9.3 z-Parameters

These are also called impedance parameters. These are obtained by expressing voltages at two ports in terms of currents at two ports. Thus, currents I_1 and I_2 are independent variables; while V_1 and V_2 are dependent variables. Thus, we have,

$$V_1 = f_1(I_1, I_2)$$

$$V_2 = f_2(I_1, I_2)$$

In equation form, above relations can be written as,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \ldots (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \ldots (2)$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

or $$[V] = [z]\,[I]$$

The individual z-parameters are defined as follows.

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$

From above equations it is clear that z_{11} and z_{22} gives the relationship between voltages and currents at port 1 - 1′ and port 2 - 2′ respectively. Such parameters are the driving point impedances. While z_{12} and z_{21} gives the relationship between voltage and currents at different ports. Such parameters are the transfer impedances.

Moreover, these parameters are defined only when the current in one of the ports is zero. This corresponds to the conditions that the one of the ports is open circuited. Hence z-parameters are named as **open circuit impedance parameters**. Hence we can write,

z_{11} = Open circuit driving point input impedance

z_{22} = Open circuit output impedance

z_{12} = Open circuit reverse transfer impedance

z_{21} = Open circuit forward transfer impedance.

The equivalent circuit of a two port network interms of open circuit impedance parameters satisfying equations (1) and (2) is as shown in the Fig. 9.3.

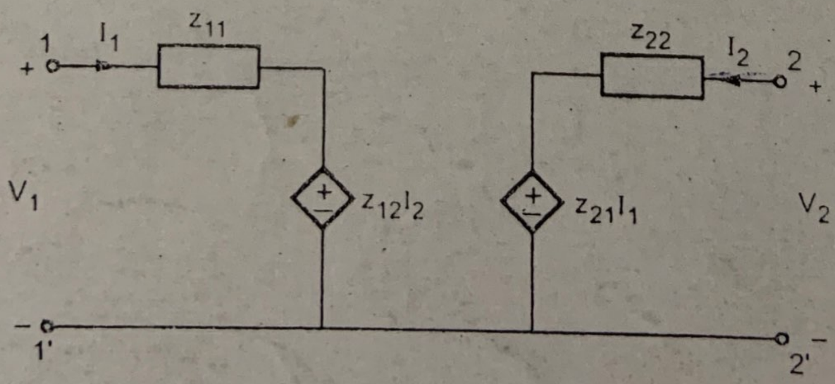


Fig. 9.3 Equivalent network of a two port network interms of z-parameters

9.4 y-Parameters

These are also called admittance parameters. These are obtained by expressing currents at two ports in terms of voltages at two ports. Thus, voltages V_1 and V_2 are independent variables, while I_1 and I_2 are dependent variables. Thus, we have

$$I_1 = f_1(V_1, V_2)$$

$$I_2 = f_2(V_1, V_2)$$

In equation form, above relations can be written as,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \text{... (1)}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \text{... (2)}$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

or

$$[I] = [y] [V]$$

The individual y-parameters are defined as follows,

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0}$$

This is called short circuit driving point input admittance.

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$$

This is called short circuit forward transfer admittance.

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

This is called short circuit reverse transfer admittance.

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$$

This is called short circuit driving point output admittance.

These parameters are defined individually only when the voltage in any one of the ports is zero. This corresponds to the condition that one of the ports is short circuited. Hence y-parameters are also called **short circuit admittance parameters.**

The equivalent circuit of a two port network interms of short circuit admittance parameters satisfying equations (1) and (2) is as shown in the Fig. 9.5.

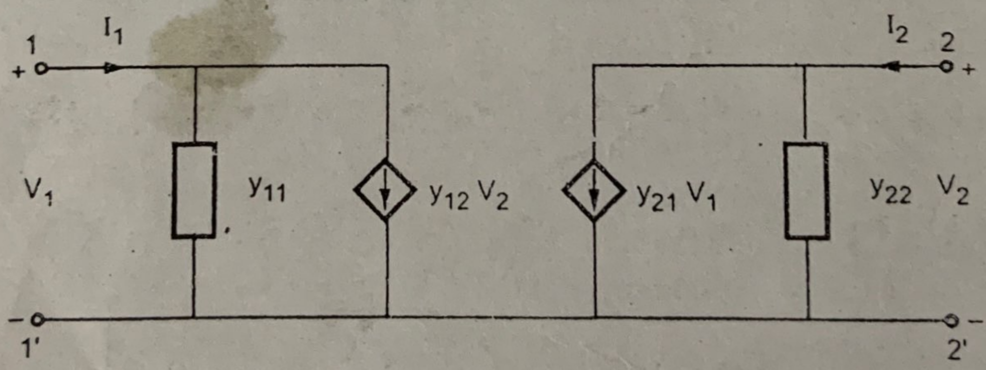


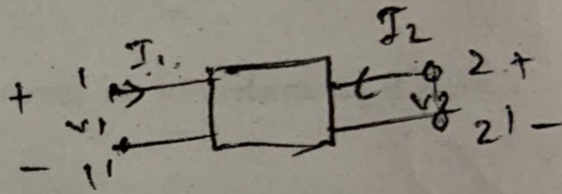
Fig. 9.5 Equivalent network of a two port network interms of y-parameters

9.5 h-Parameters

These are also called hybrid parameters. These parameters are very useful in constructing models for transistors. The transistor parameters cannot be calculated using by either short circuit admittance parameter or open circuit impedance parameter measurement. These parameters are obtained by expressing voltage at input port and the current at output port in terms of the current at the input port and the voltage at the output port. Thus, the current I_1 and voltage V_2 are independent variables ; while current I_2 and voltage V_1 are the dependent variables. Thus, we have,

$$V_1 = f_1(I_1, V_2)$$

$$I_2 = f_2(I_1, V_2)$$



In equation form, above relations can be written as,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{... (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{... (2)}$$

In matrix form, the above equations can be written as,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The individual h-parameters are defined as follows,

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2 = 0}$$

This is called short circuit input impedance.

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2 = 0}$$

This is called forward short circuit current gain.

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1 = 0}$$

This is called reverse open circuit voltage gain.

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

This is called open circuit output admittance.

All above parameters are having different units such as ohm for short circuit impedance and mho for open circuit output admittance, the name of the parameter is hybrid parameter.

The equivalent circuit of a two port network interms of hybrid parameters satisfying equations (1) and (2) is as shown in the Fig. 9.7.

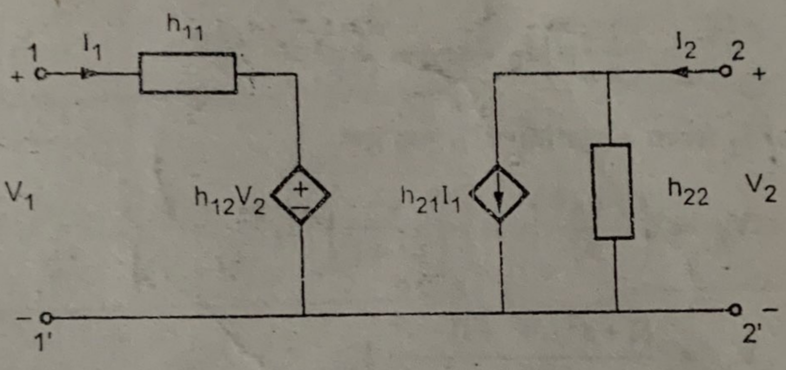


Fig. 9.7 Equivalent network of a two port network interms of h-parameters

9.6 ABCD Parameters or Transmission Parameters or Chain Parameters

These parameters are known as transmission parameters. These are generally used in the analysis of power transmission in which the input port is referred as the sending end while the output port is referred as receiving end. These are obtained by expressing voltage V_1 and current I_1 at input port interms of voltage V_2 and current I_2 at output port. Thus, voltage V_2 and current I_2 are independent variables while voltage V_1 and current I_1 are dependent variables. Thus, we have

$$V_1 = f_1(V_2, -I_2)$$

$$I_1 = f_2(V_2, -I_2)$$

Generally we have considered the currents in both the ports are entering the port and both are positive. The negative sign with I_2 indicates that, for the ABCD parameters the current I_2 is leaving the port 2.

In equation form, above relations can be written as,

$$V_1 = A V_2 + B(-I_2) \quad \text{... (1)}$$

$$I_1 = C V_2 + D(-I_2) \quad \text{... (2)}$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The individual transmission parameters are defined as follows,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}$$

This is called open circuit reverse voltage gain.

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}$$

This is called open circuit transfer admittance

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0}$$

This is called short circuit transfer impedance

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0}$$

This is called short circuit reverse current gain.

ABCD or Transmission parameters are also called **chain parameters**

9.7 Condition for Symmetry

If the impedance measured at one port is equal to the impedance measured at the other port with remaining port open circuited, the network is said to be symmetrical. The impedances at two ports can be measured as shown in Fig. 9.10 (a) and (b).

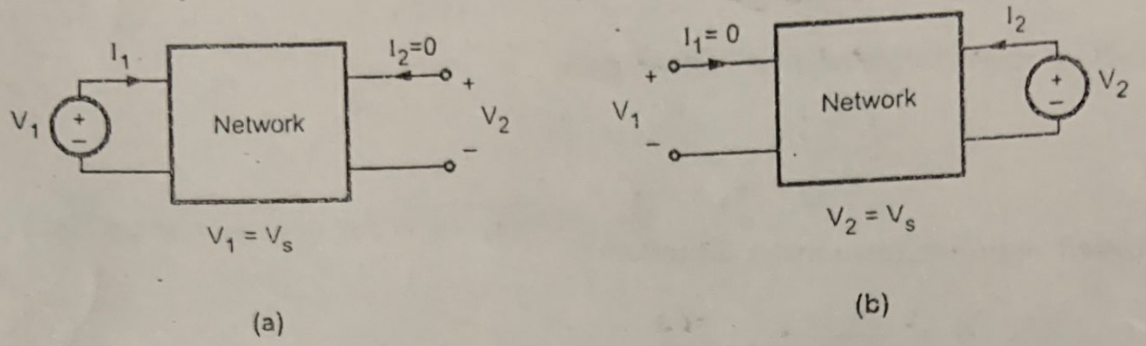


Fig. 9.10

9.7.1 Condition of Symmetry for z-Parameters

The basic equations of z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \dots (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \dots (2)$$

Consider Fig. 9.10 (a).

$$V_1 = V_s \text{ and } I_2 = 0$$

Hence equation (1) will become,

$$V_s = z_{11} I_1$$

$$\therefore \frac{V_s}{I_1} = z_{11} \quad \dots (3)$$

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Consider Fig. 9.10 (b).

$$V_2 = V_s \quad \text{and} \quad I_1 = 0$$

Equation (2) can be written as,

$$V_s = z_{22}\, I_2$$

$$\therefore \frac{V_s}{I_2} = z_{22} \qquad \dots (4)$$

For symmetrical networks, both the port impedances must be equal i.e.

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

i.e. $$z_{11} = z_{22} \qquad \text{... Condition of symmetry}$$

9.7.2 Condition of Symmetry for y-Parameters

The basic equations of y-parameters are as follows

$$I_1 = y_{11} V_1 + y_{12} V_2 \qquad \dots (1)$$

$$I_2 = y_{21} V_1 + y_{22} V_2. \qquad \dots (2)$$

Consider Fig. 9.10 (a),

$$V_1 = V_s \quad \text{and} \quad I_2 = 0$$

Equation (1) can be written as,

$$I_1 = y_{11} V_s + y_{12} V_2 \qquad \dots (3)$$

Equation (2) can be written as,

$$0 = y_{21} V_s + y_{22} V_2$$

$$\therefore V_2 = \frac{-y_{21}}{y_{22}} V_s \qquad \dots (4)$$

Substituting value of V_2 in equation (3), we have,

$$I_1 = y_{11} V_s + y_{12} \left[\frac{-y_{21}}{y_{22}}\right] V_s$$

$$\therefore I_1 = \left[y_{11} + \frac{(y_{12})(-y_{21})}{y_{22}}\right] V_s$$

$$\therefore I_1 = \left[\frac{y_{11} y_{22} - y_{12} y_{21}}{y_{22}}\right] V_s$$

$$\therefore \frac{V_s}{I_1} = \frac{y_{22}}{y_{11} y_{22} - y_{12} y_{21}} \qquad \dots (5)$$

Consider Fig. 9.10 (b),

$$V_2 = V_s \quad \text{and} \quad I_1 = 0$$

Equation (1) can be written as,

$$0 = y_{11} V_1 + y_{12} V_s \qquad \text{... (6)}$$

$$\therefore -y_{11} V_1 = y_{12} V_s$$

$$\therefore V_1 = \frac{-y_{12}}{y_{11}} V_s \qquad \text{... (7)}$$

Equation (2) can be written as,

$$I_2 = y_{21} V_1 + y_{22} V_s \qquad \text{...(8)}$$

Substituting value of V_1 in equation (8) from equation (7), we have,

$$I_2 = y_{21} \left[\frac{-y_{12}}{y_{11}}\right] V_s + y_{22} V_s$$

$$\therefore I_2 = V_s \left[y_{22} + \frac{(-y_{12})(y_{21})}{y_{11}}\right]$$

$$\therefore I_2 = V_s \left[\frac{y_{11} y_{22} - y_{12} y_{21}}{y_{11}}\right]$$

$$\therefore \frac{V_s}{I_2} = \frac{y_{11}}{y_{11} y_{22} - y_{12} y_{21}} \qquad \text{... (9)}$$

For symmetrical networks, both the port impedances must be equal i.e.

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$\therefore \frac{y_{22}}{y_{11} y_{22} - y_{12} y_{21}} = \frac{y_{11}}{y_{11} y_{22} - y_{12} y_{21}}$$

i.e. $$y_{11} = y_{22} \qquad \text{... Condition of symmetry}$$

9.7.3 Condition of Symmetry for h-Parameters

The basic equations of h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad \text{... (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad \text{... (2)}$$

Consider Fig. 9.10 (a)

$$V_1 = V_s \text{ and } I_2 = 0$$

Equation (1) can be written as,

$$V_s = h_{11} I_1 + h_{12} V_2 \qquad \text{... (3)}$$

Equation (2) can be written as,

$$0 = h_{21} I_1 + h_{22} V_2$$

$$\therefore -h_{22} V_2 = h_{21} I_1$$

$$\therefore V_2 = \frac{-h_{21}}{h_{22}} I_1 \qquad \dots (4)$$

Substituting value of V_2 in equation (3), we have,

$$V_s = h_{11} I_1 + h_{12} \left[\frac{-h_{21}}{h_{22}}\right] I_1$$

$$\therefore V_s = \left[h_{11} - \frac{h_{12} \cdot h_{21}}{h_{22}}\right] I_1$$

$$\therefore \frac{V_s}{I_1} = \frac{h_{11} h_{22} - h_{12} \cdot h_{21}}{h_{22}} \qquad \dots (5)$$

Consider Fig. 9.10 (b)

$$V_2 = V_s \quad \text{and } I_1 = 0$$

Equation (1) can be written as,

$$V_1 = h_{12} V_s \qquad \dots (6)$$

Equation (2) can be written as,

$$I_2 = h_{22} V_s$$

$$\therefore \frac{V_s}{I_2} = \frac{1}{h_{22}} \qquad \dots (7)$$

For symmetrical networks, both the port impedances must be equal i.e.

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$\therefore \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} = \frac{1}{h_{22}}$$

$$\therefore (h_{11} h_{22} - h_{12} h_{21}) = 1 \qquad \dots \text{Condition of symmetry}$$

9.7.4 Condition of Symmetry for Transmission Parameters

The basic equations for the transmission parameters are as following,

$$V_1 = A V_2 + B(-I_2) \qquad \dots (1)$$

$$I_1 = C V_2 + D(-I_2) \qquad \dots (2)$$

Consider Fig. 9.10 (a) $\quad V_1 = V_s \quad$ and $\quad I_2 = 0$

Equation (1) can be written as,

$$V_s = A V_2 \qquad \dots (3)$$

Equation (2) can be written as,

$$I_1 = C V_2 \qquad \ldots (4)$$

$$\therefore \quad V_2 = \frac{I_1}{C}$$

$$\therefore \quad V_s = A\left[\frac{I_1}{C}\right] = \frac{A}{C}[I_1]$$

$$\therefore \quad \frac{V_s}{I_1} = \frac{A}{C} \qquad \ldots (5)$$

Consider Fig. 9.10 (b)

$$V_2 = V_s \text{ and } I_1 = 0$$

Equation (1) can be written as,

$$V_1 = A \cdot V_s + B(-I_2) \qquad \ldots (6)$$

Equation (2) can be written as,

$$0 = C V_s + D(-I_2)$$

$$\therefore C V_s = D \cdot I_2$$

$$\therefore \quad \frac{V_s}{I_2} = \frac{D}{C} \qquad \ldots (7)$$

For the symmetrical network, both the port impedances must be equal.

$$\therefore \quad \frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$\therefore \quad \frac{A}{C} = \frac{D}{C}$$

$$\therefore A = D \qquad \ldots \text{Condition of symmetry}$$

9.8 Condition for Reciprocity

If the ratio of voltage at one port to the current at other port is same to the ratio if the positions of voltage and current are interchanged, then the network is said to be reciprocal. The transfer impedances can be measured as shown in Fig.9.11 (a) and (b).

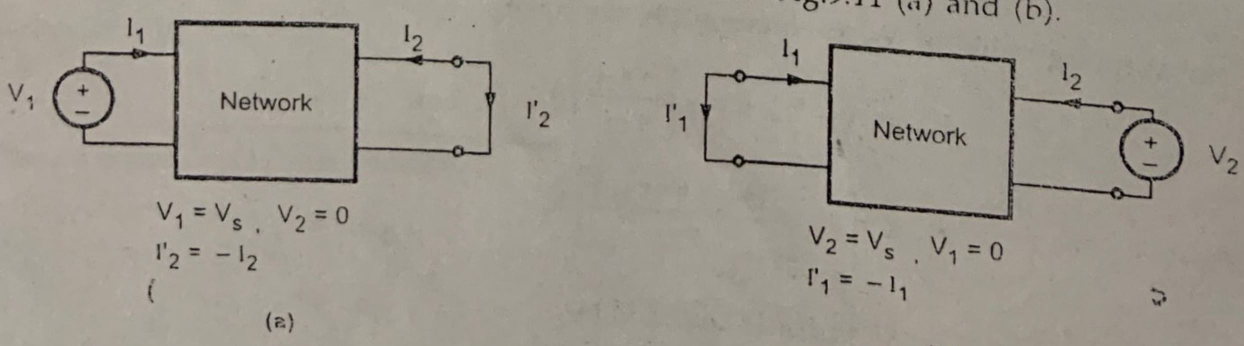

Fig. 9.11

9.8.1 Condition of Reciprocity for z-Parameters

The basic equations for z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \dots (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \dots (2)$$

Consider Fig. 9.11 (a)

$$V_1 = V_s, \quad V_2 = 0 \text{ and } I_2' = -I_2$$

Hence equation (1) can be written as,

$$V_s = z_{11} I_1 + z_{12} (-I_2') \quad \dots (3)$$

Equation (2) can be written as,

$$0 = z_{21} I_1 + z_{22} (-I_2')$$

$$\therefore -z_{21} I_1 = -z_{22} I_2'$$

$$\therefore I_1 = \frac{z_{22}}{z_{21}} I_2' \quad \dots (4)$$

Substituting value of I_1 in equation (3), we get,

$$V_s = z_{11} \left[\frac{z_{22}}{z_{21}} I_2'\right] - z_{12} I_2'$$

$$\therefore V_s = \left[\frac{z_{11} z_{22} - z_{12} z_{21}}{z_{21}}\right] I_2'$$

$$\frac{V_s}{I_2'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{21}} \quad \dots (5)$$

Consider the Fig. 9.11 (b)

$$V_2 = V_s, \quad V_1 = 0 \text{ and } I_1' = -I_1$$

Equation (1) can be written as,

$$0 = z_{11} (-I_1') + z_{12} I_2$$

$$\therefore z_{11} I_1' = z_{12} I_2$$

$$\therefore I_2 = \frac{z_{11}}{z_{12}} I_1' \quad \dots (6)$$

Equation (2) can be written as,

$$V_s = z_{21} (-I_1') + z_{22} I_2 \quad \dots (7)$$

Substituting value of I_2 in equation (7), we have,

$$V_s = -z_{21} I_1' + z_{22} \left[\frac{z_{11}}{z_{12}}\right] I_1'$$

$$\therefore \quad V_s = \frac{z_{22} z_{11} - z_{12} z_{21}}{z_{12}} I_1'$$

$$\frac{V_s}{I_1'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{12}} \quad \text{... (8)}$$

For condition of reciprocity,

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

$$\therefore \quad \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{12}} = \frac{z_{11} z_{22} - z_{21} z_{12}}{z_{21}}$$

$$\therefore \quad z_{12} = z_{21} \quad \text{... Condition of reciprocity}$$

9.8.2 Condition of Reciprocity for y-Parameters

The basic equations of y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \text{...(1)}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \text{...(2)}$$

Consider Fig. 9.11 (a)

$$V_1 = V_s, \quad V_2 = 0 \text{ and } I_2' = -I_2$$

Equation (1) can be written as,

$$I_1 = y_{11} V_s \quad \text{... (3)}$$

Equation (2) can be written as,

$$-I_2' = y_{21} V_s \quad \text{... (4)}$$

$$\therefore \quad \frac{V_s}{I_2'} = -\frac{1}{y_{21}}$$

Consider Fig. 9.11 (b)

$$V_2 = V_s, \quad V_1 = 0 \text{ and } I_1' = -I_1$$

Equation (1) can be written as,

$$-I_1' = y_{12} V_s \quad \text{... (5)}$$

$$\therefore \quad \frac{V_s}{I_1'} = -\frac{1}{y_{12}}$$

The network is said to be reciprocal if,

$$\boxed{\frac{V_s}{I_1'} = \frac{V_s}{I_2'}}$$

$$\therefore -\frac{1}{y_{12}} = -\frac{1}{y_{21}}$$

$$\therefore y_{12} = y_{21} \qquad \text{... Condition of reciprocity}$$

9.8.3 Condition of Reciprocity for h-Parameters

The basic equations of h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad \text{...(1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad \text{... (2)}$$

Consider Fig. 9.11 (a)

$$V_1 = V_s, \; V_2 = 0, \text{ and } I_2' = -I_2$$

Equation (1) can be written as,

$$V_s = h_{11} (I_1) \qquad \text{... (3)}$$

Equation (2) can be written as,

$$-I_2' = h_{21} I_1$$

$$\therefore I_1 = \frac{-1}{h_{21}} \cdot I_2' \qquad \text{... (4)}$$

Substituting value of I_1 in equation (3),

$$V_s = h_{11} \left[\frac{-1}{h_{21}} I_2' \right]$$

$$\therefore \frac{V_s}{I_2'} = \frac{-h_{11}}{h_{21}}$$

Consider Fig. 9.11 (b)

$$V_2 = V_s, \; V_1 = 0 \text{ and } I_1' = -I_1$$

Equation (1) can be written as,

$$0 = h_{11} (-I_1') + h_{12} V_s$$

$$\therefore h_{11} I_1' = h_{12} V_s$$

$$\therefore \frac{V_s}{I_1'} = \frac{h_{11}}{h_{12}} \qquad \text{... (5)}$$

The network is said to be reciprocal if,

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

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$$\therefore \frac{h_{11}}{h_{12}} = \frac{-h_{11}}{h_{21}}$$

$$\therefore h_{12} = -h_{21} \quad \text{... Condition of reciprocity}$$

9.8.4 Condition of Reciprocity for ABCD Parameters

The basic equations for ABCD parameters are as follows,

$$V_1 = A V_2 + B(-I_2) \quad \text{... (1)}$$

$$I_1 = C V_2 + D(-I_2) \quad \text{... (2)}$$

Consider Fig. 9.11 (a)

$$V_1 = V_s, V_2 = 0 \text{ and } I_2' = +I_2$$

Equation (1) can be written as,

$$V_s = +B I_2'$$

$$\therefore \frac{V_s}{I_2'} = B \quad \text{... (3)}$$

Consider Fig. 9.11 (b),

$$V_2 = V_s, V_1 = 0 \text{ and } I_1' = -I_1$$

Equation (1) can be written as,

$$0 = A V_s - B I_2 \quad \text{... (4)}$$

Equation (2) can be writen as,

$$-I_1' = C V_s - D I_2 \quad \text{... (5)}$$

From equation (4) we have,

$$-A V_s = -B I_2$$

$$\therefore I_2 = \frac{A}{B} V_s \quad \text{... (6)}$$

Substituting this value in equation (5) we have,

$$-I_1' = C V_s - D\left[\frac{A}{B}\right] V_s$$

$$\therefore -I_1' = \frac{BC - AD}{B} V_s$$

$$\therefore \frac{V_s}{I_1'} = \frac{B}{AD - BC}$$

The network is said to be reciprocal if,

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$

$$\therefore \frac{B}{AD-BC} = B$$

$$\therefore AD-BC = 1 \qquad \text{... Condition of reciprocity}$$

9.9 Interrelationships between the Parameters

We can express any parameter interms of other parameter. In this section we will discuss all such interrelationships between the parameters.

9.9.1 z-Parameters interms of other Parameters

The equations for z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \qquad \text{... (A)}$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \qquad \text{... (B)}$$

[A] Interms of y-Parameters

The equations for y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \qquad \text{... (1)}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \qquad \text{... (2)}$$

Writing equations in matrix form, we have,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Solving above equations, using Cramer's rule for V_1 and V_2. We can write,

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22} I_1 - y_{12} I_2}{y_{11} y_{22} - y_{12} y_{21}}$$

$$= \frac{y_{22}}{y_{11} y_{22} - y_{12} y_{21}} \cdot I_1 - \frac{y_{12}}{y_{11} y_{22} - y_{12} y_{21}} I_2 \qquad \text{... (3)}$$

Similarly,

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{11} I_2 - y_{21} I_1}{y_{11} y_{22} - y_{12} y_{21}}$$

$$= \frac{-y_{21}}{y_{11}\,y_{22}-y_{12}\,y_{21}} I_1 + \frac{y_{11}}{y_{11}\,y_{22}-y_{12}\,y_{21}} I_2 \qquad \text{... (4)}$$

Let $$y_{11}\,y_{22} - y_{12}\,y_{21} = \Delta y$$

Rewriting equations (3) and (4), we have,

$$V_1 = \left[\frac{y_{22}}{\Delta y}\right] I_1 + \left[\frac{-y_{12}}{\Delta y}\right] I_2 \qquad \text{... (5)}$$

and $$V_2 = \left[\frac{-y_{21}}{\Delta y}\right] I_1 + \left[\frac{y_{11}}{\Delta y}\right] I_2 \qquad \text{... (6)}$$

Comparing equations (5) and (6) to equations (A) and (B), we have

$$z_{11} = \frac{y_{22}}{\Delta y} \qquad z_{21} = \frac{-y_{21}}{\Delta y}$$

$$z_{12} = \frac{-y_{12}}{\Delta y} \qquad z_{22} = \frac{y_{11}}{\Delta y}$$

In the matrix form z-parameters can be written as,

$$[z] = \begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$$

[B] Interms of h-Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11}\,I_1 + h_{12}\,V_2 \qquad \text{... (1)}$$

$$I_2 = h_{21}\,I_1 + h_{22}\,V_2 \qquad \text{... (2)}$$

From equation (2) we can write,

$$h_{22}\,V_2 = -\,h_{21}\,I_1 + I_2$$

$$\therefore\; V_2 = \left[\frac{-h_{21}}{h_{22}}\right] I_1 + \left[\frac{1}{h_{22}}\right] I_2 \qquad \text{... (3)}$$

Substituting value of V_2 in equation (1) we have,

$$V_1 = h_{11}\,I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$\therefore\; V_1 = \left[h_{11} - \frac{h_{12}\,h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\therefore V_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] I_1 + \left[\frac{h_{12}}{h_{22}}\right] I_2 \quad \ldots (4)$$

Comparing equations (4) and (3) with equations (A) and (B) respectively, we have,

$$z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} \qquad z_{21} = \frac{-h_{21}}{h_{22}}$$

$$z_{12} = \frac{h_{12}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}}$$

In the matrix form z-parameters can be written as,

$$[z] = \begin{bmatrix} \dfrac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} & \dfrac{h_{12}}{h_{22}} \\ \dfrac{-h_{21}}{h_{22}} & \dfrac{1}{h_{22}} \end{bmatrix}$$

[C] Interms of Transmission (ABCD) Parameters

The equations for transmission parameters are as follows,

$$V_1 = A\,V_2 + B\,(-I_2) \quad \ldots (1)$$

$$I_1 = C\,V_2 + D\,(-I_2) \quad \ldots (2)$$

We can rewrite equation (2) as follows,

$$C\,V_2 = I_1 + D\,I_2$$

$$\therefore V_2 = \left[\frac{1}{C}\right] I_1 + \left[\frac{D}{C}\right] I_2 \quad \ldots (3)$$

Substituting value of V_2 in equation (1),we have,

$$V_1 = A\left[\left(\frac{1}{C}\right) I_1 + \left(\frac{D}{C}\right) I_2\right] - B\cdot I_2$$

$$\therefore V_1 = \left[\frac{A}{C}\right] I_1 + \left[\frac{AD}{C} - B\right] . I_2$$

$$\therefore V_1 = \left[\frac{A}{C}\right] I_1 + \left[\frac{AD - BC}{C}\right] I_2 \quad \ldots (4)$$

Comparing equations (4) and (3) with the equations (A) and (B) respectively, we have,

$$z_{11} = \frac{A}{C} \qquad z_{21} = \frac{1}{C}$$

$$z_{12} = \frac{AD - BC}{C} \qquad z_{22} = \frac{D}{C}$$

In the matrix form, the z-parameters can be written as,

$$[z] = \begin{bmatrix} \frac{A}{C} & \frac{AD-BC}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

9.9.2 y-Parameters interms of other Parameters

The equations for y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \qquad \text{... (A)}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \qquad \text{... (B)}$$

[A] Interms of z-parameters

The equations for z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \qquad \text{... (1)}$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \qquad \text{... (2)}$$

Writing equations in matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving above equations using Cramer's rule for I_1 and I_2, we can write.

$$I_1 = \frac{\begin{vmatrix} V_1 & z_{12} \\ V_2 & z_{22} \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{z_{22} \cdot V_1 - z_{12} \cdot V_2}{z_{11} \cdot z_{22} - z_{12} \cdot z_{21}}$$

$$= \frac{z_{22}}{z_{11}\ z_{22} - z_{12}\ z_{22}} \cdot V_1 + \frac{-z_{12}}{z_{11}\ z_{22} - z_{12}\ z_{21}} \cdot V_2 \qquad \text{...(3)}$$

Similarly,

$$I_2 = \frac{\begin{vmatrix} z_{11} & V_1 \\ z_{21} & V_2 \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{z_{11} \cdot V_2 - z_{21} \cdot V_1}{z_{11} \cdot z_{22} - z_{12} \cdot z_{21}}$$

$$= \frac{-z_{21}}{z_{11} \cdot z_{22} - z_{12}\ z_{21}} \cdot V_1 + \frac{z_{11}}{z_{11}\ z_{22} - z_{12}\ z_{21}} \cdot V_2 \qquad \text{... (4)}$$

Let $\quad \Delta z = z_{11} \cdot z_{22} - z_{12} \cdot z_{21}$

Rewriting equations (3) and (4),

$$I_1 = \left[\frac{z_{22}}{\Delta z}\right] V_1 + \left[\frac{-z_{12}}{\Delta z}\right] V_2 \qquad \ldots (5)$$

and

$$I_2 = \left[\frac{-z_{21}}{\Delta z}\right] V_1 + \left[\frac{z_{11}}{\Delta z}\right] V_2 \qquad \ldots (6)$$

Comparing equations (5) and (6) with the equations (A) and (B) respectively, we have,

$$y_{11} = \frac{z_{22}}{\Delta z} \qquad y_{21} = \frac{-z_{21}}{\Delta z}$$

$$y_{12} = \frac{-z_{12}}{\Delta z} \qquad y_{22} = \frac{z_{11}}{\Delta z}$$

In the matrix form, y-parameters can be written as,

$$[y] = \begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$$

[B] Interms of h-Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad \ldots (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad \ldots (2)$$

We can rewrite equation (1) as follows,

$$h_{11} I_1 = V_1 - h_{12} V_2$$

$$\therefore \; I_1 = \left[\frac{1}{h_{11}}\right] V_1 + \left[\frac{-h_{12}}{h_{11}}\right] V_2 \qquad \ldots (3)$$

Substituting equation (3) in equation (2), we have,

$$I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22} V_2$$

$$\therefore \; I_2 = \left[\frac{h_{21}}{h_{11}}\right] V_1 + \left[h_{22} - \frac{h_{12}\, h_{21}}{h_{11}}\right] V_2$$

$$\therefore \; I_2 = \left[\frac{h_{21}}{h_{11}}\right] V_1 + \left[\frac{h_{11}\, h_{22} - h_{12}\, h_{21}}{h_{11}}\right] V_2 \qquad \ldots (4)$$

Comparing equations (3) and (4) with the equations (A) and (B) respectively, we have,

$$y_{11} = \frac{1}{h_{11}} \qquad y_{21} = \frac{h_{21}}{h_{11}}$$

$$y_{12} = \frac{-h_{12}}{h_{11}} \qquad y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}$$

In the matrix form y-parameters can be written as,

$$[y] = \begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} \end{bmatrix}$$

[C] Interms of Transmission (ABCD) Parameters

The equations for transmission parameters are as follows,

$$V_1 = A\,V_2 + B\,(-I_2) \qquad \ldots (1)$$

$$I_1 = C\,V_2 + D\,(-I_2) \qquad \ldots (2)$$

We can rewrite equation (1) as follows,

$$-B \cdot I_2 = V_1 - A\,V_2$$

$$\therefore \; I_2 = \left[-\frac{1}{B}\right] V_1 + \left[\frac{A}{B}\right] V_2 \qquad \ldots (3)$$

Substituting value of I_2 in equation (2), we have,

$$I_1 = C \cdot V_2 + D\left[\frac{+1}{B} V_1 - \frac{A}{B}\right] V_2$$

$$\therefore \; I_1 = \left[\frac{D}{B}\right] V_1 + \left[C - \frac{AD}{B}\right] V_2$$

$$\therefore \; I_1 = \left[\frac{D}{B}\right] V_1 + \left[\frac{BC - AD}{B}\right] V_2 \qquad \ldots (4)$$

Comparing equations (4) and (3) with the equations (A) and (B) respectively, we have,

$$y_{11} = \frac{D}{B} \qquad y_{21} = \frac{-1}{B}$$

$$y_{12} = \frac{BC - AD}{B} \qquad y_{22} = \frac{A}{B}$$

In the matrix form, y-parameters can be written as,

$$[y] = \begin{bmatrix} \frac{D}{B} & \frac{BC - AD}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$$

9.9.3 h-Parameters interms of other Parameters

[A] Interms of z-Parameters

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

The equations of z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \ldots (1) \qquad \text{and} \qquad V_2 = z_{21} I_1 + z_{22} I_2 \quad \ldots (2)$$

Rewriting equation (2),

$V_1 = z_{11} I_1 + z_{12} \left[\frac{-z_{21}}{z_{22}}\right] I_1 + \frac{z_{12}}{z_{22}} V_2$

$$I_2 = \left[\frac{-z_{21}}{z_{22}}\right] I_1 + \left[\frac{1}{z_{22}}\right] V_2 \quad \ldots (3)$$

$\frac{[z_{11} z_{22} - z_{12} z_{21}]}{z_{22}} I_1 + \ldots$

Substituting value of I_2 in equation (1), we have,

$$V_1 = \left[\frac{\Delta z}{z_{22}}\right] + \left[\frac{z_{12}}{z_{22}}\right] V_2 \quad \ldots (4)$$

Comparing equations (3) and (4) with equations for h-parameters, we can write,

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$$

[B] Interms of y-parameters

The equations of y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \ldots (1) \qquad \text{and} \qquad I_2 = y_{21} V_1 + y_{22} V_2 \quad \ldots (2)$$

Substituting value of V_2 from equation (2) in equation (1) and rearranging terms, we have,

$$V_1 = \left[\frac{1}{y_{11}}\right] I_1 + \left[\frac{-y_{12}}{y_{11}}\right] V_2 \quad \ldots (3)$$

Similarly substituting value of V_1 from equation (1) in equation (2) and rearranging terms, we have,

$$I_2 = \left[\frac{y_{21}}{y_{11}}\right] I_1 + \left[\frac{\Delta y}{y_{11}}\right] V_2 \quad \ldots (4)$$

Comparing equations (3) and (4) with equations for h-parameters, we have,

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$$

[C] Interms of Transmission Parameters

The equations for transmission parameters are as follows,

$$V_1 = A\,V_2 + B\,(-I_2) \quad \ldots(1) \qquad \text{and} \qquad I_1 = CV_2 + D\,(-I_2) \quad \ldots(2)$$

Rewriting equation (2) as follows,

$$-I_2 = \left[-\frac{1}{D}\right] I_1 + \left[-\frac{C}{D}\right] V_2 \quad \ldots(3)$$

Substituting value of I_2 in equation (1) and rearranging terms, we have,

$$V_1 = \left[\frac{B}{D}\right] I_1 + \left[-\frac{AD-BC}{D}\right] V_2 \quad \ldots(4)$$

Comparing equation (3) and (4) with equations for h-parameters, we can write,

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

9.9.4 Transmission (ABCD) Parameters interms of other Parameters

The equations for transmission parameters are as follows,

$$V_1 = A\,V_2 + B\,(-I_2) \quad \ldots(A)$$

$$I_1 = C\,V_2 + D\,(-I_2) \quad \ldots(B)$$

[A] Interms of z-Parameters

The equations for z-parameters are as follows,

$$V_1 = z_{11}\,I_1 + z_{12}\,I_2 \quad \ldots(1)$$

$$V_2 = z_{21}\,I_1 + z_{22}\,I_2 \quad \ldots(2)$$

We can rewrite equation (2) as follows,

$$-z_{21}\,I_1 = -V_2 + z_{22}\,I_2$$

$$\therefore I_1 = \left[+\frac{1}{z_{21}}\right] V_2 + \left[\frac{-z_{22}}{z_{21}}\right] I_2 \quad \ldots(3)$$

Substituting value of I_1 in equation (1), we have,

$$V_1 = z_{11}\left[\left(\frac{1}{z_{21}}\right) V_2 + \left(\frac{-z_{22}}{z_{21}}\right) I_2\right] + z_{12}\,I_2$$

$$\therefore V_1 = \left[\frac{z_{11}}{z_{21}}\right] V_2 + \left[\frac{-z_{11} z_{22}}{z_{21}} + z_{12}\right] I_2$$

$$\therefore V_1 = \left[\frac{z_{11}}{z_{21}}\right] V_2 + \left[\frac{+z_{11}z_{22} - z_{12}z_{21}}{z_{21}}\right](-I_2) \qquad \ldots(4)$$

Comparing equations (4) and (3) with equations (A) and (B) respectively, we have,

$$A = \frac{z_{11}}{z_{21}} \qquad C = \frac{1}{z_{21}}$$

$$B = \frac{z_{11}z_{22} - z_{21}z_{12}}{z_{21}} \qquad D = \frac{z_{22}}{z_{21}}$$

In the matrix form, transmission parameters can be written as,

$$[T] = \begin{bmatrix} \dfrac{z_{11}}{z_{21}} & \dfrac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}} \\ \dfrac{1}{z_{21}} & \dfrac{z_{22}}{z_{21}} \end{bmatrix}$$

[B] Interms of y-Parameters

The equations for y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \qquad \ldots(1)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \qquad \ldots(2)$$

We can rewrite equation (2) as follows,

$$y_{21} V_1 = -y_{22} V_2 + I_2$$

$$\therefore V_1 = \left[\frac{-y_{22}}{y_{21}}\right] V_2 + \left[\frac{-1}{y_{21}}\right](-I_2) \qquad \ldots(3)$$

Substituting value of V_1 in equation (1), we have,

$$I_1 = y_{11}\left[\frac{-y_{22}}{y_{21}} V_2 + \frac{1}{y_{21}} I_2\right] + y_{12} V_2$$

$$\therefore I_1 = \left[y_{12} - \frac{y_{11}y_{22}}{y_{21}}\right] V_2 + \left[\frac{-1}{y_{21}}\right](-I_2)$$

$$\therefore I_1 = \left[\frac{y_{12}y_{21} - y_{11}y_{22}}{y_{21}}\right] V_2 + \left[\frac{-y_{11}}{y_{21}}\right](-I_2) \qquad \ldots(4)$$

Comparing equations (3) and (4) with the equations (A) and (B), we have,

$$A = \frac{-y_{22}}{y_{21}} \qquad C = \frac{y_{12}y_{21} - y_{11}y_{22}}{y_{21}}$$

$$B = \frac{-1}{y_{21}} \qquad D = \frac{-y_{11}}{y_{21}}$$

In the matrix form, transmission parameters can be written as,

$$[T] = \begin{bmatrix} \dfrac{-y_{22}}{y_{21}} & \dfrac{-1}{y_{21}} \\ \dfrac{y_{12}y_{21} - y_{11}y_{22}}{y_{21}} & \dfrac{-y_{11}}{y_{21}} \end{bmatrix}$$

[C] Interms of h-Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad \ldots(1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad \ldots(2)$$

We can rewrite equation (2) as follows,

$$h_{21} I_1 = I_2 - h_{22} V_2$$

$$\therefore I_1 = \left[\frac{-h_{22}}{h_{21}}\right] V_2 + \left[\frac{-1}{h_{21}}\right] (-I_2) \qquad \ldots(3)$$

Substituting value of I_1 in equation (1) we have,

$$V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2\right] + h_{12} V_2$$

$$\therefore V_1 = \left[\frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}\right] V_2 + \left[\frac{-h_{11}}{h_{21}}\right] (-I_2) \qquad \ldots(4)$$

Comparing equations (4) and (3) with the equations (A) and (B) respectively, we have,

$$A = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}} \qquad B = \frac{-h_{11}}{h_{21}}$$

$$C = \frac{-h_{22}}{h_{21}} \qquad C = \frac{-1}{h_{21}}$$

In the matrix form, transmission parameters can be written as,

$$[T] = \begin{bmatrix} \dfrac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}} & \dfrac{-h_{11}}{h_{21}} \\ \dfrac{-h_{22}}{h_{21}} & \dfrac{-1}{h_{21}} \end{bmatrix}$$

Table 9.1 gives interrelationships between all the parameters. Also Table 9.2 gives the conditions for symmetry and reciprocity.

	[z]	[y]	[h]	[T]
[z]	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta y} & -\frac{y_{12}}{\Delta y} \\ -\frac{y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
[y]	$\begin{bmatrix} \frac{z_{22}}{\Delta z} & -\frac{z_{12}}{\Delta z} \\ -\frac{z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{21}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$
[T]	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ \frac{\Delta y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Table 9.1 Interrelation between various parameters

$\Delta z = z_{11} z_{22} - z_{12} z_{21}$; $\Delta y = y_{11}y_{22} - y_{12} y_{21}$. $\Delta h = h_{11} h_{22} - h_{12} h_{21}$;
$\Delta T = AD - BC$

Parameter	Condition for reciprocity of network	Condition for symmetry of network
z	$z_{12} = z_{21}$	$z_{11} = z_{22}$
y	$y_{12} = y_{21}$	$y_{11} = y_{22}$
h	$h_{12} = -h_{21}$	$h_{11} h_{22} - h_{12} h_{21} = 1$
T	$AD - BC = 1$	$A = D$

9.10 Interconnection of Two Ports

In this section, we shall discuss various types of interconnections of two ports such as series connection of two ports, parallel connection of two ports, cascade connection of two ports etc.

9.10.1 Series Connection of Two Ports

Consider that two networks N′ and N″ are connected in series as shown in Fig. 9.13(a). When two ports are connected in series, we can add their z-parameters to get overall z-parameter of the overall series connection.

Let the z-parameters of network N′ be z'_{11}, z'_{12}, z'_{21}, z'_{22}. Let the z-parameters of network N″ be z''_{11}, z''_{12}, z''_{21}, z''_{22}. Let the overall z-parameters of series connection be z_{11}, z_{12}, z_{21} and z_{22}.

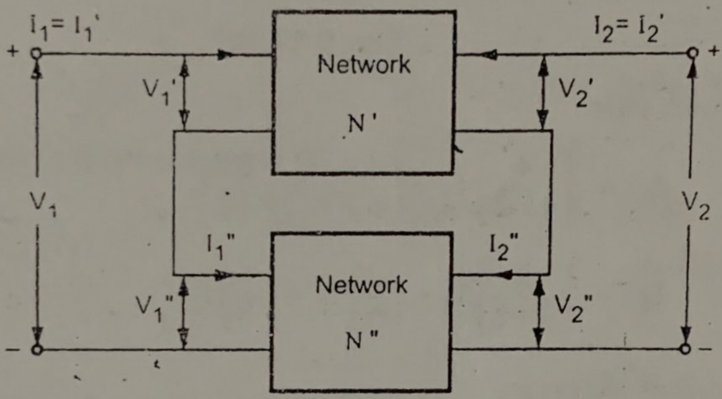

Fig. 9.13 (a) Series connection of 2 port networks

For series connection we have,

$$V_1 = V_1' + V_1'' \quad \ldots (1)$$

$$V_2 = V_2' + V_2'' \quad \ldots (2)$$

And

$$I_1 = I_1' = I_1'' \quad \ldots (3)$$

$$I_2 = I_2' = I_2'' \quad \ldots (4)$$

For network N′ , z-parameter equations are,

$$V_1' = z'_{11} I_1' + z'_{12} I_2'$$

$$V_2' = z'_{21} I_1' + z'_{22} I_2'$$

For network N", z-parameter equations are,

$$V_1'' = z_{11}'' I_1'' + z_{12}'' I_2''$$

$$V_2'' = z_{21}'' I_1'' + z_{22}'' I_2''$$

From equations (1), (2) and (3), (4) we can write,

$$V_1 = (z_{11}' + z_{11}'') I_1 + (z_{12}' + z_{12}'') I_2$$

$$V_2 = (z_{21}' + z_{21}'') I_1 + (z_{22}' + z_{22}'') I_2$$

In the matrix form, above equations can be written as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (z_{11}' + z_{11}'') & (z_{12}' + z_{12}'') \\ (z_{21}' + z_{21}'') & (z_{22}' + z_{22}'') \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Thus, overall z-parameters are,

$$[z] = \begin{bmatrix} z_{11}' + z_{11}'' & z_{12}' + z_{12}'' \\ z_{21}' + z_{21}'' & z_{22}' + z_{22}'' \end{bmatrix}$$

Hence, the z-parameters of the series connection are the sum of z-parameters of the individual network connected in series.

Example 9.7 : *Find z-parameters by using interconnection relations.*

Solution : The network shown in the Fig. 9.14 can be realized as series connection of the

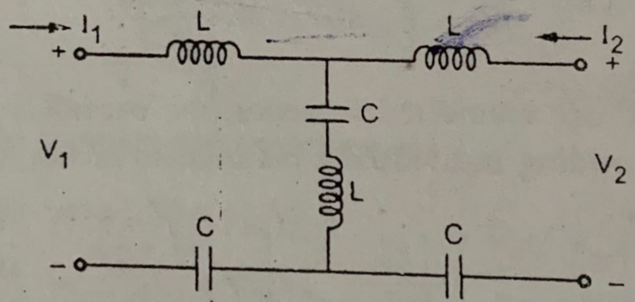

Fig. 9.14

networks N' and N" as shown in the Fig. 9.14 (a).

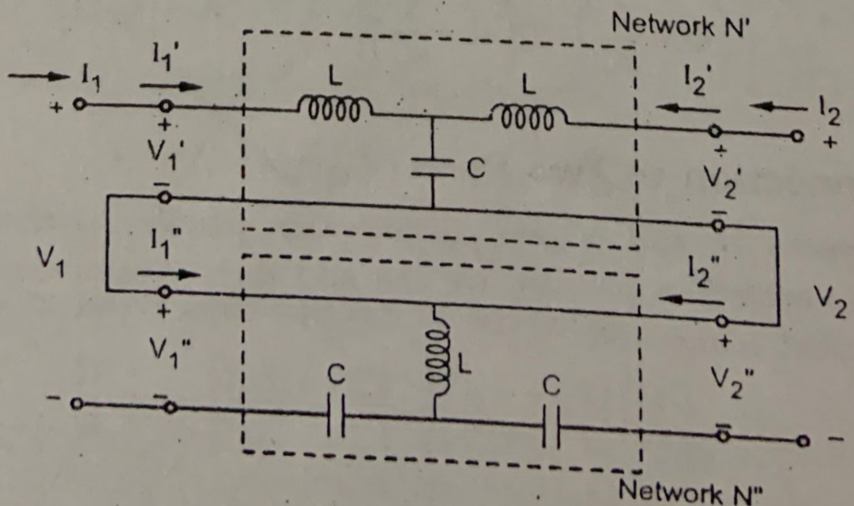

Fig. 9.14 (a)

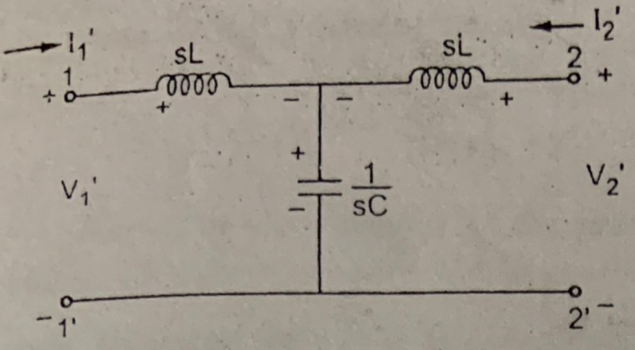

Fig. 9.14 (b)

(A) To determine z-parameters of N′ :

Consider the network N transformed into s-domain as shown in the Fig. 9.14 (b).

(1) To determine z_{11}' and z_{21}', open circuit terminals 2 - 2′ by making $I_2' = 0$.

Refer Fig. 9.14 (b) with $I_2' = 0$.

Applying KVL to the input side,

$$V_1' = I_1'\left[sL + \frac{1}{sC}\right]$$

$$\therefore \quad \frac{V_1'}{I_1'} = \left(sL + \frac{1}{sC}\right)\Omega \qquad \text{... (1)}$$

$$\therefore \text{ By definition, } z_{11}' = \frac{V_1'}{I_1'} = sL + \frac{1}{sC}$$

The voltage V_2' can be expressed in terms of I_1' as follows,

$$V_2' = \left(\frac{1}{sC}\right)I_1'$$

$$\therefore \quad \frac{V_2'}{I_1'} = \frac{1}{sC} \qquad \text{... (2)}$$

$$\therefore \text{By definition, } z_{21}' = \frac{V_2'}{I_1'} = \frac{1}{sC}\,\Omega$$

(2) To determine z_{12}' and z_{22}', open circuit terminals 1 - 1′ by making $I_1' = 0$.

Refer Fig. 9.14 (b) with $I_1' = 0$.

Applying KVL to output side,

$$V_2' = I_2'\left(sL + \frac{1}{sC}\right)$$

$$\therefore \quad \frac{V_2'}{I_2'} = sL + \frac{1}{sC} \qquad \text{... (3)}$$

$$\therefore \text{ By definition, } z_{22}' = \left(sL + \frac{1}{sC}\right)\Omega$$

The voltage V_1' is given by,

$$V_1' = \left(\frac{1}{sC}\right) I_2'$$

$$\therefore \frac{V_1'}{I_2'} = \frac{1}{sC} \qquad \text{... (4)}$$

$\therefore$ By definition, $z_{12}' = \dfrac{V_1'}{I_2'} = \dfrac{1}{sC}\ \Omega$

$\therefore$ For the network N′, z-parameters are given by,

$$[z'] = \begin{bmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{bmatrix} = \begin{bmatrix} sL + \frac{1}{sC} & \frac{1}{sC} \\ \frac{1}{sC} & sL + \frac{1}{sC} \end{bmatrix}$$

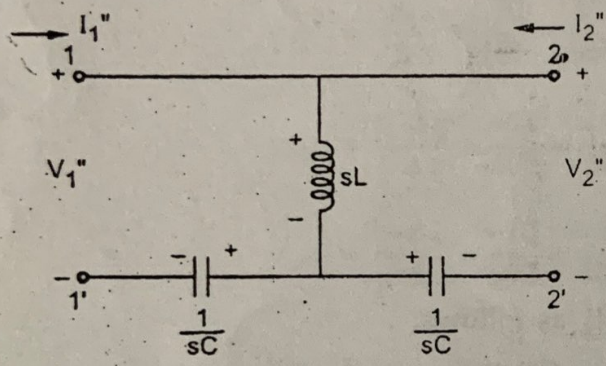

Fig. 9.14 (c)

(B) To determine z-parameters of N″ :

Consider the network N_2'' transformed into s-domain as shown in the Fig. 9.14 (c).

(1) To determine z_{11}'' and z_{21}'', open circuit terminals 2 - 2′ by making $I_2'' = 0$. Refer Fig. 9 14 (c) with $I_2'' = 0$.

Applying KVL to the input side,

$$V_1'' = \left(sL + \frac{1}{sC}\right) I_1''$$

$$\therefore \frac{V_1''}{I_1''} = \left(sL + \frac{1}{sC}\right) \qquad \text{... (5)}$$

$\therefore$ By definition, $z_{11}'' = \dfrac{V_1''}{I_1''} = \left(sL + \dfrac{1}{sC}\right)\ \Omega$

Voltage V_2'' can be expressed interms of I_1'' as follows,

$$V_2'' = (sL)\, I_1''$$

$$\therefore \frac{V_2''}{I_1''} = sL \qquad \text{... (6)}$$

$\therefore$ By definition, $z_{21}'' = sL\ \Omega$

(2) To determine z''_{12} and z''_{22}, open circuit terminals 1 - 1' by making $I''_1 = 0$.

Refer Fig. 9.14 (c) with $I''_1 = 0$.

Applying KVL to the output side,

$$V''_2 = \left(sL + \frac{1}{sC}\right) I''_2$$

$$\therefore \quad \frac{V''_2}{I''_2} = sL + \frac{1}{sC} \qquad \text{... (7)}$$

$\therefore$ By definition, $z''_{22} = \dfrac{V''_2}{I''_1} = \left(sL + \dfrac{1}{sC}\right) \Omega$

Voltage V''_1 can be expressed interms of current I''_2 as follows.

$$V''_1 = (sL)\, I''_2$$

$$\therefore \quad \frac{V''_1}{I''_2} = sL \qquad \text{... (8)}$$

$\therefore$ By definition, $z''_{12} = \dfrac{V''_1}{I''_2} = (sL)\ \Omega$

$\therefore$ For the network N", z-parameters are given by,

$$[z''] = \begin{bmatrix} z''_{11} & z''_{12} \\ z''_{21} & z''_{22} \end{bmatrix} = \begin{bmatrix} sL + \frac{1}{sC} & sL \\ sL & sL + \frac{1}{sC} \end{bmatrix}$$

As networks N' and N" are connected in series, the overall z-parameters of the series connection are obtained by adding individual z-parameters of the network.

$$[z_{overall}] = \begin{bmatrix} z''_{11} & z''_{12} \\ z''_{21} & z''_{22} \end{bmatrix} + \begin{bmatrix} z''_{11} & z''_{12} \\ z''_{21} & z''_{22} \end{bmatrix} = \begin{bmatrix} sL + \frac{1}{sC} & \frac{1}{sC} \\ \frac{1}{sC} & sL + \frac{1}{sC} \end{bmatrix} \begin{bmatrix} sL + \frac{1}{sC} & sL \\ sL & sL + \frac{1}{sC} \end{bmatrix}$$

$$= \begin{bmatrix} 2\left(sL + \frac{1}{sC}\right) & \left(sL + \frac{1}{sC}\right) \\ \left(sL + \frac{1}{sC}\right) & 2\left(sL + \frac{1}{sC}\right) \end{bmatrix} = \left(sL + \frac{1}{sC}\right) \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

9.10.2 Parallel Connection of Two Ports

Consider two networks N' and N" are connected in parallel as shown in Fig. 9.15. When two ports are connected in parallel, we can add their y-parameters to get overall y-parameters of the parallel connection.

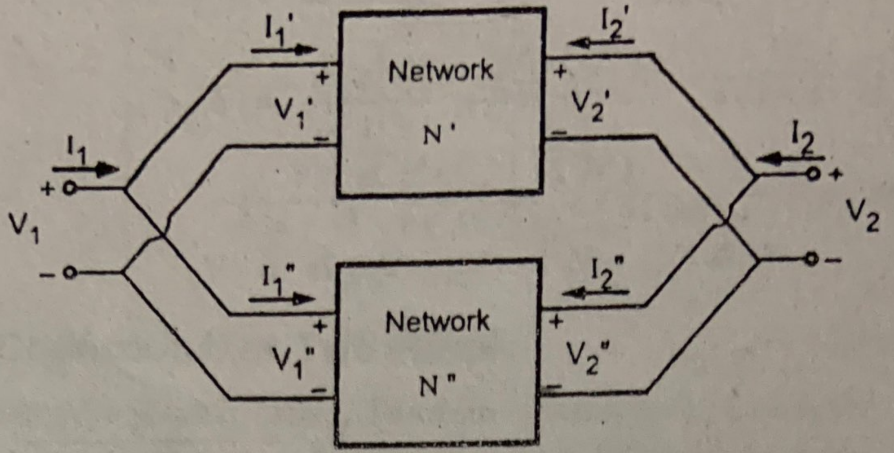

Fig. 9.15 Parallel connection of 2 two port networks

Let the y-parameters of the network N′ by y'_{11}, y'_{12}, y'_{21}, y'_{22}. Let the y-parameters of the network N″ be y''_{11}, y''_{12}, y''_{21}, y''_{22}. Let the overall y-parameters of parallel connection be y_{11}, y_{12}, y_{21}, y_{22}

For parallel connection we have,

$$I_1 = I'_1 + I''_1 \quad \text{...(1)}$$

$$I_2 = I'_2 + I''_2 \quad \text{... (2)}$$

And

$$V_1 = V'_1 = V''_1 \quad \text{... (3)}$$

$$V_2 = V_2' = V''_2 \quad \text{... (4)}$$

For network N′, the y-parameter equations are,

$$I'_1 = y'_{11}V'_1 + y'_{12}V'_2$$

$$I'_2 = y'_{21}V'_1 + y'_{22}V'_2$$

For network N″ the y-parameter equations are,

$$I''_1 = y''_{11}V''_1 + y''_{12}V''_2$$

$$I''_2 = y''_{21}V''_1 + y''_{22}V''_2$$

From equations (1), (2) and (3), (4), we can write,

$$I_1 = (y'_{11} + y''_{11})\,V_1 + (y'_{12} + y''_{12})\,V_2$$

$$I_2 = (y'_{21} + y''_{21})\,V_1 + (y'_{22} + y''_{22})\,V_2$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} (y'_{11} + y''_{11}) & (y'_{12} + y''_{12}) \\ (y'_{21} + y''_{21}) & (y'_{22} + y''_{22}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus, overall y-parameters are,

$$[y] = \begin{bmatrix} y'_{11} + y''_{11} & y'_{12} + y''_{12} \\ y'_{21} + y''_{21} & y'_{22} + y''_{22} \end{bmatrix}$$

Hence, the y-parameters of the parallel connection are the sum of y-parameters of the individual networks connected in parallel.

Example 9.8 : *The network of the Fig. 9.16 is of the type used for the "notch filter". For the element values given determine the y-parameters.*

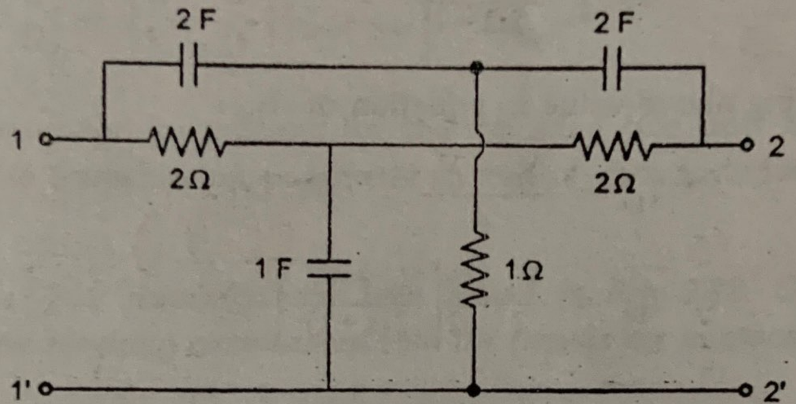

Fig. 9.16

Solution : The given network can be redrawn as, parallel combination of two networks as shown in the Fig. 9.16 (a)

$$V_1 = V_{1a} = V_{1b}$$

$$V_2 = V_{2a} = V_{2b}$$

$$I_1 = I_{1a} + I_{1b} \quad \text{... (1)}$$

$$I_2 = I_{2a} + I_{2b} \quad \text{... (2)}$$

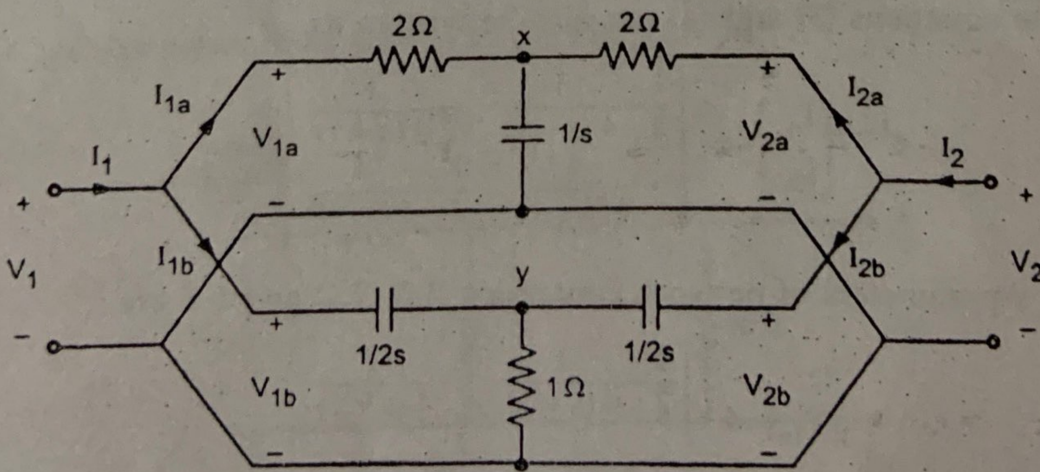

Fig. 9.16 (a) Transformed network

Applying KCL at node x,

$$I_{1a} + I_{2a} = \frac{V_x}{\left(\frac{1}{s}\right)}$$

$$\therefore \quad I_{1a} + I_{2a} = s\,V_x$$

$$\frac{V_{1a} - V_x}{2} + \frac{V_{2a} - V_x}{2} = s\,V_x$$

$$\therefore \quad \frac{V_{1a} + V_{2a}}{2} = (s+1)\,V_x$$

$$\therefore \quad V_x = \frac{V_{1a} + V_{2a}}{2(s+1)}$$

Substituting above value in equation of I_{1a}.

$$I_{1a} = \frac{V_{1a} - V_x}{2}$$

$$\therefore \quad I_{1a} = \frac{V_{1a}}{2} - \frac{1}{2}\left[\frac{V_{1a} + V_{2a}}{2(s+1)}\right]$$

$$\therefore \quad I_{1a} = \left[\frac{1}{2} - \frac{1}{4(s+1)}\right] V_{1a} - \left[\frac{1}{4(s+1)}\right] V_{2a} \qquad \ldots (3)$$

Substituting the value of V_x in equation of I_{2a} also,

$$I_{2a} = \frac{V_{2a} - V_x}{2}$$

$$\therefore \quad I_{2a} = \frac{V_{2a}}{2} - \frac{1}{2}\left[\frac{V_{1a} + V_{2a}}{2(s+1)}\right]$$

$$\therefore \quad I_{2a} = \left[-\frac{1}{4(s+1)}\right] V_{1a} + \left[\frac{1}{2} - \frac{1}{4(s+1)}\right] V_{2a} \qquad \ldots (4)$$

The equations (3) and (4) can also be written as,

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - \frac{1}{4(s+1)} & -\frac{1}{4(s+1)} \\ -\frac{1}{4(s+1)} & \frac{1}{2} - \frac{1}{4(s+1)} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

So y-parameters of network containing 2 Ω, 2 Ω and 1 F are,

$$[Y_{1a}] = \begin{bmatrix} \frac{1}{2} - \frac{1}{4(s+1)} & -\frac{1}{4(s+1)} \\ -\frac{1}{4(s+1)} & \frac{1}{2} - \frac{1}{4(s+1)} \end{bmatrix}$$

Applying KCL at node y,

$$I_{1b} + I_{2b} = \frac{V_y}{1}$$

$$\therefore \frac{V_{1b} - V_y}{\left(\frac{1}{2s}\right)} + \frac{V_{2b} - V_y}{\left(\frac{1}{2s}\right)} = V_y$$

$$2s\,(V_{1b} - V_y) + (V_{2b} - V_y)\,2s = V_y$$

$$\therefore 2s\,(V_{1b} + V_{2b}) = (1 + 4s)\,V_y$$

$$\therefore V_y = \frac{2s\,(V_{1b} + V_{2b})}{(1+4s)}$$

Substituting above value of V_y in equation of I_{1b},

$$I_{1b} = \frac{V_{1b} - V_y}{\left(\frac{1}{2s}\right)} = 2s\,(V_{1b} - V_y) = 2s\left[V_{1b} - \frac{2s\,(V_{1b} + V_{2b})}{1+4s}\right]$$

$$I_{1b} = \left[2s - \frac{4s^2}{1+4s}\right]V_{1b} - \left[\frac{4s^2}{1+4s}\right]V_{2b} \quad \ldots (5)$$

Substituting value of V_y in equation of I_{2b},

$$I_{2b} = \frac{V_{2b} - V_y}{\left(\frac{1}{2s}\right)} = 2s\,(V_{2b} - V_y) = 2s\left[V_{2b} - \frac{2s\,(V_{1b} + V_{2b})}{1+4s}\right]$$

$$\therefore I_{2b} = \left[\frac{-4s^2}{1+4s}\right]V_{1b} + \left[2s - \frac{4s^2}{1+4s}\right]V_{2b} \quad \ldots (6)$$

The equations (5) and (6) can be written as,

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} 2s - \frac{4s^2}{1+4s} & -\frac{4s^2}{1+4s} \\ -\frac{4s^2}{1+4s} & 2s - \frac{4s^2}{1+4s} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

The y-parameters can be given as,

$$[y_{1b}] = \begin{bmatrix} 2s - \frac{4s^2}{1+4s} & -\frac{4s^2}{1+4s} \\ -\frac{4s^2}{1+4s} & 2s - \frac{4s^2}{1+4s} \end{bmatrix}$$

The total y-parameters can be given as,

$$[y] = \begin{bmatrix} \left(\frac{1}{2} - \frac{1}{4(s+1)}\right) + \left(2s - \frac{4s^2}{1+4s}\right) & -\frac{1}{4(s+1)} - \frac{4s^2}{1+4s} \\ -\frac{1}{4(s+1)} - \frac{4s^2}{1+4s} & \left(\frac{1}{2} - \frac{1}{4(s+1)}\right) + \left(2s - \frac{4s^2}{1+4s}\right) \end{bmatrix}$$

9.10.3 Cascade Connection of Two Ports

The cascade connection is also called Tandem connection. Consider two networks N′ and N″ are connected in cascade as shown in Fig. 9.17. When two ports are connected in cascade, we can multiply their individual transmission parameters to get overall transmission parameters of the cascade connection.

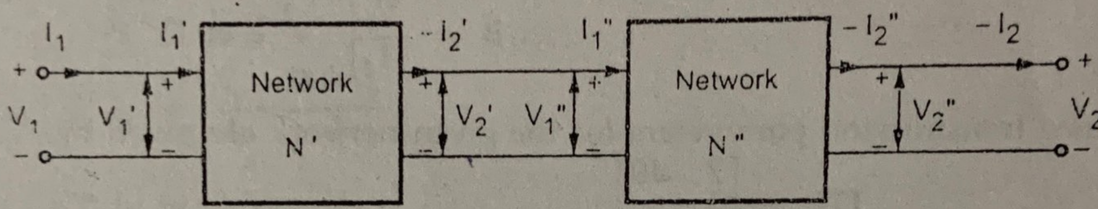

Fig. 9.17 Cascade connection of two port networks

Let the transmission parameters of network N′ be A′, B′, C′, D′. Let the transmission parameters of network N″ be A″, B″, C″, D″. Let the overall transmission parameters of cascade connection be A, B, C, D.

For cascade connection we have,

$$V_1 = V_1', V_2' = V_1'', V_2 = V_2'' \quad \ldots(1)$$

$$I_1' = I_1, -I_2' = I_1'', -I_2 = -I_2'' \quad \ldots(2)$$

For the network N′, transmission parameter equations are,

$$V_1' = A' V_2' + B'(-I_2')$$

$$I_1' = C' V_2' + D'(-I_2')$$

For the network N_1'' transmission parameter equations are,

$$V_1'' = A'' V_2'' + B''(-I_2'')$$

$$I_1'' = C'' V_2'' + D''(-I_2'')$$

The overall transmission parameters of the cascade connection can be written as,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix}$$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix}$$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

Hence, the transmission parameters for the cascaded two port network is simply the matrix product of the transmission parameter matrix of each individual two port network in cascade.

Example 9.9 : *Two networks have been shown in Fig. 9.18. Obtain the transmission parameters of the resulting circuit when both the circuits are in cascade.*

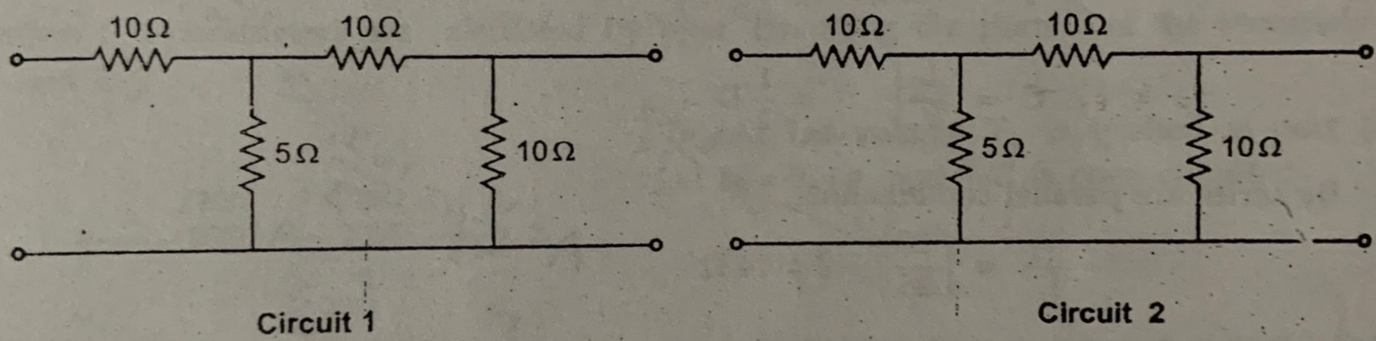

Fig. 9.18

Solution : Consider network as shown in the Fig. 9.18 (a).

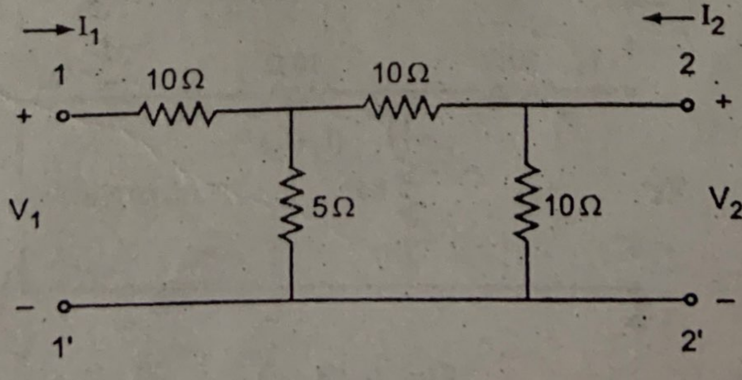

Fig. 9.18 (a)

By definition, transmission parameters are given by,

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

(A) Let $-I_2 = 0$, open circuiting port 2 as shown in the Fig. 9.18 (b).

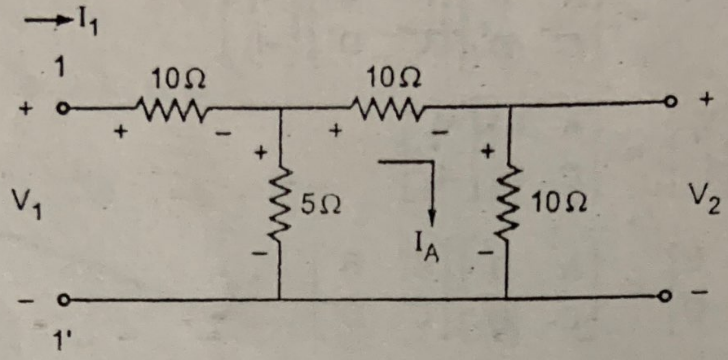

Fig. 9.18 (b)

10 IA

By current divider rule,

$$I_A = I_1\left[\frac{5}{5+10+10}\right] = \left(\frac{1}{5}\right) I_1 \qquad \text{... (1)}$$

Hence voltage at port 2 is given by,

$$V_2 = 10(I_A) = 10\left(\frac{1}{5}I_1\right) = 2I_1 \qquad \text{... (2)}$$

$$\therefore C = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = \frac{1}{2}\ \mho$$

By series are parallel combination,

$$\frac{V_1}{I_1} = \left(\frac{20}{25}\right) + 10 = 14\ \Omega \qquad \text{... (3)}$$

$A = \frac{V_1}{V_2} = \frac{14 I_1}{2 I_1}$

$A = 7$

$$\therefore A = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = \frac{V_1}{I_1}\cdot\frac{I_1}{V_2} = (14)\left(\frac{1}{2}\right) = 7$$

(B) Let $V_2 = 0$, short circuiting port 2 as shown in the Fig. 9.18 (c). Due to the short circuit 10 Ω gets shorted.

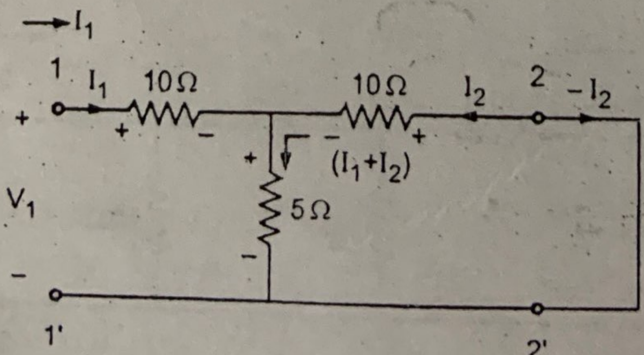

Fig. 9.18 (c)

$V_1 = +10 I_1 + 5(I_1+I_2)$

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By current divider rule,

$$-I_2 = I_1\left[\frac{5}{s+10}\right] = \frac{1}{3}I_1 \quad \text{... (4)}$$

$$\therefore D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 3$$

Applying KVL to outer loop,

$$-10I_1 - 10(-I_2) + V_1 = 0$$

$$\therefore -10I_1 - 10(-I_2) = -V_1$$

$$\therefore 10I_1 + 10(-I_2) = V_1$$

But $I_1 = 3(-I_2)$

$$\therefore 10(3)(-I_2) + 10(-I_2) = V_1$$

$$\therefore 40(-I_2) = V_1 \quad \text{.... (5)}$$

$$\therefore B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 40\ \Omega$$

Hence transmission parameters for the given network are given by,

$$[T] = \begin{bmatrix} 7 & 40 \\ \frac{1}{2} & 3 \end{bmatrix} \quad \text{... (6)}$$

Two identical networks are connected in cascade. Hence for cascade connection, the overall transmission parameters can be obtained by finding matrix product of two networks.

$$[T_{overall}] = \begin{bmatrix} 7 & 40 \\ \frac{1}{2} & 3 \end{bmatrix}\begin{bmatrix} 7 & 40 \\ \frac{1}{2} & 3 \end{bmatrix} = \begin{bmatrix} 69 & 400 \\ 5 & 29 \end{bmatrix}$$