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CSM 16A  
Spring 2021

Designing Information Devices and Systems I

Week 1

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## 1. Gaussian Elimination

**Learning Goal:** The goal of this problem is to use Gaussian Elimination to describe solutions to systems, both qualitatively and quantitatively. Please review [Note 1B](#) to understand this problem better.

Write each system as an augmented matrix, and then solve using Gaussian Elimination. Also determine whether each system has no solution, a unique solution, or a set of infinitely many solutions.

(a) Solve the following system of equations:

$$x_1 - x_2 + 2x_3 = 15$$

$$3x_2 - x_3 = 8$$

$$x_1 + 2x_3 = 21$$

(b) [WALK-THROUGH] Solve the following system of equations:

$$x_1 + 2x_2 + 3x_3 = 4$$

$$x_1 + x_3 = 0$$

$$-2x_1 + 2x_2 + x_3 = 5$$

$$x_1 + x_2 + 2x_3 = 2$$

## (c) [WALK-THROUGH]

(i) Now let us just change the third equation from the last problem to

$$-2x_1 + 2x_2 = 4.$$

The other three equations are unchanged. Do you still have a unique solution?

(ii) What if you change the third equation to

$$-2x_1 + 2x_2 = 5?$$

## (d) (PRACTICE) Solve the following system of equations:

$$2x_2 + 4x_3 = -2$$

$$-5x_3 = 10$$

$$x_1 + x_2 - 3x_3 = 8$$

## (e) (PRACTICE) Solve the following system of equations:

$$x_1 + 3x_2 - 2x_3 = -3$$

$$2x_1 + 6x_2 - 4x_3 = -5$$

## 2. Computations: Matrix-Vector Operations [WALKTHROUGH]

**Learning Goal:** The goal of this problem is to present various cases of matrix-vector operations such as addition, multiplication, and transpose. Please review [Note 2A: Section 2.3](#) and [Note 2B: Section 2.1](#) to understand this problem better.

Consider the following matrices and vectors. Complete the parts below.

$$A = \begin{bmatrix} 2 & 4 \\ 5 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 2 & -4 \end{bmatrix} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u}_2 = \begin{bmatrix} 3 \\ -4 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

- (a) What is the transpose of  $\vec{v}$ ?
  
- (b) What is  $(\vec{v} + \vec{w})^T$ ? Find  $\vec{v}^T + \vec{w}^T$  too. Compare the results.
  
- (c) What is  $2\vec{v} - 4\vec{w}$ ?
  
- (d) What is  $\vec{v}^T \vec{w}$ ?
  
- (e) What is  $A\vec{u}_1$ ? What is  $A\vec{u}_2$ ?
  
- (f) What is  $\mathbf{AB}$ ? (Do the columns of  $\mathbf{AB}$  look familiar?)
  
- (g) Find  $\mathbf{B}^T$ . Then express  $\mathbf{B}^T$  in terms of  $\vec{u}_1$  and  $\vec{u}_2$ ?

### 3. Linear or Nonlinear

**Learning Goal:** The goal of this problem is to draw a distinction between linear and non-linear functions. Please review [Section 1.4 of Note 1](#) to understand this problem better.

Determine whether the following functions are linear or nonlinear.

(a) [WALK-THROUGH]

$$f(x_1, x_2) = 3x_1 + 4x_2$$

(b)

$$f(x_1, x_2) = x_1^2 + e^{x_2}$$

(c)

$$f(x_1, x_2) = \sin(a)x_1 + e^b x_2,$$

where  $a$  and  $b$  are constants.

(d)

$$f(x_1, x_2) = x_2 - x_1 + 3$$

#### 4. Spanning Set

**Learning Goal:** The goal of this problem is to connect Gaussian Elimination and linear (in)dependence to the concept of span. Another goal is to be comfortable with the geometric representation of span.

- (a) For what values of  $b_1, b_2, b_3$  is the following system of linear equations consistent? (“Consistent” means there is at least one solution. Please see [Note 1B: Subsection 1.2.4.2](#) for more details on consistency of a system.)

$$\mathbf{A}\vec{x} = \vec{b}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- (b) What is the geometry represented by  $\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ ?

- (c) Find out if  $\vec{v}_1 = \begin{bmatrix} -3 \\ 5 \\ 0 \end{bmatrix}$  is in  $\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ . What about  $\vec{v}_2 = \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}$ ?

- (d) Reflect on your answer from part(b) and find out  $\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 0 \end{bmatrix} \right\}$ .

- (e) What is a possible choice for  $\vec{v}$  that would make  $\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \vec{v} \right\} = \mathbb{R}^3$ ?