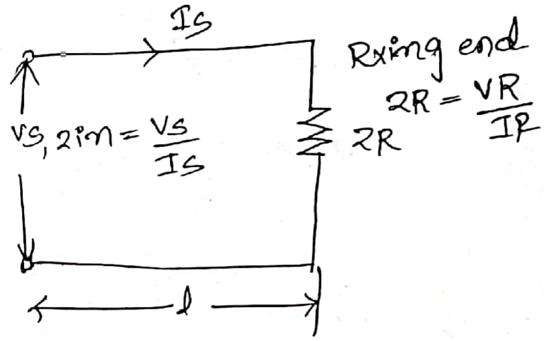
TRANSMISSION LINES - II

INPUT IMPEDANCE RELATIONS:-

The impedance at the input side is called Input Impedance of transmission line. Let us consider the finite transmission line with the input source voltage V_S and current passing through the circuit is I_S. Then, now the input impedance $\boxed{Z_{in} = \frac{V_S}{I_S}}$

Now, consider, the load with Z_R is connected at the receiving end. Hence, the voltage at the receiving end is V_R and the current is I_R.

The length of the transmission line is 'l'.



Let us consider the general transmission line equations of having constants A, B

$$V = A\cosh px + B\sinh px \quad \text{--- (1)}$$

$$I = \frac{1}{Z_0}\left[A\sinh px + B\cosh px\right] \quad \text{--- (2)}$$

At receiving end, $V = V_R$, $I = I_R$ at a distance $x = l$

Now, the above equations become

$$V_R = A\cosh pl + B\sinh pl \quad \text{--- (a)}$$

$$I_R = \frac{1}{Z_0}\left[A\sinh pl + B\cosh pl\right] \quad \text{--- (b)}$$

Now multiply Equation (a) with $\frac{\cosh pl}{Z_0}$ and Equation (b) with $\sinh pl$ on both sides

$$\frac{V_R \cdot \cosh pl}{Z_0} = \frac{A\cosh^2 pl}{Z_0} + \frac{B\sinh pl \cosh pl}{Z_0} \quad \text{--- (c)}$$

a) Multiply with $\sinh pL$ on both sides of Equation (b)

$$I_R \sinh pl = \frac{-A}{Z_0} \sinh^2 pL - \frac{B}{Z_0} \sinh pL \cosh pL \quad \text{—(d)}$$

Now Eq^n —(c) and (d) on both sides

$$\frac{V_R \cosh pl}{Z_0} + I_R \sinh pl = \frac{A \cosh^2 pL}{Z_0} + \frac{B \sinh pL \cancel{\cosh pL}}{\cancel{Z_0}}$$

$$- A \frac{\sinh^2 pl}{Z_0} - \frac{B \cosh pl \cancel{\sinh pl}}{\cancel{Z_0}}$$

$$\frac{V_R \cosh pl}{Z_0} + I_R \sinh pl = A \frac{\cosh^2 pl}{Z_0} - A \frac{\sinh^2 pl}{Z_0}$$

$$\frac{V_R \cosh pl}{Z_0} + I_R \sinh pl = \frac{A}{Z_0} \left[\cosh^2 pl - \sinh^2 pl \right] \quad \text{—(3)}$$

We know that generally $\cosh^2 pl - \sinh^2 pl = 1$ —(4)

Sub Eq^n —(4) in Eq^n —(3)

$$\frac{V_R \cosh pl}{Z_0} + I_R \sinh pl = \frac{A}{Z_0}$$

$$\frac{V_R \cosh pl + Z_0 I_R \sinh pl}{Z_0} = \frac{A}{Z_0}$$

$$\boxed{V_R \cosh pl + Z_0 I_R \sinh pl = A} \quad \text{—(5)}$$

(b) Multiply Eq^n —(a) with $\frac{\sinh pl}{Z_0}$ and Eq^n —(b) with $\cosh pl$ on both sides.

$$\frac{V_R \sinh pl}{Z_0} = \frac{A \sinh pl \cosh pl}{Z_0} + \frac{B \sinh^2 pl}{Z_0} \quad \text{—(e)}$$

$$I_R \cosh pl = \frac{-1}{Z_0} \left(A \sinh pl \cosh pl + B \cosh^2 pl \right) \quad \text{—(f)}$$

Now, add Eq^n —(e) and (f)

$$\frac{V_R \sinh pl}{Z_0} + I_R \cosh pl = \frac{-1}{Z_0} A \cancel{\sinh pl \cosh pl} - \frac{B \cosh^2 pl}{Z_0}$$

$$+ \frac{A \sinh pl \cancel{\cosh pl}}{\cancel{Z_0}} + \frac{B \sinh^2 pl}{Z_0}$$

b)

$$\frac{V_R \sinh pl}{Z_0} + I_R \cosh pl = \frac{B \sinh^2 pl}{Z_0} - \frac{B \cosh^2 pl}{Z_0}$$

$$\frac{V_R \sinh pl}{Z_0} + I_R \cosh pl = \frac{-B}{Z_0}\left[\cosh^2 pl - \sinh^2 pl\right]$$

$$\frac{V_R \sinh pl}{Z_0} + I_R \cosh pl = \frac{-B}{Z_0} \quad \left[\text{Since } \cosh^2 pl - \sinh^2 pl = 1\right]$$

$$\frac{V_R \sinh pl}{Z_0} + I_R \cosh pl = \frac{-B}{Z_0}$$

$$\frac{V_R \sinh pl + Z_0 I_R \cosh pl}{Z_0} = \frac{-B}{Z_0}$$

$$\boxed{B = -\left[V_R \sinh pl + Z_0 I_R \cosh pl\right]}$$

Finally, substitute the values of A, B in transmission line equations

$$V = A \cosh px + B \sinh px \quad \text{— (6)}$$

$$I = \frac{-1}{Z_0}\left[A \sinh px + B \cosh px\right]$$

$$\Rightarrow V = \left[V_R \cosh pl + I_R Z_0 \sinh pl\right] \cosh px + \left[-V_R \sinh pl - I_R Z_0 \cosh pl\right] \sinh px$$

$$V = V_R \cosh pl \cosh px + I_R Z_0 \sinh pl \cosh px - \sinh px \cdot V_R \sinh^2 pl - I_R Z_0 \sinh px \cosh pl$$

$$V = V_R\left[\cosh pl \cosh px - \sinh pl \sinh pl\right] + I_R Z_0\left[\sinh pl \cosh px - \sinh px \cosh pl\right]$$

$$\boxed{V = V_R\left[\cosh p(l-x) + I_R Z_0 \sinh p(l-x)\right]} \quad \text{— (7)}$$

$$I = \frac{-1}{Z_0}\left[A \sinh px + B \cosh px\right]$$

$$I = \frac{-1}{Z_0}\left[A \sinh px + B \cosh px\right] \quad \text{—(9)}$$

Substitute the values of A and B in Eq^n (9)

$$I = \frac{-1}{Z_0}\Big[\left(V_R \cosh pl + I_R Z_0 \sinh pl\right) \sinh px +$$

$$-\left[V_R \sinh pl + I_R Z_0 \cosh pl\right] . \cosh px$$

$$I = \frac{-1}{Z_0}\Big[V_R \cosh pl \sinh px + I_R Z_0 \sinh pl \sinh px$$

$$- V_R \sinh pl \cosh px - I_R Z_0 \cosh pl \cosh px\Big]$$

$$I = \frac{-1}{Z_0}\Big[-\Big[\left(V_R \sinh pl \cosh px - V_R \cosh pl \sinh px\right)$$

$$+ \left(\cosh pl \cosh px - \sinh pl \sinh px\right) I_R Z_0\Big]\Big]$$

$$I = \frac{1}{Z_0}\Big[\left(V_R \sinh pl \cosh px - V_R \cosh pl \sinh px\right) +$$

$$\left(\cosh pl \cosh px - \sinh pl \sinh pl\right) I_R Z_0\Big]$$

$$I = \frac{1}{Z_0}\left[V_R \sinh p(l-x) + I_R Z_0 \cosh p(l-x)\right]$$

$$\boxed{I = \frac{V_R}{Z_0} \sinh p(l-x) + I_R \cosh p(l-x)} \quad \text{—(8)}$$

Therefore, at Sending end, $V = V_S$, $I = I_S$, $x = 0$

Then voltage and current Equation becomes

$$V_S = V_R \cosh pl + I_R Z_0 \sinh pl$$

$$I_S = \frac{V_R}{Z_0} \sinh pl + I_R \cosh pl$$

The input impedance, $Z_{in} = \frac{V_S}{I_S}$

$$Z_{in} = \frac{V_S}{I_S} = \frac{V_R \cosh pl + I_R Z_0 \sinh pl}{\frac{V_R}{Z_0}\sinh pl + I_R \cosh pl}$$

Multiply both numerator and denominator with Z_0/I_R

$$Z_{in} = \frac{[V_R \cosh pl + I_R Z_0 \sinh pl]\left(\frac{Z_0}{I_R}\right)}{\left[\frac{V_R}{Z_0}\sinh pl + I_R \cosh pl\right]\left(\frac{Z_0}{I_R}\right)}$$

$$Z_{in} = Z_0 \frac{\left[\frac{V_R}{I_R}\cosh pl + \frac{\cancel{I_R} Z_0}{\cancel{I_R}}\sinh pl\right]}{\left[\frac{V_R}{\cancel{Z_0}}\cdot\frac{\cancel{Z_0}}{I_R}\sinh pl + \cancel{I_R}\cdot\frac{Z_0}{\cancel{I_R}}\cosh pl\right]}$$

$$Z_{in} = Z_0 \frac{[Z_R \cosh pl + Z_0 \sinh pl]}{\frac{V_R}{I_R}\sinh pl + \cosh pl \cdot Z_0}$$

$$Z_{in} = Z_0 \frac{\cancel{\cosh pl}\,(Z_R + Z_0 \sinh pl / \cosh pl)}{\cancel{\cosh pl}\,(Z_0 + Z_R \cdot \sinh pl / \cosh pl)}$$

$$\boxed{Z_{in} = Z_0 \frac{[Z_R + Z_0 \tanh pl]}{[Z_0 + Z_R \cdot \tanh pl]}}$$

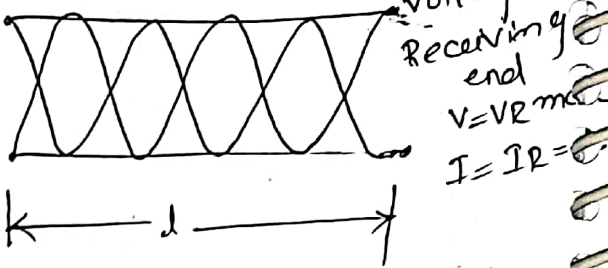
This is the input impedance for finite Transmission line with Z_i and with terminating Load impedance as Z_R.

The input impedance in terms of reflection coefficient k.

$$Z_{in} = R_0 \left[\frac{1+|K|}{1-|K|}\right] = SR_0. \quad S \rightarrow \text{Standing Wave Ratio}$$

2) OPEN CIRCUIT TRANSMISSION LINE

Open circuit Transmission line is a ~~tro~~ Finite transmission line with an open end.



In Case of finite transmission line with Z_0 at the receiving end there will not be any reflected waves, Because it acts as Infinite transmission line.

But Incase of finite transmission line without any termination at the receiving end results as reflected waves which will travel from receiving end to the transmitting end. As it is opposite the wave transmission, the impedance at receiving end should be ∞.

So, $\boxed{Z_R = \infty}$

Open Circuit Impedance (Z_{OC}):-

Since $Z_R = \frac{V_R}{I_R}$, and $Z_R = \infty$, The current at receiving I_R should be '0', and hence voltage at receiving end $V = V_R max$.

Now let us consider the equation of transmission line, $V = V_S \cosh px - I_S Z_0 \sinh px$,

At ~~$x = l, V_R = 0$~~ $\Rightarrow 0 = V_S \cosh pl$ ~~$- I_S Z_0 \sinh pl$~~

$I_S Z_0$ ~~$\sinh pl = V_R$~~ $\cosh pl$.

$V_S /$ ~~$I_S = Z_0 . \sinh pl$~~ $/ \cosh pl$.

we have $I = I_s \cosh px - V_s/Z_0 \sinh px$

at $x=L$, $V=V_{Rmax}$, $I=I_R=0$, since, $Z_R=\infty$.

$$0 = I_s \cosh pL - \frac{V_s}{Z_0}\sinh pL \Rightarrow \frac{V_s}{Z_0}\sinh pL = I_s \cosh pL$$

$$Z_0 = \frac{V_s}{I_s}\tanh pl \Rightarrow Z_0 = Z_{OC}\cdot \tanh pL \Rightarrow \boxed{Z_{OC} = Z_0 \coth pL}$$

$Z_{OC} \rightarrow$ open circuit transmission line impedance.

<u>input impedance of open circuit Transmission line</u>

we have input impedance, $Z_{in} = Z_0\left[\frac{Z_R + Z_0\tanh pl}{Z_0 + Z_R\tanh pl}\right]$

For open circuit $V=V_R$, $I=I_R=0$.

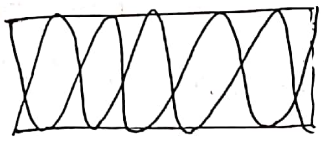
$$Z_{in} = Z_0 \cdot \frac{\cancel{Z_R}\left[1 + \frac{Z_0}{Z_R}\tanh pl\right]}{\cancel{Z_R}\left[\tanh pl + \frac{Z_0}{Z_R}\right]} = Z_0\frac{\left[1+\frac{Z_0}{Z_R}\tanh pl\right]}{\left[\frac{Z_0}{Z_R} + \tanh pl\right]}$$

Since, $Z_R = \frac{V_R}{I_R} = \frac{V_R}{0} \Rightarrow Z_R = \infty$

$$Z_{in} = Z_0\frac{\left[1+\frac{Z_0}{\infty}\tanh pl\right]}{\left[\frac{Z_0}{\infty} + \tanh pl\right]} = \frac{Z_0}{\tanh pl}$$

$$Z_{in} = \frac{Z_0}{\tanh pL} \Rightarrow \boxed{Z_{in} = Z_0 \coth pl}$$

b) SHORT CIRCUIT TRANSMISSION LINE



Rx end

$I = I_{R(max)}$

$V = V_R = 0$

In this transmission line, the receiving end is short circuited. Therefore, the signal is not opposed. That means impedance, $Z_R = \infty$, i.e $Z_R = \frac{V_R}{I_R} = \otimes$, $\cancel{V_R = V_{RE}}$ $I = I_{Rmax}$, $V_R = 0$

Now, consider the transmission line equation

$$V = V_s \cosh px - I_s Z_0 \sinh px, \quad \text{at } x=L,\ V_R = 0$$

$$0 = V_s \cosh pL - I_s Z_0 \sinh pl \Rightarrow I_s Z_0 \sinh pl = V_s \cosh pL$$

$$Z_0 = \frac{V_s}{I_s}\cdot\frac{\cosh pL}{\sinh pL} \Rightarrow Z_0 = Z_{SC}\cdot \coth pl \Rightarrow \text{where } Z_{SC} = \frac{V_s}{I_s}$$

$\boxed{Z_{sc} = Z_0 \tanh p l}$ where Z_{sc} = short circuit Impedance

Input Impedance for Short Circuit line :-

The input impedance $Z_{in} = Z_0 \dfrac{(Z_R + Z_0 \tanh p l)}{(Z_0 + Z_R \tanh p l)}$

For short circuit at the receiving end, $V_R = 0$, $I = I_{R\,max} = I_R$

So, the receiving end impedance, $Z_R = \dfrac{V_R}{I_R} = \dfrac{0}{I_R} = 0$

$\boxed{Z_R = 0}$

The input impedance, $Z_{in} = Z_0 \dfrac{(Z_R + Z_0 \tanh p l)}{(Z_0 + Z_R \tanh p l)}$

$$Z_{in} = \frac{Z_0 \cdot Z_0\left(\frac{Z_R}{Z_0} + \tanh p l\right)}{Z_0\left(1 + \frac{Z_R}{Z_0}\tanh p l\right)}$$

$\boxed{Z_{in} = Z_0 \tanh p l}$

This is the input impedance of short circuit line.

Thus we find input impedance | terminating impedance = short circuit impedance.

Input REFLECTION COEFFICIENT :-

When the transmission line is not terminated with characteristic Impedance Z_0, there will be reflected waves. The general transmission line equations that well known are,

$$V = b e^{-px} + a e^{px} \quad \text{---} (1)$$

$$I = \frac{+1}{Z_0}\left(\underset{\substack{\downarrow \\ \text{incident wave}}}{b e^{-px}} - \underset{\substack{\downarrow \\ \text{Reflected wave}}}{a e^{px}}\right) \quad \text{---} (2)$$

In the above equation, the first term represents Incident wave which travel from sending end to receiving end and second term represents reflected wave which is from receiving to sending end.

The reflection coefficient of transmission line is defined as the ratio of magnitude of reflected wave to the incident wave. It is denoted as k.

$$\boxed{\text{Reflection coefficient } (k) = \frac{\text{Reflected Wave}}{\text{Incident Wave}} = \frac{ae^{-Py}}{be^{Py}}}$$

For the voltage wave, the ratio is positive and the current wave, this is negative. There will be phase difference of 180° in case of current.

Reflection coefficient denoted with k. Replace x with -y in above equations where y is distance from receiving end.

$$V = be^{Py} + ae^{-Py}$$

$$I = \frac{+1}{Z_0}(be^{Py} - ae^{-Py})$$

At receiving end, $y=0$, $V=V_R$, $I=I_R$

$V_R = b+a$; $I_R = \frac{+1}{Z_0}(b-a) \Rightarrow b-a = I_R Z_0$.

~~$V_R + I_R = b+a+\frac{b}{Z_0} - \frac{a}{Z_0}$~~ $b+\cancel{a}+b-\cancel{a} = I_R Z_0 + V_R$

$2b = I_R Z_0 + V_R$

~~$V_R + I_R$~~

$$\boxed{b = \frac{I_R Z_0 + V_R}{2}}$$

Similarly, $\cancel{b}+a-\cancel{b}+a = V_R - I_R Z_0$

$2a = V_R - I_R Z_0 \Rightarrow \boxed{a = \frac{V_R - I_R Z_0}{2}}$

Therefore, the reflection coefficient $(k) = \frac{\text{reflected wave}}{\text{Incident wave}}$

$$k = \frac{a e^{-Py}}{b e^{Py}}$$

$$= \frac{(V_R - I_R Z_0) e^{-Py}}{(V_R + I_R Z_0) e^{Py}} = \frac{\cancel{I_R}\,(V_R/I_R - Z_0)\, e^{-Py}}{\cancel{I_R}\,(V_R/I_R + Z_0)\, e^{Py}}$$

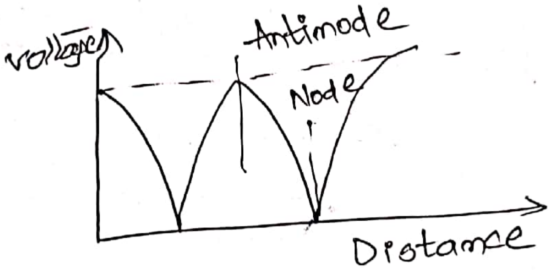
$$= \frac{(Z_R - Z_0)}{(Z_R + Z_0)} \quad \text{since } y = 0$$

So, hence, reflection coefficient $(k) = \frac{(Z_R - Z_0)}{(Z_R + Z_0)}$

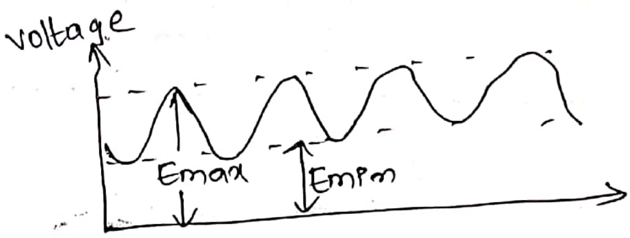
$$\boxed{k = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

STANDING WAVE RATIO

If a line is terminated in a load other than R_0, the distribution of voltage at a point along the length of the line consists maximum and minimum values of voltage as shown below



a) Standing Waves on open or shorted line



(b) Standing Waves on a line terminated in a load not equal to R_0.

When the transmission line is not terminated with Z_0 there will be reflected wave. The incident and reflected waves as shown in figure. Maxi points are obtained due to the addition of incident wave with reflected wave. Minima points of either voltage and current are obtained due to the subtraction of Incident wave and reflected wave.

The drawback of standing waves is that motion doesn't takes in transmission for standing wave

$V_{max} = |V_i| + |V_r|$, $I_{max} = |I_i| + |I_r|$

$V_{min} = |V_i| - |V_r|$, $I_{min} = |I_i| - |I_r|$

when the line is terminated with R_o the standing waves are absent, Such a line is called <u>Smooth line.</u>

$$\text{Standing Wave Ratio}(S) = \frac{|E_{max}|}{|E_{min}|} = \frac{|V_{max}|}{|V_{min}|} = \frac{|I_{max}|}{|I_{min}|}$$

The standing wave ratio s is measured by using <u>RF voltmeter</u> across the line at point.

The ratio, E_{max} to E_{min} (or) V_{max} to V_{min}, then

$$\boxed{\text{Voltage Standing Wave Ratio (VSWR)} = \frac{|E_{max}|}{|E_{min}|} = \frac{|V_{max}|}{|V_{min}|}}$$

$$VSWR = \frac{|V_i| + |V_r|}{|V_i| - |V_r|} = \frac{\cancel{V_i}\left[\left|\frac{V_i}{V_r}\right| + 1\right]}{\cancel{V_i}\left[\left|\frac{V_i}{V_r}\right| - 1\right]}, \quad |K| = \frac{V_r}{V_P}$$

$$\boxed{VSWR = \frac{1+|K|}{1-|K|}}$$

Similarly, the ratio of I_{max} to I_{min} is referred can be measured using RF ammeter in series with the line at a point. Then such ratio is called <u>Current standing Wave Ratio. (CSWR)</u>

$$ISWR = \frac{I_{max}}{I_{min}} = \frac{|I_i| + |I_R|}{|I_i| - |I_R|} =$$

$$\frac{\cancel{I_R}\left[\left|\frac{I_R}{I_i}\right| + 1\right]}{\cancel{I_R}\left[1 - \left|\frac{I_R}{I_i}\right|\right]} \qquad k = -\left|\frac{I_R}{I_i}\right|$$

$$ISWR = \frac{I_i + |I_R|}{|I_i| - |I_R|} = \frac{\cancel{I_i}\left[1 + \left|\frac{I_R}{I_i}\right|\right]}{\cancel{I_i}\left[1 - \left|\frac{I_R}{I_i}\right|\right]}$$

$$ISWR = \frac{1 - \left[-\left|\frac{I_R}{I_i}\right|\right]}{1 + \left[-\left|\frac{I_R}{I_i}\right|\right]} = \frac{1 - |k|}{1 + |k|}$$

$$\boxed{ISWR = \frac{1 - |k|}{1 + |k|}}$$

EIGHTH - WAVE LINE:-

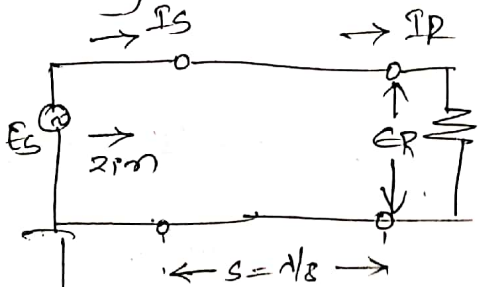
let λ be the wavelength of the transmitted frequency 'f'. Consider a transmission line of length $\lambda/8$ as shown in fig.

The generalized expression for the input impedance is given by $Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan(\beta s)}{R_0 + jZ_R \tan(\beta s)}\right]$

$\beta = 2\pi/\lambda$.

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi}{\lambda} \cdot s\right)}{R_0 + jZ_R \tan\left(\frac{2\pi}{\lambda} \cdot s\right)}\right]$$

But for a line, $s = \lambda/8$



Eigth-wave line

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}\right)}{R_0 + jZ_R \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}\right)}\right]$$

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$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan\left(\frac{\pi}{4}\right)}{R_0 + jZ_R \tan\left(\frac{\pi}{4}\right)} \right] \Rightarrow Z_{in} = R_0 \left[\frac{Z_R + jR_0}{R_0 + jZ_R} \right]$$

If a line is terminated in pure resistance, $Z_R = R_R$, then $Z_{in} = R_0 \left[\frac{R_R + jR_0}{R_0 + jR_R} \right]$ $Z_{in} \rightarrow$ Complex Quantity.

The Magnitude of Input Impedance is given by

$$|Z_{in}| = R_0 \left[\frac{\sqrt{R_R^2 + R_0^2}}{\sqrt{R_R^2 + R_0^2}} \right] = R_0.$$

$$\boxed{|Z_{in}| = R_0}$$

Application

Thus, the eighth-wave line is generally used to transform any resistance R_R to an impedance Z_{in} having its magnitude equal to the characteristic resistance R_0 of the line.

b) Quarter Wave line - Impedance Matching

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan(\beta s)}{R_0 + jZ_R \tan(\beta s)} \right], \quad Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan(\beta s)} + jR_0}{\frac{R_0}{\tan(\beta s)} + jZ_R} \right]$$

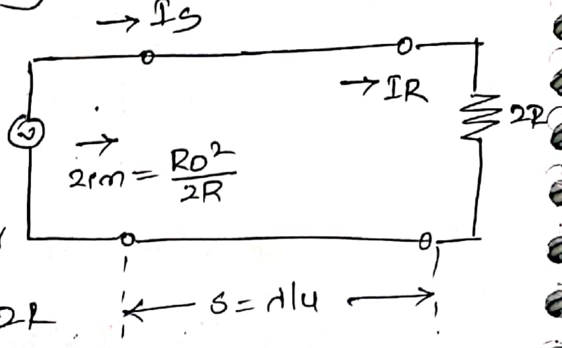
$\beta = \frac{2\pi}{\lambda}$, $Z_{in} = R_0 \left[\dfrac{\frac{Z_R}{\tan\left(\frac{2\pi}{\lambda} \cdot s\right)} + jR_0}{\frac{R_0}{\tan\left(\frac{2\pi}{\lambda} \cdot s\right)} + jZ_R} \right]$

For Quarter-wave line, $s = \lambda/4$

$$Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan\left(\frac{2\pi}{\lambda} \cdot \lambda/4\right)} + jR_0}{\frac{R_0}{\tan\left(\frac{2\pi}{4} \cdot \frac{\lambda}{4}\right)} + jZ_R} \right], \quad s = \lambda/4$$

$$Z_{in} = R_0 \left[\frac{\frac{Z_R}{\tan(\pi/2)} + jR_0}{\frac{R_0}{\tan(\pi/2)} + jZ_R} \right] \Rightarrow Z_{in} = R_0 \left[\frac{jR_0}{jZ_R} \right].$$

$$\boxed{Z_{in} = \frac{R_0^2}{Z_R}}$$



→ Thus a quarter wave line may be used as a transformer for impedance matching of load Z_R with input impedance, $Z_{in} = Z_R$.

For matching impedances Z_R and Z_{in}, the line with characteristic impedance R_0 may be selected, such that condition should be satisfied.

$$R_0 = \left| \sqrt{Z_R \cdot Z_{in}} \right|$$

A quarter wave line can transform a low impedance to a high impedance and vice versa, thus it can be considered as a wave inverter. Hence an open circuit λ/4 line gives zero input impedance while a short circuited λ/4 line gives infinite input impedance. Thus a short circuit quarter wave line behaves as an open circuit at the other end while an open circuit quarter wave line behaves as a short circuit at the other end.

Applications of Quarter Wave:-

One of the important applications is the impedance transformation in coupling a transmission line to a resistive load such as an antenna.

c) The Half - wave line

The generalised expression for the input impedance of a line is given by

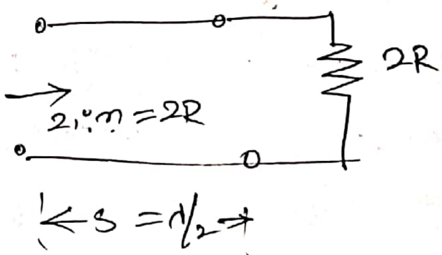
$$Z_{in} = R_0 \left[\frac{2R + jR_0 \tan(\beta s)}{R_0 + j2R \tan(\beta s)} \right] \qquad \beta = \frac{2\pi}{\lambda}$$

$$Z_{in} = R_0 \left[\frac{2R + jR_0 \tan\left(\frac{2\pi}{\lambda}\cdot s\right)}{R_0 + j2R \tan\left(\frac{2\pi}{\lambda}\cdot s\right)} \right] \qquad s = \lambda/2$$

$$Z_{in} = R_0 \left[\frac{2R + jR_0 \tan\left(\frac{2\pi}{\lambda}\cdot \frac{\lambda}{2}\right)}{R_0 + j2R \tan\left(\frac{2\pi}{\lambda}\cdot \frac{\lambda}{2}\right)} \right] \Rightarrow Z_{in} = R_0 \left[\frac{2R + jR_0 \tan(\pi)}{R_0 + j2R \tan(\pi)} \right]$$

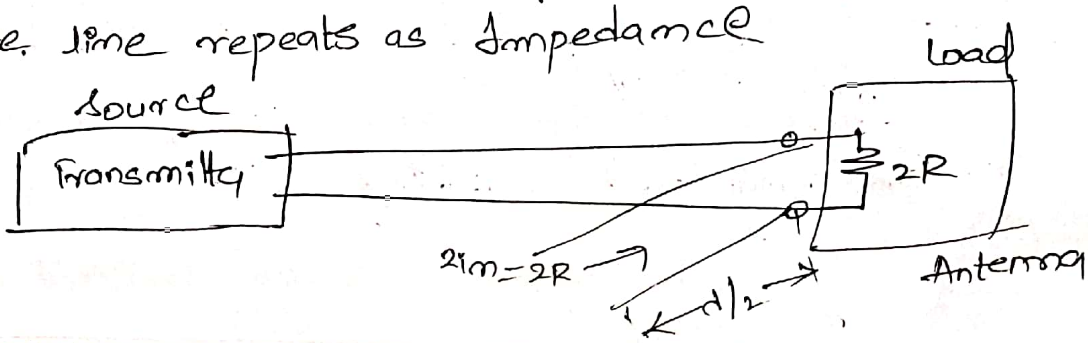
$$\therefore Z_{in} = R_0 \left[\frac{2R}{R_0} \right] \Rightarrow \boxed{Z_{in} = 2R}$$

From the Eq^n, it is clear that a half-wave line repeats its terminating impedance. In other words, the half wave line may be considered as one to one transformer.



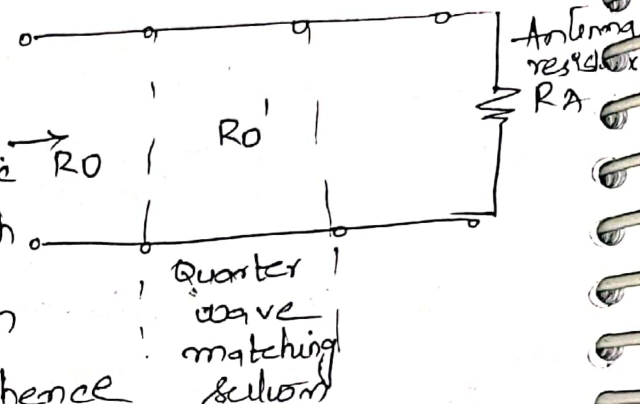
Application

The main application of a half wave line is to connect a load to a source where both of them can't be made adjacent. In such a case, we may connect a parallel half wave line repeats as impedance.



If an antenna resistance is R_A and the characteristic impedance of the line is R_o, then the Quarter wave impedance matching section is designed such that its characteristic impedance R_o' transforms antenna resistance R_A to the characteristic impedance of line R_o given by

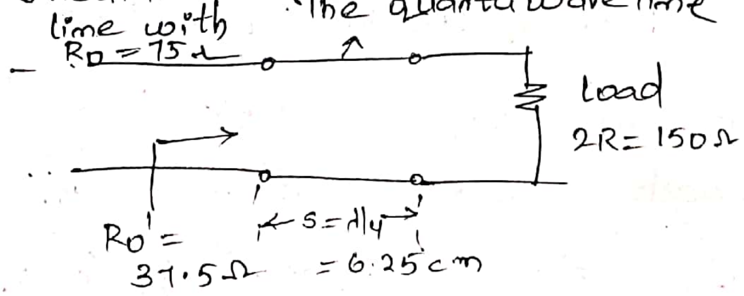
$$R_o' = \sqrt{R_A \cdot R_o}.$$



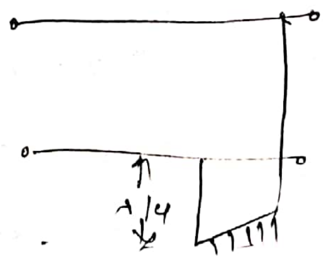
The value of the characteristic impedance R_o' is the value which gives critical coupling condition between two impedances and hence max. power transfer takes place from the line to the load.

→ Quarter wave line used to provide mechanical support to the open wire line or centre conductor of a coaxial cable.

As Quarter wave line is shorted at ground, its input impedance is very high. So, the signal passes to the receiving end, without any loss due to this mechanical support. Thus line acts as an insulator at this point. Hence such line is referred as Copper Insulator.



The quarta wave line



Quarta wavelength matching section.

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Circle Diagram for Dissipationless line

In the design of dissipationless line can be simplified significantly by drawing circle diagram, which is useful in simplifying the impedance equations. This is also called impedance circle diagram which facilitates rapid calculations for the transmission line.

a) **Constant s - circles**:- A constant s-circle represents all possible values of the resistive part ra and reactive part. A line drawn from origin to the point on the s-circle indicates normalized impedance Z_s/R_0 with real and imaginary component. i.e ra and jxa respectively.

→ When the value of βs lies in between 0 and π/2, the constant βs-circles in the positive region of the imaginary axis, i.e. xa axis. While when the value of βs lies in between π/2 and π, the constant βs-circles lie in the negative region of the imaginary axis. i.e xa axis. By Superpositioning the constant β s-circle on constant s-circles, we get final circle diagram useful to calculate input impedance rapidly.

THE SMITH CHART - THE SMITH CIRCLE DIAGRAM

The main drawback of the circle diagram is that the constant s- circles and constant βs-circles are not concentric which makes it difficult to interpolate these circles. Moreover the circle diagram can be used for the limited range of impedance values with reasonable practical chart size.

→ The Smith chart is a valuable graphical tool for solving radio frequency transmission line problems. Under the matching condition the reflection coefficient is 0, and that of VSWR is 1. In almost all the transmission line problems, the main objective is to match the impedances of line to that load.

<u>Construction of the Smith Chart</u>

Basic difference between the circle diagram and the Smith chart is that the value of resistive and reactive components are represented in rectangular form which are extended to infinity. But in the Smith chart the infinite resistive and reactive components are transferred to an area inside a circle. As the resistive and reactive components are in circular form, the Smith chart is also known as circular chart.

The Smith chart is basically a polar plot of the reflection coefficient K expressed in terms of normalized impedance.

The input impedance of the line can be expressed in terms of the characteristics impedance of the dissipationless line is given by

$$Z_{in} = Z_s = R_0 \left[\frac{1+|K|\angle\phi - 2\beta s}{1-|K|\angle\phi - 2\beta s} \right]$$

The normalized impedance is defined as,

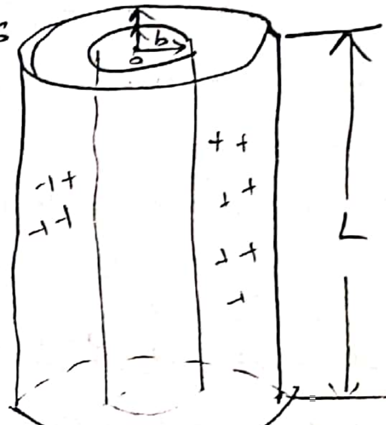
$$\frac{Z_{in}}{R_0} = \frac{1+|K|\angle\phi-2\beta s}{1-|K|\angle\phi-2\beta s}.$$

Coaxial Cable:-

Consider the two coaxial cylindrical conductors forming a coaxial cable. The radius of the inner conductors is 'a' while the radius of the outer conductor is 'b'. The Coaxial cable is b' length of the Coaxial cable is L.

The charge distribution on the outer surface of the inner conductor is having $\rho_s \, C/m^2$. The total surface area of the inner conductor is $2\pi aL$

Hence, ρ_s can be expressed interms of ρ_L.



$$\rho_L = \frac{\rho_s \times \text{surface area}}{\text{Total Length}} = \frac{\rho_s \times 2\pi aL}{L}$$

$$\boxed{\rho_L = 2\pi a \rho_s \, C/m}$$

Thus, the line charge density of the inner conductor is $\rho_L \, C/m$,

Consider the right circular cylinder of length L as the Gaussian surface. Due to the symmetry, $\bar{D}$ has only radial component. From that $a < r < b$, $D = D_r \oint ds = D_r \iint r \, d\phi \, dz$

$D = D_r \cdot 2\pi r L$. , $Q = \oint \rho$

The total charge on the inner conductor is said to be obtained by evaluating the surface charge distribution.

$$Q = \oint_s \rho_s \cdot d\bar{s}, \quad ds = r\,d\phi \cdot dz, \text{ but } r = a$$

$$ds = a\,d\phi \cdot dz \Rightarrow Q = \oint_s \rho_s \cdot a \, d\phi \cdot dz = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \rho_s \cdot a \, d\phi \cdot dz$$

$$= \rho_s \cdot a \, [z]_0^L \, [\phi]_0^{2\pi} = 2\pi a L \rho_s$$

$$D_r (2\pi r L) = 2\pi L a \rho_s \Rightarrow D_r = \frac{a \rho_s}{r}.$$

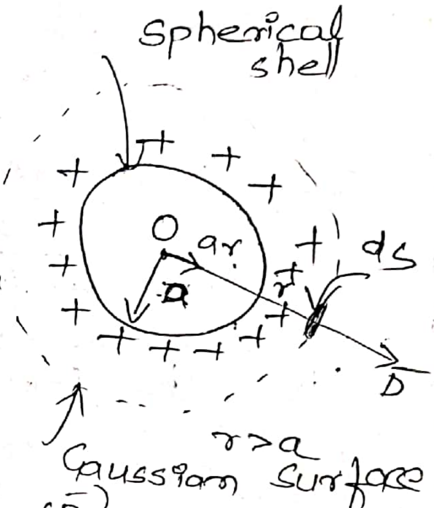
This acts along radial direction, i.e. $\bar{a}_r$

$\bar{D} = \frac{a\rho_s}{r}\bar{a}_r \Rightarrow$ but $\rho_s = \frac{\rho_L}{2\pi a} \Rightarrow \bar{D} = \frac{a \cdot \rho_L}{2\pi a \cdot r}\bar{a}_r \Rightarrow \bar{D} = \frac{\rho_L}{2\pi r}\bar{a}_r$

$\boxed{\bar{D} = \frac{\rho_L}{2\pi r}\bar{a}_r \ C/m^2}$, $\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0} \cdot \bar{a}_r$ $(a<r<b)$ V/m

2) Spherical shell of charge:-

Consider an imaginary spherical shell of radius 'a'. The charge is uniformly distributed over its surface with a density ρ_s C/m^2. Let us find $\bar{E}$ at a point P located at a distance r from the centre such that $r>a$, $r<a$, using Gauss Law.



Case:-1

Consider point P outside the shell $(r>a)$. The Gaussian surface passing through the point P is a concentric sphere of radius r. Due to spherical Gaussian surface, the flux lines are directed radially outwards are normal to the surface. Hence, electric flux density is also directed radially outwards at a point P and has component only in $\bar{a}_r$ direction. Consider a differential surface area at P normal to $\bar{a}_r$ direction hence $ds = r^2$

$ds = r^2 \sin\theta \cdot d\theta \cdot d\phi$ in spherical system

$$d\psi = \bar{D} \cdot \bar{ds} = [D_r \bar{a}_r] \cdot [r^2 \sin\theta \, d\theta \cdot d\phi \cdot \bar{a}_r]$$

$$= D_r \cdot r^2 \sin\theta \cdot d\theta \cdot d\phi$$

$$\psi = Q = \oint_s r^2 \cdot D_r \cdot \sin\theta \cdot d\theta \cdot d\phi = D_r \cdot r^2 \int_{\phi=0}^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$\Rightarrow \psi = Q = D_r r^2 (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi} = 4\pi r^2 D_r$$

$$Q = 4\pi r^2 D_r \Rightarrow D_r = \frac{Q}{4\pi r^2}$$

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2} \ C/m^2}$$

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Now, charge density is $\rho_s C/m^2$ $\therefore$ $Q = \rho_s \times$ surface area of shell

$$Q = \rho_s \times 4\pi a^2 \rightarrow \bar{E} = \frac{\bar{D}}{\varepsilon_0} = \frac{Q}{4\pi r^2} = \frac{\rho_s \times 4\pi a^2}{4\pi r^2 \cdot \varepsilon_0}$$

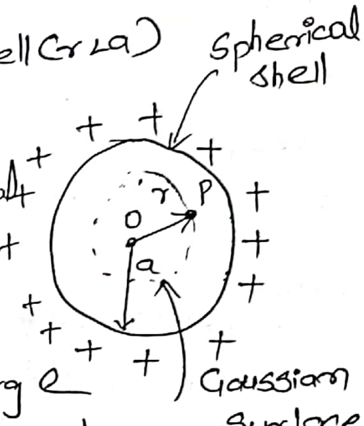
$$\boxed{\bar{E} = \frac{\rho_s a^2}{\varepsilon_0 r^2} \bar{a}_r \ V/m}$$

Case:-2 Point P is on the shell $(r=a)$

On the shell, $r=a$,
The Gaussian Surface is same as the shell itself and $\bar{E}$ can be obtained using $r=a$.

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon_0 a^2} V/m \ \& \ D = \frac{Q}{4\pi a^2} \cdot \bar{a}_r \ C/m^2}$$

Case:-3 Consider point p inside the shell $(r<a)$



The Gaussian Surface, passing through the point P is again a spherical surface with radius $r<a$.

But it can seen that the entire charge is on the surface and no charge is enclosed by the spherical shell. And when the Gaussian Surface is such that no charge is enclosed, irrespective of any charges present outside, the total charge enclosed is zero.

$$\psi = Q = \oint \bar{D} \cdot ds = 0. \rightarrow \text{As per Gauss Law}$$

$$\oint_S ds = \oint \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \cdot d\theta \cdot d\phi = 4\pi r^2 \Rightarrow \oint_S ds \neq 0.$$

Hence to satisfy the total charge enclosed is zero, ~~inside the spherical~~ shell.

$$\boxed{\bar{D} = 0 \text{ and } \bar{E} = \frac{\bar{D}}{\varepsilon_0} = 0}$$

, Thus $\bar{E}$ $\bar{D}$ and Electric field at any point inside the spherical shell is zero