

Q.1 For convection, the Schwarzschild stability

$$\text{Cond}^4: \frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \frac{dP}{dr}$$

For ideal gas with mean molecular weight at convective zone, μ then

$$P = \frac{k P T}{\mu m_H}$$

Assuming hydrodynamic equilibrium:

$$\frac{dP}{dr} = - \frac{G M_0}{r^2} \rho$$

$$\text{Now, } \frac{dT}{dr} = - \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \times \frac{G M_0}{r^2} \times \frac{\mu m_H P}{k T}$$

$$\int_{T_0}^T dT = - \frac{2}{5} \int_{r_0}^r \frac{G M_0}{r^2} \frac{\mu m_H}{k} dr$$

$$\Rightarrow T(r) - T_0 = \frac{2}{5} \frac{G M_0 \mu m_H}{k} \left(\frac{1}{r} - \frac{1}{0.7 R_0} \right)$$

$$\Rightarrow T(r) = T_0 + \frac{2}{5} \frac{G M_0 \mu m_H}{k} \left(\frac{1}{r} - \frac{1}{0.7 R_0} \right)$$

$\therefore r > 0.7 R_0 \Rightarrow$ Temp. will decrease from the base of the convective zone.

Let's calculate density: $\frac{dT}{dr} = \frac{2}{5} \frac{T}{P} \frac{dP}{dr}$

$$\frac{dP}{dr} = \frac{k}{\mu m_H} \left(T \frac{dP}{dr} + P \frac{dT}{dr} \right)$$

$$\frac{dT}{dr} = \frac{2}{5} \times \frac{T}{k P T} \cdot \mu m_H \times \frac{k}{\mu m_H} \left(T \frac{dP}{dr} + P \frac{dT}{dr} \right)$$

$$\Rightarrow \frac{dT}{dr} = \frac{2}{5P} \left(T \frac{dP}{dr} + P \frac{dT}{dr} \right)$$

$$\Rightarrow \frac{3}{2} \frac{dT}{T} = \frac{dP}{P}$$

$$\Rightarrow \ln \left(\frac{T(r)}{T_0} \right) = \ln \left(\frac{P(r)}{P_0} \right)^{2/3}$$

$$\Rightarrow P(r) = P_0 \left(\frac{T(r)}{T_0} \right)^{3/2} = P_0 \left[1 + \frac{2}{5} \frac{G M_0 \mu m_H}{k T_0} \left(\frac{1}{r} - \frac{1}{r_0} \right) \right]^{3/2}$$

$\Rightarrow T(r)$ decrease with r , $P(r)$ will also decrease with r .

Pressure calculation: From eqⁿ of hydrodynamic equilibrium,

$$\frac{dP}{dr} = - \frac{G M_0}{r^2} \rho(r)$$

$$\Rightarrow P(r) = P_0 - G M_0 \rho_0 \int_{r_0}^r \frac{\rho(r)}{r^2} dr$$

(b) $X = 0.7$, $Z = 0.02$

~~Z~~ number of particles per unit volume,

$$n = \left(2 \times 0.7 + \frac{0.02}{2} \right) \frac{\rho}{m_H} = \frac{1.401 \rho}{m_H}$$

mean molecular weight = $\frac{1}{1.401} = 0.713$

mean particle mass = $\mu m_H = 1.17 \times 10^{-27} \text{ kg}$

(c) From mass continuity eqⁿ $\frac{dM_r}{dr} = 4\pi r^2 \rho(r)$

Mass of convective zone, $M = \int_{r_0}^{R_0} 4\pi r^2 \rho(r) dr$

$$= \int_{0.7 R_0}^{R_0} 4\pi r^2 \left[1 + 2.277 \times 10^7 \left(\frac{1}{r} - \frac{1}{7 R_0} \right) \right]^{3/2} dr$$

To keep the output value in the real space $R = 6.25 \times 10^8$ has been chosen. Hence we got the mass of convective zone $1.0866 \times 10^{27} \text{ kg}$ which is 0.85% mass of sun, a good agreement.

(d) The solⁿ is not consistent becoz we have varied $\rho(r)$.

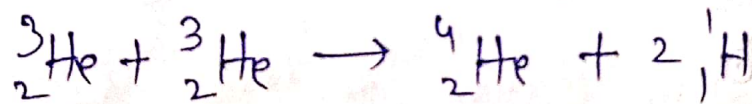
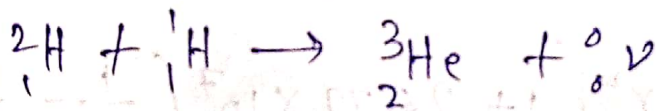
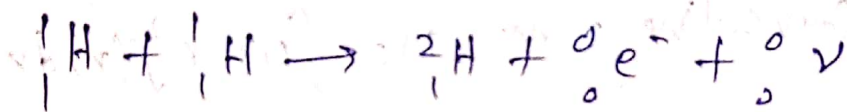
Q3) T @ center of sun : $1.56 \times 10^7 \text{ K}$

$$\rho = 1.46 \times 10^5 \text{ g/m}^3$$

$$H = 0.64, \text{ He} = 0.34, \text{ CNO} = 0.015$$

Amount of energy that is generated at the center of the sun due to the pp chain and CNO cycle.

→ For main sequence stars the most important fusion reaction fuses 4 ^1_1H atom to ^4_2He



The pp chain is most effective for temp around

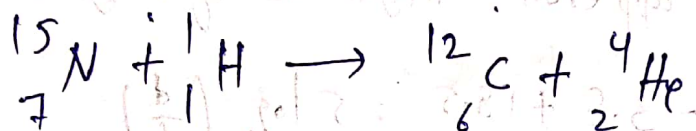
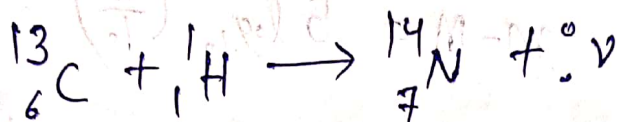
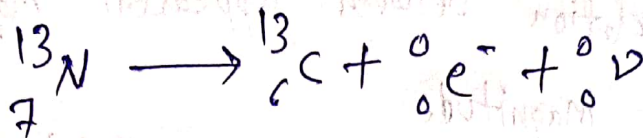
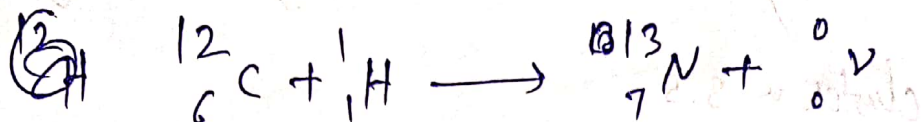
15 million $\epsilon_{pp} \propto \epsilon_{0,pp} \chi_H^2 \rho T_6^4$

$$T = 10^6 T_6 ; T_6 : \text{Temp. in millions of K}$$

$$\epsilon_{0,pp} = 1.08 \times 10^{-12} \text{ W m}^3/\text{kg}^2$$

The another reaction converting ${}^1_1\text{H}$ to ${}^4_2\text{He}$ is the

CNO cycle.



Here total reaction rate

$$\epsilon_{\text{CNO}} = \epsilon_{0, \text{CNO}} X_{\text{H}} X_{\text{CNO}} \rho T_8^{20}$$

where $\epsilon_{0, \text{CNO}} = 8.24 \times 10^{-31} \text{ W m}^3/\text{kg}^2$

$$X_{\text{CNO}} = \frac{M_{\text{CNO}}}{M}$$

Total energy by pp-chain:

for pp $E = E_0 \rho T^4$

$$= (12.86) \times 1.48 \times 10^5 \times (1.56 \times 10^7)^4$$

$$= 112.72 \times 10^{19} \text{ MeV}$$

for CNO cycle $E = E_0 \rho T^{17} = (1.2) \times (1.48 \times 10^5) \times (1.56 \times 10^7)^{17}$

$$= 4.828 \times 10^{32} \text{ MeV}$$

Q2

a) The avg. absolute visual magnitude of the globular cluster ≈ -3.5

The relation between apparent magnitude and absolute magnitude

$$m - M = 5 \log_{10} \left(\frac{d}{10} \right)$$

Average apparent magnitude of globular cluster ≈ 18.5

$$\Rightarrow -3.5 + 18.5 = 5 \log_{10} \left(\frac{d}{10} \right)$$

\Rightarrow

$$d = 10 \times 10^3 \text{ pc} \approx 10 \text{ kpc}$$

Modern distⁿ $\approx 8.5 \text{ kpc}$

good approx.

b) Clusters are generally objects which are gravitationally bound. So, their distributions are generally confined within very small radius compared to earth and sun distance and it is hard, therefore to distinguish their distances separately by parallax. So we approximate all of them at same distance.

a) (3) (e) Effective Temp. $T_{\text{eff}} \propto (L/R^2)^{1/4}$

For main sequence star, $L \propto M^4$ and $R \propto M$ which yield $T_{\text{eff}} \propto M^{1/2}$ i.e. $T_{\text{eff}} \propto M^{1/2}$ // \propto proportional const.

We can use the standard parameters of the sun to estimate the value of α which gives

$$T_{\text{eff}} = T_0 \left(\frac{M}{M_0} \right)^{1/2}$$

using Wien's displacement Law $\lambda_{\text{max}} \propto (0.2898/T_{\text{eff}}) \text{ cm k}$,

we get $\lambda_{\text{max}} = \left(\frac{0.2898}{T_0} \right) \left(\frac{M}{M_0} \right)^{-1/2} \text{ cm}$

$$= 501.6 \left(\frac{M}{M_0} \right)^{-1/2} \text{ nm}$$

For a star of $9 M_0$, $\lambda_{\text{max}} = 167.2 \text{ nm}$. Similarly

for $M = 0.25 M_0$, $\lambda_{\text{max}} = 1003.2 \text{ nm}$.

(b) $\propto 9 M_0$ $\lambda_m = 167.2 \text{ nm}$ ultraviolet regime

$\propto 0.25 M_0$ $\lambda_m = 1003.2 \text{ nm}$ infrared and microwave regime

Q4) The virial theorem $2E_T + E_g = 0$

Thermal energy

gravitational energy

$$E_{\text{Total}} = E_g + E_T = E_g/2 \Rightarrow \text{Half of the gravitational potential energy is being released.}$$

For a star of mass M , the gravitational self energy

$$\text{energy} = - \int_0^R \frac{Gm}{r} \times 4\pi r^2 \rho dr = \frac{3GM^2}{5R}$$

While

~~accelerating~~ accretion change in total energy will

$$\text{be given as: } \frac{1}{2} \times \Delta E_g$$

$$\Rightarrow \Delta E = \frac{1}{2} \times \frac{GM}{R} \Delta M$$

$$\text{So } \left| \frac{dE}{dt} \right| = L = \frac{GM}{2R} \dot{m} \quad \dot{m} = \text{mass accretion rate}$$

(b)

$$L_{\text{Eddington}} = \frac{4\pi GM c m_H}{\sigma_T}$$

$$\frac{G M \dot{M}_{\text{Edd}}}{2R} = \frac{4\pi G M c m_H}{\sigma_T}$$

$$\dot{M}_{\text{Edd}} = \frac{8\pi R c m_H}{\sigma_T}$$

$$(i) \quad R = R_0, \quad \sigma_T = 6.65 \times 10^{-29} \text{ m}^2$$

$$\dot{M}_{\text{Edd}} = 1.317 \times 10^{20} \text{ kg/s}$$

$$(ii) \quad R = 0.01 R_0, \quad \dot{M}_{\text{Edd}} = 1.317 \times 10^{18} \text{ kg/s}$$

$$(iii) \quad R = 2 \times 10^{-5} R_0, \quad \dot{M}_{\text{Edd}} = 2.635 \times 10^{15} \text{ kg/s}$$

Super Eddington Limit: for magnetars due to extremely strong magnetic field, the X-ray or X-ray burst can exceed eddington Limit due to work done by decaying magnetic field. For blackhole, since there is no limit of Eddington Limit, accreting energy could be enormous.