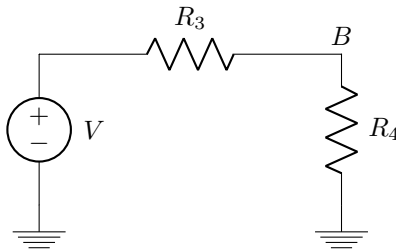
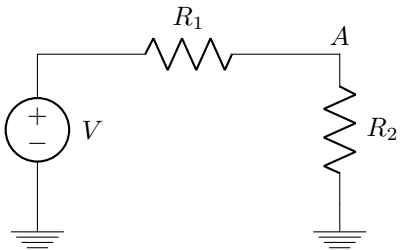


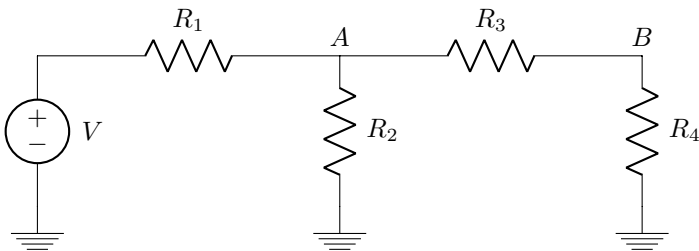
## Week 6 Worksheet

*Term:* Spring 2020*Name:***Problem 1: Kirchoff's Laws**

1. Here, you can see two standard voltage dividers. Find the voltage at the points  $A$  and  $B$  using standard nodal analysis techniques.



2. Now, let us modify this circuit to add a second voltage divider stage starting at  $A$ . You can think of this as cascading 2 voltage dividers: one which has an input of  $V$  and an output of  $V_A$ , and a second one that has an input of  $V_A$  and an output of  $V_B$ . Find the voltage at  $B$  using nodal analysis.

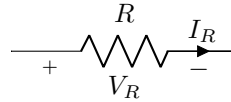


3. Compare your answer with what you get when you multiplying the amplification factors of each individual voltage divider. Are they the same or different? Does this surprise you? Why or why not?

**Problem 2: Power**

- (a) For resistors (and resistors only), we can relate the voltage drop across the resistor and the current passing through the resistor with Ohm's Law:

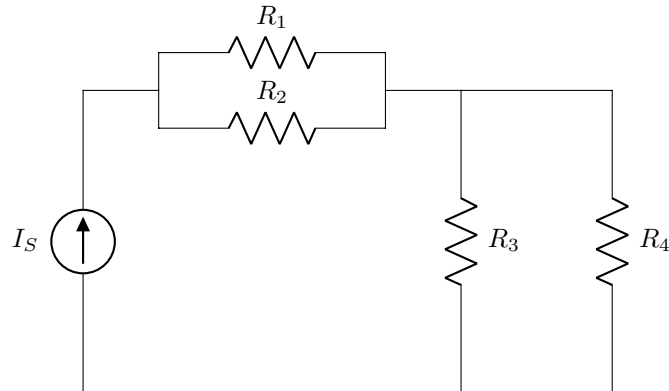
$$V_R = I_R R$$



Find an expression for the power dissipated by the resistor in terms of the following:

- i.  $V_R$  and  $I_R$
- ii.  $V_R$  and  $R$
- iii.  $I_R$  and  $R$

For the rest of this question, use the circuit below:



- (b) Which individual components have the same magnitude voltage drop across them?
- (c) Under what condition(s) will  $R_1$  and  $R_2$  dissipate the same amount of power?
- (d) Use the following values and calculate the amount of power consumed by each of the resistors  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$ .

Component	Value	Units
$R_1, R_2$	2	$\Omega$
$R_3, R_4$	6	$\Omega$
$I_S$	1	A

- (e) How much power does the current source consume? *Hint: Consider the conservation of energy.*

**Problem 3: Rubik's Cube (With Resistors!)**

In this question, we will guide you through how to apply nodes and resistance equivalence to solve a complex resistor network (like the cubic one above, assuming all resistors are identical with a resistance of  $R$ ) and find the equivalent resistance between the top back left node and the bottom front right node.

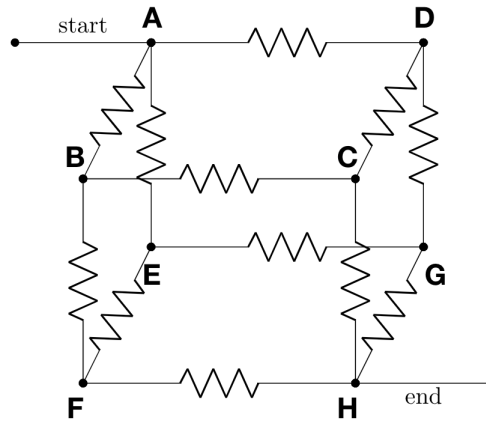


Figure 1: 3D View of the Resistor Network

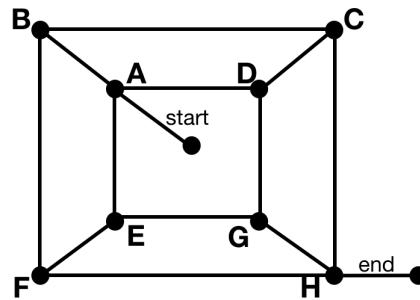


Figure 2: Flattened View of the Resistor Network,  
where each segment between 2 nodes has a resistance of  $R$

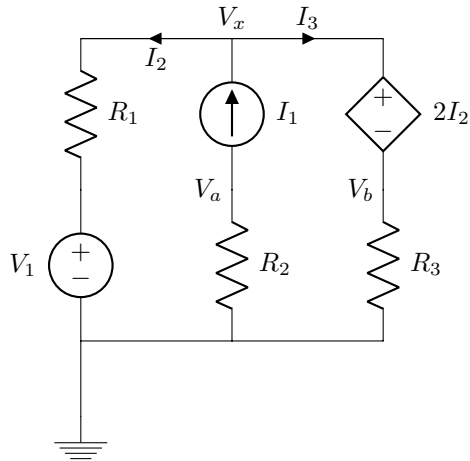
- As daunting as this question looks, fortunately we have the power of nodal analysis on our side! To get started, group the vertices from  $B$  through  $H$  based on the **distances of the shortest paths** along the edges of the cube starting from  $A$ . For example, the distance of the shortest path from  $A$  to the bottom front left vertex will be 2 (assuming each edge of the cube is 1 unit long).
- Now, consider the group of all the vertices  $v_i$  that are 1 unit away from  $A$  (the starting node in the top back left) and the resistors in and between these pairs of nodes ( $v_i, A$ ). How are they related to each other (Hint: think in terms of equivalent resistance)?
- Now, consider the group of all the vertices  $v_j$  that are 2 units away from  $A$ . Call it Group II. Let the group in the previous part be Group I. How many edges lead from a vertex in Group I to a vertex in

Group II (how many resistors are involved in this case)?

- (d) How are Group I and Group II related with each other? Is there anything we can do to simplify the resistor network so far?
- (e) Finally, How far away is  $H$  (the ending node) from  $A$  (the starting node)? How many edges are there that lead from the nodes in Group II to  $H$ ? Using equivalent resistance, is there anyway we can simplify all the circuits along the edges from the nodes in Group II to  $H$ ? (Hint: think in terms of symmetry and what you've found in part (b))
- (f) Combining all your findings together, what is the overall resistance between the starting node on the top back left and the ending node on the bottom front right in this cubic resistor network?

<b>Problem 4: Superposition</b>
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- (a) Solve the following circuit for  $V_x$  using nodal analysis. Let  $R_1 = 10\Omega$ ,  $R_2 = 5\Omega$ ,  $R_3 = 2\Omega$ ,  $V_1 = 7V$ , and  $I_1 = 6A$ .



**Note:** The diamond-shaped element in the circuit above represents a dependent voltage source. Specifically for this question, the amount of voltage it supplies depends on the current  $i_2$ :  $V_{\text{dependent}} = 2i_2$ .

- (b) Now solve the same circuit for  $V_x$  using superposition. Please note that only **independent voltage/current sources** can be removed when using superposition.