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Q① For convection, the Schwarzschild stability cond^n:

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

For ideal gas with mean molecular weight at convective zone, μ then $P = \frac{k\rho T}{\mu m_H}$

Assuming hydrodynamic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_\odot}{r^2}\rho$$

Now, $$\frac{dT}{dr} = -\left(1 - \frac{1}{5/3}\right)\frac{T}{P} \times \frac{GM_\odot}{r^2} \times \frac{\mu m_H P}{kT}$$

$$\int_{T_0}^{T} dT = -\frac{2}{5}\int_{r_0}^{r} \frac{GM_\odot}{r^2}\frac{\mu m_H}{k}\,dr$$

$$\Rightarrow T(r) - T_0 = \frac{2}{5}\frac{GM_\odot \mu m_H}{k}\left(\frac{1}{r} - \frac{1}{0.7R_\odot}\right)$$

$$T(r) = T_0 + \frac{2}{5}\frac{GM_\odot \mu m_H}{k}\left(\frac{1}{r} - \frac{10}{7R_\odot}\right)$$

$\because r > 0.7R_\odot \Rightarrow$ Temp. will decrease from the base of the convective zone.

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let's calculate density : $\frac{dT}{dr} = \frac{2}{5}\frac{T}{P}\frac{dP}{dr}$

$$\frac{dP}{dr} = \frac{k}{\mu m_H}\left(T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right)$$

$$\frac{dT}{dr} = \frac{2}{5} \times \frac{T}{k\rho T}\cdot \mu m_H \times \frac{k}{\mu m_H}\left(T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right)$$

$$\Rightarrow \frac{dT}{dr} = \frac{2}{5\rho}\left(T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right)$$

$$\Rightarrow \frac{3}{2}\frac{dT}{T} = \frac{d\rho}{\rho}$$

$$\Rightarrow \ln\left(\frac{T(r)}{T_0}\right) = \ln\left(\frac{\rho(r)}{\rho_0}\right)^{2/3}$$

$$\Rightarrow \rho(r) = \rho_0\left(\frac{T(r)}{T_0}\right)^{3/2} = \rho_0\left[1 + \frac{2}{5}\frac{GM_\odot \mu m_H}{kT_0}\left(\frac{1}{r} - \frac{10}{7R_\odot}\right)\right]^{3/2}$$

$\Rightarrow T(r)$ decrease with r, $\rho(r)$ will also decrease with r.

Pressure calculation : From eqⁿ of hydrodynamic equilibrium,

$$\frac{dP}{dr} = -\frac{GM_\odot}{r^2}\rho(r)$$

$$\Rightarrow P(r) = P_0 - GM_\odot\rho_0\int_{r_0}^{r}\frac{\rho(r)}{r^2}\,dr$$

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(b) $X = 0.7$, $Z = 0.02$

$\cancel{Z}$ number of particles per unit volume,

$$n = \left(2 \times 0.7 + \frac{0.02}{2}\right)\frac{\rho}{m_H} = \frac{1.401\,\rho}{m_H}$$

$$\text{mean molecular weight} = \frac{1}{1.401} = 0.713$$

$$\text{Mean particle mass} = \mu m_H = 1.19 \times 10^{-27}\ \text{kg}$$

(c) From mass continuity eq^n $\frac{dM_r}{dr} = 4\pi r^2 \rho(r)$

$$\text{Mass of Convective zone},\ M = \int_{r_0}^{R_\odot} 4\pi r^2 \rho(r)\, dr$$

$$= \int_{0.7R_\odot}^{R_\odot} 4\pi r^2 \left[1 + 2.277 \times 10^7 \left(\frac{1}{r} - \frac{10}{7R_\odot}\right)\right]^{3/2}$$

To keep the output value in the real space $R = 6.255 \times 10^8$ has been chosen. Hence we got the mass of convective zone 1.0866×10^{27} kg which is 0.855 % mass of sun, a good agreement.

(d) The sol^n is not consistent becoz we have varied $\rho(r)$.

Q5 T @ Center of sun : 1.56×10^7 K

$\rho = 1.46 \times 10^5$ kg/m^3

H = 0.64, He = 0.34, CNO = 0.015

Amount of energy that is generated at the center of the sun due to the pp chain and CNO cycle.

→ For main sequence stars the most important fusion reac^n fuses 4 ^1_1H atom to ^4_2He

$$^1_1H + ^1_1H \rightarrow ^2_1H + ^0_0e^- + ^0_0\nu$$

$$^2_1H + ^1_1H \rightarrow ^3_2He + ^0_0\nu$$

$$^3_2He + ^3_2He \rightarrow ^4_2He + 2\,^1_1H$$

The pp chain is most effective for temp^n around 15 millio So $E_{pp} \simeq \varepsilon_{o,pp} X_H^2 \rho T_6^4$

$T = 10^6 T_6$; T_6 : Temp. in millions of K

$\varepsilon_{o,pp} = 1.08 \times 10^{-12}$ Wm^3/kg^2

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The another reaction converting $_1^1H$ to $_2^4He$ is the CNO cycle.

$$^{12}_{6}C + ^{1}_{1}H \longrightarrow ^{13}_{7}N + ^{0}_{0}\nu$$

$$^{13}_{7}N \longrightarrow ^{13}_{6}C + ^{0}_{0}e^{-} + ^{0}_{0}\nu$$

$$^{13}_{6}C + ^{1}_{1}H \longrightarrow ^{14}_{7}N + ^{0}_{0}\nu$$

$$\vdots \qquad \vdots$$

$$^{15}_{7}N + ^{1}_{1}H \longrightarrow ^{12}_{6}C + ^{4}_{2}He$$

Here total reaction rate

$$\varepsilon_{CNO} = \varepsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20}$$

where $\varepsilon_{0,CNO} = 8.24 \times 10^{-31} \, W m^3/kg^2$

$$X_{CNO} = \frac{M_{CNO}}{M}$$

Total energy by pp-chain:

for pp $E = E_0 \rho T^4$.

$$= (12.86) \times 1.48 \times 10^5 \times (1.56 \times 10^7)^4$$

$$= 112.72 \times 10^{19} \text{ MeV}$$

for CNO cycle $E = E_0 \rho T^{17} = (1.7) \times (1.48 \times 10^5) \times (1.56 \times 10^7)^{17}$

$$= 4.828 \times 10^{32} \text{ MeV}$$

Q②

ⓐ The avg. absolute visual magnitude of the globular cluster ~ 3.5

The relation between appearent magnitude and absolute magnitude

$$m - M = 5 \log_{10} \left(\frac{d}{10}\right)$$

Average apperent magnitude of globular clusters ~ 18.5

$$\Rightarrow \quad -3.5 + 18.5 = 5 \log_{10} \left(\frac{d}{10}\right)$$

$$\Rightarrow \quad d = 10 \times 10^{3}\ pc \simeq 10\ kpc$$

Modern dist^n ≃ 8.5 kpc ⟵ good appr^n

ⓑ Clusters are generally objects which are gravitationally bound. So, their distributions are generally confined within very small radius compair to earth and sun distn. and it is hard, Therefore to distinguish their distnce saperatly by parallex. So we approximate all of them at same distance.

a (3) (a) Effective Temp. $T_{eff} \propto (L/R^2)^{1/4}$

For main sequence star, $L \propto M^4$ and $R \propto M$ which yield $T_{eff} \propto M^{1/2}$ i.e. $T_{eff} = \alpha M^{1/2}$ || α = proportional Cont.

We can use the standard parameters of the sun to estimate the value of α which gives

$$T_{eff} = T_{\odot} \left(\frac{M}{M_{\odot}}\right)^{1/2}$$

using Wien's displacement Law $\lambda_{max} \simeq (0.2898/T_{eff})$ cm k,

we get $$\lambda_{max} = \left(\frac{0.2898}{T_{\odot}}\right)\left(\frac{M}{M_{\odot}}\right)^{-1/2} \text{cm}$$

$$= 501.6 \left(\frac{M}{M_{\odot}}\right)^{-1/2} \text{nm}$$

For a star of $9 M_{\odot}$, $\lambda_{max} = 167.2$ nm. Similarly for $M = 0.25 M_{\odot}$, $\lambda_{max} = 1003.2$ nm.

(b) for $9 M_{\odot}$ $\lambda_{max} = 167.2$ nm ultraviolet regime

for $0.25 M_{\odot}$ $\lambda_{max} = 1003.2$ nm infrared and microwave regime

Q(a) The virial theorem $2E_T + E_g = 0$

$2E_T$ → Thermal energy

E_g → gravitational energy

$E_{Total} = E_g + E_T = E_g/2$ ⇒ Half of the gravitational potential energy is being released.

For a star of mass M, the gravitational ~~self~~ self energy $= -\int_0^R \frac{Gm}{r} \times 4\pi r^2 \rho \, dr = \frac{3GM^2}{5R}$

~~while~~

· While ~~accelerating~~ accreting change in total energy will be given as : $\frac{1}{2} \times \Delta E_g$

$$\Rightarrow \Delta E = \frac{1}{2} \times \frac{GM}{R} \Delta M$$

So $$\boxed{\frac{dE}{dt} = L = \frac{GM}{2R} \dot{m}}$$

$\dot{m}$ = mass accretion rate

(b) $$L_{Eddington} = \frac{4\pi G M c m_H}{\sigma_T}$$

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$$\frac{G M \dot{M}_{Edd}}{2R} = \frac{4\pi G M c m_H}{\sigma_T}$$

$$\dot{M}_{Edd} = \frac{8\pi R c m_H}{\sigma_T}$$

(i) $R = R_\odot$, $\sigma_T = 6.65 \times 10^{-29}\ m^2$

$$\dot{M}_{edd} = 1.317 \times 10^{20}\ kg/s$$

(ii) $R = 0.01\ R_\odot$, $\dot{M}_{Edd} = 1.317 \times 10^{18}\ kg\,s^{-1}$

(iii) $R = 2\times 10^{-5}\ R_\odot$, $\dot{M}_{edd} = 2.635 \times 10^{15}\ kg\,s^{-1}$

Super Eddington Limit: for magnetors due to extremely strong magnetic field, the X-ray or X-ray burst can exceed eddington Limit due to work done by decaying magnetic field. For blackhole, since there is no limit of Eddington Limit, accerting energy could be enormous.