Ratio & Proportion

Ratio

A ratio is a comparison of two quantities by division. It is a relation that one quantity bears to another with respect to magnitude. In other words, ratio means what part one quantity is of another. The quantity may be same kind or different kinds.

If a and b are two numbers, then the ratio of a to b is a/b or $a \div b$ and is denoted by a : b, The two quantities that are being compared are called terms. The first is called antecedent and second term is called consequent.

For Example, the ratio 5 : 8 represent $\frac{5}{8}$ with antecedent 5 and consequent 8.

Note:

1. A ratio is a number, so to find the ratio of two quantities, they must be expressed in the same units.
2. A ratio does not change if both of its terms are multiplied or divided by the same number. Thus, $\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$ etc.

Type of Ratios

1. **Duplicate Ratio:** The ratio of squares of two number is called the duplicate ratio of the two numbers.

   For example:

   $\frac{2^2}{5^2} = \frac{4}{25}$ is called the duplicate ratio of $\frac{2}{5}$.

2. **Triplicate Ratio:** The ratio of the cubes of two number is called the triplicate ratio of the two numbers.

   For examples,

   $\frac{2^3}{5^3}$ or $\frac{8}{125}$ is triplicate ratio of $\frac{2}{5}$.

3. **Sub-duplicate Ratio:** The ratio of the square root of two number is called the sub-duplicate ratio of two numbers for example.

   $\frac{2}{3}$ is sub-duplicate ratio of $\frac{4}{9}$.

4. **Sub-triplicate Ratio:** The ratio of the cube roots of two numbers is called the sub-triplicate ratio of two numbers.

   For example

   $\frac{3}{4}$ is sub-triplicate ratio of $\frac{27}{64}$.

5. **Inverse Ratio or Reciprocal Ratio:** If the antecedent and consequent of a ratio interchange their places, the new ratio is called the inverse ratio of the first.

   Thus, if a : b be the given ratio, then

$\frac{1}{a} : \frac{1}{b}$ or b : a is its inverse ratio.

For example,

$\frac{4}{3}$ is the inverse ratio of $\frac{3}{4}$.

6. **Compound Ratio:** The ratio of the product of the antecedent to that of the consequent of two or more given ratio is called the compound ratio. Thus if a : b and c : d are two given ratios, then ac : bd is the compound ratio of the given ratios.

For example if $\frac{1}{2} : \frac{3}{4} : \frac{5}{7}$ be the given ratios,

then their compound ratio is $\frac{1\times3\times5}{2\times4\times7} = \frac{15}{56}$

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Proportion

The equality of two ratios is called proportion if $\frac{a}{b} = \frac{c}{d}$, then a, b, c and d are said to be in proportion and we write

a : b : : c : d

This is read as "a is to be as c is to d"

For examples, since $\frac{3}{5} = \frac{15}{25}$, we write

3 : 5 : : 15 : 25 and say 3, 5, 15, 25 are in proportion.

Each term of the ratio $\frac{a}{b}$ and $\frac{c}{d}$ is called a proportion. a, b, c and d are respectively the first, second, third and fourth proportional. Here a, d are known as extreme s and b, c are known as means.

SOME BASIC FORMULAE

1. If four quantities are in proportion, then product of means = Product of Extremes for example, in proportion a : b : : c : d, we have b × c = a × d

Means
a : b : : c : d
Extremes

From this relation we see that if any three of the four quantities are given, the fourth can be determined.

2. **Fourth Proportional**

If a : b :: c : x, x is called the fourth proportional of a, b, c.

We have, $\frac{a}{b} = \frac{c}{x}$ or $x = \frac{b\times c}{a}$

Thus, fourth proportional of a, b, c, is $\frac{b\times c}{a}$

Example: Find the fourth proportional to the numbers 3, 8, 6

Solution: Let x be the fourth proportional, then

3 : 8 :: 6 : x or $\frac{3}{8} = \frac{6}{x}$

$\therefore \quad x = \frac{6\times8}{3} = 16.$

3. **Third Proportional**

If a : b :: b : x is called the third proportional of a, b

We have $\frac{a}{b} = \frac{b}{x}$ or $x = \frac{b^2}{a}$

Thus, third proportional of a, b is $\frac{b^2}{a}$.

Example: Find a third proportional to the numbers 3, 6

Solution: If third proportional is x then

$3 : 6 : 6 : x$

So $x = \frac{36}{3} = 12$

4. **Mean Proportional**

If $a : x :: x : b$, x is called the mean or second proportional of a, b.

We have, $\frac{a}{x} = \frac{x}{b}$ or $x^2 = ab$

$x = \sqrt{ab}$

$\therefore$ Mean proportional of a and b is $\sqrt{ab}$

We also say that a, x, b are in continued proportion

Example: Find the mean proportional between 4 and 64.

Solution: Let x be the mean proportional. Then

$4 : x :: x : 64$ or

$\frac{4}{x} = \frac{x}{64}$ or $x^2 = 4 \times 64$, $x = 16$

5. If $\frac{a}{b} = \frac{c}{d}$ then

(i) $\frac{a+b}{b} = \frac{c+d}{d}$ (componendo)

(ii) $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

(iii) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

(Componendo and Dividendo)

(iv) $\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$

Example: The sum of two number is c and their quotient is P/Q . Find the number.

Solution: let the number be x, y

Given $x + y = c$ ...(i)

and $\frac{x}{y} = \frac{p}{q}$ ...(ii)

So $\frac{x}{x+y} = \frac{p}{p+q} = \frac{x}{c} = \frac{p}{p+q}$ {using (i)}

$\Rightarrow x = \frac{pc}{p+c}$

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Partnership

To run a business or any organization, we need to have partnerships. There are two types of partners.

1. **Working Partner:** Those partners who invest the money and manage the business or affairs of organization are called working partners.
2. **Sleeping Partners:** Those partners who merely invest money and do not involve in business affairs are called sleeping partners.

Mainly there are four types of conditions related to distribution of profit in business.

(Here we will use following notations.
Ratio of Profit = $P_1 : P_2 : P_3 \ldots : P_n$
Ratio of Capital invested = $C_1 : C_2 : C_3 \ldots : C_n$
Ratio of Time period of investments = $t_1 : t_2 : t_3 \ldots : t_n$)

Condition I:

When time period of investment is constant then profit of partners will be divided into ratio $C_1 : C_2 : C_3 \ldots : C_n$, where $C_1, C_2, C_3 \ldots : C_n$ are investment of respective partners.

Ex.1 What will be ratio of profit if three partners A, B and C are investing 20000, 45000 and Rs. 60000 for one year.

Sol.: Ratio of profits of A, B and C will be

20000 : 45000 : 60000

$\Rightarrow$ 4 : 9 : 12 Ans.

Condition II:

When amount invested is same for different partners but time period is different then profit will be divided into ratio of their time invested i.e.

$$t_1 : t_2 : t_3 \ldots : t_n$$

Ex. 2 If three partners A, B and C are investing Rs. 20000 each for period of 1 year, 2 years and 3 years respectively then find the ratio of their profits.

Sol.: Ratio of their profits will be

$t_1 : t_2 : t_3$ i.e.

$\Rightarrow$ 1 : 2 : 3 Ans.

Condition III:

When amount of investment and time period both are different for different partners, then profit will be divided into following ratio.

$$C_1t_1 : C_2t_2 : C_3t_3 \ldots : C_nt_n$$

where $C_1, C_2, \ldots C_n$ are the respective amounts and $t_1, t_2 \ldots t_n$ are the respective time period of their investment.

Ex.1 Three partners undergo a partnership. Their investments are 30000 Rs. 45000 Rs and 50000 Rs respectively for time periods of 5 years, 2 years and 3 years respectively. Find the ratio of their profits.

Sol.: Ratio of their profit will be

$C_1t_1 : C_2t_2 : C_3t_3$

$\Rightarrow$ 35 k × 5 : 45k × 2 : 120k × 3

$\Rightarrow$ 30 × 5 : 45 × 2 : 120 × 3

$\Rightarrow$ 5 : 3 : 12 Ans.

Condition IV:

Multiple investments for different time period. In this case profit will be divided into following ratio.

$$\Sigma C_1t_1 : \Sigma C_2t_2 : \Sigma C_3t_3 \ldots : \Sigma C_nt_n$$

Ex.1 Three partners undergo a partnership with initial investment of Rs. 120000, Rs. 80000 and Rs. 150000. Next year A and B invested Rs. 80000 and Rs. 20000 respectively whereas C withdraws Rs. 50000 from the business. In third year of their partnership A, B and C invested Rs. 100000, Rs. 50000 and Rs. 100000 respectively. Find the ratio of their profits.

Sol.:

	Investment		
	A	B	C
1st year	120K	80K	150K
IInd year	+80K	+20K	–50K
IIIrd year	+100K	+50K	+100K

	Effective Investment		
	A	B	C
1st year	120K	80K	150K
IInd year	200K	100K	100K
IIIrd year	300K	150K	200K
	620K	330K	450K

Ratio of their profit will be

$\Rightarrow$ 620 : 330 : 450 $\Rightarrow$ 62 : 33 : 45 **Ans.**

Ex.2 Three partners X, Y and Z undergo a partnership with initial investment of Rs. 120000, Rs. 50000 and Rs. 150000 respectively. Next year A invested Rs. 50000 more in the business. In the third year B and C invested Rs. 50000 each in the business. They remain in partnership for five years. Find the ratio of their profits at the end of five years.

Sol.:

	Investment		
	A	B	C
1st year	120K	50K	120K
IInd year	+50K	–	–
IIIrd year	–	+50K	+50K
IV year	–	–	–
V year	–	–	–

	Effective Investment		
	A	B	C
1st year	120K	50K	150K
IInd year	170K	50K	150K
IIIrd year	170K	100K	200K
IV year	170K	100K	200K
V year	170K	100K	200K
	800	400	900

Ratio of their profit will be

$\Rightarrow$ 800 : 400 : 900 $\Rightarrow$ 8 : 4 : 9 **Ans.**

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Ratio & Proportion

Solved Examples

1. Divide Rs. 500 among A, B, C and D so that A and B together get thrice as much as C and D together, B gets four times of what C gets and C gets 1.5 times as much as D. Now the value of what B gets is
(a) 300 (b) 75
(c) 125 (d) None of these
Ans. (a)
$A + B + C + D = 500$
Here $A + B = 3(C + D)$
So $4(C + D) = 500$
$C + D = 125$
$A + B = 375$
also $B = 4C$
and C $= 1.5\ D$
$C + D = 125$
$2.5\ D = 125$
$D = 50,\ C = 75$
$\therefore$ $B = 300$

2. If $6x^2 + 6y^2 = 13xy$, what is the ratio of x to y?
(a) 2 : 3 (b) 3 : 2
(c) 4 : 5 (d) 1 : 2
Ans. (a) and (b) both
$6x^2 + 6y^2 = 13xy$
Here using options (a) and (b) will satisfy the given condition
$x : y = 2 : 3$

3. If 4 examiners can examine a certain number of answer books in 8 days by working 5 hours a day, for how many hours a day would 2 examiners have to work in order to examine twice the number of answer books in 20 days.
(a) 6 (b) 7½
(c) 8 (d) 9
Ans. (c)
Man. Day. Hour = constant
$\Rightarrow \quad 4 \times 8 \times 5 = 160$
$\Rightarrow$ Man. day. hour $= 160 \times 2$ (twice as earlier)
$2 \times 20 \times h = 160 \times 2$
$h = 8$

4. In a mixture of 40 litres, the ratio of milk and water is 4 : 1. How much water must be added to this mixture so that the ratio of milk and water becomes 2 : 3.
(a) 20 litres (b) 32 litres
(c) 40 litres (d) 30 litres
Ans. (c)
Let water is x
$4x : x$ = milk : water
$32 : 8 = 4x : x$ (since total 40 liters)
Now $\frac{32}{8+x} = \frac{2}{3}$, $x = 40$ litre

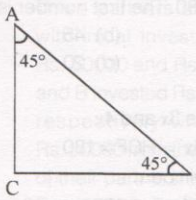
5. If three numbers are in the ratio of 1 : 2 : 3 and half the sum is 18, then the ratio of squares of the numbers is:
(a) 6 : 12 : 13 (b) 1 : 2 : 4
(c) 36 : 144 : 324 (d) None of these
Ans.(c)
Let numbers are
$x : 2x : 3x$ also it is given that
$\frac{1}{2}(x + 2x + 3x) = 18$
$6x = 36,\ x = 6$
6 : 12 : 18
ratio of squares 36 : 144 : 324: or also 1 : 4 : 9

6. The ratio between two numbers is 3 : 4 and their LCM is 180. The first number is:
(a) 60 (b) 45
(c) 15 (d) 20
Ans. (b)
Let number are $3x$ and $4x$
$3x \times 4x = \text{HCF} \times 180$
Clearly HCF will be x.
$12x^2 = x \times 180,\ x = 15$
numbers will be 45 : 60

Note: If there are two numbers N_1 & N_2 then $N_1 \times N_2$ = HCF of $(N_1, N_2) \times$ LCM of (N_1, N_2)

7. The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3. If each saves Rs. 1000, then, A's income is
(a) Rs. 3000 (b) Rs. 4000
(c) Rs. 6000 (d) Rs. 9000
Ans.(c)
Let incomes are $3x : 2x$
expenditures are $5y : 3y$
then
$3x - 5y = 1000$ ...(i) also
$2x - 3y = 1000$ ...(ii)
from (i) and (ii) we get
$6x - 10y = 2000$
$+ 6x - 9y = 3000$
$\Rightarrow \quad -y = -1000$
$y = 1000, x = 2000$
A's income is 6000

8. It the ratio of sines of angles of a triangles is $1 : 1 : \sqrt{2}$, then the ratio of square of its greatest side to sum of the squares of other two sides is
(a) 3 : 4
(b) 2 : 1
(c) 1 : 1
(d) Can't be determined
Ans. (c)
sin A : sin B : sin C = 1 : 1 : $\sqrt{2}$
sin A : sin B = 1 : 1
angle are equal
So it is a right angled triangle.



the $AB^2 : (AC + CB)^2 = 1 : 1$

9. Divide Rs. 680 among A, B and C such that A gets 2/3 of what B gets and B gets 1/4th of what C gets. Now the share of C is?
(a) Rs. 480 (b) Rs. 300
(c) Rs. 420 (d) None of these
Ans. (a)
$680 = A + B + C$
$A = \frac{2}{3}B$ and $B = \frac{1}{4}C$, $4B = C$
$A = \frac{2}{3}B$, $C = 4B$
$680 = \frac{2}{3}B + B + 4B$
$680 = \frac{17}{3}B$, $B = 120$, $C = 480$

10. The students in three batches at Made Easy are in the ratio 2 : 3 : 5. If 20 students are increased in each batch, the ratio change to 4 : 5 : 7. The total number of students in the three batches before the increase were.
(a) 10 (b) 90
(c) 100 (d) 150
Ans. (c)
Let students are $2x : 3x : 5x$ now
According to the given condition
$2x + 20 : 3x + 20 : 5x + 20 = 4 : 5 : 7$
$x = 10$
20 : 30 : 50 = 2 : 3 : 5 and after adding 20
40 : 50 : 70

11. The speeds of three cars are in the ratio 2 : 3 : 4. The ratio between the times taken by these cars to travel the same distance is
(a) 2 : 3 : 4 (b) 4 : 3 : 2
(c) 4 : 3 : 6 (d) 6 : 4 : 3
Ans. (d)
Speeds are in the ratio 2 : 3 : 4
ratio of time taken will be $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$
$\Rightarrow \frac{6}{12} : \frac{4}{12} : \frac{3}{12} = 6 : 4 : 3$

12. After an increment of 7 in both the numerator and denominator, a fraction change to 3/4. Find the original fraction.
(a) 5/12 (b) 7/9
(c) 2/5 (d) 3/8
Ans. (c)

$$\frac{x+7}{y+7}=\frac{3}{4}$$

$4x + 28 = 3y + 21$, $4x + 7 = 3y$,
only $x : y = 2/5$ satisfies the given condition.

13. The difference between two positive numbers is 10 and the ratio between them is 5 : 3. Find the product of the two numbers.
(a) 375 (b) 175
(c) 275 (d) 125
Ans. (a)

$x - y = 10$ ...(i)
$x : y = 5 : 3$ ...(ii)
$x : y = 5t : 3t$ ...(ii)

according to (i) and (iii)
$5t - 3t = 10$
$2t = 10,\ t = 5$
$x = 25, y = 15$
Product of two numbers = 375

14. The present ratio of ages of A and B is 4 : 5. 18 years ago, this ratio was 11 : 16. Find the sum total of their present ages.
(a) 90 years (b) 105 years
(c) 110 years (d) 80 years
Ans. (a)
Let ages are $4x : 5x$ then it is given that

$$\frac{4x-18}{5x-18}=\frac{11}{16}$$

$64x - 288 = 55x - 198$
$9x = 90,\ x = 10$
Ages are 40 and 50 sum = 90

15. Four numbers in the ratio 1 : 3 : 4 : 7 add up to give a sum of 105. Find the value of the biggest number.
(a) 42 (b) 35
(c) 49 (d) 63
Ans. (c)
$x : 3x : 4x : 7x$
are four numbers also
$x + 3x + 4x + 7x = 105$
$15x = 105,\ x = 7,\ 7x = 49$

16. If the ratio of the ages of Maya and Chhaya is 6 : 5 at present and fifteen years from now, the ratio will get changed to 9 : 8, then find Maya's present age.
(a) 24 years (b) 30 years
(c) 18 years (d) 33 years
Ans.(b)
Let their ages age $6x : 5x$ then, it is given that

$$\frac{6x+15}{5x+15}=\frac{9}{8}$$

$48x + 120 = 45x + 135$
$3x = 15,\ x = 5$
$6x : 5x = 30 : 25$
Maya age is 30 years

17. If Rs. 58 is divided among 150 children such that each girl and each boy gets 25 p and 50 p respectively. Then how many girls are there?
(a) 52 (b) 54
(c) 68 (d) 62
Ans. (c)
Let the number of girls and boys are x and y then

$0.25x + 0.5y = 58$ ...(i)

also

$x + y = 150$ ...(ii)

from (i) and (ii) we get
$0.5x + y = 116$
$x + y = 150$
So, $0.5x = 34,\ x = 68,\ y = 82$

18. If 391 bananas was distributed among three monkeys in the ratio 1/2 : 2/3 : 3/4, how many bananas did the first monkey get?
(a) 102 (b) 108
(c) 112 (d) 104

Ans. (a)

$\frac{1}{2}:\frac{2}{3}:\frac{3}{4}=\frac{6:8:9}{12}$

Bananas are in ratio $6x : 8x : 9x$

It is given that

$6x + 8x + 9x = 391$

$23x = 391, x = 17$

first monkey get 102 bananas

19. A mixture contains milk and water in the ratio 5 : 1. On adding 5 litres of water, the ratio of milk to water becomes 5 : 2. The quantity of milk in the mixture is:

(a) 16 litres (b) 25 litres

(c) 32.5 litres (d) 22.75 litres

Ans. (b)

Let milk and water are $5x : x$

Now, $\frac{5x}{x+5}=\frac{5}{2} \Rightarrow 10x = 5x + 25$

$5x = 25, x = 5$

milk : water = 25 : 10

20. Vijay has coins of the denomination of Re. 1, 50p and 25 p in the ratio of 12 : 10 : 7. The total worth of the coins he has in Rs. 75. Find the number of 25 p coins that Vijay has

(a) 48 (b) 72

(c) 60 (d) None of these

Ans. (d)

Coins are $12x : 10x : 7x$

It is given that $12x + 10x \times \frac{1}{2}+\frac{7}{4}x = 75$

$\frac{48x+20x+7x}{4}=75$

$75x = 4 \times 75, x = 4$

21. If two numbers are in the ratio of 5 : 8 and if 9 be added to each, the ratio becomes 8 : 11. Now find the lower number.

(a) 5 (b) 10

(c) 15 (d) None of these

Ans. (c)

Let number are $5x$ and $8x$ then it is given that

$\frac{5x+9}{8x+9}=\frac{8}{11}$

$55x + 99 = 64x + 72$

$27 = 9x, x = 3$

Number are 15 and 24

22. If x varies as y, and $y = 7$ when $x = 18$, find x when $y = 21$

(a) 36 (b) 54

(c) 72 (d) 18

Ans. (b)

x is directly proportional to y

So, $x = k.y$

$18 = 7k$

So, $k = \frac{18}{7}$

Now, $x = \frac{18}{7} \times 21 = 54$

23. A varies jointly as B and C; and A = 6 when B = 3, C = 2; find A when B = 5, C = 7.

(a) 17.5 (b) 35

(c) 70 (d) 105

Ans. (b)

Let $A = k(BC)$ then

$6 = k(3.2)$

So, $k = 1$ then

$A = k(BC)$

$= 1 \times (5 \times 7) = 35$

24. If x varies as y directly, and as z inversely and $x = 14$, when $y = 10$; find z when $x = 49$, $y = 45$.

(a) 14/10

(b) 10

(c) 10/14

(d) Cannot be determined

Ans. (d)

$x = k_1 y$ $x = \frac{k_2}{z}$

According to given values

$14 = k_1.10$

So, $k_1 = \frac{14}{10} = \frac{7}{5}$

also, $14 = \frac{k_2}{z}$

So, $k_2 = 14z$,

But value of k_2 is not given hence z cannot be determined.

25. A cask contains a mixture of 49 litres of wine and water in the proportion 5 : 2. How much water must be added to it so that the ratio of wine to water may be 7 : 4?
(a) 3.5 (b) 6
(c) 7 (d) None of these

Ans. (b)

Let wine and water are $5x$ and $2x$ litres

$5x + 2x = 49$

$\Rightarrow 7x = 49$

$\therefore x = 7$ so

$5x = 35, \quad 2x = 14$

Now $\frac{35}{14+w} = \frac{7}{4}$

$98 + 7w = 140$

$7w = 42, w = 6$

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Ratio & Proportion

I Practice Exercise: I

1. Find a fourth proportional to the numbers 60, 48, 30.
(a) 36 (b) 24
(c) 48 (d) None of these

2. Find a third proportional to the numbers 4, 42.
(a) 441 (b) 541
(c) 641 (d) None of these

3. In a ratio 11 : 14, if the antecedent is 55, the consequent is
(a) 70 (b) 90
(c) 60 (d) None of these

4. The mean proportional of 0.25 and 0.04 is
(a) 0.01 (b) 0.1
(c) $10\sqrt{10}$ (d) None of these

5. The ratio of two numbers is 3 : 4 and their sum is 420. The greater of the two numbers is
(a) 360 (b) 240
(c) 180 (d) None of these

6. Amit, Sumit and Puneet share an amount of Rs. 660 in the ratio of 3 : 4 : 5. What is the share of Puneet?
(a) Rs. 375 (b) Rs. 275
(c) Rs. 575 (d) None of these

7. The prices of a scooter and a television set are in the ratio of 3 : 2. If a scooter costs Rs. 600 more than the television set, then the price of television set is:
(a) Rs. 1800 (b) Rs. 1200
(c) Rs. 2400 (d) None of these

8. If A : B = 7 : 5 and B : C = 9 : 11, then A : B : C is equal to
(a) 55 : 45 : 63 (b) 63 : 45 : 55
(c) 45 : 63 : 55 (d) None of these

9. If 3A = 4B = 5C, then A : B : C is
(a) 16 : 20 : 18 (b) 15 : 20 : 16
(c) 20 : 15 : 12 (d) None of these

10. The ratio of money with Anju and Sanju is 4 : 5 and that with Sanju and Manju is 5 : 6. If Anju has Rs. 280, then the amount of money Manju has
(a) Rs. 320 (b) Rs. 420
(c) Rs. 640 (d) None of these

11. The sum of three number is 124. If the ratio between the first and second be 2 : 3 and

that between the second and third be 7 : 9, then the third number is:
(a) 54 (b) 64
(c) 48 (d) None of these

12. The ratio between two number is 2 : 3. If each number is increased by 4, the ratio becomes 5 : 7. The number are
(a) 8, 16 (b) 16, 24
(c) 24, 32 (d) None of these

13. The ratio of present ages of Suresh and Mahesh is 7 : 5. If after 6 years their ages will be in the ratio of 4 : 3, the present age of Mahesh is
(a) 32 years (b) 36 years
(c) 30 years (d) None of these

14. Two numbers are in the ratio of 5 : 7. If 25 be subtracted from each, they are in the ratio of 35 : 59. Find the difference of the two numbers.
(a) 48 (b) 52
(c) 24 (d) None of these

15. The value of k that must be added to 7, 16, 43, 79 so that they are in proportion, is
(a) 7 (b) 5
(c) 9 (d) None of these

16. The number that must be added to each of the numbers 8, 21, 13 and 31 to make the ratio of first two numbers equal to the ratio of last two number is
(a) 5 (b) 7
(c) 9 (d) None of these

17. The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves Rs. 1000, A's income is
(a) Rs. 5000 (b) Rs. 6000
(c) Rs. 8000 (d) None of these

18. A mixture contains alcohol and water in the ratio of 12 : 5. On adding 14 litres of water, the ratio of alcohol to water becomes 4 : 3. The quantity of alcohol in the mixture is
(a) 18 litres (b) 42 litres
(c) 26 litres (d) None of these

Solutions

1. Ans. (b)
Let x be the fourth proportional, then
$60 : 48 : 38 : x$ or $\frac{60}{48} = \frac{30}{x}$
$\therefore \quad x = \frac{30 \times 38}{60} = 24.$

2. Ans. (a)
Let x be the third proportional, then
$4 : 42 :: 42 : x$ or, $\frac{4}{42} = \frac{42}{x}$.
$\therefore \quad x = \frac{42 \times 42}{4} = 441.$

3. Ans. (a)
Let consequent be x
then $\frac{11}{14} = \frac{55}{x}$
$\therefore$ Consequent $x = 70$.

4. Ans. (b)
Let x be the mean proportional. Then,
$0.25 : x :: x : 0.04$
or $\frac{0.25}{x} = \frac{x}{0.04}$
or $x^2 = 0.01$
or $x = 0.1$

5. Ans. (b)
Let first and second number be $3x$ & $4x$ respectively. It is given that
$3x + 4x = 420$
$\Rightarrow \quad x = 60$
So first number $3x = 180$
Second number $4x = 240$

6. Ans. (b)
Let the shares of Amit, Sumeet and Puneet be
$3x$, $4x$ and $5x$ respectively
It is given that
$3x + 4x + 5x = 660$
$12x = 660$
$x = 55$
So $5x = 275$

7. Ans. (b)
Let the price of Scooter and Television be $3x$ & $2x$ respectively
It is given that $3x - 2x = 600$
$\Rightarrow x = 6000$
$\therefore$ Television's price $2x = 1200$

8. Ans. (b)
A : B = 7 : 5 ...(i)
B : C = 9 : 11 ...(ii)
B is common in eq. (i) & (ii)
To equate the ratios of B we will multiply eq. (i) by 9 and eq. (ii) by 5
Then A : B = 63 : 45
B : C = 45 : 55
$\therefore$ A : B : C = 63 : 45 : 55

9. Ans. (c)
A : B = 4 : 3 ...(i)
B :C = 5 : 4 ...(ii)
B is common in eq. (i) & (ii)
To equate ratios of B in eq. (i) and (ii) we will multiply eq. (i) by 5 & eq. (ii) by 3
then A : B = 20 : 15 ...(iii)
B : C = 15 : 12 ...(iv)
Hence A : B : C = 20 : 15 : 12

10. Ans. (b)
A : B = 4 : 5 ...(i)
B : C = 5 : 6 ...(ii)
B is common among them
So A : B : C = 4 : 5 : 6
Thus, ratio of money with Anju, Sanju and Manju is 4 : 5 : 6. Since Anju has Rs. 280, the amount of money Manju has
$= \frac{280}{4} \times 6 = \text{Rs. } 420.$

11. Ans. (a)
A : B = 2 : 3 ...(i)
B : C = 7 : 9 ...(ii)
$\Rightarrow$ A : B : C = 14 : 21 : 27
A + B + C = 124 then
C = 54

12. Ans. (b)
Let the two numbers be $2x$ & $3x$
then $\frac{2x+4}{3x+4} = \frac{5}{7}$
$\Rightarrow 14x + 28 = 15x + 20$
$\Rightarrow x = 8$
So numbers are 16 & 24.

13. Ans. (c)
Let ages of Suresh & Mahesh be $7x$ & $5x$
Then $\frac{7x+6}{5x+6} = \frac{4}{3}$ on solving
We get $x = 6$
Then present age of Mahesh i.e. $5x = 30$ years

14. Ans. (c)
Let the numbers be $5x$ and $7x$
Then $\frac{5x-25}{7x-25} = \frac{35}{59}$ on solving
We get, $x = 12$
Difference between two numbers
$7x - 5x = 2x = 24$

15. Ans. (b)
According to given condition
$7 + k : 16 + K :: 43 + k : 79 + k$
$\Rightarrow \frac{7+k}{16+k} = \frac{43+k}{79+k}$
going by options we can find that $x = 5$

16. Ans. (a)
Let x be the desired number then

$$\frac{8+x}{21+x} = \frac{13+x}{31+x}$$

going by options we can easily find that

$$x = 5$$

17. Ans. (b)
Let income be $3x : 2x$
and expenditure be $5y : 3y$
then $3x - 5y = 1000$
$2x - 3y = 1000$
$\Rightarrow \quad x = 2000$
$\Rightarrow \quad y = 1000$
So A's income will be $3x = 6000$.

18. Ans. (b)
Ratio of Alcohol & Water is 12 : 5
Let Alcohol be $12x$ then Water will be $5x$

$$\text{Now} \quad \frac{12x}{5x+14} = \frac{4}{3}$$

$$\Rightarrow \quad 36x = 20x + 56$$

$$16x = 56$$

$$\Rightarrow \quad x = \frac{7}{2}$$

So $12x = 42$ liters

□□□□

Partnership

I Practice Exercise: I

1. Aman and Pranjal enter into a partnership investing Rs. 50000 and Rs. 40000, respectively. They agree to share profits in the ratio of their capitals. Find the share of Aman in a profit of Rs. 22500 after one year.
(a) Rs. 12500 (b) Rs. 9500
(c) Rs. 10500 (d) None of these

2. Amit, Nitin and Ravindra entered into a partnership. Amit invested Rs. 16000 for 9 months. Nitin invested Rs. 12000 for 6 months and Ravindra invested Rs. 8000 for 12 months. At the end of a year there was a profit of Rs. 26000. Find the share of Nitin in the profit.
(a) Rs. 8000 (b) Rs. 7500
(c) Rs. 6000 (d) None of these

3. Sakshi starts business with Rs. 3500 and 5 months after Divya joins Sakshi as her partner. After a year the profits are divided in the ratio of 2 : 3. How much did Divya contribute?
(a) Rs. 7000 (b) Rs. 11000
(c) Rs. 9000 (d) None of these

4. Arvind began a business with Rs. 550 and was joined afterwards by Naveen with Rs. 330. When did Naveen join if the profits at the end of the year were divided in the ratio 10 : 3?
(a) After 4 months (b) After 6 months
(c) After 4.5 months (d) None of these

5. A, B and C invested capitals in the ratio 3 : 5 : 9; the timing of their investments being in the ratio 2 : 3 : 1. In what ratio would their profits be distributed?
(a) 2 : 5 : 3 (b) 3 : 2 : 5
(c) 7 : 5 : 3 (d) None of these

6. A, B and C start a business. If the ratio of their periods of investments are 2 : 3 : 6 and their profits are in the ratio of 4 : 5 : 6, then the ratio of capitals of A, B and C is
(a) 6 : 8 : 10 (b) 12 : 10: 6
(c) 10 : 12 : 6 (d) None of these

7. A, B, C and D enter into partnership. A subscribes $\frac{1}{3}$ of the capital, B $\frac{1}{4}$, C $\frac{1}{5}$ and D the rest. What is the share of D out of a profit of Rs. 6000?
(a) Rs. 2000 (b) Rs. 1600
(c) Rs. 1200 (d) Rs. 1300

8. A and B started a business with initial investments in the ratio 5 : 7. If after one year their profits were in the ratio 1 : 2 and the period for A's investment was 7 months, B invested the money for
(a) 6 months (b) 2½
(c) 10 months (d) 4 months

9. A, B, C enter into a partnership with shares in the ratio $\frac{7}{2}:\frac{4}{3}:\frac{6}{5}$. After 4 months, A increase his share by 50%. If the total profit at the end of one year be Rs. 21600, then B's share in the profit is
(a) Rs. 2100 (b) Rs. 2400
(c) Rs. 3600 (d) Rs. 4000

10. B is a sleeping partner and A working. A puts in Rs. 5000 and B puts in 6000. A received 12.5% of profit for managing the business and rest is divided in proportion to their capitals. A's share of profit in a profit of Rs. 880 is
(a) Rs. 350 (b) Rs. 400
(c) Rs. 420 (d) Rs. 460

11. A starts business with a capital of Rs. 1200. B and C join with some investments after 3 and 6 months, respectively. If at the end of a year, the profit is divided in the ratio 2 : 3 : 5 respectively, what is B's investment in the business?
(a) Rs. 2400 (b) Rs. 1800
(c) Rs. 3600 (d) Rs. 6000

Solutions

1. **Ans. (a)**
Here, $C_1 = 50000$, $C_2 = 40000$ and $P = 22500$.
Here time period of investment is constant
So ratio in which profit will be divided is
$C_1 : C_2 \Rightarrow 50k : 40k = 5 : 4$
Now Aman's share
$= \frac{5}{9} \times 22500 = \text{Rs.}12500.$

2. **Ans. (c)**
Here, $C_1 = 16000$, $C_2 = 12000$, $C_3 = 8000$,
$t_1 = 9$, $t_2 = 6$, $t_3 = 12$ and $P = 26000$.
Ratio in which profit will be divided i.e.
$P_1 : P_2 : P_3$ will be $C_1t_1 : C_2t_2 : C_3t_3$
$16000 \times 9 : 12000 \times 6 : 8000 \times 12$
$\Rightarrow P_1 : P_2 : P_3 = 6 : 3 : 4$
Nitin's share = 6000.

3. **Ans. (c)**
Let suppose Divya's contribution is x then
We have, $C_1 \times t_1 = 3500 \times 12 = 42000$ and
$C_2 \times t_2 = x \times 7 = 7x$.
Then, $\frac{\text{Profit for Sakshi}}{\text{Profit for Divya}} = \frac{C_1 \times t_1}{C_2 \times t_2}$
$\Rightarrow \frac{2}{3} = \frac{42000}{7x}$
or, $x = \frac{42000 \times 3}{2 \times 7} = \text{Rs. } 9000.$
Divya's contribution is Rs. 9000.

4. **Ans. (b)**
Let Naveen remain in the business for x months.
We have, $C_1 \times t_1 = 550 \times 12 = 6600$
$C_2 \times t_2 = 330 \times x = 330x$.
$\therefore \frac{\text{Arvind's share of profit}}{\text{Naveen's share of profit}} = \frac{C_1 \times t_1}{C_2 \times t_2}$
$\Rightarrow \frac{10}{3} = \frac{6600}{330x}$
$\Rightarrow x = \frac{6600 \times 3}{330 \times 10} = 6 \text{ months.}$

5. **Ans. (a)**
Ratio of capitals of A, B and C are 3 : 5 : 9.
Let the capitals of A, B and C be $3x$, $5x$ and $9x$, respectively.
Ratio of timing of their investments are 2 : 3 : 1. Let A, B and C invest their capitals for $2y$, $3y$ and y months, respectively.

Then, profit of A : profit of B : Profit of C
$= C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$
$= 3x \times 2y : 5x \times 3y : 9x \times y$
$= 6 : 15 : 9$ or, $2 : 5 : 3$.

6. Ans. (b)
Ratio in which profit will be divided i.e.
$P_1 : P_2 : P_3$ is $C_1t_1 : C_2t_2 : C_3t_3$
We have, $P_1 : P_2 : P_3 = 4 : 5 : 6$
and $t_1 : t_2 : t_3 = 2 : 3 : 6$.
$\therefore$ Required ratio $= \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} = \frac{4}{2} : \frac{5}{3} : \frac{6}{6}$
or, $12 : 10 : 6$
Thus, A, B and C invested their capitals in the ratio $12 : 10 : 6$.

7. Ans. (d)
D's Capital $= 1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} = \frac{13}{60}$
Profit ratio of A, B, C, D is $\frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{13}{60}$
$\therefore$ $20 : 15 : 12 : 13$
$\therefore$ Share of D $= \frac{13}{60} \times$ Rs.6000 = Rs.1300.

8. Ans. (c)
Let investments of A and B respectively be $5x$ and $7x$ and period of B's investment be y months.
Then, $\frac{(5x) \times 7}{(7x) \times y} = \frac{1}{2} \Rightarrow y = 10$.

9. Ans. (d)
Given ratio $= \frac{7}{2} : \frac{4}{3} : \frac{6}{5} = 105 : 40 : 36$
Let the initially invest Rs. 105, Rs. 40 and Rs. 36, respectively.
Ratio of investments
$=$ $[105 \times 4 + (150\% \text{ of } 105) \times 8] : (40 \times 12) : (36 \times 12)$
$=$ $1680 : 480 : 432 = 35 : 10 : 9$
B's share $=$ Rs. $\left(21600 \times \frac{10}{54}\right)$ = Rs.4000.

10. Ans. (d)
A's share for managing the business
= 12.5% of Rs. 880 = Rs. 110
Remaining profit = Rs. 770.
Profit ratio of A and B = 5 : 6
A's share $= \frac{5}{11}$ of Rs. 770 = Rs. 350.
A's total profit= Rs. 350 + Rs.110 = Rs. 460.

11. Ans. (a)
Profit ratio of A, B and C is
$(1200 \times 12) : (x \times 9) : (y \times 6) = 2 : 3 : 5$
Taking first and second terms we get
$1200 \times 12 : 9x = 2 : 3$
$\Rightarrow$ $1200 \times 12 \times 3 = 9x \times 2$
$\therefore$ $x = \frac{1200 \times 12 \times 3}{18} = 2400$

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