UNIT-II

Time Series Analysis

Def:- Arrangement of statistical data in chronological order i.e, in accordance with occurrence of time, is known as time series.

A time series finds the relationship between two variables, one of them being time.

Ex: The population (Y_t) of a country in different years (t). Temperature (Y_t) of a place on different days (t) etc.

→ Mathematically, a Time Series is defined by the functional relationship $Y_t = f(t)$ where, Y_t is the value of the variable under consideration at time t. If the values of a variable at times $t_1, t_2, \ldots t_n$ are $Y_1, Y_2 \ldots Y_n$ respectively then the series

$t : t_1, t_2, \ldots t_n$

$Y_t : Y_1, Y_2 \ldots \ldots Y_n$

constitute a time series.

Thus, a time series gives a bivariate distribution, one of the two variables being time (t) and the other being the value (Y_t) at different points of the time. The values of the time may be given yearly, monthly, weekly, daily and hourly also.

Components of Time Series

There are 3 types of components

1) Secular Trend (or) Long term movement
2) Periodic changes (or) Short term fluctuations
   (i) Seasonal variations
   (ii) Cyclic variations
3) Random (or) Irregular movements

Measurements of Trend

Trend can be measured by the following methods

1. Graphic method (or) free hand curve fitting
2. Method of semi averages
3. Method of curve fitting by principle of least squares
4. Method of ~~semi~~ moving averages

2. Method of Semi averages

Problem 1: Fit a trend line to the following data by the method of semi averages

Year	Profit (in crores)	Avg
1992	53	
1993	79	$\bar{x}_1 = 69.33$
1994	76	
1995	66	
1996	69	$\bar{x}_2 = 76.33$
1997	94	

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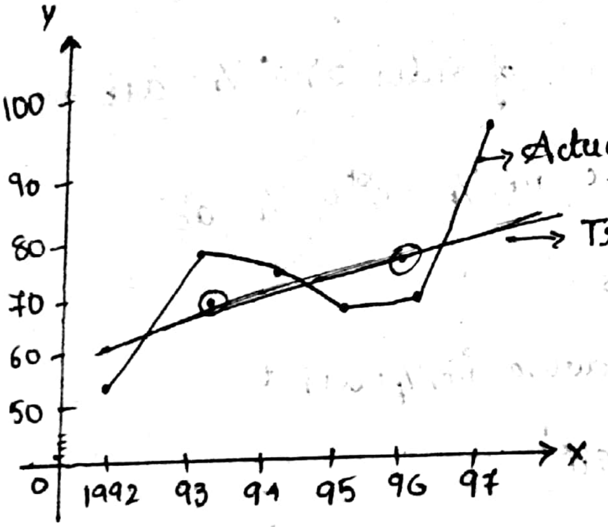
Sol: In this method we divide the total data into two equal parts and compute the arithmetic mean of each part and place those arithmetic means corresponding to middle time peroid of each part.

from the table the avg of 1st part $\bar{x}_1 = 69.33$

the line obtained on joining these two points is the required trend line and may be extended both the ways to estimate the values



Year	Profit (in crores)
1992	53
1993	79
1994	76
1995	66
1996	69
1997	94
1998	108
1999	110
2000	115

Since, $n=9$ (odd), the two parts would consist of 1992-1995 and 1997-2000, the year 1996 being omitted. Find the average of each part and plot these values corresponding to the middle point of the middle two years i.e, 1993.5 and 1998.5

3. <u>Method of Curve Fitting by principle of least squares</u>

The various types of curves that may be used to describe given data by using principle of least squares are

(1) Fitting of straight line i.e, $Y_t = a + bt$

(2) Fitting of parabola (polynomial of order 2) i.e, $Y_t = a + bt + ct^2$

(3) Fitting of exponential curve i.e, $Y_t = ae^{bt}$ or $Y_t = ab^t$

(4) Fitting of growth curves

(a) Modified exponential curve i.e, $Y_t = a + be^t$

(b) Gompertz curve i.e, $Y_t = ab^{c^t}$

(c) Logistic curve i.e, $Y_t = \dfrac{K}{1 + \exp(a + bt)}$

Fitting Of Straight Line

PCP-1

Fit a straight line to the following data by method of least squares and estimate the trend values and also draw the graph for data

Year	1976	1977	1978	1979	1980	1981	1982	1983
Profit	380	400	650	720	690	600	870	930

and also estimate the profit values during the years 1984 and 1985 also.

Aim: To fit a straight line for given data.

- To estimate trend values and draw graph.
- To estimate profit values during 1984 & 1985

Procedure and Formulae:

→ The st. line eqⁿ is in the form of $Y_t = a + bt$ — ①

t = values, Y_t = profit values. for easy calculations let us consider a variable 'u' by changing origin and scale.

$$u = \frac{t - (\text{A.M of middle two terms})}{h/2}$$

$$= \frac{t - [(1979 + 1980)/2]}{1/2}$$

$$u = 2[t - 1979.5]$$

Then the st line eqⁿ ① becomes

$$Y_t = a + bu$$

The normal eqⁿ's of st line ② are

$$\Sigma Y_t = na + b\Sigma u \text{ — (I)}$$

$$\Sigma u Y_t = a\Sigma u + b\Sigma u^2 \text{ — (II)}$$

by solving (I) & (II) we get a and b.

∴ The required eqⁿ is $\hat{Y}_t = a + bu$ — ③

By substituting corresponding 'u' values in ③ we get the trend values and future values.

Calculations :

t	Y_t	$u = 2(t-1979.5)$	uY_t	u^2
1976	380	-7	-2660	49
1977	400	-5	-2000	25
1978	650	-3	-1950	9
1979	720	1	-720	1
1980	690	1	690	1
1981	600	3	1800	~~25~~ 9
1982	870	5	4350	25
1983	930	7	6510	49
	$\Sigma Y_t = 5240$	$\Sigma u = 0$	$\Sigma uY_t = 6020$	$\Sigma u^2 = 168$

$I = 5240 = 8a$

$II = 6020 = 168b$

from the table $\Sigma Y_t = 5240$, $\Sigma u = 0$, $\Sigma u Y_t = 6020$; $\Sigma u^2 = 168$, $n = 8$

$I \Rightarrow a = \frac{5240}{8} = 655$, $II \Rightarrow b = \frac{6020}{168} = 35.83$

$\therefore$ Required straight line is $Y_t = 655 + (35.83)u$.

Trend values

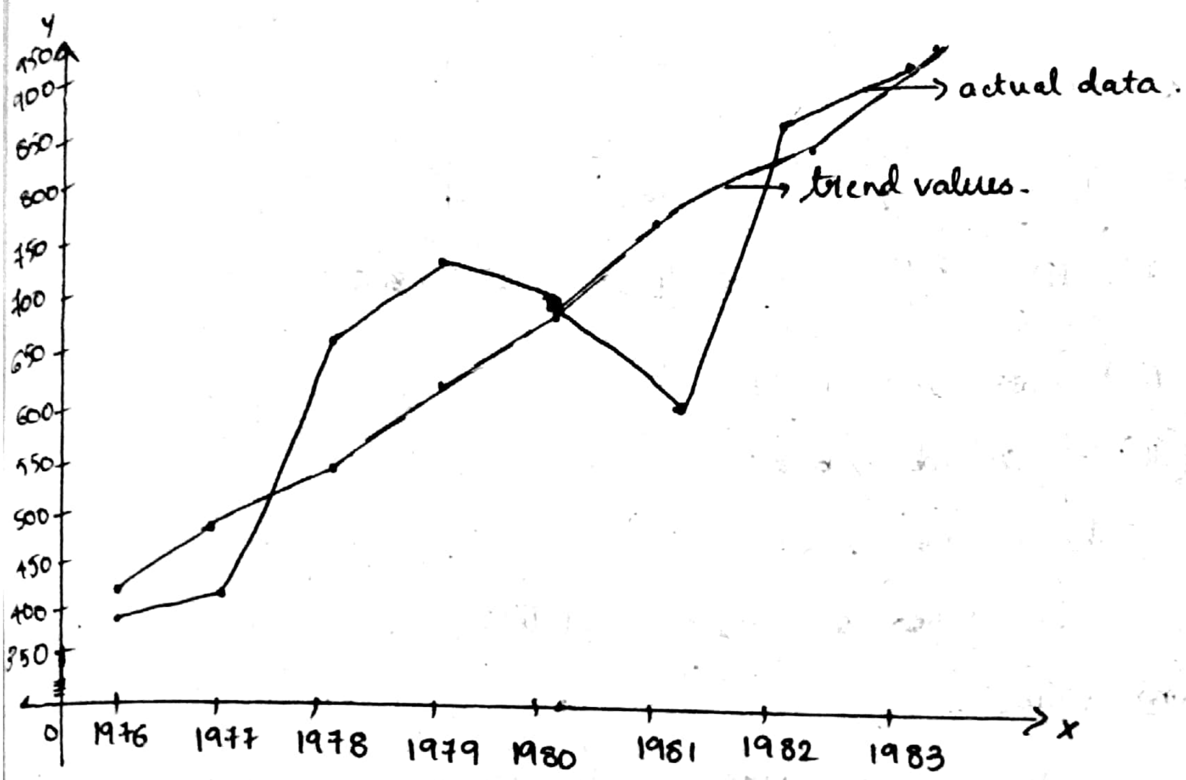
t	Y_t	$u = 2(t-1979.5)$	$\hat{Y}_t = 655 \cdot 35.83u$
1976	380	-7	404.19
1977	400	-5	475.85
1978	650	-3	547.51
1979	720	1	619.17
1980	690	1	690.83
1981	600	3	762.49
1982	870	5	834.15
1983	930	7	905.81
			5240

Profit value during the year 1984 is

$$\hat{Y}_t = 655 + 35.83\,[(t - 1979.5)\times 2]$$

$$\hat{Y}_{1984} = 655 + 35.83(2(1984 - 1979.5)) = 977.47$$

$$\hat{Y}_{1985} = 655 + 35.83(2(1985 - 1979.5)) = 1049.13.$$



PCP-2

Years	1973	1974	1975	1976	1977
Production	77	88	94	85	91

Estimate the production values during years 1980.

Sol. Since $n = 5$ (odd) $\qquad Y_t = a + bt$ — (1)

$$u = \frac{t - \text{middle term}}{h}$$

$u = t - 1975$

$Y_t = a + bu$ — (2)

$\Sigma Y_t = na + b\Sigma u$ — (I)

$\Sigma uY_t = a\Sigma u + b\Sigma u^2$ — (II)

$\hat{Y}_t = a + bu$ — (3).

t	Y_t	$u = t - 1975$	uY_t	u^2
1973	77	-2	-154	4
1974	88	-1	-88	1
1975	94	0	0	0
1976	85	1	85	1
1977	91	2	182	4
	$\Sigma Y_t = 435$	$\Sigma u = 0$	$\Sigma uY_t = 25$	$\Sigma u^2 = 10$

(I) $\Rightarrow$ $435 = 5a \Rightarrow a = 87$

(II) $\Rightarrow$ $25 = 10b \Rightarrow b = 2.5$

Subs in (3)

$\hat{Y}_t = \cancel{87+2} \; 87 + 2.5u$ – St. line

Trend values

t	Y_t	$u = t - 1975$	$\hat{Y}_t = 87 + 2.5u$
1973	77	-2	82
1974	88	-1	84.5
1975	94	0	87
1976	85	1	89.5
1977	91	2	92
			435

Production value of yr 1980 is

$Y_{1980} = 87 + 2.5(t - 1975) = 87 + 2.5(1980 - 1975) = 99.5$

. The following data relates to bank deposits during the years 1985 to 1993. Fit a st. line by least squares method for the data. Estimate the trend values. Estimate the deposits of the bank in 1995.

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
profits	27	38	44	35	51	58	50	54	62

The st. line eqn is $Y_t = a + bx$

Here n = no. of obs = ~~8~~ 9 (odd)

$$u_i = \frac{x_i - \text{middle term}}{h}$$

$u_i = x_i - 1989$

Year	Y_t	$u_i = x_i - 1989$	u_i^2	$u_i Y_t$
1985	27	-4	16	-108
1986	38	-3	9	-114
1987	44	-2	4	-88
1988	35	-1	1	-35
1989	51	0	0	0
1990	58	1	1	58
1991	50	2	4	100
1992	54	3	9	162
1993	62	4	16	248
	$\Sigma Y_t = 419$	$\Sigma u_i = 0$	$\Sigma u^2 = 60$	$\Sigma u Y_t = 223$

$\text{I} \Rightarrow 419 = 9a \Rightarrow a = 46.5556$

$\text{II} \Rightarrow 223 = 60b \Rightarrow b = 3.7167$

∴ Required st. line is $\hat{Y}_t = 46.5556 + (3.7167)u$.

Deposits of bank in 1995 is

$$\hat{Y}_t = 46.5556 + 3.7167(x_i - 1989)$$

$$= 46.5556 + 3.7167(1995 - 1989)$$

$$= 67.8558$$

Fitting Of Second Degree Parabola

PCP-3

Fit a parabolic curve of second degree to the following data and estimate the trend values and also find the sales values for the year 1984 and also draw the graph for original data and trend values.

Year :	1978	1979	1980	1981	1982
Sales :	10	12	13	10	8

Aim: To fit a second degree parabola to given data

→ To estimate trend values

→ To draw graph for original data and trend values.

Procedure: The eqⁿ of parabolic curve is given by

$$Y_t = a + bt + ct^2 \quad \text{—} ①$$

where, t = years, Y_t = Sales, a, b, c are constants for easy calculations let us introduce a new variable u by changing the origin & scale.

Since $n = 5$ (odd)

$$u = \frac{t - \text{middle term}}{h} = t - 1980$$

Eq^n of parabolic curve is given by

$$Y_t = a + bu + cu^2 \quad \text{— (2)}$$

Normal Equations are

$$\Sigma Y_t = na + b\Sigma u + c\Sigma u^2 \quad \text{— I}$$

$$\Sigma u Y_t = a\Sigma u + b\Sigma u^2 + c\Sigma u^3 \quad \text{— II}$$

$$\Sigma u^2 Y_t = a\Sigma u^2 + b\Sigma u^3 + c\Sigma u^4 \quad \text{— III}$$

By solving the above eq^{n}s we get a, b, c values.

$$\hat{Y}_t = a + bu + cu^2 \quad \text{— (3)}$$

By substituting corresponding 'u' values in (3) we get trend values.

Calculation

t	Y_t	$u = t - 1980$	u^2	u^3	u^4	uY_t	u^2Y_t
1978	10	-2	4	-8	16	-20	40
1979	12	-1	1	-1	1	-12	12
1980	13	0	0	0	0	0	0
1981	10	1	1	-1	1	10	10
1982	8	2	4	-8	16	16	32
	$\Sigma Y_t = 53$		$\Sigma u^2 = 10$	0	34	-6	94

I => $53 = 5a + 10c$

II => $-6 = 10b$

III => $94 = 10a + 34c$

$a = 12.31,\ b = -0.6,\ c = -0.85$

Substitute a, b, c in (3)

$\hat{Y}_t = 12.31 - 0.6u - 0.85u^2$ which is the parabolic curve.

Trend Values

t	Y_t	$u = t - 1980$	$\hat{Y}_t = 12.31 - 0.6u - 0.86u^2$
1978	10	-2	10.07
1979	12	-1	12.05
1980	13	0	12.31
1981	10	1	10.85
1982	8	2	7.67
			$52.95 \simeq 53$

Sales value during year 1984 is

$$\hat{Y}_t = 12.31 - 0.6(t-1980) - 0.86(t-1980)^2$$

$$\hat{Y}_{1984} = 12.31 - 0.6(1984-1980) - 0.86(1984-1980)^2$$

$$= -3.85$$

Since, sales cannot be negative, the 2nd degree parabolic curve is not fit for the given data.

2) Below is given the 8 census population of India 1901 to 1971. Fit a parabolic curve of second degree to the following data and estimate the trend values and also estimate the population values for year 1984.

Year	1901	1911	1921	1931	1941	1951	1961	1971
pop^n	238.3	252.0	251.2	278.9	318.5	361.0	439.1	547.9

Sol: $Y_t = a + bt + ct^2$ — ①

$$u = \frac{t - A.M}{h/2} = \frac{t - 1936}{5}$$

$$Y_t = a + bu + cu^2 \text{ — } ②$$

normal eq^n s are

$$\Sigma Y_t = na + b\Sigma u + c\Sigma u^2 \text{ — I}$$

$$\Sigma uY_t = a\Sigma u + b\Sigma u^2 + c\Sigma u^3 \text{ — II}$$

$$\Sigma u^2 Y_t = a\Sigma u^2 + b\Sigma u^3 + c\Sigma u^4 \text{ — III}$$

$$\hat{Y}_t = a + bu + cu^2 \text{ — } (3)$$

t	Y_t	$u = \frac{t - 1936}{5}$	u^2	u^3	u^4	uY_t	u^2Y_t
1901	238.3	-7	49	-343	2401	-1668.1	11676.7
1911	252	-5	25	-125	625	-1260	6300
1921	251.2	-3	9	-27	981	-753.6	2260.8
1931	278.9	-1	1	-1	1	-278.9	278.9
1941	318.5	1	1	1	1	318.5	318.5
1951	361	3	9	27	981	1083 ~~108.3~~	3249
1961	439.1	5	25	125	625	2195.5	10977.5
1971	547.9	7	49	343	2401	3835.3	26847.1
		0	168	0	6216	3471.7	61908.5

$\Sigma Y_t = 2686.9$

$\text{I} \Rightarrow 2686.9 = 8a + 168c$

$\text{II} \Rightarrow 3471.7 = 168b$

$\text{III} \Rightarrow 61908.5 = 168a + 6216c$

$a = 292.96,\ b = 20.66,\ c = 2.04$

$$\hat{Y}_t = (292.96) + (20.66)u + (2.04)u^2$$

Trend Values

t	y_t	$u = \frac{t-1936}{5}$	$\hat{y}_t = a + 20.7u + 2.04u^2$
1901	238.3	-7	248.3
1911	252	-5	240.66
1921	251.2	-3	249.36
1931	278.9	-1	274.34
1941	318.5	1	315.66
1951	361	3	373.3
1961	439.1	5	447.26
1971	547.9	7	537.54
			2686.4

Population value for year 1984 is

$$\hat{y}_{1984} = (292.96) + (20.66)\left(\frac{t-1936}{5}\right) + (2.04)\left(\frac{t-1936}{5}\right)^2$$

$$= 679.3024$$

~~Fitting of an Exponential Curve~~

3) Fit a second degree parabola to the following data and estimate the values of sales. Draw graph.

Year	2005	2006	2007	2008	2009
Sales	10	12	14	10	8

Aim: To fit a parabolic curve and draw graph for the given data.

Formulae: let second degree parabola be

$$y_t = a + bt + ct^2$$

$n = 5$ (odd)

$u_i = x_i - 2007$

Calculation :

Year	Sales	$u = x - 2007$	u^2	u^3	u^4	uY_t	u^2Y_t	Y_t' (trend values)
2005	10	-2	4	-8	16	-20	40	10
2006	12	-1	1	-1	1	-12	12	12.4
2007	14	0	0	0	0	0	0	12.8
2008	10	1	1	1	1	10	10	11.2
2009	8	2	4	8	16	16	32	7.6
	54	0	10	0	34	-6	94	

$$\Sigma Y_t = na + b\Sigma u + c\Sigma u^2 \qquad \Sigma uY_t = a\Sigma u + b\Sigma u^2 + c\Sigma u^3$$

$$54 = 5a + 10c \text{ --- } (1) \qquad -6 = 10b$$

$$b = -\frac{3}{5} = -0.6$$

$$94 = 10a + 34c \text{ --- } (3)$$

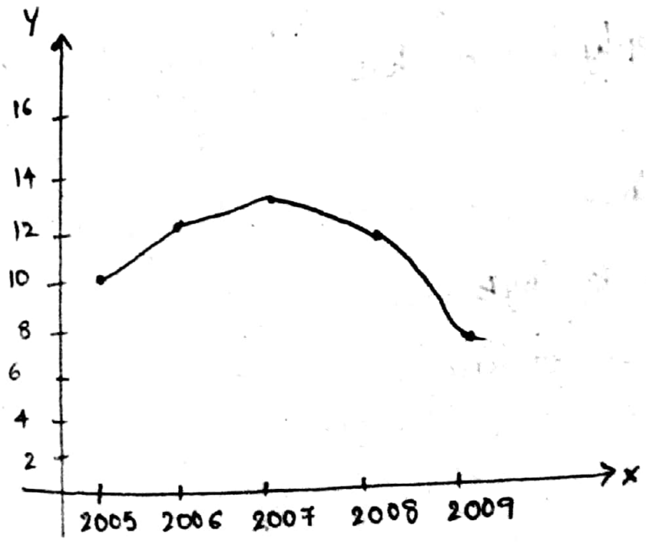
$$(1) \times 2 \Rightarrow 108 = 10a + 20c$$

$$(3) \Rightarrow \underset{(-)}{94} = \underset{(-)}{10a} + \underset{(-)}{34c}$$

$$14 = -14c$$

$$c = -1, \quad a = 12.8$$

$$Y_t = 12.8 - 0.6u - u^2$$



Fitting of an Exponential curve

• Fit an exponential curve for the following data and estimate the trend values and also estimate the population in 1981.

Year	1911	1921	1931	1941	1951	1961	1971
popl'n	25	25.1	27.9	31.9	36.1	43.9	54.7

Aim: To fit an exponential curve to the given data

To estimate trend values and also estimate the population in 1981.

Procedure and Formulae

The eqn of exponential curve is $Y_t = ab^t$ — ① where

Y_t = popl'n values at time t, t = year values, a and b are constants by changing the origin and scale

let us introduce a new variable u.

$n = 7$ (odd, $u = \frac{t - \text{middle term}}{h} = \frac{t-1941}{10}$

The exponential curve is given by

$$Y_t = ab^u \text{ — ②}$$

for easy calculations apply log on b.s

$$\log Y_t = \log (ab)^u$$

$$= \log a + u \log b$$

let $V = \log Y_t$, $A = \log a$, $B = \log B$

$V = A + Bu$ which is st. line.

normal eqns are

$$\Sigma V = nA + B\Sigma u \text{ — I}$$

$$\Sigma uV = A\Sigma u + B\Sigma u^2 \text{ — II}$$

By solving the eq^{n}s I & II we can get A, B values

since, $A = \log a \Rightarrow a = \text{Antilog}(A) = 10^A$

$B = \log b \Rightarrow b = \cancel{A} \text{Antilog}(B) = 10^B$

By substituting a, b values in ② we get exp. curve

i.e, $\hat{Y}_t = a \cdot b^u$ — ③

By substituting corresponding 'u' values in the above eq^n we get trend values.

Calculation:

t	Y_t	$u = \frac{t-1941}{10}$	u^2	$V = \log Y_t$	uV
1911	25	-3	9	1.3979	-4.1937
1921	25.1	-2	4	1.3996	-2.7992
1931	27.9	-1	1	1.4456	-1.4456
1941	31.9	0	0	1.5037	0
1951	36.1	1	1	1.557~~8~~5	1.5575
1961	43.9	2	4	1.6424	3.2848
1971	54.7	3	9	1.7379	5.2137
	244.6	0	28	10.685	1.6178

I $\Rightarrow$ $10.685 = 7A \Rightarrow A = 1.5264 \Rightarrow a = 10^A = 33.6047$

II $\Rightarrow$ $1.6178 = 28B \Rightarrow B = 0.0577 \Rightarrow b = 10^B = 1.1428$

The fitted exponential curve is

$$\hat{Y}_t = (33.6047)(1.1428)^u$$

Trend Values

t	Y_t	$u = \frac{t-1941}{10}$	$\hat{Y}_t = (33.6047)(1.1423)^u$
1911	25	-3	22.5454
1921	25.1	-2	25.7537
1931	27.9	-1	29.4184
1941	31.9	0	33.6047
1951	36.1	1	38.3866
1961	43.9	2	43.8491
1971	54.7	3	50.0888
			243.6466 $\simeq$ 244

population values for year 1981

is $\hat{Y}_{1981} = (33.6047)(1.1423)^{\frac{t-1941}{10}} = 57.2164$

2) The population of a state is given in the following table fit a curve of the form $Y_t = ae^{bt}$ and estimate the trend values and also estimate the population values in the year 1961.

Year	1881	1891	1901	1911	1921	1931	1941	1951
popl'n	3.9	5.3	7.3	9.6	12.9	17.1	23.2	30.9

Procedure and Formulae

The eq^n of exponential curve is $Y_t = ae^{bt}$ — ①

By changing origin and scale let u be new variable

$$u = \frac{t - \text{AM of middle terms}}{h/2} = \frac{t-1916}{5}$$

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The exp curve in terms of u is $Y_t = ae^{bu}$ — ②

apply log on b.s

$$\log Y_t = \log a + bu \log e$$

let $V = \log Y_t$, $A = \log a$, $B = b\log e$

$\Rightarrow V = A + Bu$ which is in the form of st line

Normal eq^n s are

~~$\log Y_t = \log a + bu \log e$~~

$$\Sigma V = nA + B\Sigma u \text{ — I}$$

$$\Sigma uV = A\Sigma u + B\Sigma u^2 \text{ — II}$$

By solving the eq^n s I and II we can get the values of A, B since $A = \log a \Rightarrow a = \text{Antilog } A \Rightarrow 10^A$

$$B = b\log e \Rightarrow b = \frac{B}{\log e} = \frac{B}{0.4343}$$

By substituting a, b in ② we get fitted exp curve

$\Rightarrow \hat{Y}_t = ae^{bu}$

By substituting corresponding u values in the above eq^n we get trend values

Calculation

t	Y_t	$u = \frac{t-1916}{5}$	u^2	$V = \log Y_t$	uV
1881	3.9	-7	49	0.5911	-4.1377
1891	5.3	-5	25	0.7243	-3.6215
1901	7.3	-3	9	0.8633	-2.5899
1911	9.6	-1	1	0.9823	-0.9823
1921	12.9	1	1	1.1106	1.1106
1931	17.1	3	9	1.2330	3.699
1941	23.2	5	25	1.3655	6.8275
1951	30.9	7	49	1.4899	10.4293
	110.2	0	168	8.3591	10.7350

$I \Rightarrow 8.359 = 8A$

$A = 1.0449 \Rightarrow a = 11.0885$

$II \Rightarrow 10.7350 = 168B$

$B = 0.0639 \Rightarrow b = \frac{B}{0.4333} = 0.1471$

Then the fitted exp curve is

$$\hat{y}_t = (11.0885)\, e^{(0.1471 * u)}$$

Trend Values

t	y_t	u	$\hat{y}_t = (11.0885)\, e^{(0.1471.u)}$
1881	3.9	-7	3.96
1891	5.3	-5	5.31
1901	7.3	-3	7.13
1911	9.6	-1	9.57
1921	12.9	1	12.84
1931	17.1	3	17.24
1941	23.2	5	23.14
1951	30.9	7	31.05
			110.2

population value for the year 1961 is

$$\hat{y}_t = (11.0885) . e^{(0.1471 \left(\frac{t-1916}{5}\right))}$$

$= 41.67.$

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3)

Years	1962	1983	1984	1985	1986	1987	1988
Sales	32	47	65	92	132	190	275

Aim: To fit an exponential curve for the given data and find the trend values.

Procedure: Let the exponential curve be

$$Y_t = ab^t \quad \text{— ①}$$

Let u be a new variable

$n = 7$ (odd) $\quad u = x - 1985$

$$\therefore Y_t = ab^u \quad \text{— ②}$$

$$\log Y_t = \log a + u \log b$$

$$V = A + Bu$$

normal eqⁿs are

$$\Sigma V = nA + B\Sigma u \quad \text{— I}$$

$$\Sigma uV = A\Sigma u + B\Sigma u^2 \quad \text{— II}$$

$a = \text{Antilog}(A)$, $b = \cancel{\log}\ \text{Antilog}(B)$ $\Rightarrow \hat{Y}_t = ab^u$ — ③

By substituting corresponding u values in ③ we get trend values.

Calculation.

Years	Sales	$u = x - 1985$	u^2	$V = \log Y$	uV	$Y_t = ab^{(x-1985)}$
1982	32	-3	9	1.5051	-4.5154	32.1528
1983	47	-2	4	1.6721	-3.3442	45.8789
1984	65	-1	1	1.8129	-1.8129	65.4646
1985	92	0	0	1.9638	0	93.4114
1986	132	1	1	2.1206	2.1206	133.2887
1987	190	2	4	2.2788	4.5576	190.1897
1988	275	3	9	2.4393	7.3179	271.3817
		0	28	13.7926	4.3236	

$I \Rightarrow 13.7926 = 7A$

$A = 1.9704$

$B = 0.1544$

$a = 93.4114, \; b = 1.4269$

$\therefore$ Required exp curve is

$\hat{y}_t = (93.4114)(1.4269)^u$

Growth Curves

The various growth curves such as the modified exponential, Gompertz and logistic curves cannot be determined by the principle of least squares.

Special techniques have been derived for fitting these curves to the given set of data.

Modified Exponential Curve & its fitting

The modified exponential curve is given by $Y_t = a + bc^t, a>0$ — (1)

where Y_t represents the time series value at the line t and a, b and c are constants called as parameters.

Taking 1st difference of (1), we get

$$\Delta Y_t = Y_{t+h} - Y_t$$
$$= (a+bc^{t+h}) - (a+bc^t)$$
$$= \cancel{a} + bc^{t+h} - \cancel{a} - bc^t$$
$$= b(c^{t+h} - c^t) \text{ — (I)} \Rightarrow bc^t(c^h - 1)$$

Similarly, $\Delta Y_{t-h} = Y_t - Y_{t-h}$

$$= \cancel{a} + bc^t - \cancel{a} - bc^{t-h}$$
$$= b(c^t - c^{t-h})$$
$$= bc^{t-h}(c^h - 1) \text{ — (II)}$$

from (I) & (II)

$$\frac{\text{(I)}}{\text{(II)}} \Rightarrow \frac{\Delta Y_t}{\Delta Y_{t-h}} = \frac{\cancel{b}c^t(\cancel{c^h-1})}{\cancel{b}c^{t-h}(\cancel{c^h-1})} = c^h$$ which is a constant value.

and h is the interval of difference

Thus, the most striking feature of modified exponential curve is that the first difference of the consecutive values of Y_t corresponding to equivalent values of t change by a constant ratio.

<u>Method of 3 selected points</u> - y_1, y_2, y_3

We take 3 ordinates corresponding to 3 equidistant value of t say t_1, t_2, t_3 respectively such that $t_2 - t_1 = t_3 - t_2$ substituting the values of t i.e t_1, t_2, t_3 in eq ①, we get respectively the following equations

$$Y_{t_1} = Y_1 = a + bc^{t_1} \quad \text{— (a)}$$

$$Y_{t_2} = Y_2 = a + bc^{t_2} \quad \text{— (b)}$$

$$Y_{t_3} = Y_3 = a + bc^{t_3} \quad \text{— (c)}$$

let us consider b-a

$$\Rightarrow Y_2 - Y_1 = \cancel{a} + bc^{t_2} - \cancel{a} - bc^{t_1} = b(c^{t_2} - c^{t_1})$$

$$Y_2 - Y_1 = bc^{t_1}(c^{t_2 - t_1} - 1) \quad \text{— (d)}$$

similarly consider c-b

$$\Rightarrow Y_3 - Y_2 = \cancel{a} + bc^{t_3} - \cancel{a} - bc^{t_2} = b(c^{t_3} - c^{t_2})$$

$$Y_3 - Y_2 = bc^{t_2}(c^{t_3 - t_2} - 1) \quad \text{— (e)}$$

dividing (e) by (d)

$$\frac{Y_3 - Y_2}{Y_2 - Y_1} = \frac{\cancel{b}c^{t_2}(c^{t_3 - t_2} - 1)}{\cancel{b}c^{t_1}(c^{t_2 - t_1} - 1)} = \frac{c^{t_2}}{c^{t_1}}\left(\frac{c^{\cancel{t_3 - t_2}} - 1}{c^{\cancel{t_2 - t_1}} - 1}\right)$$

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$$\frac{Y_3-Y_2}{Y_2-Y_1} = c^{t_2-t_1}$$

$$c = \left(\frac{Y_3-Y_2}{Y_2-Y_1}\right)^{\frac{1}{t_2-t_1}}$$

substitute the value of c in (d)

$$Y_2-Y_1 = b\left(\frac{Y_3-Y_2}{Y_2-Y_1}\right)^{\frac{t_1}{t_2-t_1}}\left(\frac{Y_3-Y_2}{Y_2-Y_1}-1\right)$$

$$= b\left(\frac{Y_3-Y_2}{Y_2-Y_1}\right)^{\frac{t_1}{t_2-t_1}}\left(\frac{Y_3-Y_2-Y_2+Y_1}{Y_2-Y_1}\right)$$

$$Y_2-Y_1 = b\left(\frac{Y_3-Y_2}{Y_2-Y_1}\right)^{\frac{t_1}{t_2-t_1}}\left(\frac{Y_3-2Y_2+Y_1}{Y_2-Y_1}\right)$$

$$b = \frac{(Y_2-Y_1)^2}{Y_3-2Y_2+Y_1}\left(\frac{Y_2-Y_1}{Y_3-Y_1}\right)^{\frac{t_1}{t_2-t_1}}$$

substitute the values of (b) and (e) in (a)

$$Y_1 = a+bc^{t_1}$$

$$Y_1 = a+\frac{(Y_2-Y_1)^2}{(Y_3-2Y_2+Y_1)}\cancel{\left(\frac{Y_2-Y_1}{Y_3-Y_2}\right)^{\frac{t_1}{t_2-t_1}}}\cancel{\left(\frac{Y_3-Y_2}{Y_2-Y_1}\right)^{\frac{t_1}{t_2-t_1}}}$$

$$a = Y_1-\frac{(Y_2-Y_1)^2}{(Y_3-2Y_2+Y_1)}$$

$$= Y_1-\frac{(Y_2^2+Y_1^2-2Y_1Y_2)}{Y_3-2Y_2+Y_1}$$

$$= \frac{Y_3Y_1-\cancel{2Y_2Y_1}+\cancel{Y_1^2}-Y_2^2-\cancel{Y_1^2}+\cancel{2Y_2Y_1}}{Y_3-2Y_2+Y_1}$$

$$a = \frac{Y_1Y_3-Y_2^2}{Y_3-2Y_2+Y_1}$$

Substitute the values of a, b and c in eq ①, we get the equation of the modified exponential curve fitted to the given time series data

Fitting of Gompertz curve

Gompertz curve is given by the following form

$$Y_t = ab^{c^t} \text{ — ①}$$

where Y_t is the time series value at time t and a, b & c are its parameters.

applying log on both sides in ①

$$\log Y_t = \log a + c^t \log b.$$

let $V_t = \log Y_t$; $A = \log a$, $B = \log b$

$V_t = A + Bc^t$ which is in the form of M.E.C and constants A, B, c can be eliminated by the method of 3 selected points as explained above.

finally, the constants of the Gompertz are given by

$a = \text{antilog } A \Rightarrow 10^A$

$b = \text{antilog } B \Rightarrow 10^B$

Method of logistic Curve

Logistic curve is a particular form of complex type of growth curves. A symmetric logistic curve is also known as Pearl Read Curve is given by

$$Y_t = \frac{k}{1+\exp(a+bt)} \quad ; b<0 \quad \text{—— (1)}$$

where Y_t is the value of time series at the time t and a, b and k are constants.

The curve (1) can also be written as

$$Y_t = \frac{k}{1+e^{a} e^{bt}} \quad \text{—— (2)}$$

Reciprocal of (2)

$$\frac{1}{Y_t} = \frac{1+e^{a}e^{bt}}{k}$$

$$= \frac{1}{k} + \frac{e^{a}e^{bt}}{k}$$

let $A = \frac{1}{k}$, $B = \frac{e^{a}}{k}$ and $c = e^{b}$

$$\frac{1}{Y_t} = A + Bc^{t}$$

~~Hence~~ Here A, B, c are constants

Thus, the reciprocal of Y_t follows M.E law

Hence, the given time series observation, Y_t will follow logistic law if the reciprocal $\frac{1}{Y_t}$ follows M.E law.

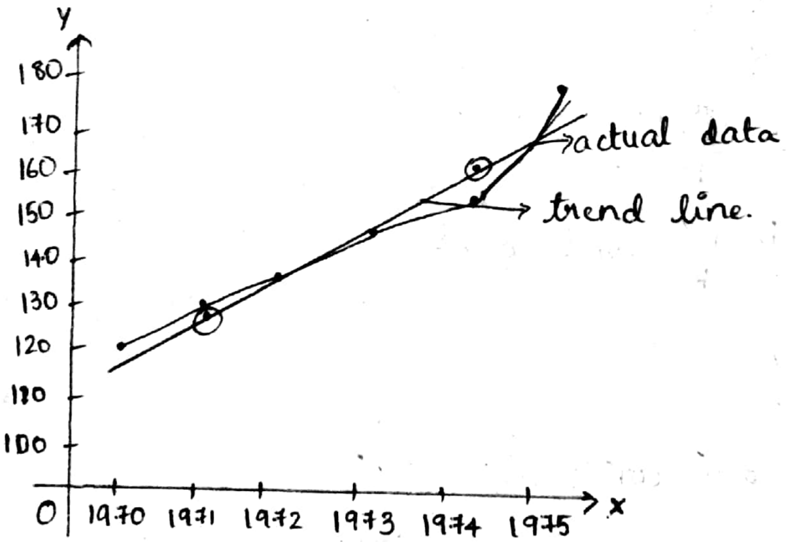
2. Method of Semi Averages (Continuation ---)

- Year 1970 1971 1972 1973 1974 1975

Producⁿ 120 130 135 145 150 175

Sol

Year	Production	Average
1970	120	$\frac{120+130+135}{3} = 128.333$
1971	130	
1972	135	
1973	145	$\frac{145+150+175}{3} = 156.6667$
1974	150	
1975	175	



~~Year~~ 1991 199

Sales

4. Method of Movi

1) Compute three y

average values fo

Year 1991 199

Sales 68 62

Sol. 3 Yearly

Year	Produc
1991	68
1992	62
1993	61
1994	63
1995	65
1996	68
1997	63
1998	67
~~1999~~	

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~~Year 1991 1992 1993 1994 1995 1996 1997 1998 1999~~

~~Sales~~

4. Method of Moving Averages

1) Compute three yearly, 4 yearly and 5 yearly moving average values for the following data.

Year	1991	1992	1993	1994	1995	1996	1997	1998
Sales	68	62	61	63	65	68	63	67

Sol. 3 Yearly moving averages

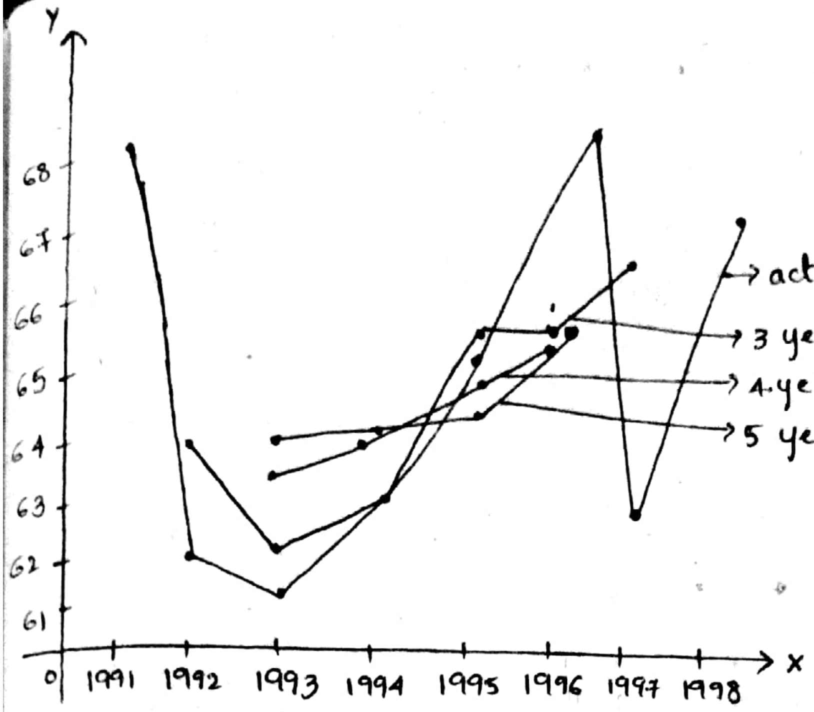
Year	Production	3 Yearly Moving Totals	3 Yearly Moving Averages
1991	68	-	-
1992	62	191	191/3= 63.6667
1993	61	186	62
1994	63	189	63
1995	65	196	65.3333
1996	68	196	65.3333
1997	63	198	66
1998	67	-	-
~~1999~~			

4 Yearly Moving Averages

Year	Production	4 Yearly mvg Totals	4 Yearly non centered M.V	4 Yearly centered moving Avgs
1991	68	–		–
1992	62	–		–
		254	$254/4 = 63.5$	
1993	61			63.125
		251	62.75 avg	
1994	63			63.5
		257	64.25	
1995	65			64.5
		259	64.75	
1996	68			65.25
		263	65.75	
1997	63	–		–
1998	67	–		–

5 Yearly Moving Averages

Year	Production	5 Yearly Moving Totals	5 Yearly Moving Averages
1991	68	–	–
1992	62	–	–
1993	61	319	$319/5 = 63.8$
1994	63	319	63.8
1995	65	320	64
1996	68	326	65.2
1997	63	–	–
1998	67	–	–



2) A study of demand (d_i) for 12 years ($t = 1$ to 12) has indicated the following

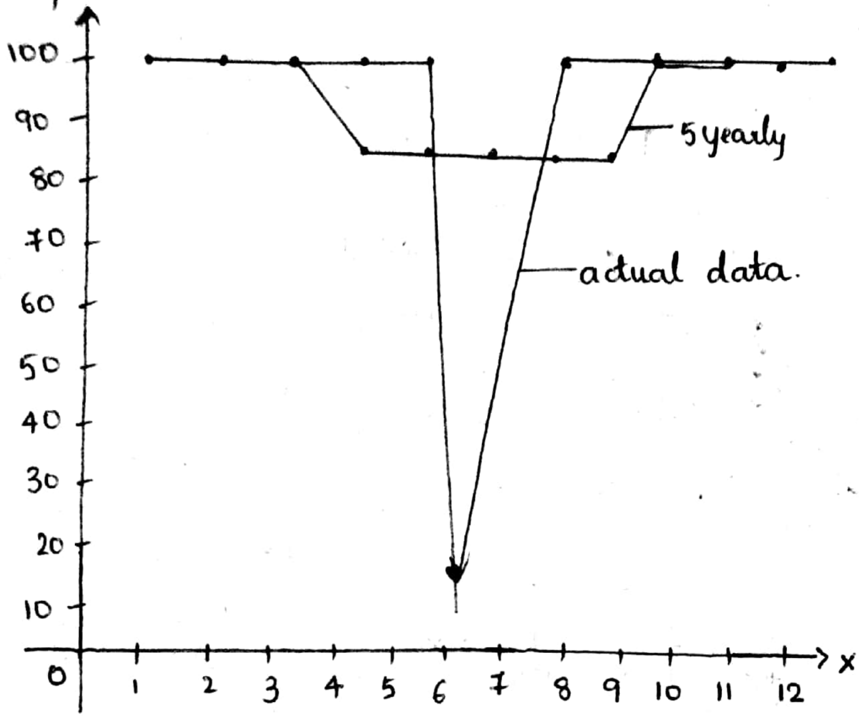
$d_i = 100 ; t = 1,2,3,4,5$

$d_i = 20 ; t = 6$

$d_i = 100 ; t = 7, 8, \ldots 12$

Compute a 5 yearly moving averages for given data

t	d_i	5 yearly moving totals	5 yrly mvg avgs
1	100	-	-
2	100	-	-
3	100	500	100
4	100	420	84
5	100	420	84
6	20	420	84
7	100	420	84
8	100	420	84
9	100	500	100
10	100	500	100
11	100	-	-
12	100	-	-



3) For the following series of observation verify that the ~~following~~ four yearly centred mov avgs are equivalent to 5 yrly weight mov averags with weights 1, 2, 2, 2, 1 respectively

Year	1960	1961	1962	1963	1964	1965
deposits	960	976	974	996	1024	1040

sol.

Year	deposits	4YMT	4YNCMA	4YCMA	5YWMT $\sum_{i=1}^{n} w_i y_i$	5YWMA $\sum_{i=1}^{n} w_i y_i / \sum w_i$
1960	960					
1961	976	3906	976.5			
1962	974			984.5	7876	984.5
1963	996	3970	992.5	1000.5	8004	1000.5
1964	1024	4034	1008.5			
1965	1040					

The weighted moving average

$$= \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i} \quad \text{where } y_i = \text{dep } \& \ w_i = \text{weights}$$

$w_1 = 1,\ w_2 = 2,\ w_3 = 2;\ w_4 = 2,\ w_5 = 1$

4)

Year	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983
Sales	2	6	1	5	3	7	2	6	4	8	3

Sol.

Year	Sales	4YMT	4YNCMA	4YCMA	5YWMT	5YWMA
1973	2					
1974	6					
		14	3.5			
1975	1			3.625	29	3.625
		15	3.75			
1976	5			3.875	31	3.875
		16	·4			
1977	3			4.125	33	4.125
		17	4.25			
1978	7			4.375	35	4.375
		18	4.5			
1979	2			4.625	37	4.625
		19	4.75			
1980	6			4.875	39	4.875
		20	5			
1981	4			5.125	41	5.125
		21	5.25			
1982	8					
1983	3					

Measurements of seasonal variations

We have 4 types of seasonal variations

1. Link relatives method
2. Ratio to moving averages method
3. Ratio to trend method.
4. Simple averages method.

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1) Link Relatives Method

1. Calculate seasonal indices by the method of link relatives for the following data.

Quarter \ Year	1979	1980	1981	1982	1983
Q_1	30	35	31	31	34
Q_2	26	28	29	31	36
Q_3	22	22	28	25	26
Q_4	31	36	32	35	33

Aim: To calculate seasonal indices for the given data by the method of link relatives.

Procedure and Formulae:

Step 1: Link relative for any quarter $= \dfrac{\text{Current Quarter fig}}{\text{Prev. Quarter fig}} \times 100$

Then find average link relative values for each quarter

Step 2: Chain relative for $Q_1 = 100$

$$\text{CR for } Q_2 = \frac{\text{LR of } Q_2 \times \text{CR of } Q_1}{100}$$

$$\text{CR for } Q_3 = \frac{\text{LR of } Q_3 \times \text{CR of } Q_2}{100}$$

$$\text{CR of } Q_4 = \frac{\text{LR of } Q_4 \times \text{CR of } Q_3}{100}$$

Step 3: Now, making the Q_4 value as base, a new chain relative for Q_1 can be obtained as

$$\text{New CR of } Q_1 = \frac{\text{Avg LR of } Q_1 \times \text{CR of } Q_4}{100}$$

Step 4: Now, we have to adjust the CR by subtracting a correction factor d from each CR

where, correction factor 'd' $= \frac{1}{4}$ [New CR of Q_1 − 100]

Then, we can get the adjusted chain relatives for Q_2, Q_3, Q_4 by subtracting 'd', '2d', and '3d' respectively.

Step 5: Finally, find the seasonal indices by using the fallowing formula.

$$\text{Seasonal Index of any Quarter} = \frac{\text{adj CR}}{\text{avg(adj CR)}} \times 100$$

Finally, the adjusted chain relatives total should be equals to 400.

Calculation:

Y\Q	Q_1	Q_2	Q_3	Q_4	
1979	–	$\frac{26}{30} \times 100 = 86.667$	84.6154	140.90	
1980	$\frac{35}{31} \times 100 = 112.9032$	80	78.5114	163.636	
1981	86.1	93.5484	96.5511	114.2857	
1982	96.8750	100	80.6452	140	
1983	97.1429	105.8624	72.2222	126.9231	
Avg L.R	98.2581	93.2195	82.5212	137.1509	
CR	100	93.2195	76.9259	105.5046	
Adj CR	100	92.3028	75.0925	102.7545	= ~~400~~ 92.5375
SI	108.0643	99.7464	81.1482	111.0409	= 400

New CR of Q_1 = $\frac{\text{Avg LR of } Q_1 \times \text{CR of } Q_4}{100}$ = $\frac{98.2581 \times 105.5046}{100}$

= 103.6670

correction factor 'd' = $\frac{1}{4}$ [New CR of Q_1 − 100]

= $\frac{1}{4}$ [103.6670 − 100]

= 0.9167

<u>Result</u>: Seasonal Index for Q_1 = 108.0643

Q_2 = 99.7464

Q_3 = 81.1482

Q_4 = 111.0409

2)

Year\Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1991	12	11	10	14	15	15	16	13	11	10	12	
1992	15	14	13	16	16	15	17	12	13	12	13	
1993	16	15	14	16	15	17	16	13	10	10	11	

<u>Aim</u>: To find seasonal indices for given data by using method of link relatives

<u>Procedure and Formulae</u>:

<u>Step 1</u>: LR for any month = $\frac{\text{Current month fig}}{\text{Prev month fig}} \times 100$

Then find LR values for each month

<u>Step 2</u>: CR for Jan = 100

CR for any month = $\frac{\text{LR for current month} \times \text{CR for prev month}}{100}$

<u>Step 3</u>: New CR for Jan = $\frac{\text{Avg LR of Jan} \times \text{CR of Dec}}{100}$

Step 4: correction factor 'd' $= \frac{1}{12}$ [New CR of Jan − 100]

Then we can get the adjusted chain relatives for Feb … Dec by subtracting d, 2d, … nd respectively.

Calculation:

Y \ M	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	
1991	-	91.667	90.9091	140	107.142	100	106.667	81.25	84.6054	90.90	120	125	
1992	100	92.8571	92.8571	123.076	100	93.75	113.33	70.5882	108.33	92.307	108.33	107.6923	
1993	114.2857 ~~93.75~~	93.75	93.3333	114.285	93.75	113.33	94.1176	81.25	76.9231	100	110	136.3636	
Avg LR	107.142	92.9167	92.3665	127.7875	100.2976	102.3611	104.7059	77.6961	89.9573	94.40	112.77	123.0186	
CR	100	92.9167	85.8239	107.95	108.27	110.83	116.04	90.16	81.11	76.5	86.35	106.2356	
Adj CR	100	91.7647	83.5199	104.49	103.6	105.07	109.13	82.10	71.89	66.2	74.8	93.56	= 90.5222
SI	100.4701	101.372	92.2645	115.4410	114.5531	116.0748	120.5641	90.67	79.42	73.136	62.67	103.3598	= 1200.0299

$d = \frac{1}{12}$ [new CR of Jan − 100]

$$\text{New CR of Jan} = \frac{\text{Avg CR of Jan} \times \text{CR of Dec}}{100} = \frac{107.1429 \times 106.2356}{100} = 113.8239$$

$$d = \frac{1}{12}[113.8239 - 100] = 1.1520$$

2) Ratio to Moving Averages

1) Calculate seasonal indices for the following data by ratio to moving averages method.

Year \ Quarter	Q_1	Q_2	Q_3	Q_4
1979	30	40	36	34
1980	34	52	50	44
1981	40	58	54	48
1982	52	76	68	62

Aim: To calculate seasonal indices for given data.

Procedure and Formulae:

1) Calculate 4 Quarters Centered Moving Averages for given data.

2) Find the ratio to moving averages by expressing the original data as percentages of centered moving averages.

i.e Ratio to moving avgs = $\frac{O.V}{C.M.A} \times 100$

3) Find the average ~~seaso~~ seasonal indices for each Quarter by using RMA values.

4) The sum of seasonal indices should be 400 for Quarterly data, otherwise we have to adjust the seasonal indices by multiplying them throughout a constant k, where $k = \frac{400}{\text{Sum of Seasonal Indices}}$

Calculation :

Year	Quarters	Q.V	M.T	M.A	C.M.A	R.M.A
1979	Q_1	30			-	-
	Q_2	40			-	-
			140	35		
	Q_3	36			35.5	101.4085
			144	36		
	Q_4	34			37.5	90.6667
			156	39		
1980	Q_1	34			40.75	83.4358
			170	42.5		
	Q_2	52			43.75	118.6571
			180	~~46.5~~ 45		
	Q_3	50			45.75	109.2896
			186	~~48~~ 46.5		
	Q_4	44			47.25	93.1217
			192	48		
1981	Q_1	40			48.5	82.4742
			196	49 ~~50~~		
	Q_2	58			49.5	117.1717
			200	50		
	Q_3	54			51.5	104.8544
			212	53		
	Q_4	48			55.25	86.8778
			230	57.5		
1982	Q_1	52			59.25	87.7637
			244	61		
	Q_2	76			62.75	121.1155
			258	64.5		
	Q_3	68			-	-
	Q_4	62			-	-

y \ Q	Q_1	Q_2	Q_3	Q_4	
1979	-	-	101.4085	90.6667	
1980	83.4358	118.85	109.2896	93.1217	
1981	82.4742	117.17	104.8544	86.8778	
1982	87.7637	121.1156	—	—	
Total (Avg)	1/3 84.5579	119.0452	105.1842	90.2221	= 399.0122
Adj SI	84.7693	119.3428	105.4472	90.4477	= 400

$$K = \frac{400}{399.0122} = 1.0025$$

2) Calculate seasonal indices for following data by ratio to moving averages method.

Year \ Quarter	Q_1	Q_2	Q_3	Q_4
1991	68	60	61	63
1992	70	58	56	60
1993	68	63	68	67
1994	65	59	56	62
1995	60	55	51	58

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Year	Quarter	Q.V	M.T	M.A	CMA	RMA
1991	Q_1	68				-
	Q_2	60	252	63	63.25	96.4427
	Q_3	61	254	63.5	63.25	99.6047
	Q_4	63	252	63	62.3750	112.2244
1992	Q_1	70	247	61.75	61.3750	94.5010
	Q_2	58	244	61	60.75	92.1811
	Q_3	56	242	60.5	61.1250	98.1595
	Q_4	60	247	61.75	63.25	107.5099
1993	Q_1	68	259	64.75	65.6250	96
	Q_2	63	266	66.5	66.1250	102.8355
	Q_3	68	263	65.75	65.25	102.6820
	Q_4	67	259	64.75	63.25	102.7668
1994	Q_1	65	247	61.75	61.1250	96.5235
	Q_2	59	242	60.5	59.875	93.5282
	Q_3	56	237	59.25	58.75	105.5319
	Q_4	62	233	58.25	57.625	104.1215
1995	Q_1	60	228	57	56.5	97.3651
	Q_2	55	224	56		-
	Q_3	51				-
	Q_4	58				

Year \ Quarter	Q_1	Q_2	Q_3	Q_4
1991	–	–	96.4427	99.6047
1992	112.2244	94.5010	92.1811	98.1595
1993	107.5099	96	102.8355	102.6820
1994	102.7668	96.5235	93.5282	105.5319
1995	104.1215	97.3651	–	–
Total	426.6226	384.3696	384.9875	405.9781
Avg	106.6557	96.0424	96.2469	101.4945
Adj SI	106.5277	95.9171	93.1314	101.3727

= 400.4895 (Avg)

= 400.0029 (Adj SI)

$$K = \frac{400}{400.4895} = 0.9988$$

3) Calculate the seasonal indices by method of ratio to moving average.

Year \ Quarter	I	II	III	IV
1994	65	58	56	61
1995	68	63	63	67
1996	70	59	56	52
1997	60	55	61	58

Sol

Year	Quarter	Q.V	M.V	M.A	C.M.A	R.M.A
1994	Q_1	65				-
	Q_2	58				-
			240	60		
	Q_3	56			60.38	92.745
			243	60.75		
	Q_4	61			61.37	99.3971
			248	62		
1995	Q_1	68			62.87	108.1597
			255	63.75		
	Q_2	63			64.5	97.6744
			261	65.25		
	Q_3	63			65.5	96.1832
			263	65.75		
	Q_4	67			65.25	102.682
			259	64.75		
1996	Q_1	70			63.87	109.5976
			252	63		
	Q_2	59			61.125	96.5235
			237	59.25		
	Q_3	56			58	96.5517
			227	56.75		
	Q_4	52			56.25	92.444
			223	55.75		
1997	Q_1	60			55.125	108.8435
			218	54.5		
	Q_2	55			55.25	99.5471
			224	56		
	Q_3	61				-
	Q_4	58				-

y \ q	I	II	III	IV
1994	-	-	92.75	99.38
1995	108.14	97.62	96.18	102.68
1996	109.58	96.52	96.55	90.44
1997	108.83	99.55 ~~99.55~~	-	-
total	326.55	293.74	285.4	294.52
avg SI	108.85	97.913	95.16	98.173
adj SI	108.82	97.893	95.14	98.153

avg SI total $= 400.096$

adj SI total $= 400.016$.

$K = \frac{400}{400.696} = 0.9998$

4) Calculate the seasonal indices for following data using ratio to moving averages method.

Year \ Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2008	12	14	15	18	20	16	15	12	18	24	18	14
2009	10	17	18	20	28	25	14	20	16	21	23	16
2010	13	13	20	22	24	19 ~~24~~	16	19	24	18	16	12

Sol

Year	Month	Month Values	Moving Values	Moving Avg	C.M.A	R.M.A
2008	Jan	12				
	Feb	14				
	Mar	15				
	Apr	18				
	May	20				
	June	16	196	16.33		
	Jul	15	194	16.16	16.24	92.3076
	Aug	12	197	16.416	16.29	73.6579
	Sep	18	200	16.666	16.54	108.8161
	Oct	24	202	16.83	16.75	143.2836
	Nov	18	210	16.6	17.[illegible]667	132.15
	Dec	14	219	17.5	17.875	78.3216

Year	Month	M.V	Moving Values	M.A	C.M.A	R.M.A
2009	Jan	10	218	18.16	18.203	54.9199
	Feb	17	226	18.83	18.5	91.8918
	Mar	18	224	18.66	18.75	96
	Apr	20	221	18.41	18.5417	107.8652
	May	28	226	18.83	18.625	150.3356
	June	25	228	19	18.9167	132.1586
	Jul	14	231	19.25	19.125	73.206
	Aug	20	227	18.91	19.083	104.8
	Sep	16	229	19.083	19	84.210
	Oct	21	231	19.25	19.1667	109.56
	Nov	23	227	18.91	19.083	120.52
	Dec	16	221	18.41	18.667	85.714
2010	Jan	13	223	18.58	18.5	70.27
	Feb	13	222	18.5	18.54	70.11
	Mar	20	230	19.16	18.633	106.19
	Apr	22	227	18.91	19.0417	115.53
	May	24	220	18.33	18.625	128.85
	Jun	19	216	18	18.1667	104.58
	Jul	16				
	Aug	19				
	Sep	24				
	Oct	18				
	Nov	16				
	Dec	12				

Year \ Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	
2008	-	-	-	-	-	-	92.30	78.65	108.81	143.28	104.85	78.32	
2009	54.91	91.89	96	107.86	150.33	132.15	73.20	104.80	84.210	109.56	120.52	85.71	
2010	70.27	70.11	106.19	115.53	128.85	104.58	-	-	-	-	-	-	
Total	125.18	162	202.19	223.39	254.91	236.73	165.5	178.45	193.02	252.84	225.37	164.03	
avg SI	65.59	81	101.69	111.69	127.45	118.36	82.75	89.22	96.51	126.42	112.68	82.015	= 1194.76
adj SI	65.852	81.35	101.53	112.18	128.01	118.88	83.11	89.61	96.93	126.97	113.18	83.37	= 1200.0763

$$K = \frac{1200}{1194.76} = 1.004$$

4) Simple Averages Method

1. Compute the SI for the following data by the method of simple averages

Quarter \ Year	1980	1981	1982	1983
Q_1	40.3	50.1	47.2	55.4
Q_2	44.8	53.1	50.1	59
Q_3	46	55.3	52.1	61.6
Q_4	48	59.5	55.2	65.3

sol (i) Arrange the data by years and quarters and compute the Quarterly wise $\bar{x}$ for $i=1,2,3,4$

Quarter \ Year	1980	1981	1982	1983	Total	Avg
Q_1	40.3	50.1	47.2	55.4	193	48.25
Q_2	44.8	53.1	50.1	59	207	51.75
Q_3	46	55.3	52.1	61.6	215	53.75
Q_4	48	59.5	55.2	65.3	228	57
						210.75

Compute the avg ($\bar{x}$) of the Quarterly averages i.e,

$$\bar{x} = \frac{1}{4}\sum_{i=1}^{4} x_i = \frac{210.75}{4} = 52.6875$$

The SI for different quarters are obtained by expressing the Quarterly averages as the percentage of $\bar{x}$.

i.e SI of i^{th} Quarter $= \frac{\bar{x}_i}{\bar{x}} \times 100$ for $i = 1,2,3,4$

i.e SI for $Q_1 = \frac{\bar{x}_1}{\bar{x}} \times 100 = \frac{48.25}{52.6875} \times 100 = 91.5777$

$Q_2 = \frac{\bar{x}_2}{\bar{x}} \times 100 = 98.2206$

$Q_3 = \frac{\bar{x}_3}{\bar{x}} \times 100 = 102.0166$

$Q_4 = \frac{\bar{x}_4}{\bar{x}} \times 100 = 108.1851$

Sum of all SI for quarterly data should equals to 400

i.e, $91.5777 + 98.2206 + 102.0166 + 108.1851 = 400$

2. Calculate the SI for following data

Year \ Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1981	15	16	18	18	23	23	20	26	29	33	33	38
1982	23	22	28	27	31	28	23	28	32	37	34	44
1983	25	25	35	36	36	30	30	24	38	47	41	53
Total	63	63	81	81	90	81	73	80	99	117	108	135
Avg	21	21	27	27	30	27	24.33	26.667	33	39	36	45

= 356.9997

$\bar{x} = \frac{356.9997}{12} = 29.75$

SI for $Q_1 = \frac{\bar{x}_1}{\bar{x}} \times 100 = \frac{21}{29.75} \times 100 = 70.5882$ ~~Q_7~~

$Q_2 = \frac{\bar{x}_2}{\bar{x}} \times 100 = 70.5882$

$Q_3 = \frac{\bar{x}_3}{\bar{x}} \times 100 = 90.7563$

$Q_4 = \frac{\bar{x}_4}{\bar{x}} \times 100 = 90.7563$

$Q_5 = \frac{\bar{x}_5}{\bar{x}} \times 100 = 100.8403$

$Q_6 = \frac{\bar{x}_6}{\bar{x}} \times 100 = 90.7563$

$Q_7 = 81.7916$

$Q_8 = 89.6360$

$Q_9 = 110.9244$

$Q_{10} = 131.0924$

$Q_{11} = 121.0084$

$Q_{12} = 151.2605$

Sum of all SI for quarterly data = 1200.

3.

Year \ Quarter	I	II	III	IV	
2003	40	45	36	42	
2004	50	54	42	48	
2005	56	58	48	52	
Total	146	157	126	142	
Avg	48.67	52.33	42	47.33	
SI	102.3	110	88.27	99.47	= 400.

$\bar{X} = 47.5825$

4. Determine Seasonal Indices by method of simple averages

Year \ Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2008	20	24	22	18	26	19	32	18	24	26	28	30
2009	38	32	26	32	38	26	46	24	28	30	30	40
2010	42	40	38	44	46	38	56	30	34	36	40	50
Total	100	96	86	94	110	83	134	72	86	92	98	120
Avg	33.33	32	28.67	31.33	36.67	27.667	44.6	24	28.67	30.6	32.6	40
SI	102.47	98.37	88.13	96.34	112.7	85.05	137.31	73.78	88.12	94.28	100.4	123

= 1200

$\bar{X} = 32.5278$

Link Relatives Method-continuation.....

- Calculate the seasonal indices by the method of link relatives.

Year \ Quarter	I	II	III	IV
2002	75	60	54	59
2003	86	65	63	80
2004	90	72	66	85
2005	100	78	72	93

Aim: To calculate seasonal indices for the given data by method of Link Relatives.

Calculation:

Chain relative of $Q_1 = 100$

Year \ Quarter	I	II	III	IV
2002	-	80	90	109.26
2003	145.76	75.58	96.92	126.98
2004	112.5	80	91.67	128.78
2005	117.65	78	92.3	129.17
Total	375.91	313.6	370.9	494.19
Avg LR	125.3	78.4	92.72	123.55
C.R	100	78.4 ~~75.27~~	72.69 ~~66.44~~	89.807
adj CR	100	75.27	66.44	80.432 = 322.1420
ST	124.18	93.47	82.5	99.879 = 400.0290

New CR of $Q_1 = \dfrac{125.3 \times 89.807}{100} = 112.5282$, $d = 3.1321$, avg (adj CR) $= 80.53$

3. Ratio to Trend Method

1) Obtain the seasonal indices to the following data by using ratio to trend method.

Year \ Quarter	I	II	III	IV
2005	35	45	41	39
2006	40	58	54	48
2007	72	88	64	96
2008	82	98	80	100
2009	112	108	86	94

Aim: To calculate SI for given data using ratio to trend method.

Procedure & Formulae:

1) Find the trend values by method of least squares using method of st. line is $y = a + bx$. Introduce a new variable

$$u = \frac{x - M.T}{h} = \cancel{x-1981}\ x - 2007 \quad \text{—} ①$$

Then the st. line eq^n in terms of u is $Y = a + bu$ —②

where Y = average of quarterly values in each year

The normal eq^{n}s of st. line are

$$\Sigma Y = na + b\Sigma u \quad \text{—— I}$$

$$\Sigma uY = a\Sigma u + b\Sigma u^2 \quad \text{—— II}$$

By solving the above eq^{n}s we can find values of a & b

then we get fitted st. line is $\hat{Y} = a + bu$ —③

By substituting a and b values in ③ we can get yearly trend values.

Quarterly trend values

from eq ③ yearly increment is b.

$Q_2 = T.V - \frac{b}{8}$

$Q_3 = T.V + \frac{b}{8}$

$Q_4 = Q_3 + \frac{b}{4}$

$Q_1 = Q_2 - \frac{b}{4}$

Trend eliminated values and SI

Find the trend eliminated values by expressing the original data as percentage of T.V.

i.e Trend eliminated value = $\frac{\text{Original Value}}{\text{Trend Value}} \times 100.$

The sum of SI for quarterly data should be equal to 400. Otherwise we have to adjust the SI by multiplying them with K. $K = \frac{400}{\text{Sum of avg SI}}$

y \ Q	I	II	III	IV	Avg (Y)	U = x-2007	U^2	UY	Y' (trend value)
2005	35	45	41	39	40	-2	4	-80	40
2006	40	58	54	48	50	-1	1	-50	56
2007	72	88	64	96	80	0	0	0	72
2008	82	98	80	100	90	1	1	90	88
2009	112	108	86	94	100	2	4	200	104
					360	0	10	160	360

I $\Rightarrow 360 = 5a \Rightarrow a = 72$

II $\Rightarrow 160 = b(10) \Rightarrow b = 16$

subs in st. line $\hat{y} = a + bu \Rightarrow 72 + 16u$

from the above equations yearly increment

$Q_2 = T.V - \frac{b}{8}$, $Q_3 = T.V + \frac{b}{8}$, $Q_1 = Q_2 - \frac{b}{4}$, $Q_4 = Q_3 + \frac{b}{4}$

Quarterly trend values

Year (x)	I	II	III	IV
2005	34	38	42	46
2006	50	54	58	62
2007	66	70	74	78
2008	82	86	90	94
2009	98	102	106	110

Trend eliminated value = $\frac{\text{actual value}}{\text{trend value}} \times 100$

Trend eliminated values

Year (x)	I	II	III	IV
2005	102.9412	118.4211	97.6190	84.7826
2006	80	107.4074	93.1034	77.42
2007	109.0909	125.7143	86.4865	123.0769
2008	100	113.9535	88.8889	106.3830
2009	114.2857	105.8824	81.1321	85.4545
Total	506.319	571.4	447.2	477.1170
Avg SI	101.2638	114.28	89.44	95.4234 = 400.4072
Adj SI	101.1625	114.1657	89.3506	95.3280 = 400.0068

$$k = \frac{400}{400.4072} = 0.9990$$

2) Calculate seasonal indices for the following data using ratio to trend method.

Year \ quarter	Q_1	Q_2	Q_3	Q_4
1979	30	40	36	34
1980	34	52	50	44
1981	40	58	54	48
1982	52	76	68	62
1983	80	92	80	82

Aim: To find seasonal indices using ratio to trend method.

Procedure:

$$u = \frac{x - M.T}{h} = x - 1981$$

$$\Sigma Y = na + b\Sigma u \text{ —— I}$$

$$\Sigma uY = a\Sigma u + b\Sigma u^2 \text{ —— II}$$

By solving I & II we get a,b.

$$Q_2 = TV - \frac{b}{8},\ Q_3 = TV + \frac{b}{8},\ Q_4 = Q_3 + \frac{b}{4},\ Q_1 = Q_2 - \frac{b}{4}$$

Calculation:

Y\Q	Q_1	Q_2	Q_3	Q_4	Avg (Y)	$u = x - 1981$	u^2	uY	Y'
1979	30	40	36	34	35	-2	4	-70	32.3
1980	34	52	50	44	45	-1	1	-45	43.95
1981	40	58	54	48	50	0	0	0	55.6
1982	52	76	68	62	64.5	1	1	64.5	67.25
1983	80	92	80	82	83.5	2	4	167	78.9
					278	0	10	116.5	278.

I ⇒ $278 = 5a \Rightarrow a = 55.6$

II ⇒ $116.5 = 10b \Rightarrow b = 11.65$

$$\hat{Y} = a + bu \Rightarrow \hat{Y} = 55.6 + (11.65)u.$$

Quarterly trend values

Year(X)	Q_1	Q_2	Q_3	Q_4
1979	27.9313	30.8438	33.7563	36.6688
1980	39.5813	42.4938	45.4063	48.3188
1981	51.2313	54.1438	57.0563	59.9688
1982	62.8815	65.7938	68.7063	71.6188
1983	74.5313	77.4438	80.8563	83.2688

Trend eliminated values

Y \ Q	Q_1	Q_2	Q_3	Q_4	
1979	107.4064	129.6857	106.6468	92.7219	
1980	85.6991	122.3708	110.1169	91.0619	
1981	78.0773	107.1221	94.6434	80.0416	
1982	82.6955	115.5724	98.9720	86.5694	
1983	107.3375	118.7958	99.5566	98.4763	
Total	461.4158	593.4868	509.9357	448.8911	
Avg SI	92.2832	118.6974	101.9871	89.7742	= 402.7419
Adj SI	91.6557	117.8903	101.2936	89.1637	= 400

$$K = \frac{400}{402.7419} \doteq 0.9932$$

3) ~~Year~~

Year \ Month	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
1981	15	16	18	18	23	23	20	28	29	33	33	38
1982	23	22	28	27	31	28	23	28	32	37	34	44
1983	25	25	35	36	36	30	30	34	38	47	41	53

Aim: To find seasonal indices using ratio to trend method.

Procedure:

$U = x - 1982$

$\Sigma Y = na + b\Sigma u$ — I

$\Sigma uY = a\Sigma u + b\Sigma u^2$ — II

$Q_2 = TV - \frac{b}{8}$, $Q_3 = TV + \frac{b}{8}$; $Q_4 = Q_3 + \frac{b}{4}$, $Q_1 = Q_2 - \frac{b}{4}$

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Calculation:

Y \ M	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Avg (Y)	$u = x - 1982$	u^2	uY	Y'
1981	15	16	18	18	23	23	20	26	29	33	33	38	24.5	-1	1	-24.5	24.3611
1982	23	22	28	27	31	28	23	28	32	37	34	44	29.75	0	0	0	30.0273
1983	25	25	35	36	36	30	30	34	38	47	41	53	35.8333	1	1	35.8333	35.5945
													90.0833	0	2	11.3333	90.0834

I $\Rightarrow 90.0833 = 3a \Rightarrow a = 30.0278$

II $\Rightarrow 11.3333 = 2b \Rightarrow b = 5.6667$ $\qquad \hat{y} = 30.0278 + (5.6667)u$

Monthly trend values.

Y \ M	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1981	21.7640	22.2362	22.7084	23.1806	23.6528	24.1250	24.5912	25.0694	25.5416	26.0138	26.4860	26.9582
1982	27.4304	27.9026	28.3748	28.8470	29.3192	29.7914	30.2636	30.73	31.2080	31.6802	32.1524	32.6246
1983	33.0968	33.5690	34.0412	34.5154	34.9856	35.4578	35.9300	36.40	36.8744	37.3466	37.8188	38.2910

Trend Eliminated values

Y \ M	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1981	68.92	71.95	79.26	77.65	97.24	95.33	81.31	111.68	113.54	126.8	124.5	140.9
1982	83.84	78.64	98.67	93.59	105.7	93.9	75.9	91.09	102.5	116.7	105.7	134.8
1983	75.53	74.47	102.8	104.3	102.8	84.6	83.4	93.4	103.0	125.8	108.4	138.4
Total	228.3 058	225.2 739	280.7	275.5	305.8	273.9	240.8	296.1	319.1	369.4	335.7	414.2
Avg SI	76.10	75.09	93.5	91.8	101.9	91.3	80.26	98.7	106.37	123.1	112.91	138.08
Adj SI	76.77	75.75	94.4	92.6	102.8	92.12	80.98	99.60	107.3	124.26	113.9	139.30

Avg SI total $= 1189.4380$

Adj SI total $= 1200.0241$

$$k = \frac{1200}{1189.4380} = 1.0225$$

Practical:

Below are given figures of production (1000 tons) of a fertilizer factory)

Year	1995	1996	1997	1998	1999	2000	2001
Producⁿ	77	88	94	85	91	98	90

i) Fit a straight line by least squares method and find trend values.

What is the monthly increment.

) Eliminate trend values assuming additive model and multiplicative model.

→ Straight line eqⁿ is $\hat{y} = a + bu$

$u = \frac{t - M.T}{h}$

normal eqⁿs are

$\Sigma Y = na + b\Sigma u$ — I

$\Sigma uY = a\Sigma u + b\Sigma u^2$ — II

→ Yearly increment $= \frac{b}{12}$

Additive Model

Assuming additive model the trend values are eliminated by using following formula

i.e $Y - \hat{y}$ (or) $Y_t - Y_e$.

Multiplicative Model

$Y/\hat{y}$ (or) Y_t/Y_e

Calculation: $u = t - 1998$

Year	Production	u	u^2	uY	$Y = 89 + 2(t-1998)$
1995	77	-3	9	-231	83
1996	88	-2	4	-176	85
1997	94	-1	1	-94	87
1998	85	0	0	0	89
1999	91	1	1	91	91
2000	98	2	4	196	93
2001	90	3	9	270	95
	623	0	28	56	~~623~~ 623

I ⇒ $623 = 7a \Rightarrow a = 89$

II ⇒ $56 = 28b \Rightarrow b = 2$

$\hat{y} = 89 + 2(t - 1998)$

Monthly Increment $= \frac{b}{12} = \frac{2}{12} = 0.1667$

Trend Eliminated Values

Year	Prod (Y)	T.V (Y')	Addi. M (Y-Y')	Mult. M (Y/Y')
1995	77	83	-6	0.9277
1996	88	85	3	1.0353
1997	94	87	7	1.0805
1998	85	89	-4	0.9551
1999	91	91	0	1
2000	98	93	5	1.0538
2001	90	95	-5	0.9474

Problems

1) The demand curve and supply curve of a commodity are given by $D = 19 - 3p - p^2$, $S = 5p - 1$. Find the equillibrium price and the quantity exchanged.

Sol. $D = 19 - 3p - p^2$

$S = 5p - 1$

$D = S$

$19 - 3p - p^2 = 5p - 1$

$19 - 3p - p^2 - 5p + 1 = 0$

$20 - 8p - p^2 = 0$

$p^2 + 8p - 20 = 0 \Rightarrow p^2 + 10p - 2p - 20 = 0$

$p(p+10) - 2(p+10) = 0$

$(p+10)(p-2) = 0$

$p = 2, -10$

$p = 2$ (since price value can't be negative)

$D = S = 9$

2) The demand functions of 2 commodities A and B are $D_A = 10 - P_A - 2P_B$, $D_B = 6 - P_A - P_B$ and corresponding supply functions are $S_A = -3 + P_A + P_B$, $S_B = -2 + P_B$ where P_A, P_B are the prices of A, B respectively.

(i) Find equillibrium prices.

(ii) Equillibrium quantities exchanged in the market.

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Sol: Consider

(i)
$$D_A = S_A$$

$$10 - P_A - 2P_B = -3 + P_A + P_B$$

$$10 - P_A - 2P_B + 3 - P_A - P_B = 0$$

$$-2P_A - 3P_B + 13 = 0$$

$$2P_A + 3P_B - 13 = 0 \quad \text{--- } (1)$$

$$D_B = S_B$$

$$6 - P_A - P_B = -2 + P_B$$

$$6 - P_A - P_B + 2 - P_B = 0$$

$$-P_A - 2P_B + 8 = 0$$

$$P_A + 2P_B - 8 = 0 \quad \text{--- } (2)$$

$$(1) \Rightarrow \cancel{2P_A} + 3P_B - 13 = 0$$

$$(2) \times 2 \Rightarrow \cancel{2P_A} + 4P_B - 16 = 0$$

$$\quad (-) \qquad (-) \qquad (+)$$

$$-P_B + 3 = 0$$

$$P_B = +3$$

$$(2) \Rightarrow P_A + 2(3) - 8 = 0$$

$$P_A - 2 = 0$$

$$P_A = 2$$

$$\therefore P_A = 2, P_B = 3$$

(iii) substituting P_A, P_B in D_A, S_A, D_B, S_B

$D_A = 10-2-2(3) = 2$

$\therefore D_A = S_A$

~~$S_A = 6-2-2$~~

$S_A = -3+2+3 = 2$

$D_B = 6-2-3 = 1$

$\therefore D_B = S_B$

$S_B = -2+3 = 1$

3) Find the time path of 'p' for dynamic equillibrium if the initial price is given to be 36 paise /kg and

$x_d = 120-2p+5\frac{dp}{dt}$ kgms/week

$x_s = 3p-30+50\frac{dp}{dt}$ kgms/week

where p is price at time 't'.

Sol. Given that the demand & supply curves are

$x_d = 120-2p+5\left(\frac{dp}{dt}\right)$

$x_s = 3p-30-50\left(\frac{dp}{dt}\right)$

when $t=0, p=36$

equillibrium equation of price is

$x_d = x_s$

$120-2p+5\left(\frac{dp}{dt}\right) = 3p-30+50\left(\frac{dp}{dt}\right)$

$50\left(\frac{dp}{dt}\right) - 5\left(\frac{dp}{dt}\right) = 120-2p-3p+30$

~~$45\left(\frac{dp}{dt}\right) = 150-5p$~~.

~~$9\left(\frac{dp}{dt}\right) = 36-p$~~.

$$\cancel{9}\left[10\left(\frac{dp}{dt}\right)-\left(\frac{dp}{dt}\right)\right] = \cancel{9}(30-p)$$

$$9\left(\frac{dp}{dt}\right) = 30-p$$

$$\frac{dp}{dt} = \frac{30-p}{9}$$

$$\frac{dp}{dt} + \frac{p}{9} = \frac{30}{9}$$

[since $\frac{dx}{dt} + ax = b$

$I.F = e^{\int a\,dt}$

$Sol^n \Rightarrow x(I.F) = \int b(I.F)\,dt + c$]

$x = p,\ a = \frac{1}{9}$

$$I.F = e^{\int \frac{1}{9} dt} = e^{\frac{t}{9}}$$

$$Sol^n \Rightarrow p(e^{t/9}) = \int \frac{30}{9}(e^{t/9})\,dt + c$$

$$\Rightarrow p.e^{t/9} = \frac{30}{9}\int e^{\frac{t}{9}}\,dt + c$$

$$\Rightarrow p.e^{t/9} = \frac{30}{\cancel{9}} \cdot \frac{e^{\frac{t}{9}}}{1/\cancel{9}} + c$$

$$\Rightarrow p.e^{t/9} = 30e^{t/9} + c$$

$$\Rightarrow p = 30 + ce^{-t/9}$$

$t = 0,\ p = 36.$

$$36 = 30 + ce^{-0/9}$$

$$36 = 30 + c \quad \Rightarrow c = 6$$

$\therefore \boxed{p = 30 + 6e^{-t/9}}$

Price Elasticity of Demand

1. If the demand function is $p = 4-5x^2$ then for what values of x the elasticity of demand will be 1 or unity.

Sol. Given that the demand function is $p = 4-5x^2$

The price elasticity of demand is

$$\eta_p = \frac{-p}{x} \cdot \frac{dx}{dp} \quad \text{--- (1)}$$

$p = 4-5x^2$

differentiating on both sides w.r.t 'p'

$\Rightarrow \quad 1 = -10x \cdot \frac{dx}{dp}$

$\Rightarrow \quad \frac{dx}{dp} = \frac{-1}{10x}.$

$\eta_p = 1, \ \frac{dx}{dp} = \frac{-1}{10x}, \ p = 4-5x^2$ substitute in (1)

$1 = \frac{+(4-5x^2)}{x} \cdot \frac{+1}{10x}.$

$1 = \frac{4-5x^2}{10x^2} \Rightarrow 10x^2 = 4-5x^2 \Rightarrow 15x^2 = 4$

$\Rightarrow x = \frac{2}{\pm\sqrt{15}}$

$\left[1 = \frac{\cancel{4}^{2}}{\cancel{10}_{5}x^2} - \frac{\cancel{5}x^2}{\cancel{10}_{2}\cancel{x^2}} \Rightarrow 1 = \frac{2}{5x^2} - \frac{1}{2} \right.$

$\Rightarrow 1 + \frac{1}{2} = \frac{2}{5x^2}$

$\Rightarrow \frac{3}{2} = \frac{2}{5x^2}$

$\Rightarrow 5x^2 = \frac{2}{3} \times 2$

$\left. \Rightarrow 5x^2 = \frac{4}{3} \Rightarrow x^2 = \frac{4}{15} \Rightarrow x = \frac{2}{\sqrt{15}} \right]$

2. The price elasticity of a demand curve $x = f(p)$ is of the form $a-bp$ where a and b are constants then find the demand curve.

Sol. given that demand curve $x = f(p)$

price elasticity of demand is of the form $a-bp$

$$\eta_p = \frac{-p}{fx} \cdot \frac{dx}{dp} = a-bp$$

$$\Rightarrow \frac{-p}{x} \cdot \frac{dx}{dp} = a-bp.$$

$$\Rightarrow a-bp + \frac{p}{x} \cdot \frac{dx}{dp} = 0$$

$$\Rightarrow \frac{p}{dp}\left[(a-bp) \cdot \frac{dp}{p} + \frac{dx}{x}\right] = 0$$

$$\Rightarrow \cancel{(a-bp)}\ a \cdot \frac{dp}{p} - b\cancel{p} \cdot \frac{dp}{\cancel{p}} + \frac{dx}{x} = 0$$

$\Rightarrow$ applying integration

$$\Rightarrow a\int \frac{1}{p}\, dp - b\int 1 \cdot dp + \int \frac{1}{x}\, dx = 0$$

$$\Rightarrow a \log p - bp + \log x = \log k$$

$$\Rightarrow a \log p - bp \log_e e + \log x = \log k$$

$$\Rightarrow \log p^a + \log e^{-bp} + \log x = \log k$$

$$\Rightarrow \cancel{\log}(p^a e^{-bp} x) = \cancel{\log} k.$$

$$\Rightarrow p^a e^{-bp} \cdot x = k$$

$$\Rightarrow x = \frac{k}{p^a e^{-bp}} \Rightarrow x = k \cdot p^{-a} e^{bp}$$

3. If the demand curve is of the form $p = \alpha e^{-kx}$ where p is the price and x is the demand. Then prove that elasticity of demand is $\frac{1}{kx}$ hence find the elasticity of demand by using the curve $p = 10e^{-x/2}$.

sol. ~~If the demand curve is of the~~

Given that demand curve $p = \alpha e^{-kx}$

price elasticity of demand is

$$\eta_p = -\frac{p}{x} \cdot \frac{dx}{dp}$$

consider $p = \alpha e^{-kx}$ — (1)

diff w.r.t 'p'

$$1 = \alpha \cdot \left(-k \cdot e^{-kx} \cdot \frac{dx}{dp}\right)$$

$$\frac{dx}{dp} = \frac{-1}{\alpha k \cdot e^{-kx}}$$

subsitute 'p and '$\frac{dx}{dp}$' in η_p

$$\eta_p = +\frac{\cancel{\alpha} \cancel{e^{-kx}}}{x} \cdot \frac{+1}{\cancel{\alpha} k \cancel{e^{-kx}}}$$

$$\eta_p = \frac{1}{kx}$$

we have $p = 10e^{-x/2}$ — (2)

from (1) & (2)

$\alpha = 10, \; k = \frac{1}{2}$ $\qquad \eta_p = \frac{1}{\frac{1}{2} \cdot x} = \frac{2}{x}$

$$\therefore \boxed{\eta_p = \frac{2}{x}}$$

1) The demand function for a commodity X is given by

$X = 300 - 0.5P_x^2 + 0.02P_o + 0.05Y$

where X is the quantity demanded of commodity X. P_x is the price of X. P_o is the price of a real related commodity and Y is the constant income. Then compute

(i) Price elasticity of demand for X.

(ii) Income elasticity of demand for X.

(iii) Cross elasticity of demand for X w.r.t P_o where $P_x = 12$, $P_o = 10$, $Y = 200$.

Sol.

(i) Price elasticity of demand for X is

$$\eta_p = -\frac{P}{x} \cdot \frac{dx}{dp}$$

$$= \left[\frac{-P_x}{300-0.5P_x^2+0.02P_o+0.05Y}\right] \times \frac{d}{dp}(300-0.5P_x^2+0.02P_o+0.05Y)$$

$$\cancel{\Rightarrow -\frac{12}{300-0.5(12)^2+0.02(10)+0.05(200)} \times \frac{d}{dp}[300-0.5(12)^2+0.02(10)+0.05(200)]}$$

$$\Rightarrow -\left[\frac{12}{300-0.5(12)^2+0.02(10)+0.05(200)}\right] \times (-0.5 \times 2P_x)$$

$$\Rightarrow \frac{-12}{238.2} \times -P_x \Rightarrow \frac{-12}{238.2} \times -12 = 0.6045$$

$$\eta_p = 0.6045$$

(ii) Income elasticity of demand for x

$$\eta_y = \left(\frac{Y}{x}\right) * \left(\frac{dx}{dY}\right)$$

$$= \left(\frac{200}{238.2}\right) * \frac{d}{dY}(300 - 0.5P_x^2 + 0.02P_o + 0.05Y)$$

$$= 0.8396 \times 0.05$$

$$= 0.0420$$

(iii) Cross elasticity of demand for x w.r.t P_o

$$\left(\frac{P_o}{x}\right) \times \frac{dx}{dP_o}$$

$$= \frac{10}{238.2} \times \frac{d}{dP_o}(300 - 0.5P_x^2 + 0.02P_o + 0.05Y)$$

$$= \frac{10}{238.2} \times \cancel{0.0008} (0.02)$$

$$= 0.0008.$$

2) If x_1 and P_1 are demand and price of tea and x_2 and P_2 are demand and price of coffee and demand functions of tea and coffee are given by $x_1 = P_1^{-1.3} \cdot P_2^{0.5}$ and $x_2 = P_1^{0.3} \cdot P_2^{-0.5}$ respectively. Show that the 2 commodities are competitive and also find 4 partial elasticities of demand.

Sol. To show that above 2 commodities are competitive we have to prove $\frac{\partial x_1}{\partial P_2} > 0$ and $\frac{\partial x_2}{\partial P_1} > 0$.

$$\frac{\partial}{\partial P_2}(P_1^{-1.3} \cdot P_2^{0.5}) = P_1^{-1.3} \cdot (0.5) P_2^{-0.5} = \frac{0.5}{P_1^{1.3} \cdot P_2^{0.5}} > 0$$

$$\frac{\partial}{\partial P_1}(P_1^{0.3} \cdot P_2^{-0.5}) = P_2^{-0.5} \cdot (0.3) P_1^{-0.7} = \frac{0.3}{P_2^{0.5} \cdot P_1^{0.7}} > 0$$

Partial elasticities:

(i) Partial elasticity of demand for tea w.r.t its own price

$$= -\left(\frac{P_1}{x_1}\right) \times \left(\frac{\partial x_1}{\partial P_1}\right)$$

(ii) Partial elasticity of demand for coffee w.r.t its own price

$$= -\left(\frac{P_2}{x_2}\right) \times \left(\frac{\partial x_2}{\partial P_2}\right)$$

(iii) Cross elasticity of demand for tea w.r.t price P_2

$$= \left(\frac{P_2}{x_1}\right) \times \left(\frac{\partial x_1}{\partial P_2}\right)$$

(iv) Cross elasticity of demand for coffee w.r.t price P_1

$$= \left(\frac{P_1}{x_2}\right) \times \left(\frac{\partial x_2}{\partial P_1}\right)$$

(i) $\Rightarrow -\dfrac{P_1}{P_1^{-1.3} \cdot P_2^{0.5}} \times \dfrac{\partial}{\partial P_1}(P_1^{-1.3} \cdot P_2^{0.5})$

$\Rightarrow -\dfrac{P_1}{P_1^{-1.3} \cdot P_2^{0.5}} \times P_2^{0.5} \cdot (-1.3) \cdot P_1^{-2.3}$

$\Rightarrow -\left[\dfrac{P_1 \cdot P_1^{1.3}}{P_2^{0.5}} \times P_2^{0.5} (-1.3) \cdot \dfrac{1}{P_1^{2.3}}\right]$

$\Rightarrow (1.3)\left[\dfrac{\cancel{P_1^{2.3}} \times \cancel{P_2^{0.5}}}{\cancel{P_2^{0.5}} \times \cancel{P_1^{2.3}}}\right]$

$\Rightarrow 1.3$

(ii) $\Rightarrow -\dfrac{P_2}{P_1^{0.3} \cdot P_2^{-0.5}} \times \dfrac{\partial}{\partial P_2}(P_1^{0.3} \, P_2^{-0.5})$

$\Rightarrow -\dfrac{P_2}{P_1^{0.3} P_2^{-0.5}} \times P_1^{0.3} (-0.5) P_2^{\cancel{-0.5}\, -1.5}$

$\Rightarrow (0.5)\left[\dfrac{P_2 \cdot P_2^{-1.5} \cdot P_1^{0.3}}{P_1^{0.3} \cdot P_2^{-0.5}}\right] = (0.5)\left[\dfrac{\cancel{P_2^{-0.5}} \cdot \cancel{P_1^{0.3}}}{\cancel{P_1^{0.3}} \cdot \cancel{P_2^{-0.5}}}\right] = 0.5$

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iii) $\Rightarrow + \dfrac{P_2}{P_1^{-1.3} \cdot P_2^{0.5}} \times \dfrac{\partial}{\partial P_2} (P_1^{-1.3} \cdot P_2^{0.5})$

$\Rightarrow + \dfrac{P_2}{P_1^{-1.3} \cdot P_2^{0.5}} \times P_1^{-1.3} \cdot (0.5) P_2^{-0.5}$

$\Rightarrow +(0.5) \left[\dfrac{P_2 \cdot P_2^{-0.5} \cdot P_1^{-1.3}}{P_1^{-1.3} \cdot P_2^{0.5}} \right]$

$\Rightarrow +(0.5) \left[\dfrac{\cancel{P_2^{0.5}} \cdot \cancel{P_1^{-1.3}}}{\cancel{P_1^{-1.3}} \cancel{P_2^{0.5}}} \right]$

$\Rightarrow +0.5$

iv) $\Rightarrow + \dfrac{P_1}{P_1^{0.3} \cdot P_2^{-0.5}} \times \dfrac{\partial}{\partial P_1} (P_1^{0.3} \cdot P_2^{-0.5})$

$\Rightarrow + \dfrac{P_1}{P_1^{0.3} \cdot P_2^{-0.5}} \times P_2^{-0.5} \cdot (0.3) P_1^{-0.7}$

$\Rightarrow + (0.3) \left[\dfrac{P_1 \cdot P_1^{-0.7} \cdot P_2^{-0.5}}{P_1^{0.3} \cdot P_2^{-0.5}} \right]$

$\Rightarrow +(0.3) \left[\dfrac{\cancel{P_1^{0.3}} \cdot \cancel{P_2^{-0.5}}}{\cancel{P_1^{0.3}} \cdot \cancel{P_2^{-0.5}}} \right]$

$\Rightarrow +0.3$

Construction of Lorenz Curve

Following table gives the income list of people as obtain[ed] from the Indian Income Tax patterns 1985. Draw Lorenz curve for this data.

Range of Annual Income (AI)	No. of assessed (p)	No. of income assessed (q)
<4200	16361	42266
4201 - 5000	43498	204483
5001 - 8400	60032	~~3873320~~ 387320
8401 - 10000	19835	179451
10001 - 15000	17583	217018
15001 - 25000	8550	161809
25001 - 40000	3779	117902
40001 - 55000	1178	54294
55001 - ~~5~~ 70000	312	19250
70001 - 85000	127	9757
85001 - 100000	60	5423
100001 - 150000	49	10623
150001 - 200001	34	6323
>200001	12	2929

Aim: To draw Lorenz Curve for given data

Procedure & Formulae:

Step 1: Compute proportion of no. of assessed (p) & total income assessed (q) in the given total.

p = No. of assessed for a particular range of AI / sum of total no. of assessed

q = total income ~~assed~~ assessed for a particular range of AI / sum of total income assessed

Step 2: Compute cumulative proportions of p & q.

Step 3: Now convert cumulative proportions into percentages

Step 4: Add an additional column with the same percentages of 'p' and 'q' as the first value for p and q as the additional column. ~~For~~ For the additional column give the heading as

p=q.

Step 5: Take the values of 1% of people on X-axis & 1% of their income on Y-axis.

Plot the points, we get the required Lorenz curve by joining all the points.

Calculations:

Σ(proportion of no. of assessed) = Σp = 171410

Σ(proportion of total income assessed) = Σp = 1414848

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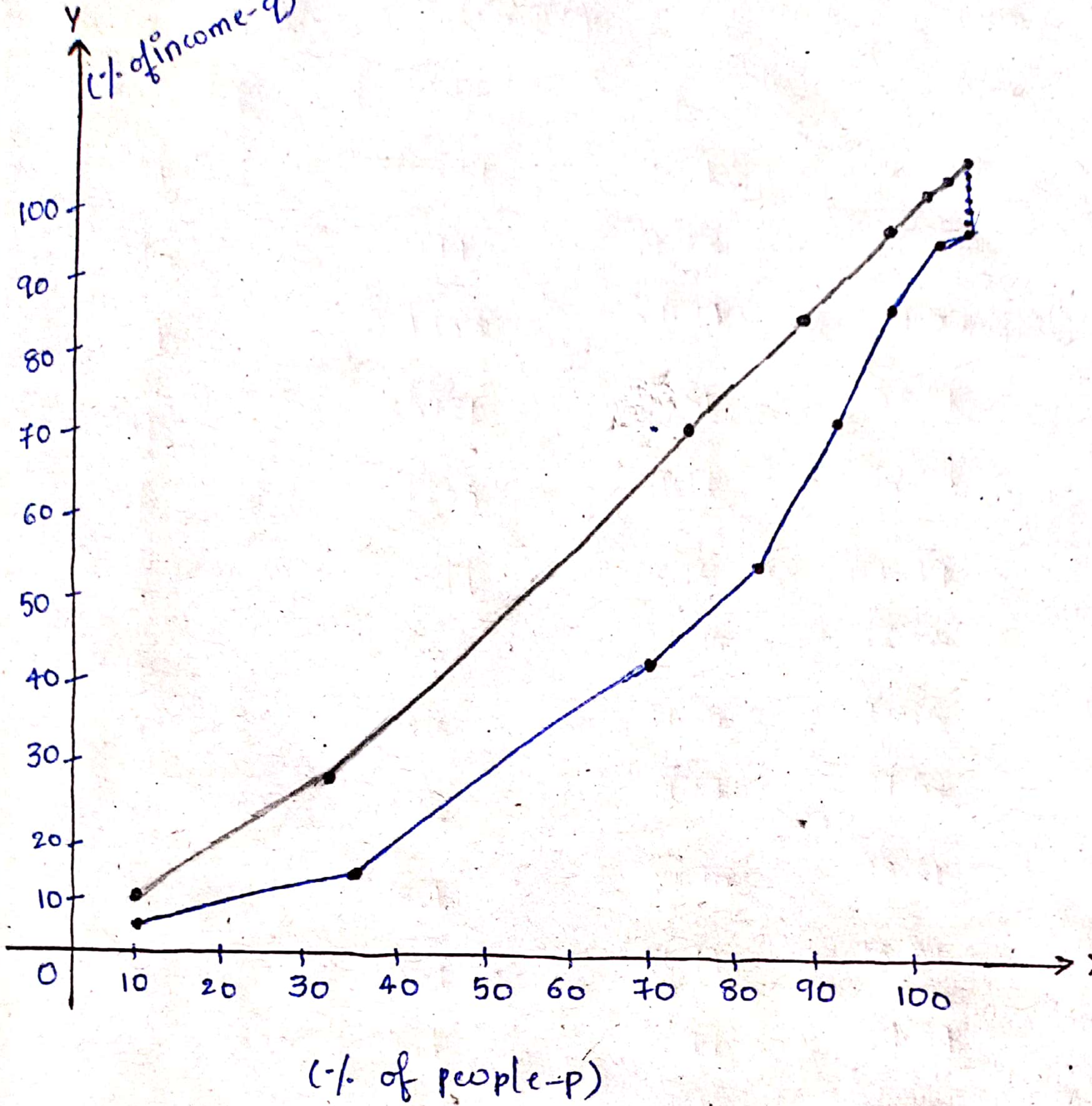
Range of AI	P	q	prop. of no. of assessed	Prop of no. of income assessed		Cumulative prop of no. of assessed	Cumulative prop of income assessed	Cumulative % of people	Cumulative % of income	(approx of P) P = q
<4200	16361	42266	0.0954	~~0.0298~~	0.0299	0.0954	0.0299	9.54	2.99	10
4201 - 5000	43498	204483	0.2538	~~0.1441~~	0.1445	0.3492	0.1744	34.92	17.44	35
5001 - 8400	60032	383320	0.3502	~~0.2730~~	0.2709	0.6994	0.4453	69.94	44.53	70
8401 - 10000	19835	179451	0.1157	~~0.1265~~	0.1268	0.8151	0.5721	81.51	57.21	82
10001 - 15000	17583	217018	0.1026	~~0.1530~~	0.1534	0.9177	0.7255	91.77	72.55	92
15001 - 25000	8550	161809	0.0499	~~0.1140~~	0.1144	0.9676	0.8399	96.76	83.99	97
25001 - 40000	3779	117902	0.0220	~~0.0831~~	0.0833	0.9896	0.9232	98.96	92.32	99
40001 - 55000	1178	54294	0.0069	~~0.0383~~	0.0384	0.9965	0.9616	99.65	96.16	100
55001 - 70000	312	19250	0.0018	~~0.0136~~	0.0136	0.9983	0.9752	99.83	97.52	100
70001 - 85000	127	9757	0.0007	~~0.0069~~	0.0069	0.9990	0.9821	99.90	98.21	100
85001 - 100000	60	5423	0.0004	~~0.0038~~	0.0038	0.9994	0.9859	99.94	98.59	100
100001 - 150000	49	10623	0.0003	~~0.0075~~	0.0075	0.9997	0.9934	99.97	99.34	100
150001 - 200000	34	6323	0.0002	~~0.0045~~	0.0045	0.9999	0.9979	99.99	99.79	100
>200 001	12	2929	0.0001	~~0.0021~~	0.0021	1	1	100	100	100
	171410	~~[illegible]~~ 1414848								


Y
(% of income-q)
100
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
X
(% of people-p)

1) Unweighted Index Numbers

These are again divided into 2 types

a) Simple aggregate method

b) Simple average of price relatives method.

a) Simple Aggregrate method

This method involvesaggrugation of prices in the current period and expressing the total as a percentage of price in base period.

i.e $P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$

where $P_0 \rightarrow$ price of commodity in base year

$P_1 \rightarrow$ price of commodity in current year.

1) Price Index for arithmetic mean

$$P_{01}^{AM} = \frac{\sum P}{N}$$

2) Price Index Value for GM

$$P_{01}^{GM} = \text{Antilog}\left[\frac{1}{N} \sum \log P\right]$$

where $P = \frac{P_1}{P_0} \times 100$ and

$P_0 \rightarrow$ Price of commodity in base year

$P_1 \rightarrow$ Price of commodity in current year.

$N \rightarrow$ No. of commodities

1. From the following data construct index data by using simple aggregate method.

Commodities	prices in 1980 (P_0)	prices in 1981 (P_1)
A	162	171
B	256	164
C	257	189
D	132	145

sol

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

$\Sigma P_1 = 669$

$\Sigma P_0 = 807$

$P_{01} = 82.8996$

2. Following are prices of different commodities in 1970 and 1975. Calculate price index numbers based on price relatives using AM and GM.

Commodities	1970 (P_0)	1975 (P_1)
A	45	55
B	60	70
C	20	30
D	50	75
E	25	90
F	120	130

sol.

$$P_{01}^{AM} = \frac{\Sigma P}{N}$$

$$P_{01}^{GM} = \text{Antilog}\left[\frac{1}{N}\Sigma \log P\right]$$

$$P = \frac{P_1}{P_0} \times 100$$

Calculation:

Commodities	1970 (P_0)	1975 (P_1)	P	log P.
A	45	55	122.2222	2.0872
B	60	70	116.6667	2.0669
C	20	30	150	2.1761
D	50	75	150	2.1761
E	25	90	360	2.5563
F	120	130	108.3333	2.0343
			1007.2222	13.0969

N = 6.

$$P_{01}^{AM} = \frac{1007.2222}{6} = 167.8703 \quad , \quad \frac{\Sigma \log P}{N} = 2.1828$$

$$P_{01}^{GM} = \text{Antilog}\left[\frac{1}{N}\Sigma \log P\right] = 152.3752$$

2) Weighted Index Numbers

These are again divided into 2 types.

a) Weighted aggregate method

b) Weighted avg. of price relative method.

a) Weighted aggregate method.

In this method appropriate weights are assigned to different types of commodities. If W is the weight ~~dedicated to~~ associated with the commodity then the weighted aggregate price index is given by

$$P_{01} = \frac{\Sigma P_1 W}{\Sigma P_0 W} \times 100$$

By the use of different types of weights

1) Laspeyre's Price Index No's

In this method the base year quantities are taken as weights. The Laspeyre's price Index No is defined as

$$P_{01}^{Las} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$$

2) Paasche's price Index No.

In this method the current year quantities are taken as weights. The paasche's price Index No. is defined as

$$P_{01}^{Pa} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100$$

3) Darbish Bowley price Index No.

This formula is the Arithmetic mean of Laspeyres & Paasche's price indices & is given by

$$P_{01}^{DB} = \frac{1}{2}\left[P_{01}^{La} + P_{01}^{Pa}\right]$$

$$= \frac{1}{2}\left[\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} + \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}\right]$$

4) Fisher's price Index No.

It is given by the Geometric mean of ~~B~~ Laspeyre's & Paasche's price indices & is given by

$$P_{01}^{F} = (P_{01}^{La} \times P_{01}^{Pa})^{1/2}$$

$$P_{01}^{F} = \left[\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}\right]^{1/2}$$

i) Marshall-EdgeWorth price Index

If we take the AM of the Base year & current year quantities as weights then the Marshall-Edgeworth formula is observed as.

$$P_{01}^{ME} = \frac{\Sigma P_1 W}{\Sigma P_0 W} \times 100 \quad ; W = \frac{q_0+q_1}{2}$$

c) Walsch price Index No.

In this case we take the GM of q_0 & q_1 as weights Then the Walsch price formula is defined as

$$P_{01}^{W} = \frac{\Sigma P_1 W_1}{\Sigma P_0 W_1} \times 100$$

where $W_1 = \sqrt{q_0 q_1}$

Problems:

1) From the following data calculate weighted aggregate price index no.'s for the year 2005 with 1995 as base.

Commodities	1995		2005	
	price	quantity	price	quantity
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Sol:

Commodities	P_0	q_0	P_1	q_1	P_1q_0	P_0q_0	P_0q_1	P_1q_1	W $\left(\frac{q_0+q_1}{2}\right)$
A									
B									
C									
D									

Commodities	P_0	q_0	P_1	q_1	P_1q_0	P_0q_0	P_0q_1	P_1q_1	$W(q_0+q_1/2)$	P_1W	P_0W	$W(\sqrt{q_0q_1})$	P_1W_1	P_0W_1
A	20	8	40	6	~~500~~ 320	160	120	240	7	280	140	6.9282	277.1230	138.5640
B	50	10	60	5	600	500	250	300	7.5	450	375	7.0711	424.2660	353.5550
C	40	15	50	15	750	600	600	750	15	750	600	15	750	600
D	20	20	20	25	400	400	500	500	22.5	450	~~400~~ 450	22.3607	447.2140	447.2140
					~~2250~~ 2070	1660	1470	1790	52	~~1880~~ 1930	~~1515~~ 1565		1898.6080	1539.333

Calculation:

1) $P_{01}^{Las} = \dfrac{\Sigma P_1q_0}{\Sigma P_0q_0} \times 100 = 124.6988$

2) $P_{01}^{Pa} = \dfrac{\Sigma P_1q_1}{\Sigma P_0q_1} \times 100 = 121.7687$

3) $P_{01}^{DB} = \dfrac{1}{2}\left[\dfrac{\Sigma P_1q_0}{\Sigma P_0q_0} + \dfrac{\Sigma P_1q_1}{\Sigma P_0q_1}\right] = 123.2338$

4) $P_{01}^{F} = \left[\dfrac{\Sigma P_1q_0}{\Sigma P_0q_0} \times \dfrac{\Sigma P_1q_1}{\Sigma P_0q_1}\right]^{1/2} \cancel{\times 100} = 123.2250$

5) $P_{01}^{ME} = \dfrac{\Sigma P_1W}{\Sigma P_0W} \times 100 = 123.3227$

6) $P_{01}^{W} = \dfrac{\Sigma P_1W_1}{\Sigma P_0W_1} \times 100 = 123.3397.$

2) Unweighted Quantity Index Numbers

These are divided into 2 types

a) Simple Aggregate Method

b) Simple Average of quantity relative method.

a) Simple Aggregate Method

This method involves aggregation of quantities in current period and expressing the total as a percentage of quantity in base period.

i.e $q_{01} = \frac{\Sigma q_1}{\Sigma q_0} \times 100.$

where q_0 = quantity of commodity in base year

q_1 = quantity of commodity in current year.

1) Quantity Index for arithmetic mean

$$q_{01}^{AM} = \frac{\Sigma q}{N}.$$

2) Quantity Index Value for GM

$$q_{01}^{GM} = \text{Antilog}\left[\frac{1}{N}\Sigma \log q\right]$$

where $q = \frac{q_1}{q_0} \times 100$

$q_1 \rightarrow$ quantity of commodity in current year

$q_0 \rightarrow$ quantity of commodity in base year.

Unweighted Quantity Index

a) Weighted aggregate method:

$$q_{01} = \frac{\Sigma q_1 W}{\Sigma q_0 W} \times 100.$$

1) Laspeyre's Quantity Index

$$q_{01}^{Las} = \frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times 100$$

2) Paasche's Quantity Index

$$q_{01}^{Pa} = \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1} \times 100$$

3) Darbish Bowley Quantity Index

$$q_{01}^{DB} = \frac{1}{2}\left[q_{01}^{La} + q_{01}^{Pa}\right]$$

$$= \frac{1}{2}\left[\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} + \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}\right]$$

~~4) Fisher's Price Index No~~

4) Fisher's Quantity Index No.

$$q_{01}^{F} = \left(q_{01}^{La} \times q_{01}^{Pa}\right)^{1/2}$$

$$= \left[\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}\right] \times 100$$

5) Marshall-Edgeworth Quantity Index

$$q_{01}^{ME} = \frac{\Sigma q_1 W}{\Sigma q_0 W} \times 100 \; ; \; W = \frac{P_0 + P_1}{2}$$

6) Walsh Quantity Index

$$q_{01}^{W} = \frac{\Sigma q_1 W_1}{\Sigma q_0 W_1} \times 100 \; ; \; W_1 = \sqrt{P_0 P_1}$$

Weighted Average of Quantity relative method

We can calculate these no.'s by using the following formula

→ Index no.'s based on the weighted AM of quantity relatives is given by

$$q_{01}^{AM} = \frac{\Sigma Wq}{\Sigma W}$$

→ Index number based on the GM of quantity relatives is given by

$$q_{01}^{GM} = \text{Antilog}\left[\frac{\Sigma W \log q}{\Sigma W}\right]$$

where W = weights and

$$q = \frac{q_1}{q_0} \times 100$$

$q_0 \rightarrow$ quantity value in base year

$q_1 \rightarrow$ quantity value in current year.

Weighted average of Price Relative method

1) $P_{01}^{AM} = \frac{\Sigma Wp}{\Sigma W}$

2) $P_{01}^{GM} = \text{Antilog}\left[\frac{\Sigma W \log p}{\Sigma W}\right]$; $p = \frac{P_1}{P_0} \times 100.$

1) Calculate for the following data quantity index no.s for the year 2005 with 2000 as base year.

Commodities	Price 2000	Price 2005	Quantity 2000	Quantity 2005
A	5	6	100	150
B	4	5	80	100
C	2.5	5	60	72
D	12	9	30	33

Commodities	P_0	q_0	P_1	q_1	q_1P_0	q_0P_0	q_1P_1	q_0P_1	$W((P_0+P_1)/2)$	q_1W	q_0W	$W_1(\sqrt{P_0P_1})$	q_1W_1	q_0W_1
A	5	100	6	150	750	500	900	600	5.5	825	550	5.4772	821.58	547.72
B	4	80	5	100	400	320	500	400	4.5	450	360	4.4721	447.21	357.768
C	2.5	60	5	72	180	150	360	300	3.75	270	225	3.5355	254.556	212.13
D	12	30	9	33	396	360	297	270	10.5	346.5	315	10.3923	342.9459	311.769
		270		355	1726	1330	2057	1570	24.25	1591.5	1450		1866.2919	1429.

3370

N = 4.

$q_{01} = 131.4815$

$q_{01}^{AM} = \frac{505}{4} = 126.25$

$q_{01}^{GM} = 125.4296.$

Wq	q	$\log q$	$W.\log q$
825	150	2.1761	11.9686
562.5	125	2.0969	9.4361
450	120	2.0792	7.7970
1155	110	2.0414	21.4347
2992.5	505	8.3936	50.6364

Weighted average

$q_{01}^{AM} = 123.4021$

$q_{01}^{GM} = 122.4898$

1) $q_{01}^{Las} = 130.4483$

2) $q_{01}^{Pa} = 131.0911$

3) $q_{01}^{DB} = 132.299$

4) $q_{01}^{F} = 130.3953$

5) $q_{01}^{ME} = 130.4483$

6) $q_{01}^{W} = 130.5659$

numbers

Mathematical Tests

→ The components of error in the construction of IN's can be broadly classified as

1) Formula Error: It arises due to the usage of different formulae.. none of which measures the price change (or) quality changes with perfection.

2) Sampling Error:

It results from the sampling of the commodities to be included in the index for

3) Homogenity Error: Change in the composition of commodities in 2 periods of comparision gives rise to homogenity error.

→ As a measure for the formula error, a no.of mathematical tests have suggested.

a) Unit Tests

This requires the index no.'s to be independent of the units in which the prices & quantities of various commodities are quoted.

b) Time Reversal Test.

This is one of the very important tests proposed by Fisher as "Tests of consistency. for good index No.

According to him any formula to be accurate & must maintain the consistency by working both forward & backward w.r.t time.

Acc... test is that the formula for calculating an index number should be such that it will give the same ratio b/w one point of comparison & the other, no matter which of the two is taken as base.

i.e. $P_{01} \times P_{01} = 1$

where P_{01} is the price IN of current year w.r.t base year

$P_{10} \rightarrow$ price IN of base year w.r.t current year.

→ Fisher's Index formula satisfies time reversal test i.e. $P_{01}^{F} \cdot P_{10}^{F} = 1$

→ It can easily verified that the IN's based on

1) The ~~sim~~ simple G.M of price relatives
2) Marshall edgeworth formula also satisfy time reversal test.
3) Laspeyre's & Paasche's formula don't satisfy time reversal test (TRT).

c) Factor Reversal Test (FRT): This is the 2nd test of consistency suggested by Fisher. In his words "the multiplication of price IN & quantity IN results value IN.

i.e. $P_{01} \cdot Q_{01} = V_{01}$.

Fishers IN satisfies FRT i.e $P_{01}^{F} \times Q_{01}^{F} = V_{01}$

where $V_{01} = \dfrac{\Sigma P_1 q_1}{\Sigma P_0 q_0} \times 100.$

It may be pointed out that none of other formulae satisfies FRT.

→ Since, Fisher's Index satisfies both TRT & FRT it is based as an Ideal Index Number.

d) Circular Test (CT)

The test is based on the ~~sustainability~~ shiftability of the base year & it is an extension of the TRT.

i.e. $P_{ab} \times P_{bc} \times P_{ac} = 1$ & $a \neq b \neq c$

Note: In verification of these tests various formulae are taken without 100.

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1) From the following data show that Fisher's Index No. is an Ideal Index No.

Commodities	Base Year Price (P_0)	Base Year Quantity (q_0)	Current Year Price (P_1)	Current Year Quantity (q_1)
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24

Sol: Time Reversal Test:

$$P_{01}^{F} \times P_{10}^{F} = 1$$

$$P_{01}^{F} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}}$$

$$P_{10}^{F} = \sqrt{\frac{\Sigma q_1 P_0}{\Sigma q_1 P_1} \times \frac{\Sigma q_0 P_0}{\Sigma q_0 P_1}}$$

Factor Reversal Test:

$$P_{01}^{F} \times Q_{01}^{F} = V_{01}$$

$$Q_{01}^{F} = \sqrt{\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}$$

$$V_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0}$$

Commodities	P_0	q_0	P_1	q_1	P_0q_0	P_1q_1	P_0q_1	P_1q_0
A	6	50	10	56	300	560	336	500
B	2	100	2	120	200	240	240	200
C	4	60	6	60	240	360	240	360
D	10	30	12	24	300	288	240	360
					1040	1448	1056	1420

TRT :-

$P_{01}^{F} = 1.3683$

$P_{10}^{F} = 0.7308$

$\therefore P_{01}^{F} \times P_{10}^{F} = 1$

FRT :-

$P_{01}^{F} = 1.3683$

$Q_{01}^{F} = 1.0175$

$P_{01}^{F} \times Q_{01}^{F} = 1.3683 \times 1.0175 = 1.3922$

$V_{01} = 1.3922.$

$\therefore P_{01}^{F} \times Q_{01}^{F} = V_{01}$

$\therefore$ The Fisher's Index No. is an Ideal Index No.

Theorum: If $L(p)$ & $P(q)$ represents Laspeyres Price Index and Paasches Quantity IN respectively then show that $\frac{L(p)}{L(q)} = \frac{P(p)}{P(q)}$

Proof: WKT Laspeyre's price IN is

$$L(p) = P_{01}^{LA} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100 \quad \text{--- (1)}$$

Laspeyre's Quantity IN is

$$L(q) = Q_{01}^{LA} = \frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times 100 \quad \text{--- (2)}$$

Paasche's Price IN is

$$P(p) = P_{01}^{PA} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100 \quad \text{--- (3)}$$

Paasche's quantity IN is

$$P(q) = Q_{01}^{PA} = \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1} \times 100 \quad \text{--- (4)}$$

We have to prove $\frac{L(P)}{L(q)} = \frac{P(P)}{P(q)}$

Let us consider LHS

$$\frac{L(p)}{L(q)} = \frac{\frac{\Sigma P_1 q_0}{\cancel{\Sigma P_0 q_0}} \times \cancel{100}}{\frac{\Sigma q_1 P_0}{\cancel{\Sigma q_0 P_0}} \times \cancel{100}}$$

$$= \frac{\Sigma P_1 q_0}{\Sigma q_1 P_0} \quad — \quad \text{I}$$

RHS,

$$\frac{P(P)}{P(q)} = \frac{\frac{\Sigma P_1 \cancel{q_1}}{\Sigma P_0 q_1} \times \cancel{100}}{\frac{\Sigma \cancel{q_1} P_1}{\Sigma q_0 P_1} \times \cancel{100}} = \frac{\Sigma q_0 P_1}{\Sigma P_0 q_1} \quad — \quad \text{II}$$

$$\therefore \quad \frac{\Sigma P_1 q_0}{\Sigma q_1 P_0} = \frac{\Sigma P_1 q_0}{\Sigma q_1 P_0}$$

$$\frac{L(P)}{L(q)} = \frac{P(P)}{P(q)}$$

Hence proved.

Chain Base Method (or) Chain Indices

The chain base method is used to calculate a series of Index no.s for each year with preceeding year as base.

i.e, $P_{01}, P_{12}, P_{23} - - - P_{(k-1)} \cdot P_k$.

where, P_{rs} represents the price IN with 'r' as base year and 's' as given year and the basic IN is obtained by the succeding multiplications of the IN's as given below

P_{01} = First link.

$P_{02} = P_{01} \times P_{12}$

$P_{03} = P_{01} \times P_{12} \times P_{23} = P_{02} \times P_{23}$

$\vdots$

$P_{0K} = P_{0(K-1)} \times P_{(K-1)K}$.

Construction of Chain Based Index No's.

Step 1: Express the figures for each period as a percentage of the preceding period to obtain the link relatives.

Step 2: These L.R. are chained together by successive multiplication to get chain indices (CI) by the following formula.

$$\text{Chain Index} = \frac{\text{Current Year L.R} \times \text{Preceding year C.I}}{100}.$$

Conversion of Chain Base Index No's to Fixed Base Index No's

The following formula may be used to convert the Chain Base IN's (CBIN) into Fixed Base IN (FBIN)

$$\text{Current Year FBIN} = \frac{\text{Current Year CBIN} \times \text{Preceding Year FBIN}}{100}$$

1. Show that for the following series of FBIN, the chain indices are as same as FBIN.

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
IN	100	120	122	116	120	120	137	136	149	156	137.

Sol.

$$\text{Chain Index} = \frac{\text{Current Year LR} \times \text{Preceding Year CI}}{100}$$

$$LR = \frac{\text{Current Year Value}}{\text{Preceding Year Value}} \times 100$$

Year	IN	LR	CI
1995	100	100	100
1996	120	120	120
1997	122	101.6667	122
1998	116	95.0820	116
1999	120	103.4483	120
2000	120	100	120
2001	137	114.1667	137
2002	136	99.2701	136
2003	149	109.5588	149
2004	156	104.6980	156
2005	137	87.8205	137

∴ The Chain Indices are same as FBIN

2. The following table given the avg whole sale prices of 4 ~~types~~ groups of commodities for the year 2001-2005. Compute CBIN.

Commodities	2001	2002	2003	2004	2005
A	20	30	40	20	70
B	30	60	90	40	30
C	40	120	200	80	160
D	50	70	180	110	220

sol: LR's based on previous Years

Commodities	2001	2002	2003	2004	2005
A	100	150	133.33	50	350
B	100	200	150	44.44	75
C	100	300	166.66	40	200
D	100	140	257.14	61.11	200
Total LR	400	790	707.13	195.55	825
Avg LR	100	197.5	176.7825	48.8875	206.25
CBIN	100	197.5	349.14	170.6858	352.0395

1) In the following table 2 prices index series are given. Splice them on base 2004. By what % did the price of the commodity rise between 2000 and 2005.

Year	Old price Index (Base 1995)	New Price Index (Base 2004)
2000	141.5	
2001	163.7	
2002	158.2	
2003	156.8	99.8
2004	157.1	100
2005		102.3

sol.

Year	1995	2004	Old PI Spliced to New PI with base 2004
2000	141.5		$=(100/157.1)\times 141.5 = 90.07$
2001	163.7		$=(100/157.1)\times 163.7 = 104.20$
2002	158.2		100.7002
2003	156.8	99.8	99.8090
2004	157.1	100	100
2005		102.3	102.3

$a_k = 157.1$ [$\because$ last value of old price index]

The % increased in the price of commodity b/w 2000 & 2005 is

~~((102.3-90.07)/90.07)×100~~ $= \dfrac{(102.3-90.07)}{90.07}\times 100$

$= 13.5783.$

Q) Splice the following 2IN series continuing series A forward and series B backward.

Year	Series A	Series B
2000	100	
2001	120	
2002	150	100
2003		110
2004		120
2005		150

Sol:

Year	Series A	Series B	Series A Forward (B to A)	Series B Backward (A to B)
2000	100		100	66.67
2001	120		120	80
2002	150	100	150	100
2003		110	165	110
2004		120	180	120
2005		150	225	150.
~~2005~~				

$\therefore a_k = 150$ [$\because$ last value of series A]

[$\because$ series A 2003 values $= \frac{150}{100} \times 110 = 165$

[$\because$ series B 2000 values $= \frac{100}{150} \times 100 = 66.67$].

Base shifting

Recast of IN of any year $= \frac{\text{Old IN of the Year}}{\text{IN of the new base Year}} \times 100$

$= \left[\frac{100}{\text{IN of new base yr}} \right]$ old IN of the year

$\therefore K = \frac{100}{\text{IN of New base year}}$

1) From the INs given below ... INs by shifting the base from 2000 to 2003.

Year	IN
2000	100
2001	76
2002	68
2003	50
2004	60
2005	70
2006	75

Sol. From the given data the IN of the new base Year 2003 is 50.

$$\text{IN with base } 2003 = \frac{100}{\text{IN of New base year}} \times \text{Old IN of the year}$$

$$K = \frac{100}{50} = 2 \quad \left[\because K = \frac{100}{\text{IN of New base year}}\right]$$

Year	IN	NBIN (New Base Index No.)
2000	100	200
2001	76	152
2002	68	136
2003	50	100
2004	60	120
2005	70	140
2006	75	150.

2) The fallowing are the index No. of prices based 1988 to 1999. Construct the Index no. by shifting the base year from 1988 to 1999.

Year	IN
1988	100
1989	110
1980	120
1991	200
1992	320
1993	400
1994	410
1995	400
1996	380
1997	370
1998	350
1999	366

Sol. From the given data the IN of the new base year 1993 is 400.

$$\text{IN with base } 1993 = \frac{100}{\text{IN of New base year}} \times \text{old IN of the Year}$$

$$K = \frac{100}{400} = 0.25$$

Year	IN	NBIN
1988	100	25
1989	110	27.5
1990	120	30
1991	200	50
1992	320	80
1993	400	100
1994	410	102.5
1995	400	100
1996	380	95
1997	370	92.5
1998	350	87.5
1999	366	91.5

1) The following table shows the avg wages (Rs) per hour of railway workers during the year 1947 to 1958 also given the customer consumer price indices for these years. Determine the real wages of the workers and also find purchasing power of a rupee.

Year	Avg wages of workers	Consumer PI
1947	1.19	100
1948	1.33	107.6
1949	1.44	106.6
1950	1.57	107.6
1951	1.75	1~~0~~6.2
1952	1.84	118.2
1953	1.89	119.8
1954	1.94	120.2
1955	1.97	119.9
1956	2.13	121.7
1957	2.28	125.9
1958	2.45	129.3

Sol:

$$\text{Real wage} = \frac{\text{Nominal wage}}{\text{Price Index}} \times 100$$

$$= \frac{\text{Avg Wages}}{\text{Consumer PI}} \times 100.$$

$$\text{Purchasing Power} = \frac{\text{Avg. wages}}{\text{Real wages.}}$$

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Calculation:

Year	Avg wages	Consumer PI	Real wages	PP of rupee.
1947	1.19	100	1.19	1
1948	1.33	107.6	1.2361	1.0760
1949	1.44	106.6	1.3508	1.0660
1950	1.57	107.6	1.4591	1.0760
1951	1.75	116.2	1.5060	1.1620
1952	1.84	118.2	1.5567	1.1820
1953	1.89	119.8	1.5476.	1.1980
1954	1.94	120.2	1.6140	1.2020
1955	1.97	119.9	1.6430	1.1990
1956	2.13	121.7	1.7502	1.2170
1957	2.28	125.9	1.8110	1.2590
1958	2.45	129.3.	1.8948	1.2930.

2) Avg. Monthly wages 'x' and cost of living Index no.s 'y' for the years 1970 to 1975 are given below. In which year the real income was the highest and the lowest.

Year.	x	y.
1970	360	100
1971	400	104
1972	480	115
1973	520	160
1974	550	210
1975	590	260.

Sol Real wage $= \frac{\text{Nominal wage}}{\text{PI}} \times 100.$

Real wage $= \frac{x}{y} \times 100$

Calculation

Year	X	Y	Real Wage.
1970	360	100	360
1971	400	104	386.6154
1972	480	115	417. 3913
1973	520	160	325
1974	550	210	261.9048.
1975	590	260	226.9231.

The real income was highest in 1972 i.e 417.3913

The real income was lowest in 1975 i.e 226.9231.

Cost of Living IN

Cost of Living IN are constructed to study the effort of changes in the prices of various commodities consumed by

1) Construct cost of living Index Number for the 2010 to the following data.

Item	price in 2006 (P_0)	price in 2010 (P_1)	weight
A	50	75	10%
B	60	75	25%
C	200	240	20%
D	80	100	40%
E	800	1000	5%

Sol.

Item	P_0	P_1	W	$P \left(\frac{P_1}{P_0} \times 100\right)$	WP.
A	50	75	10%	150	~~125~~ 15
B	60	75	25%	125	31.25
C	200	240	20%	120	24
D	80	100	40%	125	50
E	800	1000	5%	125	6.25
			100%		126.5

CLI => 126.5

3) Compute cost of living index number by aggregate ~~method~~ expenditure and family budget methods to the following data.

Commodities	Quantity in base year	Price in 2005	Price in 2008
Rice	50kgs	15	30
Wheat	4kgs	10	18
Pulses	3kgs	30	86
Ghee	1kg	47	73
Jaggery	2kgs	12	18
Sugar	5kgs	16	28
Oil	3litres	32	63
Clothing	10meters	15	27
Fuel	30litres	10	17
House Rent	-	1200	2000

Sol:

$$AE = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$$

$$FBM = \frac{\Sigma WP}{\Sigma W} \; ; \; P = \frac{P_1}{P_0} \times 100.$$

$$W = P_0 q_0$$

Calculation:

Commodities	q_0	p_0	p_1	(W) p_0q_0	p_1q_0	P	WP.
Rice	50	15	30	750	1500	200	150000
Wheat	4	~~30~~ 10	18	40	72	180	7200
Pulses	3	30	85	90	255	283.33	25499.7
Ghee	1	47	73	47	73	155.3191	7299.9977
Jaggery	2	12	18	24	36	150	3600
Sugar	5	16	28	80	140	175	14000
Oil	3	32	63	96	189	196.875	18900
Clothing	10	15	27	150	270	180	27000
Fuel	30	10	17	300	510	170	51000
House Rent	1	1200	2000	1200	2000	166.6667	200000.04
				2777	5045		504500

AE = 181.6709

FB = 181.6709

3) Calculate whole sale price index number to the following data.

Group	Weight	Index Number
Food	50	245
Fuel, Power Light	5	200
Manufactured products	33	180
Textiles	10	250
Leather products	2	220

Sol. Whole sale price index no. $= \dfrac{\Sigma WI}{\Sigma W}$

Group	W	I	WI
Food	50	245	12250
Fuel, Power light	5	200	1000
Manufactured	33	180	5940
Textiles	10	250	2500
Leather products	2	220	440
	100		22130

Whole price IN = 221.3.