Center for Theoretical Physics of the Universe

Cosmology, Gravity and Astroparticle Physics

Primordial Black Hole Mergers as a Cosmological Probe

Qianhang Ding

IBS CTPU-CGA

Reference: 2312.13728, QD
2410.02591, QD, Minxi He, Volodymyr Takhistov

100 + 9 GR&Beyond@Jeju National University
Nov 20

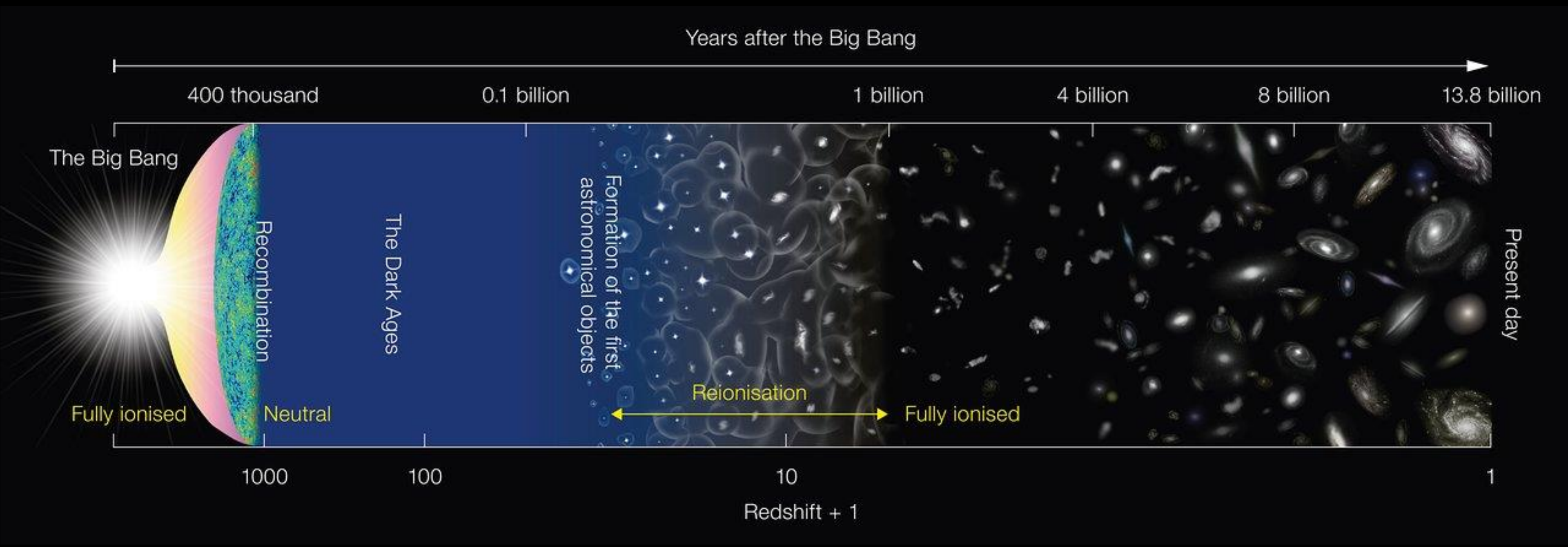
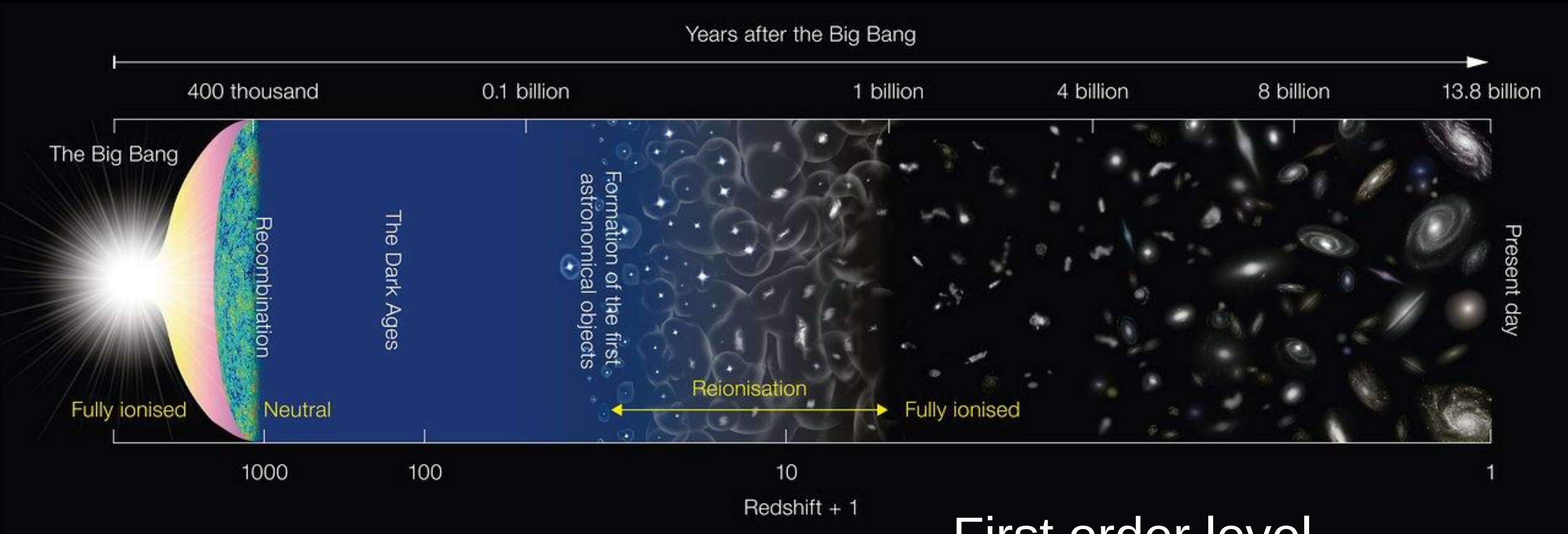


Image Credit: NAOJ

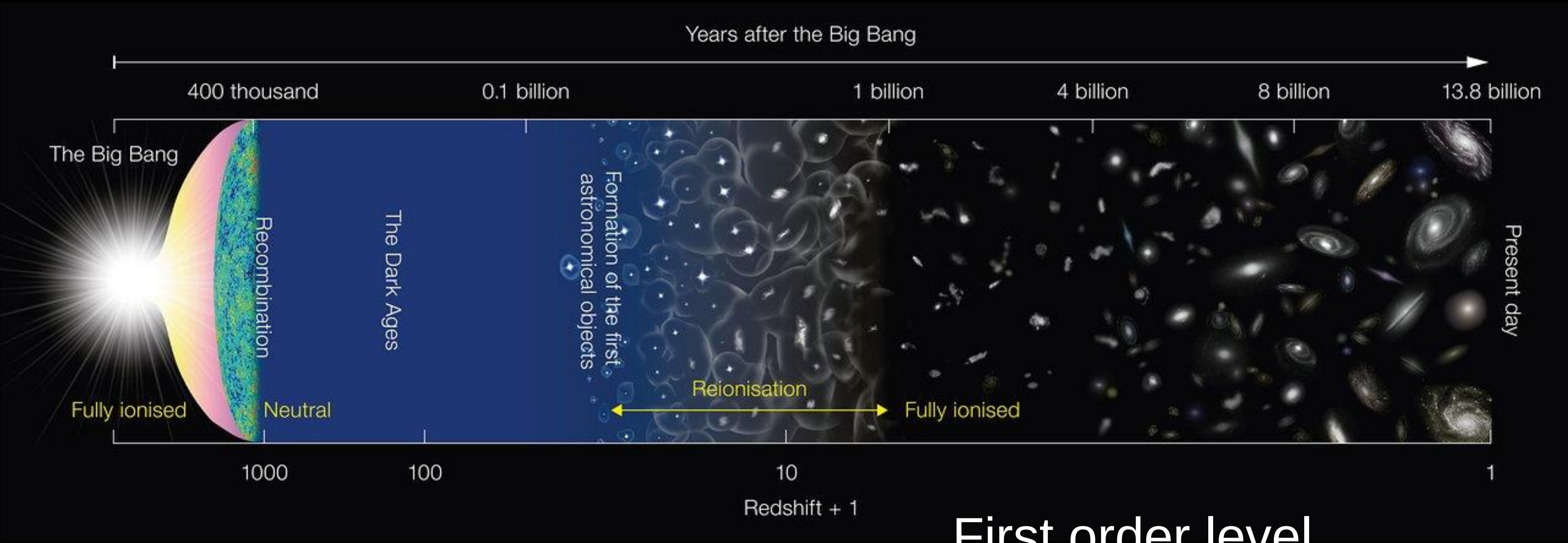
Background level
Hubble expansion rate



First order level
Structure formation

Image Credit: NAOJ

Background level
Hubble expansion rate

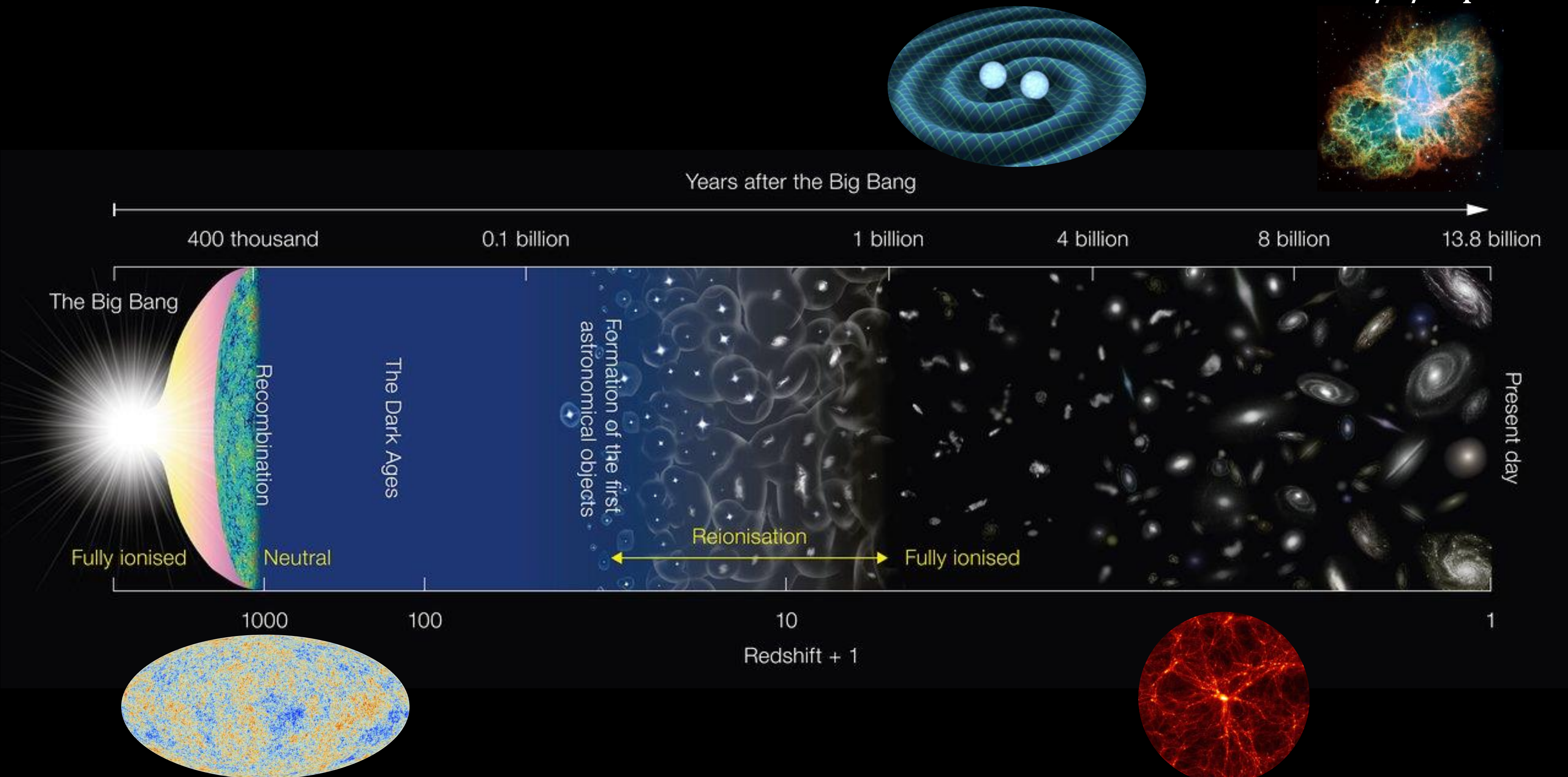


First order level
Structure formation

2312.13728, QD

Image Credit: NAOJ

73.04 ± 1.04 km/s/Mpc



67.4 ± 0.5 km/s/Mpc

Image Credit: NAOJ

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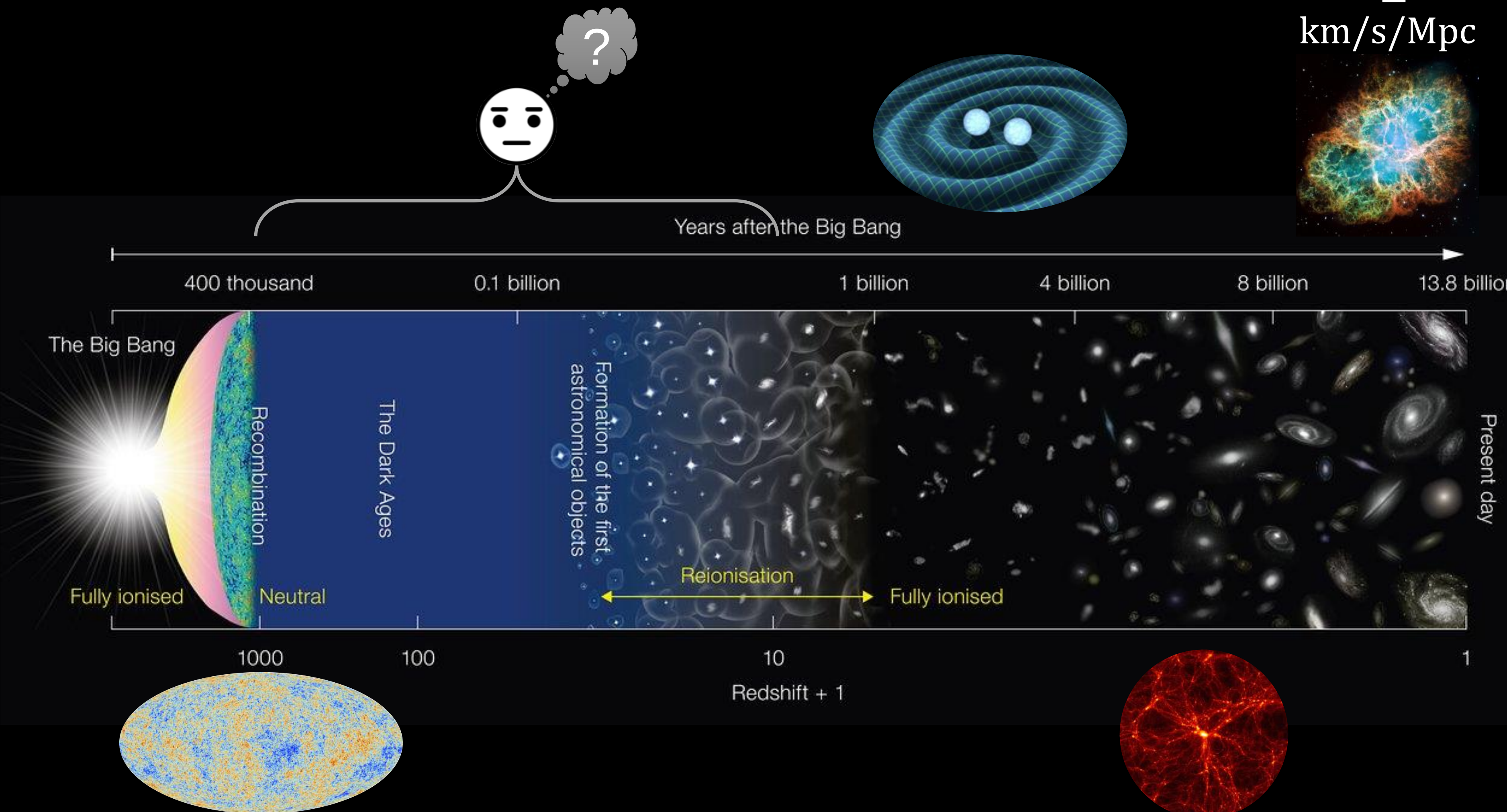

67.4 ± 0.5 km/s/Mpc

Image Credit: NAOJ

Primordial black holes as a potential candidate

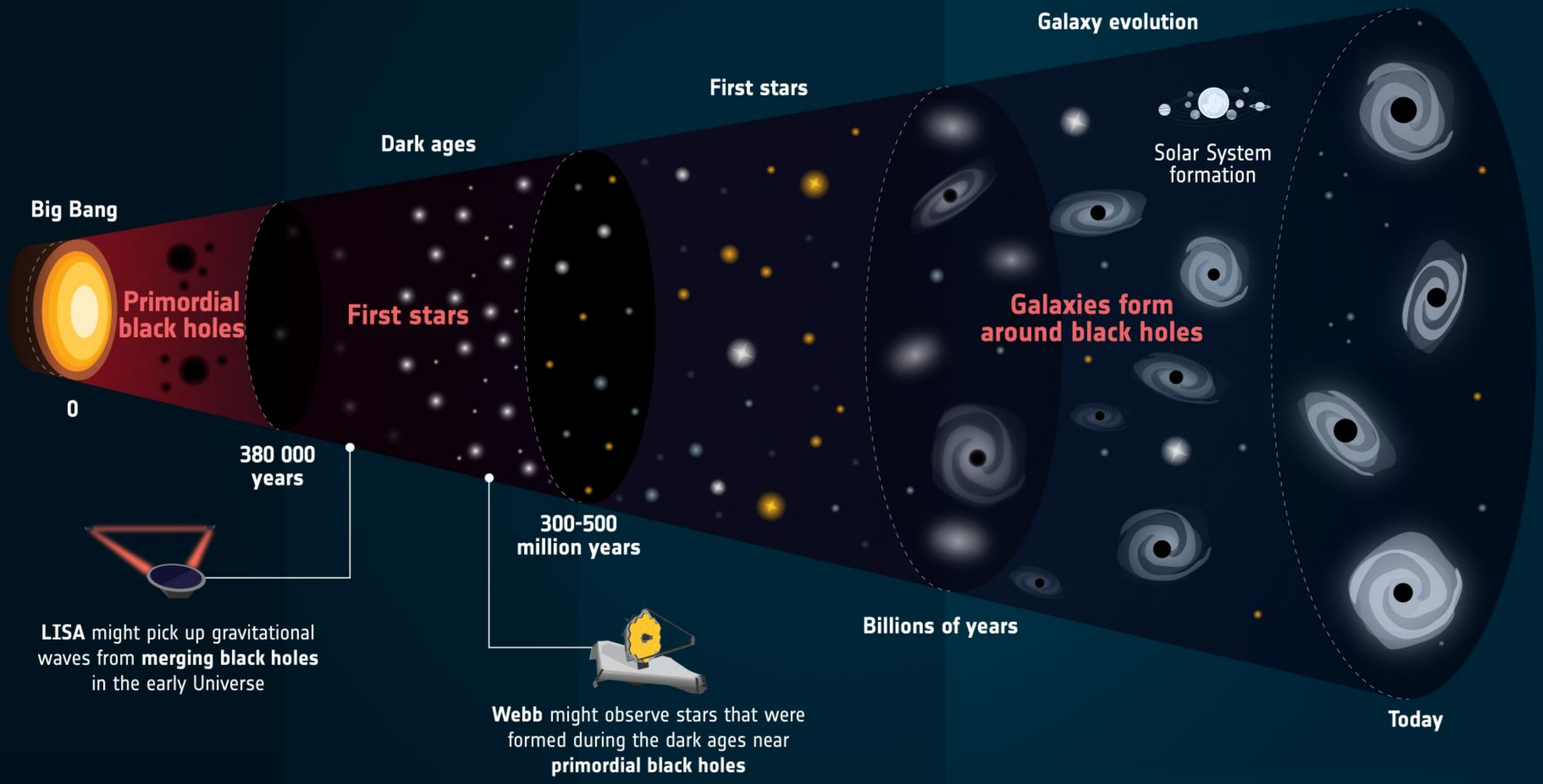

Image Credit: ESA

How to observe the signals from PBHs?

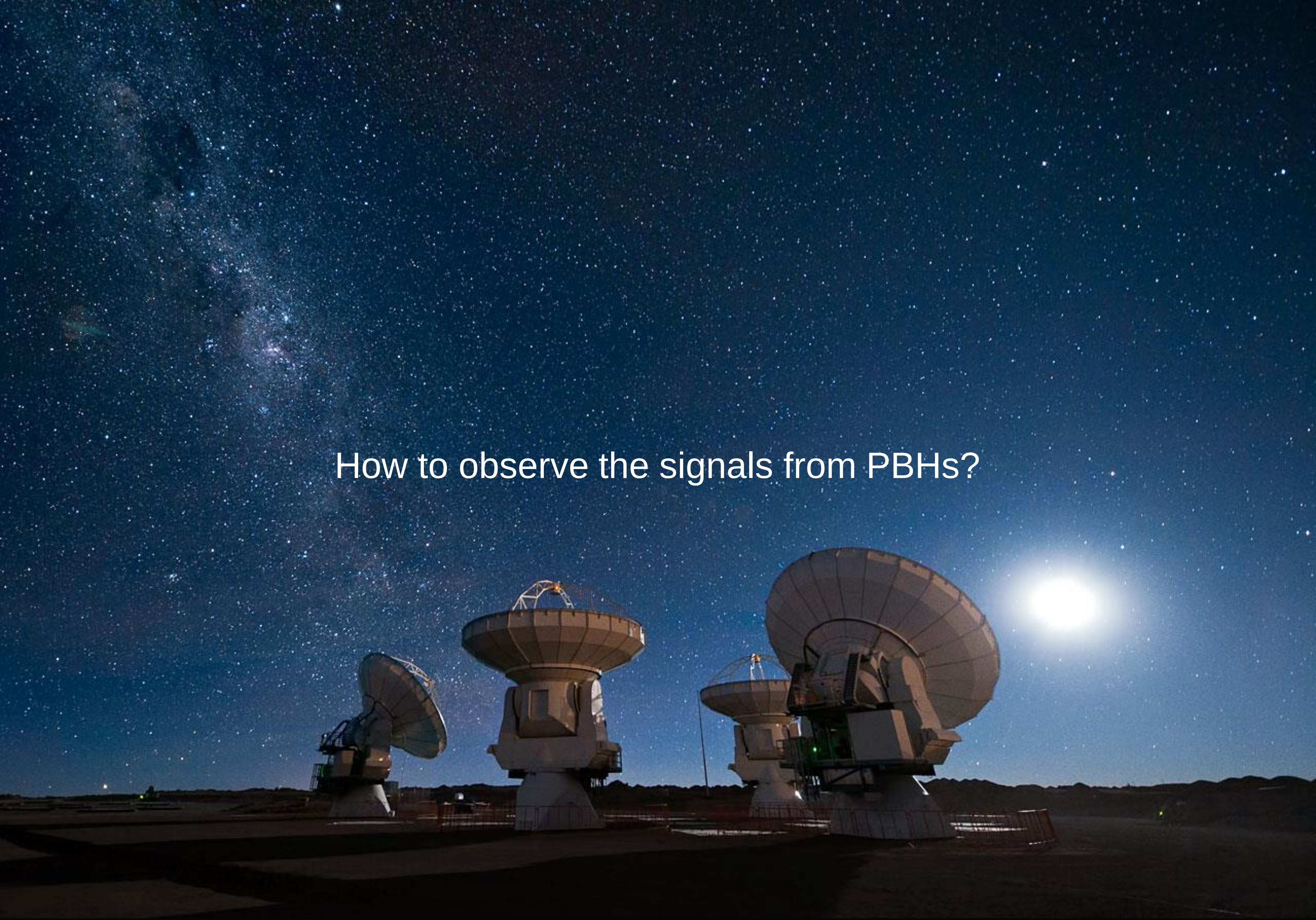

Image Credit: ESO

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$$h(t) = \frac{4}{d_L(z)}\left(\frac{G\mathcal{M}_z}{c^2}\right)^{5/3}\left(\frac{\pi f(t)}{c}\right)^{2/3}\cos\Phi(t)$$

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{c}{E(z')}dz' \qquad \mathcal{M}_z = \mathcal{M}(1+z)$$

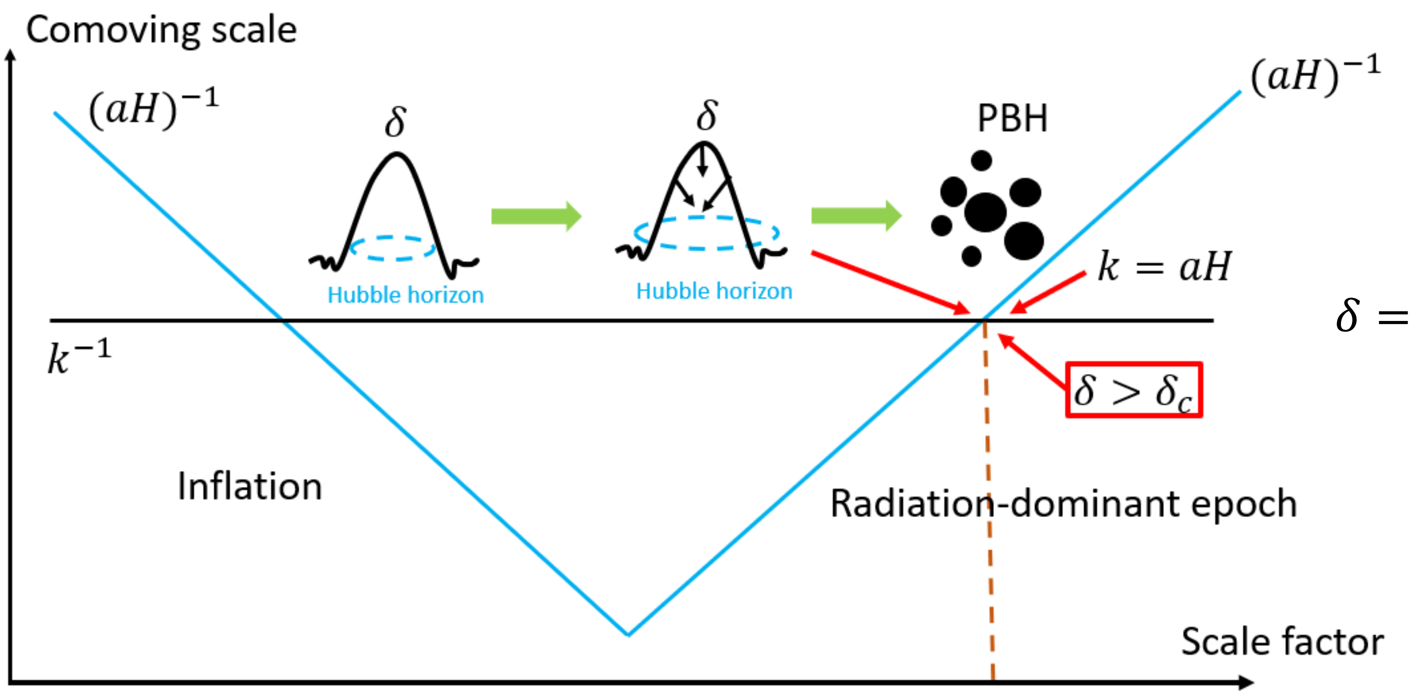
Image Credit: ESO

How to construct redshift-distance relation?

How to construct redshift-distance relation?

A statistical study on PBH binaries may help

PBH formation



$$\delta = \frac{\delta\rho}{\rho}$$

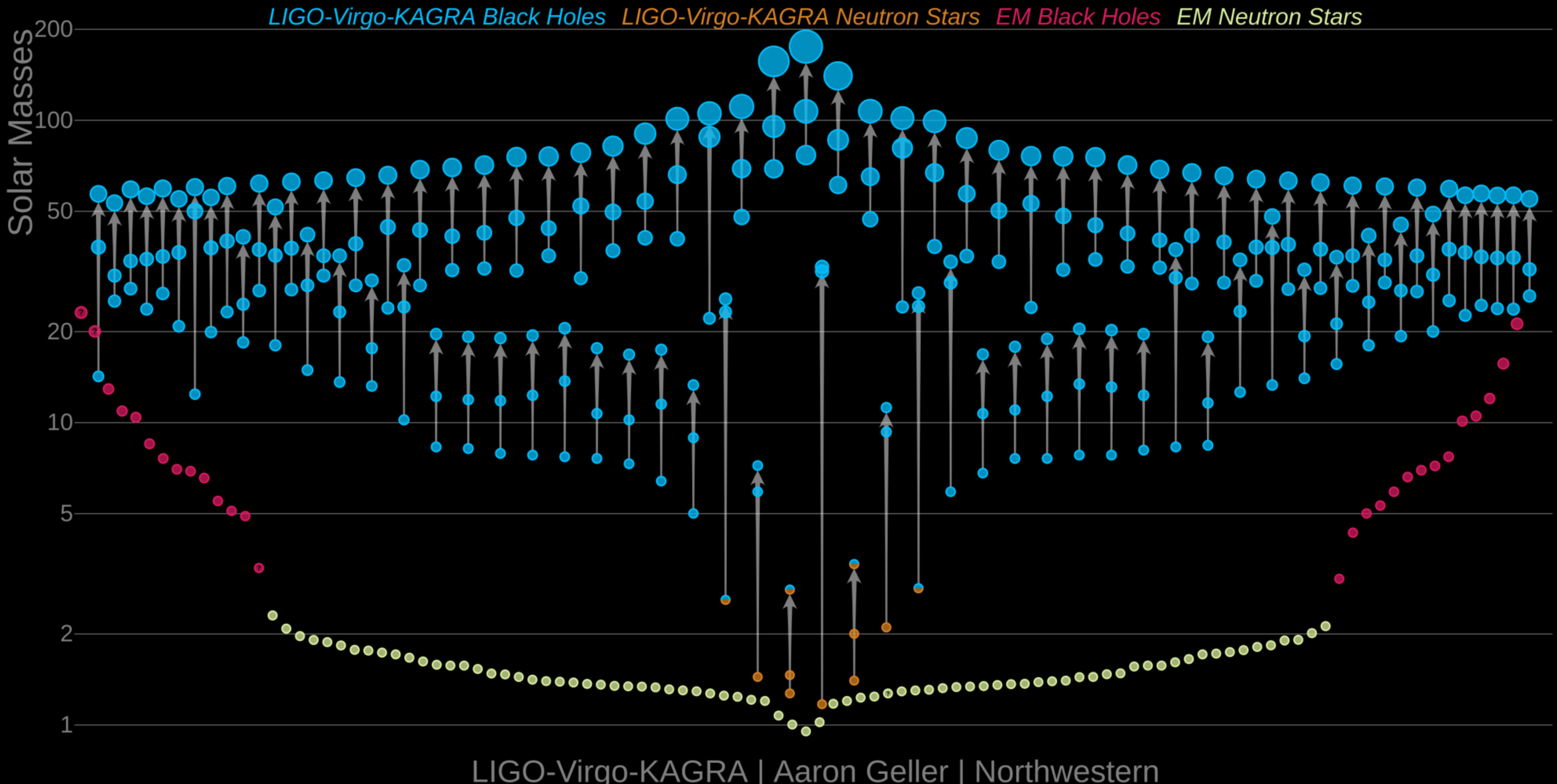
The primordial origin gives an identical primordial mass function

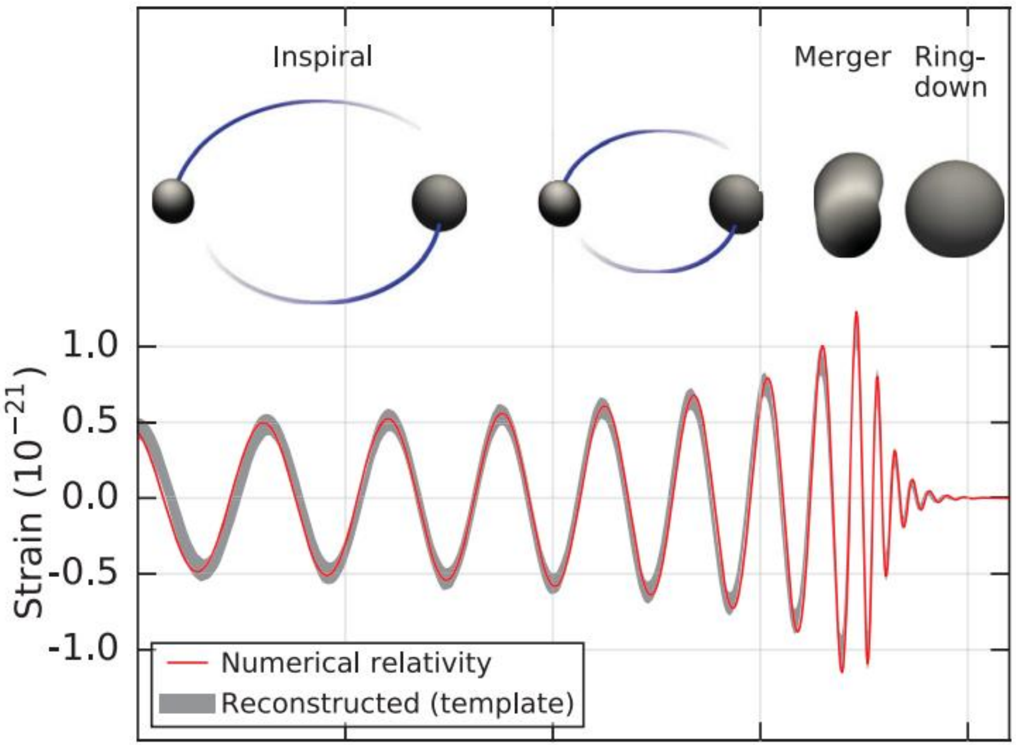
$$n(M) = \frac{f_{\mathrm{PBH}}}{\sqrt{2\pi}\sigma M} \exp\left[-\frac{\ln^2\left(M/M_{pk}\right)}{2\sigma^2}\right]$$

Merger rate of PBH binaries as a probe of Hubble parameter

PBH mass function

$$n(M) = \frac{1}{\sqrt{2\pi}\sigma M}\exp\left[-\frac{\ln^2\left(M/M_{pk}\right)}{2\sigma^2}\right]$$



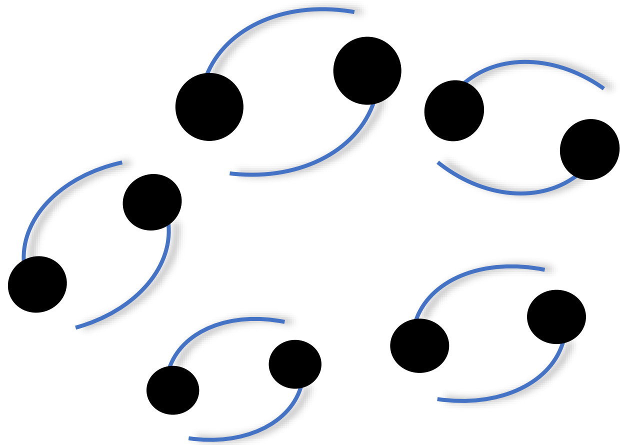


$$\mathcal{M}_z = (1+z)\mathcal{M}$$

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{c}{E(z')}dz'$$

B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. Phys. Rev. Lett., 116(6):061102, 2016.

$$h(t) = \frac{4}{d_L(z)}\left(\frac{G\mathcal{M}_z}{c^2}\right)^{5/3}\left(\frac{\pi f(t)}{c}\right)^{2/3}\cos\Phi(t)$$



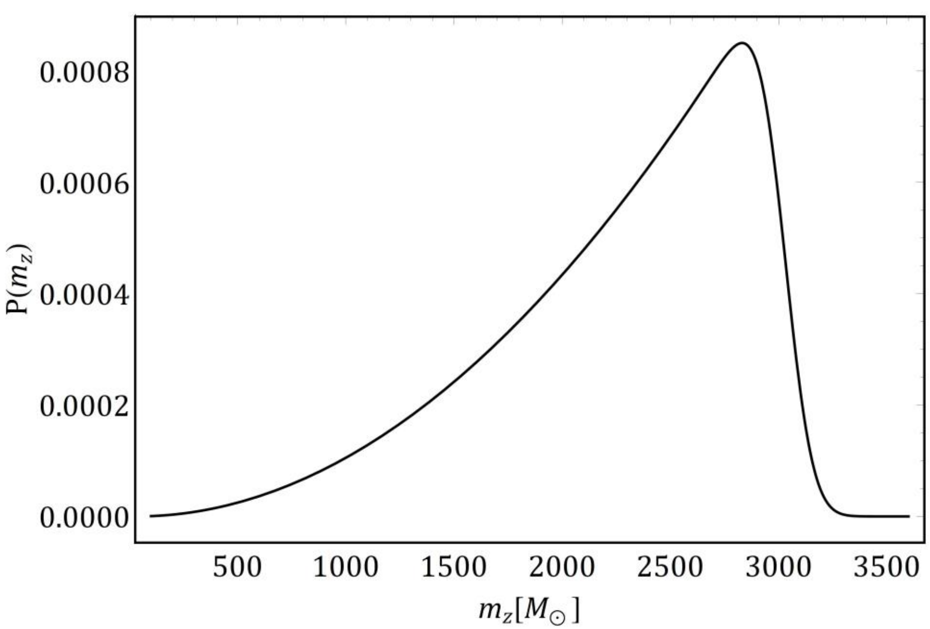
$$(m_z^i, d_L^i)$$

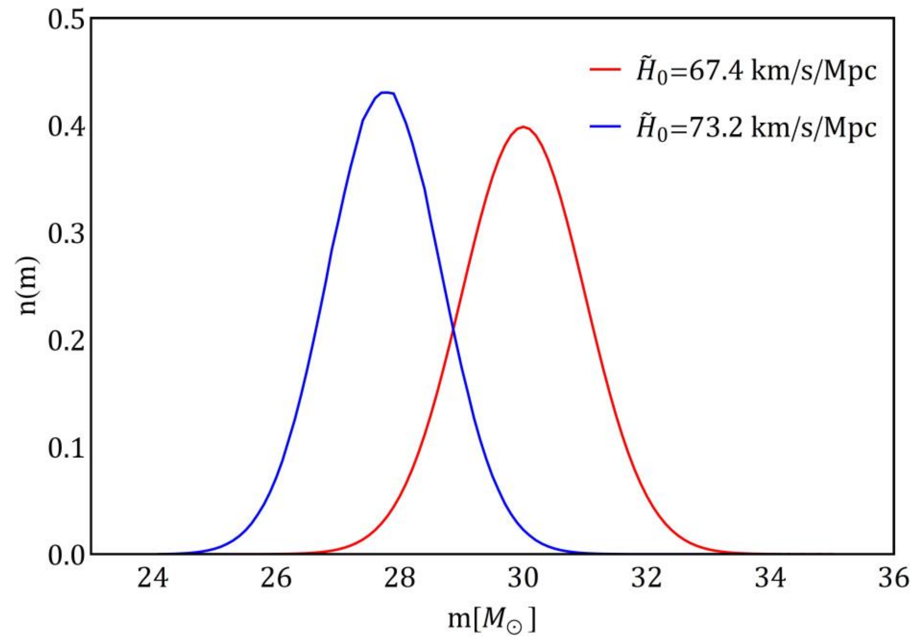
$$d_L^i = \frac{1+z_i}{H_0}\int_0^{z_i}\frac{c}{E(z')}dz'$$

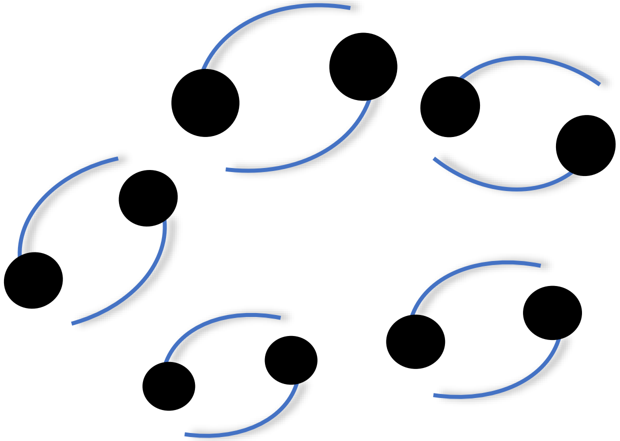
Assume a Hubble parameter $\widetilde{H}_0$

$$z_i = z(d_L^i; \widetilde{H}_0)$$

$$m^i = m_z^i/(1+z_i)$$







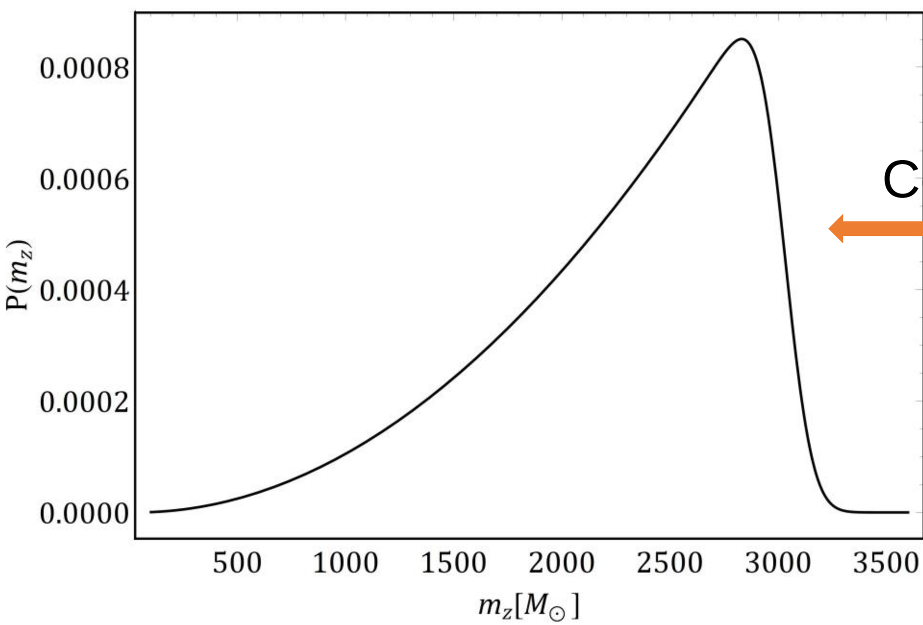
(m_z^i, d_L^i)

$$d_L^i = \frac{1+z_i}{H_0}\int_0^{z_i}\frac{c}{E(z')}dz'$$

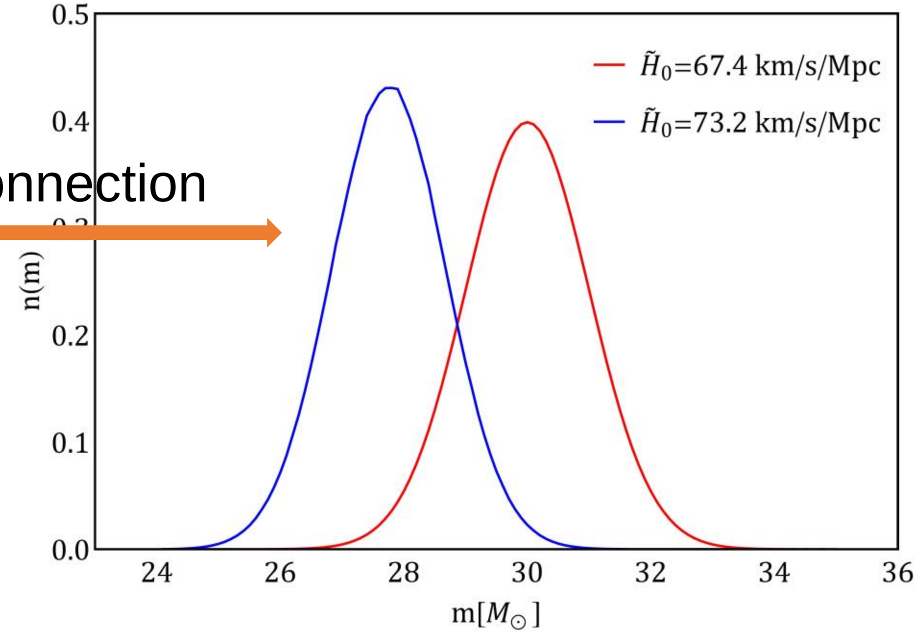
Assume a Hubble parameter $\tilde{H}_0$

$$z_i = z(d_L^i;\tilde{H}_0)$$

$$m^i = m_z^i/(1+z_i)$$



Connection



Cumulative distribution

$$C(m_1^z, m_2^z) = \frac{N(m < m_1^z, m_2^z)}{N_{\mathrm{tot}}}$$

detectable window function

$$C(m_1^z, m_2^z) = \int_0^\infty \int_0^{\frac{m_1^z}{1+z}} \int_0^{\frac{m_2^z}{1+z}} n(m_1) n(m_2) W(m_1, m_2; z) p(z)\, dm_1 dm_2 dz$$

PBH mass function

redshift distribution

Probability distribution

$$P(m_1^z, m_2^z) = \frac{1}{N_{\mathrm{tot}}} \frac{dN}{dm_1^z\, dm_2^z}$$

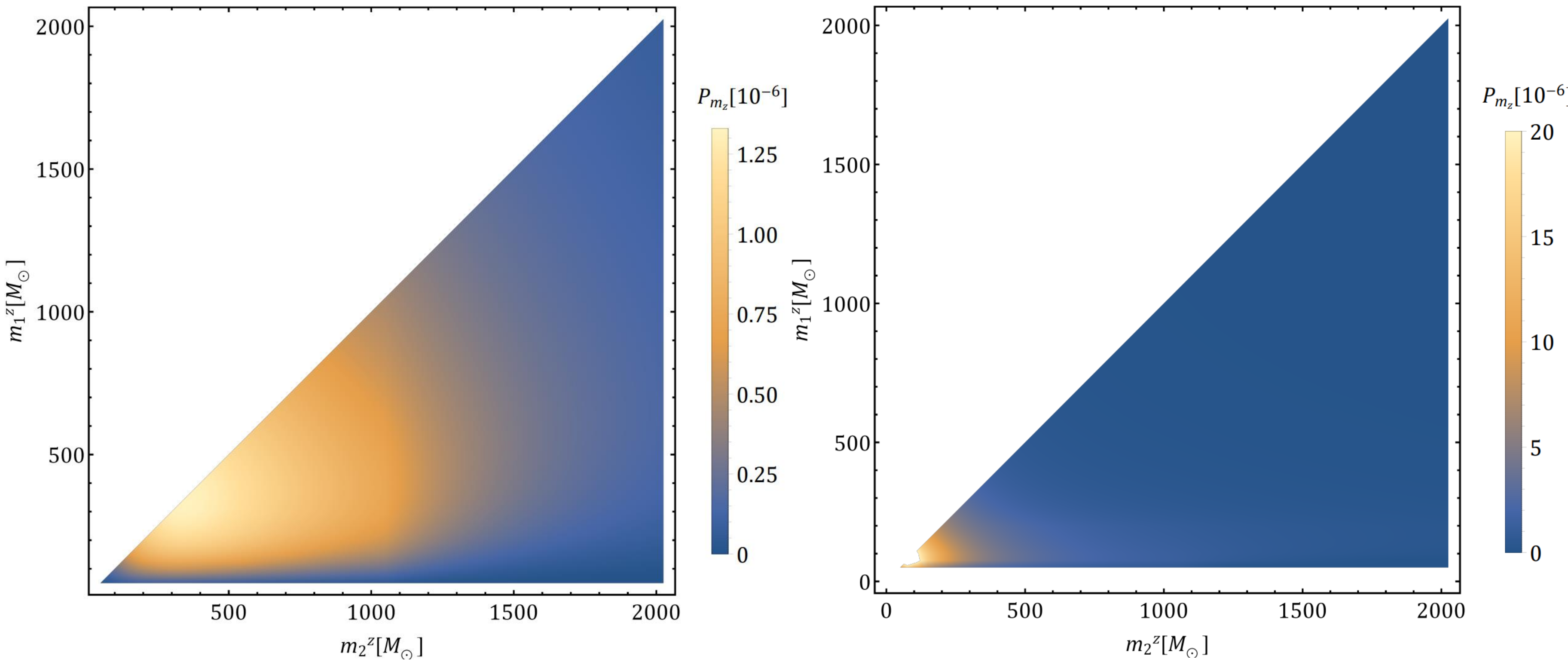
$$P(m_1^z, m_2^z) = \frac{\partial^2 C(m_1^z, m_2^z)}{\partial m_1^z \partial m_2^z}$$

$$P(m_1^z, m_2^z) = \int_0^\infty n\left(\frac{m_1^z}{1+z}\right) n\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

$$W(m_1, m_2; z) = \frac{N_{\mathrm{obs}}(m_1, m_2; z)}{N_{\mathrm{tot}}(m_1, m_2; z)} = \int_{a_{\mathrm{min}}}^{a_{\mathrm{max}}} \int_{e_{\mathrm{min}}}^{e_{\mathrm{max}}} P(a, e; z)\, dade$$

$$\mathrm{SNR} = \sqrt{4 \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \frac{\left|\tilde{h}(f)\right|^2}{S_n(f)} df} > 8 \qquad \tilde{h}(f) = \sqrt{\frac{5}{24}} \frac{(G\mathcal{M}_z)^{5/6}}{\pi^{2/3} c^{3/2} d_L} f^{-7/6}$$

$$p(z) \propto \frac{\dot{n}(z)}{1+z} \frac{dV_c}{dz} \qquad \dot{n}(z) \propto \left(\frac{t(z)}{t_0}\right)^{-34/37}$$



$$n(m) = \frac{1}{\sqrt{2\pi}\sigma m}\exp\left[-\frac{\ln^2(m/m_{pk})}{2\sigma^2}\right]$$

$$n(m) = \frac{\alpha-1}{M}\left(\frac{m}{M}\right)^{-\alpha}$$

$$P_O(m_1^z, m_2^z) = \int_0^\infty n_p\left(\frac{m_1^z}{1+z}\right) n_p\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

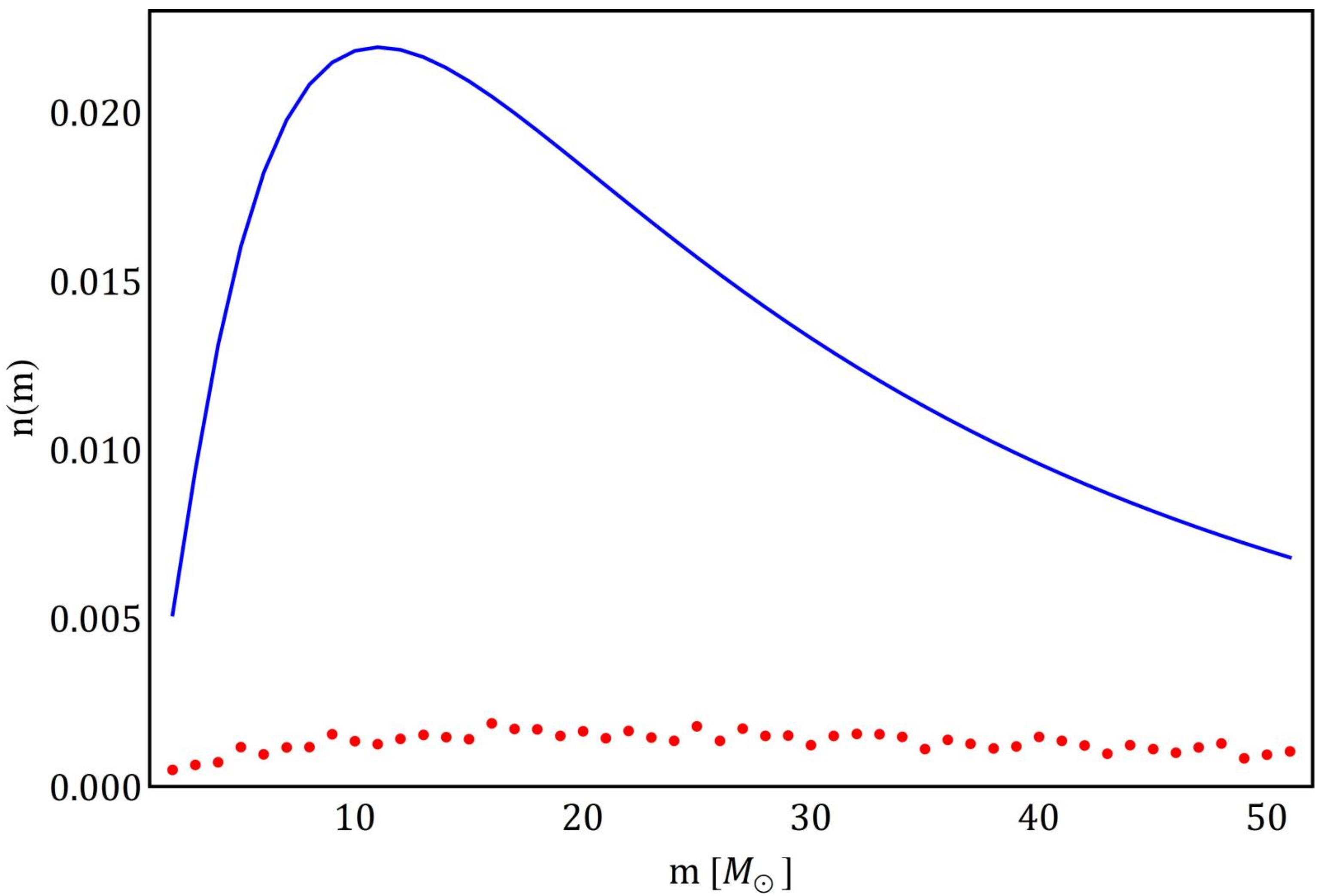
Gradient Descent Method

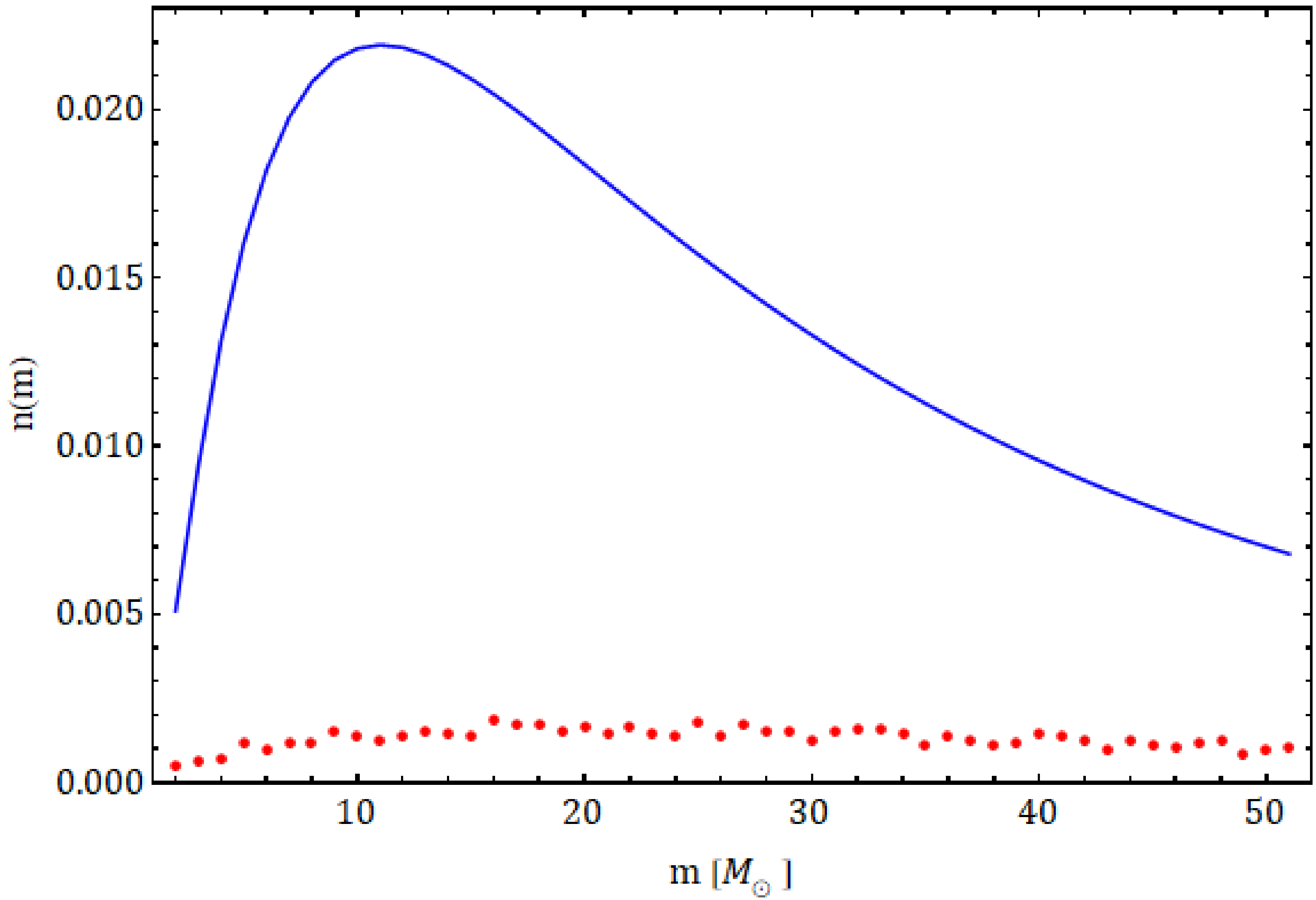
$$P_O(m_1^z, m_2^z) = \int_0^\infty n_p\left(\frac{m_1^z}{1+z}\right) n_p\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

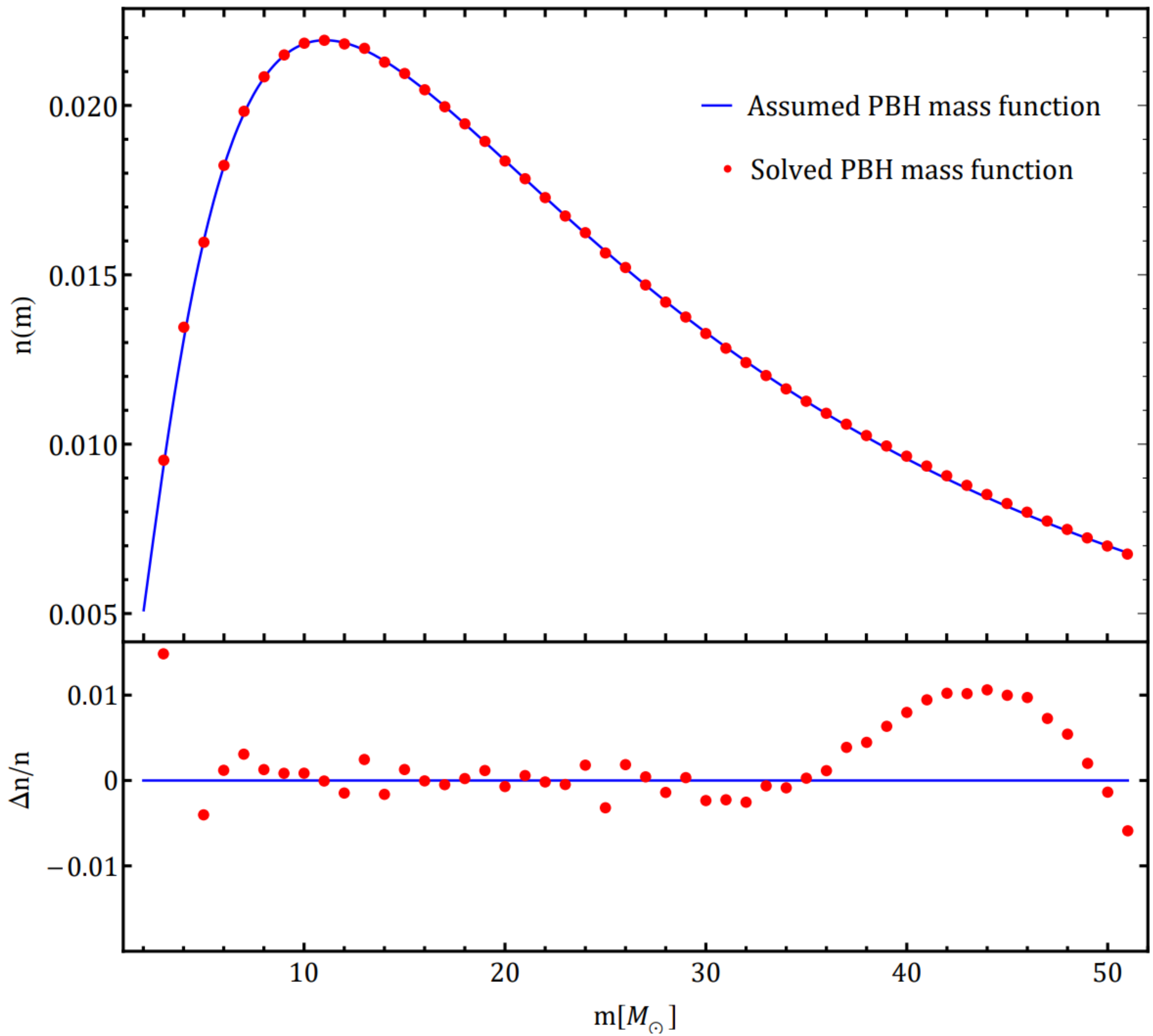
$$P_T(m_1^z, m_2^z) = \int_0^\infty n'\left(\frac{m_1^z}{1+z}\right) n'\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

$$E(n) = \sqrt{\frac{\sum_{1\le i\le j, j=1}^{j=N} [P_T(m_1^z, m_2^z) - P_O(m_1^z, m_2^z)]^2}{(1+N)N/2}}$$

$$n_{k+1}(m_i) = n_k(m_i) - \gamma \frac{\partial E(n_k)}{\partial n_k(m_i)}$$


0.020
0.015
0.010
0.005
0.000
n(m)
10
20
30
40
50
m [$M_\odot$]


0.020
0.015
0.010
0.005
0.000
n(m)
10
20
30
40
50
m [$M_\odot$]

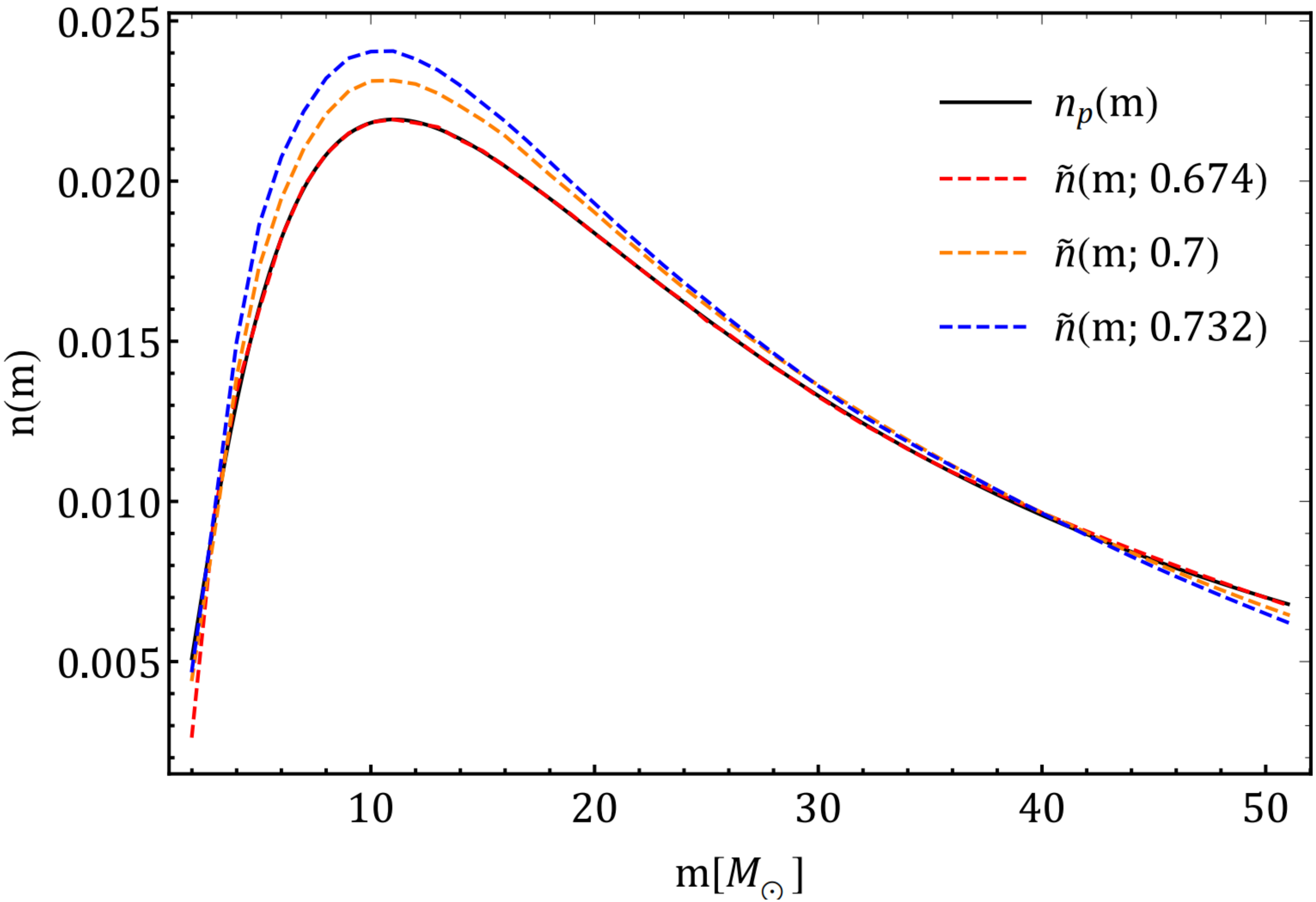


Assumed PBH mass function
Solved PBH mass function
0.020
0.015
0.010
0.005
n(m)
0.01
0
−0.01
Δn/n
10
20
30
40
50
m[$M_\odot$]

How about $p(z)$?

$$\left.\begin{array}{c} d_L^i = \dfrac{1+z_i}{H_0}\displaystyle\int_0^{z_i} \frac{c}{E(z')}\,dz' \\[2em] \text{Assume a Hubble parameter } \widetilde{H}_0 \\[2em] z_i = z(d_L^i; \widetilde{H}_0) \end{array}\right\} \quad p(z; \widetilde{H}_0)$$

$$P_O(m_1^z, m_2^z) = \int_0^\infty \tilde{n}\left(\frac{m_1^z}{1+z}\right)\tilde{n}\left(\frac{m_2^z}{1+z}\right)W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right)\frac{p(z; \widetilde{H}_0)}{(1+z)^2}\,dz$$

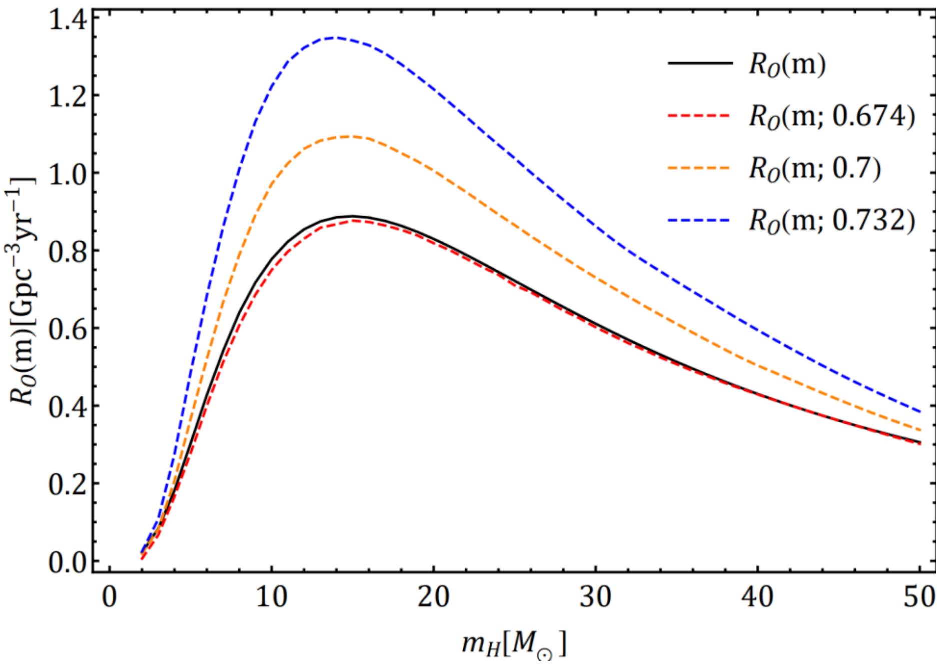


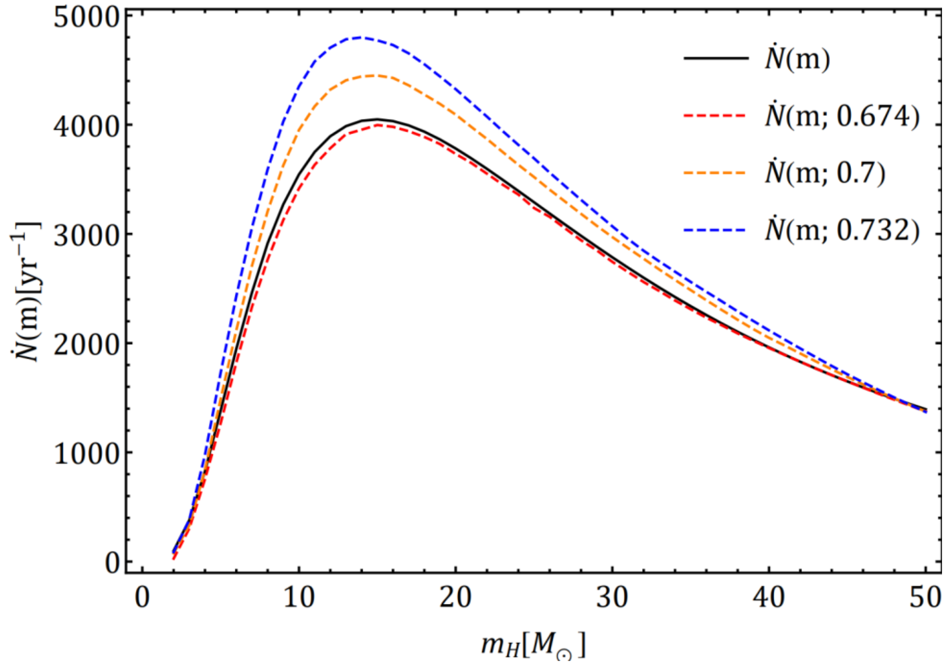
0.025
0.020
0.015
0.010
0.005
n(m)
10
20
30
40
50
m[$M_\odot$]
n_p(m)
$\tilde{n}$(m; 0.674)
$\tilde{n}$(m; 0.7)
$\tilde{n}$(m; 0.732)

However, we don’t know the PBH mass function currently.

However, we don’t know the PBH mass function currently.

Another observable related with PBH mass function
Merger rate of PBH binaries

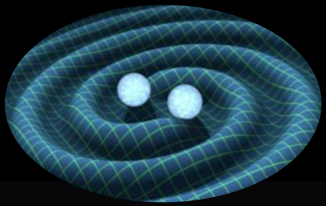




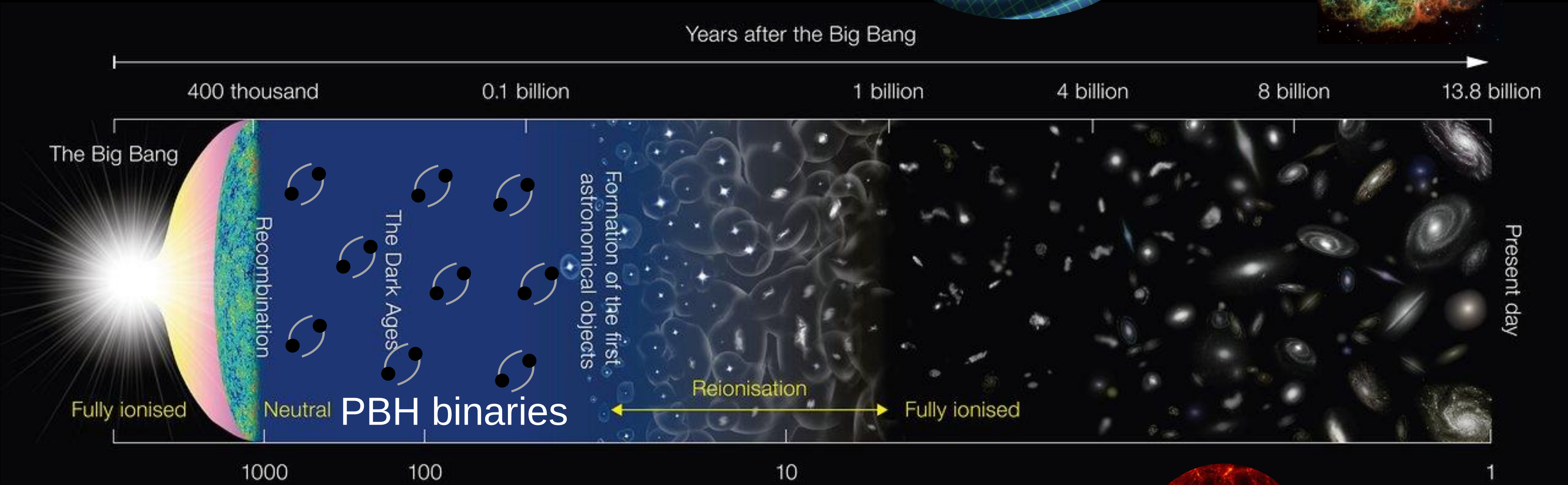
$$R_{ij} = \rho_{\mathrm{PBH}} \min\left(\frac{n(m_i)}{m_i}, \frac{n(m_j)}{m_j}\right) \Delta_m \frac{dP}{dt}$$

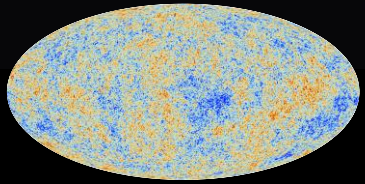
$$\dot{N} = \int_{z_{\min}}^{z_{\max}} \frac{R}{1+z} \frac{dV_c}{dz} dz$$

Background level
Hubble expansion rate



73.04 ± 1.04 km/s/Mpc





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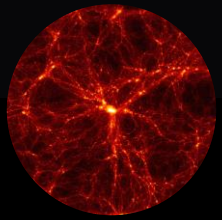
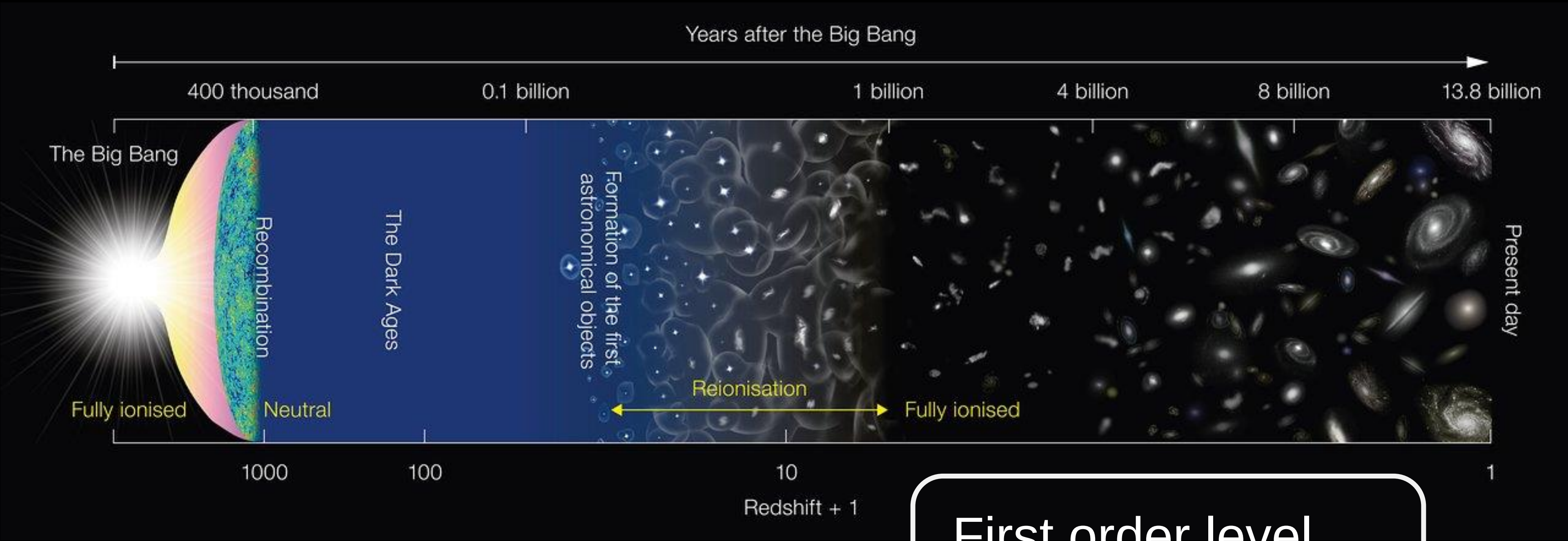


Image Credit: NAOJ

Background level
Hubble expansion rate

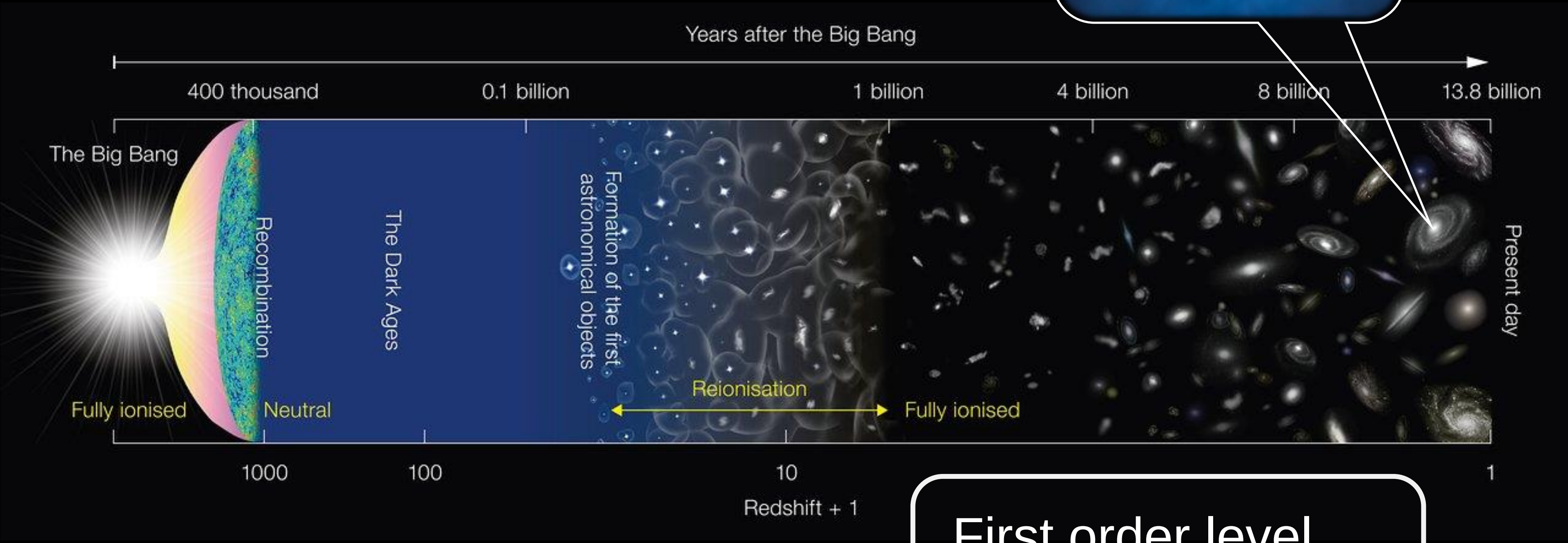


First order level
Structure formation

2410.02591, QD, Minxi He, Volodymyr Takhistov

Image Credit: NAOJ

Background level
Hubble expansion rate



First order level
Structure formation

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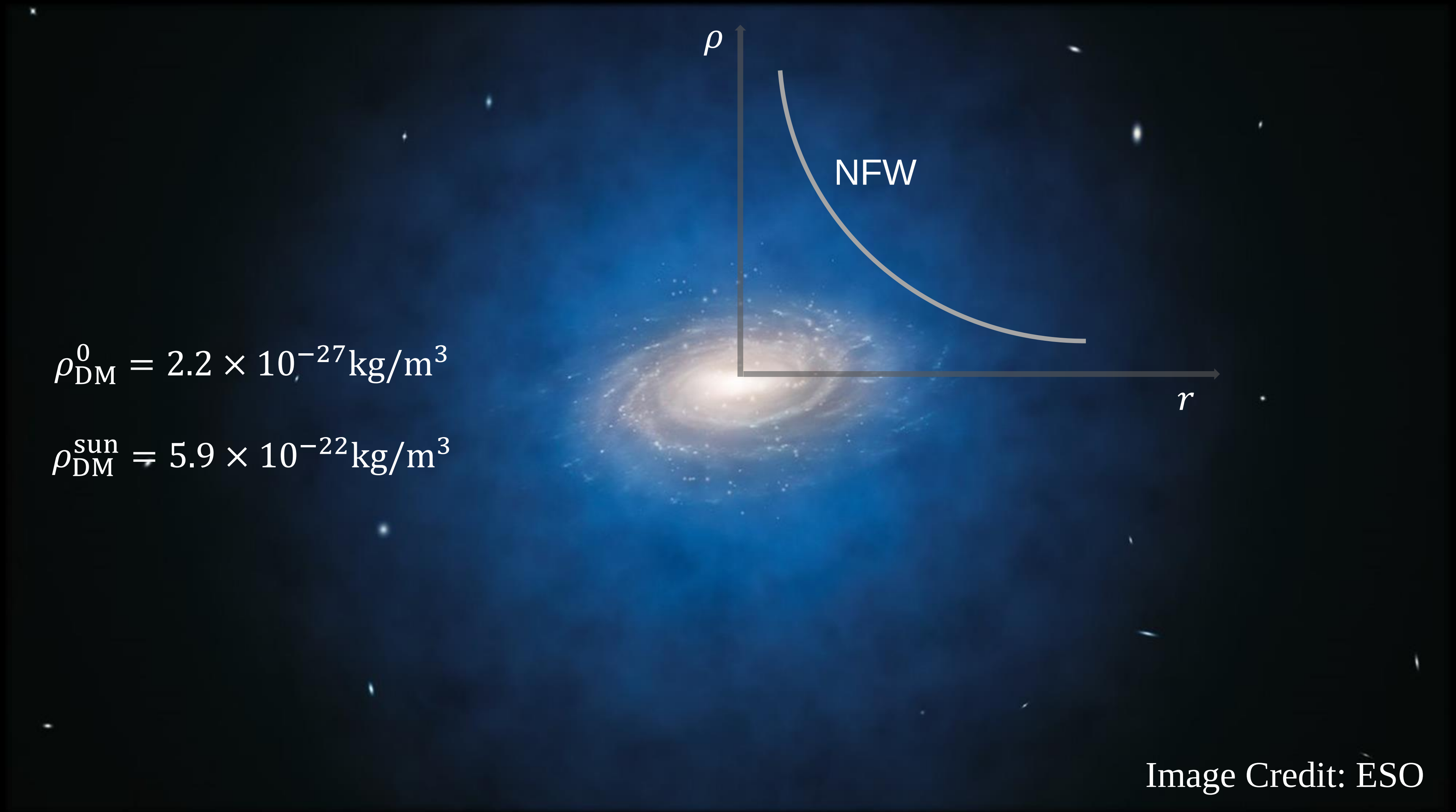
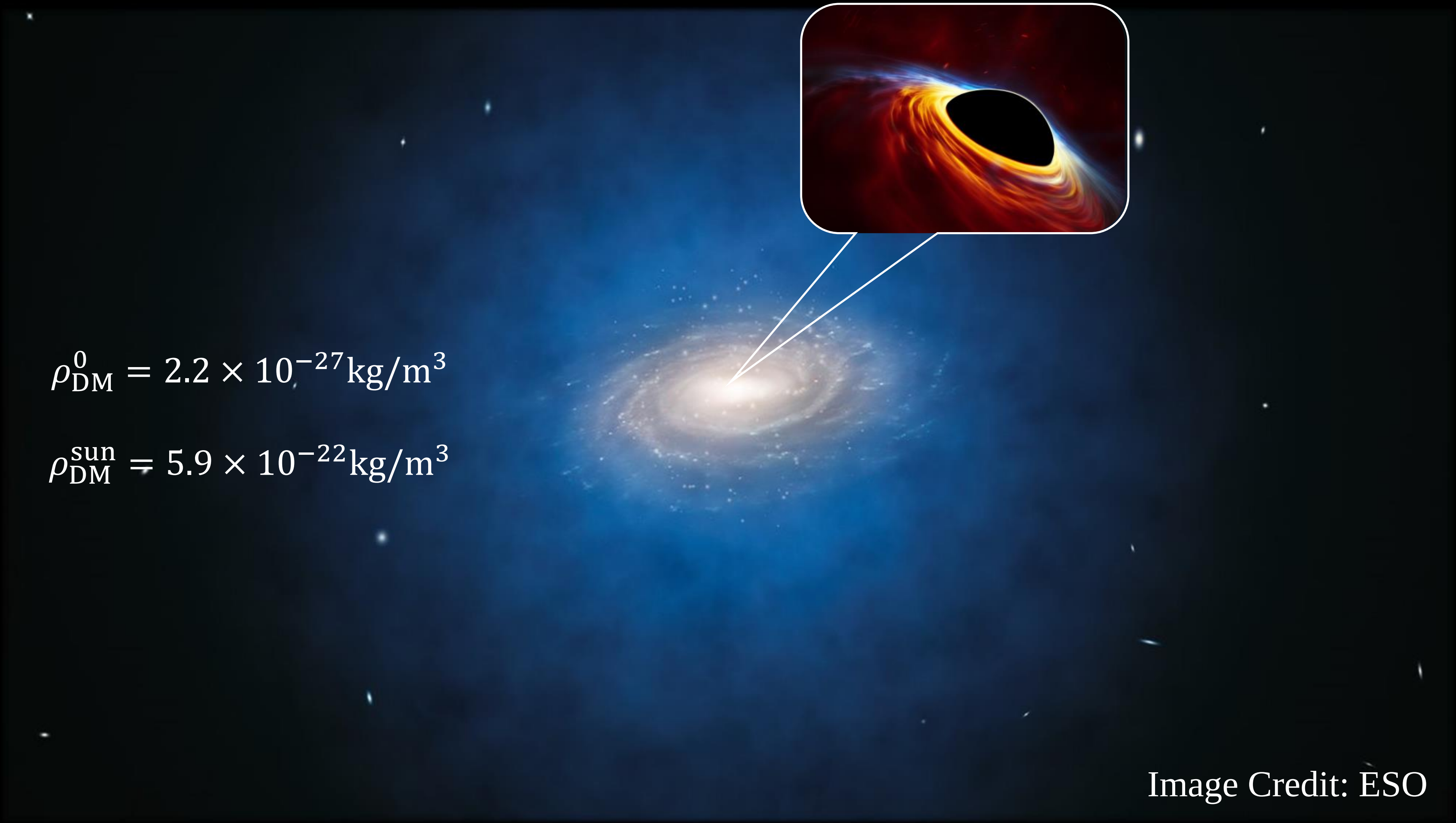
Image Credit: NAOJ



$$\rho_{\mathrm{DM}}^{0} = 2.2 \times 10^{-27}\mathrm{kg/m^3}$$

$$\rho_{\mathrm{DM}}^{\mathrm{sun}} = 5.9 \times 10^{-22}\mathrm{kg/m^3}$$

Image Credit: ESO



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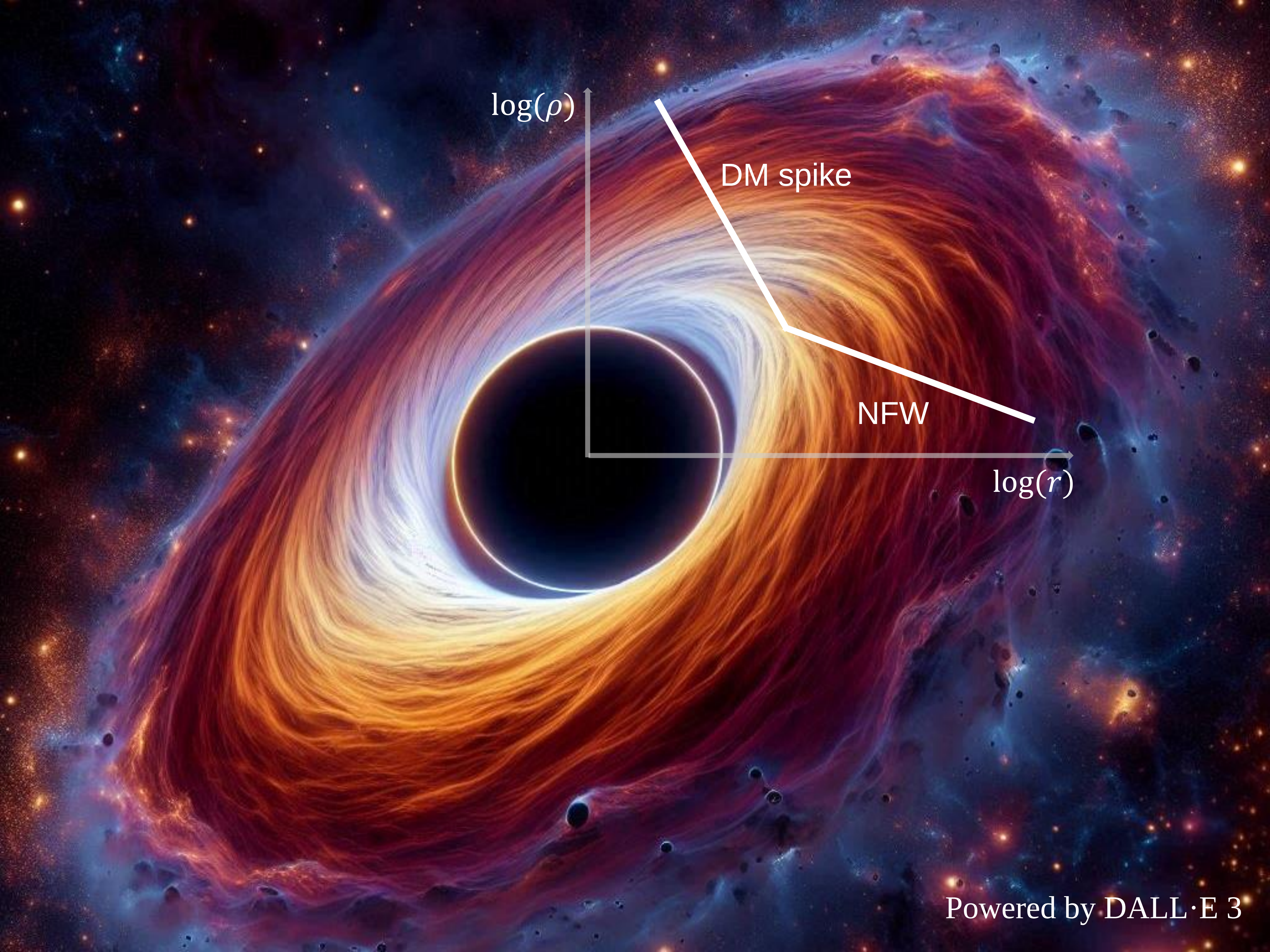
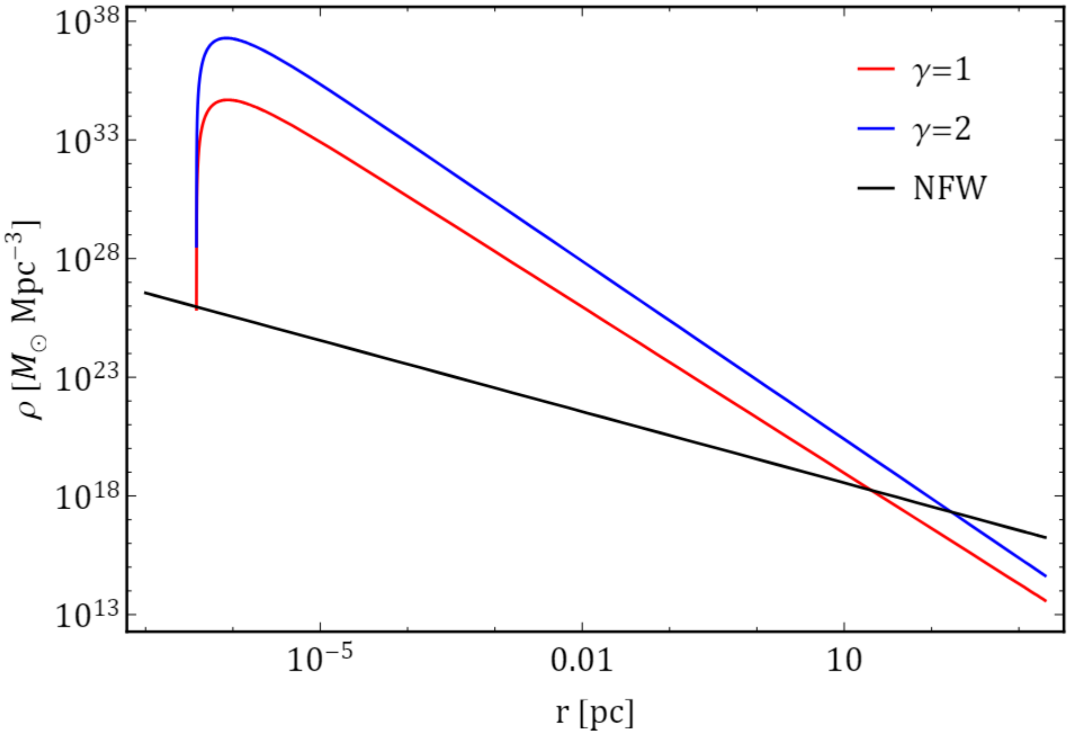

log(ρ)
DM spike
NFW
log(r)
Powered by DALL·E 3

Dark Matter Spike



Fokker-Planck Equation

$$\frac{\partial[f(E,t)g(E)]}{\partial t} = -\frac{\partial \mathcal{F}(E,t)}{\partial E},$$

$$-\mathcal{F}(E,t) \equiv D_{EE}(E)\frac{\partial f(E,t)}{\partial E} + D_E(E) f(E,t).$$

$$\rho(r) \simeq \rho_0 (r_0/r)^\gamma$$

$$\rho(r) = 4\pi \int_{\Phi(r)}^{0} dE\, f(E)\, \sqrt{2[E - \Phi(r)]}$$

$$\rho_{\rm sp}(r) = \rho_R \left(1 - \frac{4R_{\rm s}}{r}\right)^3 \left(\frac{R_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}$$

$$\gamma_{\rm sp} = \frac{9-2\gamma}{4-\gamma}$$

OJ287

$M_{\mathrm{BH}} = 1.8 \times 10^{10} M_\odot$

$P = 12.067\ yr$

$m_{\mathrm{BH}} = 1.5 \times 10^{8} M_\odot$

$\dot{P} = -0.00099$

Video Credit: NASA JPL

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The First Robust Evidence Showing a Dark Matter Density Spike Around the Supermassive Black Hole in OJ 287

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Abstract

Black hole dynamics suggests that dark matter would redistribute near a supermassive black hole (SMBH) to form a density spike. However, no direct evidence of a dark matter density spike around an SMBH has been identified. In this Letter, we present the first robust evidence showing a dark matter density spike around an SMBH. We revisit the data of the well-known SMBH binary OJ 287 and show that the inclusion of the dynamical friction due to a dark matter density spike around the SMBH can satisfactorily account for the observed orbital decay rate. The derived spike index $\gamma_{sp} = 2.351^{+0.032}_{-0.045}$ gives an excellent agreement with the value $\gamma_{sp} = 2.333$ predicted by the benchmark model assuming an adiabatically growing SMBH. This provides a strong verification of the canonical theory suggested two decades ago modeling the gravitational interaction between collisionless dark matter and SMBHs.

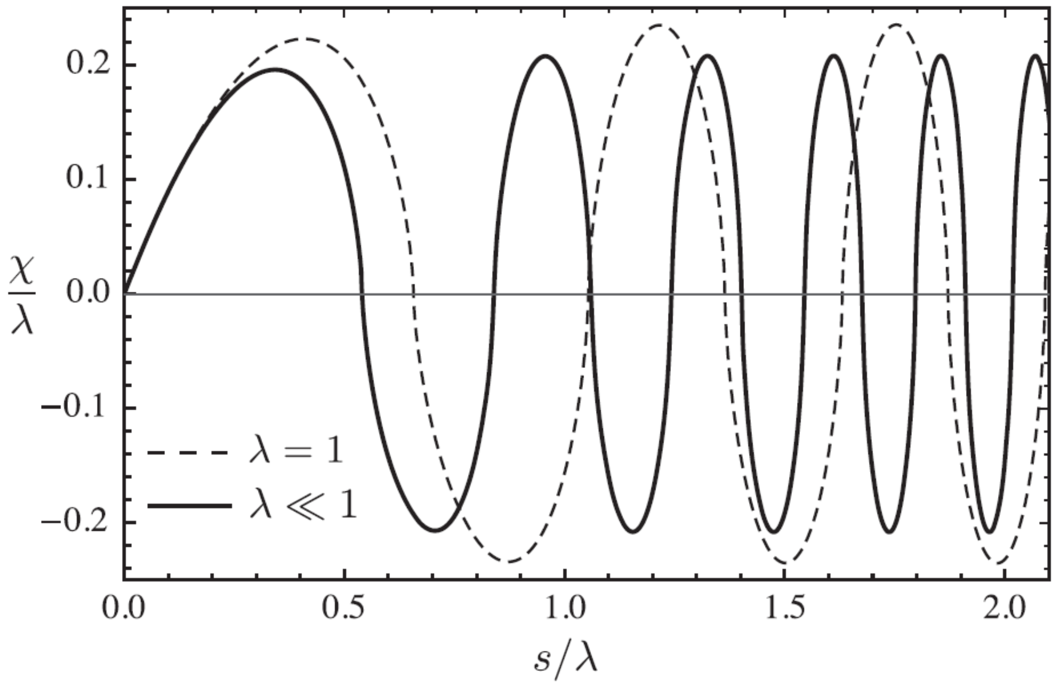
$$\gamma_{sp} = 2.333$$

How about primordial black holes in DM spike?

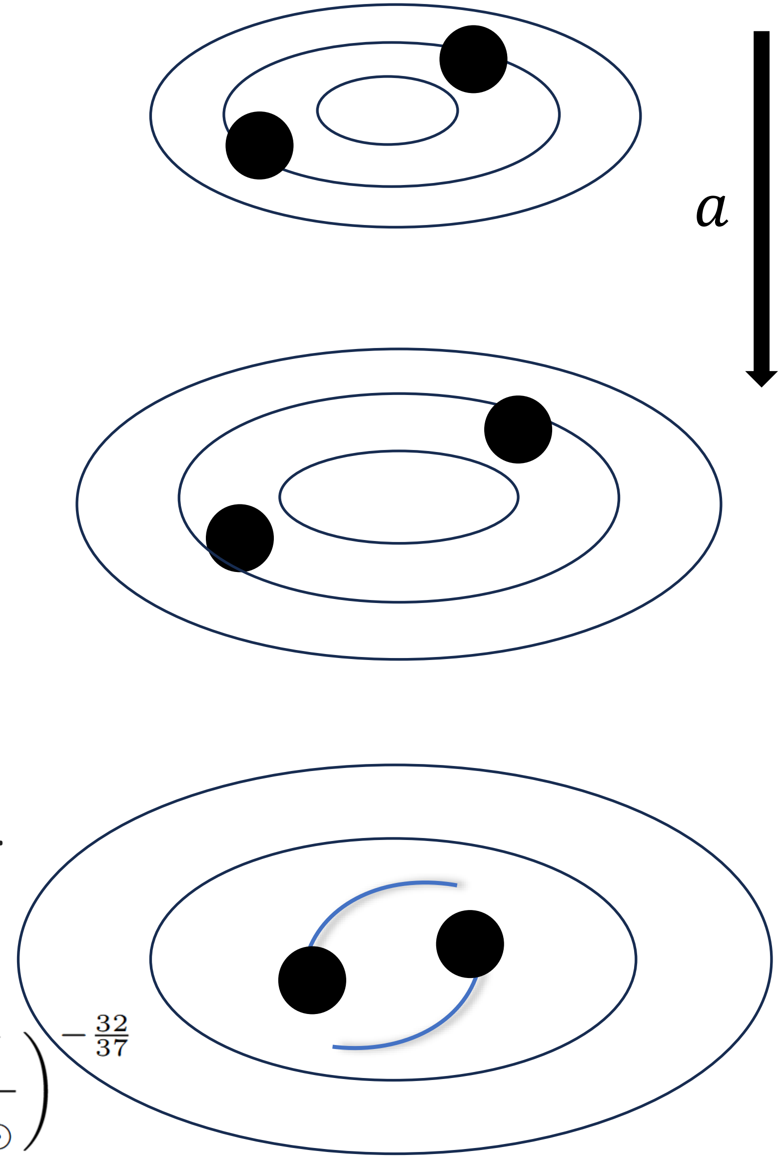
How about primordial black holes in DM spike?

PBH merger rate would be enhanced!

Early PBH Binary



Ali-Haïmoud, Yacine, Ely D. Kovetz, and Marc Kamionkowski. "Merger rate of primordial black-hole binaries." *Physical Review D* 96.12 (2017): 123523.



$$\frac{dR}{dm_1 dm_2} = \frac{1.6 \times 10^6}{\mathrm{Gpc}^3 \mathrm{yr}} f_{\mathrm{PBH}}^{\frac{53}{37}} \left(\frac{t(z)}{t_0}\right)^{-\frac{34}{37}} \eta^{-\frac{34}{37}} \left(\frac{M}{M_\odot}\right)^{-\frac{32}{37}} \times S(M, f_{\mathrm{PBH}}) \psi(m_1) \psi(m_2)$$

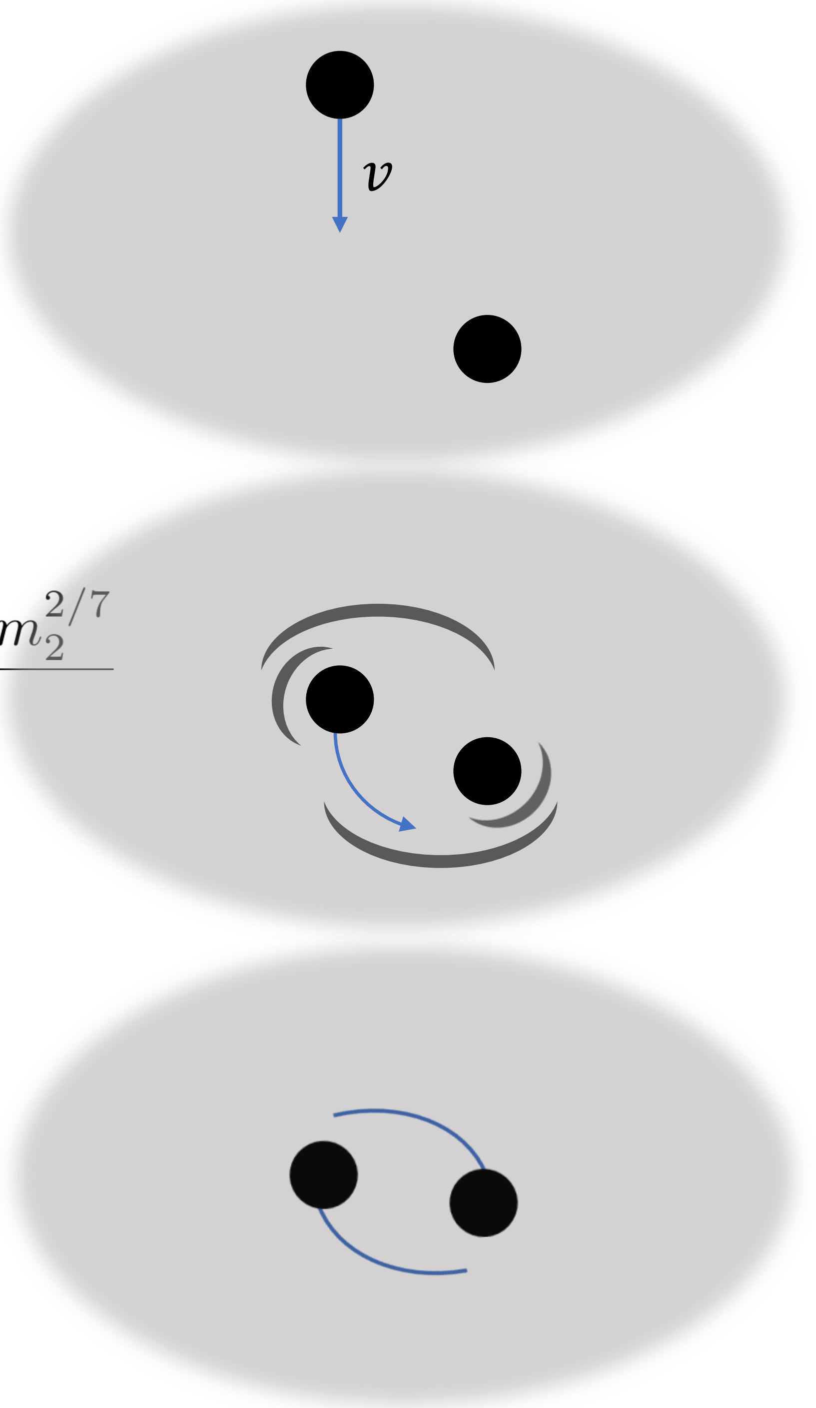
Late PBH Binary

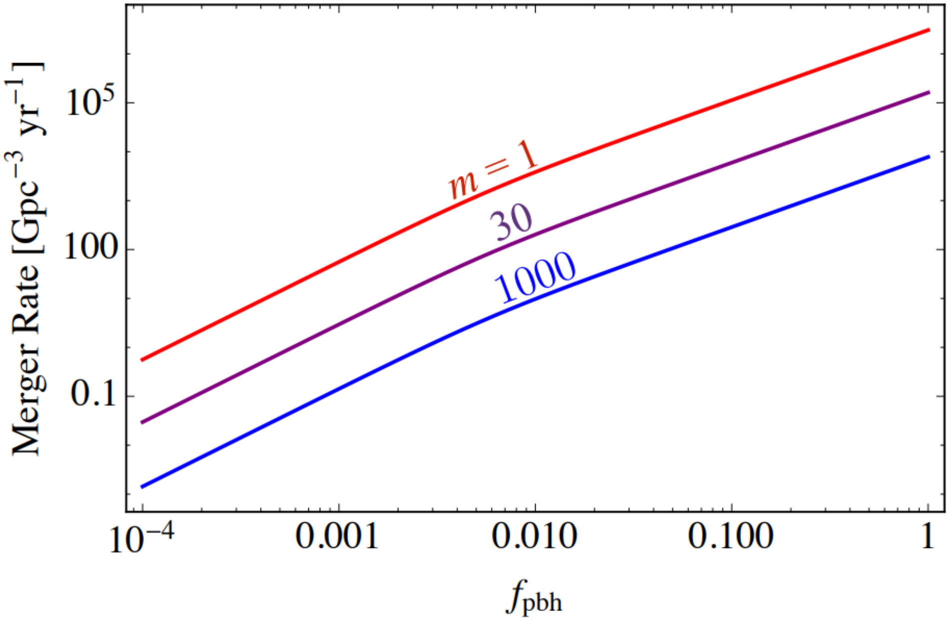
$$R_{\rm cap} = \int \frac{1}{2}\left(\frac{\rho_{\rm PBH}}{M_{\rm PBH}}\right)^2 \sigma_{\rm cap} v_{\rm rel} dV$$

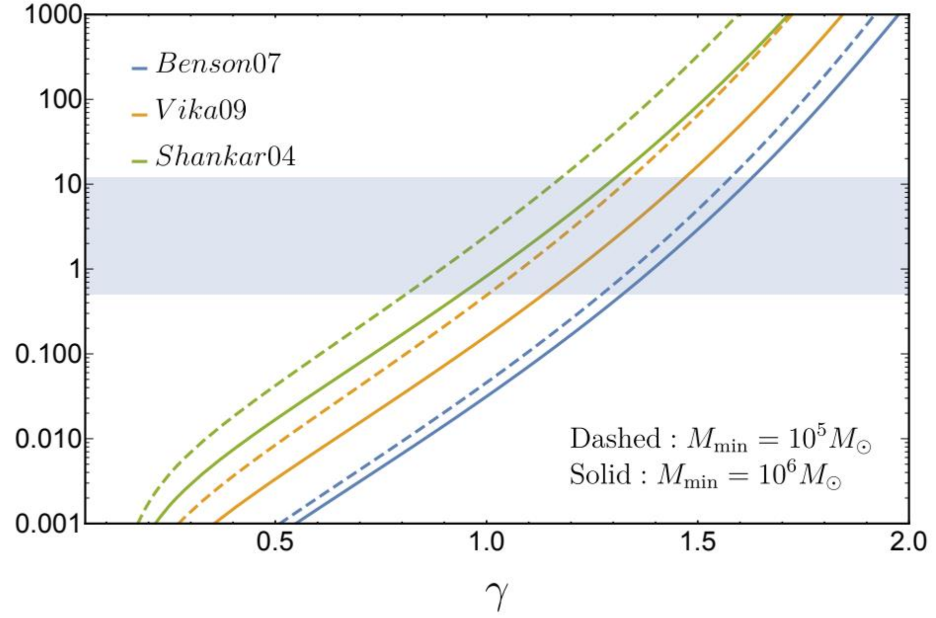
$$\sigma_{\rm cap} = 2\pi\left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7} \frac{G^2(m_1+m_2)^{10/7} m_1^{2/7} m_2^{2/7}}{c^{10/7} v_{\rm rel}^{18/7}}$$

$$t_{\rm cap} \gg t_{\rm merge}$$

$$R_{\rm merge} \simeq R_{\rm cap}$$







Ali-Haïmoud, Yacine, Ely D. Kovetz, and Marc Kamionkowski. "Merger rate of primordial black-hole binaries." *Physical Review D* 96.12 (2017): 123523.

Nishikawa, Hiroya, et al. "Primordial-black-hole mergers in dark-matter spikes." *Physical Review D* 99.4 (2019): 043533.

PBH merger rate from early binaries

PBH merger rate from late binaries in DM spike

PBH Merger Rate in DM Spike

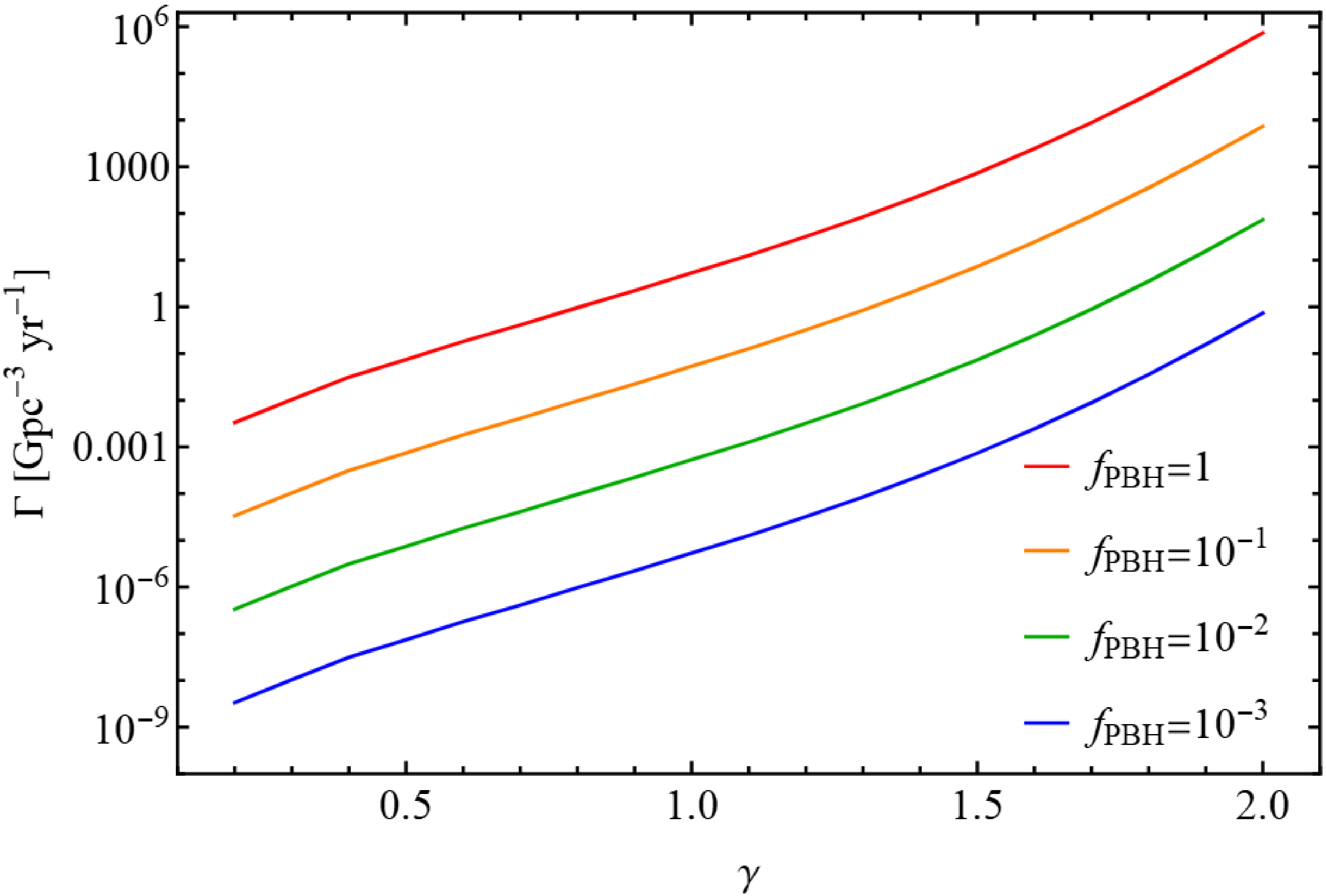
$$N_{\rm sp} = \int_{4R_s}^{R_{\rm sp}} \frac{1}{2}\left(\frac{f_{\rm PBH}\rho_{\rm sp}(r)}{M_{\rm PBH}}\right)^2 \sigma_m(r) v_{\rm rel}(r) d^3 r$$

$$\sigma_m(r) = 1.4 \times 10^{-14}\left(\frac{M_{\rm PBH}}{30\, M_\odot}\right)^2 \left(\frac{v_{\rm rel}(r)}{200\,{\rm km\,s^{-1}}}\right)^{-\frac{18}{7}} {\rm pc}^2$$

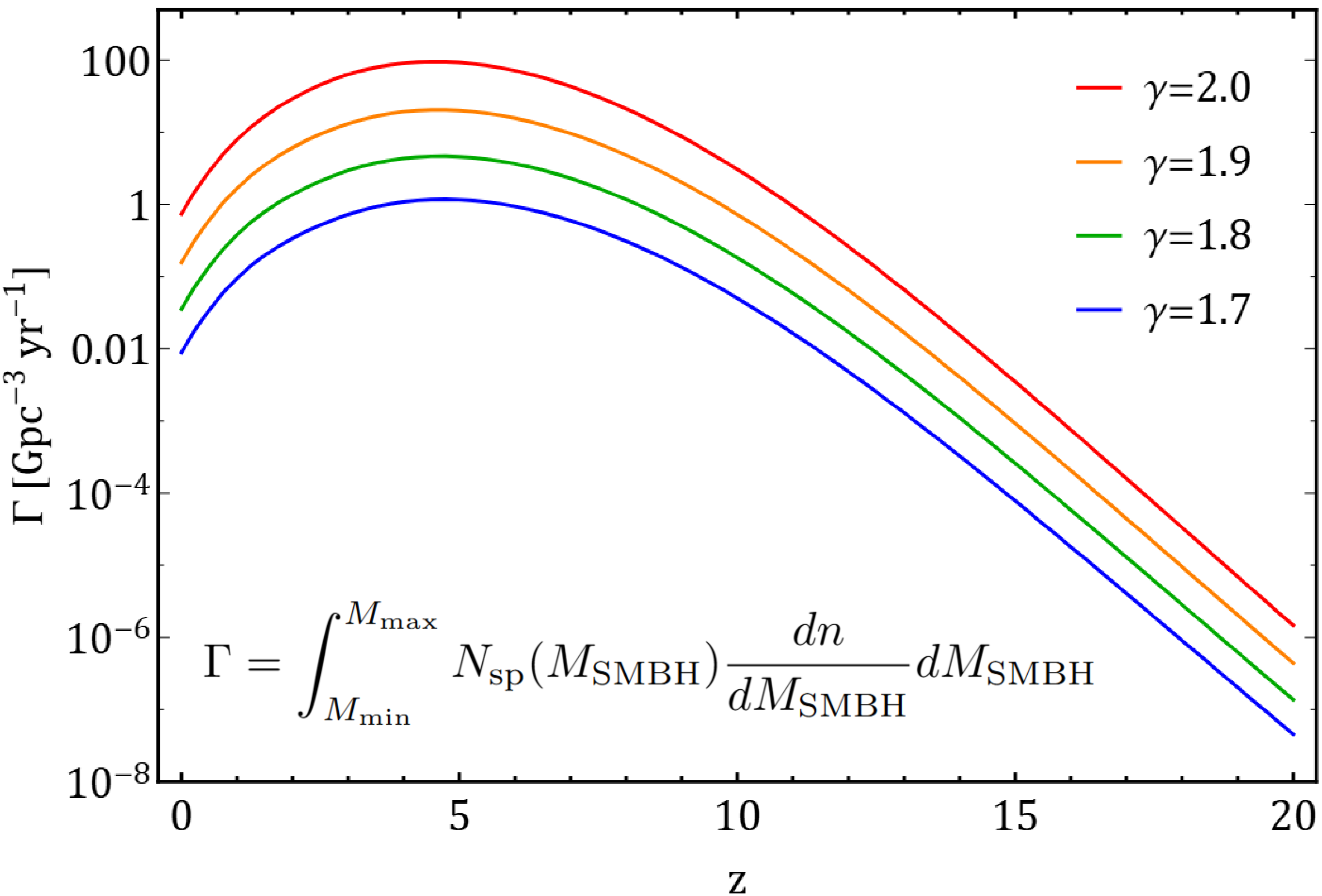
$$\Gamma = \int_{M_{\rm min}}^{M_{\rm max}} N_{\rm sp}(M_{\rm SMBH}) \frac{dn}{dM_{\rm SMBH}} dM_{\rm SMBH}$$

$$\frac{dn}{dM_{\rm SMBH}} = \frac{dn}{dM_{\rm vir}} \frac{dM_{\rm vir}}{dM_{\rm SMBH}} = f(\sigma)\frac{\rho_m}{M_{\rm vir}} \frac{d\log(\sigma^{-1})}{dM_{\rm vir}} \frac{dM_{\rm vir}}{dM_{\rm SMBH}}$$

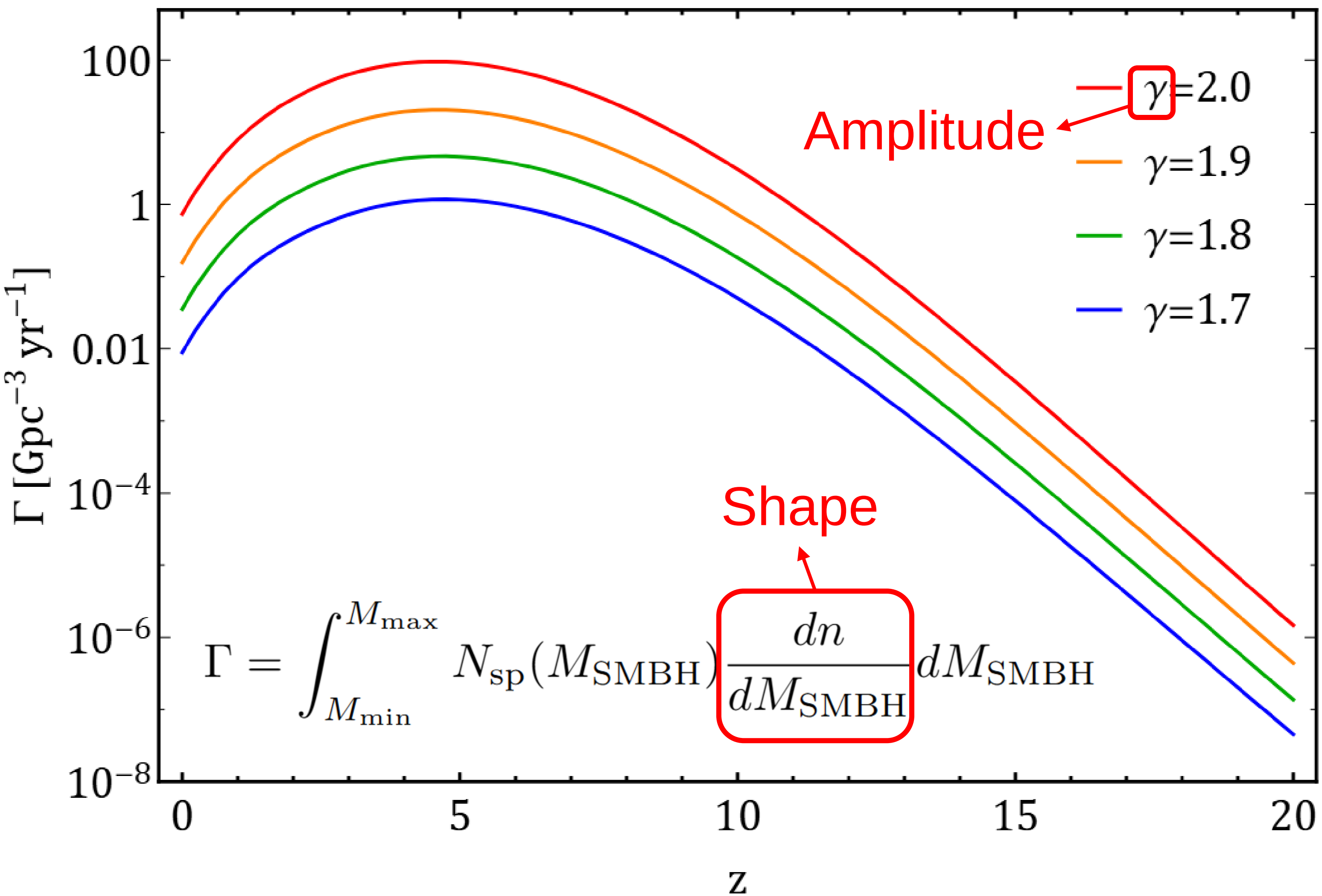
$$\log_{10}\left(\frac{M_{\rm SMBH}}{M_\odot}\right) = a + b\log_{10}\left(\frac{\sigma}{200\,{\rm km\,s^{-1}}}\right)$$



10^6
1000
1
0.001
10^{-6}
10^{-9}
$\Gamma\ [\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}]$
0.5
1.0
1.5
2.0
γ
$f_{\mathrm{PBH}}=1$
$f_{\mathrm{PBH}}=10^{-1}$
$f_{\mathrm{PBH}}=10^{-2}$
$f_{\mathrm{PBH}}=10^{-3}$

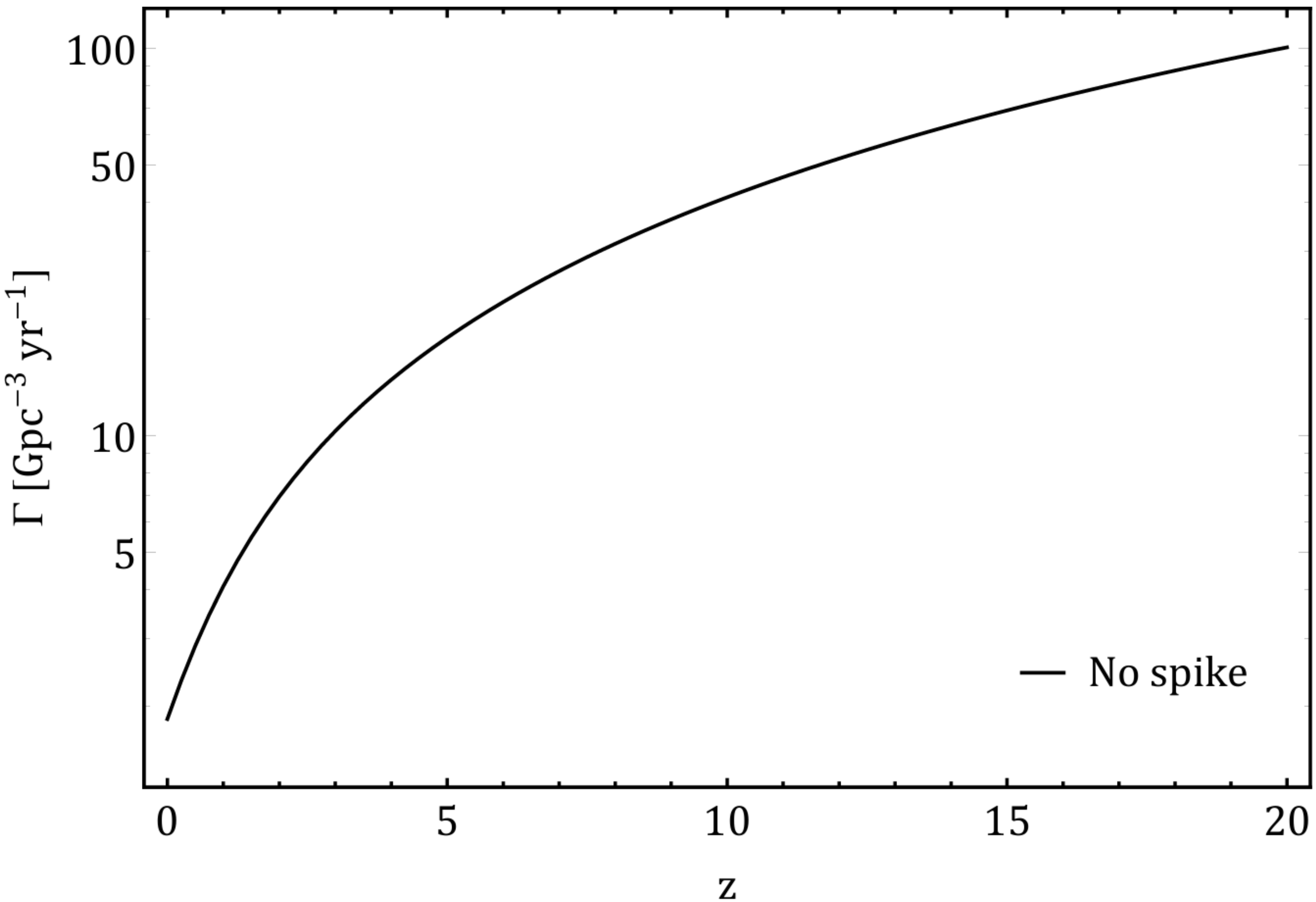


100
1
0.01
10^{-4}
10^{-6}
10^{-8}
0
5
10
15
20
z
Γ [Gpc^{-3} yr^{-1}]
γ=2.0
γ=1.9
γ=1.8
γ=1.7
$\Gamma = \int_{M_{\min}}^{M_{\max}} N_{\rm sp}(M_{\rm SMBH}) \frac{dn}{dM_{\rm SMBH}} dM_{\rm SMBH}$

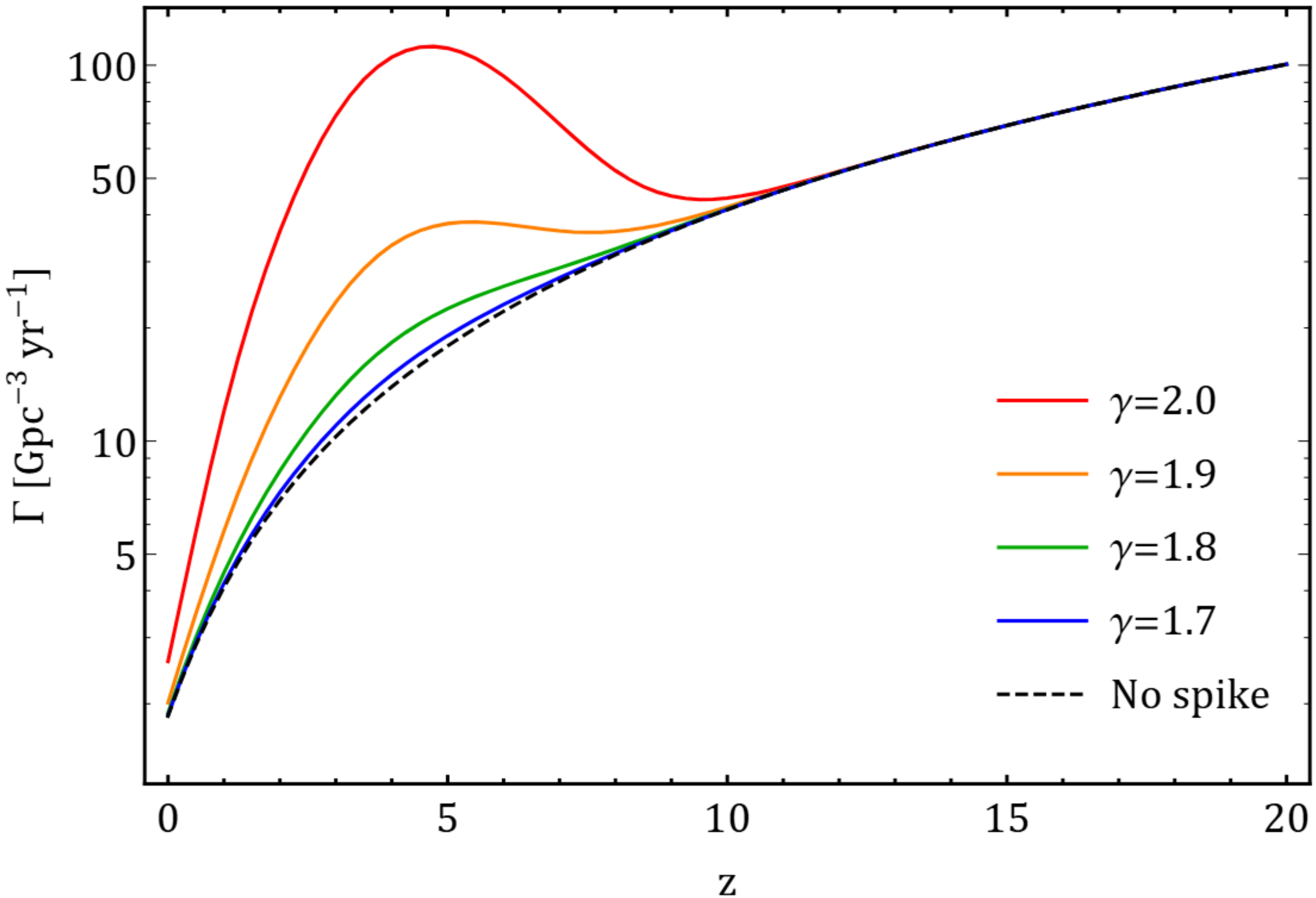


100
1
0.01
10^{-4}
10^{-6}
10^{-8}
0
5
10
15
20
z
Γ [$\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$]
γ=2.0
γ=1.9
γ=1.8
γ=1.7
Amplitude
Shape
$\Gamma = \int_{M_{\min}}^{M_{\max}} N_{\rm sp}(M_{\rm SMBH}) \boxed{\frac{dn}{dM_{\rm SMBH}}} dM_{\rm SMBH}$

PBH Merger Rate Evolution

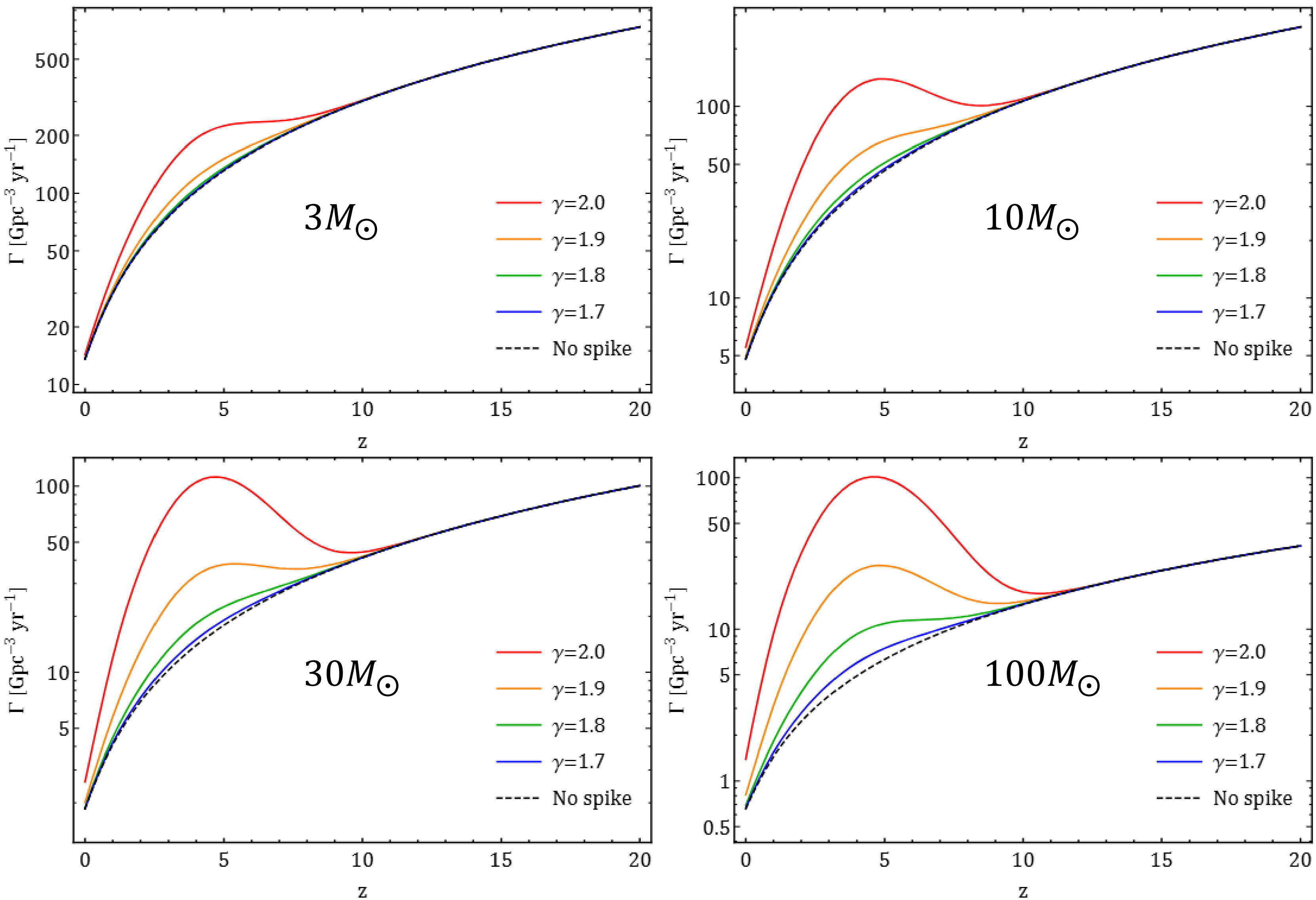


PBH Merger Rate Evolution

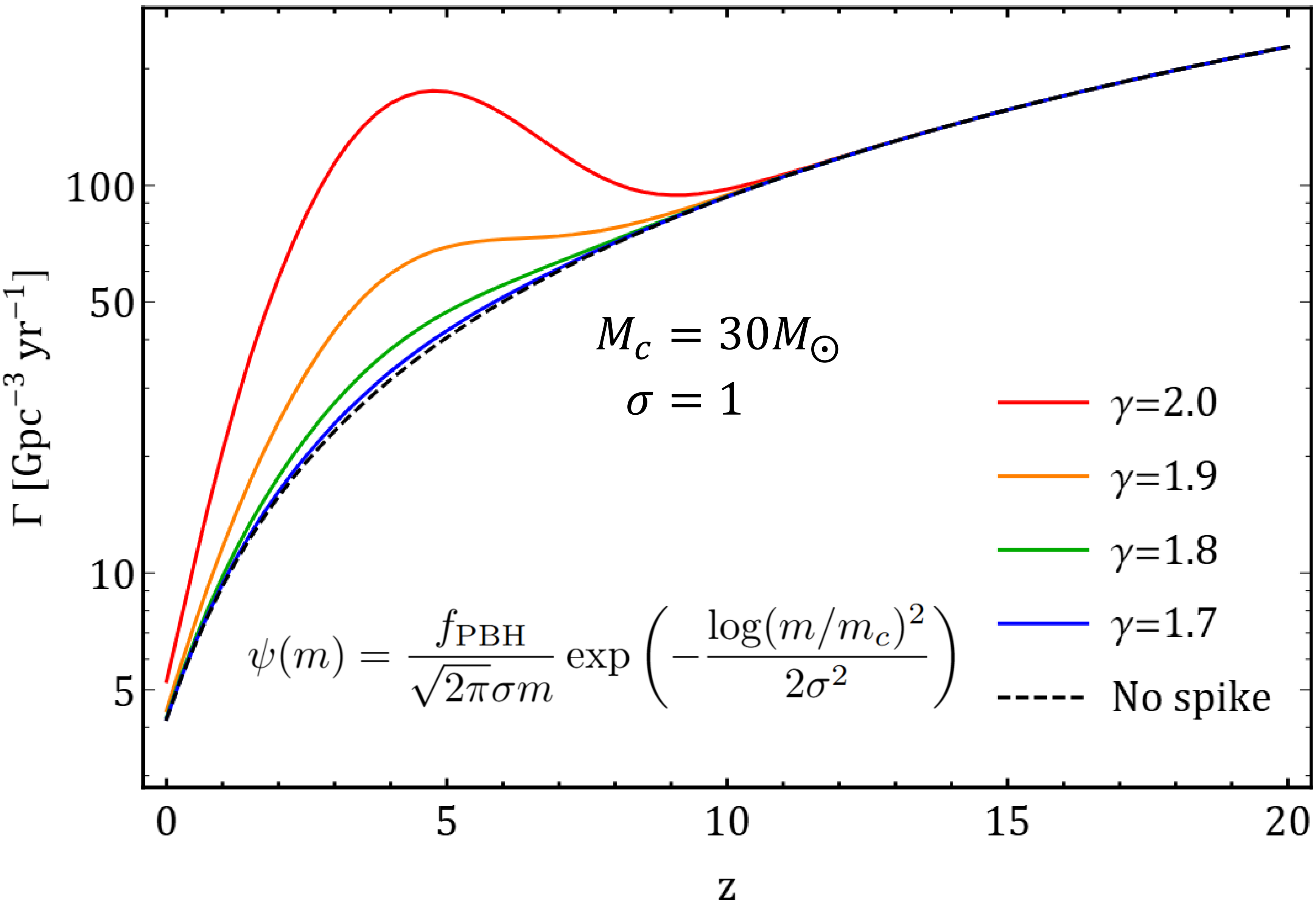


$$R_{\mathrm{tot}} = R_{\mathrm{eb}} + R_{\mathrm{sp}} + R_{\mathrm{halo}} + R_{\mathrm{cluster}} + \mathrm{etc.}$$
$$\simeq R_{\mathrm{eb}} + R_{\mathrm{sp}}$$

PBH Mass Impact

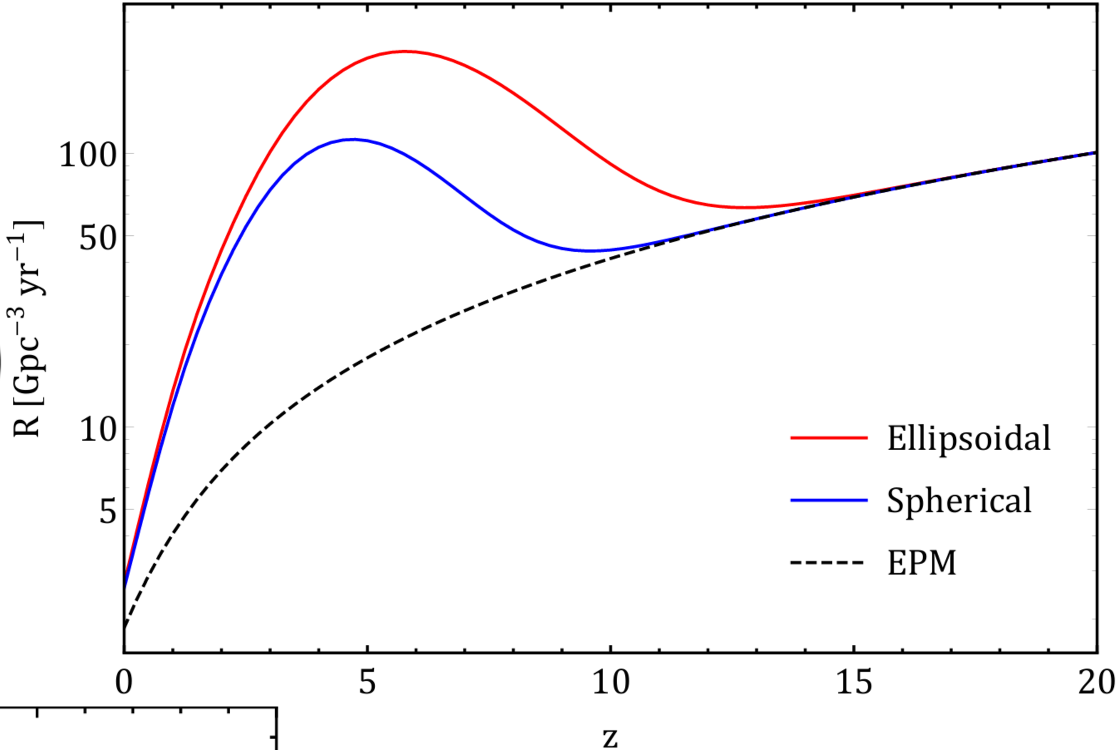


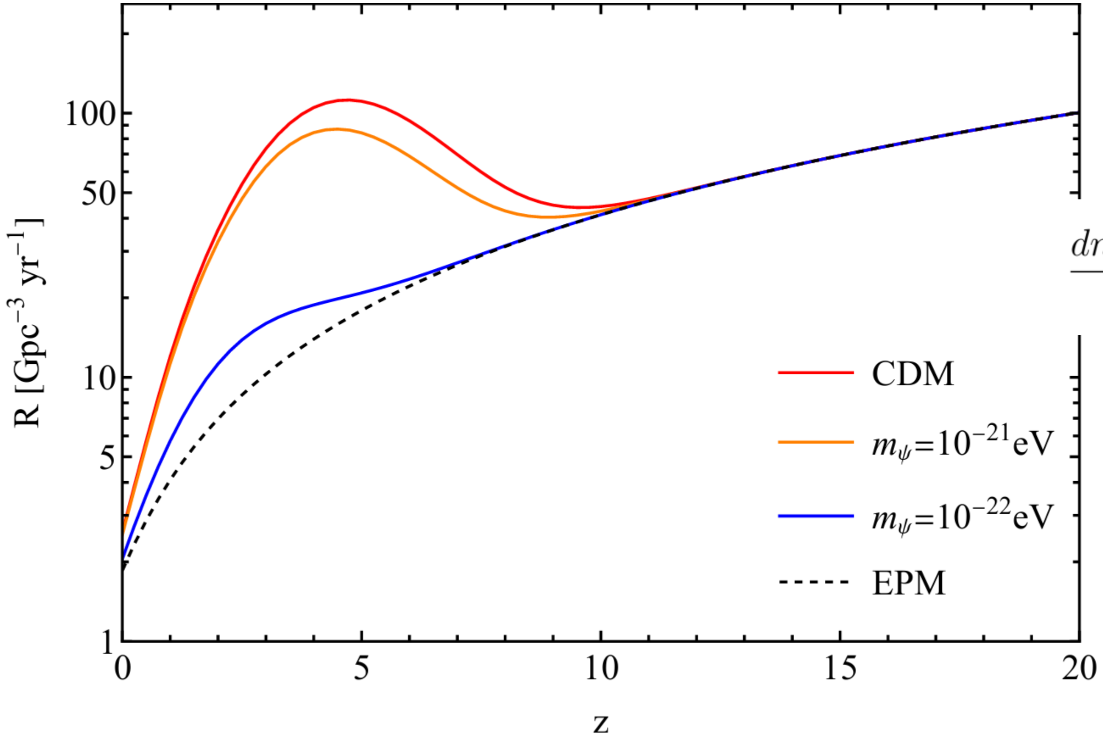
PBH Mass Impact



Halo Impact

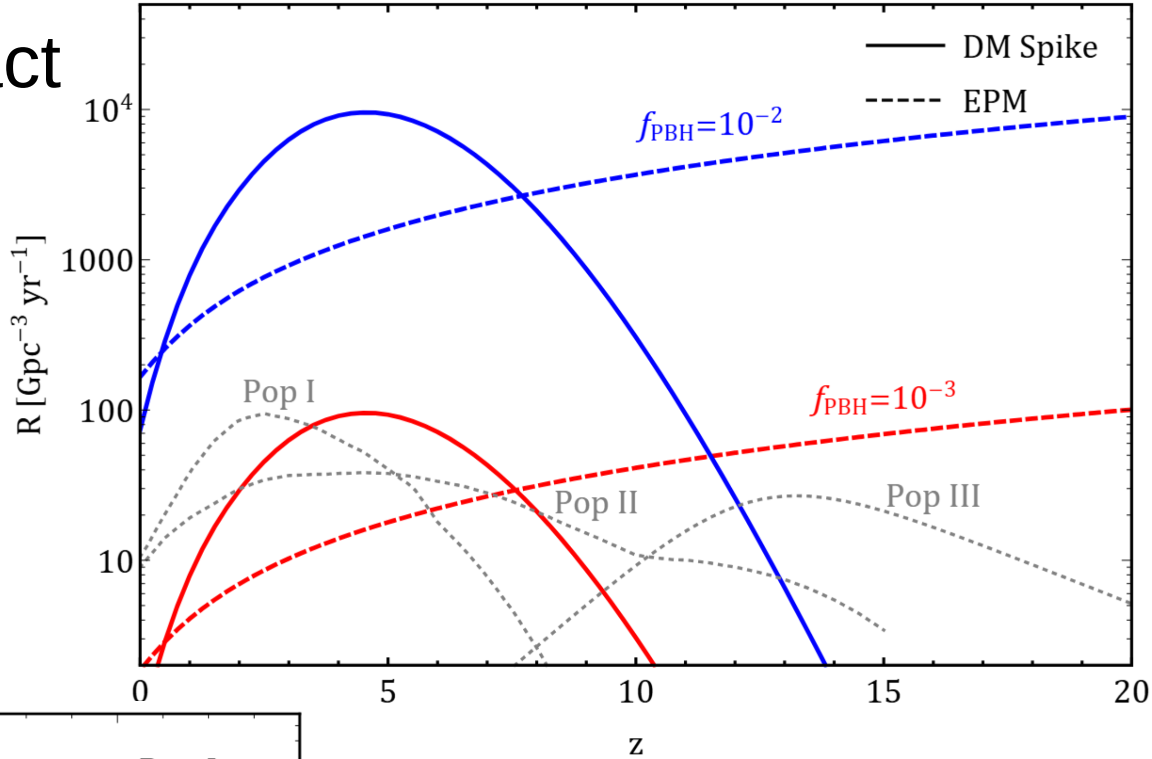
$$f_{\mathrm{ST}}(\sigma) = F\sqrt{\frac{2a}{\pi}}\left[1+\left(\frac{\sigma^2}{a\delta_c^2}\right)^p\right]\frac{\delta_c}{\sigma}\exp\left(-\frac{a\delta_c^2}{2\sigma^2}\right)$$

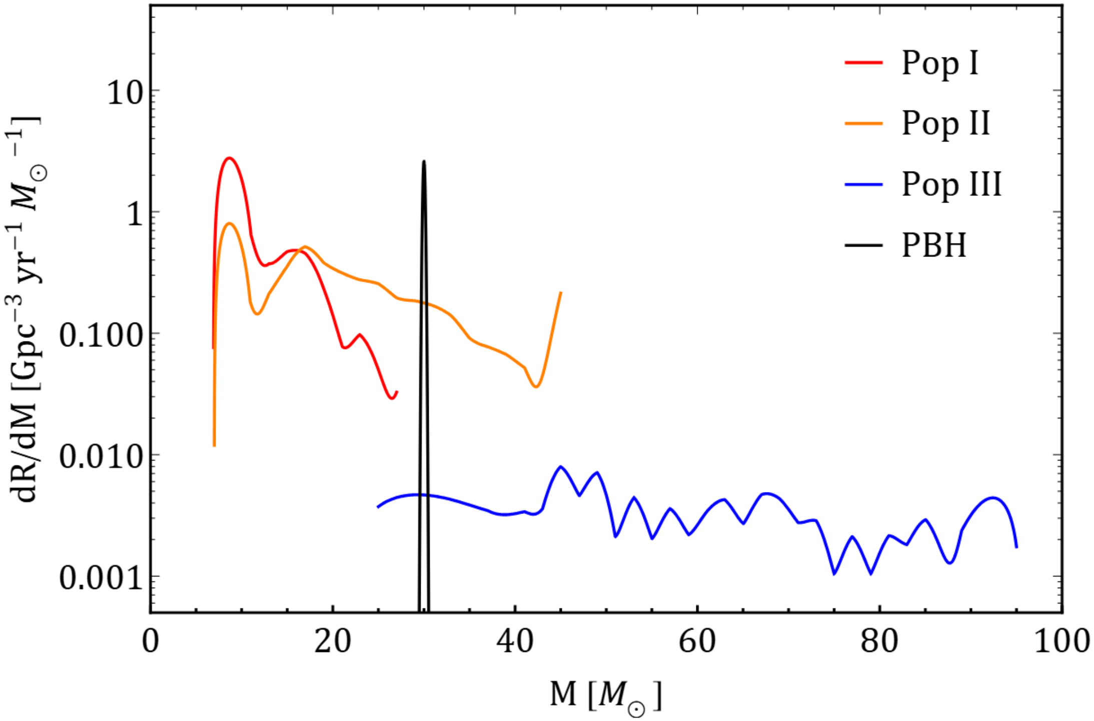




$$\left.\frac{dn(M,z)}{dM}\right|_{\psi\mathrm{DM}} = \left.\frac{dn(M,z)}{dM}\right|_{\mathrm{CDM}}\left[1+\left(\frac{M}{M_0}\right)^{-1.1}\right]^{-2.2}$$

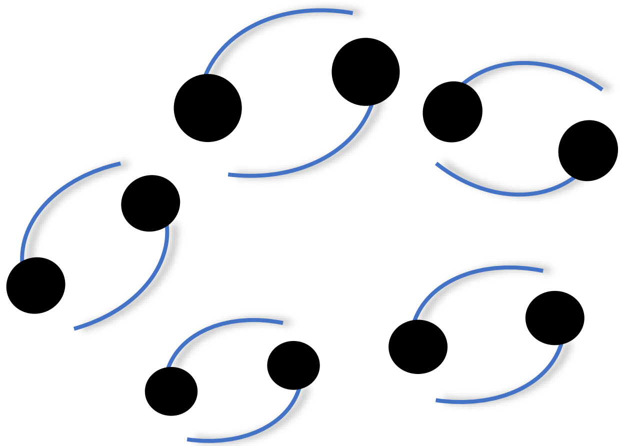
Astrophysical BH Impact



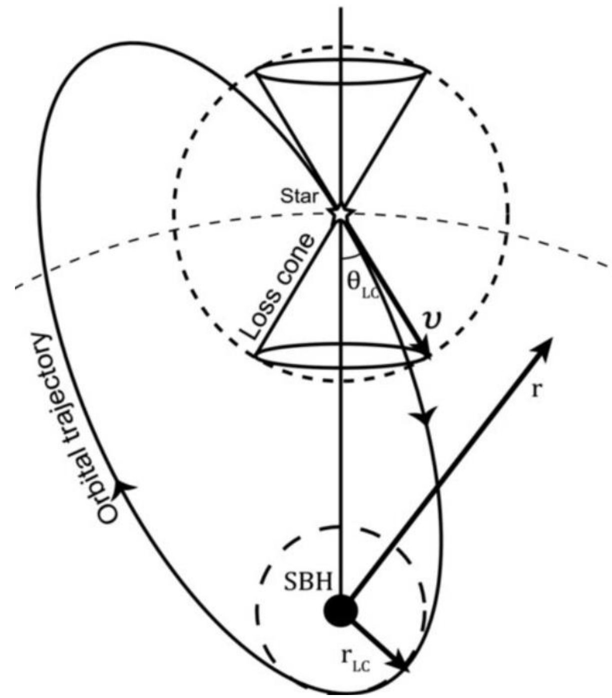


DM Spike Evolution

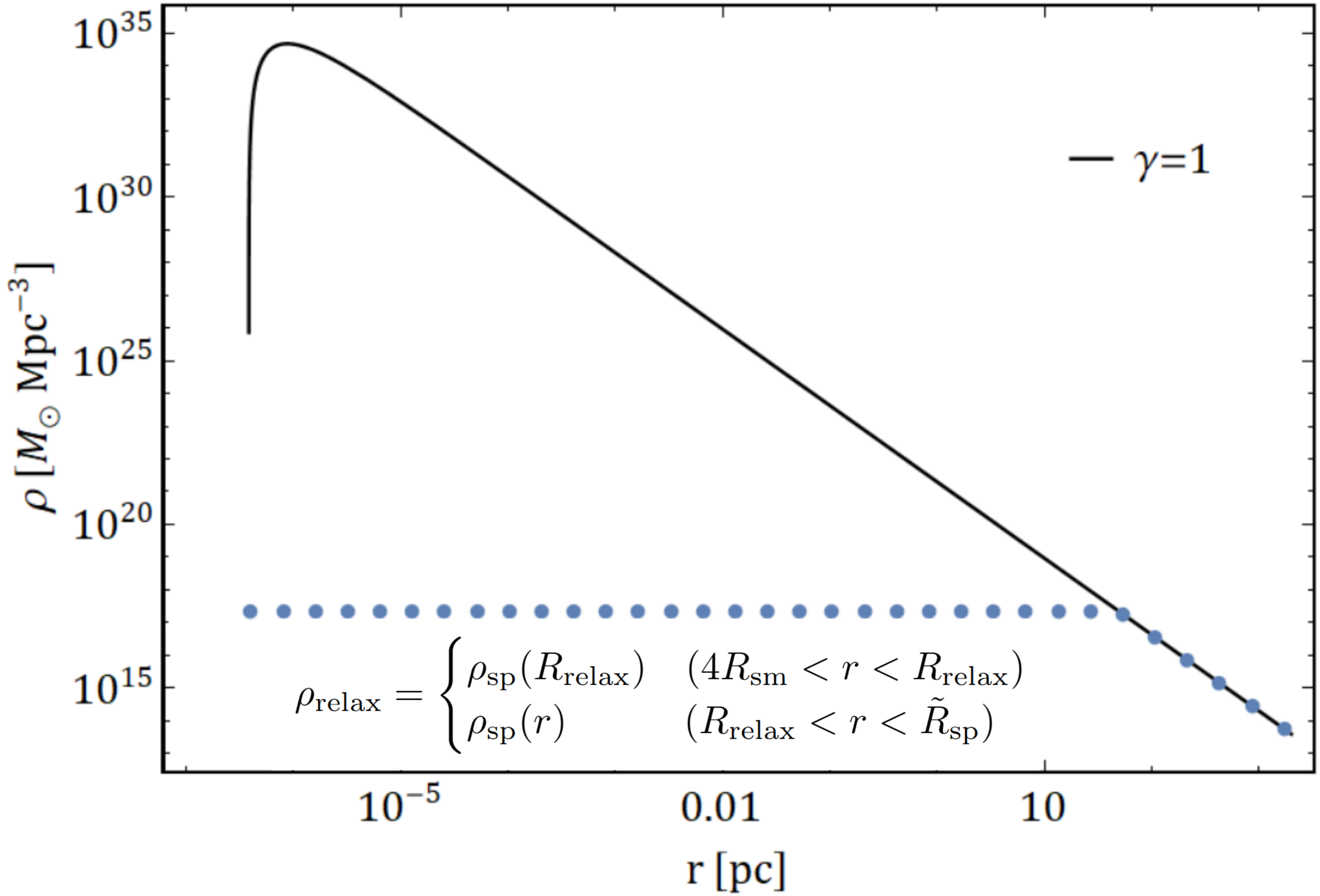
Two-body relaxation



Loss-cone refilling



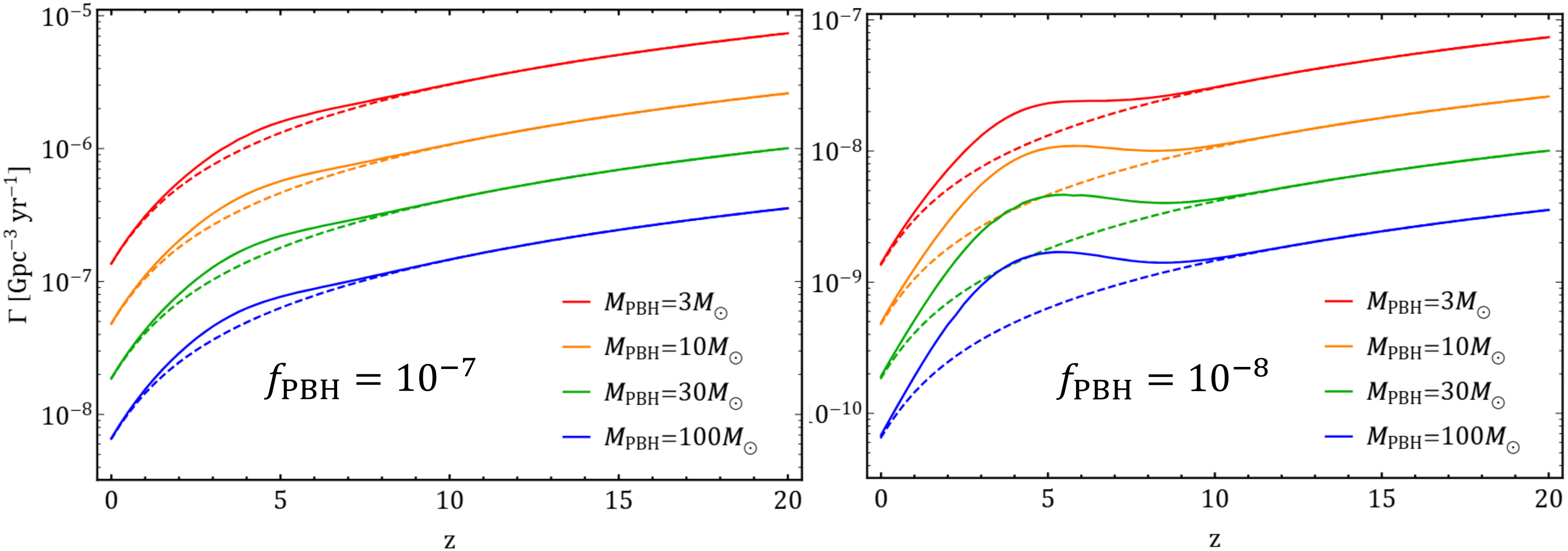
Two-Body Relaxation



$$\rho_{\mathrm{relax}} = \begin{cases} \rho_{\mathrm{sp}}(R_{\mathrm{relax}}) & (4R_{\mathrm{sm}} < r < R_{\mathrm{relax}}) \\ \rho_{\mathrm{sp}}(r) & (R_{\mathrm{relax}} < r < \tilde{R}_{\mathrm{sp}}) \end{cases}$$

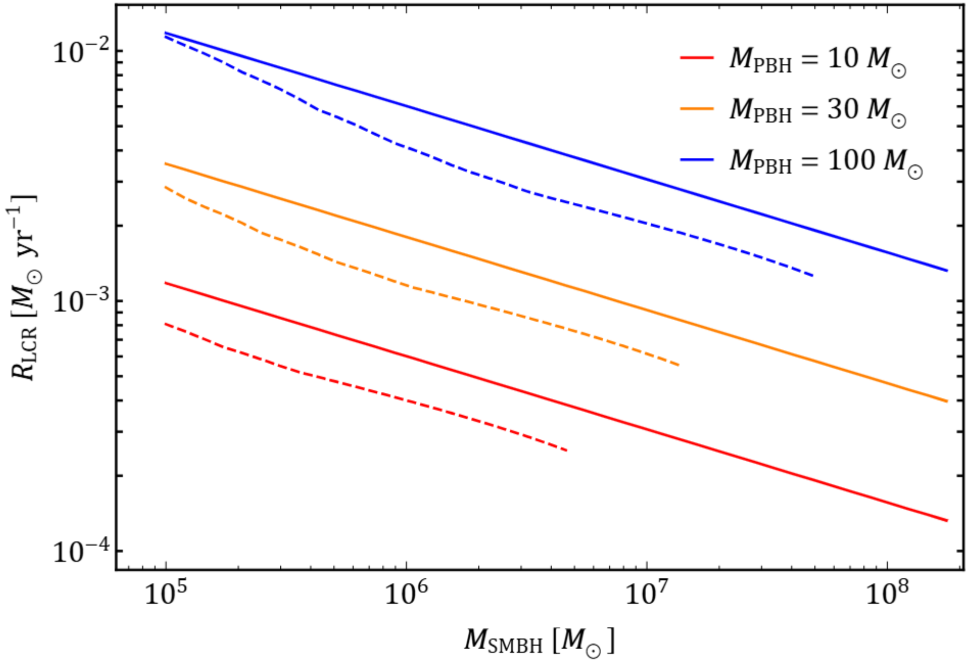
$$t_{\mathrm{relax}} = \frac{v_{\mathrm{rel}}^3(R_{\mathrm{relax}})}{8\pi G^2 M_{\mathrm{PBH}} f_{\mathrm{PBH}} \rho_{\mathrm{sp}}(R_{\mathrm{relax}}) \log(b_{\max}/b_{\min})}$$

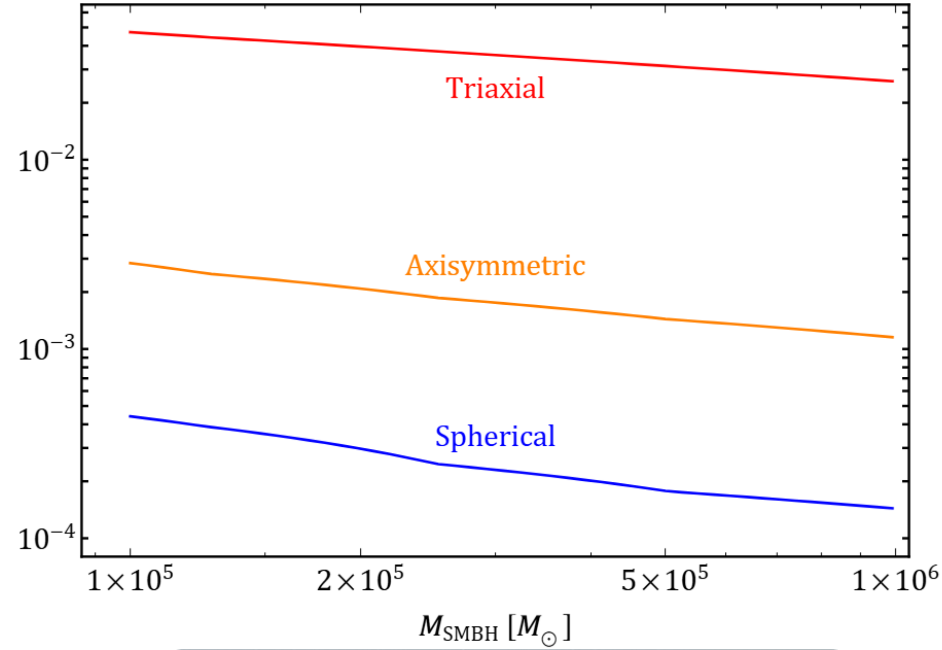
Two-Body Relaxation



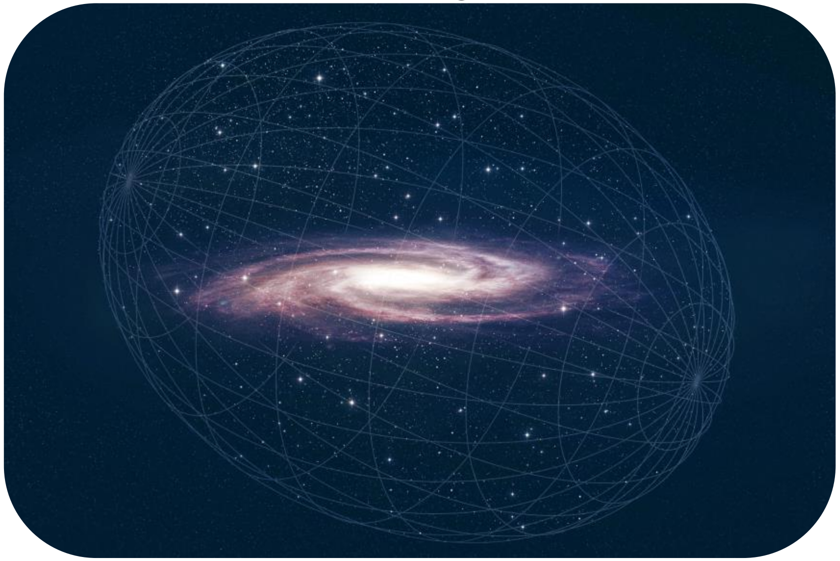
$$t_{\rm relax} = \frac{v_{\rm rel}^3(R_{\rm relax})}{8\pi G^2 M_{\rm PBH} f_{\rm PBH} \rho_{\rm sp}(R_{\rm relax}) \log(b_{\rm max}/b_{\rm min})}$$

Loss-Cone Refilling

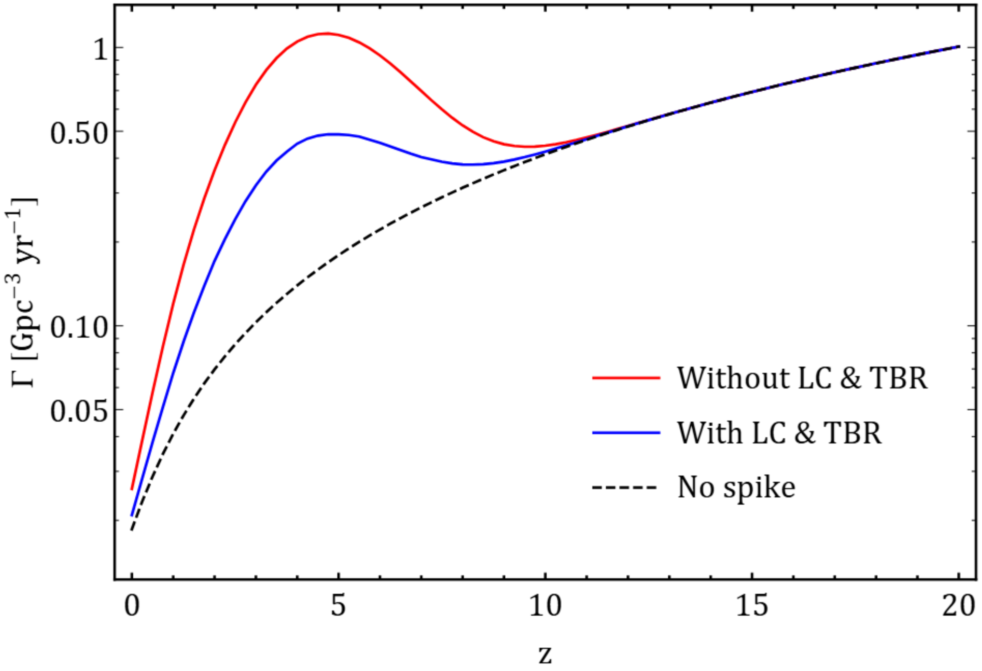


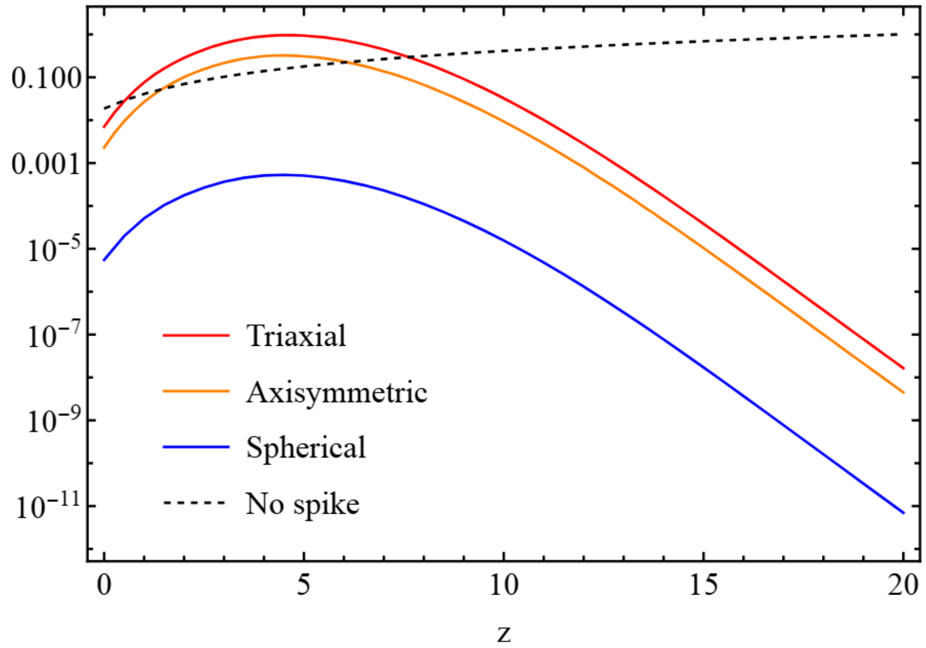


$$\frac{dM}{dt} = f_{\rm PBH}\frac{M_{\rm PBH}}{M_{\rm SMBH}}\frac{\sigma^3}{G}$$



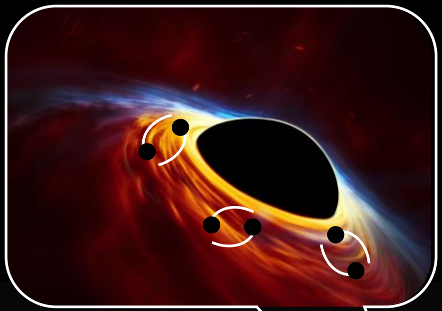
Loss-Cone Refilling

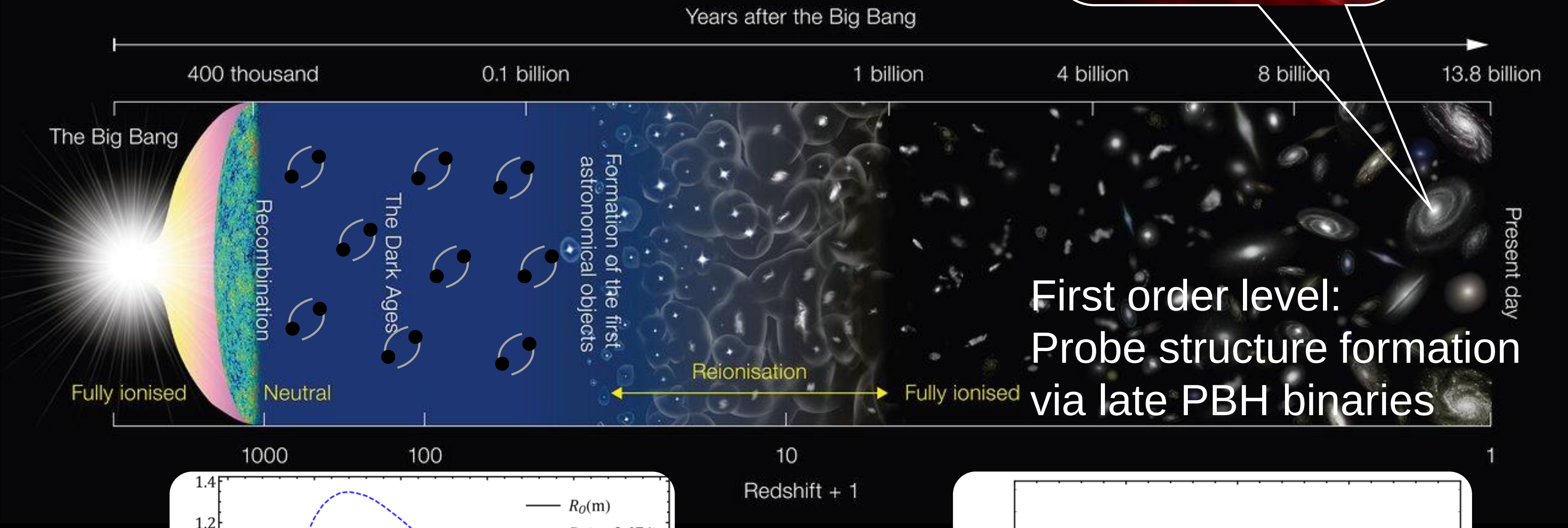




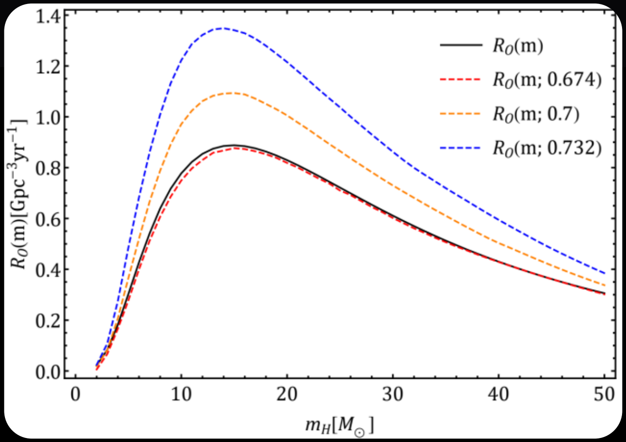
PBH Mergers as a Cosmological Probe

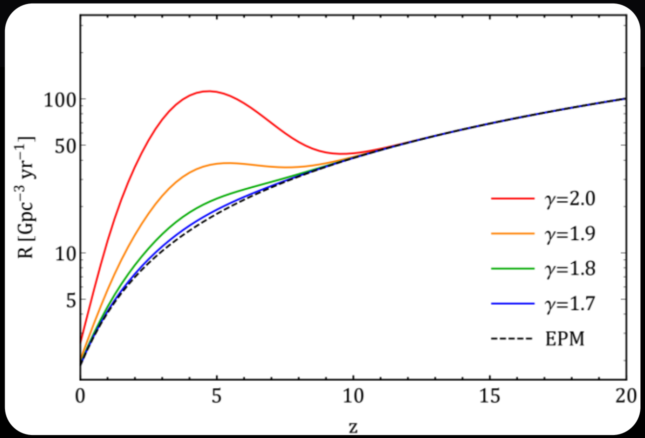
Background level:
Probe Hubble expansion rate
via early PBH binaries



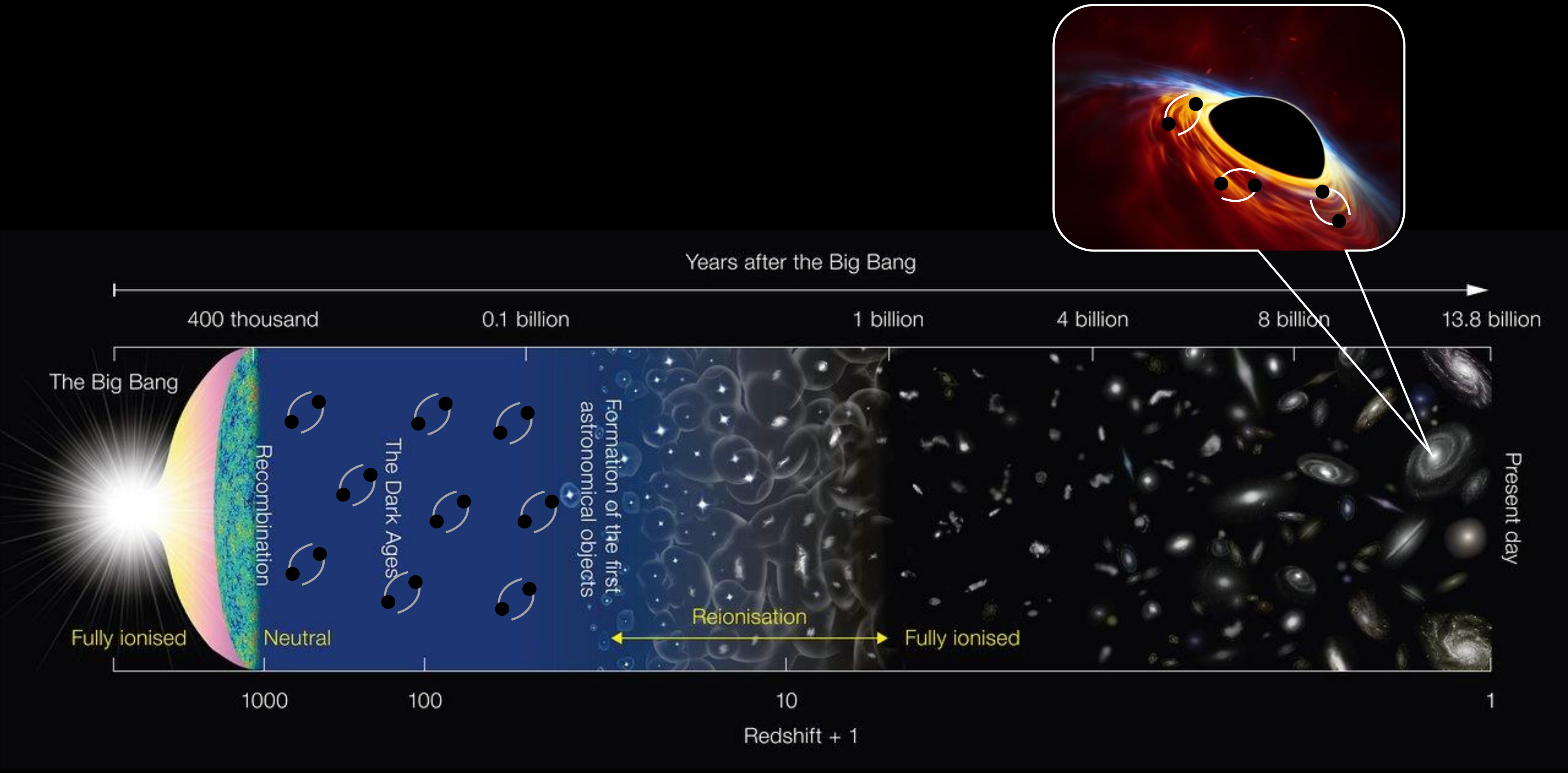


First order level:
Probe structure formation
via late PBH binaries





PBH Mergers as a Cosmological Probe



Thank you!

$$P_O(m_1^z, m_2^z) = \int_0^\infty n_z\left(\frac{m_1^z}{1+z}\right) n_z\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

$$n_z(m_z) = n_{\mathrm{i}}(m_i)\frac{dm_{\mathrm{i}}}{dm_z} = n_{\mathrm{i}}(m_i) g(z, m_{\mathrm{z}})$$

$$\frac{dm}{dt} = 4\pi\lambda\rho_m \frac{G^2 m^2}{v_{\mathrm{eff}}^3}$$

$$P_O(m_1^z, m_2^z) = \int_0^\infty n_{\mathrm{i}}\left(\frac{m_1^z}{1+z}\right) n_{\mathrm{i}}\left(\frac{m_2^z}{1+z}\right) g(z, \frac{m_1^z}{1+z}) g(z, \frac{m_2^z}{1+z})$$
$$\times W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$