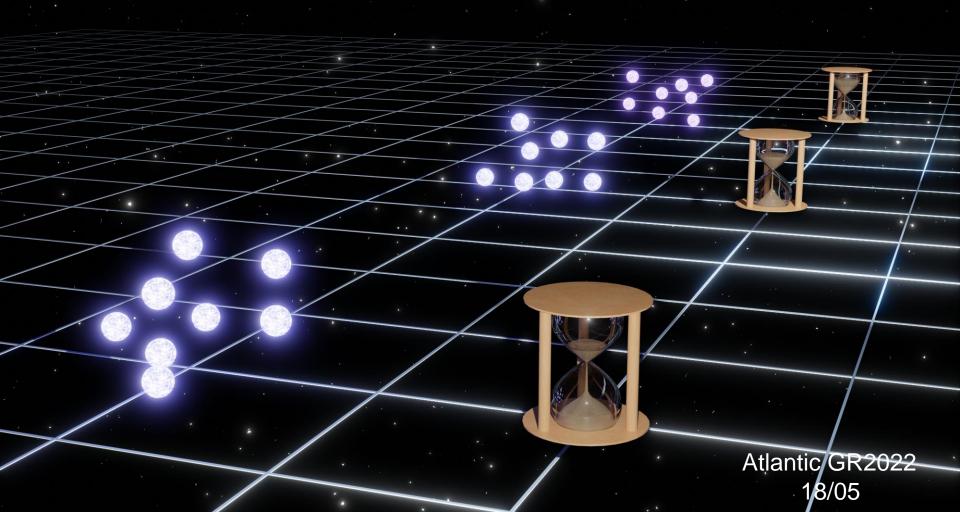
# Cosmological Standard Timer from Unstable Primordial Relic

arXiv: 2112.10422

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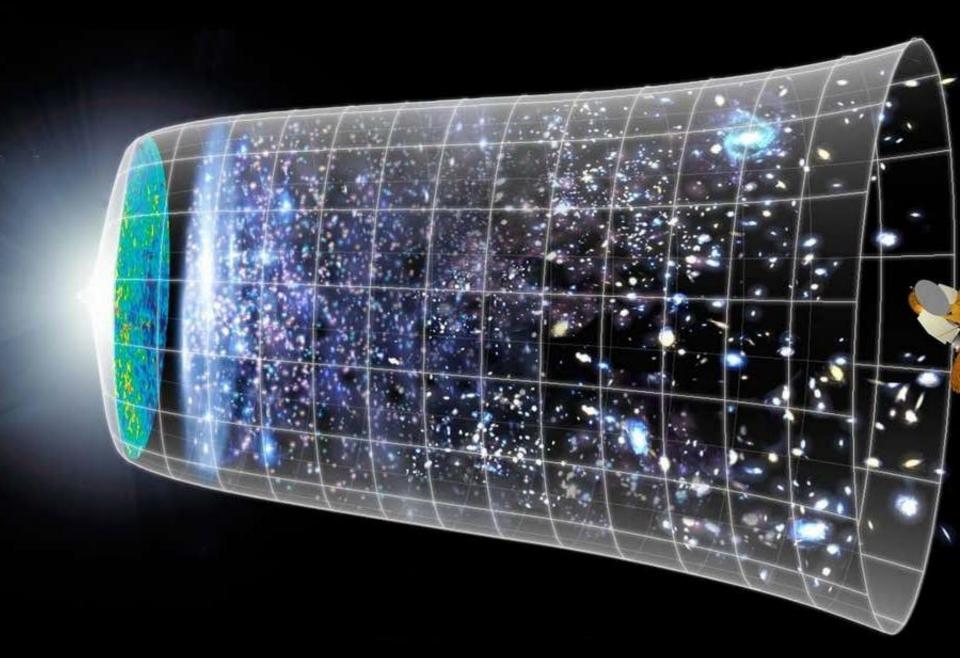


Image Credit: New Scientist

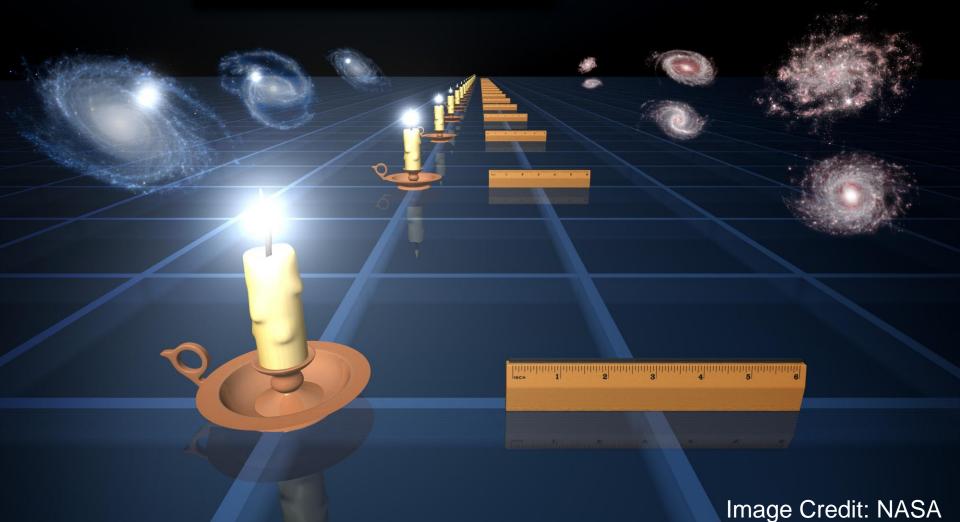


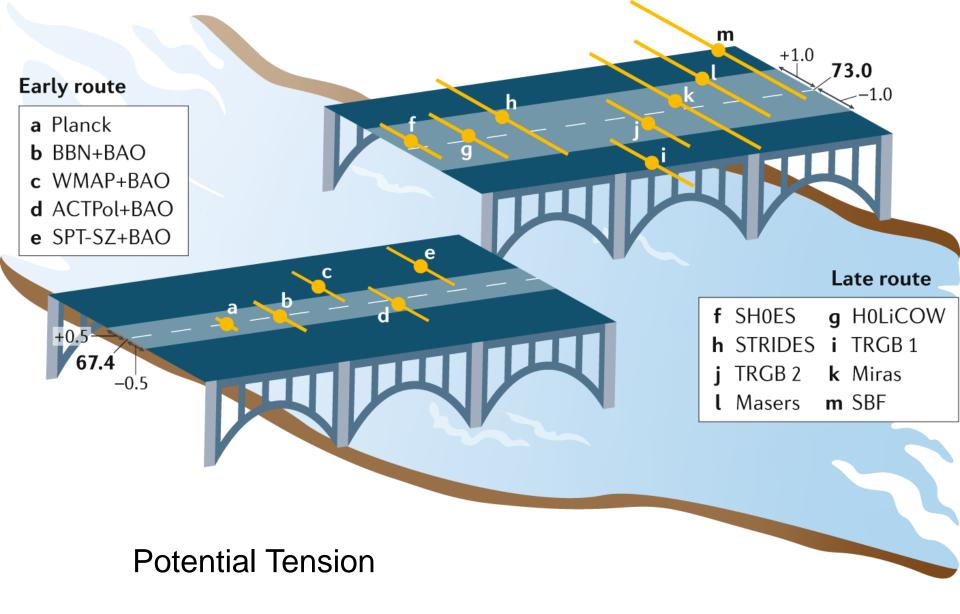
# Standard Candle

# Standard Ruler

$$F = \frac{L}{4\pi \ d_L^2(z)}$$

$$\theta = \frac{r_{S}}{D_{M}(z)}$$



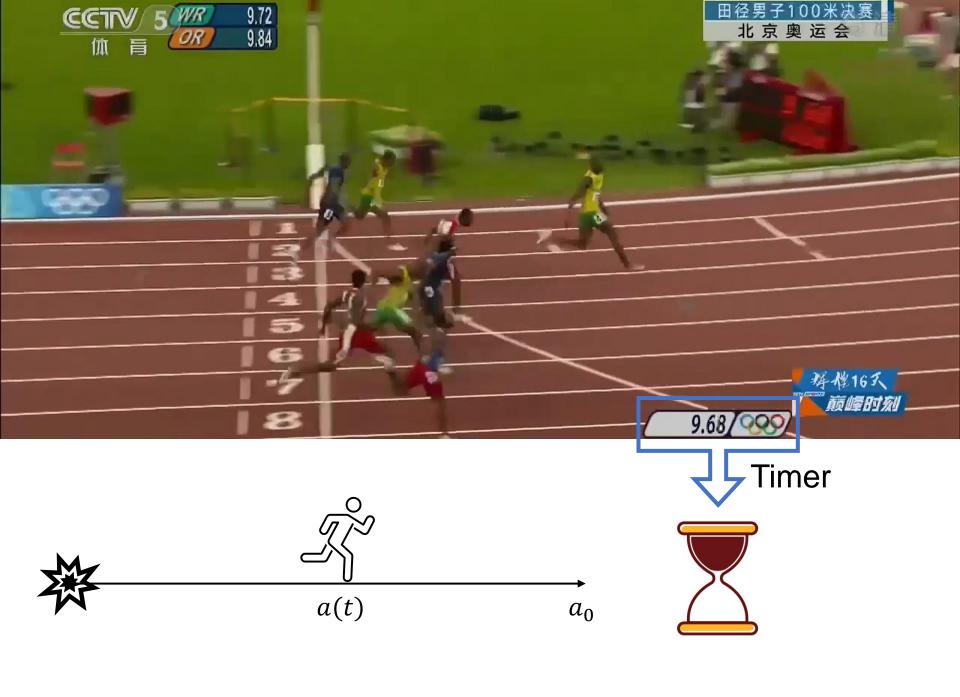


Riess, Adam G. "The expansion of the universe is faster than expected." *Nature Reviews Physics* 2.1 (2020): 10-12.



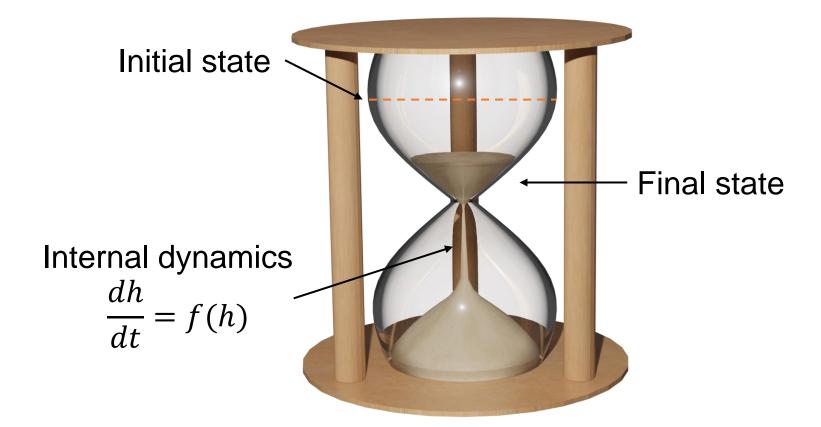
Another way to measure the Universe?





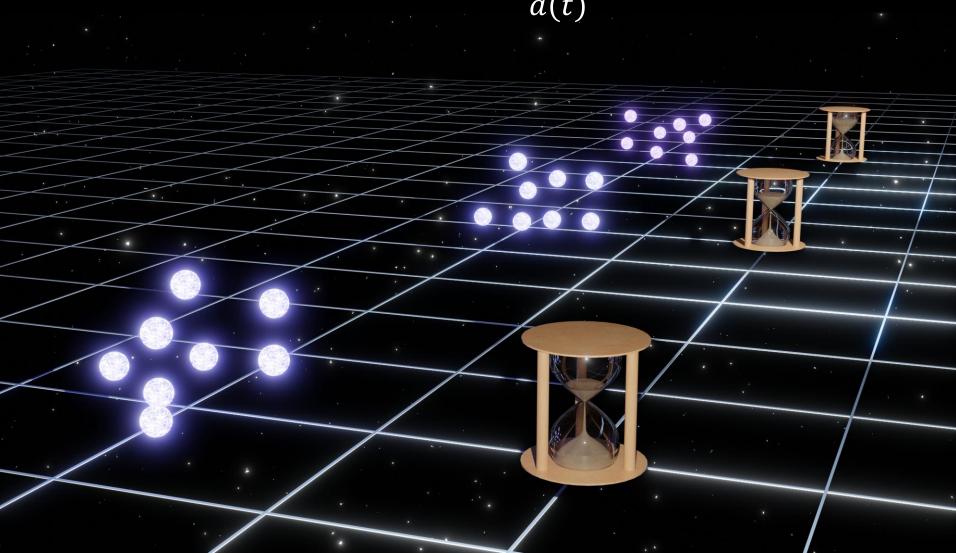
How to know the elapsed time in the timer?





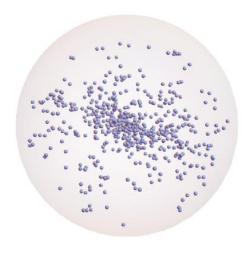
# How to obtain a(t) ?

$$1 + z(t) = \frac{a_0}{a(t)}$$



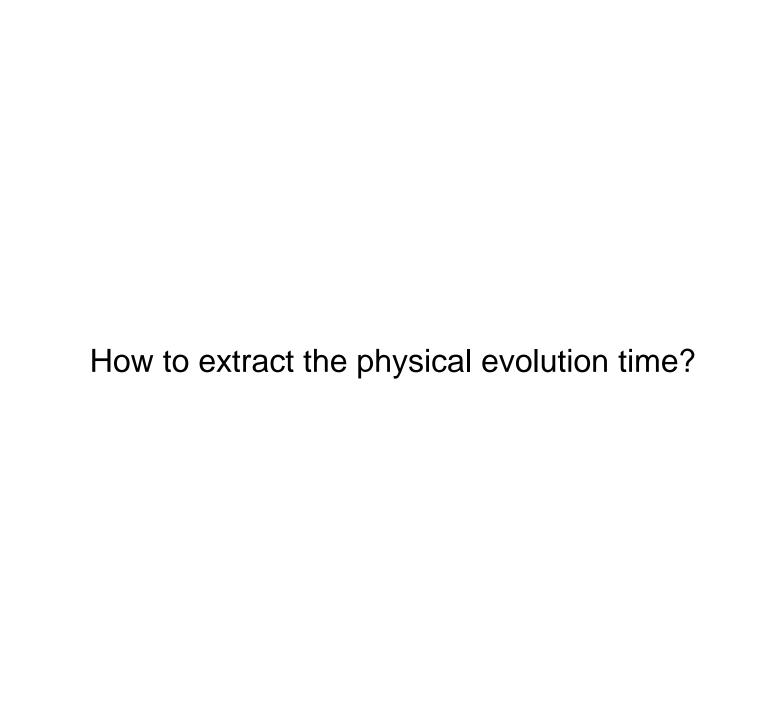
# Properties for the potential timer candidates

- A standard initial state as the standard reference
- An accurate evolution mechanism
- Redshift is encoded in the observable



- Primordial mass function
- $\dot{M} = -\hbar c^4 / 15360 \pi G^2 M^2$
- Redshift in Hawking radiation

Hawking radiation from PBH clustering



The primordial mass function of PBHs in inflationary scenario

$$n(M) = \frac{f_{PBH}}{\sqrt{2\pi}\sigma M} \exp\left[-\frac{\ln^2(M/M_{pk})}{2\sigma^2}\right]$$

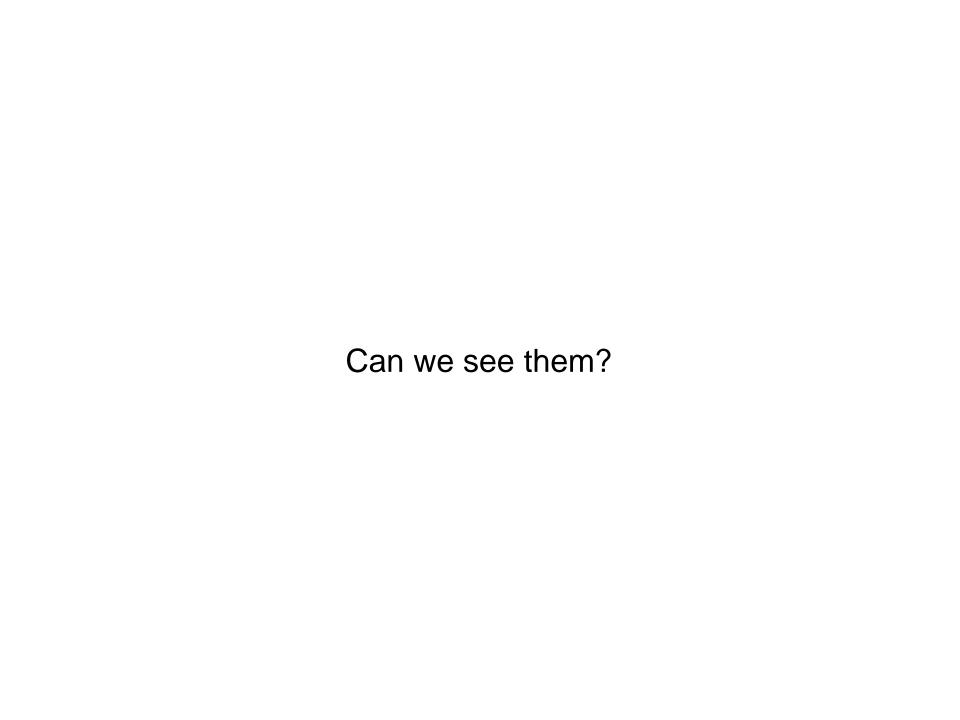
The evolution of the PBH mass function

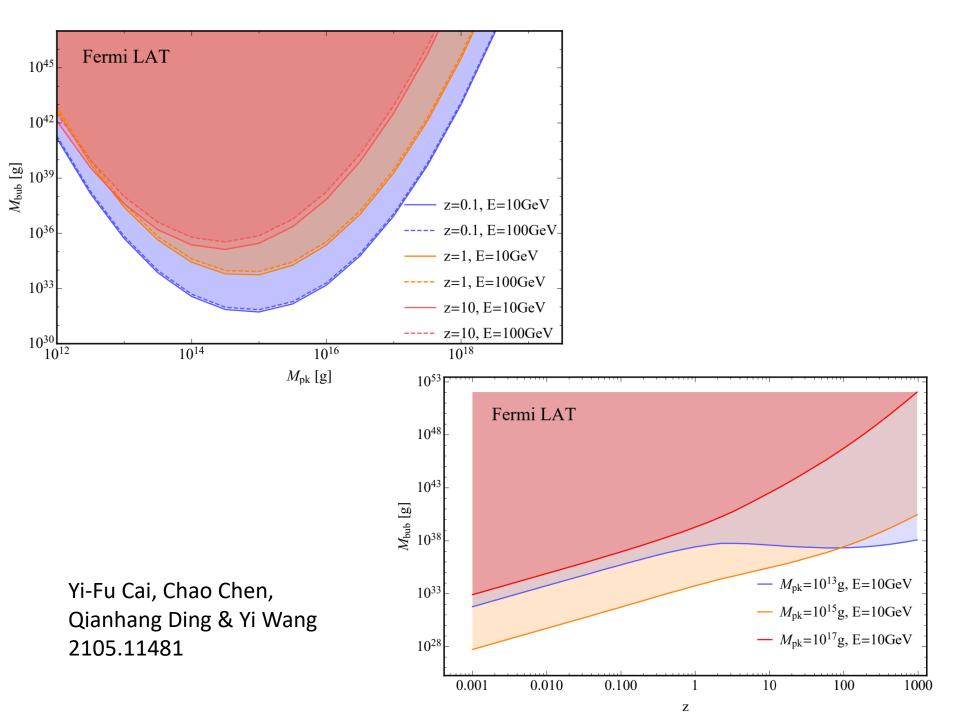
$$n(M;t) = \frac{dN}{dM} = \frac{dN}{dM_i} \frac{dM_i}{dM} = n(M;t_i) \frac{dM_i}{dM}$$

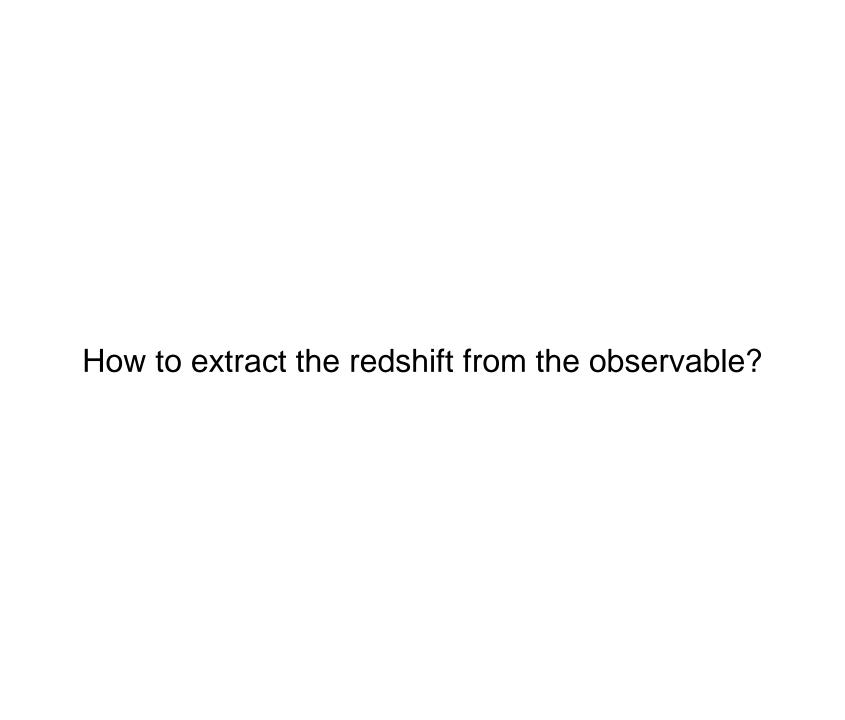
$$\frac{dM}{dt} = -\frac{\alpha}{M^2} \Longrightarrow M^3 = M_i^3 - \delta^3(\Delta t)$$

$$n(M;t) = n(M;t_i) \frac{dM_i}{dM} = n(M;t_i) \frac{M^2}{(M^3 + \delta^3(\Delta t))^{2/3}}$$

 $n(M;t) \simeq \frac{n(\delta(\Delta t);t_i)}{\delta^2(\Delta t)}M^2, \qquad M \ll \delta(\Delta t)$ 







## Primary Hawking radiation from the PBH clustering

$$P(E) = \int_0^\infty H_p(E, M) n(M) dM,$$
 
$$H_p(E, M) = \frac{1}{2\pi} \frac{\Gamma_1(E, M)}{e^{8\pi GME} - 1} \quad \Gamma_1(E, M) \propto \begin{cases} G^4 M^4 E^4, E < (8\pi GM)^{-1} \\ G^2 M^2 E^2, E > (8\pi GM)^{-1} \end{cases}$$

#### Redshift in the observed photon flux

$$F(E;z) = \frac{L(E(1+z);z)}{4\pi d_L^2(z)} \simeq \frac{(1+z)^2 E^2 V}{4\pi d_L^2(z)} \int_0^\infty H_p(E(1+z), M) n(M;z) dM$$

$$H_p(E(1+z), M) = H_p(E, M(1+z))$$

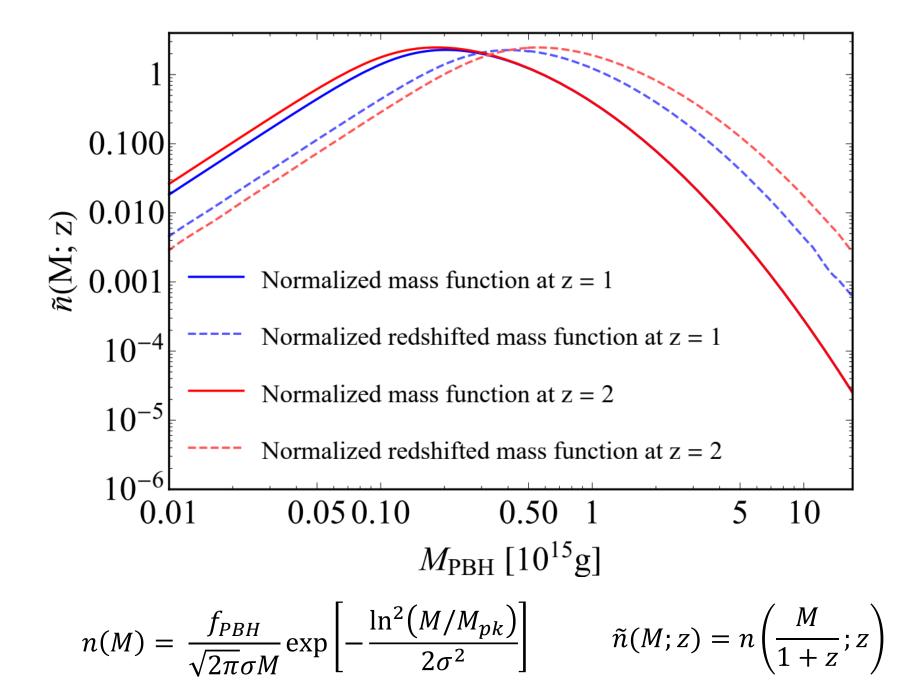
$$\frac{4\pi F(E;z)}{E^2} \simeq \int_0^\infty H_p(E, M') n\left(\frac{M'}{1+z}; z\right) \frac{(1+z)V}{d_L^2(z)} dM'$$

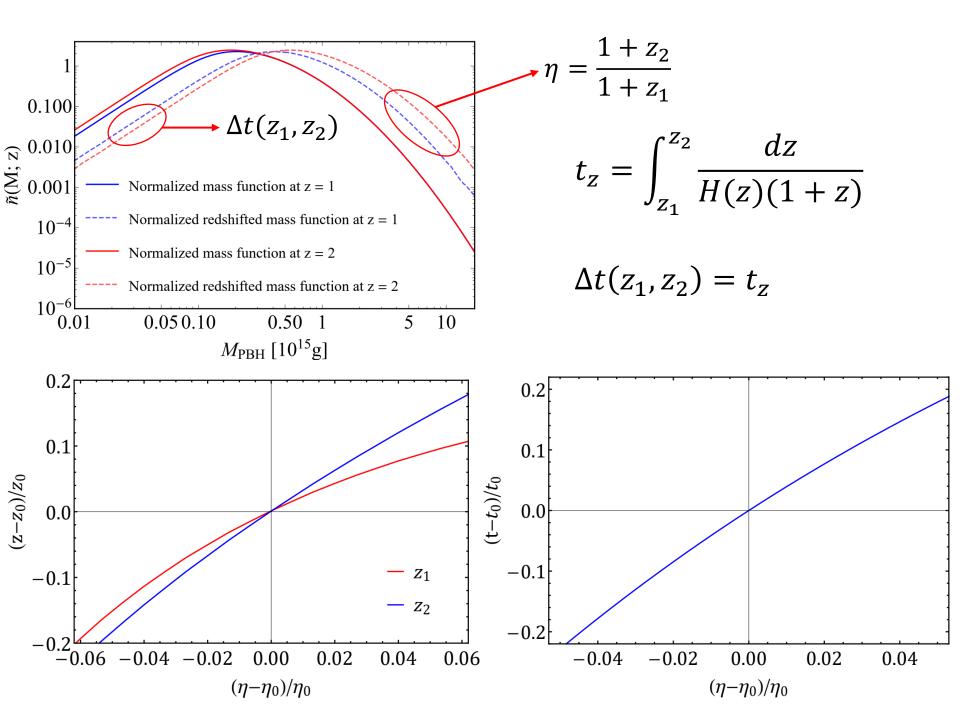
## Redshift from the inverse problem

$$P(E) = \int_0^\infty K(E, M) f(M) dM \Rightarrow f(M) = \int_0^\infty K^{-1}(E, M) P(E) dE$$

$$\frac{4\pi F(E; z)}{E^2} \simeq \int_0^\infty H_p(E, M) n\left(\frac{M}{1+z}; z\right) \frac{(1+z)V}{d_L^2(z)} dM$$

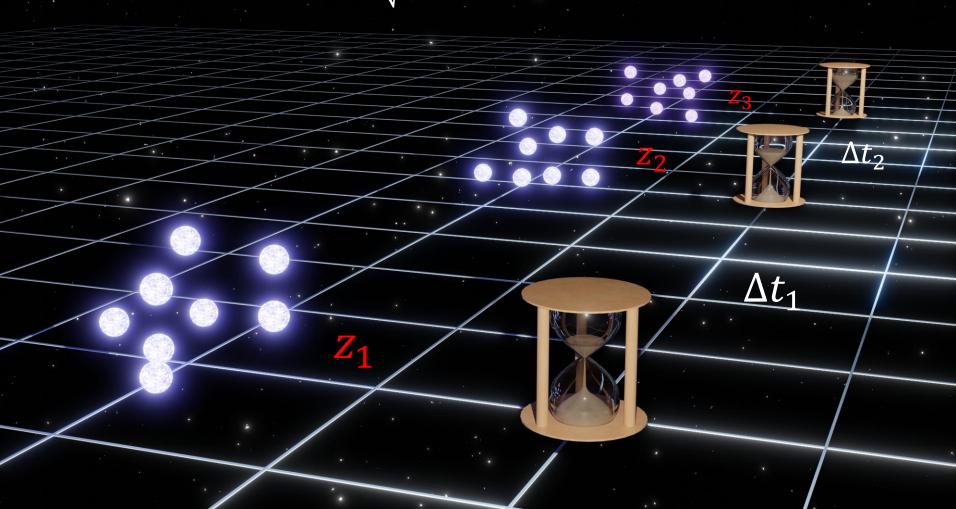
$$f(M) = n\left(\frac{M}{1+z}; z\right) \frac{(1+z)V}{d_L^2(z)}$$





$$\Delta t = \int_{z_1}^{z_2} \frac{dz}{H(z)(1+z)}$$

$$H(z) = H_0 \sqrt{\Omega_{\gamma} (1+z)^4 + \Omega_m (1+z)^3 + \Omega_{\Lambda}}$$



# Thank you!

