Stability Analysis of a Control System

SIGNALS AND SYSTEMS J - COMPONENT

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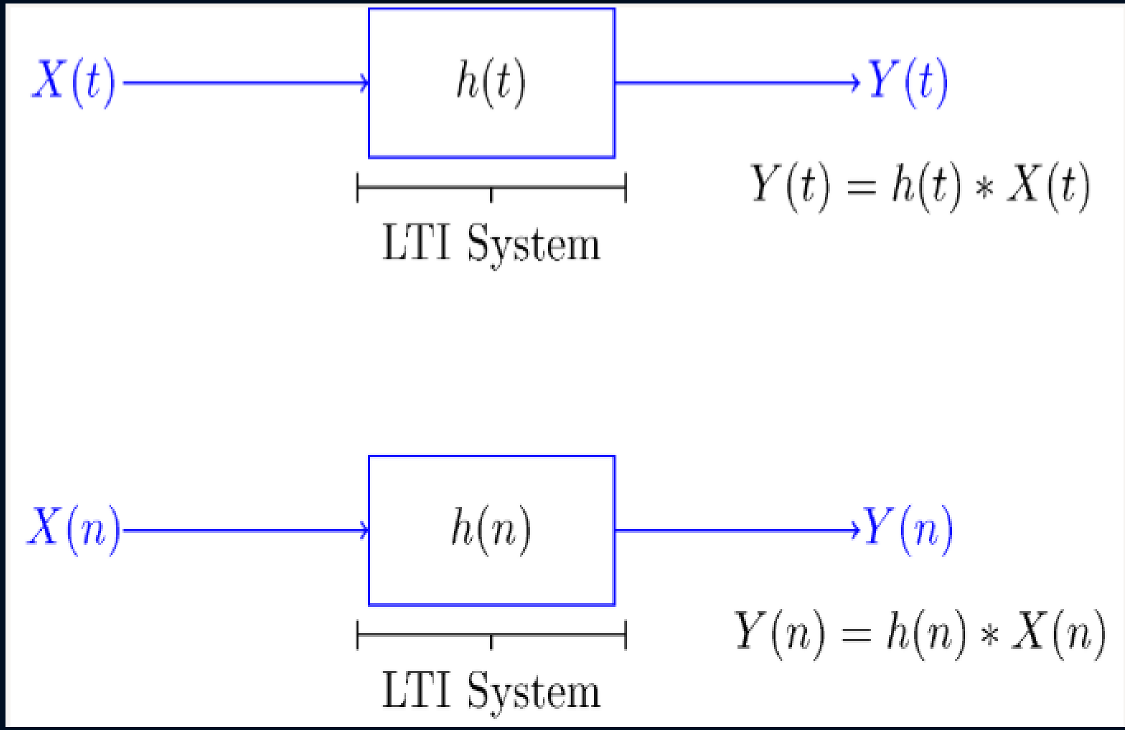
➢ ABSTRACT – LTI SYSTEMS

- Here, we take the case of LTI Systems
- Linear time-invariant system, commonly known as LTI system theory, investigates the response of a linear and time-invariant system to an arbitrary input signal.
- Trajectories of these systems are commonly measured and tracked as they move through time (e.g., an acoustic waveform), but in applications like image processing and field theory, the LTI systems also have trajectories in spatial dimensions.
- Thus, these systems are also called *linear translation-invariant* to give the theory the most general reach.

➢ ABSTRACT – LTI SYSTEMS

- In the case of generic discrete-time (i.e., sampled) systems, *linear shift-invariant* is the corresponding term.
- A good example of LTI systems are electrical circuits that can be made up of resistors, capacitors, and inductors.
- It has been used in applied mathematics and has direct applications in NMR spectroscopy, seismology, circuits, signal processing, control theory, and other technical areas.



➢ ABSTARCT - STABILITY

- In signal processing, specifically control theory, bounded-input, bounded-output (BIBO) stability is a form of stability for linear signals and systems that take inputs. If a system is BIBO stable, then the output will be bounded for every input to the system that is bounded.
- A signal is bounded if there is a finite value B> o such that the signal magnitude never exceeds B, that is

$$|y[n]| \leq B \quad \forall n \in \mathbb{Z} \text{ for discrete-time signals, or}$$
$$|y(t)| \leq B \quad \forall t \in \mathbb{R} \text{ for continuous-time signals.}$$

- A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. A stable system produces a bounded output for a given bounded input.

ABSTRACT - STABILITY

- Stability helps evaluate how a measurement device performs over time.
- Stability looks at whether the bias changes over time. If the bias does change, the device is less repeatable over time.
- Stability studies also help determine the proper increment between calibration and repair intervals.
- For finding the stability of an LTI system we first find the 'Transfer Function of the system and then we find the '**Poles'** of the system.
- Based on poles we determine the 'Stability' of the system.

➢ CONVOLUTION

- To derive the transfer function we first need y(t) i.e, **Output response**, whose Laplace Transform divided by the laplace transform of input x(t) gives the **Transfer Function** , G(s).
- To achieve y(t), we need '**Convolution Property.**'
- **Convolution** is a mathematical operation on two functions (*f* and *g*) that produces a third function expressing how the shape of one is modified by the other.
- The term *convolution* refers to both the result function and to the process of computing it.
- It is defined as the integral of the product of the two functions after one is reversed and shifted.

➢ CONVOLUTION

- The convolution of *f* and *g* is written *f*∗*g*, using an asterisk. It is defined as the integral of the product of the two functions after one is reversed and shifted. As such, it is a particular kind of integral transform.
- Convolution describes the output (in terms of the input) of an important class of operations known as *linear time-invariant* (LTI).
- In terms of the Fourier/Laplace transforms of the input and output of an LTI operation, no new frequency components are created. The existing ones are only modified (amplitude and/or phase).

$$(f * g)(t) \triangleq \int_{-\infty}^{\infty} f(\tau) g(t - \tau) \, d\tau.$$

➢ CONVOLUTION

- In other words, the output transform is the pointwise product of the input transform with a third transform (known as a transfer function).
- After application of Fourier/Laplace, the asterisk property changes to multiplication.
- Hence, we can use this property to find y(t), which is used in transfer function.
- Alternatively, we can find laplace transform of h(t), which is nothing but Transfer Function(G(s)).

The Laplace transform also has the multiplication property, i.e.

$$x(t) \overset{L}{\leftrightarrow} X(s) \qquad \text{ROC}=R_1$$

$$h(t) \overset{L}{\leftrightarrow} H(s) \qquad \text{ROC}=R_2$$

$$x(t) * h(t) \overset{L}{\leftrightarrow} X(s)H(s) \qquad \text{ROC} \supseteq R_1 \cap R_2$$

➢ TRANSFER FUNCTION

- A **transfer function** represents the relationship between the output signal of a control system and the input signal, for all possible input values.
- A block diagram is a visualization of the control system which uses blocks to represent the transfer function, and arrows which represent the various input and output signals.
- The cause and effect relationship between the output and input is related to each other through a **Transfer function**.
- In a Laplace Transform, if the input is represented by R(s) and the output is represented by C(s), then the transfer function will be

$$G(s) = \frac{C(s)}{R(s)} \Rightarrow R(s).G(s) = C(s)$$

➢ MATLAB CODE

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Stability Analysis of a Continuos Time LTI System using Laplace Transform.m | Untitled* | +

```
%Stability Analysis of a continuous Time LTI System using Laplace Tr
clear all
clc
syms t a b  m u s
x=input('Enter the input of the system:');
h=input('Enter the impulse response of the system:');
X=laplace(x);
H=laplace(h);
Y=X*H;
y=simplify(ilaplace(Y));
disp('The Laplace Transform of input function is X:');
disp(X);
disp('The Laplace Transform of impulse response is H:');
disp(H);
disp('The Laplace Transform of output function is Y:');
disp(Y);
disp('The output of the system is y:');
disp(y);
G=(Y/X);
disp('The Transfer function of the system is G(s):');
disp(G);
[n,d]=numden(G);
disp('The denominator of the Transfer function is d:');
disp(d);
p=solve(d);
n=length(p);
```

➢ MATLAB CODE:

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Stability Analysis of a Continuos Time LTI System using Laplace Transform.m | Untitled*

```
p=solve(d);
n=length(p);
disp('The poles of the system is P:');
for i=1:n
    disp(p(i));
    i=i+1;
end
m=0;
u=0;
s=0;
for j=1:n
    if(real(p(j))>0)
        u=u+1;
    end
    j=j+1;
end
if(u>0)
    disp('The system is unstable');
end
for k=1:n
    if(real(p(k))==0)
        m=m+1;
    end
    k=k+1;
end
if(u==0)
```

➢ MATLAB CODE

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Stability Analysis of a Continuos Time LTI System using Laplace Transform.m | Untitled*

```matlab
            k=k+1;
        end
        if(u==0)
            if(m>0)
                disp('The system is marginally stable');
            end
        end
        if(u==0)
            if(m==0)
                for f=1:n
                    if(real(p(f))<0)
                        s=s+1;
                    end
                    f=f+1;
                end
            end
        end
        if(u==0)
            if(m==0)
                if(s>0)
                    disp('The system is stable');
                end
            end
        end
```

➢ OUTPUT

Command Window

```
Enter the input of the system:exp(-3*t)*heaviside(t)
Enter the impulse response of the system:t^3+3*t^2+exp(-2*t)+3*sin(t)
The Laplace Transform of input function is X:
1/(s + 3)

The Laplace Transform of impulse response is H:
1/(s + 2) + 3/(s^2 + 1) + 6/s^3 + 6/s^4

The Laplace Transform of output function is Y:
(1/(s + 2) + 3/(s^2 + 1) + 6/s^3 + 6/s^4)/(s + 3)

The output of the system is y:
exp(-2*t) - (4*t)/9 - (229*exp(-3*t))/270 - (3*cos(t))/10 + (9*sin(t))/10 + (2*t^2)/3 + t^3/3 + 4/27

The Transfer function of the system is G(s):
1/(s + 2) + 3/(s^2 + 1) + 6/s^3 + 6/s^4

The denominator of the Transfer function is d:
s^4*(s^2 + 1)*(s + 2)

The poles of the system is P:
-2

0

0

0

0

-1i

1i

The system is marginally stable
>>
```

OUTPUT

Command Window

```
Enter the input of the system:t^2+3*t
Enter the impulse response of the system:exp(-t)*sin(2*t)
The Laplace Transform of input function is X:
3/s^2 + 2/s^3

The Laplace Transform of impulse response is H:
2/((s + 1)^2 + 4)

The Laplace Transform of output function is Y:
(2*(3/s^2 + 2/s^3))/((s + 1)^2 + 4)

The output of the system is y:
(22*t)/25 + (2*t^2)/5 + (64*exp(-t)*(cos(2*t) - (23*sin(2*t))/64))/125 - 64/125

The Transfer function of the system is G(s):
2/((s + 1)^2 + 4)

The denominator of the Transfer function is d:
s^2 + 2*s + 5

The poles of the system is P:
- 1 - 2i

- 1 + 2i

The system is stable
fx >>
```