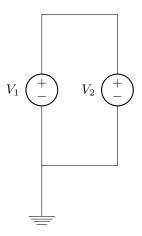
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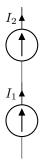
Problem 1: Introduction to Circuit Components

In this problem, we will introduce the fundamental circuit components.

- 1. What is a voltage source?
- 2. What is a current source?
- 3. What is voltage? What is a voltage drop?
- 4. Consider the figure below. If $V_1 \neq V_2$, what will happen to the circuit?



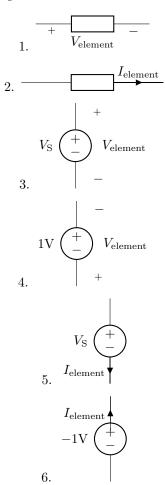
5. What happens in this case if $I_1 \neq I_2$?



- 6. What is a resistor?
- 7. What is power?

Problem 2: Passive Sign Convention

For the following components, label all the missing $V_{\rm element}$, $I_{\rm element}$, and +/- signs. Hint: The value of the voltage and current sources shouldn't affect passive sign convention—remember that voltage and current can be negative!



7. (PRACTICE)

$$I_{
m S}$$
 $V_{
m element}$ $V_{
m element}$

8. (PRACTICE)

$$I_{
m S}$$
 $V_{
m element}$ $V_{
m element}$

9. (PRACTICE)



10. (PRACTICE)



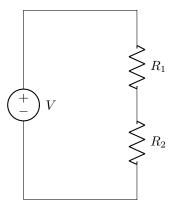
11. (PRACTICE)



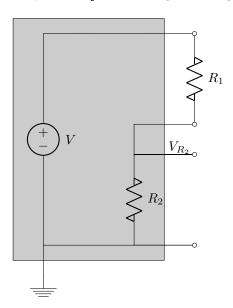
12. (PRACTICE)

Problem 3: Voltage Divider Properties

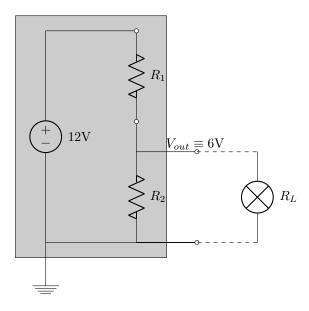
Let's take a systematic look at the voltages across a resistor, and see how other components in the circuit can affect it. Consider the following circuit:



- 1. Calculate the voltage drop across R_1 and R_2 using series resistance calculations.
- 2. Suppose we want to manipulate the voltage across R_2 , but it's locked in a box with the voltage source, as denoted below. Can we use R_1 to manipulate V_{R_2} ? What range of voltages can we achieve?



3. Now let's try using our new variable voltage source to power a light bulb with resistance R_L , where the threshold voltage for lighting the bulb is 6V. Find R_1 and R_2 so that the voltage across R_2 is this threshold voltage; that is, $V_{R_2} \equiv V_{out} = 6$ V. Assume we have a 12V voltage source.



4. Now that we found an R_1 and R_2 that seem to divide our voltage source appropriately, let's try to connect the bulb to the ends of R_2 . Remember, the bulb has a resistance R_L . Calculate the voltage across R_1 , R_2 and the light bulb when it is connected. Will the light bulb turn on?

Problem 4: Resistivity

Resistivity is a **physical property** of the material that quantifies how much it opposes the flow of electric current. Assume that in an ideal case, the cross-section and physical composition of the wire are uniform, We can find its resistivity with the equation below:

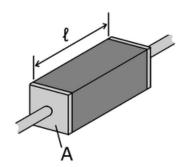
$$\rho = R \frac{A}{L}$$

Here, ρ stands for the resistivity of the wire, R stands for its resistance, A stands for the area of the cross section of the wire, and L stands for the length of the wire. Using this equation, we can also solve for the resistance of a wire:

$$R = \rho \frac{L}{A}$$

Note: Throughout the following parts of this question, we will be frequently referencing some of the following variables. In case you are confused about what these variables mean, we've included a section explaining what each variable stands for.

- A: the cross section area of a single wire.
- L: the length of a single wire.
- ρ_{cu} : resistivity for the material copper.
- ρ_{Al} : resistivity for the material aluminum.
- 1. Now, consider the rectangular copper wire below. Given that the cross-section of the wire is a square and has a cross section area of A, determine the overall resistance of the wire in terms of ρ_{cu} , L, and A.



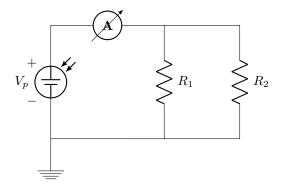
- 2. Suppose we have N such wires and align them side by side to form a mega-wire. Find the overall resistance of this mega-wire. What is this configuration similar to?
- 3. Again, with N identical wires, what's a configuration that can achieve the highest resistance possible? What is this configuration similar to?
- 4. Consider part (b) again, but this time, instead of N copper wires, we split the number evenly between aluminum wires and copper wires. First, we have N/2 copper wires on the left side, and N/2 aluminum wires on the right side, and then we push these wires together to form a new mega-wire. What's the overall resistance of this wire? (In terms of ρ_{cu} , ρ_{Al} , L, and a)

5. Instead of having all N/2 wires of **the same material** on one side before merging, now we interleave and mix every single wire together (a single copper wire can be aligned right next to a single aluminum wire), does the overall resistance of this new mega-wire change?

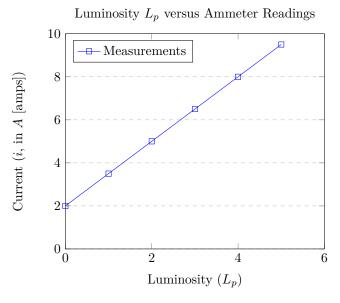
Problem 5: Signal Measurement

You are building a sensor that can detect the light in a room. Shown below is a simplified version of the main circuit for the sensor. Instead of using a normal voltage source, we are using a voltage source that changes its value based on the luminosity of the light source. Specifically, we know that the voltage V_p changes as a function of the luminosity, $V_p = f(L_p)$. However, the label on the voltage source is scratched, so we'd like to recover f.

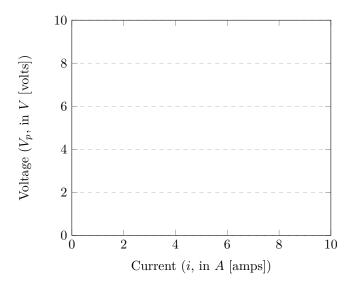
We attach an ammeter (a device that measures current) as shown in the circuit below, and we would like to recover f based on how the ammeter reading changes as we change the brightness of a light source directly pointed at the voltage source (thereby changing V_p).



- 1. To get started, label the positive and negative terminals of the resistors and the ammeter based on the passive sign convention and how we've labeled the voltage source. To ensure consistency, assume current comes out of the positive terminal of the voltage source and flows into the positive terminal of all other circuit elements.
- 2. Let the current reading measured by the ammeter be i. Express i as a function of V_p , R_1 , and R_2 .
- 3. Suppose $R_1=2\Omega$ and $R_2=3\Omega$, and we have the following scattering plot of L_p versus i based on the readings from the ammeter:



(a) In the grid below, plot out the i versus V_p graph. Make sure to label at least 2 points on the graph.



(b) Recover the expression for $V_p=f(L_p)$ based on the luminosity-v.s.-current plot.