CSM 16A Designing Information Systems and Devices I

Week 5 Worksheet Solutions

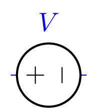
Term: **Spring 2020** *Name:*

Problem 1: Introduction to Circuit Components

In this problem, we will introduce the fundamental circuit components.

1. What is a voltage source?

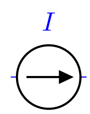
   Solution: Firstly, a voltage source is represented in this manner:

   A voltage source **guarantees** that the potential at its positive end will be V more than the potential at its negative end, no matter what.

2. What is a current source?

   Solution: A current source is represented in this manner:

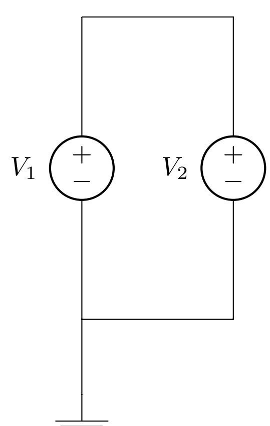
   A current source **guarantees** that the current passing through the unit in the direction of the arrow will be its designated value.

3. What is voltage? What is a voltage drop?

   Solution: For our discussion, it suffices to think of voltage as a kind of driver for current. Current is the movement of charges. A voltage difference forces current to move from the point (node) that has higher voltage, to the point that has lower voltage.

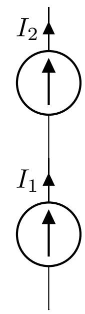
   Voltage drop is the voltage lost (decline of nodal voltage) across a circuit component.

4. Consider the figure below. If $V_1 \neq V_2$, what will happen to the circuit?

Solution: Let us designate the potential at the positive end of V_1 to be V_1^+, the potential at the negative end of V_1 to be V_1^-, the potential at the positive end of V_2 to be V_2^+, and the potential at the negative end of V_2 to be V_2^-. V_1^- and V_2^- are equal to 0 because of the ground. Then, the potential across V_1 is V_1^+, and the potential across V_2 is V_2^+. Since V_1^+ and V_2^+ are connected by a wire, they must be the same voltage; we know that a wire does not affect a circuit's behavior, so the voltage must stay constant across it. This means that $V_1^+ = V_2^+$. However, we know that the voltage potential $V_1^+ - V_1^-$ is not equal to $V_2^+ - V_1^-$ as given in the question. Hence, we see that we cannot have two voltage sources connected in this configuration.

5. What happens in this case if $I_1 \neq I_2$?



Solution: The current source at the bottom guarantees that through that wire there will be I_1 current going through, and the current source at the top guaranteed that I_2 current goes through that wire. This is a contradiction, and is not theoretically possible in a circuit.

Also, look at the point in between the two current sources. I_1 enters on one end, and I_2 leaves on the other end. This is impossible.

6. What is a resistor?

Solution: A resistor is represented in this manner:

A resistor is a circuit unit designed to 'resist' the flow of current. Following convention, there is a "voltage drop" across a resistor from the positive end to the negative end. The voltage drop across a resistor is $V_R = I_R R$, where V_R is the voltage drop, I_R is the current through the resistor and R is the resistance of the resistor.

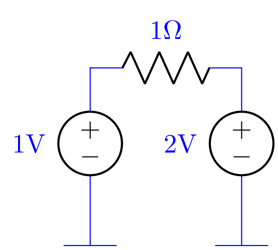
7. What is power?

Solution: Power is the rate at which work is done, where work is in terms of electrical energy.

For circuits, the power *consumed* or *dissipated* by a device is $P = IV$, where the current and voltage abide by passive sign convention.

Common Misconceptions:

- Active components do not necessarily dissipate negative power! Consider the following circuit:

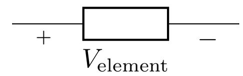


When calculating the power dissipated by the LHS voltage source, we see the current flows counterclockwise about the circuit. With passive sign convention, we calculate the power $P_{V_1} = 1\text{V} \times 1\text{A}$, which is positive! The left-side voltage source is dissipating power.

Problem 2: Passive Sign Convention

For the following components, label all the missing V_{element}, I_{element}, and +/- signs. *Hint: The value of the voltage and current sources shouldn't affect passive sign convention—remember that voltage and current can be negative!*

1. .



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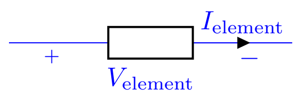

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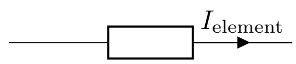

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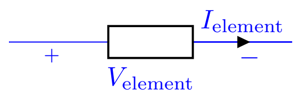
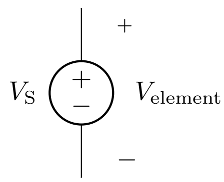


3. .



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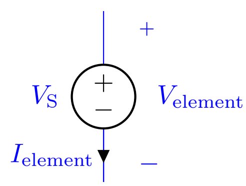
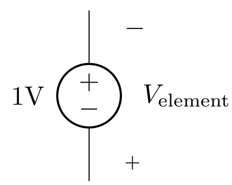


4. .



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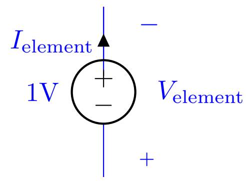

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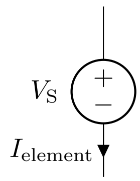

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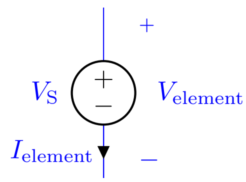

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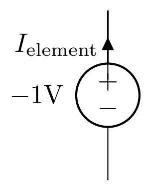

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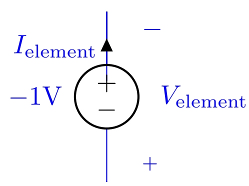

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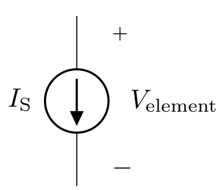

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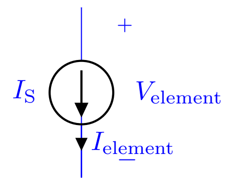

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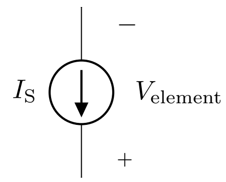

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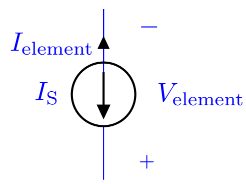

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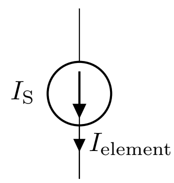

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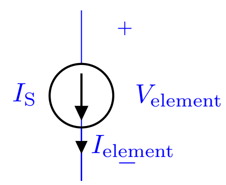

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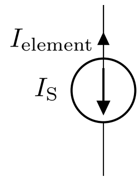

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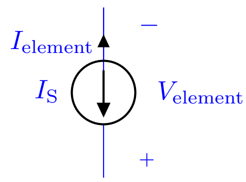

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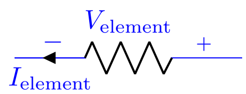

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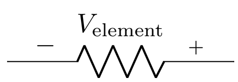

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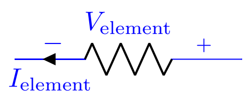
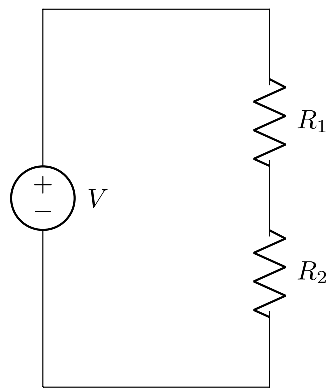


Problem 3: Voltage Divider Properties

Let's take a systematic look at the voltages across a resistor, and see how other components in the circuit can affect it. Consider the following circuit:



1. Calculate the voltage drop across R_1 and R_2 using series resistance calculations.

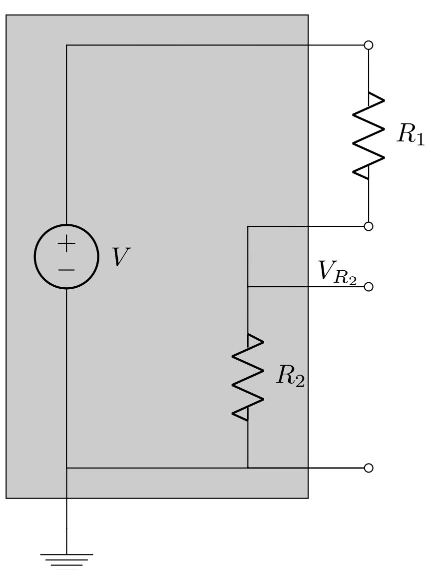
   Solution: The current out of the voltage source is given by Ohm's law:

$$i = \frac{V}{R_{eq}}$$
$$i = \frac{V}{R_1 + R_2}$$

   Again, by Ohm's law, we have that

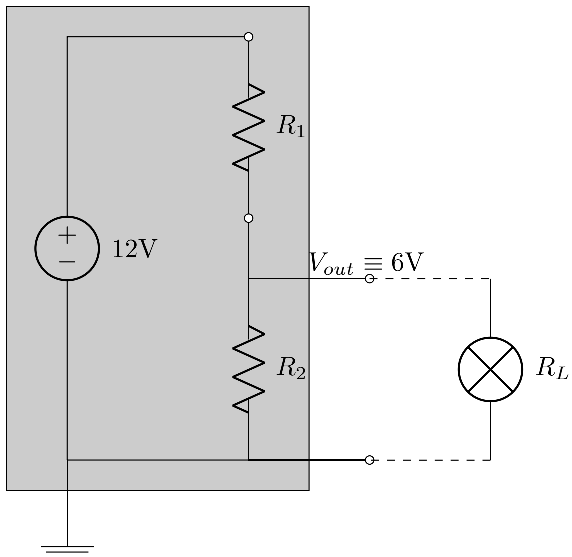
$$V_{R_1} = iR_1 = V\frac{R_1}{R_1 + R_2}$$
$$V_{R_2} = iR_2 = V\frac{R_2}{R_1 + R_2}$$

2. Suppose we want to manipulate the voltage across R_2, but it's locked in a box with the voltage source, as denoted below. Can we use R_1 to manipulate V_{R_2}? What range of voltages can we achieve?



Solution: Any voltage in the range $(0, V]$! Notice from the equations above that $V_{R_2} = V\frac{R_2}{R_{Total}}$. If we increase R_1 indefinitely, holding R_2 constant, we can make the fraction arbitrarily small. Intuitively, since the same current flows through both R_1 and R_2, they have to split the total voltage of the power source, and larger resistances correspond to larger voltage drops (by Ohm's Law). If we decrease R_1 to 0, $V_{R_2} = V$, so the voltage can be at most whatever is supplied by the power source. That the voltage source limits the achievable voltage in the circuit is a concept we will see again when we cover clipping in op-amps.

3. Now let's try using our new variable voltage source to power a light bulb with resistance R_L, where the threshold voltage for lighting the bulb is 6V. Find R_1 and R_2 so that the voltage across R_2 is this threshold voltage; that is, $V_{R_2} \equiv V_{out} = 6\text{V}$. Assume we have a 12V voltage source.



Solution: We want to split the voltage in half (from 12 to 6). Based on the voltage divider formula above, that means $R_1 = R_2 \equiv R$. Note that, under this condition, the voltage is evenly split regardless of what the actual resistance values are! While the current depends on actual resistance values, the voltage only depends on the ratio of resistances.

4. Now that we found an R_1 and R_2 that seem to divide our voltage source appropriately, let's try to connect the bulb to the ends of R_2. Remember, the bulb has a resistance R_L. Calculate the voltage across R_1, R_2 and the light bulb when it is connected. Will the light bulb turn on?

e see that once we add the resistor in parallel with R_2 the overall resistance decreases therefore our V_out will decrease as well. It might help to point out this relationship using Ohm's law: $V = IR$. This is usually the most difficult section for students to conceptualize so make sure to be very clear in the relationships between different components in a circuit.

Solution: Let's reapply the voltage divider formula, but now notice that the "second" resistor is $R_2 \| V_L$!. When we change the resistance in a voltage divider circuit, the voltage across that resistance changes as well. We find that $V_{R_1} = V\frac{R}{R+R\|R_L} = 12\text{V}\frac{R}{R+\frac{RR_L}{R+R_L}} = 12\text{V}\frac{R+R_L}{R+2R_L}$. The rest of the potential drop must be across the R_2-R_L system, so we have $V_{R_2} = V_{\text{bulb}} = 12\text{V}\left(1 - \frac{R+R_L}{R+2R_L}\right) = 12\text{V}\frac{R_L}{R+2R_L} \leq 12\text{V}\frac{R_L}{2R_L} = 6\text{V}$.

So the voltage *decreased*, and the light bulb doesn't turn on.

The takeaway from this is that, while it may seem like the voltage divider can make voltage sources of arbitrary voltages, the act of connecting a component actually changes the output of the voltage divider. We will later learn about a way to stop a light bulb (or other devices that use up power) from "affecting" the circuit that supplies power by placing a "buffer" between the two.

Problem 4: Resistivity

Resistivity is a **physical property** of the material that quantifies how much it opposes the flow of electric current.

Assume that in an ideal case, the cross-section and physical composition of the wire are uniform, We can find its resistivity with the equation below:

$$\rho = R\frac{A}{L}$$

Here, ρ stands for the resistivity of the wire, R stands for its resistance, A stands for the area of the cross section of the wire, and L stands for the length of the wire. Using this equation, we can also solve for the resistance of a wire:

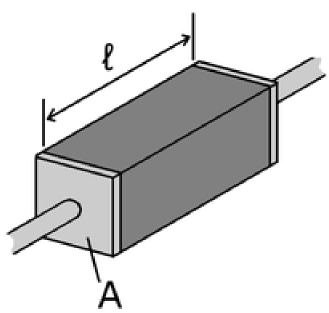
$$R = \rho\frac{L}{A}$$

Note: Throughout the following parts of this question, we will be frequently referencing some of the following variables. In case you are confused about what these variables mean, we've included a section explaining what each variable stands for.

- A: the cross section area of a single wire.
- L: the length of a single wire.
- ρ_{cu}: resistivity for the material copper.
- ρ_{Al}: resistivity for the material aluminum.

1. Now, consider the rectangular copper wire below. Given that the cross-section of the wire is a square and has a cross section area of A, determine the overall resistance of the wire in terms of ρ_{cu}, L, and A. **Solution:** By the resistivity equation, the resistance of the wire is equal to:

$$R = \rho_{cu}\frac{L}{A}$$



2. Suppose we have N such wires and align them side by side to form a mega-wire. Find the overall resistance of this mega-wire. What is this configuration similar to?

   Solution: Since we have all N wires aligned side by side, we are essentially expanding the cross-section area while merging all the wires together by their lengths. This means that the new mega-wire will have a new cross-section area of NA (since we are merging N such wires), while its length remains the same (L). Hence, the overall resistance of the mega-wire will be:

$$R_{mega} = \rho_{cu}\frac{L}{NA}$$

3. Again, with N identical wires, what's a configuration that can achieve the highest resistance possible? What is this configuration similar to?

Solution: The key of this question is to start from the resistance equation:

$$R = \rho \frac{l}{A}$$

Algebraically, we want to maximize the value of R for this question. Since ρ is just a physical constant, we can't really manipulate its value. Observing the fraction $\frac{l}{A}$, we can see that the overall length of the mega-wire should be as great as possible, while its cross section area should be kept as small as possible. How can we arrange N wires in a way so that the overall new wire is as long as possible?
We arrange in a single long line! This also helps minimize the cross section area to A, and we can't really go lower than that since we can't split up a wire into two (thereby splitting the cross section area).
Hence, for this new mega-wire, it has a length of NL and a cross section area of A. Applying the resistance equation, we have:

$$R_{mega} = \rho_{cu} \frac{NL}{A}$$

This configuration is exactly the same as a series circuit where resistors are connected in one long line. If you think in terms of the equivalent resistance for a series circuit, it also makes sense since we are summing up all the resistances.

4. Consider part (b) again, but this time, instead of N copper wires, we split the number evenly between aluminum wires and copper wires. First, we have $N/2$ copper wires on the left side, and $N/2$ aluminum wires on the right side, and then we push these wires together to form a new mega-wire. What's the overall resistance of this wire? (In terms of ρ_{cu}, ρ_{Al}, L, and a)

Solution: As we can see from part (b), when we are aligning the wires side by side, we are essentially arranging the wires to be parallel to each other. Instead of thinking in terms of individual wires, we can consider the overall mega-wire to be a mega copper wire and a mega aluminum wire in parallel! For both wires, they will have a length of L and an overall cross section area of $(N/2)A$ (since we have $N/2$ wires for each category). Hence, applying the resistance equation again, we can find that:

$$R_{cu-mega} = \rho_{cu} \frac{L}{\frac{N}{2}A} = \rho_{cu} \frac{2L}{NA}$$

$$R_{Al-mega} = \rho_{Al} \frac{L}{\frac{N}{2}A} = \rho_{Al} \frac{2L}{NA}$$

Now, since these 2 mega wires are parallel to each other, by equivalent resistance, we can find the overall resistance of the wire to be:

$$R_{overall} = \frac{1}{\frac{1}{R_{cu-mega}} + \frac{1}{R_{Al-mega}}} = \left(\frac{\rho_{cu}\rho_{Al}}{\rho_{cu} + \rho_{Al}} \right) \frac{2L}{NA}$$

If you look closely at the equation, you can actually see how the resistivities of both materials are arranged algebraically as if they are in parallel!

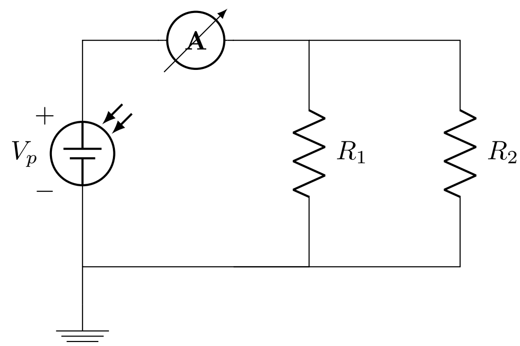
5. Instead of having all $N/2$ wires of **the same material** on one side before merging, now we interleave and mix every single wire together (a single copper wire can be aligned right next to a single aluminum wire), does the overall resistance of this new mega-wire change?

Solution: The key to this question is to realize that, we are essentially aligning all wires in parallel, so we can rearrange all the copper wires to be together on one side and all the aluminum wires to be together on another side, and now we are back to the previous question! Hence, the overall resistance of this new mega-wire doesn't change.

Problem 5: Ohm's Law

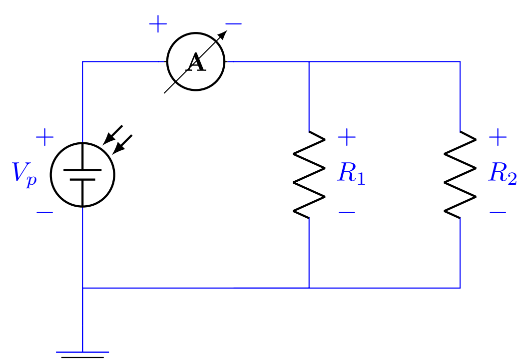
You are building a sensor that can detect the light in a room. Shown below is a simplified version of the main circuit for the sensor. Instead of using a normal voltage source, we are using a voltage source that changes its value based on the luminosity of the light source. Specifically, we know that the voltage V_p changes as a function of the luminosity, $V_p = f(L_p)$. However, the label on the voltage source is scratched, so we'd like to recover f.

We attach an ammeter (a device that measures current) as shown in the circuit below, and we would like to recover f based on how the ammeter reading changes as we change the brightness of a light source directly pointed at the voltage source (thereby changing V_p).



1. To get started, label the positive and negative terminals of the resistors and the ammeter based on the passive sign convention and how we've labeled the voltage source. To ensure consistency, assume current comes out of the positive terminal of the voltage source and flows into the positive terminal of all other circuit elements.

   Solution:

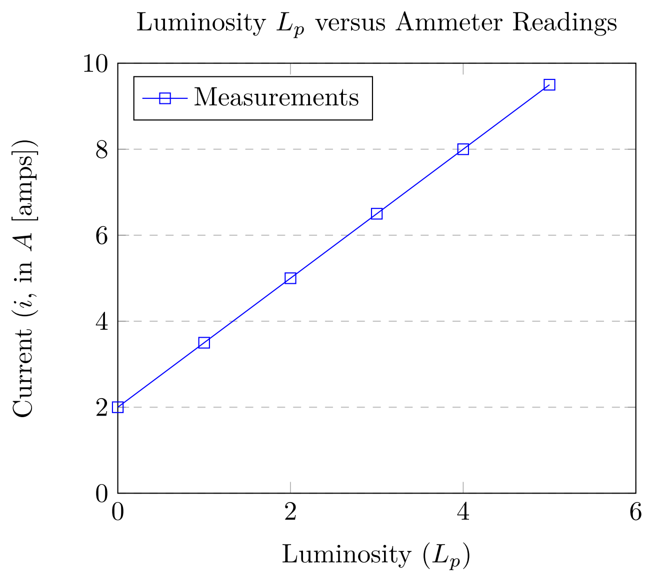


2. Let the current reading measured by the ammeter be i. Express i as a function of V_p, R_1, and R_2.

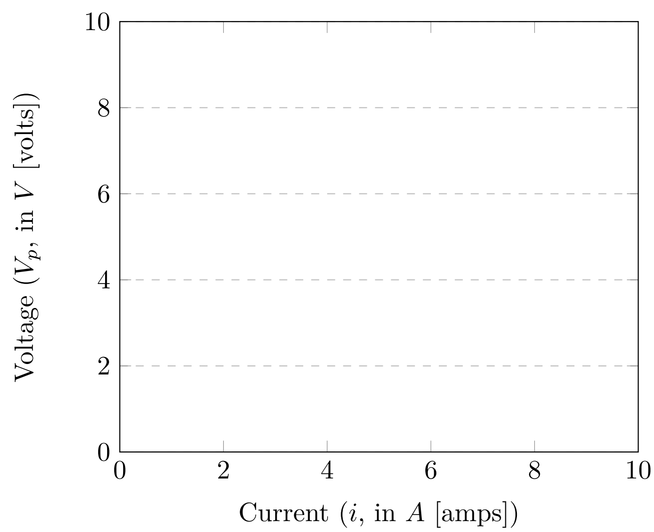
   Solution: We can see that R_1 and R_2 are in parallel, hence sharing the same voltage as V_p and act as a current divider. Using Ohm's Law, we have:

$$i = i_1 + i_2 = \frac{V_p}{R_1} + \frac{V_p}{R_2}.$$

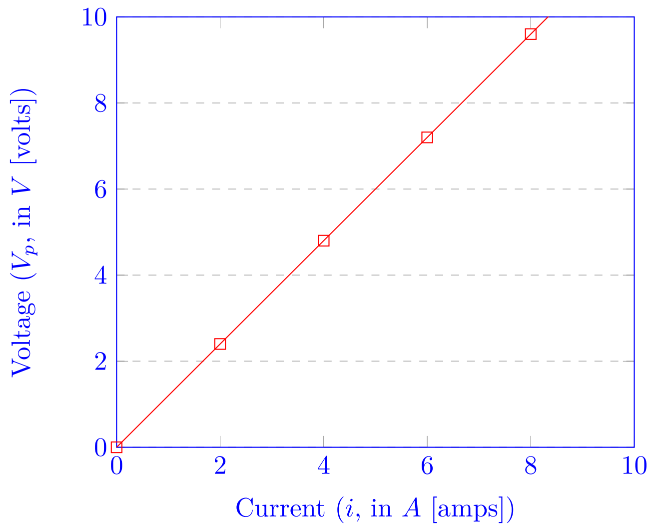
3. Suppose $R_1 = 2\Omega$ and $R_2 = 3\Omega$, and we have the following scattering plot of L_p versus i based on the readings from the ammeter:



(a) In the grid below, plot out the i versus V_p graph. Make sure to label at least 2 points on the graph.



Solution: $V_p = i \cdot \frac{R_1 R_2}{R_1 + R_2} = 1.2i.$



(b) Recover the expression for $V_p = f(L_p)$ based on the luminosity-v.s.-current plot.
Solution: Observing the plot, we can derive i as a function of L_p:

$$i = 1.5L_p + 2$$

Since we also know from analyzing the circuit in part 2 that:

$$i = \frac{V_p}{R_1} + \frac{V_p}{R_2} = \frac{V_p}{1.2}$$

Equating the two expressions, we have:

$$\frac{V_p}{1.2} = 1.5L_p + 2$$

And hence,

$$V_p = f(L_p) = 1.2(1.5L_p + 2) = 1.8p + 2.4.$$