Advances in Digital Signal Processing: Imaging and Image Processing

TECHNISCHE UNIVERSITÄT DARMSTADT

Exercise 1: Detection and Estimation Theory **Due date: 11.05.2015**

Problem 1 - Introduction to MATLAB

In this exercise you will learn the basic concepts of MATLAB. MATLAB stands for MATrix LABoratory, which makes clear that the focus is on efficiently implemented matrix operations. If you're not familiar with MATLAB, you might want to have a look at the MATLAB primer (www.math.kent.edu/~reichel/courses/num.comp.1/primer.pdf). You can access the documentation of a command by typing **help** or **doc** followed by the name of the command.

a) Create a vector t that ranges from 0 to 20 with a spacing of 0.1. What is the length N of this vector? Use the MATLAB command **length**.

Next, we want to compute $y = \sin(2\pi t)$. In many programming languages, this is achieved by a **for**-loop. MATLAB offers a different approach and we will compare both of them. First, we will consider an implementation using a **for**-loop.

b) Define y of length N and initialize the values with zero.

c) Now create a **for**-loop and for each element of t, compute the respective y value. You can plot your result using the **plot** command.

d) What do you observe from the plot regarding the minima and maxima? What would you suggest to solve this problem?

Let us now use the MATLAB-optimized implementation:

e) MATLAB is a matrix oriented language, which allows us to write down matrix operations mathematically. Compute y without using a loop.

f) Repeat the calculation of y with the vector t ranging from 0 to 10000 using the same spacing of 0.1. Use **tic** and **toc** to evaluate the runtime of both methods. What do you observe?

g) MATLAB has excellent plotting functions. Explain shortly in which cases you would choose **plot**, **surf**, **hist** or **scatter**.

Problem 2 - Ideal Observer Test

In this task, two datasets are provided containing samples of a target T and non-target (noise) N. In the following, we assume that the samples T and N follow Gaussian distributions, i.e.

$$p(x|K) = \frac{1}{\sigma_T \sqrt{2\pi}} \exp\{-\frac{(x-\mu_T)^2}{2\sigma_T^2}\}$$

and similarly

$$p(x|H) = \frac{1}{\sigma_N \sqrt{2\pi}} \exp\{-\frac{(x-\mu_N)^2}{2\sigma_N^2}\}$$

where the hypothesis K denotes a target being present and H indicates noise only.

a) Load the file **data.mat** containing samples of a target and samples of non-targets (noise) using **load**.

b) Assuming that T and N are Gaussian distributed with the parameters μ_T, σ_T^2 and μ_N, σ_N^2 find their maximum likelihood estimates. **Hint:** Use the commands **mean** and **var**.

c) Concatenate both data sets by typing **[T;N]** (for column vectors) in order to determine the maximum and minimum value x_l and x_u, respectively, of the joint sample set. Create a vector x ranging from x_l to x_u with a spacing of 0.02.

d) Use **normpdf** to estimate the probability density functions $p(x|K)$ and $p(x|H)$ of the two data sets.

e) Plot $p(x|K)$ and $p(x|H)$ in a single figure using the **hold** and **plot** commands.

f) Now, find an analytical solution of the ideal observer test

$$\frac{p(x|K)}{p(x|H)} \underset{H}{\overset{K}{\gtrless}} 1.$$

According to this test, what value should the threshold x_0 take?

Problem 3 - Receiver Operator Characteristics

Considering the probability density functions $p(x|K)$ and $p(x|H)$ from Problem 2, the receiver operator characteristics (ROC) of a simplified test is to be plotted. This test is defined by

$$x \underset{H}{\overset{K}{\gtrless}} \delta \,,$$

where δ denotes the decision threshold.

a) Define a vector D ranging from x_l to x_u with a spacing of 0.02.

b) For every threshold δ, given by the elements in D, compute the probability of false alarm $\alpha(\delta)$ and the probability of correct detection $p_d(\delta)$.

c) Plot the ROC, i.e. $p_d(\delta)$ versus $\alpha(\delta)$.

d) Now, load **data2.mat** and estimate $\alpha(\delta)$ and $p_d(\delta)$ again. What do you observe?