Assignment 1. Posted on 20.02.2022; Due on 05.04.2022 Marks 25

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1. Given below is a system of linear equations. Use Gauss-Jordon, and LU decomposition to solve it. **[5]**

$$
\begin{aligned}
19 &= a_1 - a_2 + 4a_3 + 2a_5 + 9a_6 \\
2 &= 5a_2 - 2a_3 + 7a_4 + 8a_5 + 4a_6 \\
13 &= a_1 + 5a_3 + 7a_4 + 3a_5 - 2a_6 \\
-7 &= 6a_1 - a_2 + 2a_3 + 3a_4 + 8a_6 \\
-9 &= -4a_1 + 2a_2 + 5a_4 - 5a_5 + 3a_6 \\
2 &= 7a_2 - a_3 + 5a_4 + 4a_5 - 2a_6
\end{aligned}
$$

2. Solve the following *almost* sparse system $\mathbf{A}\ \mathbf{x} = \mathbf{b}$ using LU and Jacobi. Use Jacobi, Gauss-Seidel and Conjugate Gradient to find the inverse of the matrix $\mathbf{A}$. Compare their convergence, may be using residue versus iteration steps plot. You may need storage for this matrix. Choose $\epsilon = 10^{-4}$ for convergence. **[10]**

$$
\begin{pmatrix}
2 & -3 & 0 & 0 & 0 & 0 \\
-1 & 4 & -1 & 0 & -1 & 0 \\
0 & -1 & 4 & 0 & 0 & -1 \\
0 & 0 & 0 & 2 & -3 & 0 \\
0 & -1 & 0 & -1 & 4 & -1 \\
0 & 0 & -1 & 0 & -1 & 4
\end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}
=
\begin{pmatrix} -5/3 \\ 2/3 \\ 3 \\ -4/3 \\ -1/3 \\ 5/3 \end{pmatrix}
$$

3. Find the inverse of the following matrix using Conjugate Gradient by generating the matrix on fly *i.e.* without actually storing it. It is a two-dimensional 20×20 system with periodic boundary condition Plot the residue versus iteration steps. Use $m = 0.2$ and convergence criteria to be $\epsilon = 10^{-6}$. **[10]**

$$
\mathbf{A} = \frac{1}{2}\Big(\delta_{x+\mu,y} - \delta_{x-\mu,y} + 2\delta_{x,y}\Big) + m^2\delta_{x,y}
$$