Advances in Digital Signal Processing: Imaging and Image Processing

TECHNISCHE UNIVERSITÄT DARMSTADT

Exercise 2: Radar Signal Processing **Due date: 01.06.2015**

Problem 1 - Parameter Estimation

A Gaussian mixture model can be used to model the effect of deviations from the model assumptions, such as the impulsive noise. Mathematically it is defined as

$$Q = (1 - \epsilon)P_1 + \epsilon P_2,$$

where ϵ is the contamination factor $0 < \epsilon < 1$, $P_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ is the normal distribution and $P_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ is the contaminating distribution. The following code part creates one sample from the Gaussian mixture model.

```
if rand(1) < ϵ
    sample= μ2 + σ2*randn(1);
else
    sample= μ1 + σ1*randn(1);
```

In the following questions we want to estimate the mean of a signal which is distributed as P_1, however corrupted by another signal which is distributed as P_2.

a) Using the given code part, write a function, function **samples = gmm**$(\epsilon, \mu_1, \mu_2, \sigma_1, \sigma_2, N)$ which creates N samples from the Gaussian mixture model.

b) Using the function **gmm**, create random vectors for $\epsilon = 0.1$, $\mu_1 = 0$, $\sigma_1 = 1$, $\mu_2 = 2.5$, $\sigma_2 = 3$ when $N = 10$, $N = 1000$, $N = 100000$.

c) Now use **hist** command in MATLAB and estimate the histogram of the random vectors which you found in b). How does the histogram change with the number of samples?

d) The *sample mean* estimator, $\frac{1}{N}\sum_{i=1}^{N} x_i$, is the maximum likelihood estimator for the mean μ of the Gaussian distributed signal (no contamination $\epsilon = 0$ in gmm model). Use this estimator and estimate μ_1 from the random vectors that you obtained in b).

e) Consider the *median* estimator and repeat the experiments in d). **Hint:** Use the **median** command in MATLAB.

f) For varying ϵ, $\epsilon \in \{0.2, 0.3\}$ repeat the parts b), d) and e).

g) Compare the *bias, variance* and the *mean squared error* (MSE) of both estimators for all conducted experiments. Which estimator is more suitable for the considered problem? Why? **Hint:** Expectation can be estimated by repeating and averaging the experiments.

Problem 2 - Detection in Range

In the following task, you will implement a range detection system as found in sonar or radar applications. For convenience, we consider the complex chirp signal $p(t)$

$$p(t) = a(t)\exp\{j\beta t + j\alpha t^2\}$$

where $a(t) = 1$ for $0 \leq t \leq T_p$ and zero otherwise. The duration of the pulse is denoted by T_p.

a) The instantaneous angular frequency $\omega(t)$ for $a(t) = 1$ is given by $\omega(t) = \dfrac{\mathrm{d}\left(j^{-1}\ln p(t)\right)}{\mathrm{d}t}$. Compute $\omega(t)$. What are the bounds of $\omega(t)$ and what is the center frequency ω_c?

b) Next, find an expression for β depending on ω_c, α and T_p.

c) What is the bandwidth B? Give an expression for α depending on B and T_p.

Given these values, we can start to implement this system in MATLAB. Note that the values in **problem02.m** are provided as frequencies f in [Hz] and $\omega = 2\pi f$.

d) First, we simulate the response $ee(t)$ of all targets defined by

$$ee(t) = \sum_{n=1}^{N} \frac{1}{d_n} r_t p(t - t_{d,n})$$

Compute all N distances between the target-receiver pairs d_n. The respective time delay is $t_{d,n} = 2d_n/c$ where c is the propagation speed. The target reflectivity $r_t = 0.1$ is constant for all targets.

e) Next, the signal has to be demodulated. Remember that this is easily achieved by a shift in the frequency domain by f_c.
Hint: In our case it is easier to perform this task in the time domain, i.e. $ee_b(t) = ee(t)\exp\{-j2\pi f_c t\}$.

f) The focused signal $ss(t)$ is obtained by pulse compression. For this purpose, we first define the matched filter in the base band

$$h_m(t) = p(-t)^* \exp\{-j2\pi f_c t\}$$

and convolve it with $ee_b(t)$.
Hint: Note that convolution extends the signal. Make sure that you extract the correct interval.

g) What happens when B is decreased from 8000Hz to 3000Hz? Comment on your results.