Anthony Ding

Week 10 Cribsheet

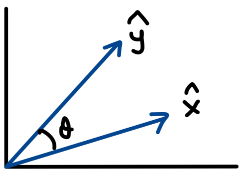
NOTES: 21, 22

MODULE 3 MATH

Inner product of $\vec{x}, \vec{y} \in \mathbb{R}^n$: $\langle \vec{x}, \vec{y} \rangle = \vec{x} \cdot \vec{y} = \vec{x}^T \vec{y} = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$

Properties: - commutative : $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$

- Scalar multiplicative: $\langle c\vec{x}, \vec{y} \rangle = c \langle \vec{x}, \vec{y} \rangle = \langle \vec{x}, c\vec{y} \rangle$
- distributive over addition: $\langle \vec{x} + \vec{y}, \vec{z} \rangle = \langle \vec{x}, \vec{z} \rangle + \langle \vec{y}, \vec{z} \rangle$



$\langle \hat{x}, \hat{y} \rangle = \cos\theta$

where $\hat{x}, \hat{y}$ are **normalized** $\vec{x}, \vec{y}$

$\vec{x}, \vec{y}$ **orthogonal** if $\langle \vec{x}, \vec{y} \rangle = 0$

(geometrically, orthogonal = 90° angle, and $\cos 90^\circ = 0$)

Norm - specifically, the 2-norm: $\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$

Generalization: the p-norm is $\|\vec{x}\|_p = \sqrt[p]{|x_1|^p + |x_2|^p + \dots + |x_n|^p}$

↳ special cases (out of scope): 1-norm $\|\vec{x}\|_1 = \sum_{i=1}^{n} |x_i|$

∞-norm $\|\vec{x}\|_\infty = \max_i |x_i|$

To "normalize" $\vec{x}$ and $\vec{y}$, set $\hat{x} = \frac{\vec{x}}{\|\vec{x}\|}$ and $\hat{y} = \frac{\vec{y}}{\|\vec{y}\|}$

Properties of Norms: - nonnegativity: $\|\vec{x}\| \geq 0$ always

- zero vector: $\|\vec{x}\| = 0 \iff \vec{x} = 0$
- Scalar mult: $\|\alpha \vec{x}\| = \|\alpha\| \, \|\vec{x}\|$
- $\triangle$ ineq: $\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$

Cauchy-Schwarz Inequality $|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\| \, \|\vec{y}\|$

CORRELATIONS

Cross-correlation $\quad \mathrm{corr}_{\vec{x}}(\vec{y})[k] = \sum_{i=-\infty}^{\infty} x[i]\, y[i-k]$