

General Principle for Sorting Runtime

$\Theta(N)$ \leq best case

worst case $\leq \Theta(N^2)$ *

* for ^{good} sorts

1. Step by Step Sorts

2 1 8 4A 6 7 9 4B

1. Insertion Sort

\downarrow
 2 1 8 4A 6 7 9 4B
 \downarrow
 2 1 8 4A 6 7 9 4B
 1 2 8 4A 6 7 9 4B
 1 2 8 4A 6 7 9 4B
 1 2 4A 8 6 7 9 4B
 1 2 4A 6 8 7 9 4B
 1 2 4A 6 7 8 9 4B
 1 2 4A 6 7 8 9 4B
 moving left $\begin{cases} 4B < 4A \\ 4B \leq 4A \end{cases} \rightarrow \text{violates stability}$
 1 2 4A 4B 6 7 8 9

Stability:

1A 2 3 1B

1B 1A 2 3

↳ not stable

Inversion:

a b
↗ ↘

Insertion Sort

Runtime:

Best

Worst

Stability

1 2 3 4 5

5 4 3 2 1

YES

1 2 3 4 5

1 2 3 4 5

$\Theta(N)$

$\Theta(N^2)$

Selection Sort

↳ pick optimal element

every loop:

iterates through every value and pick "best" number

2 1 8 4A 6 7 9 4B

1 2 8 4A 6 7 9 4B

1 2 8 4A 6 7 9 4B

1 2 4A 8 6 7 9 4B

1 2 4A 4B 8 6 7 9

1 2 4A 4B 6 8 7 9

1 2 4A 4B 6 7 8 9

1 2 4A 4B 6 7 8 9

N elements

$N \text{ picks} \left\{ \begin{array}{l} \text{During each pick:} \\ \theta(N) \end{array} \right.$
 $\theta(N \cdot N) = \theta(N^2)$

Best

Worst

Stability

Depends

Runtime :

$\theta(N^2)$

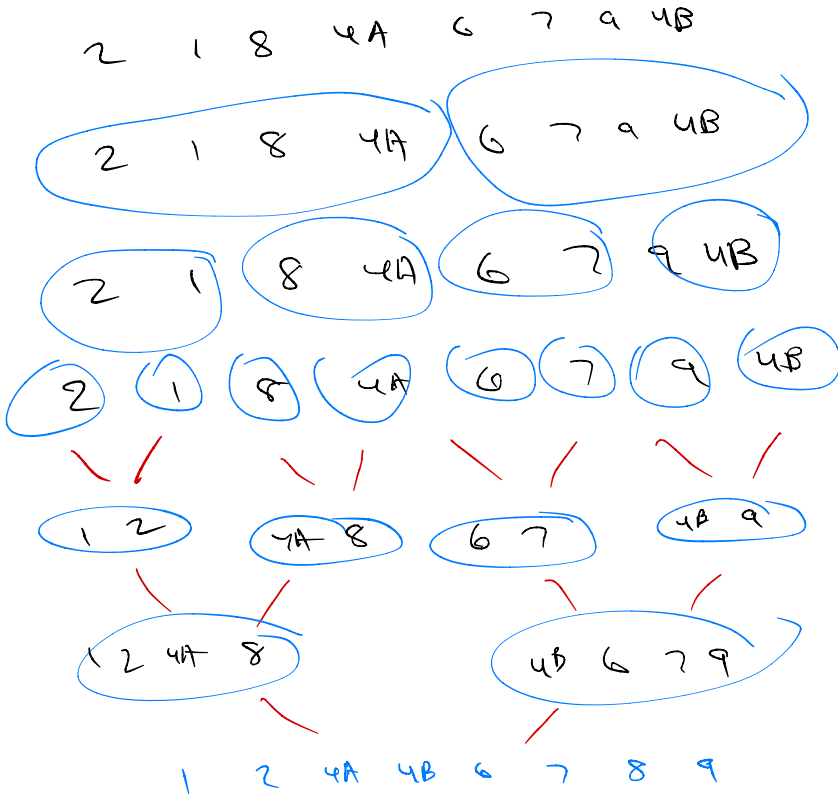
$\theta(N^2)$

Merge Sort

→ recursive

$\log N$

$\log N$



★

at every level, merge() hit every element $O(N)$

A 4A 7 B 4B 8

merge()

defaults to left value first

only works if A and B are sorted

↳ 4A 4B 7 8

Runtime?

Best

Worst

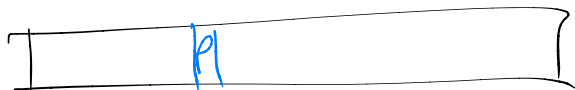
Stability

$\Theta(N \log N)$

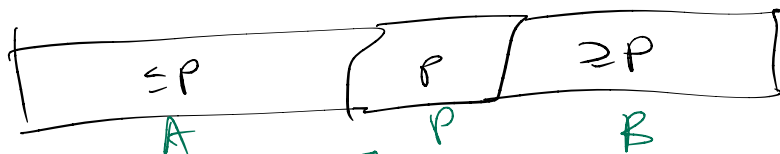
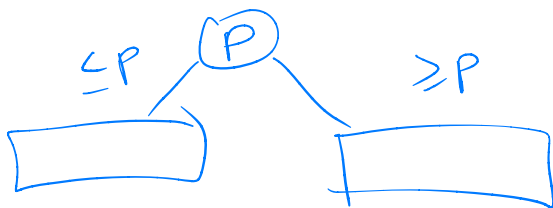
$\Theta(N \log N)$

YES

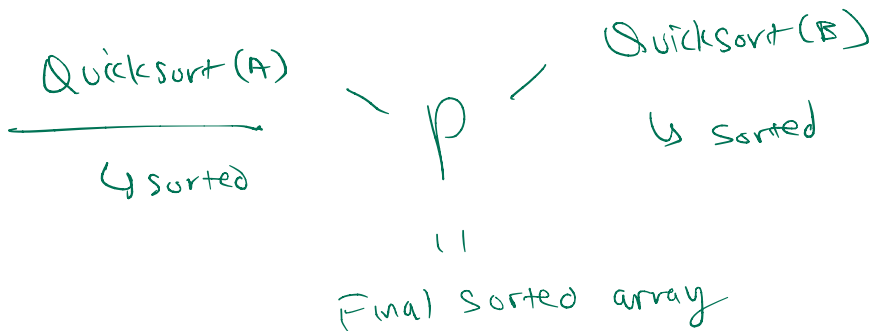
Quicksort \rightarrow recursive



pivot : choose one element based on a rule



\uparrow
P's final place



$\log N$

work per level: $\Theta(N)$

↳ "partition"
for every element

2 5 1 3 4

2 1 3 5 4

Runtime:

Best	Worst	Stability
$\Omega(N \log N)$	$\Theta(N^2)$	Depends

5 4 3 2 1

* fastest sort

3. You Choose

1. Almost / already sorted

2. 5 4 3 2 1

- 3.
- Stability
 - picking a bad pivot $\Rightarrow O(N^2)$
 - highly parallelizable

4. Quicksort — comparing to pivot

Mergesort — once merged()

Insertion — sorted elements only