Predefiniranje funkcija

Koristimo paket Riemannian Geometry & Tensor Calculus (RGTC) od Sotirios Bonanosa

In[1]:=
```
Off[General::spell]; Off[General::spell1];
```

In[2]:=
```
<< EDCRGTCcode.m // Quiet
```

Definiramo koordinatni sustav i metriku (u globalnim koordinatama). Metrika u koordinatama (τ, r, θ, φ) je dana sa (linijski element):

$$ds^2 = 2\,G\,J\,\Omega^2\,(-\left(1+r^2\right) d\tau^2 + \frac{dr^2}{1+r^2} + d\,\theta^2 + \Lambda^2\,(d\varphi + r\,d\tau)^2).$$

In[1]:=
```
Unprotect[metric];
```

```
In[2]:= coord = {τ, r, θ, φ};

In[3]:= dim = 4;

In[4]:= metric = 2 G J Ω[θ]^2 * {{- (1 + r^2 * (1 - Λ[θ]^2)), 0, 0, r Λ[θ]^2},
          {0, 1/(1 + r^2), 0, 0}, {0, 0, 1, 0}, {r Λ[θ]^2, 0, 0, Λ[θ]^2}} // FullSimplify;

In[5]:= simpRules = TrigRules;
```

Eksplicitno (treba inicijalizirati ukoliko želimo koristiti RGTC paket s eksplicitnom metrikom)

```
In[6]:= RGtensors[metric, coord]
```

$$\text{gdd} = \begin{pmatrix} 2GJ\left(-1-r^2+r^2\Lambda[\theta]^2\right)\Omega[\theta]^2 & 0 & 0 & 2GJr\Lambda[\theta]^2\Omega[\theta]^2 \\ 0 & \frac{2GJ\Omega[\theta]^2}{1+r^2} & 0 & 0 \\ 0 & 0 & 2GJ\Omega[\theta]^2 & 0 \\ 2GJr\Lambda[\theta]^2\Omega[\theta]^2 & 0 & 0 & 2GJ\Lambda[\theta]^2\Omega[\theta]^2 \end{pmatrix}$$

$$\text{LineElement} = \frac{2GJ\,d[r]^2\,\Omega[\theta]^2}{1+r^2} + 2GJ\,d[\theta]^2\,\Omega[\theta]^2 + 4GJr\,d[\tau]\,d[\varphi]\,\Lambda[\theta]^2\,\Omega[\theta]^2 + 2GJ\,d[\varphi]^2\,\Lambda[\theta]^2\,\Omega[\theta]^2 + 2GJ\,d[\tau]^2\left(-1-r^2+r^2\Lambda[\theta]^2\right)\Omega[\theta]^2$$

$$\text{gUU} = \begin{pmatrix} -\frac{1}{2GJ\left(1+r^2\right)\Omega[\theta]^2} & 0 & 0 & \frac{r}{2GJ\left(1+r^2\right)\Omega[\theta]^2} \\ 0 & \frac{1+r^2}{2GJ\Omega[\theta]^2} & 0 & 0 \\ 0 & 0 & \frac{1}{2GJ\Omega[\theta]^2} & 0 \\ \frac{r}{2GJ\left(1+r^2\right)\Omega[\theta]^2} & 0 & 0 & -\frac{-1-r^2+r^2\Lambda[\theta]^2}{2GJ\left(1+r^2\right)\Lambda[\theta]^2\Omega[\theta]^2} \end{pmatrix}$$

```
gUU computed in 0.016 sec

Gamma computed in 0. sec

Riemann(dddd) computed in 0.031 sec

Riemann(Uddd) computed in 0.031 sec

Ricci computed in 0.016 sec

Weyl computed in 0.031 sec

Einstein computed in 0.094 sec

All tasks completed in 0.234375 seconds
```

Christoffelovi simboli (1-> τ, 2 -> r, 3 -> θ, 4 -> φ)

In[7]:=
```
listaffine := Table[If[UnsameQ[GUdd[[ν, λ, μ]], 0],
    {Style[Subsuperscript[Γ, Row[{coord[[λ]], coord[[μ]]}], coord[[ν]]], 18],
     "=", Style[GUdd[[ν, λ, μ]], 14]}],
   {λ, 1, dim}, {ν, 1, dim}, {μ, 1, dim}] // FullSimplify;
```

In[8]:=
```
TableForm[Partition[DeleteCases[Flatten[listaffine], Null], 3]] // FullSimplify
```

Out[8]//TableForm=

$\Gamma^{\tau}_{\tau r}$	=	$-\frac{r\left(-2+\Lambda[\theta]^2\right)}{2\left(1+r^2\right)}$
$\Gamma^{\tau}_{\tau\theta}$	=	$\frac{\Omega'[\theta]}{\Omega[\theta]}$
$\Gamma^{r}_{\tau\tau}$	=	$-\left(r+r^3\right)\left(-1+\Lambda[\theta]^2\right)$
$\Gamma^{r}_{\tau\varphi}$	=	$-\frac{1}{2}\left(1+r^2\right)\Lambda[\theta]^2$
$\Gamma^{\theta}_{\tau\tau}$	=	$\frac{\Omega'[\theta]+r^2\left(\Omega'[\theta]-\Lambda[\theta]\left(\Omega[\theta]\Lambda'[\theta]+\Lambda[\theta]\Omega'[\theta]\right)\right)}{\Omega[\theta]}$
$\Gamma^{\theta}_{\tau\varphi}$	=	$-\frac{r\Lambda[\theta]\left(\Omega[\theta]\Lambda'[\theta]+\Lambda[\theta]\Omega'[\theta]\right)}{\Omega[\theta]}$
$\Gamma^{\varphi}_{\tau r}$	=	$\frac{1-r^2+r^2\Lambda[\theta]^2}{2+2r^2}$
$\Gamma^{\varphi}_{\tau\theta}$	=	$\frac{r\Lambda'[\theta]}{\Lambda[\theta]}$
$\Gamma^{\tau}_{r\tau}$	=	$-\frac{r\left(-2+\Lambda[\theta]^2\right)}{2\left(1+r^2\right)}$
$\Gamma^{\tau}_{r\varphi}$	=	$-\frac{\Lambda[\theta]^2}{2\left(1+r^2\right)}$
Γ^{r}_{rr}	=	$-\frac{r}{1+r^2}$
$\Gamma^{r}_{r\theta}$	=	$\frac{\Omega'[\theta]}{\Omega[\theta]}$
Γ^{θ}_{rr}	=	$-\frac{\Omega'[\theta]}{\Omega[\theta]+r^2\Omega[\theta]}$
$\Gamma^{\varphi}_{r\tau}$	=	$\frac{1-r^2+r^2\Lambda[\theta]^2}{2+2r^2}$
$\Gamma^{\varphi}_{r\varphi}$	=	$\frac{r\Lambda[\theta]^2}{2+2r^2}$
$\Gamma^{\tau}_{\theta\tau}$	=	$\frac{\Omega'[\theta]}{\Omega[\theta]}$
$\Gamma^{r}_{\theta r}$	=	$\frac{\Omega'[\theta]}{\Omega[\theta]}$
$\Gamma^{\theta}_{\theta\theta}$	=	$\frac{\Omega'[\theta]}{\Omega[\theta]}$
$\Gamma^{\varphi}_{\theta\tau}$	=	$\frac{r\Lambda'[\theta]}{\Lambda[\theta]}$
$\Gamma^{\varphi}_{\theta\varphi}$	=	$\frac{\Lambda'[\theta]}{\Lambda[\theta]}+\frac{\Omega'[\theta]}{\Omega[\theta]}$
$\Gamma^{\tau}_{\varphi r}$	=	$-\frac{\Lambda[\theta]^2}{2\left(1+r^2\right)}$
$\Gamma^{r}_{\varphi\tau}$	=	$-\frac{1}{2}\left(1+r^2\right)\Lambda[\theta]^2$
$\Gamma^{\theta}_{\varphi\tau}$	=	$-\frac{r\Lambda[\theta]\left(\Omega[\theta]\Lambda'[\theta]+\Lambda[\theta]\Omega'[\theta]\right)}{\Omega[\theta]}$
$\Gamma^{\theta}_{\varphi\varphi}$	=	$-\frac{\Lambda[\theta]\left(\Omega[\theta]\Lambda'[\theta]+\Lambda[\theta]\Omega'[\theta]\right)}{\Omega[\theta]}$
$\Gamma^{\varphi}_{\varphi r}$	=	$\frac{r\Lambda[\theta]^2}{2+2r^2}$
$\Gamma^{\varphi}_{\varphi\theta}$	=	$\frac{\Lambda'[\theta]}{\Lambda[\theta]}+\frac{\Omega'[\theta]}{\Omega[\theta]}$

Metrika je kovarijantno konstantna: $\nabla_{\sigma} g_{\mu\nu} = 0$:

In[9]:=
```
covD[metric]
```

```
Out[9]= {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
  {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}
```

Kovarijantne derivacije su definirane u paketu:

covD[xU,{1}]$[\![\mu,\nu]\!] \to \nabla_\nu x^\mu = x^\mu{}_{;\nu}$

covD[xd]$[\![\mu,\nu]\!] \to \nabla_\nu x_\mu = x_{\mu;\nu}$

Raise[covD[xd],{2}]$[\![\mu,\nu]\!] \to \nabla^\nu x_\mu = x_\mu{}^{;\nu}$

covD(xUU,{1,2})$[\![\mu,\nu,\sigma]\!] \to \nabla_\sigma x^{\mu\nu}$

covD(xdd)$[\![\mu,\nu,\sigma]\!] \to \nabla_\sigma x_{\mu\nu}$

Kod kontravarijantnih vektora (x^μ) moramo specificirati broj dignutih indeksa. Dizanje i spuštanje indeksa se vrši pomoću naredbe Raise.

U slučaju podignutog indeksa na kovarijantnoj derivaciji, moramo podignuti indeks derivacije pomoću pozadinske metrike: $\nabla^\sigma x^{\mu\nu} = g^{\sigma\rho} \nabla_\rho x^{\mu\nu}$. Analogno vrijedi i za vektore.

Lieva derivacija $\mathcal{L}_{\xi_n} g_{\mu\nu} \to$ LieD[xU, metric]

Killingovi vektori (dani u članku)

```
In[10]:= (*U(1) izometrija*)

In[11]:= ξ0 = {0, 0, 0, -1};

In[12]:= (*SL(2,ℝ) izometrije*)

In[13]:= ξ1 = {2, 0, 0, 0};
```

In[14]:= $\xi 2 = \left\{2\,\mathrm{Sin}[\tau]\,\frac{r}{\sqrt{1+r^2}},\ -2\,\mathrm{Cos}[\tau]\,\sqrt{1+r^2},\ 0,\ \frac{2\,\mathrm{Sin}[\tau]}{\sqrt{1+r^2}}\right\};$

In[15]:= $\xi 3 = \left\{-2\,\mathrm{Cos}[\tau]\,\frac{r}{\sqrt{1+r^2}},\ -2\,\mathrm{Sin}[\tau]\,\sqrt{1+r^2},\ 0,\ -\frac{2\,\mathrm{Cos}[\tau]}{\sqrt{1+r^2}}\right\};$

Komutator Killingovih vektora $\xi_a = \xi_a^\mu\,\partial_\mu$ je dan izrazom $[\xi_1, \xi_2] = \left(\xi_1^\nu\,\partial_\nu\xi_2^\mu - \xi_2^\nu\,\partial_\nu\xi_1^\mu\right)\partial_\mu$.

```
In[16]:= commutator[x_List, y_List, var_List] :=
  Module[ {dem, a, b},
  dem = Length[xIN];
  Table[Sum[
  x[[b]] D[y[[a]], xIN[[b]]] - y[[b]] D[x[[a]], xIN[[b]]],
                {b, dem}], {a, dem}]]
```

Provjera da li je matrica dobivenih jednadžbi simetrična (znamo da jest eksplicitno, a i iz toga da moramo imati zbog simetričnih rubnih uvjeta)

```
In[17]:= func[mat_?SymmetricMatrixQ] := UpperTriangularize[mat];
func[mat_?(Not@SymmetricMatrixQ[#] &)] := mat;
```

Račun centralnog naboja

Započinjemo račun centralnog naboja.

Difeomorfizmi:

In[19]:=
```
ξUm = {0, -r ⅈ m ⅇ^(-ⅈ m φ), 0, -ⅇ^(-ⅈ m φ)};
```

In[20]:=
```
ξUn = {0, -r ⅈ n ⅇ^(-ⅈ n φ), 0, -ⅇ^(-ⅈ n φ)};
```

Perturbacija pozadinske metrike $h_{\mu\nu} = \mathcal{L}_{\xi_n} g_{\mu\nu}$

In[21]:=
```
hμν = LieD[ξUn, metric] // FullSimplify
```

Out[21]=
$$\left\{\left\{-4\, i\, e^{-i n \varphi}\, G\, J\, n\, r^2 \left(-1+\Lambda[\theta]^2\right) \Omega[\theta]^2,\ 0,\ 0,\ 0\right\},\ \left\{0,\ -\frac{4\, i\, e^{-i n \varphi}\, G\, J\, n\, \Omega[\theta]^2}{\left(1+r^2\right)^2},\ 0,\ -\frac{2\, e^{-i n \varphi}\, G\, J\, n^2\, r\, \Omega[\theta]^2}{1+r^2}\right\},\ \{0,0,0,0\},\ \left\{0,\ -\frac{2\, e^{-i n \varphi}\, G\, J\, n^2\, r\, \Omega[\theta]^2}{1+r^2},\ 0,\ 4\, i\, e^{-i n \varphi}\, G\, J\, n\, \Lambda[\theta]^2\, \Omega[\theta]^2\right\}\right\}$$

In[22]:=
```
MatrixForm[hμν]
```

Out[22]//MatrixForm=
$$\begin{pmatrix} -4\, i\, e^{-i n \varphi}\, G\, J\, n\, r^2 \left(-1+\Lambda[\theta]^2\right) \Omega[\theta]^2 & 0 & 0 & 0 \\ 0 & -\frac{4\, i\, e^{-i n \varphi}\, G\, J\, n\, \Omega[\theta]^2}{\left(1+r^2\right)^2} & 0 & -\frac{2\, e^{-i n \varphi}\, G\, J\, n^2\, r\, \Omega[\theta]^2}{1+r^2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{2\, e^{-i n \varphi}\, G\, J\, n^2\, r\, \Omega[\theta]^2}{1+r^2} & 0 & 4\, i\, e^{-i n \varphi}\, G\, J\, n\, \Lambda[\theta]^2\, \Omega[\theta]^2 \end{pmatrix}$$

$h^{\mu\nu} = g^{\mu\alpha}\, g^{\nu\beta}\, h_{\alpha\beta}$

In[23]:=
```
hUμν = Raise[hμν, {1, 2}] // FullSimplify
```

Out[23]=
$$\left\{\left\{\frac{i\, e^{-i n \varphi}\, n\, r^2}{G\, J \left(1+r^2\right)^2 \Omega[\theta]^2},\ -\frac{e^{-i n \varphi}\, n^2\, r^2}{2\, G\, J \left(1+r^2\right) \Omega[\theta]^2},\ 0,\ \frac{i\, e^{-i n \varphi}\, n\, r}{G\, J \left(1+r^2\right)^2 \Omega[\theta]^2}\right\},\ \left\{-\frac{e^{-i n \varphi}\, n^2\, r^2}{2\, G\, J \left(1+r^2\right) \Omega[\theta]^2},\ -\frac{i\, e^{-i n \varphi}\, n}{G\, J\, \Omega[\theta]^2},\ 0,\ \frac{e^{-i n \varphi}\, n^2\, r \left(-1-r^2+r^2 \Lambda[\theta]^2\right)}{2\, G\, J \left(1+r^2\right) \Lambda[\theta]^2\, \Omega[\theta]^2}\right\},\ \{0,0,0,0\},\ \left\{\frac{i\, e^{-i n \varphi}\, n\, r}{G\, J \left(1+r^2\right)^2 \Omega[\theta]^2},\ \frac{e^{-i n \varphi}\, n^2\, r \left(-1-r^2+r^2 \Lambda[\theta]^2\right)}{2\, G\, J \left(1+r^2\right) \Lambda[\theta]^2\, \Omega[\theta]^2},\ 0,\ \frac{i\, e^{-i n \varphi}\, n \left(\left(1+r^2\right)^2 - r^2 \left(2+r^2\right) \Lambda[\theta]^2\right)}{G\, J \left(1+r^2\right)^2 \Lambda[\theta]^2\, \Omega[\theta]^2}\right\}\right\}$$

In[24]:=
```
hUμν // MatrixForm
```

Out[24]//MatrixForm=
$$\begin{pmatrix} \frac{i\, e^{-i n \varphi}\, n\, r^2}{G\, J \left(1+r^2\right)^2 \Omega[\theta]^2} & -\frac{e^{-i n \varphi}\, n^2\, r^2}{2\, G\, J \left(1+r^2\right) \Omega[\theta]^2} & 0 & \frac{i\, e^{-i n \varphi}\, n\, r}{G\, J \left(1+r^2\right)^2 \Omega[\theta]^2} \\ -\frac{e^{-i n \varphi}\, n^2\, r^2}{2\, G\, J \left(1+r^2\right) \Omega[\theta]^2} & -\frac{i\, e^{-i n \varphi}\, n}{G\, J\, \Omega[\theta]^2} & 0 & \frac{e^{-i n \varphi}\, n^2\, r \left(-1-r^2+r^2 \Lambda[\theta]^2\right)}{2\, G\, J \left(1+r^2\right) \Lambda[\theta]^2\, \Omega[\theta]^2} \\ 0 & 0 & 0 & 0 \\ \frac{i\, e^{-i n \varphi}\, n\, r}{G\, J \left(1+r^2\right)^2 \Omega[\theta]^2} & \frac{e^{-i n \varphi}\, n^2\, r \left(-1-r^2+r^2 \Lambda[\theta]^2\right)}{2\, G\, J \left(1+r^2\right) \Lambda[\theta]^2\, \Omega[\theta]^2} & 0 & \frac{i\, e^{-i n \varphi}\, n \left(\left(1+r^2\right)^2 - r^2 \left(2+r^2\right) \Lambda[\theta]^2\right)}{G\, J \left(1+r^2\right)^2 \Lambda[\theta]^2\, \Omega[\theta]^2} \end{pmatrix}$$

$h = g^{\mu\nu} h_{\mu\nu}$

In[25]:= `h = Sum[gUU[[a, b]] hμν[[a, b]], {a, 1, 4}, {b, 1, 4}] // Simplify`

Out[25]= 0

Možemo koristiti i multiDot iz paketa RGTC, koji kontrahira parove indeksa pojedinih tenzora.

In[26]:= `multiDot[gUU, hμν, {1, 1}, {2, 2}]`

Out[26]= 0

In[27]:= deter =

$$\sqrt{-\mathrm{Det}[\mathrm{metric}]} \;/.\; \Lambda \to \left(\frac{2\,\mathrm{Sin}[\#]}{1+\mathrm{Cos}[\#]^2}\;\&\right) \;/.\; \Omega \to \left(\frac{\sqrt{1+\mathrm{Cos}[\#]^2}}{\sqrt{2}}\;\&\right) \;//\; \mathrm{FullSimplify}$$

Out[27]= $2\sqrt{G^4 J^4 \left(1+\mathrm{Cos}[\theta]^2\right)^2 \mathrm{Sin}[\theta]^2}$

Bitno je ručno sve simplificirati jer nismo rekli *Mathematici* da su sve konstante pozitivne

In[28]:= determin = $2\,G^2 J^2 \left(1+\mathrm{Cos}[\theta]^2\right) \mathrm{Sin}[\theta]$

Out[28]= $2\,G^2 J^2 \left(1+\mathrm{Cos}[\theta]^2\right) \mathrm{Sin}[\theta]$

In[29]:= `ξdm = Lower[ξUm, {1}] // FullSimplify`

Out[29]= $\left\{-2\,e^{-i\,m\,\varphi}\,G\,J\,r\,\Lambda[\theta]^2\,\Omega[\theta]^2,\ -\frac{2\,i\,e^{-i\,m\,\varphi}\,G\,J\,m\,r\,\Omega[\theta]^2}{1+r^2},\ 0,\ -2\,e^{-i\,m\,\varphi}\,G\,J\,\Lambda[\theta]^2\,\Omega[\theta]^2\right\}$

Uzimamo definiciju k_{ξ_m} iz članka Guica et. al.

In[30]:=

$$\text{kGuica} = \text{Module}\Big[\{\sigma, \mu, \nu\}, \text{Sum}\Big[\frac{1}{4}\,\text{epsilon}[3, 4, \mu, \nu]\,\Big(-\xi\text{Um}[[\nu]]\,\text{covD}[\text{hU}\mu\nu, \{1, 2\}][[\mu, \sigma, \sigma]] + \xi\text{dm}[[\sigma]]\,\text{Raise}[\text{covD}[\text{hU}\mu\nu, \{1, 2\}], \{3\}][[\mu, \sigma, \nu]] - \text{hU}\mu\nu[[\nu, \sigma]]\,\text{covD}[\xi\text{Um}, \{1\}][[\mu, \sigma]] + \frac{1}{2}\,\text{hU}\mu\nu[[\sigma, \nu]]\,(\text{Raise}[\text{covD}[\xi\text{dm}], \{2\}][[\sigma, \mu]] + \text{covD}[\xi\text{Um}, \{1\}][[\mu, \sigma]])\Big), \{\sigma, 1, 4\}, \{\mu, 1, 4\}, \{\nu, 1, 4\}\Big]\Big]\ //\ \text{FullSimplify}$$

Out[30]=

$$\frac{i\, e^{-i\,(m+n)\,\varphi}\, n\,\left(n\,(-m+n)\,r^2 + 2\,\left(1 + r^2\right)\,\Lambda[\theta]^2\right)}{8\,G\,J\,\left(1 + r^2\right)\,\Omega[\theta]^2}$$

In[31]:=

$$\text{int2} = \text{kGuica} \Big/ e^{-i\,(m+n)\,\varphi}\ /.\ \Lambda \to \left(\frac{2\,\text{Sin}[\#]}{1 + \text{Cos}[\#]^2}\ \&\right)\ /.\ \Omega \to \left(\frac{\sqrt{1 + \text{Cos}[\#]^2}}{\sqrt{2}}\ \&\right)$$

Out[31]=

$$\frac{i\, n\,\left(n\,(-m+n)\,r^2 + \frac{8\,\left(1+r^2\right)\,\text{Sin}[\theta]^2}{\left(1+\text{Cos}[\theta]^2\right)^2}\right)}{4\,G\,J\,\left(1 + r^2\right)\,\left(1 + \text{Cos}[\theta]^2\right)}$$

In[32]:=

$$\text{INT} = \frac{2}{4\,G}\,\text{Limit}[\text{Integrate}[\text{int2} * \text{determin}, \{\theta, 0, \pi\}], r \to \infty]$$

Out[32]=

$$\frac{1}{2}\, i\, J\, n\,\left(4 - m\,n + n^2\right)$$

In[33]:= INT /. n → -m // Simplify

Out[33]=

$$-i\, J\, m\,\left(2 + m^2\right)$$

Provjera računa pomoću SurfaceCharges koda

SurfaceCharges-Code

Prije pokretanja uzimamo SurfaceCharges kod od Goeffreyja Comperea. Definiramo difeomorfizme, dane iz članaka

In[71]:=
```
Clear[m, n];
```

In[72]:=
```
Lm = {0, -r ⅈ m ⅇ^(-ⅈ m φ), 0, -ⅇ^(-ⅈ m φ)} (*ξm*)
```

Out[72]= $\{0, -\mathrm{i}\, e^{-\mathrm{i}\, m\, \varphi}\, m\, r, 0, -e^{-\mathrm{i}\, m\, \varphi}\}$

In[73]:=
```
Ln = Lm /. {m → n} (*ξn*)
```

Out[73]= $\{0, -\mathrm{i}\, e^{-\mathrm{i}\, n\, \varphi}\, n\, r, 0, -e^{-\mathrm{i}\, n\, \varphi}\}$

In[74]:=
```
Lmn = Lm /. {m → (m + n)};
```

Provjerimo da li će nam komutator dati točan izraz za Virasoro algebru $i[\xi_m, \xi_n] - (m - n)\, \xi_{m+n} = 0$

In[75]:=
```
Factor[Simplify[ I Commutator[Lm, Ln] -  (m - n) Lmn]]
```

Out[75]= {0, 0, 0, 0}

Lieva derivacija pozadinske metrike po difeomorfizmu $\mathcal{L}_{\xi_n} g_{\mu\nu}$

In[76]:=
```
cLgLn = Simplify[Liediffdd[metric, coord, Ln]]
```

Out[76]=
$$\left\{\left\{-4\,\mathrm{i}\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n\, r^2 \left(-1 + \Lambda[\theta]^2\right) \Omega[\theta]^2, 0, 0, 0\right\}, \left\{0, -\frac{4\,\mathrm{i}\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n\, \Omega[\theta]^2}{\left(1 + r^2\right)^2}, 0, -\frac{2\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n^2\, r\, \Omega[\theta]^2}{1 + r^2}\right\}, \{0, 0, 0, 0\}, \left\{0, -\frac{2\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n^2\, r\, \Omega[\theta]^2}{1 + r^2}, 0, 4\,\mathrm{i}\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n\, \Lambda[\theta]^2\, \Omega[\theta]^2\right\}\right\}$$

In[77]:=
```
cLgLn // MatrixForm
```

Out[77]//MatrixForm=
$$\begin{pmatrix} -4\,\mathrm{i}\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n\, r^2 \left(-1 + \Lambda[\theta]^2\right) \Omega[\theta]^2 & 0 & 0 & 0 \\ 0 & -\frac{4\,\mathrm{i}\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n\, \Omega[\theta]^2}{\left(1+r^2\right)^2} & 0 & -\frac{2\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n^2\, r\, \Omega[\theta]^2}{1+r^2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{2\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n^2\, r\, \Omega[\theta]^2}{1+r^2} & 0 & 4\,\mathrm{i}\, e^{-\mathrm{i}\, n\, \varphi}\, G\, J\, n\, \Lambda[\theta]^2\, \Omega[\theta]^2 \end{pmatrix}$$

Poisson bracket faktor 8π je ukomponiran

In[78]:=
```
PreCentralCharge = Series[Simplify[
   Series[ ChargesEinstein[Lm, cLgLn, -1, {1, 2}] / G, {r, ∞, 0}]], {r, ∞, 0}]
```

```
Signature of the metric: -1 .

The surface charge is inversely proportional to Newton
   constant and should be integrated on the τ , r = constant surface
```

Out[78]=
$$\frac{\mathrm{i}\, e^{-\mathrm{i}\,(m+n)\,\varphi}\, n \left(n\,(-m+n) + 2\,\Lambda[\theta]^2\right) \sqrt{G^4\, J^4\, \Lambda[\theta]^2\, \Omega[\theta]^8}}{8\, G^2\, J\, \pi\, \Omega[\theta]^2} + O\left[\frac{1}{r}\right]^1$$

Provjera s Iyer-Wald formalizmom

In[79]:=
```
Series[ChargeSuppl, {r, ∞, 0}]
```

Out[79]= $O\left[\frac{1}{r}\right]^2$

Račun centralnog naboja

In[80]:=
```
CentralCharge =
 12 Coefficient[ Factor[Normal[2 π ⅈ PreCentralCharge /. {n → -m}]], m^3]
```

Out[80]= $\frac{6\sqrt{G^4 J^4 \Lambda[\theta]^2 \Omega[\theta]^8}}{G^2 J \Omega[\theta]^2}$

In[81]:= $\frac{6 G^2 J^2 \Lambda[\theta]\, \Omega[\theta]^4}{G^2 J \Omega[\theta]^2}$

Out[81]= $6 J \Lambda[\theta]\, \Omega[\theta]^2$

Integriramo naboj po rubu

In[82]:= $\text{Integrate}\left[6 J \Lambda[\theta]\, \Omega[\theta]^2 \;/.\; \Lambda \to \left(\frac{2\,\text{Sin}[\#]}{1+\text{Cos}[\#]^2}\ \&\right) /.\; \Omega \to \left(\frac{\sqrt{1+\text{Cos}[\#]^2}}{\sqrt{2}}\ \&\right), \{\theta, 0, \pi\}\right]$

Out[82]= $12\, J$

Dobili smo dobar rezultat do na faktor $1/\hbar$.