CSM EECS 16A
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Section 3 Cribsheet

Definitions

- A set of vectors S, where each vector is from a vector space V, is a subspace of V - denoted $S \subseteq V$ - iff:
    - S contains $\vec{0}$
    - S is closed under addition: $\forall \vec{u}, \vec{v} \in S$, $\vec{u} + \vec{v} \in S$
    - S is closed under scalar multiplication: $\forall \vec{u} \in S$ and $\alpha \in \mathbb{R}$, $\alpha\vec{u} \in S$
- A set of vectors $B = \{\vec{b_1}, \vec{b_2}, \dots, \vec{b_n}\}$ where each vector is from a vector space V, forms a basis of V if:
    - $\vec{b_1}, \vec{b_2}, \dots, \vec{b_n}$ linearly independent
    - $\forall \vec{u} \in V$, $\vec{u}$ can be written as a linear combination of $\vec{b_1}, \vec{b_2}, \dots, \vec{b_n}$ (aka B spans V)
- The dimension of a basis B, $\dim(B)$, is the # of elements in B
- The column space of a matrix A, $\text{col}(A)$, is the span of A's columns
- The nullspace of a matrix $A \in \mathbb{R}^{m \times n}$, $\text{null}(A)$, is the set of all vectors $\vec{v} \in \mathbb{R}^n$ such that $A\vec{v} = \vec{0}$. A trivial nullspace only contains $\vec{0}$
- The rank of a matrix A, $\text{rank}(A)$, is $\dim(\text{col}(A))$

Theorems

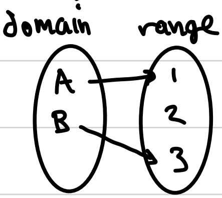
- Any nonzero $\vec{x}$ cannot be in both col(A) and null(A)
- Rank-nullity theorem: if $A \in \mathbb{R}^{n \times n}$, $\text{rank}(A) + \dim(\text{null}(A)) = n$
- $A \in \mathbb{R}^{n \times n}$ is invertible $\Leftrightarrow$

  A has a trivial nullspace $\{\vec{0}\}$ $\Leftrightarrow$

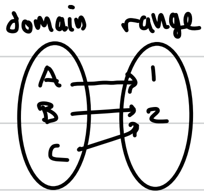
  A is full rank, i.e. $\text{rank}(A) = n$ $\Leftrightarrow$

  A's columns form a basis for $\mathbb{R}^n$ $\Leftrightarrow$

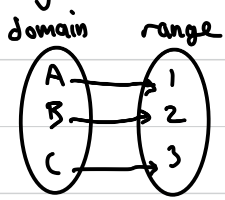
  [Extra] the transformation $\vec{x} \to A\vec{x}$ is bijective $\begin{cases} \text{one-to-one} \\ \text{onto} \end{cases}$



injective
(one-to-one)



Surjective
(onto)



bijective
(one-to-one correspondence)