magic word:
bogears

CSM 16A Section 1

GE
Matrices / vectors
✓ Span / linear (in) dependence
✓ Linearity

$$f(x_1, x_2) = ax_1 + bx_2$$

superposition

↳ $f(x_1, x_2) + f(y_1, y_2)$
$= f(x_1 + y_1, x_2 + y_2)$

multiplicative

$f(\alpha x_1) = \alpha f(x_1)$

$f_1 = 3x_1 + 4x_2$ $\qquad$ $f_2 = 3y_1 + 4y_2$

$3x_1 + 4x_2 + 3y_1 + 4y_2 = 3(x_1 + y_1) + 4(x_2 + y_2)$

Note: $\{a, b, c\}$

$\{x \in \mathbb{R} \mid x \leq 5\}$

Span

vector — 1

space — 4

"set of vectors"

$$3\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \qquad \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

$$\mathbb{R}^2 \rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \underline{a}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \underline{b}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

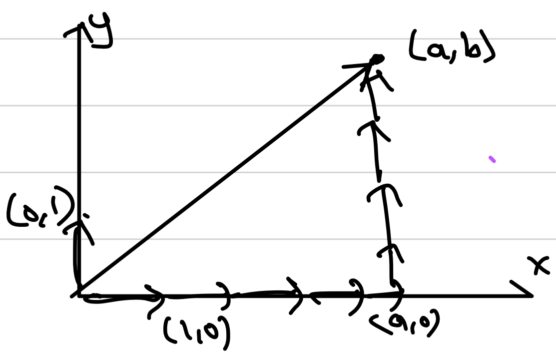
$$= \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$= \begin{bmatrix} a \\ b \end{bmatrix}$$

a times

b times

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \dots + \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \dots + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$\begin{bmatrix}1\\0\end{bmatrix} \begin{bmatrix}0\\1\end{bmatrix} \longrightarrow$ any pt in $\mathbb{R}^2$

these vectors span $\mathbb{R}^2$

Some set of vectors ________ spans ________ a vector space

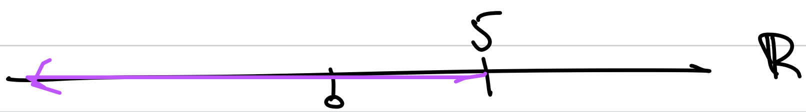
Notation

$\{$ your starting point $\mid$ constraints $\}$

$\{$ all apples owned by Carol Christ $\mid$ 1. not expired 2. Granny Smith $\}$

collection

$\{ x \in \mathbb{R} \mid x \leq 5 \}$



$\{ (x,y) \in \mathbb{R}^2 \mid x + y^y \leq 5x^x \}$

S spans $\mathbb{R}^3$

$\vec{s_1}, \vec{s_2}, \vec{s_3}$

$\alpha, \beta, \gamma \in \mathbb{R}$

for any point in $\mathbb{R}^3$ $\vec{p}$

$$\vec{p} = \alpha \vec{s_1} + \beta \vec{s_2} + \gamma \vec{s_3}$$

Spanning Set #4 on WS 1

(a) $A\vec{x} = \vec{b}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + 2x_2 \\ 2x_1 + x_2 \\ 0 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

*

$$\begin{bmatrix} | & | & | & & | \\ \vec{a}_1 & \vec{a}_2 & \vec{a}_3 & \cdots & \vec{a}_n \\ | & | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= x_1\vec{a}_1 + x_2\vec{a}_2 + \dots + x_n a_n$$

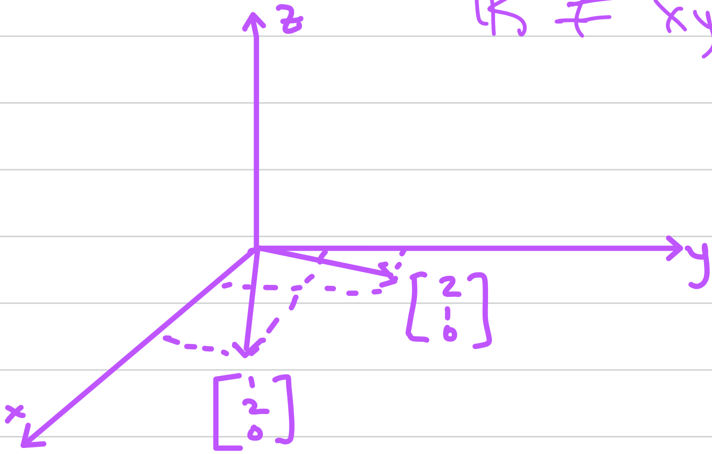
Linear combination

$$\alpha\vec{v} + \beta\vec{w}$$

$$\begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix} \rightarrow \quad b \in \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

must be $\underline{0}$ $\quad \vec{b} \in \text{range}(A)$

(b)



$\mathbb{R}^2 \neq xy\text{-plane}$

$z = 0$

(c)

$$\left[\begin{array}{cc|c} 1 & 2 & -3 \\ 2 & 1 & 5 \\ 0 & 0 & 2 \end{array}\right] \quad R_2 = R_2 - 2R_1$$

$$\left[\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -3 & 11 \\ 0 & 0 & \end{array}\right] \to 0 \overset{?!}{=} 2$$

$x_1 : \frac{13}{3}, \quad x_2 : -\frac{11}{3}$

$\vec{v}_1$

(d) $\text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 0 \end{bmatrix} \right\}$

$$0 = \frac{13}{3}\vec{a}_1 - \frac{11}{3}\vec{a}_2 - \vec{a}_3$$

span stays the same

xy-plane: $\{ \vec{v} \in \mathbb{R}^3 \mid v[3] = 0 \}$

(e)

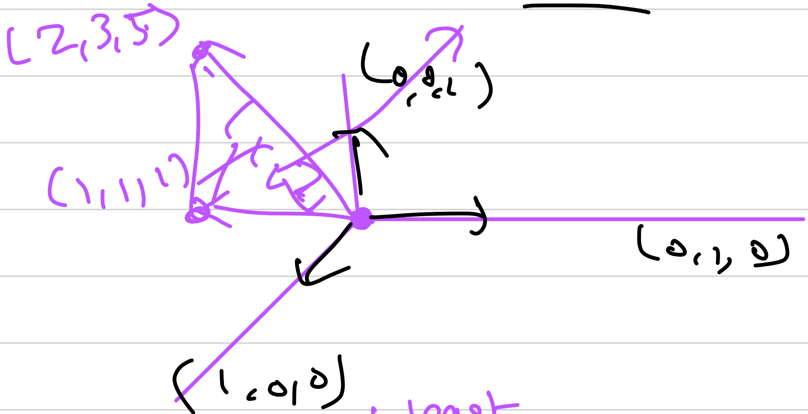
$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\{ \vec{v} \in \mathbb{R}^3 \mid v\{3\} \neq 0 \}$

2 vectors span $\mathbb{R}^3$

↳ is this possible?

$$\begin{bmatrix}1\\1\\1\end{bmatrix} \qquad \begin{bmatrix}2\\3\\5\end{bmatrix} \qquad \begin{matrix}a\\b\\c\end{matrix}$$



at least

you need 3 vectors $\mathbb{R}^3$

linearly independent

$\mathbb{R}^n \to n$ vectors