Assignment 2. Posted on 30.03.2022; Due on 12.04.2022 Marks 20

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1. The data from an experiment is given in the file `assign2fit.txt`. Try a cubic least square fit using

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Try fitting the data set with a modified basis

$$\begin{aligned} y = f(x) &= a_0\phi_0(x) + a_1\phi_1(x) + a_2\phi_2(x) + a_3\phi_3(x) \\ \text{where,} \quad & \phi_0(x) = 1, \quad \phi_2(x) = 2x - 1, \quad \phi_3(x) = 8x^2 - 8x + 1 \\ & \phi_3(x) = 32x^3 - 48x^2 + 18x - 1 \end{aligned}$$

where the above functions ϕ_α are modified Chebyshev functions. Can you explain the difference by comparing the relative merit of using different functions calculating the condition number? (You may use online condition number or norm calculator.) **[10]**

2. Generate your own random numbers in appropriate range using *multiplicative linear congruential generator* defined by

$$x_n = (a\, x_{n-1}) \bmod m$$

using (i) $a = 65$, $m = 1021$ and (ii) $a = 572$, $m = 16381$, determine the value of π both by (a) throwing points and (b) by solving the integral given below by Monte Carlo

$$\frac{\pi}{4} = \int_0^1 dx\, \sqrt{1 - x^2}$$

Comment on the choice of a, m based on your value of π. Use your own seed. **[10]**

3. Using the MLCG defined in problem 2 with the parameter set you think is best, determine by Monte Carlo method the volume of *Steinmetz solid*, the intersection of two perpendicular cylinders each of radius 1 unit. **[5]**