Statistical Quality Control and Reliability Theory.

UNIT-I, UNIT-II } SQC- Statistical Quality Control

UNIT-III → Reliability theory

UNIT-I

Statistical Quality Control (SQC)

Introduction:

SQC is one of the most important applications of statistical techniques in industry these techniques are based on theory of probability and sampling and it is being extensively used in all the industries such as Textiles, Ornaments, Rubber, Automobiles, Plastic. The most important word in SQC is Quality. The Quality is an attribute that determines its fitness for use.

Quality means a standard or level of the product which depends on 4 M's

* Machines
* Man Power
* Management
* Material

Quality of Machines: Better Quality of equipment will result in efficient work due to lack (or) scarcity of breakdowns and thus reduced the cost of defectives.

Quality of Man Power: Trained and Qualified person will increase the efficiency of the quality and also reduce production cost and waste.

Quality of Management: A good management is imperative for increase in efficiency, harmony in relations and growth of Business and Masters.

Quality of Material: Materials of good Quality will result in smooth processing there by reducing the waste and increasing output, it will also give better finish to end products

Causes of Variations

The variation in the Quality of manufactured products in the repetative process is inherent. These variations are broadly classified into two types

* Chance causes (or) Random Causes
* Assignable Causes

Chance Cause: Some stable pattern of variations is inherent in any Particular production results from many minor process that behave in Random manner. The variations due to this causes is beyond the control on human hand and cannot be prevented or eliminated in any circumstances. This variation is usually termed as allowable ~~variable~~ variation.

Assignable Cause: The second type of variation in any production process is due to non-random called Assignable cause and is termed as preventable cause.

USES OF SQC

The following are some of the benefits that result when a manufacturing process is operating in a state of statistical control.

Advantage of SQC is the control, maintainance and improvement in Quality standard.

* SQC provides protection against losses to the manufacture as well as consumer.

* It provides better quality assurance at lower inspection cost.

* Quality controls finds its application not only in the spere of production but also in other areas like packaging, scrap and spoilage, recoveries, advertising etc.

* The act of getting a process in statistical quality control involves the identification and elimination of assignable causes of variation and possibly the inclusion of good one. that is new materials or methods.

⇒ USE OF SQC IN INDUSTRIES:-

In industry one is faced with two kinds of problems.

* to check whether the process is confirming to standards laid down.

* To improve level of standard and reduce variability consistent with cost considerations.

SHEWHARTS control charts provides an answer to both the control chart is simple pictorial device for detecting unnatural patterns of variables in data resulting from repetation process that is control charts provide criteria for detecting lack of statistical control. They are simple to contruct & easy to interprit and tell us whether the sample points falls with in 3S control limits or not. Any sample point going outside the 3S control limits is an indication of lack of statistical control that is presence of some assignable causes of variation which must be traced identified and eliminated

Techniques of SQC

Techniques of SQC

- Process Control (Control charts)
  - Variables
    - $\bar{x}$
    - R
    - σ
  - Attributes
    - c
    - np
    - p
- Product Control
  - variables
    - Sampling techniques
  - Attributes

PROCESS CONTROL:-

The main objective in any production process is to control and maintain the quality of manufactured product. So that it satisfied quality specification it means the process about should be control in such a way that the number of defective products produces by the process is minimum this is known as process ~~is~~ minimum ~~this is known~~ and it can be obtained by developing and analysing control charts.

PRODUCT CONTROL :-

It aims at controlling the Quality of product and maintaining it at certain Quality level accepted by consumers regarding of what Quality level is being maintained by the produces thus process can be obtained by sampling inspection plans.

CONTROL CHARTS :-

A control chart is a device an graphical representation of the collected information it is statistical device mainly used for the study and control of a repetative process they are simple to construct and easy to interpret the construction is based on plotting 35 control limits and sequences of suitable sample statistic.

FOR EXAMPLE: Mean ($\bar{x}$), Range (R) and Standard deviation (s) etc.

Computed from a independent samples drawn at random from the production process the control charts consists of the following three horizontal lines.

* upper limit (UCL)
* Lower Control Limit (LCL)
* Control Limit (CL)

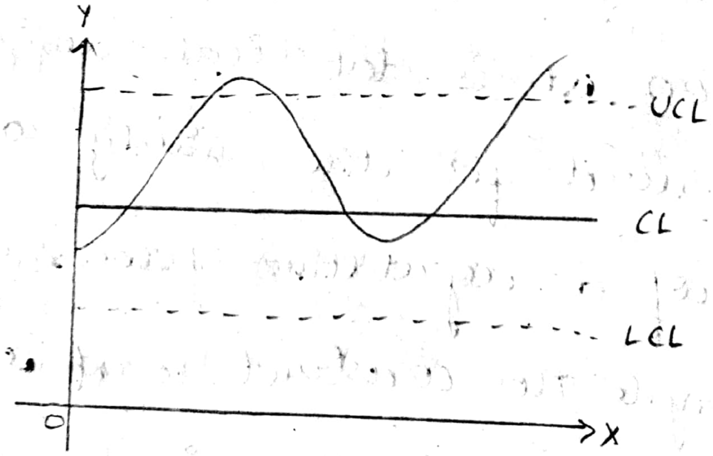
In the control chart UCL and LCL are plotted as dotted lines and CL is plotted as bold line

$$UCL = E(t) + 3\,SE(t)$$

$$CL = E(t)$$

$$LCL = E(t) - 3\,SE(t)$$

Where 't' is suitable test statistic



The sample points fall within the upper and lower control limit then the process is in statistical control. If any sample point falls outside the control limits then the process is out of control

3σ Limits

Let $x_1, x_2 \ldots x_n$ be the observations on a random sample of size n drawn from the given process with population parameter 'θ'. Let $T = t(x_1, x_2, \ldots x_n)$. Being the statistic computed from the sample with $E(t) = \mu_t$ variance $V(t) = \sigma_t^2$

If the statistic t is normally distributed then from the fundamental areas property of the Normal Distⁿ the total area within 0.9973 is 3σ limit.

$$P(\mu_t - 3\sigma_t < t < \mu_t + 3\sigma_t) = 0.9973$$

$$P(\mu_t - 3\sigma_t - \mu_t < t - \mu_t < \mu_t + 3\sigma_t - \mu_t) = 0.9973$$

$$P(-3\sigma_t < t - \mu_t < 3\sigma_t) = 0.9973$$

$$P[|t - \mu_t| < 3\sigma_t] = 0.9973$$

$$P[|t - \mu_t| > 3\sigma_t] = 1 - 0.9973$$

$$P(|t - \mu_t| > 3\sigma_t) = 0.0027$$

Hence the probability that a random value of statistic 't', computed from given sample distⁿ out side 3σ limit is too small.

$$E(t) \pm 3SE(t)$$

$= \mu_t \pm 3\sigma_t$ is with probability 1, therefore

$UCL = \mu_t + 3\sigma_t$

$LCL = \mu_t - 3\sigma_t$

$CL = \mu_t$

The computed value of statistic t line within 3σ limit the process is said to be in control. The computed value of statistic 't' lies outside 3σ limits. The process is said to be in out of control.

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CONTROL CHARTS FOR VARIABLES:-

controll charts for central tendency & variability are collectively called variable control charts the $\bar{x}$ charts is most widely use control charts for monitoring centrale tendency where as chards based on either the sample range. Sample standard deviation are used to control process variabilities.

CONTROL CHARTS FOR MEAN:- T

Step 1: Measurement

Any method of measurement in the production process has its own inherent variability errors in measurement can end into the data by following ways.

* Use of faculty instrument
* Lack of a clear cut definition of quality characteristics and method of measurements.

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* lack of experience in the handling or is of the instrument.

Step 2:- Selection of the samples.

In order to make control chart analysis effective it is essential to pay due regard to the rational selection of the sample.

Step 3:- Calculation of $\bar{X}$ and R for each subgroup.

Let k Independent Random sample of each is n. the sample means and ranges and standard deviation of each sample say.

$\bar{X}_1, \bar{X}_2 \ldots \bar{X}_K$, $R_1, R_2 \ldots R_K$ and $S_1, S_2 \ldots S_K$

respectively.

Now we find $\bar{\bar{X}}$ and $\bar{R}$ the averages of sample means and sample ranges and let $\bar{s}$ be the mean of standard deviation they are.

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_K}{}$$

$$\bar{\bar{x}} = \frac{1}{K} \sum_{i=1}^{K} \bar{x}_i$$

$$\bar{R} = \frac{1}{K} \sum_{i=1}^{K} R_i$$

$$\bar{S} = \frac{1}{K} \sum_{i=1}^{K} S_i$$

Step 4: Setting of control limits for $\bar{X}$ chart.

Case 1:- The process mean and standard deviation are known then $E(\bar{X}) = \mu$.

$V(\bar{X}) = \sigma^2/n$

$S E(\bar{X}) = \sigma/\sqrt{n}$. the control limits for $\bar{X}$ charts are given by $E(\bar{X}) \pm 3.S.E(\bar{X})$

$$\Rightarrow \mu \pm 3\sigma/\sqrt{n}$$

therefore $UCL = \mu + 3\sigma/\sqrt{n} = \mu + A\sigma$

$LCL = \mu - 3\sigma/\sqrt{n} = \mu - A\sigma$

$CL = \mu$

Case 2: When μ and σ are unknown

We know that $E(R) = d_2\sigma$

$\bar{R} = d_2\sigma \rightarrow \sigma = \bar{R}/d_2$

The 3σ control limits are $UCL = \mu + \frac{3}{\sqrt{n}} (\bar{R}/d_2)$

$$UCL = \mu + A_2 \bar{R}$$

$$A_2 = \frac{3}{\sqrt{n}\, d_2}$$

$LCL = \mu - A_2 \bar{R}$

$CL = \mu$

Since d_2 is constant depending on 'R' $A_2 = 3/d_2\sqrt{n}$ also depends only on n and its values have been computed and tabulated for different values of n from 2 to 25 and are given in the table.

Step 5: Construction of control chart for ($\bar{x}$)

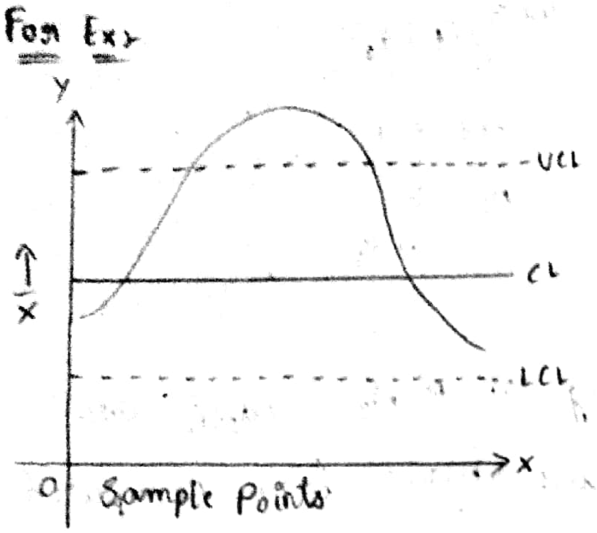
Control chart for mean is plotted on graph on x-axis sample points are plotted and statistical measures on $\bar{x}$ and y axis. The central line is drawn as bold horizontal line. LCL and UCL are plotted as dotted horizontal lines based on computed values.

Step 6: Interpretation

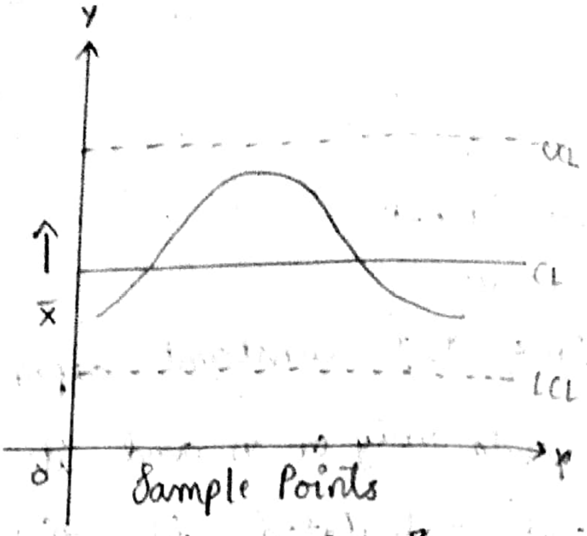
If all the points are in between UCL and LCL then the process is said to be in statistical control.

If any one of or more points lie outside the control limits is concluded as process is out of statistical control which indicates the presence of assignable causes in the process.

For Ex:-



The process is out of control



The process is in control

CONTROL CHART FOR R CHART (Range)

Step-1, Step-2, Step-3 are same as above

Step 4: Setting Of Control Limits

Case:1 When μ and σ are known

From Sampling distn of R, Sample Range we have

$E(R) \pm 3 \cdot SE(R)$

$$E(R) = d_2\sigma$$

$$SE(R) = d_3\sigma$$

Here the values of d_2 & d_3 are available in tables from 2 to 20.

When the Range R is statistic under study then 3σ control limits are given by

$$E(R) \pm 3 S.E(R)$$

$$d_2\sigma \pm 3d_3\sigma$$

$$= (d_2 \pm 3d_3)\sigma$$

$$\therefore UCL = (d_2 + 3d_3)\sigma = D_2\sigma$$

$$LCL = (d_2 - 3d_3)\sigma = D_1\sigma$$

$$CL = d_2\sigma$$

Case 2: When μ and σ are unknown

WKT, $\bar{R} = d_2\sigma \rightarrow \sigma = \bar{R}/d_2$, then $UCL = (d_2 + 3d_3)\bar{R}/d_2$

$= D_4\bar{R}$

$\left[\because D_4 = \frac{d_2+3d_3}{d_2}\right]$

$LCL = (d_2 - 3d_3)\bar{R}/d_2$

$= D_3\bar{R}$ $\quad \because D_3 = \frac{d_2 - 3d_3}{d_2}$

$CL = \frac{d_2\bar{R}}{d_2} = \bar{R}$

Where D_1, D_2, D_3, D_4 are constants depends on sample size 'n'.

Step 5: Construction of R-Chart

Sample numbers are taken on x-axis, and sample ranges on Y-axis then CL, UCL, LCL plotted on 3-horizontal lines parallel to x-axis.

Step 6: Interpretation of R-Chart

If all the plotted points lie b/w 3σ control limits, the process is in control. If any one or more points lies outside the 3σ control limits is concluded as process is out of control, which indicates the presence of assignable causes in the process.

If the sample size 7 or more that is $n \geq 7$ LCL is drawn has dotted horizontal line otherwise if $n < 7$ LCL is taken as zero.

Prattical - 1

Construct a control chart for mean and the range for the following data on the basis of fuses, samples of 5 being taken every hour (each set of 5 has been arranged in ascending order of magnitude). Comment whether the production seems to be under control, Assuming that there are the first to.

42	42	19	36	42	51	60	18	15	69	64	61
65	45	24	54	51	74	60	20	30	109	90	98
75	68	80	69	57	75	72	27	39	113	93	94
78	72	81	77	59	78	95	42	62	118	109	109
87	90	81	84	78	132	138	60	84	153	112	136

Aim: We have to find control chart for mean & range

Procedure: for $\bar{x}$- chart

$UCL = \bar{\bar{x}} + A_2 \bar{R}$

$LCL = \bar{\bar{x}} - A_2 \bar{R}$

$CL = \bar{\bar{x}}$

For $\bar{R}$ chart

$UCL = D_4 \bar{R}$

$LCL = D_3 \bar{R} = 0$

$CL = \bar{R}$

• A manufacturer wish to test the life of cells conform to standards. The data of life of cells are given below in the table. Draw $\bar{x}$, R and S charts and state whether the process is under control.

Days (Subgroups)	Life of Cells (in hours)					Mean	Range
	1	2	3	4	5		
1	27	28	25.5	26.5	23	26	5
2	23.5	27.5	26	27	29	26.6	5.5
3	27.5	27	28	26.5	24.5	26.7	3.5
4	26	26.5	27.5	28.5	27	27.5	2
5	27.5	24.5	25	26.0	27.5	26.1	3
6	26.5	26	27	27.5	26	26.6	1.5
7	21	22	28	26.5	25	24.5	7
8	25.5	24.5	25	27.5	27.5	26	3
9	28	26.5	30	29.5	27	28.2	3.5
10	25	27	26.5	24.5	23	25.2	4
11	22	26.5	27.5	23.5	25.5	25	5.5
12	26	28	27	30	29	28	4
						$\bar{x} = 316.4$	R = 47.5

$A_2 = 0.58$, ~~n=12~~ $n=5, K=12$

$\bar{X}$-Chart

$UCL = \bar{\bar{X}} + A_2\bar{R}$

$\bar{\bar{X}} = 316.4/12 = 26.3666$

$\bar{R} = 47.5/12 = 3.9583$

$UCL = (26.3666) + (0.58)(3.9583)$

$\boxed{UCL = 28.6624}$

$LCL = \bar{\bar{X}} - A_2(\bar{R})$

$= (26.3666) - (0.58)(3.9583)$

$\boxed{LCL = 24.0707}$

$\boxed{CL = 26.3666}$

R-Chart

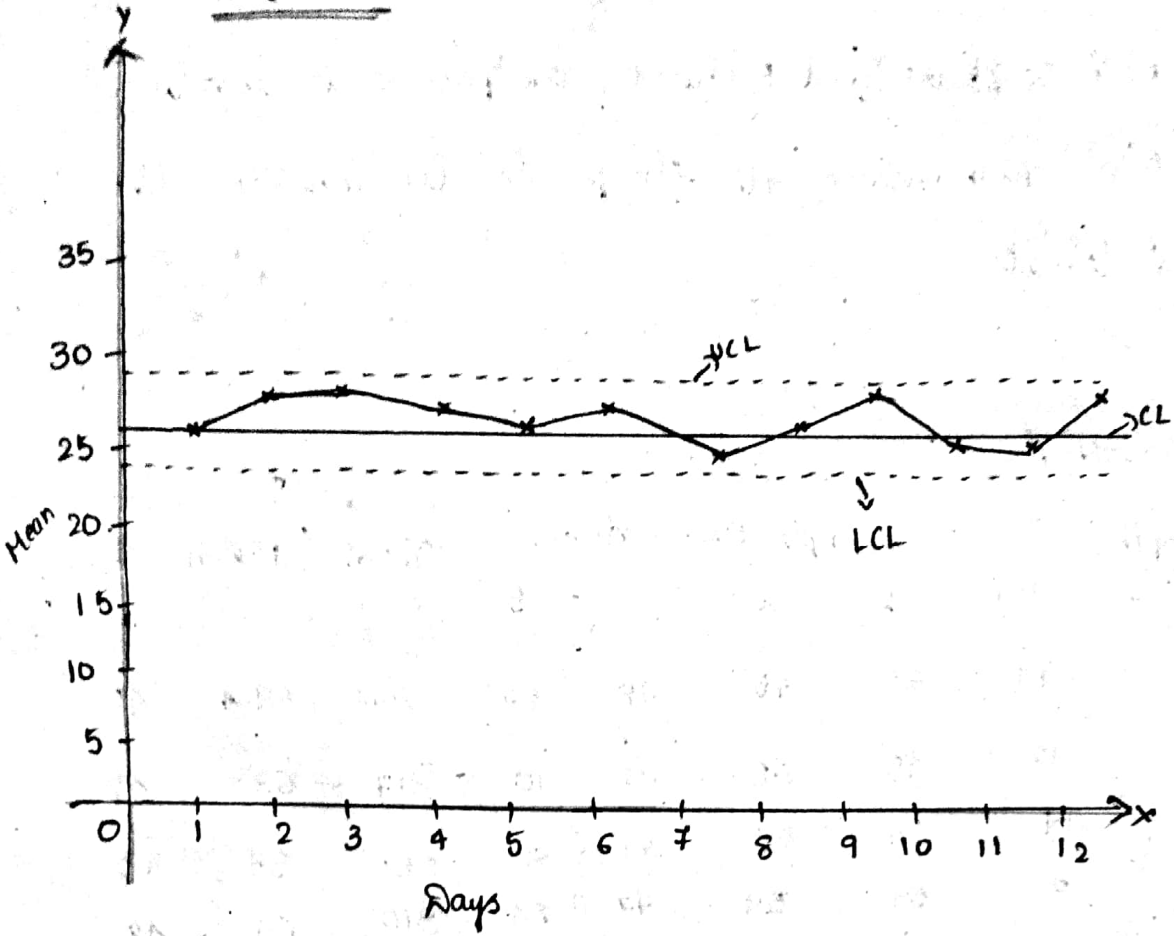
$UCL = D_4\bar{R} = 2.11 \times (3.9583)$

$\boxed{UCL = 8.3520}$

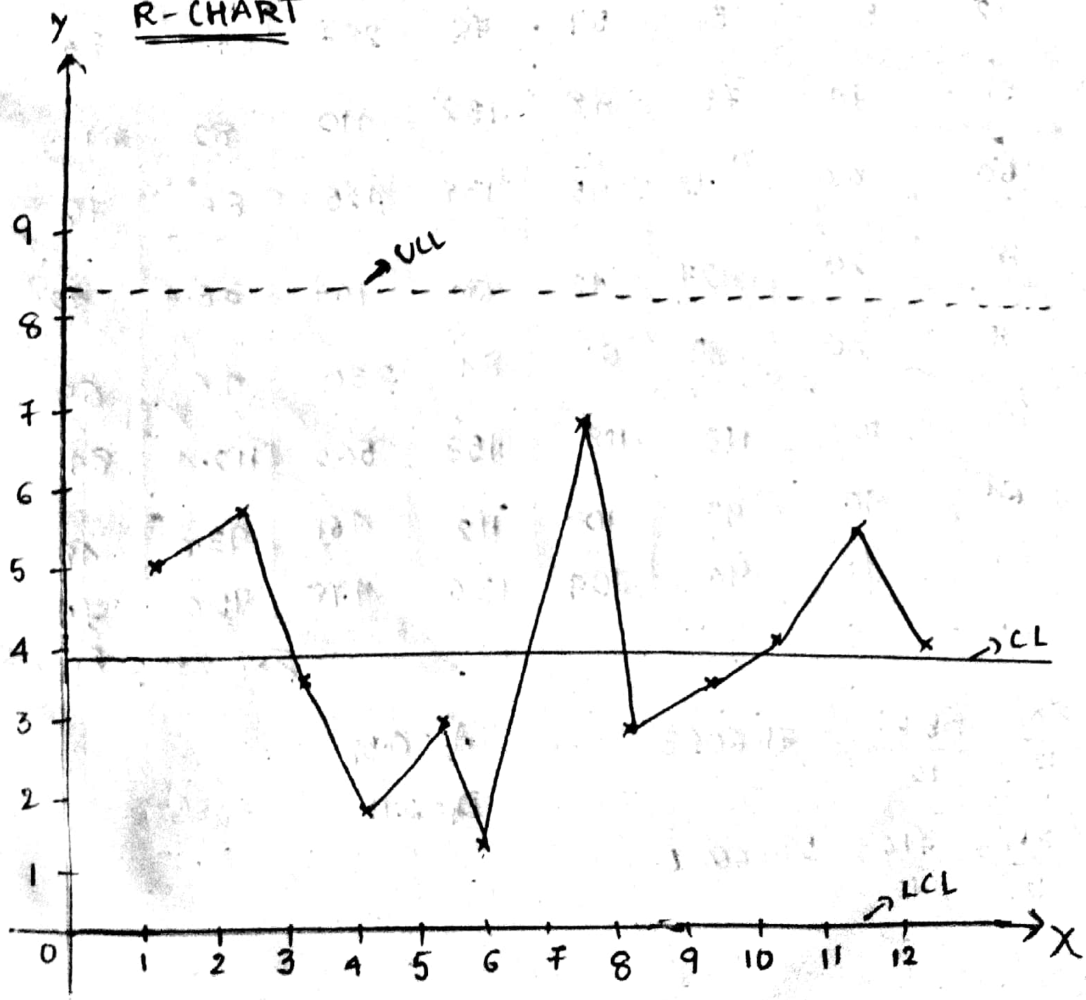
$\boxed{LCL = 0.}$ $(\because n < 7)$

$\boxed{CL = 3.9583}$

$\bar{X}$-CHART



$\bar{R}$-CHART



Result:

For both $\bar{x}$ chart and R chart, the process is under statistical control since all the points lie inside the control limits.

PCP-1

Calculation:

Sample No.	Sample Observation 1	2	3	4	5	Total	Mean	Range
1	42	65	75	78	87	347	69.4	45
2	42	45	68	72	90	317	63.4	48
3	19	24	80	81	81	285	57	62
4	36	54	59	77	84	310	64	48
5	42	51	57	59	78	287	57.4	36
6	51	74	75	78	132	410	82	81
7	60	60	72	95	138	425	85	78
8	18	20	27	42	60	167	33.4	42
9	15	30	39	62	84	230	46	69
10	69	109	113	118	153	562	112.4	84
11	64	90	93	109	112	468	93.6	48
12	61	78	94	109	136	478	95.6	75

$\Sigma x_i = 859.3$ $R_i = 716$

$$\bar{\bar{x}} = \frac{\Sigma x_i}{12} = \frac{859.3}{12} = 71.6083$$

$A_2 = 0.58$

$D_4 = 2.11$

$$\bar{R} = \frac{\Sigma R_i}{12} = \frac{716}{12} = 59.6666$$

$\bar{x}$ - Chart

$UCL = \bar{\bar{x}} + A_2\bar{R} = 71.6083 + 0.58(59.6666)$

$= 71.6083 + 34.6066 = 106.2149$

$LCL = 71.6068 - 34.6066$

$= 37.0017$

$CL = 71.6083$

For $\bar{R}$ Chart

$UCL = D_4\bar{R}$

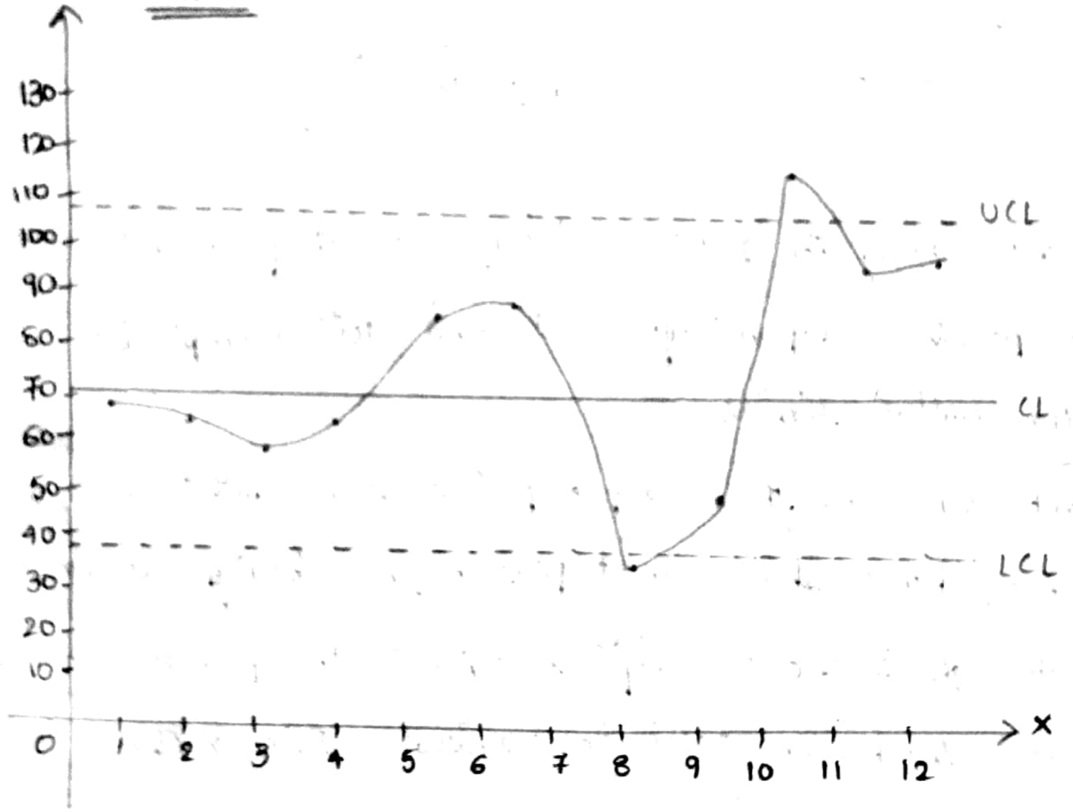
$= 2.11(59.6661)$

$= 105.8965$

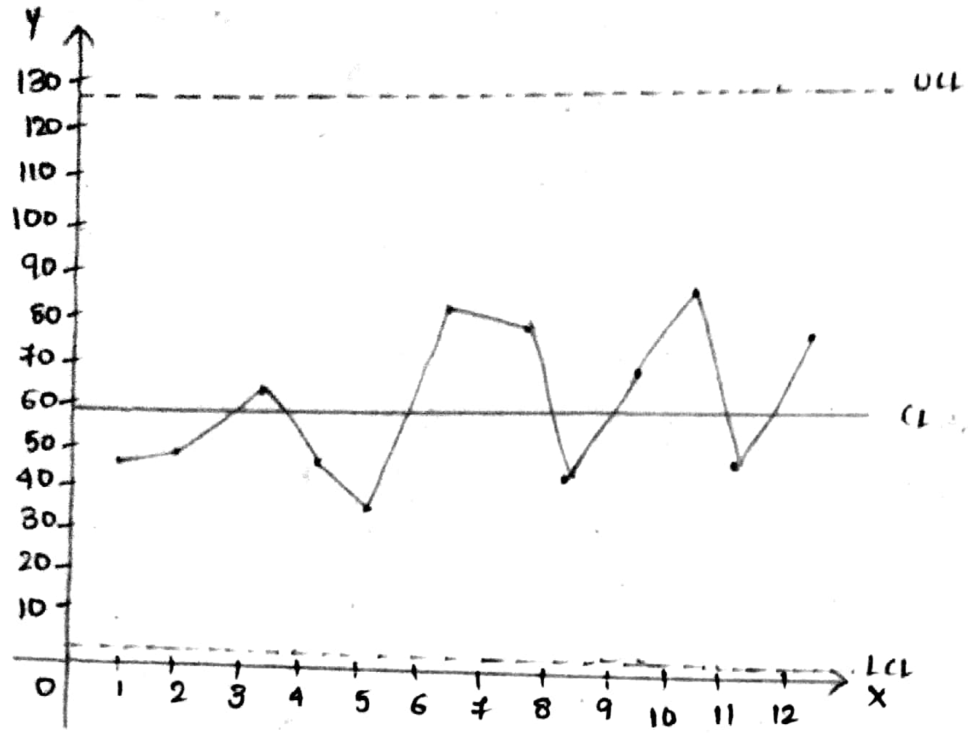
$LCL = 0$

$CL = 59.6666$

$\bar{X}$-Chart



R-Chart



Result:

For $\bar{x}$-Chart clearly the process average is out of control since the points corresponding to 8th and 10th sample lies outside the control limits.

In R-Chart since all the sample points fall within control limit this chart shows that process variability is in control although R-chart depects control the process cannot be regarded to be in statistical control since $\bar{x}$-Chart shows lack of control.

• Draw $\bar{x}$ and R- Charts for the given data.

$\bar{x}$- Chart

$UCL = \bar{\bar{x}} + A_2\bar{R}$

$LCL = \bar{\bar{x}} - A_2\bar{R}$

$CL = \bar{\bar{x}}$

R-Chart

$UCL = D_4\bar{R}$

$LCL = D_3\bar{R}$

$CL = \bar{R}$

n= no. of sample observations

k= samples

$n=6,\ k=15$

Sample No	Sample Observation x_1	x_2	x_3	x_4	x_5	x_6	Mean	Range
1	37.1	22.8	27.3	24.9	26.7	26.7	27.9167	14.3
2	20.3	28.4	26.7	21.8	24.5	27.9	24.9333	8.1
3	26.3	20.8	36.7	35.4	27.3	25.7	28.7	15.9
4	27.8	26.3	24.9	20.8	23.7	29.7	25.5333	8.9
5	19.6	20.9	23.1	24.5	26.7	26.9	23.95	9.1
6	28.2	20.7	25.8	26.3	28.6	31.4	26.83	10.7
7	34.6	32.1	31.7	29.6	26.9	29.7	30.26	9.9
8	32.4	29.7	26.3	23.9	28.7	29.3	27.98	5.8
9	34.1	38.6	26.3	22.9	28.6	29.3	29.14	15.7
10	24.6	41.2	38.4	35.1	34.3	32.7	36.34	8.5
11	42.1	36.9	39.8	24	25.2	24.3	28.45	39.7
12	41.7	36.8	32.7	28.4	23.6	27.2	29.74	13.2
13	39.3	37.8	40.7	43.1	38.1	36.4	39.24	6.8
14	27.4	26.9	31.8	22.9	41.2	42.8	32.1667	19.9
15	36.8	36.5	43.7	41.9	37.6	38.4	39.4833	6.9

Calculation:

$A_2 = 0.483$ (table value)

$D_4 = 2.004$

$\bar{X} = \Sigma(\text{Means}) = 450.673$

$R = \Sigma(\text{ranges}) = 193.4$

$$\bar{\bar{X}} = \frac{450.673}{15} = 30.04487$$

$\bar{R} = 193.4/15 = 12.8933$

$\bar{X}$- Chart

$UCL = (30.04487) + 0.483\,(12.8933)$

$UCL = 36.2723$

$LCL = (30.04487) - 0.483\,(12.8933)$

$= 23.81739$

$CL = 30.04487$

R- Chart

$UCL = (2.004).(12.8933) = 25.8382$

$LCL = 0$

$CL = 12.8933$

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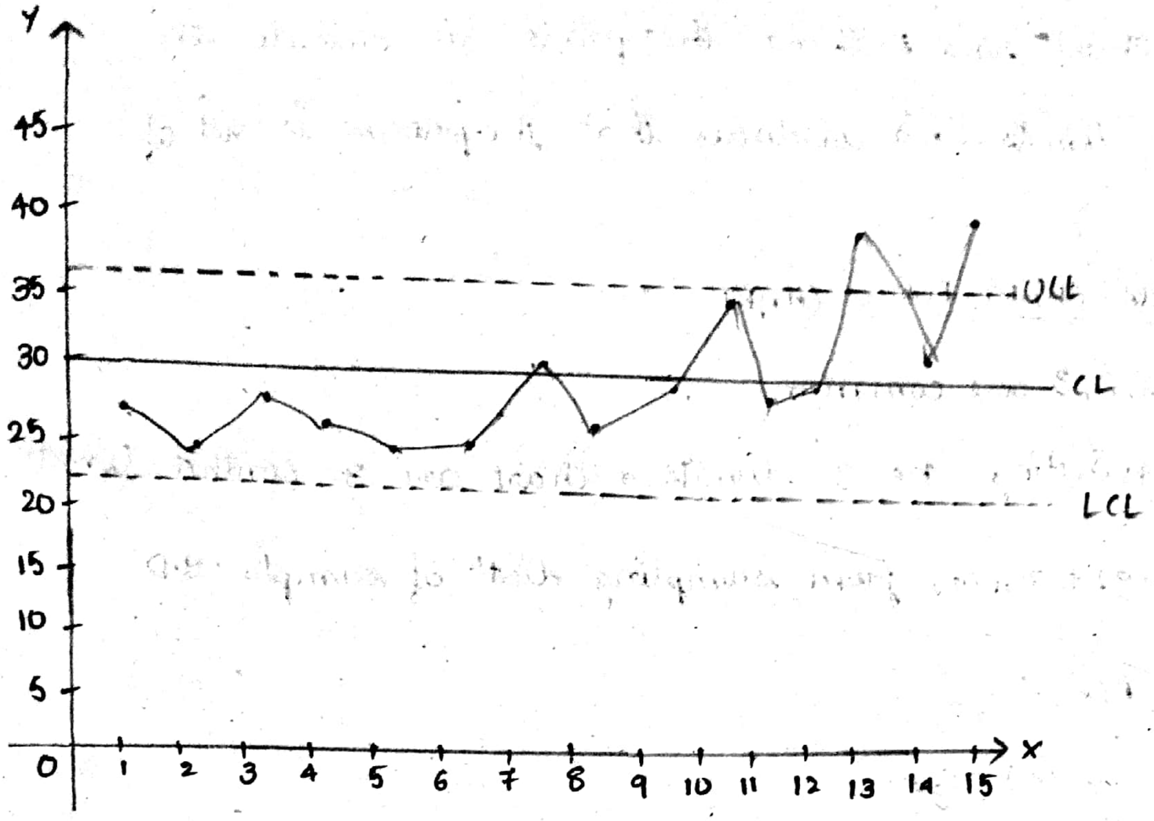
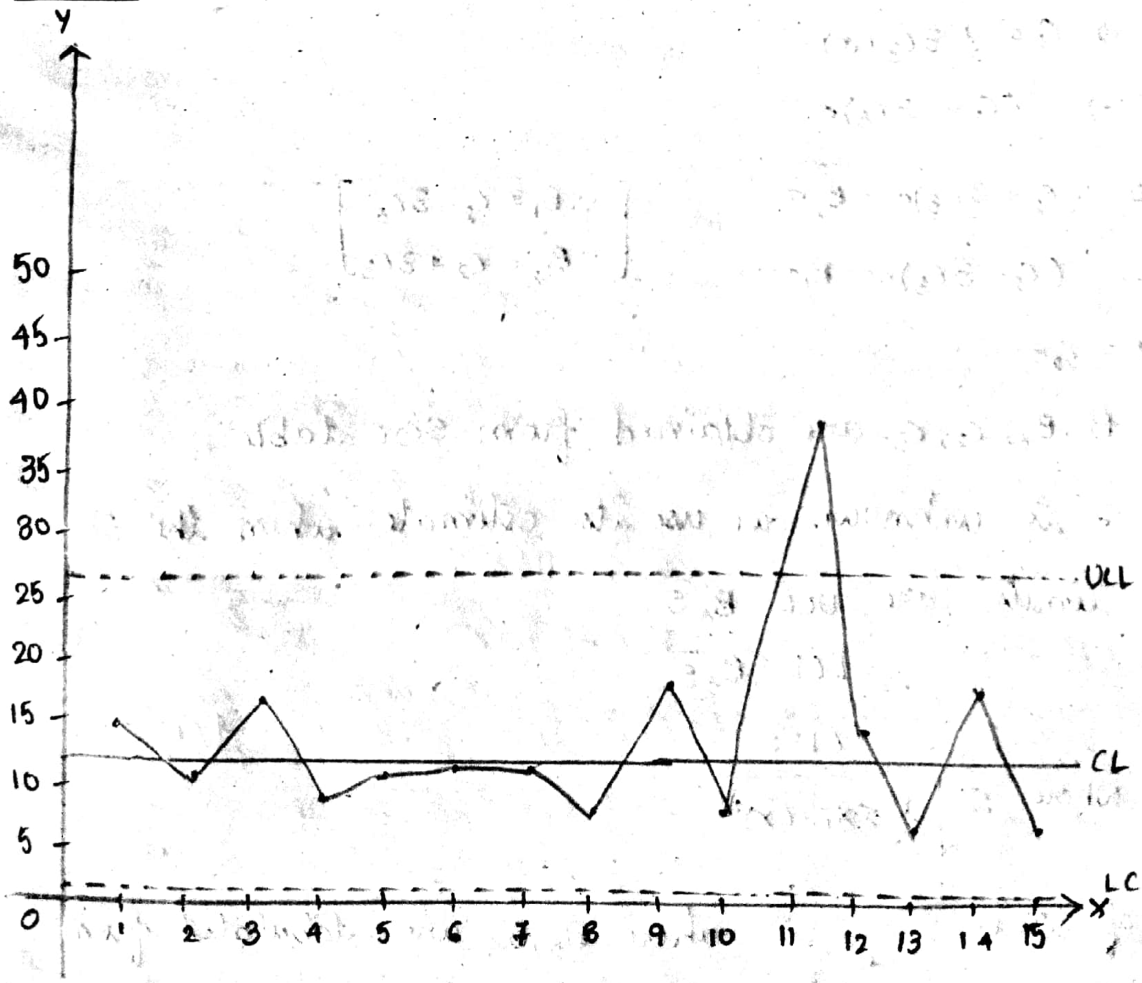
$\bar{x}$-Chart



R-Chart



Result:

In $\bar{x}$ Chart and R-Chart some the points lie outside the control limits. This concludes that the process is out of control.

CONTROL CHART FOR σ-CHART

Step-1, 2, 3 are common

Step-4: Setting the 3σ limits σ chart are 3σ control limits for $E(\sigma) \pm 3SE(\sigma)$ from sampling dist^n of sample S.D

WKT $E(\sigma) = C_2\sigma$

$SE(\sigma) = C_3\sigma$

$\therefore$ the 3σ control limits are

$E(\sigma) \pm 3S.E(\sigma)$

$\Rightarrow C_2\sigma \pm 3C_3(\sigma)$

$\Rightarrow (C_2 \pm 3C_3)\sigma$

$UCL = (C_2 + 3C_3)\sigma = B_2\sigma$

$LCL = (C_2 - 3C_3)\sigma = B_1\sigma$

$\left[\because B_1 = C_2 - 3C_3 \quad B_2 = C_2 + 3C_3\right]$

$CL = C_2\sigma$

Where B_1, B_2, C_2, C_3 are obtained from SQC table

Where σ is unknown we use its estimate when the 3σ control limits are $UCL = B_4\bar{S}$

$LCL = B_3\bar{S}$

$CL = \bar{S}$

Where $S = \sqrt{\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2}$

Mean of SD is $\bar{S} = \frac{\Sigma S_i}{K}$ where B_3, B_4 are tabulated from SQC tables.

Problem

For the given data you need to construct $\bar{x}$, R and σ chart and comment the nature of production.

S.NO	x_1	x_2	x_3	x_4	Mean	Range	SD
1	1081	363	1092	1385	980.25	1022	376.6
2	528	330	1053	945	714	723	295.9
3	964	1384	1194	456	1004.5	928	346.8
4	728	972	647	792	784.75	325	119.7
5	804	845	1132	1024	951.25	328	133.2
6	1002	80	760	1035	719.25	955	384
7	994	1023	1136	842	998.75	294	104.9
8	616	832	497	692	659.25	335	121.6
9	982	1342	1132	945	1100.25	397	156.2
10	1132	998	454	777	840.25	678	256.5
11	1134	1140	756	994	1006	384	155.7
12	749	948	1050	857	901	301	111.2

$\Sigma x_i = 10659$ $\Sigma R = 6670$ $\Sigma S_i = 2562.3$

σ-Chart

$B_4 = 2.266$

$B_3 = 0$

$\bar{S} = \frac{\Sigma S_i}{K} = 213.525$

$UCL = (2.266)(213.525)$

$UCL = 483.84$

$LCL = 0$

$CL = 213.325$

$\bar{x}$-Chart

$\bar{\bar{x}} = 888.25$

$A_2 = 0.729$

$UCL = 1293.4525$

$LCL = 483.0475$

$CL = 888.25$

R-Chart

$\bar{R} = 555.8333$

$D_4 = 2.013$

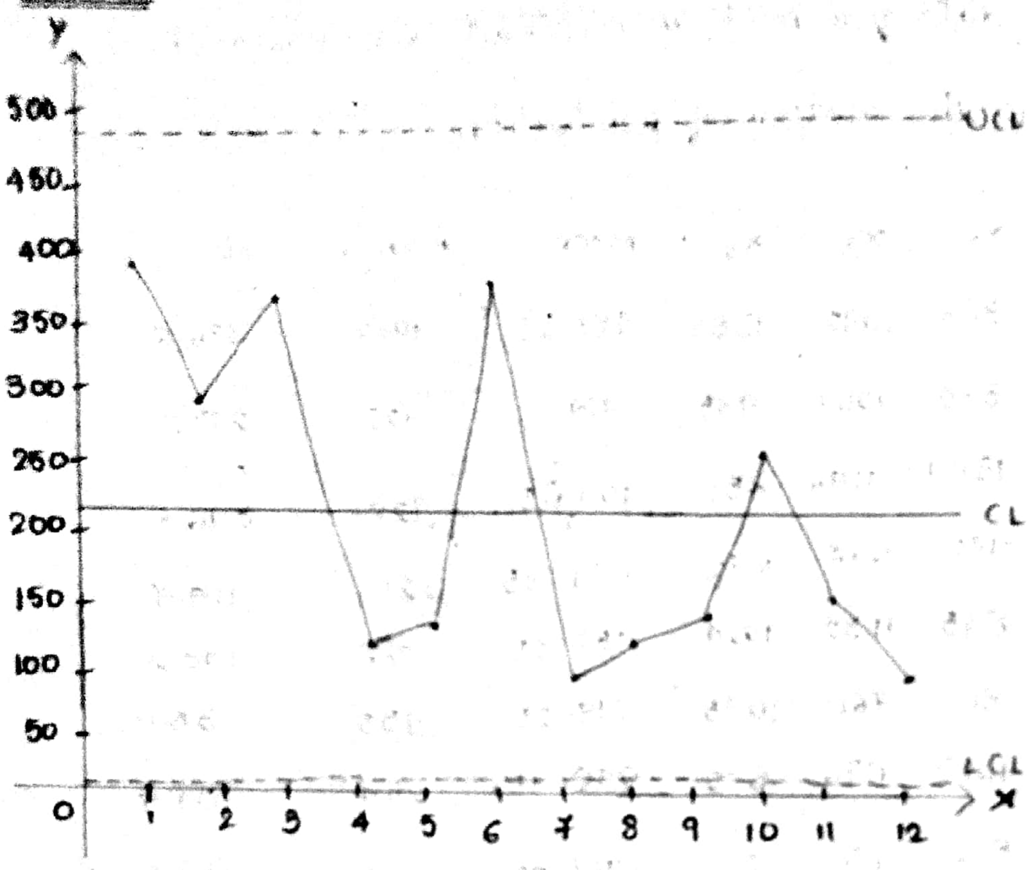
$UCL = D_4 \bar{R}$

$UCL = 1118.8924$

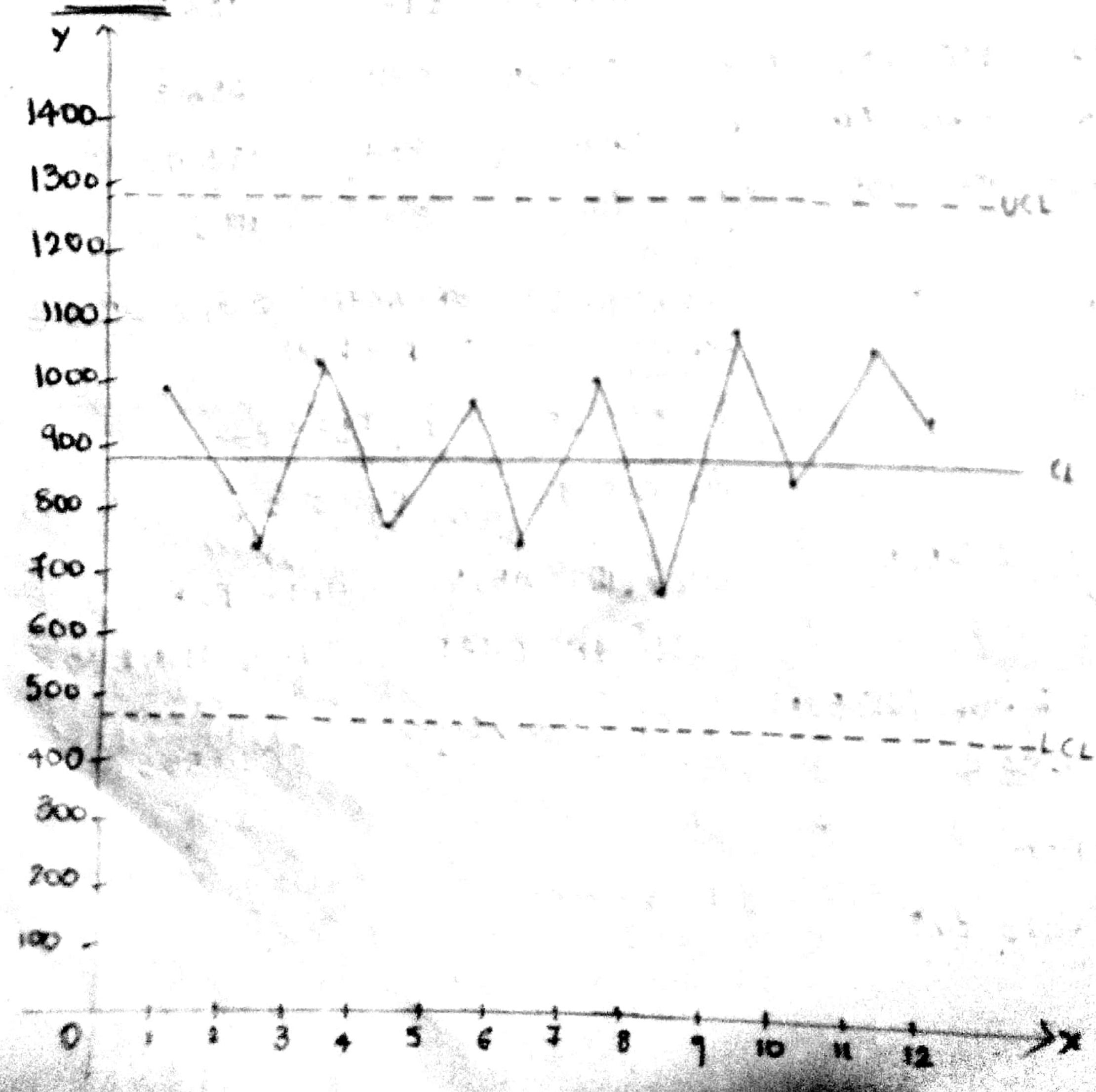
$LCL = 0$

$CL = 555.8333$

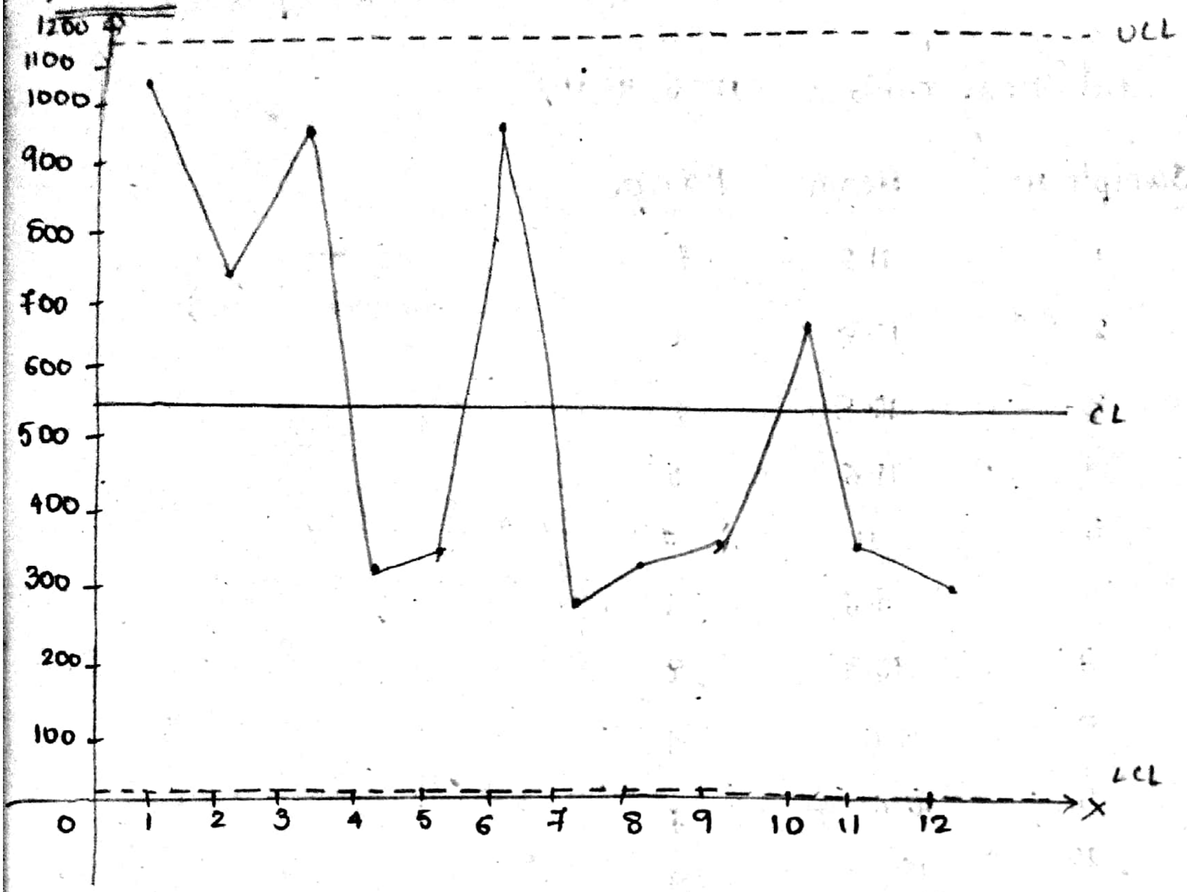
σ- Chart



$\bar{X}$- Chart



R-Chart



Result:

In $\bar{x}$-Chart, R-Chart and σ-Chart all the points lie within the control limits. So, the process is under statistical control.

• Construct $\bar{x}$ Chart, R-Chart and check whether process is under statistical control. ($n=5, K=10$)

Sample no.	Mean	Range
1	11.2	7
2	11.8	4
3	10.8	8
4	11.6	5
5	11	7
6	9.6	4
7	10.4	8
8	9.6	4
9	10.6	7
10	10	9
	106.6	63

$\bar{X}$-chart

$\bar{\bar{X}} = \frac{106.6}{10} = 10.66$

$\bar{R} = 6.3$

$UCL = \bar{\bar{X}} + A_2\bar{R}$

$A_2 = \frac{3}{\sqrt{n}.d_2} = \cancel{0.4049}$ 0.5768

$\rightarrow UCL = 14.2938$

$\rightarrow LCL = 7.0262$

$\rightarrow CL = 10.66$

R-Chart

$d_2 = 2.326$

$d_3 = 0.864$

$A_2 = \frac{3}{\sqrt{n}.d_2}$, $D_4 = \frac{d_2 + 3d_3}{d_2}$

$D_3 = \frac{d_2 - 3d_3}{d_2}$

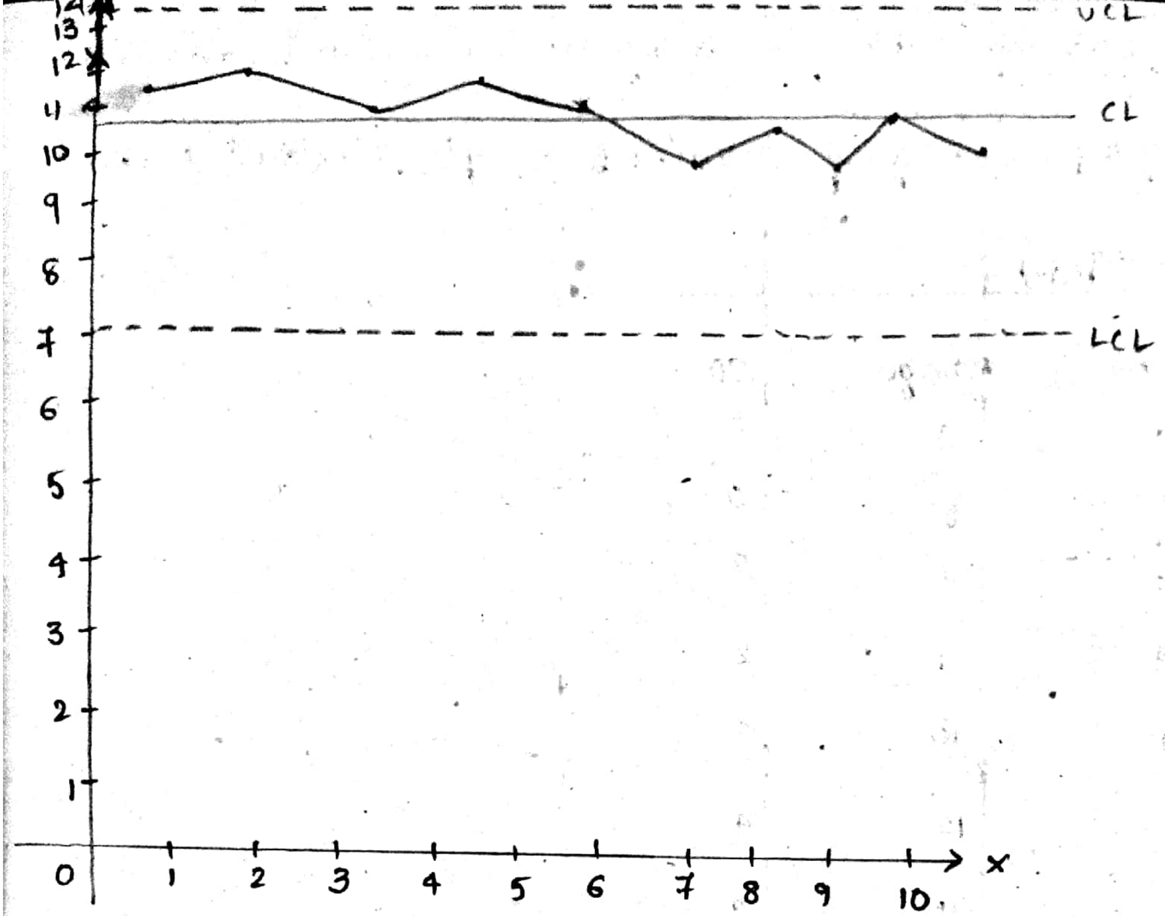
$D_4 = 2.1144$

$D_3 = -0.1144$

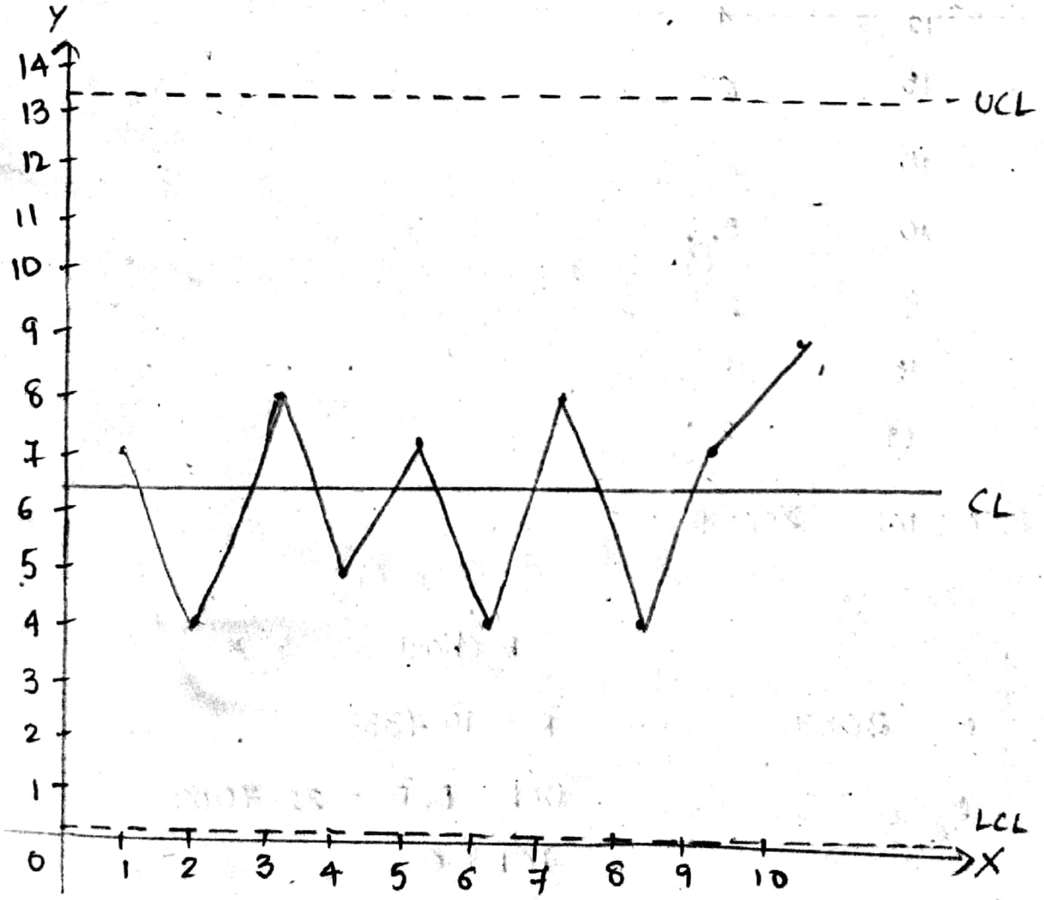
$\rightarrow UCL = D_4\bar{R} = 13.3207$

$\rightarrow LCL = D_3\bar{R} = -0.7205 = 0$

$\rightarrow CL = 6.3$


UCL
CL
LCL
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
X

R-Chart


Y
UCL
CL
LCL
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
X

• The following table gives Range and Standard deviation of 15 groups each group contains 5 samples. Construct R-Chart and σ-Chart

S. No	Range	SD
1	8	2
2	5	3
3	7	2
4	9	2
5	10	3
6	12	4
7	15	4
8	12	3
9	12	4
10	15	6
11	10	2
12	10	3
13	9	2
14	15	4
15	12	2
	$\Sigma R = 161$	$\Sigma \sigma_i = 46$

Calculation:

$n = 5,\ k = 15$

$D_4 = 2.115$ $\quad B_4 = 2.089$

$D_3 = 0$ $\quad B_3 = 0$

R-Chart

$\bar{R} = 10.7333$

$UCL = D_4\bar{R} = 22.7009$

$LCL = 0$

$CL = 10.7333$

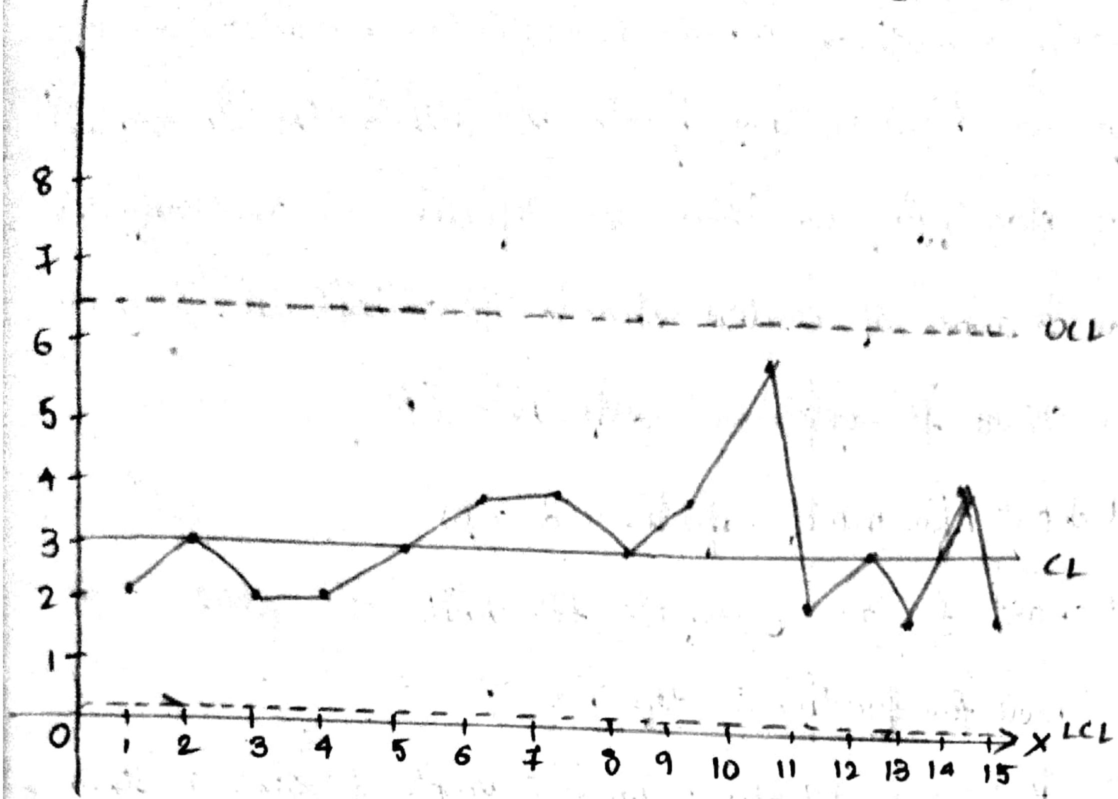
σ-chart → $\bar{S} = \frac{46}{15} = 3.0667$

$UCL = B_4\bar{S} = (2.089) \times (3.0667) = 6.4063$

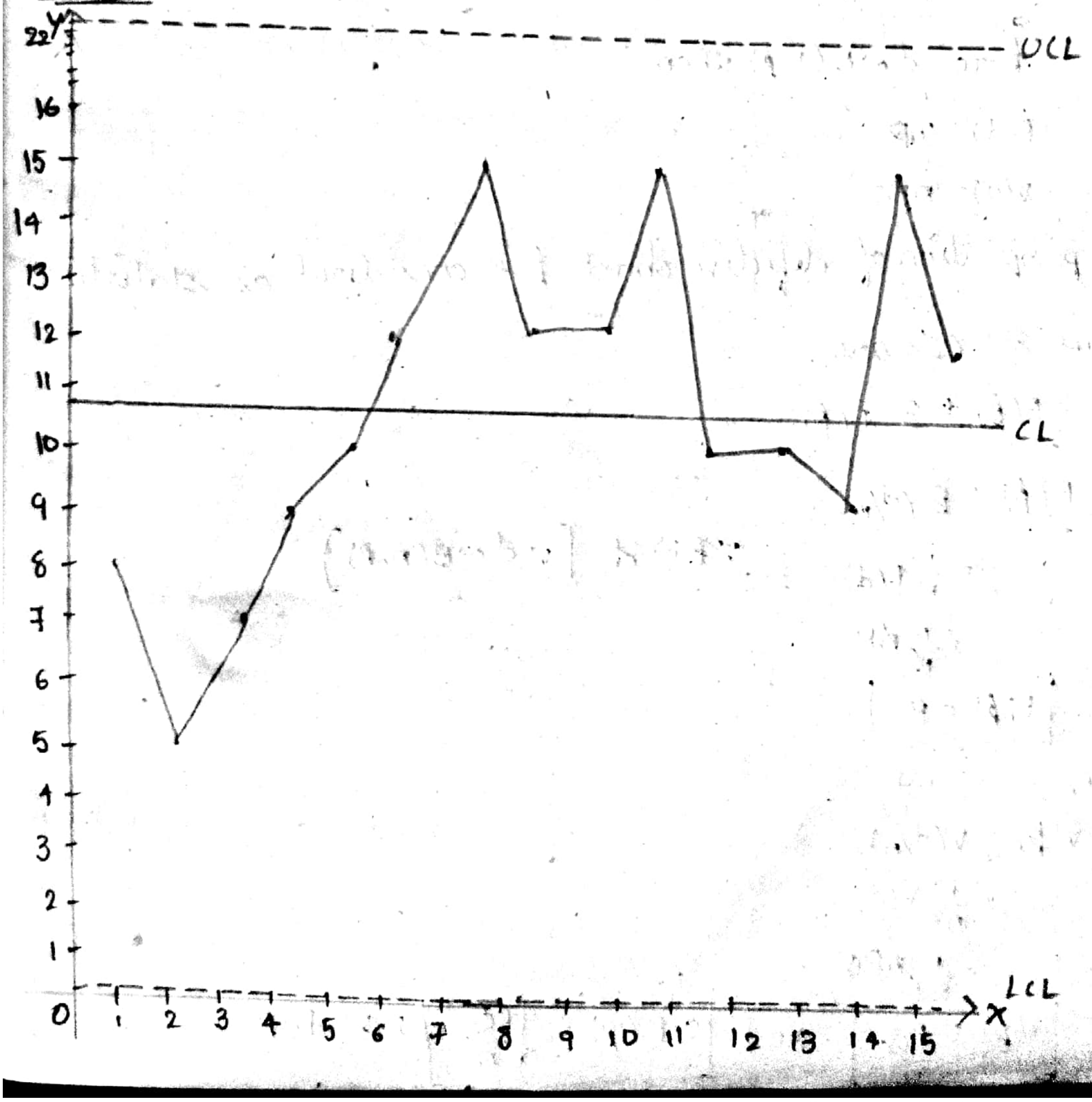
$LCL = 0$

$CL = 3.0667$

σ-Chart



R-Chart



Control charts for attributes

In certain situations where measurable characteristics are too large we have control charts for attributes. In control charts for classifying an item as defective or non-defective. There are 3 types of control charts for attributes. They are,

* Control chart for fractional defectives (p)
* Control chart for no. of defectives $(d = np)$
* Control chart for no. of defects for unit (c)

Control Chart for fractional defectives (p)

Let d be the no. of defectives in a sample of size n then proportion of defectives $p = d/n$

here $d \sim B(n,p)$ then,

$E(d) = np$

$V(d) = npq$

The proportion of defective chart p is considered as statistic then 3σ CL's are,

$$E(p) \pm 3\,SE(p)$$

Let $E(p) = E(d/n)$

$= \frac{1}{n} E(d)$ $\quad [\because d \sim B(n,p)]$

$= \frac{1}{\cancel{n}} \cancel{n} p$

$$\boxed{E(p) = p}$$

Illy,

$V(p) = V(d/n)$

$= \frac{1}{n^2} V(d)$

$= \frac{1}{n^{\cancel{2}}} \cancel{n} pq$

$\boxed{V(p) = \frac{pq}{n}}$ and $\boxed{S.E(p) = \sqrt{\frac{pq}{n}}}$; $q = 1 - p$

∴ The 3σ control limits are

$$P \pm 3\sqrt{\frac{PQ}{n}}$$

Case 1: When P & Q are known then the 3σ CL's are,

$$P' \pm 3\sqrt{\frac{P'Q'}{n}}$$

i.e,

$$UCL = P' + 3\sqrt{\frac{P'Q'}{n}}$$

$$LCL = P' - 3\sqrt{\frac{P'Q'}{n}}$$

$$CL = P'$$

Case 2: When P & Q are unknown then the 3σ control limits are,

$$p_i = d_i/n$$

$$\bar{p} = \frac{\Sigma p_i}{k}$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$$

$$CL = \bar{p}$$

Case 3: Variable sample size

Let d_i be the no of defectives in a sample of size n_i then proportion of defective,

$$p_i = \frac{d_i}{n_i}$$

$$\bar{p} = \frac{\Sigma d_i}{\Sigma n_i} \quad ; \bar{q} = 1 - \bar{p}$$

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3σ control limits for variable sample size is,

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}$$

$$CL = \bar{p}$$

Construction of p Chart

In P chart, we take the sample no. on X-axis and fractional defectives on Y-axis. The UCL & LCL are plotted lines and CL is plotted as bold line.

Interpretation of p-chart

In P-chart, if all the sample points fall within the control limits, the process is under statistical control. Otherwise, it is said to be out of control.

Problems

1. The following are the figures of defectives in 22 lots each containing 2000 rubber balls.

425, 430, 216, 341, 225, 322, 280, 306, 337, 305, 356, 402

216, 264, 126, 409, 193, 326, 280, 389, 451, 420

Draw control charts for fractional defectives and comment on the state of control of the process.

Formulae: $p_i = d_i/n$ here

$\bar{p} = \Sigma p_i / k$ $k = 22$

$\bar{q} = 1 - \bar{p}$ $n = 2000$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$$

$$CL = \bar{p}$$

Calculation:

lot no.	d_i	$p_i = d_i/n$
1	425	0.2125
2	430	0.2150
3	216	0.1080
4	341	0.1705
5	225	0.1125
6	322	0.1610
7	280	0.1400
8	306	0.1530
9	337	0.1685
10	305	0.1525
11	356	0.1780
12	402	0.2010
13	216	0.1080
14	264	0.1320
15	126	0.0630
16	409	0.2045
17	193	0.0965
18	326	0.1630
19	280	0.1400
20	389	0.1945
21	451	0.2255
22	420	0.2100

$\Sigma p_i = \cancel{3.4995}\ 3.5095$

$$\bar{p} = \Sigma p_i / k = \frac{3.5095}{22} = 0.1595$$

$\bar{p} = 0.1595$

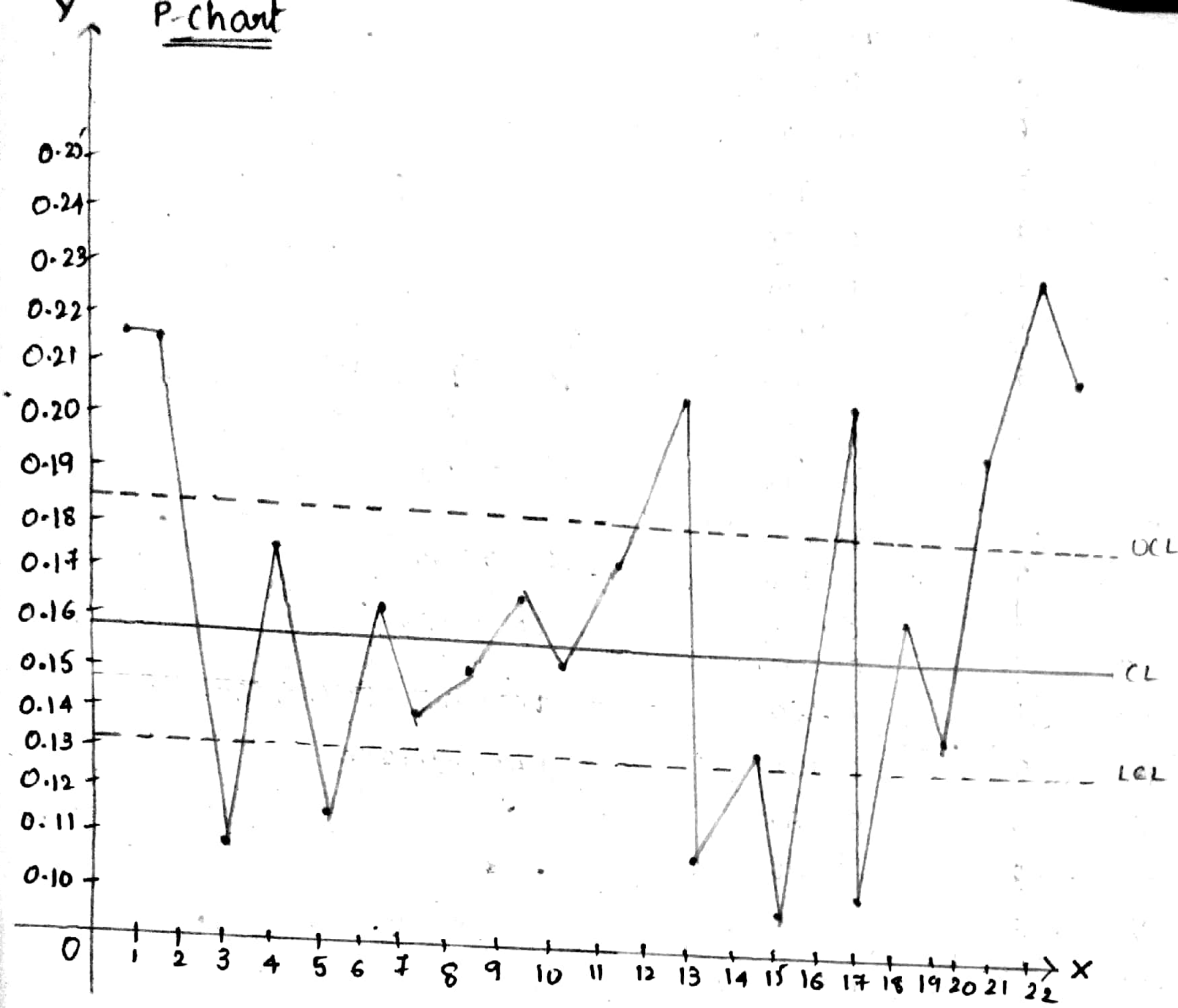
$\bar{q} = 0.8405$

UCL = ~~0.1636~~ ~~0.1481~~ 0.1841

LCL = 0.1349

Cl = 0.1595

P-Chart



Result: The process is not under statistical control.

2. Everyday a sample of items for production process was examining the no. of defectives found in each process was as follows. Construct fractional defectives control chart and comment on it. Here sample size $n=50$.

S.NO	d_i	$p_i = d_i/n$
1	1	0.0200
2	6	0.1200
3	5	0.1000
4	4	0.0800
5	2	0.0400
6	5	0.1000
7	3	0.0600
8	6	0.1200
9	1	0.0200
10	11	0.2200
11	5	0.1000
12	2	0.0400
13	4	0.0800
14	1	0.0200
15	5	0.1000
16	4	0.0800
17	2	0.0400
18	3	0.0600
19	1	0.0200
20	5	0.1000

$\Sigma p_i = 1.52$

$$\bar{p} = \frac{\Sigma p_i}{k} = \frac{1.52}{20} = 0.0760$$

$\bar{q} = 0.9240$

~~UCL = 0.5003~~

UCL = 0.1884

LCL = 0 , CL = 0.0760

~~CL = 0.07266~~

3. From the following inspection results, construct 3σ control chart for P-Chart ($n = 1000$)

$p_i = d_i/n$

~~0.0110~~
~~0.0200~~
~~0.0180~~
~~0.0160~~
~~0.0210~~
~~0.0200~~
~~0.0150~~

S.NO	defectives	$p_i = d_i/n$
1	22	0.0220
2	40	0.0400
3	36	0.0360
4	32	0.0320
5	42	0.0420
6	40	0.0400
7	30	0.0300
8	44	0.0440
9	42	0.0420
10	38	0.0380
11	70	0.0700
12	80	0.0800
13	44	0.0440
14	22	0.0220
15	32	0.0320
16	42	0.0420
17	20	0.0200
18	46	0.0460
19	28	0.0280
20	36	0.0360
21	66	0.0660
22	50	0.0500
23	46	0.0460
24	32	0.0320
25	42	0.0420
26	45	0.0450
27	30	0.0300
28	38	0.0380
29	40	0.0400
30	24	0.0240

$\Sigma p_i = 1.199$

Calculation

$$\bar{p} = \frac{\Sigma p_i}{K} = \frac{1.199}{30} = 0.0400$$

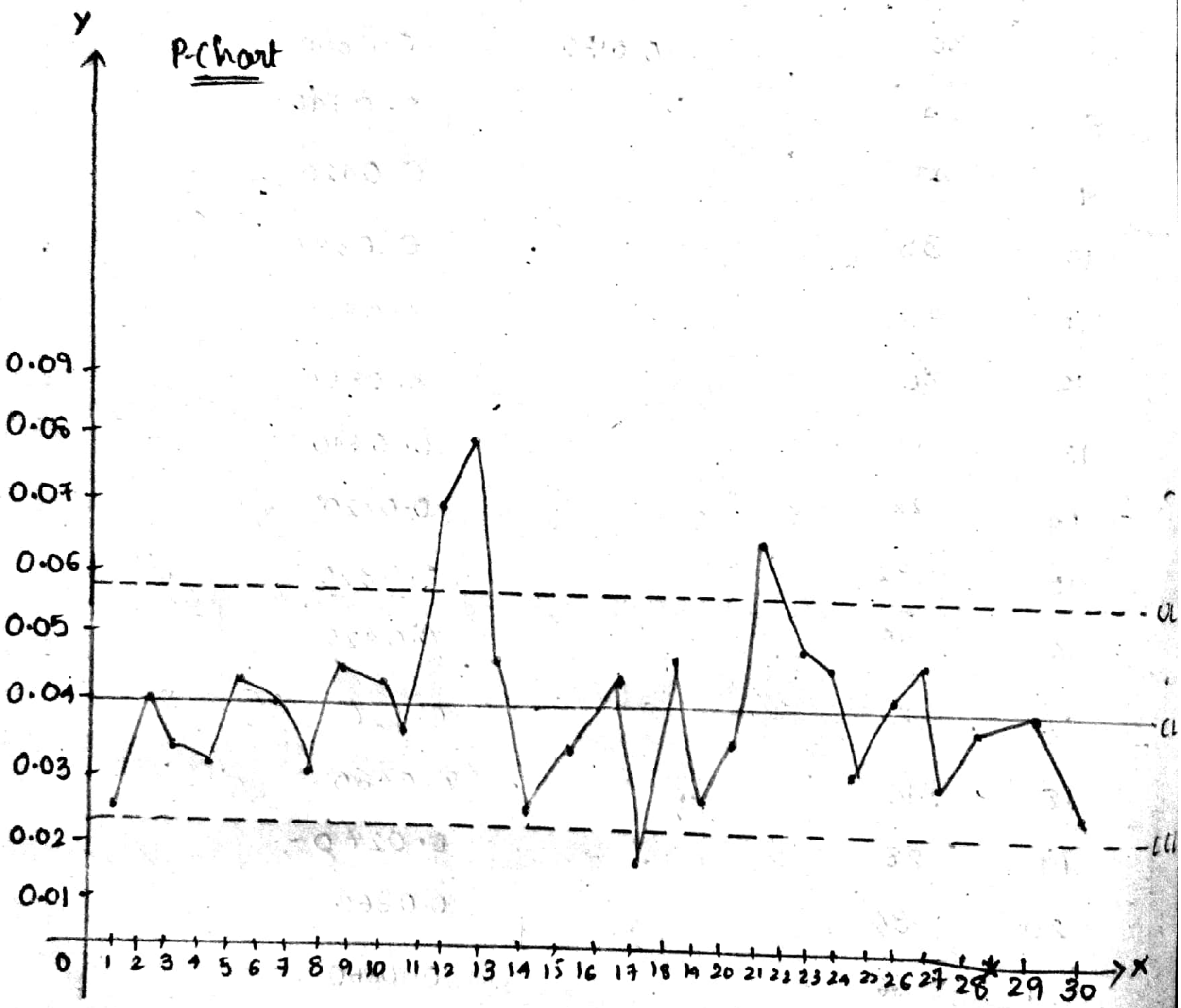
$\bar{q} = 0.9600$

$UCL = 0.0586$

$LCL = 0.0214$

$CL = 0.0400$

P-Chart



The process is not under statistical control.

4. The following data gives the no. of defectives in 10 defective samples of varying sizes from a production process.

sample no.	1	2	3	4	5	6	7	8	9	10
Sample size	2000	1500	1400	1350	1250	1760	1875	1955	3125	1575
no. of defectives	425	430	216	341	225	322	280	306	337	305

Aim: We have to find the control charts for fractional defectives (p).

Formulae:

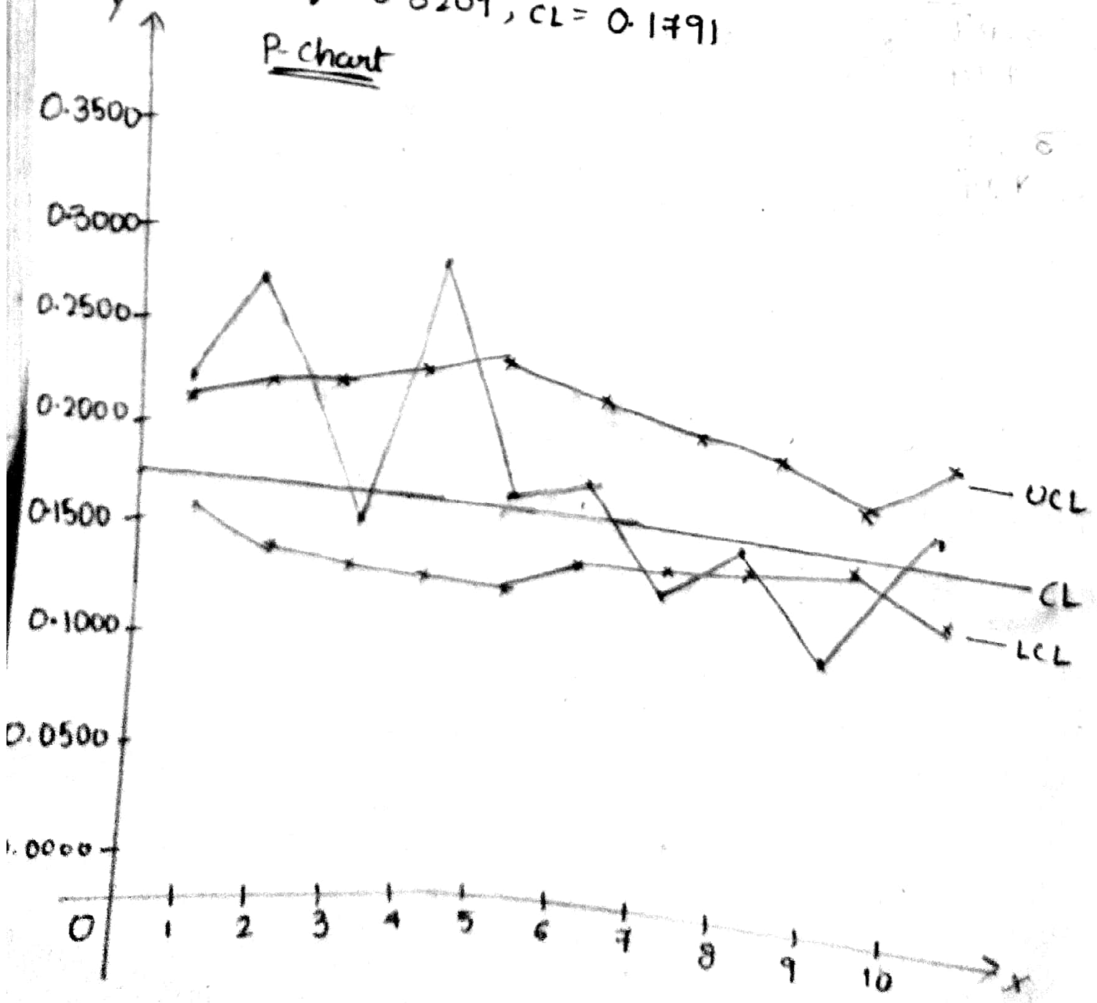
- $p_i = d_i / n_i$
- $\bar{p} = \Sigma d_i / \Sigma n_i$
- $\bar{q} = 1 - \bar{p}$
- $UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}$
- $LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n_i}}$
- $CL = \bar{p}$

Calculation

S.NO	Sample size	No. of defectives	UCL	LCL	$p_i = d_i/n_i$
1	2000	425	0.2048	0.1534	0.2125
2	1500	430	0.2088	0.1494	0.2867
3	1400	216	0.2098	0.1484	0.1543
4	1350	341	0.2104	0.1478	0.2526
5	1250	225	0.2116	0.1466	0.1800
6	1760	322	0.2065	0.1517	0.1830
7	1875	280	0.2057	0.1525	0.1493
8	1955	306	0.2051	0.1531	0.1565
9	3125	337	0.1997	0.1585	0.1078
10	1575	305	0.2081	0.1501	0.1937
	$\Sigma n_i = 17790$	$\Sigma d_i = 3187$			

$\bar{P} = 0.1791$, $\bar{q} = 0.8209$, $CL = 0.1791$

P-chart



Result:

The process is not under statistical control.

d-Chart (or) np-chart

$d \sim B(n,p)$, $E(d) = nP$, $V(d) = nPQ$, $SE(d) = \sqrt{nPQ}$

The 3σ control Limits are,

$$E(d) \pm 3 S.E(d)$$

$$E(d) = nPQ \pm 3\sqrt{nPQ}$$

Case 1: When P & Q are known

$$UCL = nP + 3\sqrt{nPQ}$$

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• The following are the figures of defectives of 22 lots each containing the 2000 rubber bolts. Draw control chart for No. of defectives and comment.

425, 430, 216, 341, 225, 322, 280, 306, 337, 305, 356, 402, 216
264, 126, 409, 193, 326, 280, 389, 451, 420.

Aim: To draw control chart for no. of defectives.

Formulae:

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}\bar{q}}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}\bar{q}}$$

$$CL = n\bar{p}$$

where $\bar{p} = \Sigma P_i / k$

$p = d_i / n$

Calculation:

S.NO	No. of defectives (d_i)	$p_i = d_i/n$
1	425	0.2125
2	430	0.215
3	216	0.108
4	341	0.1705
5	225	0.1125
6	322	0.161
7	280	0.14
8	306	0.153
9	337	0.1685
10	305	0.1525
11	356	0.169
12	402	0.201
13	216	0.108
14	264	0.132
15	126	0.063
16	409	0.2046
17	193	0.0965
18	326	0.163
19	280	0.14
20	389	0.1945
21	451	0.2255
22	420	0.21
		$\Sigma p_i = 3.5095$

From data, $n = 2000$, $k = 22$

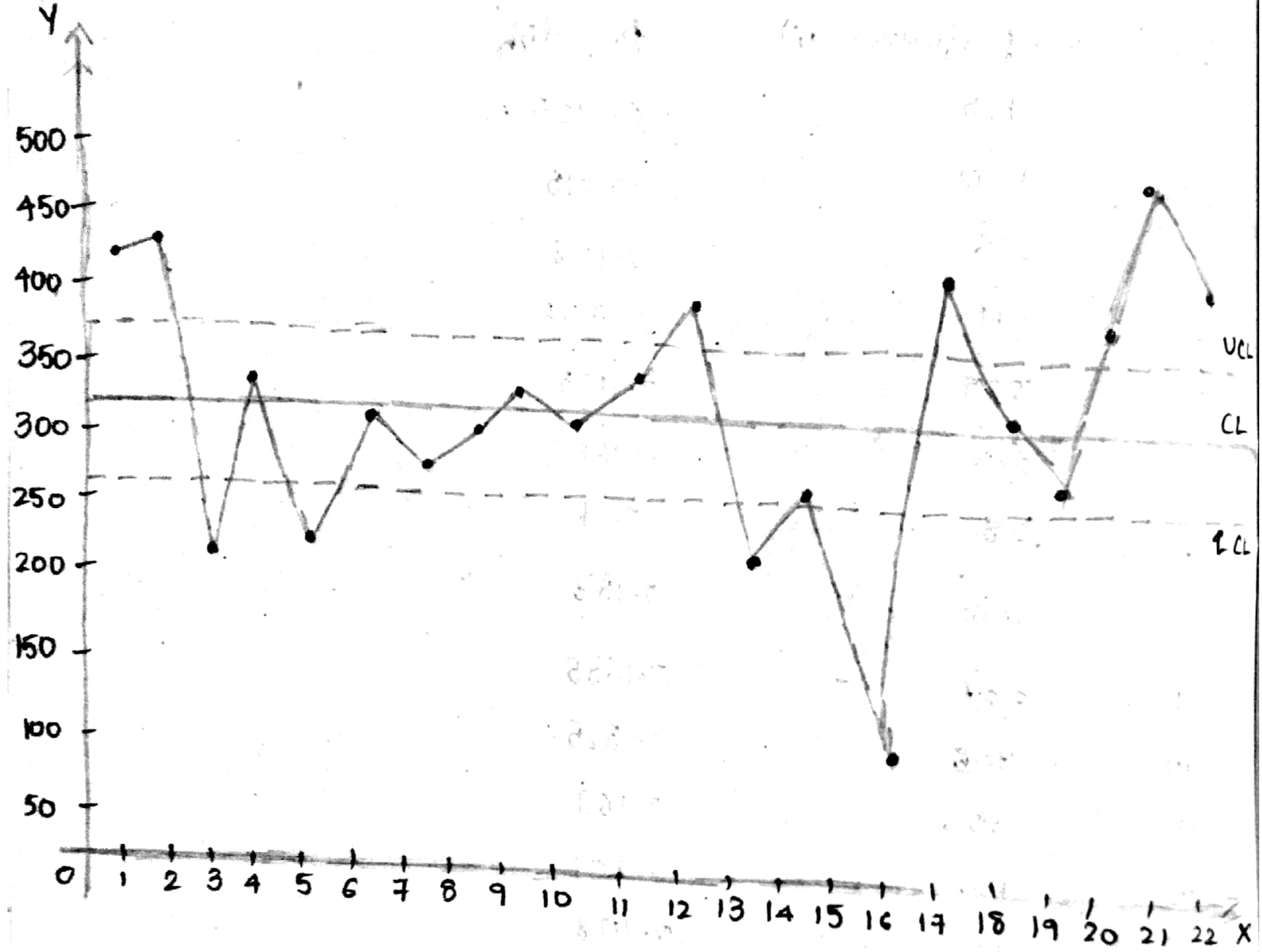
$\bar{P} = 0.1595$, $\bar{q} = 0.8405$

UCL = ~~336.9~~ 368.12

LCL = 268.87

CL = 319

d-chart



Result:

The process is not under statistical control because some points lie outsides the control limits.

• Construct np chart i.e, control chart for no. of defectives of varying sample size to the following data and state whether the process is under statistical control?

lot.no	1	2	3	4	5	6	7	8	9	10
sample size	500	400	300	150	600	450	450	800	900	1000
no. of defectives	25	42	35	16	15	40	42	81	82	100

Aim: To draw control chart for no.of defectives.

Formulae:

$$UCL = n_i\bar{p} + 3\sqrt{n_i\bar{p}\bar{q}}$$

$$LCL = n_i\bar{p} - 3\sqrt{n_i\bar{p}\bar{q}}$$

$$CL = n_i\bar{p}$$

where $\bar{p} = \frac{\Sigma d_i}{\Sigma n_i}$

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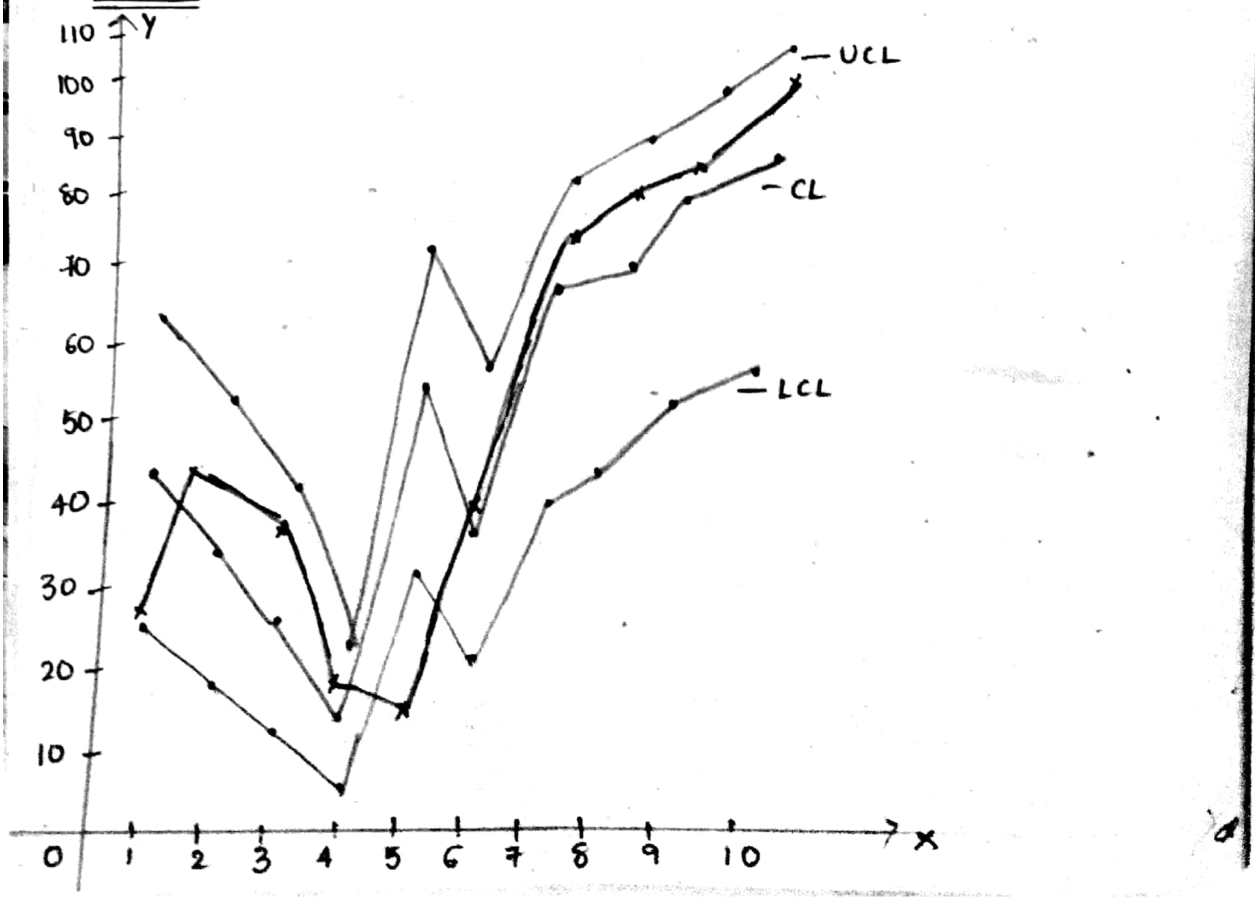
Calculations:

lot.no	samplesize	no. of defectives	~~UCL~~	UCL	LCL	CL
1	500	25	~~62.2864~~	62.2864	24.5136	43.4
2	400	42	~~60.2925~~	51.6125	17.8274	34.72
3	300	35	~~58.0293~~	40.6693	11.4106	26.04
4	150	16	~~63.7445~~	23.3645	2.6755	13.02
5	600	15	~~64.0690~~	72.7690	31.391	52.08
6	450	40	~~61.3172~~	56.9772	21.1428	39.06
7	750	72	~~66.6310~~	88.2310	41.969	65.1
8	800	81	~~67.2896~~	93.3296	45.5504	69.44
9	900	82	~~68.7384~~	103.4588	52.7812	78.12
10	1000	100	~~70.1093~~	113.5094	60.0906	86.8
	$\Sigma n_i = 5850$	$\Sigma d_i = 508$				

$$\bar{p} = \frac{\Sigma d_i}{\Sigma n_i} = \frac{508}{5850} = 0.0868$$

$$\bar{q} = 1 - \bar{p} = 0.9132$$

d-Chart



~~The process is not under statistical control because~~

Result: The process is not under statistical control because some points lie outside the control limits

~~An Inspection of 10 samples of size 400 each from 10 lots revealed~~

UNIT - II

Control chart for no. of defectives per unit

Introduction:

In many situations in industries where the ~~defects~~ data are obtained by counting.

For, example, machine idle time, defects in glass bottles, operators not working etc. also where the inspection classifies an item as defective (or) non-defective.

The defective items (or) units may be examined further and the no. of defects are controlled. The no. of defects is represented by c. In case of poisson distribution the avg no. of defects & variance are same.

Construction of C-Chart where the same size is constant

Here, the no. of defects ~~CNPD~~ $C \sim PD$ with parameter λ i.e $C \sim P(\lambda)$ then,

$$E(C) = \lambda$$

$$V(C) = \lambda$$

$$S.E(C) = \sqrt{\lambda}$$

$\therefore$ The 3σ CL's are

$$\lambda \pm 3\sqrt{\lambda}$$

$$UCL = \lambda + 3\sqrt{\lambda}$$

$$LCL = \lambda - 3\sqrt{\lambda}$$

$$CL = \lambda$$

Case(1):- When λ is known

Let $\lambda = \lambda'$ then

3σ CL's are,

$$\lambda' \pm 3\sqrt{\lambda'}$$

$$UCL = \lambda' + 3\sqrt{\lambda'}$$

$$LCL = \lambda' - 3\sqrt{\lambda'}$$

$$CL = \lambda'$$

Case 2:-

When λ is unknown

$$\hat{\lambda} = \bar{C} = \frac{\Sigma C_i}{k}$$

The 3σ CL's are

$$\bar{C} \pm 3\sqrt{\bar{C}}$$

$$UCL = \bar{C} + 3\sqrt{\bar{C}}$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}}$$

$$CL = \bar{C}$$

Construction of C-Chart

The sample points plot on x-axis & the no. of defects on Y-axis and UCL & LCL are plotted as dotted lines & CL as bold line. If all the plotted points are in b/w these 3 lines. the process is said to be under control. If the points are not in b/w the lines the process is said to be out of control.

Applications:-

1) No.of defectives per television set.
2) Accidents in industries
3) Accidents on Highway.
4) In chemical laboratories
5) No. of air bubbles observed per sq. meter in a sheet of glass.
6) No. of defects per meter in a hand woven cloth.

1) Draw a control chart for no. of defects and give your comments?

2, 4, 7, 3, 1, 4, 8, 9, 5, 3, 7, 11, 6, 4, 9, 9, 6, 4, 3, 9, 7, 4, 7, 12

Sol. **Aim**: To draw a control chart for no. of defects (C-chart) and comment on it.

Formulae:

$UCL = \bar{C} + 3\sqrt{\bar{C}}$

$LCL = \bar{C} - 3\sqrt{\bar{C}}$

$CL = \bar{C}$ where $\bar{C} = \frac{\sum C_i}{k}$; $k = 24$

Calculations:

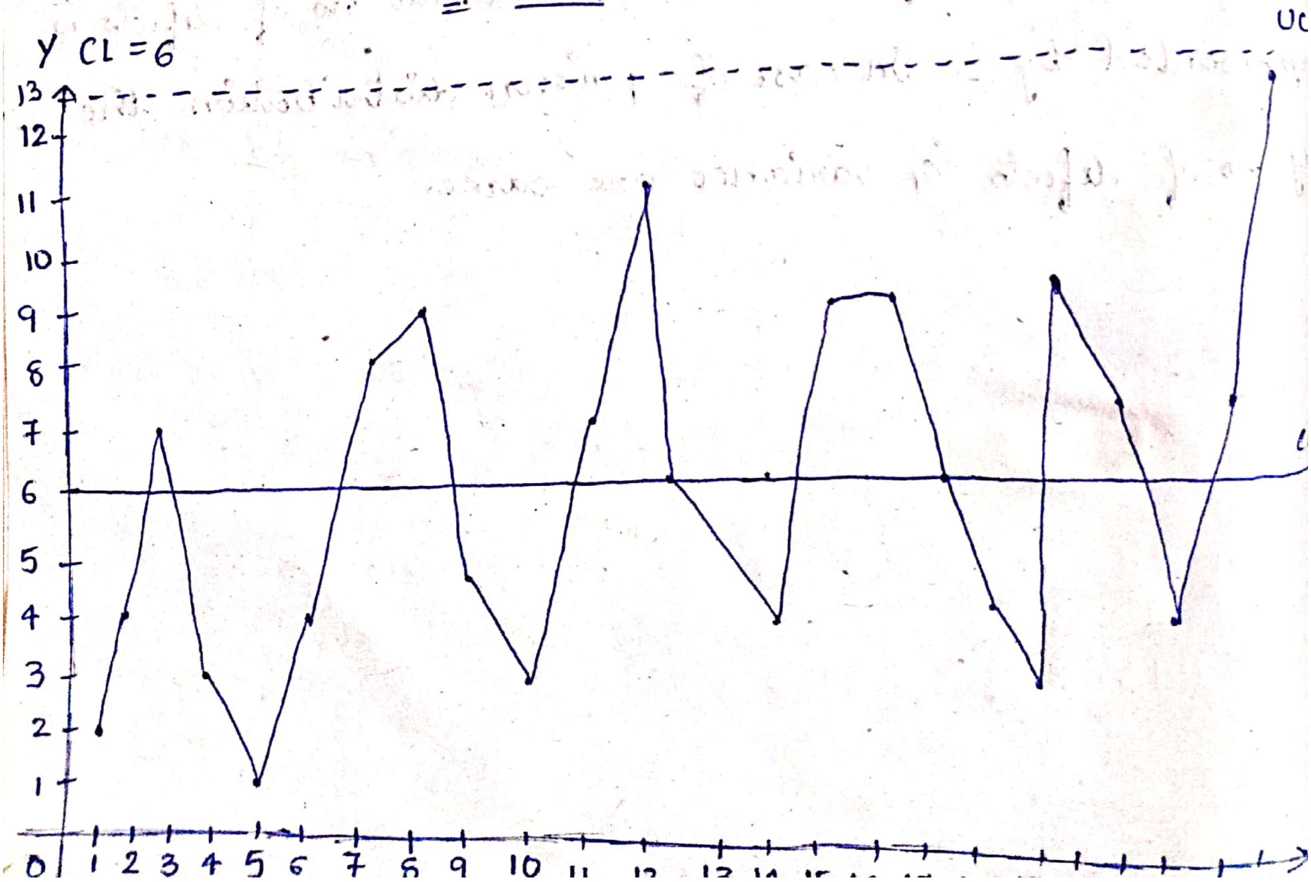
$\bar{C} = \frac{144}{24} = 6$

$\bar{C} = 6$

$UCL = 13.3485$

$LCL = -1.3485$

C-Chart



Result: The process is under statistical control, since all the points are b/w control limits.

2) Draw a control chart for no. of defectives and gives your comments.

20, 19, 25, 30, 27, 16, 18, 21, 27, 30, 15, 9, 10, 12, 14.

Sol. Aim: To construct c-chart and comment on it.

Formulae: $UCL = \bar{c} + 3\sqrt{\bar{c}}$

$LCL = \bar{c} - 3\sqrt{\bar{c}}$

$CL = \bar{c}$ where $\bar{c} = \frac{\sum c_i}{k}$; $k = 15$

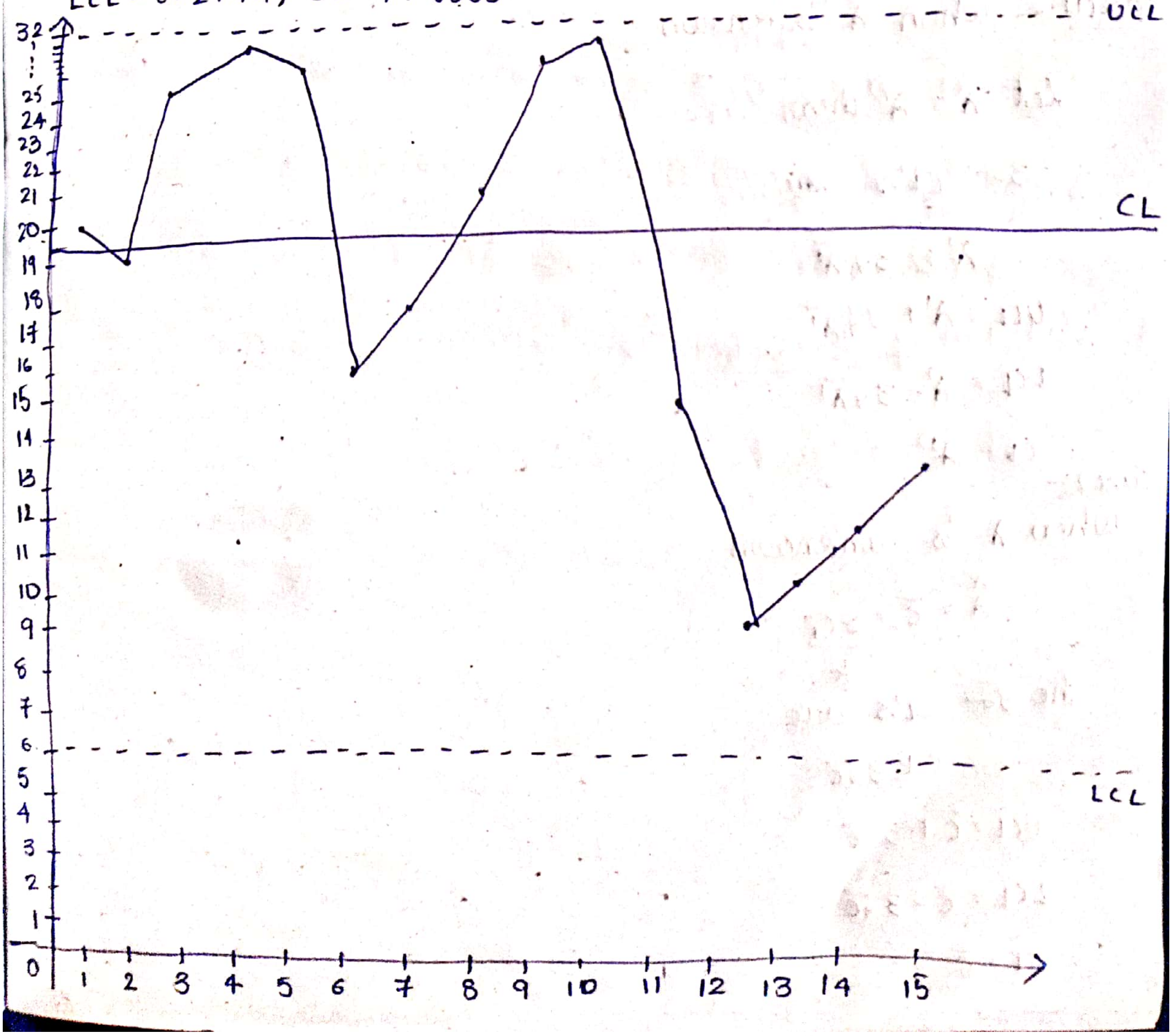
Calculations:

$\bar{c} = \frac{293}{15} = 19.5333$

$UCL = 32.7922$

$LCL = 6.2744$, $CL = 19.5333$

C-Chart



Control Chart for no. of defects for variable size (U-Chart)

If n_i is the sample size of & C_i is the total no. of defects observed in the i^{th} sample then,

$U_i = \frac{C_i}{n_i}$ where $i = 1, 2, \ldots, k$ gives the avg no. of defects per units.

$$\lambda = \bar{U} = \frac{\Sigma U_i}{k}$$

λ is the mean no. of defects per unit in the lot based on all k samples and the standard error of avg no. of defects per unit is given by

$$SE = \sqrt{\bar{U}/n_i}$$

The 3σ CL's are

$$\bar{U} \pm 3\sqrt{\frac{\bar{U}}{n_i}}$$

i.e, $UCL = \bar{U} + 3\sqrt{\bar{U}/n_i}$

$LCL = \bar{U} - 3\sqrt{\bar{U}/n_i}$

$CL = \bar{U}$

Construction of U-Chart

Same as C-Chart.

1. The following table gives the avg no. of outlets leaks per radiators each for 10 lots of 100 radiators. Draw U-Chart and whether the process is under statistical control.

Lot no	no. of outlet leak
1	15
2	17
3	12
4	16
5	14
6	5
7	14
8	11
9	9
10	10

Aim: To construct U-Chart for the given data

Procedure (Formulae)

$n = 100, \; k = 10$

3σ CL's are

$$UCL = \bar{U} + 3\sqrt{\frac{\bar{U}}{n}}$$

$$LCL = \bar{U} - 3\sqrt{\frac{\bar{U}}{n}}$$

$$CL = \bar{U} \text{ where } \bar{U} = \frac{\Sigma U_i}{k}$$

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Calculation:

Lot NO	no. of outlet leak (c)	$U = c/n$
1	15	0.15
2	17	0.17
3	12	0.12
4	16	0.16
5	14	0.14
6	5	0.05
7	14	0.14
8	11	0.11
9	9	0.09
10	10	0.10

$\Sigma U = 1.23$

$\bar{U} = 0.123$

$UCL = 0.123 + 3\sqrt{\frac{0.123}{100}} = 0.2282$

$LCL = 0.0178$

$CL = 0.123$

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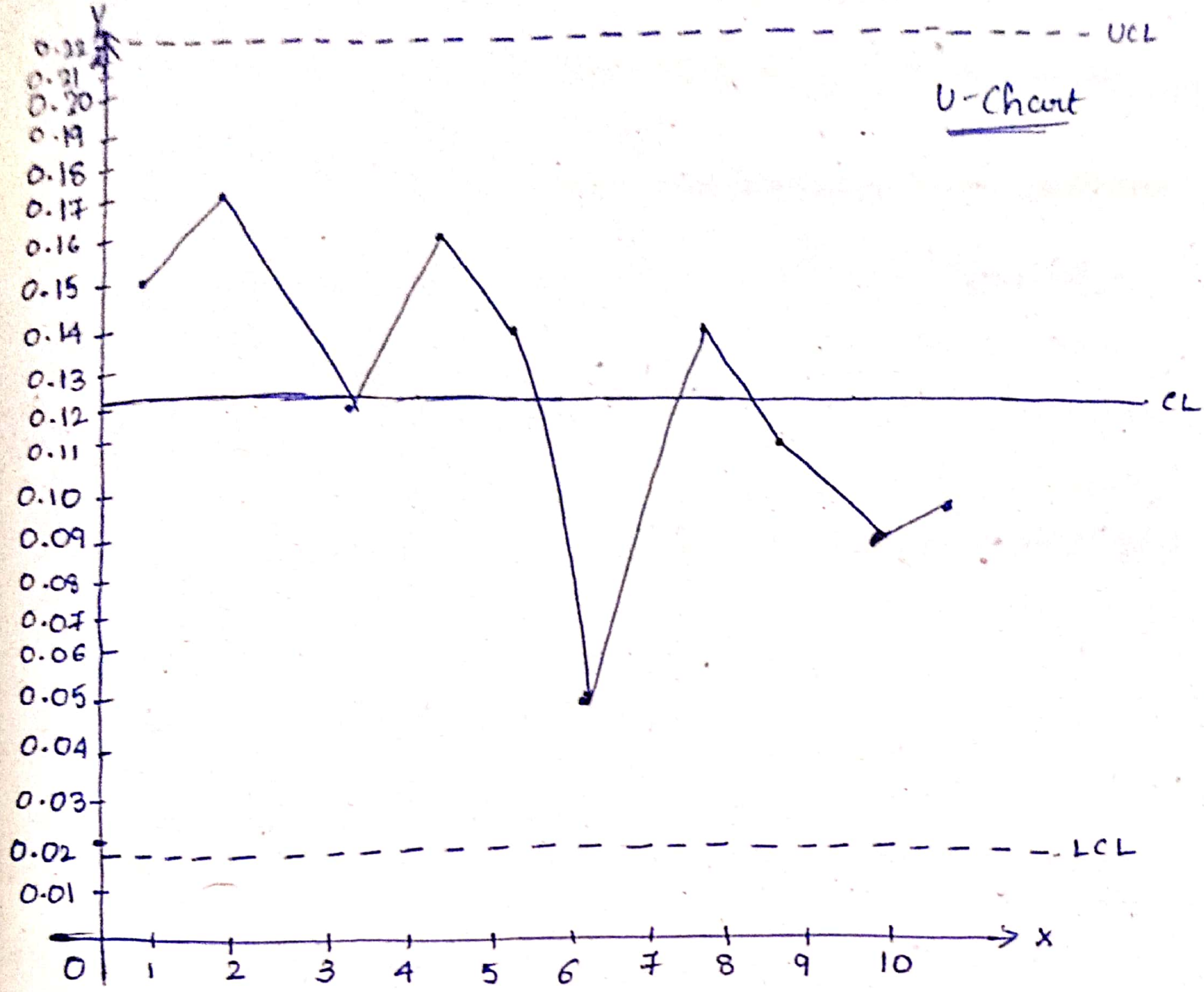


Result: The process is under statistical contral

d-Chart

1) An inspection of 10 samples of size 400 each ~~form~~ from 10 lots revealed the following no. of defectives

17, 15, 14, 26, 9, 4, 19, 12, 9, 15

Aim: To construct d-chart

Formula: $UCL = n\bar{p} + 3\sqrt{n\bar{p}\bar{q}}$

$LCL = n\bar{p} - \sqrt{3\, n\bar{p}\bar{q}}$

$n = 400,\ K = 10$

$CL = n\bar{p}$

where $\bar{p} = \Sigma P/k$, $P = d_i/n$.

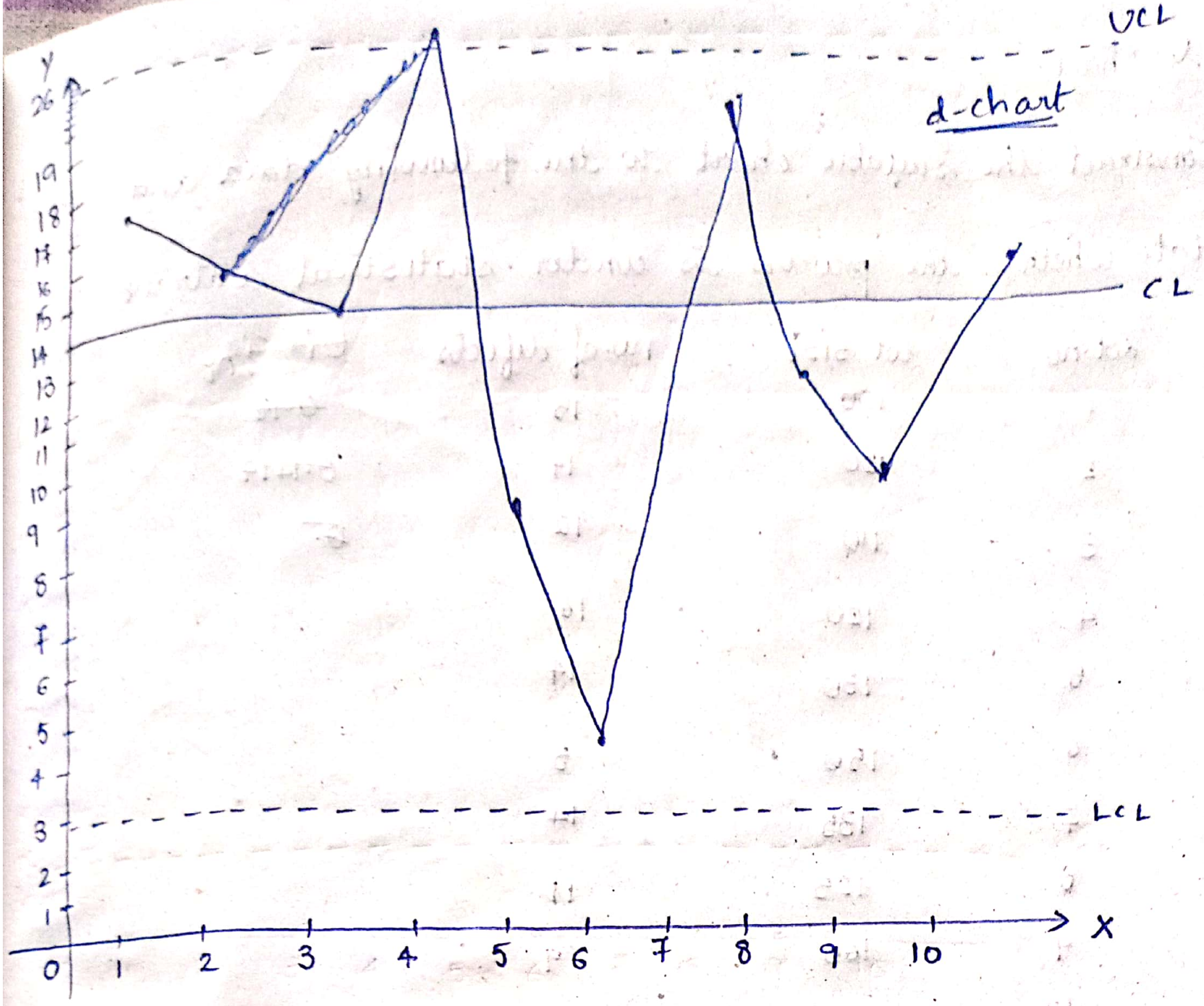
Calculation:

S.NO	d_i	$P_i = d_i/n$
1	17	0.0425
2	15	0.0375
3	14	0.0350
4	26	0.0650
5	9	0.0225
6	4	0.0100
7	19	0.0475
8	12	0.0300
9	9	0.0225
10	15	0.0375

$\Sigma P_i = 0.35$

$\bar{P} = 0.0350$, $\bar{q} = 0.9650$

$UCL = 25.0268$, $LCL = 2.9732$, $CL = 14$



Result: The process is out of Statistical Control.

U-Chart

Construct the suitable chart to the following data and state whether the process is under statistical control?

Lot.no	lot size	no. of defects	~~$U_i = C_i/n_i$~~
1	100	15	~~0.15~~
2	120	17	~~0.1417~~
3	110	12	~~0.~~
4	120	16	
5	130	14	
6	150	5	
7	135	14	
8	125	11	
9	160	9	
10	140	10	

Aim: To construct U-Chart for given data.

Formulae:

$$U_i = \frac{C_i}{n_i}$$

$$UCL = \bar{U} + 3\sqrt{\frac{\bar{U}}{n_i}}$$

$$LCL = \bar{U} - 3\sqrt{\frac{\bar{U}}{n_i}}$$

$$CL = \bar{U}$$

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Calculation

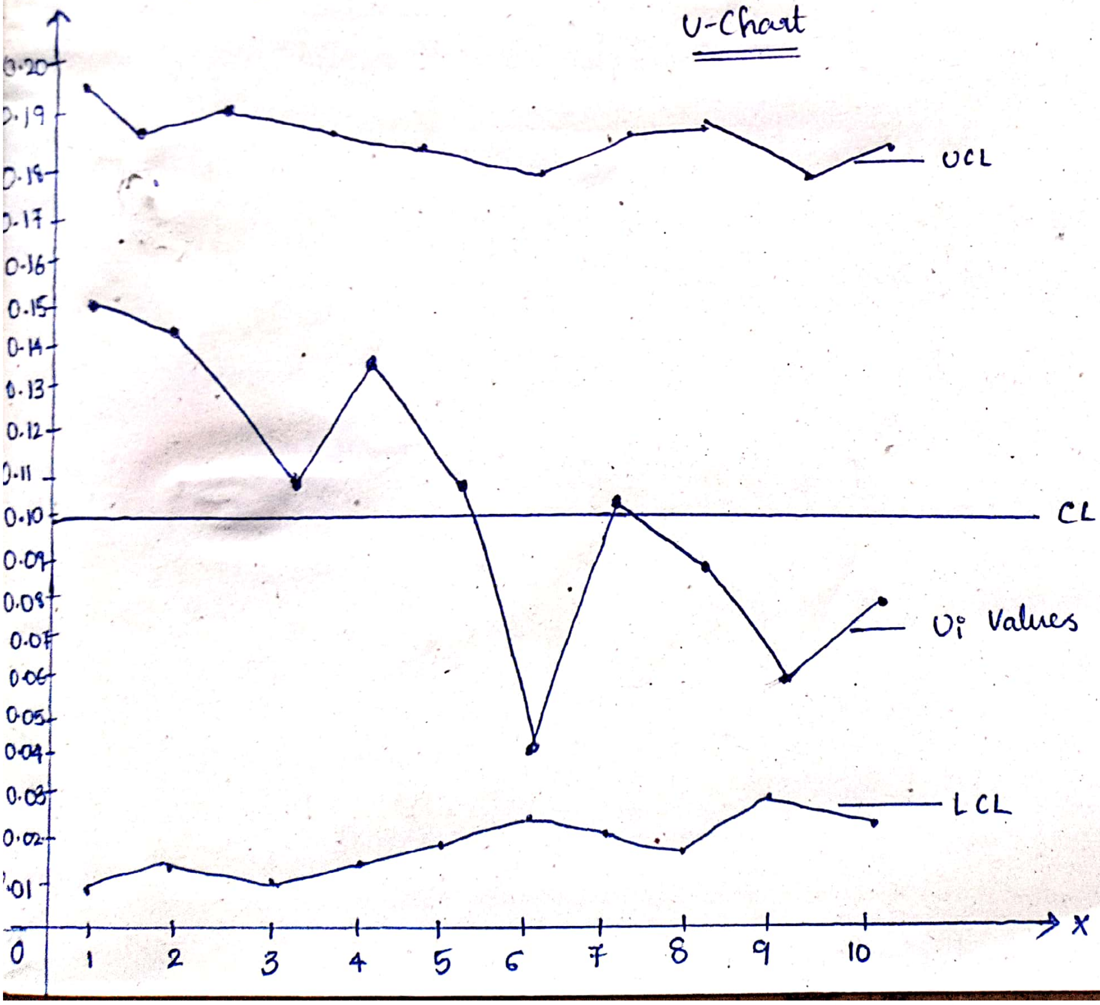
Lot no	Lot Size	no. of defects	$U_i = \frac{C_i}{n_i}$	UCL	LCL
1	100	15	0.15	0.1941	0.0049
2	120	17	0.1417	0.1859	0.0131
3	110	12	0.1091	0.1897	0.0093
4	120	16	0.1333	0.1859	0.0131
5	130	14	0.1077	0.1825	0.0165
6	150	5	0.0333	0.1768	0.0222
7	135	14	0.1037	0.1809	0.0181
8	125	11	0.0880	0.1841	0.0149
9	160	9	0.0563	0.1743	0.0247
10	140	10	0.0714	0.1795	0.0195

$\Sigma U_i = 0.9945$

$$\bar{U} = \frac{0.9945}{10} = 0.0995$$

CL = 0.0995

U-Chart



Single Sampling Plan

1. Construct OC, ASN, AOQ, ATI curves for SSP with following specifications

$N = 10,000$, $n = 100$, $c = 3$?

Sol. **Aim:** To construct OC, ASN, AOQ, ATI curves for SSP.

Procedure & Formulae:

P be the incoming lot quality.

$$\text{Probability of Acceptance} = P_a = P(d \le c)$$

$$= P(x \le c)$$

$$= P(X \le 3)$$

$$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^3}{3!}$$

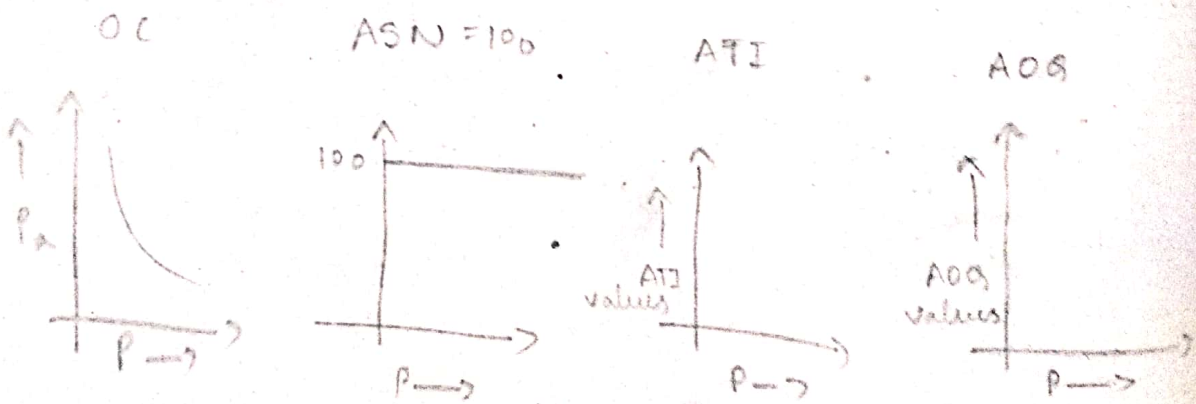
$$P_a = e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6}\right]; \lambda = np.$$

$ASN = n = 100$

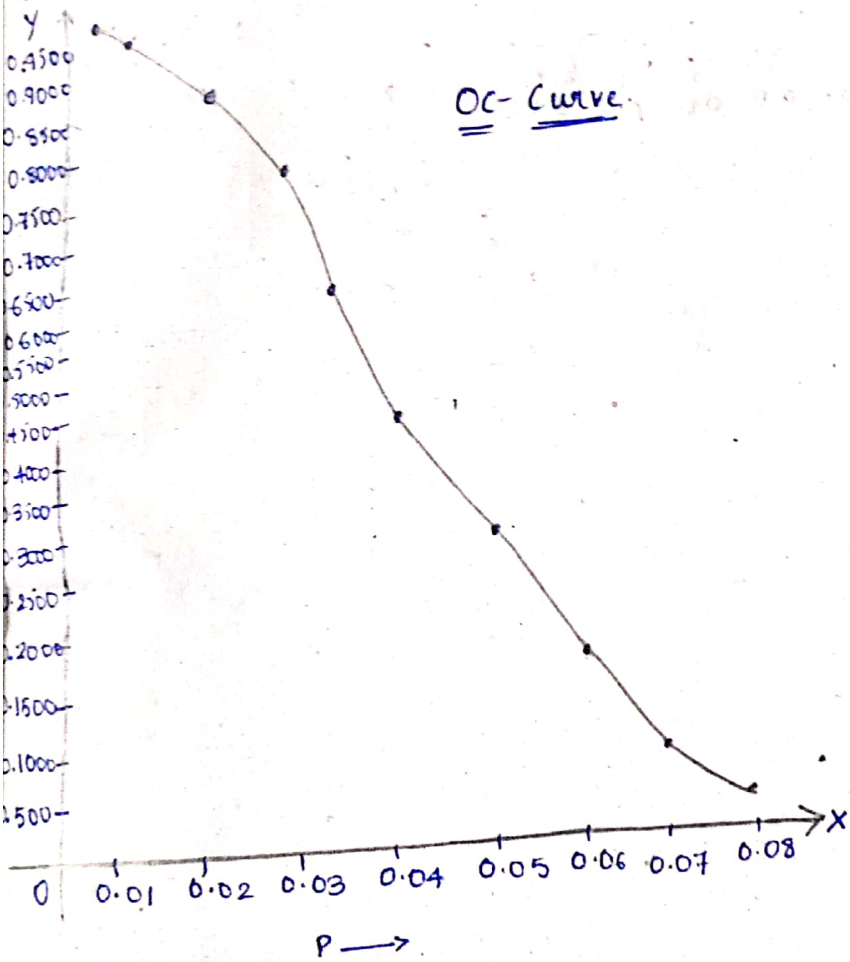
$ATI = n + (N-n)\cancel{P_a}(1-P_a)$

$= 100 + 9900\cancel{P_a}(1-P_a)$

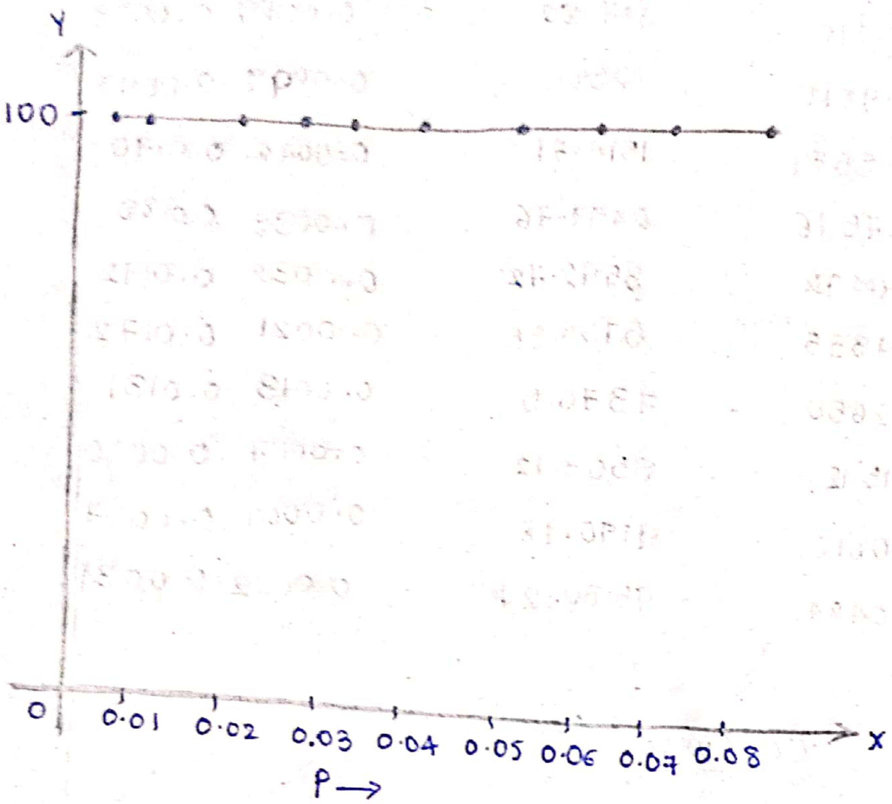
$$AOQ = \frac{p(N-n)P_a}{N} = \frac{p(9900)P_a}{10,000}$$



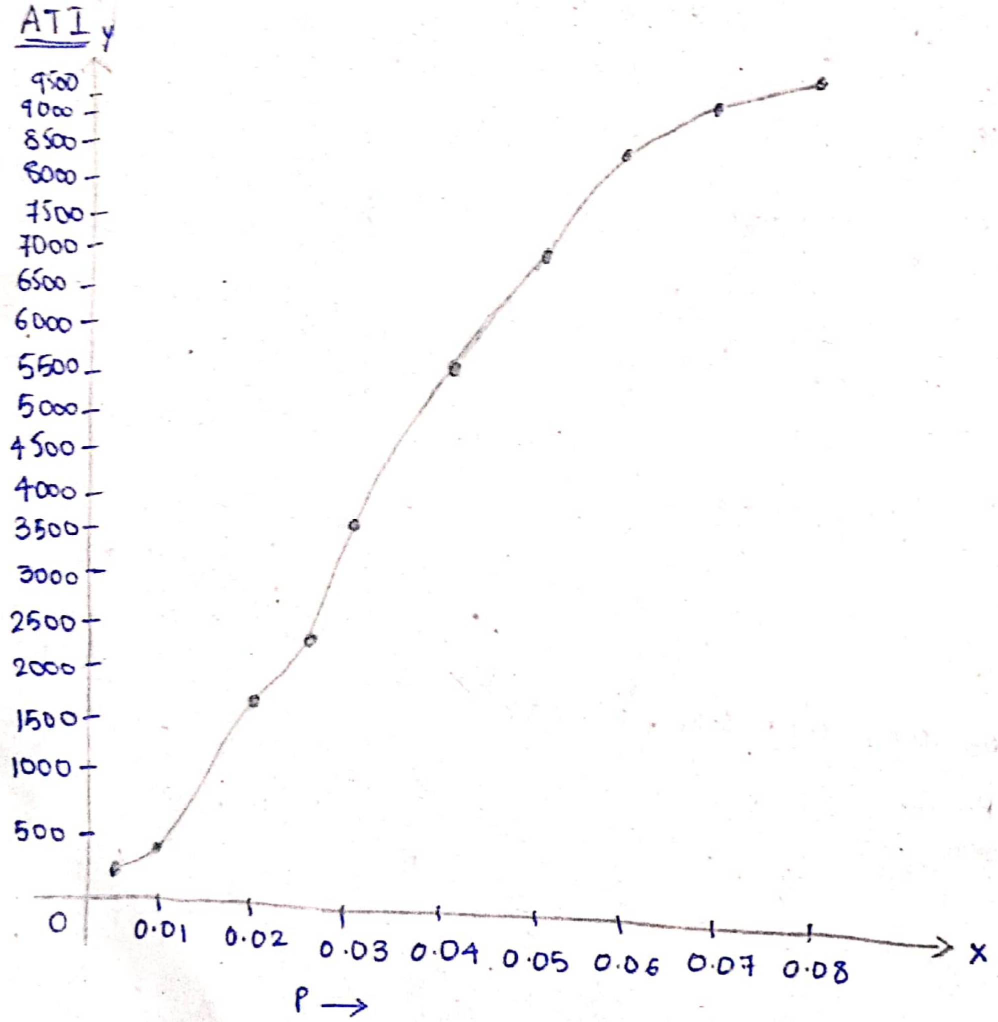
p	$\lambda = np$	Pa	ATI = 100+9900 (1-Pa)	AOQ = P(9900)Pa/10,000
0.005	0.5	0.9982	117.82	~~0.0049~~ 0.0049
0.01	1	0.9810	288.1	~~0.0049~~ 0.0099
0.02	2	0.8571	1514.71	~~0.0042~~ 0.0170
0.025	2.5	0.7576	2499.76	~~0.0038~~ 0.0188
0.03	3	0.6472	3592.72	~~0.0032~~ 0.0192
0.04	4	0.4335	5708.35	~~0.0021~~ 0.0172
0.05	5	0.2650	7376.5	~~0.0013~~ 0.0131
0.06	6	0.1512	8503.12	~~0.0007~~ 0.0090
0.07	7	0.0818	9190.18	~~0.0004~~ 0.0057
0.08	8	0.0424	9580.24	~~0.0002~~ 0.0034



ASN.



ATI



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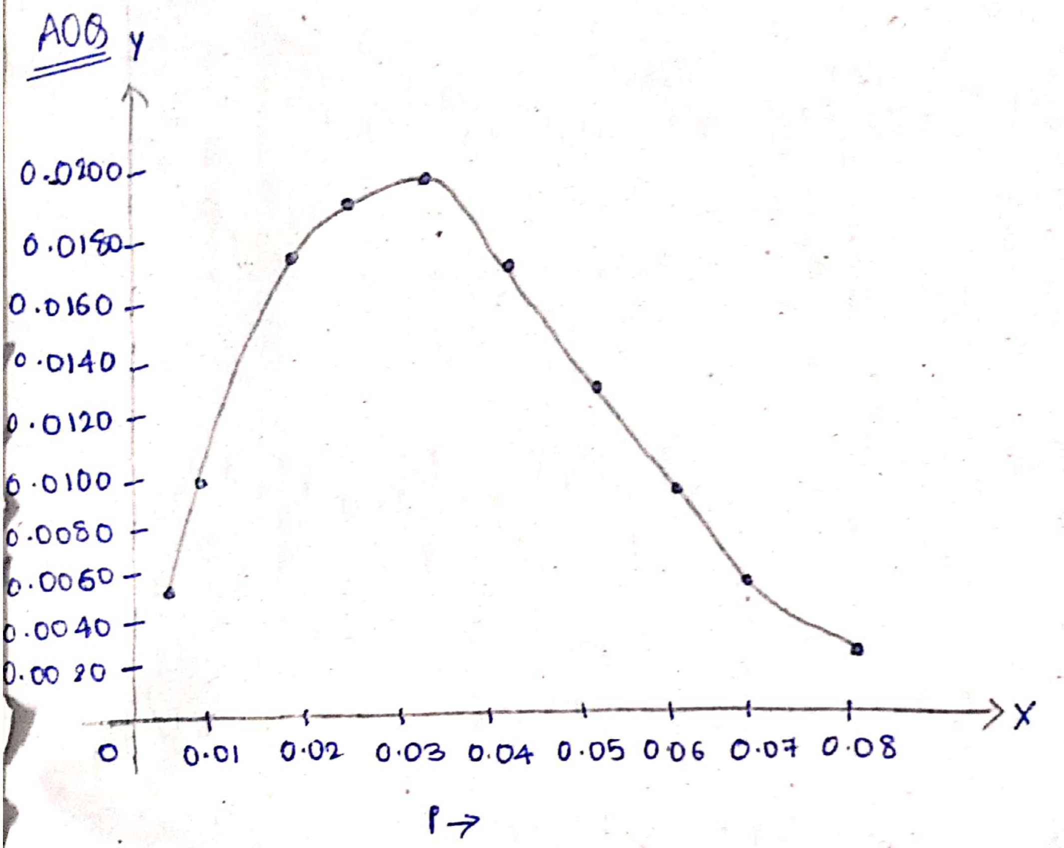


Double Sampling Plan

1) Construct OC, ASN, AOQ, ATI curves for the following for Double Sampling Plan. with the following specifications.

$N = 500, n_1 = 50, n_2 = 2n_1, c_1 = 2, c_2 = 6.$

Sol.

Aim: To construct OC, ASN, AOQ, ATI curves for DSP.

Procedure & Formulae:

AOQ Curve:

$$AOQ = \frac{P[Pa_1(N-n_1) + Pa_2(N-n_1-n_2)]}{N}$$

$$ASN = n_1 + (1-Pa)n_2$$

$$ATI = n_1 Pa_1 + (n_1+n_2)Pa_2 + N(1-Pa)$$

where $Pa = Pa_1 + Pa_2$.

$Pa_1 = P(X_1 \le c_1)$; $X_1 \sim P(\lambda)$; $\lambda_0 = n_1 P$.

$Pa_2 = \sum P(X_1 = i) \cdot P(X_2 \le c_2 - i)$; $X_2 \sim P(\lambda)$; $\lambda = n_2 P$.

$Pa_1 = P[X_1 \le c_1]$ $\left[\because P(x) = \frac{e^{-\lambda}\lambda^x}{x!}\right]$

$= P[X_1 \le 2]$ $\because X_1 \sim P(\lambda_0)$; $\lambda_0 = n_1 P$

$= P[X_1 = 0] + P[X_1 = 1] + P[X_1 = 2]$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!}$$

$$= \frac{e^{-n_1P}(n_1P)^0}{0!} + \frac{e^{-n_1P}(n_1P)^1}{1!} + \frac{e^{-n_1P}(n_1P)^2}{2!}$$

$$= e^{-n_1P}\left[1 + n_1P + \frac{(n_1P)^2}{2}\right]$$

$$Pa_1 = e^{-50P}\left[1 + 50P + \frac{2500P^2}{2}\right]$$

$$P_{a_1} = \sum_{i=c_1+1}^{c_2} P[X_1 = i] \cdot P[X_2 \leq c_2 - i]$$

$$= \sum_{i=3}^{6} P[X_i = i] \cdot P[X_2 \leq 6 - i]$$

$$= P[X_1 = 3] \cdot P[X_2 \leq 3] + P[X_1 = 4] P[X_2 \leq 2] + P[X_1 = 5] \cdot P[X_2 \leq 1] + P[X_1 = 6] \cdot P[X_2 \leq 0]$$

$$\Rightarrow \frac{e^{-n_1 p} \cdot (n_1 p)^3}{3!} \left[\frac{e^{-n_2 p} (n_2 p)^0}{0!} + \frac{e^{-n_2 p} (n_2 p)^1}{1!} + \frac{e^{-n_2 p} (n_2 p)^2}{2!} + \frac{e^{-n_2 p} \cdot (n_2 p)^3}{3!} \right] +$$

$$\frac{e^{-n_1 p} (n_1 p)^4}{4!} \left[\frac{e^{-n_2 p} (n_2 p)^0}{0!} + \frac{e^{-n_2 p} (n_2 p)^1}{1!} + \frac{e^{-n_2 p} \cdot (n_2 p)^2}{2!} \right] +$$

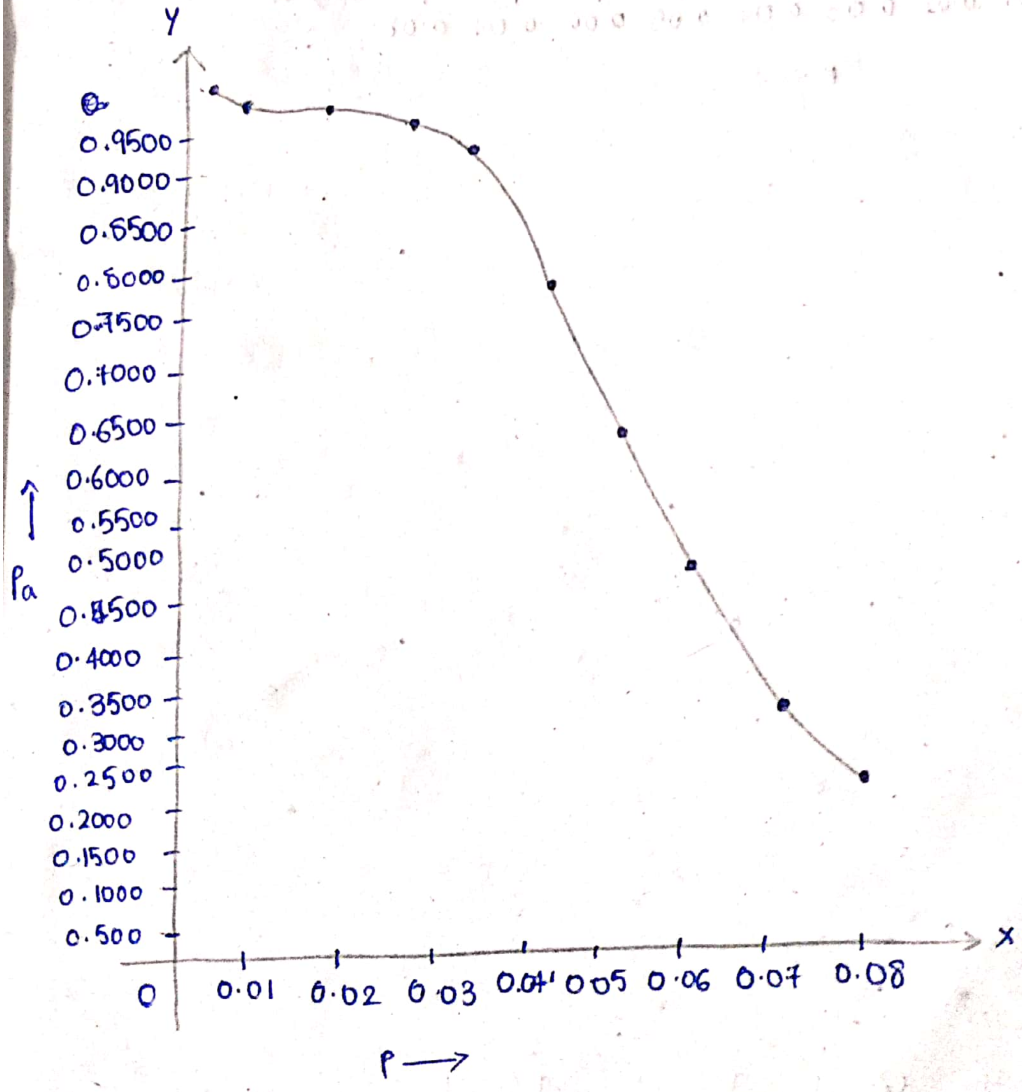
$$\frac{e^{-n_1 p} (n_1 p)^5}{5!} \left[\frac{e^{-n_2 p} (n_2 p)^0}{0!} + \frac{e^{-n_2 p} (n_2 p)^1}{1!} \right] + \frac{e^{-n_1 p} (n_1 p)^6}{6!} \left[\frac{e^{-n_2 p} (n_2 p)^0}{0!} \right]$$

$$\Rightarrow \frac{e^{-n_1 p} (n_1 p)^3}{3!} \left[e^{-n_2 p} \left(1 + n_2 p + \frac{(n_2 p)^2}{2} + \frac{(n_2 p)^3}{6} \right) \right] + \frac{e^{-n_1 p} (n_1 p)^4}{4!} \left[e^{-n_2 p} \left(1 + n_2 p + \frac{(n_2 p)^2}{2} \right) \right] + \frac{e^{-n_1 p} (n_1 p)^5}{5!} \left[e^{-n_2 p} (1 + n_2 p) \right] + \frac{e^{-n_1 p} (n_1 p)^6}{6!} \left[e^{-n_2 p} \right.$$

Calculation:

p	n_1p	n_2p	Pa_1	Pa_2	Pa	AOQ	ATI	~~ASN~~	ASN
.005	0.25	0.5	0.9978	0.0021 ~~0.4228~~	0.9999	0.0045	51.43	50.255	50.0100
.01	0.5	1	0.9856	0.0140	0.9996	0.0090	51.56		50.0400
.02	1	2	0.9197	0.0652	0.9849	0.0175	63.315		51.5100
.025	1.25	2.5	0.8685	0.0887	0.9572	0.0211	78.13		54.2800
0.03	1.5	3	0.8088	0.1041	0.9129	0.0240	99.605		58.7100
0.04	2	4	0.6767	0.1032	0.7799	0.0273	159.365		72.0100
0.05	2.5	5	0.5438	0.0763	0.6201	0.0271	288.585		87.9900
0.06	3	6	0.4232	0.0461	0.4693	0.0248	293.425		103.0700
0.07	3.5	7	0.3208	0.0243	0.3451	0.0214	347.135		115.4900
0.08	4	8	0.2381	0.0115	0.2496	0.0178	388.83		125.0400

OC Curve



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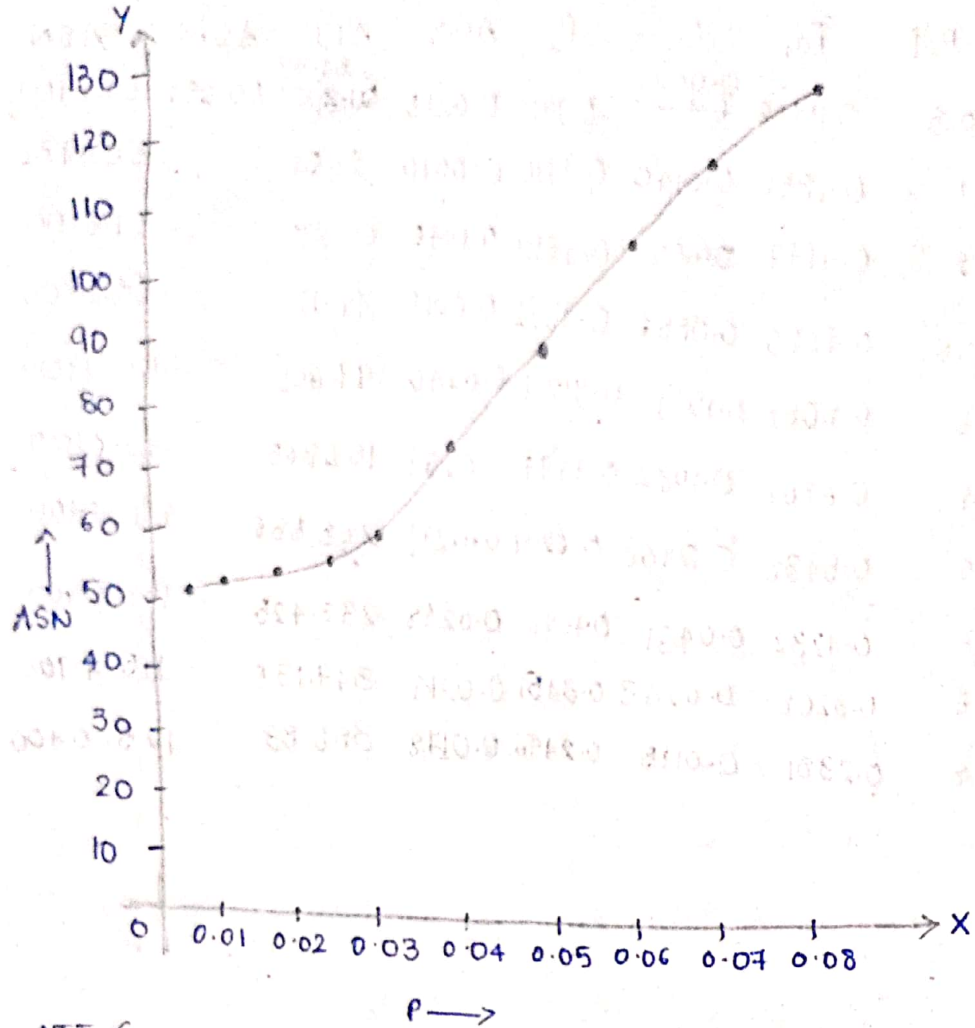
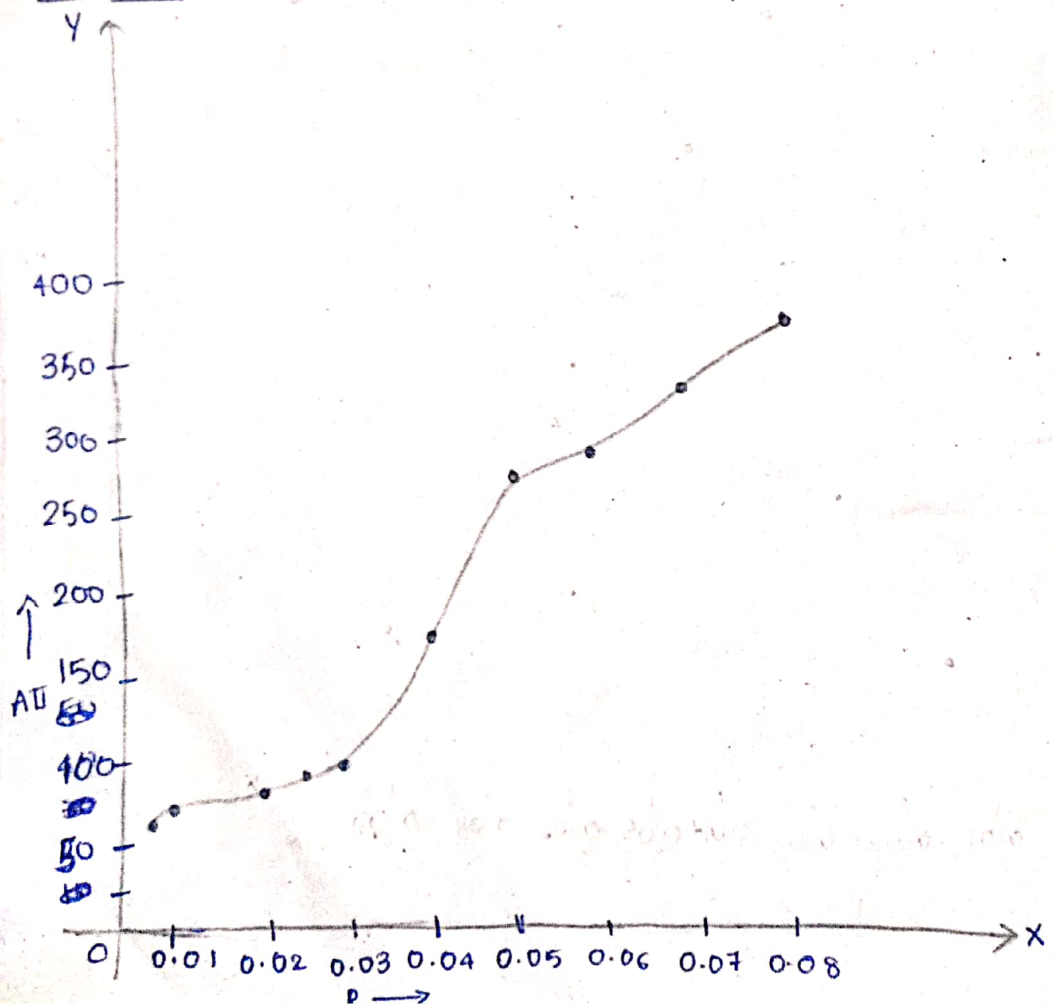
ASN-Curve



ATI Curve



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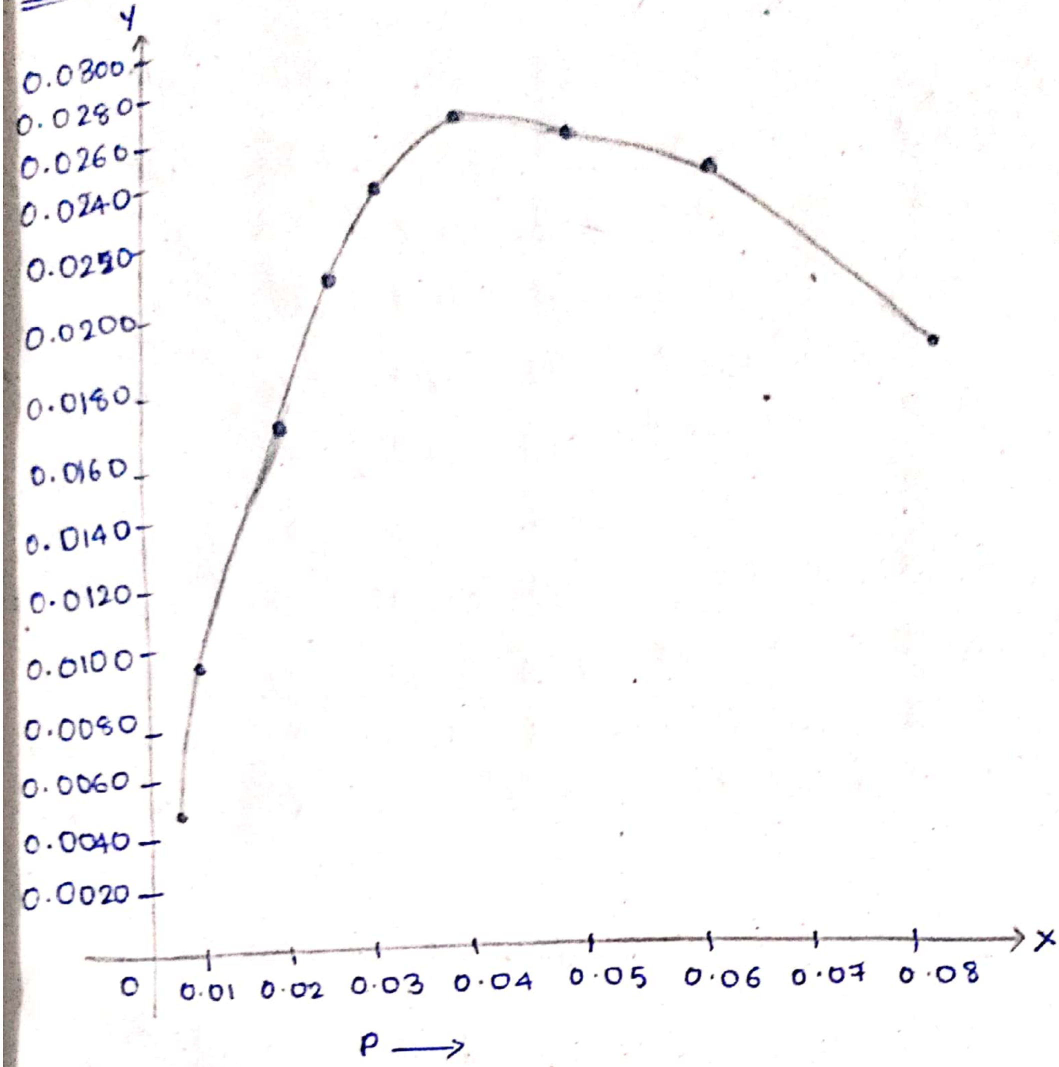
AOQ-Curve


Y
0.0300
0.0280
0.0260
0.0240
0.0200
0.0180
0.0160
0.0140
0.0120
0.0100
0.0080
0.0060
0.0040
0.0020
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
X
P