1 (a) $\theta_s = 30^\circ c$ UNIT-1

$$\boxed{\theta = \theta_s + c\,e^{-kt}} \quad -(1)$$

when $t=0$; $\theta = 100^\circ c$.

$$(1) \Rightarrow 100 = 30 + c \cdot e^{-k(0)}$$

$$\Rightarrow 100 = 30 + c \Rightarrow \underline{c = 70}$$

$$(1) \Rightarrow \theta = 30 + 70\,e^{-kt} \quad -(2)$$

when $t=2$; $\theta = 90^\circ c$

$$(2) \Rightarrow 90 = 30 + 70e^{-2k}$$

$$\Rightarrow 60 = 70e^{-2k} \Rightarrow e^{-2k} = \frac{6}{7}$$

$$\Rightarrow \log\left(\frac{6}{7}\right) = -2k \Rightarrow -k = \frac{1}{2}\log\left(\frac{6}{7}\right)$$

$$(2) \Rightarrow \theta = 30 + 70\,e^{\frac{t}{2}\log\left(\frac{6}{7}\right)}$$

$$\Rightarrow \theta = 30 + 70\,e^{\log\left(\frac{6}{7}\right)^{t/2}}$$

$$\Rightarrow \theta = 30 + 70\left(\frac{6}{7}\right)^{t/2} \quad -(3)$$

when $t=5$, $\theta = ?$

$$\theta = 30 + 70\left(\frac{6}{7}\right)^{5/2}$$

$$\theta = 30 + 70(0.68)$$

$$\theta = 30 + 47.6$$

$$\underline{\theta = 77.6^\circ c.}$$

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(b) $\theta_s = 20°c$

$$\boxed{\theta = \theta_s + ce^{-kt}} \quad - (1)$$

When $t=0$, $\theta = 100$

$(1) \Rightarrow 100 = 20 + ce^{0}$

$\Rightarrow c = 80$

$(1) \Rightarrow \theta = 20 + 80e^{-kt} \quad - (2)$

When $t=1$, $\theta = 60$

$(2) \Rightarrow 60 = 20 + 80e^{-k}$

$\Rightarrow \cancel{40} = \cancel{80}^{2} e^{-k}$

$\Rightarrow e^{-k} = \frac{1}{2} \Rightarrow -k = \log\left(\frac{1}{2}\right).$

$(2) \Rightarrow \theta = 20 + 80e^{t\log\left(\frac{1}{2}\right)}$

$\Rightarrow \theta = 20 + 80e^{\log\left(\frac{1}{2}\right)^{t}}$

$\Rightarrow \theta = 20 + 80\left(\frac{1}{2}\right)^{t} \quad - (3)$

When $\theta = 30$, $t = ?$

$(3) \Rightarrow 30 = 20 + 80\left(\frac{1}{2}\right)^{t}$

$\Rightarrow 10 = 80\left(\frac{1}{2}\right)^{t}$

$\Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^{t}$

Apply log on Both sides. $\qquad \frac{\log a}{\log b} = \log_b a$

$\Rightarrow \log\left(\frac{1}{8}\right) = t\log\left(\frac{1}{2}\right).$

$\Rightarrow t = \frac{\log(1/8)}{\log(1/2)} = \log_{(1/2)}(1/8) = \log_{(1/2)}(1/2)^3 = 3\log_{(1/2)}(1/2)$

$= 3(1)$

$= \underline{3 \text{ mins}}$

② ⓐ $(D^2+4D+4)y = 3\cos x.$

$$f(D)y = Q(x) \quad -①$$

$f(D) = D^2+4D+4 \qquad Q(x) = 3\cos x$

G.S. of ① is $y = y_c + y_p.$

$\underline{y_c}$:- consider A.E $f(m)=0.$

$\Rightarrow m^2+4m+4=0 \Rightarrow m=-2,-2$

Roots are equal,

$$\underline{y_c = (c_1+c_2x)e^{-2x}}$$

$\underline{y_p}$:- $y_p = \dfrac{Q(x)}{f(D)} = \dfrac{3\cos x}{D^2+4D+4}$

Replace D^2 with -1.

$$y_p = \frac{3\cos x}{-1+4D+4} = \frac{3\cos x}{4D+3} \times \frac{4D-3}{4D-3}$$

$$= \frac{(4D-3)(3\cos x)}{16D^2-9} \qquad D^2 \leftrightarrow -1.$$

$$= \frac{(4D-3)(3\cos x)}{-16-9}$$

$$= \frac{-(4D-3)(3\cos x)}{25}$$

$$= \frac{-1}{25}\left[12D\cos x - 9\cos x\right]$$

$$= \frac{-1}{25}\left[12(-\sin x) - 9\cos x\right]$$

$$\underline{y_p = \frac{12\sin x + 9\cos x}{25}}$$

$\therefore$ G.S is $\underline{y = (c_1+c_2x)e^{-2x} + \dfrac{12\sin x+9\cos x}{25}}.$

(b) $(D^3-1)y = e^x + \sin 3x$

$f(D)y = Q(x) \quad -(1)$

$f(D) = D^3 - 1 \quad ; \quad Q(x) = e^x + \sin 3x$

G.S of (1) is, $y = y_c + y_p$.

$\underline{y_c}$:- Consider A.E, $f(m) = 0$.

$\Rightarrow m^3 - 1 = 0 \Rightarrow (m-1)(m^2+m+1) = 0$

$\Rightarrow m = 1, \dfrac{-1 \pm i\sqrt{3}}{2}$.

$$\underline{\underline{y_c}} = e^{\frac{-x}{2}}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right) + c_3 e^x$$

y_p :- $y_p = \dfrac{Q(x)}{f(D)}$

$$= \frac{e^x + \sin 3x}{D^3 - 1} = \underbrace{\frac{e^x}{D^3-1}}_{y_{p_1}} + \underbrace{\frac{\sin 3x}{D^3-1}}_{y_{p_2}}$$

$y_{p_1} = \dfrac{e^x}{D^3-1}$

$f(D) = D^3 - 1 \Rightarrow f(1) = 1 - 1 = 0$

$f'(D) = 3D^2 \Rightarrow f'(1) = 3 = 3.$

$\Rightarrow \underline{\underline{y_{p_1}}} = \dfrac{xe^x}{3}$.

$y_{p_2} = \dfrac{\sin 3x}{D^3 - 1}$

Replace D^2 with -9

$$y_{p_2} = \frac{\sin 3x}{-9D-1} \times \frac{-9D+1}{-9D+1} = \frac{-9D \sin 3x + \sin 3x}{81D^2 - 1}$$

$D^2 \leftrightarrow -9.$

$$= \frac{-9D\sin 3x + \sin 3x}{81(-9)-1} = \frac{-9D \sin 3x + \sin 3x}{-730}$$

$$= \frac{1}{730}\left[9D\sin 3x - \sin 3x\right]$$

$$= \frac{1}{730}\left[9\cos 3x(3) - \sin 3x\right]$$

$$Y_{P_2} = \frac{27\cos 3x - \sin 3x}{730}$$

$\therefore$ G.S of ① is $y = y_c + y_{p_1} + y_{p_2}$.

$$\Rightarrow y = e^{-\frac{x}{2}}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right) + c_3 e^{x} + \frac{xe^x}{3} + \frac{27\cos 3x - \sin 3x}{730}$$

③ $(D^2+4)y = \sec 2x$. —①

$f(D)\,y = Q(x)$.

$f(D) = D^2+4$; $Q(x) = \sec 2x$.

G.S of ① is $y = y_c + y_p$.

y_c:- Consider A.E, $f(m)=0$.

$\Rightarrow m^2+4=0 \Rightarrow m = \pm 2i$

$\Rightarrow y_c = c_1\cos 2x + c_2 \sin 2x$.

$y_c = c_1 u + c_2 v$

$\Rightarrow u = \cos 2x$; $v = \sin 2x$

$\frac{du}{dx} = -2\sin 2x$; $\frac{dv}{dx} = 2\cos 2x$.

we have, $w = uv' - vu'$

$w = (\cos 2x)(2\cos 2x) - (\sin 2x)(-2\sin 2x)$

$w = 2\cos^2 2x + 2\sin^2 2x$

$w = 2(\cos^2 2x + \sin^2 2x)$

$w = 2$

$y_p = Au + Bv$, where $A = -\int \frac{vQ}{w}dx$ and $B = \int \frac{uQ}{w}dx$

$$A = -\int \frac{\sin 2x \cdot \sec 2x}{2} dx \qquad B = \int \frac{\cancel{\cos 2x} \cdot \cancel{\sec 2x}}{2} dx$$

$$= \frac{-1}{2}\int \tan 2x \cdot dx \qquad = \frac{1}{2}\int 1 \cdot dx$$

$$= \frac{-1}{2} \frac{\log|\sec 2x|}{2} \qquad = \frac{x}{2}$$

$$= \frac{-1}{4}\log(\sec 2x)$$

$$\Rightarrow y_p = \frac{-\log(\sec 2x) \cdot \cos 2x}{4} + \frac{x \sin 2x}{2}.$$

$\therefore$ G.S of ① is, $y = c_1\cos 2x + c_2 \sin 2x - \frac{\log(\sec 2x)\cos 2x}{4} + \frac{x\sin 2x}{2}$

④ ⓐ $\theta_s = 20°c.$

$$\boxed{\theta = \theta_s + ce^{-kt}} \quad —①$$

when $t = 0$, $\theta = 100°c$

① $\Rightarrow$ $100 = 20 + c \Rightarrow \underline{c = 80}$

① $\Rightarrow$ $\theta = 20 + 80e^{-kt}$ — ②

when $t = 10$, $\theta = 80°c$

② $\Rightarrow$ $80 = 20 + 80e^{-10k}$

$$\Rightarrow 60 = 80e^{-10k}$$

$$\Rightarrow e^{-10k} = \frac{\cancel{6}3}{\cancel{8}4} \Rightarrow -10k = \log\left(\frac{3}{4}\right)$$

$$\Rightarrow -k = \frac{1}{10}\log\left(\frac{3}{4}\right) \Rightarrow$$

② $\Rightarrow \theta = 20 + 80 e^{\frac{t}{10}\log\left(\frac{3}{4}\right)}$

$\Rightarrow \theta = 20 + 80 e^{\log\left(\frac{3}{4}\right)^{t/10}}$

$\Rightarrow \theta = 20 + 80\left(\frac{3}{4}\right)^{t/10}$ — ③

When $t = 20$, $\theta = ?$

③ $\Rightarrow \theta = 20 + 80\left(\frac{3}{4}\right)^{\cancel{20}/\cancel{10}\,2}$

$\Rightarrow \theta = 20 + \cancel{80}^{5}\left(\frac{9}{\cancel{16}}\right)$.

$\Rightarrow \theta = 20 + 45$

$\Rightarrow \theta = 65^\circ c$

ⓑ $(D^2 - 4D + 4)y = e^{2x} + \cos 2x$.

$f(D)y = Q(x)$

$f(D) = D^2 - 4D + 4 \qquad Q(x) = e^{2x} + \cos 2x$

y_c:- Consider A·E, $f(m) = 0$.

$m^2 - 4m + 4 = 0$

$m = 2, 2 \Rightarrow$ Roots are equal.

$\Rightarrow y_c = (c_1 + c_2 x) e^{2x}$

$y_p = \frac{Q(x)}{f(D)} = \frac{e^{2x} + \cos 2x}{D^2 - 4D + 4} = \underbrace{\frac{e^{2x}}{D^2 - 4D + 4}}_{y_{p_1}} + \underbrace{\frac{\cos 2x}{D^2 - 4D + 4}}_{y_{p_2}}$

$y_{p_1} = \frac{e^{2x}}{D^2 - 4D + 4}$

$f(D) = D^2 - 4D + 4$

$f(2) = 4 - 8 + 4 = 0$

$f'(D) = 2D - 4 \qquad f'(2) = 4 - 4 = 0$

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$f''(D) = 2 \Rightarrow f''(2) = 2.$

$$\Rightarrow y_{p_1} = \frac{x^2 e^{2x}}{2}$$

$$y_{p_2} = \frac{\cos 2x}{D^2 - 4D + 4}$$

Replace D^2 with -4.

$$y_{p_2} = \frac{\cos 2x}{\cancel{-4} - 4D + \cancel{4}}$$

$$= \frac{-1}{4}\left[\frac{1}{D}\cos 2x\right] = \frac{-1}{4}\left[\frac{-\sin 2x}{2}\right] = \frac{\sin 2x}{8}$$

$\therefore$ G.S is $y = y_c + y_{p_1} + y_{p_2}$

$$\Rightarrow y = (c_1 + c_2 x)e^{2x} + \frac{x^2 e^{2x}}{2} + \frac{\sin 2x}{8}.$$

(5) $(D^2+1)y = \sin x. \quad$ — (1)

$f(D)y = Q(x).$

$f(D) = D^2 + 1 \quad ; \quad Q(x) = \sin x.$

G.S of (1) is $y = y_c + y_p$

y_c:- Consider A.E, $f(m) = 0$,

$\Rightarrow m^2 + 1 = 0 \Rightarrow m = \pm i$

$\Rightarrow y_c = c_1 \cos x + c_2 \sin x.$

$y_c = c_1 u + c_2 v. \Rightarrow u = \cos x \quad ; \quad v = \sin x.$

$u' = -\sin x \quad ; \quad v' = \cos x.$

we have, $w = uv' - vu'$

$w = \cos^2 x + \sin^2 x$

$w = 1$

Assume, $y_p = Au + Bv$ where $A = -\int \frac{vQ}{w} dx$ & $B = \int \frac{uQ}{w} dx$

$$A = -\int \sin^2 x\,dx \qquad\qquad B = \int \cos x \cdot \sin x \cdot dx$$

$$= -\int \frac{1-\cos 2x}{2}\,dx \qquad\qquad f(x) = \sin x,\quad f'(x) = \cos x$$

$$= \int f(x)\cdot f'(x)\,dx$$

$$= \frac{-1}{2}\left[x - \frac{\sin 2x}{2}\right] \qquad\qquad = \frac{[f(x)]^2}{2} = \frac{\sin^2 x}{2}.$$

$$= \frac{-x}{2} + \frac{\sin 2x}{4}$$

$$\therefore\ y_p = \frac{-x\cos x}{2} + \frac{\sin 2x\cos x}{4} + \frac{\sin^3 x}{2}.$$

$\therefore$ G.S is $y = y_c + y_p$.

$$\Rightarrow y = c_1\cos x + c_2 \sin x - \frac{x\cos x}{2} + \frac{\sin 2x\cdot\cos x}{4} + \frac{\sin^3 x}{2}.$$

UNIT-2

1. $z = f(2x+y) + g(3x-y)$

$$p = \frac{\partial z}{\partial x} = f'(2x+y)(2) + g'(3x-y)(3)$$

$$q = \frac{\partial z}{\partial y} = f'(2x+y)(1) + g'(3x-y)(-1).$$

$$r = \frac{\partial^2 z}{\partial x^2} = 2f''(2x+y)(2) + 3g''(3x-y)(3)$$

$$\Rightarrow r = 4f''(2x+y) + 9g''(3x-y). \quad -(1)$$

$$s = \frac{\partial^2 z}{\partial x \partial y} = 2f''(2x+y)(1) + 3g''(3x-y)(-1)$$

$$\Rightarrow s = 2f''(2x+y) - 3g''(3x-y). \quad -(2)$$

$$t = \frac{\partial^2 z}{\partial y^2} = f''(2x+y) - g''(3x-y)(-1)$$

$$\Rightarrow t = f''(2x+y) + g''(3x-y). \quad -(3)$$

From (1), (2), (3),

$r + s + t = 0$

② ⓐ $(mz-ny)p+(nx-lz)q = ly-mx$

It is in the form of $Pp+Qq=R$.

consider A.E's $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\Rightarrow \frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}.$$

choose l, m, n, then each fraction is equal to,

$$\frac{dx}{mz-ny} = \frac{ldx+mdy+ndz}{l(mz-ny)+m(nx-lz)+n(ly-mx)}$$

$$\frac{dx}{mz-ny} = \frac{ldx+mdy+ndz}{\cancel{lmz}-\cancel{lny}+\cancel{mnx}-\cancel{lmz}+\cancel{lny}-\cancel{mnx}}$$

$$\Rightarrow ldx+mdy+ndz = 0.$$

I.O.B.S.

$$\Rightarrow \int ldx+\int mdy+\int ndz = 0$$

$$\Rightarrow lx+my+nz = c_1 = u$$

choose x, y, z, then each fraction is equal to,

$$\frac{dx}{mz-ny} = \frac{xdx+ydy+zdz}{\cancel{mxz}-\cancel{nxy}+\cancel{nxy}-\cancel{lyz}+\cancel{lyz}-\cancel{mxz}}$$

$$\Rightarrow \int xdx+\int ydy+\int zdz = 0.$$

$$\Rightarrow \frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2} = \frac{c}{2}$$

$$\Rightarrow x^2+y^2+z^2 = c_2 = v$$

$\therefore$ G.S is $\phi(u,v)=0$

$$\therefore \phi(lx+my+nz,\ x^2+y^2+z^2) = 0.$$

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(b) $z = p^2 + q^2$ — (1)

let $u = x + ay$.

$$\frac{\partial u}{\partial x} = 1 \quad \text{and} \quad \frac{\partial u}{\partial y} = a.$$

$$p = \frac{\partial z}{\partial x} = \frac{dz}{du} \cdot \frac{\partial u}{\partial x} = \frac{dz}{du}$$

$$q = \frac{\partial z}{\partial y} = \frac{dz}{du} \cdot \frac{\partial u}{\partial y} = a\frac{dz}{du}$$

$$(1) \Rightarrow z = \left(\frac{dz}{du}\right)^2 + \left(a\frac{dz}{du}\right)^2$$

$$\Rightarrow z = (1+a^2)\left(\frac{dz}{du}\right)^2$$

$$\Rightarrow \left(\frac{dz}{du}\right)^2 = \frac{z}{1+a^2}$$

$$\Rightarrow \frac{dz}{du} = \frac{\sqrt{z}}{\sqrt{1+a^2}}$$

$$\Rightarrow \int \frac{dz}{\sqrt{z}} = \int \frac{du}{\sqrt{1+a^2}}$$

$$\Rightarrow 2\sqrt{z} = \frac{u}{\sqrt{1+a^2}} + c$$

$$\Rightarrow 2\sqrt{z} = \frac{x+ay}{\sqrt{1+a^2}} + c.$$

(3) (a) $2z = (x+a)^{1/2} + (y-a)^{1/2} + b$ — (1)

D. p. (1) w.r.t x.

$$\Rightarrow 2\frac{\partial z}{\partial x} = \frac{1}{2}(x+a)^{-1/2} \quad \Rightarrow 4p = \frac{1}{(x+a)^{1/2}}$$

$$\Rightarrow (x+a)^{1/2} = \frac{1}{4p} \Rightarrow x+a = \frac{1}{16p^2} \Rightarrow a = \frac{1}{16p^2} - x. \quad (2)$$

D. p. (1) wrt y.

$$\Rightarrow 2\frac{\partial z}{\partial y} = \frac{1}{2}(y-a)^{-1/2} \Rightarrow 4q = \frac{1}{(y-a)^{1/2}}$$

$$\Rightarrow (y-a)^{1/2} = \frac{1}{4q} \Rightarrow y-a = \frac{1}{16q^2} \Rightarrow a = y - \frac{1}{16q^2} \quad (3)$$

(2) & (3)

$$\Rightarrow \frac{1}{16p^2} - x = y - \frac{1}{16q^2}$$

$$\Rightarrow x+y = \frac{1}{16p^2} + \frac{1}{16q^2}.$$

$$\Rightarrow 16(x+y) = \frac{p^2+q^2}{p^2q^2}$$

(b) $\phi\left(\frac{y}{x}, x^2+y^2+z^2\right) = 0.$

It is in the form of $\phi(u,v) = 0.$

$u = \frac{y}{x}$	$v = x^2+y^2+z^2.$
$\frac{\partial u}{\partial x} = \frac{-y}{x^2}$	$\frac{\partial v}{\partial x} = 2x$
$\frac{\partial u}{\partial y} = \frac{1}{x}$	$\frac{\partial v}{\partial y} = 2y$
$\frac{\partial u}{\partial z} = 0$	$\frac{\partial v}{\partial z} = 2z.$

Req. PDE is of the form $Pp + Qq = R$ where

$$P = J\left(\frac{u,v}{y,z}\right); \quad Q = J\left(\frac{u,v}{z,x}\right); \quad R = J\left(\frac{u,v}{x,y}\right).$$

$$P = \begin{vmatrix} \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \end{vmatrix} = \begin{vmatrix} \frac{1}{x} & 0 \\ 2y & 2z \end{vmatrix} = \frac{2z}{x}$$

$$Q = \begin{vmatrix} \frac{\partial u}{\partial z} & \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial x} \end{vmatrix} = \begin{vmatrix} 0 & -y/x^2 \\ 2z & 2x \end{vmatrix} = \frac{2yz}{x^2}$$

$$R = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} -y/x^2 & 1/x \\ 2x & 2y \end{vmatrix} = \frac{-2y^2}{x^2} - \frac{2x}{x}$$

$$= -2\left(\frac{y^2}{x^2}+1\right)$$

$\therefore$ Req. PDE is of the form $Pp + Qq = R$.

$$\Rightarrow \frac{2z}{x} p + \frac{2yz}{x^2} q = -2\left(\frac{y^2}{x^2}+1\right)$$

$$\Rightarrow \cancel{2}zxp + \cancel{2}yzq = -\cancel{2}(y^2+x^2)$$

$$\Rightarrow z(xp+yq) = -(x^2+y^2)$$

(4) $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u$ — (1) $\quad u(x,0) = 6e^{-3x}$.

Let $u = xy$.

$\frac{\partial u}{\partial x} = x'y$ and $\frac{\partial u}{\partial y} = xy'$

(1) $\Rightarrow x'y = 2xy' + xy$.

$\Rightarrow x'y = x(2y'+y)$

$\Rightarrow \frac{x'}{x} = \frac{2y'+y}{y} = k.$ (let)

$\int \frac{x'}{x} = \int k$

$\Rightarrow \log x = kx + c$

$\Rightarrow x = c_1 e^{kx}$

$\int \frac{2y'}{y} + \int 1 = \int k$

$\Rightarrow 2\log y + y = ky + c$

$\Rightarrow 2\log y = (k-1)y + c$

$\Rightarrow x = c_1 e^{kx}$ | $\Rightarrow \log y = \frac{(k-1)y}{2} + c_2$

$\Rightarrow y = c_2 e^{\frac{(k-1)y}{2}}$

$$u = c_1 c_2 e^{kx + \frac{(k-1)y}{2}} \quad \text{--- (2)}$$

Given $u(x,0) = 6e^{-3x}$.

$$(2) \Rightarrow u(x,0) = c_1 c_2 e^{kx + \frac{(k-1)(0)}{2}}$$

$$\Rightarrow 6e^{-3x} = c_1 c_2 e^{kx}$$

$$\Rightarrow c_1 c_2 = 6 \text{ and } k = -3.$$

Now, (2) $\Rightarrow$ $\underline{u(x,y) = 6e^{-3x-2y}}$.

5 (a) $x^2(z-y)p + y^2(x-z)q = z^2(y-x)$.

It is in the form of $Pp + Qq = R$.

consider A.E's $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{x^2(z-y)} = \frac{dy}{y^2(x-z)} = \frac{dz}{z^2(y-x)}$$

choose $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$, then each fraction is equal to

$$\frac{dx}{x^2(z-y)} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{\cancel{xz} - \cancel{xy} + \cancel{yx} - \cancel{yz} + \cancel{zy} - \cancel{xz}}$$

$$\Rightarrow \int \frac{1}{x}dx + \int \frac{1}{y}dy + \int \frac{1}{z}dz = 0.$$

$$\Rightarrow \log x + \log y + \log z = \log c_1 \Rightarrow \log xyz = \log c_1$$

$$\Rightarrow \underline{c_1} = xyz = u$$

Choose $1/x^2$, $1/y^2$, $1/z^2$ than each fraction becomes,

$$\frac{dx}{x^2(z-y)} = \frac{1/x^2\,dx + 1/y^2\,dy + 1/z^2\,dz}{\cancel{z}-\cancel{y}+\cancel{x}-\cancel{z}+\cancel{y}-\cancel{x}}$$

$$\Rightarrow \int \frac{1}{x^2}dx + \int \frac{1}{y^2}dy + \int \frac{1}{z^2}dz = 0$$

$$\Rightarrow \frac{-1}{x} - \frac{1}{y} - \frac{1}{z} = \frac{-1}{C_2}$$

$$\Rightarrow -\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \frac{-1}{C_2}$$

$$\Rightarrow C_2 = x+y+z = v$$

$$\therefore \phi(u,v) = 0 \Rightarrow \phi(xyz, x+y+z) = 0.$$

(b) $p^2+q^2 = x^2+y^2$.

$p^2 - x^2 = y^2 - q^2 = k$ (let)

$p^2 - x^2 = k$	$y^2 - q^2 = k$.
$p^2 = x^2 + k$	$q^2 = y^2 - k$
$p = \sqrt{x^2+k}$	$q = \sqrt{y^2-k}$

WKT, $dz = p\,dx + q\,dy$.

$$\Rightarrow dz = \sqrt{x^2+k}\,dx + \sqrt{y^2-k}\,dy.$$

I.O.B.S.

$$\Rightarrow \int dz = \int \sqrt{x^2+k}\,dx + \int \sqrt{y^2-k}\,dy.$$

$\left(\sqrt{x^2+(\sqrt{k})^2}\right)$ $\left(\sqrt{y^2-(\sqrt{k})^2}\right)$

$$\Rightarrow z = \frac{x}{2}\sqrt{x^2+k} + \frac{k}{2}\sinh^{-1}\left(\frac{x}{\sqrt{k}}\right) + \frac{y}{2}\sqrt{y^2-k} - \frac{k}{2}\cosh^{-1}\left(\frac{y}{\sqrt{k}}\right) + c$$

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(6) $3\dfrac{\partial u}{\partial x}+2\dfrac{\partial u}{\partial y}=0$ —(1) $\quad u(x,0)=4e^{-x}$

let $u = XY \Rightarrow \dfrac{\partial u}{\partial x} = X'Y$ and $\dfrac{\partial u}{\partial y} = XY'$.

(1) $\Rightarrow 3X'Y + 2XY' = 0 \Rightarrow 3X'Y = -2XY'$

$\Rightarrow \dfrac{3X'}{X} = \dfrac{-2Y'}{Y} = k$ (let)

$\displaystyle\int \frac{3X'}{X} = \int k$

$\Rightarrow 3\log X = kx + c$

$\Rightarrow \log X = \dfrac{kx+c}{3}$

$\Rightarrow X = e^{\frac{kx}{3}} \cdot c_1$

$\displaystyle\int \frac{-2Y'}{Y} = \int k$

$\Rightarrow -2\log Y = ky + c$

$\Rightarrow \log Y = \dfrac{-(ky+c)}{2}$

$\Rightarrow Y = e^{\frac{-ky}{2}} \cdot c_2$

$u(x,y) = c_1 c_2 \, e^{\frac{kx}{3}-\frac{ky}{2}}$ —(2)

given,

$u(x,0) = 4e^{-x}$

(2) $\Rightarrow u(x,0) = c_1 c_2 \, e^{\frac{kx}{3}}$

$\Rightarrow 4e^{-x} = c_1 c_2 \, e^{\frac{kx}{3}}$

$\Rightarrow \underline{c_1 c_2 = 4}$ $\qquad -\cancel{x} = \dfrac{k\cancel{x}}{3} \Rightarrow \underline{k=-3}$

(2) $\Rightarrow u(x,y) = 4\, e^{-x+\frac{3}{2}y}$

$\Rightarrow \underline{u(x,y) = 4\, e^{\frac{-2x+3y}{2}}}$

⑦ ⓐ $p^2 - q^2 = x - y.$

$\Rightarrow p^2 - x = q^2 - y = k$ (let).

$p^2 - x = k$

$\Rightarrow p^2 = k + x$

$\Rightarrow p = \sqrt{x+k}$

$q^2 - y = k.$

$\Rightarrow q^2 = k + y$

$\Rightarrow q = \sqrt{y+k}$

WKT, $dz = p\,dx + q\,dy.$

$\Rightarrow \int dz = \int \sqrt{x+k}\,dx + \int \sqrt{y+k}\,dy$

$\Rightarrow z = \frac{2}{3}(x+k)^{3/2} + \frac{2}{3}(y+k)^{3/2} + c.$