TRANSMISSION LINES

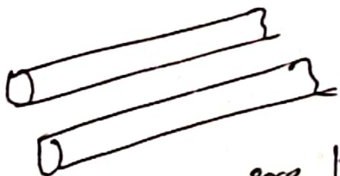
Transmission Line:- The Electrical lines which are used to transmit the electrical waves along them are called transmission lines. The practical examples of the electric waves, which are transmitted along the transmission lines are the telephone messages and electrical power signals. The transmission lines are assumed to consist of pair of wires which are uniform throughout their whole length. In the analysis of transmission line, only steady state currents and voltages are considered. The analysis includes finding the current and voltage at any point along the length of the line, when the voltage is applied continuously at one end. The end to which the voltage is applied is called sending end while the end at which the signals are received is called receiving end of the transmission line.

TYPES OF TRANSMISSION LINES:-

The various types of transmission lines are the following. a) Open wire b) Cables c) Coaxial line d) Waveguides

(a) open wire line:-

These lines are the parallel conductors open to air hence called open wire lines.

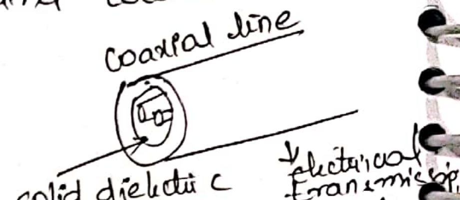


open wire line.

The conductors are seperated by air as the dielectric and mounted on the posts or the towers. The telephone lines and the electrical power transmission lines are the best examples of the open wire lines

Disadv:-

→ Requirement of telephone posts and towers are high cost



→ Affected by atmospheric conditions

→ Maintenance is diffcult.

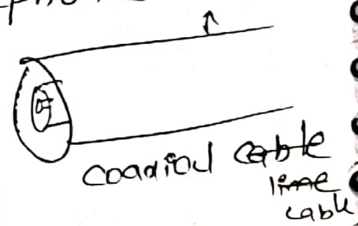
→ Possibility of shorting due to flying objects and birds.

Adv:-

Open wire line has less capacitance when compared to underground cable

2) Cables

These are underground lines. The telephone cables consist of hundred of conductors which are individually insulated with paper. These are twisted in pairs and combined together and placed inside a protective lead or plastic sheath. While underground electrical transmission cables consists of two or three large conductors which are insulated with oil impregnated paper or other solid dielectric and place inside protective lead sheath. Both these types are still considered as parallel conductors seperated by a solid dielectric.

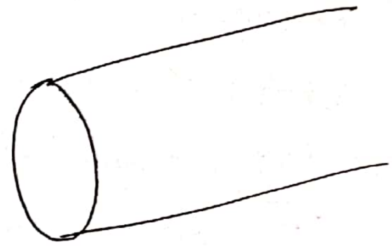


3) Coaxial Cable:-

As the name suggests, there are two conductors which are coaxially placed. One conductor is hollow and other is placed coaxillay inside the first conductor. The dielectric may be solid or gaseous. These lines are used for high voltage levels.

4) WaveGuides:-

These types of transmission lines are used to transmit the electrical waves at microwave frequencies. Constructionally these are hollow conducting tube having uniform cross section. The energy is transmitted from inner walls of the tube by the phenomenon of total internal reflection



Circular waveguide

TRANSMISSION LINE PARAMETERS

The various electric parameters associated with the transmission line are the following:-

a) Resistance b) Inductance c) Capacitance d) Conductance

a) Resistance:- Depending upon the cross sectional area of the conductors, the transmission lines associated have the resistance associated with them. The resistance is uniformly distributed all along the length of transmission line. Its total value depends on the overall length of the transmission line.

Hence its value is given per unit length of transmi--ssion line. It is denoted as R and given in ohms per unit length.

b) Inductance:-

When the conductors carry the current, the magnetic flux is produced around the conductors. It depends on the magnitude of the current flowing through the conductor. The flux linkages per ampere of current, give rise to the inductance of the transmission line. It is denoted as L and measured in henry per unit length of the transmission line.

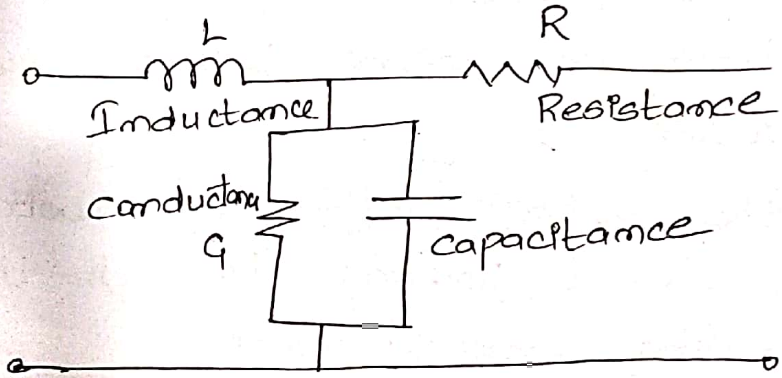
c) Capacitance:-

The transmission line consists of two parallel conductors, seperated by a dielectric like air. Such parallel conductors seperated by an insulating dielectric produces a capacitive effect. Due to this there exists a capacitance associated with the transmission line which is also distributed along the length of the conductor. It is denoted as C and measured in farads per unit length of the transmission line.

d) Conductance:-

The dielectric in between the conductors is not perfect. Hence very small amount of current flows through the dielectric called displacement current. This means that the leakage current is generated and give rise to a leakage conductance associated with the transmission line. It exists between the conductors and distributed along the length of the transmission line. It is denoted as G and measured in mho per unit length of the transmission line.

These line parameters are constants and are called Primary constants. of the transmission line. These constants are assumed to be independent of frequency for the transmission line. These primary constants can be obtained by the measurements on a sample of the transmission line.



A unit transmission line with the primary constants is shown above. If all the constants are uniform throughout the transmission line then it is called Uniform Transmission Line.

All the above parameters are called Series (or) Loop Inductance, Series or loop Capacitance etc.

→ Series inductance Units:- Henry/km

→ Series (or) Loop Resistance :- Ω/km

→ Series Capacitance :- farad/m.

→ Series Conductance :- Siemens/metre, mho's/metre.

The series Impedance will be $\boxed{Z = R + j\omega L}$

The Series Admittance will be $\boxed{Y = G + j\omega C}$

INFINITE LINE CONCEPTS

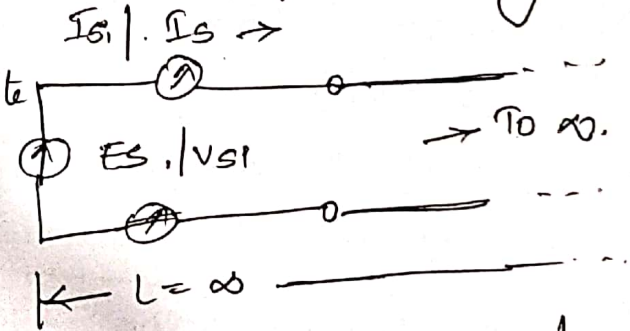
INFINITE TRANSMISSION LINE

Infinite Transmission line indicates that the length of transmission is Infinite. As the line is infinite, it has the following properities.

a) As the line has infinite length, no waves will ever reach the receiving end and hence there is no possibility of the reflection at the receiving end. Thus there cannot be any reflected waves, returning to the sending end. The complete power applied at the sending end is absorbed by the line

b) As the reflected waves are absent, the characteristic Impedance Z_0 at the sending end will decide the current flowing, when a voltage is applied to the sending end. The current will not be effected by the terminating impedance Z_R. This condition is fulfilled by the long Lines in Practice.

Consider the below infinite transmission line. The alternating voltage V_{SI} / E_S is applied to the sending end.



A finite current will flow which depends on the capacity of the line and the leakage conductance between the two wires constituting the line. This finite lite current is I_S/I_{SI}.

The ratio of applied voltage V_{SI} and the current flowing I_{SI} is the input impedance. This input impedance of the infinite line is called characteristic Impedance. Infact the characteristic impedance of any practical line is define as the impedance looking at an infinite line having same electrical properities.

We have the transmission line equation for finite system is $I = ce^{Px} + de^{-Px}$ — (1)

At Sending end:-

At sending end, distance $x=0$, current = max = I_{si}.

Now, sub the values in $I = ce^{Px} + de^{-Px}$

$$I_{si} = ce^{0} + de^{0} \Rightarrow \boxed{I_{si} = c+d} \text{ — (2)}$$

At receiving end, $I=0$, $x=\infty$, $I = ce^{Px} + de^{-Px}$

$0 = ce^{\infty} + de^{-\infty}$

$\rightarrow 0 = ce^{\infty} + de^{-\infty} \Rightarrow 0 = e^{-\infty}(e^{\infty} + e^{-2\infty})$

$c + d(0) = 0 \rightarrow \boxed{c=0}$

Then, sub $c=0 \Rightarrow$ in Eqn (2)

$I_{si} = 0 + d \Rightarrow \boxed{d = I_{si}}$.

Substitute the values of c and d in Eqn (1)

$I = ce^{Px} + de^{-Px}$

$I = 0 + de^{-Px}$

$$\boxed{I = I_{si}e^{-Px}} \text{ — (3)}$$

Similarly, we have, $V = ae^{Px} + be^{-Px}$

As $V \propto I \Rightarrow$ At Sending end, $V = V_{si}$ $x=0$

$$V_{si} = a(e^{0}) + be^{-0} \Rightarrow \boxed{V_{si} = a+b} \text{ — (4)}$$

At receiving end, $V=0$, $x=\infty$

$0 = a(e^{\infty}) + b(e^{-\infty}) \Rightarrow 0 = e^{\infty}(a + be^{-2\infty})$

$a + be^{-2\infty} = 0 \Rightarrow a + 0 = 0 \Rightarrow \boxed{a=0}$

we have $a+b=V_{si}$, sub $a=0$, then $0+b=V_{si}$

$$\boxed{b=V_{si}}$$

Substitute the values of a and b in $V=ae^{px}+be^{-px}$.

$$V=0e^{px}+V_{si}e^{-px}$$

$$\boxed{V=V_{si}e^{-px}}$$

So, the transmission line equation ~~from~~ for the infinite transmission line are the following.

$$\boxed{I=I_{si}e^{-px} \; ; \; V=V_{si}e^{-px}}$$

From the above equation, we can state that backward wave is 0, as ae^{px} component is 0. Since, there is no reflection, Infinite transmission line is also called Transmission line with no loss or Lossless Transmission line.

The above equations are used to calculate the values of voltage and current at a distance x from the Sending end of transmission line.

As there is no reflection, there is no propogation Loss.

Secondary Constants

EXPRESSION FOR CHARACTERISTIC IMPEDANCE

$$\boxed{\text{Characteristic Impedance} = Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

The characteristic impedance is a phasor quantity having the magnitude of $|Z_0|$ and an angle ϕ. Both magnitude and angle of characteristic impedance vary with frequency. Hence, the frequency.

at which the characteristic impedance must be specified while specifying the value of characteristic impedance.

Proof:- Consider from the transmission line equations for finite.

$$v-(v+dv) = (R+j\omega L).I.dx \text{ --- (1)}$$

$$-dv = (R+j\omega L).I.dx$$

$$-\frac{dv}{dx} = (R+j\omega L).I \text{ --- (1)}$$

The potential difference = (current flowing through impedance) x (R+jωL)

Substitute, the values of infinite Transmission such that they are

$$V = V_{si}e^{-Px} ; \quad I = I_{si}e^{-Px}.$$

$$-\frac{dv}{dx} = (R+j\omega L).I.(dx/dx)$$

$$-\frac{d}{dx}(v_{si}e^{-Px}) = (R+j\omega L).I_{si}.\cancel{e^{-Px}}\, e^{-Px}$$

$$v_{si}.(-(-P)e^{-Px}) = (R+j\omega L).I_{si}.\cancel{e^{-Px}}\, e^{-Px}$$

$$v_{si}(P).\cancel{e^{-Px}} = (R+j\omega L).I_{si}.\cancel{e^{-Px}}$$

$$\frac{V_{si}}{I_{si}} = \frac{R+j\omega L}{P} \text{ --- (1)}$$

we know that the propogation constant

$$P = \sqrt{(R+j\omega L)(G+j\omega C)} \text{ --- (2)}$$

Sub Eq^n (2) in Eq^n (1)

$$\frac{V_{si}}{I_{si}} = \frac{R+j\omega L}{\sqrt{R+j\omega L}.\sqrt{G+j\omega C}}$$

we know that the characteristic impedance $Z_0 = \frac{V_{si}}{I_{si}}$

$$Z_0 = 1/\sqrt{\frac{G+j\omega C}{R+j\omega L}}.$$

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$$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}} \quad , \quad Z_0 = \sqrt{Z_{OC} \cdot Z_{SC}}$$

This is the characteristic impedance.

Note:- 1) The characteristic impedance of open wire transmission line is given by.

$$Z_0 = 276 \log_{10}\left(\frac{s}{r}\right)$$

s — Spacing b/w wires, $r \rightarrow$ radius of either wire.

2) The characteristic impedance of coaxial cable is given by

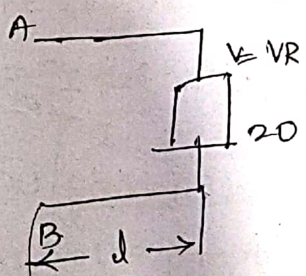
$$Z_0 = \frac{1}{2\pi}\sqrt{\mu/\varepsilon}\ \log_{10}\left(\frac{D}{d}\right)$$

$D \rightarrow$ inner diameter of outer conductor

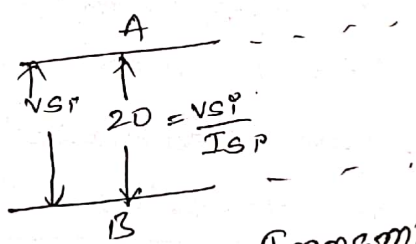
$d \rightarrow$ diameter of outer conductor.

(b) LOSSLESS | LOW LOSS CHARACTERISATION:-

finite transmission line terminated with the characteristic Impedance (Z_0)



Finite Transmission Line.



Infinite Transmission Line.

To make a finite transmission as lossless or Low loss, then finite transmission should be Converted into Infinite transmission line. For this, at the end of finite Transmission line, an infinite transmission line is connected. But this is not practically possible. Hence, the.

another way to convert finite transmission line to Infinite transmission line is that " Finite transmission line is terminated with the input impedance of Input Impedance of Infinite transmission line."

This means that the input impedance of finite Transmission line terminated with Z_0 is same (Z_0) which the Input Impedance of infinite Transmission line.

Consider the general transmission line equation at sending end.

$$V = V_S \cosh px - I_S Z_0 \sinh px \quad , I = V/Z_0 .$$

$$I = I_S \cosh px - \frac{V_S}{Z_0} \sinh px$$

Now, at receiving end, $x = L$, $V = V_R$, $I = I_R$

$$I_R = I_S \cosh pL - V_S/Z_0 \sinh pL$$

$$V_R = V_S \cosh pL - I_S Z_0 \sinh pL$$

we have characteristic impedance $(Z_0) = \frac{V_R}{I_R}$

$$Z_0 = \frac{V_S \cosh pL - I_S Z_0 \sinh pL}{I_S \cosh pL - \frac{V_S}{Z_0} \sinh pL} \quad , \text{Taking L.C.M}$$

$$\cancel{Z_0} = \frac{V_S \cosh pL - I_S Z_0 \sinh pL}{I_S Z_0 \cosh pL - V_S \sinh pL} (\cancel{Z_0})$$

$$1 = \frac{V_S \cosh pL - I_S Z_0 \sinh pL}{I_S Z_0 \cosh pL - V_S \sinh pL}$$

$$I_S Z_0 \cosh pL - V_S \sinh pL = V_S \cosh pL - I_S Z_0 \sinh pL$$

$$I_S Z_0 \cosh pL + I_S Z_0 \sinh pL = V_S \cosh pL + V_S \sinh pL$$

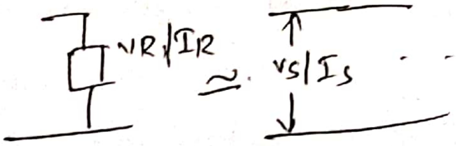
$$I_S Z_0 (\cancel{\cosh pL + \sinh pL}) = V_S (\cancel{\cosh pL + \sinh pL})$$

$$I_S Z_0 = V_S .$$

$$Z_0 I_S = V_S$$

$$\boxed{Z_0 = V_S / I_S}$$

$$\boxed{\frac{V_R}{I_R} = \frac{V_S}{I_S}}$$



This equation shows that the end impedance of in finite transmission (VR/IR) is equal to the input impedance of Infinite transmission line.

Hence, this is the sufficient condition to make the finite Transmission line. So, the finite transmission is designed according to the condition

$$\frac{V_R}{I_R} = \frac{V_S}{I_S}$$

and this makes finite transmission line as lossless.

Wavelength:- (λ):(metres)

The distance at which the wave travelled change its phase by 360°.

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

Velocity of propogation:-

Any wave may travel by anyone of the below velocity.

a) **Phase Velocity:-**

The velocity of the wave by which it propogates. These velocity is determined by the phase changes in it.

The phase velocity $\boxed{(v_p) = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}.}$

b) Group Velocity :-

When the transmitted signal is having more than one frequency it is difficult to define the phase velocity. In such cases, a group velocity is determined. Group velocity determines the velocity of wave which is combination of several waves. These generally seen in multiplexed signals.

The velocity with which the envelope of complex signal will travel along the transmission line. The group velocity is denoted as v_g.

Let us consider the two signals with frequency ω_1 and ω_2 and phase constant β.

$$v_g = \frac{\omega_2 \sim \omega_1}{\beta_2 \sim \beta_1}$$

$$\boxed{v_g = \frac{d\omega}{d\beta}.}$$

Relation between Phase and Group Velocities :-

we have phase velocity, $v_p = \omega/\beta$ $\quad \frac{u}{v}$ $\quad \frac{vu' - uv'}{d}$

$$\frac{dv_p}{d\omega} = \frac{\beta \cdot \frac{d\omega}{d\omega} - \omega \cdot \frac{d\beta}{d\omega}}{(\beta)^2}$$

$$\frac{dv_p}{d\omega}(\beta^2) = \beta - \omega \cdot \frac{d\beta}{d\omega}$$

$$\frac{dv_p}{d\omega}\beta^2 - \beta = -\omega \cdot \frac{d\beta}{d\omega} \Rightarrow -\frac{dv_p}{d\omega}\beta^2 + \beta = \omega \cdot \frac{d\beta}{d\omega}$$

Since $v_g = d\omega/d\beta$.

$$-\frac{dv_p}{d\omega}\beta^2 + \beta = \frac{\omega}{v_g}$$

$$\frac{1}{v_g} = \left[-\beta^2 \cdot \frac{dv_p}{d\omega} + \beta\right] \cdot \frac{1}{\omega}$$

$$\frac{1}{v_g} = \beta\left[-\beta \cdot \frac{dv_p}{d\omega} + 1\right]\frac{1}{\omega}$$

$$\frac{1}{v_g} = \frac{\beta}{\omega}\left[-\beta \cdot \frac{dv_p}{d\omega} + 1\right]$$

$$\frac{1}{v_g} = \frac{1}{v_p}\left[-\beta \cdot \frac{dv_p}{d\omega} + 1\right]$$

$$v_g = \frac{v_p}{\left[-\beta \cdot \frac{dv_p}{d\omega} + 1\right]}$$

$$\boxed{v_g = \frac{\omega}{\beta\left[1 - \beta \cdot \frac{dv_p}{d\omega}\right]}} \quad -(1)$$

This is relation between phase velocity and Group velocity. If we transmit only one signal then group velocity. $\frac{dv_p}{d\omega} = 0$. sub in Eqn -(1)

$$v_g = \frac{\omega}{\beta\left[1 - \beta \cdot (0)\right]} = \frac{\omega}{\beta} = v_p$$

$$\boxed{v_p = v_g}$$

DISTORSION :-

When the received signal is not the exact replica of the transmitted signal then the signal is said to be distorted. There exists of somekind of distorsions along the transmission line and they are the following.

a) Distorsion due to variation of characteristic impedance Z_0 with frequency.

b) Frequency distorsion due to the variation of attenuation constant α with frequency.

c) Phase distorsion due to the variation of phase constant β with frequency

a) Distorsion due to Z_0 varying with frequency

Generally characteristic impedance of line vary with frequency. If the wireline terminated in an impedance which doesn't vary with frequency in similar as that Z_0, it results in distorsion. The power is absorbed at certain frequencies. while it get reflected for certain frequencies. So, there exists a selective power absorption, due to this type of distorsion.

b) Frequency Distorsion:-

Frequency distorsion is function of ω generally. If all the frequency components of transmitted signal are not attenuated by the same value. then it is said to be frequency distorsion. The attenuation constant is a function of frequency. Hence different frequencies transmitted along the line will be attenuated to the different extent. For example, voice signals consists of many frequencies. And all these frequencies will not be attenuated equally along the transmission line. Such a distorsion is called frequency distorsion.

Remedy:-

Thus in high frequency radio broadcasting such frequency distorsion is eliminated by the use of equalizers. The characteristics of equalizers are frequencies and phase characteristics of equalizers are inverse to those of the line. Thus nullifying the distorsion making the overall frequency response, uniform in nature.

(c) Phase distorsion

The phase constant β also varies with frequency. Now the velocity v is given by $v = \omega/\beta$. Hence velocity of propogation also varies with frequency. Hence some waves will reach receiving end very fast while some of the waves get delayed than the others. Hence all frequencies will not have same transmission time. Thus the o/p at the receiving end will not exactly replica of the input wave at the sending end. This type of distorsion is called delay/Phase distorsion. It is not much for audio signals due to the characteristics of the human ears. But such a distorsion is very serious incase of video and Picture transmission.

Remedy:-

The remedy for this is to use co-axial cables for the picture transmission of television and video signals.

Distorsionless line

A line in which there is no phase or frequency distorsion and also it is correctly terminated, is called distorsionless line.

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \Rightarrow \gamma^2 = (RG-\omega^2 LC) + j\omega C(RC+LG)$$

It is known that for min. attenuation $L = \frac{CR}{G}$, $LG = CR$, Substituting this, we get.

$$\gamma^2 = (RG - \omega^2 LC) + j2\omega RC \quad \text{and} \quad RC = LG = \sqrt{RCLG}$$

$$\gamma^2 = RG - \omega^2 LC + j2\omega\sqrt{RCLG}$$

$$\gamma^2 = (\sqrt{RG} + j\omega\sqrt{LC})^2 \Rightarrow \gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\Rightarrow \alpha + j\beta = \sqrt{RG} + j\omega\sqrt{LC} \Rightarrow \boxed{\alpha = \sqrt{RG}, \ \beta = \omega\sqrt{LC}} \quad \text{---} (1)$$

It can be seen that from eqn-(1), α doesn't vary with frequency which eliminates the frequency distorsion

$$\beta = \omega\sqrt{LC}, \quad v = \omega/\beta = \omega/\omega\sqrt{LC} \Rightarrow v = \frac{1}{\sqrt{LC}} \text{ km/sec.}$$

Thus for the condition $LG = CR$, the velocity becomes independent of frequency. This eliminates the phase distorsion.

It is already proved that $RC = LG$, the Z_0 becomes resistive and line can be correctly terminated to eliminate distorsion due to Z_0 varying with frequency.

Thus all the distorsions are eliminated for a condition

$RC = LG \Rightarrow$ i.e, $\boxed{\frac{R}{G} = \frac{L}{C}}$ $\rightarrow$ This is condition for distorsionless line

Thus, if the primary line constants donot mutually satisfy the condition, then this condition should naturally satisfy by $\uparrow L$ or $\downarrow R$. This line if done artificially called loaded line, and the processing a line satisfied

CONDITION FOR DISTORSIONLESS / DISTORSIONLESS TRANSMISSION LINE

A line in which there is no phase or frequency distorsion is and also it is correctly terminated, is called distorsionless line.

To derive the condition for distorsionless, consider the propogation constant, $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$ —(1)

But we have propogation constant $(\alpha) = \alpha + j\beta$ —(2)

Equate (1) and (2)

$$\alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\alpha + j\beta = \sqrt{R\left(R/L + j\omega\right) L \cdot C\left(G/C + j\omega\right)}$$

We have the condition from infinite transmission line is $R/L = G/C$.

$$\alpha + j\beta = \sqrt{LC}\sqrt{(R/L + j\omega)(R/L + j\omega)}$$

$$= \sqrt{LC}\sqrt{(R/L + j\omega)^2}$$

$$\alpha + j\beta = \sqrt{LC}\left(R/L + j\omega\right) \text{—(3)} \quad (\text{or}) \quad \alpha + j\beta = \sqrt{LC}\left(j\omega + G/C\right)$$

But, we have attenuation constant $\alpha = R\sqrt{C/L}$, $\beta = \omega\sqrt{LC}$

If $\alpha \neq f(\omega)$, frequency distorsion is 0.

$\beta = \omega\sqrt{LC}$, $v = \omega/\beta = \dfrac{\omega}{\omega\sqrt{LC}}$ $\boxed{v = \dfrac{1}{\sqrt{LC}}}$ derived already

Here v is also not function of frequency. So, phase distorsion is 0. Since, v is not a function of frequency and α now, the phase and frequency distorsions are '0'. Hence, this type of transmission line is called distorsionless transmission line.

Getting α, β values

From Eqn (8)

$$\alpha + j\beta = \left(R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}\right) \text{ (or) } \left(G\sqrt{L/C} + j\omega\sqrt{LC}\right)$$

Hence, $\alpha = R\sqrt{C/L}$ (or) $G\sqrt{L/C}$, $\beta = \omega\sqrt{LC}$.

Hence, the condition of distorsionless is $LG = RC$

i.e $RC = LG \Rightarrow \boxed{\frac{R}{G} = \frac{L}{C}}$

This is condition. For such a line, received signal is exact replica of the signal at the ending end, though it is delayed by the propogation constant time and its amplitude reduces.

Characteristic Impedance (Z_0) for distorsionless line is the following.

$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{L}{C}\frac{(R/L + j\omega)}{(G/C + j\omega)}}$$ but we have $R/L = G/C$.

$$Z_0 = \sqrt{L/C\left(\frac{\cancel{G/C + j\omega}}{\cancel{G/C + j\omega}}\right)}$$

$$\boxed{Z_0 = \sqrt{L/C}}$$

CONDITION FOR MINIMUM ATTENUATION

The condition for distorsionless is $R/L = G/C$. Thus, these conditions always linked with the primary constants of the transmission line. Sometimes, the primary constants do not mutually satisfy the above condition. Then this condition will have to be ~~increased~~ satisfied artificially by increasing L or decreasing C or varying R and G.

To determine the above conditions, α, β values are determined initially itself.

DETERMINATION OF α, β VALUES (in terms of primary constants)

we have propogation constant $P = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$P^2 = (R+j\omega L)(G+j\omega C)$$

$$(\alpha+j\beta)^2 = (RG + j\omega RC + j\omega LG + j^2\omega^2 LC)$$

$$(\alpha^2 + j^2\beta^2) + 2\alpha j\beta = RG + j\omega(RC+LG) - \omega^2 LC$$

$$(\alpha^2 + j^2\beta^2) + 2\alpha j\beta = (RG - \omega^2 LC) + j\omega(RC+LG)$$

By equating the L.H.S and R.H.S

$$\alpha^2 + j^2\beta^2 = RG - \omega^2 LC, \quad 2\alpha\beta = \omega(RC+LG)$$

$$\alpha^2 - \beta^2 = RG - \omega^2 LC, \quad 2\alpha\beta = \omega RC + \omega LG \quad —(1)$$

Now, find the magnitude of propogation constant

$|P| = |\alpha+j\beta| = \sqrt{\alpha^2+\beta^2}$ —(2), sub the Eq^n (1) values in Eq^n (2),

$$|P| = |\alpha+j\beta| = \sqrt{\alpha^2+\beta^2} = \sqrt{\alpha^2+\beta^2}$$

$$P = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(RG-\omega^2 LC) + j(\omega RC + \omega LG)}$$

$$|P| = \sqrt{|R+j\omega L|\,|G+j\omega C|}$$

$$\sqrt{\alpha^2+\beta^2} = \sqrt{\sqrt{(R^2+\omega^2L^2)}\sqrt{(G^2+\omega^2C^2)}}$$

$$= \sqrt{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$$

$$|P|^2 = \left(\sqrt{\alpha^2+\beta^2}\right)^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \quad —(2)$$

From Eq^n (1) and (2), we have

$$\alpha^2-\beta^2 = RG-\omega^2 LC, \quad \alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

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a) Variable L :-

Consider L to be variable while R, C and G are the constants for the frequency under consideration.
Hence, for minimum attenuation, differentiating of α w.r.to L is 0.

i.e $\boxed{\dfrac{d\alpha}{dL} = 0.}$

Then now,

$$\cancel{\alpha^2+\beta^2}+\alpha^2-\cancel{\beta^2} = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + RG - \omega^2LC.$$

$$2\alpha^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (RG - \omega^2LC)$$

$$\alpha^2 = \frac{1}{2}\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (RG - \omega^2LC)$$

$$\boxed{\alpha = \sqrt{\frac{1}{2}\left\{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (RG - \omega^2LC)\right\}}}$$

$$\alpha^2+\beta^2-(\alpha^2-\beta^2) = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - (RG - \omega^2LC)$$

$$\cancel{\alpha^2}+\beta^2-\cancel{\alpha^2}+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - (RG - \omega^2LC)$$

$$2\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (\omega^2LC - RG)$$

$$\beta^2 = \frac{1}{2}\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (\omega^2LC - RG)$$

$$\boxed{\beta = \sqrt{\frac{1}{2}\left\{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (\omega^2LC - RG)\right\}}}$$

From the above equation, we can say that α and β both are frequency.

$\alpha = f(\omega)$, $\beta = f(\omega)$.

When this is done artificially, then the line is said to be loaded line and the process of achieving this condition is called loading of a line.

The value of α in terms of primary constants is given as below.

$$\alpha = \sqrt{\left\{ \frac{1}{2}\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (RG-\omega^2LC) \right\}}$$

Hence attenuation constant depends on the four primary constants along with the frequency under considerations. Thus to find the conditions for minimum attenuation it is necessary to vary these constants in turn.

(a) Variable L:-

Consider L to be variable while R, C and G are the constants for frequency under consideration. Hence for minimum attenuation, differentiation of α with respect to L is 0.

$$\frac{d\alpha}{dL} = 0$$

$$\frac{d\alpha}{dL} = \frac{d}{dL}\left\{ \frac{1}{2}\left[\{(R^2+\omega^2L^2)(G^2+\omega^2C^2)\}^{1/2} + (RG-\omega^2LC) \right] \right\}^{\frac{1}{2}}$$

$$= \frac{1}{2}\left\{ \frac{1}{2}\left[\{(R^2+\omega^2L^2)(G^2+\omega^2C^2)\}^{1/2} + RG - \omega^2LC \right] \right\}$$

$$\times \frac{1}{2}\left\{ \left[\frac{1}{2}\{(R^2+\omega^2L^2)(G^2+\omega^2C^2)\}^{1/2-1} \right] [2\omega^2L(G^2+\omega^2C^2)] - \omega^2C \right\}$$

$$\frac{d\alpha}{dL} = \frac{1}{2}\left\{ \frac{1}{2} \frac{(2\omega^2L(G^2+\omega^2C^2))}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} - \omega^2C \right\}$$

$$\frac{1}{2}\left[\{(R^2+\omega^2L^2)(G^2+\omega^2C^2)\} + RG - \omega^2CL \right]$$

$$\frac{d\alpha}{dL} = 0 = \frac{\frac{1}{2} \times \left[\frac{1}{2}\left\{\frac{1}{2} \cdot \frac{2\omega^2 L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} - \omega^2 C\right\}\right]}{\sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (RG-\omega^2LC)\right]}}$$

The denomination be ∞ to satisfy the above equation

$$\frac{\omega^2 L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} - \omega^2 C = 0$$

Since, numerator is 0, $\frac{d\alpha}{dL} = 0$

$$\frac{\cancel{\omega^2} L(G^2+\omega^2C^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}} = \cancel{\omega^2} C.$$

$$L(G^2+\omega^2C^2) = C\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\frac{L\sqrt{(G^2+\omega^2C^2)}\sqrt{G^2+\omega^2C^2}}{\cancel{\sqrt{G^2+\omega^2C^2}}} = C\sqrt{(R^2+\omega^2L^2)}$$

$$L\sqrt{G^2+\omega^2C^2} = C\sqrt{(R^2+\omega^2L^2)}$$ → Squaring on both sides

$$L^2(G^2+\omega^2C^2) = C^2(R^2+\omega^2L^2)$$

$$L^2G^2 + \cancel{L^2\omega^2C^2} = R^2C^2 + \cancel{C^2\omega^2L^2}$$

$$L^2G^2 = R^2C^2$$, Taking square root on both sides

$$\sqrt{(LG)^2} = \sqrt{(R^2C^2)}$$

$$LG = RC \Rightarrow \boxed{L = \frac{RC}{G} \ H/km}$$

Thus when L is variable, then the attenuation will be min. In practise, L is kept less than the above value

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(b) Variable C :-

Consider C to be variable while R, L and G are constants. Hence differentiating α with respect to C and equating it to zero, condition for minimum attenuation can be obtained. $\frac{d\alpha}{dC}=0$

$$\frac{d\alpha}{dC}=\frac{d}{dC}\left\{\frac{1}{2}\left[\left\{(R^2+\omega^2L^2)(G^2+\omega^2C^2)\right\}^{1/2}+RG-\omega^2LC\right]\right\}^{1/2}$$

$$=\frac{1}{2}\left\{\frac{1}{2}\left[\left\{(R^2+\omega^2L^2)(G^2+\omega^2C^2)\right\}^{1/2}+RG(\omega^2LC)\right]\right\}^{1/2}$$

$$\times\frac{1}{2}\left\{\left[\frac{1}{2}\left\{(R^2+\omega^2L^2)(G^2+\omega^2C^2)\right\}^{\frac{1}{2}-1}+RG-\omega^2LC\right]\right\}^{\frac{1}{2}-1}$$

$$\times\frac{1}{2}\left\{\left[\frac{1}{2}\left\{(R^2+\omega^2L^2)(G^2+\omega^2C^2)\right\}^{\frac{1}{2}-1}\right]\right.$$

$$\left.\left[2\omega^2C(R^2+\omega^2L^2)\right]-\omega^2L\right\}$$

$$\frac{d}{dC}(\alpha)=\frac{\frac{1}{2}\times\frac{1}{2}\left\{\frac{1}{2}\cdot\frac{2\omega^2C(R^2+\omega^2L^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}-\omega^2L\right\}}{\sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}+RG-\omega^2LC\right]}}=0$$

Hence, to satisfy $\frac{d\alpha}{dC}=0$, numerator should be 0.

$$\frac{\omega^2C(R^2+\omega^2L^2)}{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}-\omega^2L=0$$

$$\therefore \cancel{\omega^2}C(R^2+\omega^2L^2)=\cancel{\omega^2}L\cdot\sqrt{R^2+\omega^2L^2}\sqrt{(G^2+\omega^2C^2)}$$

$$C\sqrt{R^2+\omega^2L^2}=L\sqrt{G^2+\omega^2C^2}$$ squaring on both sides

$$C^2(R^2+\omega^2L^2)=L^2(G^2+\omega^2C^2)$$

$$R^2C^2+\cancel{C^2\omega^2L^2}=L^2G^2+\cancel{L^2\omega^2C^2}$$

$$R^2C^2=L^2G^2$$, taking root on both side

$$RC=LG$$

$$\boxed{C = \frac{LG}{R} \text{ F/km}}$$

In practice, normally larger than the value required for the min. attenuation.

(c) R and G for minimum Attenuation

We generally know, when $R=0$, there are no losses along the line while when $G=0$, there is no leakage thus in all when R and G are zero, the attenuation is 0. Hence for minimum attenuation practically R and G values must be kept as small as possible.

LOADING / TYPES OF LOADING:-

If the primary constants of a line, mutually satisfy the relation $RC = LG$, then the distortionless transmission always zero.

For a practical line, R/G is always more than L/C and hence the signal is distorted. Thus the preventive remedy is to make the condition $\frac{R}{G} = \frac{L}{C}$ satisfy artificially.

To satisfy the condition, it is necessary to reduce R/G or increase L/C. Let us consider all the possibilities. To reduce R/G, it is necessary to reduce R or increase G. The resistance R can be decreased by increase area of cross section, i.e diameter of conductor. i.e $R = \frac{\rho L}{A}$, $(R \propto \frac{1}{A})$. This increases the cost of the line. Hence this possibility is uneconomical.

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→ Next to increase G, it is necessary to use poor conductors. $(c \propto \frac{1}{G})$. To get poor insulator is easy and economical but from receiving end point of view, increase in G is very much economical. When G increase leakage increase, though it becomes distorsionless. So, quality increases but quantity decreases. Thus increase in G is quality at the cost of quantity. The signal at receiving end must activate the receivers, But if leakage is more, then the received signal becomes so weak that amplifiers are required at the intermediate stage. Then it cause the design complicated and hence the design become a worst case.

→ Now to increase L/c, it is necessary to increase L or decrease C. If C is to be reduced, then the seperation between the lines will be more. Since $C = \frac{\epsilon S}{d}$, d → seperation between lines, so, to increase the transmission line seperation, more no. brackets increased. Taller towers and posts are required and also no. of towers and posts per unit length of line will be increased. Thus for the same strength of the line, the line become very much costlier due to decrease in C. This is also not well suitable.

→ The only altarnate is to increase L. Thus the process of increasing the inductance L of a line artifically called loading of a line.

And such a line is called Loaded line.

There are two methods of loading a line which are

a) Continuous loading | krarup loading | Heavyside Loading.

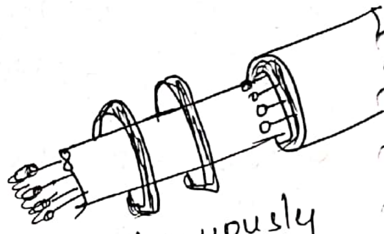
(b) Lump Loading | Pupin Loading | Coil loading.

(a) Continuous loading

In this method of loading to increase the inductance, on each conductor, the tapes of magnetic material having high permeability such as permalloy or μ-metal are wound.

The increase in the inductance for a continuously loaded line is

$$\boxed{L \approx \frac{\mu}{\frac{d}{nt}+1}} \text{ mH.}$$



Continuously loaded cable

μ – permeability of surrounding material

D – Diameter of copper conductor

t – thickness per layer of tape or iron wire

n – no. of layers.

The attenuation factor $\boxed{\alpha = \frac{R}{2}\sqrt{L/C}}$, $\boxed{\beta = \omega\sqrt{LC}}$

$$\boxed{v = \omega/\beta = \frac{1}{\sqrt{LC}}}$$

α, v is not function of frequency, i.e independent of frequency. And hence Continuously loaded Cable is distortionless

Advantages:- The advantages of continuous loading is are,

→ The attenuation to the signal is independent of frequency and it is same to all the frequencies

→ The attenuation can be reduced by increasing L, provided that R is not increased greatly.

Disadvantages:-

→ The method is very costly.

→ Existing lines cannot be modified by this method. Hence total replacement of the existing cables by the new cables wound with magnetic tapes is required. This is again costly and uneconomical

→ Extreme precision care must be taken while while manufacturing continuously loaded cable, otherwise it becomes nullified.

→ Thus, the size increased. Thus capacitance is increased. Hence ~~getting~~ benefit obtained by increasing L is partly nullified.

→ All along the conductor, there will be huge mass of iron. Thus for a.c signals there will be large eddy current and hysteresis losses The eddy current losses increase directly with square of frequency while the hysteresis losses increase directly with the frequency. Hence, overall this puts the limit to increase the inductance.

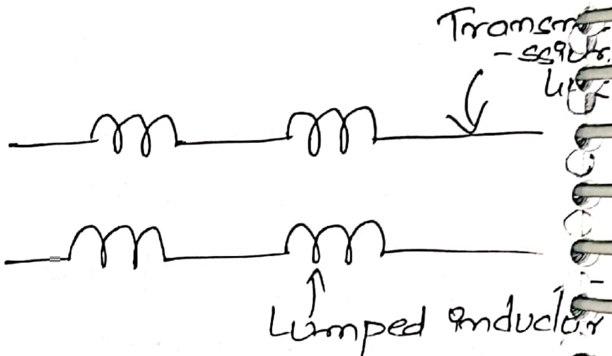
6) The value of inductance increased is fixed upto 100mH.

Applications:-

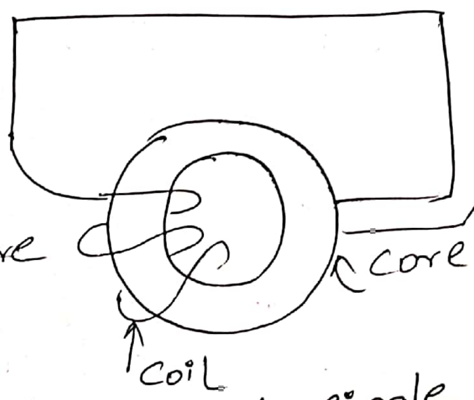
This method is used for submarine cables. For underwater circuits lumped loading is difficult to use. It is not necessary to load the submarine cable continuously while the sections of loaded cable seperated by the sections of unloaded cable, can be used. This reduces the cost, still enjoying the advantages of continuous loading. This is called Patch loading.

(b) Lumped Loading:-

In this type of loading, the inductors are introduced in lumps at the uniform distances, in the line. Such inductors are called Lumped inductors. The inductors are introduced in both the limbs to keep the line as balanced circuit. The lumped inductors are in the form of coils called loading coils. In this, the core of the coils is usually provided as torodial in shape and made of permalloy. This core type of core produces the coil of high inductance, having small dimensions, very low eddy current possible and negligible field which restrict the intaferance with neighbouring circuits.

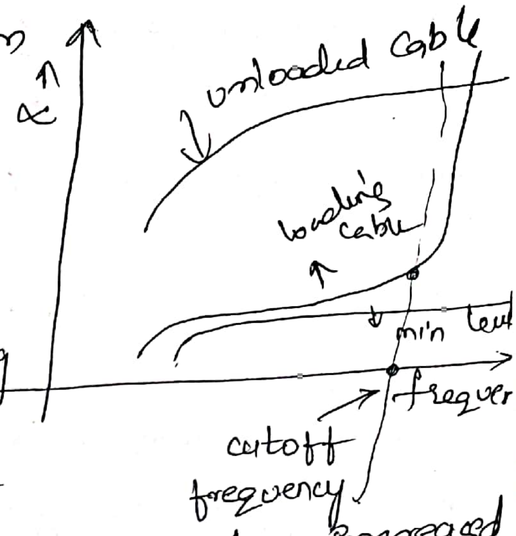


Lumped Loading



Winding of single Loading coil.

Incase of lumped loading, upto a certain frequency called cut-off frequency, attenuation constant is less. After this, the attenuation constant is increased. Hence the lumped loaded line acts as Low Pass Filter.



Advantages:-

The advantages of lumped loading are,

a) There is no practical limit the value by which the inductance can be increased

b) The cost involved is small.

c) With this method, the existing lines can be tackled and modified.

d) Hystaisis and eddy current losses are small.

Disadvantages:-

The only disadvantage of this method it acts like low pass filter. The attenuation increases considerably after the cut-off frequency. The cutoff frequency at the top of voice frequency. Hence fractional loading is used. Whatever distorsion results in ta ~~frequ~~ fractional loading is corrected using equalizers. The care must be taken while installing the lumped inductors so as to maintain exact balancing of the circuit.