UNIT- 1

Statistical Quality control :-

Introduction :-

→ SQC is the one of the most important applications of statistical techniques in industry. These techniques are based on the theory of probability, Sampling and being extensively used in all the industries such as ornaments, Auto mobiles, textile, plastic, electronics etc.

→ The most important word used in SQC is Quality.

→ The Quality is an attributes of the product that determines its fitness for use. Quality means or standard or level of the product which depends on four M's

material, Man power, Machines and Management.

Quality of material:

Materials of good Quality will result in smooth processing and finishing, there by reducing the waste and increasing the output. It also gives good finish to end product.

Quality of Man- Power:

Trained and Qualified personal will increase the efficiency of the Quality and also reduce the Production cost and waste.

Quality of Machines

Better Quality equipment will result in efficient work, due to scarcity of breakdown and thus lost of defective.

Quality of Management

A good Quality of management is important for increase in efficiency, growth of markets and business.

Causes of Variation

The Variation in quality of manufactured product in the repitation process is inheriant in nature. These variations are broadly classified due to two causes

1) chance / Random causes
2) Assignable causes.

1) chance / Random causes :-

Some stable pattern of variations is inheriant in any particular production results from many minor causes that behave in random manner. The variation due to these causes are beyond the control of human hand and cannot be prevented ~~on~~ or hand and cannot be prevented or eliminated in any circumstance. one has to allow for the variations within this stable pattern usualy termed as allowable variation.

2) Assignable causes

The second type of variation in any production process is due to non-random called assignable causes and its termed as preventable variation The Assignable causes occured from Arrival of raw material to finial delivery of the goods.

Some of the factors are :-

1) Sub standard (or) Defective raw material
2) New techniques (or) operations
3) Negligence of operators
4) Improper handling of machines
5) unskilled or In experienced techniqual sto and so on.

These causes can be identified and eliminated and to be discovered in the production process before it goes wrong
i.e, Before the production becomes defective.

Chance causes	Assignable causes
consists of many indivi-ual causes.	• consists of just a few individual causes.
Any one chance cause result in na small amount of variation	• Any one assignable cause can results in large amount of variation.
chance variation cannot nomically eliminated from process	• The presence of assignable variation can be detected and eliminated the causes
ome typical chance causes u slight variation of a achine	• Some Assignable causes as variation
lack of human perfection	• negligence of operators
voltage	• defective raw materials
variations in temperature	• faculty equipment
	• improper handling of marchines

SQC :-

SQC is a planned collection and effective use of data for studing causes of variations in quality. The main purpose of SQC is to device statistical techniques in seperating assignable causes from chances causes and thus unable us to take remedial measures whenever assignable causes are present. The elimination of these cause is described as bringing the process under control.

The production process is said to be in a state of statistical control it is having only chance cause in absence of assignable causes.

Process control and Product control :-

The main objective in any production causes is to control and maintain Satisfactory quality level of the manufactured product so that it imforms to specified Quality standards. In other words, it ensure that the proportion of defective items is not too large. This is termed as process control and is achieved through control charts prepared by wia shewart in 1924.

Product control means controlling the quality of the product by critical examination at strategic points and this acheived through compling inspection plans and proposed by H.F Dodge and H.C. Romring. It attempts to ensure that the product marketed by sales department does not contain large no. of defectives.

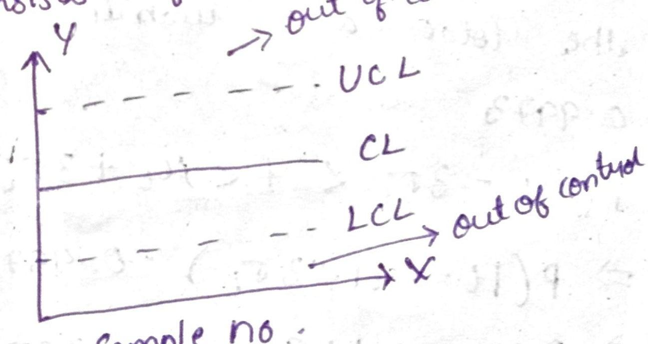
Techniques of SQC

- Process control (By control charts)
  - C.C for variables
    - $\bar{x}$ chart
    - R chart
    - σ chart
  - C.C for attributes
    - np chart
    - p chart
    - c-chart
- Product control (By sampling inspection)
  - Single sampling plan
  - Double sampling plan

<u>Control charts</u>

Control charts were develop by Dr. Walter A. Shewart of Bell laboratery in 1924. It is powerful statistical tool for discovering and correcting assignable causes of variation outside the stable pattern of chance cause and thus enable us to bring the process under control.

→ A control chart consists of following three horizontal line



1. A central line (CL) indicating desired level of standard
2. upper control limit (UCL) indicating upper limit for tolerence.
3. Lower control limit (LCL) indicating lower limit for tolerence.

→ In control chart UCL and LCL are usually ploted as dotted lines and the central lines (CL) is ploted as a bold line

Let 't' be any statistical then the 3-σ control limits are

$UCL = E(t) + 3S.E(t)$

$CL = E(t)$

$LCL = E(t) - 3S.E(t)$

Statistical basis for control charts:-

→ **Control limits**

→ Let $x_1, x_2, \ldots x_n$ be the observations on a random sample of size 'n' from the given population.

Let 't' be the statistical computed from the sample with $E(t) = \mu_t$, $V(t) = \sigma^2_{(t)}$

→ If the statistic 't' is normally distributed from fundamental area property of normal distribution the total area with in the 3σ limits is 0.9973

$$P(\mu_t - 3\sigma_t < t < \mu_t + 3\sigma_t) = 0.9973$$

$$\Rightarrow P(|t - \mu_t| < 3\sigma_t) = 0.9973$$

$$\Rightarrow P(|t - \mu_t| > 3\sigma_t) = 0.0027$$

→ Hence, the probability that a random value of a statistic 't' computed from given sample lies outside the 3σ limits is too small. Therefore, the 3σ control limits are

$UCL = E(t) + 3SE(t) = \mu_t + 3\sigma_t$

$CL = \mu_t$

$LCL = E(t) - 3SE(t)$

All the computed values of statistic 't' lies within the 3σ limits the process is said to be under control otherwise, not under control

There are two types of control charts

1) control chart for variables

a. $\bar{x}$ chart (mean chart)

b. R- chart (Range chart)

[control chart for Variables:]

i) σ- chart (S.D chart)

2) CL for attributes

a) np- chart [no. of defective chart]

b) p- chart [proportion of defective chart]

c) C- chart [no. of defect chart]

Construction of control chart for variables $\bar{x}$ and R:-

The following are the steps to construct $\bar{x}$ chart

1) Measurement :-

The work of control chart starts with the measurement. Errors in measurement can enter into data by

i) use of faulty equipment

ii) lack of experiment

iii) lack of clear cut definitions of quality characteristics etc.

Since, the conclusions are drawn from control chart. So, it is important that mistake in reading measurement should be minimized.

2) Selecting of samples :-

In order to make control chart effective. It is essential to pay attention in selection of samples or subgroups. Normally 25 samples each of size 4 or 20 samples ~~with~~ each of size 5 will give good estimates for the process average.

3) control limits for $\bar{x}$ chart :-

Let x_{ij} ; $i = 1, 2 --- k$; $j = 1, 2 --- n$ be the measurement on i^{th} sample

→ Let $\bar{x}_i$, R_i, s_i^2 denote respectively the mean Range and variance of the i^{th} sample which are given by

$$\left.\begin{array}{l}\text{Mean of} \\ i^{th} \text{ sample}\end{array}\right\} \bar{x}_i = \frac{1}{n} \sum_{i=1}^{n} x_{ij}$$

$$\left.\begin{array}{l}\text{Range of} \\ i^{th} \text{ sample}\end{array}\right\} \bar{R}_i = Max(x_{ij}) - min(x_{ij})$$

$$\left.\begin{array}{l}\text{variance of} \\ i^{th} \text{ sample}\end{array}\right\} s_i^2 = \frac{1}{n} \sum_{j=1}^{n} x_{ij}^2 - \bar{x}_i^2$$

→ also calculate

Mean of means, $\bar{\bar{x}} = \frac{1}{K} \sum_{i=1}^{K} \bar{x}_i$

Mean of samples $\bar{\bar{R}} = \frac{1}{K} \sum_{i=1}^{K} R_i$

Mean of S.D's, $\bar{s} = \frac{1}{K} \sum_{i=1}^{K} s_i$

we know; If $X \sim N(\mu, \sigma^2)$ then

$\bar{X} \sim N(\mu, \sigma^2/n)$

$\Rightarrow E(\bar{X}) = \mu$, $V(\bar{x}) = \sigma^2/n$

$S.E[\bar{x}] = \sigma/\sqrt{n}$

Then, the 3-σ control limits are,

$E(t) \pm 3 S.E(t)$

$\Rightarrow E(\bar{X}) \pm 3 S.E(\bar{X})$

$\Rightarrow \mu \pm 3\sigma/\sqrt{n}$

case 1) :- when μ and σ are known ($\mu = \mu'$ and $\sigma = \sigma'$)

the 3-σ control limits in this case are

$UCL = \mu' + 3\sigma'/\sqrt{n}$

$CL = \mu'$

$LCL = \mu - 3\sigma'/\sqrt{n}$

case 2) :- when μ and σ are unknown

we use its estimates

we known that sample mean unbaised estimator of population mean.

i.e; $E(\bar{x}) = \mu \Rightarrow \hat{\mu} = \bar{\bar{x}}$

we also known that, from sampling distribution of sample range

$\hat{\sigma} = \dfrac{\bar{R}}{d_2}$

then, the 3-σ control limits in this case are,

$\hat{\mu} \pm 3\hat{\sigma}/\sqrt{n}$

$\Rightarrow \bar{\bar{x}} \pm 3\dfrac{\bar{R}}{d_2\sqrt{n}} \Rightarrow \bar{\bar{x}} \pm A_2\bar{R}$; where $A_2 = \dfrac{3}{d_2\sqrt{n}}$

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$
$$CL = \bar{\bar{x}}$$
$$LCL = \bar{\bar{x}} - A_2\bar{R}$$

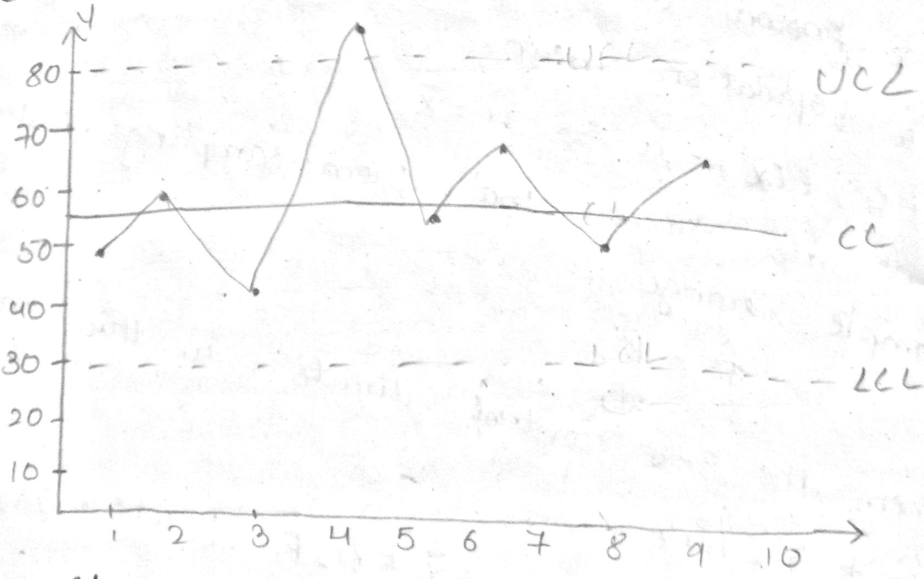
where, d_2 & A_2 are obtained from SQC tables for given 'n'.

Construction of $\bar{x}$-chart :-

$\bar{x}$-chart is constructed by taking sample number on x-axis and sample means on y-axis at 'CL' a bold line and at 'UCL' and 'LCL' two dotted lines drawn parallel to x-axis all the samples means $(\bar{X}_1, \bar{X}_2 --- \bar{X}_n)$ x ~~-axis~~ ~~all the~~ ~~samples means~~ are plotted and join them by straight lines.

Interpretation

If all the sample points on or in b/w UCL and LCL the process is under control otherwise not under control which indicates the presence of assignable causes in the process.



Note :- The 1^{st} three steps is same as $\bar{x}$-charts.

R-chart (Range chart):-

The 3σ control limits for R chart are

$E[R] \mp 3 \cdot S.E[R]$

we know from sampling distribution of sample range $E[R] = d_2\sigma$, $S.E[R] = d_3\sigma$

∴ The 3σ limits are $d_2\sigma \mp 3d_3\sigma$

case i)

where σ is known $(\sigma = \sigma')$

The 3σ limits in the case are

$UCL = d_2\sigma' + 3d_3\sigma' = \sigma'(d_2 + 3d_3) = D_2\sigma'$

$CL = d_2\sigma'$

$LCL = d_2\sigma' - 3d_3\sigma' = \sigma'(d_2 + d_3) = D_1\sigma'$

case ii)

when σ is unknown

when σ is unknown we use the to eliminate

we know from sampling distance of sample range

$$E[R] = \bar{R} = d_2\sigma$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

∴ The 3σ limits are $d_2\hat{\sigma} \pm 3d_3\hat{\sigma}$

$$UCL = d_2\frac{\bar{R}}{d_2} + 3d_3\frac{\bar{R}}{d_2}$$

$$= \bar{R}\left[1 + 3\frac{d_3}{d_2}\right] \Rightarrow D_4\bar{R}$$

$$CL = \bar{R}$$

$$LCL = d_2\frac{\bar{R}}{d_2} - 3d_3\frac{\bar{R}}{d_2}$$

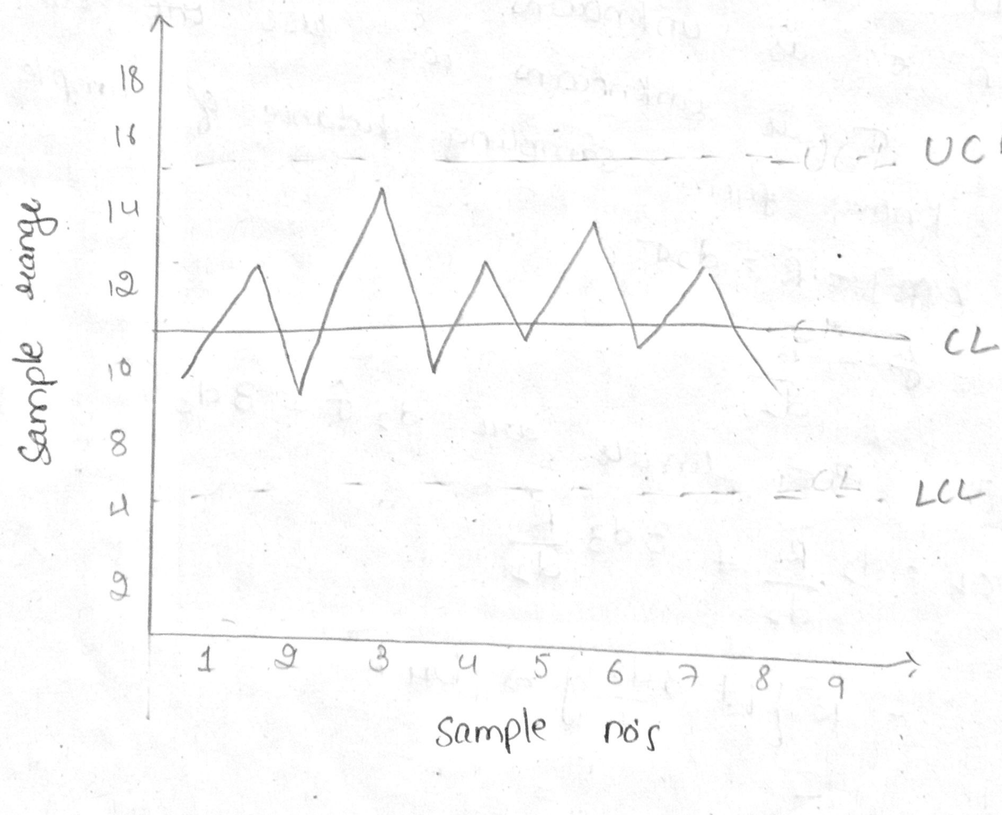
$$\bar{R} = \left[1 - 3\frac{d_3}{d_2}\right] = D_3\bar{R}$$

Construction of R-chart :-

$\bar{R}$ chart is constructed by taking sample numbers on X-axis and sample means on Y-axis at CL a bold line at UCL and LCL two dotted lines drawn parallel to X-axis all the ranges ($\bar{R}_1, \bar{R}_2 \ldots \bar{R}_k$) are plotted and join them by straight line.

Interpretation

If all the sample points on or in b/w UCL and LCL the process is under control. otherwise not under control which indicates the presence of assignable cause in the process



* Control charts:- [Practical-1]

1) Construct control chart for means and range for the following data. comment on whether the production process is under control or not sample-

Sample no	1	2	3	4	5
1	42	65	75	78	87
2	42	45	68	72	90
3	19	24	80	~~8~~ 81	81
4	36	54	69	77	84
5	42	51	57	59	78
6	51	74	75	78	132
7	60	60	72	95	138
8	18	80	27	42	60
9	15	30	39	62	84
10	69	109	113	118	153
11	64	90	93	109	112
12	61	78	94	109	136

Aim:- To construct control chart for means and range and to comment on production process.

Procedure:-

Let x_{ij} be the j^{th} observation in i^{th} sample

Then, mean of means $\bar{\bar{x}} = \frac{1}{h} \sum_{i=1}^{h} \bar{x}_i$;

$\bar{x}_i = \frac{1}{n} \sum_{j=1}^{n} x_{ij}$

Mean of ranges $\bar{R} = \frac{1}{k} \sum_{i=1}^{k} R_i$; $R_i = Max\, x_{ij} - Min\, x_{ij}$

Mean of S.D's $\bar{s} = \frac{1}{k} \sum_{i=1}^{k} s_i$; $s_i^2 = \frac{1}{n} \sum (x_{ij} - \bar{x}_i)^2$

The 3σ control units for $\bar{x}$ chart are

$UCL = \bar{\bar{x}} + A_2 \bar{R}$

$CL = \bar{\bar{x}}$

$LCL = \bar{\bar{x}} - A_2\bar{R}$

The 3σ control limits for R chart are

$UCL = D_4\bar{R}$; $CL = \bar{R}$; $LCL = D_3\bar{R}$

where A_2, D_3, D_4 are obtained from SQC tables for given sample size 'n'

Construction :- By taking sample no's on x-axis and sample means on Y-axis for $\bar{x}$ chart and sample range for R chart. A dotted straight line for UCL and LCL drawn parallel to x-axis and a bold line for CL drawn $||^{el}$ to x-axis - sample observations are plotted and joined with st. line. If all the plotted points lie b/w UCL and LCL then the process is under control or not under control.

Calculations

$n = 5$, $K = 12$.

for $n = 5$
$A_2 = 0.58$
$D_3 = 0$
$D_4 = 2.11$

Sample no	1	2	3	4	5	$\bar{x}$	R_i
							Max-Min =
1	42	65	75	78	87	$\frac{42+65+75+78+87}{5} = 69.4$	87-42 = 45
2	42	45	68	72	90	63.4	48
3	19	24	80	81	81	57	62
4	36	54	69	77	84	64	48
5	42	51	57	59	78	57.4	36
6	51	74	75	78	132	82	81
7	60	60	72	95	138	85	78
8	18	20	27	42	60	33.4	42
9	15	30	39	62	84	46	69
10	69	109	113	118	153	112.4	84
11	64	90	93	109	112	93.6	48
12	61	98	94	109	136	95.6	75
						859.2	716

Mean of Means

$$\bar{\bar{x}} = \frac{1}{K} \Sigma \bar{x}_i$$

$$= \frac{1}{12} 859.2$$

$$= 71.6$$

Mean of Range

$$\bar{R} = \frac{1}{K} \Sigma R_i$$

$$= \frac{716}{12}$$

$$= 59.6667$$

3σ ~~charts~~ control limits $\bar{x}$ chart are

$UCL = \bar{\bar{x}} + A_2 \bar{R}$

$UCL = 106.2667$

$LCL = \bar{\bar{x}} - A_2 \bar{R}$

$= 36.9933$

$CL = 71.6$

3σ control limits $\bar{R}$ chart are

$UCL = D_4 \bar{R}$

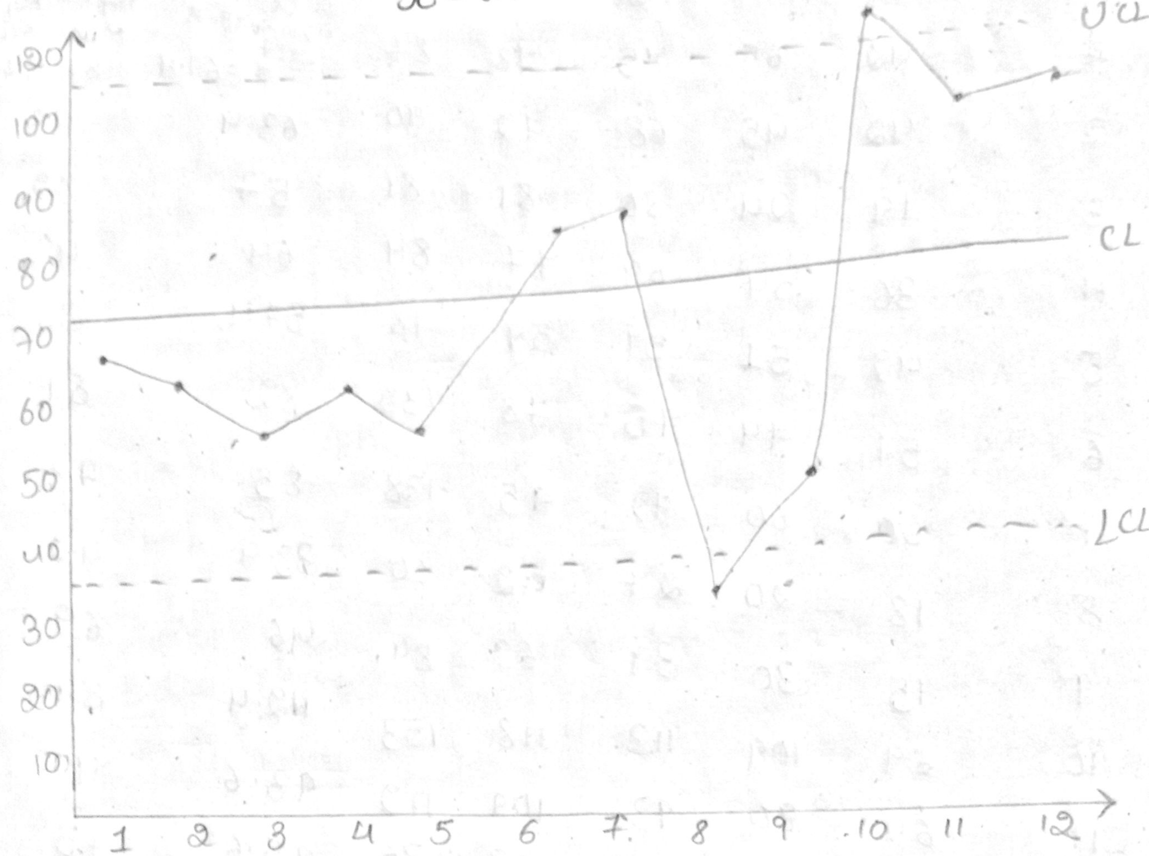
$= 125.8967$

$CL = \bar{R} = 59.6667$

$LCL = D_3 \bar{R}$

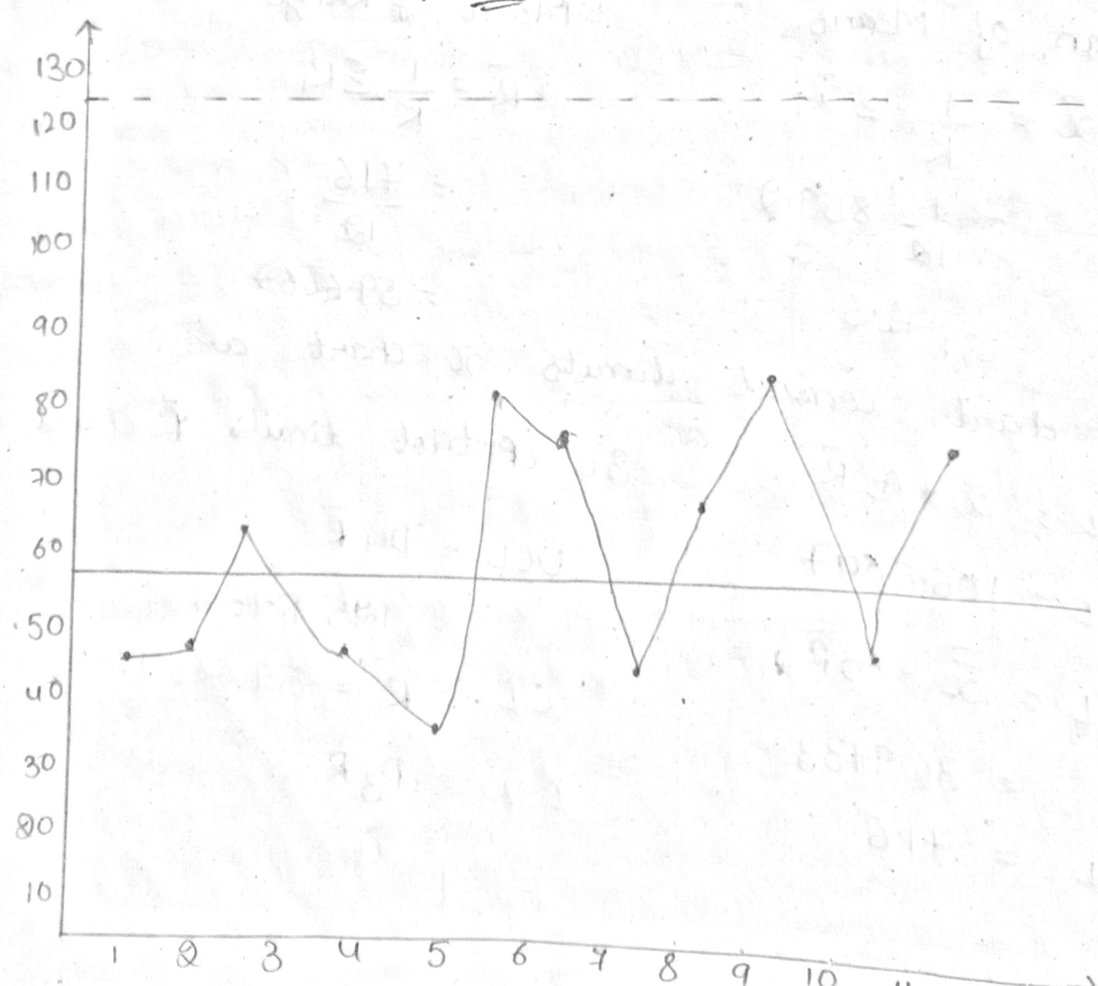
$= 0$

$\bar{x}$ - chart



The process is not under control.

R - chart



The Process is under control.

Conclusion

$\bar{x}$-chart :- The sample observations corresponding to 8^{th} sample is below LCL, 10^{th} sample above UCL so, the process is not under control.

R chart :-

Since all the plotted points lies in b/w UCL & LCL therefore the process is under control. But on the hole the process is not under control.

Pra - 2

2) A machine is set to deliver the packets of a given weights. 10 samples each of size 5 were examined and the following results were obtained. Calculate central line and control limits for mean and Range chart and comment on state of control

(for $n=5$, $d_2 = 2.326$, $D_3 = 0$, $D_4 = 2.115$)

Sample no	mean	Range
1	43	5
2	49	6
3	37	5
4	44	7
5	45	7
6	37	4
7	51	8
8	46	6
9	43	4
10	47	6
	442	58

Aim :- To construct $\bar{x}$ and R charts to the given data ~~to~~ and to comment on production process.

Procedure :- Let mean is denoted with $\bar{x}_i$ and range by R_i calculate.

mean of means $\bar{\bar{x}} = \frac{1}{k} \sum_{i=1}^{k} \bar{x}_i$

mean of ranges $\bar{R} = \frac{1}{k} \sum_{i=1}^{k} R_i$

The 3σ control limits for x-chart are

$UCL = \bar{\bar{x}} + 3 \frac{\bar{R}}{d_2\sqrt{n}}$ $\boxed{\because A_2 = \frac{3}{d_2\sqrt{n}}}$

$CL = \bar{\bar{x}}$

$LCL = \bar{\bar{x}} - 3 \frac{\bar{R}}{d_2\sqrt{n}}$

R-chart

$UCL = D_4\bar{R}$

$CL = \bar{R}$

$LCL = D_3\bar{R}$

~~where A_2 and d_2 are obtai~~

Calculation.

$\bar{\bar{x}} = \frac{1}{10} 442$

$\bar{\bar{x}} = 44.2$

$\bar{R} = \frac{1}{k} \sum R_i$

$= \frac{1}{10} (58)$

$= 5.8$

3σ control limits $\bar{x}$ chart are

$UCL = \bar{\bar{x}} + 3 \frac{\bar{R}}{d_2\sqrt{n}}$

$= 47.5454$

$LCL = 40.8546$

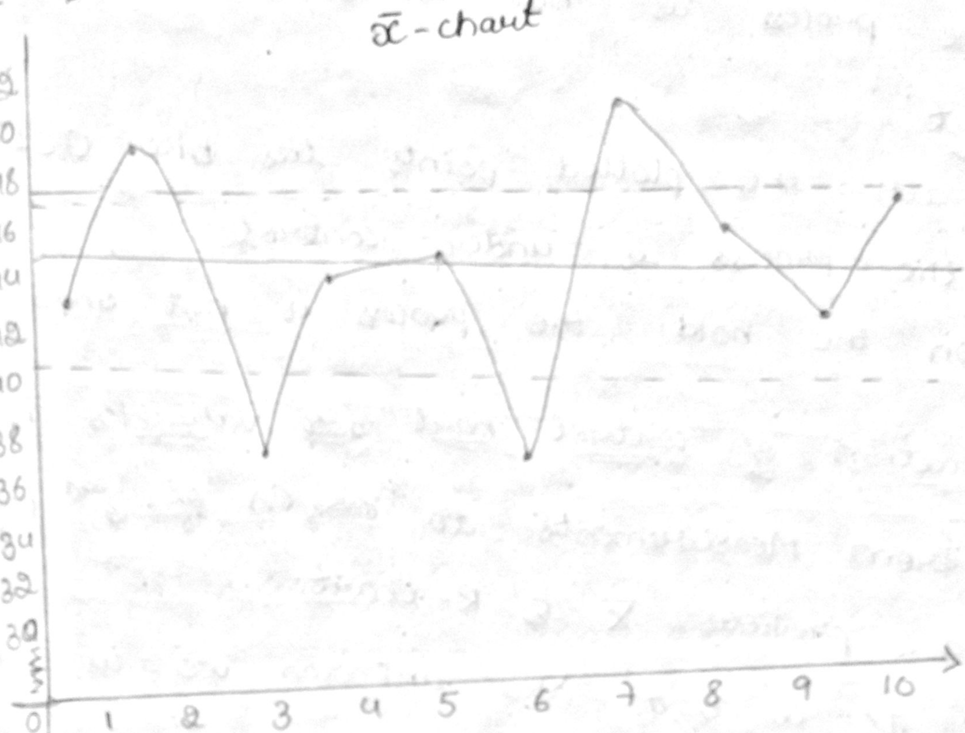
$CL = 44.2$

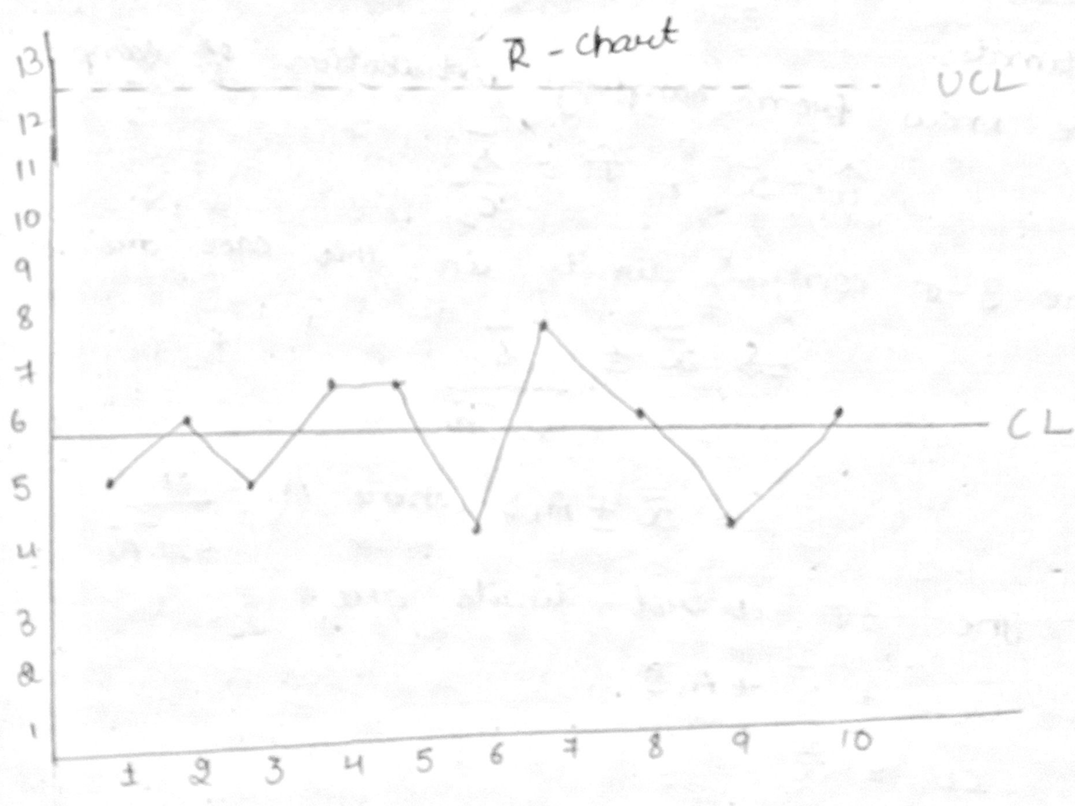
R-chart Control limit

UCL = 12.2670

CL = 5.8

LCL = 0





Conclusion $\bar{x}$ - chart

The sample observations corresponding to 2^{nd} sample is above UCL, 3^{rd} sample is below LCL, 6^{th} sample is below LCL, 7^{th} sample is above UCL.

So, the process is not under control.

R-chart

Since, all the plotted points lies b/w UCL and LCL the process is under control

But on the hold the process is not under control.

Construction of control chart for variables $\bar{x}$ & S.

Note:- from Measurements to case (i) of 3^{rd} point is same as previous $\bar{X}$ & R-chart.

case ii) :- If μ & σ are unknown we use its estimates

we know from sampling distribution of sample S.D

$$\hat{\mu} = \bar{x} \quad , \quad \hat{\sigma} = \frac{\bar{s}}{c_2}$$

The $3-\sigma$ control limits in this case are $\hat{\mu} \pm 3\frac{\hat{\sigma}}{\sqrt{n}}$

$$\Rightarrow \bar{\bar{x}} \pm \frac{3\bar{s}}{c_2\sqrt{n}}$$

$$\Rightarrow \bar{\bar{x}} \pm A_1 S \text{ where } A_1 = \frac{\bar{s}}{c_2\sqrt{n}}$$

$\therefore$ The 3σ control limits are

$$UCL = \bar{\bar{x}} + A_1\bar{S}$$

$$CL = \bar{\bar{x}}$$

$$LCL = \bar{\bar{x}} - A_1\bar{S}$$

S.D chart (s- chart) :-

The 3-σ control limits for S.D charts are

$E(S) \pm 3 \cdot SE(S)$

we know from Sampling Dist^n of sample S.D

$E(S) = C_2 \sigma$

$S.E(S) = C_3 \sigma$

The 3σ control limits in this case are

$= C_2\sigma \pm 3C_3\sigma$

case i) where $[\sigma = \sigma']$ (σ is known)

The 3σ control limits are

$UCL = C_2\sigma' + 3C_3\sigma'$

$= (C_2 + 3C_3)\sigma'$

$= B_2\sigma'$

$CL = C_2\sigma'$

$LCL = C_2\sigma' - 3C_3\sigma'$

$= (C_2 - 3C_3)\sigma'$

$= B_1\sigma'$

case ii) :- when σ is unknown

we know sample S.D is unbaised estimator of population S.D

i.e, $E(S) = \bar{S}$

Also we have $E(S) = C_2\sigma$; $S.E(S) = C_3\sigma$

$\bar{S} = C_2\sigma$

$\hat{\sigma} = \frac{\bar{S}}{C_2}$

The 3σ control limits are

$= C_2\hat{\sigma} + 3C_3\hat{\sigma}$

$= C_2\frac{\bar{S}}{C_2} + 3C_3\frac{\bar{S}}{C_2}$

$$= \left\{1 + 3\frac{c_3}{c_2}\right\}\bar{\sigma}$$

$$UCL = \left(1 + 3\frac{c_3}{c_2}\right]\bar{\sigma} = B_4\bar{\sigma}$$

$$CL = \bar{\sigma}$$

$$LCL = \left(1 - 3\frac{c_3}{c_2}\right]\bar{\sigma} = B_3\bar{\sigma}$$

Peratical-3

construct $\bar{X}$, R and S charts to the following data and comment on production process.

Sample NO	Life of Bulbs			
	1	2	3	4
1	1081	363	1092	1385
2	528	330	1053	945
3	984	1384	1194	456
4	728	972	647	792
5	804	845	1132	1024
6	1002	804	760	1035
7	994	1023	1136	842
8	616	832	497	692
9	982	1342	1132	945
10	1132	998	554	777
11	1134	1140	756	994
12	749	948	1050	857

Aim:- To construct the $\bar{X}$, R and S charts to and to comment on the ~~following~~ given data production process.

Procedure :-

Let x_{ij} be the j^{th} observation of i^{th} sample.

then calculate Mean of i^{th} sample $\bar{x}_i = \frac{1}{n}\Sigma x_i$

Range of i^{th} sample $R_i = Max(x_{ij}) - Min(x_{ij})$

S.D of i^{th} sample $s_i = \sqrt{\frac{1}{n}\Sigma x_{ij}^2 - \bar{x}_i^2}$

Also calculate Mean of Means, $\bar{\bar{x}} = \frac{1}{K}\sum_{i=1}^{k} \bar{x}$

Mean of Ranges $\bar{R} = \frac{1}{K}\sum_{i=1}^{k} R_i$

Mean of S.D's $\bar{s} = \frac{1}{K}\sum_{i=1}^{k} s_i$

The 3σ CL for $\bar{x}$ chart are

$UCL = \bar{\bar{x}} + A_2\bar{R}$

$CL = \bar{\bar{x}}$

$LCL = \bar{\bar{x}} - A_2\bar{R}$

The 3σ CL for R chart are

$UCL = D_4\bar{R}$

$CL = \bar{R}$

$LCL = D_3\bar{R}$

The 3σ CL for S.D chart are

$UCL = B_4\bar{s}$

$CL = \bar{s}$

$LCL = B_3\bar{s}$

Calculation:-

Sample No	1	2	3	4	$\bar{x}_i$	R_i	S_i
1	1081	363	1092	1385	980.25	1022	376.6493
2	528	330	1053	945	714	723	295.9451
3	984	1384	1194	456	1004.5	928	346.8440
4	728	972	647	792	784.75	325	119.6983
5	804	845	1132	1024	951.25	328	133.1679
6	1002	804	760	1035 ~~842~~	900.25	275	119.8382
7	994	1023	1136	842	998.75	294	104.9032
8	616	832	497	692	659.25	335	121.5635
9	982	1342	1132	945	1100.25	397	156.1544
10	1132	998	554	777	865.25	578	219.9106
11	1134	1140	756	994	1006	384	155.7113
12	749	948	1050	857	901	301	111.1868
					10865.5	5890	2261.5726

$$\bar{S}_i = \sqrt{\frac{1}{n}\sum x_{ij}^2 - \bar{x}_i^2}$$

$$= \sqrt{\left[\left[\frac{1}{4}\right] \times \left(1081^2 + 363^2 + 1092^2 + 1385^2\right)\right) - 980.25^2}$$

$$= 376.6493$$

for $n = 4$, $A_2 = 0.729$, $D_3 = 0$, $D_4 = 2.282$

$B_3 = 0$, $B_4 = 2.226$

$$\bar{\bar{x}} = \frac{1}{K}\sum \bar{x}_i = \frac{1}{12} \times 10865.5 = 905.4583$$

$$\bar{R} = \frac{1}{K}\sum R_i = \frac{1}{12} \times 5890 = 490.8333$$

$$\bar{S} = \frac{1}{K}\sum S_i = \frac{1}{12} \times 2261.5726 = 188.4644$$

3σ control limits of $\bar{x}$ chart

$UCL = A_2\bar{R} + \bar{\bar{x}}$

$= 905.4583 + (0.729)(490.8333)$

$= 1263.2758$

$CL = \bar{\bar{x}}$

$= 905.4583$

$LCL = \bar{\bar{x}} - A_2\bar{R}$

$= 547.6408$

3σ control limits of $\bar{R}$ chart

$UCL = D_4\bar{R}$

$= 2.282 \times \cancel{5}490.8333$

$= 1120.0816$

$CL = \bar{R}$

$= 490.8333$

$LCL = \cancel{B}\cdot D_3\bar{R}$

$= (0).\bar{R}$

$= 0$

3σ control limits of $\bar{S}$ chart

$UCL = B_4\bar{S}$

$= 2.226 \times 188.4644$

$= 419.5217$

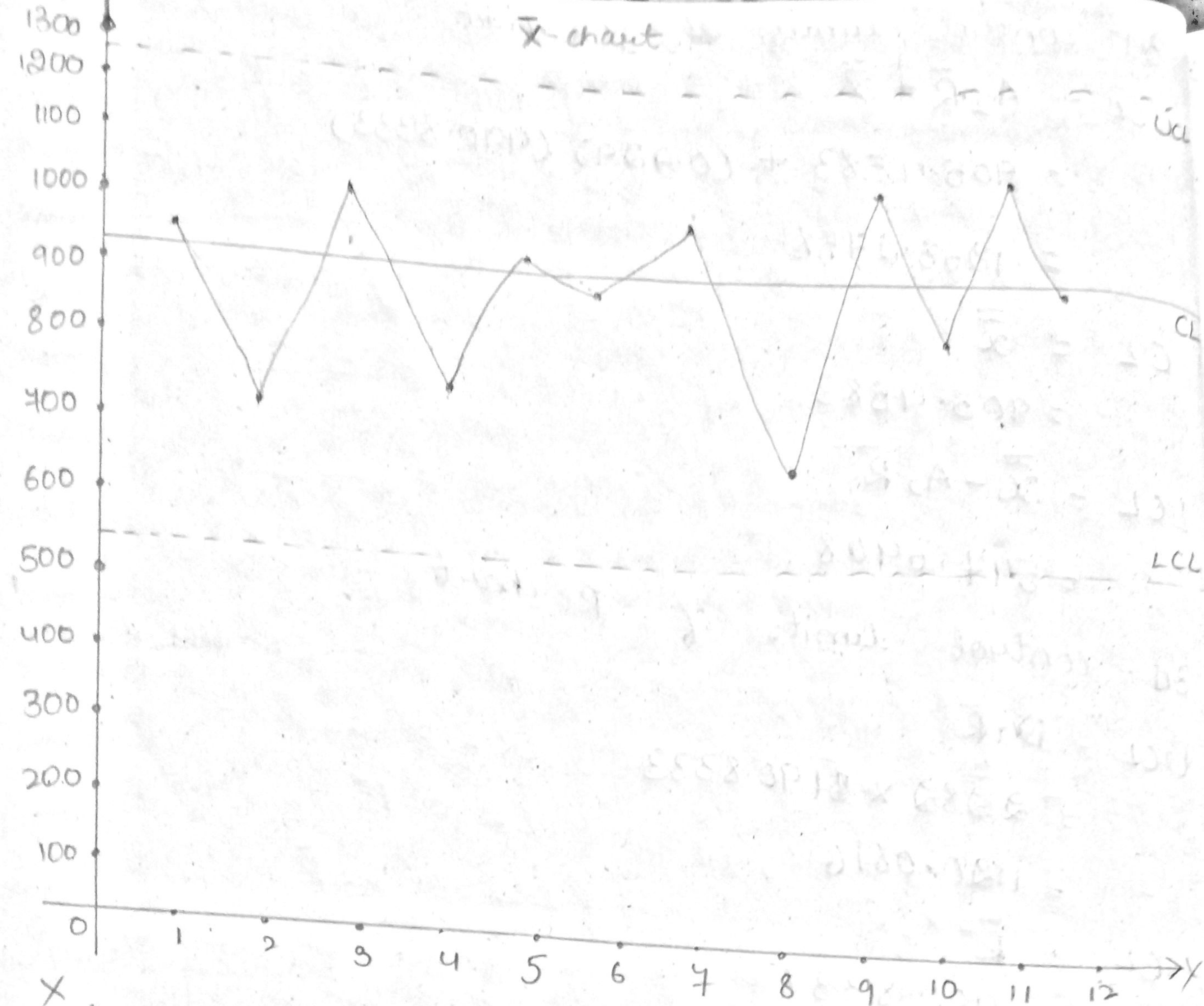
$CL = \bar{S}$

$= 188.4644$

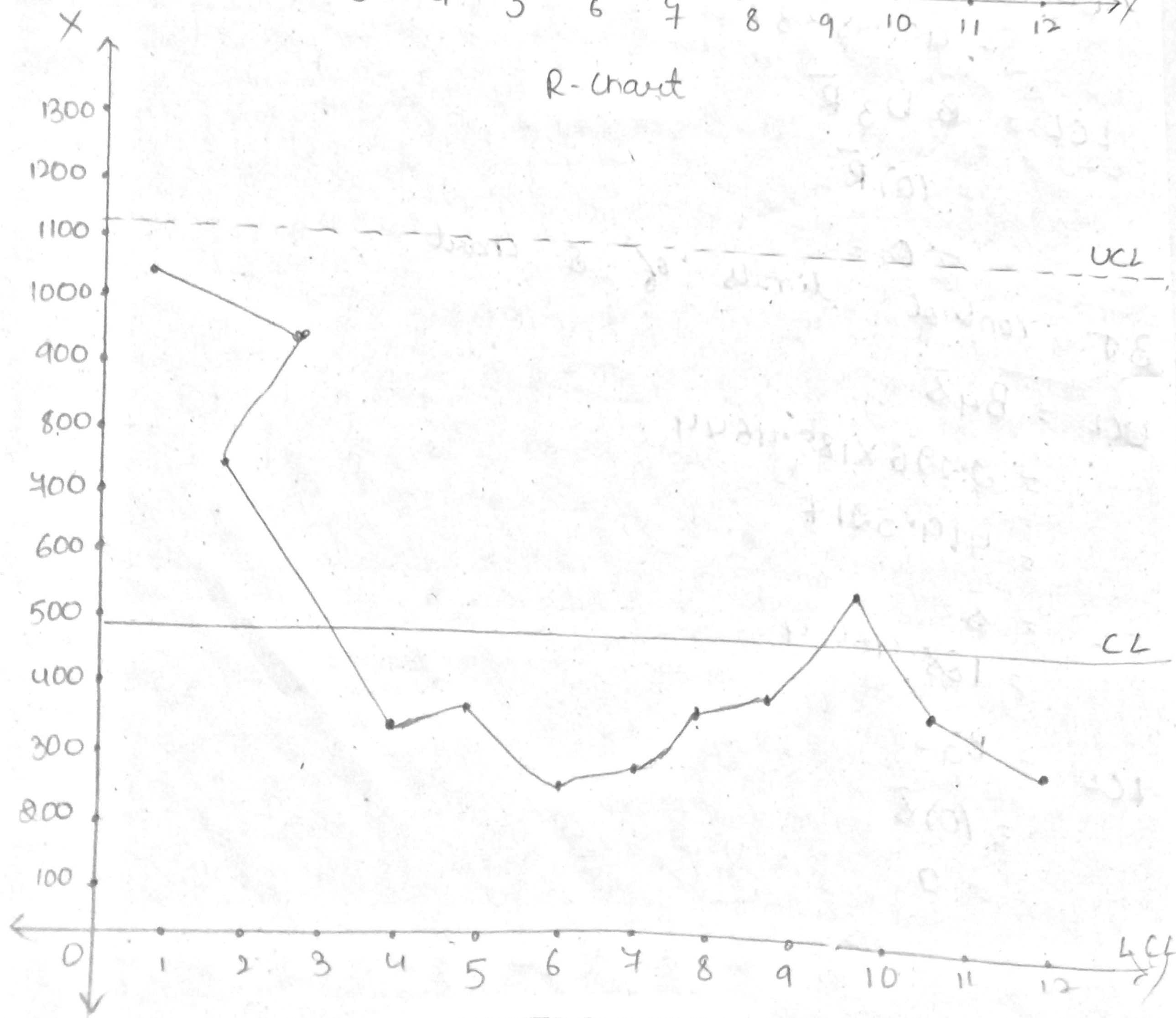
$LCL = B_3\bar{S}$

$= (0)\bar{S}$

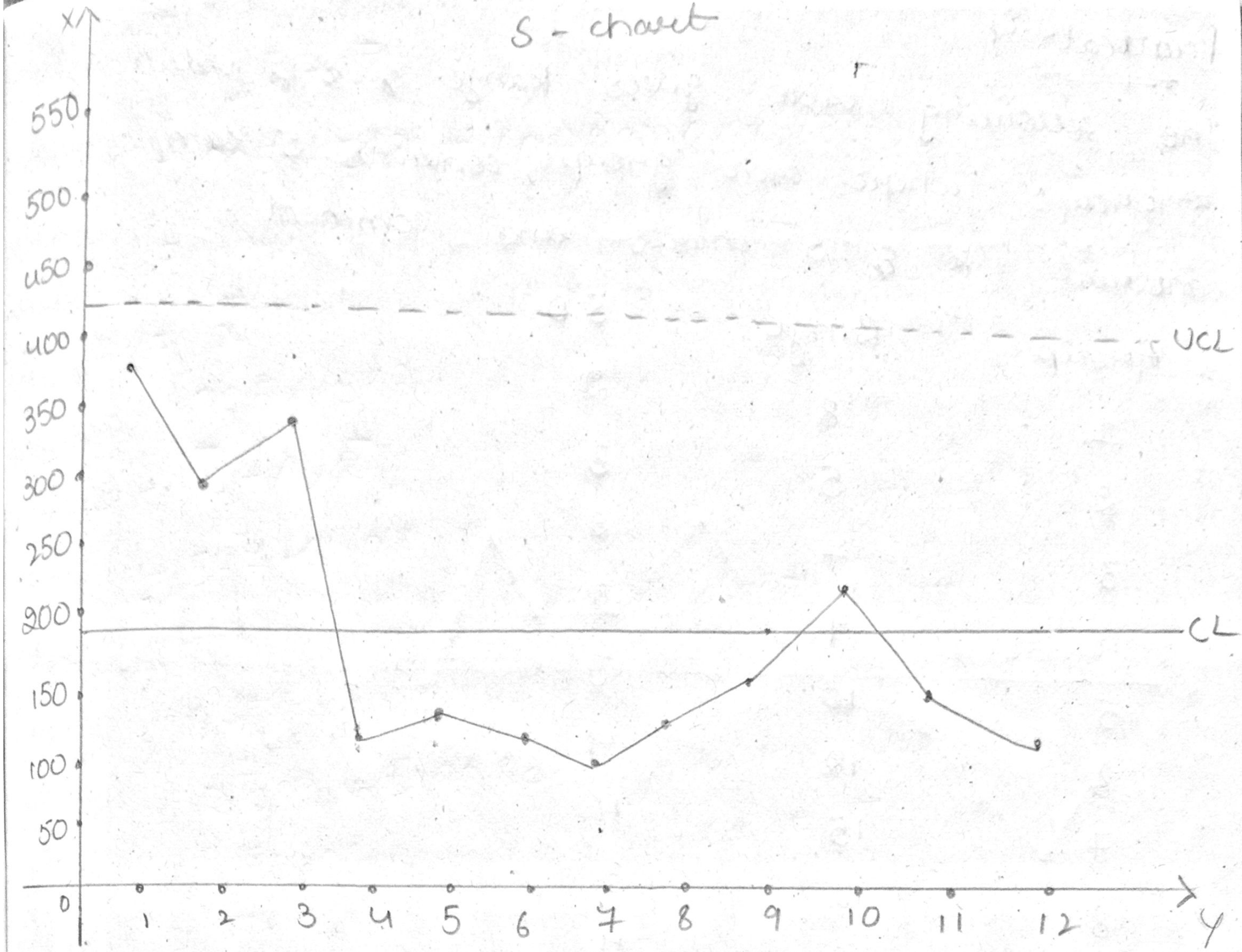
$= 0$



$\bar{X}$-chart
UCL
CL
LCL
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
1 2 3 4 5 6 7 8 9 10 11 12



R-Chart
UCL
CL
LCL
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
1 2 3 4 5 6 7 8 9 10 11 12



Conclusion:-

$\bar{X}$-chart :- Since all the points lies in b/w UCL and LCL. The process is under control.

R-chart :-

Since all the points lies in b/w UCL and LCL. The process is under control

S-chart :-

Since all the points lies in b/w UCL and LCL

The process is under control.

But on the whole the production process is under control.

Pratical- 4

The following table gives Range & S.D values of 15 groups where each group contains 5 samples construct R & S charts and comment.

Group	Range	S.D
1	8	2
2	5	3
3	7	2
4	9	2
5	10	3
6	12	4
7	15	4
8	12	3
9	12	4
10	15	6
11	10	2
12	10	3
13	9	2
14	15	4
15	12	2
	161	46

Aim:- To construct Range & σ chart to the given data

Procedure:-

Let R_i denote Range & S_i denote S.D then calculate

Mean of Ranges $\bar{R} = \frac{1}{K} \sum_{i=1}^{k} R_i$

Mean of S.D's $\bar{S} = \frac{1}{K} \sum_{i=1}^{k} S_i$

The 3-σ control limits for R-chart are

$UCL = D_4\bar{R}$

$CL = \bar{R}$

$LCL = D_3\bar{R}$

The 3-σ control limits for σ-chart (or) S-chart are

$UCL = B_4\bar{s}$

$CL = \bar{s}$

$LCL = B_3\bar{s}$

Calculation:-

for $n=5$, $K=15$, $D_3=0$, $B_3=0$, $D_4=2.115$, $B_4=2.089$

$\Sigma R_i = 161$, $\Sigma s_i = 46$

$$\bar{R} = \frac{1}{K}\sum_{i=1}^{K} R_i = \frac{161}{15} = 10.7333$$

$$\bar{s} = \frac{1}{K}\sum_{i=1}^{K} s_i = \frac{46}{15} = 3.0667$$

The 3σ control limits for R-chart are

$UCL = 2.115 \times 10.7333$

$= 22.7009$

$CL = 10.7333$

$LCL = 0$

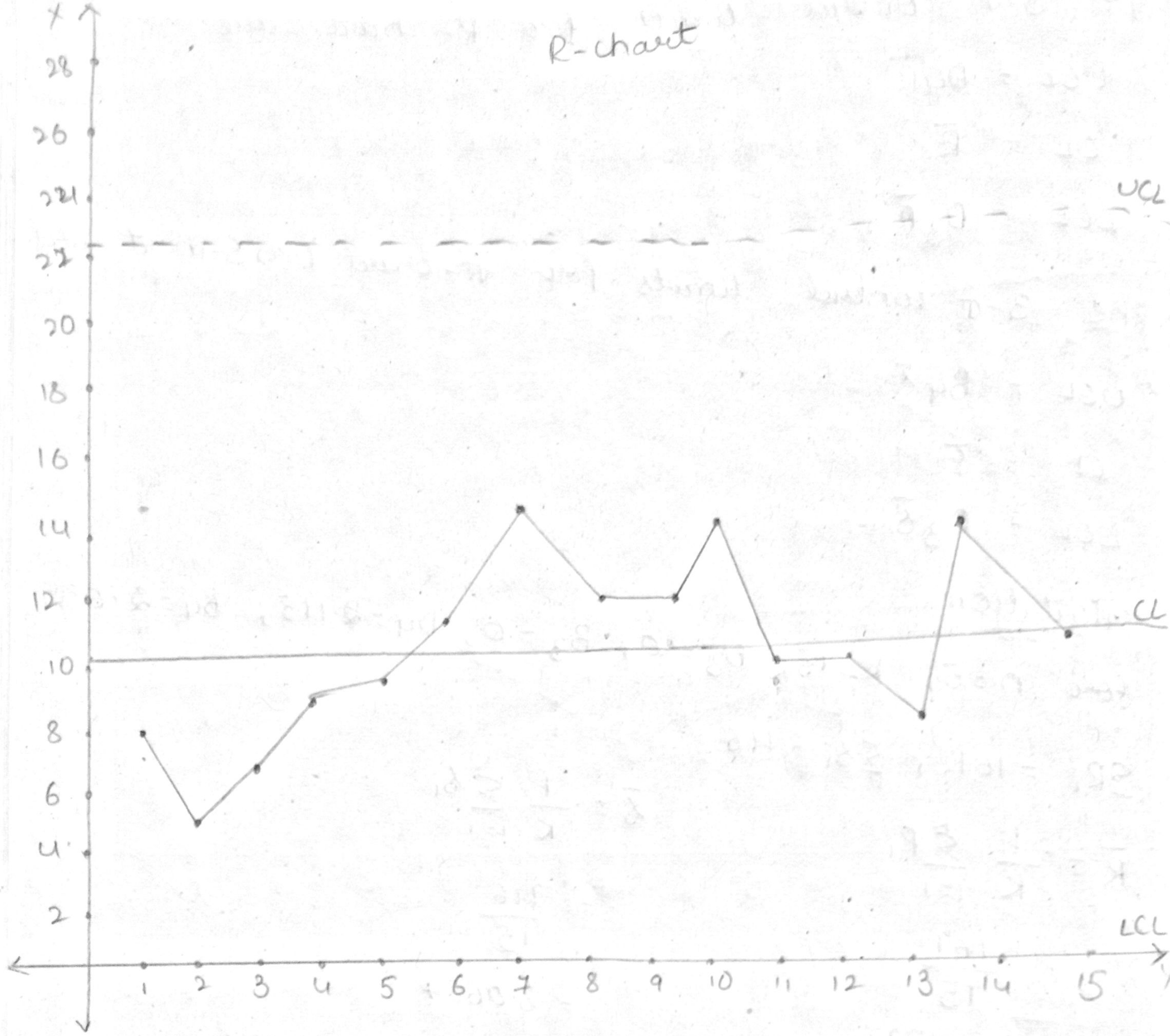
The 3σ control limits for σ-chart are

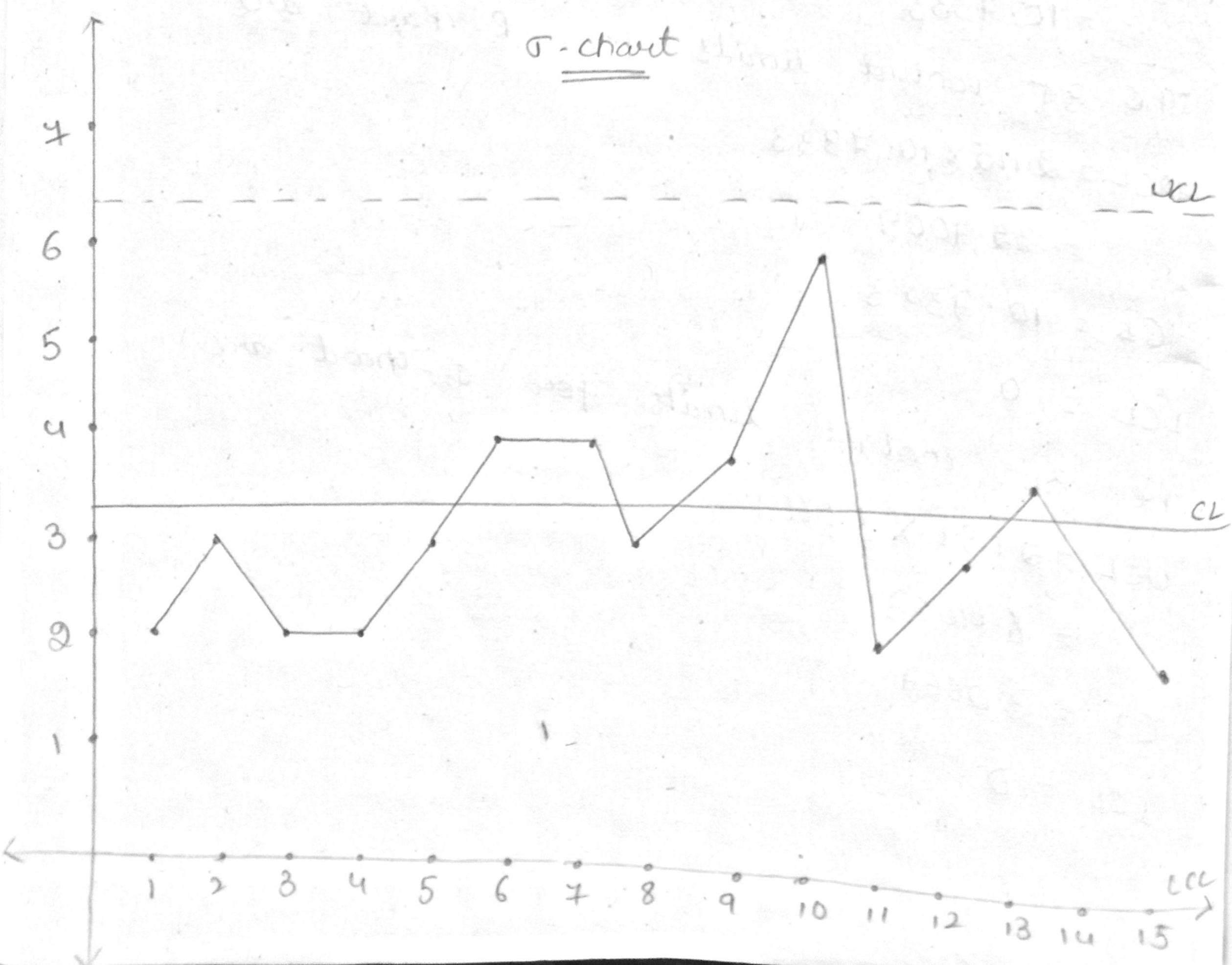
$UCL = 2.089 \times 3.0667$

$= 6.4063$

$CL = 3.0667$

$LCL = 0$


R-chart
UCL
CL
LCL
28
26
24
22
20
18
16
14
12
10
8
6
4
2
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15


σ-chart
UCL
CL
LCL
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Conclusion

1) Range chart :-

Since all the plotted points lies in b/w UCL and LCL. The process is under control for Range chart

2) σ - chart :-

Since all the plotted points lies in b/w UCL and LCL. The process is under control for σ-chart.

$\therefore$ So, on the whole the process is under control.

Control chart for attributes

Control chart for variables ($\bar{x}$, R, σ) are restricted because of the following limitations.

1) They are the charts for variables only i.e for quality characteristics which can be measured & expressed in numbers.

2) In certain situations they are impracticable and un economical. for example, if the no. of characteristics for product there we have to construct are 25 control charts.

$25 \times 2 = 50$ control charts.

As an alternatives to $\bar{x}$.R charts we have CC for attributes and they are classified as

1. CC for defectives
   a. No. of defectives chart [np (or) d chart]
   b. Proportion of defectives or fraction defective chard [P]

2. CC for no. of defects.
   a. no. of defects per unit [c-chart]
   b. no. of defects per n unit [u-chart]

1. CC for no. of defectives [np (or) d chart] - fixed Sample Size.

Let d be the no. of defectives in a sample of Size 'n' then proportion of defectives p is $p = \frac{d}{n}$

Here the no. of defectives d follows binomial distribution with parameters n, P

i.e $d \sim B(n, P)$

we know, Mean $E(d) = nP$

Variance $V(d) = nPQ$

The 3σ cl for d chart are

$$E(d) \pm 3 S.E(d)$$

$$\Rightarrow nP \pm 3\sqrt{nPQ}, \quad Q = 1 - P$$

Case - 1 when P and Q are known

($P - P'$ and $Q - Q'$)

The 3σ control limits in this case are

$$UCL = nP' + 3\sqrt{nP'Q'}$$

$$CL = nP'$$

$$LCL = nP' - 3\sqrt{nP'Q'}$$

Case - 2 :- when P and Q are unknown

Let d_i be the no. of defectives and p_i be the proportion of defectives for the i^{th} Sample. Then the population proportion P can be estimated as follows.

Let $\bar{P} = \frac{1}{K} \sum_{i=1}^{K} p_i$

$$= \frac{1}{K} \sum_{i=1}^{K} \frac{d_i}{n}$$

$$E(\bar{P}) = E\left[\frac{1}{K} \sum_{i=1}^{K} \frac{d_i}{n}\right]$$

$$E(\bar{P}) = \frac{1}{kn} \sum_{i=1}^{k} E(d_i)$$

$$= \frac{1}{kn} \sum_{i=1}^{n} np$$

$$= \frac{1}{kn} knp$$

$$E(\bar{P}) = P$$

$$\hat{P} = \bar{P}$$

Then the 3-σ control limits in this case are

$n\hat{p} \pm 3\sqrt{n\hat{p}\hat{q}}$

$UCL = n\bar{p} + 3\sqrt{n\bar{p}\bar{q}}$

$CL = n\bar{p}$

$LCL = n\bar{p} - 3\sqrt{n\bar{p}\bar{q}}$

Construction :-

np chart is constructed by taking sample no's on x-axis and defectives on y-axis. At CL a bold line and at UCL and LCL two dotted horizontal lines drawn parallel to x-axis. All the no. of defectives (d_i) points are plotted and join them by straight line.

Interpretation :-

If all plotted points lies in b/w UCL & LCL then the process is under control. O.w not under control and conclude that assignable cause is present in the process.

Practical-5

The following are the figures of defectives in 22 Lots, each containing 2000 rubber belts. Draw the control chart for no. of defectives & comment.

425, 430, 216, 341, 225, 322, 280, 306, 337, 305, 356, 402, 216, 264, 126, 409, 193, 326, 280, 389, 451, 420.

Sol) Aim :- To construct no. of defectives chart to the given data & to comment on production process.

Procedure :-

Let d_i be the no. of defectives in a sample of size 'n' then the proportion of defectives $P_i = \frac{d_i}{n}$

The 3-σ control limits for np chart are

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}\bar{q}}$$

$$CL = n\bar{p}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}\bar{q}}$$

where $\bar{p} = \frac{\sum_{i=1}^{k} P_i}{k}$; $\bar{q} = 1 - \bar{p}$

Calculation

S.No	Sample size(n)	No. of defectives(d_i)	$P_i = \frac{d_i}{n}$
1	2000	425	0.2125
2	2000	430	0.2150
3	"	216	0.1080
4	"	341	0.1705
5	"	225	0.1125
6	"	322	0.1610
7	"	280	0.1400
8	"	306	0.1530
9	"	337	0.1685
10	"	305	0.1525
11	"	356	0.1780
12	"	402	0.2010
13	"	216	0.1080
14	"	264	0.1320
15	"	126	0.0630
16	"	409	0.2045
17	"	193	0.0965
18	"	326	0.1630
19	"	280	0.1400
20	"	389	0.1945
21	"	451	0.2255
22	2000	420	0.2100
			3.5095

$$\bar{p} = \frac{1}{22}(3.5095)$$

$$= 0.1595$$

$$q = 1 - \bar{p}$$

$$= 1 - 0.1595$$

$$= 0.8405$$

The 3-σ control limits for np chart are

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}\bar{q}}$$

$$= 2000 \times (0.1595) + 3\sqrt{2000 \times 0.1595 \times 0.8405}$$

$$= 368.1231$$

$$CL = n\bar{p}$$

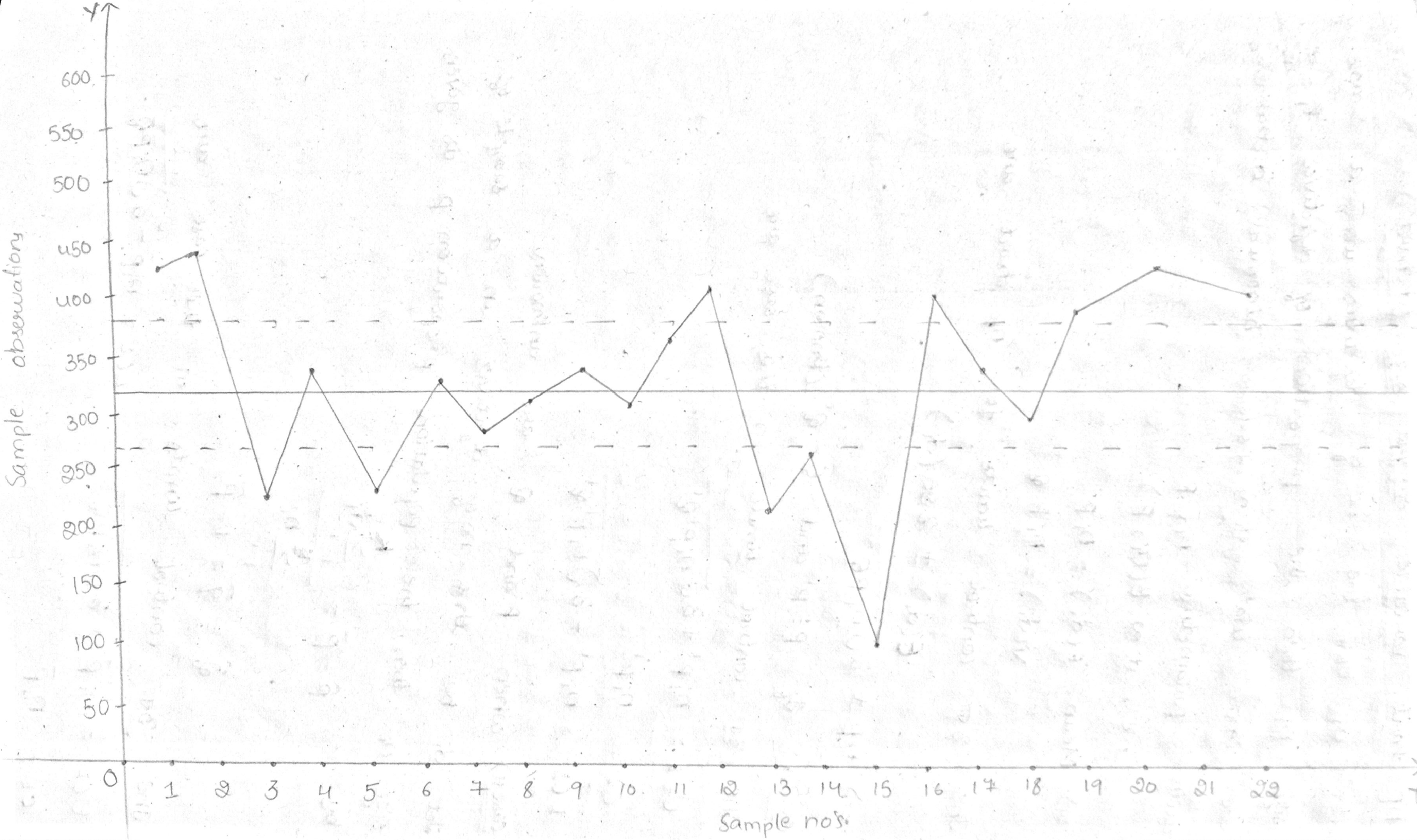
$$= 319$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}\bar{q}}$$

$$= 269.8769$$

Conclusion / Inference

Since the sample observations corresponding to 1, 2, 12, 16, 20, 21, 22 are above UCL & sample observations corresponding to 3, 5, 13, 14, 15, 17 are below LCL. Hence the process is not under control.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8
9
10
11
12
13
14
15
16
17
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19
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21
22
X
Sample observations
Sample no's

np chart variable Sample size (d chart)

Let d_i be the no. of defectives in a sample of size n_i then the proportion of defectives $P_i = \frac{d_i}{n_i}$.

we know that $d_i \sim$ follows binomial distribution with parameters n_i, P.

i.e, $d_i \sim B(n_i, P)$

$\Rightarrow$ Mean. $E(d_i) = n_i P$

$V(d_i) = n_i P \cdot Q$

The 3σ control limits for np chart are

$$E(d_i) \pm 3\,SE(d_i)$$

$\Rightarrow np \pm 3\sqrt{n_i PQ}$

case i) :- If $P = P'$ and $Q = Q'$ (known)

The 3σ control limits in this case are

$UCL = n_i p' + 3\sqrt{n_i p' Q'}$

$CL = n_i p'$

$LCL = n_i p' - 3\sqrt{n_i p' Q'}$

case ii) when P and Q are unknown

Let d_i be the no. of defectives in a sample of size n_i then the population proportion p is given by

$$\hat{P} = \bar{P} = \frac{\sum_{i=1}^{K} d_i}{\sum_{i=1}^{K} n_i}$$

$$\hat{Q} = \bar{q} = 1 - \bar{P}$$

The 3σ control limits in this case are

$UCL = n_i \bar{p} + 3\sqrt{n_i \bar{P}\bar{Q}}$

$LCL = n_i \bar{P} - 3\sqrt{n_i \bar{P}\bar{Q}}$

$CL = n_i \bar{P}$

~~control~~ construction

np chart for variable sample size is constructed by taking sample no's on x-axis and no. of defectives on y-axis. All UCL'S, LCL'S are plotted and join them with dotted lines. All ~~CL'S~~ CL'S are plotted and join them by straight line. All the no. of defectives ($d_i, d_2 --- d_K$) are plotted and join them with straight line.

Interpretation

If any di point lies above corresponding UCL (or) below corresponding LCL then then the process is not under control (or) under control.

Puatical-6

Construct np chart for the following data and comment on production process.

Sample no. ~~Size~~	Sample Size	No. of defectives
1	2000	425
2	1500	430
3	1400	216
4	1350	341
5	1250	225
6	1760	322
7	1875	280
8	1955	306
9	3125	337
10	1575	305

Aim :- To construct np chart variable sample size to the given data & to comment on production process.

Procedure :-

Let 'd_i' be the no. of defectives in a sample of size n_i then the proportion of defectives p_i

$$p_i = \frac{d_i}{n_i} \; ; \; i = 1, 2 - - - K$$

The 3σ control limits for np chart are

$$UCL = n_i\bar{p} + 3\sqrt{n_i\bar{p}\bar{q}}$$

$$CL = n_i\bar{p}$$

$$LCL = n_i\bar{p} - 3\sqrt{n_i\bar{p}\bar{q}}$$

where $\bar{p} = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i}$

$\bar{q} = 1 - \bar{p}$

Calculation

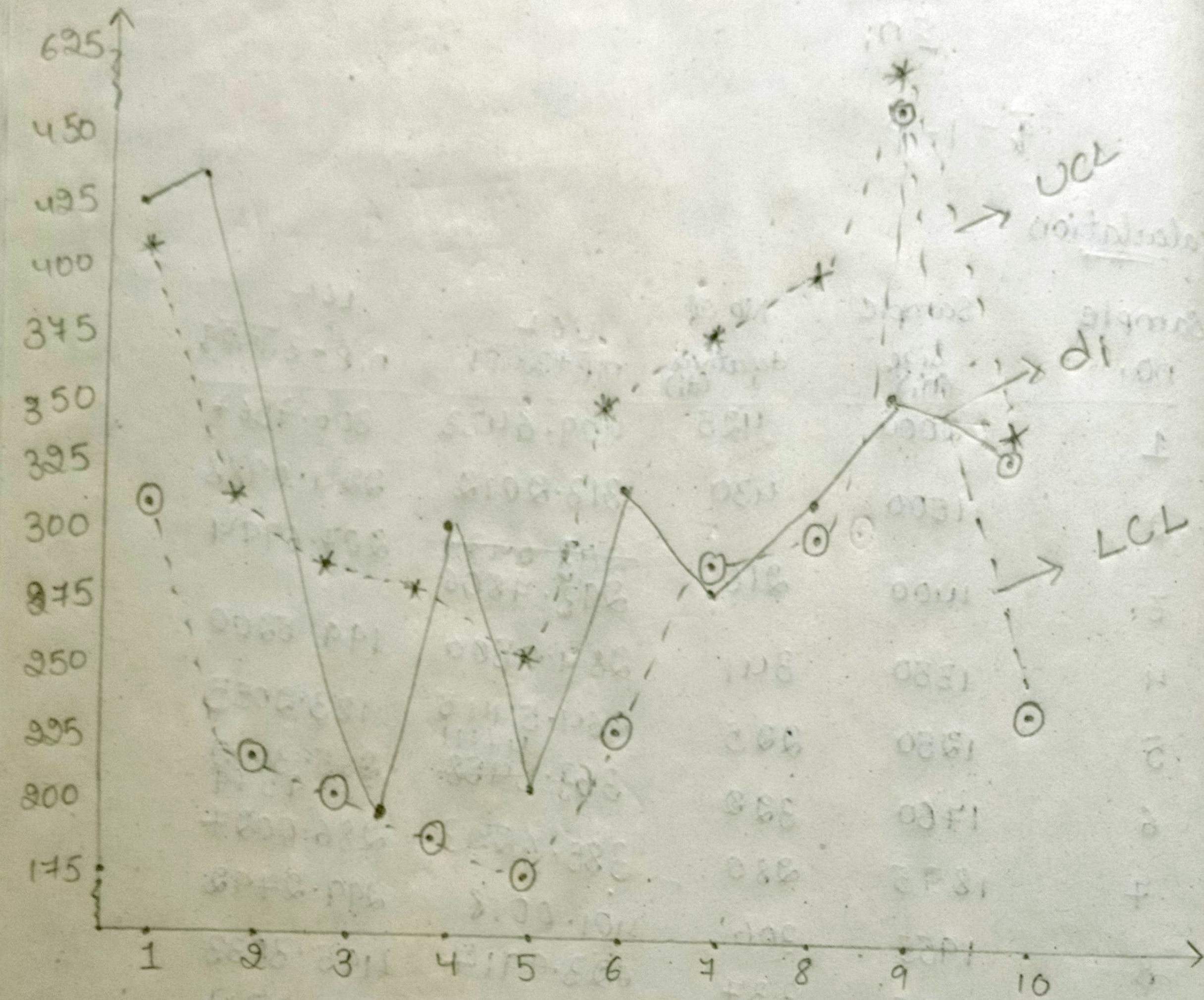
Sample no.	Sample size (n_i)	No. of defectives (d_i)	UCL $n_i\bar{p} + 3\sqrt{n_i\bar{p}\bar{q}}$	LCL $n_i\bar{p} - 3\sqrt{n_i\bar{p}\bar{q}}$
1	2000	425	409.6433	306.7567
2	1500	430	313.2012	224.0988
3	1400	216	~~293.6994~~ 293.7806	207.6994
4	1350	341	284.0500	199.5200
5	1250	225	264.5445	183.2055
6	1760	322	363.~~5458~~ 4741	~~265.3042~~ 266.9579
7	1875	280	385.6223	286.0027
8	1955	306	401.0018	299.2792
9	3125	337	623.9917	495.3833
10	1575	308	327.7339	236.4311
	17790	3187		

$\bar{p} = \frac{\sum d_i}{\sum n_i}$

$= \cancel{17790}\ \frac{3187}{17790}$

$= 0.1791$

$\bar{q} = 0.8209.$



Inference / conclusion

Since, the sample observations corresponding to the 1, 2, 4, are above corresponding UCL's and the sample observations corresponding to 7th & 9th are below LCL's

∴ The process is not under control.

P-chart [Proportion of defectives / fraction defectives chart] [Fixed sample size]

Let d be the no. of defectives in a sample of size 'n' then the proportion of defectives $p = \frac{d}{n}$

The no. of defectives d follows binomial dist. with parameters (n, P)

i.e $d \sim B(n, P)$

mean, $E(d) = nP$

variance $V(d) = nPQ$

In proportion of defectives chart p is taken as the statistic

then the 3σ control limit for P-chart are

$$E(P) \pm 3 \cdot S \cdot E(P)$$

$$E(P) = E\left(\frac{d}{n}\right)$$

$$= \frac{1}{n} E(d)$$

$$= \frac{1}{\cancel{n}} \cancel{n} P$$

$$E(P) = P$$

$$V(P) = V\left(\frac{d}{n}\right)$$

$$= \frac{1}{n^{\cancel{2}}} \cancel{n} PQ$$

$$= \frac{PQ}{n}$$

$\therefore$ The 3σ control limit are $P \pm 3\sqrt{\frac{PQ}{n}}$

case i) :- when P and Q are known (i.e, $P - P'$ & $Q - Q'$)

then the 3σ C.L are

$$UCL = p' + 3\sqrt{\frac{p'Q'}{n}}$$

$$CL = p'$$

$$LCL = p' - 3\sqrt{\frac{p'Q'}{n}}$$

case 2) :- when P and Q are unknown

Let $\bar{P} = \frac{1}{K}\sum_{i=1}^{k} P_i$

$$= \frac{1}{K}\sum_{i=1}^{k} \frac{d_i}{n}$$

$$E[\bar{P}] = E\left[\frac{1}{K}\sum_{i=1}^{k}\frac{d_i}{n}\right]$$

$$= \frac{1}{Kn}\sum_{i=1}^{k} E(d_i)$$

$$= \frac{1}{\cancel{kn}} \cancel{kn}p$$

$$E(\bar{P}) = P$$

$$\hat{P} = \bar{P}, \quad \hat{Q} = \bar{q} = 1-\bar{P}$$

The 3σ C.L in this cases are

$$UCL = \bar{P} + 3\sqrt{\frac{\bar{P}\bar{q}}{n}}$$

$$CL = \bar{P}$$

$$LCL = \bar{P} - 3\sqrt{\frac{\bar{P}\bar{q}}{n}}$$

Construction :-

P chart is constructed by taking sample no's on X-axis and p on y-axis. At CL a bold line and at UCL & LCL two dotted horizontal lines are drawn parallel to X-axis. All the

P_i points are plotted and join them by straight lines.

Interpretation

If all the plotted points lies in b/w UCL & LCL then the process is under control other wise not under control.

p- chart Variable sample size :-

Let d_i be the no. of defectives in a sample of size n_i then the proportion of defectives $p_i = \frac{d_i}{n_i}$.

WKT, the no. of defectives d_i follows binomial distribution with parameters (n_i, P)

$$\text{i.e., } d_i \sim B(n_i, P)$$

$$\text{Mean, } E(d_i) = n_i P$$

$$\text{Variance, } V(d_i) = n_i PQ$$

In P- chart the proportion of defectives p_i is taken as the statistic then the 3σ C.L are

$$E(P_i) \pm 3SE(P_i)$$

$$E(P_i) = E\left(\frac{d_i}{n_i}\right)$$

$$= \frac{1}{n_i} E(d_i)$$

$$= \frac{1}{\cancel{n_i}} \times \cancel{n_i} P$$

$$E(P_i) = P$$

$$V(P_i) = V\left(\frac{d_i}{n_i}\right)$$

$$= \frac{1}{n_i^2} V(d_i)$$

$$= \frac{1}{n_i^2} n_i PQ$$

$$= \frac{PQ}{n_i}$$

Then the 3σ C.L for chart are

$$P \pm 3\sqrt{\frac{PQ}{n_i}}$$

case i) :- when P and Q are known $(P = P' \,\&\, Q = Q')$

the 3σ CL this case are

$$UCL = p' + 3\sqrt{\frac{P'Q'}{n_i}}$$

$$CL = p'$$

$$LCL = p' - 3\sqrt{\frac{P'Q'}{n_i}}$$

case ii) :- when P and Q are unknown :

Let d_i be the no. of defectives in a sample of size n_i then, the population proportion P can be estimated as

$$\hat{p} = \bar{p} = \frac{\Sigma d_i}{\Sigma n_i}$$ and $\bar{q} = 1 - \bar{p}$ then the 3σ CL in this case are

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}$$

Conclusion :-

P-chart is constructed by taking sample no's on x-axis and p_i on y-axis. All UCL and LCL are plotted and join them with dotted lines and all P_i values and plotted and joined with straight line.

Practical-7

Each day a sample of items from production process was examined the no. of defectives found in each case was as follows. construct fraction defectives chart by taking sample size as 50.

S.NO	1	2	3	4	5	6	7	8	9	10	11
No. of deffectives	1	6	5	4	2	5	3	6	1	11	5

S.NO	12	13	14	15	16	17	18	19	20
No. of defectives	2	4	1	5	4	2	3	1	5

Aim :- To construct P-chart for the given data and to comment on production process.

Procedure :-

Let d_i be the no. of defectives in a sample of size n'

Let P_i be the proportion of defective in i^{th} sample

$P_i = \frac{d_i}{n}$

The 3σ control limits for P-chart are

$UCL = \bar{P} + 3\sqrt{\frac{\bar{P}\bar{q}}{n}}$ Here $\bar{P} = \frac{\sum_{i=1}^{K} P_i}{n}$

$CL = \bar{P}$ $q = 1-\bar{P}$

$LCL = \bar{P} - 3\sqrt{\frac{\bar{P}\bar{q}}{n}}$

Calculation

S.NO.	Sample size (n)	No. of defectives (di)	$P_i = \frac{d_i}{n}$
1	50	1	0.0200
2	50	6	0.1200
3	50	5	0.1000
4	50	4	0.08
5	50	2	0.04
6	〃	5	0.1
7	〃	3	0.06
8	〃	6	0.12
9	〃	1	0.02
10	〃	11	0.22
11	〃	5	0.1
12	〃	2	0.04
13	〃	4	0.08
14	〃	1	0.02
15	〃	5	0.1
16	〃	4	0.08
17	〃	2	0.04
18	〃	3	0.06
19	〃	1	0.02
20	〃	5	0.1
			~~0.076~~ 1.520

$\bar{p} = \frac{\Sigma P_i}{K}$

$= \frac{1.52}{20}$

$= 0.076$

$\bar{q} = 0.924$

$CL = \bar{p} = 0.076$

$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$

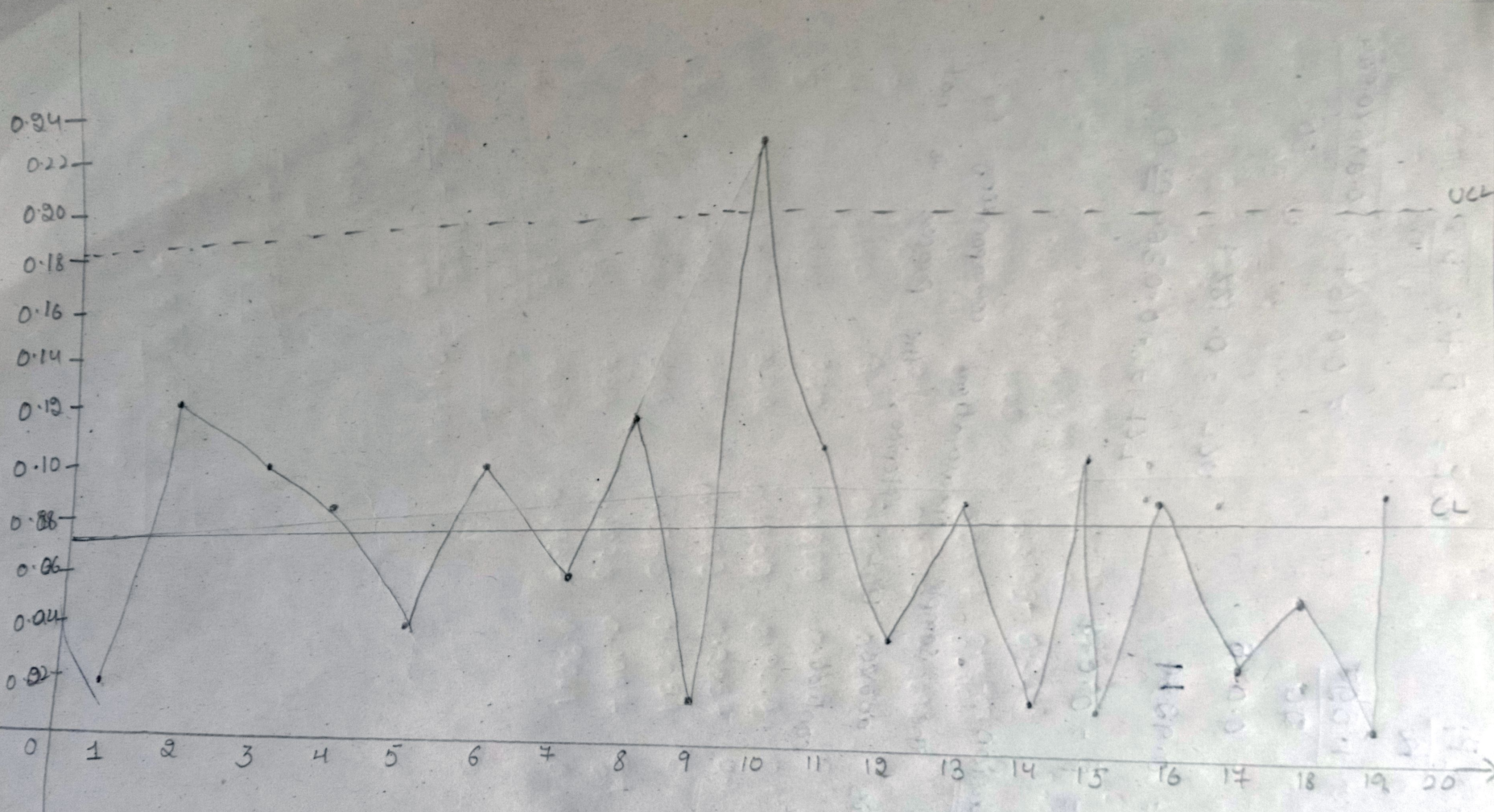
$= 0.076 + 3\sqrt{\frac{0.076 \times 0.924}{50}}$

$UCL = 0.1884$

$LCL = -0.0364 \cong 0$

Conclusion :-

Since the sample observations corresponding to 10 is above UCL. Hence, the process is not under control.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UCL
CL

Practical 8 :-

The following data gives

Drawn control chart for fraction defectives and comment on production process.

Sample NO	Sample Size	No. of defectives
1	2000	425
2	1500	430
3	1400	216
4	1350	341
5	1250	225
6	1760	322
7	1875	280
8	1955	306
9	3125	337
10	1575	305

Aim:- To construct the control chart of for fraction defectives (p-chart) and to comment on production process.

Procedure

Let d_i be the no. of defectives in a sample of size 'n_i' ; $i = 1, \dots k$

Let P_i be the proportion of defectives in i^{th} sample $P_i = \frac{d_i}{n_i}$

Then the ~~prop~~ population proportion can be calculated as

$$\hat{p} = \bar{p} = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i} \quad \text{and} \quad \bar{q} = 1 - \bar{p}$$

The 3σ control limits for p-chart are

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}$$

Calculation

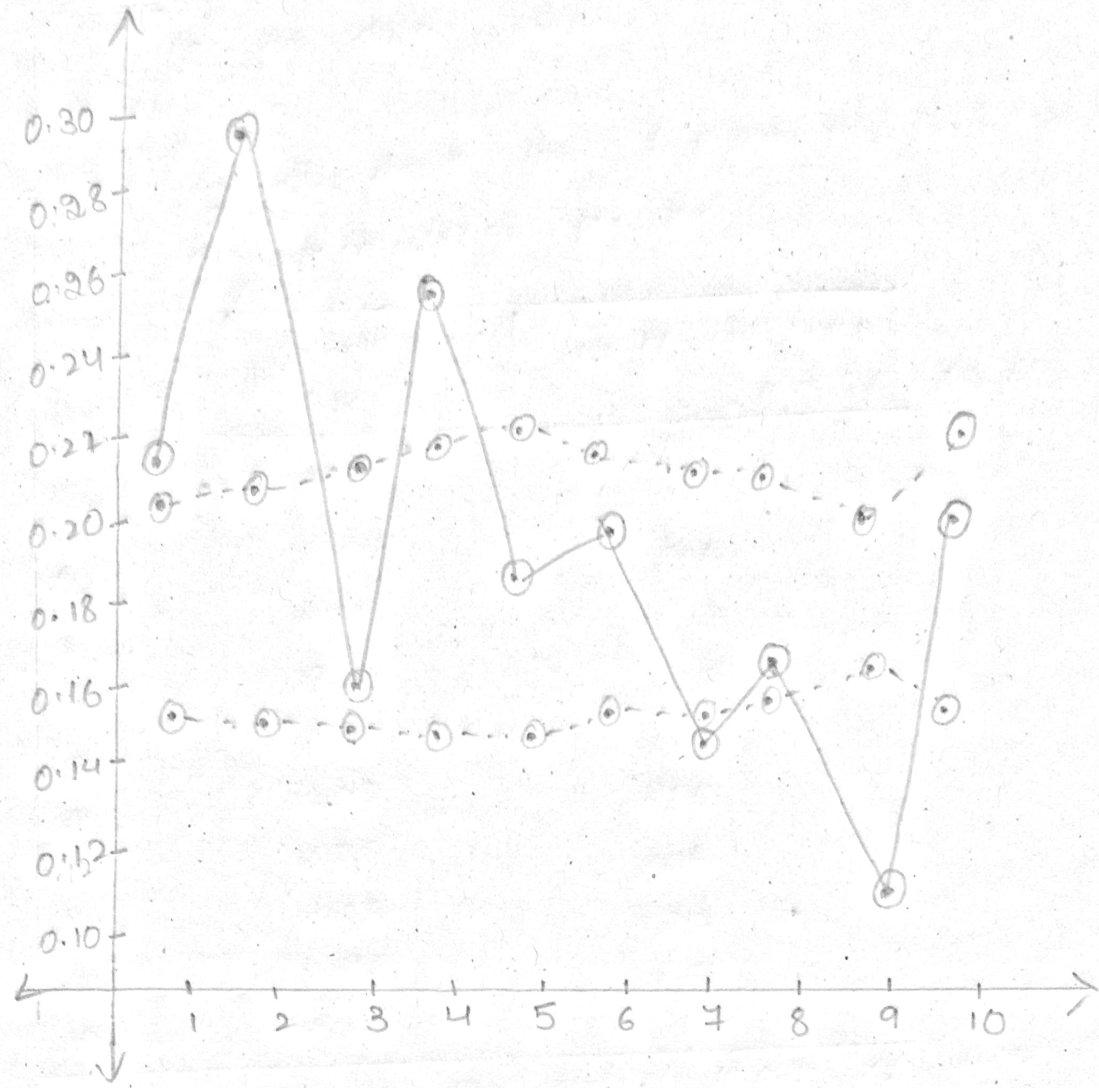
Sample No	Sample Size (n_i)	No. of defective (d_i)	$p_i = \frac{d_i}{n}$	UCL	LCL
1	2000	425	0.2125	0.2048	0.1534
2	1500	430	0.2867	~~0.1494~~ 0.2088	0.1494
3	1400	216	0.1543	0.2098	0.1484
4	1350	341	0.2526	0.2104	0.1478
5	1250	225	0.1800	0.2116	0.1466
6	1760	322	0.1830	0.2065	0.1517
7	1875	280	0.1493	0.2057	0.1525
8	1955	306	0.1565	0.2051	0.1531
9	3125	337	0.1078	0.1997	~~0.1997~~ 0.1585
10	1575	305	0.1937	0.2081	0.1501
	17790	3187			

$$\bar{p} = \frac{\Sigma d_i}{\Sigma n_i} = \frac{3187}{17790} = 0.1791$$

$$q = 0.8209$$

$$UCL = \cancel{\bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}}$$

$$= \cancel{0.1791 + 3\sqrt{\frac{0.1791 \times 0.8209}{17790}}}$$



<u>Conclusion</u>:-

Since, the sample observation corresponding to 1, 2, 4 are above corresponding UCL'S and sample observation corresponding to 7, 9 are below corresponding LCL'S so, the process is not under control.