = \iint \bar{P}.ds = \iint P_r \, ds = P_r \iint ds = P_r (4\pi r^2)$$

... lossless media, radiation is only along one direction.

$I\theta = I\phi = 0$. i.e, zero absorption $\bar{P} = P_r$ = radial component = P_{avg} density component $\bar{P}_{rad} = P_{avg}(4\pi r^2)$

$$\boxed{P_{avg} = \frac{\bar{P}_{rad}}{4\pi r^2} \; W/m^2}$$

If the area is taken as 1 unit, then $4\pi r^2 = 1$

$$P_{avg} = \frac{\bar{P}_{rad}}{1} \; W/m^2 \Rightarrow \boxed{P_{avg} = \bar{P}_{rad} \; W/m^2}$$

So, the power radiated per unit area is equal to avg power in Isotropic radiator.

Basic Antenna parameters:-

Different antennas are used in different systems in different forms. Irrespective of the antenna system and antenna applications there are some fundamental parameters which are common or same to all antennas are known as Antenna basic parameters (or) Basic Antenna parameters.

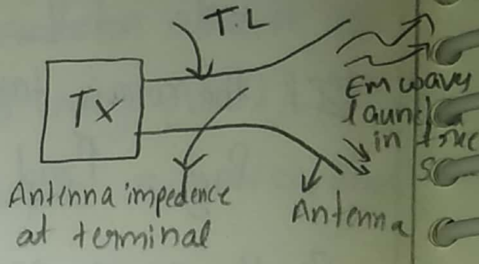
They are following

- → Radiation pattern
- → Radiation Intensity
- → Radiation Lobe
- → Directive Gain
- → Max. directive Gain (or) Directivity (D):
- → Radiation Efficiency/Antenna Efficiency (η_r):-
- → Direction gain GD (θ, ϕ)
- → Front to Back lobe Ratio (FBR)
- → Effictivity length (or) Height (Leff)
- → Aperture Area (or) Effective area (Ae)
- → Beam width (BW) (or) Half power beam width (HPBW)

→ Beam Area (B) (or) Beam solid Angle (Ω_A)
→ Resolution.

Note:- Antenna acts as terminal resistance:-

Antenna is a device which acts as interfacing medium b/w ckt and free space.



→ Antenna acts as terminal resistance to the transmission line which is commonly called Radiation resistance (R_{rad})

→ R_{rad} is not an actual resistance but it is the resistance inteface from antenna with its surrounding at the terminals.

→ the transmitting antenna radiates same power in the free space. The Rxing antenna receives two types of radiations namely Active radiations from other Antennas while passive Radiations which are reflections from the distinct objects. These radiations increases the apparent temperature T_A of the radiation resistance. The Radiation resistance and antenna temperature are important parameters which are both single valued scalar quantities.

a) RADIATION PATTERN:-

Def:- The graphical representation of antenna radiation represented as a function of direction.

→ The radiation pattern is 3-D quantity which is represented as a function of spherical coordinates as θ and ϕ both.

purpose:- practically, Antenna cannot radiate energy with same strength uniformly in all directions. It is found that the radiation is large in one direction while zero or min. in other directions. The radiation from the antenna in any direction is measured in terms of field strength at a point located at a particular distance from an antenna

The field strength can be calculated by measuring voltage at two points on an electric lines of force and then ~~of~~ dividing by distance b/w two points. Hence, unit of radiation pattern is volt/metre.

→ Radiation pattern indicates the distribution of energy radiated by an antenna in the space.

→ Thus, Radiation pattern is nothing but a graph which shows the variation of actual field strength of electromagnetic field at all points equidistance from an antenna.

→ If the radiation of an antenna is graphically represented as a function of direction, then it is so called Radiation pattern. This determines radiations in particular direction

→ Radiation pattern is 3-D quantity which is represented as a function of spherical coordinates as θ and ϕ.

→ To represent field Radiation pattern, a spherical coordinate system is most suitable.

→ Radiation pattern can be changed by following 2 factors

(a) Height of Antenna (b) Material used for conductivity of an Antenna.

Types of Radiation pattern

Field Radiation pattern

If the radiation of an antenna is expressed in terms of the field strength E(in V/m), then the graphical representation is called field Radiation pattern/ field strength pattern.

Power Radiation pattern

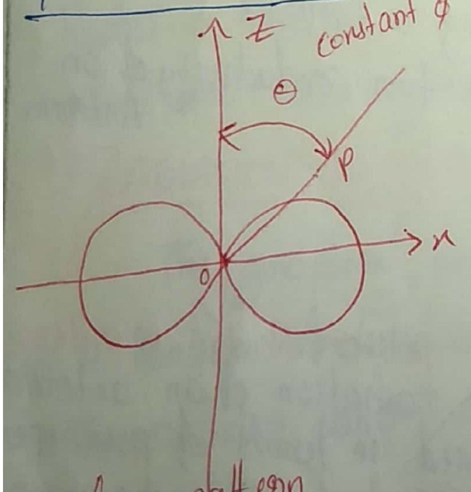
→ If the radiation of an antenna is expressed in terms of power per unit solid angle, then the graphical representation is called power Radiation pattern/ power pattern.

Principal pattern:- As the three dimensional pattern cannot be plotted in a plane, the 3-D representation is avoided. Instead of this. the polar plots of the relative magnitude of the field can be sketched. These polar plots are plotted in two planes namely 'one containing to it and the normal to it. These planes are called "principal planes" and the two plots are called principle plane patterns.

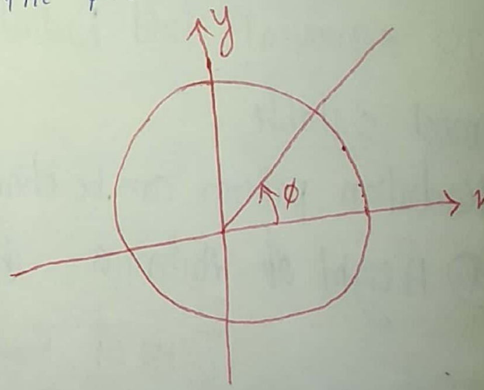
→ These pattern are obtained by plotting the normalized field strength. The normalized field pattern is defined as the ratio of field component to its max. value.

$$\boxed{E_{\theta n}(\theta,\phi) = \frac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{max}}}$$

E-plane pattern or vertical pattern:- When the magnitude of normalized field strength is plotted Vs θ with constant ϕ, the pattern is called E-plane pattern or vertical pattern. The E-plane pattern for Hertzian dipole is as shown below in fig(a). When normalized field strength is plotted Vs ϕ with $\theta = \pi/2$, the pattern is called H-plane pattern or Horizantal pattern.



E-plane pattern.



H-plane pattern.

→ Radian is the measure of plane angle.

→ Steradian is the measure of solid angle.

→ Normalized field pattern is a dimensionless quantity with max. value equal to unity/one.

→ plane angle means circle, that means around circle there are 2π radians. Steradian is a measure of solid angle means around it 4π radians. Solid angle $d\Omega = \frac{ds}{r^2} = \sin\theta \, d\theta \, d\phi$. Sr.

② RADIATION INTENSITY:- $[U(\theta,\phi)]$:- W/S = watts/steradian.

→ The radiation intensity is defined as power per unit solid angle.

→ Radiation pattern is denoted by U and doesn't depend on the distance from the antenna or radiator.

→ Radiation Intensity of an antenna is defined as

$$\boxed{U(\theta,\phi) = r^2 P_d(\theta,\phi)} \text{ — (a)} \quad P_d(\theta,\phi) \rightarrow \text{Total power.}$$

Then, the total power radiated can be expressed interms of radiation Intensity as

$$P_{rad} = \oint_S P_d(\theta,\phi)\, ds = \oint_S P_d(\theta,\phi)\,[r^2 \sin\theta \, d\theta \, d\phi]$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} [P_d(\theta,\phi) r^2] \sin\theta \, d\theta \, d\phi \text{ — (1)}$$

let $d\Omega = \sin\theta \, d\theta \, d\phi$ — (b) be the differential solid angle in steradians (Sr). Then we can write, sub eq^n - (a), (b) in eq^n (1)

then we get,

$$\boxed{P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta,\phi)\, d\Omega}$$

→ Thus radiation intensity $U(\theta,\phi)$ is expressed in watts per steradian (w/sr) and it is defined as time avg power per unit solid angle

The avg. value of Radiation Intensity is given by

$$\boxed{U_{avg} = \frac{P_{rad}}{4\pi}} \rightarrow \text{proof in next page (8)}$$

Adv:- Using radiation intensity $U(\theta,\phi)$ we can also calculate normalized power pattern as ratio of radiation Intensity $U(\theta,\phi)$ to $U(\theta,\phi)_{max}$

$$\boxed{P_d(\theta,\phi) = U(\theta,\phi) / U(\theta,\phi)_{max}}$$

power radiated in terms of Radiation Intensity:-

$$P_{rad} = \iint P_d(\theta,\phi)\, ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \underbrace{P_d(\theta,\phi)\, r^2}_{U(\theta,\phi)} \overbrace{\sin\theta\, d\theta\, d\phi}^{d\Omega}$$

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\, d\Omega.$$

Max. Radiation intensity, $U_{max} = r^2 P_{d\,max}$.

Avg. Radiation Intensity, $U_{avg} = r^2 P_{avg}(\theta,\phi)$ — (1)

we have $P_{avg}(\theta,\phi) = \frac{P_{rad}}{4\pi r^2}$ — (2)

sub eq^n (2) in eq^n (1) $U_{avg} = \cancel{r^2} \cdot \frac{P_{rad}}{4\pi \cancel{r^2}}$, $\boxed{U_{avg} = \frac{P_{rad}}{4\pi}}$

3) DIRECTIVE GAIN [$G_D(\theta,\phi)$]

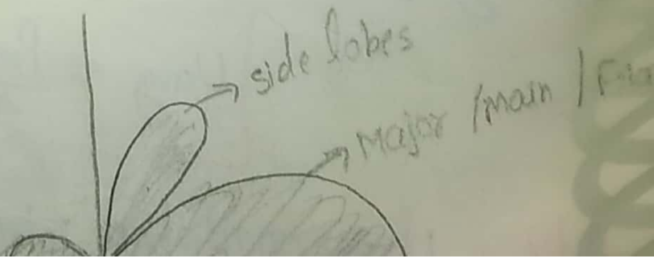
If an antenna were istotropic, it were to radiate uniformly in all direction. The avg. power density at all points on the surface of sphere will be same. The avg. power will $\boxed{P_{avg} = \frac{P_{rad}}{4\pi r^2}}$

Directive Gain:- (i) It is used to measure the concentration of radiation in particular direction.

Directive Gain $\boxed{G_D(\theta,\phi) = \frac{P_d(\theta,\phi)}{P_{avg}}}$

For Isotropic antenna, the value of directive gain is unity.

4) RADIATION LOBE:-



"The radiations by an antenna are bounded within the particular direction

5) MAX. DIRECTIVE GAIN (or) DIRECTIVITY (D) :-

→ The ratio of Max. power density to the avg. power radiated is called Max. directive gain or directivity of an antenna. It is denoted as $G_{D max}$ or D.

$$\boxed{D = G_{D max} = \frac{P_{d max}}{P_{avg}}}, \quad P_{avg} = \frac{P_{rad}}{4\pi r^2}$$

$$D = G_{D max} = \frac{P_{d max}}{P_{rad}/4\pi r^2} \Rightarrow D = \frac{P_{d max} \cdot r^2}{P_{rad}} \quad [\because U_{max} = P_{d max} \cdot r^2]$$

$$\boxed{D = \frac{U_{max} 4\pi}{P_{rad}} = G_{D max}}$$

→ The directivity of an antenna is called dimensionless quantity. The directivity can also be expressed in terms of E is.

$$D = G_{D max} = \frac{4\pi |E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E_\theta(\theta,\phi)|^2 \sin\theta \, d\theta \, d\phi}.$$

6) RADIATION EFFICIENCY: (Antenna Efficiency (η_r)) [$G_p(\theta,\phi)$] → power gain

The practical antenna is made up of a conductor having finite conductivity. Hence, we must consider the ohmic power loss of the antenna. If the practical antenna has ohmic power losses (I^2R), then the power radiated is less than the i/p power. Then we calculate efficiency (η_r)

$$\boxed{\eta_r = \text{Antenna Efficiency} = \frac{P_{rad}}{P_{in}}}$$

→ Thus, the total i/p power can be written as $P_{in} = P_{rad} + P_{loss}$

So, efficiency, $\eta_r = \dfrac{P_{rad}}{P_{rad} + P_{loss}}$

we have, $P_{rad} = I^2_{rms} R_{rad}$. $P_{loss} = I^2_{rms} \cdot R_{loss}$.

$$\boxed{\eta_r = \frac{R_{rad}}{R_{rad} + R_{loss}}}$$

Power Gain :- $G_P(\theta,\phi)$

Power Gain, $G_P(\theta,\phi) = \frac{\text{Power radiated in particular direction}}{\text{Actual power i/p}}$

$$\boxed{G_P(\theta,\phi) = \frac{P_d(\theta,\phi)}{P_{in}}}$$

Max. power Gain :- $G_{P_{max}} = \frac{\text{Max. Radiation Intensity}}{\text{Radiation intensity due to Isotropic lossless antenna}}$

$G_{P_{max}} = \frac{U_{max}}{P_{rad}}$. [$U_{avg} = P_{rad}/4\pi$

If $U_{avg} = U_{max}$, then $P_{rad} = P_{in}$, then

$U_{avg} = P_{in}/4\pi$

$G_{P_{max}} = \frac{U_{max}}{P_{rad}/4\pi}$ — (1)

we have directivity, $D = \frac{4\pi U_{max}}{P_{rad}}$ — (2)

$P_{rad} = \frac{4\pi U_{max}}{D}$ — (3)

Sub. eq^n (3) in eq^n (1), $G_{P_{max}} = \frac{U_{max}}{\frac{4\pi U_{max}}{D}}$ $\Rightarrow$ $\boxed{G_{P_{max}} = \frac{D}{4\pi}}$

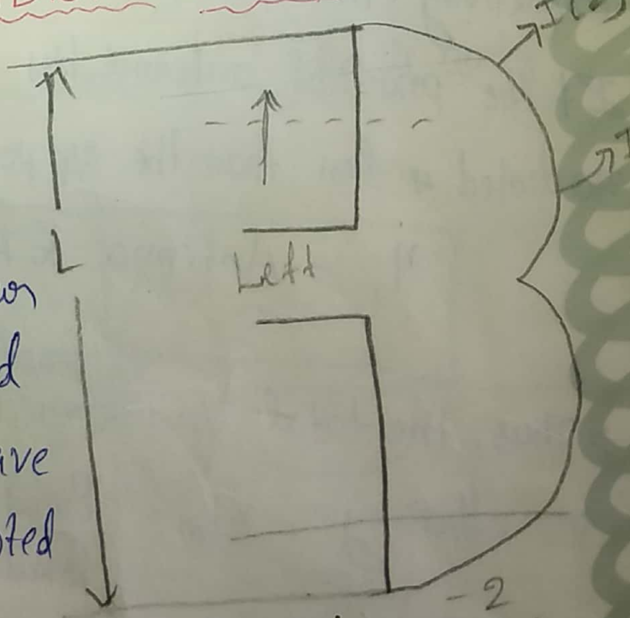
$\boxed{G_{P_{max}} = D}$ → This is for Isotropic Radiator.

→ power Gain is expressed in decibles.

→ For practical antenna, η_r may not be 100%.

$\therefore$ $\boxed{G_{P_{max}} = \eta_r \cdot D}$

7) EFFECTIVE LENGTH (or) HEIGHT (L_{eff}) :-



→ Def :- The effective length of an antenna carrying peak current I_m is defined as the length of Imaginary linear antenna with a uniformly distributed current such that both the antennas have same field in $\theta = \frac{\pi}{2}$ plane. It is denoted by l_{eff}.

Effective length of transmitting antenna lying along z-axis.

→ The effective length of an antenna used for transmitting is same as that for an antenna used for receiving.

→ The effectiveness of antenna to radiate and extract (or) receive radiation and which carries the max (or) peak currents along its effective length.

→ Generally, $E(\theta)$ is same for both antennas at far fields at effective length $L = L_{eff}$.

→ Generally for practical antenna.

$$E(\theta) = \frac{j\eta_0 \beta}{4\pi r} e^{-j\beta r} \int_{-l/2}^{l/2} I(z)\,dz \quad \text{—} ①$$

But under Effective length consideration $L = L_{eff}$

$$E(\theta) = \frac{j\eta_0 \beta}{4\pi r} e^{-j\beta r} \int_{-L_{eff}/2}^{L_{eff}/2} I(z)\,dz \quad \text{—} ②$$

→ For practical antenna, variation of current can have any distribution But for an imaginary antenna current is assumed to be uniformly distributed over the length.

For imaginary Antenna, $I(z) = I_m$, $l = L_{eff}$

$$E(\theta) = \frac{j\eta_0 \beta}{4\pi r} e^{-j\beta r} \int_{-L_{eff}/2}^{L_{eff}/2} I_m\,dz \quad \text{—} ②$$

But practical and imaginary antenna should produce same electric field at far point so, putting Eq^n equating ① & ② we get

$$\cancel{\frac{j\eta_0 \beta}{4\pi r}} \cancel{e^{-j\beta r}} \int_{-L_{eff}}^{L_{eff}} I_m\,dz = \cancel{\frac{j\eta_0 \beta}{4\pi r}} \cancel{e^{-j\beta r}} \int_{-l/2}^{l/2} I(z)\,dz.$$

$$\int_{-l/2}^{l/2} I(z)\,dz = I_m \int_{-L_{eff}/2}^{L_{eff}/2} dz$$

$$\Rightarrow \int_{-l/2}^{l/2} I(z)\,dz = I_m\,(z)_{-L_{eff}/2}^{L_{eff}/2}$$

$$= I_m\left[\frac{2L_{eff}}{2}\right]$$

$$I_m\, L_{eff} = \int_{-l/2}^{l/2} I(z)\,dz.$$

$$\boxed{L_{eff} = \frac{1}{I_m}\int_{l/2}^{l/2} I(z)\,dz}$$

where $L_{eff} \rightarrow$ Effective length of antenna

$l \rightarrow$ Total length of antenna

Effective length of Receiving Antenna :-

→ L_{eff} of receivng antenna is the ratio of open ckt voltage induced at open terminals of receiving antenna to the electric field intensity produced at the receiving point

$$L_{eff} = \frac{\text{open circuit voltage induced}}{E \text{ at Rxing end}}$$

L_{eff} of halfwave dipole antenna :-

L_{eff} for halfwave dipole antenna is calculated by taking $L = \lambda/2$, $I(z) = I_m \sin\beta\left(\frac{L}{2} - z\right)$ where $\beta = \frac{2\pi}{\lambda}$.

→ we have effective length formulae.

$$L_{eff} = \frac{1}{I_m}\int_{-L/2}^{L/2} I(z)\,dz$$

$$L_{eff} = \frac{1}{I_m}\int_{-\lambda/4}^{\lambda/4} I_m \sin\left(\frac{2\pi}{\lambda}\right)\cdot\left[\frac{\lambda}{4} - z\right]\Big]\,dz$$

$$L_{eff} = \int_{-\lambda/4}^{\lambda/4}\left[\sin\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} - \frac{2\pi}{\lambda}\cdot z\right)\right]dz$$

$$L_{eff} = \int_{-\lambda/4}^{\lambda/4}\left[\sin\left[\frac{\pi}{2} - \frac{2\pi}{\lambda}\cdot z\right]\right]dz$$

$$L_{eff} = \int_{-\lambda/4}^{\lambda/4}\cos\left(\frac{2\pi}{\lambda}\cdot z\right)dz \Rightarrow L_{eff} = \sin\left(\frac{2\pi}{\lambda}\cdot z\right)_{-\lambda/4}^{\lambda/4}\cdot\frac{1}{\frac{2\pi}{\lambda}}.$$

$$L_{eff} = \frac{\lambda}{2\pi}\left[\sin\left(\frac{2\pi}{\frac{4\lambda}{2}}\cdot\lambda\right) + \sin\left(\frac{2\pi}{\lambda}\cdot\lambda\right)\right]$$

$$L_{eff} = \frac{\lambda}{2\pi}[1+1] \Rightarrow \frac{\lambda}{2\pi}\cdot 2 = \frac{\lambda}{\pi}.$$

$$\boxed{L_{eff} = \frac{\lambda}{\pi} m} \rightarrow \text{for halwave dipole Antenna.}$$

$$L_{eff} = 0.3183\lambda m.$$

8) FRONT TO BACK RATIO (FBR):-

→ It is defined as the ratio of power radiated in the desired direction to the power radiated in opposite direction.

$$\boxed{FBR = \frac{\text{power radiated in desired direction.}}{\text{power radiated in opposite direction.}}}$$

→ obviously, FBR should be high.

→ FBR depends on frequency. So, when frequency of antenna changes also FBR changes.

→ spacing b/w the antenna elements $\propto$ FBR.

→ The method of adjusting the electrical length of parasitic element is called Tuning.

→ The FBR can be raised by diverting the backward direction response of antenna to the front or forward by the length of parasitic element.

→ practically, FBR is important incase of Rxing antennas than Txing antennas. At Rxing antenna, antennas are made in such a way to obtain max. FBR than the max. gain.

(9) ANTENNA BEAMWIDTH (OR) HALF POWER BEAM WIDTH.

It is the measure of the directivity of the antenna

→ Antenna beamwidth is angular width in degrees.

→ It is measured on a radiation pattern an major lobe

→ Antenna beamwidth is defined as the angular width in degrees b/w the two points on a major lobe of the radiation pattern where the radiated power decreases to half of its max. value.

→ The beamwidth is also called <u>Half power beamwidth</u> (HPBW) because it is measured b/w two points on the major lobe where the power is half of its max. power.

→ The beamwidth is also called 3 dB beamwidth as reduction of power to half of its max. value corresponds to the reduction of power.

→ Many times, the antenna radiation pattern is described in terms of angular width b/w first nulls and first side lobes. Then such angular beamwidth is called beamwidth b/w first nulls.

→ The directivity (D) of the antenna is related with solid angle Ω_A or beam area B through expression.

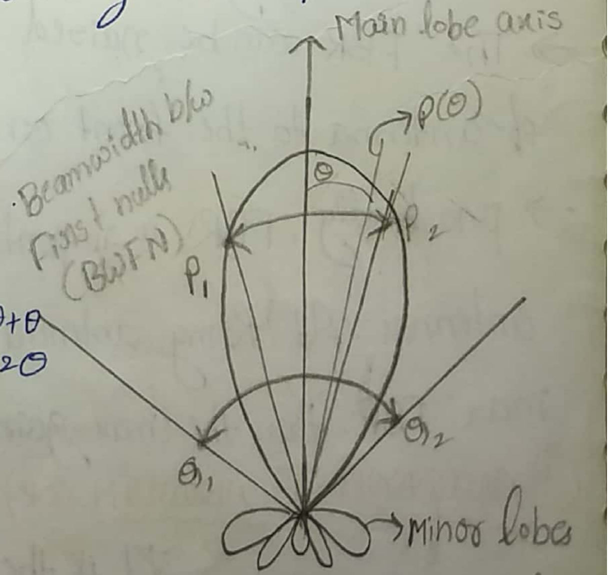
$$\boxed{D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B}}$$

Where B = Beam Area B = (HPBW) in horizantal planex (HPBW) in vertical plane.

(or) $\boxed{B = \theta_E \times \theta_H}$ ... Where θ_E & θ_H in radians.

→ The beamwidth of the antenna is affected by the shape of the radiation pattern wavelength and dimensions.

$$\boxed{P_1 = P_2 = \frac{P}{2} = \frac{P_R\,max}{2} = P_{half}}$$



Antenna power pattern in polar coordinates

<u>At Beam width</u> → $E(\theta) = \frac{1}{\sqrt{2}} = 0.707$. $B.W = \theta + \theta = 2\theta$

<u>At BWFN</u>

$E(\theta) = 0$

$BW = \theta + \theta = 2\theta$

RESOLUTION:-

→ It relates beam width and BWFN. and half power beamwidth

$$\boxed{\text{HPBW (or) BW} = \frac{\text{BWFN}}{2}}$$

BEAM AREA(B) (or) BEAM SOLID ANGLE (Ω_A)

It is defined as

$$\text{Beam Area} = \Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_d(\theta,\phi)\, d\Omega .$$

So, $\Omega_A = B = \theta_E \times \theta_H$ in radians

ANTENNA BEAM EFFICIENCY:-

To examine the quality of the transmitting and receiving antennas, the antenna beam efficiency parameter is important. For the antenna with major lobe coincident with z axis the beam efficiency is defined as

$$\boxed{B_E = \frac{\text{power transmitted (or received) within the cone angle } \theta_1}{\text{power transmitted (or received) by antenna.}}}$$

Where θ_1 is the half angle of the cone within the percentage of total power is found.

Mathematically, beam efficiency is given by,

$$B_E = \frac{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\theta_1} U(\theta,\phi) \sin\theta \, d\theta \, d\phi}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} U(\theta,\phi) \sin\theta \, d\theta \, d\phi} .$$

→ The beam efficiency can be expressed interms of the Main beam area (Ω_M) and total beam area (Ω_A). The beam efficiency can also be defined as the ratio of the main beam area to the total beam area. It is denoted by ε_M.

$$BE = \varepsilon_M = \frac{\Omega_M}{\Omega_A} .$$

Note that the total beam Area (Ω_A) is the combination of the main beam area (Ω_M) and the minor lobe area (Ω_m) i.e,

$$\boxed{\Omega_A = \Omega_m + \Omega_M} \quad (1)$$

dividing eqn (1) by Ω_A on both sides, we get $1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$

where $$\boxed{\frac{\Omega_M}{\Omega_A} = \frac{\text{Main beam area}}{\text{Total beam area}} = \varepsilon_M = \text{Beam efficiency.}}$$

and $$\boxed{\frac{\Omega_m}{\Omega_A} = \frac{\text{Minor lobe area}}{\text{Total beam area}} = \varepsilon_m = \text{stray factor}}$$

Thus, $\boxed{\varepsilon_M + \varepsilon_m = 1}$

ANTENNA BEAMWIDTH

Bandwidth of the antenna is inversely proportional to Q factor of antenna.

Thus, Bandwidth can be expressed as,

$$\text{Bandwidth} = B.W. = \Delta W = W_2 - W_1 = \frac{W_0}{Q}$$

$$\boxed{\Delta W = \frac{W_0}{Q}} \quad , \quad \boxed{\Delta f = f_2 - f_1 = \frac{f_0}{Q} \text{ Hz}}$$

Where f_0 is the centre frequency or design frequency or resonant frequency while Q factor of antenna is given by,

$$\boxed{Q = 2\pi \frac{\text{Total Energy stored by antenna}}{\text{Energy radiated per cycle.}}}$$

Thus for low frequencies, thus antenna beamwidth is high.

EFFECTIVE APERTURE (or) EFFECTIVE AREA [Ae]

→ The effective aperture is the ability of an antenna to extract energy from the EM wave. It is also called effective area.

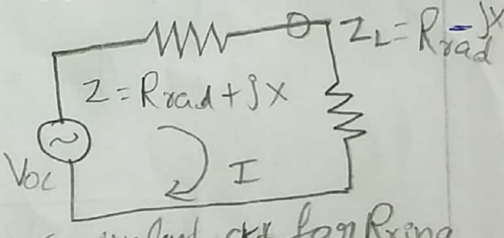
→ Effective aperture is defined as the ratio of power received in the the load to the avg. power density produced at that point.

$$\boxed{A_e = \frac{P_{received}}{P_{avg}} \; m^2} \quad ; \quad \frac{watts}{watts/m^2} = m^2.$$

→ obviously an antenna should have max. useful area to extract energy. Thus the max. effective aperture obtained when power received is max. It is denoted by A_{er}. In general, antenna have certain impedance which is made up of resistance and reactive part. The resistive part is nothing but the radiation resistant R_{rad}. This cond^n is true because the same antenna is used for transmitting and Rxing signal. Now, the power transferred is max. if the load connected to the antenna is the complex conjugate of Antenna Impedence.

Let us calculate effective aperture for the Hertizian dipole when the hertizian is used as the Rxing antenna, it extracts the power from ~~Rxing~~ Rxing antenna and delivers it to the load, producing voltage in it.

The equivalent ckt for Rxing antenna is as shown below.



Equivalent ckt for Rxing antenna

The voltage induced in antenna is given by $V_{oc} = |\bar{E}| dL$

Where $|\bar{E}|$ → Electric field intensity at the Rxing ($\because E = V/d$) point and dL is the Hertzian dipole.

Then the current flowing the load is given by, $I = \dfrac{V_{oc}}{Z + Z_L}$

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For the max. power transfer condition, load is selected as the complex conjugate of antenna Impedence $Z_L = Z^*$. Substituting the values of impedences z and z_L, the current flowing can be written as

$$I = \frac{V_{oc}}{(R_{rad} + jx) + (R_{rad} - jx)} \Rightarrow I = \frac{V_{oc}}{2R_{rad}}$$

Then the power delivered to the load is given by,

$$P_R = I^2_{rms} \cdot R_{rad} = \left[\frac{V_{oc}}{2R_{rad}/\sqrt{2}}\right]^2 R_{rad}$$

$$P_R = \frac{V_{oc}^2}{8R_{rad}} \quad -① $$

Substituting the value of $V_{oc} = |E| dl$ in eq^n - ①

$$P_R = \frac{|E|^2 dL^2}{8R_{rad}} \quad -② \quad = P_{Rmax}$$

→ Then the max. effective aperture area is given by,

$$A_{em} = \frac{\text{Max. power received}}{\text{Avg. power density}} = \frac{P_{Rmax}}{|P_{avg}|}$$

$$A_{em} = \frac{\frac{|E|^2 dL^2}{8R_{rad}}}{\frac{1}{2}\frac{|E|^2}{\eta_0}} \Rightarrow A_{em} = \frac{dL^2 \eta_0}{4R_{rad}} \quad -②$$

substituting the values of R_{rad} & η_0, we get in eq^n - ②

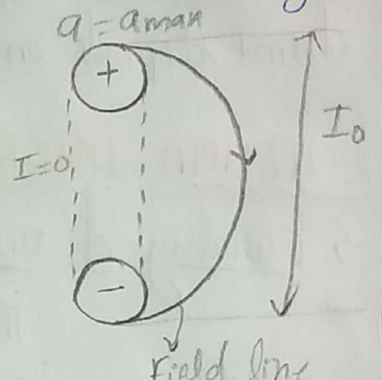
$$A_{em} = \frac{dL^2 (120\pi)}{4\left[80\pi^2\left[\frac{dL}{\lambda}\right]^2\right]}$$

$$\boxed{A_{em} = \frac{3\lambda^2}{8\pi}} \Rightarrow \boxed{A_{em} = \frac{1.5\lambda^2}{4\pi}}$$

* FIELDS FROM OSCILLATING DIPOLE :-

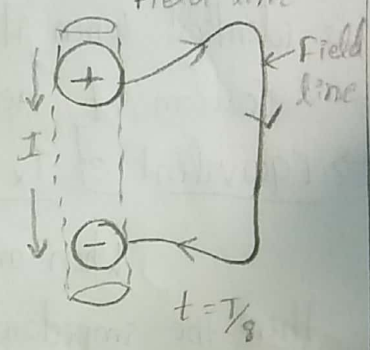
Though conductor is straight when the charge moves back and forth direction in harmonic direction/motion. (charge is oscillating) radiation results and the conductor is subjected to acceleration and deacceleration b/w the charges.

Let us consider a dipole antenna with two equal and opposite charges oscillating up and down with simple harmonic motion. Consider that l_0 be the max. separation between two equal and opposite charges. While l be an instantaneous seperation b/w charges.



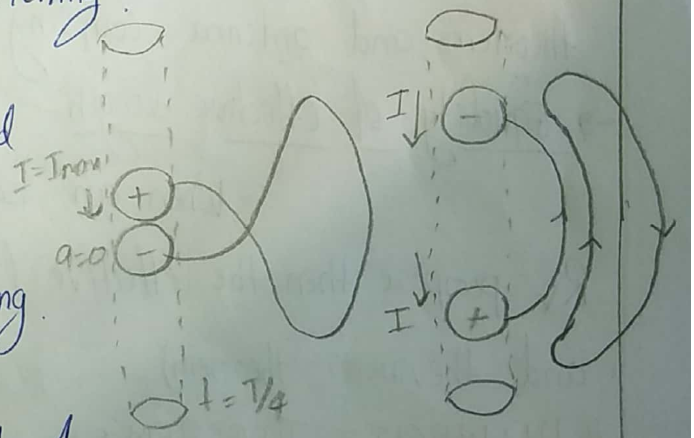
Case I :- At $t=0$, the equal and opposite charges are at max. seperation I_0. Hence, acceleration of charge is max. i.e, amax, they are in reverse direction. At that time, current is zero.

Let T be period of oscillation.



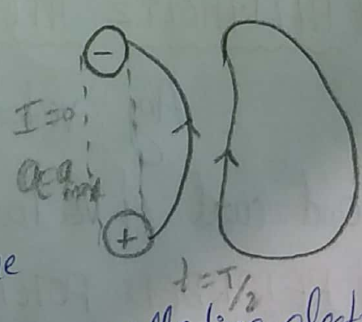
Case 2 :- Then after $t = T/8$, the charges moves towards each other.

Case 3 :- At $t = T/4$, the charges reach at midpoint of the axis of a dipole. At this instant, the field line cross each other and start ditatching.



Case 4 :- At $t = \frac{3T}{8}$, a field line is completely from the detached and released from the dipole. After detatching of field line new field lines starts developing.

Case 5 :- At $t = T/2$ Instant, the charges ends reach to the exactly opposite ends of the dipole, again the acceleration gains a = amax, current being 0.



In this way, the electric field lines are detatched (or radiated) into the free space by an oscillating electric dipole with two equal and opposite at two ends oscillating in simple harmonic motion.

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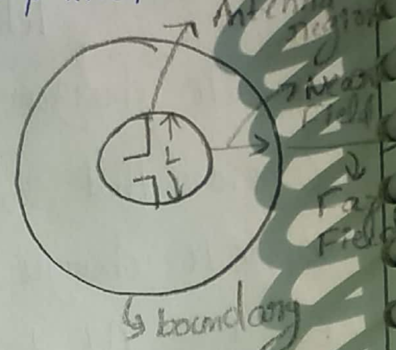
* ANTENNA FIELD ZONES :-

Around the antennas the field is divided into two principal mediums.



1) Near Field (or) Fresnel Region.
2) Far field (or) Fraunhofer region

Near field depends on distance and far field doesnot depends on distance d

ANTENNA THEOREMS

→ Equality of Directional pattern :-

The directional pattern of an antenna as Rxing antenna is identical when it is used as Txing antenna (outcome of reciprocity theorem).

→ Equivalent of Tx and Rx antenna impedance :-

When an isolated antenna used as Tx also well as Rx purposes then the impedances are indicated (outcome of max. power transfer theorem and antenna coupling.

→ Equality of effective length :-

When an isolated antenna used as Tx as well as Rx purpose then the effective lengths are identical (outcome of superposition and thevinens theorem).

* HELMHOLTZ THEOREM :-

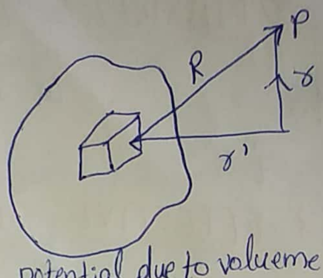
Any vector quantity can be completely / uniquely if the divergence and curl of vector fields both are known at any point.

* RETARDED POTENTIALS :-

To obtain electric field strength (or) Magnetic field strength (or) Electromagnetic field strength we should obtain potentials in terms of charges (or) current known as Retarded potentials. There are

two ways of approach
1) Heuristic Approach
2) maxwell Approach

→ HEURISTIC APPROACH:-



potential due to volueme charge density.

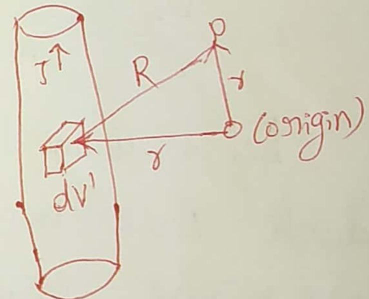
Consider uniform volume charge density. Consider the differential volume dv' at a point distance r' from the origin, where the charge density is $\rho_v(r')$.

Then the scalar electric potential V at point P can be expressed interms of static charge distribution as $V(r') = \int_V \frac{\rho_v(r')\,dv'}{4\pi\varepsilon R}$ —①

Then the fundamental electric field can be obtained by finding the gradient of scalar potential as, $\bar{E} = -\nabla . V$,

Similarly for a steady magnetic field, in homogeneous medium, the vector magnetic potential can be expressed interms of current distribution which is a constant with time as,

$$\bar{A}(r) = \int_v \frac{\mu \bar{J}(r')}{4\pi r} dv' \quad —②$$



Then the fundamental magnetic field can be obtained by finding the curl of the vector magnetic potential $\bar{A}$ as,

$\bar{B} = \nabla \times \bar{A}$

Eqn ①, ② represent the potential for static electric and magnetic fields respectively where the charge and current distributions do not vary with time

But the charge & current distributions producing the Em field vary with respect to time. Thus for time varying, we must vary the potentials represented by eqns ① & ② for time variation as,

$$V(r',t) = \frac{1}{4\pi\varepsilon} \int_v \frac{\rho_v(r',t)}{R} dv'$$

$$\bar{A}(r',t) = \frac{\mu}{4\pi} \int_v \frac{\bar{J}(r,t)}{R} dv' \quad \text{where } R = |r - r'|$$

we need to find potentials from the time varying maxwells eq^{n}s.

* EFFECTIVE AREA / RELATION B/w D & Aemax

Consider two antennas as antenna1 & antenna2. Let the distance directives of both the antennas are D1 & D2. Assume that the max. effective area is denoted by $(A_{e1})_{max}$ and $(A_{e2})_{max}$. Directivity of antenna is directly proportional to the max. effective are.

$$D_1 \propto (A_{e1})_{max}, \quad D_2 \propto (A_{e2})_{max}, \quad \frac{D_1}{D_2} = \frac{(A_{e1})_{max}}{(A_{e2})_{max}}$$

Let Antenna1 be the isotropic radiator for which the directivity is unity.

i.e. $D_1 = 1$

Hence we can write $\frac{1}{D_2} = \frac{(A_{e1})_{max}}{(A_{e2})_{max}}$.

$$D_2 = \frac{(A_{e2})_{max}}{(A_{e1})_{max}} \Rightarrow (A_{e1})_{max} = \frac{(A_{e2})_{max}}{D_2} \quad \text{--- (1)}$$

Let us assume that antenna will be a test antenna which is a short dipole. As we know for the short dipole antenna, max. effective aperture is $\left(\frac{3}{8\pi}\right)\lambda^2$ and the directivity is $3/2$. Hence, we can write,

$$(A_{e1})_{max} = \frac{(3/8\pi)\lambda^2}{(3/2)} = \frac{\lambda^2}{4\pi}$$

putting the value of $(A_{e1})_{max}$ in the expression (1)

$$D_2 = \frac{(A_{e2})_{max}}{(\lambda^2/4\pi)} = \frac{4\pi}{\lambda^2}(A_{e2})_{max}$$

Hence, in general we can write

$$D = \frac{4\pi}{\lambda^2}(A_e)_{max}.$$

But max. effective aperture can be expressed interms of effective length as,

$$\boxed{(A_e)_{max} = \frac{L_{eff}^2 \eta_0}{4 R_{rad}}}$$

Hence, putting value of Aemax in general express of D given above.

$$D = \left[\frac{4\pi}{\lambda^2}\right]\left[\frac{L_{eff}^2 \eta_0}{4 R_{rad}}\right]$$

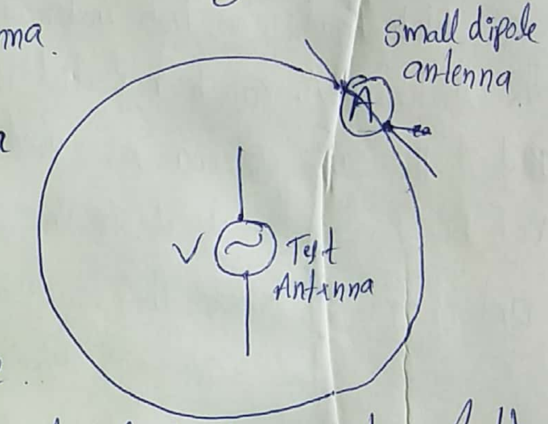
$$\boxed{D = \left[\frac{\pi}{\lambda^2}\right] L_{eff}^2 \left[\frac{\eta_0}{R_{rad}}\right]}$$

Applications of Network theorems to Antenna.

The properties of the transmitting antenna and the receiving antenna are related to each other through various antenna theorems. By using following antenna theorems, the properties of the receiving antenna can be infered from its properties as the transmitting antenna, and vice versa.

i) Equity of Directional patterns:-

<u>Statement</u>:- The directional pattern of an antenna as a receiving antenna is identical to that when used as a transmitting antenna.



→ To measure the directional pattern of an antenna as a transmitting antenna, the test antenna is kept at the centre of very large sphere and small dipole antenna is moved along the surface of this sphere.

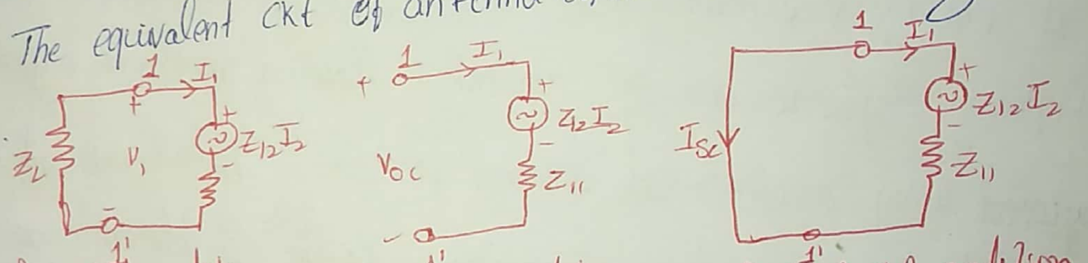
→ A voltage V is connected to the test antenna placed at the centre of the imaginary sphere and the current I flowing in short dipole antenna is measured using ammeter at different positions. This current is the measure of the electric field at different positions of the dipole antenna. Now using the concept of reciprocity theorem, the positions of the voltage excitation and current measurement are interchanged. Now the same voltage V is applied to the terminals of the small dipole antenna which is moved along the surface of the sphere and the current I is measured in the test antenna located at the centre. then the receiving pattern for the test antenna canbe obtained. But according to the reciprocity theorem for every location of the dipole antenna, the ratio of V to I is same as before obtained for the test antenna as a transmitting antenna. Thus the radiation pattern i.e, directional pattern of a receiving antenna is identical to that of the transmitting antenna.

Equivalence of Txing and Receiving Antenna Impedances.

Statement :- "The impedance of an isolated antenna used for transmitting as well as receiving purposes is identical"

Proof :- Consider two antennas, namely A_1 & A_2 are widely seperated. As the Antenna A_2 is located far away from A_1, the self impedance of the antenna A_1 can be written as, self impedance of $A_1 = Z_{S1} = \frac{V_1}{I_1} = Z_{11}$

Note that when two antennas are seperated widely, the mutual impedance Z_{12} of the Antenna A_1 can be neglected if the Antenna A_1 is used as a transmitting antenna. But if the same antenna A_1 is used as a receiving antenna the mutual impedance Z_{12} can not be neglected as it is the only parameter indicating coupling between two antennas. So consider the load Z_L is connected to the A_1 used as receiving antenna. Similarly the coupling b/w A_1 & A_2 is represented with the help of a mutual voltage $Z_{12}I_2$ which is due to the mutual impedance Z_{12} and current I_2 in the antenna A_2. The equivalent ckt of antenna used as a receiving antenna shown in fig.



a) Loaded condition b) Open ckt condition c) Short ckt condition

Since the two antennas are seperated with a large distance, the variation in the load impedance Z_L connected to the antenna A_1 will not change the current I_2 in the antenna A_2. Thus the generator of value $Z_{12}I_2$ can be considered as an ideal generator with zero internal impedance providing constant voltage at its o/p terminals.

Under the open ckt condition, the voltage measured across terminal 1-1' is given by $V_{oc} = Z_{12}I_2$.

Under the short ckt condition, the short ckt current flowing from terminal 1 to 1' is given by $I_{sc} = \frac{Z_{12}I_2}{Z_{11}}$

Hence the ratio of V_{oc} to I_{sc} is called as the transfer impedence and it is given by $\frac{V_{oc}}{I_s} = \frac{Z_{12}I_2}{\frac{Z_{12}I_2}{Z_{11}}}$ $\Rightarrow$ $\boxed{\frac{V_{oc}}{I_{sc}} = Z_{11}}$

So, it is clear that the receiving antenna impedence is equal to the transmitting antenna impedance.

Equality of Effective length. (l_{eff}):-

→ Basically the effective length (l_{eff}) of an antenna represent the effectiveness of an antenna as a radiator or collector. The effective length (l_{eff}) is defined as the length of an equivalent linear antenna which has current $I(0)$ along its length at all points radiating the field strength in direction perpendicular to the length same as actual antenna. The current $I(0)$ is the current at the antenna terminals.

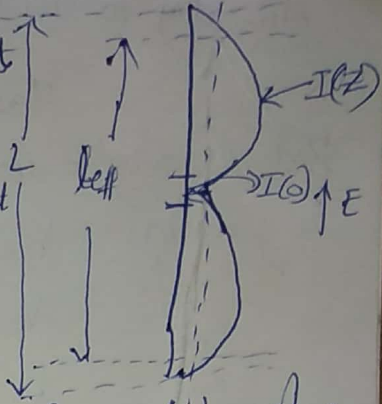


fig: Representation of a effective length.

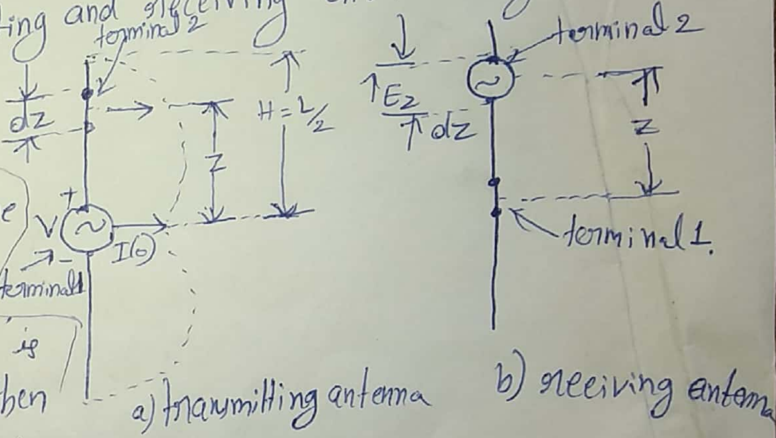
For the transmitting antenna. l_{eff} (transmitter) $= \frac{1}{I(0)}\int_{-l/2}^{l/2} I(z)\, dz$.

→ For the receiving antenna, the effective length can be defined as the ratio of the open ckt voltage developed at the antenna terminals to the given received field strength. Hence the effective length (l_{eff}) is given by.

$$l_{eff}\,(rec) = \frac{-V_{oc}}{E_z}$$

→ To show the equality of the transmitting and receiving effective lengths, let us apply the reciprocity theorem

→ First consider antenna used as a transmitting antenna. Let Za be the antenna impedance measured at the antenna terminals. Assume that voltage V is applied at the antenna terminals, then the current produced at the is given by



$$I(0) = \frac{V}{Z}$$

→ The current at all the point on z is $I(z)$.

→ Now consider that the same antenna is used as receiving antenna. Consider that the electromagnetic field E_z is incident on it this induces voltage $E\,dz$ in the element dz. At this induced voltage is independent of the current through antenna, it can be indicated as an ideal generator of voltage $E_z dz$ in series. When antenna terminals are short ckted, the ideal generators produces current I_{sc} in the antenna.

According to reciprocity theorem. $\frac{V}{I(z)} = \frac{E_z dz}{I_{sc}} \Rightarrow I_{sc} = \frac{E_z dz}{V} I(z)$

$$I_{sc} = \frac{1}{V}\int E_z I(z).dz.$$

But According to thevinin theorem, the open ckt voltage at the antenna terminals is given by $V_{oc} = -I_{sc} Z_a$.

$$V_{oc} = -\frac{Z}{V}\int E_z I(z).dz.$$

$$V_{oc} = -\frac{1}{I(0)}\int E_z I(z) dz.$$

$$V_{oc} = -\frac{E_z}{I(0)}\int I(z).dz.$$

$$-\frac{V_{oc}}{E_z} = \frac{1}{I(0)}\int I(z) dz.$$

$$\boxed{l_{eff}(rec) = \frac{1}{I(0)}\int I(z) dz}.$$

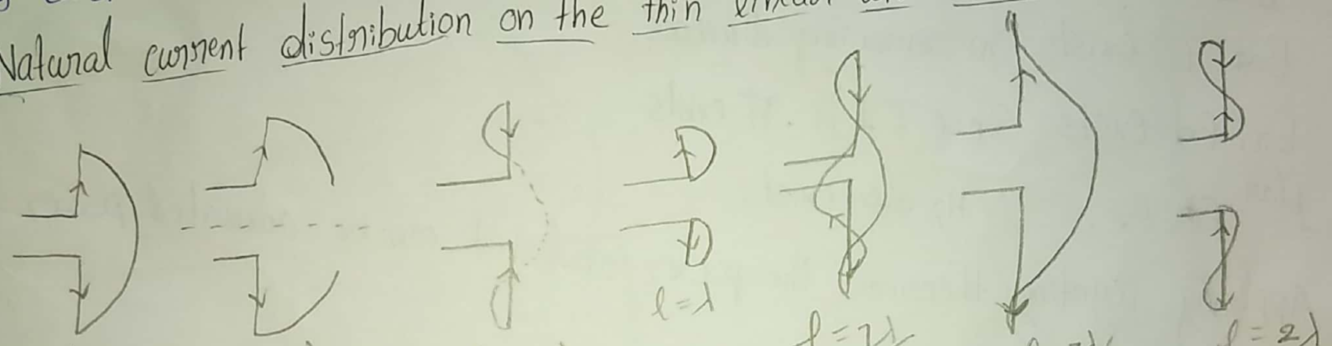
* THIN LINEAR WIRE ANTENNAS:

The antenna whose diameter is very thin such that diameter is less than $\lambda/100$ and current distribution is always constant along the length of antennas and Im is independent of length of antenna and dependent on diameter.

$$P = I_m^2 \cdot R_s, \qquad I_m - \text{constant}.$$

As Im is constant, it is easy to calculate the power radiation. Hence this is called Natural current distribution. As, the sinusoidal current is constant this is called thin linear wire antennas.

Natural current distribution on the thin linear wire antennas of varying length



$l = \lambda/2$ $l = 3\lambda/4$ $l = 5\lambda/4$ $l = \lambda$ $l = 7\lambda/4$ $l = 3\lambda/2$ $l = 2\lambda$

→ Here the current distribution is same and it is independent of length of antenna but there is change in Magnitude and Time period.

* power radiated by short dipole antenna (or) Small Electric dipole antenna (or) Alternating current Element:-

The electromagnetic fields or radiations due to short dipole in space is given by

$$E_r = \frac{2 I\,dl \cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega t'}{c r^2} + \frac{\sin\omega t'}{\omega r^3}\right]$$

similarly $$E_\theta = \frac{I dl \sin\theta}{4\pi\varepsilon}\left[\frac{-\omega \sin\omega t'}{c^2 r} + \frac{\cos\omega t'}{c r^2} + \frac{\sin\omega t'}{\omega r^3}\right]$$

& $$H_\phi = \frac{I dl \sin\theta}{4\pi}\left[\frac{-\omega \sin\omega t'}{c r} + \frac{\cos\omega t'}{r^2}\right]$$

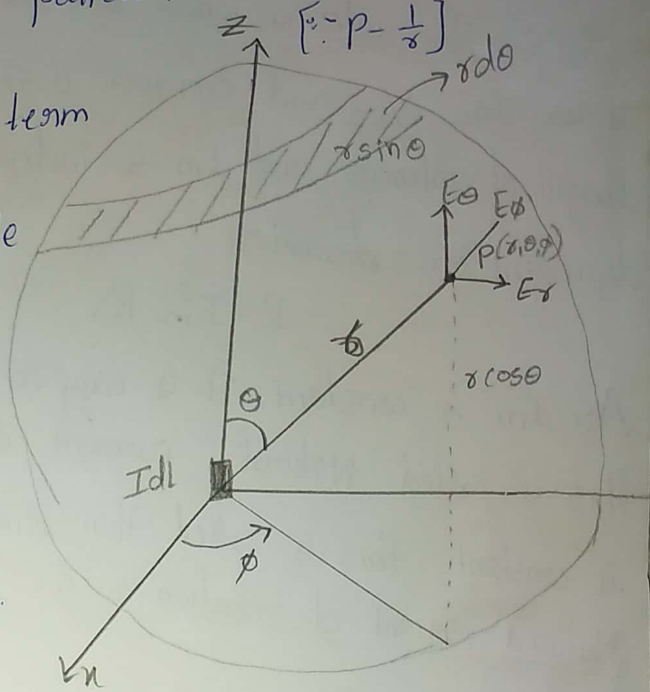
Idl - current element, ω - frequency, c - velocity of light,
t' → retarded potential r - distance from antenna to a point

$$\eta_0 = E/H.$$

Where $\frac{1}{r}$ is known as Radiation pattern.

$\frac{1}{r^2}$ - Induction term

$\frac{1}{r^3}$ - Electrostatical field term



Consider an small electric dipole at center of antenna. Here, the direction of propagation is along ϕ direction. Hence, in uniform plane,

$$\boxed{E_\phi = 0}$$

But $\bar{E}$ exists in remaining directions. E_r, E_θ exists. Since $\bar{E} \perp \bar{H}$. $\bar{H}$ exists $\perp$lar E_θ, E_r. $\therefore H_\phi$ also exists.

Applying poynting theorem, the power radiated can be calculated, power density $\bar{P} = \bar{E} \times \bar{H}$ w/m²

$\therefore$ The radial component of power in (θ, ϕ) direction is, $P_r = E_\theta \times H_\phi =$ 0, $P_r = E_\theta \times H_\phi$.

$$P_r = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[\frac{-\omega\sin\omega t'}{c^2 r} + \frac{\cos\omega t'}{c r^2} + \frac{\sin\omega t'}{\omega r^3}\right]\left[\frac{Idl\sin\theta}{4\pi}\right] \times \left[\frac{-\omega\sin\omega t'}{c r} + \frac{\cos\omega t'}{r^2}\right]$$

$$= \left[\frac{Idl\sin\theta}{4\pi}\right]^2 \cdot \frac{1}{\varepsilon}\left[\frac{(\omega\sin\omega t')^2}{c^3 r^2} - \frac{\omega\sin\omega t'\cos\omega t'}{c^2 r^3} - \frac{\omega(\sin\omega t')^2}{c\omega r^4}\right.$$

$$\left. - \frac{\omega\sin\omega t'\cos\omega t'}{c^2 r^3} + \frac{(\cos\omega t')^2}{c r^4} + \frac{\sin\omega t'\cos\omega t'}{\omega r^5}\right] \qquad \left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right.$$

$$= \frac{1}{\varepsilon}\left[\frac{Idl\sin\theta}{4\pi}\right]^2\left[\frac{\omega^2}{c^3 r^2}\left[\frac{1-\cos 2\omega t'}{2}\right] - \frac{2\omega\sin\omega t'\cos\omega t'}{c^2 r^3} - \frac{\omega}{\omega c r^4}\left[\frac{1-\cos 2\omega t'}{2c}\right]\right.$$

$$\left. + \frac{2}{2}\frac{\sin\omega t'\cos\omega t'}{\omega r^5} + \frac{(1+\cos 2\omega t')}{2c r^4}\right] \qquad \left(\because \cos 2x = 2\cos^2 x - 1\right) \quad \frac{\cos 2x + 1}{2} = \cos^2 x.$$

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$$= \frac{1}{\varepsilon}\left[\frac{Idl\sin\theta}{4\pi}\right]^2\left[\frac{\omega^2(1-\cos 2\omega t')}{2c^3r^2} - \frac{\sin 2\omega t' \cdot \omega}{c^2r^3}\right] - \frac{1}{cr^4}\left[\frac{1-\cos 2\omega t'}{2}\right]$$
$$+ \frac{\sin 2\omega t'}{2\omega r^5} + \frac{(1+\cos 2\omega t')}{2cr^4}\Bigg].$$

$$= \frac{1}{\varepsilon}\left[\frac{Idl\sin\theta}{4\pi}\right]^2\left[\frac{(1-\cos 2\omega t')}{2c^3r^2}\left[-\frac{\omega^2}{c^2} - \frac{1}{r^2}\right] + \frac{\sin 2\omega t'}{2\omega r^5} + \frac{(1+\cos 2\omega t')}{2cr^4}\right] \text{— } 1$$

In the above eq^n — 1, the avg. value of $\sin 2\omega t'$ and $\cos 2\omega t'$ are 0 over a complete circle/cycle.

→ Therefore, $\sin 2\omega t' = \cos 2\omega t' = 0$, sub in eq^n — 1 and now it is modified as

$$P_r = \frac{1}{\varepsilon}\left[\frac{Idl\sin\theta}{4\pi}\right]^2\left[\frac{\omega^2}{2c^3r^2} - \cancel{\frac{1}{2cr^4}} + \cancel{\frac{1}{2cr^4}}\right]$$

$$= \left[\frac{Idl\sin\theta}{4\pi}\right]^2 \times \frac{1}{\varepsilon}\left[\frac{+\omega^2}{2c^3r^2}\right]$$

$$\boxed{P_r = \frac{I^2dl^2\sin^2\theta\,\omega^2}{2(4\pi)^2\varepsilon\, c^3r^2}}$$

$$P_r = E_\theta \times H_\phi,\quad \eta = \sqrt{\mu/\varepsilon} = \sqrt{\mu/\varepsilon} \times \sqrt{\varepsilon/\varepsilon} = \frac{\sqrt{\mu\varepsilon}}{\varepsilon}.$$

$$\eta_0 = \frac{\left(\frac{1}{v}\right)}{\varepsilon} \qquad \left[\because v = \frac{1}{\sqrt{\mu\varepsilon}}\right]$$

$$\Rightarrow \eta_0 = \frac{1}{\varepsilon v} = \frac{1}{\varepsilon c}$$

$$P_r = \frac{I^2dl^2\sin^2\theta\,\omega^2}{2(4\pi)^2 c^2r^2(\varepsilon c)} \text{ — } 2$$, we have, $\eta_0 = \frac{1}{\varepsilon c}$ sub in eq^n — 2

$$P_r = \left(\frac{Idl\sin\theta}{4\pi}\right)^2 \times \frac{\omega^2}{2c^2r^2} \times \eta_0$$

$$P_r = \left[\frac{I\,dl\,\sin\theta}{(4\pi c^2)}\right]^2 \times \frac{\omega^2\eta_0}{2}$$

$$\boxed{P_r = \frac{\eta_0}{2}\left[\frac{Idl\,\sin\theta\,\omega}{4\pi c r}\right]^2 \ \mathrm{w/m^2}}$$

The total power radiated will be calculated by Integrating above equation over a complete sphere. i.e, total power radiated, is

$$\boxed{P_{total} = \oint_S P_r . ds}$$

Let us calculate magnitude of electric field strength and magnetic field strength considering only radiation term ($1/r$).

$$|E(\theta)| = \left| \frac{I dl \sin\theta}{4\pi\varepsilon} \left[\frac{-\omega \sin\omega t'}{c^2 r}\right] \right| \quad \text{--- } 3$$

This is because with $1/r^2$, and $1/r^3$ term, $|E(\theta)|$ will be decreases.

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi}, \quad \lambda = \frac{c}{f} = \frac{c}{\frac{\omega}{2\pi}} = \frac{c 2\pi}{\omega}$$

$$\frac{\lambda}{2\pi} = \frac{c}{\omega} \Rightarrow \frac{\omega}{c} = \frac{2\pi}{\lambda} \quad \text{Sub in eq}^n \ 3$$

$$|E(\theta)| = \left| \frac{I dl \sin\theta}{4\pi\varepsilon} \left[\frac{-\sin\omega t' (2\pi)}{c . \lambda . r}\right] \right|$$

$$|E(\theta)| = \left[\frac{I dl \sin\theta}{4\pi\varepsilon c r} \sin\omega t' \left(\frac{2\pi}{\lambda}\right)\right] \qquad \frac{2\pi}{\lambda} = \frac{\omega}{c}.$$

$$|E(\theta)| = \frac{I dl \sin\theta}{4\pi\varepsilon c r} . \sin\omega t' \left(\frac{2\pi}{\lambda}\right) \quad , \eta = \frac{1}{\varepsilon c}$$

$$\boxed{|E(\theta)| = \frac{\eta . I dl \sin\theta . \sin\omega t'}{2\lambda r}} \ V/m$$

Similarly, $\eta_0 = \left|\frac{E_\theta}{H_\phi}\right| \Rightarrow H_\phi = \frac{E_\theta}{\eta_0}$

$$\boxed{H_\phi = \frac{I dl \sin\theta \sin\omega t'}{2\lambda r} \quad A/m}$$

Total power radiated is obtained by integrating total. i.e, complete sphere.

$$P_{total} = \oint_{surface} P_r . ds.$$

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$$P_T = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\eta_0}{2}\left[\frac{I dl \sin\theta}{4\pi c r}\right]^2 \omega^2 \cdot ds$$ [∵ but ϕ azimuthal angle is constant $\int_{\phi=0}^{2\pi} d\phi = 2\pi$] (3)

$$ds = r^2 \sin\theta \, d\theta \, d\phi$$

$$P_T = \int_{\theta=0}^{\pi} \frac{\eta_0}{\cancel{2}}\left[\frac{I dl \sin\theta}{4\pi c r}\right]^2 \cdot (\cancel{2}\pi)\,(\sin\theta \, d\theta \cdot r^2)\,.\,\omega^2$$

$$P_T = \frac{\eta_0 r^2}{\cancel{}} \cdot \frac{I^2 dl^2}{(4\pi c r)^2} \int_0^{\pi} \sin^3\theta \, d\theta \cdot \omega^2$$

Using reduction formulate of reduction method

$$\int_{\theta=0}^{\pi} \sin^3\theta \, d\theta = \frac{(n-1)}{n} \pi/2 \; ; \text{ if } n \text{ is even}$$

$$= \frac{(n-1)}{n} \; ; \text{ if } n \text{ is odd.}$$

$$\left[\frac{n-1}{n}\right] = \left[\frac{3-1}{3}\right] = 2$$

$$P_T = \frac{\eta_0 \cdot I^2 dl^2 \omega^2 \cancel{r^2}}{\cancel{16}_4 \pi^2 c^2 r^2}\left[\frac{2}{3}\right]\cdot[2]$$

$3\sin^2\theta \cdot \frac{3\sin\theta}{x}$

$-\cos(x) + c$

$\frac{1}{3}\cos^3 x$

$\frac{1}{3}\cos^3$

$$P_T = \frac{\eta_0 I^2 dl^2 \omega^2}{12\pi^2 c^2}$$

$$P_T = \frac{\eta_0 I^2 dl^2}{12\pi}\left[\frac{\omega^2}{c^2}\right]$$

Since, $\frac{\omega}{c} = \frac{2\pi}{\lambda} \Rightarrow \frac{\omega^2}{c^2} = \frac{4\pi^2}{\lambda^2}$

$$P_T = \frac{\eta_0 I^2 dl^2}{12\pi}\left[\frac{4\pi^2}{\lambda^2}\right], \quad \eta_0 = 120\pi$$

$$P_T = \frac{\eta_0 I^2 dl}{12\pi}\left[\frac{4\pi^2}{\lambda^2}\right] \Rightarrow P_T = \frac{120\pi I^2 dL^2}{12\pi}\left(\frac{4\pi^2}{\lambda^2}\right)$$

$$\boxed{P_T = \frac{40 I^2 dL^2 \pi^2}{\lambda^2}}$$

Since, in thin linear wire antennas, power radiated I is constant.

$$I = I_m \simeq I_{eff}.$$

Since, we have $I_{rms} = \frac{I_{eff}}{\sqrt{2}} \Rightarrow I_{eff} = I_{rms} \cdot \sqrt{2}$.

Substituting in above eqn. $I = I_{rms} \cdot \sqrt{2}$.

$$P_T = \frac{40 dl^2 \pi^2}{\lambda^2} (I_{rms} \cdot \sqrt{2})^2 = \frac{40\pi^2 dl^2}{\lambda^2} \cdot I_{rms}^2 \cdot 2.$$

$$\boxed{P_T = I_{rms}^2 \left[80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \right]} \quad \text{— (5)}$$

Compare eqn – (5) with $P_T = I_{rms}^2 \cdot R_{rad}$ then we get Radiation resistance of halfwave dipole antenna as — (6)

$$\boxed{R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2} \; \Omega$$

→ This eqn is applicable to Thin linear antennas of length $\leq \frac{\lambda m}{10}$.

Comments :-

1) This expression is only for ideal antennas and not for practical antennas, this is because, the radiation resistance of half wave dipole is 73Ω

For practical antennas like half wave dipole antenna, the current distribution is not uniform (not same). over the length of antenna, i.e. it has Imax at centre and Imin = 0 at the ends of antennas. But for eqn – (6) we have considered current distribution is uniform. But this is not possible for practical antenna and it is possible for ideal antennas only.

Hence, $\boxed{R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2}$ (I is uniform) is valid

Only for Ideal Antennas only.

→ Half wave dipole antenna is practical Antenna $R_{rad} = 197\,\Omega$

As, Halfwave dipole Antenna is a practical Antenna, because the length of half wave dipole antenna is $\lambda/2$. then

$$R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 = 80\pi^2 \left(\frac{\lambda/2}{\lambda}\right)^2 = \frac{40\pi^2}{2}.$$

$\boxed{R_{rad} = 197\,\Omega}$

As the ideal Antenna should have the radiation resistance of 73Ω. but half wave dipole antenna is having $197\,\Omega$, these two resistances are not comparable. So, hence half wave dipole antenna is not an ideal Antenna.

→ A dipole Antenna is a vertical radiator field in the centre. It produces radiations in a plane normal to the axis. The vertical antenna of height $H = \frac{L}{2}$ can also produce similar radiations that produced by a dipole antenna. of length $L = 2H$. This antenna is known as vertical monopole Antenna.

POWER RADIATED BY HALF WAVE DIPOLE ANTENNA & QUARTER WAVE MONOPOLE ANTENNA INTERMS OF RMS CURRENT:

→ The physical length of Half wave dipole antenna is at operating frequency is $\lambda/2$ in free space. ($L = \lambda/2$; i.e, $L = 2H$, $H = \lambda/4$) which means each leg is having length $H = \lambda/4$.

→ Similarly monopole antenna which is mounted on a perfect conductor whose height $H = \frac{L}{2}$ at physical length at operating frequency is $\lambda/4$ in free space. As it is having single leg.

→ In general to radiate max. power antennas requires large currents but providing large current at radio frequency to the antenna is practically impossible. Where Hertizian dipole, small Electric dipole in which the magnitude of current assumed uniform and calculated expressions for E & $\bar{H}$. But, already we have studied the magnitude of current is not constant over the length of Antenna. As it is zero at ends and max. at centre. But, Hertizian dipole will not exists practically.

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