ANGLE MODULATION – (FM-1)

❖ Define angle modulation ?

Angle modulation is the process in which the angle of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping constant amplitude of the carrier wave.

There are two types of angle modulation:

1. **Frequency modulation**
2. **Phase modulation**

❖ Angle Modulation : Basic Concept :-

Let the modulated wave be expressed in the general form as follows:

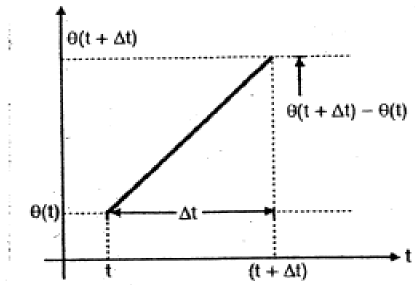
$$\boxed{s(t) = A_c \cos[\theta(t)]} \longrightarrow ①$$

Where,

$A_c \rightarrow$ carrier amplitude (Which is maintained Constant)

$\theta(t) \rightarrow$ is the angular argument which is varied in proportion with the message Signal $m(t)$.

* The variation of $\theta(t)$ due to $m(t)$ can be expressed mathematically if we know the type of angle modulation. If $\theta(t)$ changes by 2π radians then we say that a complete oscillation has occured.
* If $\theta(t)$ is increased monotonically with time as shown in figure, then the average frequency in Hz over the interval 't' to $(t+\Delta t)$ is given by:



$$\boxed{f_{\Delta t}(t) = \frac{1}{2\pi}\,\frac{\theta(t+\Delta t)-\theta(t)}{\Delta t}} \longrightarrow (2)$$

* The instantaneous frequency of the angle modulated wave $s(t)$ is given by

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0}\left[\frac{\theta(t+\Delta t)-\theta(t)}{2\pi\,\Delta t}\right]$$

$$\boxed{f_i(t) = \frac{1}{2\pi}\,\frac{d\theta(t)}{dt}} \longrightarrow (3)$$

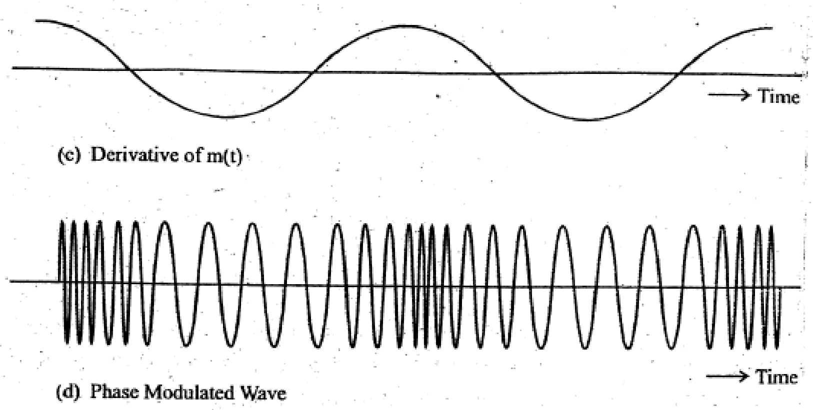
This is basic definition of derivative of a function.

* For an unmodulated carrier, the angle $\theta(t)$ is given by:

$$\boxed{\theta(t) = 2\pi f_c t + \phi_c(t)}$$

The angular frequency of the carrier is W_c, where $W_c = 2\pi f_c$, & ϕ_c is the value of $\theta(t)$ at $t=0$.

Phase Modulation (PM):-



phase modulated waves for sinusoidal signal

Phase modulation (PM) is defined as the form of angle-modulation in which the angular argument '$\theta(t)$' is varied linearly with the message signal '$m(t)$' as given below:

$$\boxed{\theta(t) = 2\pi f_c t + K_p m(t)}$$

Where,

$\omega_c t = 2\pi f_c t$ represents the angular argument of the modulated carrier &

$K_p \rightarrow$ Constant, represents the phase sensitivity of the modulator.

∴ The phase-modulated wave $S(t)$ is given by:

$$S(t) = A_c \cos[\theta(t)]$$

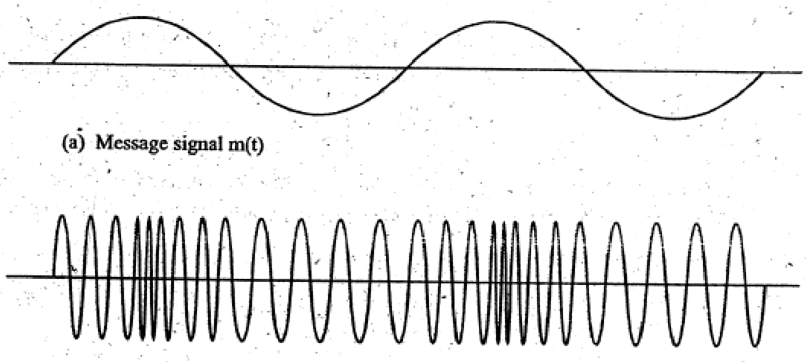
$$\boxed{S(t) = A_c \cos[2\pi f_c t + K_p m(t)]}$$

The features of phase modulation are:

i) The envelope of PM wave is a constant & equal to the amplitude of the unmodulated carrier.

2) The Zero crossings of a PM wave no longer have a perfect regularity in their spacing like AM wave. This is because instantaneous frequency of PM wave is proportional to time derivative of $m(t)$.

Frequency Modulation (FM):-



(a) Message signal m(t)

(b) Frequency modulated wave

Frequency modulation is the form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal $m(t)$ as given below:

$$\boxed{f_i(t) = f_c + k_f m(t)} \longrightarrow (1)$$

Where,

$f_c \rightarrow$ Frequency of the unmodulated carrier

$K_f \rightarrow$ Frequency Sensitivity of the modulator expressed in hertz's per volt.

W.K.T, (From eq ③)

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = 2\pi f_i(t) \longrightarrow ②$$

Integrating on both sides of eq ② w.r. to 't'

$$\theta(t) = \int_0^t 2\pi f_i(t) \cdot dt \longrightarrow ③$$

$$\theta(t) = \int_0^t 2\pi \left[f_c + K_f m(t) \right] dt$$

$$= \int_0^t 2\pi f_c \, dt + \int_0^t 2\pi K_f \, m(t) \cdot dt$$

$$= 2\pi f_c \int_0^t (1) \, dt + 2\pi K_f \int_0^t m(t) \cdot dt$$

$$\boxed{\theta(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(t) \cdot dt} \longrightarrow ④$$

* The FM wave in time domain can be written as

$$S(t) = A_c \cos\left[\theta(t)\right] \longrightarrow ⑤$$

Substituting eq ④ in eq ⑤, we get

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(t) \, dt\right]}$$

Relationship between FM & PM :-

In both FM & PM, the instantaneous angle $\theta(t)$ changes but in a different manner.

* The expressions for the FM & PM waves in the time domain are as follows:

PM Wave: $S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$

FM Wave: $S(t) = A_c \cos[2\pi f_c t + 2\pi K_f \int_0^t m(t)\, dt]$

Comparing these expressions we can conclude that an FM wave is actually a PM wave having a modulating signal $\int_0^t m(t)\, dt$ instead of $m(t)$.

Note:-

1) In FM wave, $\theta(t)$ is directly proportional to the integral of $m(t)$ i.e. $\int_0^t m(t).\, dt$

2) PM can be generated by 1st differentiating modulating signal $m(t)$ w.r. to 't' & then $\frac{d\, m(t)}{dt}$ is modulated by using a sinusoidal carrier.

❖ **Define angle modulation. Describe with the help of block diagrams schemes for generating. i. FM wave using PM ii. PM wave using FM**

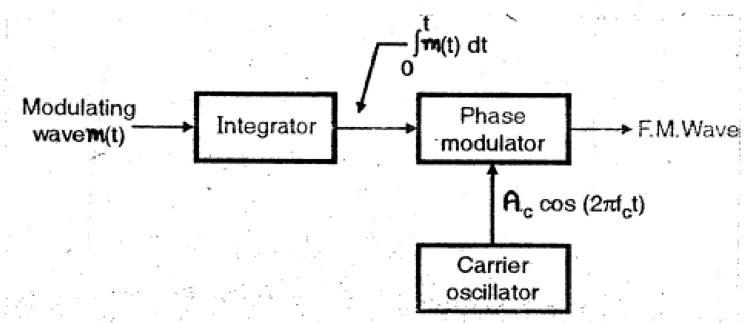
June-08,8m Jan-05,8M June-09,8M

Angle modulation is the process in which the angle of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping constant amplitude of the carrier wave.

There are two types of angle modulation:

1. **Frequency modulation**
2. **Phase modulation**

i. Generation of FM using PM (Phase Modulator):-



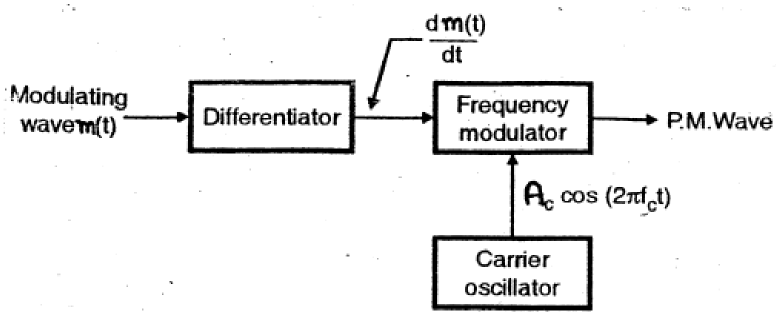
Generation of FM from phase modulator

* FM can be generated by 1st integrating m(t) & then using the result as the I/p to a phase modulator as shown in above figure.

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(t) \cdot dt\right]}$$

K_p

ii. Generation of PM using FM (Frequency Modulator):-



Generation of P.M. wave using frequency modulator

* The PM Signal can be generated by 1st differentiating m(t) & then using the result as the I/p to a frequency-modulator as shown in fig above.

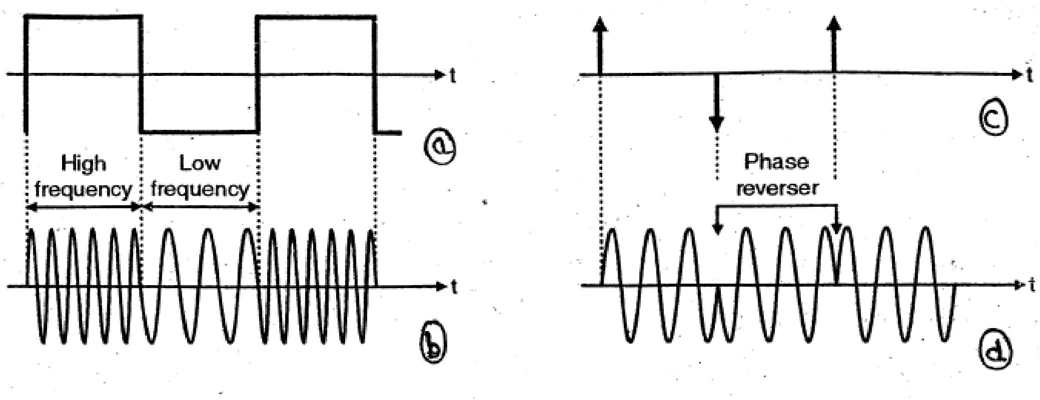
$$\therefore \quad s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t \frac{\cancel{d}}{\cancel{dt}} m(t) \cdot \cancel{dt}\right]$$

Substituting $2\pi K_f = K_p$

$$s(t) = A_c \cos\left[2\pi f_c t + \underline{2\pi K_f}\, m(t)\right]$$

$$\boxed{s(t) = A_c \cos\left[2\pi f_c t + K_p\, m(t)\right]}$$

Square Modulation:-



Frequency modulated wave Phase modulated wave

Square modulation

* Consider two full cycles of square modulating wave $m(t)$ as shown in fig ⓐ. The FM wave produced by this - modulating wave is plotted in fig ⓑ.

* The PM wave has been plotted by using the differentiated - version of $m(t)$ i.e. $\frac{d}{dt}m(t)$ as the modulating signal as shown in fig ⓒ.

* Note that $\frac{d}{dt}m(t)$ is a train of alternate (+ve & -ve) delta function.

The desired PM wave is plotted in fig ⓓ.

❖ **An FM wave is defined by**

$$S(t) = A_c \cos[10\pi t + \sin(4\pi t)]$$

find the instantaneous frequency of S(t).

Sol :- W.K.T $f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \theta(t)$ Hz &

$$S(t) = A_c \cos[\theta(t)]$$

$$\therefore \theta(t) = 10\pi t + \sin 4\pi t$$

$$f_i(t) = \frac{1}{2\pi} \frac{d}{dt}[10\pi t + \sin 4\pi t] \text{ Hz}$$

$$= \frac{1}{\cancel{2\pi}}[\overset{5}{\cancel{10\pi}} + \cos(4\pi t) \cdot \overset{2}{\cancel{4\pi}}] \text{ Hz}$$

$$\boxed{f_i(t) = 5 + 2\cos(4\pi t) \text{ Hz}}$$

❖ **An FM wave is defined by**

$$S(t) = 10\cos[2 + \sin(6\pi t)]$$

find the instantaneous frequency of S(t).

Sol :- Given : $\theta(t) = 2\pi t + \sin(6\pi t)$

W.K.T $f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \theta(t)$ Hz

$$= \frac{1}{2\pi} \frac{d}{dt}[2\pi t + \sin(6\pi t)] \text{ Hz}$$

$$= \frac{1}{\cancel{2\pi}}[\cancel{2\pi} + \cos(6\pi t) \cdot \overset{3}{\cancel{6\pi}}] \text{ Hz}$$

$$= [1 + \cos(6\pi t) \cdot 3] \text{ Hz}$$

$$\boxed{f_i(t) = [1 + 3\cos(6\pi t)] \text{ Hz}}$$

❖ **Derive an expression for single tone sinusoidal FM wave, find its spectrum**

June-10,10M

❖ **Explain single tone frequency modulation.**

* The frequency modulated wave in time domain is given by:

$$\boxed{S(t) = A_c \cos[\theta(t)]} \longrightarrow (1)$$

* The Sinusoidal modulating Signal is defined by

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (2)$$

* The Instantaneous frequency of the FM Signal is given by:

$$f_i(t) = f_c + K_f \underline{m(t)}$$

$$f_i(t) = f_c + \underline{K_f A_m} \cos(2\pi f_m t)$$

$$\boxed{f_i(t) = f_c + \Delta f \cos(2\pi f_m t)} \longrightarrow (3)$$

Where, $\Delta f = K_f A_m$ & it is called as frequency deviation

{ The quantity Δf is called the frequency deviation. The frequency deviation Δf is proportional to the amplitude of the modulating Signal & is Independent of the modulating frequency.
}

W.K.T the angular velocity $\omega_i(t)$ is the rate of change of $\theta(t)$.

$$\omega_i(t) = \frac{d}{dt}\theta(t)$$

$$2\pi f_i(t) = \frac{d}{dt}\theta(t) \longrightarrow (4)$$

Integrating eq ④ w.r.to dt

$$\int_0^t \frac{d}{\cancel{dt}}\theta(t)\,\cancel{dt} = \int_0^t 2\pi f_i(t).\,dt$$

$$\theta(t) = \int_0^t 2\pi \underline{f_i(t)}.\,dt \longrightarrow ⑤$$

Substituting eq ③ in eq ⑤

$$\theta(t) = \int_0^t 2\pi\left[f_c + \Delta f\,\cos(2\pi f_m t)\right]dt$$

$$= \int_0^t 2\pi f_c\,dt + \int_0^t 2\pi\Delta f\,\cos(2\pi f_m t)\,dt$$

$$= 2\pi f_c t + \cancel{2\pi}\Delta f.\frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}$$

$$\theta(t) = 2\pi f_c t + \frac{\Delta f}{f_m}\sin 2\pi f_m t$$

W.K.T

$$\boxed{\int_0^t \cos at\,dt = \frac{\sin at}{a}}$$

$$\boxed{\theta(t) = 2\pi f_c t + \underline{\beta}\sin(2\pi f_m t)} \longrightarrow ⑥$$

Where $\beta = \frac{\Delta f}{f_m}$

Substitution eq ⑥ in eq ①, we get

$$\therefore \boxed{s(t) = A_c\cos[2\pi f_c t + \beta\sin(2\pi f_m t)]}$$

Modulation Index (β or m_f):-

Modulation index is defined as the ratio of frequency deviation 'Δf' to the modulating frequency 'f_m'.

$$\beta = \frac{\text{Frequency deviation}}{\text{Modulating frequency}}$$

$$\boxed{\beta \text{ or } m_f = \frac{\Delta f}{f_m}}$$

NOTE:- > In FM the modulation Index can be greater than 1.

2> The modulation index is very important in FM because it decides the bandwidth of the FM wave.

3> The modulation Index also decides the number of sidebands having significant amplitude.

Frequency Deviation (Δf):-

* The Instantaneous frequency of the FM signal varies w.r.to time. The maximum change in the instantaneous frequency from the average value carrier frequency 'f_c' is known as frequency deviation.

$$\boxed{\Delta f = |K_f m(t)|_{max}}$$

❖ **Derive the equation for FM waves. Define modulation index, maximum deviation and bandwidth of a FM signal** **June-05,6M June-09,M(old)**

* Let the modulated wave be expressed in the general form as follows:

$$\boxed{S(t) = A_c \cos[\theta(t)]} \longrightarrow ①$$

* The Instantaneous frequency of the FM Signal is given by:

$$\boxed{f_i(t) = f_c + \Delta f \cos(2\pi f_m t)} \longrightarrow ②$$

WKT the angular velocity $\omega_i(t)$ is the rate of change of $\theta(t)$

$$\omega_i(t) = \frac{d}{dt}\theta(t)$$

$$2\pi f_i(t) = \frac{d}{dt}\theta(t) \longrightarrow ③$$

Integrating eq ③ w.r.to dt

$$\int_0^t \cancel{\frac{d}{dt}} \theta(t) . \cancel{dt} = \int_0^t 2\pi f_i(t) \, dt$$

$$\theta(t) = \int_0^t 2\pi \underline{f_i(t)} . dt \longrightarrow ④$$

Substituting equation ② in eq ④, we get

$$\theta(t) = \int_0^t 2\pi [f_c + \Delta f \cos(2\pi f_m t)] dt$$

$$= \int_0^t 2\pi f_c \, dt + \int_0^t 2\pi \Delta f \cos(2\pi f_m t) \, dt$$

$$= 2\pi f_c t + \cancel{2\pi} \Delta f \frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}$$

$$\theta(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t$$

$$\boxed{\theta(t) = 2\pi f_c t + \underline{\beta} \sin(2\pi f_m t)} \longrightarrow ⑤$$

Where, $\beta = \frac{\Delta f}{f_m}$.

Substituting eq ⑤ in eq ①, we get

$$\therefore \boxed{S(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]}$$

Modulation Index :-

$$\beta = \frac{\text{Frequency deviation}}{\text{Modulating frequency}} = \frac{\Delta f}{f_m}.$$

Maximum deviation :-

The maximum change in the instantaneous frequency from the average value carrier frequency 'f_c' is known as frequency deviation.

$$\boxed{\Delta f = |K_f m(t)|_{max}}$$

$$\Delta f = K_f A_m$$

Bandwidth :-

The FM wave consists infinite number of sidebands. Thus bandwidth of a FM signal is infinite.

By Carson's rule :

$$BW = 2[\Delta f + f_m]$$

❖ Types of FM or Classification of FM :-

Depending on the value of the modulation index 'β' FM wave is classified as follows:

1. **Narrow band FM (NBFM)**
2. **Wide band FM (WBFM)**

1) Narrow band FM :- property 1

* A NBFM is the FM wave with a small bandwidth. The modulation Index 'β' of NBFM is small as compared to one radian.
* Thus NBFM has a Narrow bandwidth which is equal to twice the message bandwidth.

2) Wide band FM :- property 2

* The WBFM has a much larger value of 'β' which is theoretically infinite.
* For larger value of modulation Index 'β', the FM wave ideally contains the carrier & an Infinite number of sidebands located symmetrically around the carrier.

Such a FM wave has Infinite bandwidth & hence called Wideband FM.

❖ Mention the properties of FM?

There are three properties of FM:

1. **Narrow band FM (NBFM)**
2. **Wide band FM (WBFM)**
3. **Constant average power.**

Narrow band Frequency Modulation:-

- **Describe with necessary equations and block diagram, the generation of narrow band FM.** **Jan-05,6M**
- **Narrow band FM** **June-09,5M(old)**

The time-domain expression for an FM Wave is

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \longrightarrow (1)$$

using the trignometric identity

$$\boxed{\cos(A+B) = \cos A . \cos B - \sin A . \sin B}$$

$$A = 2\pi f_c t \quad \& \quad B = \beta \sin(2\pi f_m t)$$

$$S(t) = A_c\left[\cos(2\pi f_c t).\cos(\beta \sin 2\pi f_m t) - \sin(2\pi f_c t).\sin(\beta \sin 2\pi f_m t)\right] \rightarrow (2)$$

In NBFM, β is Small, hence it possible to approximate

$$\boxed{\begin{array}{l}\cos(\beta . \sin 2\pi f_m t) \approx 1 \\ \sin(\beta \sin 2\pi f_m t) \approx \beta \sin 2\pi f_m t\end{array}} \longrightarrow (3)$$

Substituting eq (3) in eq (2), we get

$$S(t) = A_c \cos 2\pi f_c t\,(1) - A_c \sin 2\pi f_c t . (\beta \sin 2\pi f_m t)$$

$$S(t) = A_c \cos 2\pi f_c t - \beta A_c . \underline{\sin 2\pi f_c t . \sin 2\pi f_m t.} \rightarrow (4)$$

W.K.T

$$\boxed{\sin A . \sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)}$$

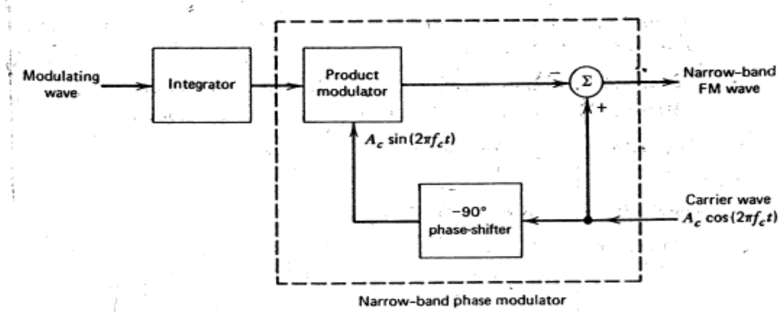
$$S(t) = A_c \cos 2\pi f_c t - \left[\frac{\beta A_c}{2}\cos 2\pi(f_c - f_m)t - \frac{\beta A_c}{2}\cos 2\pi(f_c + f_m)t\right]$$

$$S(t) = A_c \cos 2\pi f_c t - \frac{\beta A_c}{2}\cos 2\pi(f_c - f_m)t + \frac{\beta A_c}{2}\cos 2\pi(f_c + f_m)t \rightarrow (5)$$

W.K.T the amplitude modulated wave is given by

$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t \rightarrow ⑥$$

* Comparing equation ⑤ & equation ⑥. The only difference observed between NBFM wave & AM wave is the Sign reversal of the lower Sideband.
* Thus NBFM requires the Same bandwidth as that of AM.



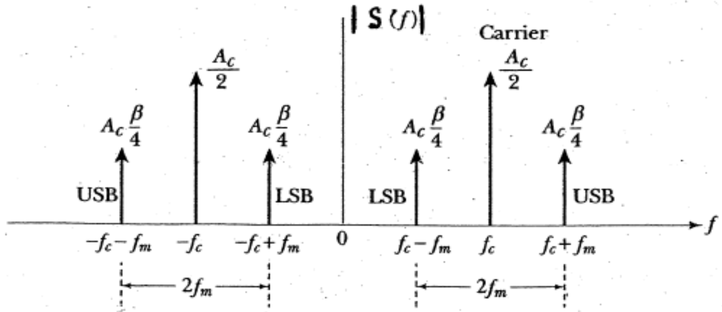
Block diagram of a method for generating a narrow-band FM signal.

Taking Fourier Transform on both Sides of eq ⑤, we get

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] - \frac{\beta A_c}{4}\left\{\delta[f-(f_c-f_m)]+\delta[f+(f_c-f_m)]\right\}$$

$$+ \frac{\beta A_c}{4}\left\{\delta[f-(f_c+f_m)]+\delta[f+(f_c+f_m)]\right\}$$

* The transmission bandwidth of a NBFM wave is $2f_m$.
* The NBFM wave & Conventional AM wave are Identical but there is No amplitude variation in FM.



Spectral content of a NBFM wave for single-tone modulation.

Complex envelope of FM wave :-

The FM Wave for Sinusoidal modulation is given by :

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \longrightarrow ①$$

by using trignometric function

$$\boxed{\cos(A+B) = \cos A . \cos B - \sin A . \sin B}$$

Where $A = 2\pi f_c t$, $B = \beta \sin 2\pi f_m t$

$$S(t) = A_c \cos(2\pi f_c t) . \cos(\beta \sin 2\pi f_m t) - A_c \sin(2\pi f_c t). \sin(\beta \sin 2\pi f_m t)$$

The inphase & Quadrature components of FM wave $S(t)$ are given by

$$S_I(t) = A_c \cos[\beta \sin(2\pi f_m t)]$$

$$S_Q(t) = A_c \sin[\beta \sin(2\pi f_m t)].$$

* The Complex envelope of the FM wave is

$$\hat{S}(t) = S_I(t) + j S_Q(t)$$

$$\hat{S}(t) = A_c \cos[\beta \sin(2\pi f_m t)] + j A_c \sin[\beta \sin(2\pi f_m t)] \rightarrow ②$$

expressing $\hat{S}(t)$ in terms of $e^{j\theta}$

i.e. $e^{j\theta} = \cos\theta + j\sin\theta$

From eq ② , $\theta = \beta \sin(2\pi f_m t)$

$$\boxed{\hat{S}(t) = A_c e^{j\beta \sin(2\pi f_m t)}}$$

Formulae & Basic Concepts :-

1) Complex number expressed as a trignometric function & exponential form $\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$

Where,

$$\text{Real part} \rightarrow R_e[e^{j\theta}] = \cos\theta$$

$$\text{Imaginary part} \rightarrow \text{Img}[e^{j\theta}] = \sin\theta$$

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2) Complex Fourier Series
$$\hat{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}$$

---

3) Complex Fourier Co-efficient

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} \hat{s}(t) \cdot e^{-j2\pi n f_m t} \cdot dt$$

---

4) Bessel Function

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \cdot dx$$

- **Derive an expression for the spectrum of FM wave with sinusoidal modulation** **Jan-05,7M**
- **Derive time-domain expression for a wideband FM wave.** **Jan-10,8M**
- **Derive the equation of a FM signal s(t), from basic principles and further analyse the sinusoidal FM wave interms of S(f) and Bessel function J_n (β).** **July-07,10M**
- **Derive an expression for spectrum of FM wave with sinusoidal modulation.** **Jan-07,10M**
- **Derive expression for the spectrum of FM wave with sinusoidal modulation** **June-09,7M**

* The FM Wave for Sinusoidal modulation is given by:

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \longrightarrow (1)$$

Taking real part of eq (1)

{ NOTE:- Equation (1) is not having imaginary part. It has only real part. $\boxed{\theta = 2\pi f_c t + \beta \sin 2\pi f_m t}$
}

$$S(t) = R_e[A_c e^{j\theta}]$$

$$S(t) = R_e[A_c e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}]$$

$$= R_e[A_c e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}]$$

$$= R_e[e^{j2\pi f_c t} \cdot A_c e^{j\beta \sin 2\pi f_m t}]$$

$$S(t) = R_e[e^{j2\pi f_c t} \cdot \hat{S}(t)] \longrightarrow (2)$$

Where, $\hat{S}(t) = A_c e^{j\beta \sin 2\pi f_m t} \longrightarrow (3)$

* $\hat{S}(t)$ is a periodic time function with a fundamental frequency 'f_m'. This can be expressed using Complex Fourier Series as:

$$\boxed{\hat{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}} \longrightarrow (4)$$

Where C_n is a complex Fourier co-efficient given by

$$\boxed{C_n = f_m \int_{-1/2f_m}^{1/2f_m} \hat{s}(t) \cdot e^{-j2\pi n f_m t} \cdot dt} \longrightarrow (5)$$

Substituting eq (3) in eq (5), we get

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} A_c e^{j\beta \sin(2\pi f_m t)} \cdot e^{-j2\pi n f_m t} \cdot dt \longrightarrow (5)$$

$$C_n = A_c f_m \int_{-1/2f_m}^{1/2f_m} e^{j[\beta \sin(2\pi f_m t) - 2\pi n f_m t]} \cdot dt$$

Let $x = 2\pi f_m t \rightarrow (a)$

differentiating eq (a) w.r.to t

$$\frac{dx}{dt} = 2\pi f_m (1)$$

$$dt = \frac{dx}{2\pi f_m}$$

Giving the limits

WKT $x = 2\pi f_m t$	
When $t = -\frac{1}{2f_m}$ $x = \cancel{2}\pi \cancel{f_m} \cdot -\frac{1}{\cancel{2}\cancel{f_m}}$ $\boxed{x = -\pi}$	When $t = \frac{1}{2f_m}$ $x = \cancel{2}\pi \cancel{f_m} \cdot \frac{1}{\cancel{2}\cancel{f_m}}$ $\boxed{x = \pi}$

$$C_n = A_c f_m \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \frac{dx}{2\pi f_m}$$

$$C_n = A_c \frac{\cancel{f_m}}{2\pi \cancel{f_m}} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} . dx$$

$$C_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} . dx$$

$$\boxed{C_n = A_c J_n(\beta)} \longrightarrow 6$$

Where, $J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} . dx$

$J_n(\beta)$ is a bessel function of the 1st kind, n^{th} order with an argument β.

Substituting eq 6 in eq 4, we get $\left\{ \hat{S}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t} \rightarrow 4 \right\}$

$$\boxed{\hat{S}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi n f_m t}} \longrightarrow 7$$

Substituting eq 7 in eq 2, we get $\left\{ S(t) = R_e[\hat{S}(t) . e^{j2\pi f_c t}] \rightarrow 2 \right\}$

$$S(t) = R_e\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \, e^{j2\pi n f_m t} . e^{j2\pi f_c t} \right]$$

$$S(t) = R_e\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \, e^{j2\pi [f_c + n f_m] t} \right]$$

W.K.T $\quad e^{j\theta} = \cos\theta + j\sin\theta$

$R_e[e^{j\theta}] = \cos\theta \quad \& \quad \theta = 2\pi(f_c + n f_m) t$

||ly

$$\boxed{s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \cos[2\pi(f_c + nf_m)t]} \longrightarrow (8)$$

Giving the values for n b/w $-\infty$ to $+\infty$

i.e. $n = 0, 1, -1, 2, -2, \cdots\cdots +\infty, -\infty$.

$$s(t) = A_c\Big[J_0(\beta)\cos 2\pi f_c t + J_1(\beta)\cos 2\pi(f_c+f_m)t + J_{-1}(\beta)\cos 2\pi(f_c-f_m)t$$

$$+ J_2(\beta)\cos 2\pi[f_c+2f_m]t + J_{-2}(\beta)\cos 2\pi[f_c-2f_m]t$$

$$+ J_3(\beta)\cos 2\pi[f_c+3f_m]t + J_{-3}(\beta)\cos 2\pi[f_c-3f_m]t + \cdots\Big] \longrightarrow (9)$$

$$s(t) = A_c\Big\{J_0(\beta)\cos 2\pi f_c t + J_1(\beta)\big[\cos 2\pi(f_c+f_m)t - \cos 2\pi(f_c-f_m)t\big]$$

$$+ J_2(\beta)\big[\cos 2\pi(f_c+2f_m)t - \cos 2\pi(f_c-2f_m)t\big]$$

$$+ J_3(\beta)\big[\cos 2\pi(f_c+3f_m)t - \cos 2\pi(f_c-3f_m)t\big] + \cdots\cdots\Big\}$$

* Thus the modulated Signal has a carrier component & an infinite number of Side frequencies $f_c \pm f_m$, $f_c \pm 2f_m$, $f_c \pm 3f_m$, $\cdots$, $f_c \pm nf_m$.

* Taking Fourier Transform on both Sides of eq (9), we get

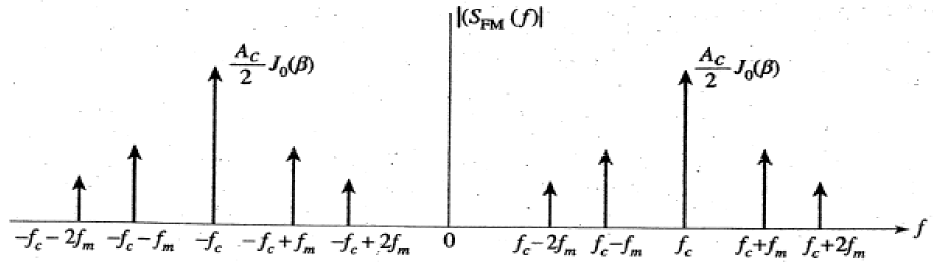
$$S(f) = \frac{A_c}{2} J_0(\beta)\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{A_c}{2} J_1(\beta)\Big\{\delta(f-[f_c+f_m]) + \delta(f+[f_c+f_m])\Big\}$$

$$+ \frac{A_c}{2} J_{-1}(\beta)\Big\{\delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)]\Big\} + \cdots\cdots$$

$$+ \frac{A_c}{2} J_n(\beta)\Big\{\delta[f-(f_c+nf_m)] + \delta[f+(f_c+nf_m)]$$

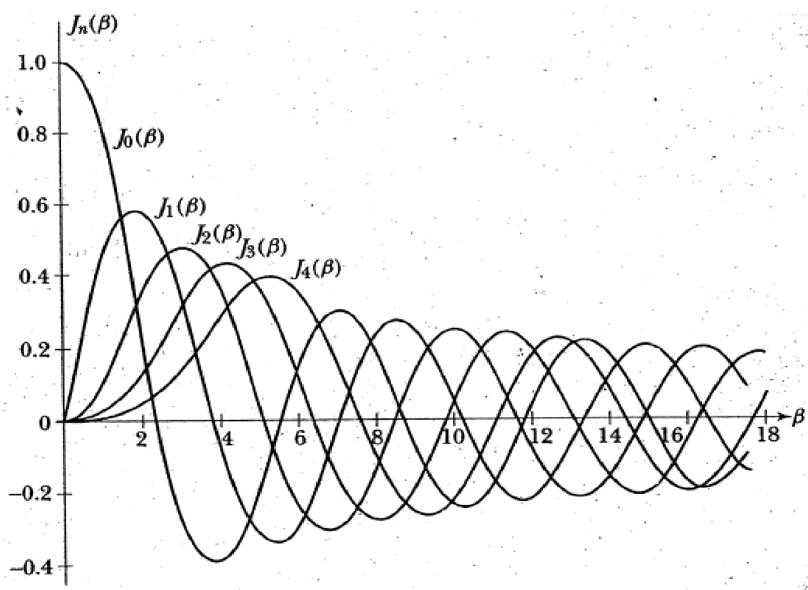
$$+ \frac{A_c}{2} J_{-n}(\beta)\Big\{\delta[f-(f_c-nf_m)] + \delta[f+(f_c-nf_m)]\Big\} \longrightarrow (10)$$

Now plotting Spectrum for above equation



Amplitude spectrum of FM signal.

* The amplitude of Side frequency component depends upon the bessel function. The Bessel variations as a function of 'β' fixing the values of 'n' as shown in figure below.



Plots of Bessel function of the first kind.

Constant average power :- (3rd Property)

* The envelope of an FM wave is constant, so that the average power of such a wave dissipated in 1-ohm resistor is also constant.

* The FM wave $s(t)$ has a constant envelope equal to A_c.

$\therefore$ power dissipation $= \dfrac{A_c^2}{2R}$

* The average power dissipated by $s(t)$ in a 1-ohm resistor is given by:

$$P = \frac{A_c^2}{2(1)}$$

$$\boxed{P = \frac{A_c^2}{2}}$$

* The average power of a single tone FM wave $s(t)$ may be expressed in the form of a corresponding series as:

$$\boxed{P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)}$$

but

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

Thus

$$P = \frac{A_c^2}{2}(1)$$

$$\boxed{P = \frac{A_c^2}{2}}$$

- **Explain difference between wideband FM and Narrow band FM** **Jan-09,4M**
- **Compare Narrowband and Wideband FM** **June-09,6M**

Sl No	parameters	NBFM	WBFM
1)	Modulation Index	$\beta < 1$	$\beta > 1$
2)	Spectrum	The Spectrum of NBFM is Same as that of AM	The Spectrum of WBFM differs from AM
3)	Maximum deviation $[\Delta f]_{Max}$	$\Delta f_{max} = 5KHz$	$\Delta f_{max} = 75KHz$
4)	Modulating freq/ Range.	30Hz - 3KHz	30Hz - 15KHz
5)	Maximum modulation Index	β may be Slightly greater than 1.	β may be between 5 to 2500
6)	Bandwidth	Small i.e. approximately Same as that of AM	Large i.e. about 15 times higher than Bw of NBFM.
7)	Pre-emphasis & De-emphasis	Needed	Needed.
8)	Applications	used in Speech transmission ex:- FM mobile Comm	Used for high quality music transmission ex:- Entertainment broadcasting

❖ Bandwidth of Angle modulated wave:-

Case i : Sinusoidal modulation :-

Theoretically FM has infinite number of sidebands. So the bandwidth required for transmission is also infinite.

* Carson generalized the bandwidth formula for an FM wave. According to him, the approximate formula for computing the bandwidth of an FM signal generated by a single tone modulating signal frequency 'f_m' is

$$\boxed{B_T \simeq 2(1+\beta) f_m} \longrightarrow (1)$$

The above formula holds good for all values of β.

* The transmission bandwidth 'B_T' can also be expressed interms of frequency deviation 'Δf'

W.K.T $\quad \beta = \dfrac{\Delta f}{f_m}$

$$\boxed{\Delta f = \beta f_m}$$

From equation (1)

$$B_T = 2(1+\beta) f_m$$
$$= 2f_m + 2\beta f_m$$
$$= 2f_m + 2\Delta f$$
$$= 2\Delta f \left[1 + \frac{f_m}{\Delta f}\right] \qquad \because \frac{1}{\beta} = \frac{f_m}{\Delta f}$$

$$\boxed{B_T = 2\Delta f \left[1 + 1/\beta\right]}$$

Case ii : Non Sinusoidal or Arbitrary modulation :-

* For an angle modulated Signal with an arbitrary modulating - Signal $m(t)$, bandlimited to 'W' Hz, we define the deviation ratio as $\boxed{D = \frac{\Delta f}{W}} \rightarrow ①$

* The deviation ratio 'D' plays the Same role for non-Sinusoidal modulation that the modulation Index 'β' plays for the case of Sinusoidal modulation.

Then, replacing 'β' by 'D' & replacing 'f_m' with 'W' we may use carson's rule

W.K.T $\boxed{B_T = 2(1+\beta)f_m} \rightarrow ②$

replacing $f_m = W$ & $\beta = D$ in eq ②

$$\boxed{B_T = 2(1+D)W}$$

The above relation is also known as Carson's formula.

Universal curve for evaluating FM bandwidth:-

Suppose, n_{max} is the largest value of Integer 'n' Such that $|J_n(\beta)| > 0.01$. Then, we define the transmission bandwidth as

$$\boxed{B_T = 2n_{max} f_m}$$

Generation of FM waves:-

❖ **Explain the methods of FM generation.** **June-10,5M**

There are two basic methods of generating FM waves:

1. **Indirect method or Armstrong method**
2. **Direct method or Direct FM**

1) Indirect Method (or) ARMSTRONG Method OR Stereo FM :-

In this method, a Narrow-band FM (NBFM) Wave is generated. Frequency multipliers are then used to increase the frequency deviation which results in Wide band-FM (WBFM).

2) Direct FM (or) Direct Method :-

In direct FM, the carrier frequency 'f_c' is directly varied in accordance with the amplitude of the modulating Signal.

{ * Direct FM is not feasible, practically as it involves - maintaining high frequency Stability of the carrier with adequate frequency deviation.
}

❖ Indirect Method or ARMSTRONG method :-

- **Explain with a neat circuit diagram, the direct method of generating FM waves. Derive an expression for instantaneous frequency of FM for sinusoidal modulation.** **June-10,10M(old)**
- **Explain the FM generation using indirect method.** **Jan-09,8M**
- **Briefly explain indirect method of generating FM wave.** **Jan-08,10M**
- **With neat block diagram, explain ARMSTRONG method of FM generation.** **July-09,7M**
- **Explain how FM wave can be generated using indirect method. Write the spectrum of FM with sinusoidal modulation along with relevant equations**
- **Explain how FM wave can be generated using indirect method.** **July-05,8M**

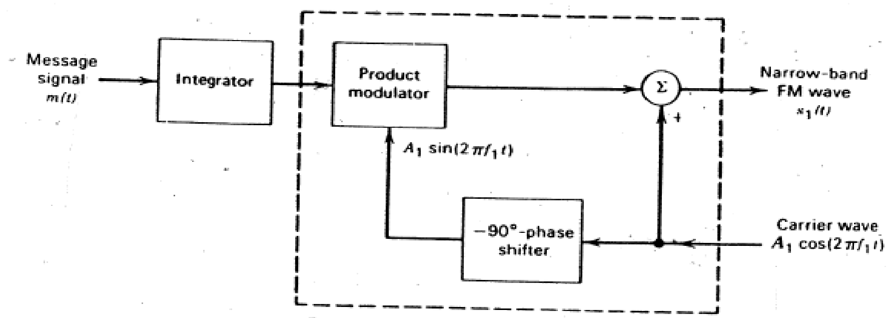


Fig ①: Narrow-band phase modulator

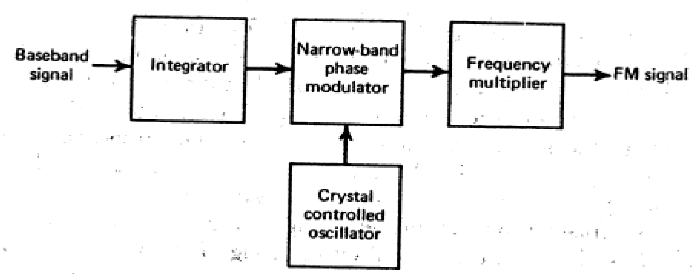


Figure ②: Block diagram of the indirect method of generating a wide-band FM signal.

Fig shows the block diagram of a Indirect FM System.

* In Indirect method, the message Signal $m(t)$ is 1st passed through an Integrator before applying it to the phase modulator, as Shown in Fig ①.

* The carrier signal is generated by using crystal oscillator because it provides very high frequency stability.

The operation of Indirect method is divided into two parts as follows:

i) Generate a NBFM wave using a phase modulator.

ii) Using the frequency multipliers & mixer to obtain the required values of frequency deviation & modulation Index. (i.e. WBFM)

* In order to minimize the distortion in the phase modulator, the maximum phase deviation or modulation Index 'β' is kept small there by resulting in a NBFM signal.

* Let $S_1(t)$ be the NBFM wave, then we have

$$\boxed{S_1(t) = A_c \cos\left[2\pi f_1 t + 2\pi K_f \int_0^t m(t) \cdot dt\right]} \longrightarrow ①$$

Where, f_c is the frequency of the crystal oscillator & K_f is the - frequency sensitivity constant in Hz/volt.

* For a single-tone modulation signal defined by

$m(t) = A_m \cos 2\pi f_m t$, then eq ① becomes

$$\boxed{S_1(t) = A_c \cos\left[2\pi f_1 t + \beta_1 \sin 2\pi f_m t\right]} \longrightarrow ②$$

Where, β_1 is the modulation Index for single-tone modulation & is kept below 0.3 radians to minimize the distortion.

The Instantaneous frequency of eq ② is

$$\boxed{f_i(t) = f_1 + K_f m(t)}$$

ii) Generation of WBFM :-

* The o/p of the Narrow band phase modulator is then multiplied by a frequency multiplier, producing the desired WBFM wave as shown in fig ③.

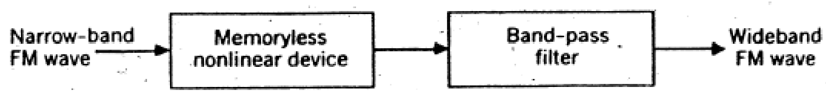


Fig ③: Frequency Multiplier.

* A frequency multiplier consists of a memoryless non-linear device followed by a BPF as shown in fig ③.

The I/p - o/p relation of such a non-linear device may be expressed in the general form.

$$\boxed{V(t) = a_1 S_1(t) + a_2 S_1^2(t) + \dots + a_n S_1^n(t)} \longrightarrow ③$$

Where $a_1, a_2, \dots, a_n$ are co-efficients & n is the highest order of non-linearity.

Substituting eq ① in eq ③ & simplifying, we find the frequency modulated wave having carrier frequencies $f_1, 2f_1, \dots, nf_1$ with frequency deviation $\Delta f_1, 2\Delta f_1, \dots, n\Delta f_1$.

The BPF has two functions to perform:

1) To pass the FM wave centered at carrier frequency nf_1 & having the frequency deviation $n\Delta f_1$.

2) To suppress all other FM spectra.

* The o/p of the frequency multiplier produces the desired WBFM wave having the following time-domain description.

$$\boxed{s(t) = A_c \cos\left[2\pi n f_1 t + 2\pi n k_f \int_0^t m(t)\,dt\right]} \longrightarrow (4)$$

Whose instantaneous frequency is

$$\boxed{f_1'(t) = nf_1 + nk_f\, m(t)}$$

Direct Method:-

❖ **Explain the direct method of generating FM waves.** **Jan-10,8M**

❖ **Explain FM generation using direct method.** **Jan-07,7M July-06,5M June-06,3M**

* In direct FM System, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal by means of a device called a "voltage controlled oscillator" (VCO).

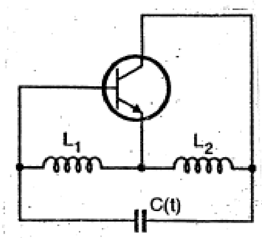
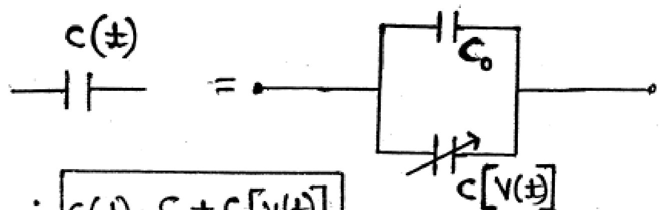


Fig ① Hartley oscillator

* Fig ① shows a hartley oscillator in which the capacitive component of the frequency determining N/w in the oscillator consists of a fixed capacitor shunted by a voltage-variable capacitor.

{



$$\therefore \boxed{c(t) = C_0 + c\left[V(t)\right]}$$

}

* The frequency of oscillation of the hartley oscillator is given by:

$$\boxed{f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}}} \longrightarrow (1)$$

Where,
$C(t) = C_0 + C[V(t)]$

$L_1 \& L_2 \rightarrow$ are the two Inductances in the frequency - determining the oscillator.

* Assume that a Sinusoidal modulating wave of frequency 'f_m', the Capacitance $C(t)$ is expressed as:

$$\boxed{C(t) = C_0 + \Delta C \cos(2\pi f_m t)} \longrightarrow (2)$$

Where, C_0 is the total Capacitance in the absence of modulation i.e. $f_m = 0$ &

ΔC is the maximum Change in Capacitance.

Substituting eq (2) in eq (1), we get

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\ C_0 + \Delta C \cos(2\pi f_m t)}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\ C_0\left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]}}$$

$$f_i(t) = f_0 \frac{1}{\sqrt{1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)}}$$

Where, $f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)\ C_0}}$, unmodulated frequency of oscillation.

$$f_i(t) = f_0 \frac{1}{\left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]^{1/2}}$$

$$f_i(t) = f_0\left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]^{-1/2}$$

Recall the binomial theorem

$$[1+x]^{-1/2} \approx 1-\frac{x}{2}$$

if $|x| \ll 1$

$$f_i(t) = f_0\left[1-\frac{\Delta C}{2C_0}\cos(2\pi f_m t)\right]$$

Let $\boxed{-\frac{\Delta C}{2C_0} = \frac{\Delta f}{f_0}}$

$$f_i(t) = f_0\left[1+\frac{\Delta f}{f_0}\cos(2\pi f_m t)\right]$$

$$f_i(t) = f_0 + \frac{\cancel{f_0}\Delta f}{\cancel{f_0}}\cos(2\pi f_m t)$$

$$\boxed{f_i(t) = f_0 + \Delta f \cos(2\pi f_m t)} \longrightarrow (3)$$

Equation (3) is the Instantaneous frequency of an FM wave, assuming Sinusoidal modulation.

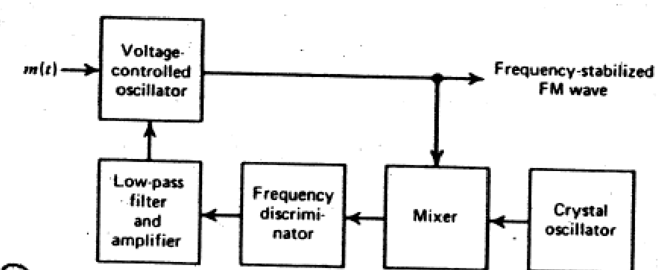


Figure (4)
A feedback scheme for the frequency stabilization of a frequency modulator.

* In order to generate a WBFM with the required frequency deviation, fig (4) is used. It consists of VCO, frequency - multiplier & mixers.

* This configuration provides good oscillator stability, constant proportionality b/w o/p frequency change to I/p voltage change, & the necessary frequency deviation to achieve WBFM.

ANGLE MODULATION – (FM-2)

Demodulation:-

Frequency demodulation is the process of recovering the original modulating wave from the frequency modulated wave.

Demodulation of FM waves:-

The FM demodulators are classified into:

1. Direct method

i. **Frequency discriminator**

ii. **Zero crossing detector.**

2. Indirect method

i. **Phase-Locked Loop.**

Requirements of FM detectors (Demodulators):-

The FM demodulator must satisfy the following requirements:

1) It must convert frequency variation into amplitude variations

2) The conversion must be linear & efficient.

3) The demodulator ckt should be insensitive to amplitude changes. It should respond only to the frequency changes.

4) It should not be too critical in its adjustment & operation.

Balanced Frequency discriminator or Balanced slope detector or Round – Travis Detector

- **Explain the detection process of FM signals using balanced frequency discriminator with relevant diagrams.** **July-09,6M(old)**
- **Draw the block diagram of balance frequency discriminator and explain it for demodulation of FM signal.** **Jan-09,8M**
- **Explain clearly how a balanced slope detector is used for FM demodulation.** **June-08,7M**
- **Explain the detection process of FM signals using balanced frequency discriminator with relevant diagrams.** **Jan-06,6M**
- **With associated diagrams and equations, explain how FM wave can be detected using ratio detector.** **July-05,7M June-09,6M**

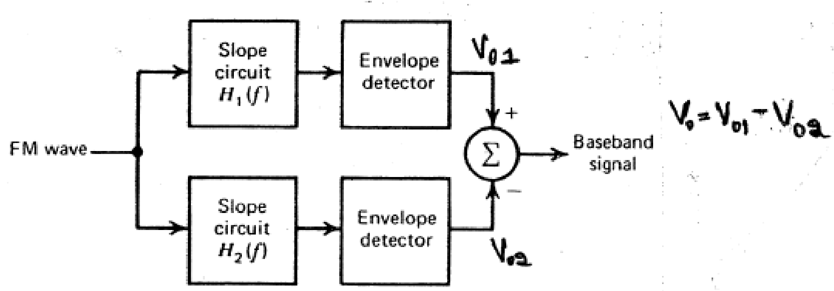


Fig: (a) Block Diagram

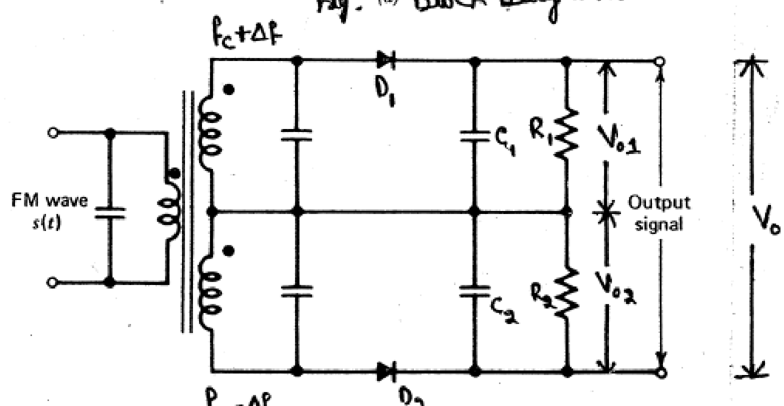


Fig (b): Ckt diagram

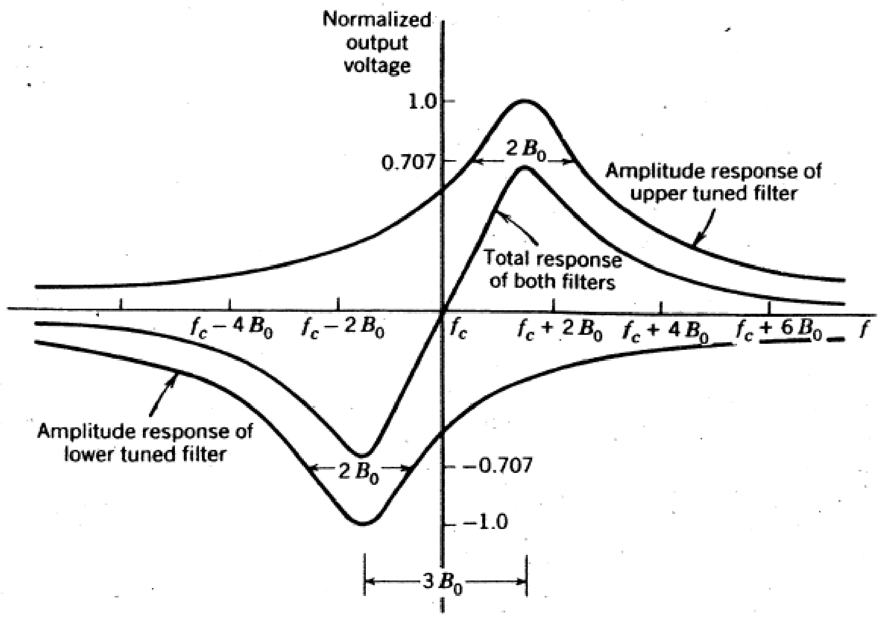


Fig ©: Frequency response

* The balanced Slope detector Consists of Two Slope detector Ckts.
* The I/p transformer has a center tapped Secondary. Hence the I/p voltages to the two Slope detectors are 180° out of phase.

* There are 3 tuned Ckts
  i) The primary is tuned to IF i.e. f_c.
  ii) The upper tuned Ckt of the Secondary (T_1) is tuned above f_c by Δf i.e. its resonant frequency is $f_c + \Delta f$.
  iii) The lower tuned Ckt of the Secondary (T_2) is tuned below f_c by Δf i.e. its resonant frequency is $f_c - \Delta f$

* R_1C_1 & R_2C_2 are the filter Ckts.
* V_{01} & V_{02} are the o/p voltages of the two Slope detectors.
* The final o/p voltage V_0 is obtained by taking the difference

of the Individual o/p voltages V_{01} & V_{02}.

i.e. $\boxed{V_0 = V_{01} - V_{02}}$

operation of the Ckt :-

We can understand the operation by dividing the I/p frequency into three ranges as follows:

i) $f_{in} = f_c$:-

When I/p frequency is equal to carrier freq 'f_c', the Induced voltage in the T_1 winding of Secondary is exactly equal to that Induced in the winding T_2.

Thus the I/p voltages to both the diodes D_1 & D_2 will be Same.

∴ The dc o/p voltages V_{01} & V_{02} will also be Identical but they have opposite polarities hence $V_0 = 0V$.

ii) $f_{in} > f_c$:-

$\boxed{f_{in} > f_c \cdot \uparrow (f_c + \Delta f)}$ i.e. $f_{in} \simeq f_c + \Delta f$

When I/p frequency is greater than 'f_c', the Induced voltage in 'T_1' winding is higher than that Induced in 'T_2'.

∴ The I/p to D_1 is higher than D_2. So +ve o/p V_{01} (of D_1) is higher than the -ve o/p V_{02} (of D_2).

Thus o/p voltage V_0 is positive. (The +ve o/p voltage increases as the I/p frequency increases towards $f_c + \Delta f$.)

iii) $f_{in} < f_c$:-

$\boxed{\text{i.e. } f_{in} \simeq f_c - \Delta f}$

When I/p frequency is less than 'f_c', the Induced voltage in 'T_2' winding is higher than in 'T_1', So I/p voltage to diode D_2 is higher than that of D_1.

Hence the -ve o/p 'V_{o2}' is greater than V_{o1}.

∴ The o/p voltage of the balanced Slope detector is -ve in this frequency range. { The -ve o/p voltage increases as f_{in} goes closer to '$f_c - \Delta f$' }

$$\therefore \boxed{V_o = \begin{cases} 0, & f_{in} = f_c \\ +ve, & f_{in} > f_c \\ -ve, & f_{in} < f_c \end{cases}}$$

Advantages :-

1) This ckt is more efficient than Simple Slope detector.

2) It has better linearity than the Simple Slope detector.

Disadvantages :-

1) This ckt is difficult to tune Since the three tuned ckts are to be tuned at different frequencies i.e. f_c, $(f_c + \Delta f)$, $(f_c - \Delta f)$.

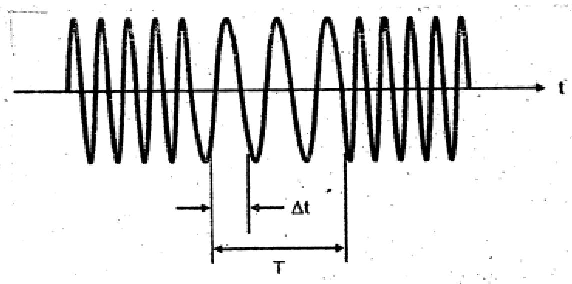
2) Amplitude limiting is not provided.

Zero – Crossing Detector:-

❖ **Explain FM demodulation using Zero crossing detector** **July-06,8M Jan-05,7M**

* The Zero Crossing detector operates on the principle that the instantaneous frequency of an FM wave is approximately given by $\boxed{f_i \simeq \frac{1}{2\Delta t}}$

Where,
Δt is the time difference b/w adjacent Zero Crossing of the FM wave as shown in fig ①.



Definitions of T and Δt for an FM wave

* The time interval 'T' is chosen in accordance with the following two conditions:

1) The interval 'T' is small compared to the reciprocal of the message bandwidth 'W' i.e. $\left(\frac{1}{W}\right)$

2) The interval 'T' is large compared to the reciprocal of the carrier frequency 'f_c' of the FM wave i.e. $(1/f_c)$.

* Let 'n_0' denote the number of Zero Crossings inside the interval 'T'. Hence Δt is the time between the adjacent Zero Crossing points given by $\boxed{\Delta t = \frac{T}{n_0}}$

$\therefore$ Instantaneous frequency is given by

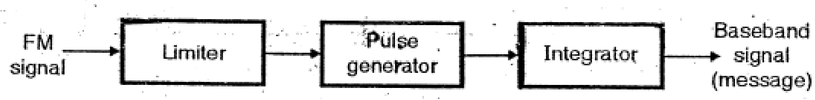
$$f_i \approx \frac{1}{2\Delta t}$$

$$f_i \simeq \frac{1}{2\frac{T}{n_0}}$$

$$\boxed{f_i \simeq \frac{n_0}{2T}}$$

* By the definition of Instantaneous frequency, W.K.T there is a linear relation b/w f_i & message signal $m(t)$. Hence we can recover $m(t)$ if n_0 is known.

* The simplified block diagram of the zero crossing detector based on this principle is shown below.



Block diagram of zero crossing detector

Phase Locked Loop:-

- **Starting from block diagram of PLL obtain its non-linear and linear model. Show that o/p of PLL is scaled version modulating signal** **June-10,12M**
- **With relevant analysis, explain the FM demodulation, using PLL** **Jan-10,10M**
- **Explain how first order PLL can be used for FM detection** **June-10,8M**
- **Explain with relevant mathematical expression the demodulation of a FM signal using PLL.** **June-09,10M**

* PLL is a -ve Feedback System that Consists of three major Components
  i) A multiplier
  ii) A loop filter
  iii) A voltage Controlled oscillator (VCO)

Connected in the form of a feedback loop as Shown in fig ①.

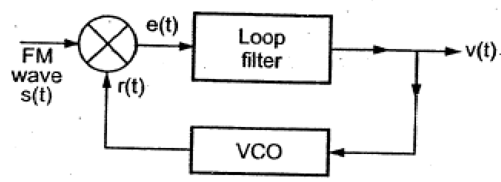


Fig ①: Block diagram of PLL

* The VCO is a Sine-wave generator whose frequency depends on the I/p Control voltage.

{ Any frequency modulator may Serve as a VCO. }

* Initially assume that VCO is adjusted So that when the Control voltage is Zero, 2 Conditions are Satisfied:
  1) The frequency of the VCO is precisely Set at the unmodulated carrier frequency 'f_c' &
  2) The VCO o/p has a 90° phase-Shift w.r.to the unmodulated carrier wave.

* Suppose that the I/p Signal applied to the PLL is an FM wave defined by $s(t) = A_c \sin[2\pi f_c t + 2\pi K_f \int_0^t m(t) \cdot dt]$

$$\boxed{s(t) = A_c \sin[2\pi f_c t + \phi_1(t)]} \longrightarrow ①$$

Where A_c is the carrier amplitude with a modulating wave $m(t)$

We have $\phi_1(t) = 2\pi K_f \int_0^t m(t)\, dt$.

Where K_f is the frequency Sensitivity of the frequency modulator.

Let the VCO o/p be defined as

$$\boxed{r(t) = A_v \cos[2\pi f_c t + \phi_2(t)]} \longrightarrow ②$$

Where 'A_v' is the amplitude of VCO o/p. If the control voltage applied to VCO is $v(t)$ then

$$\boxed{\phi_2(t) = 2\pi K_v \int_0^t v(t).\, dt}$$

Where 'K_v' is the Frequency Sensitivity Constant of the VCO having the unit Hz/v.

* When VCO I/p $v(t)$ equal to Zero i.e. $v(t)=0$, then $\phi_2(t)=0$.

* The Incoming FM wave $S(t)$ & the VCO o/p $r(t)$ are applied to the multiplier.

The o/p of the multiplier is

$$\boxed{e(t) = S(t)\, r(t)} \longrightarrow ③$$

Substituting eq ① & ② in eq ③, we get

$$e(t) = \underbrace{A_c \sin[2\pi f_c t + \phi_1(t)]}_{S(t)} \cdot \underbrace{A_v \cos[2\pi f_c t + \phi_2(t)]}_{r(t)}$$

$$e(t) = A_c A_v \sin[2\pi f_c t + \phi_1(t)] . \cos[2\pi f_c t + \phi_2(t)]$$

W.K.T $\boxed{\sin A . \cos B = \frac{1}{2}\sin[A+B] + \frac{1}{2}\sin[A-B]}$

Put $A = [2\pi f_c t + \phi_1(t)]$ & $B = [2\pi f_c t + \phi_2(t)]$

$$e(t) = \frac{A_c A_v}{2} \sin[2\pi f_c t + \phi_1(t) + 2\pi f_c t + \phi_2(t)] + \frac{A_c A_v}{2} \sin[\cancel{2\pi f_c t} + \phi_1(t) - \cancel{2\pi f_c t} - \phi_2(t)]$$

$$\boxed{e(t) = \frac{A_c A_v}{2} \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] + \frac{A_c A_v}{2} \sin[\phi_1(t) - \phi_2(t)]} \longrightarrow ④$$

$$\boxed{e(t) = K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] + K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]} \rightarrow (5)$$

Where $K_m = \frac{1}{2}$ is the multiplier gain measured in volts.

* Equation (5) is the o/p of the product modulator & it has two components

1) A high-frequency component represented by

$$K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$$

2) A low frequency component represented by

$$K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$

* The high frequency component is eliminated by the LPF.

Thus the I/p to the loop filter is given by:

$$e(t) = K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$

$$\boxed{e(t) = K_m A_c A_v \sin[\phi_e(t)]} \rightarrow (6)$$

Where, $\phi_e(t)$ is the phase error defined by

$$\boxed{\phi_e(t) = \phi_1(t) - \phi_2(t)} \rightarrow (7)$$

$$\phi_e(t) = \phi_1(t) - 2\pi K_v \int_0^t v(t).\, dt$$

* The loop filter operates on its I/p $e(t)$ to produce the o/p

$$v(t) = e(t) * h(t).$$

$$\boxed{v(t) = \int_{-\infty}^{\infty} e(\tau) . h(t-\tau)\, d\tau}$$

* Differentiating eq (7) w.r.to t, we get

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - \frac{d\phi_2(t)}{dt}$$

$$= \frac{d\phi_1(t)}{dt} - \left[\cancel{\frac{d}{dt}}\left(2\pi K_v \cancel{\int_0^t} v(t)\,\cancel{dt}\right)\right]$$

$$= \frac{d\phi_1(t)}{dt} - \left[2\pi K_v v(t)\right] \qquad \because v(t) = e(t) * h(t)$$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - \left[2\pi K_v \left(\underline{e(t)} * h(t)\right)\right] \longrightarrow 8$$

Substituting eq 6 in eq 8, we get

$$= \frac{d\phi_1(t)}{dt} - \left[2\pi K_v\left(\underline{K_m A_c A_v \sin\phi_e(t)} * h(t)\right)\right]$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi K_v K_m A_c A_v \left[\underline{\sin\phi_e(t) * h(t)}\right]$$

$$\boxed{\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin\left[\phi_e(\tau)\cdot h(t-\tau)\,d\tau\right]}$$

Where 'K_0' is a loop parameter defined by

$$K_0 = K_m K_v A_c A_v$$

Equation necessary for developing the block diagram of PLL:-

W.K.T $\quad \phi_e(t) = \phi_1(t) - \phi_2(t) \rightarrow 1$

Where, $\quad \phi_1(t) = 2\pi K_f \int_0^t m(t).\,dt$

$$\phi_2(t) = 2\pi K_v \int_0^t V(t).\,dt$$

Differentiating $\phi_2(t)$ w.r.to 't' we get

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \cancel{\frac{d}{dt}} \cancel{\int_0^t} V(t).\,\cancel{dt}$$

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \cdot V(t) \longrightarrow ②$$

W.K.T

$$\boxed{V(t) = e(t) * h(t)}$$

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \left[e(t) * h(t) \right] \longrightarrow ③$$

W.K.T

$$\boxed{e(t) = K_m A_c A_v \sin\phi_e(t)}$$

Substituting $e(t)$ in eq ③, we get

$$\frac{d\phi_2(t)}{dt} = 2\pi K_v \left[K_m A_c A_v \sin\phi_e(t) * h(t) \right]$$

$$\boxed{\frac{d\phi_2(t)}{dt} = 2\pi K_0 \sin\phi_e(t) * h(t)} \longrightarrow ④$$

Where, $K_0 = K_v K_m A_c A_v$

From equation ②, we can write

$$V(t) = \frac{1}{2\pi K_v} \frac{d\phi_2(t)}{dt} \longrightarrow ⑤$$

Substituting eq ⑤ in eq ④ we get

$$\boxed{V(t) = \frac{1}{2\pi K_v} 2\pi K_0 \sin\phi_e(t) * h(t)}$$

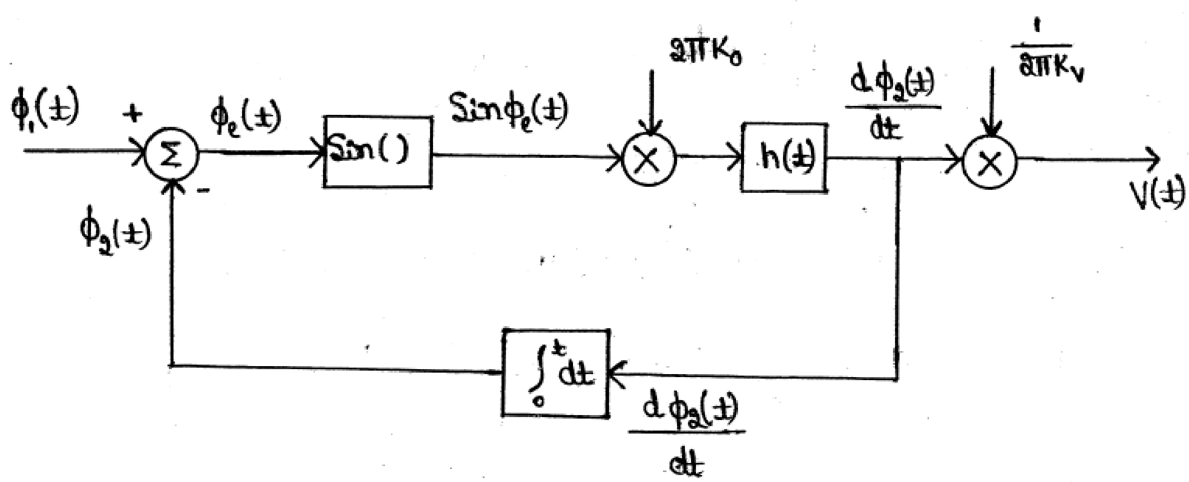


Fig ②: Non-linear model of PLL

* We can observe that the multiplier of Fig ① is replaced by a Sinusoidal non linearity & the VCO by an Integrator because of the Sinusoidal Non linearity, the above representation is known as the non-linearity representation of PLL.

Linearized PLL:- **June-07,5M**

Linearized Model :-

* When the phase error $\phi_e(t)$ is zero, the PLL is said to be in phase-locked.

* When $\phi_e(t)$ is very small compared to 0.5 radian, at all times, we may use the following approximation.

$$\boxed{\sin \phi_e(t) \approx \phi_e(t)}$$

Thus Fig ② reduces to Fig ③.

Fig ③ is known as the Linearized model of PLL.

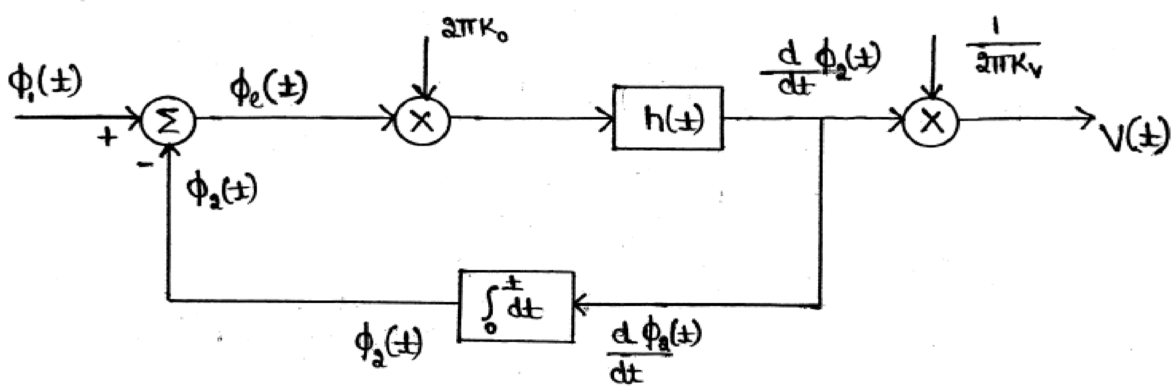


Fig ③: Linear model of PLL.

From Fig ③, we have

$$\phi_e(t) = \phi_1(t) - \phi_2(t) \longrightarrow ①$$

$$\phi_1(t) = \phi_e(t) + \phi_2(t) \longrightarrow (2)$$

differentiating both Sides of eq (2), we get

$$\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + \underline{\frac{d}{dt}\phi_2(t)}$$

$$\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + 2\pi K_0 \underline{\sin \phi_e(t)} * h(t)$$

$\because \frac{d}{dt}\phi_2(t) = 2\pi K_0 \sin\phi_e(t) * h(t)$ & W.K.T, $\sin\phi_e(t) \approx \phi_e(t)$

$$\boxed{\frac{d}{dt}\phi_1(t) = \frac{d}{dt}\phi_e(t) + 2\pi K_0 \phi_e(t) * h(t)} \longrightarrow (3)$$

Taking Fourier transform on both Sides of the eq (3), we get

$$j2\pi f \phi_1(f) = j2\pi f\, \phi_e(f) + 2\pi K_0 \phi_e(f) \cdot H(f)$$

{ <u>NOTE :-</u>

$$\frac{d}{dt}\phi_1(t) \xrightarrow{FT} j2\pi f \phi_1(f)$$

$$\frac{d}{dt}\phi_e(t) \xrightarrow{FT} j2\pi f \phi_2(f)$$

$$\phi_e(t) * h(t) \xrightarrow{FT} \phi_e(f) \cdot H(f)$$

}

$$\cancel{j2\pi f}\, \phi_1(f) = \cancel{j2\pi f}\left[\phi_e(f) + \frac{1}{jf} K_0\, \phi_e(f) \cdot H(f)\right]$$

$$\phi_1(f) = \phi_e(f) + \frac{K_0 H(f)}{jf} \cdot \phi_e(f)$$

Let $L(f) = \dfrac{K_0 H(f)}{jf}$

Then,

$$\phi_1(f) = \phi_e(f) + L(f) \cdot \phi_e(f)$$

$$\phi_1(f) = \phi_e(f)\left[1 + L(f)\right]$$

$$\boxed{\phi_e(f) = \frac{\phi_1(f)}{1+L(f)}} \longrightarrow (4)$$

Where $L(f)$ is called the open loop transfer function of the PLL.

W.K.T

$$V(t) = \frac{1}{2\pi K_v} \cdot 2\pi K_0 \underline{\sin\phi_e(t)} * h(t)$$

$\because$ $\boxed{\sin\phi_e(t) \approx \phi_e(t)}$

$$\boxed{V(t) = \frac{1}{2\pi K_v} 2\pi K_0 \phi_e(t) * h(t)} \longrightarrow (5)$$

Eq (5) is the o/p of the PLL.

* In frequency domain the o/p of the PLL is given

$$V(f) = \frac{1}{\cancel{2\pi} K_v} \cancel{2\pi} K_0 \phi_e(f) \cdot H(f)$$

$$V(f) = \frac{K_0}{K_v} \cdot \underline{\phi_e(f)} \cdot H(f) \longrightarrow (6)$$

Substituting eq (4) in eq (6), we get

$$V(f) = \frac{K_0}{K_v} \cdot \frac{\phi_1(f)}{[1+L(f)]} \cdot H(f)$$

$\because$ $\boxed{\phi_e(f) = \frac{\phi_1(f)}{1+L(f)}}$

Since $L(f) \gg 1$, we can write $1+L(f) \approx L(f)$

hence, $V(f) = \frac{K_0}{K_v} \frac{\phi_1(f)}{L(f)} H(f)$

$\because$ $\boxed{L(f) = \frac{K_0 H(f)}{jf}}$

$$V(f) = \frac{\cancel{K_0}}{K_v} \frac{\phi_1(f)}{\frac{\cancel{K_0 H(f)}}{jf}} \cdot \cancel{H(f)}$$

$$\boxed{V(f) = \frac{1}{K_v} \cdot jf\, \phi_1(f)} \longrightarrow (7)$$

$\times$lying & $\div$ing RHS of eq (7) by 2π

$$V(f) = \frac{1}{2\pi K_v} \cdot j2\pi f \phi_1(f)$$

W.K.T $\frac{d}{dt}\phi_1(t) \xrightarrow{FT} j2\pi f \phi_1(f)$

hence,
$$V(f) = \frac{1}{2\pi K_v} \frac{d}{dt}\phi_1(t) \longrightarrow ⑧$$

Substituting $\phi_1(t) = 2\pi K_f \int_0^t m(t).\, dt$ in eq ⑧, we get

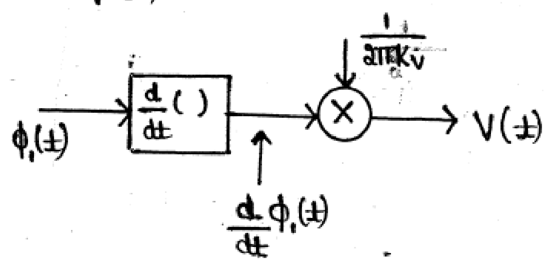
$$V(f) = \frac{1}{\cancel{2\pi} K_v} \cancel{\frac{d}{dt}}\left[\cancel{2\pi} K_f \cancel{\int_0^t} m(t).\, \cancel{dt}\right]$$

$$\boxed{V(f) = \frac{K_f}{K_v} m(t)}$$

The Corresponding time-domain relation of eq ⑧ is

$$\boxed{V(t) = \frac{1}{2\pi K_v} \frac{d}{dt}\phi_1(t)} \longrightarrow ⑨$$

From eq ⑨, we can write



Linearized Model :- | From Godse - Book

* When the phase error $\phi_e(t)$ is Zero, the PLL is Said to be in phase - locked.

* When $\phi_e(t)$ is very Small Compared to 0.5 radians, at all times, we may use the following approximation.

$$\boxed{\sin\phi_e(t) \approx \phi_e(t)}$$

Thus fig ② reduces to fig ③.

Fig ③ is Known as the Linearized model of PLL.

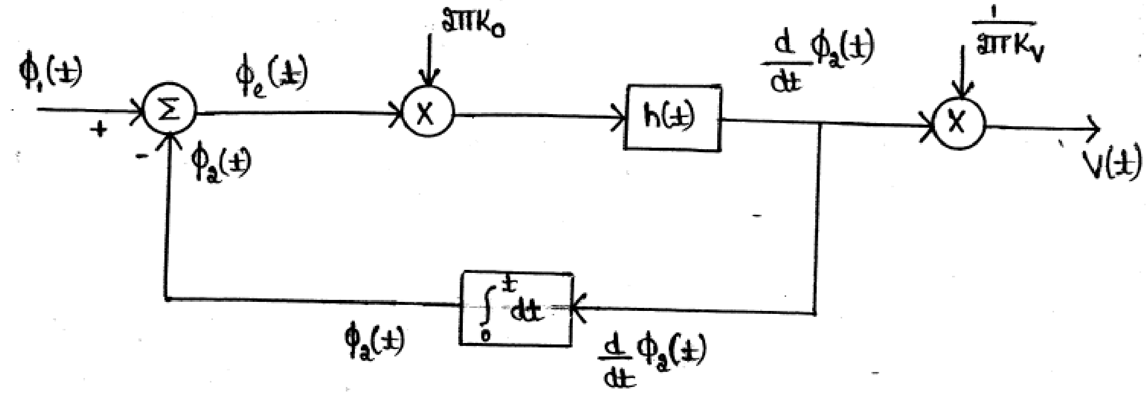


Fig ③: Linear model of PLL

W.K.T

$$\phi_1(t) = 2\pi K_f \int_0^t m(t) \cdot dt \quad \&$$

$$\phi_2(t) = 2\pi K_v \int_0^t V(t) \cdot dt.$$

From fig ③, We have

$$\phi_e(t) = \phi_1(t) - \phi_2(t). \longrightarrow ①$$

Assuming Small error So $\phi_e(t) \approx 0$

$$0 = \phi_1(t) - \phi_2(t)$$

$$\phi_2(t) = \phi_1(t) \longrightarrow ②$$

$$\cancel{2\pi} K_v \int_0^t V(t) \cdot dt = \cancel{2\pi} K_f \int_0^t m(t)\, dt \longrightarrow ③$$

Differentiating both Sides of eq ③ w.r.to time 't'

$$K_v V(t) = K_f m(t)$$

$$\boxed{V(t) = \frac{K_f}{K_v} m(t)}$$

Non- Linear effects in FM:-

❖ **Explain the non linearity and its effect in FM systems** **Jan-09,6M July-06,6M**

* Non-linearities are present in all electrical networks. Consider a communication channel having a non-linear transfer characteristic is given by

$$\boxed{V_o(t) = a_1 V_i(t) + a_2 V_i^2(t) + a_3 V_i^3(t)} \longrightarrow (1)$$

Where,

$V_i(t) \longrightarrow$ I/p Signal

$V_o(t) \longrightarrow$ O/p Signal

$a_1, a_2 \& a_3 \longrightarrow$ Constants

* Let the I/p to the channel be an FM wave given by

$$V_i(t) = A_c \cos[2\pi f_c t + \phi(t)] \longrightarrow (2)$$

Where, $\phi(t) = 2\pi K_f \int_0^t m(t).dt$

Substituting eq (2) in eq (1), we get

$$V_o(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + a_2 A_c^2 \cos^2[2\pi f_c t + \phi(t)] + a_3 A_c^3 \cos^3[2\pi f_c t + \phi(t)]$$

W.K.T

$$\boxed{\begin{aligned} \cos^2\theta &= \frac{1}{2} + \frac{\cos 2\theta}{2} \\ \cos^3\theta &= \frac{3\cos\theta}{4} + \frac{\cos(3\theta)}{4} \end{aligned}}$$

$$\boxed{\theta = [2\pi f_c t + \phi(t)]}$$

{

$$a_2 A_c^2 \cos^2[2\pi f_c t + \phi(t)] = \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \cos[4\pi f_c t + 2\phi(t)]$$

$$a_3 A_c^3 \cos^3[2\pi f_c t + \phi(t)] = \frac{3a_3 A_c^3}{4} \cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} \cos[6\pi f_c t + 3\phi(t)]$$

}

$$V_0(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} \cos[4\pi f_c t + 2\phi(t)]$$

$$+ \frac{a_3 A_c^3}{4} 3\cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} \cos[6\pi f_c t + 3\phi(t)] \longrightarrow (3)$$

* The equation (3) indicates that the channel o/p consists of a DC component & three frequency modulated signals with carrier - frequencies f_c, $2f_c$ & $3f_c$.

* The required FM wave centered at f_c is obtained by passing '$V_0(t)$' through a BPF.

* The o/p of the BPF is

$$V_0(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + \frac{a_3 A_c^3}{4} 3\cos[2\pi f_c t + \phi(t)]$$

$$\boxed{V_0(t) = \cos[2\pi f_c t + \phi(t)] \left(a_1 A_c + \frac{3}{4} a_3 A_c^3\right)} \longrightarrow (4)$$

* Equation (4) reveals that $V_0(t)$ is the original FM signal except for the change in amplitude. Thus, amplitude non-linearities of the channel does not affect an FM signal (unlike in amplitude modulation).

* For this reason, FM is widely used in Microwave & Satellite Communication.

Compare FM with AM and PM

June-06,6M

Sl No	FM	AM
1)	The equation for FM wave is: $S(t)_{FM} = A_c \sin[\omega_c t + \beta \sin\omega_m t]$	The equation for AM wave is $S(t)_{AM} = A_c[1 + \mu \sin\omega_m t]\sin\omega_c t$.
2)	The modulation Index can - have any value i.e. less than 1 or more than 1.	The modulation Index is always in between 0 and 1.
3)	All the transmitted power is useful	Carrier power and one Sideband power are useless.
4)	$P = \frac{A_c^2}{2R}$	$P_T = P_c[1 + \mu^2/2]$
5)	The modulation Index determines the number of Sidebands in an FM Signal	In AM, only two Sidebands are produced, irrespective of the modulation Index.
6)	$BW = 2[\Delta f + f_m]$. The BW depends on modulation Index	$BW = 2f_m$. The BW does not depends on modulation Index
7)	For FM, $\text{\% Modulation} = \frac{\text{Actual freq deviation}}{\text{Max allowed freq deviation}} \times 100$	For AM, $\text{\% modulation} = \frac{A_m}{A_c} \times 100$
8)	The main advantage of FM over AM is its noise Immunity.	The AM System is more - Susceptible to Noise & more affected by Noise than FM.

Sl No	FM	AM
9)	The BW required to transmit FM Signal is much larger than the BW of AM (i.e $\approx$ 200KHz)	The BW required to transmit AM Signal is much less than that of FM (i.e. $\approx$ 10KHz)
10)	FM transmission & reception equipment are more Complex	AM equipments are less Complex.
11)	FM transmission is expensive than AM transmission	AM transmission is Cheaper than FM transmission.
12)	Used for Short distance Comm	Used for long distance Comm

Sl No	FM	PM
1)	The equation for FM Wave is $S(t)_{FM} = A_c \cos[W_c t + 2\pi K_f m(t)]$	The equation for PM Wave is $S(t)_{PM} = A_c \cos[W_c t + K_p m(t)]$
2)	Amplitude of FM Wave is Constant	Amplitude of PM Wave is Constant
3)	Frequency deviation is proportional to modulating voltage.	phase deviation is proportional to the modulating voltage.
4)	The modulation Index of an FM Signal is the ratio of the frequency deviation to the modulating frequency.	The modulation Index is proportional to the maximum amplitude of the modulating Signal.
5)	Noise Immunity is better than AM & PM	Noise Immunity is better than AM but worse than FM.

6)	Signal to Noise ratio is better than that of PM	Signal to Noise ratio is Inferior to that of FM.
7)	FM is widely used	PM is used in Some mobile - Systems
8)	It is possible to receive FM on a PM receiver.	It is possible to receive PM on a FM receiver.
9)	FM is direct method of producing FM Signal	PM is Indirect method of producing FM.
10)	Noise is better Suppressed in FM Systems as Compared to PM System	Noise Immunity is Inferior to that of FM.
11)	To have better quality of - transmission & reception of higher audio frequencies, pre-emphasis & de-emphasis Circuits are used.	The amount of frequency Shift produced by a phase modulator increases with the modulating frequency. Hence an audio equalizer is required to - compensate this.
12)	FM is mainly used for FM broadcasting. i.e. Entertainment purpose	PM is used in mobile Comm System

FM FORMULAE

1. **Carrier Frequency**	$W_c = 2\pi f_c$,	$f_c = W_c / 2\pi$
2. **Modulating Frequency**	$W_m = 2\pi f_m$,	$f_m = W_m / 2\pi$
3. **Modulation Index (β or m_f)**	$\beta = \Delta f / f_m$	
4. **Power dissipation**	$P = A^2{}_c / 2R$	
5. **Frequency deviation**	$\Delta f = K_f A_m$ $\Delta f = \beta f_m$	
6. **Frequency sensitivity**	$K_f = \Delta f / A_m$	
7. **Deviation ratio**	$D = \Delta f_{max} / f_{max}$	
8. **Highest frequency reached**	$(f_i)_{max} = f_c + \Delta f$	
9. **Lowest frequency reached**	$(f_i)_{min} = f_c - \Delta f$	
10. **Carrier Swing**	$(f_i)_{max} - (f_i)_{min}$	
11. **Carrier Swing**	$2 x \Delta f$	
12. **Frequency deviation**	$\Delta f = \text{Carrier Swing} / 2$ $\Delta f = (f_i)_{max} - f_C$	
13. **Bandwidth** (**Carson rule**)	$BW = 2(\Delta f + f_m)$ or $BW = 2\Delta f\,(1 + 1/\beta)$	
14. **Message signal**	$S(t) = A_c \cos[W_{ct} + \beta \sin W_m t]$ $S(t) = A_c \sin[W_{ct} + \beta \sin W_m t]$ $S(t) = A_c \cos[2\pi f_{ct} + 2\pi k_f \int m(t)\, dt]$ $S(t) = A_c \cos[W_{ct} + 2\pi k_f \int m(t)\, dt]$ $S(t) = A_c \cos[W_{ct} + 2\pi k_f\, m(t)]$	

PM FORMULAE

1. **Phase deviation**	$\Delta P = K_p A_m f_m$
2. **Bandwidth**	$BW = 2(\Delta f + f_m)$ or
(Carson rule)	$BW = 2\Delta f\,(1+1/\,\beta)$
3. **Message signal**	$S(t) = A_c \cos[W_c t + K_p m(t)]$
	$S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$

1. The equation for a FM wave is $S(t) = 10\cos[5.7\times10^8 t + 5\sin(12\times10^3)t]$.

Calculate. i. Carrier frequency ii. Modulating frequency

iii. Modulation index iv. Frequency deviation

v. Power dissipated in a 100Ω resistor load.

June-10,6M July-09,5M (old) June-08,10M June-06,6M

Sol:-

$$S(t) = 10\cos[5.7\times10^8 t + 5\sin(12\times10^3)t] \longrightarrow ①$$

Compare eq ① with Standard equation for FM

$$S(t) = A_c\cos[\omega_c t + \beta\sin\omega_m t] \longrightarrow ②$$

$A_c = 10V$, $\omega_c = 5.7\times10^8$, $\beta = 5$ & $\omega_m = 12\times10^3$

i) Carrier Frequency $f_c = \frac{\omega_c}{2\pi} = \frac{5.7\times10^8}{2\pi}$

$$\boxed{f_c = 90.7183\ MHz}$$

ii) Modulating Frequency $f_m = \frac{\omega_m}{2\pi} = \frac{12\times10^3}{2\pi}$

$$\boxed{f_m = 1.909\ KHz}$$

iii) Modulation Index

$$\boxed{\beta = 5}$$

iv) Frequency deviation $\Delta f = \beta f_m = 5\times1.909\ KHz$

$$\boxed{\Delta f = 9.545\ KHz}$$

v) power dissipated in a 100Ω resistor load

$$P = \frac{A_c^2}{2R} = \frac{10^2}{2\times100}$$

$$\boxed{P = 0.5\ W}$$

2. A FM signal has sinusoidal modulation with f_m=15KHz and modulation index β=2. Using carson's rule, find the transmission bandwidth and deviation ratio. Assume Δf =75 KHz. **June-10,6M**

Given :-

$f_m = 15 KHz$, $\beta = 2$, $\Delta f = 75 KHz$

BW = ? & Deviation ratio 'D' = ?

* $BW = 2(\Delta f + f_m) = 2(75KHz + 15KHz) = \underline{180 KHz.}$

* $D = \frac{\Delta f}{f_m} = \frac{75KHz}{15KHz} = \underline{5}$

---

3. A sinusoidal modulating voltage of amplitude 5V and frequency 1 KHz is applied to frequency modulator. The frequency sensitivity of modulator is 40 Hz/V. The carrier frequency is 100KHz. Calculate i. Frequency deviator ii. Modulation index **June-10,5M**

Given :- $A_m = 5V$, $f_m = 1KHz$, $K_f = 40 Hz/V$ & $f_c = 100KHz$.

i) Frequency deviator $\Delta f = K_f A_m = 40 \times 5 = \underline{200 Hz}$

ii) Modulation Index 'β' $= \frac{\Delta f}{f_m} = \frac{200}{1000} = \underline{0.2}$

---

4. A sinusoidal modulating waveform of amplitude 10V and a frequency of 1 KHz is applied to FM generator that has a frequency sensitivity constant of is 40 Hz/V. Determine the i. Frequency deviation and ii. Modulation index **Jan-10,4M**

Given : $A_m = 10V$, $f_m = 1KHz$, $K_f = 40 Hz/V$.

i) Frequency deviation $\Delta f = K_f A_m = 40 \times 10 = 400\,Hz$

ii) Modulation Index $\beta = \frac{\Delta f}{f_m} = \frac{400\,Hz}{1\,KHz} = 0.4$

---

5. A carrier wave of 100 MHz is frequency modulated by a 100 KHz sinewave of amplitude 20V, the sensitivity of the modulator is 25 KHz/V.

i. Determine the frequency deviation and bandwidth of the modulated signal using Carson's rule.

ii. Repeat your calculation for PM wave, assume $k_p = k_f$ **June-10,6M(IT)**

Given: $f_c = 100\,MHz$, $f_m = 100\,KHz$, $A_m = 20V$, $K_f = 25\,KHz/V$.

i) $BW = 2[\Delta f + f_m]$

$\Delta f = K_f A_m = 25\,KHz \times 20 = \underline{500\,KHz}$

$BW = 2[500\,KHz + 100\,KHz]$

$\boxed{BW = 1200\,KHz}$

OR

$BW = 2f_m(1+\beta)$

$\beta = \frac{500\,KHz}{100\,KHz} = \underline{5}$

$BW = 2 \times 100\,KHz(1+5)$

$\boxed{BW = 1200\,KHz}$

ii) Assuming that $K_p = K_f$ for PM wave

$\Delta f = K_p A_m f_m = 25\,KHz \times 20 \times 100\,KHz$

$\boxed{\Delta f = 50000\,KHz}$

---

6. A single tone FM signal is given by: $S(t) = 10 \sin[16\pi \times 10^6 t + 20 \sin 2\pi \times 10^3)t]$.

Calculate. i. Modulation index **ii. Modulation Frequency**

iii. Frequency deviation **iv. Carrier frequency**

v. Power of the FM signal. **Jan-09,8M**

Sol :- $S(t) = 10 \sin[16\pi \times 10^6 t + 20 \sin 2\pi \times 10^3 t] \longrightarrow ①$

Comparing eq ① with Standard equation for FM

$$S(t) = A_c \sin[\omega_c t + \beta \sin \omega_m t] \longrightarrow ②$$

we get, $A_c = 10V$, $\omega_c = 16\pi \times 10^6$, $\omega_m = 2\pi \times 10^3$, $\beta = 20$

i) Modulation Index $\beta = 20$.

ii) Modulating frequency $f_m = \frac{\omega_m}{2\pi} = \frac{2\pi \times 10^3}{2\pi} = 1KHz$

iii) Frequency deviation $\Delta f = \beta f_m = 20 \times 1 \times 10^3 = 20KHz$

iv) Carrier frequency $f_c = \frac{\omega_c}{2\pi} = \frac{16\pi \times 10^6}{2\pi} = 8MHz$

v) Power 'p' $= \frac{A_c^2}{2R} = \frac{10^2}{2R} = \frac{50}{R} W$

---

7. An angle modulated signal is defined by S(t) = 10 sin[$2\pi \times 10^6 t$ + 0.2 sin(2000π)t] volts. Find the following:

i. Power in the modulated signal **ii. Frequency deviation**

iii. Phase deviation **iv. Approximate transmission bandwidth.**

Jan-08,10M

Given :- $S(t) = 10 \cos[2\pi \times 10^6 t + 0.2 \sin(2000\pi t)] \longrightarrow ①$

Comparing eq ① with Standard equation for FM

$$S(t) = A_c \cos[\omega_c t + \beta \sin \omega_m t] \longrightarrow ②$$

We get, $A_c = 10V$, $\beta = 0.2$, $\omega_m = 2000\pi$, $\omega_c = 2\pi \times 10^6$

* $f_m = \frac{\omega_m}{2\pi} = \frac{2000\pi}{2\pi} = 1KHz$

* $f_c = \frac{\omega_c}{2\pi} = \frac{2\pi \times 10^6}{2\pi} = 1MHz$

i) $P = \frac{A_c^2}{2R} = \frac{10^2}{2 \cdot R} = \frac{50}{R} W$

ii) $\Delta f = \beta f_m = 0.2 \times 1000 = \underline{200Hz}$

iii) Phase deviation $\Delta\theta = \beta = \frac{\Delta f}{f_m} = \frac{200Hz}{1KHz} = \underline{0.2}$

iv) $BW = 2(\Delta f + f_m) = 2(200 + 1000) = \underline{2400Hz}$

OR

$BW = 2\Delta f\left(1 + \frac{1}{\beta}\right) = 2 \times 200\left(1 + \frac{1}{0.2}\right) = \underline{2400Hz}$

---

8. A given angle modulated signal is S(t) given by the equation:
$S(t) = 12\cos(12\pi 10^8 t + 200\cos 2\pi 10^3 t)$. Find its bandwidth.

June-07,5M

Given :-

$$S(t) = 12\cos(12\pi 10^8 t + 200\cos 2\pi 10^3 t) \longrightarrow (1)$$

Compare eq (1) with Standard equation for FM

$$S(t) = A_c \cos(\omega_c t + \beta \sin \omega_m t) \longrightarrow (2)$$

We get, $A_c = 12V$, $\omega_c = 12\pi 10^8$, $\beta = 200$, $\omega_m = 2\pi 10^3$

* $f_m = \frac{\omega_m}{2\pi} = \frac{2\pi 10^3}{2\pi} = 1KHz$

* $\Delta f = \beta f_m = 200 \times 1KHz = 200KHz$

* $BW = 2(\Delta f + f_m) = 2(200KHz + 1KHz) = 402KHz.$

---

9. A modulated signal $5\cos 2\pi\, 15\times10^3 t$, angle modulates a carrier $A\cos w_c t$. Find the modulation index and the bandwidth for the FM system. Determine the change in the bandwidth and modulation index if f_m is reduced to 5 KHz. What is the conclusion of the two results?
Assume $k_p = k_f = 15KHz/Volt$.

Jan-07,13M

Given :- $A_m = 5V$, $f_m = 15KHz$, $K_p = K_f = 15KHz/V$.

For FM System:

i) Frequency deviation $\Delta f = K_f A_m = 15KHz \times 5 = 75KHz$

ii) Modulation Index $\beta = \frac{\Delta f}{f_m} = \frac{75KHz}{15KHz} = 5$

iii) $BW = 2(\Delta f + f_m) = 2(\Delta f + f_m) = 2(75KHz + 15KHz) = \underline{180KHz.}$

When f_m is reduced to 5KHz i.e. now $\boxed{f_m = 5KHz}$

i) $\Delta f = K_f A_m = 15KHz \times 5 = 75KHz$

ii) $\beta = \frac{\Delta f}{f_m} = \frac{75KHz}{5KHz} = 15$

iii) $BW = 2(\Delta f + f_m) = 2(75KHz + 5KHz) = \underline{160KHz}$

Conclusion:

Bandwidth changes only Slightly with modulating - frequency 'f_m'.

---

10. Find the carrier and modulating frequencies along with modulation index and maximum deviation of the FM wave represented by deviation of the FM wave represented by the voltage equation: $V = 12 \sin(6\times10^8 t + 5 \sin 1250t)$. What power will the FM wave dissipated in a 10Ω resistor? **July-05,5M**

Given :- $S(t) = 12 \sin(6\times10^8 t + 5 \sin 1250t) \longrightarrow ①$

Comparing eq ① with Standard equation for FM

$S(t) = A_c \sin(\omega_c t + \beta \sin \omega_m t) \longrightarrow ②$

We get, $A_c = 12V,\ \omega_c = 6\times10^8,\ \beta = 5,\ \omega_m = 1250$

i) $f_c = \frac{\omega_c}{2\pi} = \frac{6\times10^8}{2\pi} = \underline{95.5MHz}$

ii) $f_m = \frac{\omega_m}{2\pi} = \frac{1250}{2\pi} = \underline{199 Hz}$

iii) $\beta = \underline{5}$

iv) $\Delta f = \beta f_m = 5 \times 199 = \underline{995 \ Hz}$

v) $P = \frac{A_c^2}{2R} = \frac{12^2}{2 \times 100} = \underline{7.2W}$

---

11. An angle modulated signal is described by $S(t) = 10\cos[2\pi(10^6)t + 0.1\sin(10^3)t$. Find the message signal m(t).

i. Considering S(t) is PM with k_p=10.

ii. Considering S(t) is FM with k_f=5. **Jan-05,5M**

Sol:- The equation for PM wave is given by:

$$S(t) = A_c \cos[\omega_c t + K_p m(t)]$$

Comparing this equation with given equation, we have

$$K_p m(t) = 0.1 \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{K_p} \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{10} \sin(10^3)\pi t$$

$$\boxed{m(t) = 0.01 \sin(10^3)\pi t}$$

ii) The equation for FM Wave is given by

$$S(t) = A_c \cos[\omega_c t + 2\pi K_f m(t)]$$

Comparing this equation with given equation, we have

$$2\pi K_f m(t) = 0.1 \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{2\pi K_f} \sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{\cancel{2\pi} \times 5} \sin(10^3)\cancel{\pi} t$$

$$\boxed{m(t) = 0.01 \sin(10^3)\, t}$$

An angle modulated Signal is described by **Aug-2000**

$x_c(t) = 10\cos\left[2\pi(10^6)t + 0.1\sin(10^3)\pi t\right]$ Considering $x_c(t)$ as a PM Signa With $K_p = 10$. Find $m(t)$.

Sol:- The equation For PM wave is given by

$$S(t) = A_c \cos\left[\omega_c t + K_p m(t)\right]$$

Comparing this equation with the given equation, we have

$$K_p m(t) = 0.1\sin(10^3)\pi t$$

$$m(t) = \frac{0.1}{K_p}\sin(10^3)\pi t$$

$$= \frac{0.1}{10}\sin(10^3)\pi t$$

$$\boxed{m(t) = 0.01\sin(10^3)\, t}$$

* In the block diagram shown in Fig. find out the carrier frequency, frequency deviation and modulation index at the points A and B. Assume that at the output of the mixer, the additive frequency component is being selected.

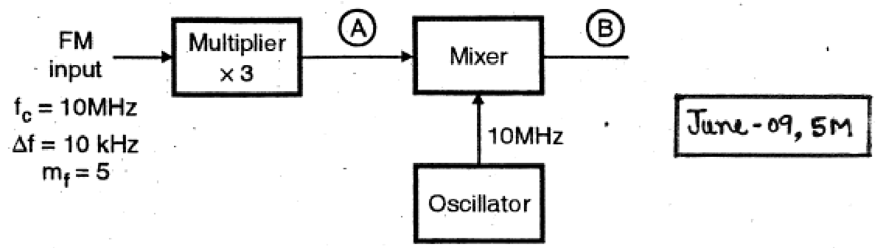


Fig.

Soln. :

(i) At point (A) :

$$\text{The carrier } f_c = 3 \times 10\,\text{MHz} = 30 \text{ MHz.}$$

$$\text{The frequency deviation } \delta = 3 \times 10\,\text{kHz} = 30 \text{ kHz and modulation index } m_f = 3 \times 5 = 15.$$

$$\text{The minimum frequency } f_{min} = 30\,\text{MHz} - 30\,\text{kHz} = 29.970 \text{ MHz}$$

$$\text{The maximum frequency } f_{max} = 30\,\text{MHz} + 30\,\text{kHz} = 30.030 \text{ MHz.}$$

(ii) At point (B) :

$$\text{Carrier frequency } f_c = 30 \text{ MHz} + 10 \text{ MHz} = 40 \text{ MHz.}$$

$$\text{Maximum frequency } f_{max} = 30.03 + 10 = 40.03 \text{ MHz}$$

$$\text{Minimum frequency } f_{min} = 29.970 + 10 = 39.970 \text{ MHz.}$$

As there is no change in deviation due to mixing, the modulation index will remain same i.e. $m_f = 15$.

Determine the bandwidth of FM Signal, if the maximum value of frequency deviation Δf is fixed at 75KHz for commercial FM broadcasting by radio & modulation frequency is W = 15KHz.

$\boxed{\text{Aug - 2001}}$

Sol:- Given : $\Delta f = 75 KHz$

$W = 15 KHz$ or f_m

i) Deviation ratio : $D = \frac{\Delta f}{W} = \frac{75KHz}{15KHz} = 5$

ii) Using Carson rule

$$B_T = 2[1+D]W$$

$$= 2[1+5] 15\times10^3$$

$$B_T = 180KHz$$

$\boxed{OR}$

$$* \quad \beta = D = \frac{\Delta f}{W} = \frac{75KHz}{15KHz} = 5$$

$$* \quad B_T = 2[1+\beta] f_m$$

$$= 2[1+5] 15KHz$$

$$B_T = 180KHz.$$

$\boxed{OR}$

$$* \quad B_T = 2\Delta f + 2f_m$$

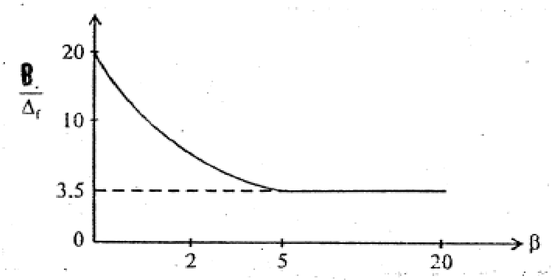
$$= 2(75KHz) + 2(15KHz)$$

$$B_T = 180KHz$$

A carrier wave of 100MHz is frequency modulated by a sine wave of amplitude 20 volts & frequency 100KHz. The frequency sensitivity of modulator is 25 KHz/V. Determine

i) Transmission bandwidth using Carson's rule.

ii) Transmission bandwidth using universal rule (The universal graph is as shown below).



Sol:- Given: $f_c = 100MHz$, $A_m = 20V$, $f_m = 100KHz$, $K_f = 25 KHz/volt$.

ii) Modulation Index

$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{f_m} = \frac{25\times10^3 \times 20V}{100\times10^3} = 5$$

From the given universal graph the value of $\frac{B}{\Delta f}$ for $\beta = 5$ is

$$\frac{B}{\Delta f} = 3.5 \;\&$$

$$\Delta f = K_f A_m = (25\times10^3)(20) = 500KHz$$

$$B = \Delta f \times 3.5$$

$$= 500 KHz \times 3.5$$

$$\therefore \boxed{B = 1750 KHz}$$

i) The transmission BW using Carson's rule is

$$B_T = 2[\Delta f + f_m] = 2[500KHz + 100KHz]$$

$$\boxed{B_T = 1200KHz}$$

Sol:- Given: $f_c = 100MHz$, $f_m = 100KHz$, $A_m = 20V$, $K_f = 25KHz/V$

i) $B_T = 2[\Delta f + f_m]$

$\Delta f = K_f A_m = (25KHz/\cancel{V}) \times 20\cancel{V} = 500KHz$

$B_T = 2[500KHz + 100KHz]$

$\boxed{B_T = 1.2MHz}$

ii) $\beta = \frac{\Delta f}{f_m} = \frac{500KHz}{100KHz} = 5$

From universal curve, for $\beta = 5$, We have $\boxed{\frac{B}{\Delta f} = 3.2}$

$B = \Delta f \times 3.2$

$B = 500KHz \times 3.2$

$\boxed{B = 1.6MHz}$

iii) Modulating voltage is doubled $= 2 \times 20V = 40V$.

* $\Delta f = K_f A_m = (25KHz/\cancel{V})(40\cancel{V}) = 1MHz.$

* BW using Carson's rule

$B_T = 2[\Delta f + f_m]$

$= 2[1MHz + 100KHz]$

$\boxed{B_T = 2.2MHz}$

* $\beta = \frac{\Delta f}{f_m} = \frac{1MHz}{100KHz} = 10$

From universal curve, for $\beta = 10$, We have $\frac{B}{\Delta f} = 3$

A carrier wave frequency 100MHz is frequency modulated by a sinusoidal wave of amplitude 20V & frequency 100KHz. The frequency sensitivity of the modulator is 25KHz per volt.

i) Determine the approximate bandwidth of the FM signal, using Carson's rule.

ii) Determine the bandwidth by transmitting only those side frequencies whose amplitude exceed 1 percent of the unmodulated carrier amplitude. Use the universal curve of fig ① for this calculation.

iii) Repeat the calculation, assuming that the amplitude of the modulating signal is doubled.

iv) Repeat the calculations, assuming the modulation freq is doubled

July - 2008, 8M

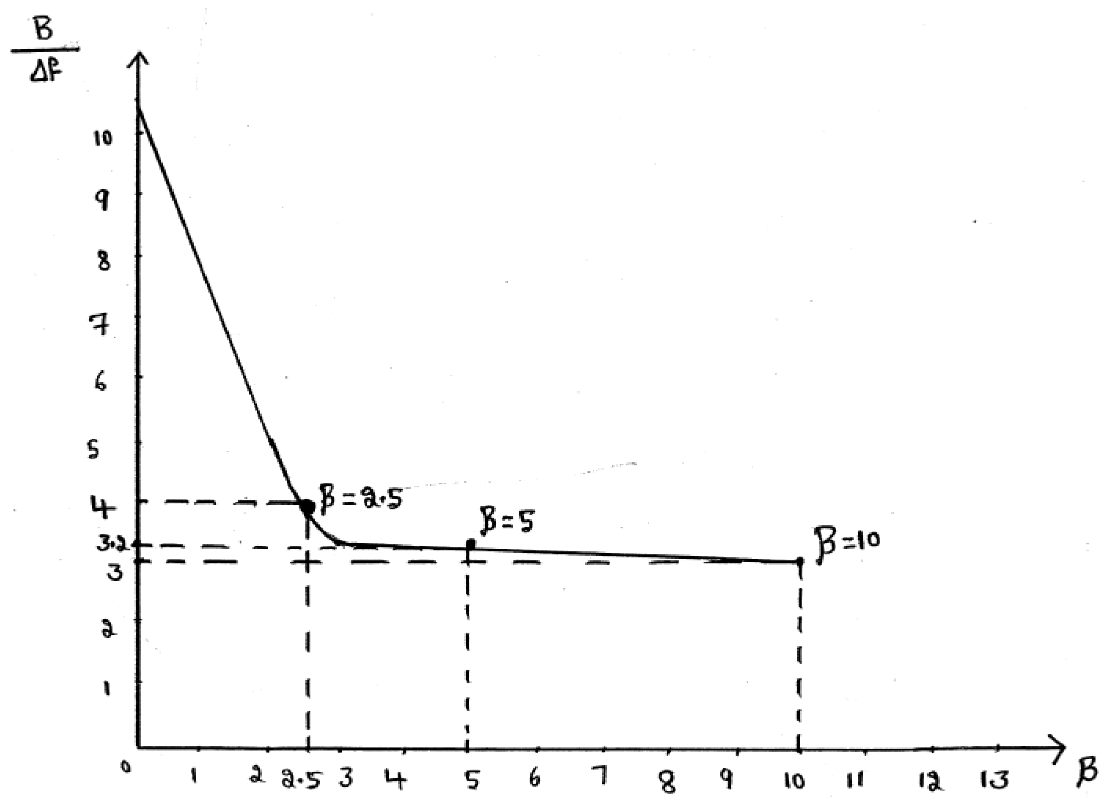


Figure ①: Universal Curve

$$B = \Delta f \times 3 = 1\,MHz \times 3$$

$$\boxed{B = 3\,MHz}$$

iv) Given : $f_m = 200\,KHz$, $A_m = 20V$

* Frequency deviation $\Delta f = K_f A_m$

$$= (25\,KHz/v) \times 20v$$

$$\boxed{\Delta f = 500\,KHz}$$

* Bandwidth using Carson's rule

$$B_T = 2[\Delta f + f_m]$$

$$= 2[500\,KHz + 200\,KHz]$$

$$\boxed{B_T = 1.4\,MHz}$$

$$\beta = \frac{\Delta f}{f_m} = \frac{500\,KHz}{200\,KHz} = 2.5$$

* From universal curve, for $\beta = 2.5$,

We have, $\frac{B}{\Delta f} = 4.0$

$$B = \Delta f \times 4.0$$

$$= 500\,KHz \times 4$$

$$\boxed{B = 2\,MHz}$$

Sketch the variations of the frequency of the resulting FM & PM signal as a function of time where a carrier signal is modulated by a modulating signal $m(t) = \frac{A}{T_0} t \quad 0 \le t \le T_0$, which is periodic with period T_0. Assume the following:

Carrier frequency $f_c = 100\ KHz$, $A = 5$ volts, $T_0 = 1$ msec, $K_p = 0.2\pi$ rad/sec & $K_f = 2KHz/V$.

Derive the equation for the PM & FM signals & draw the relevant block diagram.

$\boxed{\text{Jan - 2006, 11M}}$

Sol:-

i) FM Wave :-

W.K.T the FM wave is given by:

$$S(t) = A_c \cos\left[\omega_c t + 2\pi K_f \int_0^t m(t)\cdot dt\right] \longrightarrow ①$$

$$= A_c \cos\left[2\pi f_c + 2\pi K_f \int_0^t \frac{A}{T_0} t \cdot dt\right]$$

$$= 5 \cos\left[2\pi \times 10^5 t + (2\pi \times 2\times 10^3)\left\{\frac{A}{T_0} \frac{t^2}{2}\right\}\right]$$

$$= 5 \cos\left[2\pi \times 10^5 t + \frac{4\pi \times 10^3}{2}\left(\frac{5}{1\times 10^{-3}} \cdot t^2\right)\right]$$

$$\boxed{S(t) = 5 \cos\left[2\pi \times 10^5 t + 10\pi \times 10^6 t^2\right]} \longrightarrow ②$$

Eq ② is the FM modulated wave for the given values.

* The Instantaneous frequency is given by

$$f_i = f_c + K_f m(t)$$

$$= 100 \times 10^3 + 2\times 10^3 \left(\frac{A}{T_0} t\right)$$

$$= 10^5 + 2\times10^3 \times \frac{5}{1\times10^{-3}} t$$

$$f_i = 10^5 + 2\times10^3 (5000t)$$

$$\boxed{f_i = 10^5 + 10\times10^6 t}$$

at $t = 1$ msec

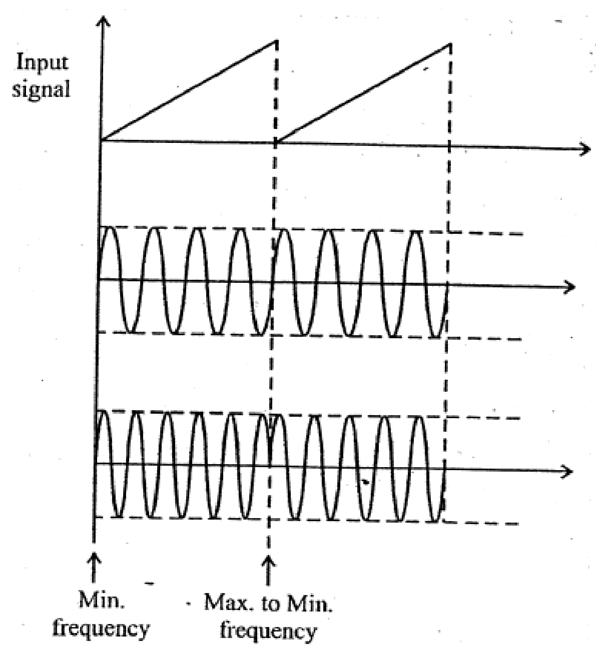
$$f_i = 10^5 + 10\times10^6 (1\times10^{-3})$$

$$\boxed{f_i = 110\text{KHz}}$$

t in msec	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$m(t) = \left(\frac{A}{T_0} t\right)$ in V	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5

Where, $T_0 = 1$ msec

$A = 5V$



WKT the PM Wave is given by

$$S(t) = A_c \cos[\theta(t)]$$

$$S(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

$$S(t) = 5 \cos\left[2\pi \times 10^5 t + 0.2\pi \left(\frac{A}{T_0} t\right)\right]$$

$$S(t) = 5 \cos\left[2\pi \times 10^5 t + 0.2\pi \frac{5}{1\times 10^{-3}} t\right]$$

$$\boxed{S(t) = 5 \cos[2\pi \times 10^5 t + \pi(1\times 10^3) t]} \longrightarrow 3$$

Equation 3 is the modulated equation of PM Wave.

* The Instantaneous Frequency of the phase modulated wave is given b

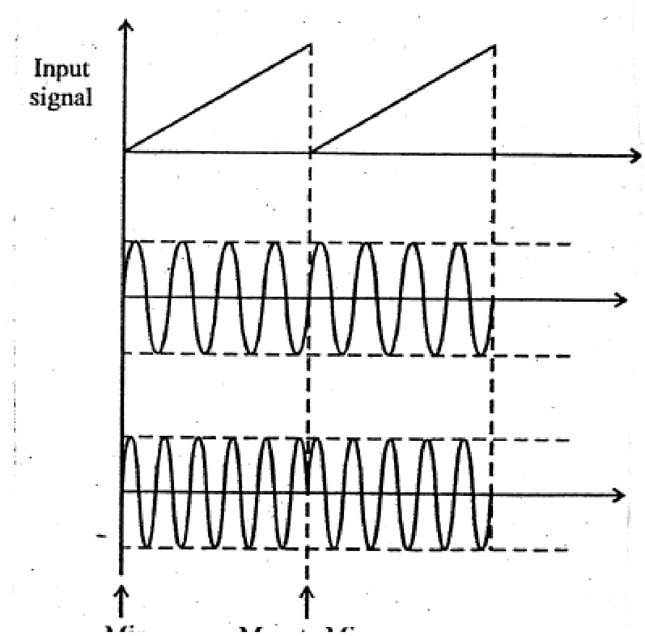
$$\omega_i(t) = \frac{d}{dt}\theta(t) \rightarrow 4$$

From eq 3, $\theta = [2\pi \times 10^5 t + \pi(1\times 10^3) t]$

$$2\pi f_i(t) = \frac{d}{dt}[2\pi \times 10^5 t + \pi(1\times 10^3) t]$$

$$f_i(t) = \frac{1}{2\pi}\left[2\pi \times 10^5 + \frac{1000\pi}{500}\right]$$

$$\boxed{f_i(t) = [1\times 10^5 + 500] \text{ Hz}}$$



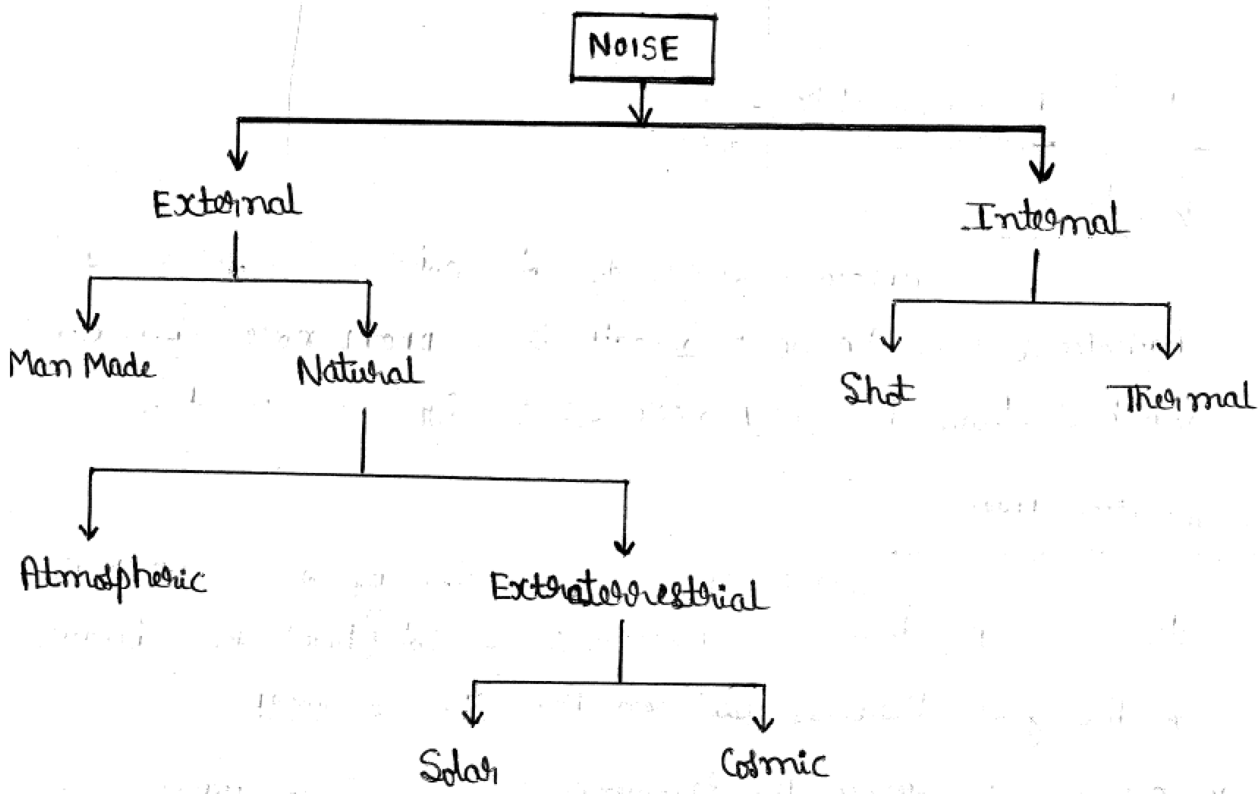
NOISE

NOISE :-

Noise is an unwanted Signal. Noise is random in nature & interferes with the desired Signal.

* Noise disturb the proper reception & reproduction of transmitted Signals.

Classification of Noise :-



PRE-EMPHASIS and DE-Emphasis

- Pre-emphasis: The noise suppression ability of FM decreases with the increase in the frequencies. Thus increasing the relative strength or amplitude of the high frequency components of the message signal before modulation is termed as Pre-emphasis. The Fig3 below shows the circuit of pre-emphasis.

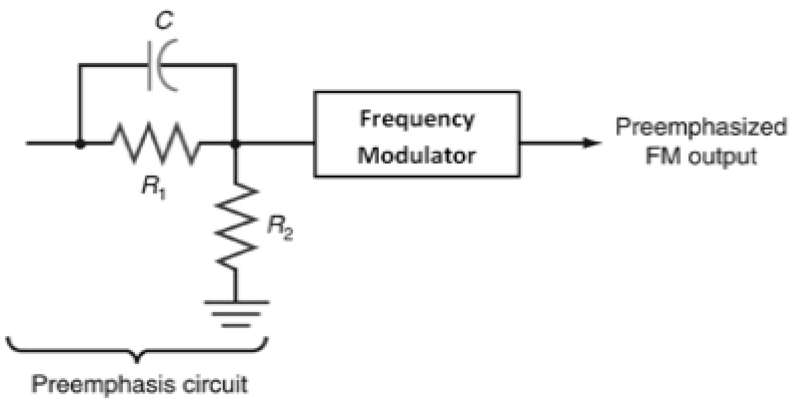


Fig3. Pre-emphasis circuit

- **De-emphasis:** In the de-emphasis circuit, by reducing the amplitude level of the received high frequency signal by the same amount as the increase in pre-emphasis is termed as De-emphasis. The Fig4. below shows the circuit of de-emphasis.

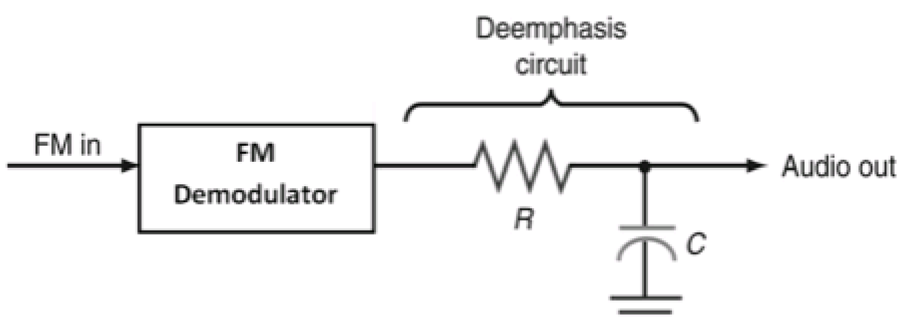


Fig4. De-emphasis circuit

- The pre-emphasis process is done at the transmitter side, while the de-emphasis process is done at the receiver side.
- Thus a high frequency modulating signal is emphasized or boosted in amplitude in transmitter before modulation. To compensate for this boost, the high frequencies are attenuated or de-emphasized in the receiver after the demodulation has been performed. Due to pre-emphasis and de-emphasis, the S/N ratio at the output of receiver is maintained constant.

- The de-emphasis process ensures that the high frequencies are returned to their original relative level before amplification.
- Pre-emphasis circuit is a high pass filter or differentiator which allows high frequencies to pass, whereas de-emphasis circuit is a low pass filter or integrator which allows only low frequencies to pass.