Newton's law of cooling: The rate of change of temp of a body is proportional to difference of the temp of body & that of surrounding media.

Let 'θ' be the temperature of the body at the time 't' and 'θ_0' be the temperature of its surrounding medium (air) then by the Newton's law of cooling we have

$$\frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

Now by variable seperable method

$$\frac{d\theta}{\theta - \theta_0} = -Kdt$$

On Integrating $\int \frac{d\theta}{\theta - \theta_0} = -k \int dt$

$$\boxed{\log(\theta - \theta_0) = -kt + \log c}$$

(Q) A body is originally at 80°c and cools down at 60° in 20min if the temp of the air is 40°, find the temp of the body after 40 min.

Sol: Let 'θ' be the temp of the body at tym 't'

Given: $\theta_0 = 40°$

By Newton's law of cooling we have

$$\log(\theta - \theta_0) = -kt + \log c \rightarrow ①$$

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$t = 0$, $\theta = 80°$

$t = 20$, $\theta = 60°$

$t = 40$, $\theta = ?$

At $t = 0$ $\theta = 80°$

sub t, θ in eq. ①

$$\log(80-40) = -k(0) + \log c$$

$$\log 40 = \log c$$

$$c = 40$$

sub c in eq. ①

$$\log(\theta - 40) = -kt + \log 40 \rightarrow ②$$

At $t = 20$, $\theta = 60°$

sub t, θ in eq. ②

$$\log(60-40) = -k(20) + \log 40$$

$$k = \frac{1}{20}\log 2$$

At $t = 40$ $\theta = ?$

$$\log(\theta - 40) = -\frac{\cancel{40}^{2}}{\cancel{20}}\log 2 + \log 40.$$

$$= \log 40 - \log 4$$

$$= \log 10$$

$$\theta - 40 = 10$$

$$\theta = 50°C$$

(Q) If the air is maintained at 15°C & the temp of the body drops from 70° to 40° in 10 min what will be its temp after 30 min

$\theta_0 = 15°C$ $t = 0$ $\theta = 70°C$

$t = 10$ $\theta = 40$ min.

$t = 30$ $\theta = ?$

$\log(\theta - 15^\circ) = -k(0) + \log c$

$\theta - 15 = c$

$70 - 15 = c$

$\Rightarrow c = 55^\circ$

now sub $c = 55^\circ$ $t = 10$ & $\theta = 40$ in eq. ①

$\log(40 - 15^\circ) = -k(10) + \log 55$

$\log 25 = -10k + \log 55$

$10k = \log(11/5)$

$k = \frac{1}{10} \log(11/5)$

when $t = 30$ $\theta = ?$

$\log(\theta - 15) = -\frac{30}{10} \log\left(\frac{11}{5}\right) + \log(55)$

$= -3 \log\left(\frac{11}{5}\right) + \log(55)$

$= \log 55 - \log\left(\frac{11}{5}\right)^3$

$= \log\left(\frac{55}{1} \times \frac{5^3}{11^2}\right)$

$\log(\theta - 15)$

$= 4 \log 5$

$\cancel{\theta - 15 = 5^4}$

$\cancel{\theta = 225 + 15}$

$\cancel{= 240}$

$\log(\theta - 15) = \log(5^4/11^2)$

$\theta - 15 = 5^4/11^2 = \frac{625}{121}$

$= 5.16$

$\Rightarrow \theta = 20.16$

Q) A body's temp is changing from 100° to 70° in 15 minutes find when the temperature will be 40°. If the temp of the air is 30°

$$\theta_0 = 30$$

$$t = 0 \qquad \theta = 100°c$$

$$t = 15 \qquad \theta = 70°c$$

$$t = ? \qquad \theta = 40°c$$

$$\log(70) = -k(0) + \log c$$

$$c = 70$$

$$\log(70-30) = -kt + \log 70$$

$$\log 40 = -kt + \log 70$$

$$kt = \log 7/4$$

$$k = \frac{1}{15}\log(7/4)$$

$$\log(40-30) = -\frac{t}{15}\log(7/4) + \log 70$$

$$\log(10) = -t/15 \log(7/4) + \log 70$$

$$t/15 \log(7/4) = \log 70/10$$

$$\cancel{t/15 \log(7/4) = 15\log\left(\frac{70}{40}\times 40\right)}$$

$$\cancel{= 15\log 7/4 + \log 40}$$

$$t \log(7/4) = 15\log(7)$$

$$t = \frac{15\log 7}{\log(7/4)} = \frac{12.67}{0.24}$$

$$= 52.15 \text{ min}$$

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(Q) A body kept in air with temp 25°c cools from 140°c to 80°c in 20 min. Find when the body cools down to 35°c

$\theta_0 = 25°c$

$t = 0$ $\quad \theta = 140°c$

$t = 20$ $\quad \theta = 80°c$

$t = ?$ $\quad \theta = 35°c$

$\log(140-25) = -k(0) + \log c$

$115 = c$

$t = 20$ $\quad \theta = 80°c$

~~$\log(140-\cancel{80}^{25}) = -k(20) + \log(115)$~~

~~$\log(60) = -20k + \log 115$~~

~~$20k = \log \frac{\cancel{115}^{23}}{\cancel{60}_{12}}$~~

~~$k = \frac{1}{20}\log\left(\frac{23}{12}\right)$~~

$\log(80-25) = -20k + \log 115$

$\log(55) = -20k + \log 115$

$20k = \log 115/55$

$k = \frac{1}{20}\log(23/11)$

$\log(10) = \frac{-1}{20}\log(23/11)t + \log 115$

$\frac{1}{20}\log(23/11)\, t = \log 115 - \log 10$

$= \log 11.5$

$t = \frac{(2.24)20}{0.32} = 66.22$

Q) If the temp of air is 20° and temp of the body drops from 100° to 80° in 10min. What will be its temp after 20 min. When will be temp 40°

sol: $\theta_0 = 20$

$t = 0 \quad \theta = 100°c$

$t = 10 \quad \theta = 80$

$t = 20 \quad \theta = ?$

$t = ? \quad \theta = 40$

$\log 80 = -K(0) + \log c$

$C = 80$

When $t = 10 \quad \theta = 80$

$\log(80 - 20) = -10K + \log 80$

$10k = \log \frac{\cancel{80}^{4}}{\cancel{60}_{3}}$

$k = \frac{1}{10} \log 4/3$

$t = 20 \quad \theta = ?$

$\log(\theta - 20) = -\frac{1}{10} \log 4/3 (20) + \log 80$

$= \log 80 - \log 16/9$

$= \log\left(\frac{\cancel{80}^{5}}{\cancel{16}_{2}} \times 9\right)$

$\theta - 20 = 45$

$\theta = 65$

When $\theta = 40 \quad t = ?$

$\log 20 = -\frac{t}{10} \log 4/3 + \log 80 \Rightarrow t = 10 \frac{\log 4}{\log 4/3}$

$= \frac{6.02}{0.12} = 50.4 \quad 48.1$

Law of natural Growth & decay:

Let $x(t)$ be the amount of some substance at a time 't'. A law of chemical conversion states that the rate of change of among $x(t)$ of a chemically changing substance is proportional to the amount of substance available at that time

i.e $\frac{dx}{dt} \alpha k$.

i.e $\frac{dx}{dt} = -kt$ - Decay

$\frac{dx}{dt} = kt$ - Growth

The formula is $\log x = -kt + \log c$

(Q) The number 'N' of bacteria in a culture grow at a rate proportional to 'N'. The value of N was initially 100 & increased to 332 in 1hr. What was the value of 'N' after 1½ hrs.

sol: Let 'N' be the growth of bacteria in a culture as the bacteria is growing we have

$$\frac{dN}{dt} = kt$$

Initially Given $t = 0$ $N = 100$

$t = 1hr$ $N = 332$

$= 60 min$

$t = 1\frac{1}{2} hr$ $N = ?$

$= 90 min$

$\log N = kt + \log c$

$\log N + k(0) + \log c$

$\log 10^2 = \log c$ $\Rightarrow C = 100$

$\log N = kt + \log c$

$\log 332 = ~~90~~ k + \log 100$

$\log 3.32 = 6~~9~~0k$

$k = \frac{1}{6~~9~~0} \log 3.32$

when $t = 90$

$\log N = \frac{1}{2~~6~~0} \log 3.32^{\frac{3}{(90)}} + \log 100$

$= \log\left(\frac{332}{100}\right)^{3/2} + \log 100$

$= \log\left(\left(\frac{2\sqrt{83}}{10}\right)^{3} + \log 100\right.$

$= \log\left(\frac{(2\sqrt{83})^3}{10^3} \cdot 10^2\right)$

$\log N = \log \frac{(2\sqrt{83})^3}{10}$

$N = \frac{(2\sqrt{83})^3}{10}$

$= 604.93$

(Q) A bacterial culture ~~gets~~ growing exponentially increases from 200 to 500 gms in the period from 6 am to 9 a.m How many gms will be present at noon (afternoon)

$t = ~~0900~~\ 0 \qquad N = 200$

$t = ~~180~~\ 3hr \qquad N = 500$

$t = 6hr \qquad N = ?$

$\log N = kt + \log c$

~~using~~ $\log 200 = k(0) + \log c$

$\log c = \log 200$

$\log(500) = 3k + \log 200$

$3k = \log 5/2 \Rightarrow k = \frac{1}{3} \log 5/2$

$\log N = \frac{1}{3}\log(5/2)(6) + \log 200$

$= \log(5/2)^2 + \log 200$

$= \log\left(\frac{25}{4} \times 200\right)$

$N = 1250 gms$

(8) Bacteria in a culture grows exponentially so that the initial num has doubled in 3hrs. How many times the initial num will be present after 9hrs.

Ans. Let us take $N = N_1$

$t = 0$ $N = N_1$

$t = 3hr$ $N = 2N_1$

$t = 9hrs$ $N = ?$

$\log N_1 = k(0) + \log c$

$\log c = \log N_1$

$\log(2N_1) = 3k + \log N_1$

$3k = \log 2 \Rightarrow k = \frac{1}{3}\log 2$

$\log N = \frac{9}{3}\log 2 + \log N_1$

$= \log 8 + \log N_1$

$N = 8N_1$

(9) If radio active $_{14}C$ has half-life of 5750 year what will remain of 1gm after 3000 year.

sol: The problem is related to decay so we use the formula

$$\log N = -Kt + \log c$$

$t = 0$ $N = 1gm$

$t = 5750$ yr $N = \frac{1}{2}gm$

$t = 3000$ yrs $N = ?$

$$\log N = -K(0) + \log c.$$

$$\log c = 0$$

$$c = 1$$

$$\log \tfrac{1}{2} gm = -K(5750) + 0$$

$$-\cancel{\log \tfrac{1}{2}} = -K(5750)$$

$$K = \frac{1}{5750} \log 2$$

$$\log 2^{-1} = -K(5750)$$

$$-\log 2 = -K(5750)$$

$$K = \frac{1}{5750} \log 2$$

$$\log N = -\frac{3000}{5750} \log 2 + 0.$$

$$\log N = -0.15$$

$$N = 10^{-0.15}$$

$$= 0.70$$

(Q) The rate at which bacteria multiplied is α to N. If the original value is doubled in two hours? When it will be tripled

sol: let $N = N_1$

$t = 0$ $N = N_1$

$t = 2h$ $N = 2N_1$

$t = 3h$ $N = 3N_1$

$$\log N_1 = Kt + \log c$$

$$\log N_1 = \log c$$

$$\log(2N_1) = K(2) + \log N_1$$

$$K = \cancel{\log N_1}\ \frac{1}{2}\log 2$$

$$\cancel{\log N = t\log N_1 = \log N_1\ \frac{3}{2}\log 2 + \log N_1}$$

$$\cancel{\log N = \log 2^{3/2} N_1}$$

$$\cancel{N = 2^{3/2} N_1}$$

$$\log(3N_1) = \frac{t}{2}\log 2 + \log N_1$$

$$\log 3 = t/2 \log 2$$

$$t = \frac{2\log 3}{\log 2} = 3.16 \text{ hrs}$$

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