# Class 1: Equations of stellar structure I

## Reading project

### • Formats:

- ◆ While there is more pressure in the **presentation** format, it is also easier in the sense that it requires much less delivery of content. However, you must be able to answer any question, and do so "on the fly".
- ◆ The **paper** format requires you to use the MNRAS template, which may be a bit tricky. Particularly if you use Latex and natbib. These are not strictly required by MNRAS, but they (and I) strongly suggest using Latex, at least.
- ◆ The website format is kind of in between, in that it allows you the flexibility of a talk, but does not require you to do anything on the fly. However, you should make good use of the features found in a normal website, such as links between the sections or to external references, and movies or animations if appropriate.

- <u>Structure</u> (I have mostly papers in mind here but can be generalized to other formats as well)
  - ◆ Title: Catchy and smart is okay, or you could just use your topic as the title
    - Example: "Magnetic fields in galaxies"

#### + Abstract (<200 words):

- → about 1 sentence introducing the problem (why is it relevant?)
  - Example: "Magnetic fields in the interstellar media of nearby spiral galaxies are in approximate energy equipartition with thermal motions, turbulence and cosmic rays, making them dynamically important"
- → about 1 sentence giving background (what was already known?)
  - "The most basic features of these fields can be understood in the context of dynamo models."
- → about 1 sentence stating the problem (what is needed?)

- "However, quantitative comparison between theory and observation — needed to validate or reject theoretical models — is still lacking."
- → about 1 sentence on what the paper is about (what did I do?)
  - "Here I review the literature in this field to better map out current observational and theoretical knowledge, as well as the existing comparisons between them."
- → about 1 sentence on results (what did I find?)
  - "I find that estimates of magnetic field properties from observations and theory rely on different assumptions, which complicates their comparison."
- about 1 sentence on what the future holds (what is needed?)
  - "This problem could be alleviated in the future if theoretical models employ methods of averaging that better approximate the implicit averaging in observations"

#### + Introduction

- → History of the field (what came before?)
- Motivation for studying this topic (why is this subfield relevant for researchers in other subfields of astrophysics?)
- → Aim of the present work (what am I trying to accomplish with this work?)

#### + Methods

- → Observational techniques (in the example of magnetic fields in galaxies, I would explain how observations are done, and how they are analyzed to get information about the magnetic fields)
- → Theoretical techniques (here I would discuss the basics of dynamo theory, as applied to galaxies. Equations and derivations are needed here.)
- ◆ **Results** (if you choose to omit the discussion section, you would want to include more discussion here, interspersed between the reporting of various results)

- Explanation of current observational results (here I would summarize the key findings from the observations, e.g. showing a histogram of magnetic field strength for nearby galaxies, from a review paper)
- → Explanation of current theoretical results (here I would summarize various galactic dynamo models in the literature)

#### **→ Discussion** (optional)

→ Discussion about what we know from theory and observation together (here I would discuss comparisons between the two, and what is learned from those comparisons)

#### **+** Conclusions:

- → Restate the motivation and aim
- **→** Summary of methods
- → Summary of key results
- → Sentence or two that ties everything together
- → Statement about what is needed in the future

## Choudhuri problem 2.2/ Rybicki & Lightman problem 1.1

1.1—A "pinhole camera" consists of a small circular hole of diameter d, a distance L from the "film-plane" (see Fig. 1.14). Show that the flux  $F_{\nu}$  at the film plane depends on the brightness field  $I_{\nu}(\theta, \phi)$  by

$$F_{\nu} = \frac{\pi \cos^4 \theta}{4f^2} I_{\nu}(\theta, \phi),$$

where the "focal ratio" is f = L/d. This is a simple, if crude, method for measuring  $I_{\nu}$ .

## Rybicki & Lightman problem 1.2

1.2—Photoionization is a process in which a photon is absorbed by an atom (or molecule) and an electron is ejected. An energy at least equal to the ionization potential is required. Let this energy be  $h\nu_0$  and let  $\sigma_{\nu}$  be the cross section for photoionization. Show that the number of photoionizations per unit volume and per unit time is

$$4\pi n_a \int_{\nu_0}^{\infty} \frac{\sigma_{\nu} J_{\nu}}{h\nu} d\nu = c n_a \int_{\nu_0}^{\infty} \frac{\sigma_{\nu} u_{\nu}}{h\nu} d\nu,$$

where  $n_a$  = number density of atoms.

## Equations of stellar structure

- Assume spherical symmetry valid?
- Assume that the star is in equilibrium, so no time-dependence valid?
- Let m(r) be the mass interior to radius r
- Then the mass of an infinitesimal shell is  $dm = 4\pi r^2 dr \rho(r)$ . Then

$$\frac{dm}{dr} = 4\pi r^2 \rho \tag{1}$$

• Special case of the continuity equation of hydrodynamics (homework)

• Spherical symmetry is not perfect owing to the rotation of the star, but it is not a bad approximation

- Consider a small portion of a shell between r and r + dr
- Pressure force on inward surface at radius *r* is *P dA*
- Pressure force on outward surface at radiuas r + dr is -(P + dP) dA
- Net force is -dP dA
- Since star is in equilibrium, this force is balanced by gravity:

$$-dP dA - \frac{Gm(r)}{r^2} \rho(r) dr dA = 0$$

• Thus, we can write

$$\frac{dP}{dr} = -\frac{Gm(r)}{r^2}\rho(r) \tag{2}$$

- Equation of hydrostatic equilibrium
- Special case of the Euler equation of hydrodynamics (homework)
- Equation of HE can be written as

$$\frac{dP}{dr} = -\rho g$$

with 
$$g(r) = \frac{Gm(r)}{r^2}$$