Hypothesis Testing

① A factory has a machine that fills 80 ml of baby medicine in a bottle. An employee believes that average amount of baby medicine is not 80 ml. Using 40 samples he measures the average amount dispensed by machine is 78 ml with a std deviation of 2.5.

i) State Null and alternate hypothesis.

ii) At 95% C.I is there enough evidence to support the machine is not working properly?

A) :- $n = 40$, $\bar{x} = 78$ ml, $s = 2.5$

Step 1 :-

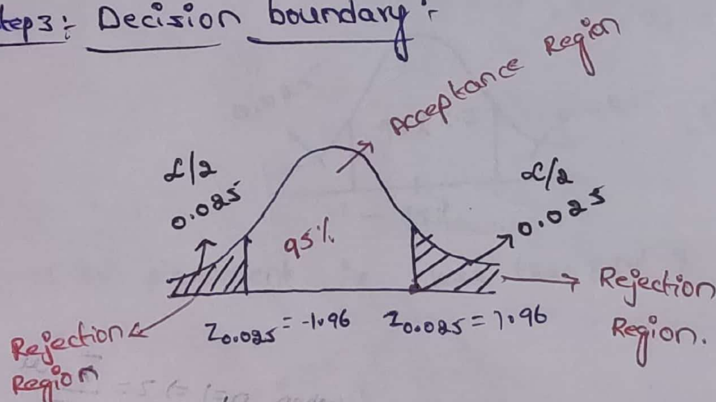
Null hypothesis $(H_0) = \mu = 80$

alternate hypothesis $(H_1) = \mu \neq 80$

Step 2 :-

$\alpha = 0.05$, C.I = 95%

Step 3 :- Decision boundary :-



$Z_{0.025} = 1 - 0.025$
$= 0.9750$

Why Z-test

① sample size ≥ 30

② population std or sample standard deviation should be given.

Step 4 :- Calculate test statistics

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$\bar{x}$ - sample mean
μ - population mean
s - sample std deviation

n = sample size
$\frac{s}{\sqrt{n}}$ = standard error.

$$= \frac{78-80}{2.5/\sqrt{40}} = \frac{-2 \times \sqrt{40}}{2.5} = \frac{-2}{2.5} \times 6.32 = -5.05$$

Step 5 :- State the results :-

Decision Rule :- If $z = -5.05$ is less than -1.96 or greater than 1.96, then reject the null hypothesis with 95% C.I.

Reject H_0 null hypothesis {There is some fault in the machine}.

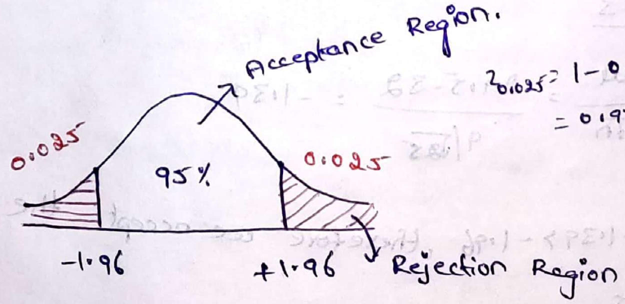
② In the population the average IQ is 100 with a standard deviation of 15. A team of of scientists wants to test a new medication to see if it has a +ve or -ve effect, or no effect at all. A sample of 30 participants who have taken the medication has a mean of 140. Did the medication effect the intelligence?

A :- $\sigma = 15$ $n = 30$ $\bar{x} = 140$

① $H_0 : \mu = 100$
$H_1 : \mu \neq 100$

② $\alpha = 0.05$, C.I = 95%.

③ Decision Boundary :



$z_{0.025} = 1 - 0.025$
$= 0.9750$

④ Test statistics

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{140 - 100}{15/\sqrt{30}} = 14.607$$

⑤ $14.60 > 1.96$ Reject the Null hypothesis

medication has +ve effect on IQ.

③ A complain was registered, the boys in the muncipal primary school are underfed. Average weight of boys of age 10 is 32 kgs with S.D = 9 kgs. A sample of 25 boys was selected from the muncipal school and the average weight was found to be 29.5 kgs ? with C.I = 95% check whether it is true or false ?

A) $\mu = 32$ kgs $\sigma = 9$ kg, $n = 25$, $\bar{x} = 29.5$ $\alpha = 0.05$

1) $H_0 : \mu = 32$

$H_1 : \mu < 32$

2) $\alpha = 0.05$, C.I = 95%.

3) Decision boundary

= 1 - 0.025

= 0.9750

Z table ⇒ 1.96

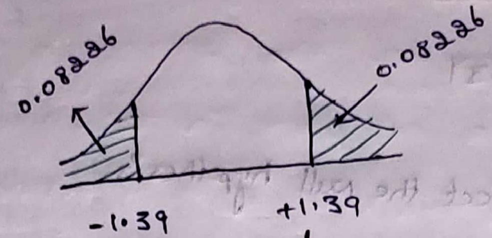
4) Test statistics

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{29.5 - 32}{9/\sqrt{25}} = -1.39$$

5) Conclusion: $-1.39 > -1.96$, therefore we accept the null hypothesis.

→ so, the boys are not underfed.

p-value



$\alpha = 0.05$
$\Downarrow$
p value

↳ 0.91774

↳ we got this value from z-table.

$= 1 - 0.91774$

$= 0.08226$

Acceptance region $= 1 - 0.08226 - 0.8226$

$= 0.8384$

p-value $= 0.08226 + 0.08226 = 0.1645$

$0.1645 > 0.05$
p-value > significance value } ⇒ Accept the null hypothesis.

→ The average weight of all residents in town XYZ is 168 lbs. A nutritionist believes the true mean to be different. she measured the weight of 36 individuals and found the mean to be 169.5 lbs with a standard deviation of 3.9.

a) At 95% C.I is there enough evidence to discard the null hypothesis

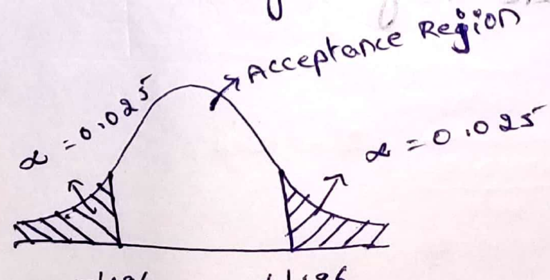
$n = 36, \ \bar{x} = 169.5 \quad s = 3.9$

① $H_0: \mu = 168$

$H_1: \mu \neq 168$

② C.I $= 95\%$, $\alpha = 0.05$

③ Decision Boundary

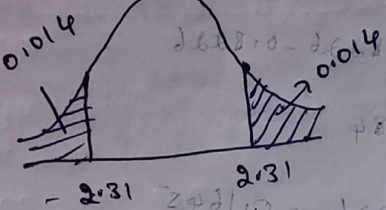


④ calculate test statistics

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = 2.31$$

$2.31 > 1.96$ Reject the null hypothesis.

p-value



0.9896 → From z-table

= 1 - 0.986

z_i = 0.014

p-value = 0.014 + 0.014

= 0.028

0.028 < 0.05 ⇒ Reject the null hypothesis.

→ A company manufactures bike batteries with an average life span of 2 or more years. A engineer believes this value to be less. using 10 samples, he measures the average life span to be 1.8 years, with a standard deviation of 0.15

a) state the null and alternate hypothesis.

b) At 99% CI, is there enough evidence to discard the Ho?

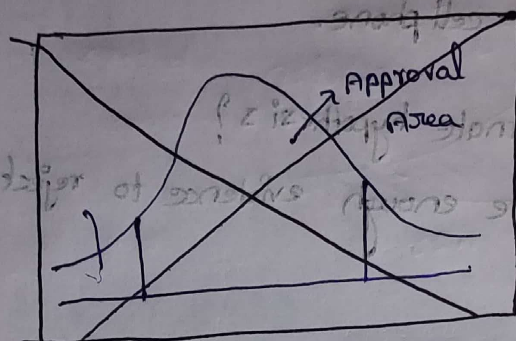
a) $n = 10$, $\bar{x} = 1.8$, $s = 0.15$

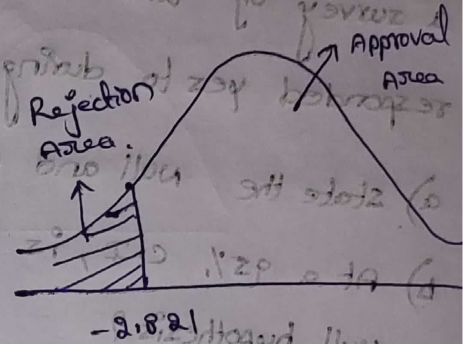
(1) $H_0 : \mu \geq 2$

$H_1 : \mu < 2$

(2) $\alpha = 0.01$, C.I $= 99\%$

(3) Decision Boundary:





Degree of freedom $= n - 1$

$= 10 - 1$

$= 9$

(4) calculate t-test statistics :-

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1.8 - 2}{0.15/\sqrt{10}} = \frac{-0.2}{0.15/3.1622} = -4.216$$

(5) conclusion

$-4.216 < -2.821$ Reject the null hypothesis.

Z-test with proportions:-

Q) A tech company believes that the percentage of residents in town XYZ that owns a cell phone is 70%. A marketing manager believes that this value to be different. He conducts a survey of 200 individuals and found that 130 responded yes to during a cell phone.

a) state the Null and Alternate hypothesis?

b) At a 95% C.I, is there enough evidence to reject the Null hypothesis?

Ans) $n = 200$ $x = 130$

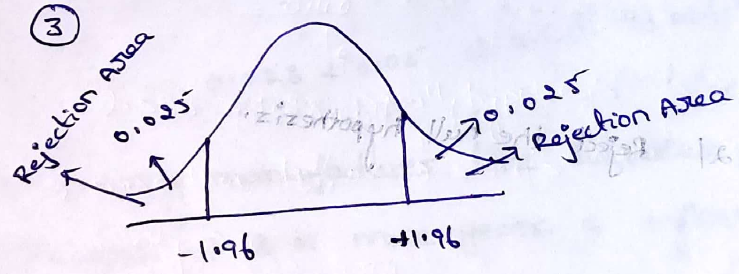
1) $H_0 : P_0 = 0.70$

$H_1 : P_0 \neq 0.70$

$\hat{p} = \frac{x}{n} = \frac{130}{200} = \frac{13}{20} = 0.65$

2) $\alpha = 0.05$, C.I = 95%

3)



4) Z-test

$$\text{Z-test} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

$q_0 = 1 - p_0$

$= 1 - 0.70$

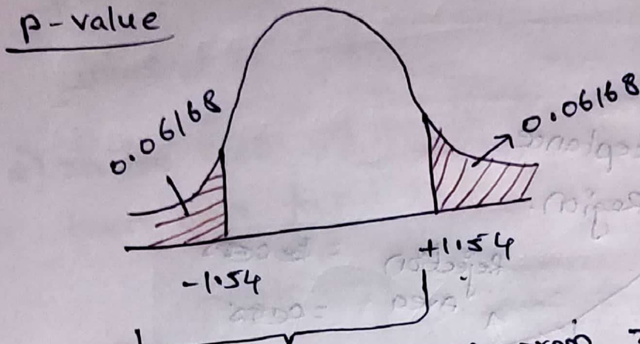
$= 0.30$

$$= \frac{0.65 - 0.70}{\sqrt{\frac{0.7 \times 0.3}{200}}} \approx -1.54$$

At 95% C.I there is

$-1.54 > -1.96$, so we accept the null hypothesis.

p-value



$\hookrightarrow$ 0.93822 from z-table we got this value.

$= 1 - 0.93822$

$= 0.06168$

p-value $= 0.06168 + 0.06168$

$= 0.1236$

$0.1236 > 0.05$, so we accept the null hypothesis.

* A car company believes that the percentage of residents in city ABC that owns a vehicle is 60% or less. A sales manager disagrees with this. He conducts a hypothesis testing surveying 250 residents and found that 170 responded yes, to owing a vehicle.

a) state the null and alternate hypothesis.

b) At 10% significance level, is there enough evidence to support the idea that vehicle ownership in city ABC is 60% or less?

A) 1) $H_0: P_0 \leq 0.60$

$H_1: P_0 > 0.60$

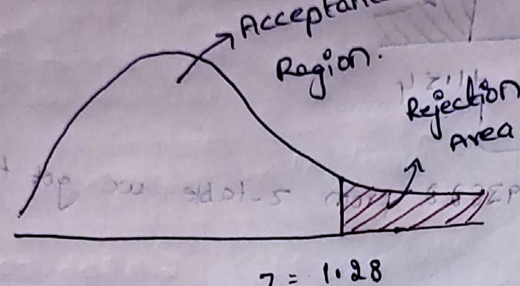
2) $\alpha = 0.$[redacted], $CI = 0.9$[redacted]

Z-value at $\alpha = 0.1$ is 1.28

$n = 250 \quad x = 170$

$$\hat{p} = \frac{x}{n} = \frac{170}{250} = 0.68$$

step 3:-



= 0.005

= 0.005

z 1-0.1

= 0.9

step-4 z-test with proportion

$$z\text{-test} = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0 q_0}{n}}} = \frac{0.68 - 0.60}{\sqrt{\frac{0.6 \times 0.4}{250}}} = 2.582$$

$q_0 = 1 - P_0$

$= 1 - 0.60$

$= 0.40$

$2.588 > 1.28 \Rightarrow$ Null hypothesis is rejected.

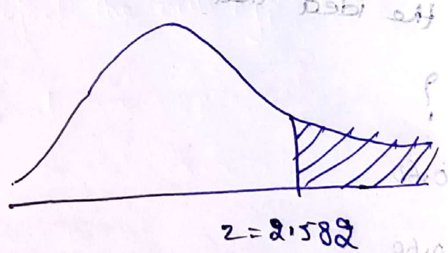
→ more than 60% of residents own a vehicle in city ABC.

using p-value test

at $z = 2.582$

p-value $= 1 - 0.99506$

$= 0.00494$



p-value $< \alpha$

$0.0049 < 0.1$

→ Reject the Null hypothesis.