UNIT-I

ELECTROSTATICS

Electrostatics:- The force experienced by either due to +ve charges (∵ test charge) under static electric field is called Electrostatic field.

Electrical charging:- It is the process of removal of valen electrons from an atom or addittion of $\bar{e}$ to the neutra atoms of a material.

Applications of Electrostatics:-

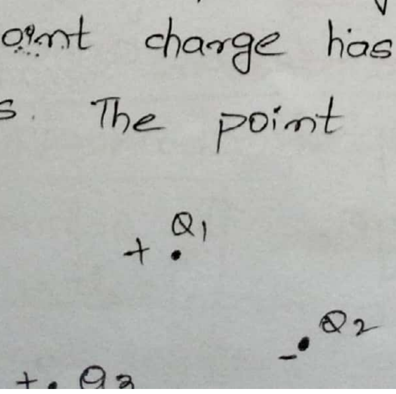
→ To obtain electron beam deflection beam in CRO.
→ Electric transmission line.
→ In L.C.D's
→ In FET's and Capacitors.
→ In L.C.D's

Different charge distributions /charge density:-

There are four types of charge distributions.

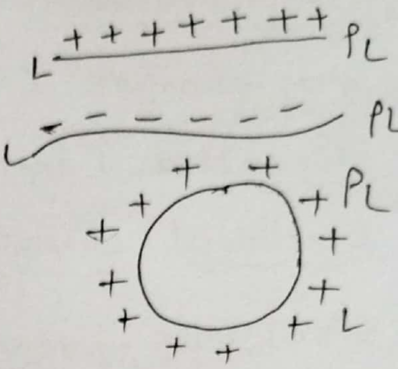
(a) **Point charge distribution:-**

It is seen that if the dimensions of a Surface carrying charge are very very small compared to the region it then the Surface can be treated to be a point. The corresponding charge is called point charge. The point charge has a position but not the dimensions. The point charge can be positive or negative.



(b) Line charge:-

The charge may be spreaded along a line, which may be finite or infinite. Such a uniformly distributed along a line is called Line charge. The charge density of a line charge is denoted as ρ_L and is defined as charge per unit length.



$$\rho_L = \frac{\text{Total charge in columb}}{\text{Total length in metres}}.$$

ρ_L is constant. all along the length L of the line carrying the charge.

Integrating the charge dQ on dl, for the entire length dl of the line. Then by Integrating the charge dQ on dl, for the entire length, the total charge Q is obtained. Such an Integral is called Line Integral.

$dQ = \rho_L \cdot dl$ = charge on differential length dl,

(since $\rho_L = \frac{dQ}{dl} \Rightarrow dQ = \rho_L \cdot dl$)

$$Q = \int_L dQ = \int_L \rho_L \cdot dl.$$

If the line of length is closed path, then Integral is called closed countor Integral, and called

$$\boxed{Q = \oint_L \rho_L \cdot dl}$$

→ Ex:- A sharp beam in a cathode ray tube or a charged circular loop of conductor are the examples of line charge. The charge distributed may be positive or negative along the line.

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b) Surface charge:-

If a charge is distributed over a two dimensiona Surface then it is called surface charge or sheet of charge. The two dimensional surface charge has area in its square metres. Then Surface charge density denoted as ρ_s and is defined as charge per unit surface area.

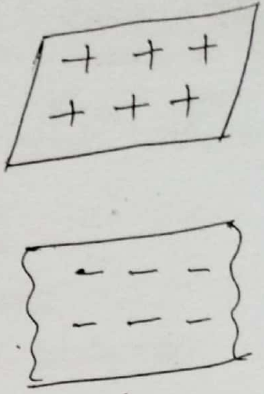


fig:- surface charge distribution

$$\rho_s = \frac{\text{Total charge in coulomb}}{\text{Total area in square mts.}} \; (C/m^2).$$

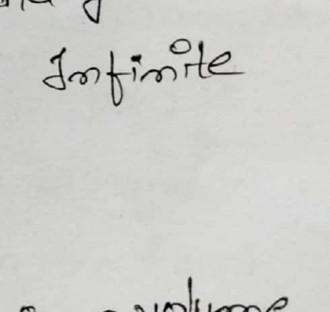
$\rho_s \rightarrow$ Constant over the surface carrying the charge

$$Q = \int_S dQ = \int_S \rho_s \, ds.$$

Eg:- The plate of a charged parallel plate capacitor is an example of Surface charge distribution. If the dimensions of sheet of charge are very large compared to the distance at which the effects of charge are to be considered then distribution is called Infinite sheet of charge.

c) Volume charge:-

If the charge distributed uniformly in a volume. Then it is called Volume charge distribution



$\rho_v \rightarrow$ Volume charge density

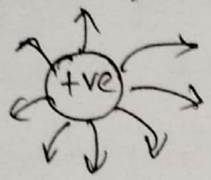
$$\rho_v = \frac{\text{Total charge in coulomb}}{\text{Total volume in cubic metres}} \left[\frac{C}{m^3}\right]$$

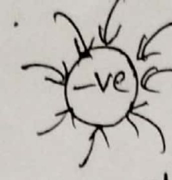
$$\boxed{Q = \int_{vol} \rho_v \, dv}$$

fig:- Volume charge distribution

Eg:- Ionospheric region, Electron cloud in Vacuum tube called Electric field.

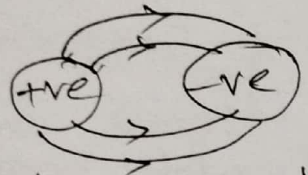
<u>Electric lines of force</u>;- Direction of lines of force of unit charge kept in the field.





(a) field due to +ve charge (b) field due to -ve charge

Electric flux always originate from the positive charge into all directions and finally it terminates (converges) at negative charge in all directions.



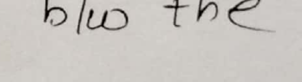
attractive force b/w unlike charges with electric field

Repulsive force b/w unlike charges shown in electric field.

(1) COULOMB'S LAW;-

Unit of charge is called columb. One coulomb of charge is defined as the charge possessed by 6×10^{8} no. of electrons. Coulumb is a scientist and he explains the attractive or repulsive force b/w the like or unlike charges.

Consider the two point charges Q_1 and Q_2 seperated by the distance R. The charge Q_1 exerts a force on Q_2 while Q_2 exerts a force on Q_1. The force acts along the line joining Q_1 & Q_2. The force exerted b/w them is repulsive if the charge are of same polarity or attractive if they are with different polarity



* Coulombs law states that force b/w two point charges Q_1 and Q_2,

→ acts along the line joining the two point charges

→ is directly proportional to the product $(Q_1 Q_2)$ of the two charges.

→ Inversely proportional to the Square of the distance between them.

Mathematically, it was represented as

$$\boxed{F \propto \frac{Q_1 Q_2}{R^2}}$$

Columbs law also states that this force also depends on the medium in which point charge is located. The effect of medium is introduced in the equation of force as a constant of proportionality denoted as k.

$$F = k \cdot \frac{Q_1 Q_2}{R^2} \quad , k = \text{Constant of proportionality}$$

where $k = \frac{1}{4\pi\varepsilon}$, $\varepsilon = \varepsilon_0 \varepsilon_r$.

ε_0 = Permittivity of the free space or vacuum

ε_r = relative permittivity (or) dielectric constant of the medium with respect to free space , $(\varepsilon_r = 1)$

ε = Absolute permittivity.

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{R^2} \quad , \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}.$$

$k = 9 \times 10^9$ m/F.

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot R^2}$$

. This is the force b/w the two point charges in located in free space or vacuum.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{Q_1 Q_2}{R^2}\right) \cdot \bar{a}_r$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{R^2} \cdot \frac{\vec{r}}{|r|} \text{ N}$$

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→ To find force between the two charges [partly cut off]
→ To find force between the two fixed charge.
→ To find potential at a point due to fixed charge
$(\because E = F/Q,\ V = -\int_a^b E \cdot \overline{dl},\ \overline{D} = \varepsilon_0 \cdot \overline{E}$ (or) $\varepsilon\overline{E})$
→ To find electric field at a point due to fixed
→ To find Electric potential and Electric field due to any type of charge distribution.
→ To find charge when force and electric fields are known.
→ To find electric flux density.

Conditions of Coulombs Law:-

→ The two charges must be point charges and must be at rest.
→ The distance b/w the charges must be large as compared to the dimensions of point charges.

Limitations of Coulombs Law:-

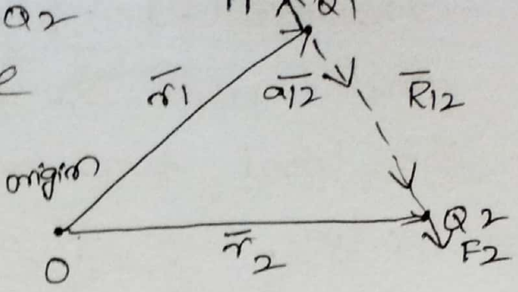
When the two charges are of orbitrary shape it is difficult to find the exact centres of them. Hence we may not know the accurate distance b/w the charges. So, Coulumbs law is not applicable.

Vector form of Coulumbs Law:-

The force exerted b/w the point charges has a fixed direction which is a straight line joining the two charges. Hence the force exerted between the two charges can be expressed in vector form.

Consider the two point charges Q_1 and Q_2 located at the two points having position vector $\overline{r_1}$ and $\overline{r_2}$.

Then the force exerted by Q_1 on Q_2 acts along the direction $\overline{R_{12}}$ where $\overline{a_{12}}$ is the unit vector along $\overline{R_{12}}$. Hence, the force in the vector can be expressed as.



F on Q_2 by Q_1

$$\overline{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot \overline{R_{12}^2}} \overline{a_{12}}$$

$\overline{a_{12}}$ = unit vector along $\overline{R_{12}} = \frac{\text{Vector}}{\text{Magnitude of vector}}$

$$\overline{a_{12}} = \frac{\overline{R_{12}}}{|R_{12}|} = \frac{\overline{r_2} - \overline{r_1}}{|R_{12}|} = \frac{\overline{r_2} - \overline{r_1}}{|\overline{r_2} - \overline{r_1}|}.$$

Since, $|\overline{R_{12}}| = R$ = distance b/w to two charges.

→ force on Q_1 due to Q_2.

$$\overline{F_1} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot \overline{R_{21}^2}} \cdot \overline{a_{21}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot R_{21}^2} \cdot \frac{\overline{R_{21}}}{|R_{21}|} = \frac{\overline{r_1} - \overline{r_2}}{|r_1 - r_2|}$$

$$F_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot R_{12}^2} \cdot \frac{(\overline{r_1} - \overline{r_2})}{R} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot R^3} \cdot (\overline{r_1} - \overline{r_2}) \text{ — (1)}$$

$$F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot R_{21}^2} \cdot \frac{(r_2 - r_1)}{R} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot R^3} (\overline{r_2} - \overline{r_1}) \text{ — (2)}$$

Compare, Eq^n — (1) and Eq^n — (2),

$$\boxed{\overline{F_1} = -F_2}$$

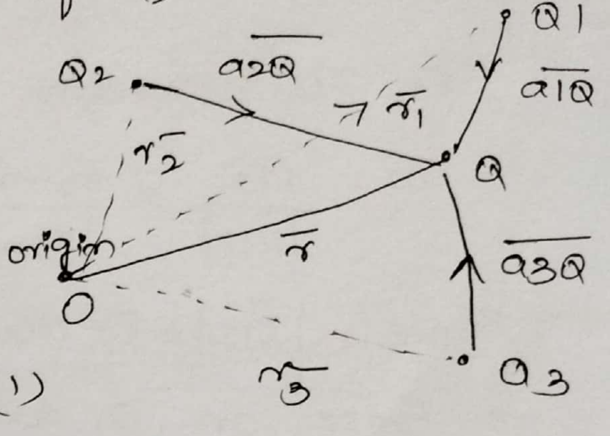
Hence, by this, the force experienced by both the charges will be Same in magnitude but opposite in direction.

Force on point charge due to 'N' other point charges

If there are more than two point charges, then each will exert force on the other, then the net force on any charge can be obtained by the principle of Superposition.

other point charges ...

The total force on charge Q in such a way is a vector sum of all the forces exerted due to each of other point charges. Q_1, Q_2 and Q_3. Consider force exerted on Q due to Q_1. At this time, according to principle of superposition effects of Q_2 and Q_3 can be suppressed



→ Force on Q due to Q_1 is

$$\bar{F}_{Q_1Q} = \frac{Q_1 Q}{4\pi\varepsilon_0 \cdot R_{1Q}^2} \cdot \overline{a_{1Q}}$$

$$F_{1Q} = \frac{Q_1 Q}{4\pi\varepsilon_0 \cdot R_{1Q}^2} \frac{(\bar{r}-\bar{r}_1)}{|\bar{r}-\bar{r}_1|} \quad -(1)$$

→ Force on Q due to Q_2 is

$$\bar{F}_{Q_2Q} = \frac{Q_2 Q}{4\pi\varepsilon_0 \cdot R_{2Q}^2} \cdot \overline{a_{2Q}} = \frac{Q_2 Q}{4\pi\varepsilon_0 \cdot R_{2Q}^2} \cdot \frac{(\bar{r}-\bar{r}_2)}{|\bar{r}-\bar{r}_2|} \quad -(2)$$

→ Similarly, force on Q due to Q_3 is

$$\bar{F}_{3Q} = \frac{QQ_3}{4\pi\varepsilon_0 \cdot R_{3Q}^2} \cdot \overline{a_{3Q}} = \frac{QQ_3}{4\pi\varepsilon_0 \cdot R_{3Q}^2} \cdot \frac{(\bar{r}-\bar{r}_3)}{|\bar{r}-\bar{r}_3|} \quad -(3)$$

In general, the net force or total force

$$\bar{F}_Q = \bar{F}_{Q_1Q} + \bar{F}_{Q_2Q} + \bar{F}_{Q_3Q}$$

For n charges, the force exerted on Q due to all other charges.

$$\bar{F}_Q = F_{Q_1Q} + F_{Q_2Q} + \dots + \bar{F}_{Q_nQ}$$

$$= \frac{Q_1 Q}{4\pi\varepsilon_0 R_{1Q}^2} \cdot \frac{(\bar{r}-\bar{r}_1)}{|(r-\bar{r}_1)|} + \frac{Q_2 Q}{4\pi\varepsilon_0 \cdot R_{2Q}^2} \frac{(\bar{r}-\bar{r}_2)}{|\bar{r}-\bar{r}_2|} + \dots + Q_n$$

$$+ \dots + \frac{Q_n Q}{4\pi\varepsilon_0 \cdot R_{2n}^2} \cdot \frac{(\bar{r}-\bar{r}_n)}{|\bar{r}-\bar{r}_n|}$$

$$\boxed{\bar{F}_Q = \frac{Q}{4\pi\varepsilon_0} \cdot \sum_{i=1}^{n} \frac{Q_i}{R_{iQ}^2} \frac{(\bar{r}-\bar{r}_i)}{|\bar{r}-\bar{r}_i|}}$$

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2) Electric field Intensity | Electric field | Electric field strength:-

Def:- The force experienced by a unit positive charge kept in the field is known as electric field intensity at that point. / Force on unit test charge at that point which we want to find value of $\bar{E}$.

(2) It is defined as the columb force per unit positive charge in the field.

$$\boxed{\bar{E} = \frac{\bar{F}}{Q}}$$ Units : N/C.

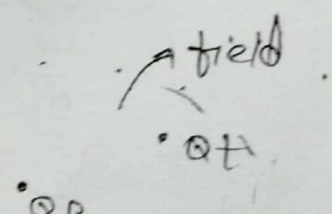
(3) The negative gradient of electric potential is known as electric field Intensity.

$$\boxed{\bar{E} = -\nabla V}$$ units: V/m.

(4):- When the electric field intensity is same at all the points in the static field then it is called Uniform / Homogeneous electric field. If $\bar{E}$ is different, then it is non uniform (or) non Homogeneous electric field:-

Eg:- Uniform → Parallel plate capacitor.
Non Uniform → Solenoid.

(a) $\bar{E}$ due to point charge:-



let us consider the two point charges. Qt = test point charge C
Qf = fixed point charge.

$\bar{r}t$ = position / location of Qt, $\bar{r}f$ = position / location of Qf

Now, E due to fixed on test charge.

$$F_{tf} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_t Q_f}{r}$$

$$\boxed{E = \frac{F_t}{Q_t}}$$

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$$\bar{F}_{ft} = \frac{Q_t Q_f}{4\pi\varepsilon_0} \cdot \frac{1}{r_{tf}^2} \cdot \overline{r_{tf}}$$

$$\bar{F}_{ft} = \frac{Q_t Q_f}{4\pi\varepsilon_0 \cdot r_{tf}^2} \cdot \frac{(\bar{r}_t - \bar{r}_f)}{|r_t - r_f|}$$

$$\bar{E} = \frac{F_t}{Q_t} = \frac{\bar{F}_{ft}}{Q_t} = \frac{\cancel{Q_t} Q_f}{4\pi\varepsilon_0 \cdot r_{tf}^2} \cdot \frac{(\bar{r}_t - \bar{r}_f)}{|r_t - r_f|} \cdot \frac{1}{\cancel{Q_t}}$$

$$\bar{E} = \frac{Q_f (\bar{r}_t - \bar{r}_f)}{4\pi\varepsilon_0 \cdot |r_t - r_f|^3} = \frac{Q_f \cdot (\bar{r}_t - \bar{r}_f)}{4\pi\varepsilon_0 \cdot r_{tf}^3}$$

$$\boxed{\bar{E} = \frac{Q_f (\bar{r}_t - \bar{r}_f)}{4\pi\varepsilon_0 \cdot r_{tf}^3}} \quad N/C.$$

* In general E at any point in freee space due to point charge Q is written as

$$\bar{E} = \frac{Q_f (\bar{r}_t - \bar{r}_f)}{4\pi\varepsilon_0 \cdot r_{tf}^3} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{\bar{r}}{|r|^2} \; N/C.$$

→ E at any point due to n point charges is vector sum of field Intensities at that point due to point charge.

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \dots + \overline{E_n}$$

Case: ii) $\bar{E}$ due to line charge density

$$\rho_l = \frac{dQ}{dl} \Rightarrow dQ = \rho_l \cdot dl \Rightarrow \int \rho_l \cdot dl = \int dQ \Rightarrow \boxed{Q = \int \rho_l \cdot dl}$$

$$\bar{E} = \frac{\bar{F}}{Q} = \frac{\int \rho_l \cdot dl}{4\pi\varepsilon_0} \cdot \frac{\bar{r}}{r^2} \Rightarrow \left[\because \bar{F} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{\bar{r}}{r^2}\right].$$

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a) $\bar{E}$ due to Infinite line charge density

Consider an infinitely long straight line carrying line charge having density ρ_L c/m. let this line lies along z-axis from $-\infty$ to $+\infty$ and hence called infinite line charge. let point P is on y axis at which electric field intensity is to be determined. The distance of point P from origin is 'r'.

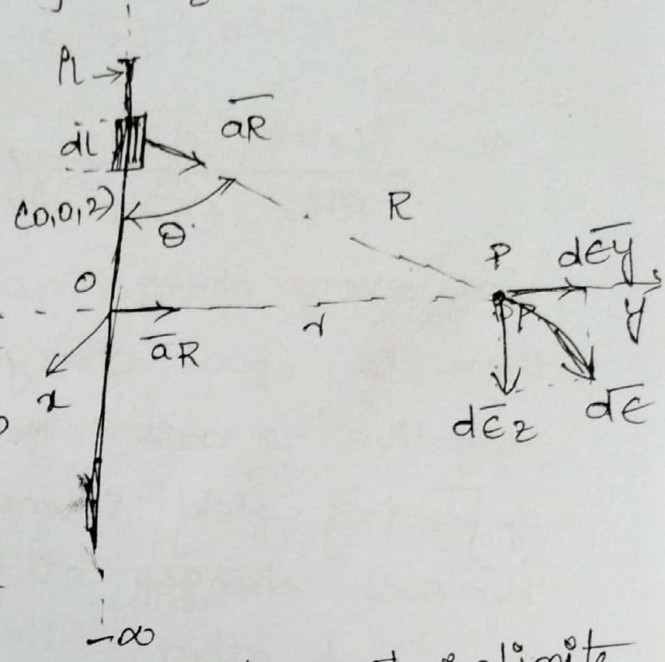


fig:- Field due to infinite line charge

Consider a small differential line length dl carrying a charge dQ, along the line as shown in fig. It is along z axis hence $dl = dz$. The coordinates of dQ are (0,0,z) while the coordinates of point P are (0,r,0). $dQ = \rho_L . dl = \rho_L . dz$. Hence the distance vector $\bar{R} = r_P - \bar{r}_{dl}$

hypotenuese + opposite = adjacent, triangle law

$$\bar{R} = [|r_P| \bar{a_y} - |r_{dL}| \bar{a_z}]$$

$$\bar{R} = [r . \bar{a_y} - z . \bar{a_z}], \quad |\bar{R}| = \sqrt{r^2 + z^2}$$

$$\bar{a_R} = \frac{\bar{R}}{|R|} = \frac{r . \bar{a_y} - z . \bar{a_z}}{\sqrt{r^2 + z^2}} \quad \text{--- (1).}$$

$$d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 . R^2} . \bar{a_R} \quad \text{--- (2)}, \quad dQ = [\rho_L . dL]$$

$$\bar{a_R} \Rightarrow d\bar{E} = \frac{\rho_L . dL}{4\pi\varepsilon_0} \left[\frac{r . \bar{a_y} - z . \bar{a_z}}{\sqrt{r^2 + z^2}}\right]$$

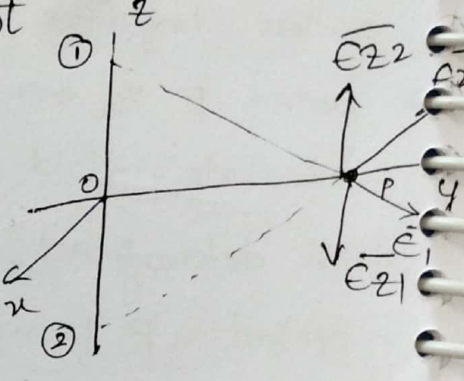
$dl = dz$

$$d\bar{E} = \frac{\rho_L \cdot dz}{4\pi\varepsilon_0 (\sqrt{r^2+z^2})^2} \left[\frac{r\bar{a_y} - z\cdot\bar{a_z}}{\sqrt{r^2+z^2}}\right]$$

$$d\bar{E} = \frac{\rho_L \cdot dz}{4\pi\varepsilon_0 \cdot (r^2+z^2)^{3/2}} \left[r\cdot\bar{a_y} - z\cdot\bar{a_z}\right] \quad \text{—— (3)}$$

$|\bar{E_{z1}}| = |\bar{E_{z2}}|$
Equal and opp.
hence cancel

For every charge on positive z axis there is equal charge present on negative z axis. Hence z component of electric field intensities produced by such charges at point P will cancel each other. Hence effectively there will not be any z component of $\bar{E}$ at P.



Hence the equation of $d\bar{E}$ can be written by eliminating $\bar{a_z}$ component,

$$d\bar{E} = \frac{\rho_L \cdot dL}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \cdot \frac{r\bar{a_y}}{\sqrt{r^2+z^2}}$$

Now, integrating $d\bar{E}$ over z axis from $-\infty$ to $+\infty$ we obtain total point P

$$\bar{E} = \int_{-\infty}^{\infty} \frac{\rho_L \cdot dL}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \cdot r \cdot dz, \bar{a_y}$$

$\tan\theta = \frac{z}{r} \Rightarrow z = r\tan\theta \Rightarrow dz = r\sec^2\theta \cdot d\theta.$

$$\bar{E} = \int_{-\infty}^{\infty} \frac{\rho_L \cdot dz}{4\pi\varepsilon_0 \cdot (r^2+z^2)^{3/2}} \cdot r\cdot\bar{a_y} \cdot dz.$$

for $z = -\infty \Rightarrow \theta = \tan^{-1}(-\infty) = -\pi/2 = -90°$

$z = +\infty \Rightarrow \theta = \tan^{-1}(\infty) = \pi/2 = +90°$

$$\bar{E} = \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_L \cdot dz}{4\pi\varepsilon_0 \cdot (r^2+z^2)^{3/2}} \; r\cdot\bar{a_y} \cdot \cancel{dz}\, d\theta$$

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$$\bar{E} = \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_L \cdot r \cdot \sec^2\theta \cdot r \; d\theta}{4\pi\varepsilon_0 (r^2 + r^2\tan^2\theta)^{3/2}} \cdot r \cdot \bar{a_y}$$

$$\bar{E} = \int_{-\pi/2}^{\pi/2} \frac{\rho_L \cdot r \cdot \sec^2\theta \cdot d\theta}{4\pi\varepsilon_0 \cdot (r^2(\sec^2\theta)^{3/2})} \cdot r \cdot \bar{a_y}$$

$$\bar{E} = \int_{-\pi/2}^{\pi/2} \frac{\rho_L \cdot r \cdot \sec^2\theta \cdot r \; d\theta}{4\pi\varepsilon_0 \cdot r^3 \cdot \sec^3\theta} \bar{a_y}$$

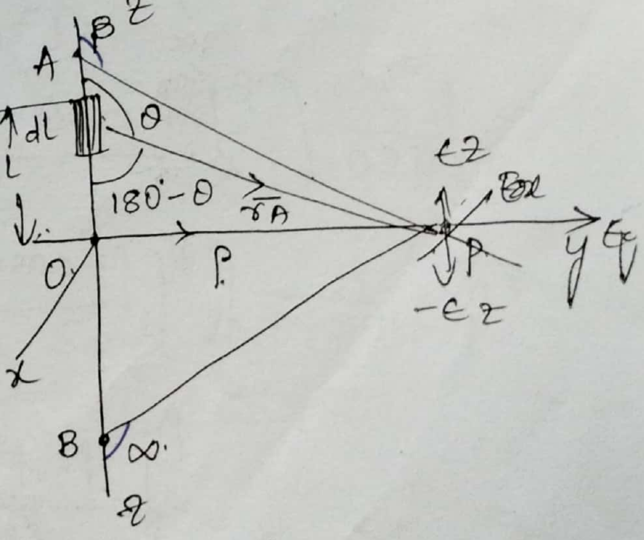
$$\bar{E} = +\int_{-\pi/2}^{\pi/2} \frac{\rho_L}{4\pi\varepsilon_0} \cdot \frac{1}{r\sec\theta} \cdot \bar{a_y} \cdot d\theta$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 \cdot r} \int_{-\pi/2}^{\pi/2} \cos\theta \cdot \bar{a_y} \cdot d\theta = \frac{\rho_L}{4\pi\varepsilon_0 \cdot r} \left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 \cdot r} [1-(-1)] \bar{a_y} = \frac{\rho_L}{4\pi\varepsilon_0 r} \times 2\bar{a_y}$$

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 \cdot r} \cdot \bar{a_y} \ V/m}$$

ii) $\bar{E}$ due to finite line charge density :-



$$\vec{dE_A} = \frac{\rho_L \cdot dL}{4\pi\varepsilon_0 \cdot r_A^2} \cdot \vec{r_A}$$

$$L \cdot \hat{z} + \vec{r_A} = \rho \cdot \hat{\rho}$$

$$\vec{r_A} = -L \cdot \hat{z} + \rho \cdot \hat{\rho}$$

$$|\vec{r_A}| = \sqrt{L^2 + \rho^2}$$

$$\vec{dE_A} = \frac{\rho_L \cdot dL}{4\pi\varepsilon_0 \sqrt{(L^2+\rho^2)}} \cdot (-L\hat{z} + \rho \cdot \hat{\rho})$$

Integrating on both sides.

$$\tan(180^\circ - \theta) = \frac{\rho}{L} \Rightarrow \frac{\rho}{L} \Rightarrow \rho = L\tan(180^\circ - \theta)$$

$$L = \frac{\rho}{\tan(180^\circ - \theta)} = \frac{\rho}{-\tan\theta} \Rightarrow L = -\rho\cot\theta$$

$$dL = \rho \operatorname{cosec}^2\theta \cdot d\theta$$

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$$d\vec{E_A} = \frac{\rho_L \cdot dl}{4\pi\varepsilon_0 \sqrt{(l^2+\rho^2)}} \cdot \vec{a_R}$$

$$E_A = \int_{\alpha}^{\beta} \frac{\rho_L \cdot \rho \cdot \operatorname{cosec}^2\theta \cdot d\theta}{4\pi\varepsilon_0 \cdot \sqrt{(l^2+\rho^2)}} \frac{(-l\hat{z} + \rho \cdot \hat{\rho})}{(l^2+\rho^2)}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \cdot \left[\int_{\alpha}^{\beta} \frac{\rho \cdot \operatorname{cosec}^2\theta \cdot d\theta}{(l^2+\rho^2)^{3/2}} (-l\hat{z} + \rho \cdot \hat{\rho}) \right]$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \left[\int_{\alpha}^{\beta} \frac{\rho \cdot \operatorname{cosec}^2\theta \cdot d\theta \cdot -l\hat{z}}{(l^2+\rho^2)^{3/2}} + \int_{\alpha}^{\beta} \frac{\rho \cdot \hat{\rho}}{(l^2+\rho^2)^{3/2}} \cdot \operatorname{cosec}^2\theta \cdot d\theta \right]$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \left[\int_{\alpha}^{\beta} \frac{\rho \cdot \operatorname{cosec}^2\theta \cdot d\theta\ (-l\hat{z})}{(l^2 + (-l\tan\theta)^2)^{3/2}} + \int_{\alpha}^{\beta} \frac{\rho\ \hat{\rho} \cdot \operatorname{cosec}^2\theta \cdot d\theta}{[l^2 + (-l\tan\theta)^2]^{3/2}} \right]$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \left[\int_{\alpha}^{\beta} \frac{\rho \cdot \operatorname{cosec}^2\theta \cdot d\theta \cdot (-l\hat{z})}{(l^2 + l^2\tan^2\theta)^{3/2}} + \int_{\alpha}^{\beta} \frac{\rho \cdot \hat{\rho}\ \operatorname{cosec}^2\theta\ d\theta}{(l^2 + l^2\tan^2\theta)} \right]$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \left[\int_{\alpha}^{\beta} \frac{\rho \cdot \operatorname{cosec}^2\theta \cdot d\theta\ (-(-\rho)\cot\theta) \cdot \hat{z}}{(l^2 \sec^2\theta)^{3/2}} + \int_{\alpha}^{\beta} \frac{\rho \cdot \hat{\rho} \cdot \operatorname{cosec}^2\theta \cdot d\theta}{(l^2 \sec^2\theta)^{3/2}} \right]$$

$l = -\rho\cot\theta$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \left[\int_{\alpha}^{\beta} \rho^2 \cot\theta \cdot \operatorname{cosec}^2\theta \cdot d\theta \cdot \hat{z} \right.$$

$$\vec{E_A} = \frac{\rho_L}{4\pi\varepsilon_0} \left[\int_{\alpha}^{\beta} \frac{\rho \cdot \operatorname{cosec}^2\theta \cdot d\theta\,(-(-\rho) \cdot \cot\theta) \cdot \hat{z}}{(\rho^2\cot^2\theta + \rho^2)^{3/2}} + \int_{\alpha}^{\beta} \frac{\rho^2\ \hat{\rho} \cdot \operatorname{cosec}^2\theta \cdot d\theta}{(\rho^2\cot^2\theta + \rho^2)^{3/2}} \right]$$

$$\vec{E_A} = \frac{\rho_L}{4\pi\varepsilon_0} \left[\int_{\alpha}^{\beta} \frac{\rho \cdot \operatorname{cosec}^2\theta \cdot d\theta \cdot \rho \cdot \cot\theta \cdot \hat{z}}{(\rho^2 \operatorname{cosec}^2\theta)^{3/2}} + \int_{\alpha}^{\beta} \frac{\rho^2\ \hat{\rho} \cdot \operatorname{cosec}^2\theta\ d\theta}{} \right]$$

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$$\vec{E_A} = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_{\alpha}^{\beta} \frac{\cancel{\rho^2} \cdot \cancel{cosec^2\theta} \cdot d\theta \cdot \cancel{\rho}\cot\theta \cdot \hat{z}}{\cancel{\rho^3} \cdot cosec^3\theta} + \int_{\alpha}^{\beta} \frac{\cancel{\rho^2} \cdot \hat{\rho} \cdot \cancel{cosec^2\theta} \cdot d\theta}{\cancel{\rho^3}\, cosec^3\theta}\right]$$

$$\vec{E_A} = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_{\alpha}^{\beta} \frac{\cancel{\rho} d\theta \cdot \cot\theta}{\rho \cdot cosec\theta} \cdot \hat{z} + \int_{\alpha}^{\beta} \frac{\hat{\rho} \cdot \cancel{\cos} \cdot d\theta}{\rho \cdot cosec\theta}\right]$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\left[\frac{1}{\rho}\int_{\alpha}^{\beta} \frac{\cos\theta}{\sin\theta} \cdot \sin\theta \cdot d\theta \cdot \hat{z} + \int_{\alpha}^{\beta} \cdot \frac{\hat{\rho} \cdot d\theta \cdot \sin\theta}{\rho}\right]$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 \cdot \rho}\left[\int_{\alpha}^{\beta} \cos\theta \cdot d\theta \cdot \hat{z} + \int_{\alpha}^{\beta} \sin\theta \cdot \hat{\rho} \cdot d\theta\right]$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 \cdot \rho}\left[\int_{\alpha}^{\beta} (\sin\theta)_{\alpha}^{\beta} \cdot \hat{z} + \int_{\alpha}^{\beta} \left[-\cos\theta\right]_{\alpha}^{\beta} \cdot \hat{\rho}\right]$$

$$\boxed{\vec{E_A} = \frac{\rho_L}{4\pi\varepsilon_0 \cdot \rho}\left[(\sin\beta - \sin\alpha)\hat{z} + (\cos\alpha - \cos\beta)\hat{\rho}\right] \text{ N/C}}$$

(a) If line charge density is infinite, α becomes $\alpha = 0°$, $\beta = 180°$,

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0 \cdot \rho}\left[\left[\sin(180°) - \sin 0\right] + \left[\cos 0° - \cos 180°\right]\right]\hat{\rho}$$

$$\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0 \cdot \rho}\left[0 + 2 \cdot \hat{\rho}\right]$$

$$\boxed{E = \frac{\rho_L}{2\pi\varepsilon_0 \cdot \rho} \cdot \hat{\rho} \cdot \text{N/C}}$$

(b) If line charge density is Semi finite, then $\alpha = \pi/2$, $\beta = \pi$, then

$$\boxed{\bar{E} = \frac{\rho_L}{4\pi\varepsilon_0 \cdot \rho}(\hat{\rho} - \hat{z})}$$

(c) If line charge density is $\perp^{lar}$ to a boundary then $\beta = 180° - \alpha$ (or) $\alpha = 180° - \beta$, then

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 \cdot \rho}(\cos\alpha) \cdot \hat{\rho}}$$

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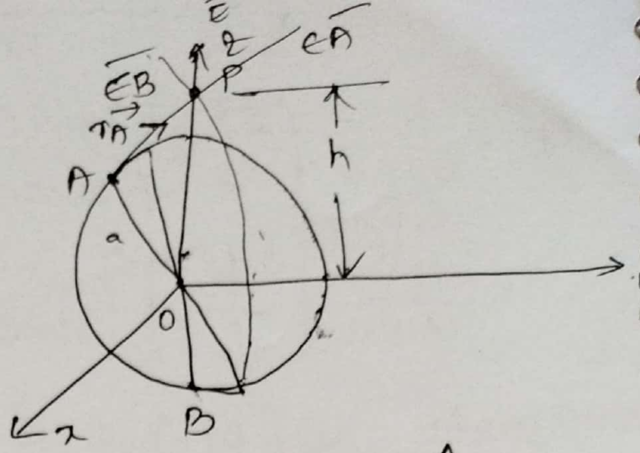
~~$a\bar{a}+h.z$~~ $a.\bar{a}+\vec{r_A}=h.\hat{z}$

$\bar{r_a}=-a.\bar{a}+h.\hat{z}$
$\downarrow$ horizontal $\downarrow$ vertical

horizontal Compoment gets Cancelled.



$\bar{r_a}=h.\hat{z}$

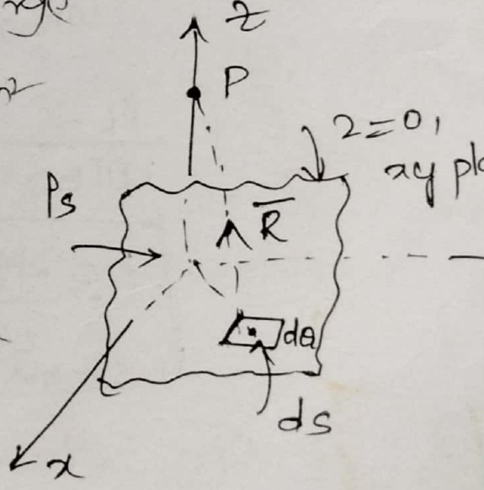
$$d\bar{E}=\frac{\rho_L.dL.\vec{r_a}}{4\pi\varepsilon_0.(a^2+h^2)^{3/2}}=\frac{\rho_L.dL}{4\pi\varepsilon_0(a^2+h^2)^{3/2}}h.\hat{z}$$

$$\bar{E}=\int_0^a\frac{\rho_L\,dL}{4\pi\varepsilon_0(a^2+h^2)^{3/2}}\cdot h.\hat{z}$$

$$\boxed{\bar{E}=\frac{a.\,\rho_L.h.\,\hat{z}}{4\pi\varepsilon_0.(a^2+h^2)^{3/2}}}$$

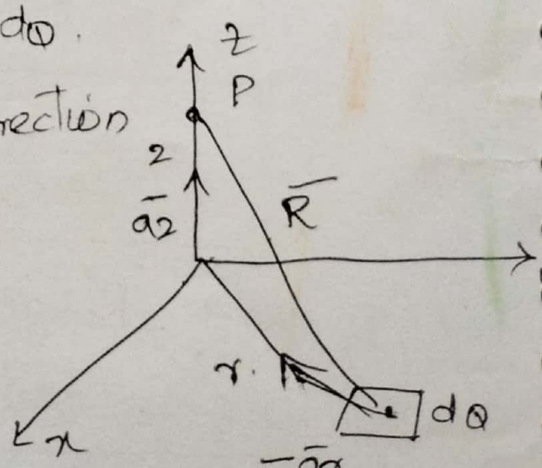
(v) Electric field due to Infinite Sheet of charge (Surface)

consider an infinite sheet of charge having uniform charge density ρ_s c/m² placed in xy plane. let us consider cylindrical coordinates.



The point P at which E to be calculated is placed on z-axis.

Carrying the differential Surface area ds carrying a charge dQ. The normal direction to ds is z direction hence ds normal to z direction is $r\,dr\,d\phi$, $dQ=\rho_s.ds$.



$$d\bar{E_A}=\frac{\rho_s\,ds}{4\pi\varepsilon_0.r_A^2}.\vec{r_A}$$

$\bar{R}=\bar{a_z}.z+(\bar{r}\,\hat{a_r})$

$\Rightarrow |R|=\sqrt{r^2+z^2}$

$$d\vec{E_r} = \frac{\rho_s\, ds}{4\pi\varepsilon_0\, r_A^{\,2}}\, \vec{a_{rA}} = \frac{\rho_s\, ds}{4\pi\varepsilon_0(\sqrt{r^2+z^2})^2}\, (-r\,\bar{a_r} + z\,\bar{a_z})$$

$$d\vec{E_A} = \frac{\rho_s\, r\, dr\, d\phi}{4\pi\varepsilon_0\,(\sqrt{r^2+z^2})^2}\left[\frac{-r\bar{a_r} + z\,\bar{a_z}}{\sqrt{r^2+z^2}}\right]$$

$$\bar{E} = \int_{\phi=0}^{2\pi}\int_{r=0}^{\infty} \frac{\rho_s\, r\, dr\, d\phi}{4\pi\varepsilon_0\,(r^2+z^2)^{3/2}}\,(z\,\hat{a_z})$$

Since, all radial compoments of $\bar{E}$ are going to cancel out each other and net $\bar{E}$ will not have any radial compoment.

$$\bar{E} = \frac{\rho_s}{4\pi\varepsilon_0} \int_{r=0}^{\infty} \frac{r\, dr}{(r^2+z^2)^{3/2}}\, .\, z\, \hat{a_z} \int_{\phi=0}^{2\pi} d\phi$$

$r^2+z^2 = u^2 \Rightarrow 2r\,dr = 2u\,du \Rightarrow r\,dr = u\,du$

$r = 0 \Rightarrow u = z$

$r = \infty \Rightarrow u = \infty$

$$\bar{E} = \int_0^{2\pi}\left[\int_{u=z}^{\infty} \frac{du}{(u^2)^{3/2}}\, \hat{a_z}\right] . z\, d\phi \;\frac{\rho_s}{4\pi\varepsilon_0}$$

$$= \int_0^{2\pi}\left[\int_{u=z}^{\infty} \frac{du\, \cancel{u}}{u^{\cancel{3}}}\, \hat{a_z}\, z\, d\phi\right] \frac{\rho_s}{4\pi\varepsilon_0}$$

$$\frac{\rho_s}{4\pi\varepsilon_0}\left(\int_0^{2\pi}\int_{u=z}^{\infty} \frac{du}{u^2}\, \hat{a_z}\, z\, d\phi\right) = \frac{\rho_s}{4\pi\varepsilon_0}\left[\int_0^{2\pi}\int_z^{\infty} \frac{du}{u^2}\, z\, \bar{a_z}\, d\phi\right]$$

$$= \frac{\rho_s}{4\pi\varepsilon_0}\left[\frac{-1}{u}\right]_z^{\infty} \int_{\phi=0}^{2\pi} d\phi\, .\, z\, \bar{a_z}$$

$$= \frac{\rho_s}{4\pi\varepsilon_0}\, .\, z\, \bar{a_z}\, (\phi)_0^{2\pi}\left[\frac{-1}{\infty} - \left(\frac{-1}{z}\right)\right]$$

$$= \frac{\rho_s}{4\pi\varepsilon_0}\, \bar{a_z}\, 2\pi\left[\frac{+1}{\cancel{z}}\right] . \cancel{z}$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0}\, \hat{a_z}}$$ for xy plane

(a) If charge sheet lies in Y-Z plane, then E at a point on X-axis

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \cdot \bar{a}_i}$$

(b) If chargesheet lies in X-Y plane, then E at a point in z axis

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \cdot \bar{k}}$$

(c) If chargesheet lies in Z-X plane, then E at a point in XY plane.

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \cdot \hat{j}}$$

(d) Generally, $\bar{E}$ at a point on axis which is normal to charge sheet planes is $\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \cdot \hat{n}}$

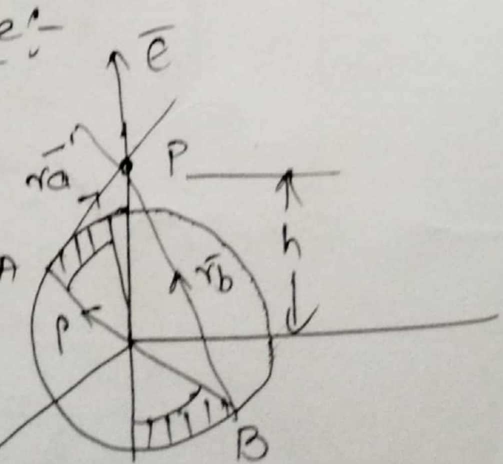
(e) Consider parallel plate capacitor, then $\bar{E}$ b/w the two plates is $E = E_1 + E_2$, $E_1 = \frac{\rho_s}{2\varepsilon_0} \cdot \hat{n}$ due to +ve plate

$E_2 = \frac{-\rho_s}{2\varepsilon_0} \cdot (-\hat{n})$ due to -ve plate

$$E = e_1 + e_2 = \frac{\rho_s}{2\varepsilon_0}\hat{n} + \frac{\rho_s}{2\varepsilon_0}(\hat{n}) = \frac{2\rho_s}{2\varepsilon_0} \cdot \hat{n}$$

$$\boxed{\bar{E} = \frac{\rho_s}{\varepsilon_0} \cdot \hat{n}}$$ because of two plates of equal and opposite charge.

1) $\bar{E}$ due to circular Surface charge:-



$$\overrightarrow{dE_A} = \frac{\rho_s \cdot ds}{4\pi\varepsilon_0 \cdot r_A^2} \hat{r_A}$$

$$\overrightarrow{r_A} + \rho \cdot \hat{\rho} = h \cdot \hat{z} \Rightarrow \overrightarrow{r_A} = h \cdot \hat{z} - \rho \cdot \hat{\rho}$$

$$\overrightarrow{dE_A} = \frac{\rho_s \cdot ds}{4\pi\varepsilon_0 \cdot r_A^3} \overrightarrow{r_A}, \quad l = 0.$$

$$\overrightarrow{E_A} = \int\int \frac{\rho_s \cdot ds}{4\pi\varepsilon_0 (h^2+\rho^2)^{3/2}} \cdot (h\hat{z} - \rho \cdot \hat{\rho})$$

$$\vec{E_A} = \int_0^a \int_0^{2\pi} \frac{\rho\, d\rho\, d\phi\, \rho_s\, h\, \hat{z}}{4\pi\epsilon_0 (r_A)^3} = \int_0^a \int_0^{2\pi} \frac{\rho\, d\rho\, d\phi\, \rho_s\, h\, \hat{z}}{4\pi\epsilon_0 (h^2+\rho^2)^{3/2}}$$

$$\vec{E_A} = \frac{\rho_s\, h}{4\pi\epsilon_0} \int_0^a \frac{\rho\, d\rho}{(h^2+\rho^2)^{3/2}} \int_0^{2\pi} d\phi\ \hat{z} \quad \text{——(1)}$$

$\rho^2 + h^2 = x^2 \Rightarrow 2\rho\, d\rho = 2x\, dx \Rightarrow \rho\, d\rho = x\, dx$

~~on~~ $\rho = 0 \Rightarrow x = h$, $\rho = a \Rightarrow a^2 + h^2 = x^2 \Rightarrow x = \sqrt{a^2+h^2}$

$$\Rightarrow \vec{E_A} = \frac{\rho_s h}{4\pi\epsilon_0} \int_h^{\sqrt{a^2+h^2}} \frac{x\, dx}{(x^2)^{3/2}} (\hat{z}) \int_0^{2\pi} d\phi$$

$$\vec{E_A} = \frac{\rho_s \cdot h}{4\pi\epsilon_0} \cdot \hat{z} \int_h^{\sqrt{a^2+h^2}} \frac{dx}{x^2} \cdot \int_0^{2\pi} d\phi$$

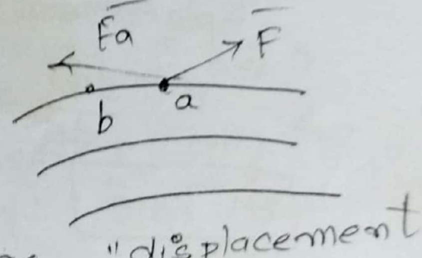
$$= \frac{\rho_s \cdot h}{4\pi\epsilon_0} \cdot \hat{z} \left(\frac{-1}{x}\right)_h^{\sqrt{a^2+h^2}} \cdot 2\phi$$

$$= \frac{\rho_s \cdot h}{2\pi\epsilon_0} \cdot \hat{z} \left(\frac{+1}{h} + \frac{(-1)}{\sqrt{a^2+h^2}}\right)$$

$$\boxed{\vec{E_A} = \frac{\rho_s \cdot h}{2\pi\epsilon_0} \left(\frac{+1}{h} + \frac{-1}{\sqrt{a^2+h^2}}\right)\hat{z}} = \frac{\rho_s \cdot h}{2\pi\epsilon_0}\left[\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right]\hat{z}$$

Workdone :- (W) :-

$$w = \int_a^b \bar{E_a} \cdot \bar{dl}$$



Electric flux :-

Electric flux is also known as "displacement Electric flux". (ψ)

$\psi = \int_s \bar{D} \cdot \bar{ds}$, $\bar{D}$ = Electric flux density,

Units : Coulomb

~~Magnitude~~ of Electric lines of force is called Electric flux. Scalar Quantity.

$$\boxed{\psi = Q \text{ Coulombs}}$$

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Consider the two point charges. The flux lines originating from +ve charge and terminating at -ve charge in the form of tubes.

Consider a unit surface area as shown in fig. The net flux passing normal through the unit surface area is called electric flux density. It is denoted as $\bar{D}$.

Consider a sphere with a charge Q placed at its centre. There are no other charges present around. The total flux distributes radially around the charge is $\psi = Q$. This flux distributes uniformly over the surface of the sphere.

ψ = total flux, S = total surface area of the sphere

$$\boxed{\text{Electric flux density} = D = \frac{\psi}{S} \text{ in magnitude}}$$

displacement density.

$\bar{D} = \frac{d\psi}{ds} \bar{a}_n$ C/m², differential surface area.

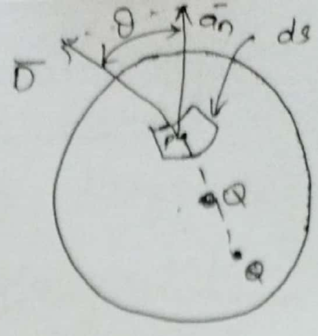
(a) $\boxed{D = \varepsilon_0 \cdot \bar{E}}$, $\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{r}$

$$D = \varepsilon_0 \cdot \frac{Q}{4\pi\varepsilon_0 \cdot r^2} \hat{r}$$

$$\boxed{D = \frac{Q}{4\pi r^2} \cdot \hat{r}}$$ d.

$\bar{ds} = ds \cdot \hat{a}_n$

GAUSS LAW



Consider a sphere of radius r and a point charge +Q placed at its centre. Then the total flux radiated outwards and passing through the total surface area of the sphere is same as the charge +Q, which is enclosed by the sphere.

Now replace the point charge by a line charge, such that the portion of the line charge enclosed by the sphere consists of same charge +Q as before. In this case also, the total flux radiating outwards remains same as Q which is the charge on the line enclosed by the sphere.

Now ~~ins~~ if point charge +Q or a part of the line carrying +Q are moved inside the sphere anywhere, still the total flux radiating outwards from the surface of the sphere remains same as Q.

Now, instead of sphere, consider any irregular closed surface area is considered with total charge enclosed as +Q in any form.

Thus, Faraday's experiment lead to Gauss law.

Statement:- Irrespective of the shape of the closed surface area and irrespective of type of charge distribution, the total flux passing through the closed surface is the total charge enclosed by that surface.

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Integral form of ... is enclose...

Proof:- consider a sphere of radius r is enclosed / a point charge at its centre.

let s be the total surface area of sphere, $s = 4\pi r^2$

$\overline{ds} = ds \cdot \overline{a_n}$, direction of vector field is normal to the surface area.

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 \cdot r^2} \cdot \overline{a_r} \quad , \overline{D} = \varepsilon_0 \cdot \overline{E} \quad , D = \frac{Q}{4\pi r^2} \cdot \overline{ds} \cdot \hat{a_n}$$

$$\overline{D} \cdot \overline{ds} = \frac{Q}{4\pi r^2} \cdot \overline{a_r} \cdot ds \cdot \overline{a_n}$$, direction of ar and an are same

$$\overline{D} \cdot \overline{ds} = \frac{Q}{4\pi r^2} \cdot ds .$$

Now, net flux, $\oint_s \overline{D} \cdot ds = \oint_s \frac{Q}{4\pi r^2} \cdot ds = \frac{Q}{4\pi r^2} \oint_s ds$

$$\oint_s \overline{D} \cdot ds = \frac{Q}{4\pi r^2} (4\pi r^2) = Q$$

$$\boxed{\oint_s \overline{D} \cdot ds = Q}$$

Gauss law is applicable when the charge distribution is symmetry like point charge, Infinite surface charge and line charge, Infinite line and surface charge distributions.

Gaussian surface:- If $\overline{E}$ is constant at all the points and direction is always normal to surface.

Dipole:- Combination of two equal and opposite charges separated with min. distance is called dipole.

→ Dipole moment is a vector quantity which always directs from -ve to +ve charge which is opposite to direction of $\overline{E}$

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Differential form / Point form of Gauss Law

From the Integral form of Gauss Law

$$\oint_S \bar{D}\cdot ds = Q_{enclosed} = \psi .$$

$Q = \int_v \rho_v \cdot dv$, $Q_{enclosed}$ is volume Integral.

$$\oint_S \bar{D}\cdot ds = \int_v \rho_v \cdot dv \quad \text{——(1)}$$

Applying divergence theorem, $\oint_S \bar{D}\cdot \bar{ds} = \int_v (\nabla\cdot\bar{D})\, dv$ —(2)

Compare Eq^n—(1) & (2)

$$\int_v (\nabla\cdot\bar{D})\, dv = \int_v \rho_v \cdot dv \quad \text{—(3)}$$

Equating —(3) on both sides

$$\boxed{\nabla\cdot\bar{D} = \rho_v}$$

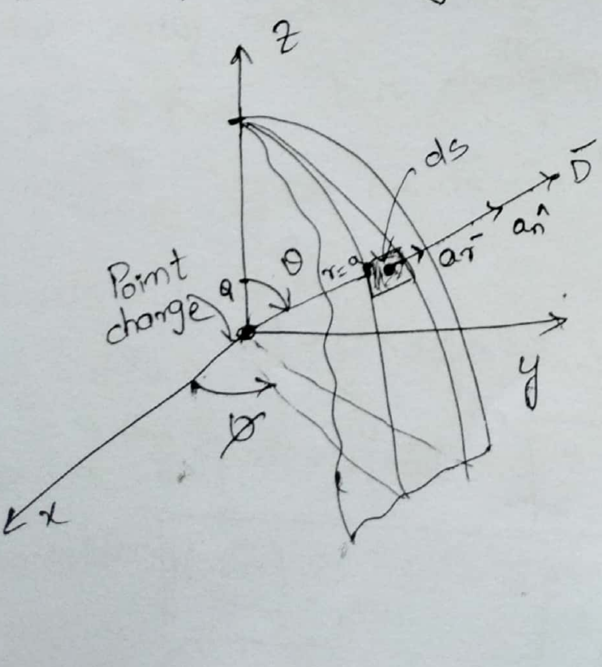
* This is called Maxwells first eq^n of electrostatics

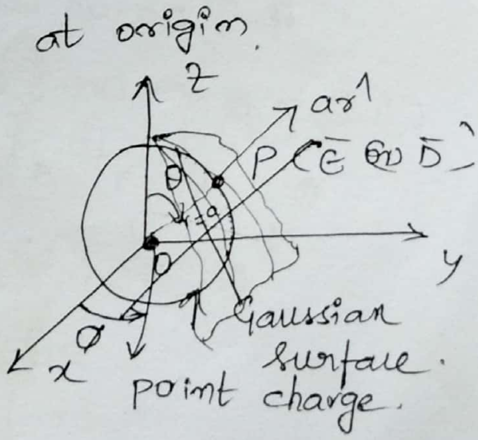
Applications of Gauss Law:-

→ To find Electric field Intensity
→ To find Unknown charge
→ Electric flux density.

(a) $\bar{E}$ due to point charge:-

Consider point charge Q is located at origin.





To determine $\bar{D}$ and to apply Gauss Law, consider a spherical surface around Q with centre as origin. This spherical surface is Gaussian surface and it satisfies required condition. The $\bar{D}$ is always directed radially outwards along $\bar{a}_r$ which is normal to the spherical surface at any point P on the surface.

Consider a differential surface area ds as shown. The direction normal to the surface ds is $\bar{a}_r$, considering spherical coordinate system. The radius of sphere is $r=a$. In spherical coordinate system, ds normal to radial direction $\bar{a}_r$

$\bar{ds} = ds.\bar{a}_n$. At point 'P', $\bar{D} = D_r.\bar{a}_n$

Applying coulombs law to surface. $\psi = \oint_S \bar{D}.\bar{ds} = Q$

$$ds = r^2 \sin\theta . d\theta . d\phi .$$

$$\psi = \oint_S \bar{D}.\bar{ds} = \oint_S D_r.\bar{a}_r . ds.\bar{a}_n = \oint_S D_r . ds.$$

$\because \bar{a}_r, \bar{a}_n$ are in same direction $\bar{a}_r.\bar{a}_n = 1)$

$$\psi = \oint_S D_r . ds = Q$$

$$Q = D_r \oint_S ds = D_r \oint_S r^2 \sin\theta . d\theta . d\phi = D_r . r^2 \int_0^{\pi} \sin\theta . d\theta \int_0^{2\pi} d\phi$$

$$Q = D_r . r^2 (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi}$$

$$Q = D_r . r^2 (2)(2\pi)$$

$$D_r = \frac{Q}{4\pi r^2}$$

Electric flux density, $\bar{D} = D_r . \bar{a}_r$

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2} . \bar{a}_r}$$

$\Rightarrow D = \varepsilon_0 \bar{E} \Rightarrow \bar{E} = \frac{D}{\varepsilon_0}$

$$\boxed{\bar{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \bar{a}_r} \quad N/C.$$

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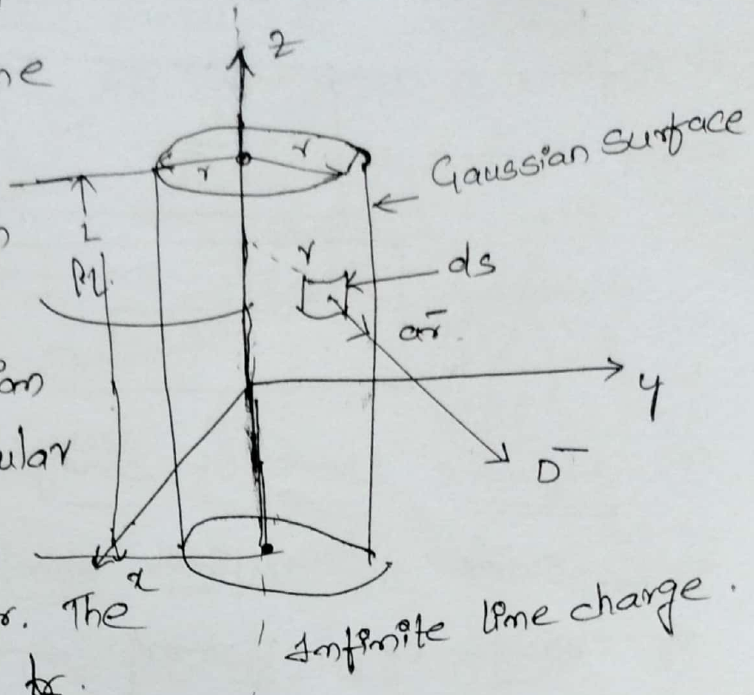
(b) Infinite line charge:-

Consider an infinite line charge of density ρ_L c/m lying along z axis from $-\infty$ to $+\infty$.



Consider the Gaussian Surface as the right circular cylinder with z axis as its x-axis and radius r. The length of cylinder is L.

The flux density at any point on the surface is directed radially outwards in the $\bar{a_r}$ direction, according to cylindrical coordinate system. Consider differential surface area ds as shown which is at radial distance r from the line charge. The direction normal to ds is $\bar{a_r}$.

As the line charge is along z-axis, there cannot be any component of D in z direction. So D has only radial component.

$\psi = Q = \oint_S \bar{D}\cdot\bar{ds}$, but charge $(Q) = \int_0^L \rho_L \cdot dL = \rho_L(L)$ — (1)

$\oint_S \bar{D}\cdot\bar{ds} = \oint_S D_r \cdot \bar{a_r} \cdot \bar{ds} \cdot \bar{a_n} = \oint_S D_r \cdot ds = \oint_S D_r \cdot ds$

but $ds = \rho \cdot d\phi \cdot dz$

$\oint_S D_r \cdot ds = D_r \oint_S \rho \cdot d\phi \cdot dz = D_r \cdot \rho \cdot \int_0^L \int_0^{2\pi} d\phi \cdot dz$

$= D_r \cdot \rho \int_0^L dz \cdot \int_0^{2\pi} d\phi = D_r \cdot \rho \cdot [L][2\pi]$ — (2)

Equating (1) and (2) $\rho_L \cdot \cancel{L} = D_r \cdot \rho \cdot \cancel{L} \, 2\pi$

$$\boxed{D_r = \frac{\rho_L}{2\pi\rho}}$$

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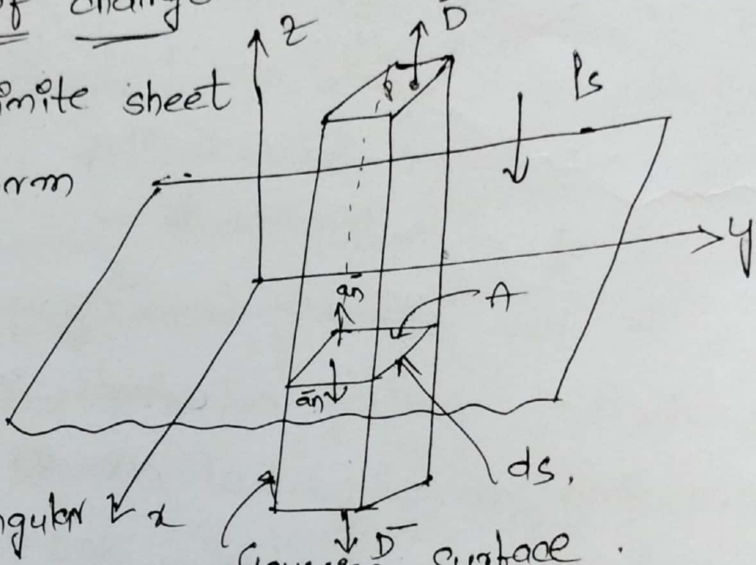
Hence, $\bar{D} = D_{r0}\bar{a}_r$

$$\boxed{\bar{D} = \frac{\rho_L}{2\pi\rho}\cdot\bar{a}_r \ C/m^2} \quad D = \varepsilon_0 E \Rightarrow E = \frac{D}{\varepsilon_0}$$

Hence, $$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0\cdot\rho}\cdot\bar{a}_r} \ N/C.$$

(c) **Infinite sheet of charge**



Consider the infinite sheet of charge of uniform charge density ρ_s C/m^2, lying in the $z=0$ plane. i,e xy plane.

Consider a rectangular box of Gaussian Surface which is cut by the sheet of charge to give $ds = dx\,dy$.

$\bar{D}$ acts normal to the plane, i,e $\bar{a}_n = \bar{a}_z$, $\bar{a}_n = -\bar{a}_z$

$$\varphi = \oint_S D\cdot\bar{ds} = \oint_S \bar{D_z}\cdot\bar{ds} \Rightarrow \bar{D_z} = D_z\cdot a_z$$

$$\Rightarrow \varphi = \oint_S D_z\cdot\bar{a}_z\cdot ds\cdot\bar{a}_n = \oint_S D_z\cdot ds = D_z\oint_S ds$$

$$= D_z\left[\int_{\text{top plate}} ds + \int_{\text{bottom plate}} ds\right] = D_z\left[A + A\right] = 2AD_z$$

$$\varphi = Q = 2AD_z \quad \text{—(1)}$$

$$\text{charge} = Q = \oint_S \rho_s\,ds = \rho_s\oint_S ds = \rho_s\cdot[A] = A\rho_s \quad \text{—(2)}$$

Equating (1) and (2)

$$2AD_z = A\cdot\rho_s \Rightarrow \boxed{D_z = \frac{\rho_s}{2}}, \quad \bar{D} = D_z\cdot\bar{a}_z$$

$$\boxed{\bar{D} = \frac{\rho_s}{2}\cdot\bar{a}_z}, \quad \bar{D} = \varepsilon_0\bar{E} \Rightarrow \bar{E} = \frac{\bar{D}}{\varepsilon_0} \Rightarrow \boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0}\cdot N/C}$$

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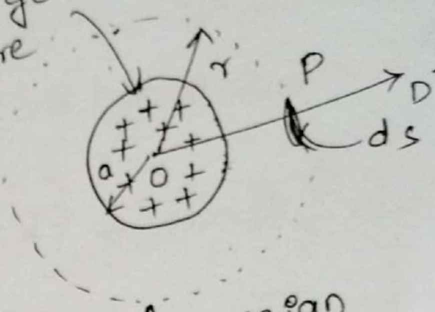
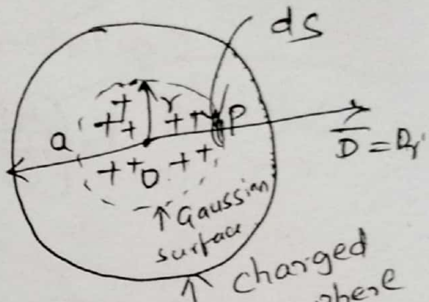
(d) **Uniformly charged sphere**

Consider a sphere of radius 'a' with a uniform charge density of ρ_v C/m³. Let us find $\bar{E}$ at point P located at a radial distance r from centre of the sphere $r \le a$ and $r \ge a$ using Gauss law.



Case I:- $r \le a$ (have charge upto r only)

Applying Gauss Law. $\psi = \oint_s \bar{D} \cdot ds = Q$.



$$Q_{enclosed} = \int_v \rho_v \cdot dv = \rho_v \int_v dv$$

$$= \rho_v \int_0^{\pi} \int_0^{2\pi} \int_0^{r} r^2 \sin\theta \cdot dr \cdot d\theta \cdot d\phi$$

$$= \rho_v \left[\int_0^{r} r^2 \cdot dr \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \cdot d\theta \right]$$

$$= \rho_v \left[\left(\frac{r^3}{3}\right)_0^{r} (\phi)_0^{2\pi} (-\cos\theta)_0^{\pi} \right]$$

$$= \rho_v \left[\frac{r^3}{3}\right][2][2\pi] = \rho_v \left[\frac{r^3}{3}\right] 4\pi \quad \text{—— (1)}$$

$$\oint_s \bar{D} \cdot \bar{ds} = \oint_s D_r \cdot \bar{a_r} \cdot ds \cdot \bar{a_n} = \oint_s D_r \cdot ds = D_r \oint_s ds$$

$$= D_r \int_0^{\pi} \int_0^{2\pi} r^2 \cdot \sin\theta \cdot d\theta \cdot d\phi = D_r \cdot r^2 \int_0^{\pi} \sin\theta \cdot d\theta \cdot \int_0^{2\pi} d\phi$$

$$= D_r \cdot r^2 (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi} = D_r \cdot r^2 (4\pi^2) \quad \text{—— (2)}$$

Equating (1) and (2); Since, $Q_{enclosed} = \oint_s \bar{D} \cdot ds$

$$\rho_v \left(\frac{r^{\cancel{3}}}{3}\right) (4\cancel{\pi}) = D_r \cdot \cancel{r^2} (4\cancel{\pi^2})$$

$$\Rightarrow \rho_v \left(\frac{r}{3}\right) = D_r, \text{ then } \bar{D} = D_r \cdot \bar{a_r}$$

$\boxed{\bar{D} = \rho_v \left(\frac{r}{3}\right) \bar{a_r}}$, $\bar{D} = \varepsilon_0 \bar{E} \Rightarrow \boxed{\bar{E} = \frac{\rho_v \cdot r}{\ } \cdot \bar{a_r}}$

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Case 2: $r \geq a$ charge

$$Q_{enclosed} = \int_v \rho_v \, dv = \rho_v \int_v dv = \rho_v \iiint r^2 \sin\theta \, dr \ldots$$

$$= \rho_v \int_0^a r^2 \, dr \cdot \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$Q_{enclosed} = \left[\frac{a^3}{3}\right] [4\pi] \rho_v \quad \text{--- (1)}$$

$$\oint_s \bar{D} \cdot \bar{ds} = \oint_s D_r \cdot \bar{a_r} \, ds \cdot \bar{a_n} = \oint_s D_r \cdot ds = D_r \oint_s ds$$

$$= D_r \iint r^2 \sin\theta \cdot d\theta \cdot d\phi = D_r \cdot r^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$\oint_s \bar{D} \cdot \bar{ds} = D_r \cdot r^2 (4\pi)$$

Equating (1) and (2)

$$\left[\frac{a^3}{3}\right] (\cancel{4\pi}) \rho_v = D_r \cdot r^2 \cdot (\cancel{4\pi})$$

$$D_r = \frac{a^3}{3r^2} \cdot \rho_v$$

$$\bar{D} = D_r \cdot \bar{a_r} \Rightarrow \boxed{\bar{D} = \frac{a^3}{3r^2} \cdot \rho_v}$$

$$D = \varepsilon_0 \cdot \bar{E} \Rightarrow \boxed{\bar{E} = \frac{a^3}{3\varepsilon_0 r^2} \cdot \rho_v}$$

* **Workdone:-**

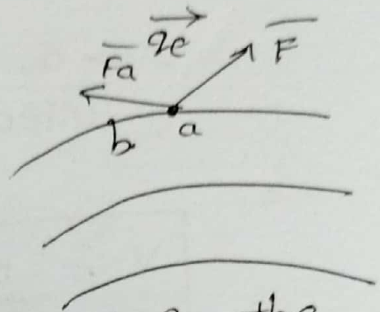
Consider an electric field due to positive charge. If a unit test positive charge Q_t is placed at any point in this field, it experiences a repulsive force and tends to move in the direction of the force.

But if a positive test charge Q_t is to be moved towards the positive base charge Q, then it is required to be moved against the electric field of the charge Q, i.e against the repulsive force exerted by charge Q on the test charge Q_t.

hile doing so, an external source

o move the test charge at . against the electric field. This movement of charge requires to expend the energy. This work done becomes the potential energy of the test charge . Qt , at the point at which l is moved.

Thus work is said to be done when test charge s moved against the electric field.



Total work done $= \int_a^b Fa \cdot dl$.

To keep the charge in equilibrium , it is ecessary to apply the force which is equal and opposite to the force exerted by the field in the direction dl.

$$F_{applied} = -\bar{F_t} \quad , \bar{Fa} = -\bar{F} \quad , \text{since } \bar{e} = \frac{F}{Q}.$$

$$W = -\int_a^b \bar{F} \cdot dl = -\int_a^b \bar{e} \cdot dl \cdot q$$

$$\boxed{W = -q \int_a^b \bar{E} \cdot dl}$$

$$E = Ex \cdot \hat{i} + Ey \cdot \hat{j} + Ez \cdot \bar{k}$$

$$dl = dx \cdot \bar{i} + dy \cdot \bar{j} + dz \cdot \bar{k}.$$

$$\boxed{W = -q \int_{(x_1, y_1, z_1)}^{(x_2, y_2, z_2)} (Ex \cdot dx + Ey \cdot dy + Ez \cdot dz)}$$ Joules.

POTENTIAL / ELECTRIC POTENTIAL (V ; J/C or Volts) / POTENTIAL DIFFERENCE

The workdone inorder to bring the unit charge from the infinite distance to a point in the field is known as Electric potential.

$$\boxed{V = \frac{\text{Workdone}}{Q} \; J/C.}$$

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The potential is said to be absolute potential when a unit +ve charge is brought into field from infinite distance.

$$W = -q\int_a^b \bar{E}\cdot dl \quad , \quad V = \frac{W}{Q} = \frac{-q}{q}\int_a^b \bar{E}\cdot dL \quad , \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V = -\int_a^b \bar{E}\cdot dl = -\int_{(\infty,0,0)}^{(r,0,0)} E_x\cdot dx = -\int_{(\infty,0,0)}^{(r,0,0)} \frac{Q}{4\pi\varepsilon_0 x^2}\cdot dx$$

$$= -\frac{Q}{4\pi\varepsilon_0}\int_{(\infty,0,0)}^{(r,0,0)} \frac{1}{x^2}\cdot dx = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{x}\right]_{(\infty,0,0)}^{(r,0,0)} = \frac{-Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}+\frac{1}{\infty}\right]$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}} \; V.$$

Above expression shows potential at a point P in electric field due to point charge 'Q' which is located at origin.

(a) If fixed charge q_1 is not located at origin and its position is at r_1 distance from origin.

$$V = \frac{Q}{4\pi\varepsilon_0 |r - r_1|} \text{ volts.}$$

(b) Electric potential at 'r' distance in a field due to 'N' point charges will be $V = V_1 + V_2 + V_3 + \dots + V_n$

$$V = \frac{Q}{4\pi\varepsilon_0|r-r_1|} + \frac{Q}{4\pi\varepsilon_0|r-r_2|} + \dots + \frac{Q_n}{4\pi\varepsilon_0|r-r_n|}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{n}\frac{Q_k}{|r-r_k|} \text{ volts.}}$$

Potential difference:- Work done per unit charge in moving unit charge from B to A in the field E is called potential difference between the points B and A.

$$\boxed{V_{AB} = -\int_B^A \bar{E}\cdot dL} \quad , \quad \boxed{V_{AB} = V_B - V_A}$$

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Note:-

(1) Potential at ∞' distance $V = \frac{Q}{4\pi\epsilon_0 r}$ $r \to \infty$, $\boxed{V = \frac{1}{\infty} = 0}$

(2) $V_A > V_B$, $V_{AB} = V_B - V_A = V_B - 0 \Rightarrow \boxed{V_{AB} = -V_A}$ Some external force is required to do work.

(3) $V_A < V_B$, $V_{AB} = V_B - V_A \rightarrow \boxed{V_{AB} = V_B}$

Relation between $\bar{E}$ & V | Second Maxwell Eq^n

Potential difference between two points A and B independent of the path taken.

$$V_{AB} = +V_B - V_A = -\int_A^B \bar{E} \cdot dl \quad \text{--- (1)}$$

$$V_{BA} = V_A - V_B = \int_A^B \bar{E} \cdot dl \quad \text{--- (2)}$$

$$\boxed{V_{BA} + V_{BA} = \oint_l \bar{E} \cdot dl = 0}$$

Applying stokes theorem,

$$\oint_l \bar{E} \cdot dl = \int_S (\nabla \times \bar{E}) ds \quad , \left(\because \oint_l \bar{E} \cdot dl \right)$$

$$0 = \int_S (\nabla \times \bar{E}) \cdot ds.$$

$$\boxed{\nabla \times \bar{E} = 0}$$

This is called conservative nature of electrostatic field

Relation b/w $\bar{E}$ & V

We have $V = -\int \bar{E} \cdot dl$

differentiating on both sides, $dV = -\bar{E} \cdot dl$

$$dV = \frac{d}{dx} V \cdot d\hat{x} + \frac{d}{dy} V \cdot d\hat{y} + \frac{d}{dz} V \cdot d\hat{z} \quad \text{--- (1)}, \quad \bar{E} = E_x \cdot \bar{dx} + E_y \cdot \bar{dy} + E_z \cdot \bar{dz}$$

$$\bar{dl} = dx \cdot \bar{a_x} + dy \cdot \bar{a_y} + dz \cdot \bar{a_z}$$

$$\bar{E} \cdot dl = (E_x \cdot \bar{dx} + E_y \cdot \bar{dy} + E_z \cdot \bar{dz})(dx \cdot \bar{a_x} + dy \cdot \bar{a_y} + dz \cdot \bar{a_z})$$

$$-\bar{E} \cdot \bar{dl} = -(E_x \cdot dx + E_y \cdot dy + E_z \cdot dz) \quad \text{--- (2)}$$

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Equating

$dv = -\int \bar{e} \cdot dL$

$\frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} \cdot dz = -(e_x \cdot dx + e_y\, dy + e_z\, dz)$

$e_x = -\frac{\partial v}{\partial x}, \quad e_y = -\frac{\partial v}{\partial y}, \quad e_z = -\frac{\partial v}{\partial z}$

But, $\bar{e} = e_x \cdot \bar{a_x} + e_y \cdot \bar{a_y} + e_z \cdot \bar{a_z}$

$\bar{e} = -\left[\frac{\partial v}{\partial x} \cdot \bar{a_x} + \frac{\partial v}{\partial y} \cdot \bar{a_y} + \frac{\partial v}{\partial z} \cdot a_z\right].$

$\bar{e} = -v\left[\frac{\partial}{\partial x} \cdot \bar{a_x} + \frac{\partial}{\partial y} \cdot \bar{a_y} + \frac{\partial}{\partial z} \cdot a_z\right].$

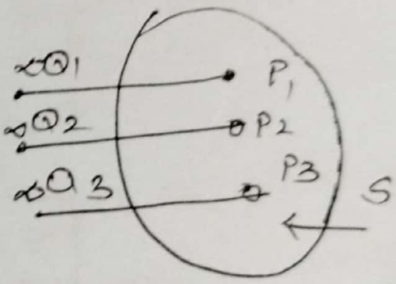
$\boxed{\bar{e} = -\nabla v}$

The above expression indicates the direction of $\bar{e}$ is opposite to the increment of potential in static field.

ENERGY DENSITY OF STATIC FIELD:-

Consider three charges.

First consider initially the region is charge free region.



Total workdone, $W = W_1 + W_2 + W_3$.

Consider an empty space where there is no electric field at all. The charge Q_1 is moved from infinity to a point in the space say P_1. This requires no work as there is no $\bar{e}$ present. Now the charge Q_2 is to be placed at point P_2 in the space. But now there is an electric field due to Q_1 and Q_2 is required to be moved against the field of Q_1. Hence the work is required to be done.

Total workdone, $W = W_1 + W_2 + W_3$

where V_{21} = potential at P_2 due to Q_1

V_{31} = potential at P_3 due to Q_1

V_{32} = potential at P_3 due to Q_2

Since, it is a chargeless region, $W_1 = 0$, $V = \frac{W}{Q}$,

$V_1 = 0$ — (1), $W = V.Q$.

$W = W_1 + W_2 + W_3$.

$W_2 = Q_2 (V_{21})$, $W_1 = 0$,

$W_3 = Q_3 (V_{31}) + Q_3 (V_{32})$.

$W = W_1 + W_2 = 0 + Q_2 V_{21} + Q_3 V_{31} + Q_3 V_{32}$ ——— (2)

$W_a = Q_2 V_{23} + Q_3 V_{31} + Q_3 V_{32}$ — (3)

Reversing the order and position

$W_E = 0 + Q_2 V_{23} + Q_1 (V_{31} + V_{21})$

$W_E = Q_2 V_{23} + Q_1 V_{31} + Q_1 V_{21}$ ——— (4)

(3) + (4), $2W_E = Q_2 V_{21} + Q_3 V_{31} + Q_3 V_{32} + Q_2 V_{23} + Q_1 V_{31} + Q_1 V_{21}$

$2W_E = Q_1 (V_{31} + V_{21}) + Q_2 (V_{21} + V_{23}) + Q_3 (V_{31} + V_{32})$

$2W_E = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$

$$\boxed{W_E = \frac{1}{2} \left[Q_1 V_1 + Q_2 V_2 + Q_3 V_3 \right]}$$

V_1, V_2, V_3 are total potentials at P_1, P_2, P_3

In general, $\boxed{W_E = \frac{1}{2} QV \text{ Joules.}}$

Similarly, in the case of 'n' no. of charges

$$\boxed{W_E = \frac{1}{2} \sum_{k=1}^{n} Q_K V_K \text{ Joules.}}$$

→ Energy stored in due to line charge density,

$$W_E = \frac{1}{2} \int \rho_L . dl . V.$$

→ due to surface charge density, $W_E = \frac{1}{2} \iint \rho_S . ds . V$

→ due to volume charge density, $W_E = \frac{1}{2} \int \rho_v . dv . V$

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Energy stored in terms of D and E

Using maxwell first equation, $\nabla \cdot D = \rho v$.

$$W_E = \frac{1}{2}\int_v \rho_v \cdot dv \, V = \frac{1}{2}\int_v (\nabla \cdot \overline{D}) dv \cdot V$$

$* \ (\nabla \cdot \overline{D})V = \nabla \cdot V\overline{D} - \overline{D} \cdot \nabla V$

using vector identity.

$$W_E = \frac{1}{2}\int_v (\nabla \cdot V\overline{D}) \, dv - \frac{1}{2}\int \overline{D} \cdot \nabla v \cdot dv$$

applying divergence theorem $\cdot \nabla v = +E$

$$W_E = \frac{1}{2}\oint_s V \cdot \overline{D} \cdot ds \cdot d - \frac{1}{2}\int_v \overline{D} (-E) \, dv \text{ — (1)}$$

Since, $V = \frac{Q}{4\pi\varepsilon_0 r}$, $V \propto \frac{1}{r}$, $D = \frac{Q}{4\pi r^2}$, $\overline{D} \propto \frac{1}{r^2}$

As the surface is too large, $r \to \infty$

$V \cdot \overline{D} \propto \frac{1}{0} \Rightarrow V \cdot \overline{D} \cdot ds = 0$

So, substitute in Eq^n — (1), $\frac{1}{2}\oint_s V \overline{D} \, \overline{ds} = 0$

$$W_E = \frac{1}{2}\int_v \overline{D} \cdot \overline{E} \cdot dv$$

differentiating on both sides

$$\frac{dW_E}{dv} = \frac{1}{2}\overline{D} \cdot \overline{E}, \text{ but } \overline{D} = \varepsilon_0 \cdot \overline{E}$$

Energy density,

$$W_E = \frac{\varepsilon_0 \cdot \overline{E} \cdot \overline{E}}{2} = \frac{\varepsilon_0 \cdot \overline{E}^2}{2} \text{ (or) } \frac{\overline{D}^2}{2\varepsilon_0}$$

$$\boxed{W_E = \frac{\varepsilon_0 \cdot \overline{E}^2}{2} \text{ (or) } \frac{\overline{D}^2}{2\varepsilon_0}}$$

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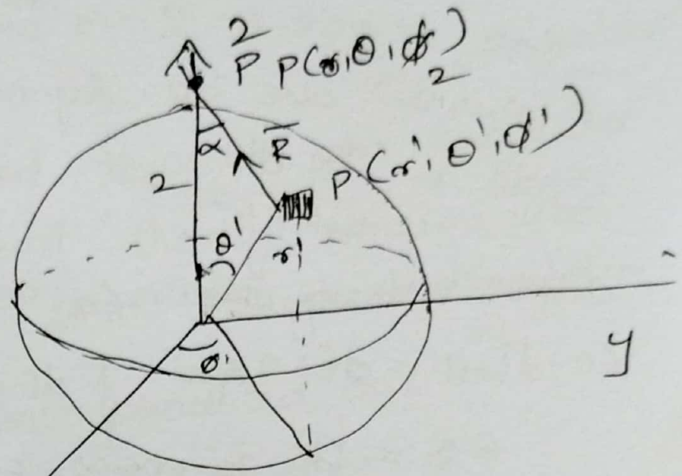
Consider a sphere of radius 'a' centered at the origin. let the volume of the sphere will be fully filled with volume charge density ρ_v (in C/m^3). The charge dQ associated with differential element volume dv is $\boxed{dQ = \rho_v . dv}$

$$Q = \int \rho_v . dv = \rho_v \int dv = \rho_v \frac{4\pi a^3}{3}$$



According to cosine rule.

$$R^2 = z^2 + r'^2 - 2zr' . \cos\theta' \quad -(1)$$

$$r'^2 = z^2 + R^2 - 2zR . \cos\alpha \quad -(2)$$

From (2)

$$2zR\cos\alpha = z^2 + R^2 - r'^2$$

$$\cos\alpha = \frac{z^2 + R^2 - r'^2}{2zR} \quad -(3)$$

From Eqn -(3)

$$2zr'\cos\theta' = R^2 - z^2 + r'^2 - R^2$$

$$\cos\theta' = \frac{z^2 + r'^2 - R^2}{2zr'} \quad -(4)$$

Differentiating Eqn (2) w.r.to θ, keeping z and r' constant

$$\cos\theta' = \frac{z^2 + r'^2 - R^2}{2zr'} \Rightarrow -\sin\theta' d\theta' = \frac{-2R . dR}{2zr_1}$$

$$\sin\theta' d\theta' = \frac{2R . dr}{2zr_1} \Rightarrow \boxed{\sin\theta' d\theta = \frac{R . dr}{zr_1}} \quad -(5)$$

Consider point P is outside of the sphere. R varies from $(z - R_1')$ to $(z + r')$, $\theta' \to 0$ to 2π

$$dv = r'^2 \sin\theta' \, dr' \, d\theta' \, d\phi'$$

$$\rightarrow d\bar{E}_A = \frac{dQ}{4\pi\varepsilon_0 R^2} \hat{a_r} \Rightarrow E_A = \frac{Q}{4\pi\varepsilon_0 R^2}$$

$$d\bar{E}_A = \frac{\rho_v . dv}{4\pi\varepsilon_0 . R^2} \hat{a_z} \Rightarrow d\bar{E}_A = \frac{\rho_v . r'^2 \sin\theta' . d\theta' \, d\phi'}{4\pi\varepsilon_0 R^2}$$

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$$\bar{e}_A = \int_{\phi=0}^{2\pi}\int_{r=0}^{a}\int_{\theta=0}^{\phi} \frac{\rho_0 \cdot d\theta}{4\pi\epsilon_0 \cdot R^2} \cdot r'^2 \sin\theta' \, dr' \, d\phi' \, d\theta' \, . \cos\alpha$$

$$= \int_{\phi=0}^{2\pi} d\phi' \int_{r=0}^{a} \int_{R=2-r'}^{2+r'} r'^2 \frac{R.dR}{2.r'} \cdot dr' \cdot \frac{2^2+R^2-r'^2}{2\,2R} \quad \text{—(1)}$$

~~con~~ since, $\bar{E}$ is outside the sphere at $P(0,0,2)$ due to symmetry volume charge is

$$d\bar{e} = \frac{\rho_0 . dv}{\epsilon_0 4\pi R^2} \, a_R \quad , \text{ where } a_R = \cos\alpha \, a_z + \sin\alpha \, a_\rho .$$

Contribution with E_x and E_y component is 0.

so, $dE_z = dE \cdot a_z = \int dE \cdot a_z . \cos\alpha \Rightarrow de_z = de . a_z . \cos\alpha$

$$E_z = \int dE . a_z . \cos\alpha . = \frac{\rho_0}{4\pi\epsilon_0} \int \frac{dv . \cos\alpha}{R^2} \quad \text{—(2)}$$

$$E_z = \frac{\rho_0 . 2\pi}{8\pi\epsilon_0 . 2^2} \int_{r=0}^{a} \int_{R=2-r'}^{2+r'} r' \left[1 + \frac{2^2 - r'^2}{R^2} \right] dR \, dr'$$

$$= \frac{\rho_0 . \pi}{4\pi\epsilon_0 2^2} \int_0^a r' \left[R - \frac{(2^2 - r'^2)}{R} \right]_{2-r'}^{2+r'} dr$$

$$= \frac{\rho_0 . \pi}{4\pi\epsilon_0 2^2} \int_0^a 4r'^2 dr' = \frac{1}{4\pi\epsilon_0} \frac{1}{2^2} \left(\frac{4}{3}\pi a^3 \rho_0 \right)$$

$$\boxed{E_z = \frac{Q}{4\pi\epsilon_0 2^2} . \bar{a}_z}.$$

$\bar{E}$ due to volume charge distribution, at $P(r, \theta, \phi)$ where point P lies on z axis is

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a_r} \Rightarrow \bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} . a_z$$

ELETROSTATICS - II

Electric Current:- Flow of charge carrier per unit time in a specific medium

- Conventional
- Conductional
- displacement.

(a) Conventional Current:-

When beam of $\bar{e}$ passed through an ~~insulator~~ conductor Conventional current. Current through vacuum tube doesn't have any air. /water Current flow through an insulator.

(b) Conductional Current:- current produced due to Flow of Conductors.

a) Current through Copper wire.
b) It obey's ohms law.

(c) Displacement current:- Current produced due to time varying electric field in dielectric is called displacement current.

a) Current produced through the capacitor when A.C voltage is applied. b) It obeys ohms law.

conductional > displacement > Conventional current.

Current density ($\bar{J}$)

Current flowing through unit area is called Current density. It is a vector quantity,

$$\boxed{\bar{J} = \frac{\Delta I}{\Delta s}} \; A/m^2.$$

$$I = \int_S \bar{J} \cdot \bar{ds} \Rightarrow \Delta I = \bar{J} \cdot \bar{ds}.$$

~~Current~~

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a) **Conventional Current density**:- Consider a filament. If there is a flow of charge of density ρ_v, at velocity v ($\vec{v}$),

$$\bar{J} = \rho_v \cdot v, \quad \rho_v = n\bar{e}$$

$$\bar{J} = \frac{\Delta I}{\Delta S} \Rightarrow I = \int_S J \cdot \Delta S$$

The Current density through any closed surface area is equal to the total current flowing through that area.

b) **Conductional Current density**:-

$$\boxed{\bar{J} = \sigma \bar{E}}$$ also known as point form of ohms law.

$\sigma = \frac{ne^2\tau}{m}$, $\tau \to$ avg. time b/w collision of $\bar{e}$.

$n \to$ no. of $\bar{e}$, $m \to$ mass,

* JOULE'S LAW

When current is flowing through any conductor heat energy is produced.

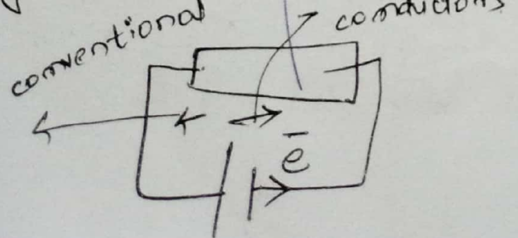
Heat Energy $= \frac{I^2Rt}{J_1}$ $J_1 \to$ Joules Constant

$$P = VI = VI\frac{(\cancel{A}l)}{\cancel{Al}} = VI \cdot \frac{(Al)}{(Al)} = \left(\frac{V}{l}\right)\left(\frac{I}{A}\right) \cdot Al$$

$$P = \bar{E} \cdot \bar{J}$$

$$\boxed{P = \int_v (E \cdot \bar{J})\, dv}$$

Therefore, Joules law states that the power dissipated per unit second is equal to volume integral of dot product of $\bar{E}$ & $\bar{J}$.



Dielectric:-

Dielectric is the material in which few free $\bar{e}$ is available for conduction and it is capable of storing energy for short duration. Ideally conductivity of dielectric is 0. Hence, perfect dielectric is known as Insulator. But practically no dielectric is ideal.

When the electric field is applied to dielectric, all +ve and -ve charges are seperated and all the -ve charges are accumulated one side and -ve charges at another. At that moment, the dipole exists from -ve to +ve.

Dielectric strength:-

The min. electric field required to occur breakdown in dielectric and to flow current in it is known as dielectric strength.

→ dielectric constant > 1, breakdown occurs, conduction takes place

⇒ $\varepsilon_r < 1$, conduction doesn't takes place. It acts as an Insulator.

Dielectric constant

We know that $\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$.

$\varepsilon = \varepsilon_0 \varepsilon_r \Rightarrow \boxed{\varepsilon_r = \varepsilon/\varepsilon_0}$ $\quad \therefore \varepsilon_r = 1 + \chi_e = \varepsilon/\varepsilon_0$

ε_r = relative permittivity / dielectric constant

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Classification of dielectrics

Linear dielectric

$D = \varepsilon_0 \bar{E}$

If $\bar{E}$ varies linearly with $\bar{D}$, then so called linear dielectric

Homogeneous dielectric

ε_0 is same (constant) at all points & independent of (x, y, z) variables.

~~Permit~~ Isotropic

Permittivity (ε_0) is constant with respect to directions, then it is Isotropic & $\bar{D}, \bar{E}$ are in same direction.

EQUATION OF CONTINUITY / CONTINUITY EQUATION

It works on "charge conservative principle".

Consider a closed surface S with a current density $\bar{J}$, then the total current I crossing the surface S is given by

$Q_{en} = \int_v \rho_v \, dv$

$$\boxed{I = \oint_S \bar{J} \cdot \overline{ds}}$$

, $I_{out} = \oint_S J \cdot ds \Rightarrow \frac{-d}{dt} \int \rho_v \, dv$

$\int_v (\nabla \cdot J) dv = -d/dt \int \rho_v \, dv$

$$\boxed{I_{out} = \frac{-d}{dt} Q_{enclosed}}$$

Hence the outward rate of flow of positive charge gets balanced by the rate of decrease of charge inside the closed surface.

This negative sign indicates decrease in charge.

This indicates no charge accumulate at any point.

For steady currents, $\boxed{\nabla \cdot \bar{J} = 0}$, rate of flow is constant

$$\boxed{\nabla \cdot \bar{J} = \frac{-d}{dt} \rho_v}$$

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In two dimensional

$\nabla^2 V = \frac{\partial^2}{\partial x^2} V + \frac{\partial^2}{\partial y^2} V$

Solution is $V(x,y) = \frac{1}{2\pi r} \oint_{circle} V \cdot dl$

(c) In three dimensional

$$\nabla^2 V = \frac{\partial^2}{\partial x^2} V + \frac{\partial^2}{\partial y^2} V + \frac{\partial^2}{\partial z^2} V$$

$$V_P(x,y,z) = \frac{1}{4\pi r^2} \oint_S V \cdot ds.$$

* Application of laplacian and poissons Equation is to calculate electric field intensity and current density in the region of interest.

CAPACITOR :-

Capacitor is formed between two conductors with an insulator or the dielectric media. The property of dielectric that it stores the electrical energy.

When potential is applied between two conductors of Capacitor, then Electric field induces in dielectric.

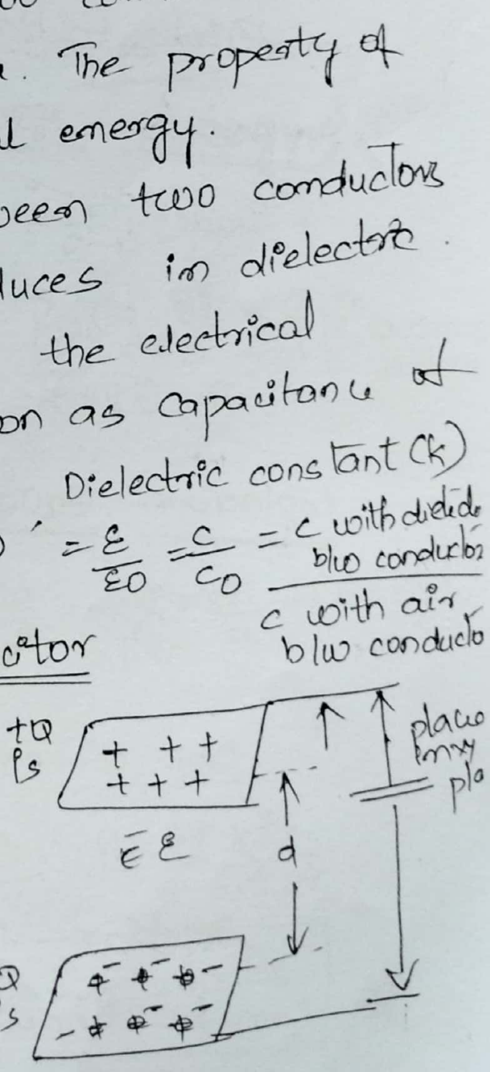
∴ Ability of dielectric that stores the electrical energy b/w two conductors is known as capacitance of a Capacitor.

$C = \frac{Q}{V}$, units : farads. (F)

Dielectric constant (k) $= \frac{\varepsilon}{\varepsilon_0} = \frac{C}{C_0} = \frac{C \text{ with dielectric b/w conductors}}{C \text{ with air b/w conductors}}$

Capacitance of parallel plate Capacitor

$C = \frac{Q}{V}$, $\rho_s = \frac{Q}{S} \Rightarrow Q = \rho_s . S$



(a) find V

$V = -\int \bar{E} \cdot d\bar{l}$

$\bar{E} = \frac{\rho_s}{2\varepsilon_0} \cdot \bar{a}_z$, due to +ve plate

RELAXATION TIME

From the equation of Continuity, $\nabla \cdot \bar{J} = \frac{-d}{dt}(\rho_v)$

$\nabla \cdot \bar{D} = \rho_v \Rightarrow \nabla \cdot \varepsilon_0 \cdot \bar{E} = \rho_v \Rightarrow \nabla \cdot \bar{E} = \rho_v / \varepsilon$ — (1)

$\bar{J} = \sigma \bar{E} \rightarrow E = \bar{J}/\sigma$ — (2), Sub Eq^n — (2) in Eq^n — (1)

$\nabla \cdot \bar{J}/\sigma = \rho_v / \varepsilon \Rightarrow \nabla \cdot \bar{J} = \sigma \rho_v / \varepsilon$ — (3)

Sub Eq^n — (3) in 4.

$-d/dt\, \rho_v = \sigma\, \rho_v / \varepsilon$

$\sigma\, \rho_v / \varepsilon + d/dt\, \rho_v = 0$ — (4)

For good conductors, relaxation time is less, that charge disappears at interior at surface area.

Soln of Eq^n — (4) is

$$\boxed{\rho_v = \rho_{v0} \cdot e^{-t/T_r}}, \quad T_r = \varepsilon / \sigma$$

Relaxation time (T_r):-

Time taken by the charge to place in the interior of the material to drop into 36.8% of its initial value

POISSON'S EQUATION

Gauss Law in point form

$\nabla \cdot \bar{D} = \rho_v \Rightarrow \nabla \cdot \varepsilon \bar{E} = \rho_v \Rightarrow \nabla \cdot \bar{E} = \rho_v / \varepsilon$, $\bar{E} = -\nabla V$

$\nabla \cdot (-\nabla V) = \rho_v / \varepsilon \Rightarrow \boxed{\nabla^2 V = -\rho_v / \varepsilon} \rightarrow$ Poissons Equation

Laplacian Equation:-

$\rho_v = 0$

$\nabla^2 V = -\rho_v / \varepsilon \Rightarrow \boxed{\nabla^2 V = 0} \rightarrow$ Laplacian Eq^n.

(i) One dimensional

$\nabla^2 V = 0$, $\nabla^2 = d^2/dx^2 + d^2/dy^2 + d^2/dz^2$

Solution is $V = mx + a$.

Can be known from Coulombs / Gauss Law, when charge distribution is given | $\bar{E} = -\nabla V$, when potential is known

when " " is not given. at particular region of interest due to -ve plate $\Rightarrow \bar{E} = -\frac{\rho_s}{2\varepsilon_0}(-\hat{a_z}) = +\frac{\rho_s}{2\varepsilon_0}$

Total E due to parallel plates = E due to +ve plate + E due to -ve plate

$= \frac{\rho_s}{2\varepsilon_0} + \frac{\rho_s}{2\varepsilon_0} = \frac{\rho_s}{\varepsilon_0}\cdot \hat{a_z}$ — (1)

$\Rightarrow V = -\int \bar{E}\cdot dl = -\int \frac{\rho_s}{\varepsilon}\cdot \hat{a_z}\cdot dz\cdot \bar{a_z} = -\int_{z=d}^{0} \frac{\rho_s}{\varepsilon}\cdot dz = -\frac{\rho_s}{\varepsilon}[0-d]$

$$\boxed{V = \frac{\rho_s}{\varepsilon}\cdot d}$$

Then $C = \frac{Q}{V} = \frac{Q}{\frac{\rho_s\cdot d}{\varepsilon}} = \frac{Q\varepsilon}{\rho_s\cdot d}$, $\rho_s = \frac{Q}{S}$

$C = \frac{Q\varepsilon}{\frac{Q}{S}\cdot d} = \frac{S\varepsilon}{d}$.

$$\boxed{C = \frac{\varepsilon S}{d}}$$

$\varepsilon \rightarrow$ permittivity of dielectric
$S \rightarrow$ surface area
$d \rightarrow$ min. seperation b/w two plate

Energy density in parallel plate capacitor:

From the concept of electrostatics, energy stored in electrostatics, $W_E = \frac{1}{2}\int_v E^2\varepsilon\cdot dv$, $E = -\frac{\rho_s}{\varepsilon}\cdot \bar{a_z}$

$W_E = \frac{1}{2}\int_v \left(\frac{\rho_s}{\varepsilon}\right)^2 \varepsilon\cdot \bar{a_z}\cdot dv = \frac{1}{2}\int_v \frac{\rho_s^2}{\varepsilon}\cdot dv = \frac{1}{2}\int \frac{\rho_s^2}{\varepsilon}$ (area x distance)

$= \frac{1}{2}\frac{\rho_s^2}{\varepsilon}\cdot d\int_s ds = \frac{1}{2}\frac{\rho_s^2 d}{\varepsilon}(S)$, $\rho_s = \frac{Q}{S}$

$W_E = \frac{1}{2}\frac{Q^2}{S^{\cancel{2}}}\cdot\frac{d}{\varepsilon}\cdot \cancel{S} = \frac{1}{2}\frac{Q^2 d}{\varepsilon S} = W_E$, $C = \frac{\varepsilon S}{d} \rightarrow S = \frac{C\cdot d}{\varepsilon}$

$W_E = \frac{1}{2}\frac{Q^2 \cancel{d}\cdot\cancel{\varepsilon}}{C\cdot \cancel{d}\cdot\cancel{\varepsilon}} = \frac{Q^2}{2C}$

$$\boxed{W_E = \frac{Q^2}{2C} \text{ Joules}}$$

(or) $C = \frac{Q}{V}$, $W_E = \frac{\cancel{Q^2}}{V\cdot 2C}$

~~$W_E = \frac{Q^2\cdot V}{2\cdot Q}$~~, $W_E = \frac{C\cancel{V}V^2}{2\cancel{C}}$ $\Rightarrow$ $\boxed{W_E = \frac{CV^2}{2} \text{ Joules}}$

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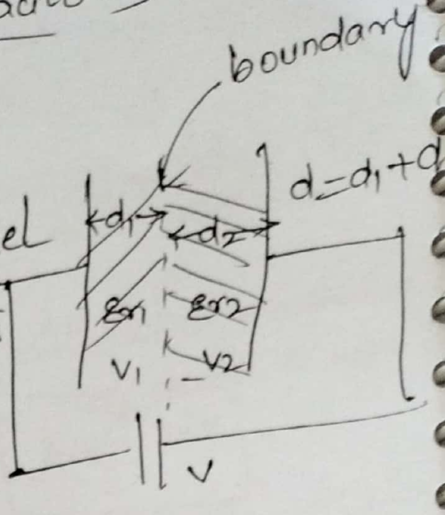
Capacitance of parallel plate capacitor —
media (Composite parallel plate capacitor)

Case:- Dielectric - Dielectric Interface

Media is in series $d = d_1 + d_2$

When boundary is ~~normal to~~ parallel to the plates the normal components of D are equal.



i.e, $D_{n1} = D_{n2} = D$

$E = \frac{V}{d} \Rightarrow V = dE$

$D = \varepsilon E \Rightarrow E_1 = \frac{D_1}{\varepsilon_1}$

$V = V_1 + V_2$

$V = E_1 \cdot d_1 + E_2 \cdot d_2$

$= \frac{D_1}{\varepsilon_1} \cdot d_1 + \frac{D_2}{\varepsilon_2} \cdot d_2 = \frac{D}{\varepsilon_1} \cdot d_1 + \frac{D}{\varepsilon_2} d_2$

$D = \frac{Q}{S}$

$V = \frac{Q}{S\varepsilon_1} \cdot d_1 + \frac{Q}{S\varepsilon_2} \cdot d_2 = \frac{Q}{S}\left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right]$

$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{S}\left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right]} = \frac{S}{d_1/\varepsilon_1 + d_2/\varepsilon_2} = \frac{S}{d_1/\varepsilon_0\varepsilon_{r1} + d_2/\varepsilon_0\varepsilon_{r2}}$

$C = \frac{\varepsilon_0 \cdot S}{d_1/\varepsilon_{r1} + d_2/\varepsilon_r}$, In case of n dielectrics

$$\boxed{C = \frac{\varepsilon_0 \cdot S}{\sum_{i=1}^{N} d_i/\varepsilon_{ri}}}$$

$C = \frac{S}{d_1/\varepsilon_1 + d_2/\varepsilon_2}$

$C = \frac{1}{d_1/S\varepsilon_1 + d_2/S\varepsilon_2} = \frac{1}{1/C_1 + 1/C_2} = \frac{C_1 C_2}{C_1 + C_2}$.

It notes that when the boundary is parallel to the plates, the resultant capacitance is nothing but the two capacitors are connected in series

$$\boxed{C = \frac{C_1 C_2}{C_1 + C_2}}$$

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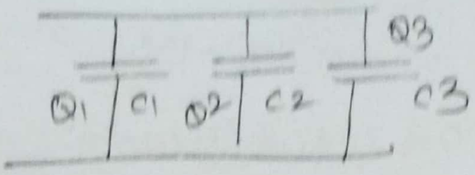
pacitance in parallel:-

$Q = Q_1 + Q_2 + Q_3$



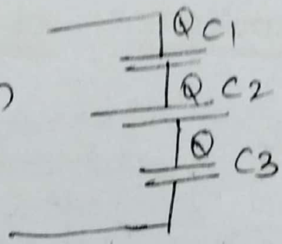
f capacitance is connected in parallel, the voltage across them s constant. $C = \frac{Q}{V} \Rightarrow Q_1 = C_1 \cdot V,\ Q_2 = C_2 \cdot V$

$Q = C_1 V + C_2 \cdot V = V(C_1 + C_2)$

$Q = C_{eq} \cdot V$ for n capacitors $\boxed{C_{eq} = C_1 + C_2 + C_3 + \dots + C_n}$

Capacitance in Series:-

For all the capacitors in series, the charge across them s constant, but voltage is different $C = \frac{Q}{V}$



$\cancel{C = C_1 + C_2 + C_3}$ $\cancel{Q = Q_1 + Q_2 + Q_3}$

$V = V_1 + V_2 + V_3$

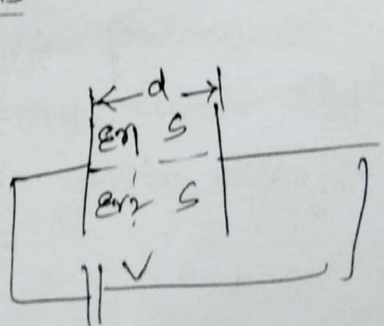
$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \dots + \frac{Q}{C_n}$

$= Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \frac{1}{C_n}\right] = Q C_{eq}$

where $\boxed{C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}}$ for n capacitors

Case:2:- Dielectric-Dielectric Intaphase normal to plates (parallel)

When the boundary is normal to the plates, the tangential components of ~~D~~ E are equal



$E_{t1} = E_{t2} = E$

When the surface area is divided, the charge distribute is divided.

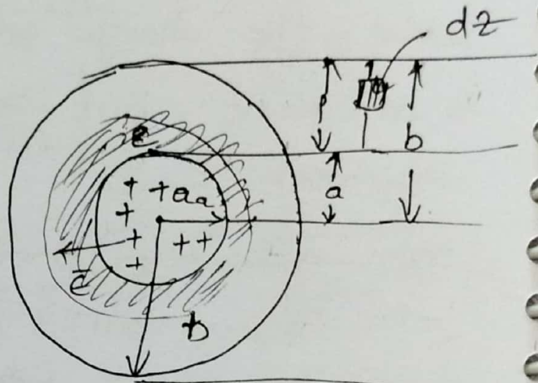
Case:2 dielectric - dielectric interphase normal to plates

$Q = Q_1 + Q_2$ $\quad D = \frac{Q}{S}$, $S = \frac{V}{d}$

$Q = D_1 S_1 + D_2 \cdot S_2$.

$Q = \frac{\varepsilon S}{d} \Rightarrow Q = \frac{\varepsilon_1 S_1}{d} + \frac{\varepsilon_2 S_2}{d}$ $\quad C = \frac{Q}{V}$

Case 3:- Capacitance of Co-axial Cable (or) coaxial Capacitor:-



Consider a coaxial cable or coaxial Capacitor. Let a = Inner radius

b = outer radius

The two concentric conductors are seperated by dielectric of permittivity ε. The length of Cable is L mts.

The inner conductor carries a charge density $+\rho_L$ C/m on its surface then equal and opposite charge density $-\rho_L$ C/m exists on outer conductor

$Q = \rho_L \cdot L$ —— (1)

Assuming cylindrical coordinate system, $\bar{E}$ will be radial from inner to outer conductor and for infinite line charge it is given by.

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon r} \cdot \bar{a}_r$$

$\bar{E}$ is directed from inner conductor to the outer conductor. The potential difference is workdone in moving charge against $\bar{E}$ from $r=b$ to $r=a$.

To find potential difference, consider $\bar{dL}$ in radial direction which is $dr\,\bar{a}_r$.

$$V = -\int_{-}^{+} \bar{E} \cdot \bar{dL} = -\int_{r=b}^{r=a} \frac{\rho_L}{2\pi r \varepsilon}\,\bar{a}_r \cdot dr\,\bar{a}_r = \frac{-\rho_L}{2\pi\varepsilon}\left[\int_{r=b}^{a} \frac{1}{r}\,dr\right]$$

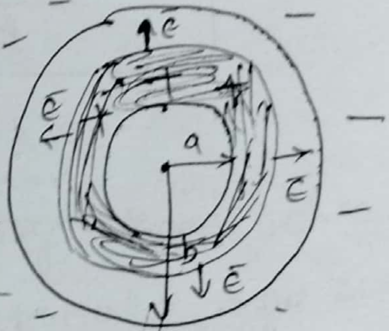
$$V = \frac{-\rho_L}{2\pi\varepsilon}\left[\log r\right]_b^a = \frac{-\rho_L}{2\pi\varepsilon}\left[\log a - \log b\right] = \frac{\rho_L}{2\pi\varepsilon}\log\left[b/a\right]$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{\rho_L}{2\pi\varepsilon}\log(b/a)} = \frac{\rho_L \cdot L}{\rho_L/2\pi\varepsilon \,\log(b/a)} = \frac{2\pi\varepsilon L}{\log(b/a)}$$

$$\boxed{C = \frac{2\pi\varepsilon L}{\log(b/a)}}$$

d) **Spherical Capacitor Capacitance:-**

Consider a spherical capacitor formed of two concentric spherical conducting shells a and b. The radius of outer sphere is b' while that of inner sphere is 'a'. Thus b>a. The region between the two spheres is with a dielectric of permittivity ε. The inner sphere is given a positive charge +Q while the outer sphere is -Q.



Consider a sphere of radius r with Gaussian surface, it can be obtained that $\bar{E}$ is in radial direction and is given by

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\,\bar{a}_r \ \text{V/m}.$$

$$V = -\int_{-}^{+} \bar{E} \cdot dL = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2}\,\bar{a}_r \cdot dl$$

$$V = -\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon r^2} \cdot a_r \cdot dr \cdot a_r \qquad dl = dr \cdot a_r$$

$$= -\int^{a} \frac{Q}{4\pi\varepsilon r^2} \cdot dr$$

$$= \frac{-Q}{4\pi\varepsilon} \int_b^a \frac{1}{r^2} \cdot dr = \frac{-Q}{4\pi\varepsilon} \left[\frac{-1}{r}\right]_b^a = \frac{-Q}{4\pi\varepsilon} \left[\frac{-1}{a} + \frac{1}{b}\right]$$

$$V = \frac{-Q}{4\pi\varepsilon} \left[\frac{1}{b} - \frac{1}{a}\right] = \frac{Q}{4\pi\varepsilon} \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]} = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]} \; F$$

$$\boxed{C = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]}}$$

$$\boxed{C = \frac{4\pi\varepsilon ab}{b-a} \; F}$$

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