

$$1. \quad \frac{\tilde{d}\rho_a}{dt} = \sum_{b \in A} \frac{m_b}{\rho_b} (\rho_a \tilde{\mathbf{u}}_{ab} + (\rho (\tilde{\mathbf{u}} - \mathbf{u}))_{ab}) \cdot \nabla_a W_{ab}$$

$$2. \quad \frac{\tilde{d}\mathbf{u}_a}{dt} = - \sum_{b \in A} m_b \left[\left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2} \right) \mathbf{I} - \left(\frac{\boldsymbol{\sigma}'_a}{\rho_a^2} + \frac{\boldsymbol{\sigma}'_b}{\rho_b^2} + \Pi_{ab} \mathbf{I} \right) \right] \cdot \nabla_a W_{ab} + \mathbf{g}_a + \frac{1}{m_a} \sum_{b \in B} \mathbf{F}_{a \leftarrow b}^{\text{contact}}$$

$$3. \quad \nabla \mathbf{u}_a = - \sum_{b \in A} \frac{m_b}{\rho_b} (\mathbf{u}_a - \mathbf{u}_b) \otimes (\nabla_a W_{ab})$$