## UNIT - I

## INTRODUCTION

**Concepts of Control systems:** Open loop and closed loop control systems and their differences, Different examples of control systems, classification of control systems, Feedback characteristics, effects of feedback, Mathematical models - Differential equations, Impulse response and transfer functions.

**Transfer function Representation:** Block diagram representation of systems considering electrical systems as examples - Block diagram algebra, Representation of signal flow graph, Block diagram reduction using Mason's gain formula.

To understand the meaning of the word control system, first we will define the word system and then we will try to define the word control system.

**System :** A system is a combination or an arrangement of different physical components which act together as an entire unit to achieve certain objective.

Every physical object is actually a system. A classroom is a good example of physical system. A room along with the combination of benches, blackboard, fans, lighting arrangement etc. can be called a classroom which acts as an elementary system.

Another example of a system is a lamp. A lamp made up of glass, filament is a physical system. Similarly a kite made up of paper and sticks is an example of a physical system.

Similarly system can be of any type i.e. physical, ecological, biological etc.

**Control system :** To control means to regulate, to direct or to command. Hence a control system is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself or some other system.

For example if in a classroom, professor is delivering his lecture, the combination becomes a control system as; he tries to regulate, direct or command the students in order to achieve the objective which is to input good knowledge to the students. Similarly if lamp is switched ON or OFF using a switch, the entire system can be called a control system. The concept of physical system and a control system is shown in the Fig.1.1 and Fig.1.2

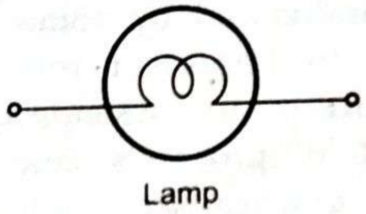


**Fig. 1.1 Physical system**

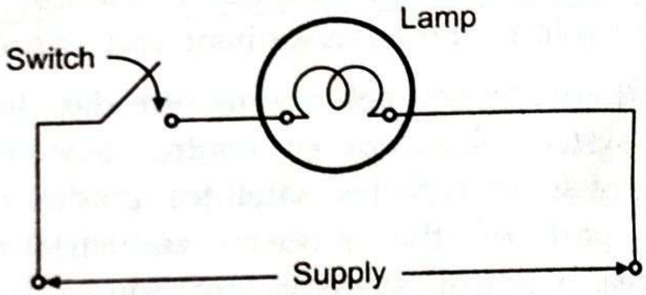


**Fig. 1.2 Control system**

When a child plays with the kite, he tries to control it with the help of string and entire system can be considered as a control system.

In short, a control system is in the broadest sense, an interconnection of the physical components to provide a desired function, involving some kind of controlling action in it.

**Plant :** The portion of a system which is to be controlled or regulated is called the **plant or the Process.**

**Controller :** The element of the system itself or external to the system which controls the plant or the process is called **controller**.

For each system, there must be an excitation and system accepts it as an input. And for analyzing the behaviour of system for such input, it is necessary to define the output of a system.

**Input :** It is an applied signal or an excitation signal applied to a control system from an external energy source in order to produce a specified output.

**Output :** It is the particular signal of interest or the actual response obtained from a control system when input is applied to it.

**Disturbances :** Disturbance is a signal which tends to adversely affect the value of the output of a system. If such a disturbance is generated within the system itself, it is called an **internal disturbance**. The disturbance generated outside the system acting as an extra input to the system in addition to its normal input, affecting the output adversely is called an **external disturbance**.

Control systems may have more than one input or output. From the information regarding the system, it is possible to well define all the inputs and outputs of the systems.

The input variable is generally referred as the **Reference Input** and Output is generally referred as the **Controlled output**.

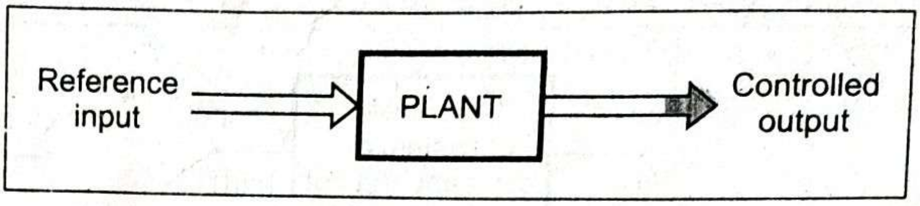


**Fig. 1.3**

Cause and effect relationship between input and output for a plant can be shown as in the Fig. 1.3.

## 1.3 Classification of Control Systems

Broadly control systems can be classified as,

1) **Natural Control Systems :** The biological systems, systems inside human being are of natural type.

**Ex.1 :** The perspiration system inside the human being is a good example of natural control system. This system activates the secretion glands, secreating sweat and regulates the temperature of human body.

2) **Manmade Control Systems :** The various systems, we are using in our day to day life are designed and manufactured by human beings. Such systems like vehicles, switches, various controllers etc. are called manmade control systems.

**Ex.2 :** An automobile system with gears, accelerator, braking system is a good example of manmade control system.

3) **Combinational Control Systems :** Combinational control system is one, having combination of natural and manmade together: i.e. driver driving a vehicle. In such system, for successful operation of the system, it is necessary that natural systems of driver alongwith systems in vehicles which are manmade must be active.

But for the engineering analysis, control systems can be classified in many different ways. Some of the classifications are given below.

4) **Time Varying and Time - Invariant Systems :** Time varying control systems are those in which parameters of the systems are varying with time. It is not dependent on whether input and output are functions of time or not. For example, space vehicle whose mass decreases with time, as it leaves earth. The mass is a parameter of space vehicle system. Similarly in case of a rocket, aerodynamic damping can change with time as the air density changes with the altitude. As against this if even though the inputs and outputs are functions of time but the parameters of system are independent of time, which are not varying with time and are constants, then system is said to be time invariant system. Different electrical networks consisting of the elements as resistances, inductances and capacitances are time invariant systems as the values of the elements of such system are constant and not the functions of time. The complexity of the control system design increases considerably if the control system is of the time varying type. This classification is shown in the Fig.1.4.

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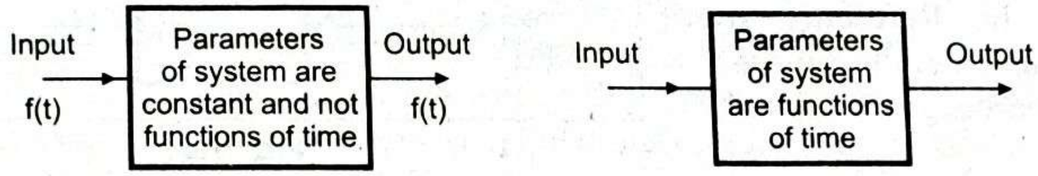



**(a) Time invariant system** **(b) Time variant system**

**Fig. 1.4**

5 **Linear and Nonlinear Systems** : A control system is said to be linear i superposition principle applies to it. For linear systems the response to severa forcing functions can be calculated by considering one forcing function at a tim and adding the results.

The system is said to be linear if it satisfies following two properties,

i) Additive property that is for any x and y belonging to the domain of th function f, we have

$$f(x + y) = f(x) + f(y)$$

ii) Homogeneous property that is for any x belonging to the domain of the functio f and for any scalar constant $\alpha$, we have.

$$f(\alpha x) = \alpha \cdot f(x)$$

These two properties together constitute a principle of superposition.

Hence the transformation, operation, function which satisfies above two propertie is called linear in nature.

The function $f(x) = x^2$ is nonlinear as,

$$(x_1 + x_2)^2 \neq x_1^2 + x_2^2$$

and $$(\alpha x)^2 \neq \alpha (x)^2$$

It is very difficult to have a linear system satisfying the above two properti perfectly. All the physical systems are nonlinear to some extent. But if the presence certain nonlinearity is not affecting the performances of system much, as per the abo two properties and deviation of system from the principle of superposition negligible, the presence of nonlinearity is neglected and the system can be assumed be linear from the analysis point of view.

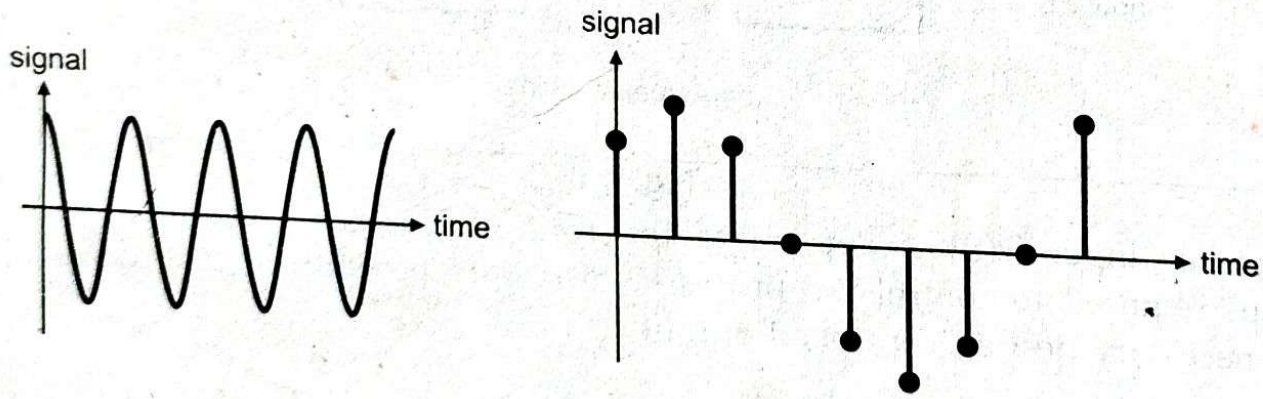
In practice most of the physical systems are nonlinear in nature because different non-linearities present in the system i.e. saturation, friction, deadzone. Such systems are nonlinear systems for which principle of superposition cannot applied. Procedures for finding the solutions of nonlinear system problems complicated and time consuming. Because of this difficulty generally nonlin systems are treated as linear systems for a limited range of operation with s approximation. Then number of linear methods can be applied for analysis of s systems.

6) **Continuous Time and Discrete Time Control Systems :** In a continuous time control system all system variables are the functions of a continuous time variable 't'. The speed control of a d.c. motor using a tachogenerator feedback is an example of continuous data system. At any time 't' they are dependent on time. In discrete time systems one or more system variables are known only at certain discrete intervals of time. They are not continuously dependent on the time. Microprocessor or computer based systems use such discrete time signals. The reasons for using such signals in digital controllers are,

1) Such signals are less sensitive to noise.
2) Time sharing of one equipment with other channels is possible.
3) Advantageous from point of view of size, speed, memory, flexibility etc.

The systems using such digital controllers or sampled signals are called sampled data systems.

Continuous time system uses the signals as shown in the Fig. 1.5(a) which are continuous with time while discrete system uses the signals as shown in the Fig.1.5(b).




**(a) Continuous signal** **(b) Discrete signal**

**Fig. 1.5**

7) **Deterministic and Stochastic Control Systems :** A control system is said to be deterministic when its response to input as well as behaviour to external disturbances is predictable and repeatable. If such response is unpredictable, system is said to be stochastic in nature.

8) **Lumped Parameter and Distributed Parameter Control Systems :** Control system that can be described by ordinary differential equations is called lumped parameter control system. For example electrical networks with different parameters as resistance, inductance, etc. are lumped parameter systems. Control systems that can be described by partial differential equations are called distributed parameter control systems. For example, transmission line having its parameters resistance and inductance totally distributed along it. Hence description of transmission line characteristics is always by use of partial differential equations.

9) **Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) Systems** : A system having only one input and one output is called single input single output system. For example a position control system has only one input (desired position) and one output (actual output position). Some systems may have multiple type of inputs and multiple outputs, these are called multiple input multiple output systems.

10) **Open loop and Closed Loop Systems** : This is another important classification. The features of both these types are discussed in detail in coming sections.

## 1.4 Open Loop System

**Definition** : A system in which output is dependent on input but controlling action or input is totally independent of the output or changes in output of the system, is called an Open Loop System.

In a broad manner it can be represented as in Fig. 1.6.

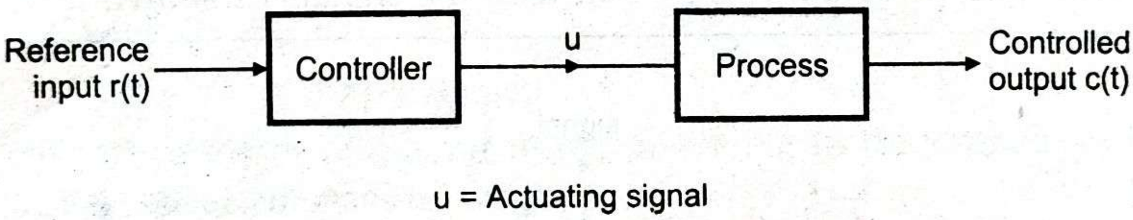



**Fig. 1.6**

Reference input [r(t)] is applied to the controller which generates the actuating signal (u) required to control the process which is to be controlled. Process is giving out the necessary desired controlled output c(t).

### 1.4.1 Advantages

The advantages of open loop control system are,

1) Such systems are simple in construction.
2) Very much convenient when output is difficult to measure.
3) Such systems are easy from maintenance point of view.
4) Generally these are not troubled with the problems of stability.
5) Such systems are simple to design and hence economical.

### 1.4.2 Disadvantages

The disadvantages of open loop control system are,

1) Such systems are inaccurate and unreliable because accuracy of such systems are totally dependent on the accurate precalibration of the controller.
2) Such systems give inaccurate results if there are variations in the external environment i.e. such systems cannot sense environmental changes.

3) Similarly they cannot sense internal disturbances in the system, after the controller stage.

4) To maintain the quality and accuracy, recalibration of the controller is necessary, time to time.

To overcome all the above disadvantages, generally in practice closed loop systems are used.

The good example of an open loop system is an electric switch. This is open loop because output is light and switch is controller of lamp. Any change in light has no effect on the ON-OFF position of the switch, i.e. its controlling action.

Similarly automatic washing machine. Here output is degree of cleanliness of clothes. But any change in this output will not affect the controlling action or will not decide the operation time or will not decide the amount of detergent which is to used. Some other examples are traffic signal, automatic toaster system etc.

## 1.5 Closed Loop System

**Definition :** A system in which the controlling action or input is someho dependent on the output or changes in output is called closed loop system.

To have dependence of input on the output, such system uses the feedba property.

**Feedback :** Feedback is a property of the system by which it permits the output be compared with the reference input so that appropriate controlling action can l decided.

In such system, output or part of the output is fedback to the input f comparison with the reference input applied to it.

Closed loop system can be represented as shown in the Fig. 1.9.

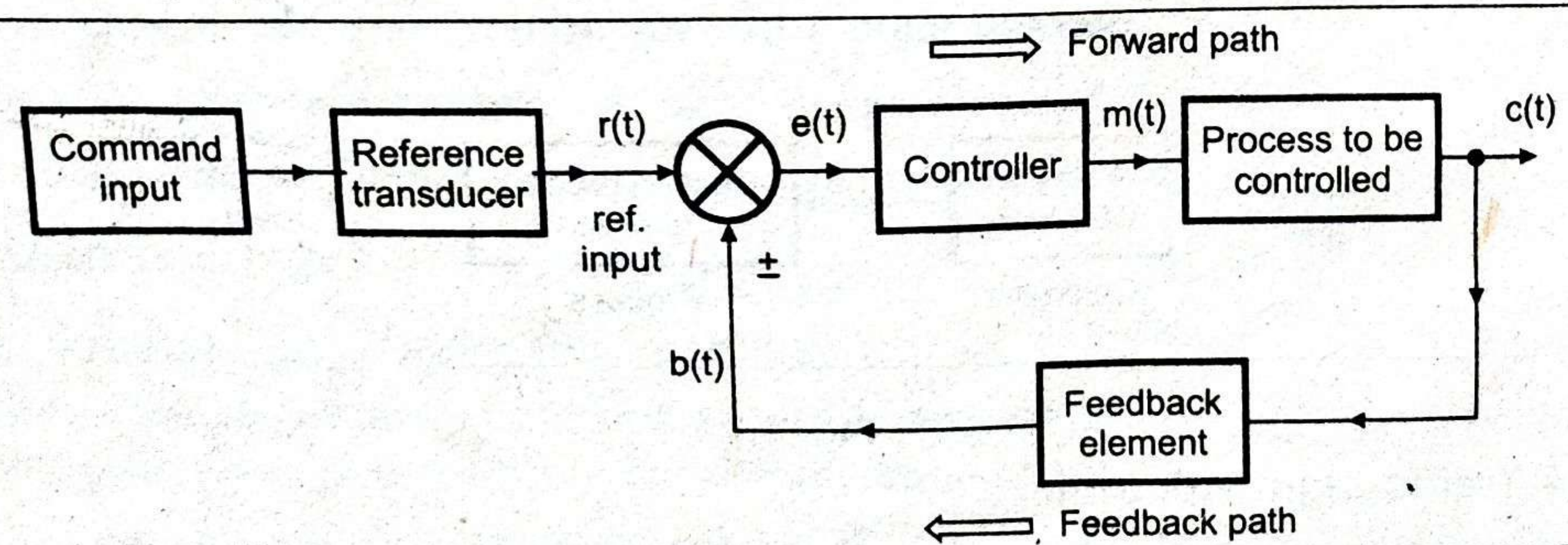



**Fig. 1.9**

r(t) = Reference input e(t) = Error signal

$c(t)$ = Controlled output   $m(t)$ = Manipulated signal   $b(t)$ = Feedback signal

It is not possible in all the systems that available signal can be applied as input to the system. Depending upon nature of controller and plant it is required to reduce it or amplify it or to change its nature i.e. making it discrete from continuous type of signal etc. This changed input as per requirement is called **reference input** which is to be generated by using reference transducer. The main excitation to the system is called its **command input** which is then applied to the reference transducer to generate reference input.

The part of output, which is to be decided by feedback element is fed back to the reference input. The signal which is output of feedback element is called '**feedback signal**' $b(t)$.

It is then compared with the reference input giving error signal $e(t) = r(t) \pm b(t)$

When feedback sign is positive, systems are called positive feedback systems and if it is negative systems are called negative feedback systems.

This error signal is then modified by controller and decides the proportional manipulated signal for the process to be controlled.

This manipulation is such that error will approach to zero. This signal then actuates the actual system and produces an output. As output is controlled one, hence called **controlled output** $c(t)$.

### 1.5.1 Advantages

The advantages of closed loop system are,

1) Accuracy of such system is always very high because controller modifies and manipulates the actuating signal such that error in the system will be zero.
2) Such system senses environmental changes, as well as internal disturbances and accordingly modifies the error.
3) In such system, there is reduced effect of nonlinearities and distortions.
4) Bandwidth of such system i.e. operating frequency zone for such system is very high.

### 1.5.2 Disadvantages

The disadvantages of closed loop system are,

1) Such systems are complicated and time consuming from design point of view and hence costlier.
2) Due to feedback, system tries to correct the error time to time. Tendency to overcorrect the error may cause oscillations without bound in the system. Hence system has to be designed taking into consideration problems of instability due to feedback.

## .6 Comparison of Open Loop and Closed Loop Control System

| | Open Loop | | Closed Loop |
|---|---|---|---|
| 1) | Any change in output has no effect on the input i.e. feedback does not exists. | 1) | Changes in output, affects the input which is possible by use of feedback. |
| 2) | Output measurement is not required for operation of system. | 2) | Output measurement is necessary. |
| 3) | Feedback element is absent. | 3) | Feedback element is present. |
| 4) | Error detector is absent. | 4) | Error detector is necessary. |
| 5) | It is inaccurate and unreliable. | 5) | Highly accurate and reliable. |
| 6) | Highly sensitive to the disturbances. | 6) | Less sensitive to the disturbances. |
| 7) | Highly sensitive to the environmental changes. | 7) | Less sensitive to the environmental changes. |
| 8) | Bandwidth is small. | 8) | Bandwidth is large. |
| 9) | Simple to construct and cheap. | 9) | Complicated to design and hence costly. |
| 10) | Generally are stable in nature. | 10) | Stability is the major consideration while designing |
| 11) | Highly affected by nonlinearities. | 11) | Reduced effect of nonlinearities. |

Examples of Control systems:

1. Traffic control system:

Traffic control by means of traffic signals operated on a time basis are called open loop control systems. The sequence of control signals is based on a time slot given for each signal. The time slots are decided based on a traffic study. The system will not measure the density of the traffic before giving the signals. It gives the signals in sequence as per the setting irrespective of the actual traffic. Since the time slots do not change according to the density, the system is an open loop system.

This open loop system can be made as a closed loop system if the time slots of the signals are based on the density of traffic. In a closed loop control system, the density of the traffic is measured in all the sides and the information is fed to a computer. Now the timings of control signals are decided by the computer based on the traffic density. Since the closed loop system dynamically changes the timings, the flow of vehicles will be better than in that of an open loop system.

2. Room heating system:

A room heater without any temperature sensing device is an example of open loop control system. In this an electric furnace is used to heat the room. The output is the desired room temperature. The temperature of the room is risen by the heat generated by the heating element. The output temperature depends on the time during which the supply to the heater remains ON. ON-OFF time will be set as per some calculation. After set time, whatever may be temperature, the heater will be OFF. The actual temperature is not compared with the reference temperature and the difference is not used for correction.

The above system becomes a closed loop system if a thermostat is provided to measure the actual temperature and it is compared with the reference temperature and the difference is used to control the timing for which the heater is ON.

3. Washing machine:

A washing machine without any cleaniness measuring system is an example of open loop CS. In this, the soaking, washing and rinsing in the washer operate on a time basis. The machine ON time is set based on some calculation. The machine does not measure the output signal, i.e. cleaniness of the clothes. once the set ON time is over, the machine will automatically stop, whatever may be the level of cleaniness.

This can be modified as closed loop, if the level of cleanliness can be measured and compared with the desired cleaniness and the difference is used to control the washing time of the machine.

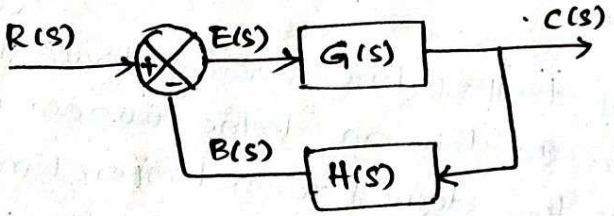
Feedback & its effects:

The purpose of the feedback is to reduce the error between the reference input and the system output.

In general, we can state that whenever a closed sequence of cause-and-effect relationship exist among the variables of a system, feedback is said to exist.

Feedback effect the system performance characteristics such as Stability, bandwidth, overall gain, external disturbance & sensitivity.

Consider a single loop control system shown in Fig below.




Feedback system with one feedback loop.

From the Fig.

$$G(S) = \frac{C(S)}{E(S)} \Rightarrow C(S) = G(S)E(S) \quad \text{—} ①$$

$$H(S) = \frac{B(S)}{C(S)} \Rightarrow B(S) = H(S)\,C(S) \quad \text{—} ②$$

$$E(S) = R(S) - B(S) \quad \text{—} ③$$

From ② & ③

$$E(S) = R(S) - H(S)\,C(S) \quad \text{—} ④$$

From ① & ④

$$E(S) = R(S) - H(S)G(S)E(S)$$

$$E(S) + H(S)G(S)E(S) = R(S)$$

$$E(S)\left[1 + G(S)H(S)\right] = R(S) \quad \text{—} ⑤$$

But $E(S) = \frac{C(S)}{G(S)}$ — ⑥

From eqn ⑤ & ⑥

$$\frac{C(S)}{G(S)}\left[1 + G(S)H(S)\right] = R(S)$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)} \quad \text{—} ⑦$$

∴ input-output relation of a single loop control system is

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)} \quad \text{i.e.} \quad \frac{c}{r} = \frac{G}{1 + GH} \quad \text{—} ⑧$$

## Effect of ~~bad~~ feedback on Overall gain:

Feedback affects the open loop gain G by a factor $\frac{1}{1+GH}$. Since a minus sign is assigned to the feedback signal, the system said to have negative feedback. The quantity GH may itself include a -ve sign, so the effect of feedback is that it may increase or decrease the gain.

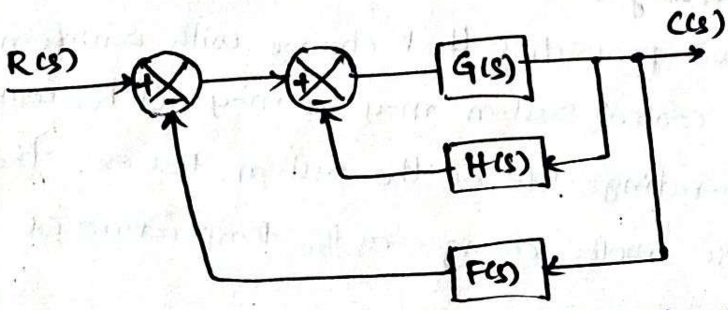
## Effect of feedback on stability:

Stability is a parameter that describes whether the system will be able to follow the input command. A system is said to be unstable if its output is out of control.

From eqn (8) if $GH = -1$, the output of the system is infinite for any input and hence the system is unstable.

One of the advantage of incorporating feedback is that it can stabilize an unstable system. (For the case $GH = -1$ in eqn 8)

If another feedback loop is introduced with -ve feedback gain of F as shown below.




Input-output relation of the overall system is given by

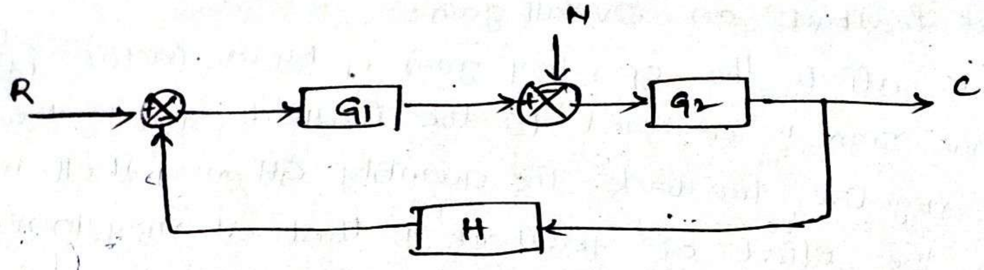
$$\frac{C}{R} = \frac{G}{1+GH+GF} \quad \text{——— (9)}$$

Since $GH = -1$, inner loop feedback system is unstable. But the overall system can be stable by proper selection of the outer loop feedback gain F. So we can conclude that feedback can improve stability or be harmful to stability if is not applied properly.

## Effect of feedback on external disturbance or Noise:

All the physical systems are subject to some types of extraneous signals or noise during the operation. Therefore, in design of a control system, consideration should be given so that the system is insensitive to noise and disturbance and sensitive to input commands.

The effect of feedback on noise and disturbance depends greatly on where these extraneous signals occurs in the system.




In many systems feedback can reduce the effect of noise and disturbance on system performance.

From the figure, in the absence of feedback, $H=0$, the output c due to noise acting alone is

$$C = G_2 n$$

with the presence of feedback, the system output due to n acting alone is

$$C = \frac{G_2}{1+G_1G_2H} n$$

i.e, noise component in the output of a system is reduced by a factor $1+G_1G_2H$

4. Effect of feedback on sensitivity :

All the physical elements have properties that change with environment and age. The parameters of a control system may or may not be completely stationary over the entire operating life of the system. For ex. the winding resistance of an electric motor changes as the temperature of the motor rises during operation.

A good control system should be very insensitive to parameter variations but sensitive to input commands.

The sensitivity of the gain of the overall system (M) to the variation in G is defined as

$$S_G^M = \frac{\partial M/M}{\partial G/G} = \frac{\%\text{ change in } M}{\%\text{ change in } G} = \frac{G}{M}\cdot\frac{1}{1+GH}$$

If GH is +ve, the magnitude of the sensitivity function can be made arbitrarily small by increasing GH, provided that the system remains stable.

## Mathematical models of control systems:

A control system is a collection of physical components connected together to serve an objective. The input output relations of various physical components of a system are governed by differential equations.

The mathematical model of a control system constitutes a set of differential equations. The response of the system can be studied by solving the differential equations for various input conditions.

The mathematical model of a system is linear if it obeys the principle of superposition and homogenity. This principle implies that if a system has responses $y_1(t)$ and $y_2(t)$ to the inputs $x_1(t)$ and $x_2(t)$ respectively, then the system response to the linear combination of these inputs $a_1x_1(t)+a_2x_2(t)$ is given by linear combination of the individual outputs $a_1y_1(t)+a_2y_2(t)$ where $a_1, a_2$ are constants

The transfer function of a system is defined as the ratio of Laplace transform of output to the laplace transform of input with zero initial conditions

$$\text{Transfer function} = \left.\frac{\text{Laplace Transform of output}}{\text{Laplace Transform of input}}\right|_{\text{with zero initial conditions}}$$

## Mechanical Translational Systems:

The model of mechanical translational systems can be obtained by using three basic elements mass, spring and dashpot.

The weight of mechanical system is represented by the element mass and it is assumed to be concentrated at the centre of the body. The elastic deformation of the body can be represented by a spring. The friction existing in rotating mechanical system can be represented by the dashpot.

When a force is applied to a translational mechanical system, it is opposed by opposing forces due to mass, friction and elasticity of the system. The force acting on a mechanical body are governed by Newton's second law of motion. Newton's second law states that the sum of applied forces is equal to the sum of opposing forces on a body.

## List of symbols used in mechanical translational system:

$x$ = displacement, m

$v = \frac{dx}{dt} \rightarrow$ velocity, m/sec

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ = acceleration, m/sec$^2$

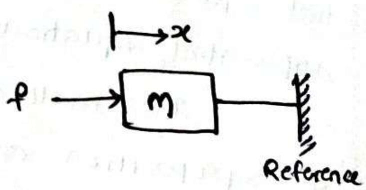
$f$ = applied force, Newtons

$f_m$ = opposing force due to mass of the body

$f_k$ = opposing force offered by the elasticity of the body
$f_b$ = opposing force offered by the friction of the body
M = mass, kg
K = Stiffness of spring, N/m
B = Viscous friction co-efficient, N-sec/m

Force balance equations idealized elements

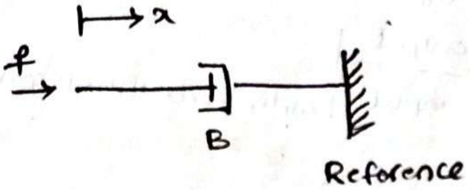
consider an Ideal mass element which has negligible friction and elasticity. let a force be applied on it. The mass will offer an opposing force which is proportional to acceleration of the body.



Ideal mass element

$$\therefore \; f_m \propto \frac{d^2x}{dt^2} \Rightarrow f_m = M\frac{d^2x}{dt^2}$$

By Newton's second law $f = f_m = M\frac{d^2x}{dt^2}$



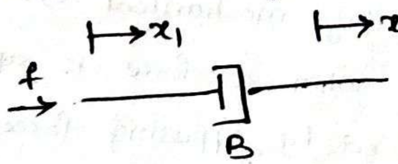
Ideal dashpot with one end fixed to refference

Consider an ideal frictional element dashpot which has negligible mass and elasticity. Let a force be applied on it. The dashpot will offer an opposing force which is proportional to the velocity of the body

$$\therefore \; f_b \propto \frac{dx}{dt} \Rightarrow f_b = B\frac{dx}{dt}$$

By Newton's second law $f = f_b = B\frac{dx}{dt}$

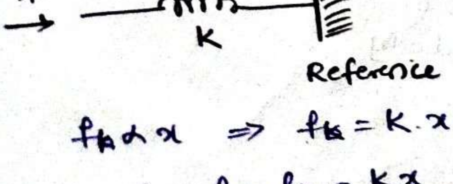
when the dashpot has displacement at both ends, Then the opposing force is proportional to differential velocity

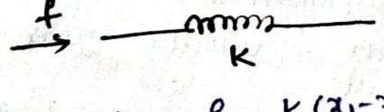



$$f_b \propto \frac{d}{dt}(x_1 - x_2)$$

$$\Rightarrow f = f_b = B\frac{d}{dt}(x_1 - x_2)$$

Consider an ideal elastic element spring which has negligible mass and friction. Let a force be applied on it. The spring will offer an opposing force which is proportional to displacement of body



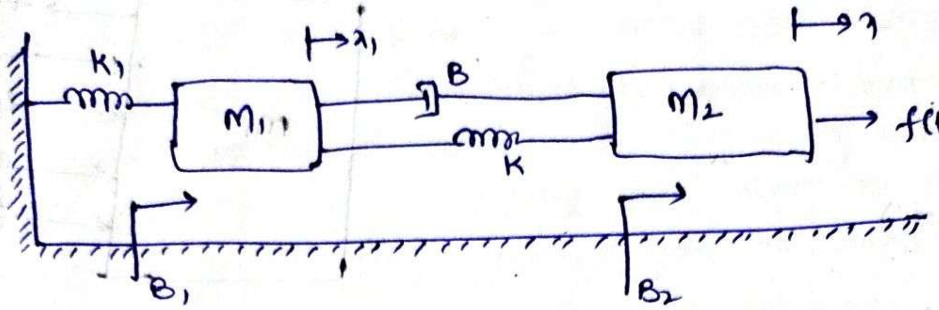

$$f_k \propto x \Rightarrow f_k = K.x$$

$$\therefore \; f = f_k = Kx$$

$$f_k \propto (x_1 - x_2) \Rightarrow f_k = K(x_1 - x_2)$$

$$\therefore \; f = f_k = K(x_1 - x_2)$$

2. Write the differential equations governing mechanical system shown in Figure. Determine the transfer Function.




Soln: In the given system, applied force $f(t)$ is the input and displacement $x$ is the output

Hence the required transfer function is $\frac{X(s)}{F(s)}$

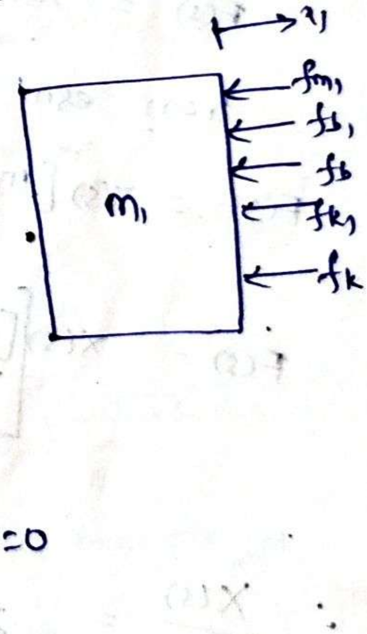
The System has two nodes and they are mass $M_1$ & $M_2$. The differential equations governing the system are given by force balance equations at these nodes

The free body diagram of mass $M_1$ is shown in fig below. The opposing forces acting on $M_1$ are marked as $f_{m_1}, f_{b_1}, f_b, f_{k_1}, f_k$

$$f_{m_1} = M_1 \frac{d^2 x_1}{dt^2} \qquad f_{b_1} = B_1 \frac{dx_1}{dt}$$

$$f_{k_1} = K_1 x_1$$

$$f_b = B \frac{d(x_1 - x)}{dt} \qquad f_k = k(x_1 - x)$$




By Newton's Second law

$$f_{m_1} + f_{b_1} + f_{k_1} + f_b + f_k = 0$$

$$\therefore M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B \frac{d}{dt}(x_1 - x) + K(x_1 - x) = 0$$

Taking Laplace Transform

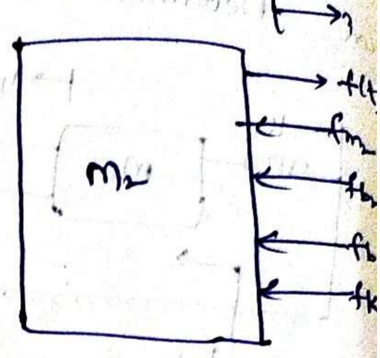
$$M_1 s^2 X_1(s) + B_1 s X_1(s) + K_1 X_1(s) + Bs\left[X_1(s) - X(s)\right] + k\left[X_1(s) - X(s)\right] = 0$$

$$X_1(s)\left[M_1 s^2 + (B_1 + B)s + (K_1 + K)\right] - X(s)\left[Bs + k\right] = 0$$

$$X_1(s)\left[M_1 s^2 + (B_1 + B)s + (K_1 + K)\right] = X(s)\left[Bs + k\right]$$

$$X_1(s) = X(s) \frac{Bs + K}{M_1 s^2 + (B_1 + B)s + (K_1 + K)} \quad \text{--- (1)}$$

The free body diagram of mass $M_2$ is shown below. The opposing forces acting on $M_2$ are marked as $f_{m_2}$, $f_{b_2}$, $f_b$ and $f_k$




$$f_{m_2} = M_2 \frac{d^2x}{dt^2}$$

$$f_{b_2} = B_2 \frac{dx}{dt}$$

$$f_b = B \frac{d}{dt}(x - x_1)$$

$$f_k = K(x - x_1)$$

By Newtons second law

$$f(t) = f_{m_2} + f_{b_2} + f_b + f_k$$

$$f(t) = M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1)$$

Take laplace Transform

$$F(s) = M_2 s^2 X(s) + B_2 s X(s) + Bs[X(s) - X_1(s)] + K[X(s) - X_1(s)]$$

$$F(s) = X(s)[M_2 s^2 + (B_2 + B)s + K] - X_1(s)[Bs + K] \quad \text{--- (2)}$$

from eqns (1) & (2)

$$F(s) = X(s)[M_2 s^2 + (B_2 + B)s + K] - X(s) \frac{(Bs + K)^2}{M_1 s^2 + (B_1 + B)s + (K_1 + K)}$$

$$F(s) = X(s)\left[\frac{[M_2 s^2 + (B_2 + B)s + K][M_1 s^2 + (B_1 + B)s + (K_1 + K)] - (Bs + K)^2}{M_1 s^2 + (B_1 + B)s + (K_1 + K)}\right]$$

$$\therefore \frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1 + B)s + (K_1 + K)}{[M_1 s^2 + (B_1 + B)s + (K_1 + K)][M_2 s^2 + (B_2 + B)s + K] - (Bs + K)^2}$$

* Mechanical Rotational Systems:

The model of rotational mechanical systems can be obtained by using three elements: moment of inertia (J), dashpot with rotational frictional coefficient (B) and a spring with stiffness K.

The weight of the rotational mechanical system is represented by the moment of inertia of the mass. The elastic deformation of the body can be represented by a spring. The friction existing in rotational mechanical system can be represented by the dash pot.

When a torque is applied to a rotational mechanical system, it is opposed by opposing torques due to moment of inertia, friction & elasticity of the system. According to the Newton's second law of motion, sum of the applied torques is equal to the sum of opposing torques on a body.

List of symbols used in mechanical rotational systems

$\theta$ - angular displacement, rad

$\frac{d\theta}{dt}$ - angular velocity, rad/sec

$\frac{d\theta}{dt^2}$ - angular acceleration, rad/sec$^2$

T - applied torque, N-m

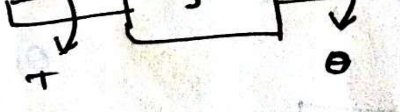
J - Moment of inertial, kg-m$^2$/rad

B - rotational friction coefficient, N-m

K - stiffness of the spring, N-m/rad

Torque-balance equations of idealized elements.

① $T_j \propto \frac{d^2\theta}{dt^2} \Rightarrow T_j = J\frac{d^2\theta}{dt^2}$

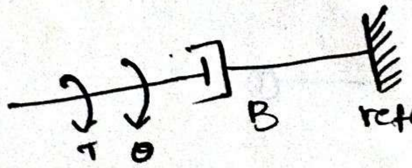



By Newton's 2nd law

$$T = T_j = J\frac{d^2\theta}{dt^2}$$

Ideal rotational mass element

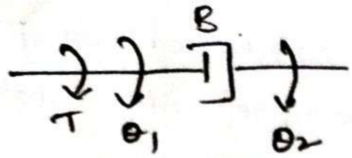
② $T_b \propto \frac{d\theta}{dt} \Rightarrow T_b = B\frac{d\theta}{dt}$




By Newton's 2nd law

$$T = T_b = B\frac{d\theta}{dt}$$

Dashpot with one end fixed to reference



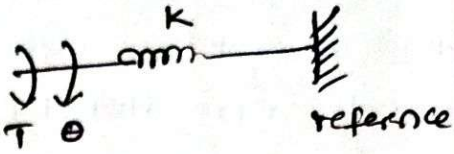

$T_b \propto \frac{d}{dt}(\theta_1-\theta_2) \Rightarrow T_b = B\frac{d}{dt}(\theta_1-\theta_2)$

By Newton's 2nd law

$$T = T_b = B\frac{d}{dt}(\theta_1-\theta_2)$$

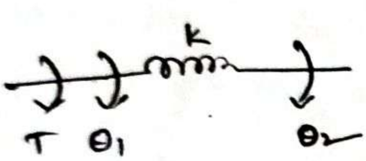
(3) $T_K \propto \theta$

$\therefore T_K = K\theta$




By Newton's second law

$$T = T_K = K\theta$$




$T_K \propto (\theta_1-\theta_2)$

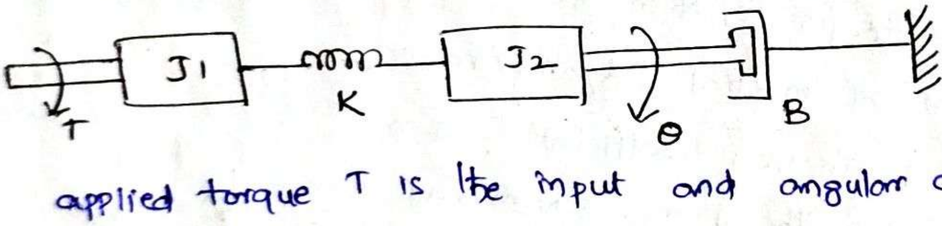
$\Rightarrow T_K = K(\theta_1-\theta_2)$

By Newton's second law

$$T = T_K = K(\theta_1-\theta_2)$$

Problems:

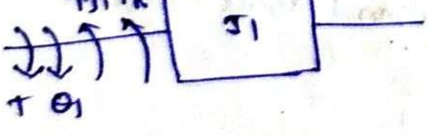
(1) Write the differential equations governing the mechanical rotational system shown in figure below. Obtain the transfer function of the system.




Soln: applied torque T is the input and angular displacement $\theta$ is the output

The required transfer function is $\frac{\theta(s)}{F(s)}$

Free body diagram of mass with moment of inertia $J_1$ is shown below




$\theta_1$ is the angular displacement of mass with moment of inertia $J_1$

$T_{J1} = J_1 \frac{d^2\theta_1}{dt^2}$ $\quad T_K = K(\theta_1-\theta)$

By Newton's 2nd law $T = T_{J1} + T_K$

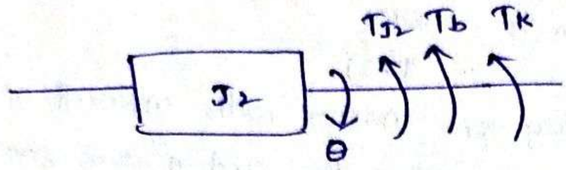
$$T = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1-\theta) \quad \text{---} (1)$$

Apply Laplace Transform

$$T(s) = J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s)$$

$$T(s) = \theta_1(s)\left[J_1 s^2 + K\right] - K\theta(s) \quad \text{--- (2)}$$

Free body diagram of mass with moment of inertia $J_2$ is shown below




$$T_{J_2} = J_2 \frac{d^2\theta}{dt^2}$$

$$T_b = B\frac{d\theta}{dt}$$

$$T_K = K(\theta - \theta_1)$$

By Newton's 2nd law

$$T_{J_2} + T_b + T_K = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0 \quad \text{--- (3)}$$

Apply Laplace Transform

$$J_2 s^2 \theta(s) + Bs\theta(s) + K\theta(s) - K\theta_1(s) = 0$$

$$\theta(s)\left[J_2 s^2 + Bs + K\right] = K\theta_1(s) \quad \text{--- (4)}$$

$$\theta_1(s) = \frac{\theta(s)\left[J_2 s^2 + Bs + K\right]}{K} \quad \text{--- (5)}$$

Substituting $\theta_1(s)$ in eqn (2)

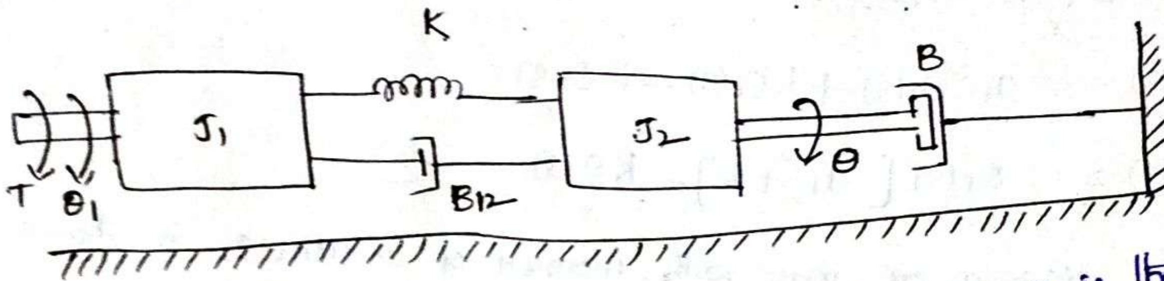
$$T(s) = \frac{\theta(s)\left[J_2 s^2 + Bs + K\right]\left[J_1 s^2 + K\right]}{K} - K\theta(s)$$

$$T(s) = \frac{\theta(s)\left(\left[J_2 s^2 + Bs + K\right]\left[J_1 s^2 + K\right] - K^2\right)}{K}$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{K}{\left[J_2 s^2 + Bs + K\right]\left[J_1 s^2 + K\right] - K^2}$$

↳ Required Transfer Function.

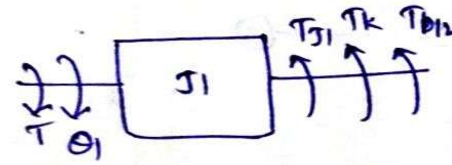
②




applied torque T is the input, angular displacement $\theta$ is the output.
hence the required transfer function is $\frac{\theta(s)}{T(s)}$.

The system has two nodes and they are masses with moment of inertia $J_1$ and $J_2$. The differential equations governing the system are given by torque balance equations at these nodes.

The free body diagram of mass with moment of inertia $J_1$ is shown below




$$T_{J1} = J_1 \frac{d^2\theta_1}{dt^2}$$

$$T_k = K(\theta_1 - \theta)$$

$$T_{b12} = B_{12} \frac{d}{dt}(\theta_1 - \theta)$$

By Newton's 2nd law

$$T = T_{J1} + T_k + T_{b12}$$

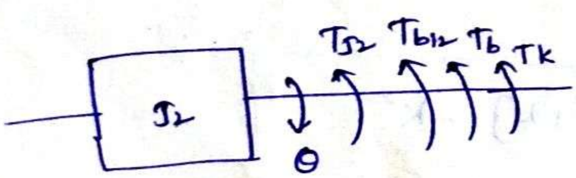
$$T = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) + B_{12}\frac{d}{dt}(\theta_1 - \theta) \quad \text{—— ①}$$

Apply laplace Transform

$$T(s) = J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) + B_{12} s\left[\theta_1(s) - \theta(s)\right]$$

$$T(s) = \theta_1(s)\left[J_1 s^2 + K + B_{12}s\right] - \theta(s)\left[K + B_{12}s\right] \quad \text{—— ②}$$

The free body diagram of mass with moment of inertia $J_2$ is shown below




$$T_{J2} = J_2 \frac{d^2\theta}{dt^2}$$

$$T_{b12} = B_{12}\frac{d}{dt}(\theta - \theta_1)$$

$$T_b = B\frac{d\theta}{dt}$$

$$T_k = K(\theta - \theta_1)$$

By Newton's 2nd law

$$T_{J_2}+T_{b12}+T_b+T_k=0$$

$$J_2\frac{d^2\theta}{dt^2}+B_{12}\frac{d}{dt}(\theta-\theta_1)+B\frac{d\theta}{dt}+K(\theta-\theta_1)=0 \quad \text{—③}$$

Apply laplace Transform

$$J_2s^2\theta(s)+B_{12}s\left[\theta(s)-\theta_1(s)\right]+Bs\theta(s)+K\theta(s)-K\theta_1(s)=0$$

$$\theta(s)\left[J_2s^2+B_{12}s+Bs+K\right]=\theta_1(s)\left[B_{12}s+K\right]$$

$$\theta_1(s)=\frac{\left[J_2s^2+s(B_{12}+B)+K\right]}{B_{12}s+K}\theta(s) \quad \text{—④}$$

Substituting $\theta_1(s)$ in eqn ②

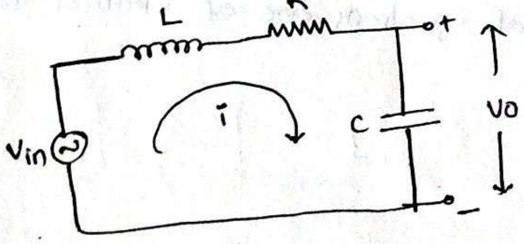
$$T(s)=\frac{\left[J_2s^2+s(B_{12}+B)+K\right]\theta(s)\left[J_1s^2+B_{12}s+K\right]}{B_{12}s+K}-\theta(s)\left[B_{12}s+K\right]$$

$$T(s)=\theta(s)\left[\frac{\left[J_2s^2+s(B_{12}+B)+K\right]\left[J_1s^2+B_{12}s+K\right]-(B_{12}s+K)^2}{B_{12}s+K}\right]$$

$$\therefore \frac{\theta(s)}{T(s)}=\frac{sB_{12}+K}{\left[J_1s^2+B_{12}s+K\right]\left[J_2s^2+s(B_{12}+B)+K\right]-(sB_{12}+K)^2}$$

Electrical Systems:

The resistor, inductor and capacitors are the three basic elements of electrical circuits. These circuits are analyzed by the applications of Kirchoff's voltage and current laws.




Consider a series R-L-C circuit using Kirchoff's voltage law

$$V_{in}=V_L+V_R+V_C$$

$$V_L=L\frac{di}{dt}$$

$$V_R=iR$$

$$V_C=\frac{1}{C}\int i\,dt$$

electric charge $q = \int i\,dt$ $\qquad i = \frac{dq}{dt}$

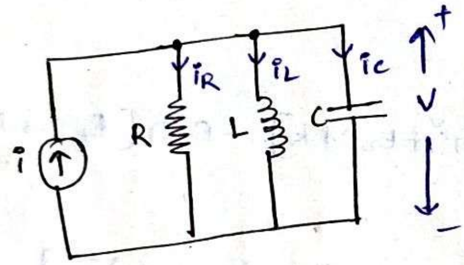
$\therefore \quad V_{in} = V_R + V_L + V_C$

$V_{in} = iR + \frac{1}{C}\int i\,dt + L\frac{di}{dt}$

$V_{in} = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}\int \frac{dq}{dt}\,dt$

$$\boxed{V_{in} = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q}$$

Similarly for the R-L-C parallel circuit shown in Fig below




using Kirchoff's current law

$i = i_R + i_L + i_C$

$i = \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt}$

magnetic flux linkage is given by

$\phi = \int V\,dt \Rightarrow V = \frac{d\phi}{dt}$

$\therefore \quad i = \frac{1}{R}\frac{d\phi}{dt} + C\frac{d}{dt}\left(\frac{d\phi}{dt}\right) + \frac{1}{L}\phi$

$$\therefore \boxed{i = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi}$$

Analagous Systems:

The systems whose differential equations are of identical form are called analogous systems

Analogous quantities in Force (Torque) - voltage analogy

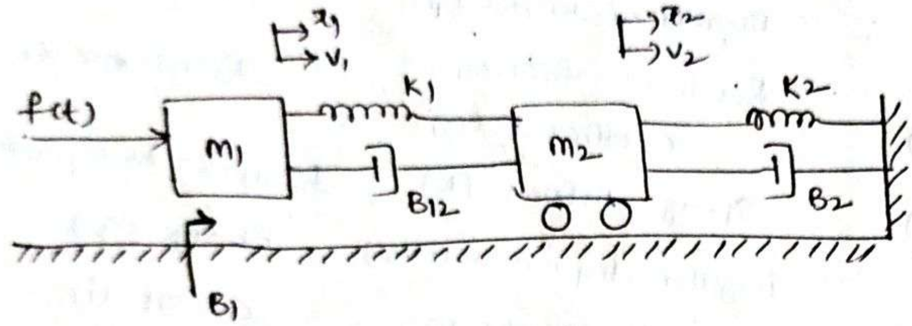
| Mechanical Translational system | Mechanical Rotational system | Electrical system |
|---|---|---|
| Force (F) | Torque (T) | voltage (V) |
| Mass (M) | Moment of inertia (J) | inductance (L) |
| Viscous friction co-efficient (B) | Rotational friction co-efficient (B) | Resistance (R) |
| Spring stiffness (K) | Spring stiffness (K) | Reciprocal of capacitance $(\frac{1}{C})$ |
| Displacement (x) | Angular displacement (θ) | charge (q) |
| Velocity (v) | Angular velocity ω | current (i) |

Analogous quantities in Force (Torque) – current analogy

| Mechanical Translational System | Mechanical Rotational System | Electrical system |
|---|---|---|
| Force (F) | Torque (T). | current (i) |
| Mass (M) | Moment of Inertia (J) | Capacitance C |
| Viscous Friction coefficient (B) | Viscous Friction Coefficient (B) | Reciprocal of Resistance $(\frac{1}{R})$ |
| Spring stiffness (K) | Spring stiffness (K) | Reciprocal of inductance $(\frac{1}{L})$ |
| Displacement (x) | Angular displacement (θ) | Magnetic flux linkage φ |
| Velocity (v) | Angular velocity (ω) | Voltage (V) |

Problems:

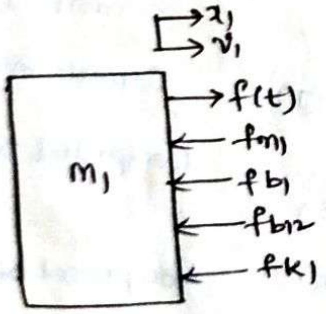
① Write the differential equations governing the mechanical system shown in fig below. Draw the Force-voltage and force-current electrical analogous circuits and verify by writing mesh and node equations.




Solution: The given system has two nodes

Let the displacements of masses $M_1$ and $M_2$ be $x_1$ & $x_2$ and the corresponding velocities are $v_1$ & $v_2$

The free body diagram of $M_1$ is shown below




$$f_{m1} = M_1 \frac{d^2x_1}{dt_1}$$

$$f_{b1} = B_1 \frac{dx_1}{dt}$$

$$f_{b12} = B_{12} \frac{d}{dt}(x_1 - x_2)$$

$$f_{k1} = K_1 (x_1 - x_2)$$

By Newton's 2nd law

$$f(t) = f_{m1} + f_{b1} + f_{b12} + f_{k1}$$

$$f(t) = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1 (x_1 - x_2) \quad \text{—①}$$

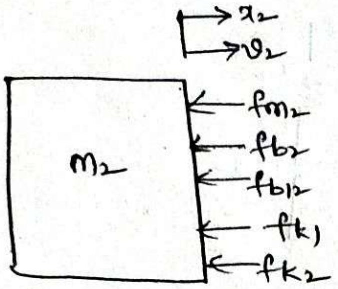
But we know that

$$v = \frac{dx}{dt} \quad \& \quad \frac{dv}{dt} = \frac{d^2x}{dt} \quad \& \quad x = \int v\,dt$$

∴ eqn ① becomes

$$f(t) = M_1 \frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + K_1 \int (v_1 - v_2)\,dt \quad \text{—②}$$

The free body diagram of mass $M_2$ is shown below




$$f_{m2} = M_2 \frac{d^2x_2}{dt^2} \quad , \quad f_{b2} = B_2 \frac{dx_2}{dt}$$

$$f_{b12} = B_{12} \frac{d}{dt}(x_2 - x_1)$$

$$f_{k1} = K_1(x_2 - x_1)$$

$$f_{k2} = K_2 x_2$$

By Newton's 2nd law

$$f_{m2} + f_{b2} + f_{b12} + f_{k1} + f_{k2} = 0$$

$$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1) + K_2 x_2 = 0 \quad \text{--- (3)}$$

But $\frac{d^2x}{dt^2} = \frac{dv}{dt}$ , $\frac{dx}{dt} = v$ & $x = \int v\, dt$

$\therefore$ eqn (3) becomes

$$M_2 \frac{dv_2}{dt} + B_2 v_2 + B_{12}(v_2 - v_1) + K_1 \int (v_2 - v_1)\, dt + K_2 \int v_2\, dt = 0 \quad \text{--- (4)}$$

Force - voltage analogous circuit

From the table, the electrical analogous elements for the elements of mechanical system are

$f(t) \Rightarrow V(t)$ $\quad M \rightarrow L \quad K \rightarrow \frac{1}{C}$

$v \rightarrow i$ $\quad B \rightarrow R$

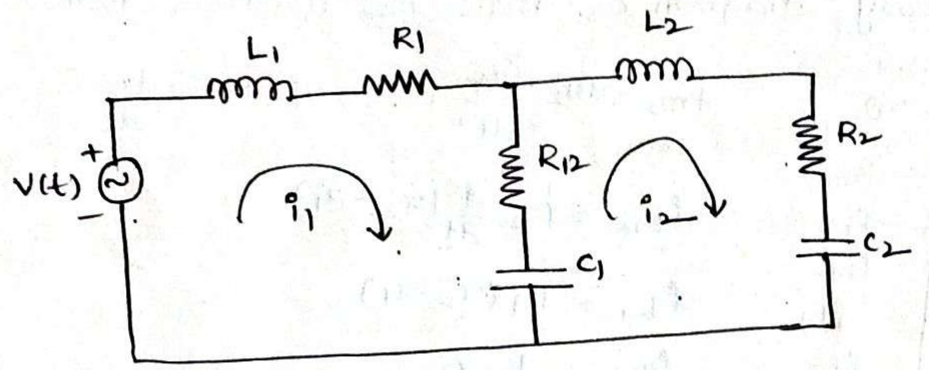
$\therefore$ eqn (2) becomes

$$V(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\, dt \quad \text{--- (5)}$$

Ily eqn (4) becomes

$$L_2 \frac{di_2}{dt} + R_2 i_2 + R_{12}(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1)\, dt + \frac{1}{C_2}\int i_2\, dt = 0 \quad \text{--- (6)}$$

Force - voltage electrical analogous circuit for the given mechanical system is shown below



Force - Current analogous circuit

The electrical analogous elements for the elements of mechanical system are given below.

$f(t) \rightarrow i(t)$ $\quad M \rightarrow C$ $\quad K \rightarrow \frac{1}{L}$

$v \rightarrow V$ $\quad B \rightarrow \frac{1}{R}$

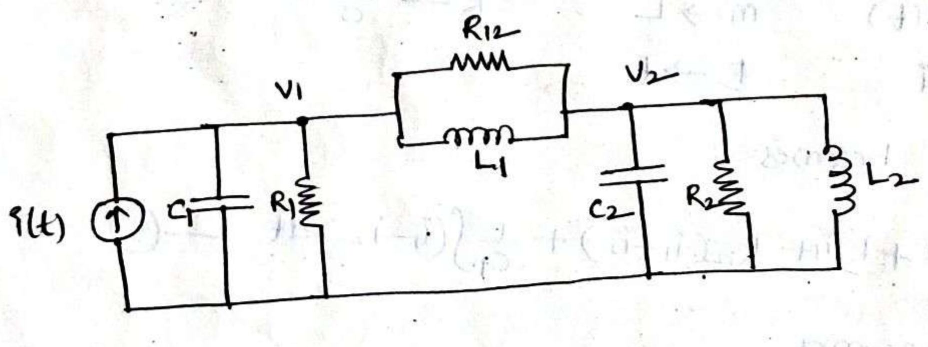
$\therefore$ eqn (2) becomes

$$i(t) = C_1 \frac{dV_1}{dt} + \frac{1}{R_1} V_1 + \frac{1}{R_{12}} (V_1 - V_2) + \frac{1}{L_1} \int (V_1 - V_2)\, dt \quad \text{—(7)}$$

Illy eqn (4) becomes

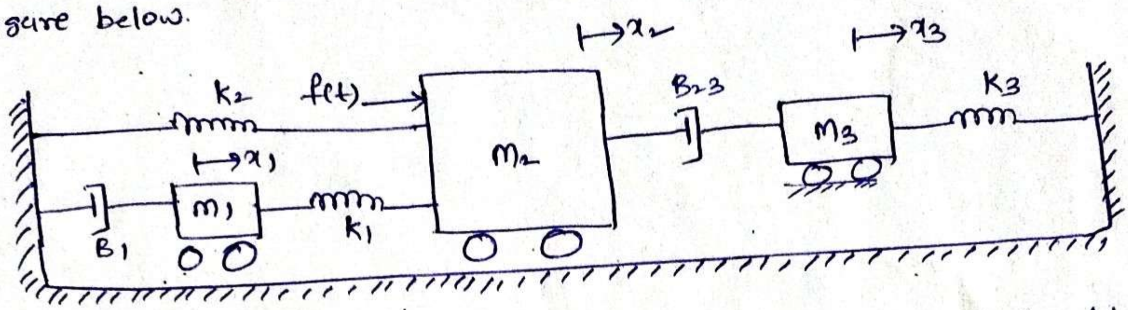
$$C_2 \frac{dV_2}{dt} + \frac{1}{R_2} V_2 + \frac{1}{L_2} \int V_2\, dt + \frac{1}{R_{12}} (V_2 - V_1) + \frac{1}{L_1} \int (V_2 - V_1)\, dt = 0 \quad \text{—(8)}$$

Force current electrical analogous circuit for the given mechanical system shown below.

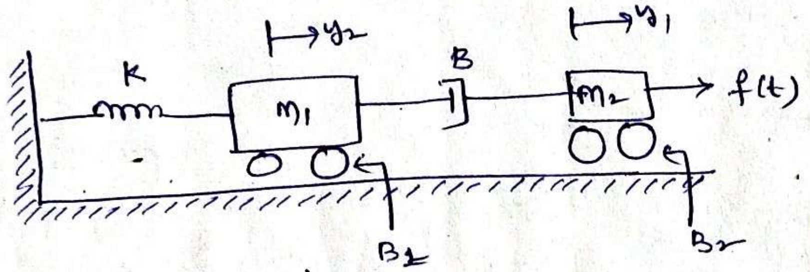


Excercise problems:

① Obtain the mathematical model of the mechanical system shown in figure below.

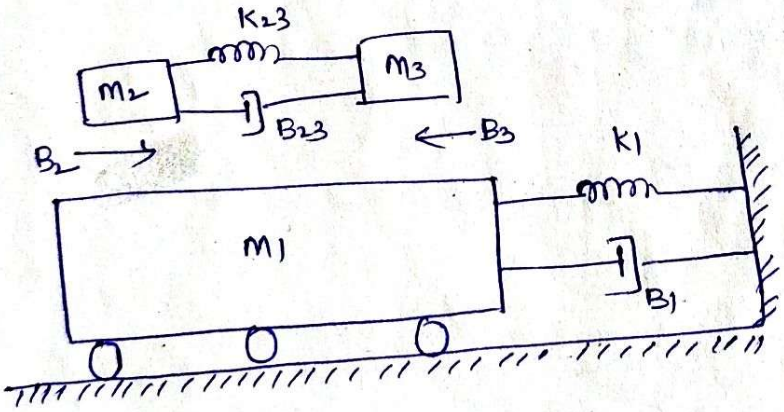



② Write the differential equation for the mechanical n/w and obtain transfer functions $\frac{Y_1(S)}{F(S)}$ & $\frac{Y_2(S)}{F(S)}$

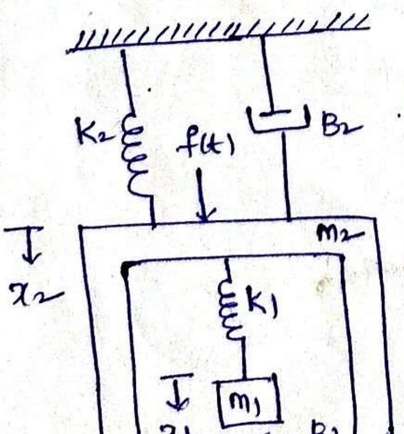



③ Write the differential equations governing a mechanical system shown in figure below.

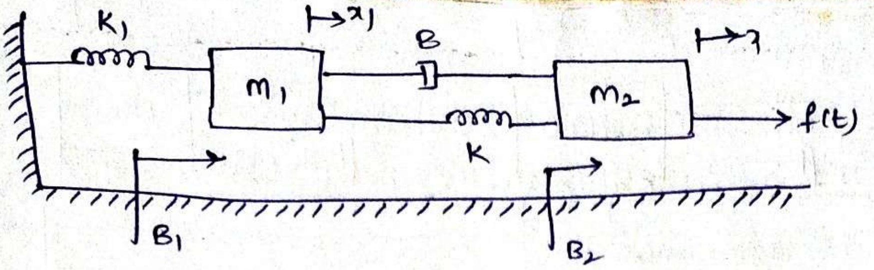
ⓐ




ⓑ

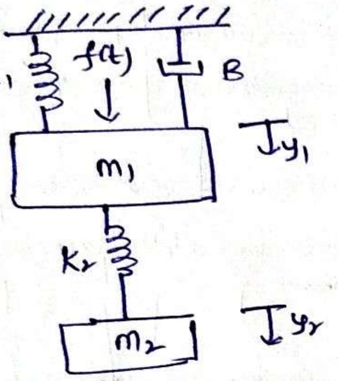


1.




$$\frac{X(s)}{F(s)} = \frac{m_1 s^2 + (B_1+B)s + (K_1+k)}{\left[m_1 s^2 + (B_1+B)s + (k_1+k)\right]\left[m_2 s^2 + (B_2+B)s + k\right] - (Bs+k)^2}$$

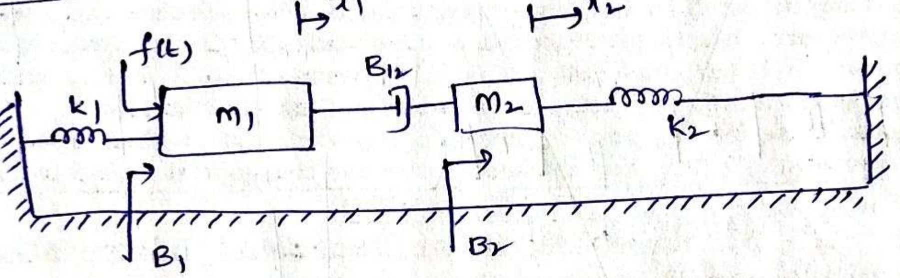
2.




$$\frac{Y_2(s)}{F(s)} = \frac{K_2}{\left[m_1 s^2 + Bs + (k_1+k_2)\right]\left[m_2 s^2 + k_2\right] - K_2^2}$$

3.

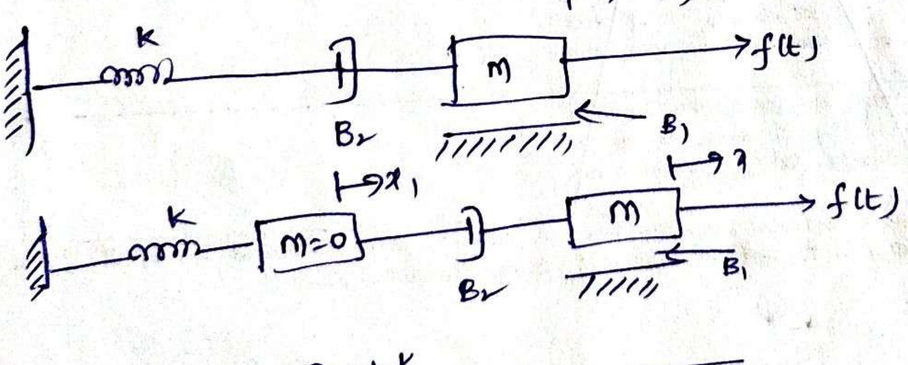
Find $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$




$$\frac{X_1(s)}{F(s)} = \frac{m_2 s^2 + (B_2+B_{12})s + k_2}{\left[m_1 s^2 + (B_1+B_{12})s + k_1\right]\left[m_2 s^2 + (B_2+B_{12})s + k_2\right] - (B_{12}s)^2}$$

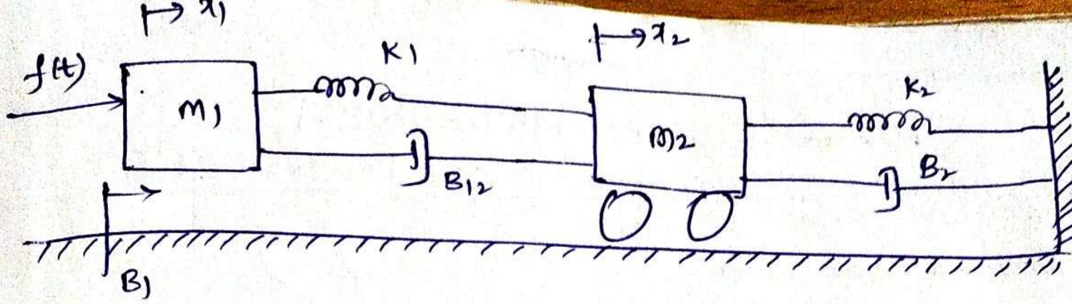
$$\frac{X_2(s)}{F(s)} = \frac{B_{12}s}{\left[m_2 s^2 + (B_2+B_{12})s + k_2\right]\left[m_1 s^2 + (B_1+B_{12})s + k_1\right] - (B_{12}s)^2}$$

4.

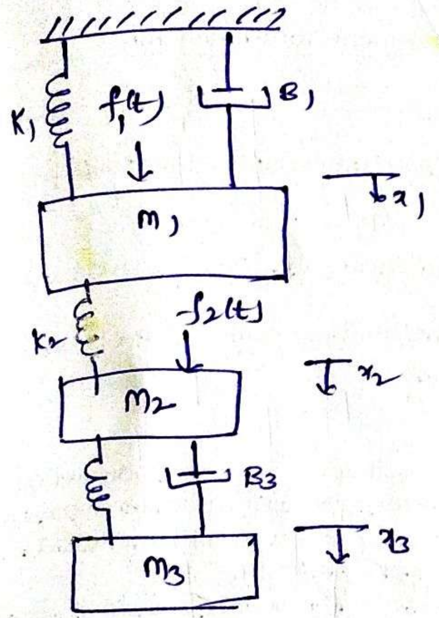



$$\frac{X(s)}{F(s)} = \frac{B_2 s + K}{\left[Ms^2 + (B_1+B_2)s\right](B_2 s + k) - (B_2 s)^2}$$

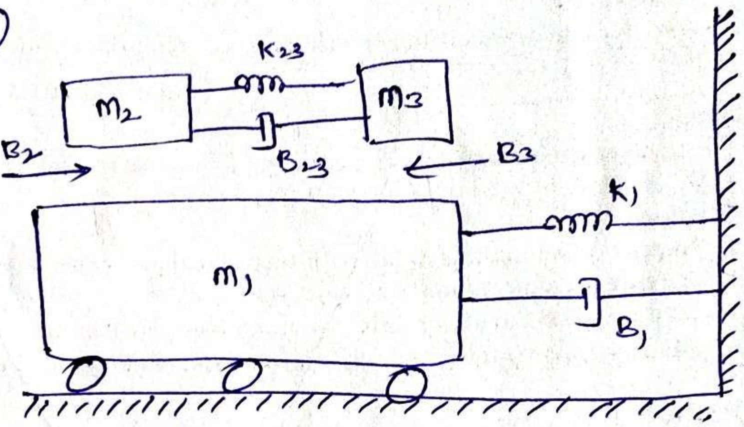
1



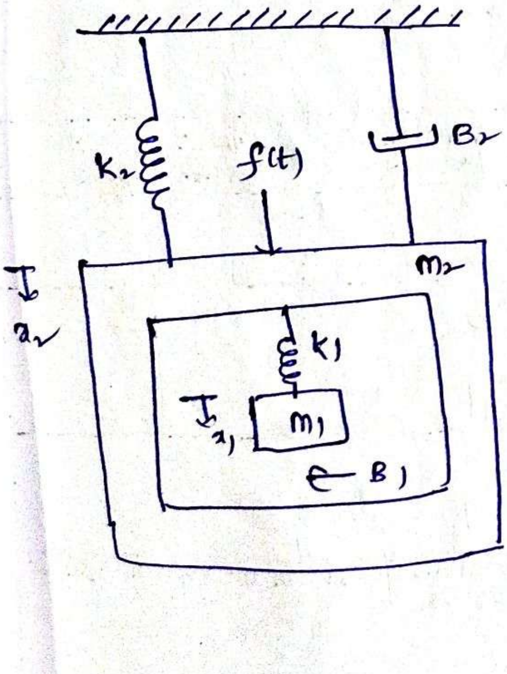

2



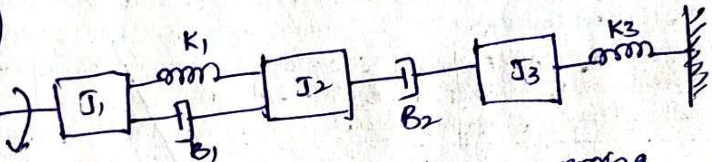

3




4




5



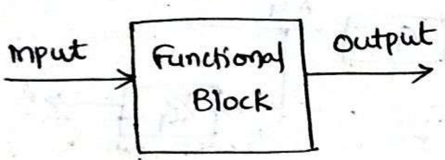

Write the differential equations governing the mechanical rotational system shown in figure. Draw the torque voltage and torque current electrical analogous circuits

Block diagrams:

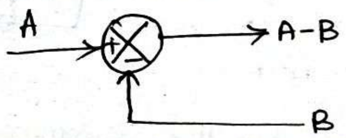
A block diagram is a pictorial representation of the functions performed by each component and the flow of signals between the components.

The elements of a block diagram are block, branch point and summing point.

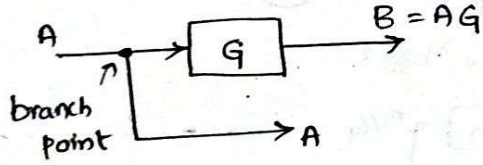
Block: In a block diagram all the system variables are linked to each other through functional blocks. The functional block or simply block is a symbol for the mathematical operation on the input signal to the block that produces the output.




Summing Point: Summing points are used to add two or more signals in the system. A circle with a cross symbol that indicates a summing operation. The plus or minus sign at each arrowhead indicates whether the signal is to be added or subtracted.




Branch point: A branch point is a point from which the signal from a block goes concurrently to other blocks or summing points

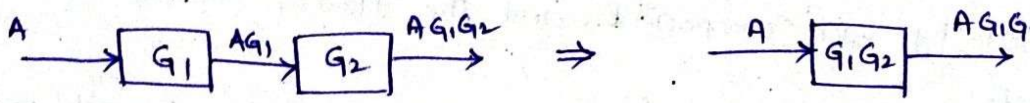



Block diagram reduction:

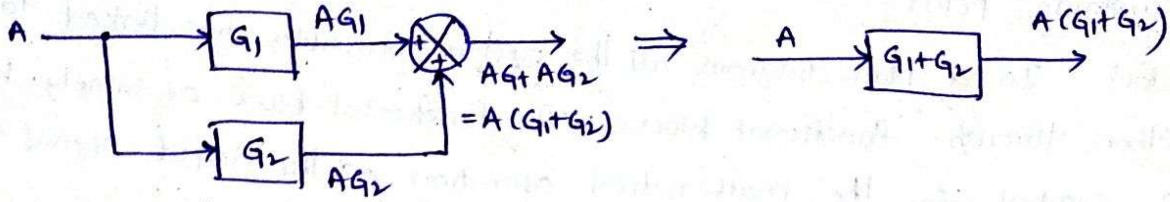
The block diagram can be reduced to find the overall transfer function of the system. The following rules can be used for block diagram reduction.

## BLOCK DIAGRAM ALGEBRA :

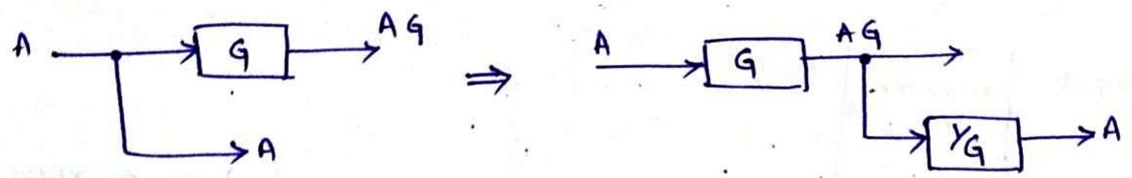
① Combining the blocks in cascade



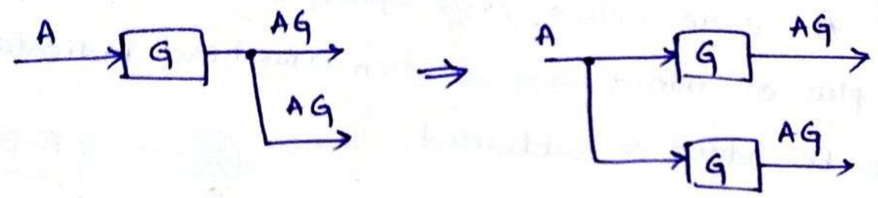

② Combining parallel blocks:



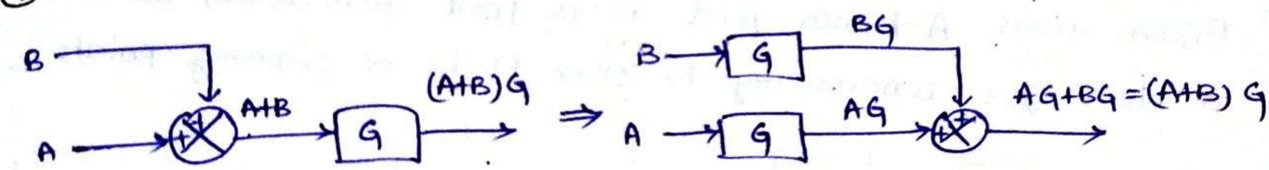

③ Moving the branch point ahead of the block.



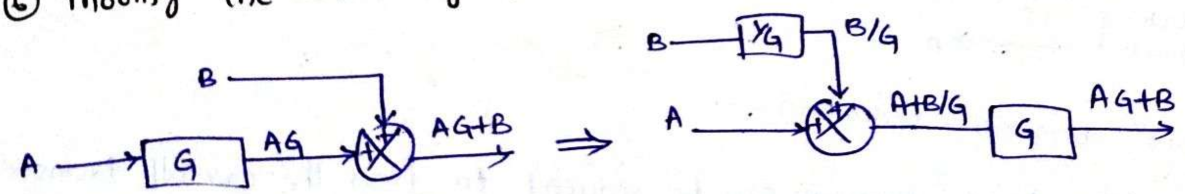

④ Moving the branch point before the block



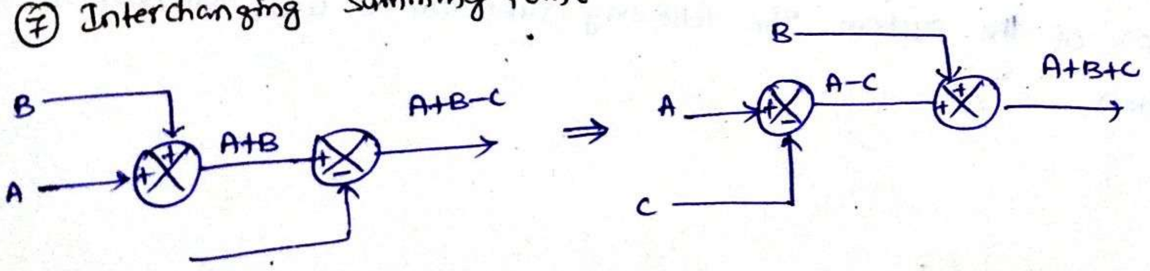

⑤ Moving the summing point ahead of the block:



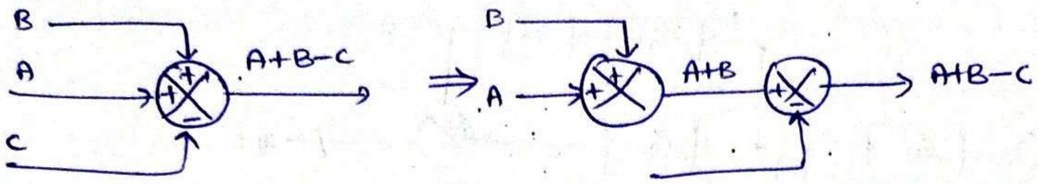

⑥ Moving the summing point before the block:



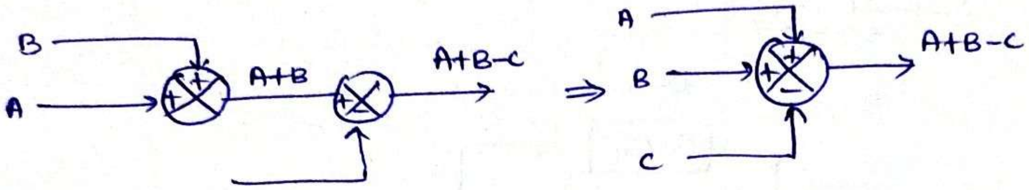

⑦ Interchanging summing point



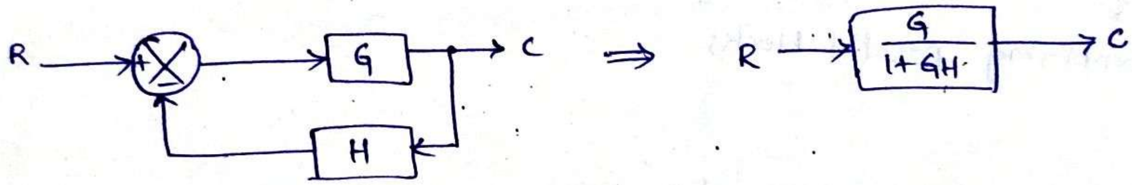
8. Splitting summing points



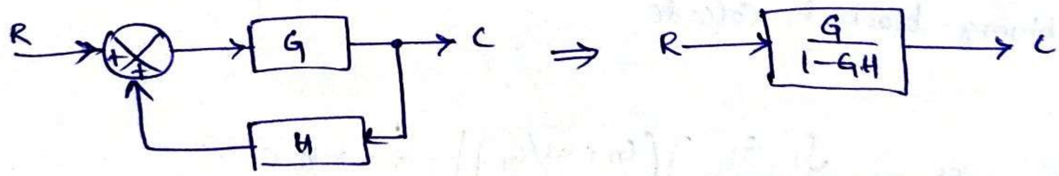

9. Combining summing points




10. Elimination of −ve feedback loop

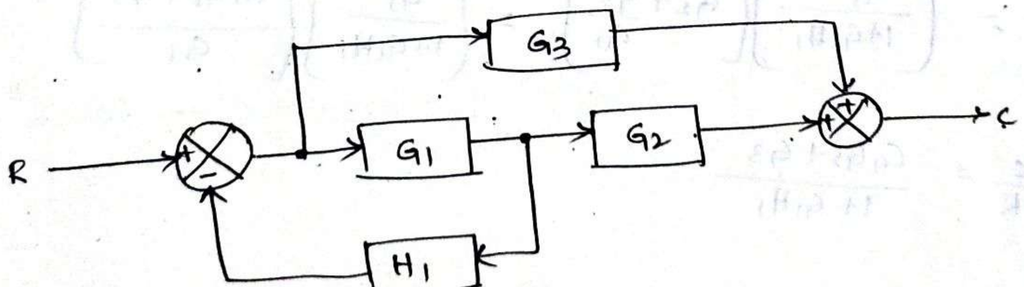



11. Elimination of +ve feedback loop

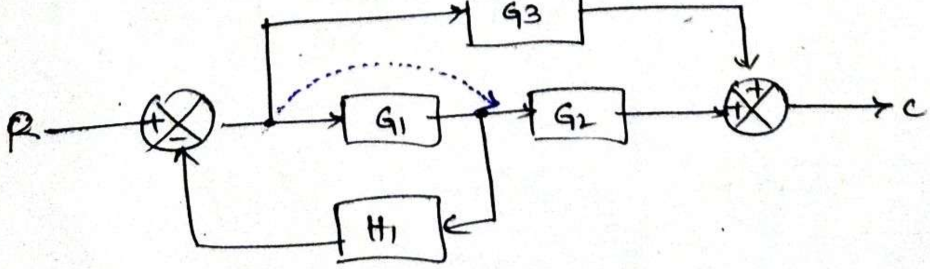



Problems.

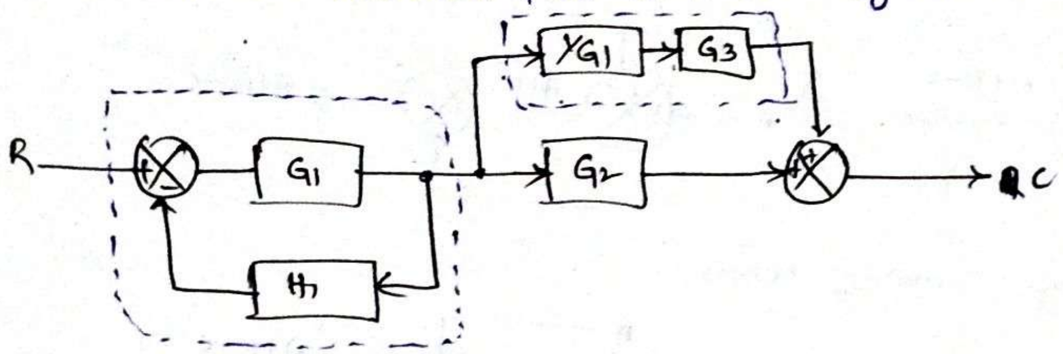
① Reduce the block diagram shown in fig below and find $\frac{C}{R}$



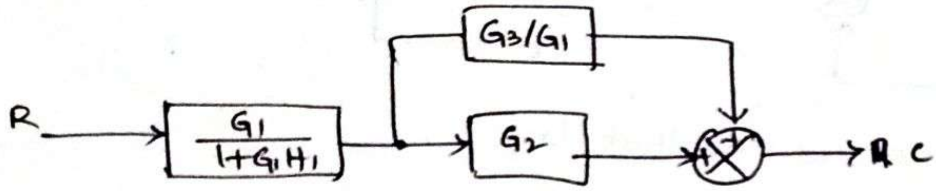

Solution: Move the take off point after the block



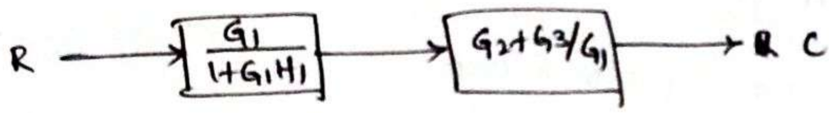
Eliminate the feedback path and combing blocks in cascade



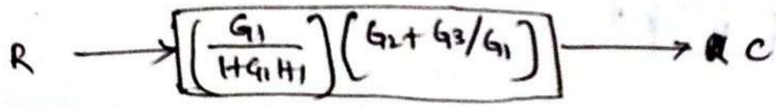

⇓




Combining parallel blocks

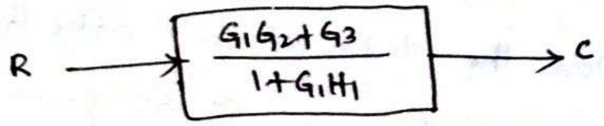



Combining blocks in Cascade

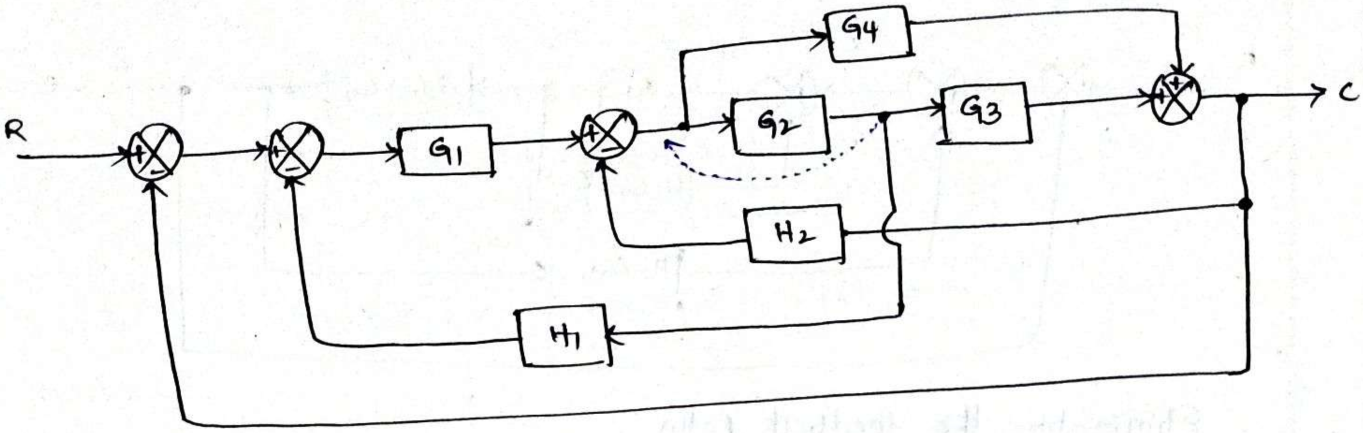



$$\frac{C}{R} = \left(\frac{G_1}{1+G_1H_1}\right)\left(G_2+\frac{G_3}{G_1}\right) = \left(\frac{G_1}{1+G_1H_1}\right)\left(\frac{G_1G_2+G_3}{G_1}\right)$$

$$\frac{C}{R} = \frac{G_1G_2+G_3}{1+G_1H_1}$$

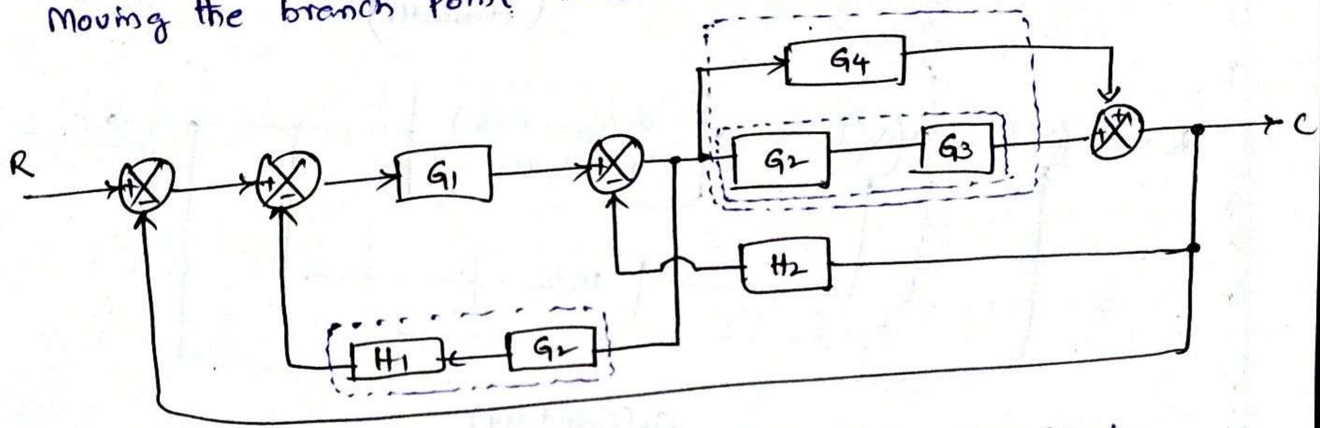


② Using block diagram reduction technique find closed loop transfer function of the system shown in figure below.

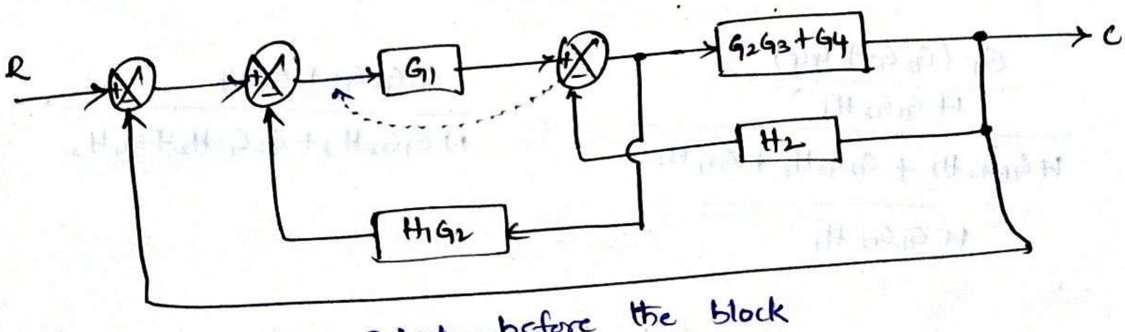



Solution

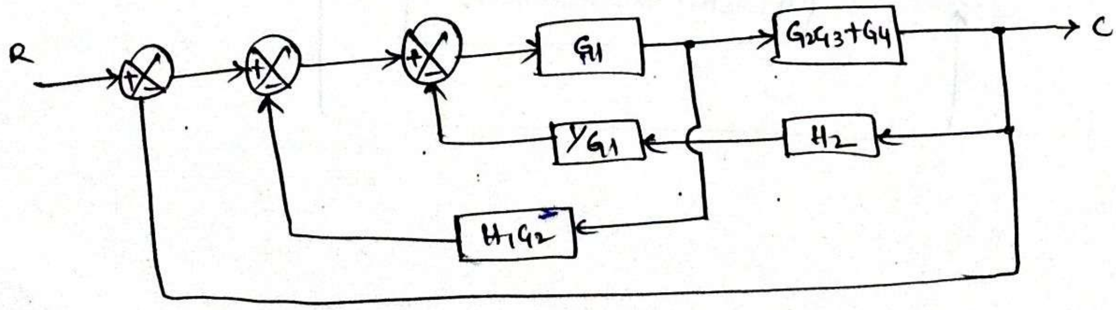
Moving the branch point before the block G2



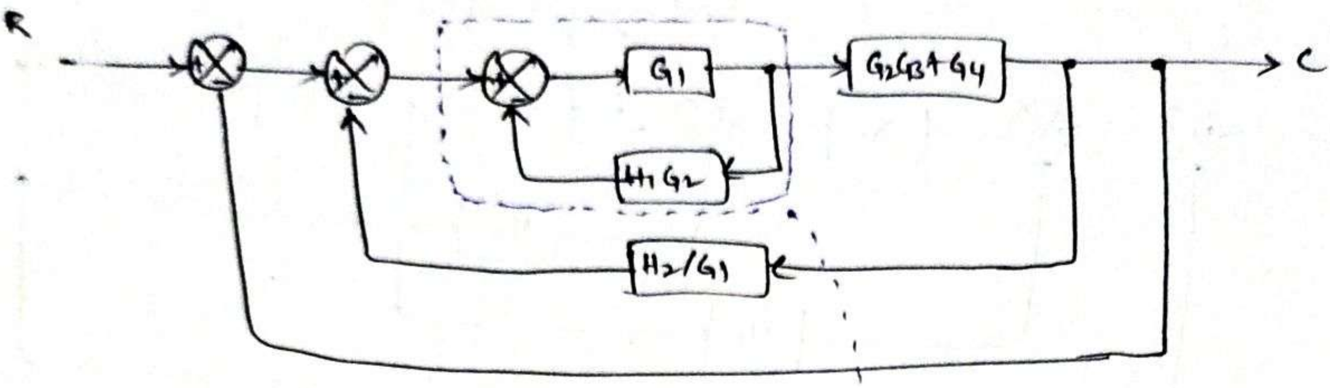

Combine the blocks in cascade and eliminate parallel blocks




Moving summing point before the block

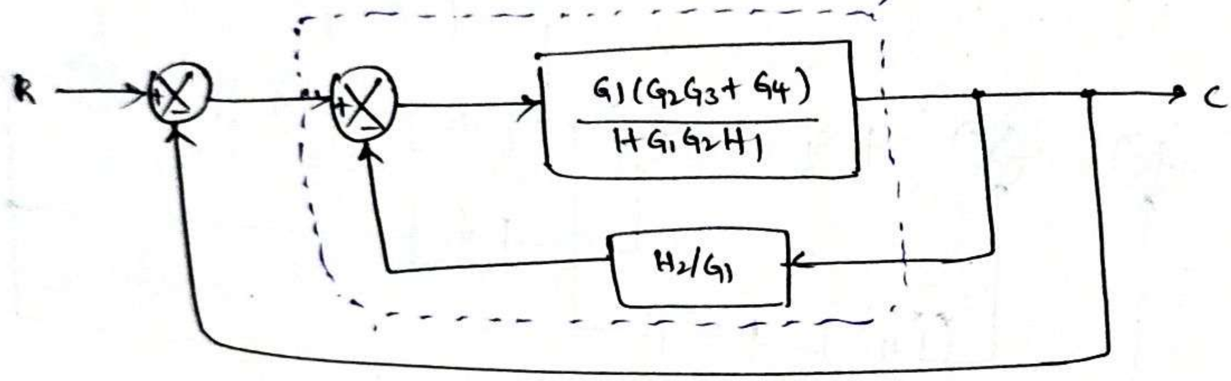


Interchanging summing points and modifying branch points



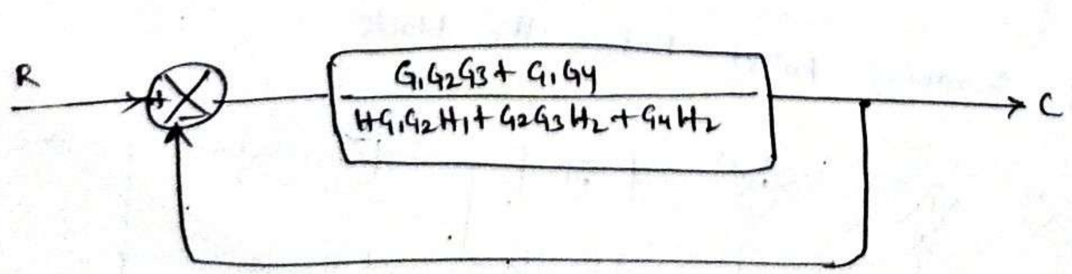

Eliminating the feedback paths

$$\left(\frac{G_1}{1+G_1G_2H_1}\right)(G_2G_3+G_4)$$




$$\rightarrow \frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{\cancel{G_1}(G_2G_3+G_4)}{1+G_1G_2H_1}\cdot\dfrac{H_2}{\cancel{G_1}}}$$

$$\Rightarrow \frac{\dfrac{G_1(G_2G_3+G_4)}{\cancel{1+G_1G_2H_1}}}{\dfrac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}{\cancel{1+G_1G_2H_1}}} = \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$



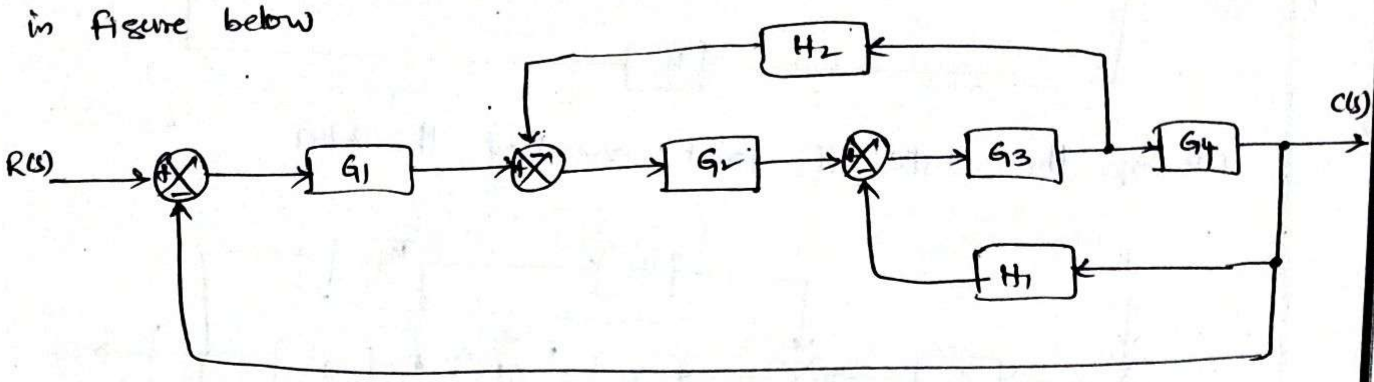
Eliminating the feedback path

$$\frac{C}{R} = \frac{\dfrac{G_1G_2G_3 + G_1G_4}{1+G_1G_2H_1 + G_2G_3H_2 + G_4H_2}}{1+ \dfrac{G_1G_2G_3 + G_1G_4}{1+G_1G_2H_1 + G_2G_3H_2 + G_4H_2}}$$

$$\frac{C}{R} = \frac{G_1G_2G_3 + G_1G_4}{1+G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4}$$

Problem (3)

Determine the overall transfer function $\frac{C(s)}{R(s)}$ for the system shown in figure below

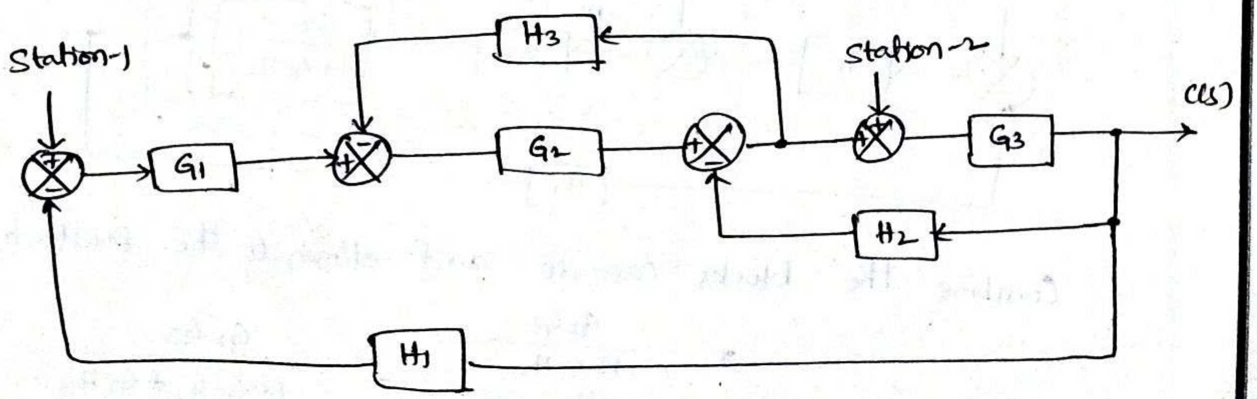



Ans:

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+ G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3\cancel{+}G_4}$$

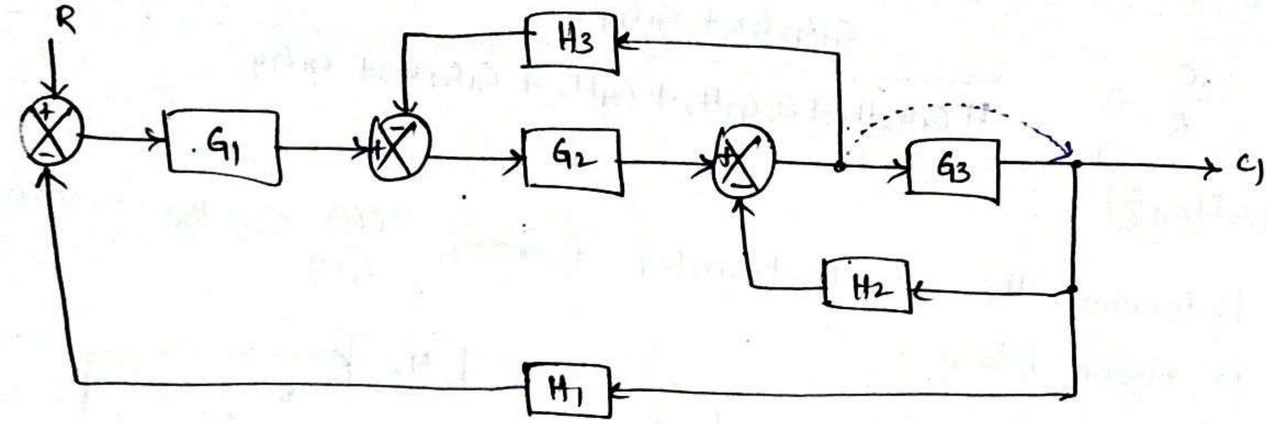
Problem (4)

For the system represented by the block diagram shown in fig below Evaluate the closed loop transfer function when the input R is (i) at station-1 (ii) at station-2

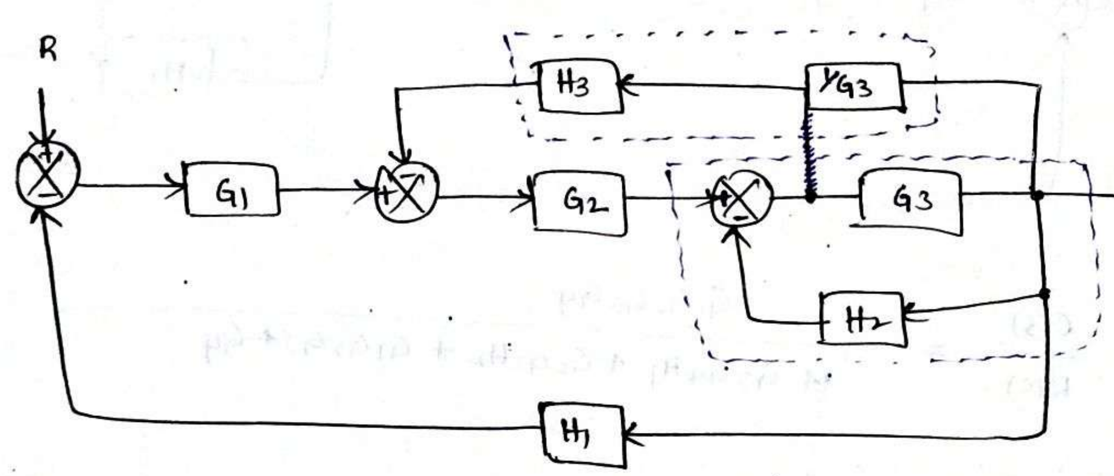


Solution:

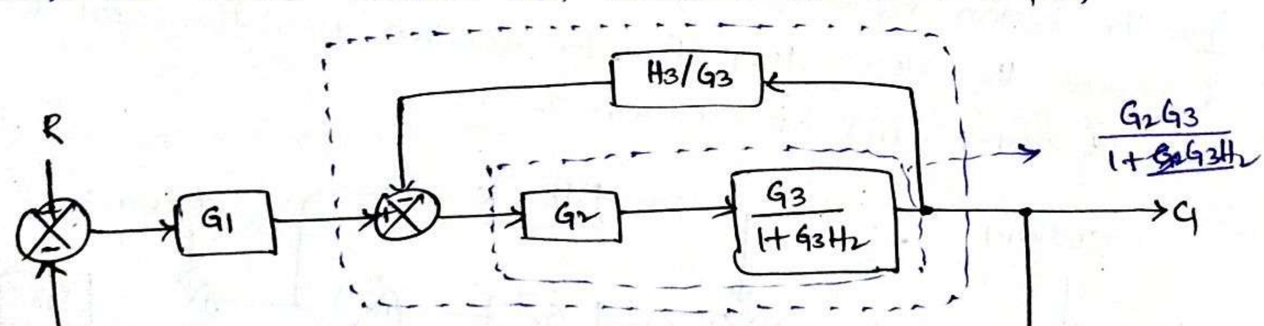
i) Consider the input R is at station-1 and so the input at station-2 is made zero. Let the output be $C_1$. Since there is no summing point at station-2 that summing point can be removed and resulting block diagram is shown below.




Moving the take-off point ahead of the block

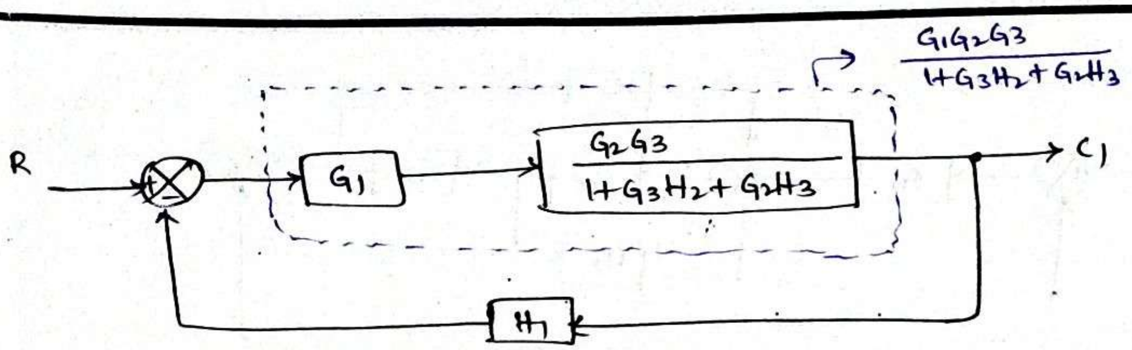



Combine the blocks cascade and eliminate the feedback path

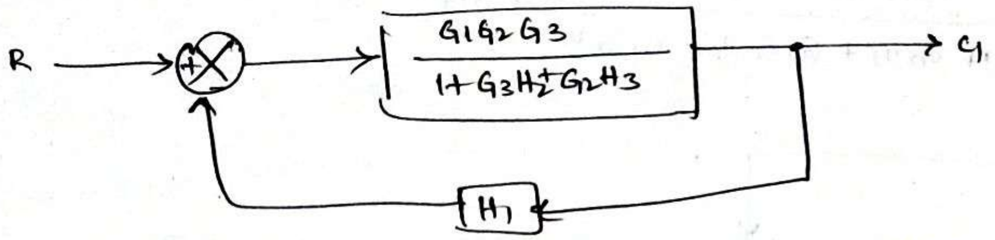



Combine the blocks cascade and eliminate the feedback path

$$\frac{\frac{G_2G_3}{1+G_3H_2}}{1+\frac{H_3}{\cancel{G_3}}\,\frac{G_2\cancel{G_3}}{1+G_3H_2}} = \frac{G_2G_3}{1+G_3H_2+G_2H_3}$$




Eliminate the feedback path.



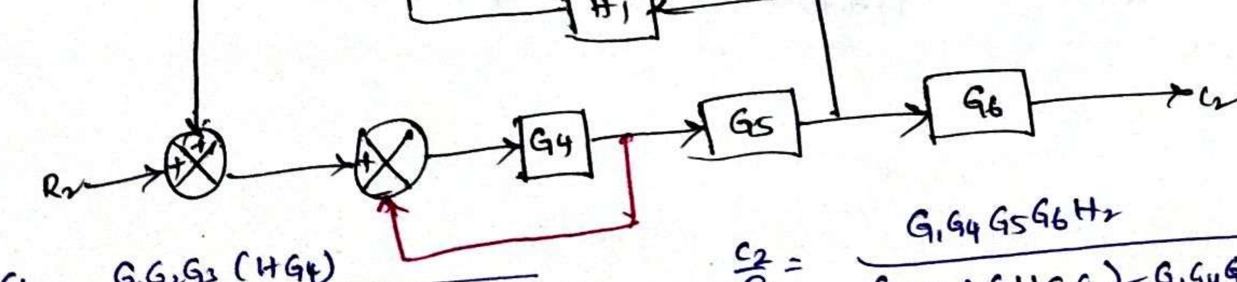

$$\frac{C_1}{R} = \frac{\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3}}{1+H_1\frac{G_1G_2G_3}{1+G_3H_2+G_2H_3}} = \frac{\frac{G_1G_2G_3}{\cancel{1+G_3H_2+G_2H_3}}}{\frac{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}{\cancel{1+G_3H_2+G_2H_3}}}$$

$$\frac{C_1}{R} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

at Station - 2:

$$\frac{C_2}{R} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3(G_1G_2H_1+H_2)}$$

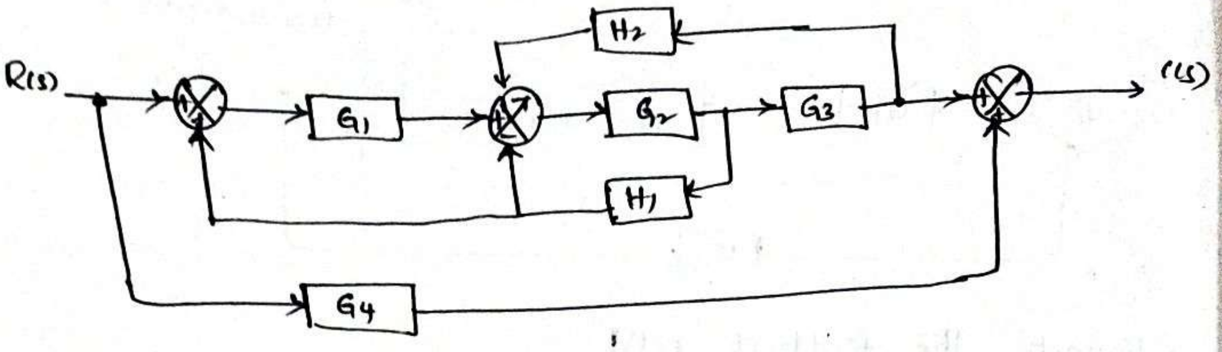
(5) Determine $C_1/R_1$ and $C_2/R_1$




$$\frac{C_1}{R_1} = \frac{G_1G_2G_3(1+G_4)}{(1+G_1G_2)(1+G_4)-G_1G_4G_5H_1H_2}$$

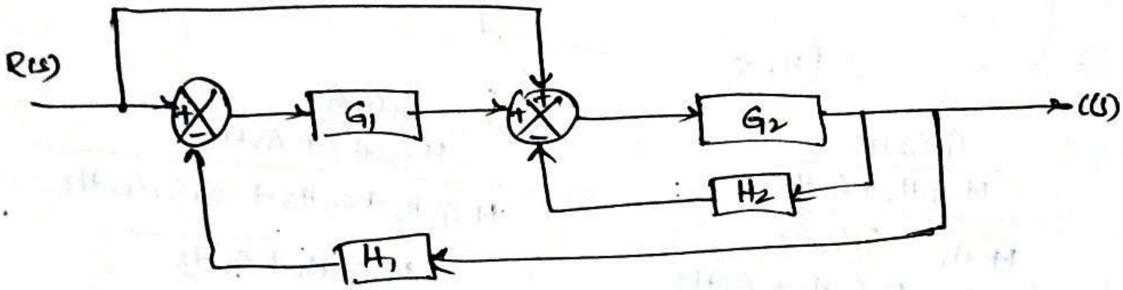
$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{(1+G_4)(1+G_1G_2)-G_1G_4G_5H_1H_2}$$

(6)



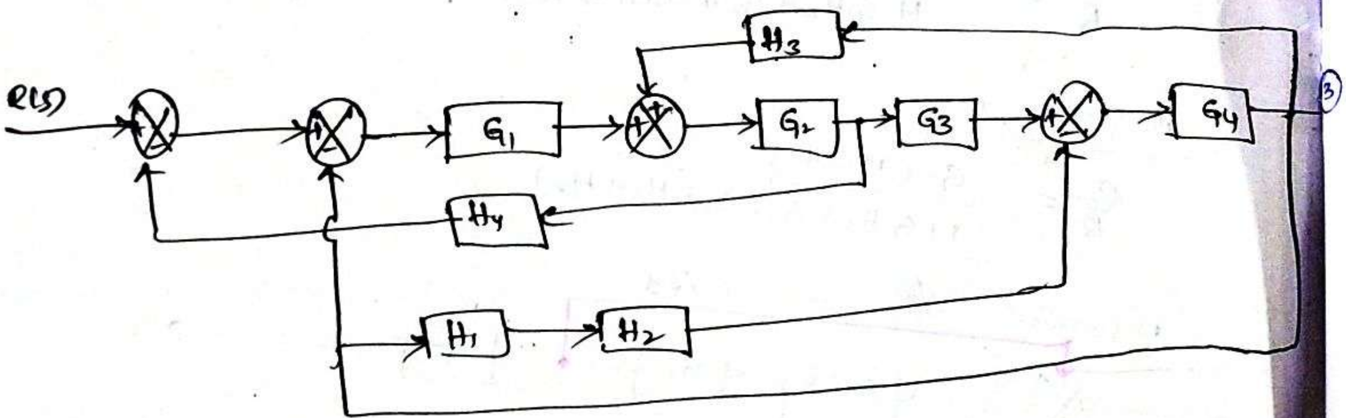

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1} + G_4$$

(7)



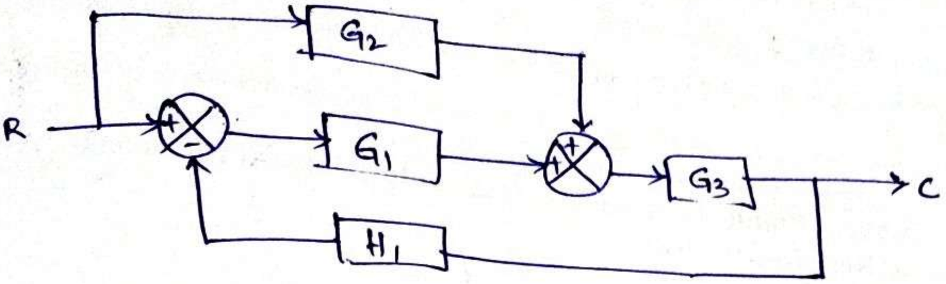

$$\frac{C(s)}{R(s)} = \frac{G_1G_2+G_2}{1+G_2H_2+G_1G_2H_1}$$

8

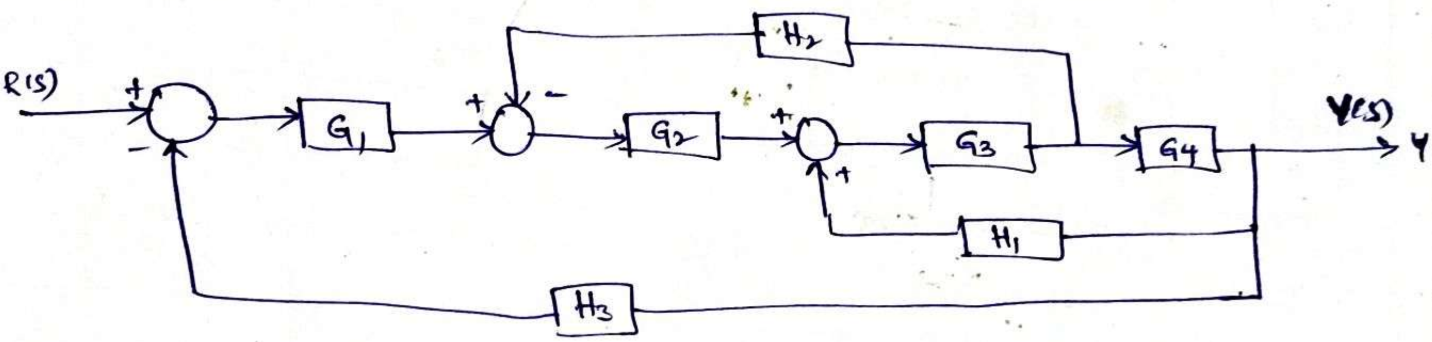



$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+H_1H_2(G_4+G_1G_2G_4H_4)+G_1G_2(H_4+G_3G_4)-G_2G_3G_4H_3}$$

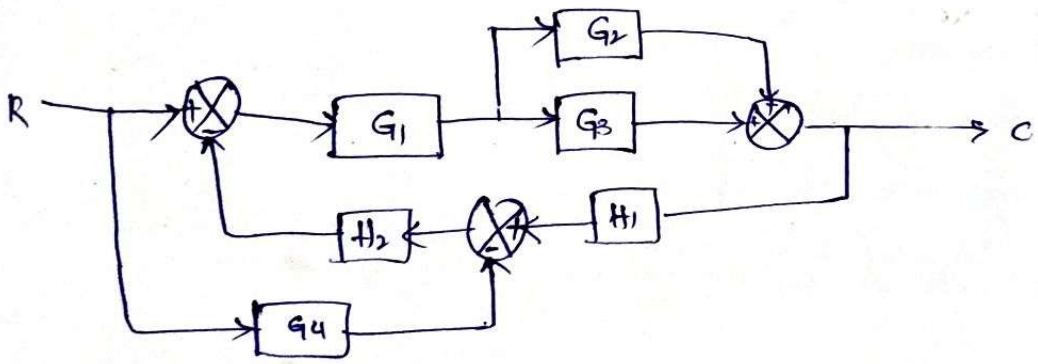
1) Determine the transfer function $C(s)/R(s)$ for the following block diagram. (Mar-2017)



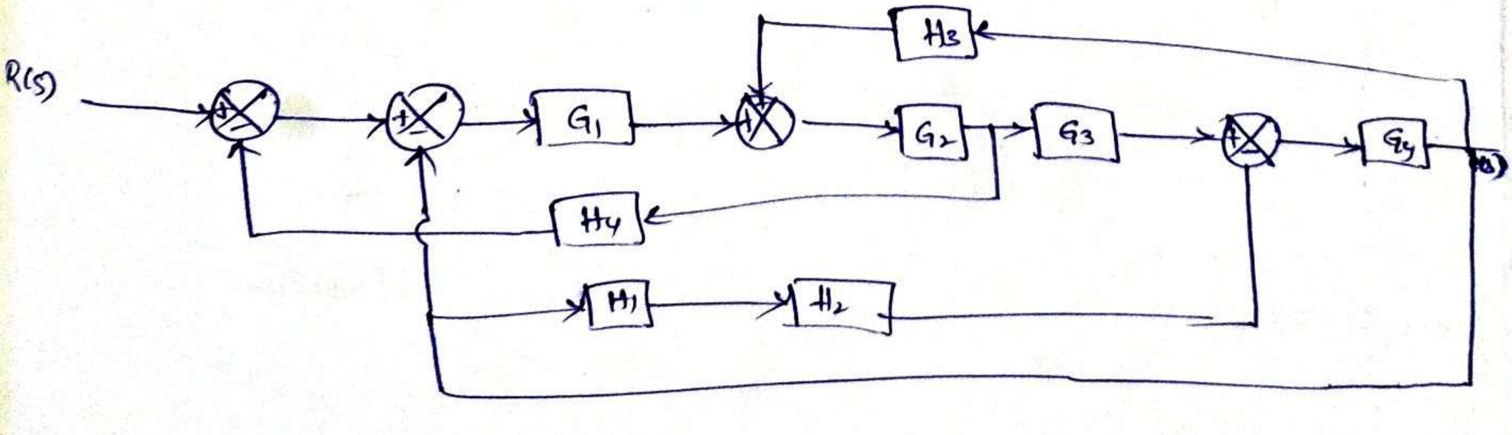

2) Obtain the transfer function $\frac{Y(s)}{R(s)}$ for the following block diagrams (Nov/Dec-2016)




3)




4)



Advantages of Block diagram:

① Very simple to construct the block diagram for complicated systems.

② The function of individual element can be visualised from block diagram.

③ Individual as well as overall performance of the system can be studied by using transfer functions shown in the block diagram.

④ Overall closed loop T.F can be easily calculated by using block diagram reduction rules

Disadvantages:

① The block diagram does not include any information about the physical construction of the system.

② Source of energy is generally not shown in the block diagrams. So no. of different block diagrams can be drawn depending upon the point of view of analysis. So block diagram for the given system is not unique.

## Signal flow Graph:

A signal flow graph is a graphical representation of the relationship between the variables of a set of linear algebraic equations written in the form of cause-and-effect relations.

A signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable and each branch connected between nodes acts as a signal multiplier. Each branch has a gain or transmittance. When the signal pass through a branch, it gets multiplied by the gain of the branch.

In a signal flow graph, the signal flows only in one direction. The direction of the signal flow is indicated by an arrow placed on the branch and the gain is indicated along the branch.

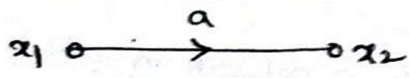
### Signal flow Graph Terminology:

1. Node: A node is a point representing a variable.
2. Branch: A branch is directed line segment joining two nodes
3. Transmittance: The gain acquired by the signal when it travels from one node to the other node is called transmittance. It can be real or complex.
4. Input node (source): It is a node that has only outgoing branches.
5. Output node (Sink): It is a node that has only incoming branches.
6. Mixed node: It is a node that has both incoming & outgoing branches
7. Path: A path is a traversal of connected branches in the direction of the branch arrows. It should not cross a node more than once.
8. Open path: A open path starts at a node and ends at another node.
9. closed path: Closed path starts and ends at same node.
10. Forward path: It is a path from input node to an output node that does not cross any node more than once.
11. Forward path gain: It is the product of the branch transmittances (gains) of a forward path.
12. Individual loop: It is a closed path starting from a node and after passing through a certain part of a graph arrives at same node without crossing any node more than once
13. Loop gain: It is the product of the branch transmittances of a loop
14. Non-touching loops: If the loops does not have a common node then they are said to be non-touching loops.
15. Self loop: A self loop is a loop consisting of a single branch.

## Signal flow Graph Algebra:

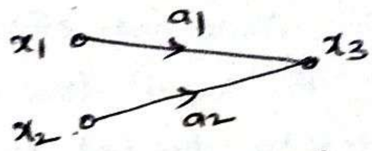
**Rule ①** Incoming signal to a node through a branch is given by the product of a signal at previous node, and the gain of the branch.

Ex ①




$x_2 = a x_1$

Ex ②

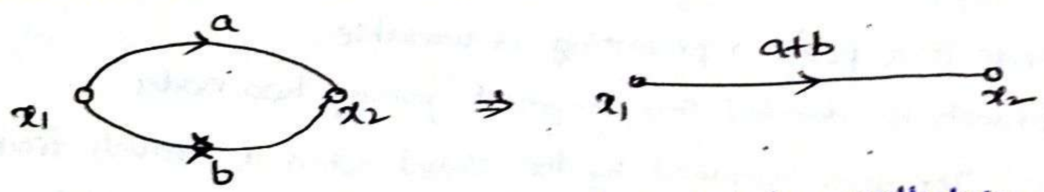



$x_3 = a_1 x_1 + a_2 x_2$

**Rule ②:** Cascaded branches can be combined to give a single branch whose transmittance is equal to the product of individual branch transmittance.

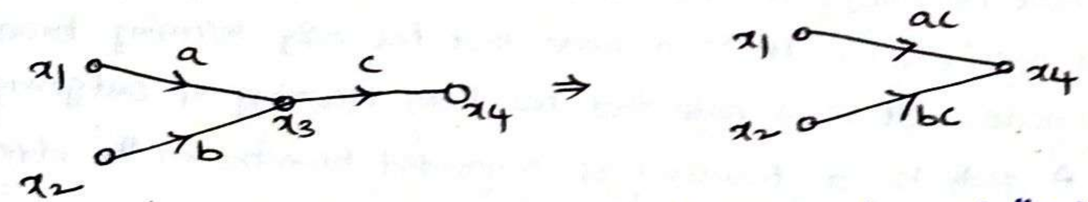
Ex:


**Rule ③:** Parallel branches can be represented by a single brach whose transmittance is the sum of individual branch transmittances.



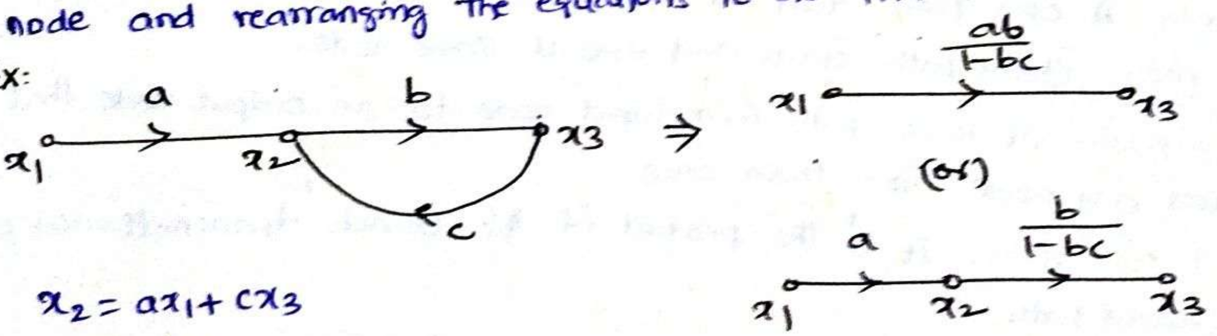

**Rule ④:** A mixed node can be eliminated by multiplying the gain of out-going branch to the ~~gr~~ gain of all incoming branches to the mixed node.

Ex:




**Rule ⑤** A loop may be eliminated by writting equations at the i/p and o/p node and rearranging the equations to the find the ratio of output to input

Ex:




$x_2 = a x_1 + c x_3$

$x_3 = b x_2$

Subst $x_2$ in $x_3$ $\therefore$ $x_3 = b[a x_1 + c x_3] = ab x_1 + bc x_3$

$x_3 = ab x_1 + bc x_3$

$x_3(1-bc) = ab x_1$

$$\frac{x_3}{x_1} = \frac{ab}{1-bc}$$

## Mason's Gain Formula:

S.J Mason has developed a simple procedure to determine the transfer function of the system represented as a signal flow graph.

Using the Mason's Gain Formula, Overall Gain is given by

$$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$$

where $T = T(s) = \frac{C(s)}{R(s)}$ → Transfer Function of the system.

$P_K$ → Forward path gain of $K^{th}$ forward path

$K$ → no. of forward paths in the signal flow graph

$\Delta$ → 1 − (Sum of individual loop gains)

+ (Sum of gain products of all possible combinations of two non-touching loops)

− (Sum of gain products of all possible combinations of three non-touching loops)

+ ......

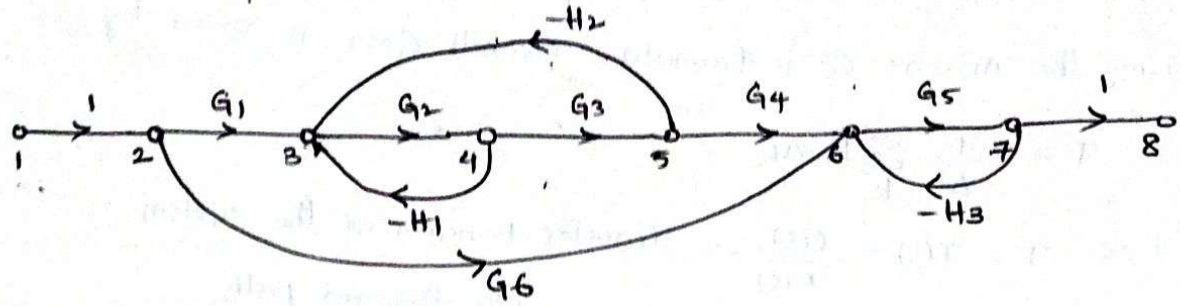
$\Delta_K$ = $\Delta$ for that part of the graph which is not touching $K^{th}$ forward path

### Procedure for converting block diagrams to signal flow Graph:

1. Assume nodes at input, output, at every summing point, at every branch point and in between cascaded blocks.
2. Draw the nodes separately as small circles and number the circles in the order 1,2,3,4,....
3. From the block diagram find the gain between each node in the main forward path and connect all the corresponding circles by straight line and mark the gain between the nodes.
4. Draw the feed forward paths between various nodes and mark the gain of feed forward path along with sign.
5. Draw the feedback paths between various nodes and mark the gain of feedback paths along with sign.

Problems: (1)

Find the overall transfer function of the system whose signal flow graph is shown in figure below



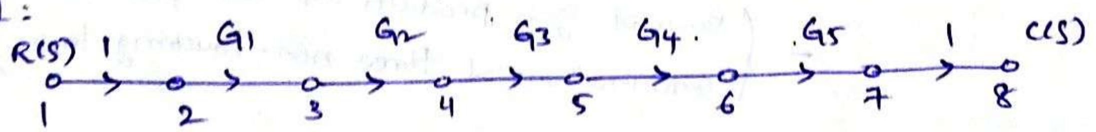

Soln:

Forward path Gains ($P_k$):

There are two forward paths $\therefore k=2$

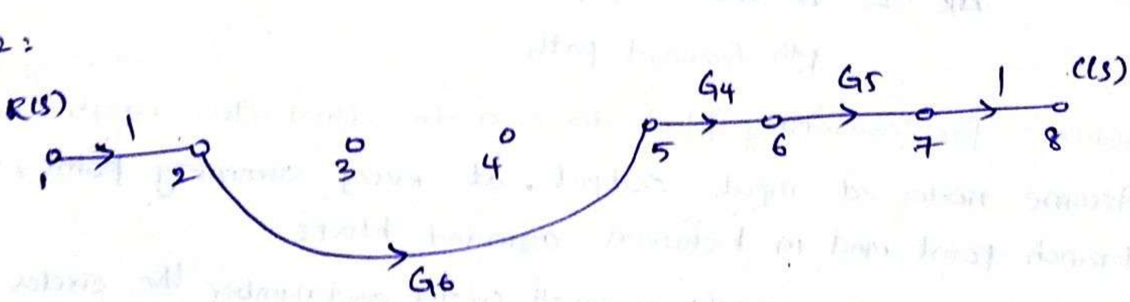
and let the forward path gains are $P_1$ & $P_2$

FP1:




$\therefore P_1 = G_1G_2G_3G_4G_5$

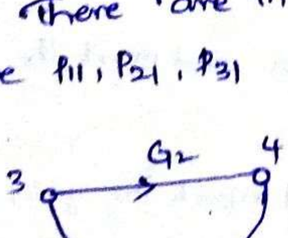
FP2:

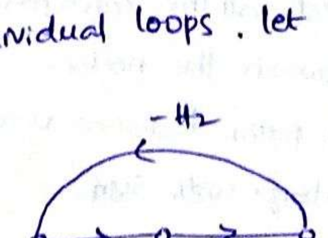


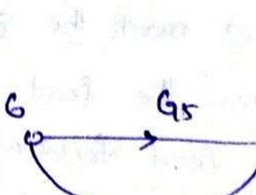

$\therefore P_2 = G_4G_5G_6$

Individual loop gains

There are three individual loops. let the individual loop gains be $P_{11}, P_{21}, P_{31}$



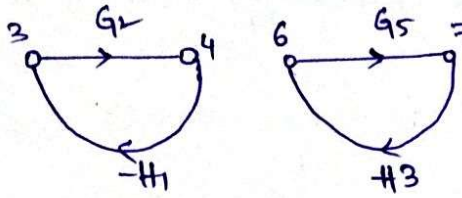

$P_{21} = -G_2G_3H_2$ $\quad$ $P_3 = -G_5H_3$

$P_{11} = -G_2H_1$

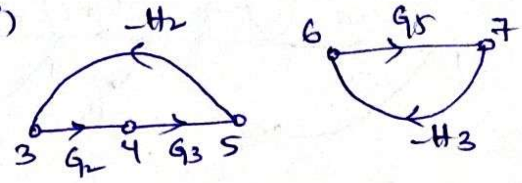
Gain products of two non-touching loops.

Two non-touching loops

i)




ii)




let the gain products of two non-touching loops be $P_{12}$ and $P_{22}$

$\therefore P_{12} = P_{11}P_{31} = (-G_2H_1)(-G_5H_3) = G_2G_5H_1H_3$

$P_{22} = P_{21}P_{31} = (-G_2G_3H_2)(-G_5H_3) = G_2G_3G_5H_2H_3$

Calculation of $\Delta$ and $\Delta k$

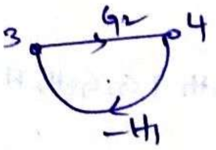
$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$

$\Delta = 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$

$\Delta_1 = 1 - 0 = 1$

Since there is no part of graph which is not touching with 1st FP.

The part of graph which is non-touching with 2nd FP is




$\therefore \Delta_2 = 1 - P_{11} = 1 - (-G_2H_1) = 1 + G_2H_1$

Transfer Function:

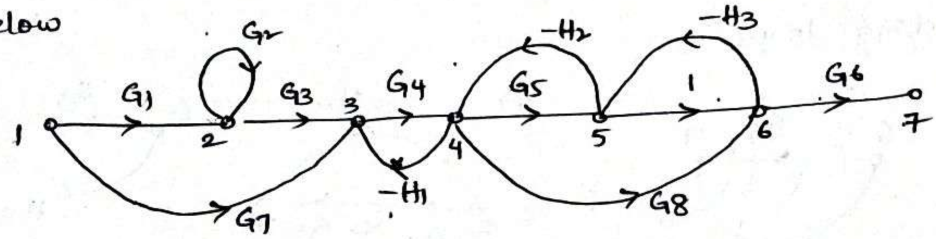
Using Mason's Gain formula, T is given by

$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$ here $K = 2$

$\therefore T = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$

$$T = \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1+G_2H_1)}{1+G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

(2) Find the overall Transfer Function of the signal flow graph shown in Figure below




Forward paths : 4.

$FP1 = 1-2-3-4-5-6-7$ $\therefore P_1 = G_1G_3G_4G_5G_6$

$FP2 = 1-3-4-5-6-7$ $\therefore P_2 = G_7G_4G_5G_6$

$FP3 = 1-2-3-4-6-7$ $\therefore P_3 = G_1G_3G_4G_8G_6$

$FP4 = 1-3-4-6-7$ $\therefore P_4 = G_7G_4G_8G_6$

Individual loops:

$L_1 = G_2$ ; $L_2 = -G_4H_1$ ; $L_3 = G_8H_2H_3$ , $L_4 = -G_5H_2$ , $L_5 = -H_3$

Two non touching loops

$L_1L_2 = -G_2G_4H_1$

$L_1L_3 = G_2G_8H_2H_3$

$L_1L_4 = -G_2G_5H_2$

$L_1L_5 = -G_2H_3$

$L_2L_5 = G_4H_1H_3$

Three non touching loops

$L_1L_2L_5 = G_2G_4H_1H_3$

$\therefore \Delta = 1-(L_1+L_2+L_3+L_4+L_5) + (L_1L_2+L_1L_3+L_1L_4+L_1L_5+L_2L_5) - L_1L_2L_5$

$\therefore \Delta = 1-(G_2-G_4H_1+G_8H_2H_3-G_5H_2-H_3) + (-G_2G_4H_1+G_2G_8H_2H_3-G_2G_5H_2 + -G_2H_3+G_4H_1H_3) - G_2G_4H_1H_3$

$\Delta_1 = 1-0 = 1$ ; $\Delta_2 = 1-G_2$ ; $\Delta_3 = 1-0$ ; $\Delta_4 = 1-G_2$

Using Mason's Gain Formula

$T = \frac{1}{\Delta}\sum_{K} P_K\Delta_K$ here $k=4$ : $T = \frac{1}{\Delta}\left[P_1\Delta_1+P_2\Delta_2+P_3\Delta_3+P_4\Delta_4\right]$

$$\therefore T = \frac{G_1G_3G_4G_5G_6 + G_7G_4G_5G_6(1-G_2) + G_1G_3G_4G_8G_6(1) + G_7G_4G_8G_6(1-G_2)}{1-G_2+G_4H_1-G_8H_2H_3+G_5H_2+H_3-G_2G_4H_1+G_2G_8H_2H_3-G_2G_5H_2-G_2H_3+G_4H_1H_3-G_2G_4H_1H_3}$$

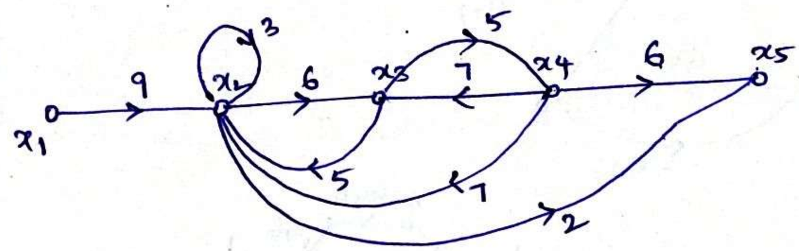
③ Draw the signal flow Graph of the following system which is described by the algebraic equations

$x_2 = 5x_3 + 7x_4 + 3x_2 + 9x_1$ $\qquad$ $x_4 = 5x_3$

$x_3 = 6x_2 + 7x_4$ $\qquad$ $x_5 = 6x_4 + 2x_2$

and find the overall system function.

Soln:




Forward paths:

FP1 : $x_1 - x_2 - x_3 - x_4 - x_5$ $\quad \therefore P_1 = 9 \times 6 \times 5 \times 6 = 1620$

FP2 : $x_1 - x_2 - x_5$ $\quad \therefore P_2 = 9 \times 2 = 18$

Loops:

Loop1 : $x_2$ $\quad L_1 : 3$

Loop2 : $x_2 - x_3 - x_2$ $\quad \therefore L_2 = 6 \times 5 = 30$

Loop3 : $x_3 - x_4 - x_3$ $\quad \therefore L_3 = 7 \times 5 = 35$

Loop4 : $x_2 - x_3 - x_4 - x_2$ $\quad \therefore L_4 = 6 \times 5 \times 7 = 210$

Two non-touching loops:

$L_1 L_3 = 3 \times 5 \times 7 = 105$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_3 = 1 - (3 + 30 + 35 + 210) + 105 = -172$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - L_3 = 1 - 35 = -34.$

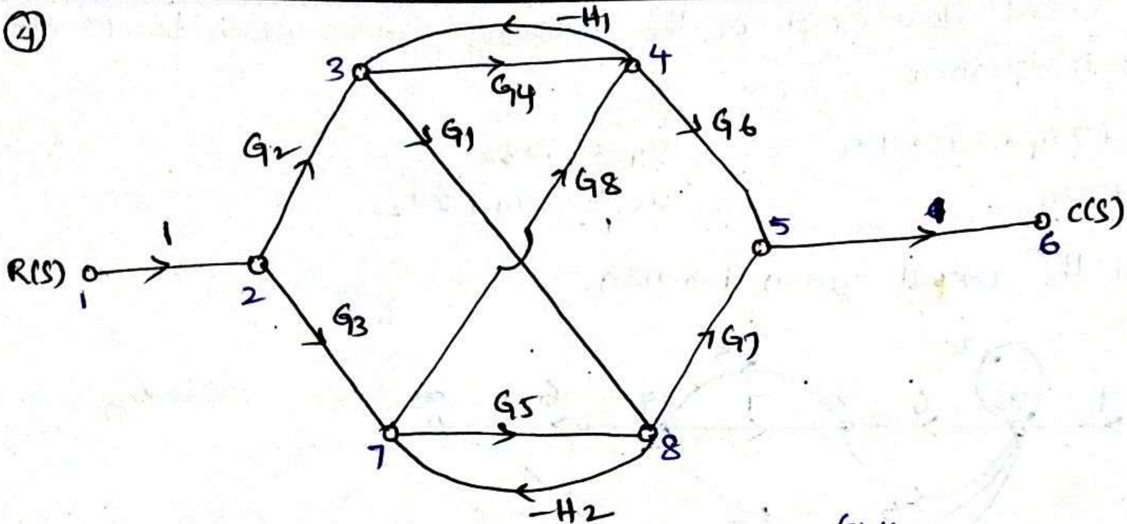
Mason's Gain Formula.

$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$ $\quad$ here $K = 2$

$\therefore T = \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2]$

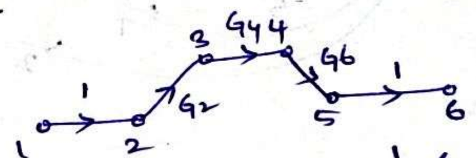
$$T = \frac{1620(1) + 18(-34)}{-172} = -5.8604$$

(4)

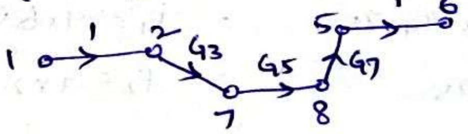



Forward paths :

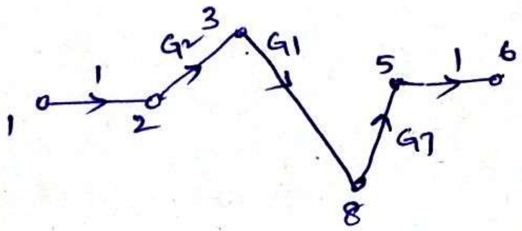
FP1 : 1-2-3-4-5-6



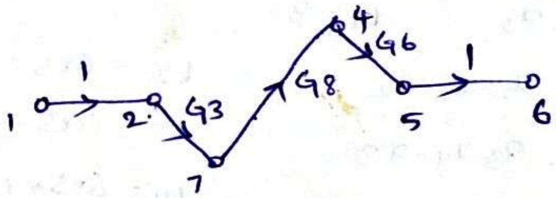
FP2 : 1-2-7-8-5-6



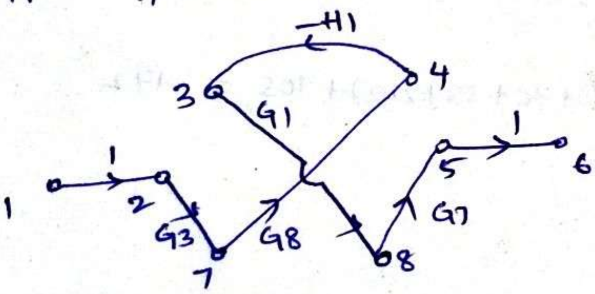
FP3 : 1-2-3-8-5-6



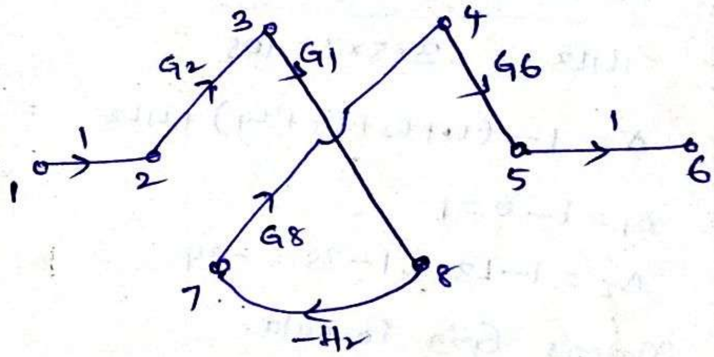
FP4: 1-2-7-4-5-6



FP5: 1-2-3-7-4-3-8-5-6



FP6: 1-2-3-8-7-4-5-6



$\therefore P_1 = G_2G_4G_6$

$P_2 = G_3G_5G_7$

$P_3 = G_1G_2G_7$

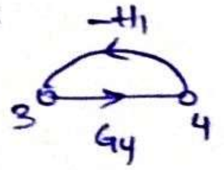
$P_4 = G_3G_6G_8$

$P_5 = -G_1G_3G_7G_8H_1$

$P_6 = -G_1G_2G_6G_8H_2$

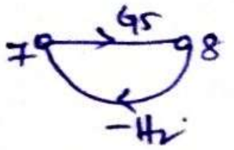
Individual loops:

L1 : 3-4-3



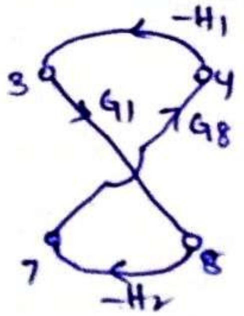

$L_1 = -G_4H_1$

Loop2 : 7-8-7




$L_2 = -G_5H_2$

Loop3 : 4-3-8-7-4




$L_3 = G_1G_8H_1H_2$

Two non-touching loops

$$L_1L_2 = G_4G_5H_1H_2$$

$$\Delta = 1-(L_1+L_2+L_3) + L_1L_2$$

$$\Delta = 1-(-G_4H_1-G_5H_2+G_1G_8H_1H_2) + G_4G_5H_1H_2$$

$$\Delta = 1+G_4H_1+G_5H_2-G_1G_8H_1H_2+G_4G_5H_1H_2$$

$$\Delta_1 = 1-L_2 = 1-(-G_5H_2) = 1+G_5H_2$$

$$\Delta_2 = 1-L_1 = 1-(-G_4H_1) = 1+G_4H_1$$

$$\Delta_3 = 1; \quad \Delta_4 = 1; \quad \Delta_5 = 1; \quad \Delta_6 = 1$$

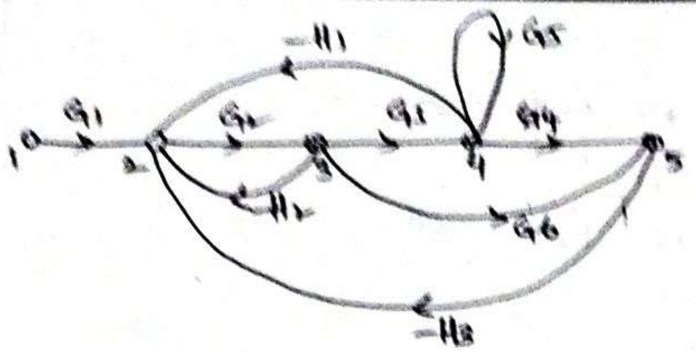
Mason's Gain Formula

$$T = \frac{1}{\Delta}\sum_K P_K \Delta_K \qquad \text{here } k=6$$

$$T = \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6\right]$$

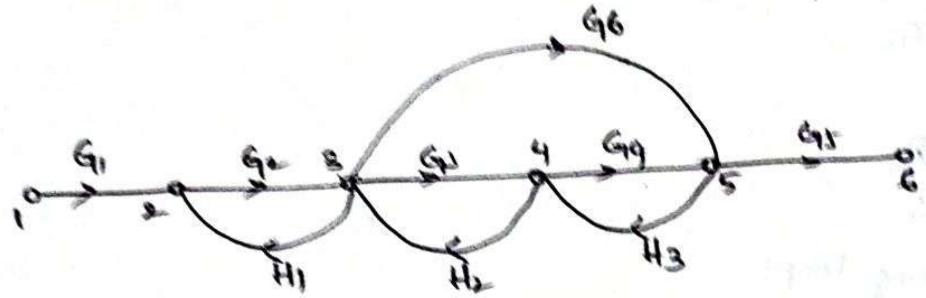
$$T = \frac{G_2G_4G_6(1+G_5H_2) + G_3G_5G_7(1+G_4H_1) + G_1G_2G_7 + G_3G_6G_8 - G_1G_3G_7G_8H_1 - G_1G_2G_6G_8H_2}{1+G_4H_1+G_5H_2-G_1G_8H_1H_2+G_4G_5H_1H_2}$$

(5)



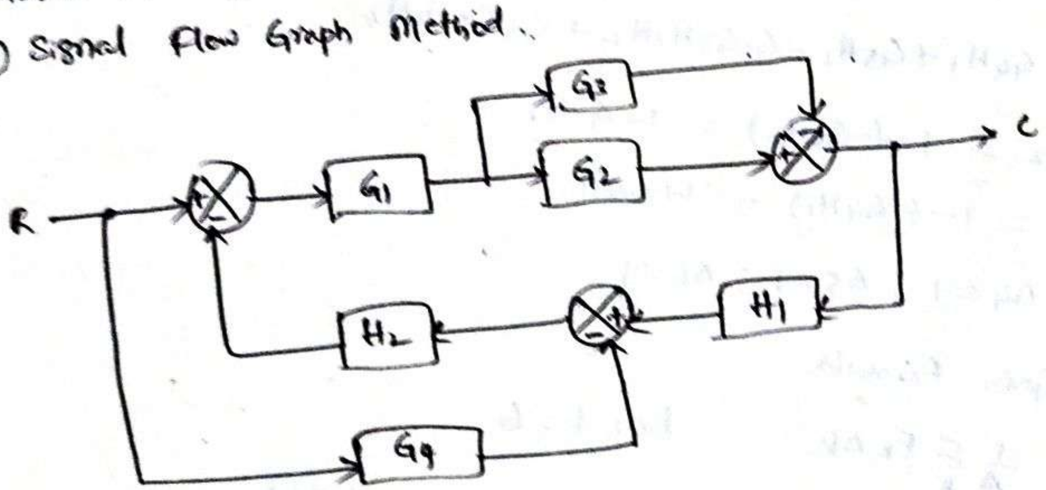

Ans: $$T = \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1+G_2G_3H_1 + H_2G_2 + G_2G_3G_4H_3 - G_5 + G_2G_6H_3 - G_2H_2G_5 - G_2G_5G_6H_3}$$

(6)



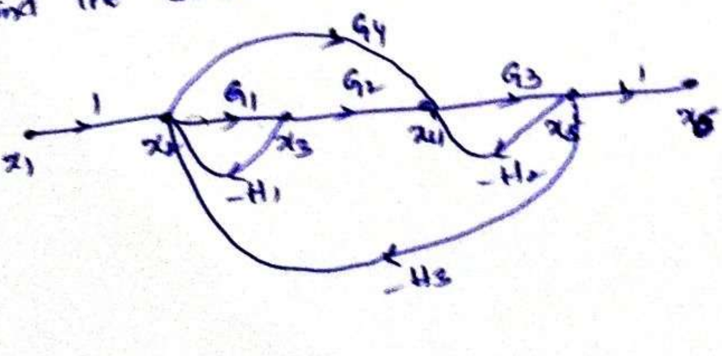

Ans: $$T = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_5G_6}{1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3}$$

(7) Obtain the Transfer function of the control system whose block diagram is shown in fig below by (a) Block diagram Reduction technique
(b) Signal Flow Graph method.

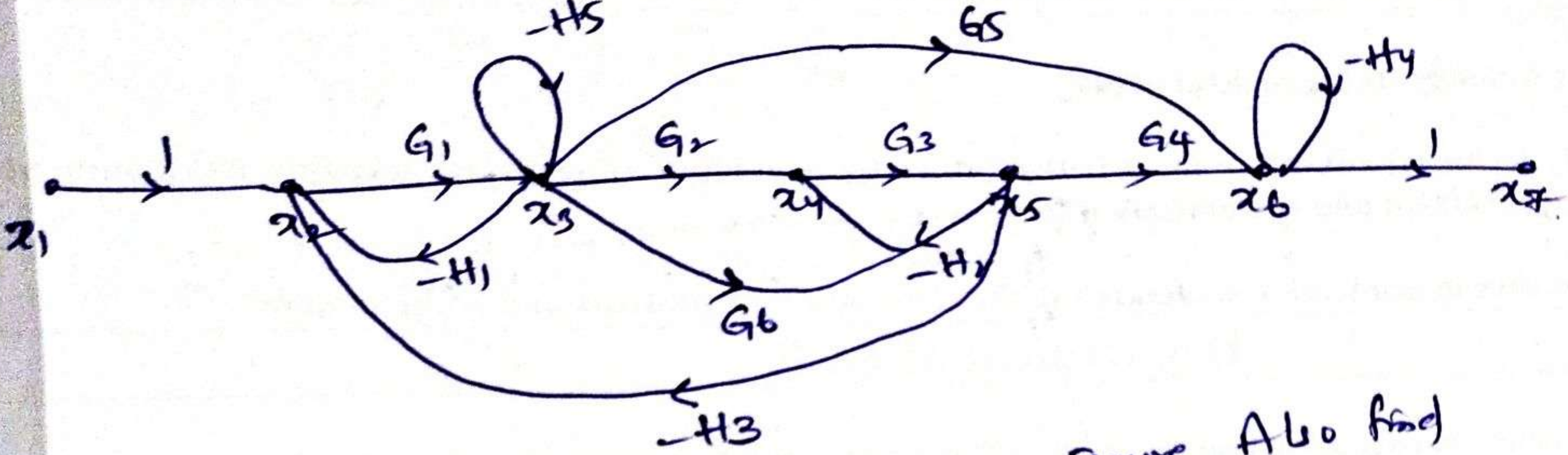



Soln:

(8) Apply the gain formula to the signal flow graph shown in figure below.
Find the transfer functions : $\frac{x_5}{x_1}$, $\frac{x_4}{x_1}$, $\frac{x_2}{x_1}$, $\frac{x_5}{x_2}$, $\frac{x_4}{x_2}$



(9)

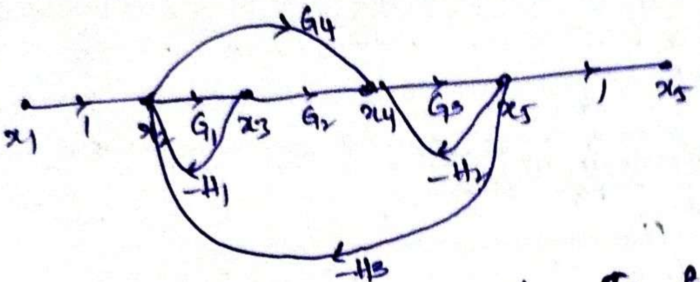



Obtain T.F of the SFG shown in Figure. Also find

$\frac{x_2}{x_1}$, $\frac{x_4}{x_1}$, $\frac{x_7}{x_2}$, $\frac{x_4}{x_2}$ & $\frac{x_7}{x_4}$

Apply the gain formula to the signal flow graph shown in figure to find the following transfer functions:

$$\frac{x_5}{x_1}, \frac{x_4}{x_1}, \frac{x_2}{x_1}, \frac{x_5}{x_2}, \frac{x_4}{x_2}$$




find $\frac{x_5}{x_1}$:

FP1: $x_1 - x_2 - x_3 - x_4 - x_5 - x_6 \Rightarrow P_1 = G_1G_2G_3$

FP2: $x_1 - x_2 - x_4 - x_5 - x_5 \Rightarrow P_2 = G_3G_4$

loop1: $x_2 - x_3 - x_2 \Rightarrow L_1 = -G_1H_1$

loop2: $x_4 - x_5 - x_4 \Rightarrow L_2 = -G_3H_2$

loop3: $x_2 - x_3 - x_4 - x_5 - x_2 \Rightarrow L_3 = -G_1G_2G_3H_3$

loop4: $x_2 - x_4 - x_5 - x_2 \Rightarrow L_4 = -G_3G_4H_3$

Non-Touching loops:

$L_{12} = G_1G_3H_1H_2$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_{12}$

$= 1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + G_3G_4H_3 + G_1G_3H_1H_2$

$\Delta_1 = 1 - 0$

$\Delta_2 = 1 - 0$

T.F is $\frac{x_5}{x_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$$\frac{x_5}{x_1} = \frac{G_1G_2G_3 + G_3G_4}{\Delta}$$

To find $\frac{x_4}{x_1}$:

FP1: $x_1 - x_2 - x_3 - x_4 \Rightarrow P_1 = G_1G_2$

FP2: $x_1 - x_2 - x_4 \Rightarrow P_2 = G_4$

$\Delta_1 = 1 - 0$, $\Delta_2 = 1 - 0$

T.F is $\frac{x_4}{x_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$$= \frac{G_1G_2 + G_4}{\Delta}$$

To find $\frac{x_2}{x_1}$:

FP1: $x_1 - x_2 \Rightarrow P_1 = 1$

$\Delta_1 = 1 - L_2 = 1 + G_3H_2$

TF is given by

$$\frac{x_2}{x_1} = \frac{P_1\Delta_1}{\Delta} = \frac{1 + G_3H_2}{\Delta}$$

To find $\frac{x_5}{x_2}$

$$\frac{x_5}{x_2} = \frac{x_5/x_1}{x_2/x_1} = \frac{\frac{G_1G_2G_3 + G_3G_4}{\Delta}}{\frac{1 + G_3H_2}{\Delta}}$$

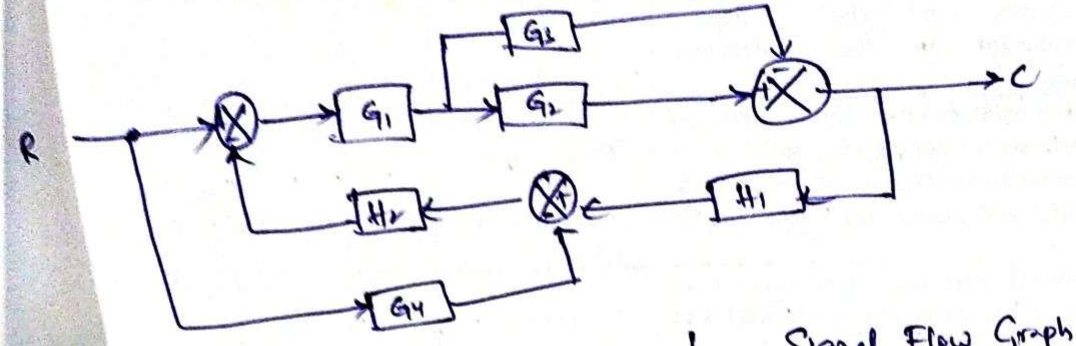
$$= \frac{G_1G_2G_3 + G_3G_4}{1 + G_3H_2}$$

To find $\frac{x_4}{x_2}$

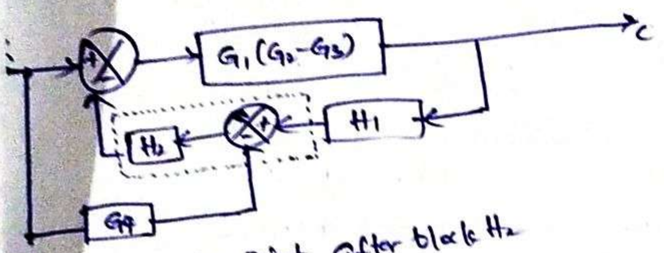
$$\frac{x_4}{x_2} = \frac{x_4/x_1}{x_2/x_1} = \frac{G_1G_2 + G_4/\Delta}{\cancel{G_1G_2G_3 + G_3G_4}\ 1 + G_3 / \Delta}$$

$$= \frac{G_1G_2 + G_4}{1 + G_3}$$

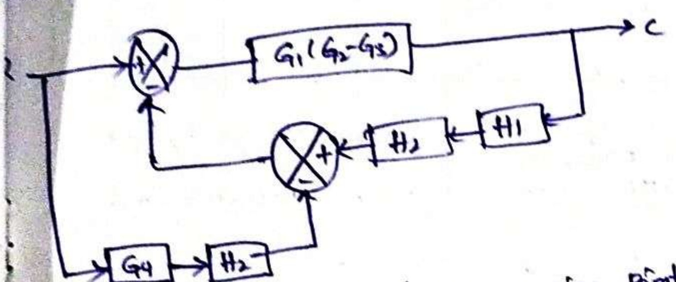
btain the Transfer function of the control system whose block diagram is shown in figure by using (i) Block diagram Reduction technique (ii) Signal flow graph method



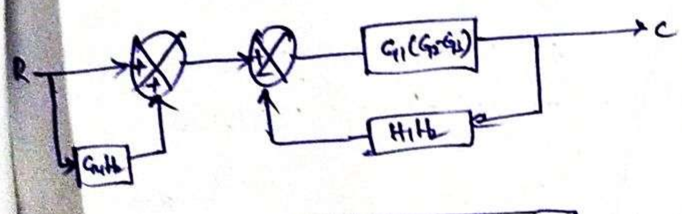
Block diagram Reduction Technique

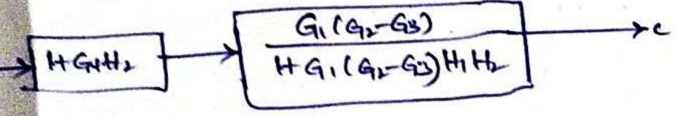


Move the summing point after block $H_2$



Combine series blocks & Interchange summing points

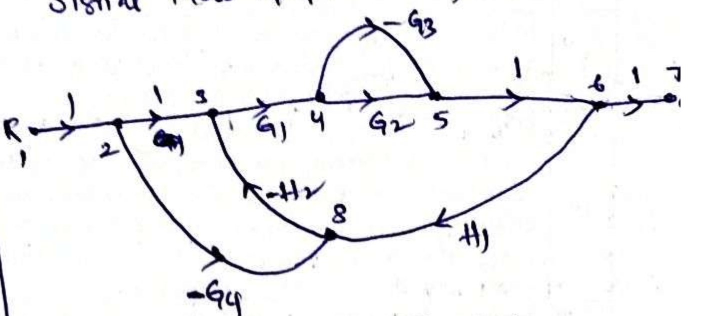




$$\frac{C}{R} = \frac{(1+G_4H_2)[G_1(G_2-G_3)]}{1+G_1(G_2-G_3)H_1H_2}$$

$$= \frac{G_1G_2 - G_1G_3 + G_1G_2G_4H_2 - G_1G_3G_4H_2}{1+G_1G_2H_1H_2 - G_1G_3H_1H_2}$$

Signal Flow Graph Method



FP1 : 1-2-3-4-5-6-7 $\quad P_1 = G_1G_2$

FP2 : 1-2-8-3-4-5-6-7 $\quad P_2 = +G_1G_2G_4H_2$

FP3 : 1-2-3-4-5-6-7 $\quad P_3 = -G_1G_3$

FP4 : 1-2-8-3-4-5-6-7 $\quad P_4 = -G_1G_3G_4H_2$

loop1 : 3-4-5-6-8-3

$L_1 = -G_1G_2H_1H_2$

loop2 : 3-4-5-6-8-3

$L_2 = G_1G_3H_1H_2$

$$\Delta = 1-(L_1+L_2)$$
$$= 1-(-G_1G_2H_1H_2 + G_1G_3H_1H_2)$$
$$= 1+G_1G_2H_1H_2 - G_1G_3H_1H_2$$

$\Delta_1 = 1-0$

$\Delta_2 = 1-0$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_2 + G_1G_2G_4H_2 - G_1G_3 - G_1G_3G_4H_2}{1+G_1G_2H_1H_2 - G_1G_3H_1H_2}$$

Comparison of block diagram & Signal flow graph methods

| Block diagram method | Signal flow graph method |
| --- | --- |
| ① It is a pictorial representation of the functions performed by each component and of the flow of signals | ① It is graphical repⁿ of a relation b/w variables of a set of linear algebraic eqn's written in the form of cause and effect relations |
| ② It can be used to represent linear as well as non-linear systems | ② It can be used to represent only linear systems |
| ③ No direct formula is available to find the overall T.F of the system | ③ Mason's gain formula is available to find the overall T.F of the system |
| ④ Step-by-step procedure is to be followed to find the T.F | ④ T.F. can be obtained in one-step |
| ⑤ It is not a symmetrical method | ⑤ It is a symmetrical method |
| ⑥ It indicates more realistically the signal flows of the system than the original system itself | ⑥ It is constrained by more rigid mathematical rules than a block diagram |
| ⑦ For a given system, the block diagram is not unique. Many dissimilar and unrelated systems can be represented by the same block diagram | ⑦ For a given system, the signal flow graph is not unique. |