Sphere

1) Find the equation to the sphere thro' the points $(0,0,0)$, $(0,1,-1)$, $(-1,2,0)$, $(1,2,3)$

2) Find the equation of the sphere thro' the four points $(4,-1,2)$, $(0,-2,3)$, $(1,-5,-1)$, $(2,0,1)$

3) Obtain the equation of the sphere which passes thro' the three points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and has its radius as small as possible

4) Obtain the sphere having its centre on the line $5y+2z=0=2x-3y$ and passing thro' the 2 points $(0,-2,-4)$, $(2,-1,-1)$

5) P.T the plane section of a sphere is a circle.

6) Find the centre and radius of the circle $x+2y+2z=15$,

$x^2+y^2+z^2-2y-4z=11$

7) Find the equation of the sphere thro' the circle and the point $(1,2,3)$

$x^2+y^2+z^2=9$, $2x+3y+4z=5$

8) Find the equation of the sphere thro' the circle $x^2+y^2+z^2+2x+3y+6=0$, $x-2y+4z-9=0$ and the centre of the sphere

$x^2+y^2+z^2-2x+4y-6z+5=0$.

9) Show that the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z-2=0$;

$x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lie on the same sphere and find its equation.

10) Obtain the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as the great circle.

11) S.T the plane $lx+my+nz=p$ will touch the sphere

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ if

$(ul+vm+wn+p)^2=(l^2+m^2+n^2)(u^2+v^2+w^2-d)$

12) Find the two tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane $2x+2y=z$

13) Find the equation of the sphere which touches the sphere $x^2+y^2+z^2-x+3y+2z-3=0$ at the point $(1,1,-1)$ and passes thro' the Origin.

14) Find the eqns of the sphere thro' the circle $x^2+y^2+z^2=1$, $2x+4y+5z=6$ and touching the plane $z=0$

15) Find the eqns of the two tangent planes to the sphere $x^2+y^2+z^2=9$, which pass thro' the line $x+y=6$, $x-2z=3$.

16) If the tangent plane to the sphere $x^2+y^2+z^2=r^2$ makes intercepts a, b, c on the coordinate axes, ST $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{r^2}$

17) ST the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z=3$ and find the point of contact.

18) Find the equation of the tangent plane to the sphere $3(x^2+y^2+z^2)-2x-3y-4z-22=0$ and the point $(1,2,3)$

19) Condition for Orthogonality of two spheres.

20) Find the eqn of the sphere that passes thro' the circle $x^2+y^2+z^2-2x+3y-4z+6=0$, $3x-4y+5z-15=0$ and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11=0$ orthogonally.

21) Find the eqn of the sphere that passes thro' the two points $(0,3,0)$, $(-2,-1,-4)$ and cuts orthogonally the two spheres $x^2+y^2+z^2+x-3z-2=0$, $2(x^2+y^2+z^2)+x+3y+4=0$.

22) Find the limiting points of the coaxal system defined by the spheres $x^2+y^2+z^2+3x-3y+6=0$, $x^2+y^2+z^2-6y-6z+6=0$.

23) Find the limiting points of the coaxal system of spheres $x^2+y^2+z^2-20x+30y-40z+29+\lambda(2x-3y-4z)=0$.

Cones

1) Theorem: Find the equation of the cone whose vertex is the point (α, β, γ) and whose generators intersect the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = 0$.

2) Find the equation of the cone whose vertex is (α, β, γ) and base $ax^2 + by^2 = 1, z = 0$

3) Find the equation of a cone whose vertex is (α, β, γ) and base $y^2 = 4ax, z = 0$.

4) Enveloping cone of a sphere: Theorem. Find the eqn of the cone whose vertex is at the point (α, β, γ) and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$

5) Find the enveloping cone of the sphere $x^2 + y^2 + z^2 - 2x + 4z = 1$ with its vertex at $(1, 1, 1)$

6) ST the general eqn to the cone which passes thro' the three axes is $fyz + gzx + hxy = 0$.

7) Find the equation to the cone which passes thro' the three coord axes as well as the two lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$, $\frac{x}{3} = \frac{y}{-1} = \frac{z}{1}$.

8) Find the eqn of the quadric cone which passes thro' the three coord axes and the three mutually perpendicular lines $\frac{1}{2}x = y = -z$, $x = \frac{1}{3}y = \frac{1}{5}z$, $\frac{1}{8}x = -\frac{1}{11}y = \frac{1}{5}z$

9) ST the equation $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$ represents a cone with vertex $(-1, -2, -3)$

10) PT the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$

11) P.T the Eqn $2x^2+2y^2+7z^2-10yz-10zx+2x+2y+26z-17=0$ represents a Cone with Vertex at $(2,2,1)$

12) S.T the Eqn $x^2-2y^2+3z^2-4xy+5yz-6zx+8x-19y-2z-20=0$ represents a Cone with Vertex $(1,-2,3)$

13) Find the Eqns to the lines in which the plane $2x+y-z=0$ cuts the Cone $4x^2-y^2+3z^2=0$ [Similar problems also]

14) P.T the Cones $ax^2+by^2+cz^2=0$ and $\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=0$ are reciprocal.

15) P.T the tangent planes to the Cone $x^2-y^2+2z^2-3yz+4zx-5xy=0$ are perpendicular to the Generators of the Cone $17x^2+8y^2+29z^2+28yz-46zx-16xy=0$.

16) P.T the Cones $fyz+gzx+hxy=0$, $\sqrt{fx}+\sqrt{gy}+\sqrt{hz}=0$ are reciprocal.

17) Theorem Right circular Cone Find the Eqn of the right circular Cone whose Vertex is the point (α,β,γ) and whose axis is the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ and Semi Vertical angle θ.

18) Find the Eqn to the right Circular Cone whose Vertex is $P(2,-3,5)$, axis PQ which makes equal angles with axes and Semi vertical angle is 30°.

19) P.T $x^2-y^2+z^2-4x+2y+6z+12=0$ represents a right circular Cone whose Vertex is the point $(2,1,-3)$, whose axis is parallel to OY and whose Semi Vertical angle is 45°.

* All Similar problem on Right Circular Cone.

20) Cylinder:

* To find Eqn of the cylinder - Theorem.

* Enveloping cylinder - Theorem

* Right circular cylinder - Theorem

* All problem based on the above three theorems are very very important.