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who has worked for the year 2019-20 in the Computer Laboratory.

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Head of Department

Date :________________

Examiner

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SEM II

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25.11.19

PRACTICAL - 1

Aim :- Limits & Continuity.

1.) $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

2.) $$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$$

3.) $$\lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$$

4.) $$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$$

5.) Examine the continuity of the following function:

(i) $$f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & 0 < x \le \pi/2 \\ \dfrac{\cos x}{\pi - 2x}, & \pi/2 < x < \pi \end{cases} \quad \text{at } x = \pi/2$$

(ii) $$f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & 0 < x < 3 \\ x+3, & 3 \le x < 6 \\ \dfrac{x^2-9}{x+3}, & 6 \le x < 9 \end{cases}$$

at $x = 3$
& $x = 6$

6.) Find values of K, so that the function $f(x)$ is continuous at the indicated point.

(i) $f(x) = \frac{1-\cos 4x}{x^2}$, $x<0$
$= K$, $x=0$ $\Big\}$ at $x=0$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$, $x \neq 0$
$= K$, $x=0$ $\Big\}$ at $x=0$

(iii) $f(x) = \frac{\sqrt{3}-\tan x}{\pi - 3x}$, $x \neq \pi/3$
$=$, $x = -\pi/3$ $\Big\}$ at $x = \pi/3$

7.) Discuss the continuity of the following function which of these function have removable discontinuity? Redefine funⁿ so as to remove the discontinuity.

(i) $f(x) = \frac{1-\cos 3x}{x\tan x}$, $x \neq 0$
$= 9$, $x=0$ $\Big\}$ at $x=0$

(ii) $f(x) = \frac{(e^{3x}-1)\sin x}{x^2}$, $x \neq 0$
$= \pi/60$, $x=0$ $\Big\}$ at $x=0$

8.) If $f(x) = \frac{e^{x^2}-\cos x}{x^2}$ for $x \neq 0$ is continuous at $x=0$ find $f(0)$

9.) If $f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$, for $x \neq \pi/2$.

is continuous at $x = \frac{\pi}{2}$.

find $f(\pi/2)$

SOLUTIONS :-

1.) $\lim\limits_{x\to a} \left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$\Rightarrow \lim\limits_{x\to a} \left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$

$\lim\limits_{x\to a} \frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$

$\lim\limits_{x\to a} \frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$

$\lim\limits_{x\to a} \frac{\cancel{(a-x)}(\sqrt{3a+x}+2\sqrt{x})}{3\cancel{(a-x)}(\sqrt{a+2x}+\sqrt{3x})}$

$\frac{1}{3}\lim\limits_{x\to a} \left(\frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}\right)$

$\Rightarrow \frac{1}{3}\left(\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}\right)$

$\Rightarrow \frac{1}{3}\left(\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}\right)$

$\Rightarrow \frac{1}{3}\left(\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}\right)$

$\Rightarrow \frac{1}{3}\left(\frac{\cancel{4}\sqrt{a}}{\cancel{2}\sqrt{3a}}\right) \Rightarrow \frac{2\sqrt{a}}{3\sqrt{3a}} \Rightarrow \boxed{\frac{2\sqrt{a}}{3\sqrt{3a}}}$

2.) $$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$\Rightarrow \lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$\lim_{y\to 0}\left(\frac{\cancel{a}+y-\cancel{a}}{(y\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}\right)$$

$$\lim_{y\to 0}\left(\frac{\cancel{y}}{(\cancel{y}\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}\right)$$

$$\lim_{y\to 0}\left(\frac{1}{(\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}\right)$$

$$\lim_{y\to 0}\left(\frac{1}{(\sqrt{a+y})(\sqrt{a+y}+\sqrt{a})}\right)$$

$$\Rightarrow \frac{1}{(\sqrt{a+0})(\sqrt{a+0}+\sqrt{a})}$$

$$\Rightarrow \frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$$

$$\Rightarrow \frac{1}{\sqrt{a}(2\sqrt{a})}$$

$$\Rightarrow \boxed{\frac{1}{2a}}$$

3.) $\lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3} \sin x}{\pi - 6x} \right]$

$= \lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3} \sin x}{\pi - 6x} \right]$ By substituting

$x - \pi/6 = h \Rightarrow x = h + \pi/6$

where $h \to 0$

$\lim_{h \to 0} \left(\frac{\cos(h + \pi/6) - \sqrt{3} \sin(h + \pi/6)}{\pi - 6(h + \pi/6)} \right)$

using $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

& $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\lim_{h \to 0} \left(\frac{\cos h \cdot \cos \pi/6 - \sin h \cdot \sin \pi/6 - \sqrt{3} (\sin h \cdot \cos \pi/6 + \cos h \cdot \sin \pi/6)}{\pi - \cancel{6} (6h + \pi/\cancel{6})} \right)$

$\lim_{h \to 0} \left(\frac{\cos h \cdot \sqrt{3}/2 - \sin h \cdot 1/2 - \sqrt{3} (\sin h \cdot \sqrt{3}/2 + \cos h \cdot 1/2)}{\cancel{\pi} - 6h + \cancel{\pi}} \right)$

$\lim_{h \to 0} \left(\frac{\cancel{\cos h \cdot \sqrt{3}/2} - \sin h/2 - \sin h \cdot 3/2 - \cancel{\cos h \cdot \sqrt{3}/2}}{-6h} \right)$

$\lim_{h \to 0} \frac{\cancel{-} \sin 4h/2}{\cancel{-} 6h}$

$\lim_{h \to 0} \frac{\sin h}{3h} = \frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = \boxed{\frac{1}{3}}$

4) $$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$$

= By rationalizing Num. & Denom. both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right]$$

$$\lim_{x\to\infty}\left(\frac{(\cancel{x^2}+5-\cancel{x^2}+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\cancel{x^2}+3-\cancel{x^2}-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right)$$

$$\lim_{x\to\infty}\ \frac{\overset{4}{\cancel{8}}}{\cancel{2}}\left(\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right)$$

$$4\lim_{x\to\infty}\left(\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}\right)$$

$$\therefore\ \infty = 1/0$$

$$\therefore\ 4\lim_{x\to\infty}\left(\frac{\cancel{x^2}}{\cancel{x^2}}\ \frac{\sqrt{(1+3/x^2)}+\sqrt{1+1/x^2}}{\sqrt{1+5/x^2}+\sqrt{1-3/x^2}}\right)$$

$$\therefore = 4\times\frac{1+1}{1+1}$$

$$= 4\times\frac{\cancel{2}}{\cancel{2}}$$

$$= \boxed{4}$$

5) i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, $0 < x \le \pi/2$
$= \dfrac{\cos x}{\pi - 2x}$, $\pi/2 < x < \pi$ $\Big\}$ at $x = \pi/2$

$\because f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \pi/2$ define

$\Rightarrow \lim\limits_{x\to\pi/2^+} f(x) = \lim\limits_{x\to\pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting method

$x - \pi/2 = h \Rightarrow x = h + \pi/2$
where $h \to 0$

$\lim\limits_{h\to 0} \dfrac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$

$\lim\limits_{h\to 0} \dfrac{\cos(h+\pi/2)}{\cancel{\pi} - 2h - \cancel{\pi}}$

$\lim\limits_{h\to 0} \dfrac{\cos(h+\pi/2)}{-2h}$

$\lim\limits_{h\to 0} \dfrac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$ using $\cos(A+B)$ formula.

$\lim\limits_{h\to 0} \dfrac{\cos h \cdot 0 - \sin h}{-2h}$

$\lim\limits_{h\to 0} \dfrac{\cancel{-}\sin h}{\cancel{-}2h}$

$\dfrac{1}{2} \lim\limits_{h\to 0} \dfrac{\sin h}{h} \Rightarrow \dfrac{1}{2} \cdot 1 = \dfrac{1}{2}.$

$$\Rightarrow \lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

using $\sin 2x = 2\sin x . \cos x$

$$\lim_{x \to \pi/2^-} \frac{2\sin x . \cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x \to \pi/2^-} \frac{2\cancel{\sin x} . \cos x}{\sqrt{2}\, \cancel{\sin x}}$$

$$\lim_{x \to \pi/2^-} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$$

$$= 0$$

$\therefore$ LHL $\neq$ RHL

$\therefore$ f is not continous at $x = \pi/2$

5 ii)
$$f(x) = \left.\begin{array}{ll} \frac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \leq x \leq 6 \\ = \frac{x^2-9}{x+3} & 6 \leq x < 9 \end{array}\right\} \text{at } x = 3, \& \ x = 6$$

$$\Rightarrow \circ \ f(3) = \frac{x^2-9}{x-3} = 0$$

f at $x=3$ define

$$\circ \ \cancel{\lim_{x \to 3^+} f(x)} = \lim_{x \to 3^+} x+3$$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \lim_{x\to 3^-} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$= \lim_{x\to 3^-} (x+3) = x+3 = 3+3 = 6$

$\therefore$ LHL = RHL.

f is continous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = 3$

$\lim_{x\to 6^+} \frac{x^2-9}{x+3} = \lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6^+} \cancel{x-3} = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 6+3 = 9$

$\therefore$ LHL $\neq$ RHL

f is not continous at $x=6$

6.) i) $\because$ f is continous at $x=0$

$$\lim_{x\to0} f(x) = f(0)$$

$$\lim_{x\to0} \frac{1-\cos4x}{x^2} = K$$

$$\lim_{x\to0} \frac{2\sin^2 2x}{x^2} = K$$

$$2\lim_{x\to0} \frac{\sin^2 2x}{x^2} = K$$

$$2\lim_{x\to0} \left(\frac{\sin2x}{x}\right)^2 = K$$

$$\therefore \quad 2(2)^2 = K$$

$$\boxed{8 = K}$$

ii) $\because$ f is continous at $x=0$

$$\lim_{x\to0} f(x) = f(0)$$

$$\lim_{x\to0} (\sec^2 x)^{\cot^2 x} = K$$

using $\tan^2 x - \sec^2 x = 1$

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& $\cot^2 x = \frac{1}{\tan^2 x}$

$\therefore \lim_{x \to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

we know that

$$\lim_{x \to 0} (1+px)^{\frac{1}{px}} = e$$

$\therefore = e.$

$\boxed{K = e}$

iii) $\therefore f$ is continous at $x = \pi/3$

$\therefore \lim_{x \to \pi/3} f(x) = f(\pi/3)$

$$\lim_{x \to \pi/3} \frac{\sqrt{3} - \tan x}{\pi - 3x} = K$$

$\therefore \quad x - \pi/3 = h \quad \Rightarrow x = h + \pi/3$

where $h \to 0$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan(h+\pi/3)}{\pi - 3(h+\pi/3)}$$

using $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$

$$= \lim_{h \to 0} \frac{\sqrt{3} - \frac{\tan h + \tan \pi/3}{1 - \tan h . \tan \pi/3}}{\cancel{\pi} - 3h - \cancel{\pi}}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{3}(1 - \tan \pi/3 . \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 . \tan h}}{-3h}.$$

using $\tan \pi/3 = \tan 60 = \sqrt{3}$

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$$= \lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3}\times\sqrt{3}\,.\tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\,.\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\cancel{-}\,4\tan h}{\cancel{-}\,3h\,(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0} \frac{\tan h}{h} \quad \lim_{h\to 0} \frac{1}{1 - \sqrt{3}\tan h}$$

$$= \frac{4}{3} \cdot \frac{1}{1 - \sqrt{3}(0)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) \quad = \frac{4}{3}$$

$$\boxed{K = 4/3}$$

7i) $\therefore f(x) = \dfrac{1 - \cos 3x}{x \tan x}$

$$\lim_{x\to 0} \frac{2 \sin^2 3/2\, x}{x \tan x}$$

$$\lim_{x\to 0} \frac{2\, \frac{\sin^2 3x/2}{x^2} \times x^2}{x \cdot \frac{\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x\to 0} \frac{(3/2)^2}{1} = \cancel{2} \times \frac{9}{\cancel{4}} = \frac{9}{2}$$

$$\therefore \lim_{x\to 0} f(x) = \frac{9}{2}. \quad \therefore \quad 9 = f(0)$$

$\therefore$ f is not continous at $x=0$

Redefined function

$$f(x) = \left.\begin{array}{ll} \dfrac{1-\cos 3x}{x \tan x} & x \neq 0 \\ = \dfrac{9}{2} & x = 0 \end{array}\right\} x = 0$$

Now $\lim_{x\to 0} f(x) = f(0)$

f has remouable discontinuity at $x=0$

ii) $\therefore \lim_{x \to 0} (e^{3x} - 1) \frac{\sin(\pi x/180)}{x^2}$

$\lim_{x \to 0} \frac{e^{3x} - 1}{x} \cdot \lim_{x \to 0} \frac{\sin(\pi x/180)}{x}$

$\lim_{x \to 0} 3 \cdot \frac{e^{3x} - 1}{3x} \cdot \lim_{x \to 0} \frac{\sin(\pi x/180)}{x}$

$3 \lim_{x \to 0} \frac{e^{3x} - 1}{3x} \cdot \lim_{x \to 0} \frac{\sin(\pi x/180)}{x}$

$\cancel{3} \log_e \frac{\pi}{\cancel{180}} = \frac{\pi}{60} = f(0)$

$\therefore$ f is continous at $x = 0$

8.) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$, $x = 0$

is continuous at $x = 0$

Given,

f is cont. at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$$\lim_{x\to0} \frac{e^{x^2}-\cos x-1+1}{x^2}$$

$$\lim_{x\to0} \frac{(e^{x^2}-1)+(1-\cos x)}{x^2}$$

$$\lim_{x\to0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to0} \frac{(1-\cos x)}{x^2}$$

$$\log e + \lim_{x\to0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply $\cancel{2}$ on num. & Denom.

$$= 1 + \cancel{2}\times\frac{1}{\cancel{4}_2}$$

$$= \frac{3}{2} = f(0)$$

9.) $$f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$$

f(0) is continous at $x=\pi/2$

$$\lim_{x\to\pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$\lim_{x\to\pi/2} \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

S.P

$$\lim_{x\to\pi/2} \frac{1+\sin x}{\cos^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to\pi/2} \frac{1+\sin x}{1-\sin^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to\pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to\pi/2} \frac{1}{(1-\sin x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \frac{1}{2\left(\sqrt{2}+\sqrt{2}\right)}$$

$$= \frac{1}{2\left(2\sqrt{2}\right)}$$

$$= f(\pi/2) = \frac{1}{4\sqrt{2}} \;//$$

02/12/19

2.12.19

PRACTICAL - 2.

Topic :- Derivative.

Q.1) Show that the following function defined from R to R are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x-a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)(\tan x \tan a)}$$

put $x - a = h$

$x = h + a$ as $x \to a$, $h \to 0$

$$Df(h) = \lim_{x \to a} \frac{\tan a - \tan(a+h)}{(\cancel{a} + h - \cancel{a})(\tan(a+h) \tan a)}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

formula : $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A \tan B)$

$$= \lim_{h\to 0} \frac{\tan(\cancel{a} - \cancel{a} - h) - (1 + \tan a \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} -\frac{\tanh}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cancel{\cos^2} a} \times \frac{\cancel{\cos^2} a}{\sin^2 a}$$

$$= -\cos^2 a$$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in R$

ii) cosec x

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$

$x = a + h$ as $x \to a \Rightarrow h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(\cancel{a} + h - \cancel{a})(\sin a \sin(a+h))}$$

formula :

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos(2a+h/2)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= -\frac{1}{\cancel{2}} \times \frac{\cancel{2}\cos(2a+0/2)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

$\therefore$ $Df(a) = -\cot a \operatorname{cosec} a$

$\therefore$ f is differentiable $\forall a \in R$.

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iii) $\sec x$

$= f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)(\cos a \cos x)}$$

put $x - a = h$.

$x = a + h$ as $x \to a \Rightarrow h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

formula : $-2\sin((C+D/2)) \sin((C-D/2))$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \sin -\frac{h}{2}}{\cos a \cos(a+h) \times -\frac{h}{2}} \times -\frac{1}{2}$$

$$= \frac{-\frac{1}{\cancel{2}} \times -\cancel{2}\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$= \tan a \sec a$

$\therefore Df(a) = \tan a \sec a$

$\therefore f$ is differentiable $\forall a \in R$.

Q.2) If $f(x) = 4x+1$, $x \le 2$
$= x^2+5$, $x > 0$ at $x = 2$, then find function is differentiable or not.

Solution :

LHD =

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$\therefore Df(2^-) = 4$

RHD =

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2^+} x + 2$$

$$= 2 + 2 = 4$$

$\therefore \; Df(2^+) = 4$

$\therefore$ RHD = LHD

$\therefore$ f is differentiable at $x = 2$

Q.3) If $f(x) = 4x + 7 \quad , \quad x < 3$

$\quad\quad\quad\;\; = x^2 + 3x + 1 \quad , \quad x \geq 3$

at $x = 3$, then find f is differentiable or not.

Solution:

RHD :

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 \cdot 3 + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$= \lim_{x \to 3^+} x+6 = x+6 = 3+6 = 9$$

$Df(3^+) = 9$

LHD : $Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$\therefore Df(3^-) = 4$

$\therefore$ RHD $\neq$ LHD

$\therefore$ f is not differentiable at $x=3$

Q.4) If $f(x) = 8x-5$, $x \leq 2$

$= 3x^2 - 4x + 7$, $x > 2$ at $x=2$.

then find f is differentiable or not

Solution: $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD:

$$Df(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3\times 2+2 = 8$$

$$Df(2^+) = 8$$

LHD:

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 8$$

$\therefore Df(2^-) = 8$

$\therefore$ LHD = RHD

$\therefore f$ is differentiable at $x=3$

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PRACTICAL - 3

Topic :- Applications of Derivation.

1) Find the intervals in which function is increasing or decreasing :-

i) $f(x) = x^3 - 5x - 11$
ii) $f(x) = x^2 - 4x$
iii) $f(x) = 2x^3 + x^2 - 20x + 4$
iv) $f(x) = x^3 - 27x + 5$
v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards :-

i) $y = 3x^2 - 2x^3$
ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$
iii) $y = x^3 - 27x + 5$
iv) $y = 69 - 24x - 9x^2 + 2x^3$
v) $y = 2x^3 + x^2 - 20x + 4$

Solution :-

Q.1 i) $f(x) = x^3 - 5x - 11$

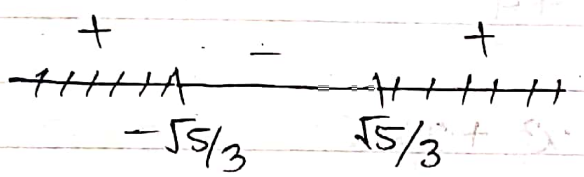
$\therefore \quad f'(x) = 3x^2 - 5$

$\because$ f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x + 5/3)(x - 5/3) > 0$



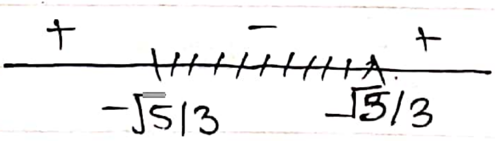
$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$

and f is decreasing iff $f(x) < 0$

$\therefore \quad 3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$(x - 5/3)(x + 5/3) < 0$



$x \in (-\sqrt{5}/3, \sqrt{5}/3)$

ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore$ f is increasing iff $f(x) > 0$

$\therefore \quad 2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

$\therefore \quad x \in (2, \infty)$

and f is decreasing iff $f(x) < 0$

$\therefore \quad 2x - 4 < 0$

$\therefore \quad 2(x-2) < 0$

$x - 2 < 0$

$\therefore \quad x \in (-\infty, 2)$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing iff $f(x) > 0$

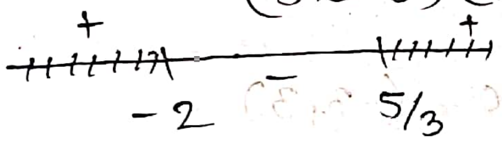
$\therefore \quad 6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(3x-5)(x+2) > 0$



$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f(x) < 0$

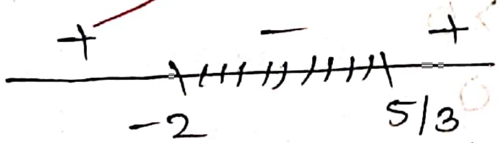
$6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(3x-5)(x+2) < 0$



$x \in (-2, 5/3)$

iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because$ f is increasing iff $f(x) > 0$

$$3x^2 - 27 > 0$$
$$3(x^2 - 9) > 0$$
$$(x-3)(x+3) > 0$$

$x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f(x) < 0$

$$3x^2 - 27 < 0$$
$$3(x^2 - 9) < 0$$
$$(x-3)(x+3) < 0$$

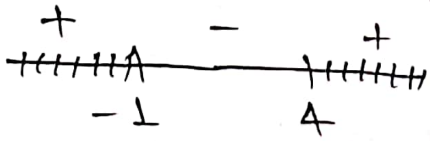
$\therefore x \in (-3, 3)$

v) $f(x) = 2x^3 - 9x^2 - 24x - 69$

$f'(x) = 6x^2 - 18x - 24$

$\because$ f is increasing iff $f(x) > 0$

$$6x^2 - 18x - 24 > 0$$
$$6(x^2 - 3x - 4) > 0$$
$$(x^2 - 4x + x - 4) > 0$$
$$x(x-4) + 1(x-4) > 0$$
$$(x-4)(x+1) > 0$$

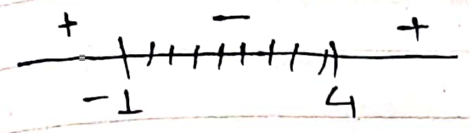


$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f(x) < 0$

$$6x^2 - 18x - 24 < 0$$
$$x^2 - 3x - 4 < 0$$
$$x^2 - 4x + x - 4 < 0$$

$$x(x-4)+1(x-4)<0$$
$$(x-4)(x+1)<0$$



$\therefore\ x \in (-1,4)$

Q.2) i) $y = 3x^2 - 2x^3$

$\therefore\ f(x) = 3x^2 - 2x^3$
$f'(x) = 6x - 6x^2$
$f''(x) = 6 - 12x$

f is concave upward iff $f''(x) > 0$
$$(6-12x) > 0$$
$$-12(-6/12 + x) > 0$$
$$(-1/2 + x) > 0$$
$$(x - 1/2) > 0$$
$$x > 1/2$$
$\therefore\ f''(x) > 0 \quad \therefore\ x \in (1/2, \infty)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$
$f'(x) = 4x^3 - 18x^2 + 24x + 5$
$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$
$$12x^2 - 36x + 24 > 0$$
$$12(x^2 - 3x + 2) > 0$$
$$x^2 - 3x + 2 > 0$$
$$x^2 - 2x - x + 2 > 0$$
$$x(x-2) - 1(x-2) > 0$$
$$(x-1)(x-2) > 0$$

CA:

$+ \quad - \quad +$

$1 \quad 2$

$\therefore x \in (-\infty, 1) \cup (2, \infty)$

iii) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore \quad 6x > 0$

$x > 0$

$\therefore \quad x \in (0, \infty)$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore \quad 12x - 18 > 0$

$12(x - 18/12) > 0$

$x - 3/2 > 0$

$x > 3/2$

$\therefore \quad x \in (3/2, \infty)$

v) $y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$12x + 2 > 0$

$12x + 2 > 0$

$$12(x + 2/12) > 0$$

$$x + 1/6 > 0$$

$$x < -1/6$$

i.e. $f''(x) > 0 \quad \therefore x \in (-1/6, \infty)$

$\therefore$ ~~Therefore~~ interval exists

& It is concave upwards.

AR
6/12/19

PRACTICAL - 4

Topic :- Applications of Derivation & Newton's Method

Q.1) Find maximum & minimum value of following function :-

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$ be minima = 1 maxima = 5

iii) $f(x) = x^3 - 3x^2 + 1$ in $[-\frac{1}{2}, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2) Find the root of following equation by Newton's method (Take 4 iteration only) correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Solution :-

Q.1) i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider , $f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$= 2 + 96/x^4$

$f''(2) = 2 + 96/(2)^4$

$= 2 + \frac{96}{16}$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore$ $f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore$ Function reaches minimum value at $x = 2$ & $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider,

$$f'(x) = 0$$

$$-15x^2 + 15x^4 = 0$$

$$15x^4 = 15x^2$$

$$x^4 = x^2$$

$$x^2 = 1$$

$$x = \pm 1$$

$f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0$ $\therefore f$ has minimum value at $x = 1$

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum value at $x = 1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$.

iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x.$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x - 2 = 0$ or $3x = 0$

$\therefore x = 2$ or $x = 0$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore f$ has maximum value at $x = 0$

$f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6x - 6$

$= 12 - 6$

$= 6 > 0$

f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore f$ has maximum value 1 at $x = 0$ &

f has minimum value -3 at $x = 2$

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iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x+1)(x-2) = 0$

$x = 2, -1.$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$ $\therefore f$ has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -18 < 0$ f has maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

f has max. value 8 at $x = -1$ &

f has min. value -18 at $x = 2$

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Q.2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$$x_1 = 0.1727$$

$$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$
$$= -0.0829$$

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$
$$= -55.9467$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0829}{55.9467}$$

$$= 0.1712$$

$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$
$$= 0.0011$$

$$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$$
$$= -55.9393$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.1712 - \frac{0.0011}{55.9393}$$

$$= 0.1712$$

$\therefore$ The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_0 = 3$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{23}$$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 0.5960$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 18.5096$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7392 - \frac{0.5960}{18.5096}$$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$
$= 17.9851$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7071 - \frac{0.0102}{17.9851}$$

$$= 2.7015$$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$
$= 17.8943$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.7015 + \frac{0.0901}{17.8943}$$

$$= 2.7065$$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= -2.2$

Let $x_0 = 2$ be the initial approximation

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{2.2}{5.2}$$

$$= 1.5770$$

$$f(x_1) = (1.5770)^3 - 1.8(1.5770)^2 - 10(1.5770) + 17$$

$$= 0.6755$$

$$f'(x_1) = 3(1.5770)^2 - 3.6(1.5770) - 10$$

$$= -8.2164$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5770 + \frac{0.6755}{8.2164}$$

$$= 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592) - 10(1.6592) + 17$$

$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$

$$= -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.6592 + \frac{0.0204}{7.7143}$$

$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - (1.8)(1.6618)^2 - 10(1.6618) + 17$$

$$= 0.0004$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618)^2 - 10$$

$$= -7.6977$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.6618 + \frac{0.0004}{7.6977}$$

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$$= 1.6618$$

$\therefore$ The root of the equation is 1.6618

PRACTICAL - 5

Q.1 Solve the following Integration

1.) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ d

$= \int \frac{1}{\sqrt{x^2+2x(1)-3}} dx$

$= \int \frac{1}{\sqrt{x^2+2x(1)+(1)^2-(1)^2-3}} dx$ [Add. & sub. $(1)^2$]

$= \int \frac{1}{\sqrt{x^2+2x(1)+(1)^2-4}} dx$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$ [by $(a+b)^2 = a^2+2ab+b^2$]

$= \int \frac{1}{\sqrt{(x+1)^2-(2)^2}} dx$

by $\int \frac{1}{\sqrt{x^2+a^2}} dx = \log|x+\sqrt{x^2+a^2}|+c$

$= \log|x+1+\sqrt{(x+1)^2-(2)^2}|+c$

$= \log|x+1+\sqrt{x^2+2x+1-4}|+c$

$= \log|x+1+\sqrt{x^2+2x-3}|+c$

2) $\int (4e^{3x} + 1)\, dx.$

$= \int (4e^{3x} + x^{0})\, dx.$

$= 4\int e^{3x}\, dx + \int x^{0}\, dx.$

$= \frac{4e^{3x}}{3} + \frac{x^{0+1}}{0+1} + c$

$= \frac{4e^{3x}}{3} + x + c$

3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx.$

$= 2\int x^2\, dx - 3\int \sin x\, dx + 5\int x^{1/2}\, dx$

$= 2\frac{x^3}{3} - 3(\cos x) + \frac{5x^{1/2+1}}{1/2+1} + c$

$= \frac{2x^3}{3} + 3\cos x + \frac{5x^{3/2}}{3/2} + c$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x^{3/2}}{3} + c$

4) $\int \frac{x^3 + 3x+4}{\sqrt{x}}\, dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\, dx.$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\, dx.$

$$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}} \, dx.$$

$$= \int x^{5/2} \, dx + \int 3x^{1/2} \, dx + \int \frac{4}{x^{1/2}} \, dx.$$

$$= \frac{x^{5/2+1}}{5/2+1}$$

$$= \frac{2x^{7/2}}{7} + 2x^{3/2} + 8\sqrt{x} + c$$

5) $\int t^7 \times \sin(2t^4) \, dt$

$$I = \int t^7 \sin(2t^4) \, dt$$

Let $t^4 = x$.

$$4t^3 \, dt = dx.$$

$$= \frac{1}{4} \int 4t^3 . t^4 \sin(2t^4) \, dt$$

$$= \frac{1}{4} \int x . \sin(2x) \, dx.$$

$$= \frac{1}{4} \left[x \int \sin 2x - \int \left[\int \sin 2x . \frac{d}{dx} x \right] \right].$$

$$= \frac{1}{4} \left[-\frac{x \cos 2x}{2} + \frac{1}{2} \int \cos 2x . 1 \right]$$

$$= \frac{1}{4} \left[-\frac{\cos 2x}{2} + \frac{1}{4} \sin 2x \right] + c.$$

$$= \frac{-1}{8} x \cos 2x + \frac{1}{16} \sin 2x + c.$$

$$= \frac{-1}{8} t^4 \cos(2+4) + \frac{1}{16} \sin(2t4) + c$$

6) $\int \sqrt{x}\,(x^2-1)\,dx.$

$= \int \sqrt{x}\;x^2 - \sqrt{x}\;dx.$

$= \int x^{1/2}.\,x^2 - x^{1/2}\,dx.$

$= \int x^{5/2} - x^{1/2}\,dx.$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx.$

$= \frac{2x^{7/2}}{7} - \frac{2}{3}x^{3/2} + c$

7) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx.$

$t = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx.$

Let $\frac{1}{x^2} = t$

$x^{-2} = t$

$-\frac{2}{x^3}\,dx = dt$

$I = -\frac{1}{2}\int \frac{-2}{x^3}\sin\left(\frac{1}{x^2}\right)dx.$

$= \frac{-1}{2}\int \sin t$

$= \frac{-1}{2}(-\cos t) + c$

$= \frac{1}{2}\cos t + c$

Resubsitiution $t = 1/x^2$

$$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$$

8) $\int \frac{\cos x}{3\sqrt{\sin^2 x}} dx.$

$$I = \int \frac{\cos x.}{\sqrt[3]{\sin^2 x}} dx.$$

Let $\sin x = t$

$\cos x \, dx = dt$

$$I = \int \frac{dt}{\sqrt[3]{t^2}}$$

$$= \int \frac{dt}{t^{2/3}}.$$

$$= \int t^{-2/3} dt$$

$$= 3t^{1/3} + c$$

$$= 3 (\sin x)^{1/3} + c.$$

$$= 3. \sqrt[3]{\sin x} + c$$

9) $\int e^{\cos^2 x} \sin 2x \, dx.$

$$I = \int e^{\cos^2 x} \sin 2x \, dx.$$

Let $\cos^{2x} = t$

$-2 \cos x . \sin x \, dx = dt$

$-2 \sin 2x \, dx = dt$

$$I = \int - \sin 2x . e^{\cos 2x} dx.$$

$= -\int e^t \, dt$

$= -e^t + c$

Resubstituting $t = \cos^2 x$.

$I = -e^{\cos^2 x} + c.$

10) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx.$

$I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^{2+1}} \right) dx.$

Let $x^3 - 3x^2 + 1 = t$

$3(x^2 - 2x)\, dx = dt$

$(x^2 - 2x)\, dx = dt/3$

$I = \int 1/t \, dt$

$= 1/3 \int dt/t$

$= \frac{1}{3} \log t + c$

$= 1/3 \log (x^3 - 3x^2 + 1) + c.$

AK
08/01/2020

PRACTICAL - 6

Topic :- Applications of Integration & Numerical Integration.

Q. Find the length of the following curve :-

1) $x = t - \sin t$
$y = 1 - \cos t \qquad t \in [0, 2\pi]$

$$\frac{dx}{dt} = 1 - \cos t$$

$$\frac{dy}{dt} = \sin t$$

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\; dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\; dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\; dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\; dt$$

$$= \int_0^{2\pi} \sqrt{2(1-\cos t)}\; dt$$

$$= \int_0^{2\pi} \sqrt{2 \times 2 \sin^2 t/2}\; dt$$

$$= \sqrt{4 \sin^2 t/2}$$

$$= \int_0^{2\pi} 2\,|\sin t/2|\; dt \qquad \therefore \sin^2 \frac{t}{2} = \frac{1-\cos t}{2}$$

$$= \int_0^{2\pi} 2 \sin \frac{t}{2}\; dt$$

$$= 8\left(-4\cos(t/2)\right)_0^{2\pi}$$

$$= (-4\cos\pi) - (-4\cos 0)$$

$$= 4+4$$

$$= 8$$

2) $y = \sqrt{4-x^2}$ $\quad x \in [-2,2]$

$$\frac{dy}{dx} = \frac{-2x}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$$

$$l = 2\int_0^2 \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_0^2 \sqrt{\frac{1+x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}(x/2)\right)_0^2 = 2\pi$$

3) $y = x^{3/2}$ in $[0,4]$

$$f'(x) = \frac{3}{2}x^{1/2}$$

$$[f'(x)]^2 = \frac{9}{4}x$$

$$l = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$$

$$= \int_0^2 \sqrt{1+\frac{9}{4}x}\, dx$$

Put $u = 1 + \frac{9}{4}x$, $du = \frac{9}{4}dx$

$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right)-1\right]$

4. $x = 3\sin t \quad , y = 3\cos t \quad t \in [0, 2\pi]$

$\rightarrow \frac{dx}{dt} = 3\cos t \quad dy \quad \frac{dy}{dt} = -3\sin t$

$l = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 + d\cos^2}\, dt$

$= \int_0^{2\pi} 3\cancel{\sqrt{x}}\, dt$

$= 3 \int_0^{2\pi} x\, dt$

$= 3\,[x]_0^{2\pi} \quad = 3(2\pi - 0)$

$= 6\pi$ units.

5. $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1,2]$

$\rightarrow$ $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_1^4 \sqrt{1 + \frac{(y^4-1)}{4y^4}}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{2}\right]$

$= \frac{1}{2}\left(\frac{17}{8}\right) = \frac{17}{12}$ units.

Q2. solve the foll using simpson rule.

1. $\int_0^2 e^{x^2} dx = 16.04526$

in each case the width the sub interval be $x = \frac{2-0}{4} = \frac{1}{2}$

and so the sub intervals will be [0, 0.5] [0.5, 1]

[1, 1.5] [1.5, 2]

by simpson rule.

$$\int_0^2 e^{x^2} dx = \frac{1/3}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

$= 17.3536$

2. $\int_0^4 x^2 dx \; n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$$\int_a^b f(x) \, dx = \frac{\Delta x}{3} [y_0 + 4y_1 + 2y_2 + 4y_3 + y_4]$$

$= \frac{1}{3} [0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2]$

$= \frac{64}{3}$

03

3. $\int_{}^{\pi/3} \sqrt{\sin x}\, dx = n=6$

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\pi/8$	$2\pi/8$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.41	0.58	0.70	0.80	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$= \pi/18 \; (0 + (0+4167 + 0.72 + 0.875)$
$+ 0.930$

$= 2(0.5857 0.801) + 0.930$

$= 0.682$

09/07/2020

PRACTICAL - 7

Topic: Differential Equation.

1] Solve the following differential Equation:-

1) $x \frac{dy}{dx} + y = e^x$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

3) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

4) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

5) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

6) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

7) $\frac{dy}{dx} = \sin^2 (x \quad y + 1)$

8) $\frac{dy}{dx} = \frac{2y + 3y - 1}{6x + 9y + 6}$

Solutions :-

1) $x \frac{dy}{dx} + y = e^x$

$\therefore \quad \frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

Comparing with $\frac{dy}{dx} + P(x) y = Q(x)$

$P(x) = \frac{1}{x} \quad ; \quad Q(x) = \frac{e^x}{x}$

$IF = e^{\int 1/x \, dx}$

$= e^{\log x}$

$IF = x$

$y(IF) = \int Q(x)(IF) \, dx$

$y(x) = \int \frac{e^x}{x} \cdot x \, dx$

$y(x) = \int e^x \, dx$

$xy = e^x + c$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\therefore \quad e^x \left(\frac{dy}{dx} + 2y \right) = 1$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 2$; $Q(x) = \frac{1}{e^x}$; $IF = e^{\int 2dx}$
$= e^{2x}$

$$y(IF) = \int Q(x)\,(IF)\,dx$$

$$ye^{2x} = \int \frac{1}{e^x} \cdot e^{2x}\,dx$$

$$ye^{2x} = \int e^{-x} \cdot e^{x}\,dx$$

$$ye^{2x} = e^x + c$$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$\therefore x\frac{dy}{dx} + 2y = \frac{\cos x}{x}$$

$$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$

comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = \frac{2}{x}$; $Q(x) = \frac{\cos x}{x^2}$; $IF = e^{\int 2/x\,dx}$
$= e^{\log x^2}$
$= x^2$

$$y(IF) = \int Q(x) \cdot (IF)\,dx$$

$$yx^2 = \int \frac{\cos x}{x^2} \cdot (x^2)\,dx$$

$$x^2y = \int \cos x\,dx$$

$$x^2y = \sin x + c$$

4) Sa $x\dfrac{dy}{dx} + 2y = \dfrac{\sin x}{x^2}$

$$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^3}$$

comparing with $\dfrac{dy}{dx} + P(x)y = Q(x)$

$P(x) = 3x^{-1}$; $Q(x) = \dfrac{\sin x}{x^3}$;

$$IF = e^{\int 3/x\,dx}$$
$$= e^{3\log x}$$
$$= e^{\log x^3}$$
$$= x^3$$

$$y(IF) = \int Q(x).(IF)\,dx$$

$$x^3 y = \int \frac{\sin x}{x^3}.(x^3)\,dx$$

$$x^3 y = \int \sin x\,dx$$

$$x^3 y = -\cos x + c$$

5.) $e^{2x}\dfrac{dy}{dx} + 2e^{2x}y = 2x$

$$e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$$

$$\therefore \frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore$ $P(x) = 2$; $Q(x) = \frac{2x}{e^{2x}}$; $IF = e^{\int 2\,dx}$
$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx$

$e^{2x}y = \int \frac{2x}{e^{2x}} \cdot e^{2x}\,dx$

$ye^{2x} = \int 2x\,dx$

$ye^{2x} = x^2 + c$

6) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\therefore$ $\sec^2 x \tan y\,dx = -\sec^2 y \tan x\,dy$

$\therefore$ $\frac{\sec^2 x}{\tan x}\,dx = -\frac{\sec^2 y}{\tan y}\,dy$

$\therefore$ $\int \frac{\sec^2 x}{\tan x}\,dx = \int -\frac{\sec^2 y}{\tan y}\,dy$

$\log|\cancel{\tan x}| = -\log|\tan y| + c$

$\log|\tan x| + \overset{\log}{\hat{}}|\tan y| = c$

$\therefore$ $\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = 1 - \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$1 - \sin^2 v = \frac{dv}{dx}$

$dx = \frac{dv}{1-\sin^2 v}$

$\int dx = \int \sec^2 v \, dv$

$x = \tan v + c$

But $v = x+y-1$

$\therefore x = \tan(x+y-1) + c$

8.) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

$\cancel{\frac{dy}{dx}}$ put $2x+3y = v$

$2x + 3\frac{dy}{dx} = \frac{dv}{dx}$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\therefore \quad \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)} = dx$$

$$\frac{1}{3}\int \frac{(v+1+1)}{(v+1)}\,dv = \int dx$$

$$\frac{1}{3}\int 1 + \frac{1}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\left(v + \log(v+1)\right) = x + c$$

But $v = 2x+3y$

$$\therefore \quad 2x+3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c$$

PRACTICAL - 8

Topic :- Euler's Method.

1.) $\frac{dy}{dx} = y + e^{x} - 2$, $y(0) = 2$, $h = 0.5$, find $y(2)$

2.) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$, find $y(1)$

3.) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$, find $y(1)$

4.) $\frac{dy}{dx} = 3x^2 + 1$, $y(0) = 2$, find $y(2)$
for $h = 0.5$, $h = 0.25$

5.) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$, find $y(1.2)$ with
$h = 0.2$; $x_0 = 1$

Solutions :-

1) $\frac{dy}{dx} = y + e^x - 2$

$f(x,y) = y + e^x - 2$, $y(0) = 2$, $x_0 = 0$; $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.2831		

$y_{n+1} = y_n + h f(x_n, y_n)$

∴ By Euler's formula,

$y(2) = 9.8215$

2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$, $h = 0.2$, find $y(1)$, when

$y_0 = 0$, $x_0 = 0$, $h = 0.2$

By Euler's formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.104	0.408

2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

$\therefore$ $y(1) = \underline{1.2942}$

3.) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ where $y_0 = 1$, $x_0 = 0$, $h = 0.2$

By Euler's formula

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	01	0	01
1	0.2	01	0.4472	1.0894
2	0.4	1.0894	0.6059	1.21 06
3	0.6	1.21 06	0.7016	1.3509
4	0.8	1.3509	0.7695	1.5048
5	1	1.5048		

$\therefore$ $y(1) = \underline{1.5048}$

4) $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$ find $y(2)$, $h = 0.5$ &

$y_0 = 2$, $x_0 = 1$, $h = 0.25$

By Eulers Formula

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.8750
2	2	7.8750		
3	2.5			

$\therefore y(2) = 7.8750$

when $y_0 = 2$, $x_0 = 1$, $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.7500	6.3594
3	1.75	6.3594	10.1875	8.9063
4	2	8.9063		

$\therefore y(2) = 6.3594$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y_1 = 1$, $h = 0.2$

$x_0 = 1$ $y_0 = 1$, $h = 0.2$

By Euler's Formula

$$y_{n+1} = y_n + h f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1		1.6
1	1.2	1.6	1.3856	1.8771
2	1.4	1.8771	1.6211	2.2013
3	1.6	2.2013		

$y(1.2) = \underline{1.6}$

PRACTICAL - 9

Topic :- Limits and Partial Order Derivative.

1) Evalute the following limits :-

1) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$

2) $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2 + y^2 z^2}{x^3 - x^2 yz}$

3) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

2) Find f_x, f_y for each of the following f.

1) $f(x,y) = xy\, e^{x^2+y^2}$

2) $f(x,y) = e^x \cos y$

3) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

3) Using definition find values of f_x, f_y at $(0,0)$ for
$f(x,y) = \dfrac{2x}{1+y^2}$

4) Find all second order partial derivatives of f.
Also verify whether $f_{xy} = f_{yx}$.

1) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

3) $f(x,y) = \sin(xy) + e^{x+y}$

5) Find the linearization of $f(x,y)$ at given point

1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

2) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

3) $f(x,y) = \log x + \log y$ at $(1,1)$

Q.1 (i) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$, Denoninaler $\neq 0$

$\therefore$ By applying limit

$$= \frac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$$

$$= \frac{-64+3+1-1}{4+5}$$

$$= \frac{-61}{9}$$

(ii) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit,

$$= \frac{(0+1)((2)^2+0-4(2))}{2+0}$$

$$= \frac{1(4+0-8)}{2}$$

$$= \frac{-4}{2}$$

$$= -2.$$

(iii) $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

At (1,1,1) Denominator = 0

$\therefore \lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{\cancel{(x-yz)}(x+yz)}{x^2\cancel{(x-yz)}}$

$= \lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x^2}$

On Applying Limit

$= \dfrac{1+1(1)}{(1)^2}$

$= 2.$

Q2. (i).

$f(x, y) = x y e^{x^2} + y^2$

$\therefore f x = \frac{\partial}{\partial x} (f(x, y))$

$= \frac{\partial}{\partial x} (x y e^{x^2 + y^2})$

$= y e^{x^2 + y^2} (2x).$

$\therefore f x = 2 x y e^{x^2 + y^2}.$

$f y = \frac{\partial}{\partial y} (f(x, y))$

$= x e^{x^2 + y^2} (2y)$

$\therefore f y = 2 y x e^{x^2 + y^2}$

(ii) $f(x, y) = e^x \cos y$

$f x = \frac{\partial}{\partial x} (f(x, y))$

$= \frac{\partial}{\partial} (e^x \cos y)$

$\therefore f x = e^x \cos y$

$f y = \frac{\partial}{\partial y} (f(x, y))$

$= \frac{\partial}{\partial y} (e^x \cos y)$

$\therefore f y = - e^x \sin y.$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$$f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$$

$$\therefore f_x = 3x^2y^2 - 6xy$$

$$f_y = \frac{\partial}{\partial y}(f(x,y))$$

$$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$$

$$\therefore f_y = 2x^3y - 3x^2 + 3y^2.$$

Q.3. (i)

$$f(x,y) = \frac{2x}{1+y^2}.$$

$$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2\frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2 - 0}{(1+y^2)^2}$$

$$= \frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y)^2}$$

$$= \frac{2}{1+y^2},$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial z}(x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$,

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

4. (j)

$$f(x;y) = \frac{y^2 - xy}{x^2}$$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (-x^2 y - 2xy + 2x^2 y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^3y - 2xy + 2x^2y)}{x^6} \quad \text{--- (1)}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2-0}{x^2} = \frac{2}{x^2} \quad \text{--- (2)}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad \text{--- (3)}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \frac{\partial}{\partial x}(2y - x) - 2y - x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}. \quad \text{--- (4)}$$

from (3) & (4);

$f_{xy} = f_{yx}$.

(ii) $f(x, y) = x^3 + 3x^2 y^2 - \log(x^2 + 1)$

$$f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2 + 1))$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2 + 1}$$

$$f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2 + 1))$$

$$= 0 + 6x^2y - 0$$

$$= 6x^2y$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{x^2 + 1\,\frac{\partial(2x)}{\partial x} - 2x\,\frac{\partial(x^2)}{\partial x}}{(x^2 + 1)^2}\right)$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \quad \text{--- (1)}$$

$$f_{yy} = \frac{\partial}{\partial y}(6x^2 y)$$

$$= 6x^2 \quad \text{--- (2)}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12xy - 0$$

$$= 12xy \quad \text{--- (3)}$$

$$f_{yx} = \frac{\partial}{\partial x}(6x^2 y)$$

$$= 12xy \quad \text{--- (4)}$$

from (3) & (4),

$\therefore f_{xy} = f_{yx}$.

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y \cos(xy) + e^{x+y}(1)$ $\qquad$ $f_y = x \cos(xy) + e^{x+y}(1)$

$= y \cos(xy) + e^{x+y}$ $\qquad$ $= x \cos(xy) + e^{x+y}$

$$\therefore f_{xx} = \frac{\partial}{\partial x}\left(y \cos(xy) + e^{x+y}\right)$$

$$= -y \sin(xy) \cdot (y) + e^{x+y}(1)$$

$$= -y^2 \sin(xy) + e^{x+y} \quad \text{--- (1)}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(x \cos(xy) + e^{x+y}\right)$$

$$= -x \cdot \sin(xy)(x) + e^{x+y}(1)$$

$$= -x^2 \sin(xy) + e^{x+y} \quad \text{--- (2)}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(y\cos(xy) + e^{x+y}\right)$$

$$= -y^2\sin(xy) + \cos(xy) + e^{x+y} \quad \text{--- (3)}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right).$$

$$= -x^2\sin(xy) + \cos(xy) + e^{x+y} \quad \text{--- (4)}$$

$\therefore$ from (3) & (4)

$f_{xy} \neq f_{yx}$.

Q.5 (i)

$f(x, y) = \sqrt{x^2 + y^2}$ at $(1, 1)$

$\rightarrow f(1, 1) = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x) \qquad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$$

$$= \frac{x}{\sqrt{x^2+y^2}} \qquad = \frac{y}{\sqrt{x^2+y^2}}$$

f_x at $(1, 1) = \dfrac{1}{\sqrt{2}}$ $\qquad$ f_y at $(1, 1) = \dfrac{1}{\sqrt{2}}$

$\therefore L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x - 1) + \frac{1}{\sqrt{2}}(y - 1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x - 1 + y - 1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x + y - 2)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

(ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

$$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$$

$f_x = 0 - 1 + y\cos x$ $\quad$ $f_y = 0 - 0 + \sin x$

f_x at $\left(\frac{\pi}{2}, 0\right) = -1+0$ $\quad$ f_y at $\left(\frac{\pi}{2}, 0\right) = \sin\frac{\pi}{2} = 1$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$$

$$= 1 - \cancel{\frac{\pi}{2}} - x + \frac{\pi}{2} + y$$

$$= 1 - x + y$$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$$f(1,1) = \log(1) + \log(1) = 0$$

$f_x = \frac{1}{x} + 0$ $\quad$ $f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\quad$ f_y at $(1,1) = 1$

$$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= 0 + 1(x-1) + 1(y-1)$$

$$= x - 1 + y - 1$$

$$= x + y - 2$$

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PRACTICAL - 10

TOPIC : DERIVATES, VECTORS, MAX. & MIN. TANGENT

Q.1 Find the directional derivative of the following function at given points in the direction of given vector.

i) $f(x,y) = x+2y-3$ $a=(1,-1)$ $u=3i-j$

Here, $u \cdot 3i-j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{\cancel{-4} + \cancel{h}/\sqrt{10} + \cancel{4}}{\cancel{h}}$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

ii) $f(x) = y^2 - 4x + 1$ $\quad a = (3,4) \quad u = i + 5j$

Here, $u = i + 5j$ is not a unit vector

$$|\vec{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{(x,y)}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + \cancel{8} - \cancel{5}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

iii) $2x + 3y$ $\quad a = (1,2)$, $u = 3i + 4j$

Here,

$u = 3i + 4j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$$f(a) = f(1,2) = 2(1) + 3(2) = 8$$

$$f(a + hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$$

$$= f\left(\frac{1+3h}{5}, 2 + \frac{4}{5}\right)$$

$$f(a+hu) = 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8$$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{18\cancel{h}}{5} \cancel{+8} \cancel{-8}}{\cancel{h}}$$

$$= \frac{18}{5}$$

Q.2 Find gradient vector for the following function at given point.

i) $f(x,y) = x^y + y^x \qquad a = (1,1)$

$fx = y \cdot x^{y-1} + y^x \log y$

$fy = x^y \log x + xy^{x-1}$

$\nabla f(x,y) = (fx, fy)$

$= (yx^{y-1} + y^2 \log y, x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1} x) \cdot y^2 \qquad a = (1,-1)$

$fx = \frac{1}{1+x^2} \cdot y^2$

$fy = 2y \tan^{-1} x$

$\nabla f(x,y) = (fx, fy)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{\cancel{4}} \frac{(-\cancel{2})}{2}\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

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iii) $f(x,y,z) = xyz - e^{x+y+z}$ $a = (1,-1,0)$.

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$f(1,-1,0) = ((-1)(0) - e^{(1-1+0)}, (1)(0) - e^{(1+(-1)+0)}, (1)(-1) - e^{(1-1+0)})$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q.3 Find the equation of tangent & normal to each of the following using curves at given points.

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy}.y$

$f_y = x^2(-\sin y) + e^{xy}.x$

$(x_0, y_0) = (1,0) = \therefore x_0 = 1, y_0 = 0$

equation of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \, 2(1) + e^0 \, 0$

$= 1(2) + 0$

$= 2$

$$f_y(x_0, y_0) = (1)^2 (-\sin 0) + e^0 . 1$$
$$= 0 + 1.1$$
$$= 1$$

$$2(x-1) + 1(y-0) = 0$$
$$2x - 2 + y = 0$$
$$2x + y - 2 = 0$$ → It is the required eqⁿ of tangent.

equation of Normal

$$= ax + by + c = 0$$
$$= bx + ay + d = 0$$

$$\therefore \quad 1(1) + 2(y) + d = 0$$
$$\therefore \quad 1 + 2y + d = 0$$
$$1 + 2(0) + d = 0$$
$$d + 1 = 0$$
$$d = -1$$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$$f_x = 2x + 0 - 2 + 0 + 0$$
$$= 2x - 2$$
$$f_y = 0 + 2y - 0 + 3 + 0$$
$$= 2y + 3$$

$(x_0, y_0) = (2, -2)$ $\therefore$ $x_0 = 2$, $y_0 = -2$

$$f_x(x_0, y_0) = 2(2) - 2 = 2$$
$$f_y(x_0, y_0) = 2(-2) + 3 = -1$$

Equation of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ → It is required eqⁿ of tangent.

Equation of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q.4) Find the equation of tangent and normal line to each of the following surface.

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$f_y = +2z + 3$

$f_z = 0 - 2y + 0 + x$
$f_z = -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $\therefore$ $x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$
$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$
$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$
$= 4(x-2) + 3(y-1) + 0(z-0) = 0$
$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$ $\rightarrow$ This is required eq^n of tangent

Equation of Normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

ii) $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x (x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y (x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z (x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Equation of tangent

$= -7(x-1) + 5(y+1) - 2(z-2) = 0$

$= -7x + 7 + 5y + 5 - 2z + 4 = 0$

$= -7x + 5y - 2z + 16 = 0$ → This is required eq^n of tangent

Equation of Normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q.5) Find the local maxima and minima for the following functions :-

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

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$$2x - y + 2 = 0$$
$$2x - y = -2 \quad \text{--- (1)}$$

$f_y = 0$

$$2y - 3x - 4 = 0$$
$$2y - 3x = 4 \quad \text{--- (2)}$$

Multiply equation 1 with 2

$$\therefore \quad 4x - 2y = -4$$
$$2y - 3x = 4$$
$$x = 0$$

Substituting value of x in equation (1)

$$2(0) - y = -2$$
$$+y = +2$$
$$\therefore \quad y = 2$$

Critical points are $(0,2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

There, $r > 0$

$$= rt - s^2$$
$$= 6(2) - (-3)^2$$
$$= 12 - 9$$
$$= 3 > 0$$

$\therefore$ f is has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \quad \text{——} \ (1)$

$f_y = 0$

$\therefore 3x^2 - 2y = 0 \quad \text{——} \ (2)$

Multiply eqn (1) with (3) & (2) with (4)

$$\begin{array}{r} \cancel{12x^2} + 9y = 0 \\ -\cancel{12x^2} - 8y = 0 \\ + \\ \hline \end{array}$$

$y = 0$

Substitute value of y in eqn (1)

$4x^2 + 3(0) = 0$

$$4x^2 = 0$$
$$x = 0$$

critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x = 0$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0$

$= 0$

$r = 0$ & $rt - s^2 = 0$

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 - 0$

$= 0$

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$ $\quad \therefore 2x + 2 = 0$

$x = \frac{-2}{2}$ $\quad \therefore x = -1$

$f_y = 0$

$\therefore -2y + 8 = 0$

$y = \cancel{\frac{-8}{-2}}\ 4$ $\quad \therefore y = 4$

$\therefore$ critical point is $(-1,4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$$rt - s^2 = 2(-2) - (0)^2$$
$$= -4 - 0$$
$$= -4 < 0$$

$f(x,y)$ at $(-1,4)$

$$= (-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$$
$$= 1 + 16 - 2 + 32 - 70$$
$$\Rightarrow 17 + 30 - 70$$
$$\Rightarrow 37 - 70 = -33$$

AV
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